Exploring the quantum dynamics of atoms and photons in cavities

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Experiments in which single atoms and photons are manipulated in high Q cavities are modern realizations of the thought experiments of the founders of quantum theory. They provide textbook illustrations of quantum concepts such as entanglement, complementarity and decoherence, allow us to demonstrate basic procedures of quantum information (realization of quantum gates) and provide ideal illustrations of measurement theory.

Outline of Lectures

1. Entangling atoms and photons in a cavity
2. Entanglement, complementarity and decoherence: an introduction to Schrödinger cat states
3. Quantum non-demolition photon counting and quantum jumps of light
4. Reconstruction of non-classical trapped fields and movie of their decoherence
CQED experiments are modern - and real - versions of the thought experiments imagined by Einstein and Bohr...

A box stores a photon which can be released on demand at a well defined time....

Weighing the box before and after the release of the photon determines its energy.
Is this violating Heisenberg relation $\Delta E \cdot \Delta T > \hbar$?

Here, the field is « weighed » and manipulated by atoms...
The ENS photon box (latest version)

In its latest version, the cavity has a damping time in the 100 millisecond range. Atoms cross it one at a time.

Cavity half-mounted...

...and fully-mounted
Lecture 1:
Entangling atoms and photons in a cavity

«Two-level» Rydberg atoms interact one at a time with the microwave field stored in a very high Q superconducting cavity, before being detected.

Tests of entanglement and demonstration of quantum logic.
1A
Principles and methods of
Cavity Quantum Electrodynamics (CQED)
A cavity with very reflecting mirrors keeps photons during long time intervals. Atoms cross the cavity one by one, or are trapped in it. This is the physics of atom-light coupling at the level of single particles (one atom and one photon). We deal here with the microwave domain only.
CQED: A story about oscillators and spins

Towards classical domain...

Mesoscopic field
(n~5-100)

Microscopic field

The spin:
a 2 level atom

The oscillator:
a cavity mode

Standard Hamiltonian
\[ H = \hbar \omega_a \frac{\sigma_z}{2} + \hbar \omega_c a^\dagger a + \hbar \Omega \left[ \sigma_+ a + \sigma_- a^\dagger \right] \]

Strong coupling

\[ \Omega \gg \frac{1}{T_c}, \frac{1}{T_a} \]

Optical
20 MHz, 5 MHz, 5 MHz

Microwaves
50 kHz, 1 kHz

The oscillator:
a cavity mode

The spin:
a 2 level atom

\[ n=0 \]

\[ e \]

\[ g \]
General scheme of experiments

Rev. Mod. Phys. 73, 565 (2001)
**Two essential ingredients**

\[ n = 50 \]

**Circular Rydberg atoms**
- Large circular orbit
- Strong coupling to microwaves
- Long radiative lifetimes (30ms)
- Level tunability by Stark effect
- Easy state selective detection
- Quasi two-level systems

\[ e \]
\[ n = 51 \]
\[ g \]
\[ n = 50 \]

**Superconducting microwave cavity**
- Gaussian field mode with 6mm waist
- Large field per photon
- Long photon life time (from 0.16ms in 1996 up to 130 ms now)
- Easy tunability
- Possibility to prepare Fock or coherent states with controlled mean photon number

**The « oscillator »**

**The « spin »**
The atom-cavity coupling: vacuum Rabi frequency

Giant atomic dipole \( D \approx qa_0 n^2 \approx 2000 qa_0 \)

Vacuum field fluctuations in mode volume \( V \)

\[
E_0 = \sqrt{\frac{\hbar \omega}{2 \varepsilon_0 V}} \approx 1.5 \text{ mV} / \text{m}
\]

\[
\Omega = \frac{D.E_0}{\hbar} = 50 \times 2\pi \text{ kHz}
\]
The path towards circular states: an adiabatic process involving 53 photons

Static magnetic field (18 Gauss) lifts the $\sigma^+/\sigma$ degeneracy
Controlling the atom-cavity interaction time by selecting the atom’s velocity via Doppler effect sensitive optical pumping.

Rubidium level scheme with transitions implied in the selective depumping and repumping of one velocity class in the $F=3$ hyperfine state.

In green, velocity distribution before pumping, in red velocity distribution of atoms pumped in $F=3$, before they are excited in circular Rydberg state.
Detecting circular Rydberg states by selective field ionization

Adjusting the ionizing field allows us to discriminate two adjacent circular states with a fidelity close to 100%. The global detection efficiency is > 90%.
Coherent state of light

Produced by coupling the cavity to a source (classical oscillating current)

A Poissonian superposition of photon number states:

$$|\alpha\rangle = \sum_n C_n |n\rangle, \quad C_n = e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}},$$

$$\bar{n} = |\alpha|^2, \quad P(n) = |C_n|^2 = e^{-\bar{n}} \frac{n^{-n}}{n!}$$

Complex plane representation

Amplitude $|\alpha|$ 

Uncertainty circle

$\varphi = \varphi_0 - \omega_c t$
**Resonant Rabi flopping**

\[
\cos\left(\frac{\Omega t}{2}\right)|e,0\rangle + \sin\left(\frac{\Omega t}{2}\right)|g,1\rangle
\]

*Spontaneous emission and absorption involving atom-field entanglement*

When \(n\) photons are present, the oscillation occurs faster (stimulated emission):

\[
\cos\left(\sqrt{n+1}\frac{\Omega t}{2}\right)|e,n\rangle + \sin\left(\sqrt{n+1}\frac{\Omega t}{2}\right)|g,n+1\rangle
\]

