
Outline of Presentation

Part I. Tutorial Introduction to Nonlinear Optics

Part II. Recent Research in Quantum Nonlinear Optics

Weak Values and Direct Measurement of the Quantum Wavefunction

Robert W. Boyd

University of Ottawa
University of Rochester
University of Glasgow

Collaborators:

University of Ottawa: Jonathan Leach,* Jeff Z. Salvail, Megan Agnew,
Allan S. Johnson, Eliot Bolduc, Filippo M. Miatto

University of Rochester: Mehul Malik, Mohammad Mirhosseini,
Omar Magana, Zhimin Shi,** Andrew Jordan, and Justin Dressel

University of Glasgow: Martin P. J. Lavery, Miles J. Padgett,

*Currently at Heriot-Watt University

**Currently at the University of South Florida

How the Result of a Measurement of a Component of the Spin of a Spin- $\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

*Physics Department, University of South Carolina, Columbia, South Carolina 29208, and
School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel*

(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin- $\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz

standard expectation value: $\langle A \rangle = \langle \Psi | \hat{A} | \Psi \rangle$

weak value: $A_w \equiv \langle \psi_f | A | \psi_{in} \rangle / \langle \psi_f | \psi_{in} \rangle$.

Why are weak values important?

can lead to amplification of small signals

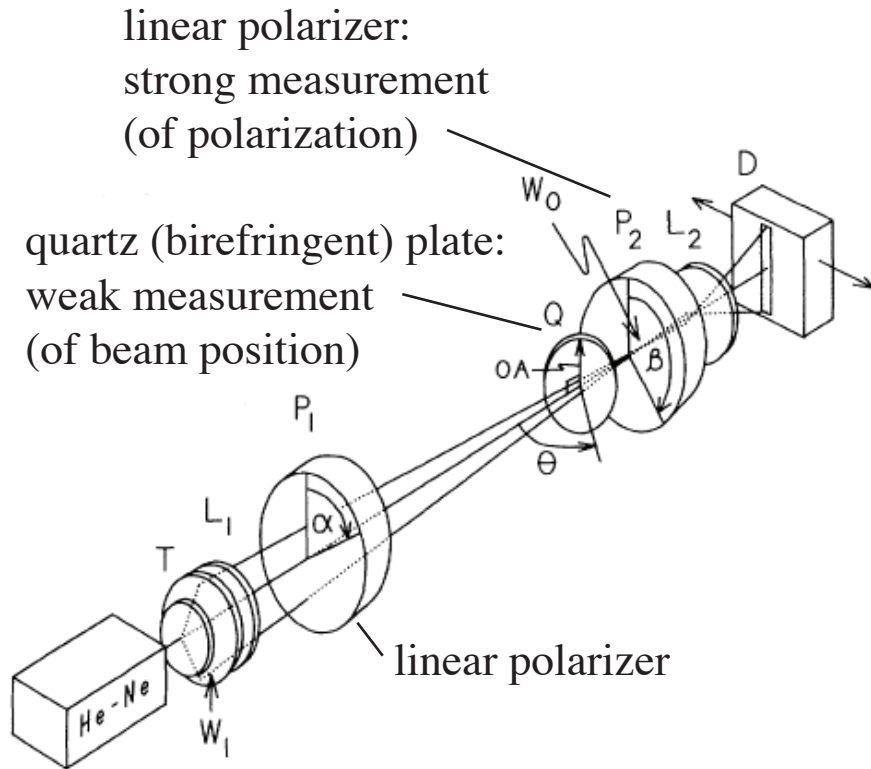
can lead to direct measurement of the quantum wavefunction

Realization of a Measurement of a “Weak Value”

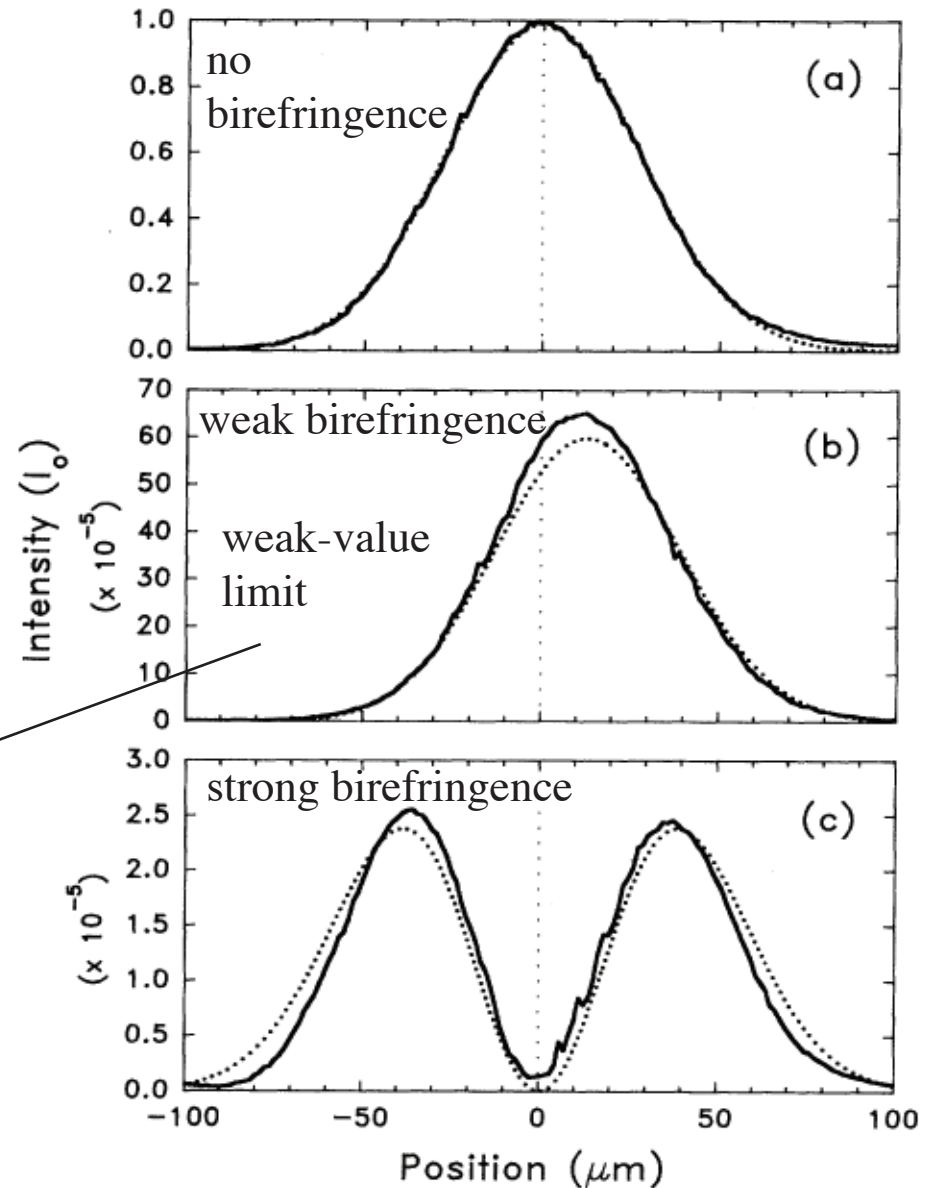
N. W. M. Ritchie, J. G. Story, and Randall G. Hulet

Department of Physics and Rice Quantum Institute, Rice University, Houston, Texas 77251-1892

(Received 7 December 1990)



Birefringence separates polarized beams by $0.64 \mu\text{m}$, but gaussian in (b) is displaced by $12 \mu\text{m}$.





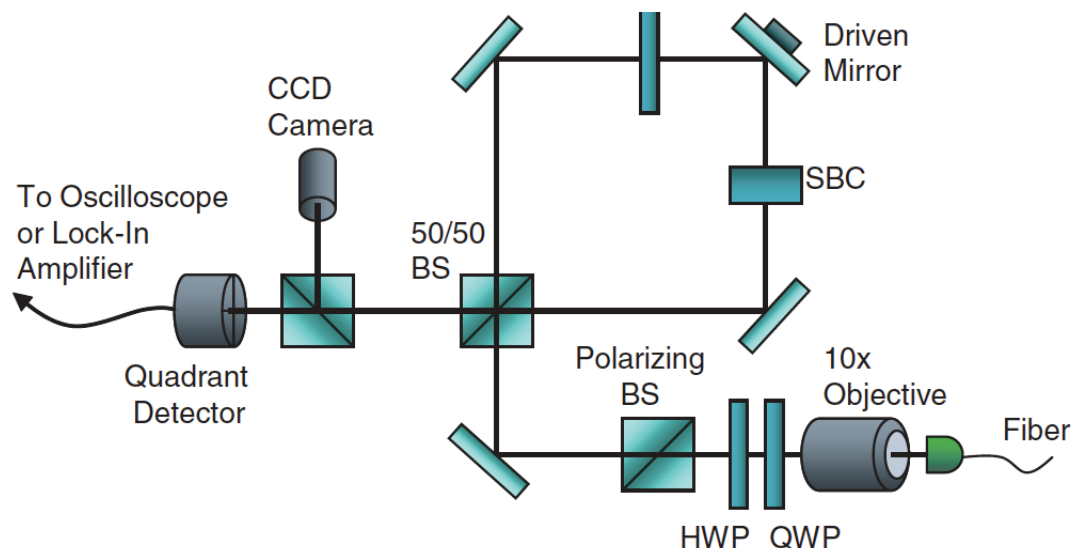
Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification

P. Ben Dixon, David J. Starling, Andrew N. Jordan, and John C. Howell

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA

(Received 12 January 2009; published 27 April 2009)

We report on the use of an interferometric weak value technique to amplify very small transverse deflections of an optical beam. By entangling the beam's transverse degrees of freedom with the which-path states of a Sagnac interferometer, it is possible to realize an optical amplifier for polarization independent deflections. The theory for the interferometric weak value amplification method is presented along with the experimental results, which are in good agreement. Of particular interest, we measured the angular deflection of a mirror down to 400 ± 200 frad and the linear travel of a piezo actuator down to 14 ± 7 fm.



Direct Measurement of the Quantum Wavefunction

The wavefunction of quantum mechanics is notoriously difficult to measure

Measurement at any one position causes the entire wavefunction to “collapse”

(The problem is related to the Heisenberg Uncertainty Relation; if you measure position you cannot also know momentum.)

Historically, the wavefunction has been measured only indirectly and inefficiently, using “quantum state tomography.”

Recent work has demonstrated how to measure the wavefunction directly.

The idea is to perform a “weak measurement” on one variable (which thus only minimally disturbs the system), followed by a “strong measurement.”

J. Lundeen et al., Nature 474, 188 (2011)

J. Z. Salvail et al., Nature Photonics, 10.1038 (2013)

Direct measurement of the quantum wavefunction

Jeff S. Lundeen¹, Brandon Sutherland¹, Aabid Patel¹, Corey Stewart¹ & Charles Bamber¹

$$\langle A \rangle_W = \frac{\langle c|A|\Psi \rangle}{\langle c|\Psi \rangle}$$

Returning to our example of a single particle, consider the weak measurement of position ($A = \pi_x = |x\rangle \langle x|$) followed by a strong measurement of momentum giving $P = p$. In this case, the weak value is:

$$\langle \pi_x \rangle_W = \frac{\langle p|x \rangle \langle x|\Psi \rangle}{\langle p|\Psi \rangle} \quad (2)$$

$$= \frac{e^{ipx/\hbar} \Psi(x)}{\Phi(p)} \quad (3)$$

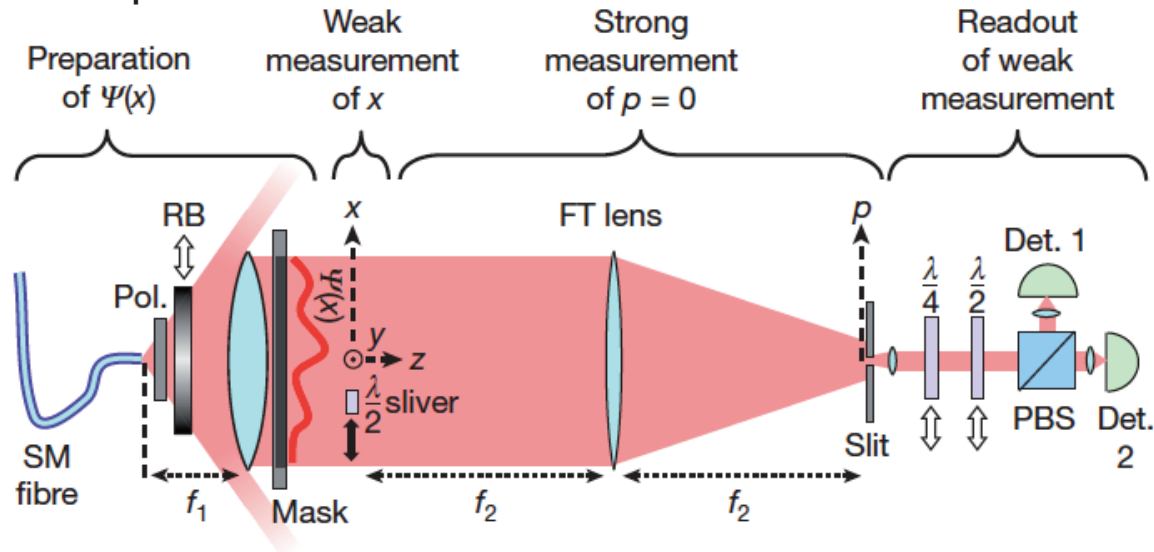
In the case $p = 0$, this simplifies to

$$\langle \pi_x \rangle_W = k \Psi(x) \quad (4)$$

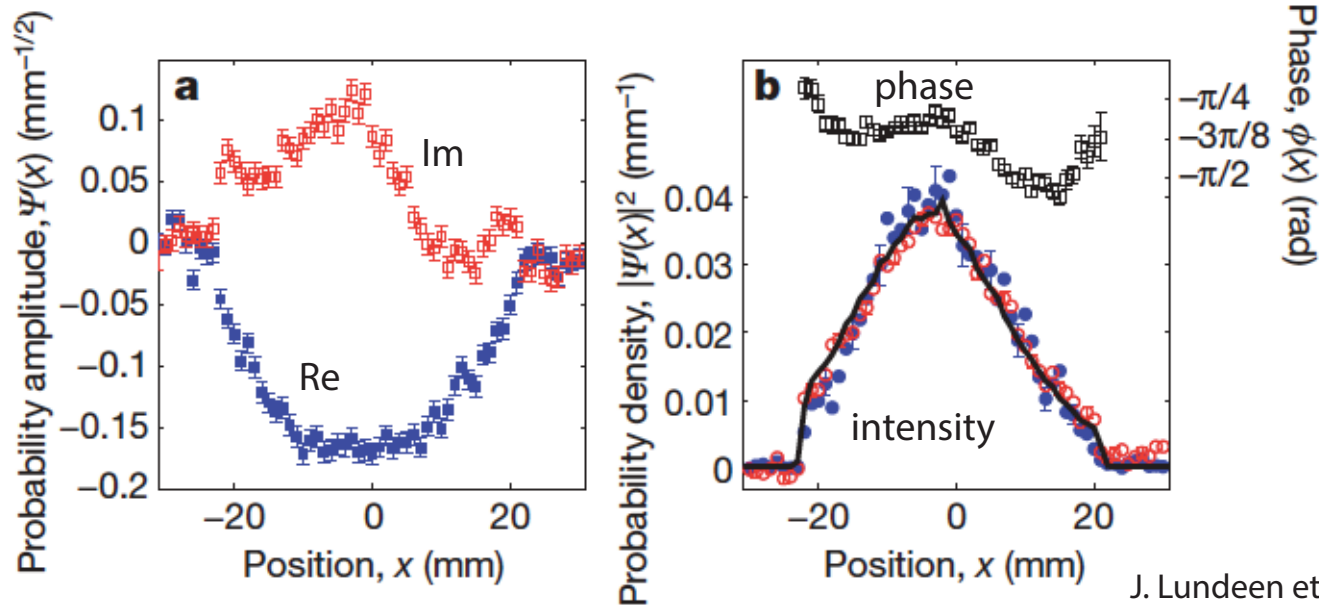
where $k = 1/\Phi(0)$ is a constant (which can be eliminated later by normalizing the wavefunction). The average result of the weak mea-

Direct Measurement of the Photon "Wavefunction"

Measurement setup



Typical results



Wavefunction of the Photon?

Many people feel it is inaccurate to speak of the “wavefunction” of the photon.

I personally try to avoid the term wavefunction of the photon. A photon is an excitation of a mode of the field. I prefer to distinguish the photon from the mode in which it lives.

Lundeen et al. use the term “wavefunction of the photon,” but comment that it is sometimes called the “spatial mode of the photon.”

See also Iwo Bialynicki-Birula, *The Photon Wave Function, Coherence and Quantum Optics VII*, Eds. J. H. Eberly, L. Mandel, and E. Wolf Plenum, NY 1996, p. 313

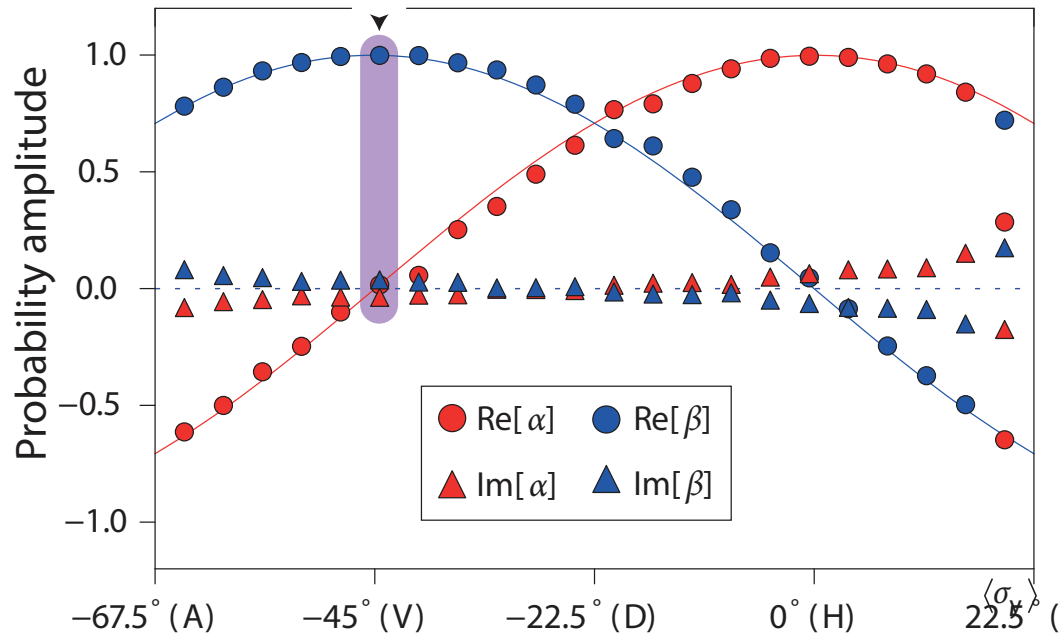
Work of My Own Group

1. We have made a direct measurement of the state of polarization of the photon. This is thus the first direct measurement of a qubit.
2. We have measured the statevector of a state imbedded in a 27-dimensional OAM Hilbert space. One expects direct measurement to be increasingly useful with increasing size of the Hilbert space.

