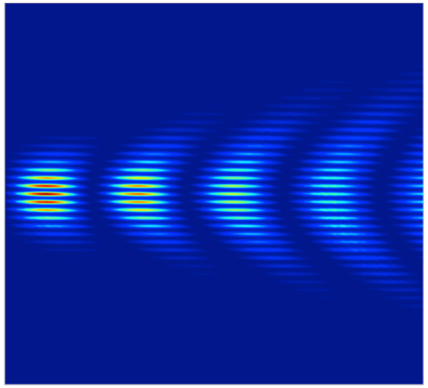
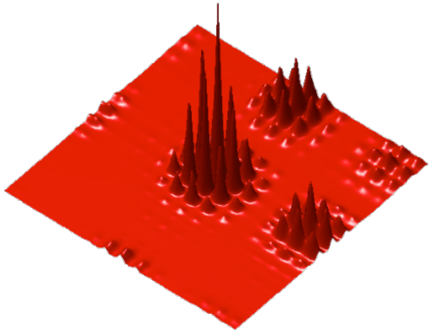
Symmetries in Optics

Demetri Christodoulides



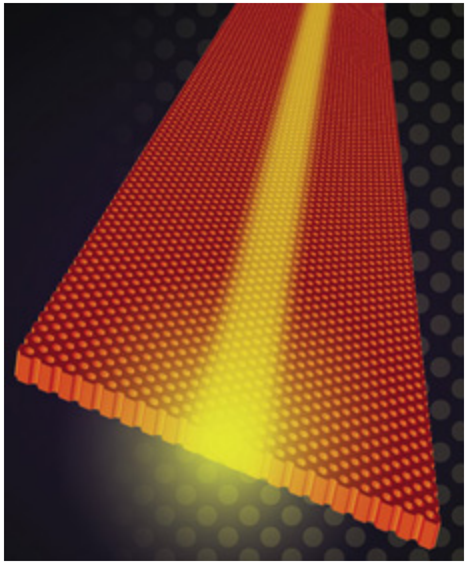


CREOL - The College of Optics and Photonics

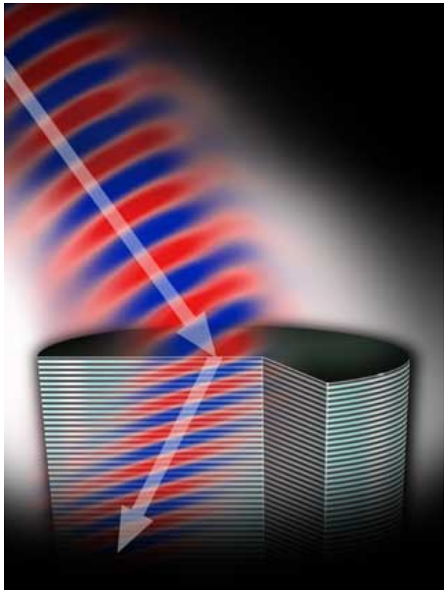
$\mathcal{PT}$ symmetry in Optics

Recent developments in new optical structures and materials

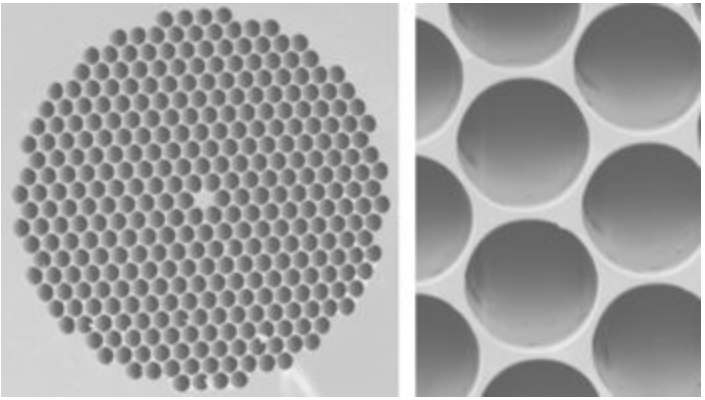
Photonic crystals



Negative index materials

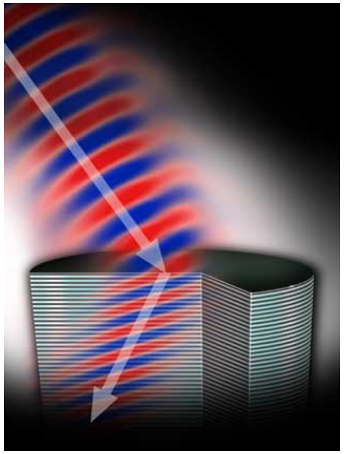


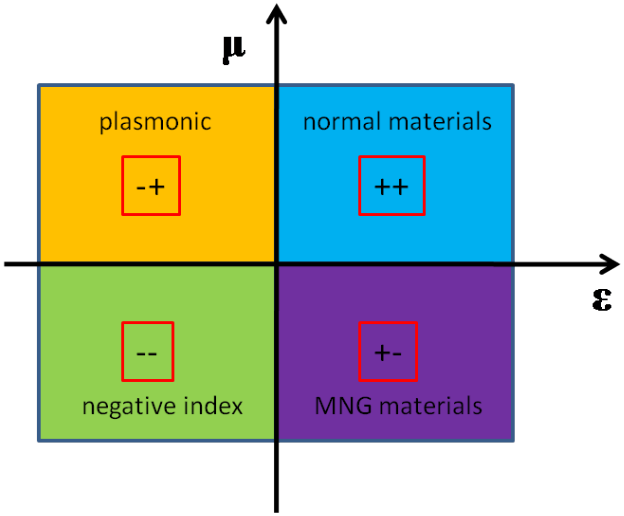
Photonic crystal fibers



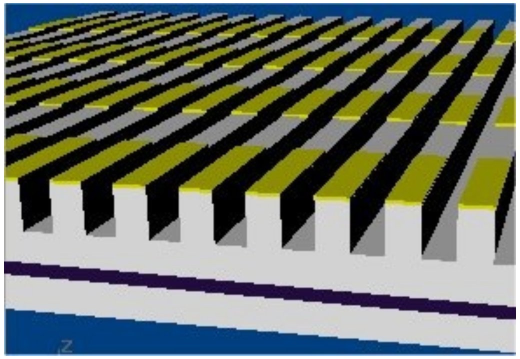
Recent developments in new optical structures and materials

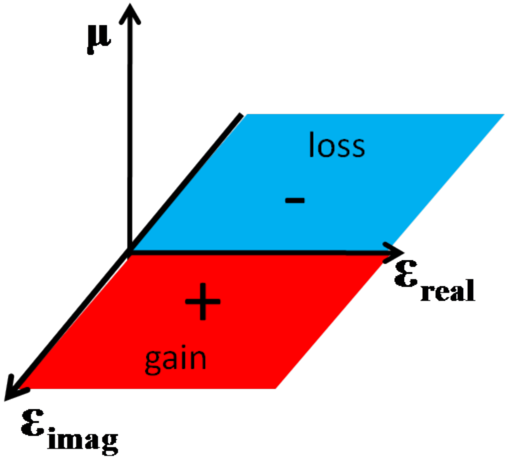
Negative index materials



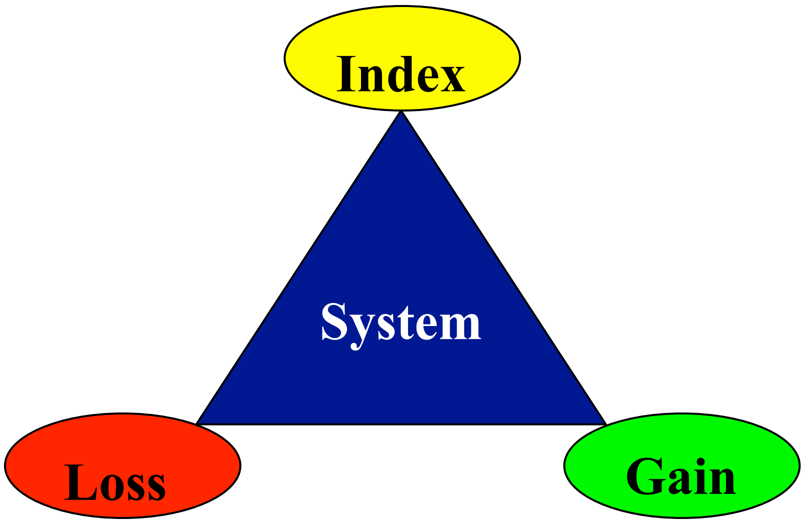


The structures proposed here exploit the ± imaginary part of the dielectric constant





Basic optical "ingredients"



$\mathcal{PT}$-symmetric potentials in Quantum Mechanics

Quantum mechanics is based on Hermitian operators associated with real eigenvalues. The question is:

Should a Hamiltonian be Hermitian in order to have real eigenvalues?

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Real Spectra in Non-Hermitian Hamiltonians Having $\mathcal{PT}$ Symmetry

Carl M. Bender[1] and Stefan Boettcher[2,3]
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(Received 1 December 1997; revised manuscript received 9 April 1998)

The condition of self-adjointness ensures that the eigenvalues of a Hamiltonian are real and bounded below. Replacing this condition by the weaker condition of $\mathcal{PT}$ symmetry, one obtains new infinite classes of complex Hamiltonians whose spectra are also real and positive. These $\mathcal{PT}$ symmetric theories may be viewed as analytic continuations of conventional theories from real to complex phase space. This paper describes the unusual classical and quantum properties of these theories. [S0031-9007(98)06371-6]

Parity-time ($\mathcal{PT}$) symmetric Hamiltonian share common eigenfunctions with the $\mathcal{PT}$ operator. As a result they can exhibit entirely real spectra.

