The S-Z Effect & Its Use as a Probe

Yoel Rephaeli

Tel Aviv University & UC San Diego

Astrophysics of Galaxy Clusters

July 17, 2008
1. Theory of the S-Z effect
2. The effect as a probe
3. Nonthermal phenomena in clusters
The Effect as a Probe

* Intracluster gas properties
* Mass of a cluster in hydrostatic equilibrium
* Velocities from the kinematic component
* $H_0$ from a sufficiently large cluster sample
* Global parameters from induced CMB anisotropy and cluster surveys
* Form and isotropy of $T(z)$ & constraints on non-standard cosmological models
Radial velocities

- Measurements at three spectral bands, including the crossover frequency, yield both $\tau$ & $v_r$

SuZIE:
- 140, 218, 270 GHz

SuZIE II:
- 145, 221, 355 GHz

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Radial velocity, km/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1689</td>
<td>170 (+815, -630)</td>
</tr>
<tr>
<td>A2163</td>
<td>490 (+1370, -880)</td>
</tr>
<tr>
<td>A1835</td>
<td>-225 (+1370, -880)</td>
</tr>
<tr>
<td>A2261</td>
<td>-1400 (+1725, -1050)</td>
</tr>
<tr>
<td>A2390</td>
<td>1950 (+6275, -2675)</td>
</tr>
<tr>
<td>Cl0016</td>
<td>-4050 (+2900, -1775)</td>
</tr>
<tr>
<td>Zw3146</td>
<td>-650 (+3550, -1875)</td>
</tr>
<tr>
<td>MS0451</td>
<td>750 (+1500, -1125)</td>
</tr>
</tbody>
</table>

Holzapfel & 97, Benson & 03
• Measurement errors dominate (at present); systematic uncertainties include:
  • gas modeling
  • primary anisotropy (has same spectral dependence as $\Delta I_k$) at a level of $\pm 400$ km/s
  • sub-millimetric emission from galaxies
Measurement of $H_0$

- Need to use other independent methods to control systematics
- CMB anisotropy yields a global average: $71.9^{+2.6}_{-2.9} \text{ km/s Mpc}$
  
  - Advantages of the SZX method:
    - has a simple, well understood physical basis
    - direct, not based on a distance ladder
    - independent of redshift
    - can test isotropy of the expansion

Komatsu ea (06)
• The different density dependences of thermal emissivity and scattering allow extraction of the angular diameter distance, $d_A$

Cavaliere & 77, Gunn 78

• Need these observables:
  • X-ray surface brightness, $S_x$, & $T_e$
  • S-Z intensity change, $\Delta I$
  • density and temperature profiles, core radius, $\theta_c$, & $z$

\begin{align*}
S_x &= \frac{1}{4\pi(1+z)^3} \int n_e^2 \Lambda(\varepsilon, T_e) dl, \quad l = d_A \theta \\
\Delta I &= I_0 \sigma_T \int n_e \Psi(x, T_e) dl \\
d_A &= \frac{N_{SZ}^2}{N_x} \frac{\Lambda_0}{4\pi(1+z)^3 \Psi_0^2 I_0^2 \sigma_T^2}
\end{align*}
• $H_0$ is determined from

\[
d_A = \frac{c}{H_0 (1 + z)_0^\infty} \left[ \Omega_m (1 + z)^3 + \Omega_\Lambda \right]^{-1/2} d\Omega
\]
\[
\Omega_m + \Omega_\Lambda = 1
\]

• Optimal implementation of SZX method requires high spatial resolution, multi-frequency measurements of a large sample of (‘regular’) nearby and distant clusters
Systematic uncertainties:

- sphericity
  - in a large sample, expect small mean deviation
  - can employ deprojection algorithms in joint analyses of S-Z & lensing data
- isothermality?
  - can select well-studied clusters with measured temperature profile
- clumpiness, \( C = \langle n_e^2 \rangle / \langle n_e \rangle^2 \)
  - may lead to overestimation of \( H_0 \) by \( \sim C \)
  - if in isobaric equilibrium, impact of clumps will be weaker
• velocity
• CMB anisotropy
• nonthermal Comptonization

