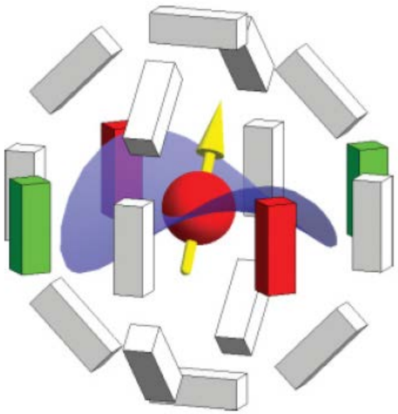
Holonomic quantum computing with heavy hole spin qubits

Varenna Summer School on
Quantum Spintronics and related phenomena

Varenna, 19 - 29 June 2012



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What is it?

- Holonomic QC is encoded in an n-fold degenerate Hilbert space of $H_{\square}$ with control parameters □.

- Cyclic change of these parameters around a loop C during time T such that $\square_{in} = \square_{out}$. □

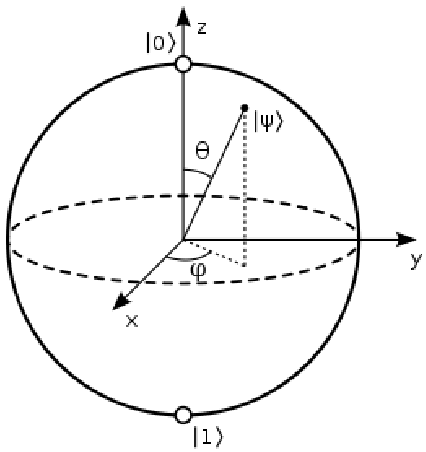
$$\left|\Psi\right\rangle_{out} = e^{i\varepsilon_0 T}\,\Gamma\left(C\right)\left|\Psi\right\rangle_{in}$$

dynamical phase

non-Abelian Berry phase

Motivation I

do any desired **single-qubit operation**

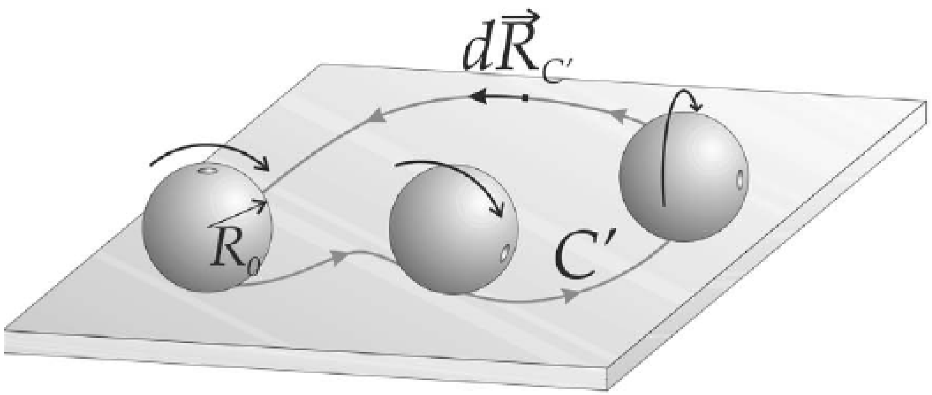


$$U\left(\hat{n},\varphi\right) = \exp\left(i\varphi\frac{\hat{n}\vec{\sigma}}{2}\right)$$

