The Sunyaev-Zel’dovich Effect as a cosmic thermometer

Methods, results, future prospects

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T_{CMB}(z): why measure it?

- **Observational test of the standard model:** $T_{CMB}(z) = T_0(1+z)$
  $T_0 = (2.725 \pm 0.002)K$ solar system value measured by COBE/FIRAS (Mather et al. 1999)

- **Test of the nature of redshift**

- **Constraints on alternative cosmological models** (which rely on the physics of the matter and radiation content of the Universe):
  - $\Lambda$–decaying models
    (Overduin and Cooperstock, Phys.Rev.D, 58 (1998));
  - Decaying scalar field cosmologies

- **Possible constraints on the variation of fundamental constants** over cosmological time
\( T_{\text{CMB}}(z) \): measurements

Measurements of CMB temperature traditionally through the study of excitation temperatures in high redshift molecular clouds. First attempt pioneered by (Bahcall and Wolf, 1968)

- Many high redshift estimates of \( T_{\text{CMB}} \) at redshift of absorbers (Songaila et al 1994; Lu et al. 1996; Ge et al 1997; Roth and Bauer, 1999; Srianand et al 2000; JoSecco et al. 2001; Levshakov et al. 2002; Molaro et al. 2002; Cui et al. 2005)

- Systematics:
  - CMB is not the only radiation field populating the energy levels, from which transitions occur.
  - Detailed knowledge of the physical conditions in the absorbing clouds is necessary (Combes and Wiklind, 1999; Combes, 2007)

(LoSecco et Al. Phys. Rev. D, 64, 123, 2002)
The Sunyaev Zel’dovich Effect (SZE) (I)

Secondary CMB anisotropy

Comptonization of the CMB by electrons in the hot gas of galaxy clusters.

\[ \Delta I = \frac{2kT_{\text{CMB}}}{h^2c^2} \frac{x^4e^x}{(e^x-1)^2} \sigma_T \int n_e dl [\theta f_1(x) - \beta + R(x, \theta, \beta)] \]

\[ x = h\nu/kT \]
\[ \theta = kT_e/mc^2 \]
\[ \beta = V/c \]

R function = relativistic corrections

The Sunyaev Zel’ dovich Effect (SZE) (II)

- **Properties:**
  - unique spectral shape
  - Redshift independent
  - \( \propto \) electron pressure in cluster atmospheres

![Spectral distortion of the CMB due to the SZE](image)

- **Galaxy clusters Physics:**
  - \( \tau \) optical depth
  - \( T_e \) electronic temperature
  - \( v_{pec} \) peculiar velocity

- **Cosmology:**
  - \( H_0 \)
  - \( \Omega_B \)
  - evolution of abundance of clusters
  - \( T_{CMB}(z) \)
\( \Delta l_{\text{SZ}} \) depends on frequency \( \nu \) through the nondimensional ratio \( h\nu/kT \):

\[
x = \frac{h\nu(z)}{kT_{\text{CMB}}(z)} = \frac{h\nu_0(1+z)}{kT_0(1+z)} = \frac{h\nu_0}{kT_0}
\]

redshift–invariant only for standard scaling of \( T(z) \)

In all the other non standard scenarios, the "almost" universal (remember rel. corrections!) dependence of thermal SZ on frequency becomes \( z \)-dependent, resulting in a small dilation/contraction of the SZ spectrum on the frequency axis.

Ex: \( T_{\text{CMB}}(z) = T_{\text{CMB}}(0)(1+z)^{1-\alpha} \)

(Lima et al. 2000)

\[
x' = \frac{h\nu_0(1+z)}{kT_0(1+z)^{(1-\alpha)}} = \frac{h\nu_0}{kT_{\text{CMB}}^*}
\]

where \( T_{\text{CMB}}^* = T_{\text{CMB}}(0)(1+z)^{-\alpha} \)
\( T_{\text{CMB}}(z) \) from SZE (II)

- Steep frequency dependency of \( \Delta l_{\text{SZ}} \) to \( T_{\text{CMB}}(z) \)
- Ratio of \( \Delta l_{\text{SZ}}(\nu_1)/\Delta l_{\text{SZ}}(\nu_2) \) weakly dependent on cluster properties (\( \tau, T_e, v_{\text{pec}} \))

Relative variation of SZ signal, evaluated for a typical 10^{-4} comptonization parameter at a \( T_e=10\text{keV}, V_{\text{pec}}=300\text{km/s} \) along l.o.s.
COMA observations

