Universal Spectra of Coherent Atoms in a Recurrent Random Walk

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Why study recurrence?

• Polya’s theorem $\leftrightarrow$ Mermin-Wagner
• Anderson Localization
  - Zero-spin, non-interacting electrons in disordered media.
• Anomalous diffusion
• Spectral dimension $d_s$ of random walk:
  $P(T) \sim T^{(d_s/2)}$
Outline

• Very brief introduction to EIT

• Diffusion: spatial effects (images)

• Diffusion: spectral effects (random Ramsey narrowing and the universal regime)
Introduction to EIT
Electromagnetically Induced Transparency (EIT)

Pump off

Active atoms (^{87}\text{Rb})

Suscetibility

Pump off

Detuning from atomic resonance

Real (refraction)

Imaginary (absorption)
Electromagnetically Induced Transparency (EIT)

low absorption + strong dispersion

Raman (two-photon) detuning:
\[ \Delta_R = \omega_{HF} - (\omega_1 - \omega_2) \]

probe

pump

Active atoms
\(^{87}\text{Rb}\)

Buffer gas atoms

vapor cell

 susceptibilities

\[ \text{Real (refraction)} \]
\[ \text{Imaginary (absorption)} \]

Pump on

Detuning from atomic resonance

\[ \omega_1 \]

\[ \omega_2 \]

\[ \omega_{HF} \]
Dark States

Coupling Hamiltonian (pump=probe):

\[ V = -\hbar \Omega_1 |3\rangle\langle 1| - \hbar \Omega_1 |3\rangle\langle 2| + \text{h.c} \]

A dark state:

\[ |\text{dark}\rangle = \frac{1}{\sqrt{2}} (|1\rangle - |2\rangle) \]

\[ \langle 3|V|\text{dark}\rangle = 0 \]

A specific super-position of the lower levels
Storage of light using EIT

Buffer gas atoms

narrow line-width = long coherence = long storage duration
Diffusion during storage-of-light

Storage:

\[ \rho_{1,2}^{\text{initial}}(\mathbf{r}) = \frac{\Omega_2^* \Omega_1}{|\Omega_1|^2 + |\Omega_2|^2} \approx -\frac{g}{\Omega_2} \hat{E}_{\text{probe}}(\mathbf{r}) \]

Strong pump
Diffusion during storage:

\[ \rho_{1,2}^{\text{initial}}(r) = \frac{\Omega_2^* \Omega_1}{|\Omega_1|^2 + |\Omega_2|^2} \approx -\frac{g}{\Omega_2} \hat{E}_{\text{probe}}(r) \]

Storage:

\[ \dot{\rho}_{1,2}(r,t) \]

Diffusion during storage:

\[ \hat{E}_{\text{retrieved}}(r) = -\frac{\Omega}{g} \rho_{1,2}(r, \tau_{\text{storage}}) \]
Storing an optical vortex

\[ e^{i\Theta} \text{ phase singularity is topologically stable} \]

flat phase beam is not stable
Storing helical beams

\[ \hat{E}_m^{\text{helical}}(r, \theta) = A_m(r, w_0) e^{-im\theta} \]

Diffusion during storage:

\[ w_f^2 = w_0^2 + 4D\tau \]

\[ \hat{E}_m^{\text{retrieved}}(r, \theta) \propto A_m(r, w_f) e^{-im\theta} \]

Helical modes are self-similar under diffusion

R. Pugatch, M. Shuker, O. Firstenberg et. al,
PRL 98, 203601 (2007)
Self-Similarity of Gaussian modes

Gaussian (m=0)

LG (m=1)

LG (m=2)

Diffusion (imaginary time)

\[ \frac{\partial}{\partial t} E = D \nabla^2 \perp E \]

Paraxial Diffraction (schrodinger)

\[ \frac{\partial}{\partial z} E = i \frac{1}{2q} \nabla^2 \perp E \]

Expansion Ratio (Scaling)

All modes expand at the same rate

\[ s(t) = 1 + \frac{4Dt}{w_0^2} \]
Conclusions (so far)

• Stored light exhibit *complex-field diffusion*

• Paraxial modes (e.g. a helical beam) remains self-similar under diffusion
  - Vortex charge is a valid qubit space
Diffusion:

Spectral effects
Transit (diffusion) time broadening

Ideal spectrum: atoms see the beams from $-\infty$ to $+\infty$

Finite beams, thermal atoms

$\rightarrow$ transit time broadening $\sim 1/\text{transit time}$.