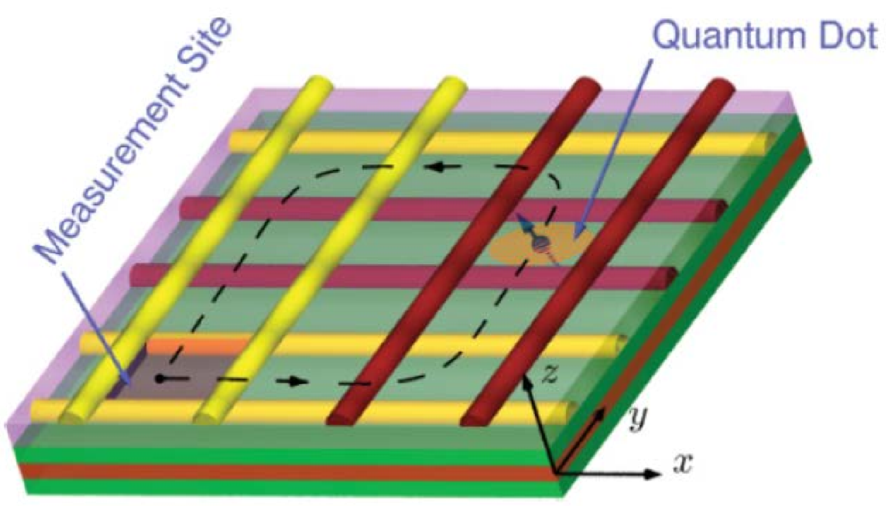
by a time-dependent **electric quadrupole field**
acting on a heavy hole spin qubit