**SZ on A1656 by MITO**
\[ \Delta T_0 (143\text{GHz}) = (-184 \pm 39) \mu\text{K} \]
\[ \Delta T_0 (214\text{GHz}) = (-32 \pm 79) \mu\text{K} \]
\[ \Delta T_0 (272\text{GHz}) = (+172 \pm 36) \mu\text{K} \]
\[ \Rightarrow \tau_0 = (5.05 \pm 0.84) \cdot 10^{-3} \]


**SZ on A1656 by OVRO+WMAP+MITO**

First SZ spectrum with 6 frequencies
\[ \Rightarrow \tau_0 = (5.35 \pm 0.67) \cdot 10^{-3} \]

T_{CMB}(z) from SZE: first application

Fit of measured SZ signals ratios with the expected values by changing T(z)/\(1+z\) as in the following:

\[
\frac{\Delta S_i}{\Delta S_j} = \frac{G_i}{G_j} A \Omega_i A \Omega_j \int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2} \left\{ \int d \tau \left[ \theta f_1(x) - \beta + R(x, \theta, \beta) \right] \right\} \cdot \epsilon_i(\nu) d \nu
\]

\[
\frac{\Delta S_i}{\Delta S_j} = \frac{G_i}{G_j} A \Omega_i A \Omega_j \int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2} \left\{ \int d \tau \left[ \theta f_1(x) - \beta + R(x, \theta, \beta) \right] \right\} \cdot \epsilon_j(\nu) d \nu
\]

\(G_i\): Responsivity

\(A \Omega_i\): Throughput

\(\epsilon_i(\nu)\): Transmission efficiency

- independent of absolute calibration uncertainties (\(T_{\text{planet}}\));
- independent of \(\tau\), if KIN–SZ removed or \(\beta\) negligible;
- dependent on precise knowledge of \(A \Omega_i\) and \(\epsilon_i(\nu)\);
- Ratios have non gaussian distributions and introduce correlations
$T_{\text{CMB}}(z)$ from SZE: first results

$T_{\text{CMB}}(z = 0) = 2.725^{+0.02}_{-0.02} \text{ K}$

$T_{A1656}(z = 0.0231) = 2.789^{+0.080}_{-0.065} \text{ K}$

$T_{A2163}(z = 0.203) = 3.377^{+0.101}_{-0.102} \text{ K}$

\[ T(z) = T_0 (1 + z) \]

\[ T(z) = T_0 (1 + z)^{1-\alpha} \]

\[ T(z) = T_0 [1 + (1 + \gamma)z] \]

$\alpha = -0.16^{+0.34}_{-0.32} (95\% \text{ c.l.})$

$d = -0.17 \pm 0.36 (95\% \text{ c.l.})$

Molecular microwave transitions

CONSISTENT

Standard Model

CONSISTENT
SZ measurements of 14 clusters by different experiments expressed in central thermodynamic temperature

<table>
<thead>
<tr>
<th>Cluster</th>
<th>OVRO+BIMA(^a)</th>
<th>SuZIE II(^b)</th>
<th>SCUBA(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Delta T_{30\text{GHz}}) (mK)</td>
<td>(\Delta T_{145\text{GHz}}) (mK)</td>
<td>(\Delta T_{221\text{GHz}}) (mK)</td>
</tr>
<tr>
<td>A520</td>
<td>-0.66 (\pm) 0.09 (^d)</td>
<td>-0.44 (\pm) 0.13</td>
<td>0.14 (\pm) 0.14</td>
</tr>
<tr>
<td>A697</td>
<td>-1.22 (\pm) 0.12</td>
<td>-0.93 (\pm) 0.13</td>
<td>0.41 (\pm) 0.16</td>
</tr>
<tr>
<td>A773</td>
<td>-1.08 (\pm) 0.11</td>
<td>-0.91 (\pm) 0.16</td>
<td>0.04 (\pm) 0.25</td>
</tr>
<tr>
<td>A1689</td>
<td>-2.06 (\pm) 0.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A1835</td>
<td>-2.90 (\pm) 0.21</td>
<td>-1.74 (\pm) 0.26</td>
<td>0.14 (\pm) 0.41</td>
</tr>
<tr>
<td>A2204</td>
<td>-3.22 (\pm) 0.32</td>
<td>-0.65 (\pm) 0.10</td>
<td>0.21 (\pm) 0.10</td>
</tr>
<tr>
<td>A2261</td>
<td>-1.36 (\pm) 0.14</td>
<td>-1.56 (\pm) 0.18</td>
<td>0.21 (\pm) 0.50</td>
</tr>
<tr>
<td>A2390</td>
<td>-</td>
<td>-0.91 (\pm) 0.10</td>
<td>-0.10 (\pm) 0.17</td>
</tr>
<tr>
<td>CL0016+16</td>
<td>-1.44 (\pm) 0.09</td>
<td>-0.57 (\pm) 0.23</td>
<td>0.66 (\pm) 0.46</td>
</tr>
<tr>
<td>MS0451-03</td>
<td>-1.48 (\pm) 0.09</td>
<td>-0.779 (\pm) 0.065</td>
<td>-0.21 (\pm) 0.10</td>
</tr>
<tr>
<td>RXJ1347</td>
<td>-5.15 (\pm) 0.60</td>
<td>-3.22 (\pm) 0.39</td>
<td>-0.10 (\pm) 0.39</td>
</tr>
<tr>
<td>ZW3146</td>
<td>-2.02 (\pm) 0.25</td>
<td>-1.56 (\pm) 0.39</td>
<td>-0.25 (\pm) 0.48</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cluster</th>
<th>OVRO(^d)</th>
<th>MITO(^e)</th>
<th>SCUBA(^c)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(\Delta T_{32\text{GHz}}) (mK)</td>
<td>(\Delta T_{143\text{GHz}}) (mK)</td>
<td>(\Delta T_{214\text{GHz}}) (mK)</td>
</tr>
<tr>
<td>A1656</td>
<td>-0.520 (\pm) 0.093</td>
<td>-0.179 (\pm) 0.037</td>
<td>0.033 (\pm) 0.080</td>
</tr>
</tbody>
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<td></td>
<td>(\Delta T_{30\text{GHz}}) (mK)</td>
<td>(\Delta T_{142\text{GHz}}) (mK)</td>
<td>(\Delta T_{217\text{GHz}}) (mK)</td>
</tr>
<tr>
<td>A2163</td>
<td>-1.89 (\pm) 0.17</td>
<td>-1.011 (\pm) 0.098</td>
<td>-0.21 (\pm) 0.16</td>
</tr>
</tbody>
</table>

