Application of Laser-Plasma Accelerators

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Varenna Enrico Fermi Summer School 2011

Outline of lectures

- Lasers
- High Power Lasers
- Plasma
- Accelerators
- Applications
- Large and small accelerators
- How can we use a laser-driven plasma wave to produce coherent electromagnetic radiation?
- Laser driven wakes
- Ultra-short bunch electron production using wakefield accelerators
- Synchrotron, free-electron laser and betatron sources
- Conclusion and outlook
Selected review articles


Basic Reference Material

- A. Chao and M. Tigner “Handbook of Accelerator Physics and Engineering”
Lasers: High power, short pulse and high intensity

Power:
- TW = 10^{12} W
- GW = 10^9 W
- MW = 10^6 W
- KW = 10^3 W

Intensity:
- I = 10^{18} W cm^{-2}

Focus to a spot less than thickness of a human hair:
- Spot diameter 30 μm

Average Sunshine: 120 W m^{-2}
- Peak: 1.3 kW m^{-2}
- 98 PW reaches the earth

Chirped Pulse Amplification

See Wiggins lectures
High Power Lasers: TW to PW

Power amplifiers

VULCAN Laser at Rutherford Appleton Laboratory

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Lots of applications of lasers

- material processing
  - welding, cutting, drilling
  - soldering,
  - marking,
  - surface modification

- laser displays (→ RGB sources)

- remote sensing (e.g. LIDAR)

- medical (e.g. Surgery)
- military (e.g. anti-missile weapons)
- fundamental science (e.g. particle acceleration)
- laser-induced nuclear fusion (e.g. in the NIF project)

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Accelerators in use

- High energy accelerators (> 1GeV) >112
- Biomedical accelerators
  - Radiotherapy >4000
  - Research including biomedical >800
  - Medical radioisotope >200
- Accelerators in industry >1500
- Ion implanters >2000
- Surface processing >1000
- Synchrotron sources >50
- FELs >10
- Laser and beam driven plasma accelerators >5
- TOTAL >10,000

Lots of uses of accelerators

- Study the structure of matter: colliders in the search for elementary particles
- Electromagnetic sources:
  - X-rays for oncology (most widespread),
  - synchrotron light sources and
  - free-electron lasers
- Medical uses:
  - Radiation therapy
  - Radio-isotope production (for PET)
  - Biological research
- Industrial uses:
  - Ion implantation
  - Photo-lithography
  - Materials processing
Accelerators come in many guises

- Low energy – used to drive x-ray sources etc.
- Intermediate energy
  - Light sources: Synchrotrons and free-electron lasers
  - Energy amplifiers: accelerator driven sub-critical reactors: ADSR
  - Material analysis – Rutherford backscattering, particle induced X-ray emission, nuclear reactions, elastic recoil, charged particle activation, accelerated mass spectroscopy
- High energy – Nuclear physics – probing nuclear structure
- Very high energy – energy frontier and astrophysics – Tevatron, LHC, muons as a source of neutrinos
- Secondary sources: gamma rays, neutrons, muons etc

A Brief History of Accelerators

- Rolf Widerøe born in Oslo (1902-1996)
  - betatron and resonance accelerator
  - betatron – based on magnetic induction
  - resonance – based on resonant electric fields in cavities
- Ernest Walton born in Ireland (1903-1995)
  - Linear accelerator – split atom
- Robert J. van de Graaff (1901-1967)
  - Static field accelerator
- Ernest Lawrence US (1901-1958)
  - Cyclotron
Particle and nuclear physics

- Energy creation
- Astrophysics
- Elementary composition of matter
- Origins of the universe
- Hadron colliders – to find new particles
- Lepton colliders – precise measurement of particles after their discovery

Medical application

- Treatment of cancer
  - X-rays, electrons, protons and ions
- Isotope generation – for PET
- Biological research – crystallography of large molecules: proteins, viruses etc.
- X-ray imaging: macroscopic and microscopic

Spectroscopy and Imaging in Alzheimer research
Atomic physics, condensed matter, material science and chemistry

- Atomic collisions
- Ionisation
- X-ray production
- Scattering
- Neutron and ion beams
- Elemental mapping
- Charged particle activation
- Radiation chemistry and radiolysis

Industrial applications

- Studies of the structure of matter: chemistry and biology
- Trace element analysis
- Ion implantation
- Radiation processing (e.g. hardening)
- Food preservation and sterilisation
- Microlithography
- Precision machining – ion beam milling
- Cross-linking, polymerisation, depolymerisation, grafting
Military applications

• Radar (microwaves from beams)
• Countermeasures (energy delivery, jamming)
• Beams (weapons)
• Remote sensing (surveillance and stewardship of nuclear material)

Power and energy

• Energy amplifiers (subcritical reactors)
• Nuclear fusion
• Power engineering
• Treatment of nuclear waste and control of nuclear proliferation (transmute dangerous and long-lived isotopes to short-lived isotopes)
Energy amplifier and waste transmutation

• Reactors produce poisonous by products: actinides – half-lives geological time scales
• Public worries about accidents, terrorist attack and proliferation
• Need high current proton accelerators
• Energy amplifier – produces energy while transmuting waste – possibility of using thorium to get around the proliferation problem
• Need about 1 GeV, 10 mA (10 MW beam)
• Choice of accelerator crucial
  – options: high current cyclotrons, synchrotrons, FFAG etc.

Fusion

• Options for fusion:
  – Magnetic confinement
  – Laser inertial confinement
    • Fast ignition
    • Indirect drive
  – Ion fusion – indirect drive – 10 GeV heavy ion accelerators
Neutron sources

- Neutrons are complimentary to X-rays and electrons
- Condensed matter
- Molecular structure
- Penetrate deep into bulk (interact with nuclei)
- Less damage to biological material

Livingston plot

New ideas lead to new technologies

Acceleration gradient currently limits maximum energy

Laser wakefield accelerators?
Modern Livingston Plot

Collider energy plotted as equivalent laboratory energy of particles colliding with a proton at rest to reach the same centre of mass energy

Different technologies = New particles + Higher energies

A decade increase every 6 years

Limitations of present accelerators

RF technology reaches its limit
E<100 MV/m
B<10 Tesla
Synchrotron radiation (e-)

Test of new concepts:
Plasma based accelerators
Synchrotrons and free-electron laser light sources: tools for scientists & industrialists

Synchrotron – size and cost determined by accelerator technology

Diamond

DESY undulator

Free-electron laser – coherent radiation: XFEL

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Large accelerators depend on superconducting
Radio Frequency cavities and magnets

SLAC

7 TeV in 27 km
7 MV/m

CERN – LHC
27 km circumference

50 GeV in 3.3 km
20 MV/m

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Conventional accelerators are based on RF cavities

- Conventional accelerator
  - RF Cavities

Laser-plasma wakefield Accelerator is:
- x1000 smaller
- x1000 more acceleration
- x1000 less expensive

Small accelerator based on plasma

Centimetre long plasma based accelerator
- 200 GeV/m
Laser-plasma accelerator development and application lab at Strathclyde: ALPHA-X

Generate radiation pulses on the ALPHA-X Beam Line

Waves in plasma

\[ E(\omega, t) = E_0 e^{-i(\omega t - kz)} \]

Dispersion relationship

\[ \omega^2 = \omega_p^2 + k^2 c^2 \]

Warm plasma: Bohm-Gross dispersion

\[ \omega_p^2 = \omega_p^2 + 3k_B^2 v_{TH}^2 \]

Phase velocity: \[ v_\varphi = \frac{\omega}{k} \]

Group velocity: \[ v_g = \frac{d\omega}{dk} \]
Relativistic effects

The threshold for relativistic regime is \( a = 1 \) where the field intensities are greater than \( 10^{18} \) W/cm\(^2\) for 1 \( \mu \)m lasers. The momentum of the electron increases due to the relativistic mass increase: \( p > m_e c \).