Full characterization of polarization states of light via direct measurement

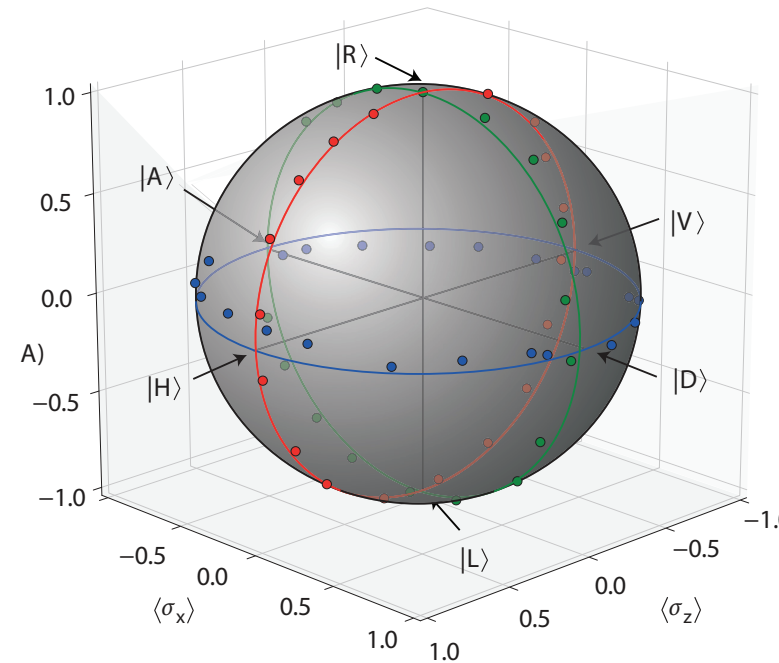
Jeff Z. Salvail^{1*}, Megan Agnew¹, Allan S. Johnson¹, Eliot Bolduc¹, Jonathan Leach¹
and Robert W. Boyd^{1,2}

$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle.$$

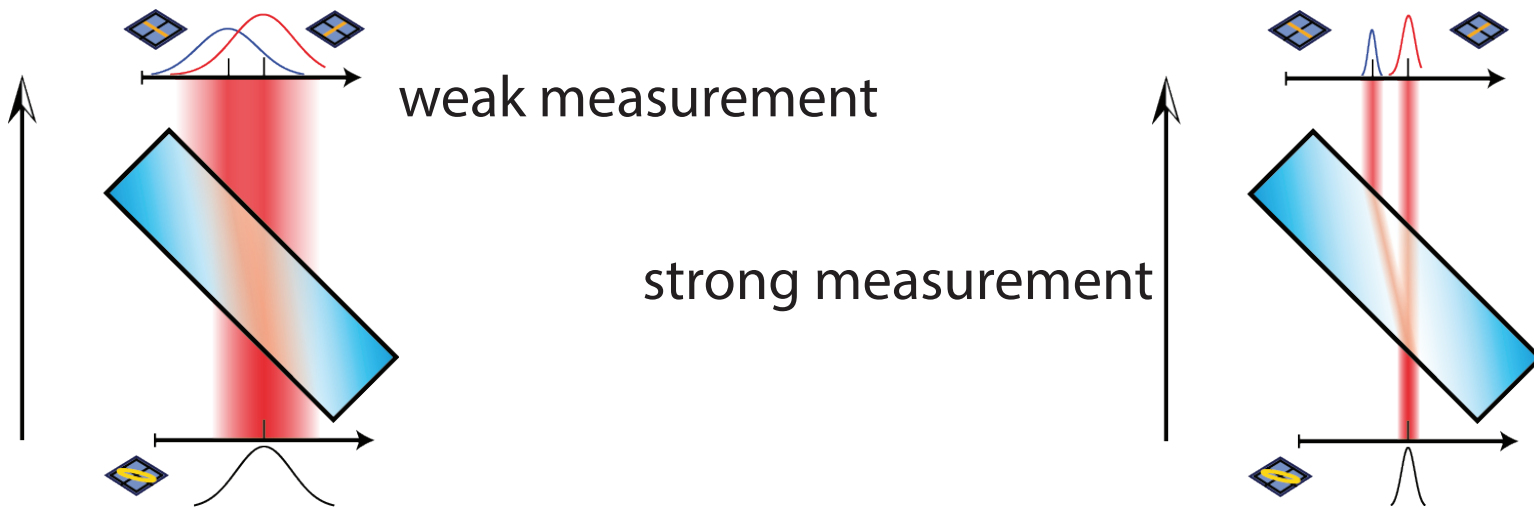
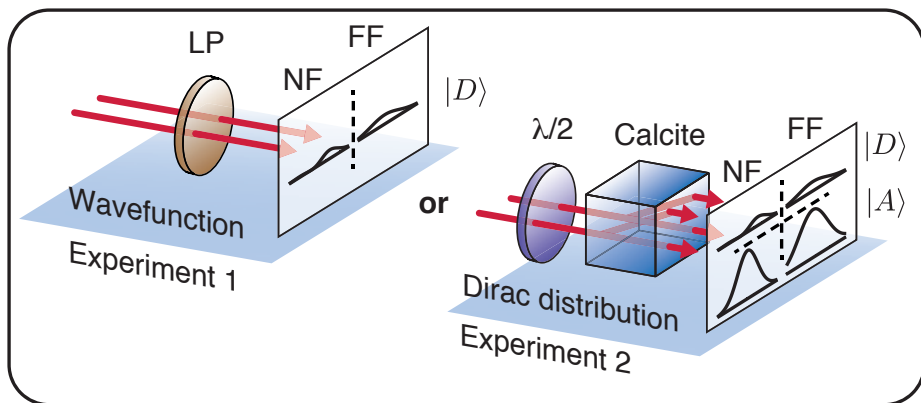
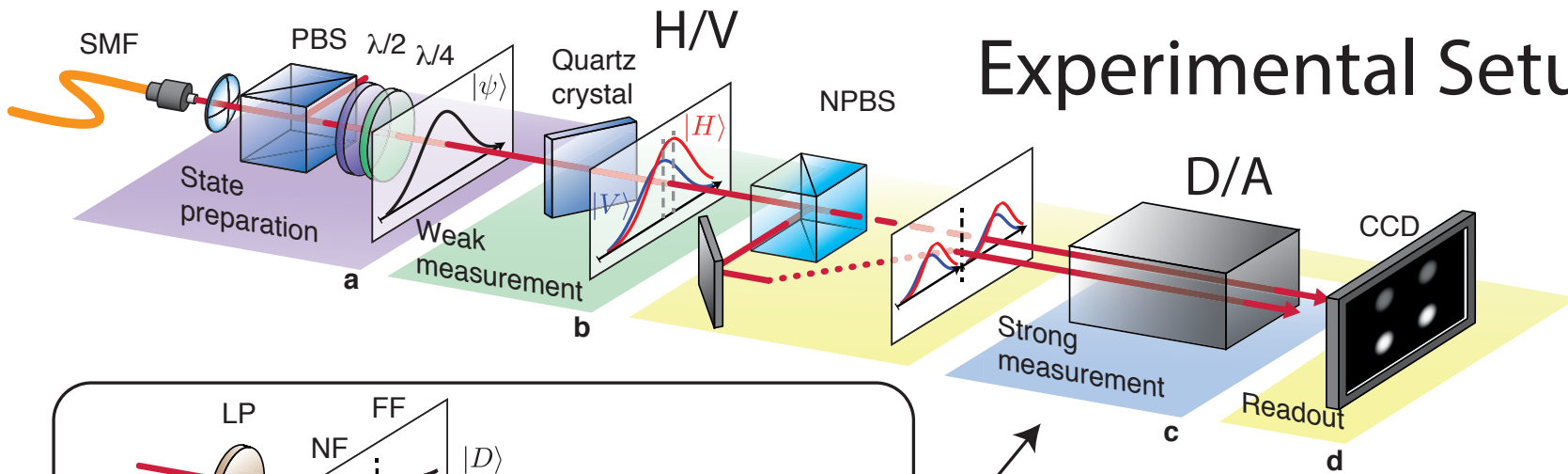


Half-wave plate angle (input polarization state)

points on Poincaré sphere



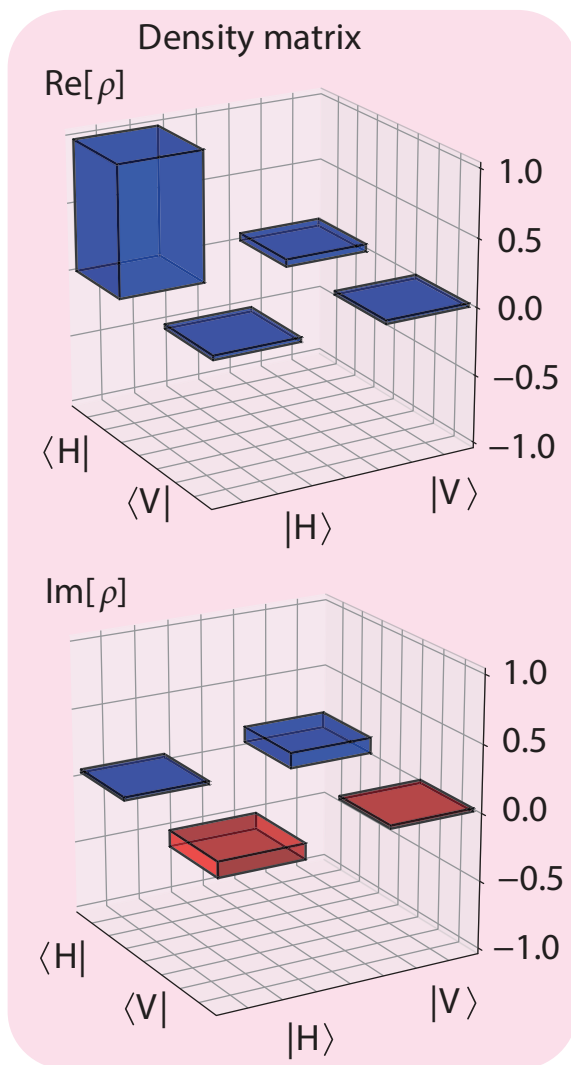
Experimental Setup



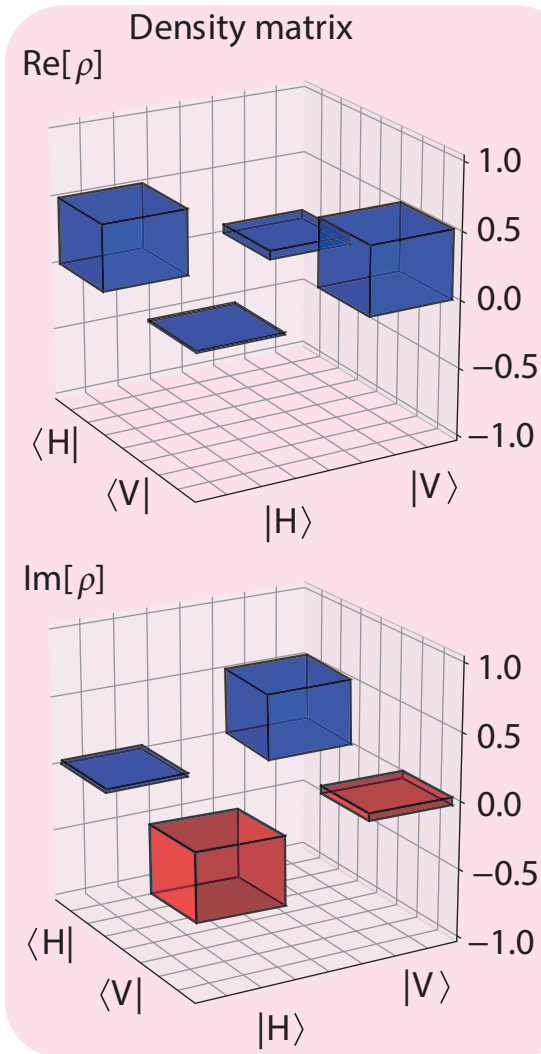
Direct Measurement of the Full Density Matrix

Measurement provides enough information to determine density matrix*

Horizontal polarization



Left-hand-circular polarization



* J.S. Lundeen & C. Bamber, Phys. Rev. Lett. 108, 070402 (2012).

News Accounts of Our Work

physicsworld.com

First weak measurements made on optical polarization states

Mar 11, 2013 [4 comments](#)

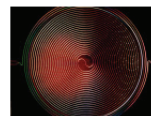
Physicists in Canada and the US claim to be the first to make a direct measurement of the polarization quantum state of light – a feat that at first glance appears to defy Heisenberg's uncertainty principle. The technique, which relies on a process known as weak measurement, could help in fundamental studies on quantum mechanics or in the development of quantum computing.

NATIONAL POST

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JEFF SALVAIL

Canadian researchers take a sneak peek at Schrödinger's Cat and a step toward a quantum computer



In an Ottawa lab, scientists have succeeded in side-stepping an obstacle of Heisenberg's Uncertainty Principle, a strange law of the quantum world

[Lawrence Krauss: The Large Hadron Collider helps science do a quantum leap](#)

[Neil Turok: Furthering our understanding of the universe](#)

[Freezing antimatter could allow scientists to study the strangest stuff in existence: Canadian researcher](#)

SCIENCE RECORDER

BIOLOGY

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Scientists discover a way around Heisenberg's Uncertainty Principle

A new technique for quantum mechanics.

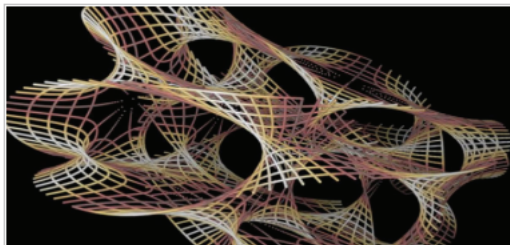


Photo credit:

Science Recorder | Mark Petillo | Monday, March 04, 2013

It could be big news of the world of quantum physics. According to a pair of scientists from the University of Rochester and the University of Ottawa, there may be a way around Heisenberg's famous Uncertainty Principle. According to a report published this week in *Nature Photonics*, a recently developed technique that allows scientists to directly measure the polarization states of light could be the key... **To continue reading, subscribe to Science Recorder today.**

Work of My Own Group

1. We have made a direct measurement of the state of polarization of the photon. This is thus the first direct measurement of a qubit.
2. We have measured the statevector of a state imbedded in a 27-dimensional OAM Hilbert space. One expects direct measurement to be increasingly useful with increasing size of the Hilbert space.

Malik, Mirhosseini, Lavery, Leach, Padgett, Boyd, in review

High Dimensional QKD Protocol

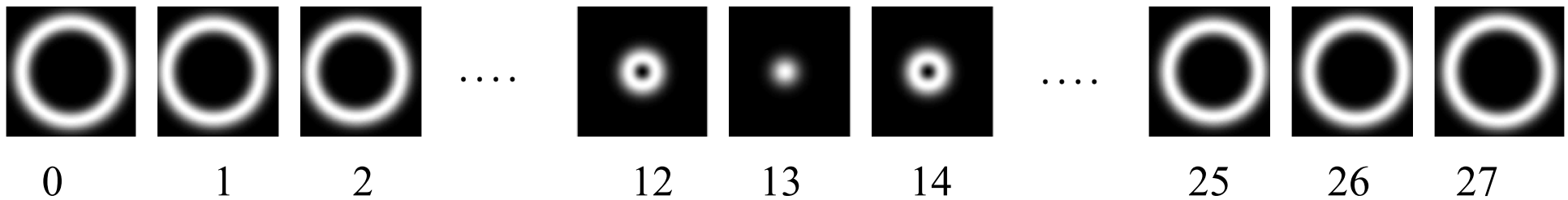
We are constructing a QKD system in which each photon carries many bits of information

We encode in states that carry OAM such as the Laguerre-Gauss states

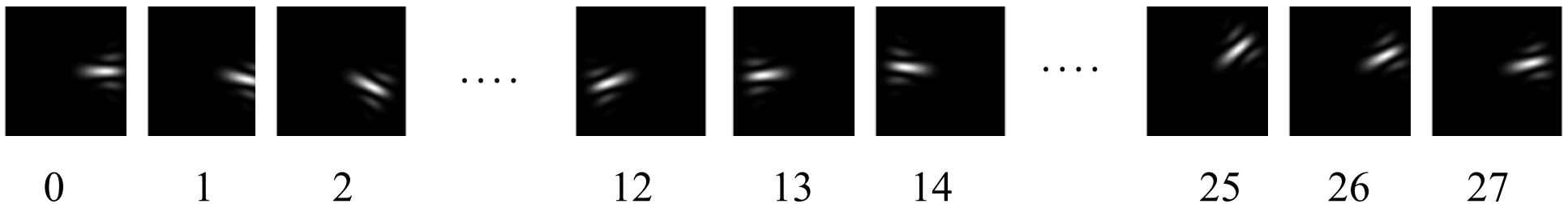
As a diagnostic, we need to be able to measure the statevector of OAM states

Single Photon States

Laguerre-Gaussian Basis $\ell = -13, \dots, 13$

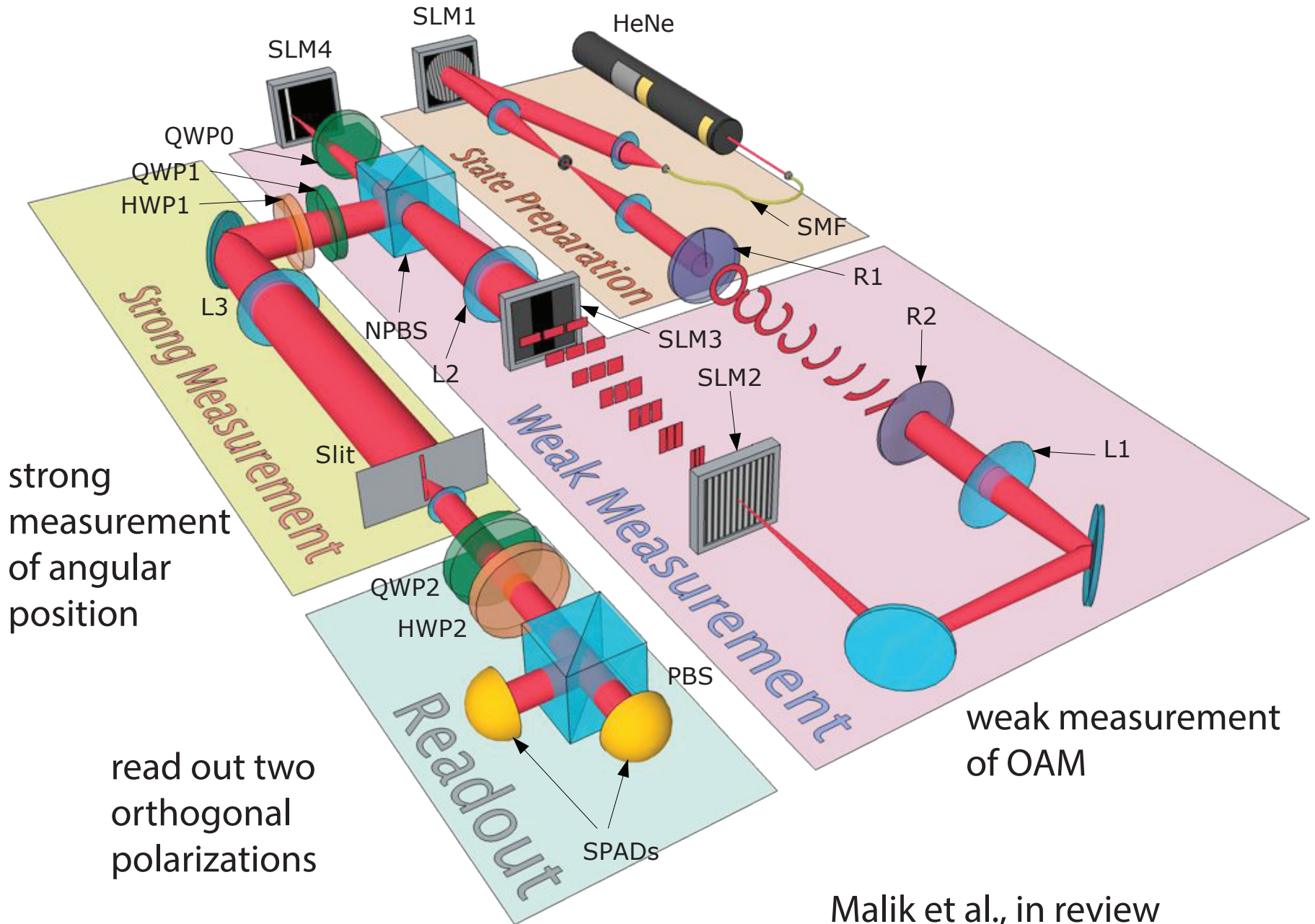


“Angular” Basis (mutually unbiased with respect to LG)

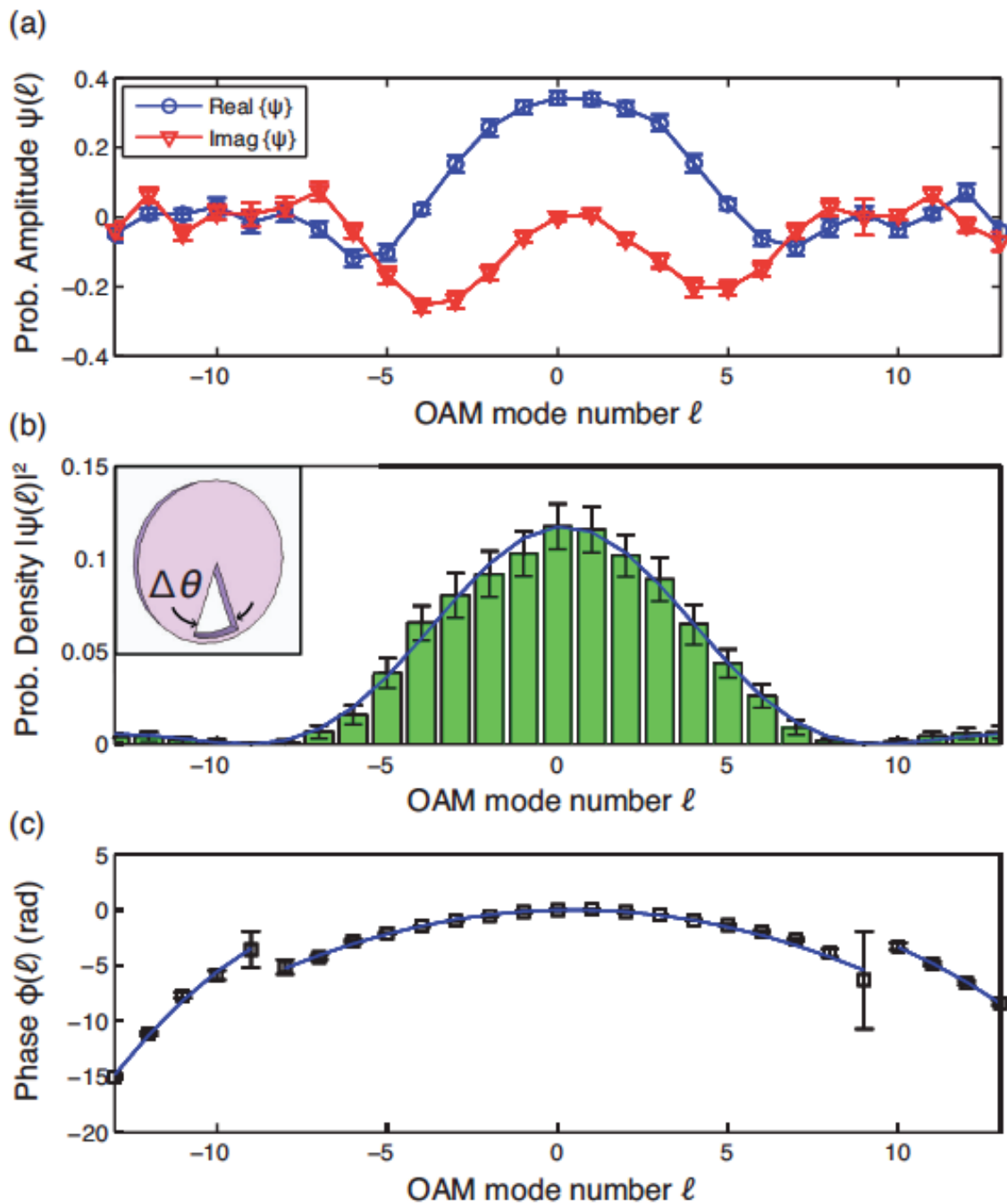


$$\Psi_{AB}^N = \frac{1}{\sqrt{27}} \sum_{l=-13}^{13} \text{LG}_{l,0} \exp(i2\pi Nl/27)$$