• Current estimated level of overall systematic uncertainty is \(\approx 20\%\)

• Errors due to asphericity, velocity and CMB primary anisotropy can be substantially reduced by averaging over a large sample

• Overall measurement uncertainty in \(H_0\) that seems to have already been attained is \(\approx 5\%\)
• BIMA, OVRO and Chandra measurements of 38 clusters, $0.14 < z < 0.89$

Models:

♦ $\Omega_m = 0.3, \Omega_{\Lambda} = 0.7$

♦ M1: Markov chain Monte Carlo estimation of all DM (NFW profile) and gas parameters; HE

♦ M2: isothermal $\beta$, no HE

♦ M3: isothermal $\beta$, excluding central 100 kpc; no HE

$$H_0 [\text{km / (s Mpc)}] = \begin{cases} 76.9^{+3.9+10.0}_{-3.4-8.9}, & M1 \\ 73.7^{+4.6+9.5}_{-3.8-7.6}, & M2 \\ 77.6^{+4.8+10.1}_{-4.3-8.2}, & M3 \end{cases}$$
**S-Z anisotropy & cluster counts**

- Important probe of the LSS on cluster scales, cluster mass function, evolution, and global parameters
- Extensively explored in theoretical models and hydrodynamic simulations
- Wide range of predicted power spectra and cluster number counts reflecting uncertainties in values of global, LSS and cluster quantities
Cluster properties:

- polytropic gas, $T_e(r) \propto n(r)^{\gamma - 1}$, usually assumed to have $\beta$ density profile
- gas mass $M_g = f_g M$ ($M$ is total mass within $R_V$)

Profile of thermal $S$-$Z$ component:

$$
\varphi(\theta) = \chi(\theta)^{-2\mu} \varepsilon(\theta) \, _2F_1[1/2, \mu, 3/2, -\varepsilon(\theta)^2 / \chi(\theta)^2]
$$

$$
\chi(\theta) = \left(1 + \frac{\theta^2}{\theta_c^2}\right)^{1/2}, \quad \varepsilon(\theta) = \left(\frac{R_v^2}{r_c^2} - \frac{\theta^2}{\theta_c^2}\right)^{1/2}, \quad \mu = \frac{3 \beta \gamma}{2}
$$

$$
\varphi(\theta) = \tan^{-1}\left[\frac{\varepsilon(\theta) / \chi(\theta)}{\chi(\theta)}\right], \quad \beta = \frac{3}{2}, \quad \gamma = 1
$$
Cluster population:

- mass range, $M_{\text{min}} \sim 10^{13} h^{-1} M_\odot$, $M_{\text{max}} \sim 10^{16} h^{-1} M_\odot$
- gas evolution, when included, taken in the general form $f(M,z) \propto M^{\eta \xi}$ (e.g., $\eta=0.2$, $\xi=1.45$ inferred from EMSS)
- mass function either Press-Schechter,

\[
\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} \frac{\Omega_m \rho_c \delta}{M \sigma^2} \left( -\frac{d\sigma}{dM} \right) e^{-\frac{\delta^2}{2\sigma^2}}
\]

\[
\sigma^2 = \frac{1}{2\pi^2} \int_0^\infty P(k) k^2 W^2(kR) dk, \quad P(k) = A k^n T^2(k)
\]

\[
\sigma = \sigma_8 \left( \frac{M}{M_8} \right)^{-\left(\frac{n+3}{6}\right)}, \quad M_8 = 8.6 \cdot 10^{14} \Omega_m h_{70}^{-1} M_\odot
\]

or various variants, e.g., Lee & Shandarin, Sheth & Tormen; $\rho_c$ & $\delta=\delta(z)$ are the critical density and relative density contrast, and $W$ & $T$ are the window and transfer functions.
Power spectrum:

- angular correlation function of the temperature anisotropy is

\[ C(\theta) = \left\langle \frac{\Delta T}{T} (\hat{n}) \frac{\Delta T}{T} (\hat{n} + \theta) \right\rangle = \frac{1}{4\pi} \sum_{l} (2l + 1)C_{l}P_{l}(\cos \theta) \]