The ratio of these two values defines the normalised vector potential

\[
a = \frac{P}{m_e c} = 0.85 \times 10^{-3} \sqrt{I} \lambda
\]

where \( I \) is given in W/cm\(^2\) and the wavelength in \( \mu \)m.

The force and energy equations are given by the Lorentz equation:

\[
\mathbf{F} = \frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})
\]

and relativistic energy equation:

\[
\frac{d\gamma}{dt} = \frac{d(m_e c^2)}{dt} = -e\mathbf{E} \cdot \mathbf{v}
\]

From the Lorentz force electron motion becomes relativistic the electron describes a figure of eight path.

This due to the fact that \( \mathbf{v} \times \mathbf{B} \propto \mathbf{E} \times \mathbf{B} \propto E^2 \lambda^2 \)

Self-focussing due to relativistic and ponderomotive effects

Plasma behaves as a dielectric medium with a dispersion relation governed by the plasma frequency

\[
\omega_p = \left( \frac{n_e e^2}{\varepsilon_0 m_e} \right)^{1/2}
\]

\( \gamma \) is the time averaged relativistic factor \( \gamma = (1+a^2/2)^{1/2} \).

The higher the laser power the lower the plasma frequency.

The phase velocity in the plasma gives the refractive index

\[
\eta = \left[ 1 - \frac{\omega_p^2}{\omega_0^2} \right]^{-1/2}
\]

When the laser frequency \( \omega_0 \) is less than the plasma frequency the field is evanescent and does not propagate in the plasma, thus the plasma frequency defines a cut-off frequency below which no transmission occurs.

Higher intensities lead to lower plasma frequencies and therefore lower cut-off frequencies. This means that a laser will penetrate further into an expanding plasma plume. The effect is sometimes termed relativistically induced transparency.
Relativistically induced guiding

The increase of the refractive index due to the relativistic causes relativistic self-focusing. The radial profile of a laser beam has a maximum on axis and therefore the refractive index will vary radially and act as a waveguide for the laser beam. When the focussing exactly matches diffraction, the laser beam will be guided.

![Image: intensity and phase fronts]

The initially planar wavefront deforms in the plasma because the phase velocity $v_{ph} = c/\eta$ is smaller on axis, causing the plasma to act as a positive lens and focus the beam. Guiding occurs when self-focussing is matched by diffraction. This occurs when $P > P_c$.

Relativistically induced transparency

An intense laser pulse with a frequency $\omega$ propagates in plasma with a phase and group velocity that is determined by the dispersion relation $\omega^2 = \omega_p^2 + k^2c^2$, which depends on $\gamma$ through $\omega_p$.

![Image: dispersion relationship]

In dense plasma, with $\omega < \omega_p$, light cannot propagate and is reflected from the surface. However, at relativistic intensities with a large $\gamma$, the plasma can become transparent.
Self-steepening

Because the group velocity, \( v_{gr} = \frac{c}{\eta} \), increases where the intensity is highest, the peak outruns the lower intensity parts of the laser pulse leading to self-steepening.

This leads to an optical shock, which is closely related to self-phase modulation of long laser pulses.

Because the front steepens, the ponderomotive force at the head of the pulse increases. This means that even though a laser may initially start with a low intensity, it can compress to become a very intense pulse, nearly an order of magnitude more intense.

Wakefield accelerator

UCLA: Tajima + Dawson 1979

The ponderomotive force is given by the gradient of the light pressure

\[
F_{\text{pond}} = -\frac{e^2}{4m\omega^2} \frac{dE^2}{dz} = -mc^2 \frac{d}{dz} \left( |a|^2 \right)
\]

The electrons are pushed out of high intensity regions by the ponderomotive force

Group Lorentz factor

\[
\gamma_g = \sqrt{1 / \left( 1 - \frac{v_g^2}{c^2} \right)} = \frac{\omega_p}{\omega_p^*}
\]

Critical density for 800 nm:

\[
n_e = 1.75 \times 10^{21} \text{ cm}^{-3}
\]

\[
\omega_p = \sqrt{n_e e^2 / \epsilon_0 m_e}
\]
Particles accelerated by electrostatic fields of plasma waves

\[ E[V/cm] \approx e \sqrt{n} \]

\[ \gamma_{\text{max}} \approx \frac{2\gamma^2a}{3} \]

Accelerators:
Surf a 10’s cm long microwave – conventional technology
Surf a 10’s μm long plasma wave – laser-plasma technology

Laser & Electron Wakes

Nonlinear wakes are similar with laser or particle beam drivers:

3-D PIC OSIRIS Simulation (self-ionized gas)

Laser & Electron Wakes
Laser Wake
Electron beam Wake

[Simulations courtesy of Silva & Mori – Lisbon/IST & UCLA]
Wakefield acceleration: accelerating and focusing fields

- Laser pulse
- Envelope
- Electrostatic wakefield
- Bunch density
- Energy density of wakefield

Laser pulse envelope dynamics: ponderomotive wakefield excitation - electron bunch acceleration - phase slippage - beam loading
Laser pulse envelope dynamics and energy conservation

laser pulse amplitude: \( a_0 \)
laser pulse energy depletion rate: \( \omega_d \sim a_0^2 \omega_s \)

- **Linear regime:** \( a_0^2 \ll 1, \omega_d \ll \omega_s \); pulse energy loss through photon deceleration without envelope modulation, static wakefield, low energy efficiency
- **Nonlinear regime:** \( a_0^2 \sim 1, \omega_d \sim \omega_s \); pulse energy loss through photon deceleration and strong envelope modulation, dynamic wakefield, better energy efficiency

\[
\Delta W = eE_z L_{\text{acc}}
\]

- **Diffraction:**
  \[
z_R = \frac{\pi^2 w_0^2}{\lambda}
  \]
  ~ millimeters!
  (but can be avoided using plasma channels or relativistic self-focusing)
  
- **Dephasing:**

- **Pump Depletion:**

  For small \( a_0 \)
  For \( a_0 \geq 1 \)

  \[
  L_{\text{dp}} \gg L_{\text{dph}}
  L_{\text{dp}} \sim L_{\text{dph}}
  \]

  \[
  E \text{ [GeV]} \sim 0.038 \left( \frac{P \text{ [TW]}}{P_{cr}} \right)^{-2/3}
  P_{cr} \sim 17 \gamma_g^2 \text{ [GW]}
  \]

