Nonuniversal dynamic conductance fluctuations

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From the First Talk by Wölfle

- Quasi one dimensional systems are in the localized or delocalized regime, depending on length. On delocalized side the conduction fluctuations are universal.

-Dr. Wölfie

- Essentially, for quasi 1-D systems localization will occur at some length, $\xi$, and all lengths greater then $\xi$.

- Lengths below the localization length will result in diffusive scattering.
  - Universal Conductance Fluctuations (UCF) are prominent in diffusive samples.
From the beginning: Conductance in an electronic system

The ensemble average of conductance in an electronic system can be described as:

$$\langle G \rangle = \left( \frac{e^2}{h} \right) \langle g \rangle$$

Where $G$ is the conductance, $e$ is the electron charge and $h$ is the Planck constant; $g$ is the dimensionless conductance.
What is Conductance for classical waves?

In classical waves dimensionless conductance is found to be the transmission sum of all input modes $a$ and output modes $b$

$$g \equiv T = \sum_{ab} T_{ab}$$

For a random system deviations of conductance from its average value arise from the interference of wavefunctions in the system, it is here where conductance fluctuations come into play:

$$\delta g = g - \langle g \rangle$$
Conductance Fluctuations

These fluctuations are known as universal conductance fluctuations (UCF). They occur for:

- any metal
- are independent of sample size
- independent of disorder strength.

The UCF can be expressed in terms of the dimensionless conductance $g$ as:

$$\langle \delta g^2 \rangle \sim 1$$
Cont.

\[ \langle \delta g^2 \rangle \sim 1 \]

Measurements for UCF have been well studied for steady state conductance cases.


**Light:** Phys. Rev. Lett. 81, 5800 (G. Maret et. al.)

**Microwaves:** Phys. Rev. Lett. 92, 173901

**Reviews:** *Mesoscopic Physics of Electrons and Photons* (2007), Rev. Mod. Phys. 71, 313
Question: How do these conductance fluctuations vary for static samples in the time domain?

• Or more specifically,
  – What is the magnitude of the fluctuations in the *dynamic* case, \( g(t) \)? (Using a pulsed excitation)
  – To what extent are the fluctuations universal?
Theory

- Using diagrammatic theory Nicholas Cherroret and Sergey E. Skipetrov found an expression for the time dependent fluctuations of conductance when considering a Gaussian pulse, they found:

\[
\text{var}[s(t)] = \frac{2}{15\langle g^2 \rangle} x \frac{5\sqrt{2}t_P}{\pi^3 \sqrt{\pi} t_D} (\frac{t}{t_D})^3
\]

Where \( t_P \) is the duration of the pulse, \( t_D \) is the diffusion time and \( t \) is the delay time. Here \( \text{var}[s(t)] \) is the variance of the normalized transmittance (i.e. variance of conductance).
Question posed: Can you find this experimentally?

\[
\text{var}[s(t)] = \frac{2}{15 \langle g^2 \rangle} x \frac{5 \sqrt{2} t_P}{\pi^3 \sqrt{\pi} t_D} \left( \frac{t}{t_D} \right)^3
\]

• Yes!

BUT, IT REQUIRES A STORY!
Origin of UCF: Intensity Correlations

- Feng, Kang, Lee and Stone (Phys. Rev. Lett. 61, 834, 1988) found that there exists an intensity correlation function for random systems.

\[ C_{aba'b'} = \langle \delta \Gamma_{ab} \delta \Gamma_{a'b'} \rangle \]

- The intensity correlation function is an expansion using \( 1/<g> \) as the expansion parameter:

\[ C_{aba'b'} = C_1 \left( \frac{1}{\langle g^0 \rangle} \right) + C_2 \left( \frac{1}{\langle g^1 \rangle} \right) + C_3 \left( \frac{1}{\langle g^2 \rangle} \right) \]

- Where \( C_1, C_2 \) and \( C_3 \) correspond, respectively to the short, long and infinite range correlation.
Feng et. al. found variance of transmittance is essentially given by the last term:

\[
\langle \delta g^2 \rangle \sim 1 = C_{aba'b'} = C_3 \left( \frac{1}{\langle g \rangle} \right)^2
\]

Thus, \( C_3 \) corresponds to any fluctuations of conductance.
How is this useful?

In 2004 Chabanov, et al. (Phys. Rev. Lett. 92, 17) found the description of the correlation function using polarization angles of a source and detector. They found:

\[
C(\Delta \theta_S, \Delta \theta_D) = C_1 \cos^2 \Delta \theta_S \cos^2 \Delta \theta_D + \\
C_2 (\cos^2 \Delta \theta_S + \cos^2 \Delta \theta_D) + C_3
\]

Here $\Delta \theta_S$ and $\Delta \theta_D$ are the difference in polarization angles for the source and detector.
What this means

\[ C(\Delta \theta_s, \Delta \theta_D) = C_1 \cos^2 \Delta \theta_s \cos^2 \Delta \theta_D + \]
\[ C_2 (\cos^2 \Delta \theta_s + \cos^2 \Delta \theta_D) + C_3 \]

Varying the polarization of the source and detector allows for isolation of correlation terms!
To illustrate this better:

- By measuring $I(0,0)$, $I(0,90)$ and $I(90,90)$ we can find the three correlation terms.

\[
\begin{align*}
<I(0,0)I(0,90)> & = C(0,90) = C_2 + C_3 \\
<I(0,0)I(90,90)> & = C(90,90) = C_3 \\
<(I(0,90)I(90,90)> & = C(90,0) = C_2 + C_3 \\
<(I(0,0)I(0,0)> & = <I^2(0,0)> = C(0,0) = C_1 + 2C_2 + C_3
\end{align*}
\]

\[
C(\Delta \theta_S, \Delta \theta_D) = C_1 \cos^2 \Delta \theta_S \cos^2 \Delta \theta_D + C_2 (\cos^2 \Delta \theta_S + \cos^2 \Delta \theta_D) + C_3
\]

Here $I(x_1,x_2)$ correspond to the polarization of the source and detector respectively.
So we have learned:

• By measuring intensities at different polarizations we can find correlation terms describing short, long and infinite range correlation.
  – To find any information about conductance fluctuations we must find $C_3$. We can do this by measuring the transmitted intensity for two different polarization setups of an ensemble. Specifically $I(0,0)$ and $I(90,90)$. 

