### Optical Airy beams and bullets



### **Demetri Christodoulides**



CREOL & FPCE THE COLLEGE OF OPTICS AND PHOTONICS



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Medical applications

Lasers



**Military** 



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#### **Nanolasers**



**Integrated optics** 





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**Optical fibers** 



### Light travels in straight lines

#### Euclid of Alexandria 325-265 BC









### Optica

# Yet, can light bend?



### **Can light bend?**









### Are curved light trajectories possible?

#### science fiction



arts









### **Bending light**

#### **Negative refraction**



Cloaking





#### **Gravitational lensing**



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# **Optical diffraction**



Francesco Grimaldi 1613-1663

# Diffraction

Diffraction is defined as "any deviation of light rays from rectilinear paths that can not be interpreted as reflection or refraction".



### **Diffraction of a Gaussian beam**





### Diffraction



A beam from a green laser pointer, 1 mm in diameter, will have a diameter of 1500 km when it reaches the moon.

#### That will be the size of Texas !



# Why optical beams diffract?

Intuitively one may say that all plane waves components comprising the beam tend to walk-off from each other and a as a result the beam diffracts.



# Non-diffracting beams & waves

# **Diffraction-free patterns?**



$$E = E_0 \exp[i(k_x x + k_z z)] + E_0 \exp[i(-k_x x + k_z z)]$$
$$E = 2E_0 \cos(k_x x) \exp[ik_z z]$$
$$I = |E|^2 = 4E_0^2 \cos^2(k_x x)$$

# Non-diffracting beams - conical plane wave superposition



### 4-waves







### Non-Diffracting Beams

<u>A non-diffracting beam</u> remains intensity invariant during propagation.

Bessel



Mathieu



Non-diffracting beams share two common characteristics:

Non-diffracting beams (like plane waves) are known to convey **infinite power or energy.** 

All the known non-diffracting beams can be generated through conical superposition of plane waves.

On the other hand, finite energy beams/ pulses are known to eventually diffract or disperse.

 $k_{x,i}^2 + k_{y,i}^2 = const.$ 



J. Durnin, J. J. Miceli, and J. H. Eberly, PRL 58, 1499 (1987).
J. C. Gutiérrez-Vega, M. D. Iturbe-Castillo, and S. Chávez-Cerda, Opt. Lett. 25, 1493-1495 (2000)

One-dimensional non-diffracting accelerating beams:

Airy-beams

### Non-spreading Airy wavepackets

$$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = 0$$

<u>Free particle</u> <u>1D Schrödinger equation</u>

The Airy wavepacket is a unique, non-spreading and accelerating solution!



# **The Airy function**



George Biddell Airy 1801-1892



 $\frac{d^2 y}{dx^2} = xy$ 



### Rainbow

### Why do Airy packets freely accelerate?



### Equivalence principle & quantum mechanics

D. M. Greenberger, Am. J. Phys. 48, 256 (1980)

# Uniqueness

# It can be formally shown that the Airy state is the only non-dispersing wavepacket in 1D.

K. Unnikrishnan and A. R. P. Rau, Am. J. Phys. 64, 1034 (1996)

### Finite Energy Optical Airy Beams



Finite power optical Airy beams have recently been suggested\*:

 $\varphi(x, z = 0) = Ai(x / x_0) \exp[x / w]$ 



\* G. A. Siviloglou and D. N. Christodoulides, Opt. Lett. 32, 979 (2007)

### Acceleration dynamics of Airy Beams

Finite energy Airy stripe beams propagate according to\* :

$$I = \left|\varphi(x,z)\right|^{2} = \left|Ai\left[x/x_{0} - \frac{1}{4k^{2}x_{0}^{4}}z^{2} + i\frac{1}{kwx_{0}}z\right]\right|^{2} \exp\left(2x/w\right)\exp\left(-\frac{1}{k^{2}wx_{0}^{3}}z^{2}\right)$$





\* G. A. Siviloglou and D. N. Christodoulides, Opt. Lett. **32**, 979 (2007)

### Optical analog of projectile ballistics

The Airy beam moves on a parabolic trajectory very much like a body under the action of gravity!