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Capillary: preformed plasma waveguide

\[ n(r) = n_0 + \delta n \frac{r^2}{r_0^2} \]

\[ r_0 = 150 \mu m \text{ capillary} \]

\[ n(0) = 10^{18} \text{ cm}^{-3} \]

\[ I > 10^{17} \text{ W/cm}^2 \]

Plasma formed by electrical discharge between electrodes

Electrodes

Plasma capillary waveguide

Phase-matching: tapered capillary

Plasma waveguide formation

\[ \frac{1}{r} \frac{d}{dr} \left( r \kappa_{\perp} \frac{dT}{dr} \right) + \sigma_T E^2 = 0 \]

Solution of the heat flow equation yields scaling relation for matched spot size:

\[ \omega_0 \left[ \mu m \right] = 1.5 \times 10^5 \frac{\sqrt{a \left[ \mu m \right]}}{\left( \pi \left[ \text{cm}^{-1} \right] \right)^{1/4}} \]

Plasma-filled capillary waveguide electron density profiles

- Electron density measured with Mach-Zender interferometer
- H₂ pressure is 63 mbar.
- Central section **parabolic** electron density profile: matched spot size 37 μm.
- >90 % transmission over 4 cm channel
- > 10⁵ shots

D. J. Spence & S. M. Hooker
Phys. Rev. E [63](2000) 015401

Manufacture of plasma capillaries for laser wakefield accelerators

- Developed at Strathclyde
- 300 μm
- After one year…. (Jaroszynski et al., Royal Society Transactions, 2006)

This method of manufacture is now used by all groups using plasma capillaries
Guiding in plasma capillaries

\[ \text{Strathclyde} \]

\[ \text{~ 60 } \mu\text{m dia.} \]

\[ \text{~ 60 } \mu\text{m dia.} \]

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**Equations of motion in harmonic potential**

For *relativistic* particles, the trajectory is governed by the equations

\[ \frac{dr}{dt} = v, \quad \frac{dP}{dt} = F, \quad P = \gamma mv \quad \text{where} \]

\[ \gamma = \frac{1}{\sqrt{1 - v^2 / c^2}} = \sqrt{1 + P^2 / (mc)^2} \quad \text{is the Lorentz factor.} \]

If force can be written as a gradient of a potential \( F = -\nabla_r V(r) \nb \text{then the equations are Hamiltonian} \]

\[ \frac{dr}{dt} = \nabla_r H, \quad \frac{dP}{dt} = -\nabla_r H. \]

with conserved energy \( H = \gamma mc^2 + V \) as Hamiltonian

(Theory lectures based Strathclyde undergraduate lectures)
Hamiltonian: pendulum equation

\[ \frac{d^2 \theta}{dt^2} + \frac{g}{l} \sin \theta = 0 \]

\[ \Rightarrow H = \frac{1}{2} \left( \frac{d\theta}{dt} \right)^2 - \frac{g}{l} \cos \theta \]

Examples of 1-D wakefields: Linear and non-linear

wakefield profiles in the frame moving with the laser pulse: linear and nonlinear regimes

red: intensity \( I \)
green: electric field \( E_z \)
blue: density perturbation \( \delta n \)
Efficiency

- Effect of bunch wakefield = beam loading
- Important for wake-to-bunch energy transfer
- Finite charge required for energy absorption from the wakefield

- ideal (almost 100%) conversion of wake energy into bunch energy
- all electrons accelerated
- → bunch slips out of ideal position
- → large spread of accelerating field induces large energy spread

• slight loss of energy from bunch to wake
• most electrons decelerated
• complicated structure of accelerating field along electron bunch

Wakefield efficiency

- At low amplitude, the LWFA scheme suffers not only from a lower energy gain, but also from low efficiency.
- Energy transfer from plasma wave to electron limited by dephasing – electron runs into decelerating region due to $v_a < c$ - this determines the acceleration length.
- Energy transfer from laser pulse to plasma wave is governed by feedback (instability), which develops faster at high $a_0$.

Example: 1-D simulation results for Gaussian laser pulse with resonant pulse length, plasma density $10^{18}$ cm$^{-3}$, and 4 different pulse amplitudes.
Examples 2-D plasma wakefields

**linear regime, uniform plasma**

\[ k \cdot x \]

**nonlinear regime, uniform plasma**

\[ E_z \]

**linear regime, plasma channel**

\[ \delta n \]

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**Travelling electromagnetic waves**

Maxwell’s equations reformulated with potentials \((A, \phi)\)

\[
\begin{align*}
\partial_t E - c \nabla \times B &= 0, \\
\nabla \cdot B &= 0, \\
\partial_t B + c \nabla \times E &= 0, \\
\nabla \cdot E &= 0
\end{align*}
\]

\[
E = -\left[ \nabla \phi / c + \nabla \times A \right], \\
B = \nabla \times A \\
\Rightarrow \left[ \nabla^2 - c^2 \nabla^2 \right] (A, \phi) = 0.
\]

in Lorentz gauge:

\[
\partial_t \phi + c \nabla \cdot A = 0
\]

Plane-wave solutions: transverse wave

\[
A_x = A_0 \cos \left( k z - \omega t \right), \quad \phi = \phi_0 \cos \left( k z - \omega t \right)
\]

\[
\omega = ck, \quad A_y = \phi_0 \Rightarrow E_x = -\left( \partial_t A_y / c + \partial_y \phi \right) = 0.
\]

**No acceleration from transverse E-field**

- Need RF waveguide – TM modes have longitudinal E-field component
- Need wave in a medium – e.g. structured solid (crystal) or plasma

Tesla superconducting cavities
Acceleration in travelling wave

\[ \frac{dP}{dt} = F_0 \sin(kz - \omega t) = F_0 \sin(k \int v(t') dt' - v_\phi t) \]

Acceleration: \( \frac{dP}{dc} = 0 \)
Deceleration: \( \frac{dP}{dt} = 0 \)

Constant force - resonance condition: \( \int v(t') dt' = v_\phi t + C \) at all values of \( t \),
only possible if \( v_\phi \) time-dependent, else \( \frac{dP}{dt} = 0 \) (no acceleration).

Motion of particles with speed close to \( c \) in a wave travelling at speed \( v_\phi < c \).

Dephasing limits acceleration time to advance over \( \frac{1}{2} \) wavelength:

\[ v_z \approx c \]
\[ v_\phi < c \]

\[ \begin{align*}
  r = 0 & \quad \text{ Motion of particles with speed close to } c \\
  r = t_1 & \quad \text{ Dephasing limits acceleration time to advance over } \frac{1}{2} \text{ wavelength}
\end{align*} \]

Acceleration in travelling wave continued...

