Concluding remarks

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Measuring neutrino mass

Varenna, June 27, 2008
E. Fermi’s estimation, 1934: $m < 0.1 \, m_e$

W. Pauli’s original idea 1930: ``...of the order of electron mass’’

It took more than 70 years since ... 

work of several generations of theoreticians and experimentalists

at least one neutrino mass is in the range (0.05 – 0.20) eV

``neutrino time’’
B. Pontecorvo
``Mesonium and antimesesonium’’
[Sov. Phys. JETP 6, 429 (1957)] translation

mentioned a possibility of neutrino mixing and oscillations

Results of Wu experiment, 1957:
Parity violation $\rightarrow$ V-A theory,
two-component massless neutrino

Lesson...  ```non-standard process’’ $\rightarrow$ discovery
In what follows

- Phenomenology of neutrino mass
- Analyzing results
- Nature of neutrino mass
- Masses and mixing
- Predicting neutrino masses
- Implications for fundamental physics
Phenomenology of neutrino mass
Neutrino Masses

- Kinematics
- Beta decay
- Peaks, kinks
- Cosmology
- LSS
- $Z^0$-bursts
- BBN
- Leptogenesis
- Helicity flip
- Neutrino decays
- Double beta decay
- Supernova neutrinos dispersion
- $\gamma$-bursts
What else?
Cosmological bounds

Large scale structure of the Universe

SDSS
Cosmological bound

G. Fogli et al., 2008, 0805.2517

CMB: WMAP-5 + ACBAR + VSA + CBI + BOOMERANG

HST = Hubble Space Telescope

BAO = Baryonic Acoustic Oscillations
Relaxing constraint

Dependence of bound on DE equation of state

S. Hannestad, astro-ph/0505551
2 neutrino double beta decay:

\[ Z \to (Z + 2) + e^- + e^- + \bar{\nu} + \nu \]

H-M, NEMO \( \sim 200\,000 \) events

Neutrinoless double beta decay

\[ Z \to (Z + 2) + e^- + e^- \]

Spectrum total energy of the electron pair

Majorana mass of the electron neutrino

Rate \( \sim |m_{ee}|^2 \)

\[ m_{ee} = \sum_k U_{ek}^2 m_k e^{i\phi(k)} \]
Double beta decay

Heidelberg-Moscow experiment

$^{76}\text{Ge} \rightarrow ^{76}\text{Se} + e^- + e^-$

neutrinoless double beta decay

Evidence of the effect
Cosmology? Mechanism?

evidence

Fifth detector
A. Strumia, F. Vissani

**Present and future limits**

Cuoricino (90%)

NEMO (90%)

IGEX (99%)

GERDA II

CUORE

Comments

NEMO: $^{100}$Mo

Cuoricino, CUORE: $^{130}$Te

GERDA: $^{76}$Ge
For degenerate spectrum

$\Sigma = \Sigma_i m_i$

Interplay

Double beta decay $M_{ee}$

Majorana? Contributions unrelated to light neutrinos

Beta decay $M_{ee}$

Cosmology $\Sigma$

- Non-standard neutrino properties, (fast decay)
- Degeneracy of cosmological parameters
- Non-standard cosmology
Non-radiative decays in vacuum

\[ \nu_i \rightarrow \nu_j + \phi \]

\[ \Gamma \sim m_i \]

Majoron decay

\[ \nu_i \rightarrow \bar{\nu}_j + \phi \]

\[ \Gamma \sim m_i \]

Radiative decay

\[ \nu_2 \rightarrow \nu_1 + \gamma \]

\[ \Gamma \sim m_2^5 \text{ or } \Gamma \sim m_2^3 \]

Non-radiative decays in vacuum

\[ \nu_2 \rightarrow \nu_1 + \bar{\nu}_1 + \nu_1 \]

\[ \Gamma \sim m_2^5 \]

Z\(^0\)-burst

\[ \nu + \bar{\nu} \rightarrow Z^0 \rightarrow \text{hadrons} \]

