Nonlinear dynamics of waves and transport in the atmosphere and oceans

Harry L. Swinney
University of Texas at Austin

Joel Sommeria
Coriolis Lab, CNRS, University of Grenoble

Steven Meyers
University of South Florida

Tom Solomon
Bucknell University (Pennsylvania)

Eric Weeks
Emory University (Atlanta, Georgia)

Brendan Plapp
U.S. Department of Homeland Security

Charles Baroud
Ecole Polytechnique (Paris)

Sunny Jung
Virginia Polytech

Jori Ruppert-Felsot
finance (Tokyo)

Hepeng Zhang
U. Texas & Shanghai Jaiotong University

Ben King
University of Texas

Bruce Rodenborn
University of Texas
Persistent coherent structures: JETS & VORTICES

isobars at $p=1/2$ atm
Feb 17

JET STREAM
NORTH POLE
LOW
HIGH
Jupiter: from Voyager I (1978)
Kurioshio Current

Japan Aerospace Exploration Agency

JAPAN

Sea surface temperature

10 °C  Sea surface temperature  30 °C
Antarctic circumpolar current
How do jets and vortices affect transport and climate?
Climate prediction: Global Climate Models

- Model rather than simulate lengths 1 cm to 100 km (diffusion: < 1 cm).
- Double processor speed every 18 months for 50 years ⇒ 6X resolution increase.
Rise in mean global temperature from 8 of the 30+ global climate models
Navier-Stokes equations

- fields: velocity $u(r,t)$, density $\rho(r,t)$, pressure $p(r,t)$
- inertial reference frame
  $$\rho \left[ \frac{\partial \tilde{u}}{\partial t} + (\tilde{u} \cdot \nabla)\tilde{u} \right] = -\nabla p + \rho \nu \nabla^2 \tilde{u}$$
  incompressible fluid: $\nabla \cdot \tilde{u} = 0$

Reynolds number $Re = \frac{\text{inertia}}{\text{dissipation}} = \frac{(\tilde{u} \cdot \nabla)\tilde{u}}{\nu \nabla^2 \tilde{u}} \sim \frac{U^2}{L} \frac{U}{vU} = \frac{UL}{v}$

In atmosphere and oceans typically $Re \sim 10^7 - 10^{12}$
**Effect of earth’s rotation \( \Omega \)**

\[
\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} - \Omega \times (\Omega \times \vec{r}) - 2\Omega \times \vec{u}
\]

The centrifugal term, \(-\Omega \times (\Omega \times \vec{r}) = -\nabla \left( \frac{1}{2} \Omega^2 r_{axis}^2 \right)\), can be incorporated into pressure and doesn’t change the dynamics.

But the Coriolis term has an enormous effect on the dynamics:

Atmosphere and oceans: \( U \sim 2 \text{ m/s}, \quad \Omega=4\pi \times 10^{-5} \text{ rad/s}, \quad L \sim 100 \text{ km} \)

Our lab experiments: \( U \sim 0.3 \text{ m/s}, \quad \Omega=4\pi \text{ rad/s}, \quad L \sim 0.3 \text{ m} \)

So in both cases:

\[
\frac{\text{Coriolis}}{\text{inertia}} \approx \frac{2\Omega L}{U} = \frac{1}{\text{Rossby number}} \sim 10
\]
Water swirls down a sink drain

See [youtube.com/watch?v=ZBVntSA-qoQ](https://youtube.com/watch?v=ZBVntSA-qoQ)

- Counter-clockwise in northern hemisphere
- Clockwise in the southern hemisphere

**In a sink**

$U \sim 0.05 \text{ m/sec}$, $\Omega_{EARTH} = 1 \text{ rev/(86400 s)}$, $L \sim 0.02 \text{ m}$

$$\frac{\text{Coriolis}}{\text{inertia}} = \left| \frac{2\Omega \times \vec{u}}{(\vec{u} \cdot \nabla) \vec{u}} \right| \sim \frac{2\Omega L}{U} \sim 10^{-5}$$

Moreover, at the equator: $\vec{\Omega} \times \vec{u} = 0$

Thus Coriolis effect is negligible
- for small length scales
- near equator
Taylor-Proudman theorem

\[ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} - 2\vec{\Omega} \times \vec{u} \]

slow change  rapid rotation  small dissipation

Then with \( \vec{\Omega} = \hat{z} \Omega \)

\[ \frac{\partial \vec{u}(x, y, z, t)}{\partial z} = 0 \quad \text{flow is 2-dimensional} \]
2-dimensionality of oceanic and atmospheric flow due to

• Rapid rotation (Taylor-Proudman)

• Thinness:
  ocean & troposphere ~10 km thick, while horizontal scales ~1000 km

• Stratification:
  – suppresses vertical motions
  – leads to internal gravity waves
Lab experiments on rotating flow
University of Texas

- **Rotating annulus**
- **SINKS** ring of 120 holes
- **SOURCES** ring of 120 holes
- **b-effect:** sloping bottom mimics variation of earth’s Coriolis force with latitude
- **water**
- **20 cm**
- **86 cm**
- **Lab experiments on rotating flow**
- **University of Texas**
Weak radial pumping produces strong jet stream

\[ u_\theta \gg u_r \]

**STRONG JET:**

Pump outward: *anti-cyclonic* (counter-rotating) jet

Pump inward: *cyclonic* (co-rotating jet)
Measure 2-dimensionality of flow
Velocity measured with two probes displaced in z (axially) by 16 cm:
almost no z-dependence $\Rightarrow$ quasi-2-dimensional flow