Calculation of maximum \( \Delta P \):

- position \( z = z_1 + ct \)
- force at \( t = 0 \) is \( F_0 \sin(kz_1) = 0 \) if \( \sin(kz_1) = 0 \)
- advance \( \frac{1}{2} \) wavelength - force returns to 0 at \( t = t_1 = \pi/(kc - kv_\phi) \)
- equal to \( 2\pi \gamma_\phi^2 kc \) if \( v_\phi \approx c \)

\[ P = P_0 + \left( \lambda \gamma_\phi^2 F_0 / c \right) \int_0^c \sin(\alpha) d\alpha = P_0 + 2\lambda \gamma_\phi^2 F_0 / c \]

Phase space diagram illustrates motion of particles with \( v_z \approx c \)

\[ \gamma / \gamma_\phi \]

\[ (z - v_\phi t) / \lambda \]
Electrons in harmonic potential: ponderomotive or Coulomb potential

Assume that the electrons are travelling very close to the phase velocity of the field, $v = \omega/k$ and that electrons are close to the trough of the potential then $\sin k\zeta = k\zeta$

and $\zeta = -\frac{e}{m}E_k\zeta$ which describes simple harmonic motion with a bounce frequency $\omega_B = \frac{eE_k}{\sqrt{m}}$

Wave-breaking occurs when the bounce frequency approaches the plasma frequency, i.e. when $\omega_B \approx \omega_p$ Gauss Law: $E_0 = \frac{en\delta z}{e_0} = \frac{en\lambda_g}{e_0}$

i.e. when $E_{mb} = E_0\sqrt{\gamma(Y_g - 1)}$ where $E_0 = (m_e\omega_p/e) = 0.69V_{\text{cm}}^{-1}$ V/cm

$\gamma_g$ is the relativistic factor corresponding to the plasma phase velocity, $v_g$.

$E_{mb}$ can be obtained from Gauss’s law, $eE_0 \sim mc\omega_p$.

When the wave breaks electrons suddenly increase their energy spread and cause the electromagnetic wave to lose energy. Plasma wave grows and becomes non-sinusoidal and steepens resulting in a singularity in the electron density.

Electron acceleration: catching the wave

Electrons are accelerated in wakefield if their initial velocity is sufficiently close to the phase velocity of the wakefield for trapping occurs i.e.

The electron motion: velocity and acceleration:

$$\frac{ds}{dt} = v_z - v_g,$$

$$\frac{dP_z}{dt} = -mc^2\frac{dy}{ds}.$$
Electron motion

\[ \gamma mc^2 \] corresponds to an electron moving at the phase velocity of the wakefield, given by \( \psi = e^{g/s} \).

Acceleration in potential: dephasing for weakly nonlinear case

Energy gain limited by dephasing, caused by difference between velocities of electron and wakefield

\[ v_{\text{el}} = c > v_{\text{w}} = v_s \]

Scaling \( \Delta \gamma \approx E \times L_{\text{dip}} \approx n_p^{1/2} n_p^{3/2} \approx n_p^{-1} \) favours low plasma density

Note: logarithmic energy scale
Efficiency

- ideal (almost 100%) conversion of wake energy into bunch energy
- all electrons accelerated
- wakefield to zero at rear part of bunch
  - bunch slips out of ideal position
  - large spread of accelerating field
    - induces large energy spread
  - slight loss of energy from bunch to wake
  - most electrons decelerated
  - complicated structure of accelerating field along electron bunch

Beam loading & energy spread

- Electron bunch induces wakefield.
- Total wakefield is sum of laser wakefield and bunch wakefield.
- Amplitude difference is a measure of energy transfer.

Beam loading sets charge limit. For acceleration, amplitude of bunch wakefield must remain smaller than amplitude of wakefield.

Fully compressed bunch with finite charge induces jump in accelerating gradient.
Energy spread and stability: measurements at Strathclyde

Mean energy drops with increase in charge:

Beam loading

Energy spread increases with charge

Stability:

![Image](chart.png)
r.m.s. spread of mean energy 2.8%

Bubble scaling: highly relativistic case

- Bubble radius is approximately equal to the relativistic plasma wavelength: $R = \frac{\lambda_p}{\pi}$

- $\lambda_p$ becomes $\sqrt{\mu_0 \lambda_p}$ giving $R = \frac{\sqrt{\mu_0} \lambda_p}{\pi}$

- The dephasing length is determined by the bubble radius: electrons catch up with the bubble centre.

- The laser propagates at the group velocity but plasma piles up in front of it causing a barrier and an increase in plasma density.

- This causes the laser to etch back at a velocity: $V_{\text{etch}} = \frac{c}{\gamma_g}$

Use expansion $(1-x)^{1/2} \approx 1-x/2$
Dephasing and depletion lengths: highly relativistic case (Bubble regime)

Dephasing length can be calculated from
\[ L_d = \frac{c}{c - v_g} R \]

Therefore
\[ L_d = \frac{2\gamma^2}{3} R \]

We can also estimate the pump depletion length: i.e. over which the laser pulse is etched away.

This is given by
\[ L_{\text{etch}} = \frac{c}{v_{\text{etch}}} \tau_{\text{FWHM}} = \frac{c\tau_{\text{FWHM}}}{\gamma_g^2} \]

The laser spot size should be matched to the bubble diameter i.e. \( W_0 \approx R \)

Relativistic self-guiding occurs if
\[ P \geq P_c \quad \text{or} \quad \frac{a_0^2}{8} \geq \left( \frac{k_p W_0}{\gamma} \right)^2 \]

Energy gain can be easily calculated:
\[ \gamma_{\text{max}} = \frac{2}{3} \left( \frac{\omega_b}{\omega_p} \right)^2 a_0 \]

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ALPHA-X all-optical injection experiments on ASTRA

10^{18} Wcm^{-2} in 25 μm spot

\[ a_0 \approx 0.7 - 1 \]

800 nm

\[ n_e \approx 2 \times 10^{19} \text{ cm}^{-3} \]

360 – 540 mJ

40 fs

F/16 mirror

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Imperial/RAL/Strathclyde

S. Mangles et al. Nature 2004

\( \tau \approx 5 - 10 \text{ fs} \)

I \approx 5 \text{ kA}
Jena measurements

- Thomson scattering of the channel above the gas nozzle
- Channel location in the plasma profile
- Spectrum on x-ray film
- Sauerbrey, Schwoerer, MPQ and ALPHA-X team

LBNL - Oxford campaign (ALPHA-X) team: GeV beams from capillary

- Robust capillary channels manufactured using methods developed at Strathclyde (Jaroszynski et al., Royal Society Transactions, 2006)
- Pre-formed plasma channels – Spence & Hooker (PRE 2001)
1 GeV beams


**Energy spread and emittance**

- Non-ideal electron beams degrade radiation properties: reduces brilliance and lowers FEL gain.

- Emittance: area in transverse phase space \( \varepsilon_{x,y} = \sigma_{x,y} \sigma'_{x,y} \)

- Approx. beam size times RMS beam divergence.

- Normalised emittance: \( \varepsilon_n = \gamma \beta \varepsilon \) needs to be 1 \( \pi \) mm mrad.

- Energy spread: \( \frac{\sigma_{x,y}}{\gamma} \) needs to be 0.1% for a free-electron laser.