UHE cosmic neutrinos on relic neutrinos

CC-interaction with radioactive nuclei

\[ \nu + ^3\text{H} \rightarrow e^- + ^3\text{He} \]

\[ \sigma \sim m_\nu \]
Even with high statistics and sophisticated method difficult to test masses below 1 eV

Time delay: \[ \Delta t = 5.1 \, \text{ms} \left( \frac{L}{10 \, \text{kpc}} \right) \left( \frac{10 \, \text{MeV}}{E} \right)^2 \left( \frac{m}{1 \, \text{eV}} \right)^2 \]

Effects: - time delay with respect to certain benchmarks (e.g. neutronization burst, lightest neutrino arrival, gravitational waves)
- increase of the length of the burst
- smoothing fine structures of the burst (neutronization peak, initial steep rise, abrupt interruption of signal in the case of black hole)
- energy ordering
- spread of the wave packets

E. Nardi J. I. Zuluaga

(1). use high energy part to reconstruct time dependence of spectra, (2). analyze whole spectrum

G. T. Zatsepin
D Schramm
T. Lodredo,
D. Q. Lamb,
J. Bahcall,
D Fargione
T. Totani,
J Beacom,
R. Boyd,
A. Mezzacappa
Analyzing results

three consequences
Lower bound on the heaviest neutrino mass:

\[ m_h > \sqrt{\Delta m_{32}^2} \]

MINOS: \[ \Delta m_{32}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2 \]

\[ m_h > 0.049 \text{ eV} \]

From global fit:

Cosmology: \[ m_h < 0.2 \text{ eV} \]

At least one neutrino \[ 0.05 < m_h < 0.2 \text{ eV} \]

G. Fogli et al., 2008
Lower bound on the ratio of neutrino masses

\[
\frac{m}{m} = \sqrt{\frac{\Delta m_{32}^2}{\Delta m_{21}^2}}
\]

\[
\Delta m_{32}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2
\]

\[
\Delta m_{21}^2 = (7.66 \pm 0.35) \times 10^{-5} \text{ eV}^2
\]

Neutrinos have the weakest mass hierarchy (if any) among fermions

Related to the large lepton mixing?

G. Fogli et al., 2008
Mass Ratios

Regularities?

\[ m_u m_t = m_c^2 \]

\[ V_{us} V_{cb} \sim V_{ub} \]

Gatto –Sartori-Tonin relation

\[ \sin \theta_c \sim \sqrt{m_d/m_s} \]

Koide relation?
Spectrum can be completely (quasi) degenerate partially degenerate hierarchical but not non-degenerate and non-hierarchical $\Delta m_{ij} \sim m_i$
Bounds on mass parameters

G. Fogli et al., 2008
$^{76}\text{Ge} \rightarrow ^{76}\text{Se} + e^- + e^-$

$Q_{ee} = 2039\text{ keV}$
5 detectors, 71.7 kg yr

$T_{1/2} = 1.19 \times 10^{25}\text{ y}$
$T_{1/2} = (0.69 - 4.18) \times 10^{25}\text{ y}$
(3σ range)

$m_{ee} = 0.24 - 0.58\text{ eV (3σ)}$

$m_{ee} = 0.16 - 0.52\text{ eV (2σ)}$

4.2σ – evidence $\rightarrow$ 6σ

New experimental claim
New values of matrix elements

G.L. Fogli et al
Bounds on $m_{ee}$

G. Fogli et al., 2008

All cosmologica data
1. Smallness of neutrino mass indicates that most probably we are touching something qualitatively new

\[
\frac{m_3}{m_\tau} \sim (0.3 - 1) \times 10^{-10}
\]

whereas

\[
\frac{m_\tau}{m_t} \sim 10^{-2}
\]

Scale of neutrino mass \( m = (0.05 - 0.20) \) eV

Other closest scale: dark energy in the Universe \( \sim 10^{-3} \) eV?