Pseudo-Hermitian quantum mechanics?

PT-symmetric potentials

$$\hat{P} \equiv \begin{cases} \hat{p} \to -\hat{p} \\ \hat{x} \to -\hat{x} \end{cases} \qquad \hat{T} \equiv \begin{cases} \hat{p} \to -\hat{p} \\ \hat{x} \to \hat{x} \\ i \to -i \end{cases}$$

$$PT - Hamiltonian \Leftrightarrow V^*(x) = V(-x)$$

In Quantum Mechanics

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi$$

Schrödinger equation

$$PT - Potential \Leftrightarrow V^*(x) = V(-x)$$

Real part: even
Imaginary part: odd

stationary solution

$$\psi(x,t) = U(x)\exp(-iEt/\hbar)$$

eigenvalue

A complex PT-potential, below threshold, has real eigenvalues!

Some peculiar $\mathcal{PT}$ algebra

PT systems are described by a special algebra (PT algebra). The mathematical properties of the inner products (bra-kets), Hamiltonians, etc has to be reworked and redefined!

$$i\frac{\partial\Psi}{\partial z}+\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi=0, \qquad \Psi(x,z)=u(x)\exp(i\lambda z) \qquad \frac{\partial^2 u}{\partial x^2}+V(x)u=\lambda u$$

Examples:

If u_n, u_m are different eigenmodes of a $\mathcal{PT}$-potential, the <u>**new**</u> orthogonality condition is:

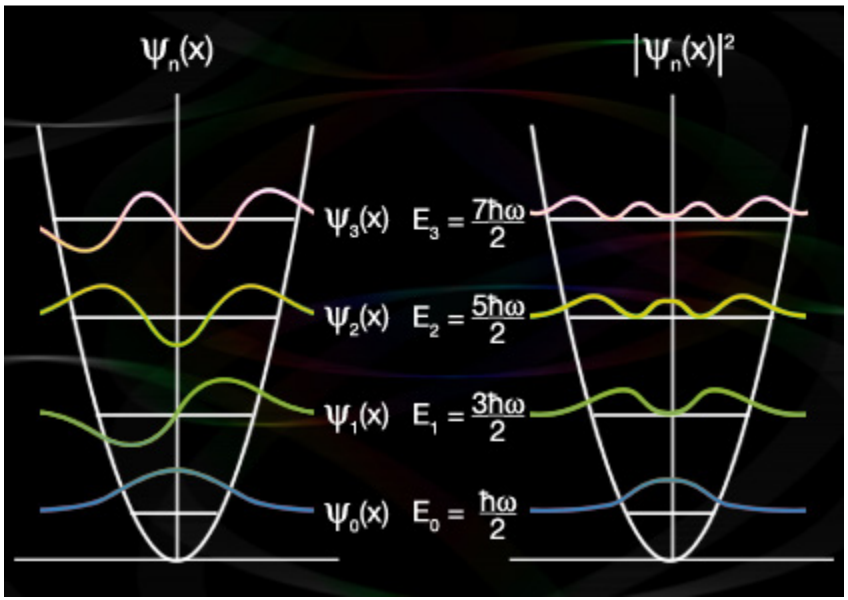
(a) $$\int_{-\infty}^{+\infty} u_m^*(-x)\,u_n(x)\,dx = d_n\delta_{n,m}$$ $$d_n=\{\pm 1\}$$ **instead of** $$\int_{-\infty}^{\infty} u_m^*(x)u_n(x)dx=\delta_{mn}$$

(b) $$\int_{-\infty}^{\infty}\Psi^*(-x)\Psi(x)dx=\text{constant}$$ **as opposed to** $$\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=\text{constant}$$

C. M. Bender, D. C. Brody, and H. F. Jones, . Phys. Rev. Lett. 89, 270401 (2002)

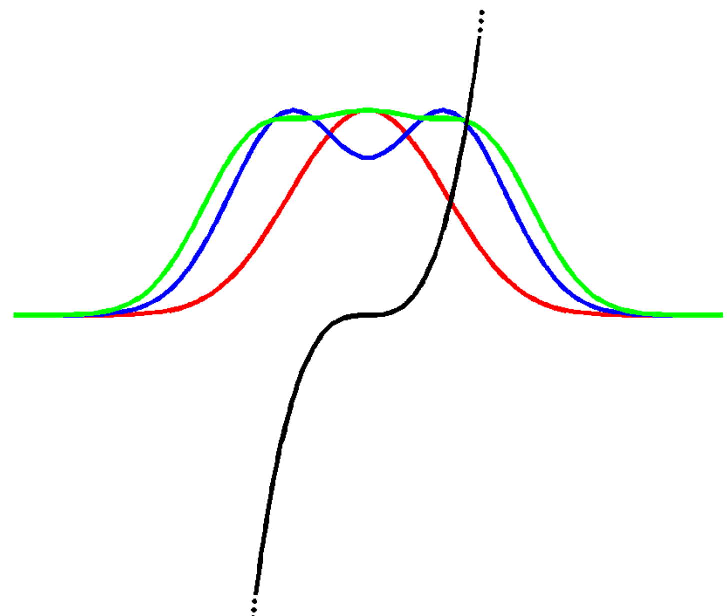
$\mathcal{PT}$ symmetric potentials $(ix)^N$

$$V = x^2$$



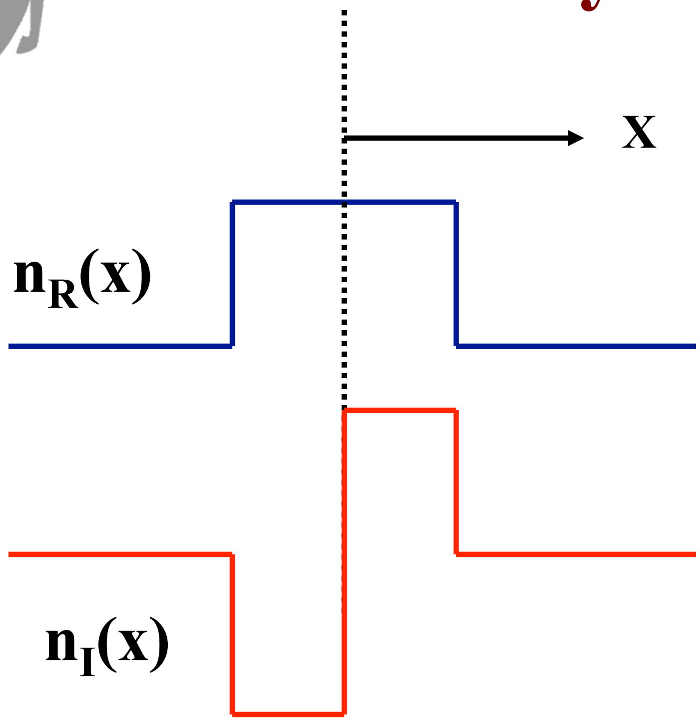
Quantum mechanical oscillator

$$V = ix^3$$



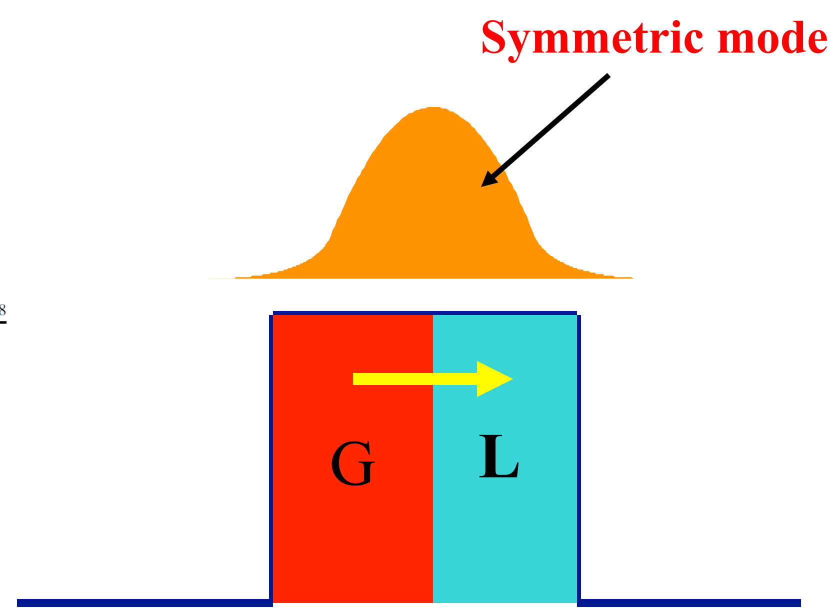
A $\mathcal{PT}$ or pseudo-Hermitian oscillator

$\mathcal{PT}$ symmetry in optics



Symmetric index profile

Anti-symmetric gain distribution



Transparent $\mathcal{PT}$ waveguide

PRL **100,** 103904 (2008) PHYSICAL REVIEW LETTERS week ending 14 MARCH 2008

Beam Dynamics in $\mathcal{PT}$ Symmetric Optical Lattices

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LETTERS
PUBLISHED ONLINE: 24 JANUARY 2010 | DOI: 10.1038/NPHYS1515
nature physics

