Exploring the quantum dynamics of atoms and photons in cavities

Lecture 2
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Entanglement, complementarity and decoherence: an introduction to Schrödinger cat states

2A. Complementarity tests in CQED
2B. Mesoscopic superpositions of field states (Schrödinger cats) and their decoherence
2C. Towards non-local cats stored in two cavities
2A.

Complementarity tests in CQED

Which slit did a given particle go through?

Wave or particle??
A thought experiment about complementarity

- Microscopic slit: recoils due to impact of scattered particle. Path Information: no fringes

- Macroscopic slit: insensitive to scattered particle (negligible recoil). No path information: fringes are visible

- Wave and corpuscular aspects are complementary for a quantum system

Einstein-Bohr discussion at Solvay

particle-slit entanglement!
A modern version of the Bohr-Einstein thought experiment with a Mach-Zehnder

Interference between two paths.
How to get which-path information?
A Mach-Zehnder with a moving beam-splitter

Heavy beam-splitter: negligible recoil, no path information and fringes

Microscopic beam-splitter: important recoil, path information available and no fringes
Complementarity and entanglement: quantitative analysis

Initial state of beam-splitter (h.o) \( |0\rangle \)

Final state corresponding to path b (coherent state) \( |\alpha\rangle \)

State of “particle + beam-splitter”
\[
|\Psi\rangle = |\Psi_a\rangle |0\rangle + |\Psi_b\rangle |\alpha\rangle
\]

Interference amplitude linked to entanglement
\[
\langle \Psi_a | \Psi_b \rangle \langle 0 | \alpha \rangle
\]

Small mass, important recoil
entanglement and no fringes
\[
\langle 0 | \alpha \rangle = 0
\]

large mass, weak recoil
No entanglement and fringes
\[
\langle 0 | \alpha \rangle = 1
\]
Complementarity and decoherence

Entanglement with another system destroys interference
- “Explicit” Detector (moving beam-splitter/ external detector)
- Uncontrolled “measurements” by environment (decoherence)

Complementarity, decoherence and entanglement are intimately linked
More realistic system: Ramsey interferometry

- Two resonant $\pi / 2$ classical pulses on an atomic transition e/g

Which path information?
Atom emits one photon in $R_1$ or $R_2$

Ordinary macroscopic fields
(heavy beam-splitter)
Field state not appreciably affected. No "which path" information

Mesoscopic Ramsey field
(light beam-splitter)
Addition of one photon changes the field. "which path" info

FRINGES

NO FRINGES
A system at the quantum/classical boundary

Coherent field in a cavity

- State produced by a classical source coupled for finite time to the cavity mode: field defined by complex amplitude $\alpha$

- A picture in phase space (Fresnel plane)

From quantum to classical

- Vacuum or small field:
  - Large quantum fluctuations. A field at the single-photon level is a quantum object

- Large field
  - Small quantum fluctuations. A field with more than 10 photons is almost a classical object
Bohr’s experiment with a Ramsey interferometer

Store one Ramsey field in a high Q cavity

Atom-cavity interaction time tuned for $\pi / 2$ pulse
Possible even if C empty

Initial cavity state $|\alpha\rangle$

- Intermediate atom-cavity state
  - Ramsey fringes contrast
  
  $$|\psi\rangle = \frac{1}{\sqrt{2}} \left( |e, \alpha_e\rangle + |g, \alpha_g\rangle \right)$$

- Large field $|\alpha_e\rangle \approx |\alpha_g\rangle \approx |\alpha\rangle$ FRINGES
- Small field $|\alpha_e\rangle = |0\rangle, |\alpha_g\rangle = |1\rangle$ NO FRINGE
Quantum/classical limit for an interferometer

Fringes contrast versus photon number $N$ in first Ramsey field

Fringes vanish for quantum field

 photon number plays the role of the beam-splitter's "mass"

An illustration of the $\Delta N \Delta \Phi$ uncertainty relation:

• Ramsey fringes reveal field pulses phase correlations.

• Small quantum field: large phase uncertainty and low fringe contrast

2B.