Motivation II

do **holonomic quantum computation**
on the basis of **solid state spin qubits**

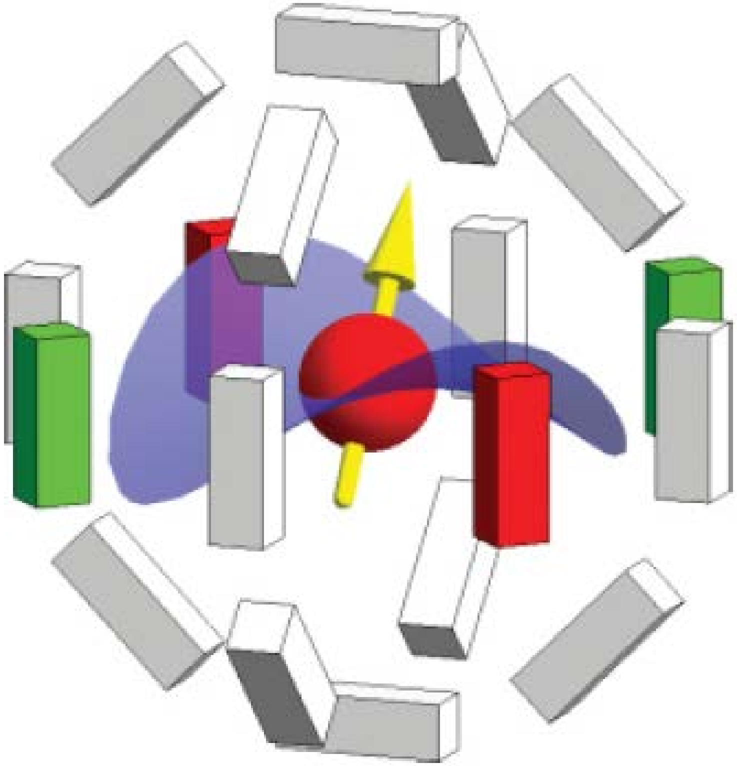


San-Jose, Scharfenberger, Schön, Shnirman, Zarand PRB 2008

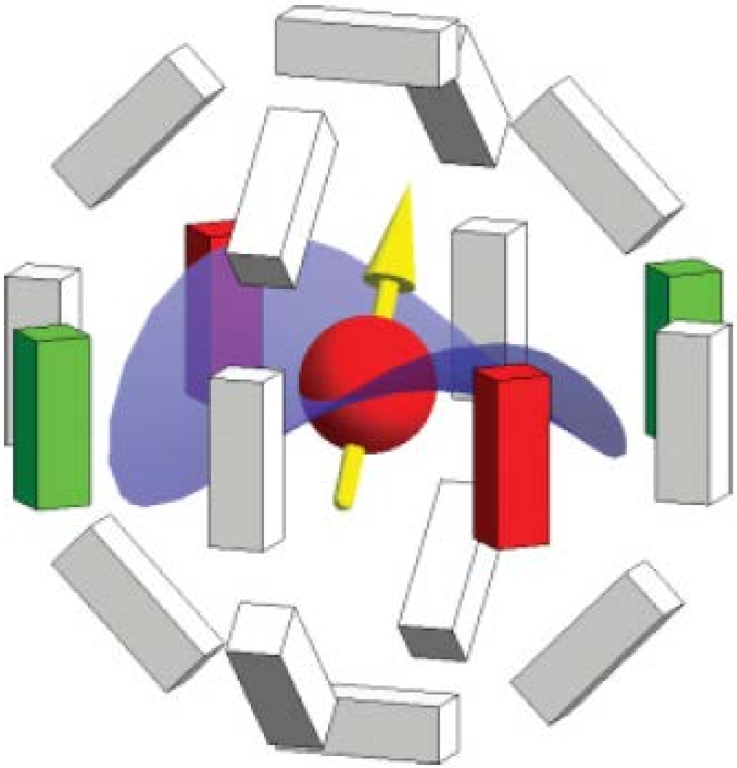


Golovach, Borhani, Loss PRA 2010

Simpler setup



Simpler setup



Not even all gates have to work. Four of them can be broken!

Outline

- Kato's connection □ non-Abelian geometric phase
- Qubit control via quadrupole fields
- Estimation of experimental parameters

Adiabatic time evolution

time-dependent Schrödinger equation

$$\frac{d\Psi_t}{dt} = -iH_t\Psi_t$$

- This equation has in general no stationary solution.
- **Adiabatic theorem:** If the change of H_t is made infinitely slow, the system, when started from a stationary state of H_0, passes through the corresponding stationary states of H_t for all times t.

Kato’s connection

$$\Psi_t = U_{t,0}\Psi_0 = \tilde{U}_{t,0} e^{-i\int_0^t H(\tau)d\tau} \Psi_0$$

unitary transformation
on the degenerate subspace
of EV with EW E_0

$$\tilde{U}_{t,0} = T\exp\left(-\int_0^t A(s)\,ds\right)$$

Kato‘s connection

$$A(s) = -\left[\frac{dP(s)}{ds}, P(s)\right]$$

time-dependent projector
on the degenerate subspace

Kato JPSJ 1950; *Wilczek & Zee* PRL 1984

Geometric interpretation

time-dependent SG: $$\left(d_t + iH_t\right)\Psi_t = 0$$

▯ adiabatic equation for Kato‘s connection: $$\left(d_t + A(t)\right)\tilde{\Psi} = 0$$

$$PA = 0 \text{ since } P\dot{P}P = 0$$ ▯ $$Pd = 0$$

no in-plane change
in the degenerate subspace of EV

Kato JPSJ 1950

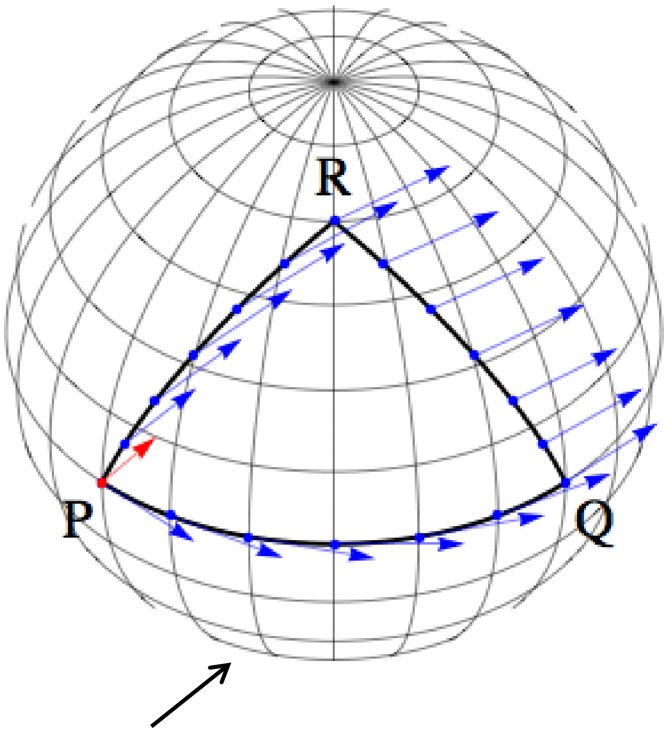
Relation to parallel transport

covariant derivative

$$\left(d_t + A\left(t\right)\right) \triangleq \nabla_{\dot{\gamma}}$$

Parallel transport:

$$\nabla_{\dot{\gamma}} X = 0$$



Parallel transport of a vector around a closed loop on the sphere

Kato vs. Berry

- Kato‘s connection allows us to derive the correct final state after an adiabatic time evolution on the basis of a given degenerate subspace of initial states.

- Kato‘s connection is basis-independent.

- Berry‘s phase is derived for a given set of (instantaneous) eigenstates and eigenvalues of H_t.

- The Abelian Berry‘s phase of a closed loop in parameter space is gauge-independent.

- The non-Abelian Berry‘s phase of a closed loop in parameter space depends on the choice of basis at the initial and final point.

Outline

- Kato's connection → non-Abelian geometric phase
- Qubit control via quadrupole fields
- Estimation of experimental parameters

Schedule of this part

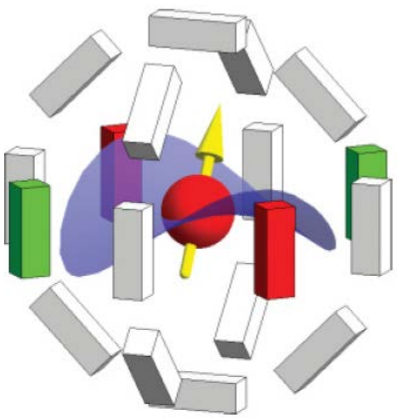
- Symmetry of a general quadrupole Hamiltonian.

- Derivation of the adiabatic time evolution of an initial HH eigenstate.

- Identification of single-qubit operation by time-dependent electric quadrupole field.

Quadrupole coupling

Hamiltonian of a spin-3/2 particle coupled to an electric quadrupole field:



$$H\left(Q\right) = J_i Q^{ij} J_j$$

angular momentum operator

quadrupole tensor of the applied field

Q: real, symmetric, traceless matrix with basis $\{Q_{\square}\}_{\square}$

Quadrupole Hamiltonian

$$H\left(Q\right) = H\left(x^\mu Q_\mu\right) = x^\mu J_i Q_\mu^{ij} J_j \equiv x^\mu \Gamma_\mu$$

where the basis Hamiltonians Γ_μ obey the SO(5) Clifford algebra:

$$\left\{\Gamma_\mu, \Gamma_\nu\right\} = 2\delta_{\mu\nu}$$

Basis Hamiltonians

$$\Gamma_\mu = J_i Q_\mu^{ij} J_j$$

with orthogonal basis:

$$Q_0 = \frac{1}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, Q_1 = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, Q_2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$Q_3 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, Q_4 = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Symmetry of H(Q)

- All possible H(Q) are unitarily related by a Spin(5) rotation.
- The ten generators of the corresponding SO(5) symmetry group are:

$$\left\{V_i\right\}_i = \left\{\Gamma_\mu \Gamma_\nu\right\}_{\mu<\nu} \quad \text{with } \mu,\nu \in 0,\ldots,4$$

Next step: use the generators to describe a cyclic time evolution
(That is the big advantage of spin 3/2 qubits!)