- power spectrum of the uncorrelated (Poissonian) part of the anisotropy is

\[ C_{l} = \int \frac{dV}{dz} dz \int \frac{dn(M,z)}{dM} \zeta_{l}^{2}(M,z) dM \]

\( \zeta_{l} \) is the Fourier transform of \( \varphi(\theta) \)

- the full power spectrum includes smaller contributions from the correlated part, and the kinematic component which can be similarly calculated when the velocity field is specified
**Number counts:**

- the number of clusters with (S-Z flux change) $\Delta F$ above a limiting value is

$$
N(\Delta F_m) = \int \frac{dV}{dz} dz \int_{\Delta F_m} b(M, z) \frac{dn(M, z)}{dM} dM
$$

$$
b(M, z) = \begin{cases} 
1, & \Delta F \geq \Delta F_m \\
0, & \Delta F < \Delta F_m 
\end{cases}
$$

**Cosmological model:**

- range of models explored; results shown here are for the currently favored flat $\Lambda CDM$, non-Gaussian and EDE models
Power spectra

Cooray (01)
Power spectrum: $\sigma_8$

$C_\ell \propto \sigma_8^7$

Komatsu & Seljak 02
Power spectrum: global parameters

\[ C_l \propto (\Omega_b h)^2 \]

Komatsu & Seljak 02
Power spectrum: gas evolution

Sadeh & Rephaeli 04
Power spectrum: mass functions

gas evolution assumed

Sadeh & Rephaeli 04
Power spectrum: polytropes

Sadeh & Rephaeli 04
Number counts
Number density

\[ n(M) \propto M^n \]

\[ \sigma_8 = 0.9 \]

--- \( \Omega_\Lambda, \Omega_m = 0.5, 0.5 \)

--- \( \Omega_\Lambda, \Omega_m = 0.3, 0.7 \)

\[ M > 10^{14} h^{-1} M_\odot \]

\[ M > 10^{15} h^{-1} M_\odot \]

Carlstrom & 02
Enhanced power on cluster scales?

- Detection of $z \sim 4$ system with $\sigma > 300 \, \text{km/s}$ (Miley 04)
- Apparent slow evolution of cluster X-ray luminosity function
- CMB power at high $l$ as measured by CBI, ACBAR, & BIMA
Spergel & 06
Enhanced power on cluster scales?

- Detection of $z \sim 4$ system with $\sigma > 300 \text{ km/s}$ (Miley 04)
- Apparent slow evolution of cluster X-ray luminosity function
- CMB power at high $l$ as measured by CBI, ACBAR, & BIMA
- Scale-dependent non-Gaussian density fluctuation field (e.g., Mathis, Diego, & Silk 04)
- Early Dark Energy (e.g., Ratra & Peebles 98)
Scale-dependent non-Gaussianity?

- Theoretical motivation: multiple scalar field inflation with isocurvature initial perturbations (Peebles 97)
- Probability density has a $\chi^2$ distribution (Koyama et al. 99)
- Model provides a more consistent framework for the presence of high $z$ systems, and slow evolution of cluster $L_X$ function than the standard $\Lambda$CDM with Gaussian density field (Mathis, Diego, & Silk 04)
• Degree of non-Gaussianity characterized by

\[ \zeta = \frac{\int_{-\infty}^{\infty} P(\delta, M) d\delta}{3\sigma} \]

\[ \frac{3\sigma}{\int_{-\infty}^{\infty} P_G(\delta, M) d\delta} \]

• Observational limits are either from CMB primary anisotropy, which (so far) probed linear regime where \( \zeta \) is small, or from low-\( z \) clusters, whereas \( \chi^2 \) model predicts enhanced cluster number at high \( z \).
Early Dark Energy?

- Theoretical motivation: scalar field that describes DE has (attractor) solutions, such that DE tracks matter component. Current low DE reflects long age; nonnegligible level at early epochs (Ratra & Peebles 98; Caldwell 02)

- EDE lowers linear growth rate of structure; for a given \( \sigma_8 \), higher density fluctuation amplitude is predicted at high \( z \).

