A global analysis of neutrino masses, mixings and phases: entering the new era of leptonic CP violation searches

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Outline

1. Why global analyses?
2. From first hints to evidence of $\theta_{13} > 0$
3. Global $3\nu$ analysis: methodological issues
4. Global $3\nu$ analysis: results
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   4.2 $(\theta_{13}, \delta_{\text{CP}})$ correlations
5. Conclusions

Mainly based on recent and earlier work done in collaboration with:
E. Lisi, A. Marrone, D. Montanino, A. Palazzo, A.M. Rotunno
1. Why global analyses?
In general, global analyses of data from different experiments are necessary when some physical parameters are not (precisely) measured by any single experiment.

In this case, the parameters may be at least constrained by a joint, careful comparison of various datasets.

Even when measurements become available, global analyses often remain useful to perform consistency tests of theoretical scenarios, where possible “tensions” may eventually emerge from the comparison of different datasets.

Of course, such analyses have obvious limitations: they cannot replace the experimental measurements!
2. From first hints to evidence of $\theta_{13} > 0$
A few years ago (2008), the good agreement of solar and KamLAND data in 2ν analyses was one of the main highlights ...

... agreement obtained assuming

\[ \nu_e = \cos \theta_{12} \nu_1 + \sin \theta_{12} \nu_2 \]

But the agreement could be even improved by going beyond the 2ν approximation and allowing 3ν mixing ...

For 3ν:

\[ \nu_e = \cos \theta_{13} (\cos \theta_{12} \nu_1 + \sin \theta_{12} \nu_2) + e^{-i \delta} \sin \theta_{13} \nu_3 \]

[figure taken from the official KamLAND site (2008)]

mixing angle $\theta_{13}$

possible CP phase $\delta_{CP}$
Indeed, nonzero $\theta_{13}$ generally improves the convergence of the two data sets.

$$\sin^2\theta_{13} = 0$$

$$\sin^2\theta_{13} = 0.03$$

[GLF, Lisi, Marrone, Palazzo, Rotunno, Proc. of “NO-VE 2008”]
The solar+KamLAND hint for $\theta_{13} > 0$ can be plotted in the plane of the two mixing angles, where the different correlations of the two datasets are more evident:

![Diagram of mixing angles](image)

- **Best fit more than 1 sigma away from zero**
- $\sin^2 \theta_{13} = 0.021 \pm 0.017$ (Solar + KamLAND)

2008: first hint of $\nu_e$ as a superposition of three states.


Note: even though statistically weak (~1.5 $\sigma$), this hint was quite “clean”, as it involved very simple oscillation physics.
In addition, the solar + KL hint was consistent with a previous preference for $\theta_{13} > 0$, found from our $3\nu$ analysis of atmospheric + LBL + Chooz data ...


... mainly due to subleading “solar term” effects which help fitting atmospheric electron event data (especially sub-GeV).

[However, atmospheric $\nu$ physics and data analysis are admittedly more involved; statistical significance of the atmospheric hint somewhat debated in the literature.]
Our summary in 2008 ...

“Hints of $\theta_{13} > 0$ from global neutrino data analysis”

Hint significance: 90% C.L.

$\sin^2 \theta_{13} = 0.016 \pm 0.010$

Large literature and debate 2008-2011!
June 2011 breakthrough: new T2K and MINOS appearance results

Both experiments favored $\sin^2 \theta_{13} \sim \text{few}\%$

$P_{\mu e} = \sin^2 \theta_{23} \sin^2 (2\theta_{13}) \sin^2 (\Delta m^2 L/4E_\nu) + \text{subleading solar & CP terms (wiggles in the plots)}$

So, it makes sense to combine these with all the other oscillation data...
In conclusion, evidence for $\sin^2 \theta_{13} > 0$ at $>3\sigma$
(with small changes for new/old reactor fluxes
assumed in the fit)

\[ \sin^2 \theta_{13} = 0.021 \pm 0.007 \quad \text{("old" reactor fluxes)} \]
\[ \sin^2 \theta_{13} = 0.025 \pm 0.007 \quad \text{("new" reactor fluxes)} \]

Note:
ATM + LBL + CHOOZ now more significant than Solar + KamLAND

Very good agreement of two totally independent sets of data on $\sin^2 \theta_{13}$

2011 data: “Evidence of $\theta_{13} > 0$ from global neutrino data analysis”
[GLF, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028]
The 2011 evidence for $\sin^2 \theta_{13} > 0$ ...