- The rms emittance gives the minimum beam size that can be achieved.
RMS emittance

The rms emittance gives the minimum beam size that can be achieved

\[ \varepsilon_{u,rms} = \sqrt{u_{rms}^2 \cdot u_{rms}'^2 - (uu_{rms}')^2} \]

The normalised emittance is defined as:

\[ \varepsilon_{u,n} = \beta \gamma \cdot \varepsilon_{u'} \]

which is conserved for accelerating beams.

Transverse Brightness

- Quality of electron beam given by normalised brightness 
  \[ B_\varepsilon = \frac{2J}{\varepsilon_{nx}\varepsilon_{ny}} \]
- \( J \) = peak current, \( \varepsilon_{nx,ny} \) = normalised emittance
- Pencil shaped beam \( \varepsilon_{nx} = \varepsilon_{ny}, J = 1/\sigma_r^2 \)
- Beam waist \( \sigma_r = \frac{\varepsilon_{nx,ny}}{\sqrt{\gamma}} \)
- Beta function \( \beta \) (analogue of Rayleigh range)

\[ B_\varepsilon = \frac{2J}{(\sigma_r \gamma)^2} = \frac{2J \sigma_r^2}{\varepsilon_r^2} \]
Experimental Results - beam pointing

- 500 consecutive shots
- narrow divergence (~2 mrad) beam
- wide divergence halo
- $\theta_X = (7 \pm 3)$ mrad, $\theta_Y = (3 \pm 2)$ mrad
- 8 mrad acceptance angle for EMQs
- 25% pointing reduction with PMQs installed

Emittance of Strathclyde beam

- divergence 6 mrad
- hole size correction
- limited by detection system
- $\epsilon_{N,x} < (7.8\pm1)$ $\pi$ mm mrad

A measure of the upper limit of emittance

- divergence 4 mrad
- $\epsilon_{N,x} < (5.5\pm1)$ $\pi$ mm mrad
**Experimental Results - emittance**

- Second generation mask with hole $\phi \sim 25 \mu m$ and improved detection system
- divergence 1 - 2 mrad for this run with 125 MeV electrons
- average $\varepsilon_N = (2.2 \pm 0.7)\pi$ mm mrad
- best $\varepsilon_N = (1.0 \pm 0.1)\pi$ mm mrad
- Elliptical beam: $\varepsilon_{N,X} > \varepsilon_{N,Y}$

- E. Brunetti, et al., PRL 2010

---

**Strathclyde: Measured energy spread**

- Shot to shot variation in mean energy: $\frac{\Delta\gamma}{\gamma} \sim 6\%$
- $\sigma_{\gamma} = 0.7\%$
- beam loading simulations $\rightarrow$
- Further accelerate to 1 GeV would give $\frac{\Delta\gamma}{\gamma} < 0.05\%$

---

[Graphs and data showing experimental results and energy spread data.]
Experimental Results - energy spectra III

\[ \sigma_{\gamma}/\gamma \text{ MEAS} = 0.7\% \]

\[ \sigma_{\gamma}/\gamma \text{ MEAS} = 0.4\% \]

- Indicates fixed absolute energy spread ~ 600 KeV

Experimental Results - charge

Imaging Plate

LANEX 2

Cross-calibration in progress

Cross-calibration in progress
**Extending to higher energy:**

**Strathclyde plasma media**

- Extend energy range to multi GeV
- Study plasma media – extend length relativistic self focussing, gas cells and channels
- Stable electron beam generation

**Typical high energy spectra: RAL-Gemini experiment using plasma channel 85% of shots**
Radiation sources: Synchrotron and Free-electron laser (FEL):
a potential compact light source

- Use output of wakefield accelerator to drive compact synchrotron light source or FEL
- Take advantage of electron beam properties
- Coherent spontaneous emission: prebunched FEL \( I \sim I_0(N+N(N-1)f(k)) \)
- Ultra-short duration electron bunches: \( I > 10 \) kA
- Operate in superradiant regime: FEL X-ray amplifier (self-similar evolution)

**Potential compact future synchrotron source and x-ray FEL**

Need a low emittance GeV beam with < 10 fs electron beam with \( I > 10 \) kA
Operate in superradiant regime SASE alone is not adequate: noise amplifier
- Need to consider injection (from HHG source) or pre-bunching

---

**Synchrotrons**

- 1-8 GeV – X-rays to THz interact with electronic structure
  - THz – low frequency modes
  - VUV – molecular structure
  - X-ray spectroscopy
  - Extended X-ray Absorption Fine Structure – EXAFS
  - X-ray diffraction
  - Imaging and microscopy

- Bending magnets, undulators and wigglers

---

Varenna Enrico Fermi Summer School 2011
**Undulator**

![Undulator Diagram]

- Magnetic field
- Permanent magnet
- Light pulse
- Electron trajectory

**Synchrotron Radiation Properties I**

- Bending magnet:
  \[ \hbar \omega_c = \frac{3eB\gamma^2}{2m} \]

![Graph of Synchrotron Radiation Properties I]

- Angle = 0.01 mrad, 3.9 GeV, 400 mA, 1.27 γ

![Lorentz Transformation Diagram]
### Synchrotron Properties II

**Wiggler and undulator**

\[ h \omega = \hbar k_c 2 \gamma_i^2 \]

where \( k_c = \frac{2 \pi}{\lambda_0} \)

\[ \gamma_i = \frac{\gamma_0}{1 + \frac{a_u^2}{2} + \gamma_0^2 \theta^2} \]

where \( a_u = \frac{eB}{k_m c} \)

and \( \theta \) is observation angle or beam angle

\[ n_i = \frac{3a_u}{4} \left( 1 + \frac{a_u^2}{2} \right) \] critical harmonic

\[ P_i = \frac{e a_u^2 \gamma^2 N_i k_i}{6 \epsilon_0} \]

\[ \theta = \frac{a_u}{\gamma} \] total angle

\[ \theta = \frac{1}{\gamma \sqrt{N_i}} \]

where \( N_i \) is the current

and \( N_u \) is the number of periods

\[ \lambda = \frac{\lambda_u}{2 \pi^2} (1 + a_u^2/2 + \gamma^2 \theta^2); \]

and \( \frac{\Delta \lambda}{\lambda} \approx \frac{1}{2 N_u} \)

\( \lambda_u \) – undulator wavelength

**Compact synchrotron source**

- Undulator radiation emitted into cone \( \theta \approx \frac{1}{\gamma} \)

- Wavelength: \( \lambda = \frac{\lambda_u}{2 \pi^2} (1 + a_u^2/2 + \gamma^2 \theta^2); \) and \( \frac{\Delta \lambda}{\lambda} \approx \frac{1}{2 N_u} \)

\[ a_u = \frac{eB \lambda_u}{2 \pi mc} \]

\[ \frac{1}{\sqrt{N_u \gamma}} \]
First compact wakefield driven synchrotron source