Cyclic time evolution of H(Q)

Starting point: $$H\left(t=0\right)=\Gamma_0$$ $$\left|\mathbf{x}\right|=1$$

Cyclic time evolution is given by a 2π SO(5) rotation on the space of quadrupole fields:

$$t \mapsto H\left(t\right)=e^{t\hat{a}\vec{V}/2}\Gamma_0 e^{-t\hat{a}\vec{V}/2} \ \text{with} \ t\in\left[0,2\pi\right]$$

ten-component unit vector

Our choice of H(t=0)

$$H\left(t=0\right)=J_iQ_0^{ij}J_j \;\; \text{with } \mathrm{Q}_0 = \frac{1}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$H\left(t=0\right)=J_z^2-\frac{1}{3}J^2$$

J_z good quantum number □ easy initialization

Initialization: HH eigenstate

$$P_0^+ \left|\Psi(0)\right\rangle = \left|\Psi(0)\right\rangle$$

Projector on HH and LH subspaces:

$$P_0^\pm = \frac{1}{2}\left(1 \pm \Gamma_0\right)$$

$$\Gamma_0 = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

in the basis $\left\{\left|\frac{3}{2},+\frac{3}{2}\right\rangle, \left|\frac{3}{2},+\frac{1}{2}\right\rangle, \left|\frac{3}{2},-\frac{1}{2}\right\rangle, \left|\frac{3}{2},-\frac{3}{2}\right\rangle\right\}$

Adiabatic time evolution I

$$\tilde{U}_{t,0} = \lim_{n\to\infty} \tilde{U}_{t,0}^{(n)}$$

$$\tilde{U}_{t,0}^{(n)} = P^{+}(t)\, P^{+}\left(\frac{(n-1)t}{n}\right) \ldots P^{+}\left(\frac{2t}{n}\right) P^{+}\left(\frac{t}{n}\right) P_{0}^{+}$$

time-dependent projector
on HH subspace

How can we prove this?

$$t_i = \frac{i\varepsilon}{n}$$

$$\tilde{U}_{t,0} = T \exp\left(-\int_0^t A(s)\, ds\right) = \prod_{i=1}^{n} e^{-A(t_i)\varepsilon}$$

$$\varepsilon = \frac{t}{n}$$

$$A(s) = -\left[\frac{dP(s)}{ds}, P(s)\right]$$

Adiabatic time evolution II

$$\tilde{U}_{t,0} = \lim_{n\to\infty} \tilde{U}_{t,0}^{(n)}$$

$$\tilde{U}_{t,0}^{(n)} = P^{+}\left(t\right) P^{+}\left(\frac{\left(n-1\right)t}{n}\right) \ldots P^{+}\left(\frac{2t}{n}\right) P^{+}\left(\frac{t}{n}\right) P_{0}^{+}$$

Take the advantage of the SO(5) rotation:

$$P^{+}\left(t\right) = e^{t\hat{a}\vec{V}/2} P_{0}^{+} e^{-t\hat{a}\vec{V}/2}$$

$$\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^{n} = e^{-x} \quad \Rightarrow \quad \tilde{U}_{t,0} = e^{t\hat{a}\vec{V}/2} e^{-tP_{0}^{+}\left(\hat{a}\vec{V}/2\right)P_{0}^{+}}$$

(-1) for t=2π

defines for t=2π desires holonomy

Relation to single qubit operation

$$U\left(\hat{n},\varphi\right) = \exp\left(i\varphi\frac{\hat{n}\vec{\sigma}}{2}\right)$$

$$\tilde{U}_{2\pi,0} = -e^{-2\pi P_0^+\left(\hat{a}\vec{V}/2\right)P_0^+}$$

four relevant generators:

$$P_0^+\Gamma_0\Gamma_\mu P_0^+ = 0 \text{ for } \mu \neq 0$$
$$P_0^+\Gamma_4\Gamma_1 P_0^+ = i\sigma_x$$
$$P_0^+\Gamma_1\Gamma_3 P_0^+ = i\sigma_y$$
$$P_0^+\Gamma_1\Gamma_2 P_0^+ = i\sigma_z$$

Pauli matrices on HH subspace

$$V_0 = \Gamma_0\Gamma_1, V_1 = \Gamma_4\Gamma_1, V_2 = \Gamma_1\Gamma_3, V_3 = \Gamma_1\Gamma_2$$

$$\varphi = 2\pi\left(1-\sqrt{1-a_0^2}\right)$$
$$\hat{n} = \frac{\left(a_1,a_2,a_3\right)}{\left|\left(a_1,a_2,a_3\right)\right|}$$