\[ \sin^2 \theta_{13} = 0.021 \pm 0.007 \quad \text{(old reactor fluxes)} \]
\[ \sin^2 \theta_{13} = 0.025 \pm 0.007 \quad \text{(new reactor fluxes)} \]

... should be compared with the very recent (2012) SBL reactor data:

- **Double Chooz (far detector):** $\sin^2 \theta_{13} = 0.022 \pm 0.013$
- **Daya Bay (near + far detectors):** $\sin^2 \theta_{13} = 0.024 \pm 0.004$
- **RENO (near + far detectors):** $\sin^2 \theta_{13} = 0.029 \pm 0.006$

we find a spectacular agreement!
Status of 3ν oscillation parameters, one year ago (2011)

Global data analysis in terms of $N\sigma = \sqrt{\Delta \chi^2}$
(the more linear and symmetric, the more gaussian errors)

“small” $\delta m^2$ ("solar" $\nu$)

mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$

“large” $\Delta m^2$ ("atmospheric" $\nu$)


Still open problems (2011), apart from $\delta_{CP}$ ...

1. 3ν mass-mixing hierarchy  ➔ no hints so far

2. $\sin^2 \theta_{23}$: $\nu_\mu$-$\nu_\tau$ mixing, maximal or not? ➔ weak hints ?...

- SK@Neutrino2010
- Gonzalez-Garcia et al. 2010
- Our analysis, 2011

Nearly maximal...

Slightly nonmaximal...

Slightly more nonmaximal...

It includes now SK I+II+III data with both $\theta_{13} \neq 0$ and $\delta m^2 \neq 0$

$\sin^2 \theta_{23}$

1. $\nu_\mu$-$\nu_\tau$ mixing, maximal or not?

$\sin^2 \theta_{23}$

$\sin^2 \theta_{23}$
3. Global $3v$ analysis: methodological issues
There is an interesting interplay between LBL appearance + disappearance data, SBL reactor data, and octant degeneracy, as pointed out long ago [GLF & Lisi, Phys. Rev. D54 (1996) 3667, hep-ph/9604415]

For (slightly) non-maximal mixing, LBL appearance + disappearance data give rise to two quasi-degenerate solutions with a (slight) anticorrelation between $\theta_{13}$ and $\theta_{23}$.

[In our 1996 notation: $\varphi = \theta_{13}$, $\psi = \theta_{23}$]
On the other hand, SBL reactor data, or any other constraint on $\theta_{13}$ (eg, solar + KamLAND data), may “select” or at least “prefer” one of the two solutions, lifting (part of) the degeneracy.

As we shall see in a moment, this hypothetical 1996 scenario seems to emerge from 2012 data ...

... even though only at the level of hints!
This interplay is simply an effect of $3\nu$ oscillation physics: in first approximation, $\Delta m^2$-driven vacuum oscillation probabilities are generalized as $(2\nu \rightarrow 3\nu)$:

\[
\begin{align*}
P_{\alpha\beta} &\approx \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E}\right) \\
P_{\alpha\alpha} &\approx 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E}\right)
\end{align*}
\]

\[
\rightarrow \quad P_{\alpha\beta} \approx 4|U_{\alpha3}|^2 |U_{\beta3}|^2 \sin^2 \left(\frac{\Delta m^2 L}{4E}\right) \\
P_{\alpha\alpha} \approx 1 - 4|U_{\alpha3}|^2 (1 - |U_{\alpha3}|^2) \sin^2 \left(\frac{\Delta m^2 L}{4E}\right)
\]

**LBL appearance:** $P_{\mu e} = \sin^2 \theta_{23} \sin^2(2\theta_{13}) \sin^2(\Delta m^2 L/4E_{\nu}) + \text{corrections}$

**NOT** octant symmetric, anticorrelates $\theta_{23}$ and $\theta_{13}$: the lower $\theta_{23}$, the higher $\theta_{13}$

**SBL reactors:** $P_{ee} = 1 - \sin^2(2\theta_{13}) \sin^2(\Delta m^2 L/4E_{\nu}) + \text{corrections}$

So, they may distinguish “high” from “low” $\theta_{13}$

---

3$\nu$ combination of LBL accelerator and SBL reactor data may already provide some slight preference for one $\theta_{23}$ octant versus the other.
In order to reveal a similar effect, one needs a full $3\nu$ analysis of LBL accelerator data (appearance + disappearance).