Mesoscopic superpositions of field states
(Schrödinger cats)
Non-resonant atom-field coupling

At perturbative limit ($\Omega \sqrt{n+1} \ll \delta$), the atom-cavity energy states undergo $2^{nd}$ order shifts in $\Omega$ (quantized light shifts in $n$).

On a coherent field, the coupling results in a phase shift proportional to the interaction time. Atoms in $g$ and $e$ produce opposite shifts.

$$|g\rangle \otimes |\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |g,n\rangle \rightarrow e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{i \int dt \frac{\Omega^2(t)n}{4\delta}} |g,n\rangle = |g\rangle \otimes |\alpha e^{i\varphi}\rangle$$

Phase shift of a coherent field (a single atom index!)

$$\varphi = \int dt \frac{\Omega^2(t)}{4\delta}$$

Atom in $g$:

$$|\alpha e^{i\phi}\rangle$$

Atom in $e$:

$$|\alpha e^{-i\phi}\rangle$$
Measuring the phase of a coherent field by homodyne method

To the signal field to be measured, we add a reference field of same amplitude and variable phase $\phi_r$. When $\phi_r = \phi + \pi$, the resulting field cancels in C (destructive classical interference).

After injecting the reference, a probe atom resonant with C and prepared in state $g$ is sent across C. Its final state is measured. The probability $P_g$ for finding it in $g$ is maximum for $\phi_r = \phi + \pi$. By repeating the experiment many times for different $\phi_r$, we reconstruct $P_g(\phi_r)$ whose variations reproduce the phase distribution of the signal field.

Measuring the phase shift induced by single atom (a one particle index effect)

- A coherent field is injected in C and a single non-resonant atom, prepared in e or g, phase-shifts this field. A reference field with a variable phase is then added into C and a second atom, prepared in state g and resonant with the field, is sent across C and finally detected. The sequence is repeated many times for each value of the reference field’s phase. Using the probe signal given by the second atom, we reconstruct the phase distribution of the field perturbed by the first atom.
**Phase shift induced by a single atom on a mesoscopic field**

Coherent field with 29 photons on average.

\( \delta/2\pi = 50 \text{ kHz} \)

**Phase-shift**
\( \pm 39^\circ \)

The coherent field is a « meter » whose phase points towards two different directions depending upon the state of the atom: a simple model for a measuring device in quantum physics.
Preparation of a photonic Schrödinger cat

Dispersive coupling of a coherent state with an atom prepared in a state superposition.

Classical source (initial injection of a coherent field in state |\alpha\rangle).

Field acquires two phases «at the same time», each component being correlated to an atomic state: entanglement between a mesoscopic system (field) and a microscopic object (atom).

\[
\left( |g\rangle + |e\rangle \right) \otimes |\alpha\rangle \rightarrow |g\rangle \otimes |\alpha e^{i\varphi}\rangle + |e\rangle \otimes |\alpha e^{-i\varphi}\rangle
\]
Detecting atom-field entanglement: again complementarity at work

Ramsey interferometer with Schrödinger cat trapped inside: the field is a «meter» informing about the atom’s path. If the final field states are quasi-orthogonal (disconnected uncertainty circles), the atomic Ramsey fringes vanish. The fringe contrast reveals the degree of entanglement between atom and field.

Ramsey signal for three different phase splittings $\phi$ between the cat components ($\phi$ is increased by decreasing $\delta$)

Fringe contrast versus $\phi$: the fringe vanishing signals the separation of the two field components in phase space (Schrödinger cat).

Fringe phase-shift versus $\phi$. The slope yields an absolute calibration of mean photon number (9.5 here). This is a QND measurement of photon number (1st lecture).
Schrödinger cat’s decoherence: entanglement and complementarity again

Entanglement with environment (outside modes coupled with $C$ by scattering):

\[
\left( |\alpha e^{i\varphi}\rangle + |\alpha e^{-i\varphi}\rangle \right) \otimes |E_0\rangle \rightarrow |\alpha e^{i\varphi}\rangle \otimes |E_+(t)\rangle + |\alpha e^{-i\varphi}\rangle \otimes |E_-(t)\rangle
\]