Translation into t-dependent quadrupole field

$$H(t) = e^{t\hat{a}\vec{V}/2} x^{\mu}(0)\,\Gamma_{\mu} e^{-t\hat{a}\vec{V}/2} = \left[e^{t\hat{a}\vec{W}} \mathbf{x}(0) \right]^{\mu} \Gamma_{\mu}$$

SO(5) generators in the representation acting on the five-component vector **x**

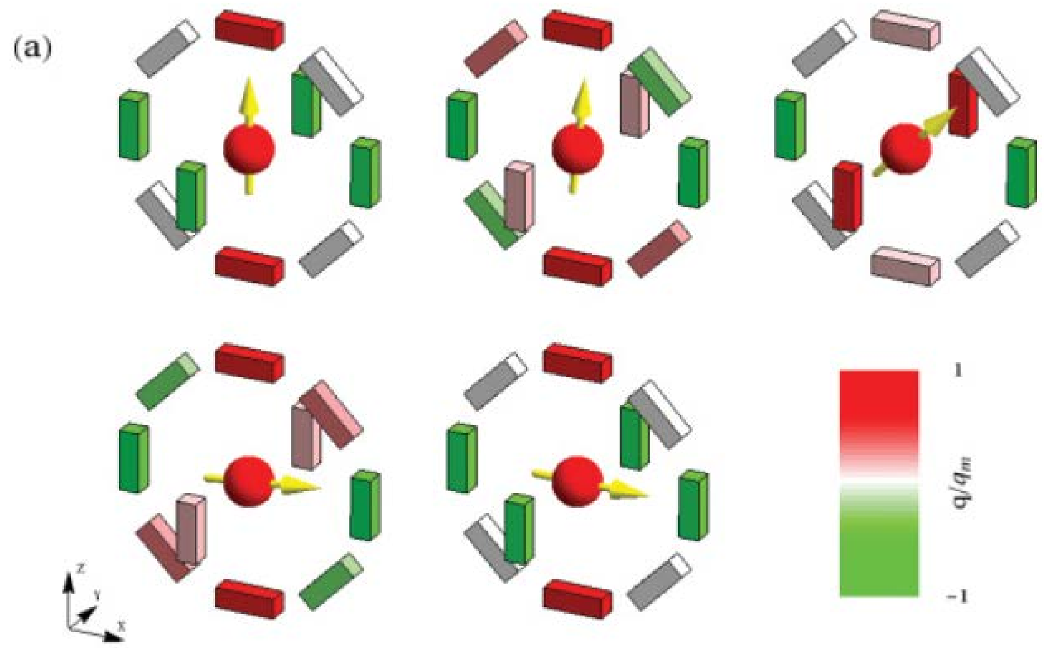
 time-dependent quadrupole field associated with the loop in direction **a**

$$Q(t) = x^{\mu}(t)\, Q_{\mu} = \left[e^{t\hat{a}\vec{W}} \mathbf{x}(0) \right]^{\mu} Q_{\mu}, \quad t \in [0, 2\pi]$$

Example

$$U\left(-\hat{e}_y, \frac{\pi}{2}\right)$$

rotation
from **z** to **x**



$$Q(t) = \left[e^{t\left(\sqrt{7}/4\right)W_0 - \left(3/4\right)W_2}\mathbf{e_0}\right]^{\mu} Q_{\mu}, \quad t \in \left[0, 2\pi\right]$$

(only 10 of
18 gates needed)

Outline

- Kato’s connection non-Abelian geometric phase

- Qubit control via quadrupole fields

- Estimation of experimental parameters

Schedule of this part

- Up to now: $\square E = |\mathbf{x}|$ has been treated as a free parameter.

- We want to estimate the quadrupole induced splitting for GaAs quantum dots.

- Strain will become important to reduce confinement-induced HH-LH splitting.