Strathclyde & Jena

Wakefield accelerator undulator


Strathclyde collaboration with IOQ Jena

Supported by EPSRC, E.U. Laserlab and EuroLEAP

dino@phys.strath.ac.uk  Varenna Enrico Fermi Summer School 2011

Electron and radiation spectra

Strathclyde, Jena & Stellenbosch

\[ N_{\text{ph}} \approx 300,000 \text{ photons} \]
\[ Q = 28 \text{ pC} @ 65 \text{ MeV} \]
\[ \varphi = 2 \text{ mrad} \]
\[ \delta \lambda / \lambda = 55 \text{ nm (FWHM)} \]
\[ B = 6.5 \times 10^{16} \text{ ph/sec/mrad}^2/\text{mm}^2/0.1\% \text{BW} \]
\[ \delta \gamma / \gamma < 0.5 \left[ \left( \delta \lambda / \lambda \right)_{\text{measured}}^2 - \left( \delta \varphi^2 \right)^2 - 1/N_{\text{ph}}^2 \right]^{1/2} \]

Sets an upper limit to total energy spread (≈1%) and emittance (1 π mm mrad)


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Scaling with energy

Strathclyde, Jena & Stellenbosch

\[ \lambda = \frac{\lambda_u}{2\hbar\gamma} \left( 1 + \frac{\alpha_u^2}{2} + \gamma^2 \varphi^2 \right) \]

Extrapolate to 1 GeV:
400 eV photons
B = 2 \times 10^{23} \text{ ph/sec/mrad}^2/mm^2/0.1\%BW

World first undulator radiation from LWFA demonstration

- Strathclyde, Jena, Stellenbosch collaboration
- 55 - 70 MeV electrons
- VIS/IR synchrotron radiation

- Measured \( \sigma_\gamma/\gamma \sim 2.2 - 6.2\% \)
- Analysis of undulator spectrum and modelling of spectrometer
  \( \sigma_\gamma/\gamma \) closer to 1%

Electron and optical spectra: undulator as an electron spectrometer

Strathclyde, Jena & Stellenbosch

Scaling with energy

Strathclyde, Jena & Stellenbosch

\[ \lambda = \frac{\lambda_u}{2h\gamma^2} \left( 1 + \frac{\alpha_u^2}{2} + \gamma^2 \sigma^2 \right) \]

- \( h \) – harmonic number
- Extrapolate to 1 GeV:
  - 400 eV photons
  - \( B = 2 \times 10^{23} \) ph/sec/mrad²/mm²/0.1%BW
Free-electron laser driven by wakefield accelerator

- Combination of undulator and radiation fields produces ponderomotive force which bunches electron on a wavelength scale
  \[ \lambda = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{a_u^2}{2} \right) \]
- Laser field grows exponentially at a rate governed by the electron beam gain coefficient: \( \rho = 1.1 \gamma^{-1} B_u \lambda_u^{-2/3} I_{pk}^{1/3} \epsilon_n^{-2/3} \)
- Gain length \( L_g = \frac{\lambda_u}{2\pi\sqrt{3}\rho} \)
- Need slice energy spread \( \delta\gamma/\gamma < \rho \) and slice emittance \( \epsilon_n < 4\lambda\beta\gamma\rho/\lambda_u \) or \( \epsilon_n < \gamma/\lambda \) (matched)
- Slice length: cooperation length: 30 – 100 periods i.e. <1 fs for x-rays – several pC

See Bonifacio et al. (Nuova Cimento 1990, 1992)

---

Synchrotron and FEL radiation peak brilliance

\[ I(k) \sim I_0(k)(N+N(N-1)f(k)) \]
- \( \lambda_u = 1.5 \text{ cm} \)
- \( \epsilon_n = 1 \pi \text{ mm mrad} \)
- \( \tau_e = 10 \text{ fs} \)
- \( Q = 100 – 200 \text{ pC} \)
- \( I = 25 \text{ kA} \)
- \( \delta\gamma/\gamma < 1\% \)

FEL: Brilliance 5 – 7 orders of magnitude larger
Probes of matter

Synchrotron application of X-rays

- X-ray diffraction: surface, bulk, large molecule, and interface
  - Structure of macromolecules: enzymes, proteins etc.
  - Solid state: Crystal, semiconductor and non-crystalline structure – new materials, superconductors etc
- X-ray spectroscopy
  - Fluorescence, dichroism
  - photo-emission
- X-ray microscopy
  - Microfocus, spectromicroscopy
- X-ray tomography
Biology

- Protein and enzyme structure
- Basic biological structure
- Drug development
- Physiology

Betatron motion

- Transverse oscillations in focusing regions are called betatron oscillations
- Betatron frequency and amplitude depends on longitudinal phase and energy

\[ \omega_b \]

\[ \lambda_p \]

\[ v_g \]

\[ z \]
Betatron radiation

Increase in wiggler parameter

$$n_{\text{crit}} \approx \frac{3a_\beta^3}{8}$$

$\alpha = 0.6$

$\alpha = 1$

$\alpha = 2$

$\alpha = 3$
Betatron radiation as a gamma ray source

• Transverse oscillation in ion channel:
  • wiggler-like emission

Ion channel radiation: Whittum et al., PRL 1990

---

Plasma wiggler

• Wiggler motion – electron deflection angle $\theta \sim (p_x/p_z)$ is much larger than the angular spread of the radiation ($1/\gamma$)

$\Omega = 1/\gamma \quad \gamma > a_\beta > 1$

Deflection angle – $a_\beta / \gamma$

• Only when $k$ & $p$ point in the same direction do we get a radiation contribution.
• Spectrum very rich in harmonics – peaking at $n_{\text{peak}} = \frac{3a_\beta^3}{8}$
X-ray generation in a plasma wake

- Strong radial forces cause synchrotron or betatron oscillations of electron beam
- Ion channel radius: \( r_i = r_e\sqrt{\frac{n_e}{n_p}} \)  electrostatic field \( E = \frac{n_e r_e}{2\varepsilon_0} \)
- Dense pencil electron beam – radius \( r_e \ll r_i \) and \( r_e \ll 1/k_p \)
- Restoring force given by Gauss law: \( F = -\frac{1}{2} m_o \omega_p^2 \mathbf{r}_e \)
- Oscillation at the betatron frequency (in bubble): \( \omega_B = \frac{\omega_p}{\sqrt{2\gamma}} \)
- Emitted synchrotron radiation viewed in the lab frame
  \[ \lambda_h = \frac{\lambda_p}{h} \left( 1 + \frac{a_p^2}{2} + (\gamma, \phi)^2 \right) = \frac{\sqrt{5\pi c}}{h\omega_p \gamma^2} \left( 1 + \frac{a_p^2}{2} + (\gamma, \phi)^2 \right) \]
- \( h \) – harmonic number
- \( h_{\text{crit}} \) - maximum intensity at harmonic \( h_{\text{crit}} = \frac{3a_p}{8} \)
- Undulator/wiggler deflection parameter \( a_p = \gamma \beta \approx 2\pi \sqrt{\gamma / 3} \frac{r_e}{\lambda_p} \)