Very fast decay of $\langle E_+(t)| E_-(t)\rangle$ (information leaking into environment) corresponds to the transformation of cat into a statistical mixture (vanishing of interferences between cat’s components):

\[
|\alpha e^{i\varphi}\rangle + |\alpha e^{-i\varphi}\rangle \xrightarrow{\langle E_+(t)| E_-(t)\rangle \rightarrow 0} |\alpha e^{i\varphi}\rangle \langle \alpha e^{i\varphi} | + |\alpha e^{-i\varphi}\rangle \langle \alpha e^{-i\varphi} | \\
\text{(Tracing over } E) 
\]

Decoherence time:

\[
T_D \sim T_C / \bar{n}
\]

Decoherence becomes more and more efficient as $n$ increases
Observing decoherence by two atom Ramsey interferometry


After $R_1$ and $C$, atom 1 is entangled with field. It is then subjected to a second classical $\pi / 2$ pulse in $R_2$:

$$\left| g \right\rangle \otimes \left| \alpha e^{i\varphi} \right\rangle + \left| e \right\rangle \otimes \left| \alpha e^{-i\varphi} \right\rangle \xrightarrow{R_2} \left( \left| g \right\rangle - \left| e \right\rangle \right) \otimes \left| \alpha e^{i\varphi} \right\rangle + \left( \left| e \right\rangle + \left| g \right\rangle \right) \otimes \left| \alpha e^{-i\varphi} \right\rangle$$

Atom 1 is finally detected in $|g\rangle$ or $|e\rangle$, projecting field into one of the two «Schrödinger cat» states:

$$\left| \Psi^\pm_{\text{chat}} \right\rangle = \left| \alpha e^{i\varphi} \right\rangle \pm \left| \alpha e^{-i\varphi} \right\rangle \quad \begin{cases} \text{(sign + : atom 1 in } g; \\ \text{sign - : atom 1 in } e \end{cases}$$

Assume atom 1 detected in $g$. State $\left| \Psi^+_{\text{chat}} \right\rangle$ undergoes decoherence during time $t$, then atom 2 («probing quantum mouse ») is sent across same set-up and is in turn detected….
When atom 2 enters in \( C \), the «field + E + atom 2» system’s state is:

\[
|\Xi\rangle = (|e\rangle + |g\rangle)_{2} \otimes \left( |ae^{iq}\rangle \otimes |E_{+}(t)\rangle + |ae^{-iq}\rangle \otimes |E_{-}(t)\rangle \right)
\]

After atom 2 has crossed \( C \), this state becomes:

\[
|\Xi'\rangle = e_{2} \otimes |\alpha\rangle \otimes |E_{+}(t)\rangle + e_{2} \otimes |ae^{-2iq}\rangle \otimes |E_{-}(t)\rangle + g_{2} \otimes |ae^{2iq}\rangle \otimes |E_{+}(t)\rangle + g_{2} \otimes |\alpha\rangle \otimes |E_{-}(t)\rangle
\]

2 paths lead to the same final state \(|\alpha\rangle\) of field (in the 2 cases, 2nd atom undoes the phase-shift induced by the 1st):

The two final states correspond to different states of \( E \).
After atom 2 has crossed \( R_2 \), its states are mixed with equal weights, leading to two final states associated to the \( |\alpha\rangle \) field state:

\[
|\alpha\rangle \otimes \left( |e_2\rangle + |g_2\rangle \right) \otimes |E_+(t)\rangle \\
|\alpha\rangle \otimes \left( |g_2\rangle - |e_2\rangle \right) \otimes |E_-(t)\rangle
\]

The « field + atom 2 + E » system, after detecting atom 1 in \( g \), is in state:

\[
|\Xi_{g_1}\rangle = |\alpha\rangle \otimes \left[ \left( |e_2\rangle + |g_2\rangle \right) \otimes |E_+(t)\rangle + \left( |g_2\rangle - |e_2\rangle \right) \otimes |E_-(t)\rangle \right] + \text{terms en } |\alpha e^{2i\varphi}\rangle
\]