2. Pattern of neutrino mixing with two large angles
Nature of neutrino mass

- Majorana-Dirak
- hard-soft (VEV)
- effective

Smallness may indicate on something different or on importance of contribution

Qualitatively the same as quark masses?
Smallness of mass is due to some mechanism, that involves the EW scale and probably some higher scale(s) of nature.

In the context of the see-saw mechanism: $M_R$ (heaviest) $\sim M_{GUT}$ is an interesting possibility.

Difference of the quark and lepton mass spectra and mixing patterns is related to the smallness of neutrino mass.

Neutrinos are Majorana particles?

In general: $m_\nu = m_{\text{hard}} + m_{\text{soft}}(E,n)$

medium-dependent soft component

Bounds on $m_{\text{soft}}$
``Soft'' neutrino mass?

Are neutrino masses usual?

Exchange by very light scalar

\[ m_\phi \sim 10^{-8} - 10^{-6} \text{eV} \]

\( f = e, u, d, \nu \)

\( \lambda_f \sim \phi / M_{Pl} \)

In the evolution equation:

\[ m_{\text{vac}} \rightarrow m_{\text{vac}} + m_{\text{soft}} \]

generated by some short range physics (interactions) EW scale VEV

medium and energy dependent mass

\[ m_{\text{soft}} = \lambda_\nu \lambda_f n_f / m_\phi \]
Hidden sector (HS) e.g., gauge theory with fermions and coupling $g$

If $g^* \gg 1$ $\rightarrow$ appearance of composite (confined) states of the HS particles (described by operators $O_U$)

Particles of HS couple with SM particles via exchange of messenger field(s) with mass $M$

Scale invariance $\rightarrow$ continuous mass spectrum of confined states, Each has infinitesimal coupling with SM particles. Integral is finite

Individual (mass) modes: negligible effect
Unparticle effects

Effects in solar neutrinos

Neutrino decay: $\nu_i \rightarrow \nu_j \, U$

Unparticle exchange: modify matter potential and effective neutrino mass $\rightarrow$ modify survival probability

L. Anchordoqui, H. Goldberg

M.C. Gonzalez-Garcia, P.C. de Holanda, R. Zukanovich-Funchal

$M$ – mass of messenger

$d_{H}$ - dimension of operator in hidden sector,

$d$ – dimension of unparticle operator

$\Lambda_{U}$ - infrared fixed point

$\Lambda_{U}=1 \, \text{TeV} \quad d_{UV}=3 \quad \alpha_{S,V}=10^{-3}$

Scalar

$m_{1}=0 \, \text{eV}$

Vector

$P_{\text{SUN}}(\nu_{e}\rightarrow \nu_{e})$

Tensor

$E_{\nu} \,(\text{MeV})$

$M=8.5 \times 10^{4} \, \text{TeV}$

$M=1.5 \times 10^{6} \, \text{TeV}$

$M=3 \times 10^{7} \, \text{TeV}$
Small Dirac masses due to ``overlap suppression’’

Mass term: $m \bar{f}_L f_R + \text{h. c.}$

If left and right components are localized differently in extra dimensions $\rightarrow$ suppression:

$m \epsilon f^I_L f^I_R + \text{h. c.}$

amount of overlap in extra D

Arkani-Hamed, Dvali, Dimopoulos

Large extra D + 3D brane
In Randall-Sundrum (non-factorizable metric)

Setting: 1 extra D $S^1/Z_2$

RH neutrinos - bulk zero mode localized on the hidden brane

In warped extra D

Grossman
Neubert
Huber, Shafi...
Masses and mixing
Inverted mass hierarchy

Normal mass hierarchy

Unknown:

- 1-3 mixing
- mass hierarchy
- CP violation phase
- absolute scale of neutrino mass
- additional neutrino states
Test equalities

Oscillations vs. double beta decay

among observables

1. Normal mass hierarchy: \( m_2 >> m_1 \)

   \[ m_{ee} = \sin^2 \theta_{\text{sol}} \sqrt{\Delta m_{\text{sol}}^2} \]