\(^a\)(Bonamente et al. 2006); \(^b\)(Benson et al. 2003; Benson et al. 2004); \(^c\)(Zemcov et al. 2007); \(^d\)(Herbig et al. 1995; Mason et al. 2001); \(^e\)(De Petris et al. 2002; Savini et al. 2003); \(^f\)(Holzapfel et al. 1997a).
**$T_{\text{CMB}}(z)$ from SZE: improved statistical analysis**

Two main approaches:

- **Ratios** of SZ intensity change (RI)
  \[\Delta I_{\text{SZ}}(\nu_1)/\Delta I_{\text{SZ}}(\nu_2)\]
  - weakly dependent on IC gas properties if $\beta$ negligible.

- **Directly $\Delta I_{\text{SZ}}$ measurements** (DI)
  - easier control of systematics
  - more complex structure in the parameter space

**Priors:**
- $P(T_{ei}) = N(E(T_{ei}), \sigma(T_{ei}))$
- $P(\nu_p) = N(0 \text{ km/s}, 1000 \text{ km/s})$
- $P(T_{\text{CMB}}) = \text{flat}$
- $P(\tau) = \text{flat (only for DI)}$
- $P(C) = N(1, 0.1)$

**T_{CMB}(z) from SZE: Ratios approach (RI)**

**Likelihood of intensity ratios:**
- weak dependence on cluster parameters (no marginalization on $\tau$)
- not considered measurements at the crossover frequency (Cauchy tail)
- bias due to arbitrariness in selecting the intensity change used in denominator of ratios
- more precise SZ measurements or larger dataset $\rightarrow$ bias removed

**Probability function for ratio $r$:**
Two measurements $\rho_1$ e $\rho_2$ with gaussian errors $\sigma_1$ e $\sigma_2$:

$$P(r) = \frac{1}{2\pi \sigma_1 \sigma_2} \int_{x=-\infty}^{\infty} xe^{-\frac{(x-\rho_1)^2}{2\sigma_1^2}} e^{-\frac{(xr-\rho_2)^2}{2\sigma_2^2}} dx.$$  

Extended to the case of n ratios
$T_{\text{CMB}}(z)$ from SZE: Direct approach

**DI multicluster likelihood:** (2D numerical integration, Vp and C integration analytical)

$$P(d|\alpha, \tau, T_e, v_p, C) = \prod_i P(d_i|\alpha, \tau, T_e, v_p, C)$$

$$P(\alpha|d) \propto \int P(d|\alpha, \tau, T_e, v_p, C) P(\alpha) P(\tau) P(T_e) P(v_p) P(C) d\tau \ dT_e \ dv_p \ dC$$

**DI single likelihood for each cluster:**

$$P(d|\tau, T_e, v_p, T_{\text{CMB}}, C) \propto \exp\left[-\chi^2(\tau, T_e, v_p, T_{\text{CMB}}, C)/2\right]$$