$\Gamma_{\text{transit}} = \frac{\langle v \rangle}{a} \text{ (ballistic)} \sim \frac{(300 \text{ m/s})}{(1\text{ mm})} = 300 \text{ kHz}$

$\Gamma_{\text{transit}} = \frac{D}{a^2} \text{ (diffusion)} = \frac{10 \text{ cm}^2/\text{s}}{(1\text{ mm})^2} = 1 \text{ kHz}$
Diffusion induced broadening

Naively: coherence time limited by time to leave the beam.

⇒ Lorenzian with line width: \( \Gamma_{\text{Time-of-flight}} \sim \frac{1}{\tau} = \frac{D}{\Delta x^2} \)

In practice:
“good” atoms can diffuse back into the beam
... and “Ramsey” narrowing

Sum **coherences** over trajectories:
\( \text{ln} + \text{in/out/in} + \text{in/out/in/out/in} + \ldots \)  
(long trajectories suffer from decay)

**Coherence** of each trajectory:  
“Ramsey” fringes with period \( \sim 1/T \)

Y. Xiao et.al., PRL 96, 043601 (2006)
differential view:

\[
\left[ -i\Delta + \frac{\Gamma_0}{2} - D\nabla^2 + \Gamma_p (r) \right] R_{21} (\mathbf{r}) = -n_0 \Gamma_p (\mathbf{r})
\]

universal regime:

Replace \( \Gamma_p(r) \rightarrow \delta(r) \)

\[
\left[ -i\Delta + \frac{\Gamma_0}{2} - D\nabla^2 + \Gamma_p(r) \right] R_{21}(r) = -n_0 \delta(r)
\]

\[
S(\Delta) = C \int d^d r \ \Gamma_p(\mathbf{r}) \left[ 1 + 2R_{21}(\mathbf{r}) / n_0 \right]
\]

Prepare, propagate, measure

Measured spectrum $S(\Delta)$ is given by:

\[
\int d^2 r' \int d^2 r \int dt \quad \Gamma_p(r) G(r, r', t) e^{i \Delta t - \Gamma_0 t / 2} \Gamma_p(r')
\]

Propagate (outside the beam)
+ Dark-bright oscillation and decay
Experimental setup

RF modulation @ 3.4 GHz AND 97 kHz

Vapor cell with buffer gas

PD

VCSEL

DAQ

Lock-In Amp.
Remark: Kramers-Kronig relation
Prediction: $s = -i\Delta + \Gamma_0 / 2$

diffusion in $d=1$: $S(s) = P(0,s) = s^{-1/2}$

no diffusion ($D=0$): $S(s) = s^{-1}$ (lorentzian)

$\beta_{1d} = 0.56 \pm 0.01$

$\beta_{D=0} = 0.97 \pm 0.01$
remarks:

diffusion in d=1: $S(s) = P(0,s) = s^{-1/2} \Rightarrow P(0,t) = t^{-1/2}$
mean return time diverges..

In d=2 $S(s) = K_0(bs^{1/2}) \sim \ln(s^{1/2}) + \text{const } (s \text{ small})$
“phase diagram”
**Imaginary time Anderson Localization**

What is the fate of dark state atoms in the presence of traps?

What is the spectrum?

\[ \Gamma_p (r) \]

\[ P(s) = \sum_n \frac{V_n}{s + E_n} \]

\[ V_n \sim \text{area below peaks} \]

\[ E_n = \text{Eigen-energy of d=2 Anderson Hamiltonian} \]
Prospect

• Devices
  - Miniature atomic clocks, paraffin coated cells.
  - Spintronic devices?

• Basic
  - Imaginary time Anderson Localization (Diffusion with traps).
  - New type of random laser.
  - Atomic billiards.
  - Measure First Passage times
Conclusions

- Diffusion keeps paraxial diffraction modes invariant.

- Complex EIT spectrum in the universal regime is the Laplace transform of the return probability.
Thank you

OPTICAL VORTEXES
A lasting twist

A Laguerre–Gauss-mode light beam, also known as an optical-vortex beam, is ‘twisted’ so that any two diametrically opposed points at the centre of the beam have a π-phase shift, resulting in destructive interference and a dark centre. Now researchers in Israel have demonstrated that this optical vortex can be used to transport information.