Competing energy scales

Hamiltonian due to quadrupole potential

HH-LH splitting due to parabolic confinement potential

$$H' = H(Q) + \frac{\Delta E_0}{2}\tau_z$$

▯ spectrum:

$$E = \pm\frac{1}{2}\sqrt{(\Delta E_0)^2 + 4|\mathbf{x}|^2}$$

Quantum dot model

envelope functions of
lowest LH and HH subbands

$$H_{\alpha\beta}^{QW} = \int dz f_{\alpha}^{\dagger}(z) H_L f_{\beta}(z)$$

Luttinger Hamiltonian for □$_8$ bands
including strain corrections

+ in-plane confinement potential

$$e\Phi_1 = -0.15eV\left(\frac{r}{R_{\max}}\right)^2$$

□ reduces the symmetry
□ HH-LH splitting □E_0

Luttinger Hamiltonian

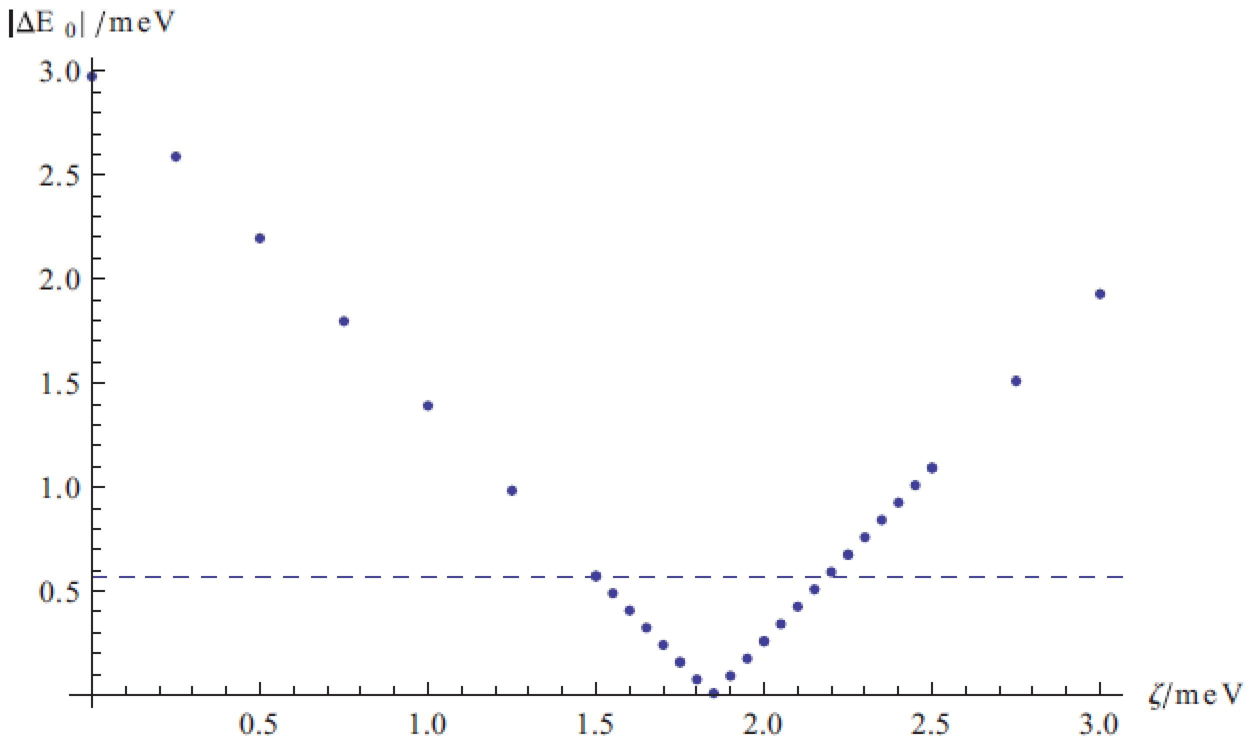
$$H_L = -\begin{pmatrix} P+Q & -S & R & 0 \\ -S^\dagger & P-Q & 0 & R \\ R^\dagger & 0 & P-Q & S \\ 0 & R^\dagger & S^\dagger & P+Q \end{pmatrix}$$