### **Experimental Set-up**

$$= \exp(-ak^2)\exp(\frac{i}{3}k^3)$$

We can synthesize a truncated Airy wave by imposing <u>a cubic phase on a Gaussian beam</u> and then taking its <u>Fourier transform</u> using a lens

 $P_0(k)$ 







### Phase Mask for 2D



25

# Other possibilities-2D cubic phase masks



**U. of Arizona** 



Papazoglou et al, PRA A 81, 061807(R) (2010)

f = 500 mm

200 µm

f=100 mm

f = 200 mm

### Ballistic dynamics of Airy beams

An Airy beam can move on parabolic trajectories very much like a cannonball under the action of gravity!





### Parabolic deflection of the beam:

$$x_d = \theta z + z^2 / (4k^2 x_0^3)$$

\* G. A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christodoulides, Optics Letters 33, 207 (2008)

# Ballistic dynamics of Airy beams



Beam's center of gravity moves on straight lines



In agreement with Ehrenfest's theorem

Transverse electromagnetic momentum is conserved !

### 2-D Airy Beams

Similarly, we can introduce 2D finite energy Airy beams:

$$\phi(x, y, z = 0) = Ai(x / x_0) Ai(y / y_0) \exp[(x / w_1) + (y / w_2)]$$



**Example:**  $x_0 = y_0 = 50 \,\mu m$   $w_1 = w_2 = 0.5 \,mm$ 

## **Diffraction of the main lobe**



The main lobe launched in isolation has experienced a 5-fold increase in the beam width, while the peak intensity has dropped to 5% of its initial value.

### Possibilities



Airy beams can "heal" themselves during propagation.

# Airy beams can circumvent opaque obstacles.



Spatio-temporal **optical bullets** resisting both <u>diffraction</u> and <u>dispersion</u> effects.

### Self-healing of Airy Beams





Airy beams are robust and self-reconstruct even under severe perturbations

Broky, Siviloglou, Dogariu, and Christodoulides, "Self-healing properties of optical Airy beams," Opt. Express 16, 12891 (2008)

# **Self-healing**





#### Regeneration





### Airy Beams in adverse environments: I. Scattering media



Diameter = 1.5 µm Severe Scattering



Silica microspheres (n=1.47) suspended in water (n=1.33) Concentration: 0.2 % w/w

### **Optical nano-particle manipulation using curved Airy beams**





Baumgartl, Mazilu & Dholakia Nature Photonics 2, 675 - 678 (2008)

# Airy Beams in adverse environments: Turbulent media

Airy beam

Gaussian beam





Airy beams are resilient under turbulence while a comparable Gaussian beam is badly deformed.
## Scintillation dynamics of Airy beams under turbulence

#### Scintillation of Airy beam arrays in atmospheric turbulence

Yalong Gu\* and Greg Gbur

Department of Physics and Optical Science, University of North Carolina at Charlotte, Charlotte, North Carolina 28223, USA \*Corresponding author: ygu4@uncc.edu

> Received August 9, 2010; revised September 14, 2010; accepted September 17, 2010; posted September 27, 2010 (Doc. ID 133186); published October 12, 2010

We investigate the scintillation properties of Airy beam arrays in atmospheric turbulence. By utilizing the "selfbending" propagation property of Airy beams, the constituent beamlets propagate through relatively independent regions of turbulence but still largely overlap at the on-axis detector. Through numeric simulations, it is shown that the scintillation of an Airy beam array is significantly reduced and close to the theoretical minimum. © 2010

Optics Letters vol. 35, pp. 3456, (2010)