In particular it is important that T2K & MINOS abandon their (no longer defensible) $2\nu$ approximation in disappearance mode:

We suggest:

On the $x$-axis, put $\sin^2\theta_{23}$, NOT $\sin^22\theta_{23}$

On the $y$-axis, put some $3\nu$ (NOT $2\nu$) definition of $\Delta m^2$

We use:

$$\Delta m^2 = \frac{1}{2} (\Delta m^2_{31} + \Delta m^2_{32})$$

in both normal and inverted hierarchy (the sign just flips)
... it is also important that T2K and MINOS combine their own disappearance + appearance data in a "full-fledged" 3ν analysis without approximation or assumption a priori.

These LBL contours may be shifted to the left (right) for higher (lower) θ_{23}, due to the anti-correlation effect seen before ...
... it is also important that T2K and MINOS combine their own disappearance + appearance data in a “full-fledged” $3\nu$ analysis without approximation or assumption a priori.

These LBL contours may be shifted to the left (right) for higher (lower) $\theta_{23}$, due to the anti-correlation effect seen before ...

... this introduces obvious consequences for the comparison with $\theta_{23}$-independent SBL reactor data

**Suggestion:** never assume maximal mixing a priori, let the data decide!
Final “methodological note”:

We prefer to group LBL accelerator data with solar + KamLAND data, since the latter provide the “solar parameters” needed to calculate the full 3ν LBL probabilities in matter.

So, the sequence of constraints will be shown as:

(LBL + Solar + KamLAND) + (SBL reactor) + (SK atm)
4. Global $3\nu$ analysis: results *

(updated including the data presented at Neutrino 2012 by Daya Bay, Reno, T2K and MINOS)

4.1 \((\theta_{13}, \theta_{23})\) correlations

Let us remind the 1996 hypothetical example [hep-ph/9604415], in which ...

... two quasi-degenerate solutions from LBL app + disapp. data ...

... are solved in favor of higher \(\theta_{13}\) and 1st \(\theta_{23}\) octant by SBL reactor data ...
From 2012 LBL appearance + disappearance data plus solar + KamLAND data:

we obtain for both hierarchies, NH & IH:

- two quasi-degenerate and anticorrelated solutions (merging above 1σ)

Moreover:

- a weaker constraint on $\theta_{13}$ for inverted hierarchy (as already known from T2K, MINOS)
Adding SBL reactor data (Chooz, Double Chooz, Daya Bay, RENO):

- **Normal hierarchy**
  - Including v 2012 data

- **Inverted hierarchy**

For both NH & IH, large $\theta_{13}$ preferred!

For **NH**:
- Further preference for the solution with:
  - Higher $\theta_{13}$
  - Lower $\theta_{23}$ (1st octant)

For **IH**:
- Only a marginal preference for 1st octant
Adding SK atm data; some preference for $\theta_{23}$ in the 1st octant

Note: overall goodness of fit very similar in NH and IH. No hint about hierarchy yet...
We find a significant role of SK atm data in favoring $\theta_{23}$ in the 1st octant.

To this regards let us note that:

- The SK collaboration had found no significant "hint" of non-maximal mixing in their official $3\nu$ data analysis of 2010. [ditto for Forero, Tortola & Valle 2012, which however is not a full $3\nu$ for atmospheric data].

- However, another recent, full $3\nu$ data analysis also find a (weaker) preference for the 1st octant in atmospheric data: [T. Schwetz with Gonzalez-Garcia and Maltoni]

Therefore, concerning (non)maximal $\theta_{23}$:

The possible preference for the 1st octant is still a "fragile" feature, but worth of further studies in both LBL and ATM data analyses, provided that full $3\nu$ oscillation probabilities are included!
4.2 \((\theta_{13}, \delta_{\text{CP}})\) correlations

We reproduce well the T2K and MINOS contours under their same assumptions:

- Let us note that an analysis of \((\theta_{13}, \delta_{\text{CP}})\) has been performed by Minakata et al. using the recent LBL + SBL reactor data (Machado, Minakata, Nunokawa, Zukanovich Funchal, arxiv 1111.3330 and update 2012). [We agree under the same inputs and priors.]

- Similarly Forero, Tortola & Valle 2012 [but ignoring \(\delta_{\text{CP}} \neq 0\) and \(\delta m^2 \neq 0\) effects in atmospheric data].