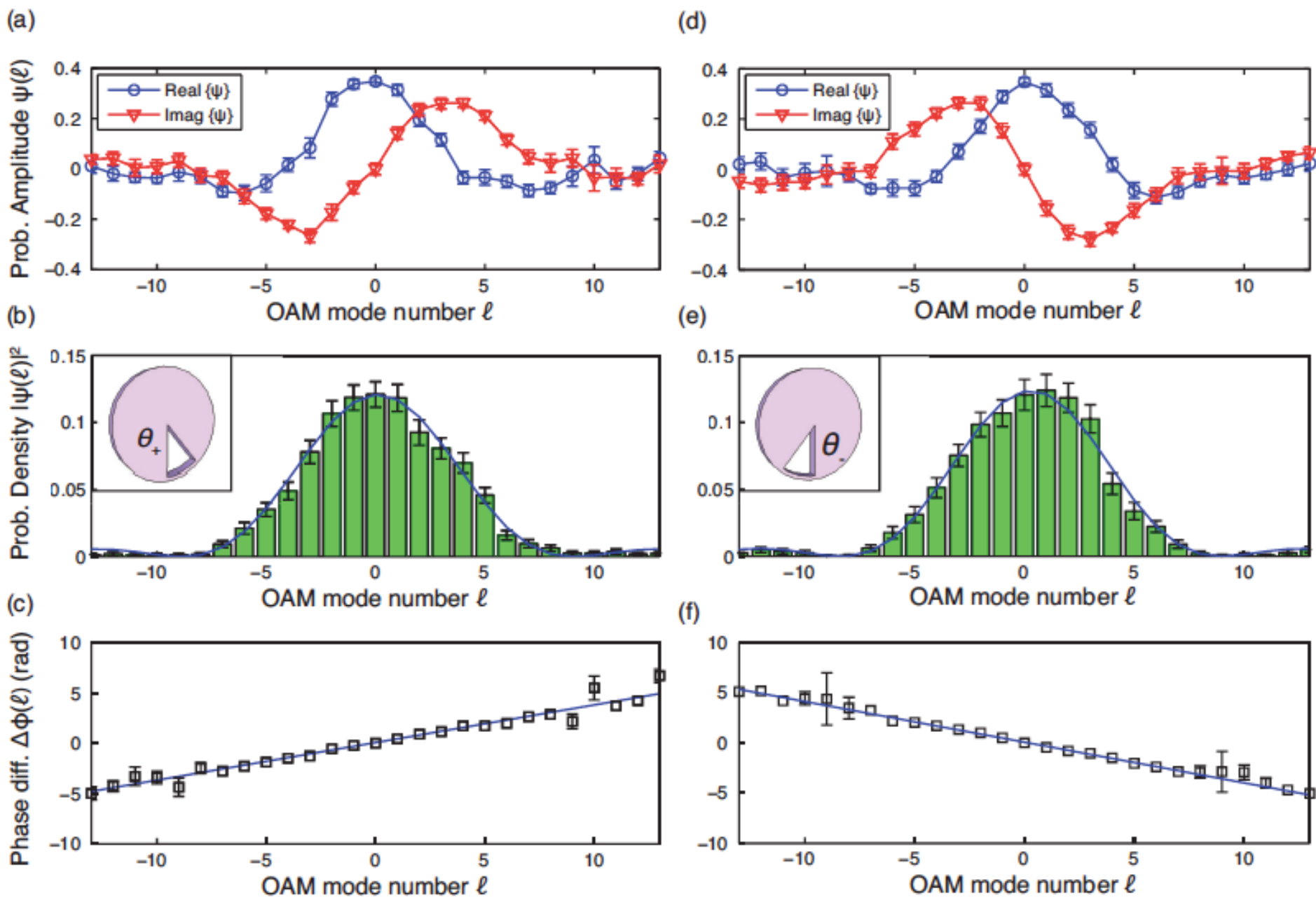
Direct Measurement of a High-Dimensional OAM State



Results: Direct Measurement of a High-Dimensional OAM State



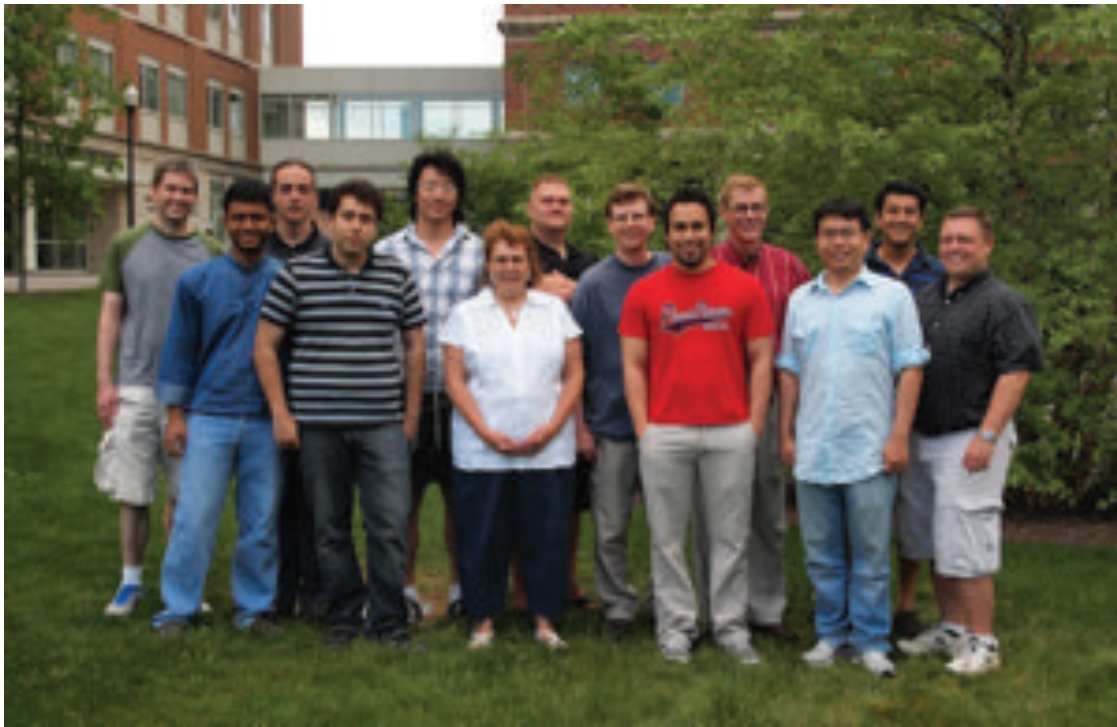
Direct Measurement Procedure Properly Measures Phase



Summary

Weak values and weak measurements offer important opportunities in quantum optics, including the ability to “amplify” weak signals and the ability to perform direct measurements of the quantum state vector.

Thanks To My Research Groups



Rochester



Ottawa

Quantum Photonics

(Encompassing nanophotonics and quantum nonlinear optics)

Robert Boyd

Canada Excellence Research Chair in Quantum Nonlinear Optics



Our research interests include:

Nanophotonics

Plasmonics

Photonic crystals

Photonic device

Applications of slow and fast light

Quantum nonlinear optics

Optical methods for quantum information

Biophotonics

Nonlinear optics of atomic vapors

Optical chirality and structure surfaces

University of Ottawa Nonlinear Photonics Programme - Senior Staff



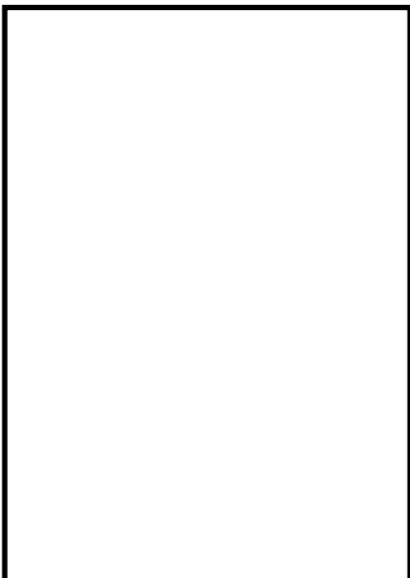
Robert Boyd
CERC



Ksenia Dolgaleva
Asst. Prof.



Jeff Lundeen
Asst. Prof.



????



Gerd Leuchs
Adjunct Prof.



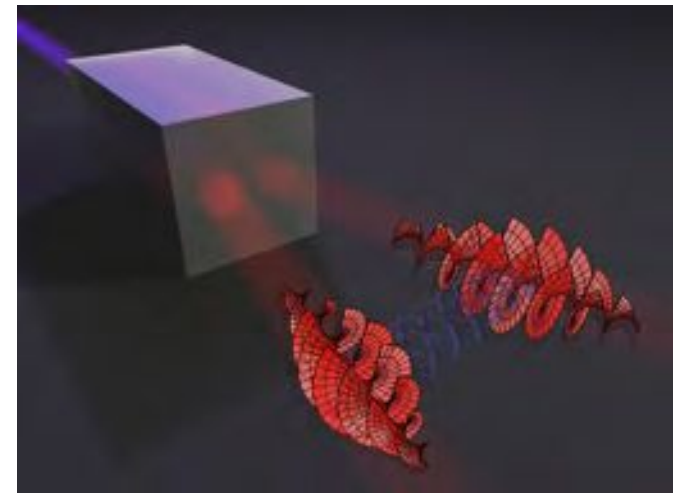
Jeremy Upham
Staff Scientist

Our Quantum Information Program

A Quantum Key Distribution (QKD) System Utilizing
Orbital Angular Momentum (OAM) States of Light
to Carry Many Bits of Information per Photon.

Role of Light Beams Carrying OAM in Quantum Information Science

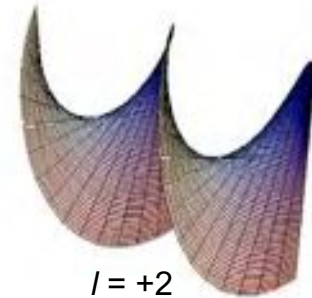
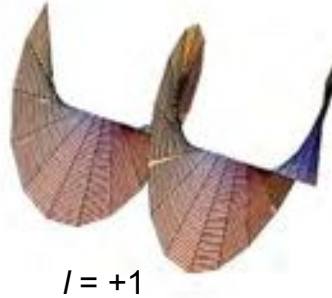
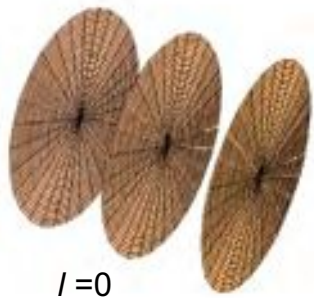
- Brief overview of orbital angular momentum (OAM) of light
- Use of OAM states for high capacity quantum key distribution (QKD)
 - Can we build a QKD system that carries many bits per photon?
 - In traditional telecom, one uses many photons per bit!
- If time permits, description of some related work



What Are the OAM States of Light?

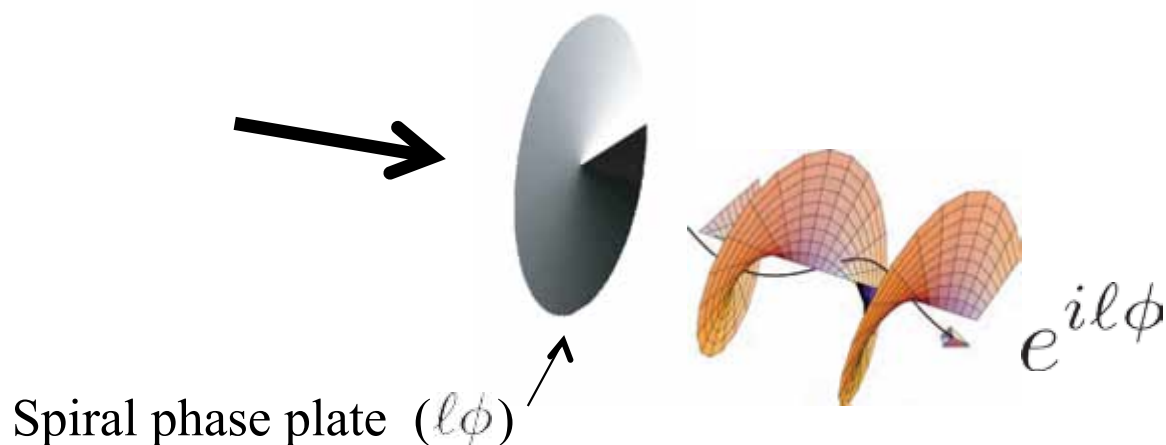
- Light can carry spin angular momentum (SAM) by means of its circular polarization.
- Light can also carry orbital angular momentum (OAM) by means of the phase winding of the optical wavefront.
- A well-known example are the Laguerre-Gauss modes. These modes contain a phase factor of $\exp(il\phi)$ and carry angular momentum of $l\hbar$ per photon. (Here ϕ is the azimuthal coordinate.)

Phase-front structure of some OAM states

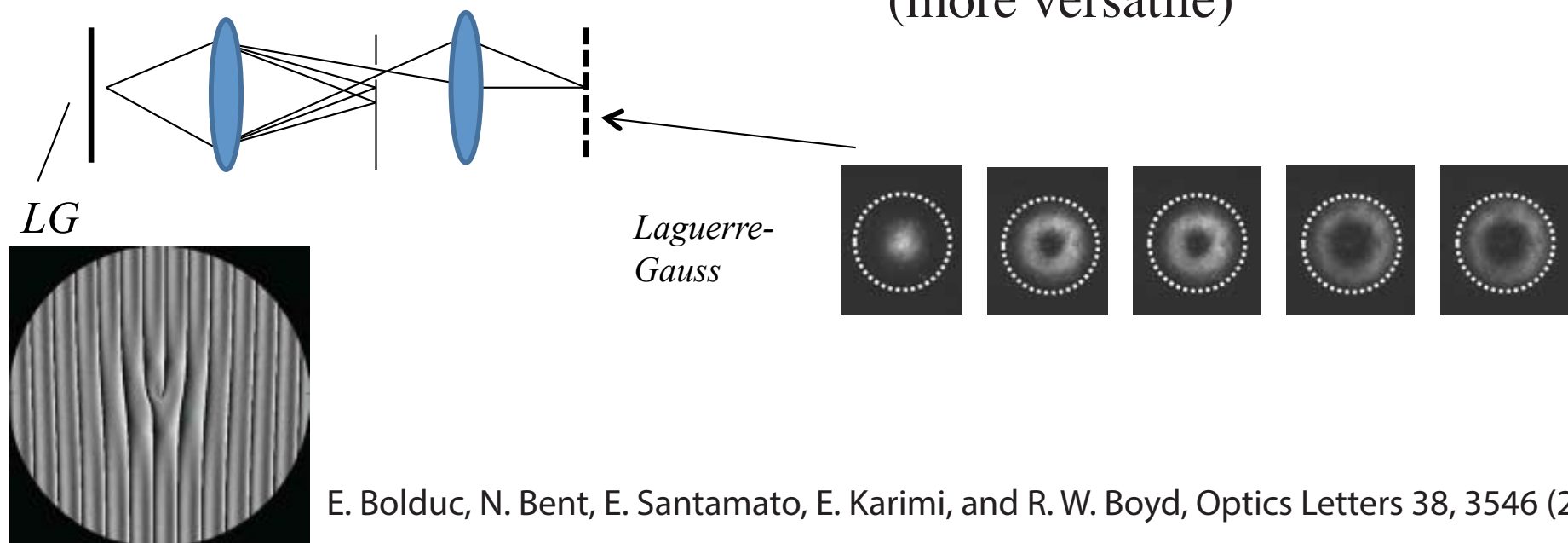


How to create a beam carrying orbital angular momentum?

Pass beam through a spiral phase plate



Use a spatial light modulator acting as a computer generated hologram
(more versatile)

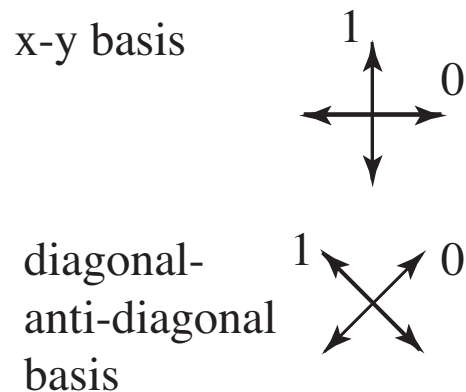


OAM States and Quantum Key Distribution

- The most widely studied protocol is that of Bennett and Brassard (1984), known as the BB84 protocol. It makes use of measurements performed on a single photon, but in more than one set of bases.
- Our work involves an extension of the BB84 protocol by making use of the OAM states of light. One motivation is to increase the data transfer rate by impressing more than one bit per photon.
- Let us begin by reviewing the BB84 protocol.

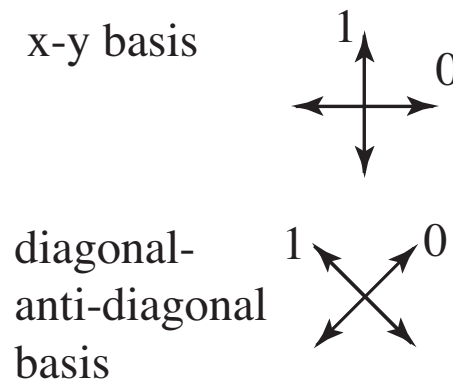
The BB84 QKD Protocol – Polarized Light Implementation

Alice sends an individual photon in one of two polarization bases, chosen at random



transmission

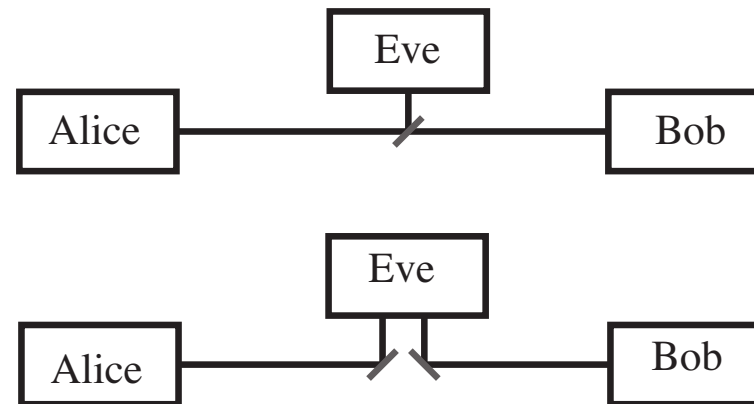
Bob receives in one of two polarization bases, which he chooses at random



After sending the entire string of numbers that constitutes the key, Alice and Bob openly divulge the basis that they used for each measurement. If they chose different bases, they discard the result of that measurement. (The remaining data is known as sifted data.)

Why Is This Protocol Secure?

- Suppose that an eavesdropper (Eve) intercepts the transmission. Since only one photon was transmitted, Bob will know that the message was intercepted, because he does not receive Alice's photon.
- To avoid divulging her presence in such an obvious manner, Eve can resend the photon after she intercepts it. But Eve has no guarantee that she will be sending the photon in the same basis as that used by Alice. And if she chooses wrong, Alice and Bob will realize that there is a problem.