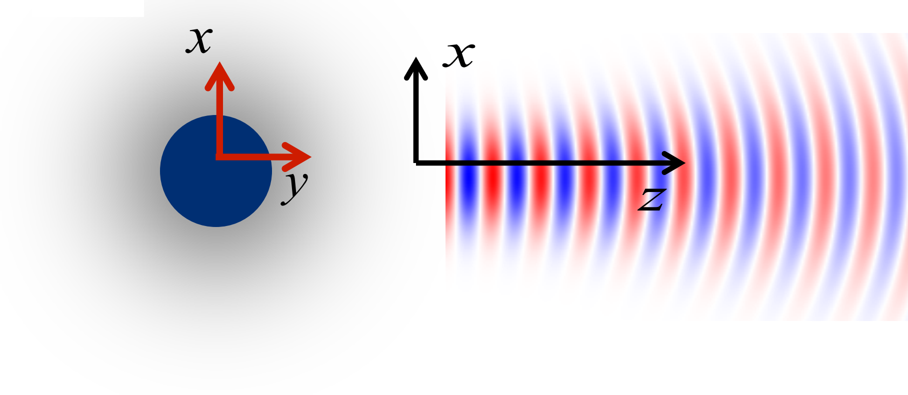
Observation of parity–time symmetry in optics

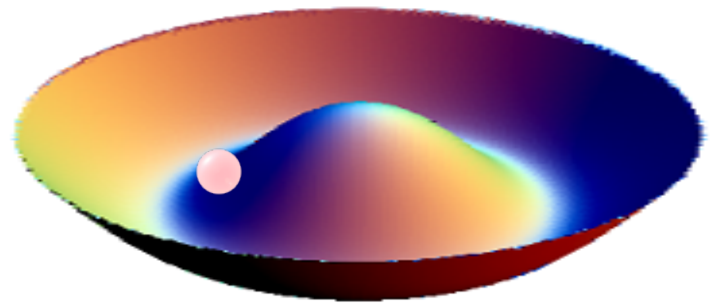
Christian E. Rüter[1], Konstantinos G. Makris[2], Ramy El-Ganainy[2], Demetrios N. Christodoulides[2], Mordechai Segev[3] and Detlef Kip[1]*

$\mathcal{PT}$ symmetry in optics

Paraxial Optics

QM





Paraxial Wave equation — **Schrodinger equation**

$$i\frac{\partial E}{\partial z}+\frac{1}{2k}\frac{\partial^2 E}{\partial x^2}+k_0\left[n_R(x)+in_I(x)\right]E=0$$

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi$$

Propagation constants — **Energy eigenvalues**

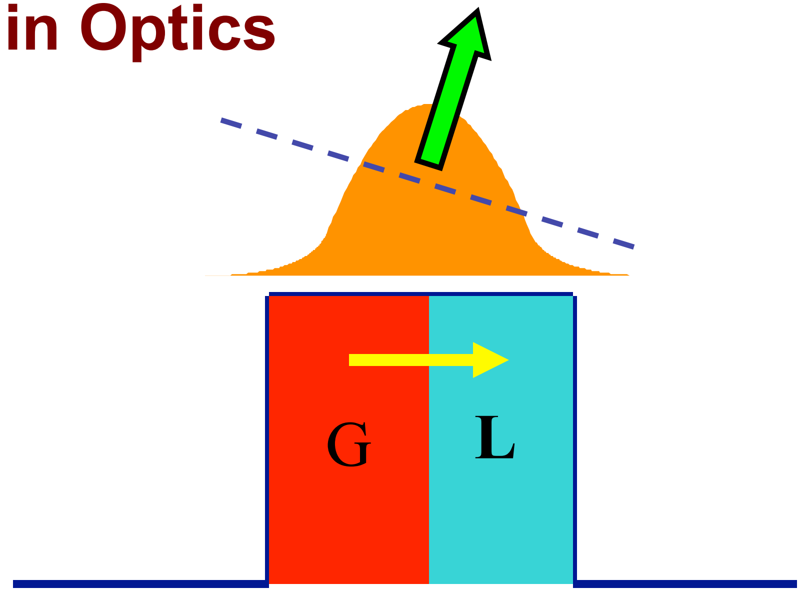
$n_R(x)=n_R(-x)$ Index profile

$n_I(x)=-n_I(-x)$ Gain - loss profile

$V_R(x)=V_R(-x)$

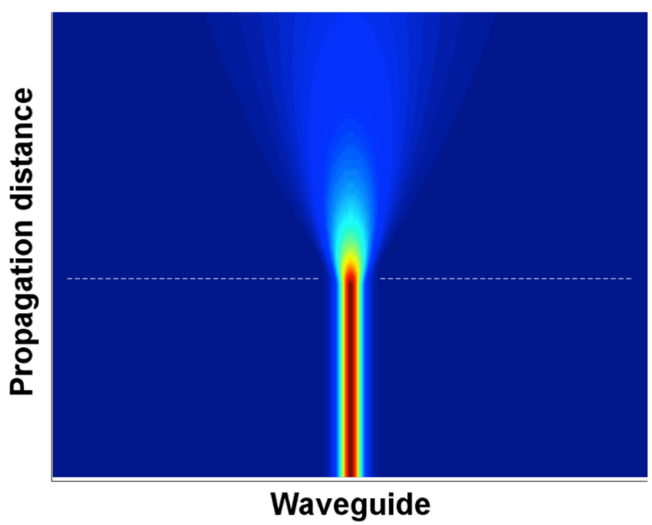
$V_I(x)=-V_I(-x)$

PT-invariant potentials in Optics

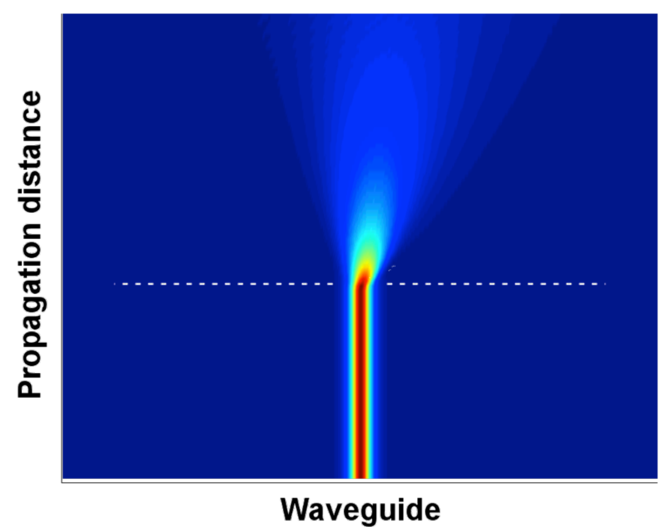


The mode wave fronts are tilted

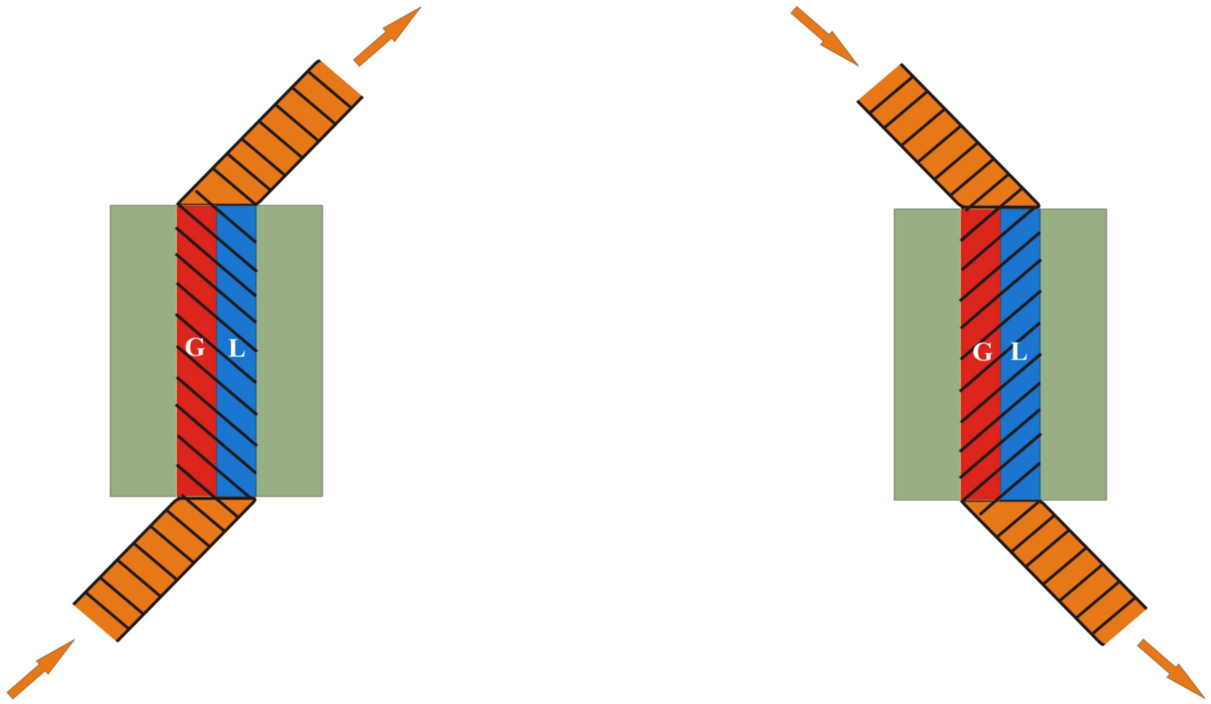
Diffraction from a normal waveguide



Diffraction from a $\mathcal{PT}$ waveguide



Symmetry breaking in $\mathcal{PT}$ synthetic materials



Light can distinguish left from right.