   Also implies:

   \( U_{e3}^2 \ll 0.04 \)

2. Inverted mass hierarchy

   \[ m_{ee} = \cos 2\theta_{\text{sol}} \sqrt{\Delta m_{\text{atm}}^2} \]

   opposite CP phases

   \[ m_{ee} = \sqrt{\Delta m_{\text{atm}}^2} \]

   the same CP phases

3. Degenerate mass spectrum

   \[ m_{ee} = m_e \]

   the same CP phases

   \[ m_{ee} = \cos 2\theta_{\text{sol}} m_e \]

   opposite CP phases
Salient features:

Small neutrino masses related? Strange mixing pattern

indicate that most probably we are touching something qualitatively new
Flavor symmetry

Quark-lepton complementarity

``bi-maximal - CKM''

GST-approach mass-mixing

The same principle as in quark sector

Extension to quarks?

- subject of RGE
- no relation to masses?
- additional ambiguities (CP-phases)

Flavor symmetry

structure which produces bi-maximal mixing – symmetry?

Large mixing is related to weak mass hierarchy of neutrinos

Quark-lepton symmetry, GUT
$U_{tbm} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ \sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ \sqrt{1/6} & -\sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$

- maximal 2-3 mixing
- zero 1-3 mixing
- no CP-violation

$\sin^2\theta_{12} = 1/3$ in agreement with 0.315

$U_{tbm} = U_{23}(\pi/4)U_{12}$

$\nu_3$ is bi-maximally mixed
$\nu_2$ is tri-maximally mixed

- is in flavor basis...
- relation to masses?
- implies non-abelian symmetry
Possible implications

$U_{tbb} = U_{mag} U_{13} (\pi/4)$

$U_{mag} = \begin{pmatrix}
1 & 1 & 1 \\
1 & \omega & \omega \\
1 & \omega^2 & \omega
\end{pmatrix}$, $\omega = \exp(-2i\pi/3)$

tetrahedron

Symmetry:

$A_4$  

symmetry group of even permutations of 4 elements

representations: $3$, $1'$, $1''$

Other possibilities:

$T_7, D_4, S_4, \Delta(3n^2)$ ...

Extended higgs sector, Auxiliary symmetries, vacuum alignment, Extra dimensions?

Relation to masses?  
No analogy in the quark sector?  
Unification?
\[ \theta_{12}^l + \theta_{12}^q \sim \pi/4 \]

\[ \theta_{23}^l + \theta_{23}^q \sim \pi/4 \]

Difficult to expects exact equalities but qualitatively:

- 2-3 leptonic mixing is close to maximal because 2-3 quark mixing is small
- 1-2 leptonic mixing deviates from maximal substantially because 1-2 quark mixing is relatively large
Quark-lepton symmetry

Existence of structure which produces bi-maximal mixing

``Lepton mixing = bi-maximal mixing – quark mixing’’

In the lowest approximation:

\[ V_{\text{quarks}} = 1, \quad V_{\text{leptons}} = V_{\text{bm}} \]

\[ m_1 = m_2 = 0 \]

See-saw?

Properties of the RH neutrinos
Establish that spectrum is hierarchical
Put upper bound on the lightest neutrino mass
Confirm scenario which predict, e.g. the hierarchical mass spectrum
  - "νSM" by Asaka – Shaposhnikov
  - some GUT models...
Measure different processes, e.g. neutrino decay, further study of the LSS of the Universe
LHC: uncover the electroweak mechanisms e.g. double charged Higgs bosons, measuring its couplings
Coincidences...
Predicting neutrino mass
Koide relation


\[ \left( \sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau} \right)^2 = \frac{2}{3} \]

was obtained in attempt to explain all three families are involved: no perturbation approach!

\[ \tan \theta_C = \sqrt{3} \left( \frac{\sqrt{m_\mu} - \sqrt{m_e}}{2 \sqrt{m_\tau} - \sqrt{m_\mu} - \sqrt{m_e}} \right) \]