### Airy beams in optical filamentation studies



#### Curved plasma channel generation using ultra-intense Airy beams in air



#### **U. of Arizona /CREOL**

Science, 324, 5923 (2009)

#### Curved plasma channel generation using ultra-intense Airy beams in air



#### Curved plasma channel generation using ultra-intense Airy beams in air



#### Intensity distribution of the forward emission along the filament, for different values of pulse energy. At energies above 5 mJ, the distribution develops two peaks consistent with the experimentally observed emission patterns



## Abruptly auto-focusing waves



Efremidis, Christodoulides, Opt. Lett. 35, 4045-4047 (2010)

### Autofocusing waves versus Gaussian beams



### Experimental observation of auto-focusing waves





Fused silica ablation





#### Experimental results: Tzortzakis, FORTH Crete, OL vol. 36, p. 1842 (2011)

# **Optical Airy Bullets**

# **Optical Bullets**

An spatio-temporal optical wave propagating under the influence of diffraction and dispersion obeys:

$$i\frac{\partial E}{\partial z} + \frac{1}{2k} \left( \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \right) - \frac{k_0^{"}}{2} \frac{\partial^2 E}{\partial \tau^2} = 0$$
  
**Diffraction Dispersion**  
 $k_0^{"} > 0$  : Normal dispersion

 $k_0^{"} < 0$  : Anomalous dispersion

#### **Broadening in both space and time occurs**



## Nonlinear optical bullets

In the presence of nonlinearity one finds that:

$$i\frac{\partial E}{\partial z} + \frac{1}{2k}\left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2}\right) - \frac{k_0^{"}}{2}\frac{\partial^2 E}{\partial \tau^2} + k_0 n_2 |E|^2 E = 0$$

If the dispersion and diffraction lengths are <u>equal</u> and if the dispersion is <u>anomalous</u>:

**Spherical optical bullet** 

$$L_{dispersion} = L_{diffraction} = \frac{\tau_0^2}{\left|k_0''\right|} = \frac{w_0^2}{k}$$

$$i\frac{\partial\psi}{\partial Z} + \frac{1}{2}\left(\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2}\right) + \frac{1}{2}\frac{\partial^2\psi}{\partial T^2} + \left|\psi\right|^2\psi = 0$$

Y. Silberberg, Optics Letters 15, 1282 (1990)



# Nonlinear optical bullets

- They demand equalization of dispersion and diffraction lengths (they are spherical)
- They only exist under anomalous dispersive conditions
- They need high power levels
- Nonlinear optical bullets are highly unstablethey <u>implode/explode</u> during propagation !

Never observed experimentally except in phase matched chi-2 crystals and only in 2+1 D (Frank Wise group-Cornell)



## Are linear optical bullets possible?

$$i\frac{\partial\psi}{\partial Z} + \frac{1}{2}\left(\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2}\right) + \frac{1}{2}\frac{\partial^2\psi}{\partial T^2} = 0$$

**Anomalous dispersion** 

Yes, as long as the diffraction and dispersion lengths are again equal!

$$\psi = \frac{\sin \sqrt{X^2 + Y^2 + T^2}}{\sqrt{X^2 + Y^2 + T^2}} \exp(i\mu Z)$$

More complicated bullets are possible as well.

Т





- Very difficult to synthesize
- Complex spectra
- Need dispersion +diffraction equalization

Y

Possible under anomalous dispersion

## Are linear optical bullets possible?

$$i\frac{\partial\psi}{\partial Z} + \frac{1}{2}\left(\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2}\right) - \frac{1}{2}\frac{\partial^2\psi}{\partial T^2} = 0$$

**Normal dispersion** 

Again, as long as the diffraction and dispersion lengths are equal!

$$\psi = \frac{1}{\sqrt{X^2 + Y^2 + (iT + c)^2}}$$
**X-waves**

- Very difficult to synthesize
- Complex spectra
- Need diffraction +dispersion equalization
- Possible under normal dispersion

Lu and Greenleaf, Ultrasonics 39 (1992). Di Trapani et al, PRL 1994. Christodoulides, Efremidis, Optics Letters **29**, 1446 (2004).