- Lower critical density for collapse
(Sadeh, Rephaeli & Silk 07)
Early Dark Energy?

• Theoretical motivation: scalar field that describes DE has (attractor) solutions, such that DE tracks matter component. Current low DE reflects long age; nonnegligible level at early epochs \(\text{(Ratra & Peebles 98; Caldwell 02)}\)

• EDE lowers linear growth rate of structure; for a given \(\sigma_8\), higher density fluctuation amplitude is predicted at high \(z\).

• Lower critical density for collapse leads to a higher abundance of virialized systems at \(z \sim 1\) than in \(\Lambda\)CDM \(\text{(Bartelmann, Doran & Wetterich 06)}\).
S-Z observables in $\chi^2$ and EDE models

- Contrast between Gaussian and non-Gaussian models larger at high $z$
- Due to the $z$-independence of the S-Z effect, power is sensitive to high $z$ abundance of clusters
- Comparison of S-Z power spectrum and cluster number counts in Gaussian, $\chi^2$, and EDE models

(Sadeh, Rephaeli & Silk 06, 07)
Global models:

- \( \Lambda \)CDM

EDE:

\[
\Omega_e = 0.0008, \quad w(z) = \frac{w_0}{1 + u \log(1 + z)}, \quad w_0 = -0.99, \quad u = u(w_0, \Omega_e, \Omega_m)
\]

Cluster population:

- Press-Schechter mass function with

  \[
  M_{\text{min}} \sim 10^{13} \, h^{-1} \, M_\odot, \quad M_{\text{max}} \sim 10^{16} \, h^{-1} \, M_\odot
  \]

  normalized to have same cumulative density at \( z=0 \) in all three models

- \( \Lambda \)CDM, \( n=1, \sigma_8 = 0.74 \pm 0.06 \)

  non-Gaussian, \( n=-1.8 \)

- gas mass fraction = 0.1 (no evolution)
(Sadeh, Rephaeli & Silk 07)
σ = 0.9

ν = 31 GHz
PDF of Einstein radius, $\theta_E$

$R_V = 2.63$

$R_V = 3.1$

$2\sigma$ of $\Omega_m$, $n$, & $\sigma_8$

A1689-like cluster ($z = 0.183$)
Motivated by need to verify $T = T_0(1+z)$ relation, isotropy, and for setting constraints on non-standard cosmologies

Standard atomic and molecular spectroscopic method plagued by substantial systematic uncertainties

$T(z)$ from multi-frequency S-Z measurements

\[ \frac{\Delta I}{\Delta I} \frac{i_0}{\tau} = \frac{x^4 e^x}{(e^x - 1)^2} \left[ \theta f(x) - \beta + R(x, \theta, \beta) \right] \]

\[ i_0 = 2(kT)^3 / (hc)^2, \quad \tau = \sigma_T \int n_e dl \]

$\Delta I(\nu_1)/\Delta I(\nu_2)$ is essentially independent of cluster properties: can determine $T(z)$ (given that $\nu \propto l + z$)
• Optimize selection of $v_i$ to reduce observational uncertainties due to primary CMB anisotropy, kinematic component & dust emission

• Method applied recently to determine the parameters $a$ & $d$ in the proposed alternative scaling laws

$$T(z) = T_0 (1+z)^{1-a}, \quad T(z) = T_0 [1+(1+d)z]$$

from multi-frequency MITO & SuZIE measurements of Coma & A2163
S-Z Spectrum of Coma
• Optimize selection of $\nu_i$ to reduce observational uncertainties due to primary CMB anisotropy, kinematic component & dust emission

• Method applied recently to determine the parameters $a$ & $d$ in the proposed alternative scaling laws

$$T(z) = T_0 (1+z)^{1-a}, \quad T(z) = T_0 [1+(1+d)z]$$

from multi-frequency MITO & SuZIE measurements of Coma & A2163:

$$a \approx -0.1 \pm 0.3, \quad d \approx 0.1 \pm 0.4$$

Battistelli & 02