![Diagram of a nanorod with polarization angles labeled 0° and 90°.](image)
The experiment: Procedure

1. Measure the transmission field in frequency domain for polarizations of (0,0) and (90,90). (~15,000 configurations)
2. Multiply the field by a Gaussian envelope of certain width.
3. Fourier transform the frequency data to obtain pulse response in the time domain.
4. Calculate the correlation contribution $C_3$ in the time domain.

Do this entire procedure for different lengths, scale $C_3$, and hope experimental data agrees with theoretical fits!
Experimental Setup

- We have a network analyzer that produces and receives microwaves connected to polarized horns.
- A waveguide filled with scattering spheres.
The sample

• 1/4” diameter Al₂O₃ Scatterer placed inside a 5/8” diameter styrofoam.
  – Index of refraction is 3.14 (strongly scattering)
  – Frequency range is away from 2ⁿᵈ Mie Resonance at ν₀ = 19.2 Ghz (λ ~ 1.5 cm)
  – Al₂O₃ Filling fraction of waveguides is 0.03, mean free path from Mie theory is 3.76 cm.
Results

We found $C_3$ for all lengths using pulse times of 1.8 (Blue), 1.2 (Green) and 0.9 ns (Red).

The growth of the experimental conductance fluctuations fit theoretical predictions (Solid lines).
Finding the diffusion time, $t_D$ from Diffusion theory

- Three different waveguide lengths: 61 cm, 76.2 cm and 91.4 cm.
  - Each sample had a diffusion time of $t_D = 17.5$, 27.3 and 39.6 ns for the respective lengths. This was done using a three parameter diffusion equation involving absorption time, $t_a = 39.7$ ns and the total time transmitted intensity (measured).

$$\tau_D = \frac{L^2}{\pi^2 D}$$
In addition

\[ \text{var}[s(t)] = \frac{2}{15\langle g^2 \rangle} x \frac{5\sqrt{2}t_p}{\pi^3 \sqrt{\pi t_D}} \left( \frac{t}{t_D} \right)^3 \]

Also, the experimental data for C_3 allowed us to calculate experimental values of conductance, \( g \), for each sample. Remember, for diffusion \( g > 1 \).

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>Experimental Conductance</th>
<th>Theoretical Conductance</th>
</tr>
</thead>
<tbody>
<tr>
<td>61</td>
<td>4.17</td>
<td>4.33</td>
</tr>
<tr>
<td>76.2</td>
<td>3.21</td>
<td>3.33</td>
</tr>
<tr>
<td>91.4</td>
<td>1.85</td>
<td>2.81</td>
</tr>
</tbody>
</table>
Results

If we normalize the y-axis by the time scale $\tau_p = t_p / t_D$, we find all three curves for a single length collapse on a single plot, in agreement with the theory.

The graph shows the variance of the dynamic conductance increases as $(t/t_D)^3$. The inset is the same figure in a log-log scale showing $\text{var}[s(t)] \propto \tau^3$. 
One final result!

In 2004, Skipetrov has also found how $C_2$ should behave in time (Phys. Rev. Lett. 93, 233901):

$$C_2(t) \propto \frac{1}{\langle g \rangle} \frac{t_p}{t_D}\left(\frac{t}{t_D}\right)^2$$

While we know $C_3$ behaves as:

$$\text{var}[s(t)] \propto \frac{1}{\langle g^2 \rangle} \frac{t_p}{t_D}\left(\frac{t}{t_D}\right)^3$$

The two correlators are equal at time:

$$t_q \equiv t = \sqrt{g t_D}$$
Significance of $t_q$

- This time is between the Thouless time and the Heisenberg time:

$$t_D < t_q < \langle g \rangle t_D$$

$$t_{Th} \sim t_D$$

$$t_H \sim \langle g \rangle t_D$$

- In has appeared in the context of weak localization in classically chaotic systems and disordered systems, G. Casati et. al. (Phys. Rev. E 56, R6240; 1997) and A.D. Mirlin (Phys. Rep. 326, 259; 2000) but its role in the analysis of fluctuations of transport properties had not yet been identified.

  - While this might indicate new physics taking place at $t > t_q$, the experimental data still fits well with theoretical predictions presented by Cherroret and Skiptrov at longer times then $t_q$. 
Conclusions

• The dynamic conductance fluctuations are nonuniversal as opposed to the steady state case. The collaboration was able to provide theoretical and experimental work describing the infinite range correlator as a function of time.
  – By depending on diffusion time, the infinite range correlator, $C_3$, is nonuniversal among samples. $C_3$ also grows as a third power of delay time from a pulse.

$$\text{var}[s(t)] \propto \frac{1}{\langle g^2 \rangle} \frac{t_P}{t_D} \left( \frac{t}{t_D} \right)^3$$

• One can obtain a value for the dimensionless conductance for a sample length by knowing the infinite range correlator, $C_3$, for a diffusive sample.

• We identified a time where the long and infinite range correlator are equal to each other:

$$t_q = \sqrt{g} t_D$$

• These results should apply to a system including electronic transport as well.
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