Direct laser acceleration

- Transverse oscillations in the ponderomotive potential occurs at the betatron frequency \( \omega_B = \frac{\omega_p}{\sqrt{2\gamma}} \)
- When the laser field observed in the moving frame of reference is resonant with the betatron frequency \( \omega_B = \frac{\omega_p}{2\gamma^2} \) where \( \gamma = \frac{\gamma_0}{\sqrt{1 + a_p^2 / 2}} \)
- Then the electron can experience a constant force and gain a huge amount of transverse momentum.
- The Lorentz force \( \mathbf{v} \times \mathbf{B} \) with the laser magnetic field can lead to a transfer of transverse to longitudinal momentum without adding further energy

Extending to higher energies:

\[ a_\beta = \gamma_c k_\beta r_c = \frac{\sqrt{2}\gamma_c \pi r_c}{\lambda_p} \]

\[ n_{crit} \approx \frac{3a_\beta^3}{8} \]

Dependence on initial electron distribution

\[ \beta \approx \text{crit} \]

\[ \text{calculation} \]

\[ \text{X-ray emission [arb. unit]} \]

\[ \text{X-ray energy [KeV]} \]

\[ \tau_c = 3 \mu m \]

\[ \tau_c = 8 \mu m \]

\[ \tau_c = 14 \mu m \]

\[ \lambda = 2 \gamma \pi \lambda_c \]

\[ \lambda_c = \text{electron beam width} \]

\[ \tau_c = \text{electron beam radius} \]

\[ \beta \approx \text{crit} \]

\[ \lambda_c = \text{electron beam wavelength} \]

\[ \tau_c = \text{electron beam duration} \]

\[ \lambda_c = \text{electron beam coherence length} \]

\[ \beta \approx \text{crit} \]

\[ \lambda_c = \text{electron beam spatial separation} \]

\[ \tau_c = \text{electron beam temporal separation} \]

\[ \lambda_c = \text{electron beam spectral separation} \]

\[ \beta \approx \text{crit} \]

\[ \lambda_c = \text{electron beam interferometric separation} \]

\[ \tau_c = \text{electron beam diffraction separation} \]

\[ \lambda_c = \text{electron beam spatial coherence} \]

\[ \beta \approx \text{crit} \]

\[ \lambda_c = \text{electron beam temporal coherence} \]

\[ \tau_c = \text{electron beam spectral coherence} \]

\[ \beta \approx \text{crit} \]

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\[ \tau_c = \text{electron beam diffraction spatial diffraction coherence} \]

\[ \beta \approx \text{crit} \]

\[ \lambda_c = \text{electron beam temporal spatial diffraction coherence} \]

\[ \tau_c = \text{electron beam spectral spatial diffraction coherence} \]
Positron source (UCLA –SLAC) using betatron radiation

Johnson, Clayton et al. 2005

\[ \lambda = \left( \frac{2}{\beta^2} \right)^{1/2} \lambda_p \]

Betatron period Number of photons

Strength parameter \( K = a_u \)

\[ N_x = 5.6 \times 10^7 N_0 K \]

X-ray sources Synchrotron radiation in a mm-scale plasma

First beam of x-rays using lasers

New pioneering tool for ultrafast x-ray science

<table>
<thead>
<tr>
<th>Broad spectrum</th>
<th>x-ray absorption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collimation</td>
<td>improved focusing efficiency on sample</td>
</tr>
<tr>
<td>Tunable</td>
<td>x-ray diffraction</td>
</tr>
</tbody>
</table>

Betatron X-rays

- No electrons - Plasma wave weak
- Electron spectrum
- Signal on X-ray CCD

<table>
<thead>
<tr>
<th>Broken wave regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energetic electrons</td>
</tr>
</tbody>
</table>

- Kim TA PHUOC & Antoine ROUSSE

Betatron Radiation Properties

How to produce coherent radiation?  
the free-electron laser

- Coherent, very high brightness beam
- Tuneable: THz-IR-UV-X-rays
- Short pulse
- Synchronised with other sources (lasers, particle sources) – can be aggregated to create toolbox
- LCLS, XFEL, SCSS, PSI-XFEL etc.

Undulator for SCSS: Spring-8 Compact BASE Source

Energy recover linac FEL

Undulator

magnetic field

permanent magnet

light pulse

e- trajectory

\( \lambda_u \)

\( \lambda_g \)
FEL

• Tuning range the same as undulator or wiggler radiation

Coherence due to bunching enhances brilliance by many orders of magnitude

Free-electron laser

• Combination of undulator and radiation fields produces ponderomotive force which bunches electron on a wavelength scale

\[ \lambda = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{a_u^2}{2} \right) \]

• Laser field grows exponentially at a rate governed by

• Matched electron beam gain coefficient: \( \rho = 1.1 \gamma^{-1} B_u \lambda_u^{4/3} I_{pk}^{1/3} \epsilon_n^{-1/3} \)

• Gain length

\[ L_e = \frac{\lambda_u}{2\pi\sqrt{3}\rho} \]

• Need slice energy spread \( \delta\gamma/\gamma < \rho \) and slice emittance \( \epsilon_n < 4\lambda\beta\gamma\rho/\lambda_u \) or \( \epsilon_n < \gamma\lambda \) (matched)

• Slice length: cooperation length: 30 – 100 periods i.e. \( \ll 1 \) fs for x-rays – several pC

See Bonifacio et al. (Nuova Cimento 1990, 1992)
Free-electron laser

$\lambda \sim 1.5$ nm x-ray FEL requires $\sim 1$ GeV

- Growth of an injected or spontaneous field in a FEL amplifier is given by $I = I_0 \exp(gz)$,
- $g = 4\pi \rho 3^{1/2}/\lambda_u$ – small signal gain,
- $\rho$ - FEL gain parameter - a function of the beam energy, current and emittance.
- $\varepsilon_n = \gamma \sigma' \sigma$ is the normalised slice emittance of the beam.
- $\rho \sim 0.001$ for electron beam parameter but need slice energy spread $\delta\gamma/\gamma < \rho$ i.e. $<0.1\%$

LWFA-driven FEL

- High FEL gain criteria: $\varepsilon_n < \lambda\gamma/4\pi$ & $\sigma\gamma < \rho$
- Experimental $\varepsilon_n \leq 0.8\pi$ mm mrad & $\sigma\gamma / \gamma \leq 0.007$
- For fixed $\sigma\gamma = 0.6$ MeV, $\sigma\gamma / \gamma$ reduces at short $\lambda$

$$\rho = \frac{1}{2\gamma} \left[ \frac{I_p (\lambda_u a_u)}{2\pi \sigma} \right]^{2^{1/3}}$$

R. Bonifacio et al., 1984

<table>
<thead>
<tr>
<th>Electron energy (MeV)</th>
<th>Radiation $\lambda$ (nm)</th>
<th>Emittance criterion (x mm mrad)</th>
<th>Gain parameter $\rho$</th>
<th>Relative energy spread $\delta\gamma / \gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>261</td>
<td>3</td>
<td>0.011</td>
<td>0.007</td>
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<tr>
<td>150</td>
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<td>0.004</td>
</tr>
<tr>
<td>500</td>
<td>8</td>
<td>0.6</td>
<td>0.002</td>
<td>0.001(?</td>
</tr>
</tbody>
</table>