Hence, the conditional probabilities \( P_{g/g} \) et \( P_{e/g} \) for finding 2\(^{nd} \) atom in \( g \) or \( e \) provided the first atom has been found in \( g \) take the form:

\[
P_{g/g} = a + b \text{Re}\left\langle E_+ (t) | E_-(t) \right\rangle \\
P_{e/g} = a - b \text{Re}\left\langle E_+ (t) | E_-(t) \right\rangle \\
(a=1/2 \text{ et } b=1/4)
\]

We construct an atomic correlation signal \( \eta = P_{g/g} - P_{e/g} = Re \left\langle E_+ (t) | E_-(t) \right\rangle / 2 \) proportional to the overlap of the \( E \) final states, which measures the quantum interference between the components of the cat created by atom 1. Observing the decay of \( \eta \) versus delay between the atoms amounts to witnessing the decoherence process.
Variation of $\eta$ versus delay between the two atoms for two different values of $\phi$. The maximum of $\eta$ (ideally 0.5) is reduced to 0.18 by imperfections.

First experimental decoherence signal


How the states of the meter loose their coherence in a quantum measurement

For a more detailed and more direct observation of decoherence of larger cats, wait Lecture 4
Orders of magnitude of cats’ sizes

Cat must lose its coherence slower than it is prepared!

Decoherence time:

\[ T_D \sim \frac{T_C}{\bar{n}} \]

Preparation time \( T_p \) (cavity crossing time):

\[ \frac{\Omega^2 T_p}{4\delta} \sim 1 \quad \rightarrow \quad T_p \sim \frac{4\delta}{\Omega^2} \geq \frac{4\sqrt{\bar{n}}}{\Omega} \]

Hence condition:

\[ T_D \geq T_p \quad \rightarrow \quad \bar{n} \leq \left( \frac{\Omega T_c}{4} \right)^{2/3} \]

\textbf{progress of cavities}

\begin{align*}
\text{en 1996 : } & \quad T_C \sim 0.16 \text{ ms} \quad \rightarrow \quad \bar{n} \leq 5 \\
1999 – 2005 : & \quad T_C \sim 1 \text{ ms} \quad \rightarrow \quad \bar{n} \leq 16 \\
2006 – 2007 : & \quad T_C \sim 130 \text{ ms} \quad \rightarrow \quad \bar{n} \leq 400
\end{align*}
Towards non-local cats stored in two cavities

Davidovich et al, PRA 53, 1295 (1996)
Two-cavity entanglement experiment

Cavities $C_1$ and $C_2$

Set-up (almost) ready
Tricks to play with fields in two identical cavities \((R_2 \text{ not used})\)

1. Prepare state \(|\beta,\beta\rangle\) by injecting same field in \(C_1\) and \(C_2\)

\[
|\beta,\beta\rangle
\]

2. Send atom in state \(|e\rangle + |g\rangle\) inducing \(\pm \pi/2\) phase-shift. Define \(\alpha = i\beta\). Detecting the atom in \(e\) or \(g\) projects field in entangled state of \(C_1\)-\(C_2\):

\[
|\alpha,\alpha\rangle \pm |\alpha,-\alpha\rangle \over \sqrt{2}
\]

3. Injecting amplitude \(\alpha\) in \(C_1\) and \(-\alpha\) in \(C_2\) transforms state into another entangled state:

\[
|2\alpha,0\rangle \pm |0,-2\alpha\rangle \over \sqrt{2}
\]

4. Performing QND measurement of total photon number with a stream of atoms collapses field into « NOON state »:

\[
|n,0\rangle \pm |0,n\rangle \over \sqrt{2}
\]

Non-local Schrödinger cat states at quantum-classical boundary for Bell's inequality tests and decoherence studies
Conclusion of lectures 1 and 2

Rydberg atoms and microwave photons in a superconducting cavity constitute an ideal system to test fundamental concepts of quantum physics (entanglement, complementarity, decoherence) and to demonstrate essential steps of quantum information processing (quantum gates).

Recent advances in cavities technology (leading to a two order of magnitude increase of their damping time) open the way to new experiments: repeated non-demolition measurement of photons, full reconstruction of photonic states, study of large size cat states, non-local mesoscopic superpositions of fields in two cavities...

Last two lectures