Our analysis includes all data and treats all 3ν parameters as free (no prior assumptions).
With only LBL + solar + KamLAND data:

normal hierarchy

including \( \nu \) 2012 data

inverted hierarchy

no significant sensitivity to \( \delta_{CP} \) yet
Adding SBL reactor data:

- Normal hierarchy
- Inverted hierarchy

Inverting normal hierarchy at most $\sim 1\sigma$ sensitivity

Inverting inverted hierarchy not yet sensitivity at $\sim 1\sigma$

Including 2012 data
Adding SK atmospheric data:

We find a ~ $1\sigma$ preference for $\theta \sim \pi$ as in the early analysis of hep-ph/0506083.
We note that $\delta \sim \pi$ is compatible with previous SK atmospheric official results presented at “Neutrino 2010” by Y. Takeuchi,

Let us remind you our interpretation, proposed in hep-ph/0506083:

$\delta \sim \pi$ enhances the interference oscillation term and gives extra electron appearance for atmospheric events $O(\text{GeV})$, “explaining” part of the persisting SK electron excess.

Once more: this is a “fragile” feature, but worth further studies with current and near-future data.
5. Conclusions
Previous hints of $\theta_{13} > 0$ are now measurements! (and basically independent of old/new reactor fluxes)

Some hints of $\theta_{23}$ in the 1st octant are emerging at $\sim 2\sigma$, worth exploring by means of atm. and LBL+reac. data

A possible hint of $\delta_{CP} \sim \pi$ is emerging from atm. data [Is the PMNS matrix real?]

So far, no hints for NH $\leftrightarrow$ IH
Numerical $1\sigma$, $2\sigma$, $3\sigma$ ranges:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best fit</th>
<th>$1\sigma$ range</th>
<th>$2\sigma$ range</th>
<th>$3\sigma$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta m^2 / 10^{-5}$ eV$^2$ (NH or IH)</td>
<td>7.54</td>
<td>7.32 – 7.80</td>
<td>7.15 – 8.00</td>
<td>6.99 – 8.18</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12} / 10^{-1}$ (NH or IH)</td>
<td>3.07</td>
<td>2.91 – 3.25</td>
<td>2.75 – 3.42</td>
<td>2.59 – 3.59</td>
</tr>
<tr>
<td>$\Delta m^2 / 10^{-3}$ eV$^2$ (NH)</td>
<td>2.43</td>
<td>2.33 – 2.49</td>
<td>2.27 – 2.55</td>
<td>2.19 – 2.62</td>
</tr>
<tr>
<td>$\Delta m^2 / 10^{-3}$ eV$^2$ (IH)</td>
<td>2.42</td>
<td>2.31 – 2.49</td>
<td>2.26 – 2.53</td>
<td>2.17 – 2.61</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13} / 10^{-2}$ (NH)</td>
<td>2.41</td>
<td>2.16 – 2.66</td>
<td>1.93 – 2.90</td>
<td>1.69 – 3.13</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13} / 10^{-2}$ (IH)</td>
<td>2.44</td>
<td>2.19 – 2.67</td>
<td>1.94 – 2.91</td>
<td>1.71 – 3.15</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23} / 10^{-1}$ (NH)</td>
<td>3.86</td>
<td>3.65 – 4.10</td>
<td>3.48 – 4.48</td>
<td>3.31 – 6.37</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23} / 10^{-1}$ (IH)</td>
<td>3.92</td>
<td>3.70 – 4.31</td>
<td>3.53 – 4.84 $\oplus$ 5.43 – 6.41</td>
<td>3.35 – 6.63</td>
</tr>
<tr>
<td>$\delta / \pi$ (NH)</td>
<td>1.08</td>
<td>0.77 – 1.36</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\delta / \pi$ (IH)</td>
<td>1.09</td>
<td>0.83 – 1.47</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Note: above ranges obtained for “old” reactor fluxes. For “new” fluxes, ranges are shifted (by $\sim 1/3 \sigma$) for two parameters only: $\Delta \sin^2 \theta_{12} / 10^{-1} \approx +0.06$ and $\Delta \sin^2 \theta_{13} / 10^{-2} \approx +0.10$

**Fractional $1\sigma$ accuracy** [defined as $1/6$ of $\pm 3\sigma$ range]

<table>
<thead>
<tr>
<th>$\delta m^2$</th>
<th>$\sin^2 \theta_{12}$</th>
<th>$\sin^2 \theta_{13}$</th>
<th>$\sin^2 \theta_{23}$</th>
<th>$\Delta m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6%</td>
<td>5.4%</td>
<td>10%</td>
<td>14%</td>
<td>3.0%</td>
</tr>
</tbody>
</table>

We were already in the precision era for $\nu$ physics!
Thanks for your attention!