*Simple dynamics of a two-level system (|e,n>, |g,n+1>)*
Rabi flopping in vacuum or in small coherent field: a direct test of field quantization

\[ P_e(t) = \sum p(n) \cos^2 \left( \frac{\Omega \sqrt{n + 1} t}{2} \right) ; \quad p(n) = e^{-\frac{n}{n!}} \]

\[ n = 0 \quad (n_{th} = 0.06) \]
\[ n = 0.40 \quad (\pm 0.02) \]
\[ n = 0.85 \quad (\pm 0.04) \]
\[ n = 1.77 \quad (\pm 0.15) \]

Useful Rabi pulses
(quantum « knitting »)

Initial state

$|e,0\rangle \rightarrow |e,0\rangle + |g,1\rangle$

$\pi / 2$ pulse
creates atom-cavity
entanglement

$|e,0\rangle \rightarrow \cos\left(\frac{\Omega t}{2}\right)|e,0\rangle + \sin\left(\frac{\Omega t}{2}\right)|g,1\rangle$

Brune et al, PRL 76, 1800 (96)
Useful Rabi pulses (quantum knitting)

\[ |e,0\rangle \rightarrow |g,1\rangle \]
\[ |g,1\rangle \rightarrow |e,0\rangle \]
\[ |g,0\rangle \rightarrow |g,0\rangle \]
\[ (|e\rangle + |g\rangle)|0\rangle \rightarrow |g\rangle (|1\rangle + |0\rangle) \]

\[ |e,0\rangle \rightarrow \cos\left(\frac{\Omega t}{2}\right) |e,0\rangle + \sin\left(\frac{\Omega t}{2}\right) |g,1\rangle \]

Brune et al, PRL 76, 1800 (96)
Useful Rabi pulses (quantum knitting)

\[ |e,0\rangle \rightarrow -|e,0\rangle \]
\[ |g,1\rangle \rightarrow -|g,1\rangle \]
\[ |g,0\rangle \rightarrow |g,0\rangle \]

2\pi pulse:
conditional dynamics and quantum gate

\[ |e,0\rangle \rightarrow \cos\left(\frac{\Omega t}{2}\right)|e,0\rangle + \sin\left(\frac{\Omega t}{2}\right)|g,1\rangle \]

Brune et al, PRL 76, 1800 (96)
1B.

Entanglement experiments and quantum gate in CQED
Atom pair entangled by photon exchange

Electric field $F(t)$ used to tune atoms 1 and 2 in resonance with $C$ for times $t$ corresponding to $\pi/2$ or $\pi$ Rabi pulses

Hagley et al, P.R.L. 79,1 (1997)
Ramsey interferometer with phase controled by field in the cavity

Resonant classical $\pi/2$ pulses in auxiliary cavities $R_1$-$R_2$ (with adjustable phase offset $\phi$ between the two) prepare and analyse atom state superpositions.

The phase of the atomic fringes and their amplitude depend upon the state of field in $C$, which affect in different ways the probability amplitudes associated to states $e$ and $g$.

The probabilities $P_e$ (or $P_g = 1 - P_e$) for finding atom in $e$ or $g$ oscillate versus $\phi$.

$$P(g) = \frac{A}{2} (1 + \cos(\phi))$$
Effect of $2\pi$ Rabi flopping on Ramsey signal

Cavity $C$ resonant with $e$-$g$ (51-50) transition.

Ramsey $R_1$-$R_2$ interferometer resonant with $g$-$i$ (50-49) transition.

$2\pi$ Rabi flopping on transition $e$-$g$ in 1 photon field induces a $\pi$ phase shift between the $g$ and $i$ amplitudes.

$g$-$i$ fringes are inverted when photon number in $C$ increases from 0 to 1.
Ramsey fringes conditioned to one photon in $C$

With proper phase choice, atom is detected in $g$ if $n = 0$, in $i$ if $n = 1$:

quantum gate with photon (0/1) as control qubit and atom (i/g) as target qubit

---

Experiment with 1$^{\text{st}}$ atom acting as source emitting 1 photon with probability 0.5 ($\pi/2$ pulse on $e$-$g$ transition) and 2$^{\text{nd}}$ probe atom undergoing Ramsey interference on g-i transition
The quantum gate with photon as control bit realises a quantum non-demolition measurement (QND) of field

Control bit (photon): $a = 0/1$

Target bit (atom): $b = 0 \ (g) / 1 \ (i)$

The atom carries away information about field energy without altering the photon number (2$\pi$ Rabi pulse). This is very different from usual photon detection, which is destructive


Repetitive QND measurement of photons stored in super high Q cavity.
Third lecture (tomorrow)
Combining Rabi pulses for entanglement knitting

First atom prepares 1 photon with 50% probability (pulse $\pi/2$) and second atom reads photon by QND (pulse $2\pi$).

Third atom absorbs field (pulse $\pi$), producing a three atom correlation (GHZ state).

Three particle engineered entanglement (Rauschenbeutel et al, Science, 288, 2024 (2000))
Conclusion of first lecture

The resonant interaction between atom and cavity mode can be used to entangle atoms and photons and atoms with atoms and to realize quantum gates and elementary steps of quantum information.

The non-resonant coupling of atoms with the cavity is also useful to achieve atom-field entanglement in a dispersive way. We will see in Lecture 2 that it can be used to prepare Schrödinger cat states of the field (which are superposition of coherent field states with different phases) and to study their decoherence.

We will also see in the last two lectures that the non-resonant atom-field coupling is very useful to achieve repetitive quantum non-destructive measurements of light.