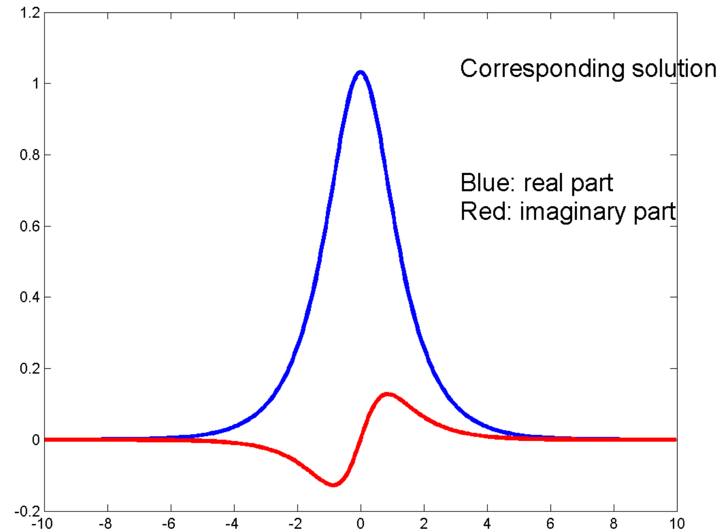
Non-reciprocal propagation!

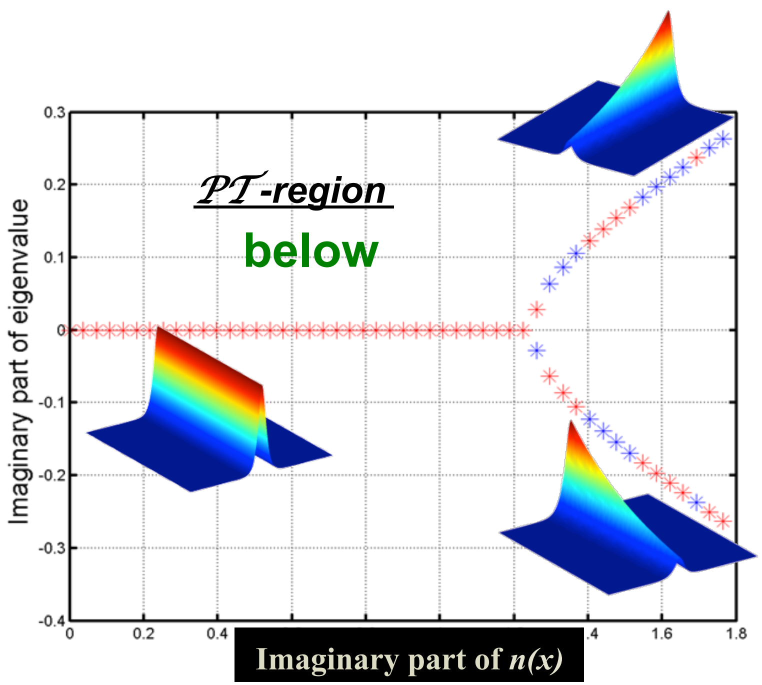
Non-reciprocity in $\mathcal{PT}$ synthetic structures

Optical non-reciprocity is possible irrespective of the state of polarization

$\mathcal{PT}$ Phase transitions

$$n(x) = n_R(x) + i n_I(x)$$





Complex conjugate modes

Above phase transition

Index guiding VS gain guiding

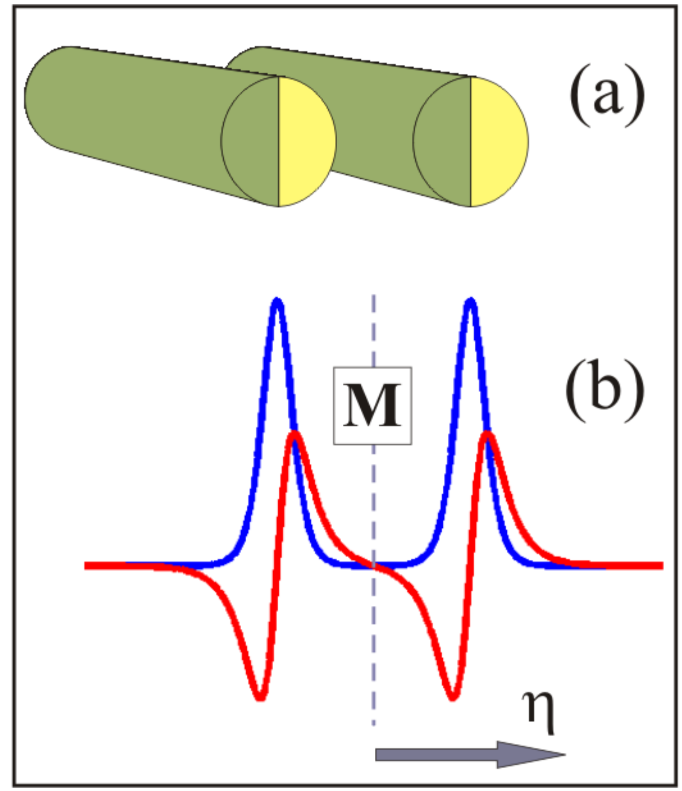
Coupled mode theory of $\mathcal{PT}$ symmetric systems

The $\mathcal{PT}$-eigenmodes have a unique algebra, as a result of their non-orthogonality. For this reason the conventional coupled mode equations cannot describe the $\mathcal{PT}$-coupler. New equations must be derived by using a **Lagrangian density approach**.

$$L = \frac{i}{2}\left[\phi(\eta)\phi_{\xi}^{*}(-\eta) - \phi_{\xi}(\eta)\phi^{*}(-\eta)\right] + \phi_{\eta}(\eta)\phi_{\eta}^{*}(-\eta) - V(\eta)\phi(\eta)\phi^{*}(-\eta)$$

$$\begin{cases} V_1(\eta) = V_1^{*}(-\eta) \\ V_2(\eta) = V_2^{*}(-\eta) \end{cases}$$

Every individual cell is $\mathcal{PT}$-symmetric



Modal coefficient

$$U(\eta,\xi) = \left[a(\xi)u_1(\eta) + b(\xi)u_2(\eta)\right]\exp(i\beta\xi)$$

Local mode   Local mode

R. El-Ganainy et al., "Theory of coupled optical PT-symmetric structures," Opt. Lett. 32, 2632-2634 (2007)

Wave propagation in $\mathcal{PT}$-synthetic waveguide arrays and lattices

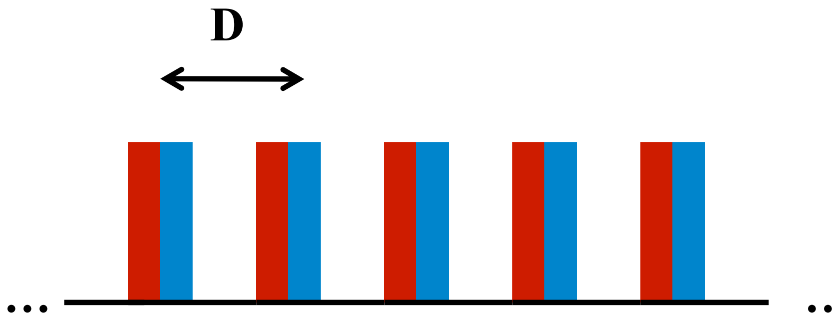
Real periodic potentials and Floquet-Bloch theorem