RESEARCH HIGHLIGHTS

Fuzzy figures
Capture the complex patterns of photons that make up several numerical images in a vapour of rubidium atoms at 52 K, and those images will degrade as the atoms diffuse (pictued right). But Moshe Shlizer of the Technion-Israel Institute of Technology and his colleagues have found a way to store images and then regenerate the original light beam. The numbers were created by projecting a laser beam through a stencil.

They stored images comprising sets of three parallel lines for 2, 10, 20 or 30 microseconds (picted far right and in descending order) using a ‘phase shift’ technique to counteract the effect of diffusion (shown near right). The technique involves manipulating the phase of the input image, which controls the quantum phases of the atoms. The phases of the atoms that diffuse away from an image’s lines are at 180° to one another, and so cancel each other out in the restored image.

Thirty microseconds is a thousand-fold increase over the previous record. The work has potential applications in many fields, including quantum information processing.
Backup slides
Diffusion propagator:

\[ P(\vec{r}, t) = (2\pi Dt)^{-d/2} e^{-r^2 / 4Dt} \]

**note**

\[ P(\vec{r} = 0, t) = (2\pi Dt)^{-d/2} \]

Average number of visits at origin = \( \int P(0, t)dt = \infty \) (d<3)

Polya's theorem
Alternatively

\[ P(\vec{r}, t) = \delta_{\vec{r}, 0} \delta_{t, 0} + \int_0^t dt' \ FPT(\vec{r}, t') P(0, t - t') \]

This is what we measure

\[ FRT(s) = 1 - \frac{1}{P(0, s)} \]

intuition

Taking the Laplace transform:
Levy\(\frac{1}{2}\) distribution function

First passage time from origin to \(r \neq 0\)

\[
FPT (s, r) = e^{-r \sqrt{s/D}}
\]

Can be inverted analytically

\[
FPT (r, t) = \frac{e^{-r^2/4Dt}}{\sqrt{2\piDt}^{3/2}}
\]
Dicke narrowing experiment

Residual Doppler broadening: 250 KHz
Measured Dicke-Doppler width: 2 KHz
Dicke-EIT narrowing parameter: \(~125\)

The model captures the functional dependence of both the Amplitude and Width.
Visualizing the angular dependence

Off-resonance transmission

EIT transmission

divergent probe

Probe transmission vs. θ

- Spectroscopic measurement
- Imaging measurement
- Theory (maximum fitted)

Vapor cell

Probe

Pump
Storing helical beams

Diffusion during storage:

\[ \hat{E}_m^{\text{helical}}(r, \theta) = A_m(r, w_0) e^{-im\theta} \]

\[ w_f^2 = w_0^2 + 4D\tau \]

\[ \hat{E}_m^{\text{retrieved}}(r, \theta) \propto A_m(r, w_f) e^{-im\theta} \]

R. Pugatch et al., PRL 98, 203601 (2007)
Diffusion: spectral effects 1

Doppler broadening and Dicke narrowing
velocity-changing coherence-preserving collisions can decrease the Doppler width

\[
\text{FWHM} = 2\Gamma_D \times 2.8 \left( \frac{\Lambda}{\lambda} \right)
\]

Doppler width
mean free path
optical wavelength
Dicke parameter
Doppler broadening in EIT

When $q_1 \neq q_2$, the pump and the probe exhibit different Doppler shifts.

Thermal velocity distribution ($v_{th}$) cause broadening

$$\Gamma_{D}^{\text{res}} = |q_1 - q_2| v_{th} = (\omega_1 - \omega_2) v_{th} / c \approx 9\text{KHz}$$

But the measured EIT line is much narrower
Dicke narrowing in EIT

\[ S_{\text{EIT}}(\Delta_R) = \frac{|\Omega_2|^2}{\left[ \Gamma_1 + q_1 (q_1 - q_2) \frac{\nu_h}{\gamma} \right]^2} \times \frac{\Gamma_{\text{dec}} + \eta \Gamma_{\text{res}}^D}{\Delta_R^2 + \left[ \Gamma_{\text{dec}} + \eta \Gamma_{\text{res}}^D \right]^2} \]