QKD System Carrying Many Bits Per Photon

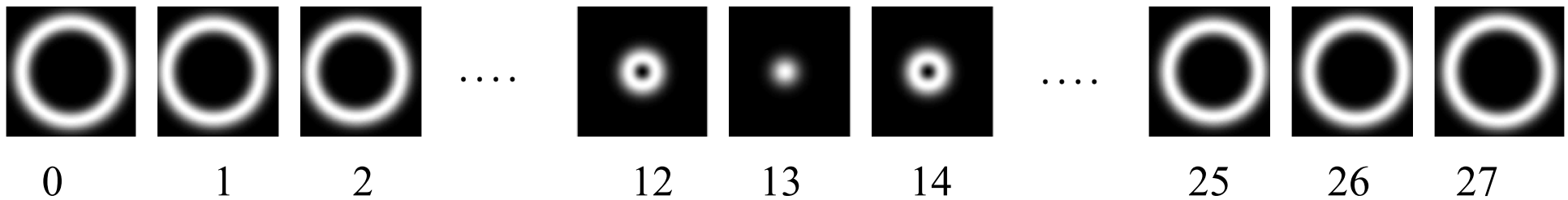
We are constructing a QKD system in which each photon carries many bits of information

We encode in states that carry OAM such as the Laguerre-Gauss states

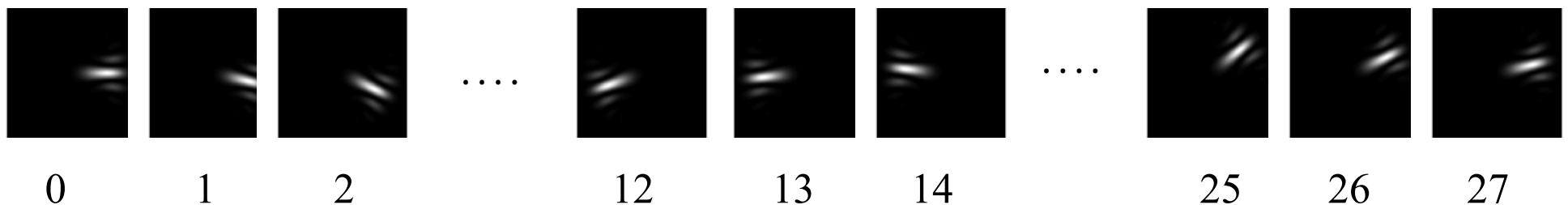
As a diagnostic, we need to be able to measure the statevector of OAM states

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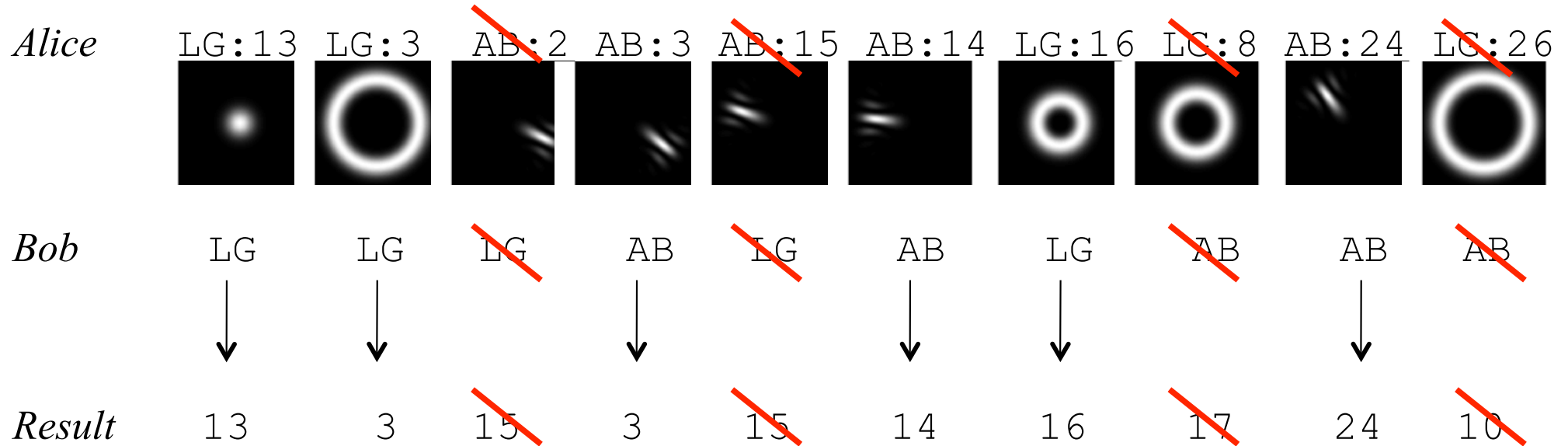


“Angular” Basis (mutually unbiased with respect to LG)



$$\Psi_{AB}^N = \frac{1}{\sqrt{27}} \sum_{l=-13}^{13} \text{LG}_{l,0} \exp(i2\pi Nl/27)$$

Protocol



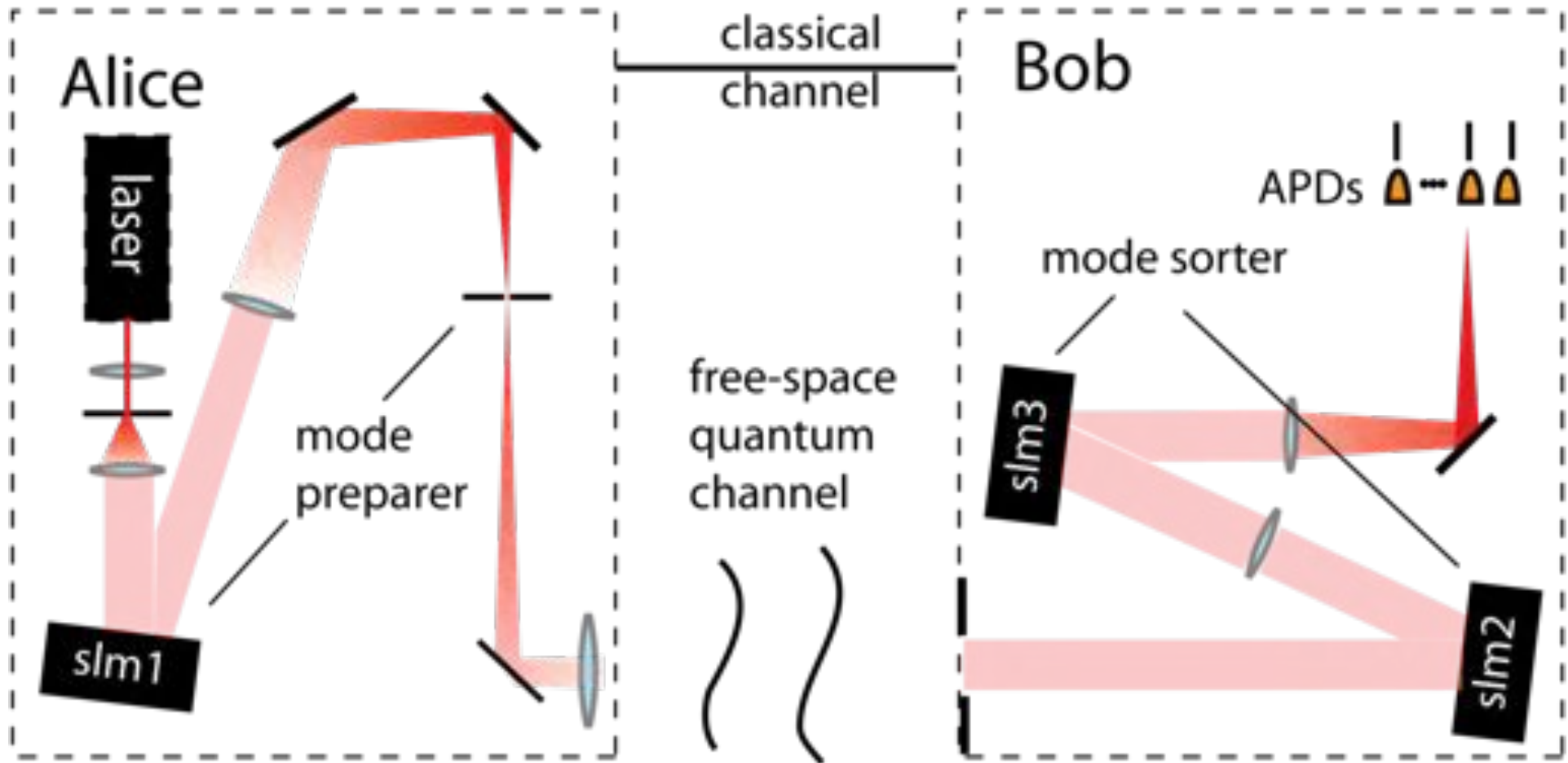
Sifted Key 13 3 3 14 16 24 ... ← in principle contains no errors unless eavesdropper is present.

In any real system, Bob's key will have errors due to system imperfections.

1. Error Correction (Cascade Protocol)
2. Privacy Amplification

Under many conditions, these protocols can be successfully implemented if Alice/Bob share more bits of information than Alice and Eve.

Spatially-Based QKD System



Source

Weak Coherent Light
Heralded Single Photon

Protocol

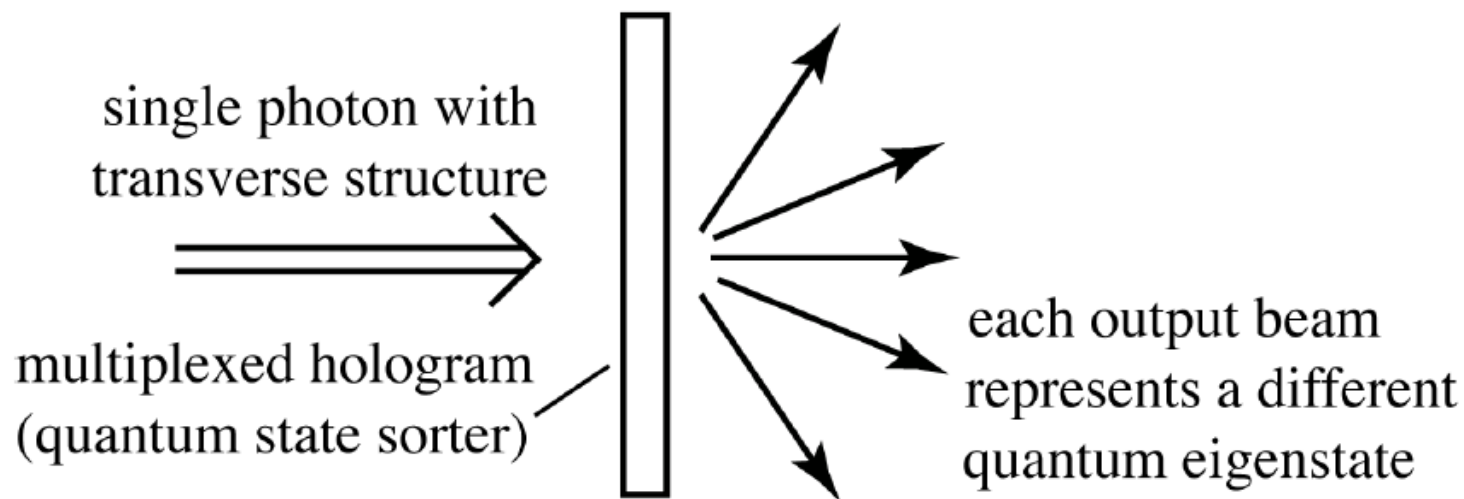
Modified BB84 as
discussed

Challenges

1. State Preparation
2. State Detection
3. Turbulence

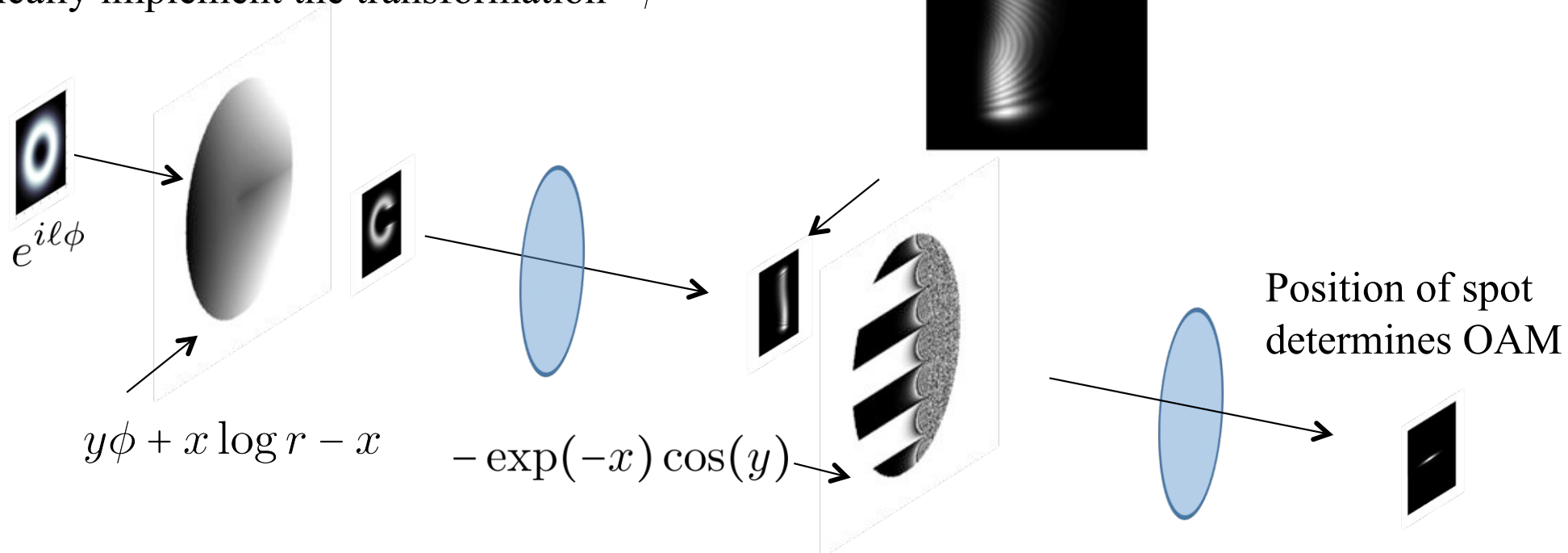
Mode Sorting

A mode sorter

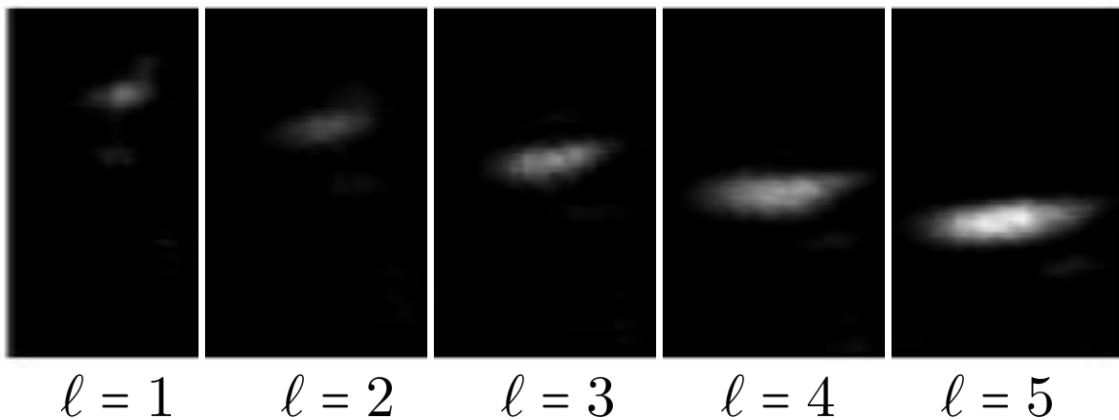


Sorting OAM using Phase Unwrapping

Optically implement the transformation $\phi \rightarrow x$



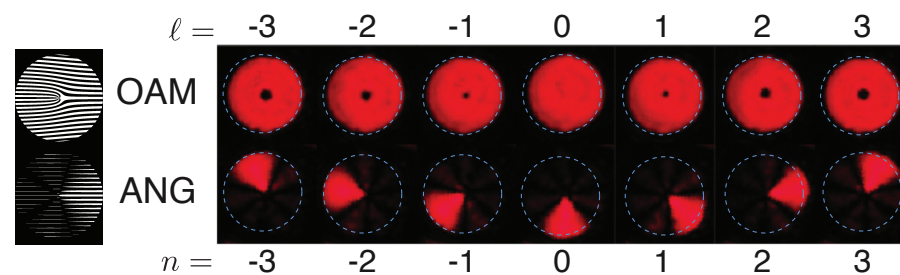
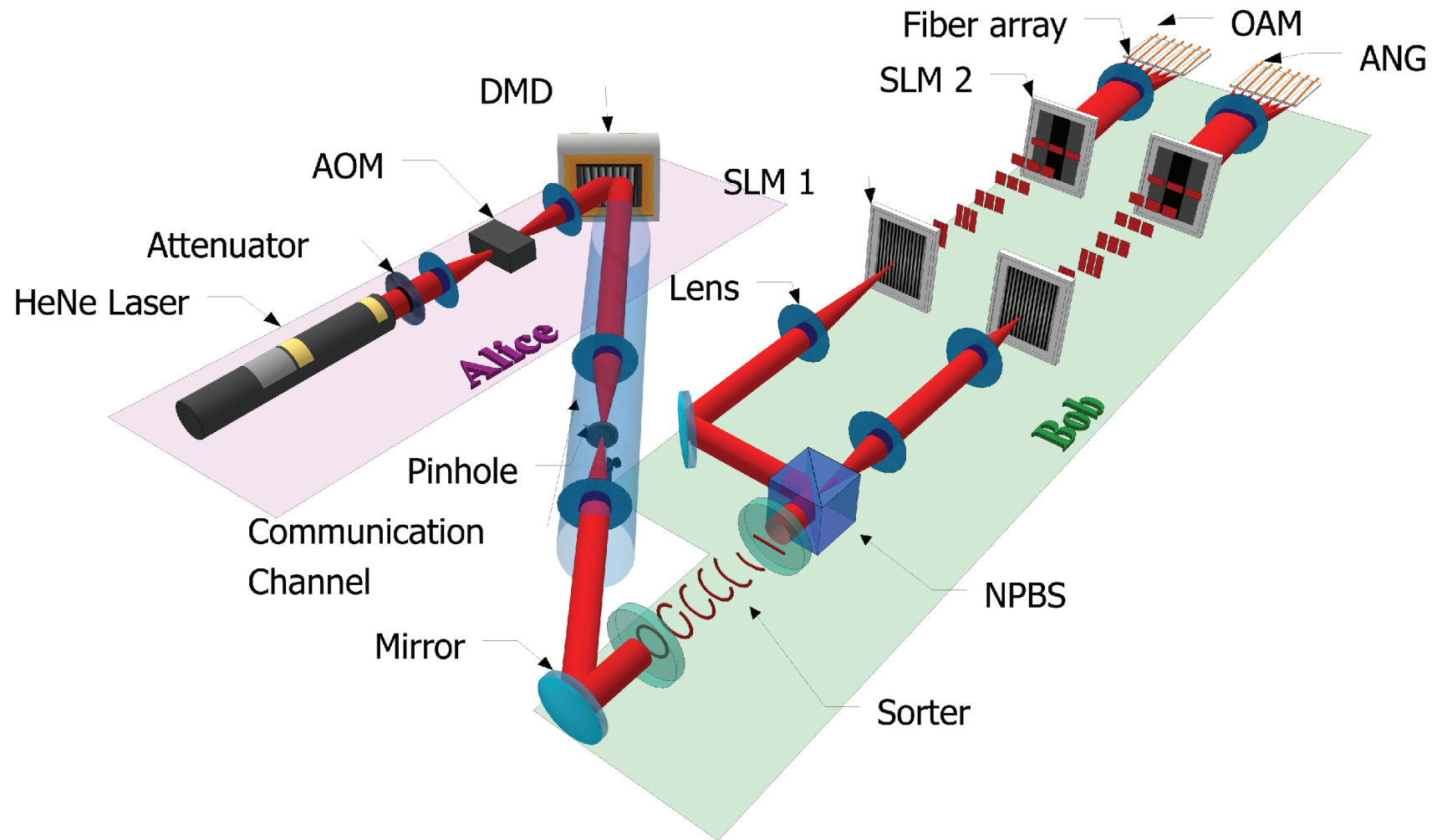
Experimental Results (CCD images in output plane)



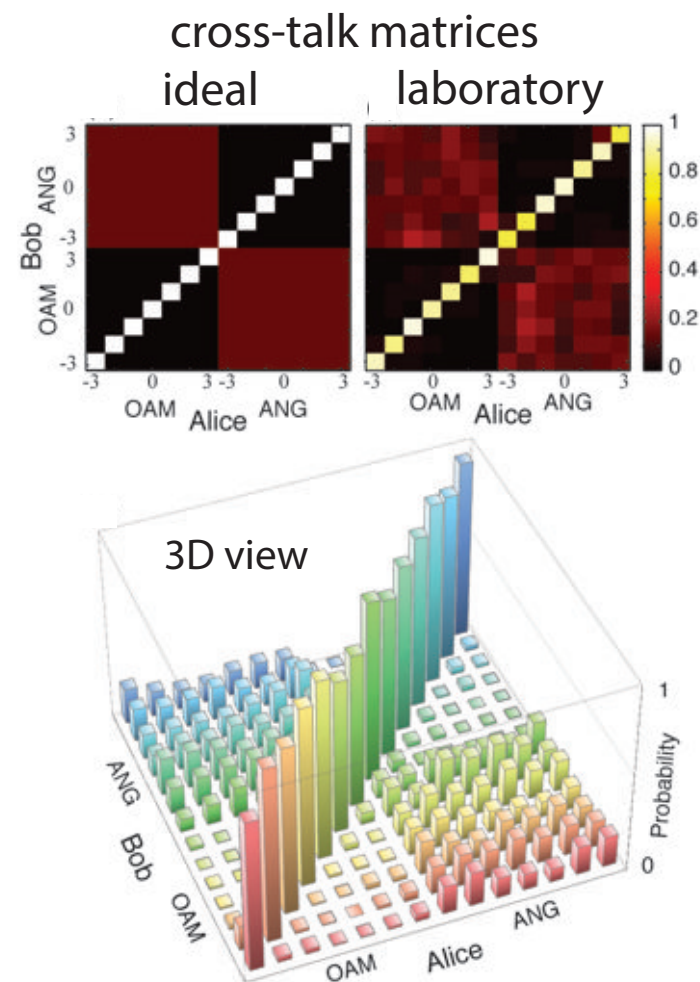
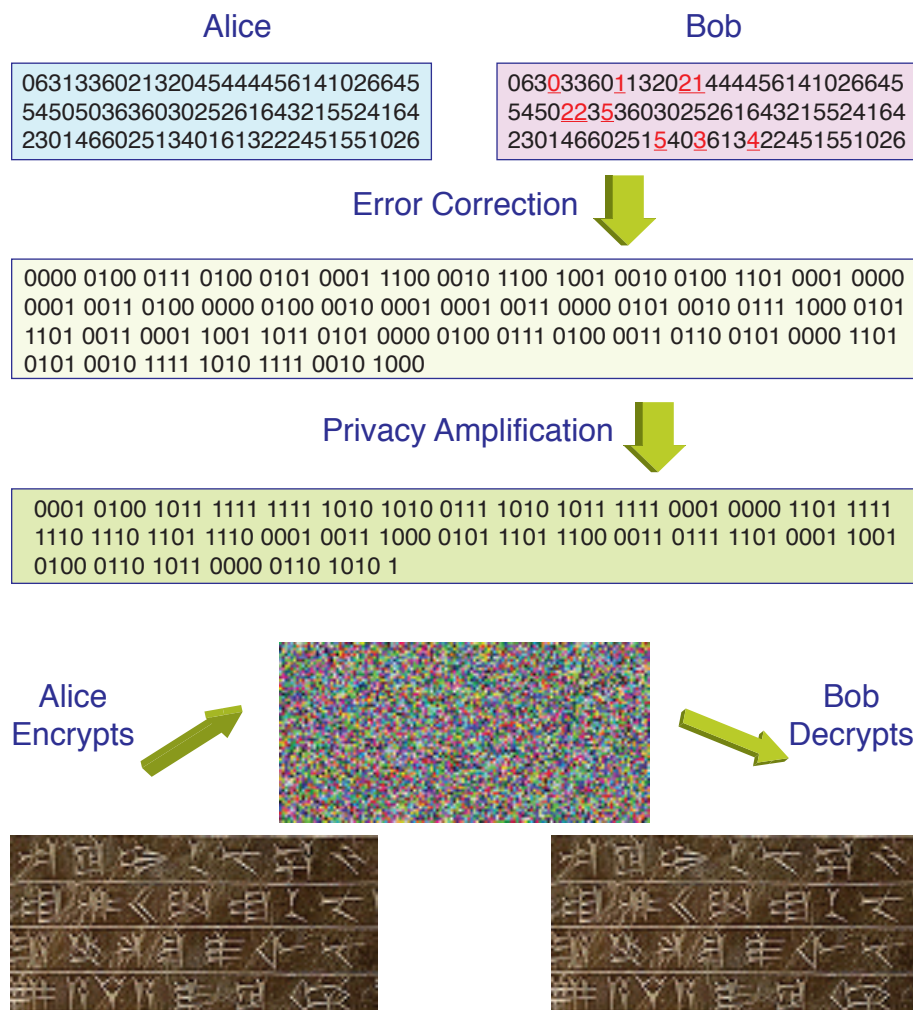
- Can also sort angular position states.
- Limited by the overlap of neighboring states.