$$i\frac{\partial U}{\partial z}+\frac{\partial^2 U}{\partial x^2}+V(x)U=0$$

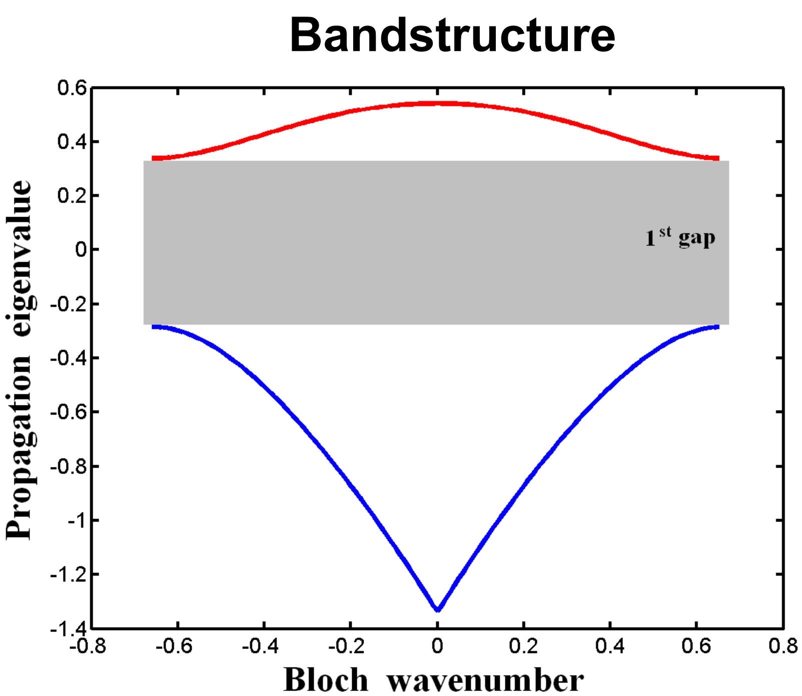
Floquet Bloch mode

$$\phi_{k\,n}(x)\exp[i\beta_n(k)z]$$

k **: Bloch wavenumber**
n **: nuber of band**



Periodic PT potential

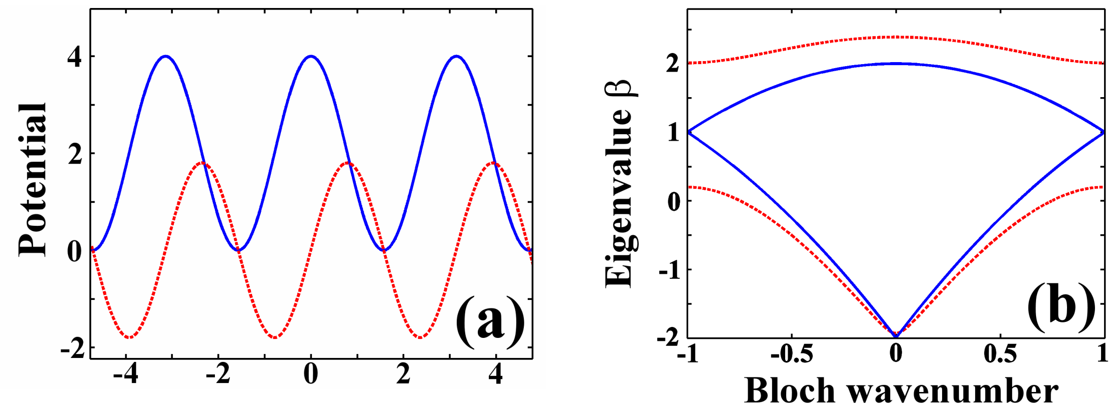


$$V(x)=V(x+D)$$

Band-stucture of a $\mathcal{PT}$ optical lattice

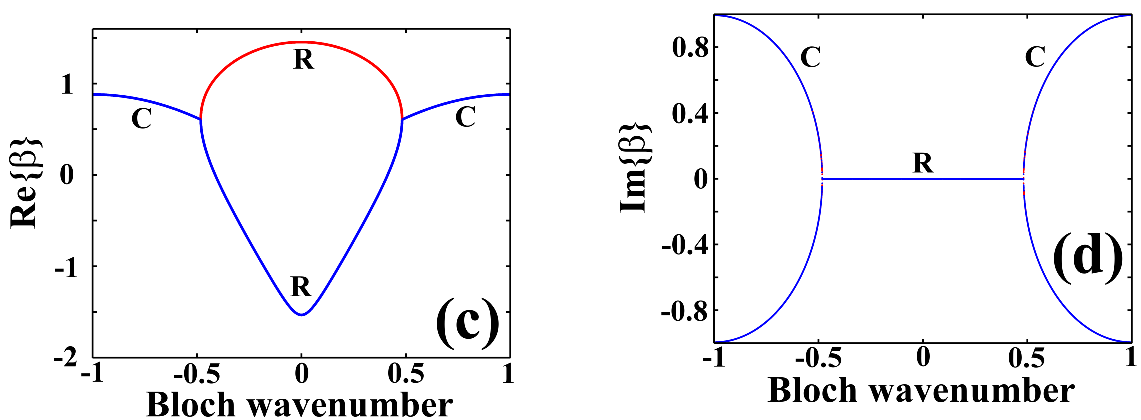
$$V(\eta) = 4\left[\cos^2(\eta) + iV_0\sin(2\eta)\right]$$

$$\begin{cases} If \quad V_0 \leq 0.5 \quad real \quad eigenvalues \\ If \quad V_0 > 0.5 \quad complex \quad eigenvalues \end{cases}$$



Before phase transition

After phase transition



FB mode

$$\phi_{k\,n}(\eta)\exp\left[i\beta_n(k)\xi\right]$$

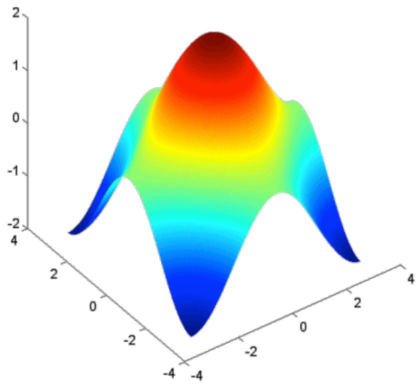
$$\beta_n(k) = \beta_n(-k)$$
$$\phi_{-k\,n}(\eta) \neq \phi^{*}_{k\,n}(\eta)$$

Non-reciprocal bandstructure

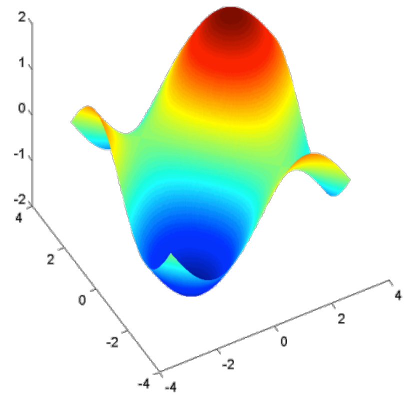
K. G. Makris et al, Phys. Rev. Lett. 100, 103904 (2008)

2D *PT* optical lattices

$$V(x,y)=\cos(x)+\cos(y)+iV_0(\sin(x)+\sin(y)) \qquad V(x,y)=V^*(-x,-y)$$

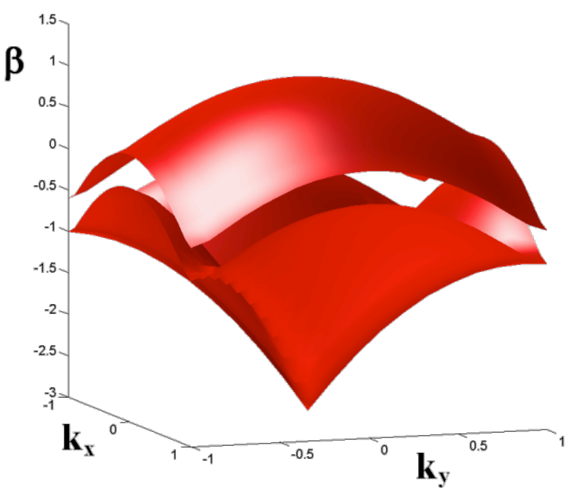


Real part of cell

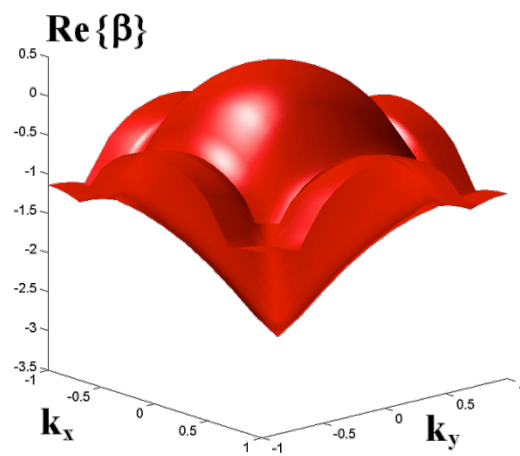


Imaginary part of cell

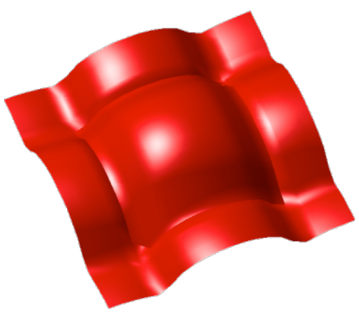
$$\begin{cases} If \quad V_0 \le 1 \quad real \quad eigenvalues \\ If \quad V_0 > 1 \quad complex \quad eigenvalues \end{cases}$$