$$P = \gamma_1\left(k_x^2 + k_y^2\right) + k_z\gamma_1 k_z + P_\varepsilon$$
$$Q = \gamma_2\left(k_x^2 + k_y^2\right) - k_z\gamma_2 k_z + Q_\varepsilon$$
$$R = \sqrt{3}\left[-\gamma_2\left(k_x^2 - k_y^2\right) + 2i\gamma_3 k_x k_y\right]$$
$$S = \sqrt{3}\left(k_x - ik_y\right)\left\{\gamma_3, k_z\right\}$$

with uniaxial strain corrections:

$$P_\varepsilon = -a\left(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\right)$$
$$Q_\varepsilon = -b\left(\varepsilon_{xx} + \varepsilon_{yy} - 2\varepsilon_{zz}\right)$$

HH-LH splitting ΔE_0



<u>Estimation:</u>

pressure of 1kbar $\triangleq$ $\zeta\left(=Q_\varepsilon\right)=2.61\text{ meV}$

Quadrupole-induced splitting

quadrupole potential: $e\Phi_4 = \vec{r}^{\,T} Q \vec{r}$

associated with the quadrupole tensor of four Coulomb charges □q at equal radius R in the (x,y) plane:

$$Q = \frac{1}{4\pi\varepsilon}\frac{6eq}{R^3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

<u>Our estimation:</u>

$$e\Phi_4\left(r = 50\mathrm{nm}\right) = 50\mathrm{eV} \;\Rightarrow\; 2\left|\mathbf{x}\right| \approx 0.57\mathrm{meV}$$

How fast am I allowed to operate the device?

$$\Delta E_Q \approx 0.57 \text{ meV} \triangleq 6.6 \text{ K} \triangleq 0.87 \text{ THz}$$

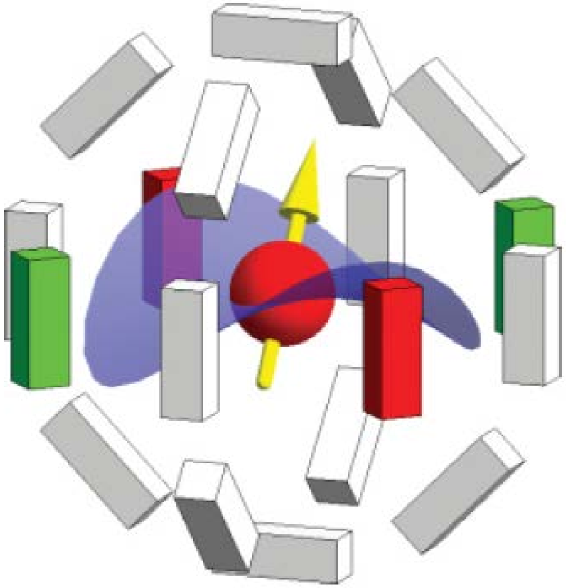
Manipulations need to be done below this frequency – because of the adiabatic approximation – but still faster than typical dephasing times.

$$T_2 \approx 10^{-6} s \gg t_{op} \gg \left(\Delta E_Q\right)^{-1} \approx 10^{-12} s$$

de Greve, …, Yamamoto Nature Phys. 2011

Summary

- 14 out of 18 gates needed to perform an arbitrary SU(2) transformation
- Strain-engineering important in GaAs QDs
- Explore the third dimension of QD control



Budich, Rothe, Hankiewicz & Trauzettel PRB **85**, 205425 (2012)

Advantage of our proposal

- Adiabatic manipulation of 4 dimensional Hilbert space (HH+LH+both spin directions).

- In principle, the same manipulation on the HH and LH subspace is possible.

- This is not the case in a previous proposal based on the electric Stark effect, where the resulting holonomy on the HH subspace is Abelian and only the one on the LH subspace is non-Abelian.

Bernevig & Zhang PRB 2005