Amplitude

Width (Lorentzian)

EIT-Dicke parameter: \( \eta = 2\pi \frac{\Lambda}{\lambda_{\text{EIT}}} \)

\( \lambda_{\text{EIT}} = 2\pi / |q_1 - q_2| \)

Firstenberg et. al., quant-ph 0701008
Angular dependence of the Dicke-Doppler width

For $\omega_1=\omega_2$ and $q_1 \parallel q_2$

EIT-Dicke parameter: $\eta = \frac{2\pi \Lambda}{\lambda} \theta$

Residual Doppler width: $\Gamma_D^{\text{res}} = \Gamma_D \theta$

$$S_{\text{EIT}}(\Delta_R) = \frac{\left| \Omega_2 \right|^2}{\left[ \Gamma_1 + \frac{\pi \Lambda}{\lambda} \Gamma_D \theta^2 \right]^2} \times \frac{\Delta^2 + \left[ \Gamma_{\text{dec}} + \frac{2\pi \Lambda}{\lambda} \Gamma_D \theta^2 \right]^2}{\Delta_R^2 + \left[ \Gamma_{\text{dec}} + \frac{2\pi \Lambda}{\lambda} \Gamma_D \theta^2 \right]^2}$$

The Doppler width is quadratic in the angular deviation.
EIT line-shapes for different θ’s

Shuker et al., quant-ph 0702254
Spatial-spectral equivalence in ballistic motion

Doppler ≡ Time-of-flight broadening

\[ V_0 \sim \frac{1}{k \perp \Delta} \]

Transverse shape in real-space

\[ \Delta k_\perp \sim \frac{1}{\Delta x} \]

k projection in the transverse plane

\[ \Gamma_{\text{Time-of-flight}} \sim \frac{1}{\tau} = \frac{V_0}{\Delta x} \sim V_0 \cdot \Delta k_\perp \sim \Gamma_{\text{Doppler}} \]
Spatial-spectral equivalence in diffusion

Dicke $\equiv$ Time-of-flight broadening

Transverse shape in real-space

$k$ projection in the transverse plane

$\Gamma_{\text{Time-of-flight}} \sim \frac{1}{\tau} = \frac{D}{\Delta x^2} \sim V_{th} \Lambda \cdot \Delta k_{\perp}^2 \sim \Gamma_{\text{Dicke}}$

(remember that $D \approx v_{th} \Lambda$)
Self-Similarity of Gaussian modes

Data collapse

\[ s(t) = 1 + \frac{4Dt}{w_0^2} \]
Expansion Ratio (Scaling)

All modes expand at the same rate

\[ s(t) = 1 + \frac{4Dt}{w_0^2} \]
Diffusion of atoms limits the resolution

Imaging a resolution chart

Experiment  Calculation

2μs storage
10μs storage
20μs storage
30μs storage
Line visibility

In the $\pi$-shifted image there is almost no loss of visibility
Self-Similarity of Gaussian modes

Data collapse

\[ s(t) = 1 + \frac{4Dt}{w_0^2} \]
Path integral

\[ G(r-0, s) = \sum \int dT_1 \ldots \int dT_n \int dt_1 \ldots \int dt_n e^{i \Delta \sum t_i - \Gamma_p \sum t_i} \prod FPT(t_i) e^{i \Delta \sum T_i - \Gamma_0 \sum T_i} \prod FRT(T_i) = \]

\[ \sum_{n} FPT^n(s_{in}) FPT^n(s) = \frac{FPT(s_{in})}{1 - FPT(s_{in}) FRT(s)} \]

In the limit of low power and small beams \( FPT(s) \to 1 \) and we obtain

\[ G(0, s) = \sum_{n} FRT^n(s) = \frac{1}{1 - FRT(s)} = P(0, s) \]
Random Ramsey

\[
\begin{align*}
FPT(s_{in}) &\quad FRT(s) FPT(s_{in}) FRT(s) FPT(s_{in}) \\
&= s_T \\
FRT(s) &= \int dT \ e^{i\Delta T + \Gamma_0 T/2} FRT(T) = FRT(s) \\
FPT(s) &= \int dt_{in} e^{i\Delta t_{in} + \Gamma pt_{in}/2} FPT(t_{in}) = FPT(s_{in}) \\
&= s_{in} t
\end{align*}
\]