*Berkhout *et al.* *PRL* **105**, 153601 (2010).
 O. Bryngdahl, *J. Opt. Soc. Am.* **64**, 1092 (1974).

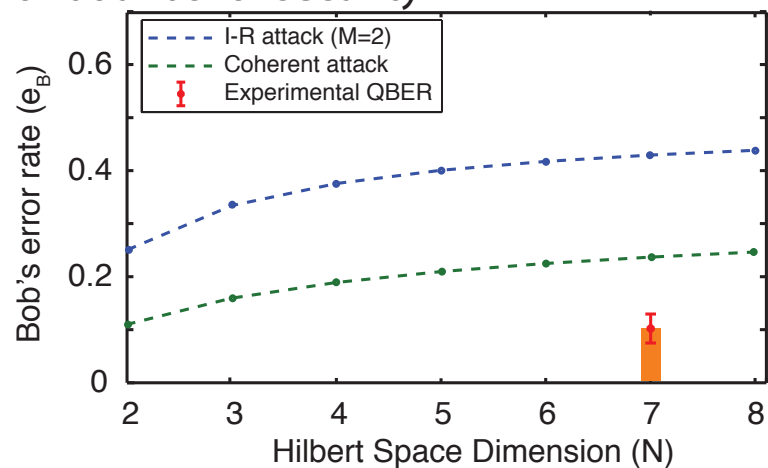
Laboratory Setup



Laboratory Results - OAM-Based QKD



- error bounds for security



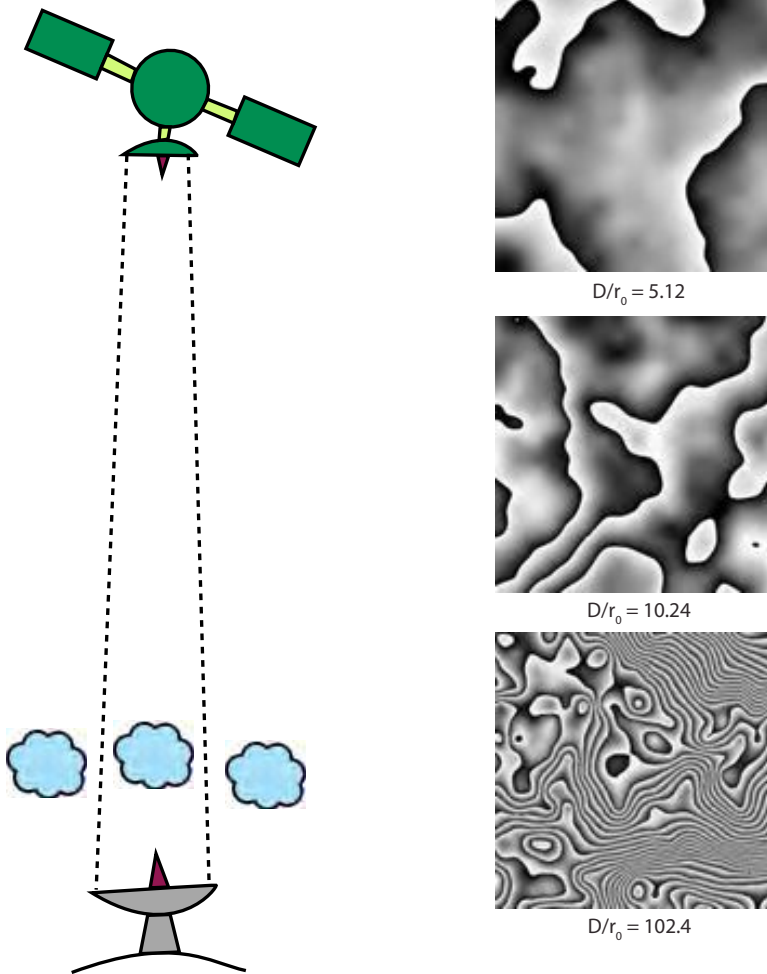
We use a 7-letter alphabet, and achieve a channel capacity of 2.1 bits per sifted photon.

We do not reach the full 2.8 bits per photon for a variety of reasons, including dark counts in our detectors and cross-talk among channels resulting from imperfections in our sorter.

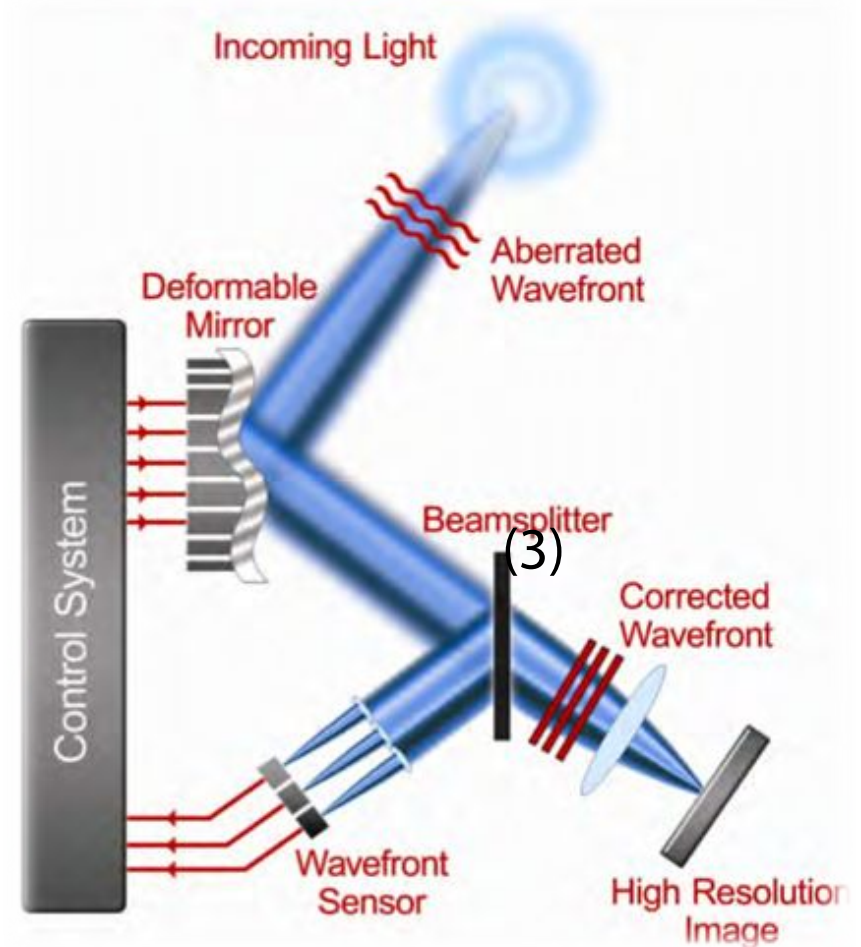
Nonetheless, our error rate is adequately low to provide full security,

Turbulence and Adaptive Optics

Atmospheric Turbulence Model



Our Adaptive Optics System



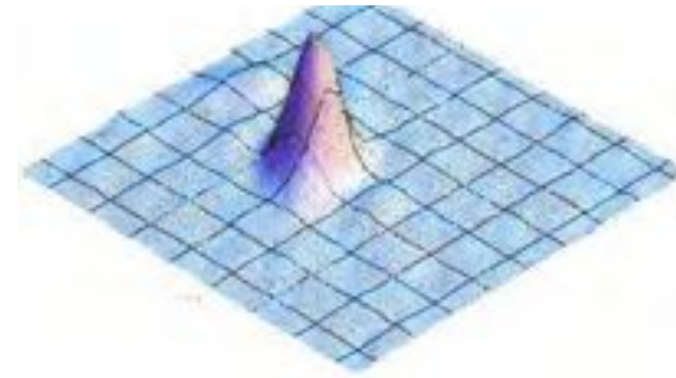
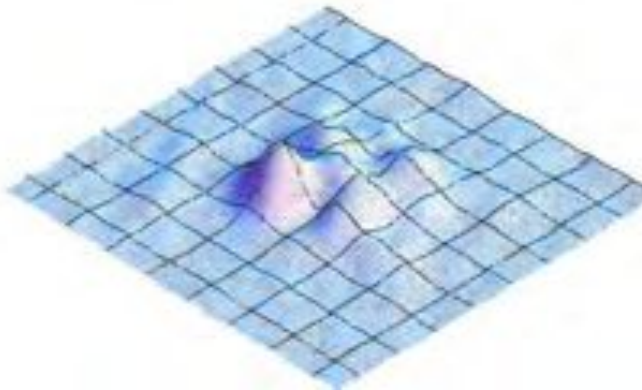
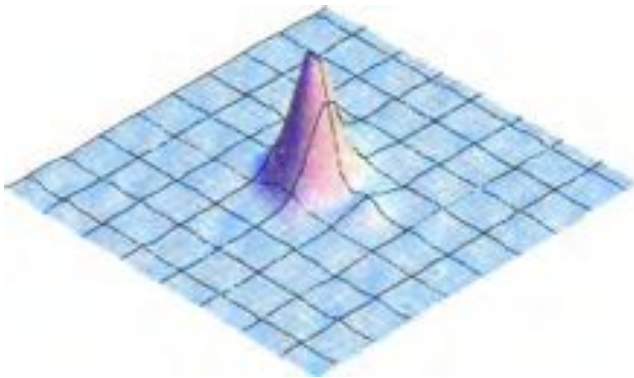
Improved QKD Performance Using Adaptive Optics

Before turbulence

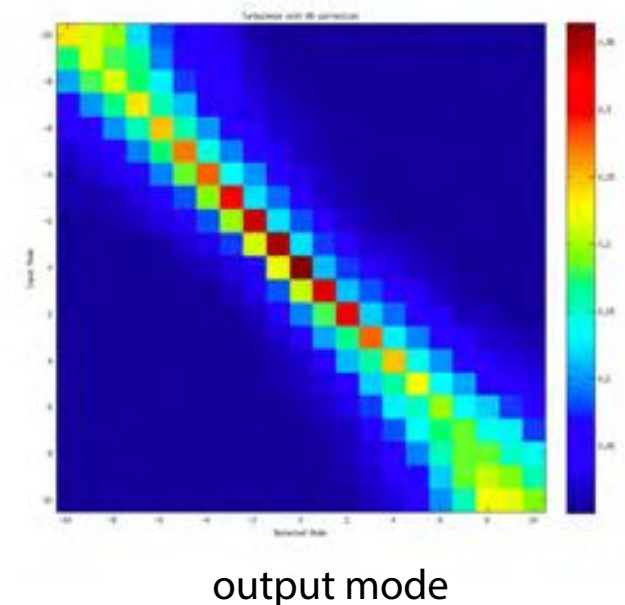
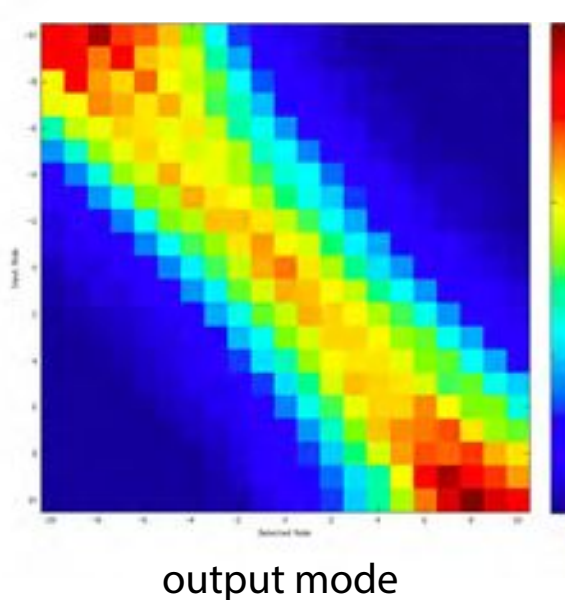
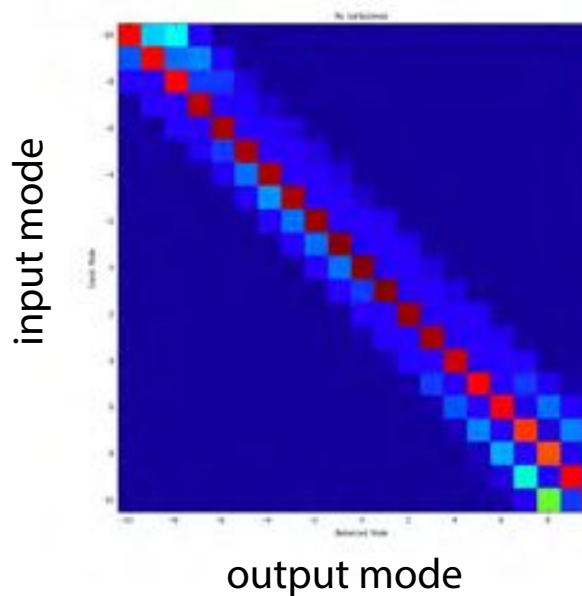
After turbulence

After adaptive optics correction

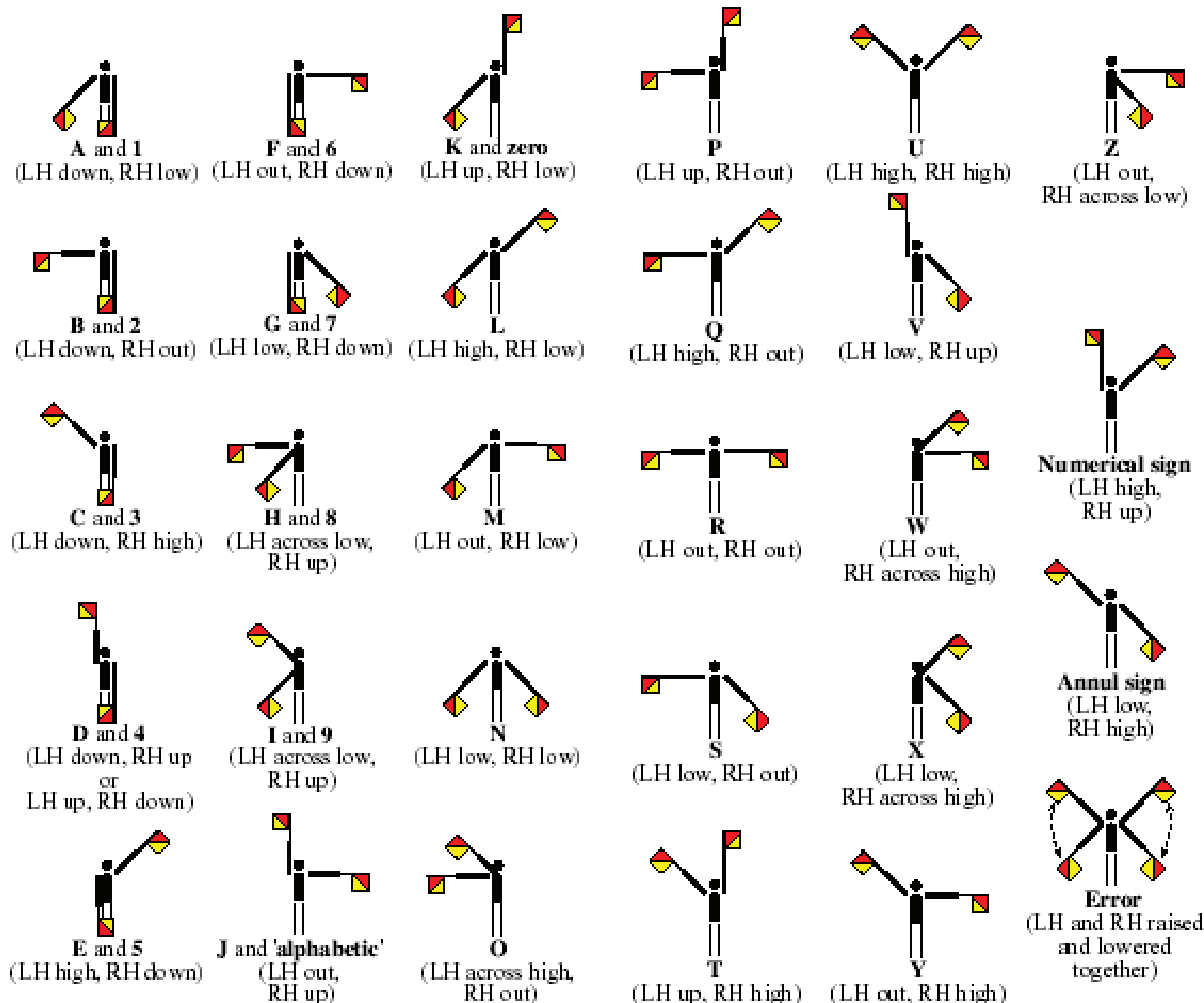
- focal plane distribution



- OAM cross talk

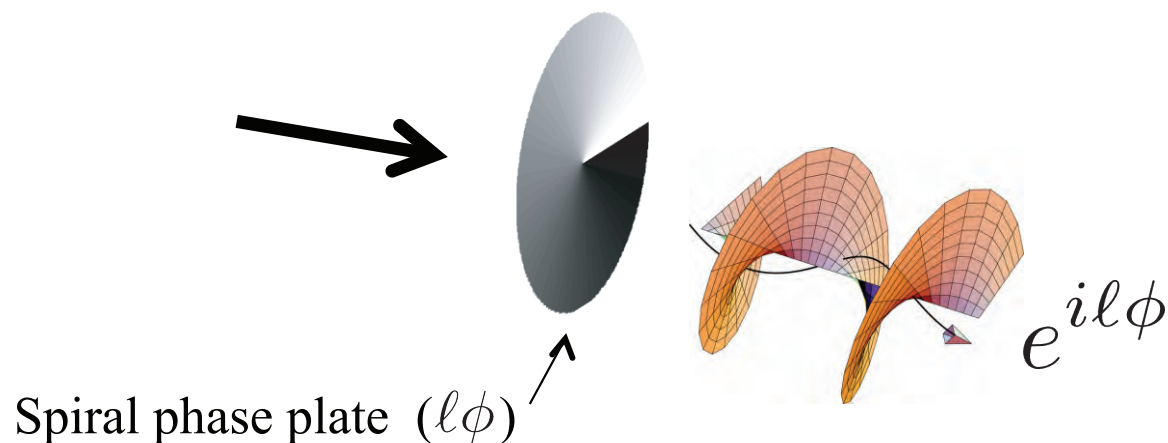


Free-Space Optical Telecommunication based on Transverse Field Structures

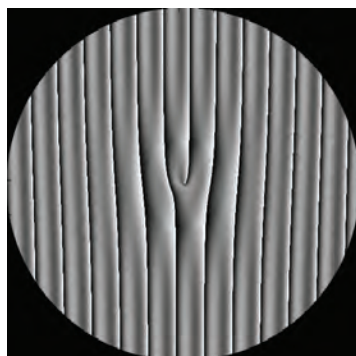
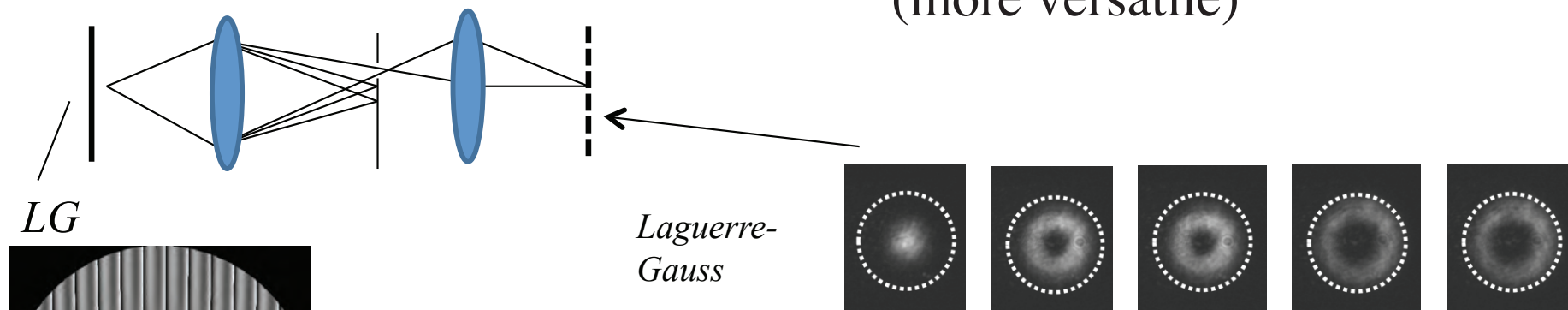


How to create a beam carrying orbital angular momentum?