ALPHA-X Undulator

STEADY STATE SIMULATION RESULTS (100 MeV electrons)
- Saturation power (1st harmonic): 20 GW
- @ saturation distance: 1.8 m

UNDULATOR RADIATION EXPERIMENTS
- In progress for improving beam transport and observing gain
Synchrotron radiation and SASE FEL for same parameters

**Synchrotron:**
- Peak Brilliance $B = 3 \times 10^{25}$ ph/s/°mrad²/mm²/0.1% b.w. for 10 Hz
- Average brilliance $B = 2.5 \times 10^{11}$
- With laser improvements: 1 kHz: rep rate: average brilliance $B > 10^{13}$

$\sigma_p/\gamma = 0.1\%$  
$I_{pk} = 12$ kA  
$\epsilon_n = 1$ π mm mrad  
$N_x = 200$  
$\tau_e < 10$ fs

**SASE FEL**

Predicted SASE FEL Power growth

$E = 1$ GeV  
$Q = 100$ pC  
$I_{pk} = 30$ kA  
$\epsilon_n = 1$ π mm mrad  
$\delta\gamma/\gamma = 0.1\%$  
$\beta = 0.5$ m  
$\rho = 0.0065$  
$E_{ph} = 422$ eV  
$B_{pk} = 6 \times 10^{11}$ phot/sec/mm²/°mrad²/0.1% BW

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Varenna Enrico Fermi Summer School 2011
Tuning range

FELs give huge increase in brilliance

More applications
Fourier transform holography

- X-ray pinhole camera with a shutter speed of a few femtoseconds
- Holograms of nanoscale objects
- “Coded apertures” can image without focusing optics
- X-rays transmitted through array of holes interferes with radiation through (micron sized) sample
- Hadamard transform to deconvolve image
- Resolution determined by photon energy and scale of array - very high resolution possible

X-ray diffraction

- FLASH – DESY experiments
- Need Ångström wavelength X-rays
Medical applications

- Treatment of cancer
  - X-rays, electrons, protons, neutrons and ions
- Isotope generation for PET (Positron Emission Tomography) and SPECT (Single Photon Emission Computed Tomography)
- Biological research – crystallography of large molecules: proteins, viruses etc.
- X-ray imaging: macroscopic and microscopic

Radiotherapy I

- >1 m people develop cancer every year in Western Europe
- Radiotherapy treatment goes back to 1890's - Röntgen and Curie
  - Effective and physically selective - geometry
- 45% of cancers can be cured
  - 50% of these by radiation therapy alone, or combined with chemotherapy and/or surgery.
- X-rays from 5-20 MeV linacs used in most cases
  - 3-D Conformation therapy – multiple beams improve treatment
- Electron beam therapy (used for 10-25% patients)
  - Superficial tumours
  - Intra-Operative-Radiation-Therapy (IORT) used in surgery
- Proton and ion therapy – in development
Radiotherapy II

- Radiotherapy damages DNA: Double-break DNA strands
- Directly – photon, electron, protons, neutron or ion beams
- Most common: Indirectly ionising DNA – by ionising water to form free radicals (hydroxyl radicals), which damages DNA.
- Cancer cells are undifferentiated (unspecialised) and stem-cell-like – reproduce rapidly but have diminished ability to repair sub-lethal damage compared with healthy differentiated cells.
- DNA damage is inherited through cell division. Accumulated damage causes cell death or reduction in repro...

X-ray, Electron, Proton and Ion Therapy

Relative depth dose curves in water (a) electron beams (b) photon beams (from – Review of Oncology Radiation Physics, 2003) and (c) protons and (d) ions showing Bragg peak (from P. Bryant – CERN)
Diseased and normal tissue

• Modern techniques – critical normal tissues e.g. brain, eyes, spinal cord, kidneys, liver, etc. may be completely avoided or irradiated at low levels
• Unless tumour is close to sensitive tissue e.g. brain for brain tumours, spinal cord for cervical or mediastinal tumours

Protons and carbon ions

• Charged particles interact by inelastic collisions with electrons in the medium
• Bragg peak: Energy loss vs distance Bethe-Bloch formula

\[ \frac{dE}{dx} = \frac{4\pi e^4 Z_i^2}{m_i v^2} Z_j \left[ \ln \left( \frac{2mv^2}{\langle I \rangle} \right) - \ln(1 - \beta^2) - \beta^2 \right] + \text{(relativistic term)} \]

- $Z_i$ atomic number of ion
- $Z_j$ atomic number of target
- $\langle I \rangle$ mean ionisation potential
Ion radiation therapy

• HIT – Heidelberg Ion Radiation Therapy Centre (€119m) – opened in 2009
• Treat brain tumours
• 670 ton gantry – 3 MW power demand – football pitch footprint
• 5 minute treatment time + 20 minutes preparation
(1300 patients per year)

Comparison of treatment: how to localize dose

Glandula parotid cancer

Universitätsklinik für Strahlentherapie und Strahlenbiologie, AKH, Wien
Cure Rates

- Patients with localised tumour at the first consultation (65%)
- cured by surgery 22%
- cured by radiotherapy 12%
- cured by combination of surgery and radiotherapy 6%
- Patients with inoperable or metastatic disease at first consultation (35%) – 5% cure rate
- Total cure rate: 45%

Oncology
Gantries are huge

• Conical (IBA) and cylindrical (GSI) gantries for ion transport and focussing (from Bryant – CERN)
• 100’s tons

Nuclear Gamma Imaging

• PET and SPECT scans -radioactive isotopes to determine cellular/tissue change.
• Radionuclides are absorbed by healthy tissue at a different rate to diseased tissue.
• A deviation in absorption indicates abnormal metabolic activity.
• X-rays, CT Scans, and MRI can only image structure (e.g. anatomy), not function or metabolism.
PET scans

- Positron Emission Tomograph PET used by cardiologists, neurologists, and oncologists.
- PET uses positron emitters (F-18, C-11,...), this positrons annihilate with electrons to produce two gamma-rays emitted 180° apart.
- PET imaging maps biological function and detects subtle metabolic changes to determine active or dormant disease and whether a tumour is benign or malignant.
- PET - inject a radionuclide tracer specific to the function or metabolism to be investigated.
- Tracer concentrates in a specific area of the body.
- Detectors in a 360-degree ring detect gamma rays emitted from internal body tissues.
- Data processed to produce cross-sectional images.
- PET requires advanced computer processing, a cyclotron, and trained specialists.

SPECT

- Single Photon Emission Computed Tomography (SPECT) provides information on blood flow to tissue.
- A SPECT isotope (Tc-99m) only emits one gamma ray.
- Sensitive diagnostic tool for detecting stress fracture, spondylosis, infections and tumours.
- Analyzing blood flow to organ helps determine its functioning.
- Radionuclide injected intravenously.
- Tissues absorb radionuclide as it is circulated in the blood.
- Camera rotates around patient and detects gamma photons.
- The images are vertical and/or horizontal cross-sections of the body part and can be rendered into 3-D format.
Medical Radioisotopes