Below PT threshold

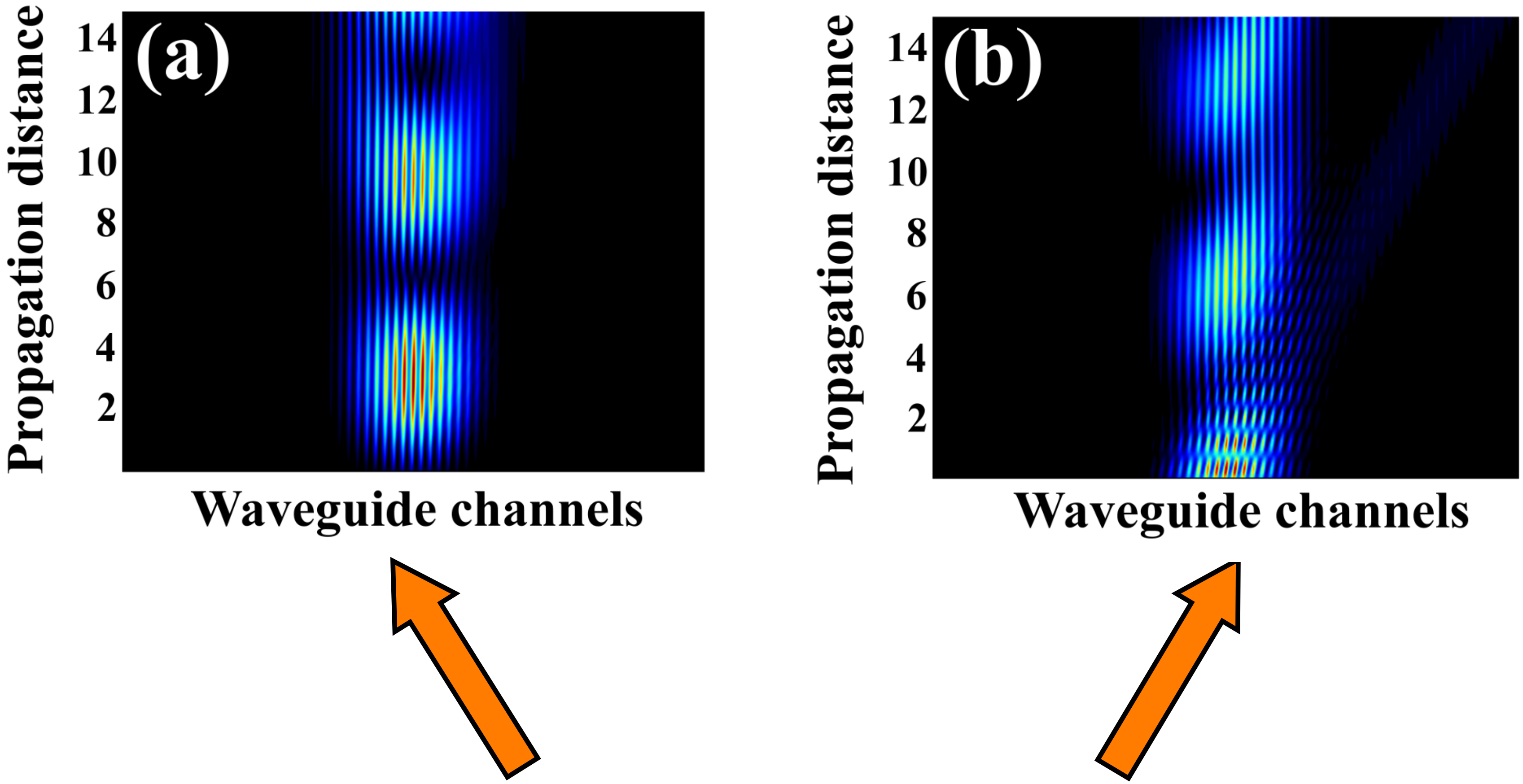


Above PT threshold



(top view)

Non-reciprocity in $\mathcal{PT}$-waveguide arrays

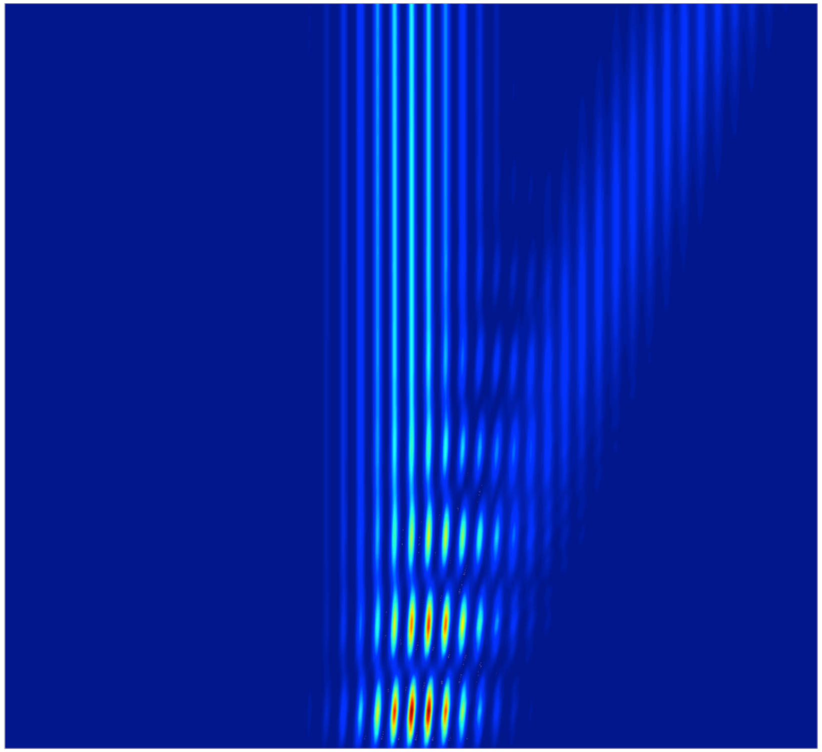


Wide beam excitation at +θ

Wide beam excitation at - θ

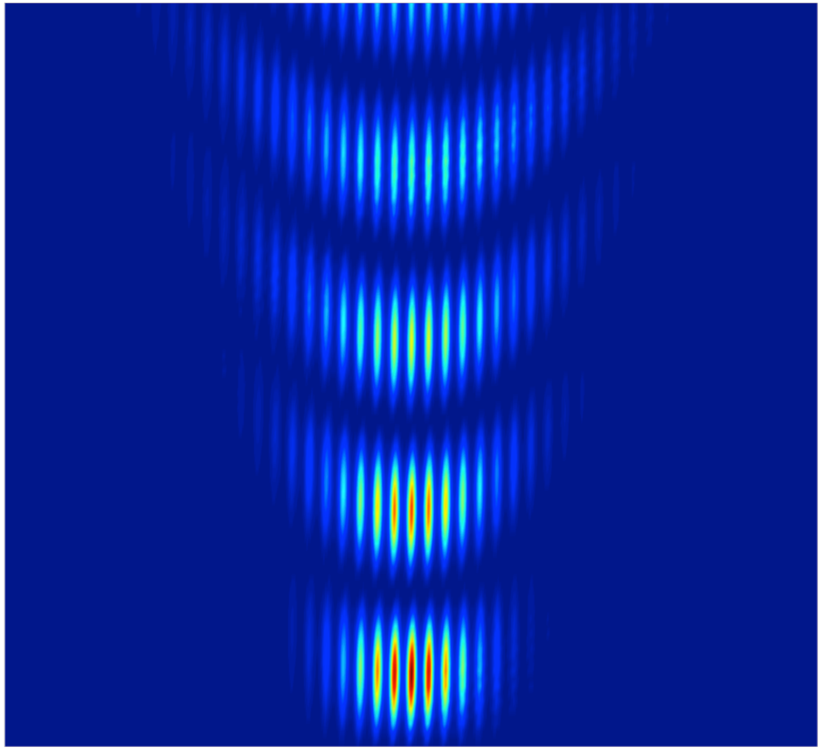
Beam dynamics *PT* waveguide arrays

Double refraction



Wide beam at normal incidence

Power Oscillation

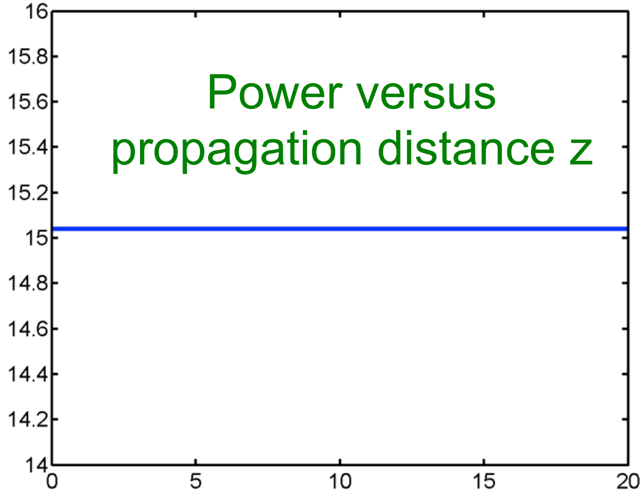


Wide beam at an angle

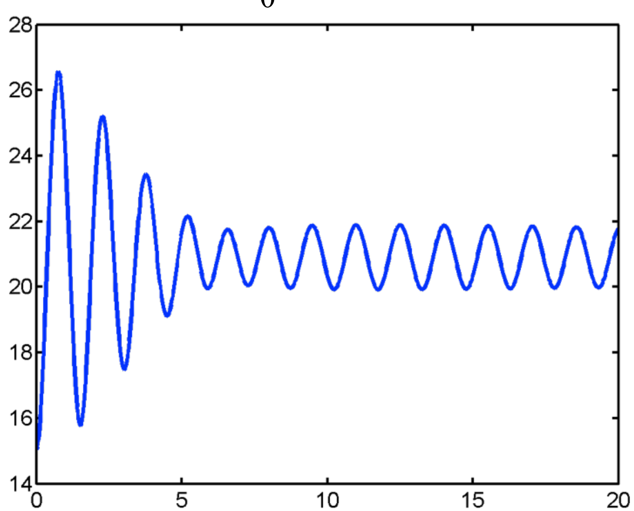
Power oscillations and eigenfunction unfolding