Pass beam through a spiral phase plate



Use a spatial light modulator acting as a computer generated hologram
(more versatile)



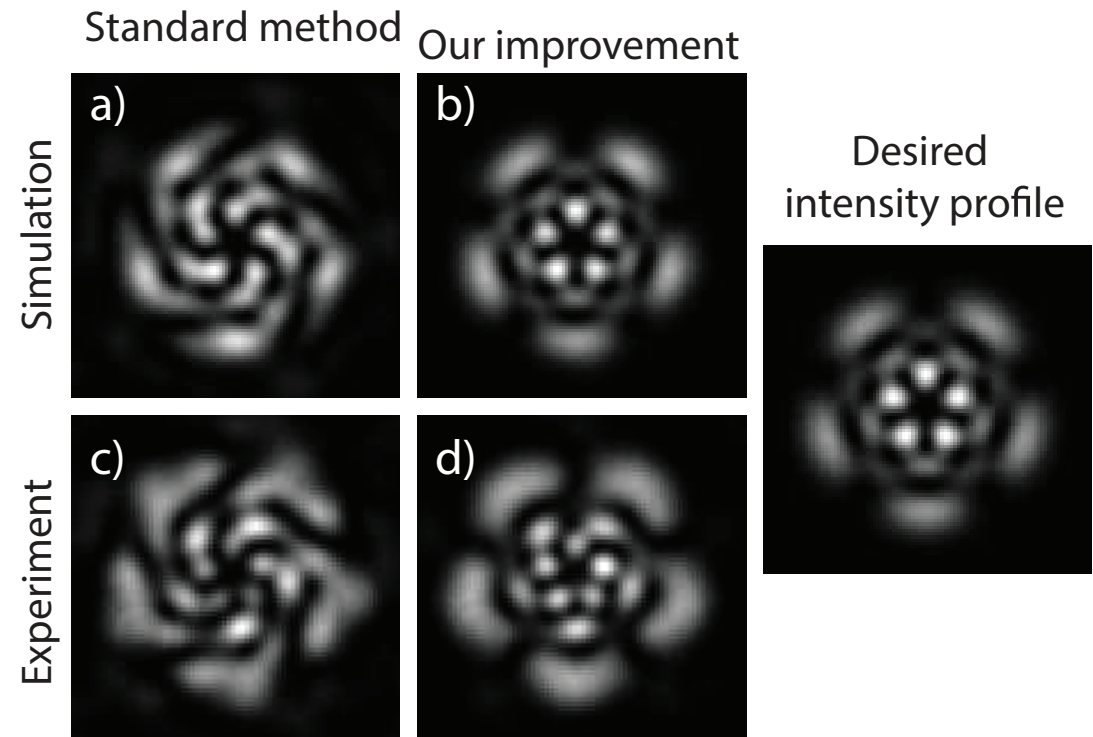
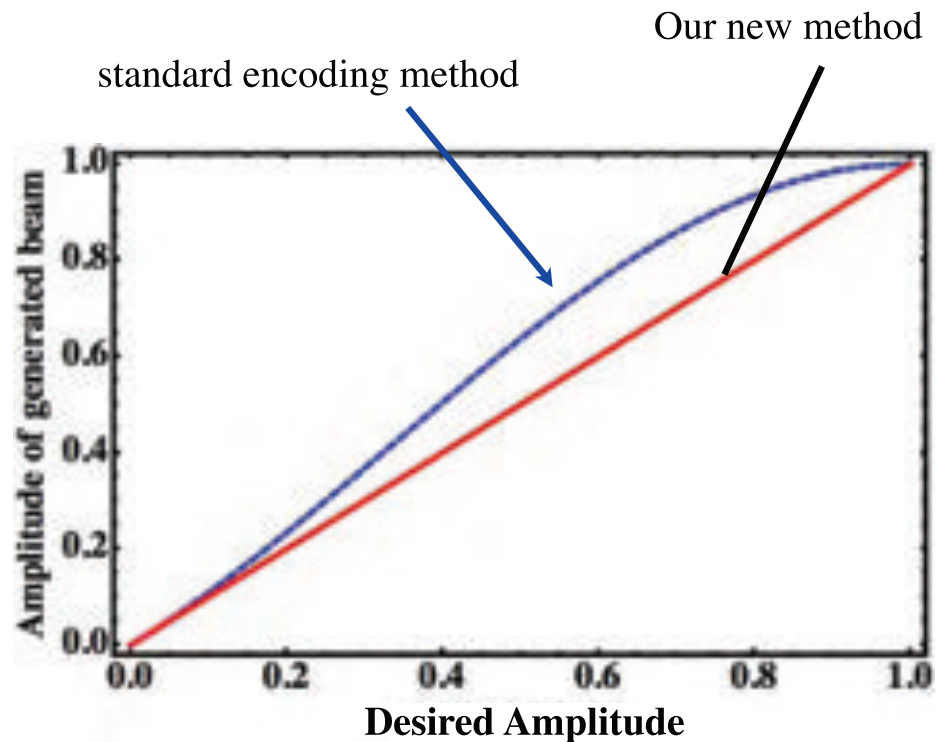
Exact solution to simultaneous intensity and phase masking with a single phase-only hologram, E. Bolduc, N. Bent, E. Santamato, E. Karimi, and R. W. Boyd, Optics Letters 38, 3546 (2013).

Exact Solution to Simultaneous Intensity and Phase Masking with a Single Phase-Only Hologram

We believe that previously workers used good but only approximate algorithms to encode holograms onto SLMs.

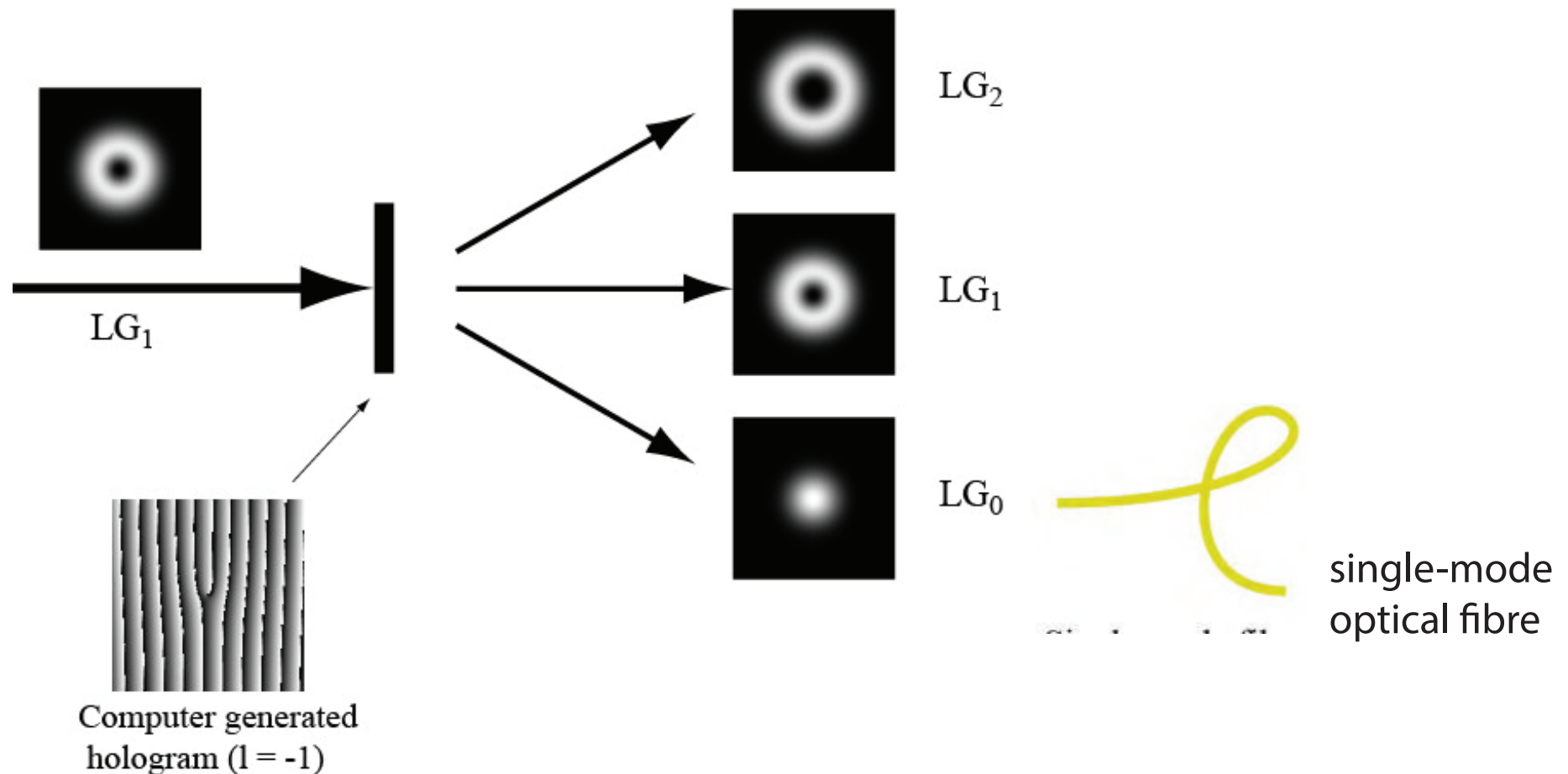
We have found an encoding algorithm that is mathematically exact.

We are currently investigating the conditions under which the use of the approximate encoding method could lead to significant errors.



How to Measure the OAM Azimuthal Quantum Number of a Laguerre-Gauss Beam

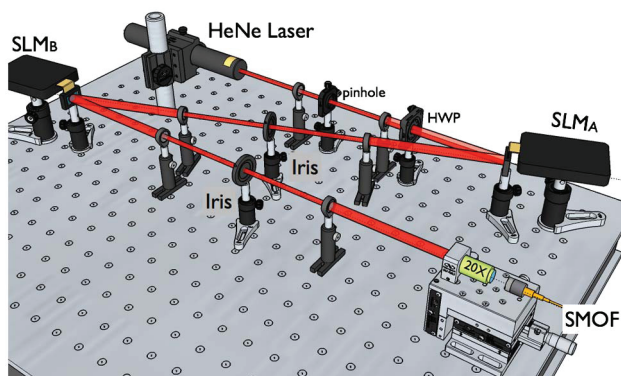
Standard Method – Projective Measurement



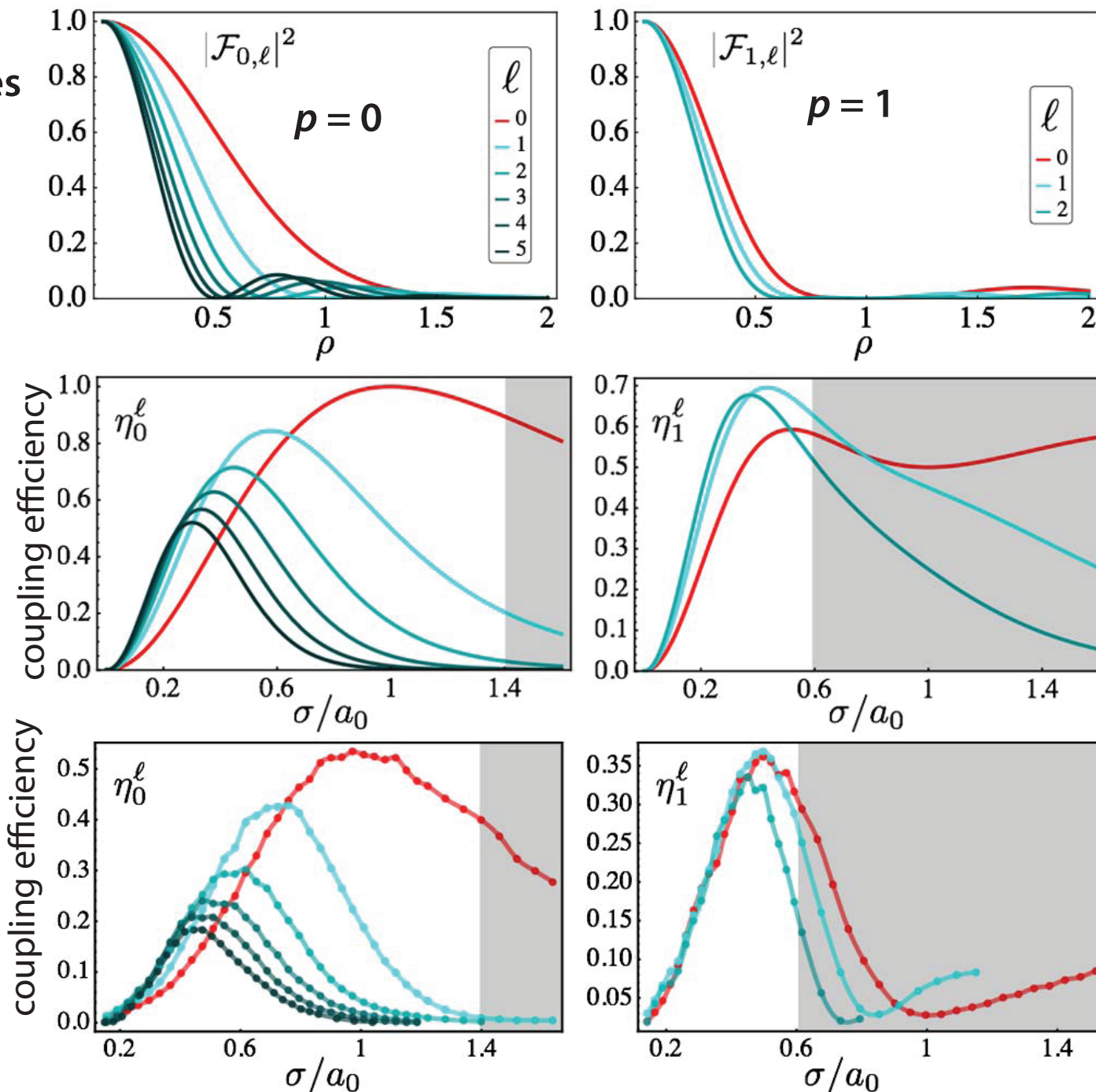
Seems almost too good to be true

We Have Quantified the Level of Inaccuracy Resulting from this Procedure

mode profiles



theory

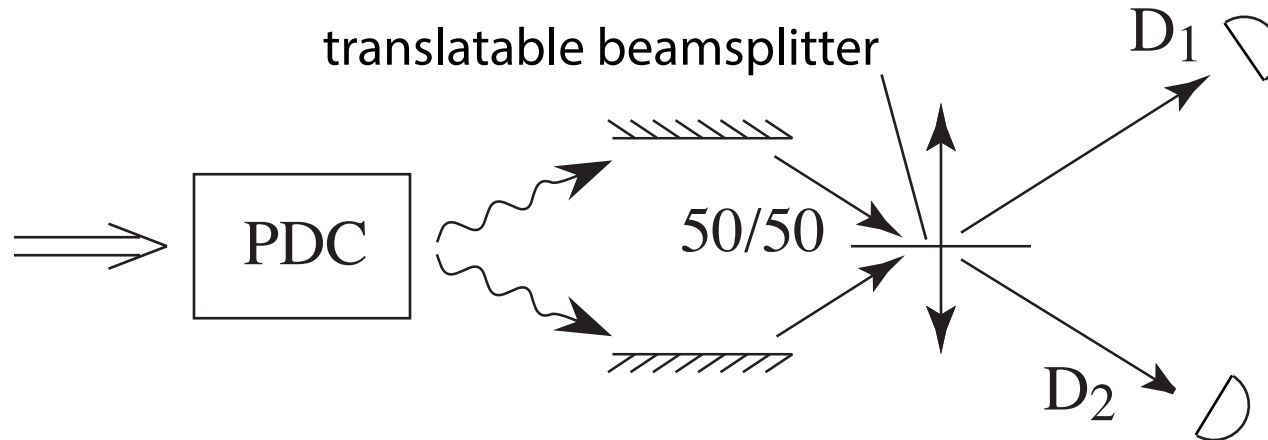


experiment

Conclusion: coupling efficiency depends on mode indices, properties of fibre, and on degree of focussing

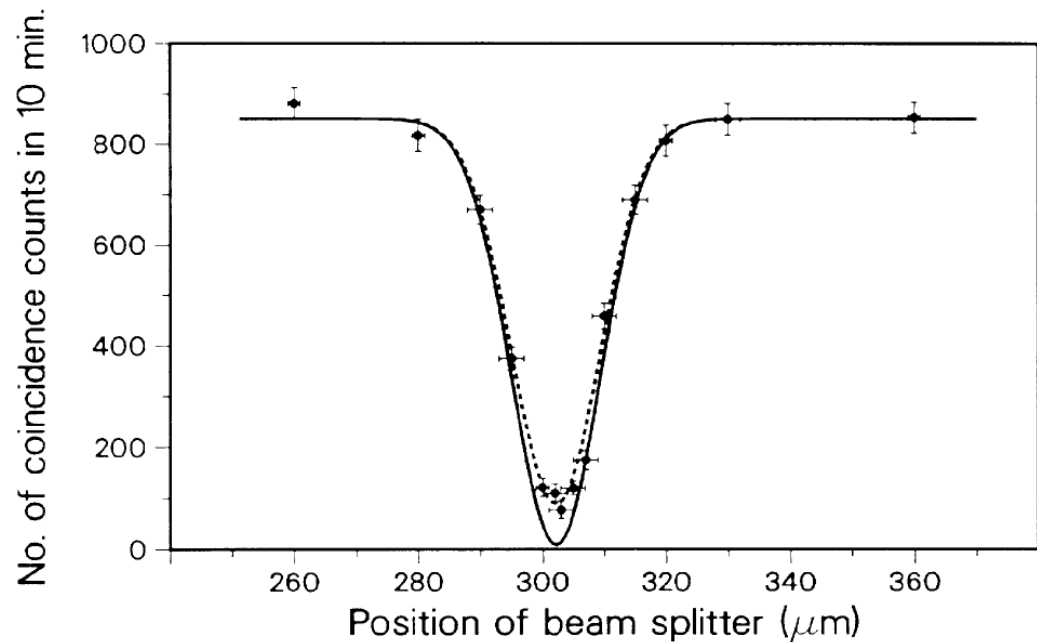
σ is the beam waist radius of the SMOF Gaussian mode.
 $a_0 = (\sqrt{2}\lambda f)/(\pi w_0)$

The Hong-Ou-Mandel Effect



- measure coincidences between D1 and D2
- coincidence rate drop to zero when the photons are indistinguishable

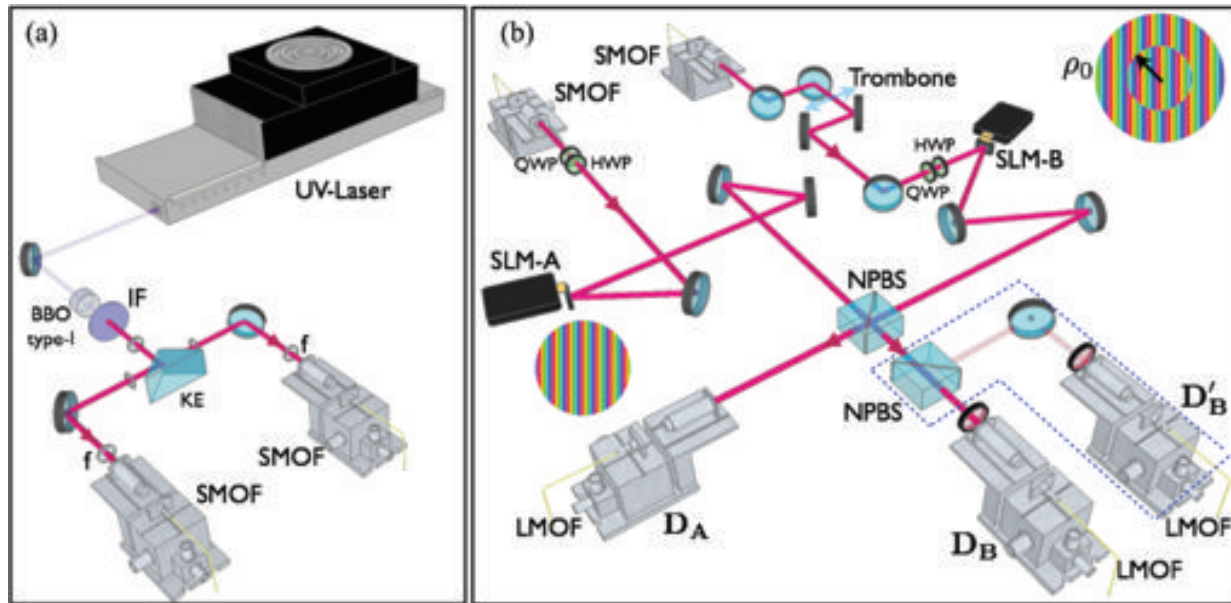
• Hong, Ou, Mandel, PRL, 1987



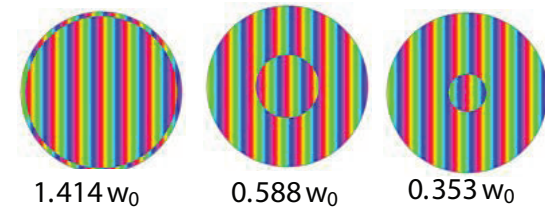
Hong-Ou-Mandel Effect and and LG Beams

- We can control the degree of distinguishability through the radial degree of freedom of a LG beam.
- Note that the radius is a continuous variable; azimuthal index is a discrete variable.