- MCMC algorithm
  (Metropolis–Hastings sampling; Gelman–Rubin test for convergence)
- Posteriors for all parameters
- Study of correlations
- Main degeneracies:
  - $T(z)$ vs $\tau$
  - $T(z)$ vs $v_p$ always evident

\[\]
**$T_{\text{CMB}}(z)$ from SZE: Results**

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$T(z)$ (K)</th>
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<tr>
<td>A1656</td>
<td>2.72 ± 0.10</td>
</tr>
<tr>
<td>A2204</td>
<td>2.90 ± 0.17</td>
</tr>
<tr>
<td>A1689</td>
<td>2.95 ± 0.27</td>
</tr>
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<td>A520</td>
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<td>a2163</td>
<td>3.36 ± 0.20</td>
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<tr>
<td>A773</td>
<td>3.85 ± 0.64</td>
</tr>
<tr>
<td>A2390</td>
<td>3.51 ± 0.25</td>
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<tr>
<td>a1835</td>
<td>3.39 ± 0.26</td>
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<td>A697</td>
<td>3.22 ± 0.26</td>
</tr>
<tr>
<td>ZW3146</td>
<td>4.05 ± 0.66</td>
</tr>
<tr>
<td>RXJ1347</td>
<td>3.97 ± 0.19</td>
</tr>
<tr>
<td>CL0016+16</td>
<td>3.69 ± 0.37</td>
</tr>
<tr>
<td>MS0451</td>
<td>4.59 ± 0.36</td>
</tr>
</tbody>
</table>

Flat prior $a \in [0,1]$ (theoretical motivation)  
(Lima et al 2000)

All limits are at 68% probability level

RI $\alpha \leq 0.092$

DI (JL) $\alpha \leq 0.059$

DI (SL–MCMC) $\alpha \leq 0.12$ (including lines $\alpha \leq 0.079$)

**Consistency with std. Model**

Posterior for $\alpha$
$T_{\text{CMB}}(z)$ from SZE: simulations

Simulated observations of 50 well known clusters
mock dataset analyzed to recover input parameters of the cluster

Analysis: MCMC
allows to explore the full space of the cluster parameters and the $T_{\text{CMB}}(z)$

$P(v_p) = \mathcal{N}(0 \text{ km/s}, 1000 \text{ km/s})$

$P(v_p) = \mathcal{N}(0 \text{ km/s}, 100 \text{ km/s})$

$P(T_e) = \mathcal{N}(6.50 \text{KeV}, 0.14 \text{KeV})$
SZE experiments

- Ongoing and near future surveys with ACT, APEX–SZ, SPT, Planck and detailed mapping of a sample of nearby clusters with MAD and OLIMPO experiments will provide much more precise and uniform datasets:
  - bias in the ratio approach largely removed
  - reduced skweness in $\tau$ and $T_{\text{cmb}}(z)$ distributions (DI)
Future SZE experiments

- Accurate spectroscopic observations from space towards a limited number of clusters (like the proposed SAGACE satellite) would allow to control a large part of the degeneracies between $T_{\text{CMB}}(z)$ and cluster parameters.
SAGACE is a space-borne differential spectrometer, coupled to a 3m telescope:

- able to cover the frequency ranges 100-450 and 720-760 GHz;
- with angular resolution ranging from 4.2 to 0.7 arcmin;
- with photon-noise limited sensitivity (~ 1.5 Jy per second of integration for a 1 GHz resolution element, 50 mJy per second for 30 GHz resolution element)
- Designed to operate for 2 years

Its main goal is to observe the SZ effect on galaxy clusters with 15 GHz resolution, providing, for the first time, precisely calibrated SZ spectral maps of clusters over its wide frequency range (no intercalibration of different experiments).

It will map and measure the physical parameters of the hot gas extended to the outskirts of a sample of ~200 clusters, and to exploit the full potential of the SZ effect as a probe for cosmology:

- \( H_0 \) with a method completely independent of others
- Dark Matter and Dark Energy
- \( T_{\text{CMB}}(z) \) with a method completely independent of others
The scientific impact of SAGACE

The high accuracy of the spectral measurements allows to control a large part of the existing degeneracies between the cluster parameters.
Conclusions

• Original tool to perform an observational test of the standard scaling of $T_{\text{CMB}}$ and its isotropy up to the redshift of galaxy clusters and to put constraints on alternative cosmological models.

• Information extracted *almost* for free from spectral SZ datasets.

• In the near future, SZE experiments will allow precise measurements of the $T_{\text{CMB}}(z)$ scaling law to set constraints on the variation of fundamental constants over cosmological time.