$u_\theta(t)$

Rossby number = 0.1
Flow in rotating tank for low pumping

Solution of Navier-Stokes for ANY system at low Re:

- unique
- symmetry of boundary conditions
- globally stable
Low pumping: axisymmetric jet

time exposure of tracer particles

Rossby # = 0.02
Reynolds # = 370
$\Omega = 6.28$ rad/s
 Flux = 5.6 cm$^3$/s
Pump faster: axisymmetric jet becomes unstable

5-fold wave
(Rossby wave)
Dye visualization of the Rossby waves

$m = 3$

$m = 6$
Linear stability analysis of Navier-Stokes eq.: axisymmetric jet $\rightarrow$ Rossby waves


particle streak lines in co-rotating frame of the Rossby wave

$m = 7$
Onset of instability of circular jet: compare experiment with theory

Solomon, Holloway, Swinney
29 OCT. 2006: NASA’s Casini spacecraft reveals “bizarre 6-sided feature encircling the north pole of Saturn”

http://saturn.jpl.nasa.gov/home/index.cfm

Each side 13,800 km

Period 10h 39min 24s

Latitude 78° North
2-dimensional Hamiltonian dynamics:

KAM tori as barriers to transport in planetary flows
What does Hamiltonian dynamics have to do with the flow of a viscous fluid?
Particles in a 2D incompressible flow

Fluid velocity \( \mathbf{u}(x,y,t) \) in terms of stream function \( \psi(x,y,t) \):

\[
\begin{align*}
  u_x &= -\frac{\partial \psi}{\partial y} \\
  u_y &= \frac{\partial \psi}{\partial x}
\end{align*}
\]

Then the position \((x,y)\) of a passive scalar particle is given by integrating

\[
\begin{align*}
  \dot{x} &= \frac{dx}{dt} = -\frac{\partial \psi}{\partial y} \\
  \dot{y} &= \frac{dy}{dt} = \frac{\partial \psi}{\partial x}
\end{align*}
\]

\( \Rightarrow \) tracer particle motion is described by Hamiltonian \( \psi \)

\[\text{HAMILTON'S EQUATIONS (1883)} \quad \text{with} \quad H = \psi\]
Kolmogorov-Arnold-Moser (KAM) theorem

Quasi-periodic motions of Hamiltonian dynamical systems persist under small perturbations

• 1954: discussed by Kolmogorov

• 1962: proved by Moser for smooth twist maps

• 1963: proved by Arnold for analytic Hamiltonian systems

“KAM tori”: tori that are deformed yet persist under weak perturbation
In the co-rotating frame of the 5 waves,

$$\psi(r, \theta, t) = \psi_0(r, \theta) + \varepsilon \psi_P(r, \theta, t)$$

**Model Hamiltonian for particle dynamics**

Zonal flow with 5 waves: 
the unperturbed (integrable) system

\[ \psi_0 = UL \{ \tanh(\frac{r-r_0}{L}) + \alpha \left[ \cosh^{-2}(\frac{r-r_0}{L}) \right] \cos(5\theta) \} \]

Time-independent flow in wave co-rotating frame

Particles initially along these lines remain on invariant curves (tori)
$\psi = \psi_0 + \varepsilon \psi_p$

$(\varepsilon = 0.01)$

with

$$\psi_p = UL \left[ \cosh^{-2} \left( \frac{r-r_0}{L} \right) \right] \cos \left[ 6\theta - (\Delta\omega) t \right]$$

where $\Delta \omega = \omega_2 - \left( \frac{m_2}{m_1} \right) \omega_1$
5-fold jet with 5% perturbation by 6-fold wave

CHAOTIC SEA

particles initially along these lines

persistent KAM TORI

Behringer, Meyers, Swinney
**Jet**: barrier to transport

Inject dye

Ghost of the KAM invariant torus


Broken torus: *Cantorous* (Percival, 1979)
Ozone hole: contained by a KAM torus

Ozone concentration: red: high blue: low

polar night jet (KAM torus)

south pole
First report on Jupiter’s Great Red Spot

The Ingenious Mr. Hook did, some moneths since, intimate to a friend of his, that he had, with an excellent twelve foot Telescope, observed, some days before, he than spoke of it, (videlicet on the ninth of May, 1664, about 9 of the clock at night) a small Spot in the biggest of the 3 obscurer Belts of Jupiter, and that, observing it from time to time, he found, that within 2 hours after, the said Spot had moved from East to West, about half the length of the Diameter of Jupiter.
Great Red Spot of Jupiter

Voyager 2 photo (1979)
Eastward-westward jets on Jupiter

from Cassini spacecraft
http://saturn.jpl.nasa.gov/photos/
Merger of Jupiter’s three White Ovals

Hubble Space Telescope WFPC2

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Laboratory model of Jupiter’s Great Red Spot

1. low Rossby number (strong Coriolis effect)
2. turbulent flow
3. strong shear
4. Beta effect:
   Coriolis effect varies with latitude
Merger of two laboratory vortices

Oceanic and Atmospheric Flows

- **Coriolis force** \((-2\Omega \times u)\) makes atmospheric and oceanic turbulence different
  - long-lived *jets and vortices* – jet stream, high and low pressure systems, Gulf stream, meddies
  - flow is approximately *2-dimensional*
  - jets are *barriers to transport*
  - *Vortex dynamics*:
    - shedding and merging of vortices
Wednesday, 30 June 2010

- Lévy statistics transport
- Lagrangian Coherent Structures

Thursday, 1 July

- Ocean circulation currents
- Resonant internal waves
- Potential climatic effects