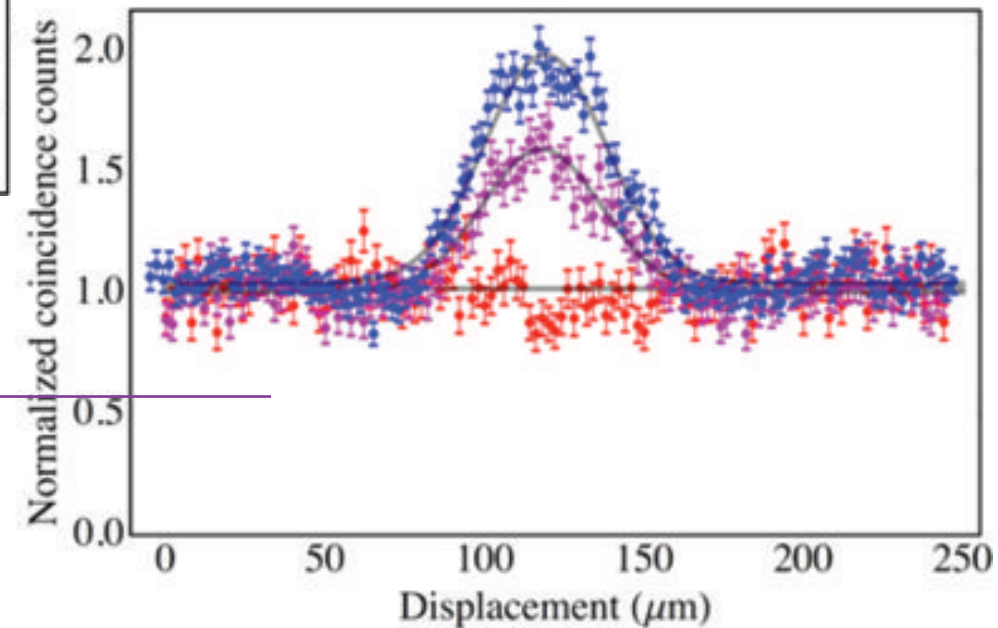
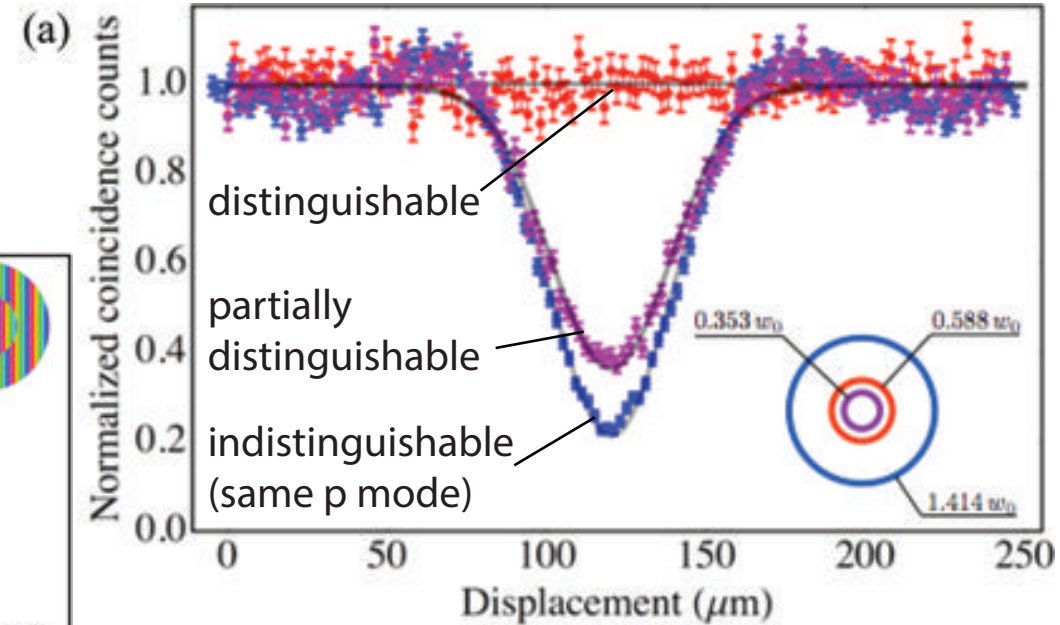
Experimental setup



- coincidences between D_B and D'_B
- measurement of coalescence



coincidences between D_A and D_B



How to Fit a Square Peg into a Round Hole

(Really, how to couple an elliptical beam into an optical fiber.)



ARTICLE

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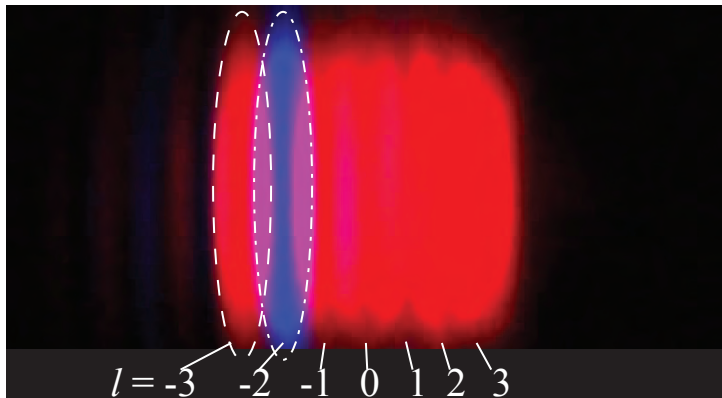
DOI: 10.1038/ncomms3781

Efficient separation of the orbital angular momentum eigenstates of light

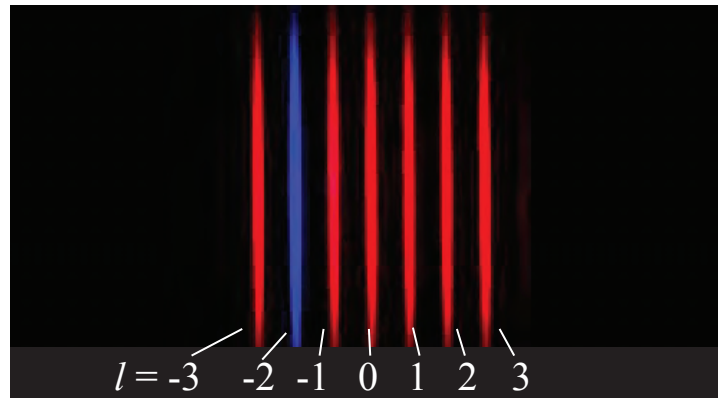
Mohammad Mirhosseini¹, Mehul Malik^{1,2}, Zhimin Shi^{1,3} & Robert W. Boyd^{1,4}

- Output of Glasgow sorter is elliptical, and ellipticity made worse by a 1D fanout.
- Solution: use a 2-D fanout, implemented on an SLM at present

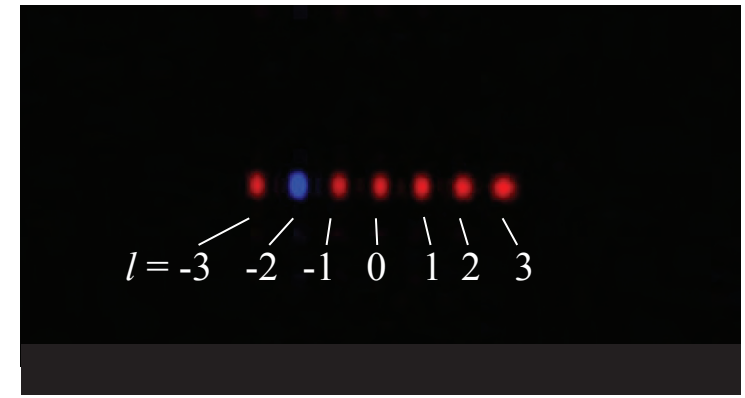
Output of sorter



Output after 7X fanout

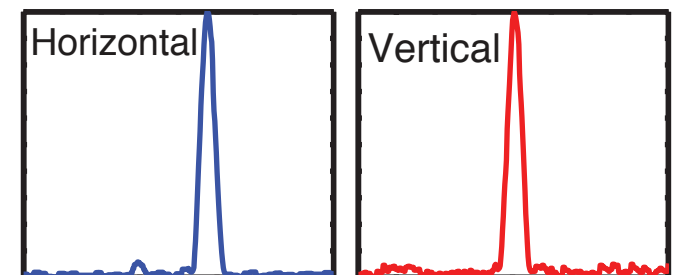


Output after 3 by 9 fanout



output nearly round

Experimental separation efficiency > 92%

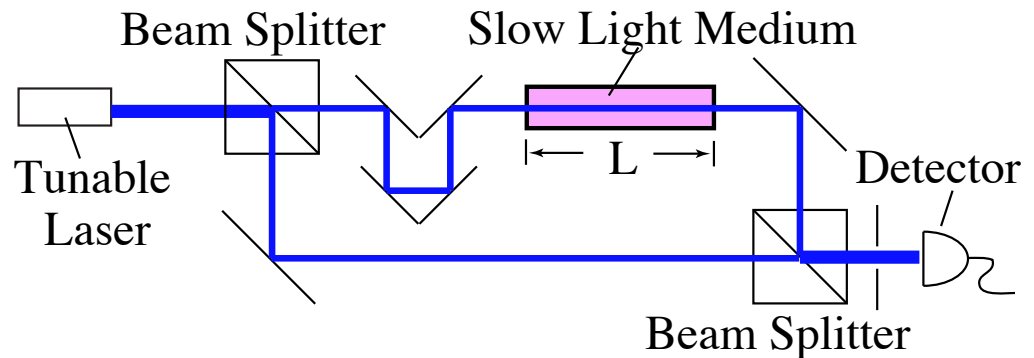


Our Program in Nanophotonics

Nanofabrication Goal: Chip-Scale Slow Light Spectrometer

- The spectral sensitivity of an interferometer is increased by a factor as large as the group index of a material placed within the interferometer.
- We want to exploit this effect to build chip-scale spectrometers with the same resolution as large laboratory spectrometers
- Here is why it works:

Slow-light interferometer:

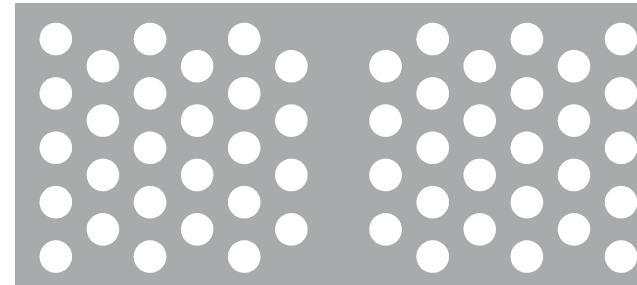


Simple analysis

$$\frac{d \Delta\phi}{d\omega} = \frac{d}{d\omega} \frac{\omega n L}{c} = \frac{L}{c} \left(n + \omega \frac{dn}{d\omega} \right) = \frac{L n_g}{c}$$

- We use line-defect waveguides in photonic crystals as our slow light mechanism

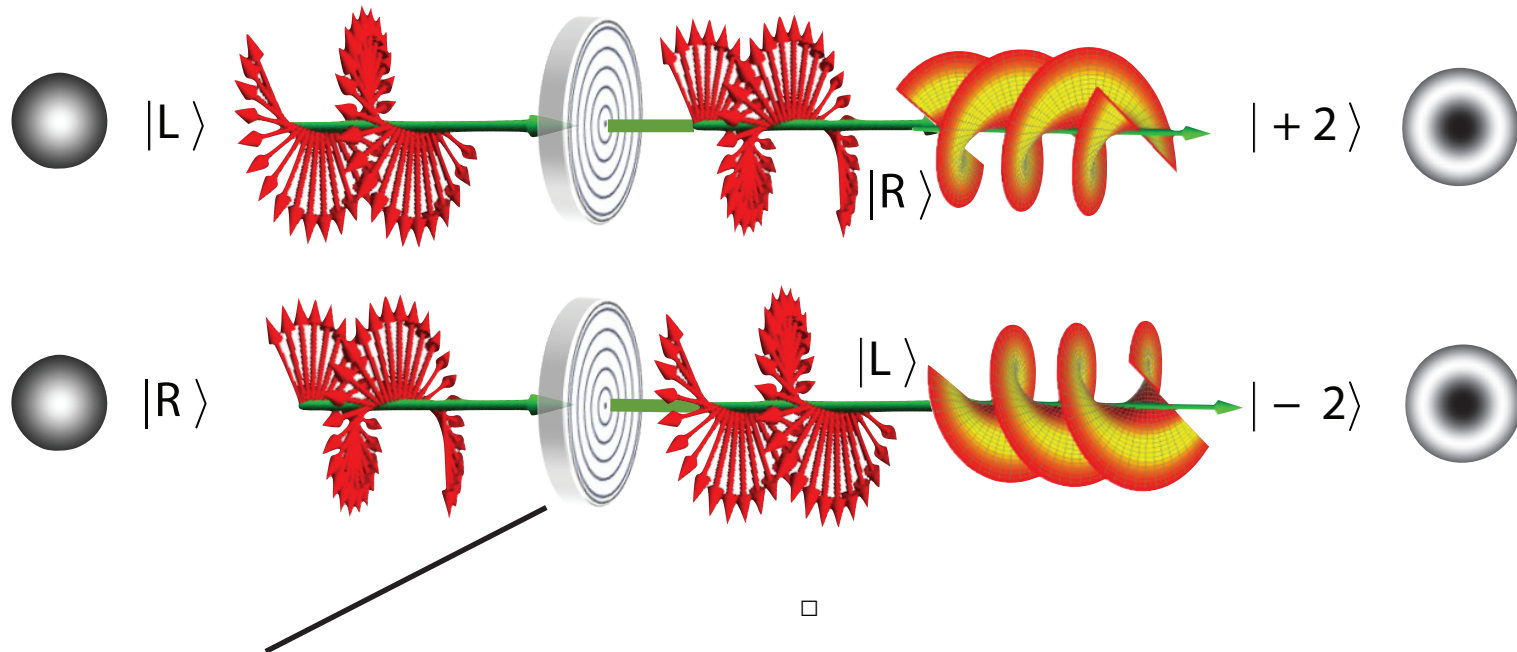
Slow-down factors of greater than 100 have been observed in such structures.



Photonic Q-Plates

Spin angular momentum can be transferred to OAM through use of a Q-plate

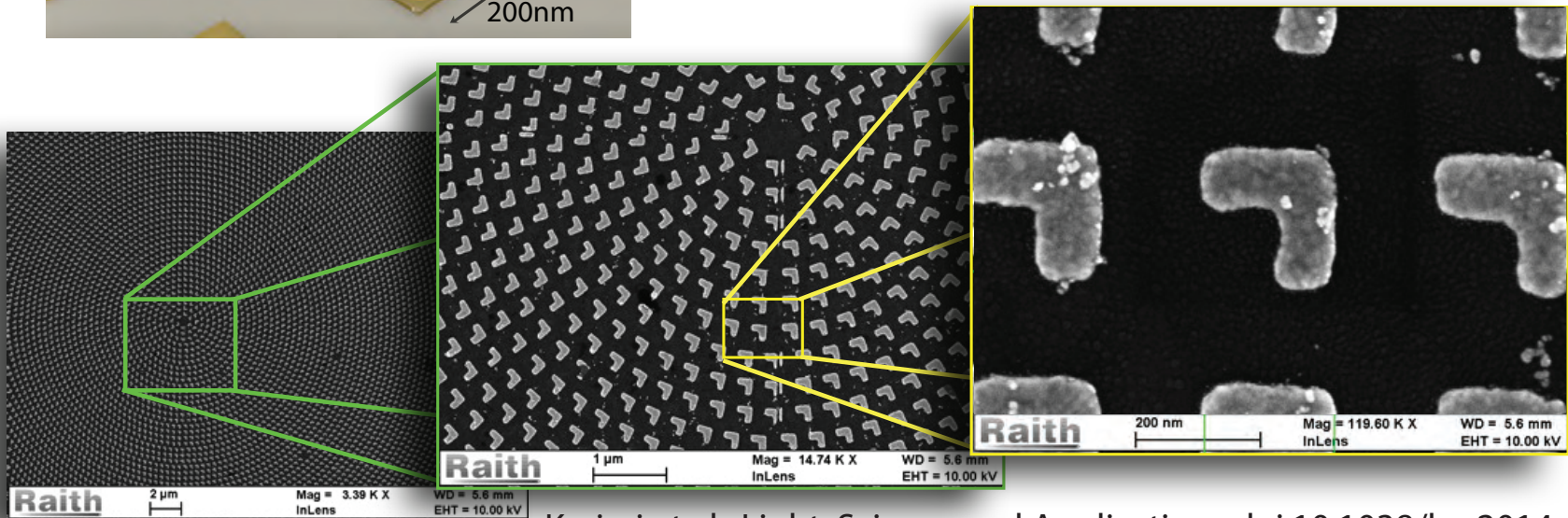
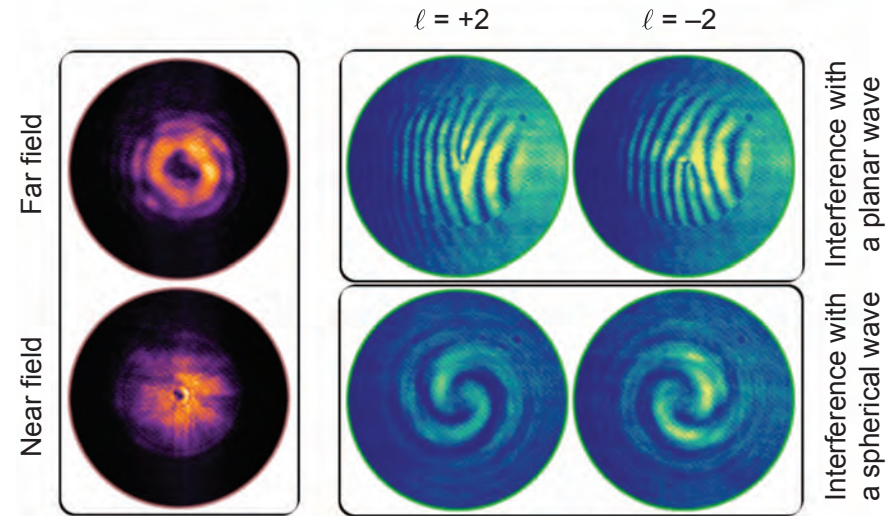
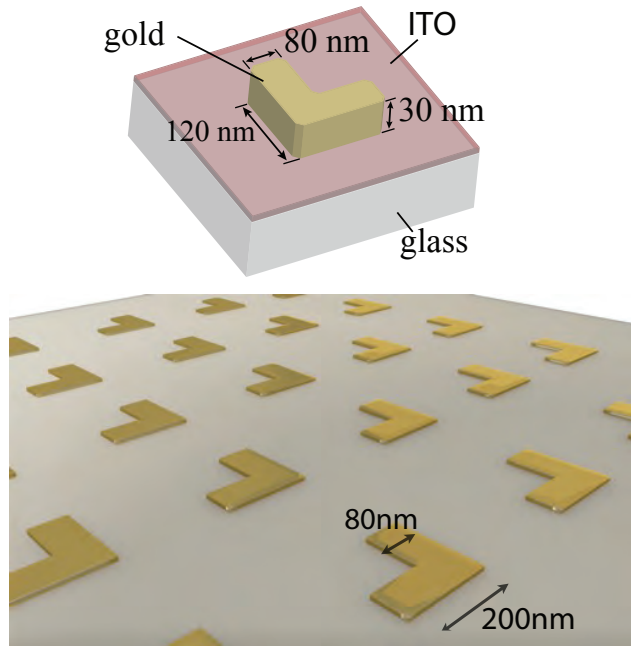
Ability to change basis of encoding useful for quantum information processing



Q-plate. Usually a carefully constructed liquid-crystal cell

Fabrication of a Nano-Scale q-Plate

- A q-plate is a device that converts spin angular momentum into orbital angular momentum.
- It functions as a quantum interface.
- Fabricated device is only 30-nm thick and thus suitable for use in integrated quantum circuits.