$$V(\eta) = 4\left[\cos^2(\eta) + iV_0 \sin(2\eta)\right]$$

$$V_0 = 0$$



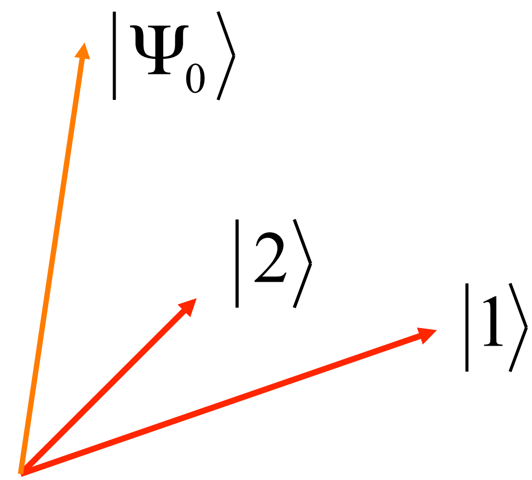
$$V_0 = 0.49$$



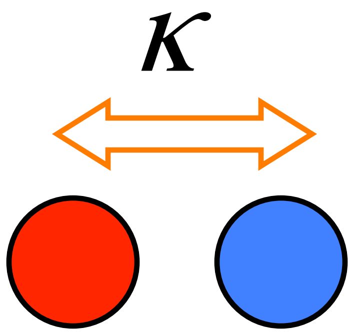
$$|\Psi_0\rangle = |1\rangle \exp(i\beta_1 z) + |2\rangle \exp(i\beta_2 z)$$

$$\langle \Psi_0 | \Psi_0 \rangle = \langle 1|1\rangle + \langle 2|2\rangle +$$
$$+ \langle 2|1\rangle \exp\left(i(\beta_1 - \beta_2)z\right) + \langle 1|2\rangle \exp\left(i(\beta_2 - \beta_1)z\right)$$

Because of pseudo-Hermiticity the vector space is skewed:



Two-level $\mathcal{PT}$ systems

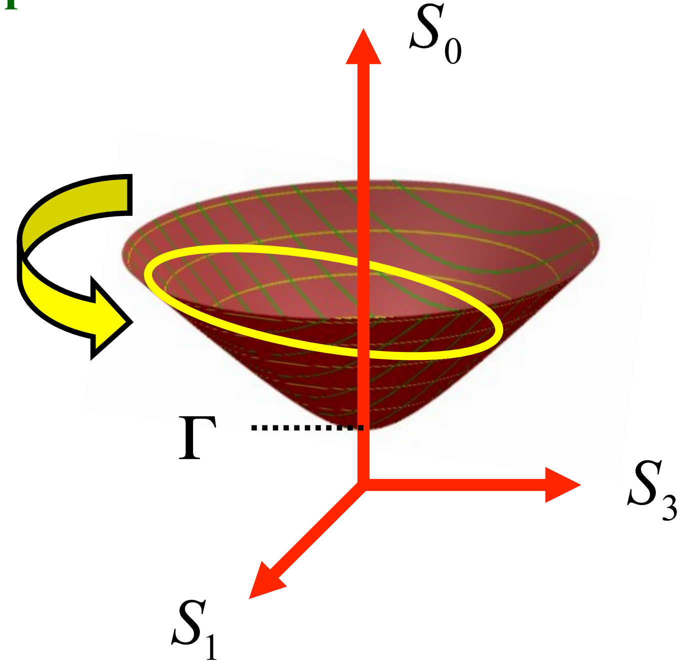


Gain **loss**

$$i\frac{da}{dz} - i\frac{g}{2}a + \kappa\, b = 0$$

$$i\frac{db}{dz} + i\frac{g}{2}b + \kappa\, a = 0$$

Hyperboloid



Bender C.M. et al, Faster than Hermitian quantum mechanics, Phys. Rev. Lett. 98 (2007)

Two-level $\mathcal{PT}$ systems-supermodes below phase transition

$$i\frac{da}{dz} - i\frac{g}{2}a + \kappa\, b = 0$$

$$i\frac{db}{dz} + i\frac{g}{2}b + \kappa\, a = 0$$

$$g/2\kappa = \sin\theta < 1$$

$$\theta = \sin^{-1}(g/2\kappa)$$

$$Z = \kappa z$$

$$i\frac{da}{dZ} - i\sin\theta\, a + b = 0$$

$$i\frac{db}{dZ} + i\sin\theta\, b + a = 0$$

$$\lambda_{1,2} = \pm\cos\theta$$

$$|1\rangle = \begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix} \quad , \quad |2\rangle = \begin{pmatrix} 1 \\ -e^{-i\theta} \end{pmatrix}$$

$$|\Psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} = c_1\begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix} e^{iZ\cos\theta} + c_2\begin{pmatrix} 1 \\ -e^{-i\theta} \end{pmatrix} e^{-iZ\cos\theta}$$

From intial conditions c_1 and c_2 can be determined

Two-level $\mathcal{PT}$ systems-supermodes below phase transition

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\cos\theta} \begin{pmatrix} \cos(Z\cos\theta - \theta) & i\sin(Z\cos\theta) \\ i\sin(Z\cos\theta) & \cos(Z\cos\theta + \theta) \end{pmatrix} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$$

Example: Let $Z\cos\theta + \theta = \pi/2$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2\sin\theta & i \\ i & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$$

Two-level $\mathcal{PT}$ systems-supermodes above phase transition

$$i\frac{da}{dz} - i\frac{g}{2}a + \kappa\, b = 0$$

$$i\frac{db}{dz} + i\frac{g}{2}b + \kappa\, a = 0$$

$$|1\rangle = \begin{pmatrix} 1 \\ ie^{-\theta} \end{pmatrix} \quad , \quad |2\rangle = \begin{pmatrix} 1 \\ ie^{\theta} \end{pmatrix}$$

$$g/2\kappa = \cosh\theta > 1$$

$$Z = \kappa z$$

$$|\Psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ ie^{-\theta} \end{pmatrix} e^{Z\sinh\theta} + c_2 \begin{pmatrix} 1 \\ ie^{\theta} \end{pmatrix} e^{-Z\sinh\theta}$$

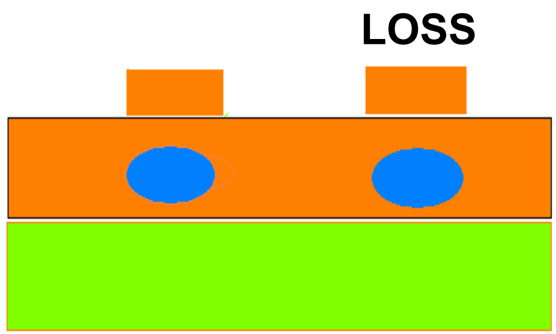
$$i\frac{da}{dZ} - i\cosh\theta\, a + b = 0$$

$$i\frac{db}{dZ} + i\cosh\theta\, b + a = 0$$

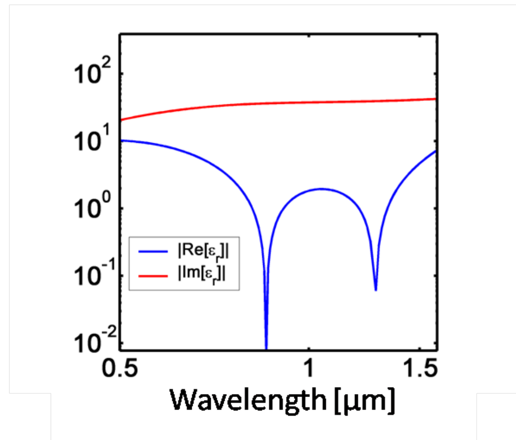
From intial conditions c_1 and c_2 can be determined