# **Optical bullets**

# To overcome these problems one has to disengage space and time- e.g. use separation of variables!

Given that 3 can be written only as:

2+1=3 1+1+1=3

A 1D non-dispersing packet is absolutely necessary as a building block.

# **Spatio-Temporal Airy Bullets**

$$i\frac{\partial\psi}{\partial Z} + \frac{1}{2}\left(\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2}\right) + \varepsilon\frac{1}{2}\frac{\partial^2\psi}{\partial T^2} = 0$$

$$L_{dispersion} \neq L_{diffraction}$$

$$\psi = A i(\alpha T) J_0(\beta r)$$



### **Airy-Bessel Bullet**

- Easy to synthesize
- Does not need diffraction +dispersion equalization
- Possible under any dispersion conditions

G. A. Siviloglou and D. N. Christodoulides, Opt. Lett. 32, 979 (2007)

## **Airy-Bessel Optical bullets**





#### **Cornell-CREOL**

500

Delay (fs)

1000

Frank Wise's group

Self-healing in time after 6  $L_d$ 

## **Airy+Bessel optical bullets**



12 diffraction + 6 dispersion lengths 90 micros-90fs 1020 nm

Cornell / CREOL 54

## **Airy-Bessel optical bullets**

**Two-photon fluorescence** 





**Bessel-Airy (90 fs main pulse lobe)** 

# **Spatio-Temporal Airy Bullets**

Airy is the only non-dispersing wavepacket in 1-D



(a) Airy-Bessel (b) 3D Airy and (c) Airy-X optical non-dispersing bullets.

# **Airy Bullets**







Papazoglou et al, PRL 105, 253901 (2010)

# Airy plasmons

$$\nabla^{2}E_{y} + k_{0}^{2}\varepsilon E_{y} = 0$$

$$\downarrow$$

$$\begin{bmatrix}E_{y}(x, y, z) = A(x, z)e^{ik_{z}z}e^{-\alpha y}\\\alpha_{d}^{2} = k_{z}^{2} - k_{0}^{2}\varepsilon_{d}\\k_{z} = k_{0}\sqrt{\varepsilon_{d}\varepsilon_{m}}/(\varepsilon_{d} + \varepsilon_{m})$$

$$\downarrow$$

$$\frac{\partial^{2}A}{\partial x^{2}} + 2ik_{z}\frac{\partial A}{\partial z} = 0$$



# Airy plasmon propagation.



$$A(x,z) = \operatorname{Ai}\left[\frac{x}{x_0} - \left(\frac{z}{2k_z x_0^2}\right)^2 + i\frac{az}{k_z x_0^2}\right] \exp\left[i\left(\frac{x + a^2 x_0}{2x_0} \frac{z}{k_z x_0^2} - \frac{1}{12}\left(\frac{z}{k_z x_0^2}\right)^3\right)\right] \exp\left[a\frac{x}{x_0} - \frac{a}{2}\left(\frac{z}{k_z x_0^2}\right)^2\right]$$

A. Salandrino and D. Christodoulides, Opt. Lett. 35, 2082-2084 (2010)

# Airy plasmons

• Selfelfealitig rbehavior pagation



## **Experimental observation of Airy plasmons**



Mask
Plasmonic Airy beam



Kivshar's group-Australia: PRL 107, 116802 (2011). Zhang's group, Opt. Lett. 36, 3191 (2011). Zhu's group-Nanjing University PRL (2011).