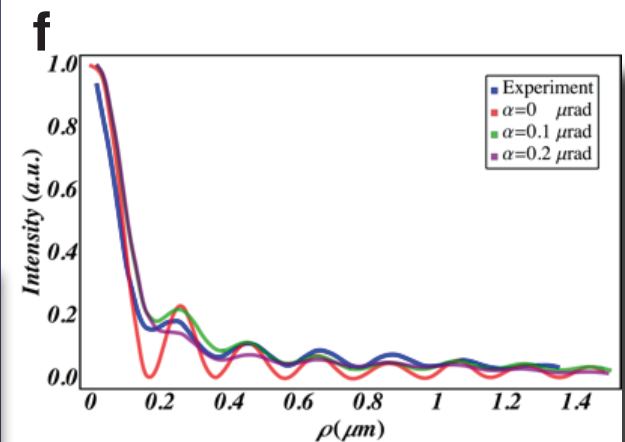
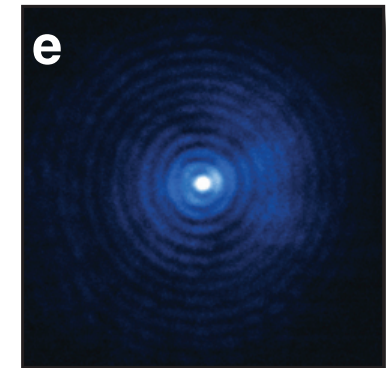
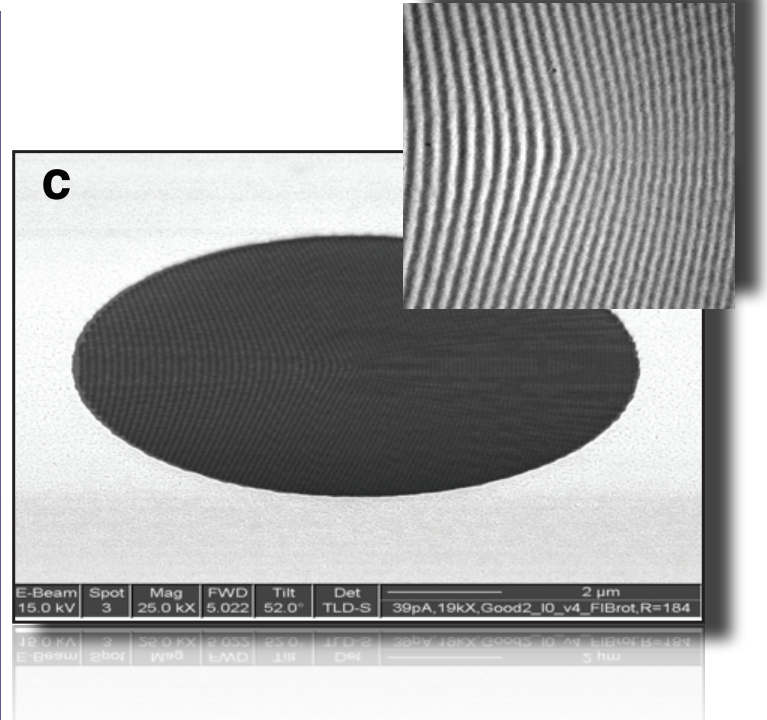
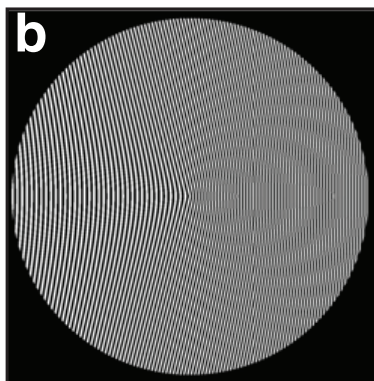
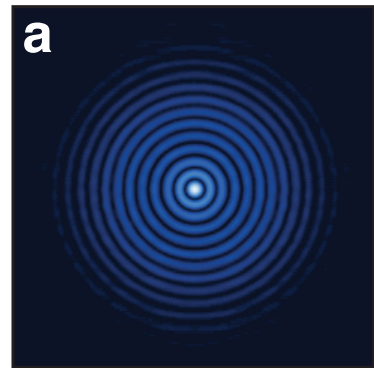
Demonstration of an Electron Bessel Beam

- Huge implications for electron microscopy!

Theoretical modeling

Hologram for electrons

Lab results

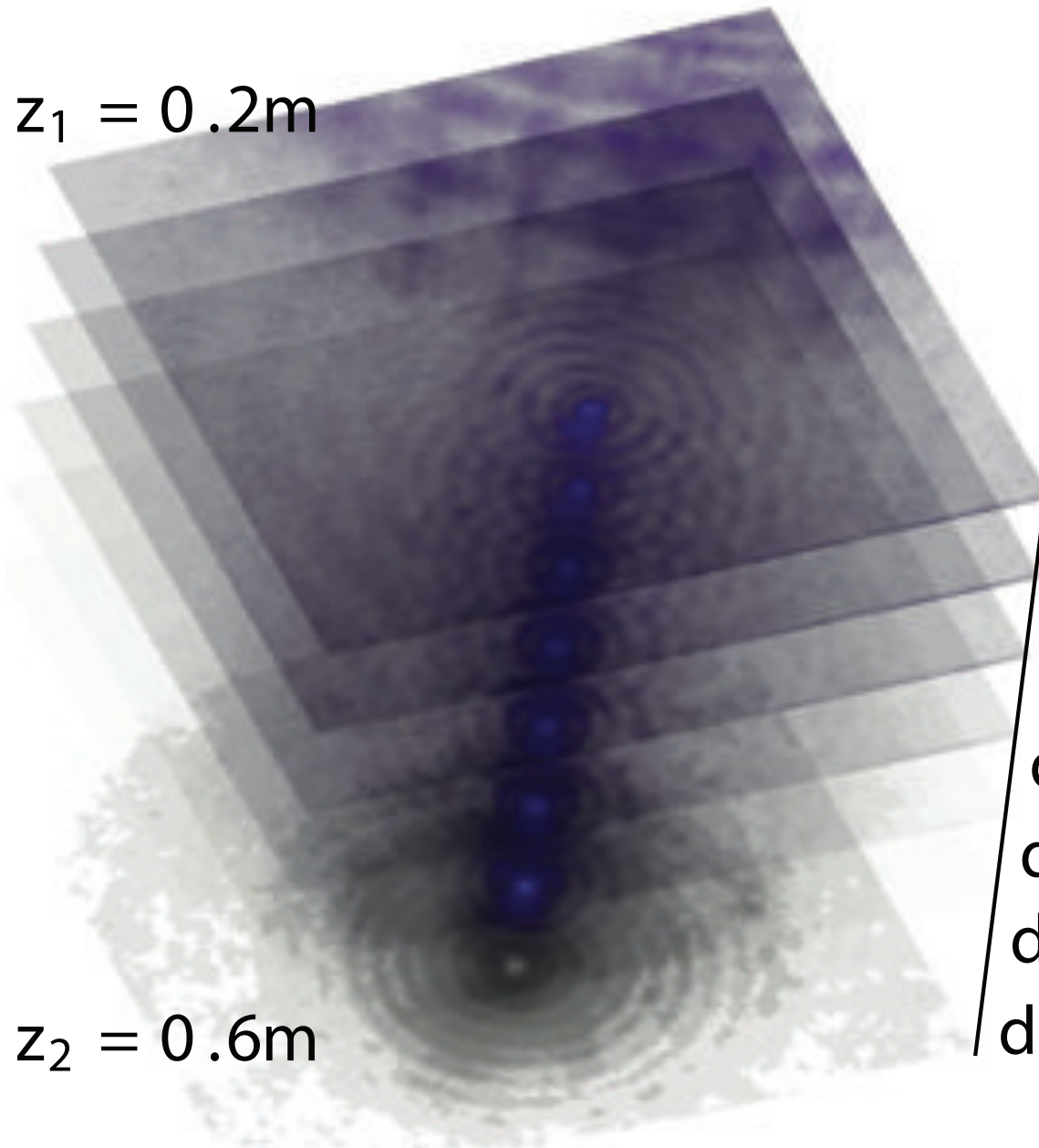


Propagates 0.8 Metres Without Diffractive Spreading!

$z_1 = 0.2\text{m}$

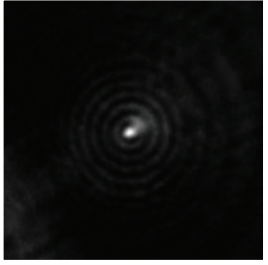
$z_2 = 0.6\text{m}$

$d = 240\text{ nm}$
 $d = 240\text{ nm}$
 $d = 285\text{ nm}$
 $d = 255\text{ nm}$
 $d = 300\text{ nm}$
 $d = 310\text{ nm}$
 $d = 275\text{ nm}$

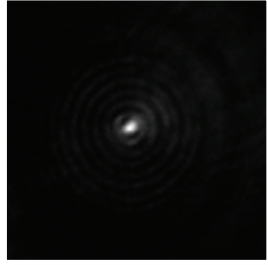


The Geometry

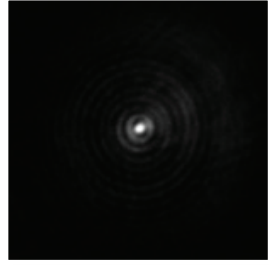
$z = 0.25\text{m}$



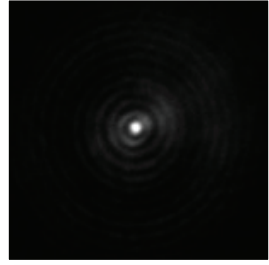
$z = 0.32\text{m}$



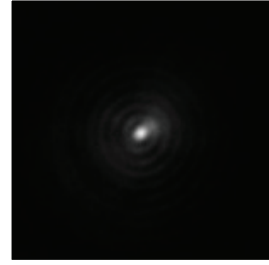
$z = 0.34\text{m}$



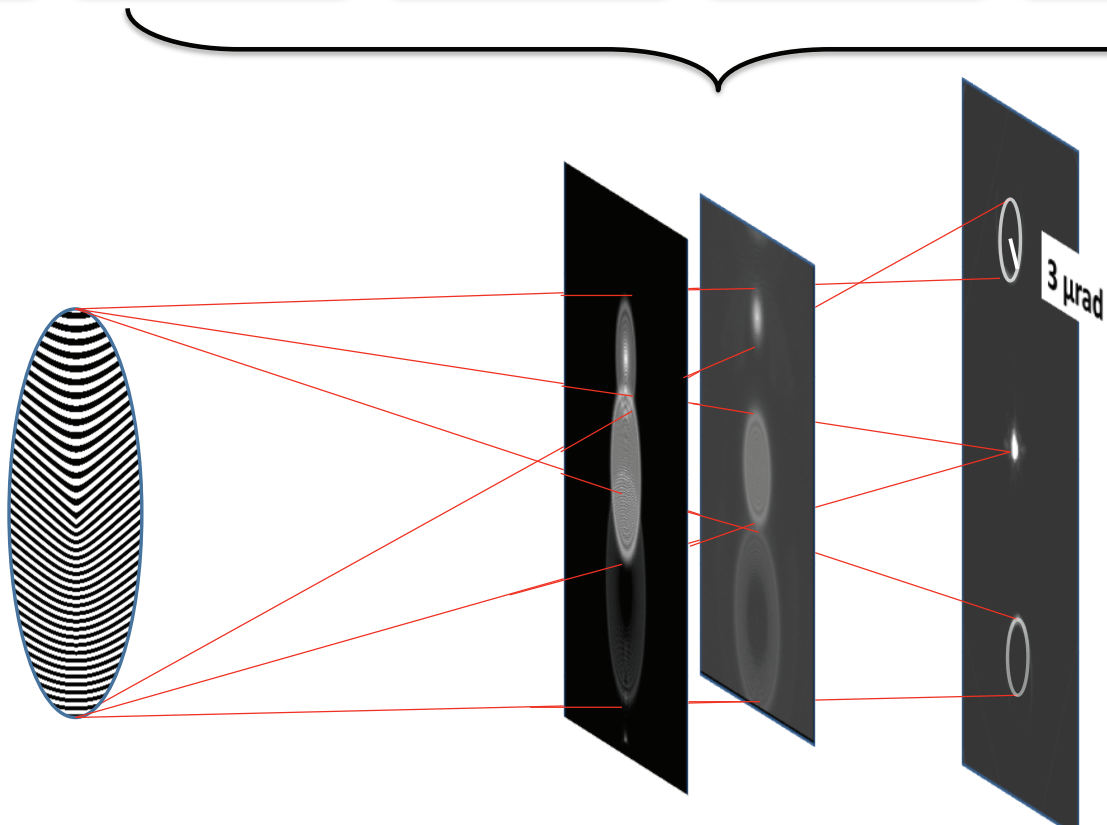
$z = 0.42\text{m}$



$z = 0.56\text{m}$



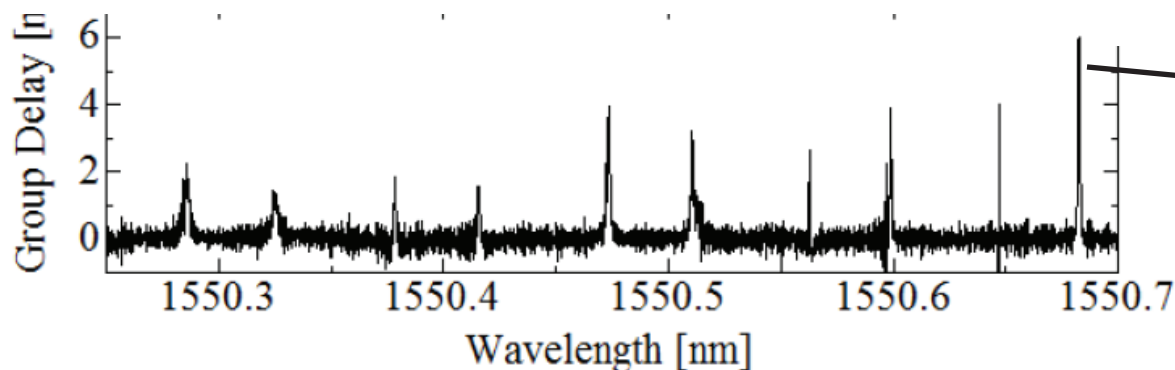
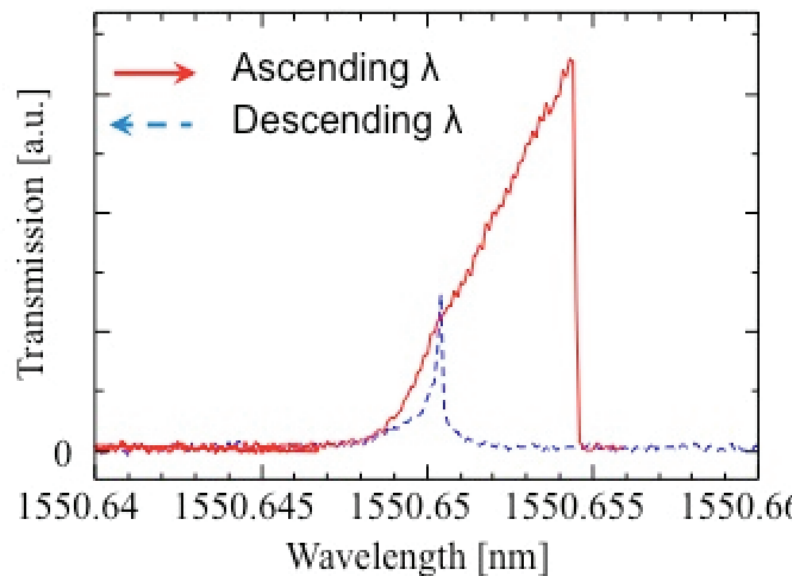
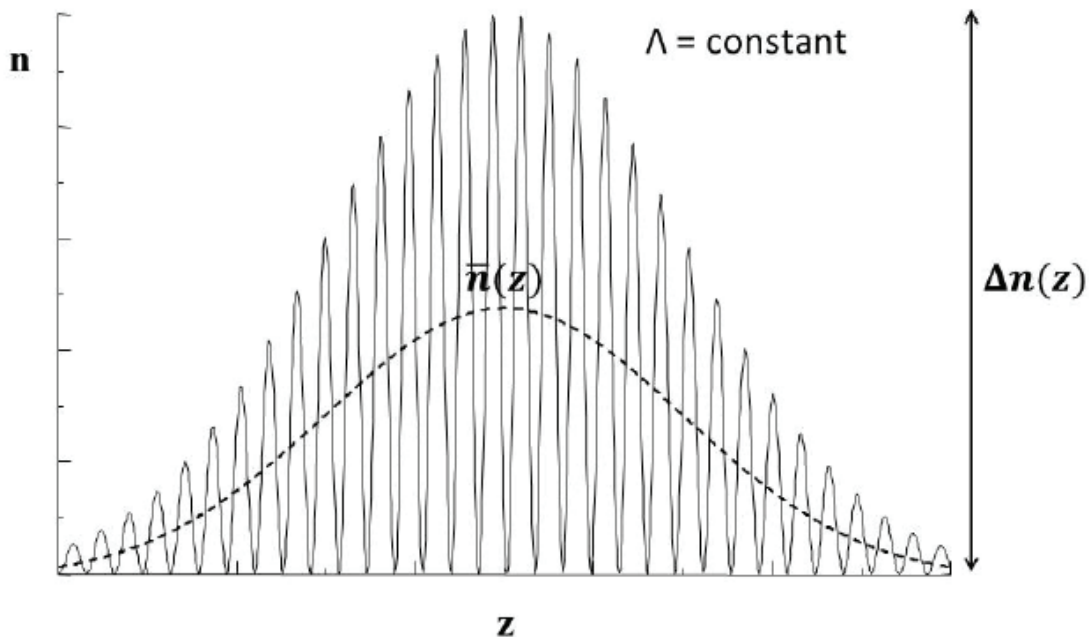
$z = 0.62\text{m}$



Improved Slow-Light Fiber Bragg Grating (FBG) Structure

Much larger slow-down factors possible with a Gaussian-profile grating

Observation of (thermal) optical bistability at mW power levels



group index
approximately 140

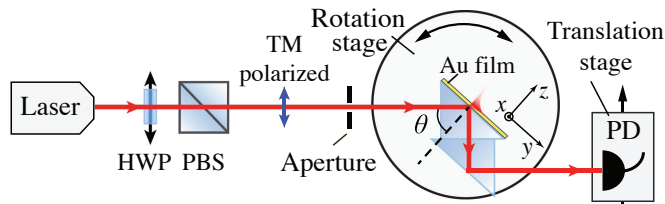
H. Wen, M. Terrel, S. Fan and M. Dignonnet, IEEE Sensors J. 12, 156-163 (2012).

J. Upham, I. De Leon, D. Grobnic, E. Ma, M.-C. N. Dicaire, S.A. Schulz, S. Murugkar, and R.W. Boyd, Optics Letters 39, 849-852 (2014).

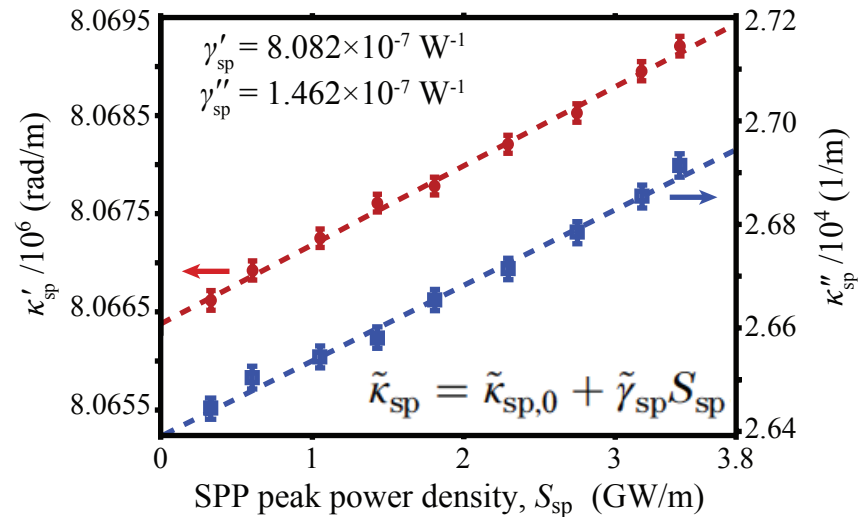
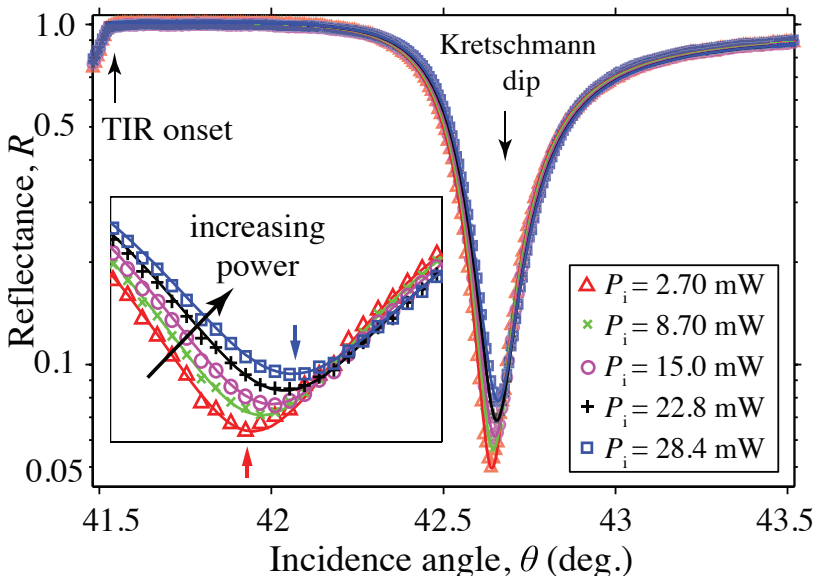
NLO in Plasmonics (Photonics Using Metals)

Is there an intrinsic nonlinear response to surface plasmon polaritons (SPPs)?

- A nonlinear response would be useful for photonics applications
- Metals are highly nonlinear (n_2 is 10^6 times that of silica)
- High (sub-wavelength) field confinement in an SPP
- But SPPs tend to show high loss



Measure intensity dependence of the Kretschmann angle



$$\tilde{\chi}_{\text{Au}}^{(3)} = (4.67 + i3.03) \times 10^{-19} \text{ m}^2/\text{V}^2$$

power varies from 2 to 18 mW;
intensity varies from 2 to 22 GW/cm
laser wavelength is 796.5 nm
laser pulse duration is 100 fs

Thank you for your attention!