Both relations can be reproduced if

\[ m_i = m_0 (z_i + z_0)^2 \]

\[ \Sigma_i z_i = 0, \quad z_0 = \sqrt{\frac{\Sigma_i z_i^2}{3}} \]

C A Brannen

Neutrinos, hierarchical spectrum

Non-abelian flavor symmetry, VEV alignment

Related to TBM?
for neutrinos

\[ \frac{m_1 + m_2 + m_3}{(-m_1 + \sqrt{m_2} + \sqrt{m_3})^2} = \frac{2}{3} \]

minus sign is crucial

Neutrinos have hierarchical spectrum with

\[ m_1 = 3.9 \times 10^{-4} \text{ eV} \]

Related to TBM?
1. Smallness of neutrino mass is related to the Majorana nature of neutrinos

**Majorana: neutrino = antineutrino**

allowed by neutrality of neutrinos

2. See-saw scenario

small = \frac{(normal)^2}{Large}

small = masses of usual neutrinos

normal = electroweak scale ~ 100 GeV

Large = masses of ``Right'' neutrino

The same mechanism explains large lepton mixing?
Energy scales of new physics

Physics behind neutrino masses is not identified

- Neutrino masses
- $m_e$
- $m_P$
- $V_{EW}$
- LHC
- $M_{GUT}$
- $M_{Pl}$

Planck scale mechanisms

GUT scale mechanisms

Intermediate Scale mechanisms

EW scale mechanisms

Low scale mechanisms

kev-GeV scale mechanisms?

$M_{3} \sim M_{GUT}$

$M_{RH}$ scale of RH neutrino masses

many $O(100)$ RH neutrinos

$M_{3} \sim M_{GUT}$

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$M_{3} \sim M_{GUT}$
RH neutrino components have large Majorana mass

\[ m_v = - m_D^T \frac{1}{M_R} m_D \]

\[ M_R \sim \begin{cases} 
M_{\text{GUT}} \\
\sqrt{\frac{M_{\text{GUT}}^2}{M_{\text{Pl}}}} 
\end{cases} \text{ in the presence of mixing} \]

\[ M_{\text{GUT}} \sim 10^{16} \text{ GeV} \quad \text{- possible scale of unification of EM, strong and weak interactions} \]

Neutrino mass as an evidence of Grand Unification?

Leptogenesis:
the CP-violating out of equilibrium decay

\[ N \rightarrow l + H \]

→ lepton asymmetry
→ baryon asymmetry of the Universe
Why $m_{\nu}$ is important?

- Determination of other neutrino parameters
- Implications for fundamental physics
- Searches for new physics

Type of neutrino mass spectrum

Mechanism of neutrino mass generation, smallness

Identification of the underlying physics

Scale of new physics

Origin and nature of neutrino mass

Flavor symmetry

Difference of mixing patterns of the quarks and leptons

Comparison of values of neutrino mass determined in different processes
<table>
<thead>
<tr>
<th>Types of spectra</th>
<th>Normal mass ordering</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Degenerate:</strong></td>
<td>$m_1 \approx m_2 \approx m_3$</td>
</tr>
<tr>
<td>$m_2 &gt; 0.1$ eV</td>
<td>Non-abelian flavor symmetry</td>
</tr>
<tr>
<td><strong>Partially degenerate:</strong></td>
<td>$m_1 \approx m_2 \approx m_3$</td>
</tr>
<tr>
<td>$m_2 = 0.02 - 0.06$ eV</td>
<td>Pseudo-Dirac pair</td>
</tr>
<tr>
<td><strong>Non-degenerate:</strong></td>
<td>$m_1 \sim m_2 \sim m_3$</td>
</tr>
<tr>
<td>$m_2 = 0.01 - 0.02$ eV</td>
<td>No symmetry</td>
</tr>
<tr>
<td><strong>Hierarchical:</strong></td>
<td>$m_1 \ll m_2 \ll m_3$</td>
</tr>
<tr>
<td>$m_2 &lt; 0.01$ eV</td>
<td>U(1) Similarity to other fermions</td>
</tr>
</tbody>
</table>
### Inverted mass ordering