# Super-continuum generation using Airy pulses

P. P. C. C. C.	
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Polynkin et al. U. of Arizona, Phys. Rev. Lett. 107, 243901 (2011)

## **New directions**



Sending femtosecond pulses along curved trajectories

John Dudley's group



# Can we identify similar accelerating beams which are non-paraxial?



To do so we must use a non-paraxial formulation based on Helmholtz equation

$$(\nabla^2 + k^2) \{ \vec{E}, \vec{H} \} = 0$$

## **New directions**

# Non-paraxial accelerating Bessel wave-packets, solutions to Maxwell's equations

$$\vec{E} = \hat{y} J_{\nu}(kr) \exp(i\nu\theta) \exp(-i\omega t)$$



Kaminer, Bekenstein, Nemirovsky, and Segev, PRL 108, 163901 (2012).

## Non-paraxial Airy and Bessel beams



For a FWHM of 500 nm, the main lobe can bent  $50^{0}\,after\,10\;\mu\text{m}$ 



 For a FWHM of 500 nm, the main lobe can bent 70<sup>0</sup> after 10 μm

### Are there any other accelerating vectorial solutions of Maxwell's equations ?

0.75 0.5

Elliptic Helmholtz equation

$$\begin{bmatrix} \frac{2}{f^2(\cosh 2u - \cos 2v)} \left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2}\right) + k^2 \end{bmatrix} \psi_z = 0$$

$$x = f \cosh u \cos v; \quad y = f \sinh u \sin v$$

$$0 \le u < \infty; \quad 0 \le v \le 2\pi$$

$$\operatorname{or} E_z = R(u)S(v) \cdot \begin{bmatrix} \frac{d^2}{dv^2} + (a - 2q\cos 2v) \end{bmatrix} S(v) = 0$$

$$\begin{bmatrix} \frac{d^2}{du^2} - (a - 2q\cosh 2u) \end{bmatrix} R(u) = 0$$

 $q = f^2 k^2 / 4$  and parameter *a* can be obtained from sequence of eigenvalues  $a_m (m = 1, 2, ...)$  from differential equation on S(v).

$$\psi_{z}^{m}(u,v;q) = Ac_{e,m}(v;q)Mc_{m}^{(1)}(u;q) + iBs_{e,m}(v;q)Ms_{m}^{(1)}(u;q)$$
Even Radial/Angular Mathieu Functions Odd Radial/Angular Mathieu Functions

\* P. Aleahmad, M. A. Miri, M. S. Mills, I. Kaminer, M. Segev and D. N. Christodoulides, *Phys. Rev. Lett.* 109, 203902 (2012)
\* P. Zhang, Y. Hu, T. Li, D. Cannan, X. Yin, R. Morandotti, Z. Chen, and X. Zhang, *Phys. Rev. Lett.* 109, 193901 (2012)

### **Future directions**



Aleahmad, Miri, Mills, Kaminer, Segev, and Christodoulides PRL 109, pp. 203902 (2012)

### **Prolate Spheroidal Coordinates**



## **Diametric drive acceleration**



#### warp drive ??



#### diametric drive

#### **Diametric drive acceleration: Newton's third law**



Experimental observation of diametric drive acceleration



Max Planck Erlangen-CREOL, Nature Physics, pp. 780-784, 2013 In principle Airy beams can be used in:

•Optics

•Microwaves

Acoustics-Ultrasonics


# Airy beams and pulses: applications

## **Biophotonics**



St. Andrews

## Filamentation



Arizona/UCF

## Plasmonics



Berkeley/ANU/Nanjing

# Super-continuum generation



Micromachining



J. Europ. Opt. Soc. Rap. Public. 8, 13019 (2013)

#### Franche-Comté

U. Of Arizona



# Airy beams and pulses:applications

## Light-sheet microscopy using Airy beams



Higher contrast and resolution 10X FOV

St. Andrews Nat. Methods, April 2014

Stochastic optical reconstruction microscopy (STORM) using Airy point spread function



Harvard Nature Photonics, April 2014

**Electron Airy beams** 



**Tel-Aviv** 

Nature, 331 (2013)

# **Caustics are everywhere**



















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