<table>
<thead>
<tr>
<th>Type</th>
<th>Conditions</th>
<th>Implications</th>
</tr>
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<tr>
<td>Hierarchical:</td>
<td>$m_3 &lt; 0.01 \text{ eV}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_2 - m_1 &gt;&gt; m_3$</td>
<td></td>
</tr>
</tbody>
</table>

Spectrum is always degenerate: partially or completely
Different source, unrelated to the Dirac mass matrices of the charged leptons and quarks

Higgs triplet
seesaw type II

Screening of Dirac structure

\[
m = \begin{pmatrix}
0 & m_D & 0 \\
m_D & 0 & M_D \\
0 & M_D & M
\end{pmatrix}
\]

\[M_D = A m_D\]

\[M \sim I\]

3x3 matrices

\[m_v = m_D M_D^{-1} M M_D^{-1} m_D \sim A^{-2} I\]
In general framework

**Standard neutrino scenario**

- $m_\nu$ - key element of the picture related to the type of spectrum

**Searches for physics beyond standard scenario**

- New neutrino states $\rightarrow$ kinks
  - NSI
  - New dynamics
- Violation of fundamental symmetries

**Understanding fermion masses**

- Properties of neutrino mass

**Unification**
Applications?

Cosmology

Determination of other cosmological parameters

- Neutrino structure of the Universe
  - neutrino stars/clouds
  - halos
  - clusters

Possible detection of relic neutrinos

- local concentration

Kinematics

Enters probabilities of the processes irrelevant: too small?

- Rare processes
  - neutrino decay
  - threshold-less processes

Instabilities condensation fluidity....
In conclusion of concluding remarks ...
What is the mass of the heaviest neutrino?

What is the type of neutrino mass spectrum? Degenerate, partially degenerate, Hierarchical?

What is the nature of neutrino mass? Majorana vs. Dirac? Hard or soft? What is the soft contributions? Mass induced by interaction?

How light is the lightest neutrino?

Is mixing related to mass?

Can we establish the absolute mass scale without direct measurements of neutrino masses?

What are fundamental limitations of sensitivity of different experimental techniques?
Additional slides
\[ U_{bm} = U_{23}^m U_{12}^m \]

Two maximal rotations

\[ U_{bm} = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} & 0 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}
\end{pmatrix} \]

as dominant structure?
Zero order?

- maximal 2-3 mixing
- zero 1-3 mixing
- maximal 1-2 mixing
- no CP-violation

contradicts data at (5-6)\(\sigma\) level
MINOS: \[ \Delta m_{31}^2 = (2.43 +/- 0.13) \times 10^{-3} \text{ eV}^2 \] 3.36 \times 10^{20} \text{ p.o.t.}

Global fit:
\[ \Delta m_{31}^2 = (2.38 +/- 0.27) \times 10^{-3} \text{ eV}^2 \]
\[ \Delta m_{21}^2 = (7.66 +/- 0.35) \times 10^{-5} \text{ eV}^2 \]  
\text{G.L. Fogli et al 2008}

\[ |m_2/m_3| > 0.18 \] the weakest mass hierarchy

Cosmology: \[ \Sigma_i m_i < 0.42 \text{ eV} \text{ (95\% C.L.)} \] \text{U. Seljak et al}
\[ \Sigma_i m_i < 0.17 \text{ eV} \text{ (95\% C.L.)} \] \text{S. Hannestadt}

At least for one neutrino: \[ m \sim 0.05 - 0.10 \text{ eV} \]

Heidelberg-Moscow: \[ m_{ee} = 0.16 - 0.52 \text{ eV} \text{ (95\% C.L.)} \]
The diagram illustrates the mass of the heaviest neutrino, $m_{h\text{, eV}}$, in units of electron-volts (eV). The scale indicates various limits and bounds:

- **Lower limit of the KATRIN sensitivity**
- **Cosmological upper bound**
- **Lower limit of the quasi-degenerate spectrum**
- **Non-degenerate spectrum**
- **Hierarchical spectrum**

The scale ranges from 0 to 0.25 eV, with markers indicating specific values and regions corresponding to different types of neutrino mass spectra.
\( \theta_{12} + \theta_C \sim \pi/4 \)

\[ U_{QLC_1} = U_C U_{bm} \]

\[ U_{tbm} = U_{tm} U_{m_{13}} \]

give almost the same 12 mixing
* Non-zero central value (Fogli, et al): Atmospheric neutrinos, SK spectrum of multi-GeV e-like events

* MINOS lead to stronger the bound on 1-3 mixing (G-G, M.)
* in agreement with maximal, though all complete 3ν - analyses show shift
* shift of the bfp from maximal is small
* still large deviation is allowed: \( \frac{0.5 - \sin^2 \theta_{23}}{\sin \theta_{23}} \sim 40\% \) 

\[ 2\sigma \]
Leptons versus quarks

Leptons

\[ \nu_f = U_{PMNS} \nu_{mass} \]

Quarks

\[ U_d = U_{CKM}^+ U \]

\[ U = (u, c, t) \]

zero 1-3 mixing?

maximal mixing

large 1-2 mixing
Neutrinoless double beta decay

Kinematic searches, cosmology

Both cosmology and double beta decay have similar sensitivities

\[ m_{ee} = \sum_k U_{ek}^2 m_k e^{i\phi(\kappa)} \]
### Kinematic methods:

<table>
<thead>
<tr>
<th>Location</th>
<th>Bound</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Troitsk</td>
<td>$m_e &lt; 2.05$ eV (95%)</td>
<td>after &quot;anomaly&quot; subtraction</td>
</tr>
<tr>
<td>Mainz</td>
<td>$m_e &lt; 2.3$ eV (95%)</td>
<td>updated, 2004</td>
</tr>
</tbody>
</table>

**Future: KATRIN**

- $m_e < 0.2$ eV (90%) (upper bound)
- Discovery potential: If $m_e = 0.35$ eV, $5\sigma$ (statistical) from 0

### From neutrinoless double beta decay:

If the effective Majorana mass $m_{ee}$ is measured:

$m > m_{ee}$
Solar neutrinos: degeneracy of 1-2 and 1-3 mixing

\[ \sin^2 \theta_{13} = 0.017 \pm 0.26 \]
Future determinations?

S. Goswami, A.S.

\[ \frac{F_{pp}}{F_{pp}^{SSM}} \]

CC/NC

1.5% +/- 0.01 3%
... on the fat brane

Arkani-Hamed, Schmaltz

wave functions

3D brane

overlap

$f^R$, $f^L$
General picture

Standard Model

Planck scale physics
Sterile Neutrinos

Contours of constant induced mass and bounds

thermalization line: above if $S$ are in equilibrium

For benchmark parameters $S$ were thermalized

Bounds - in non-equilibrium region

R. Zukanovic-Funchal, A.S.
New neutrino states

Light sterile neutrino
\[ R_\Delta = \frac{\Delta m_{01}^2}{\alpha \Delta m_{21}} \]
fixing angle

Dip in the survival probability:
- reduces the Ar-production rate
- suppresses the upturn of the boron spectrum

Motivation for the low energy solar neutrino experiments
BOREXINO, KamLAND
MOON, LENS ...
Neutrino mass matrix in the flavor basis:
For charged leptons: $D = 0$

$$
\begin{pmatrix}
A & B & B \\
B & C & D \\
B & D & C \\
\end{pmatrix}
$$

Can both features be accidental?

Often related to equality of neutrino masses

Discrete symmetries $S_3, D_4$

$\nu_\mu - \nu_\tau$

permutation symmetry

Are quarks and leptons fundamentally different?