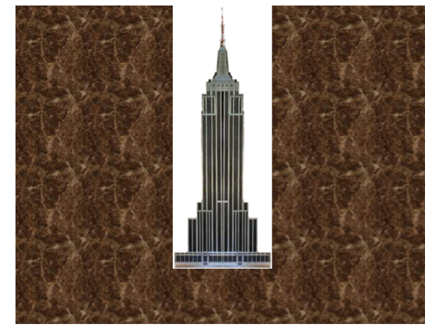
$$\lambda_{1,2} = \pm\sinh\theta$$

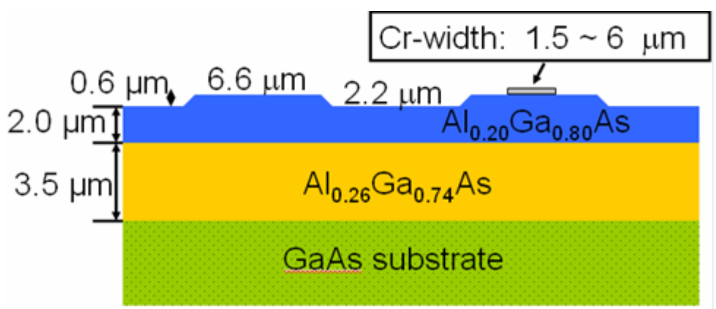
$\mathcal{PT}$–passive systems

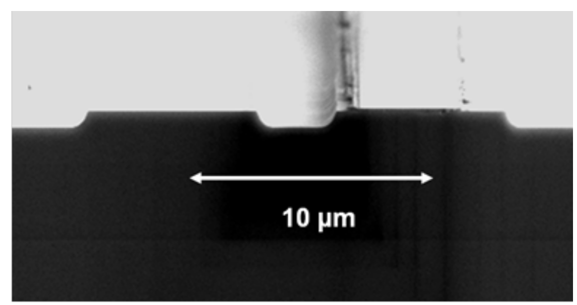


A passive PT directional coupler





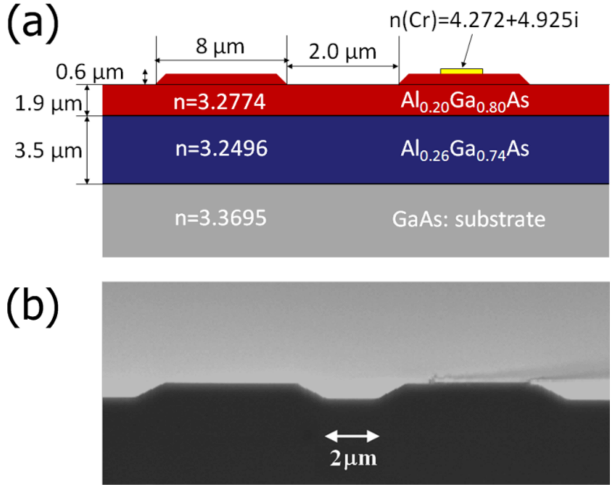


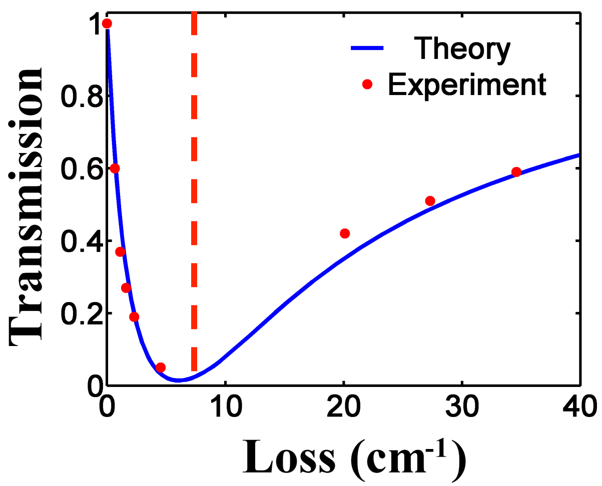


Kramers-Kronig relations must be taken into account

Experimental observation of spontaneous passive $\mathcal{PT}$-symmetry breaking.

The transmission of a passive $\mathcal{PT}$ complex system after the $\mathcal{PT}$–symmetry breaking inceases even though the absorption in the lossy waveguide arm is higher.

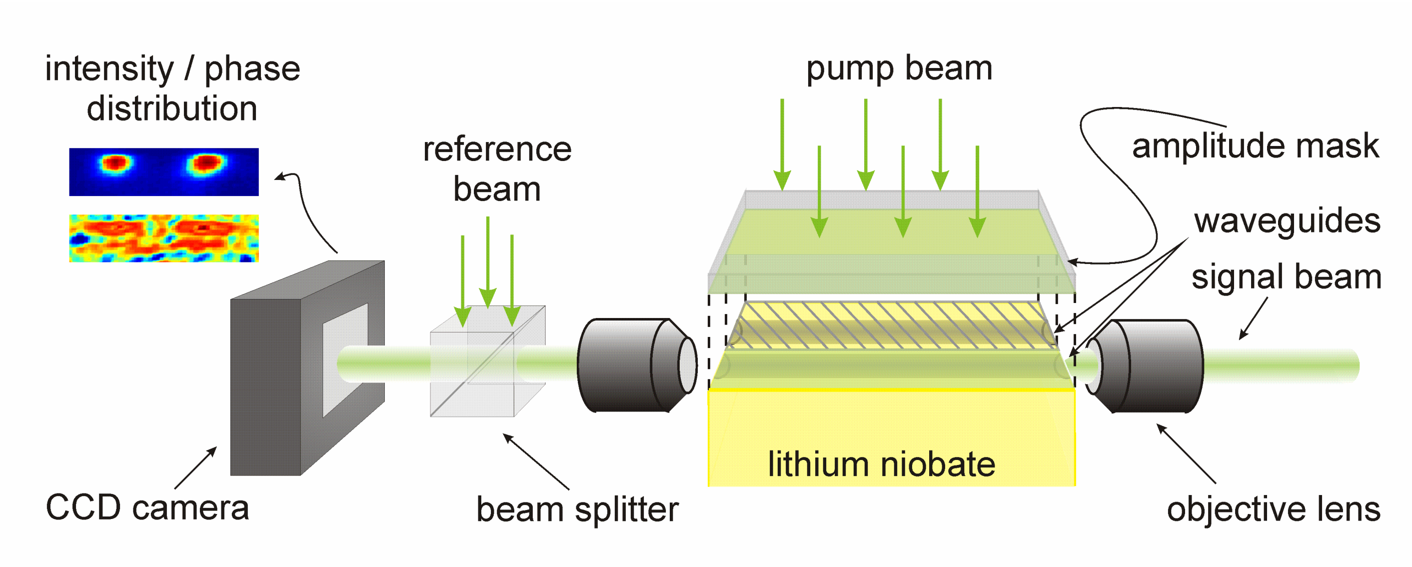


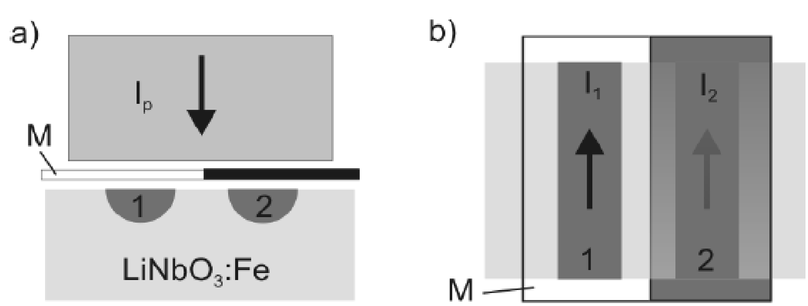


$\mathcal{PT}$–symmetry breaking point

Guo, Salamo, Morandotti, Siviloglou, and Christodoulides, PRL 103, 093902 (2009)

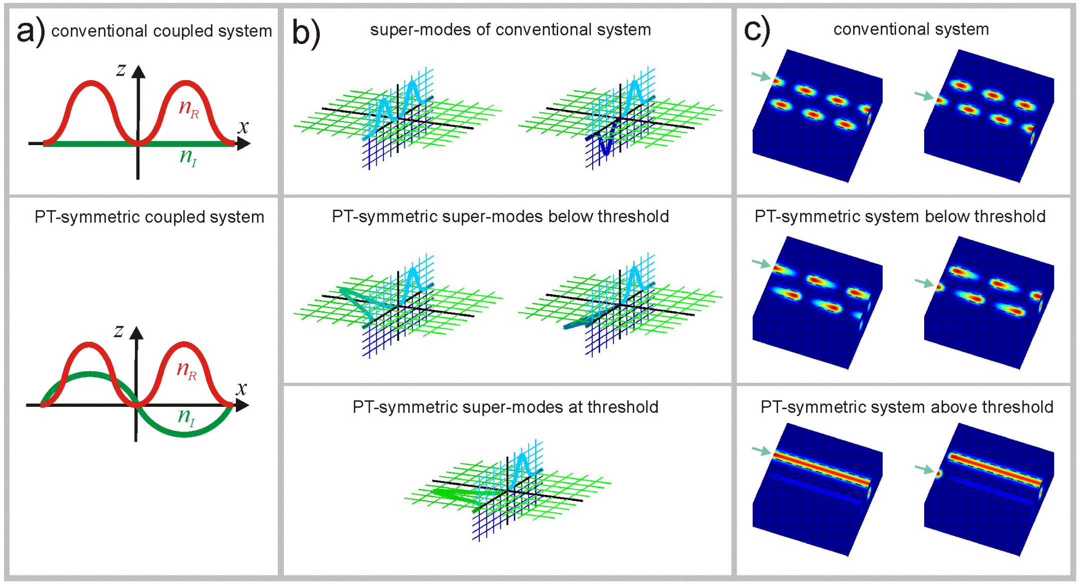
Active PT structures: Two- wave mixing gain





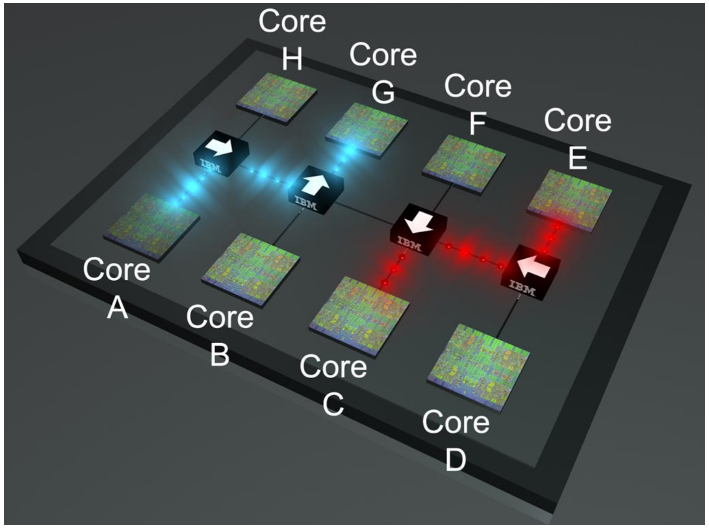
Ruter, Kip, Makris, El-Ganainy, Segev, Christodoulides, Nature Physics 6, 192 - 195 (2010)

Active PT structures: Two- wave mixing gain



Optical isolators and circulators

High-bandwidth
Integrated Photonic Circuits

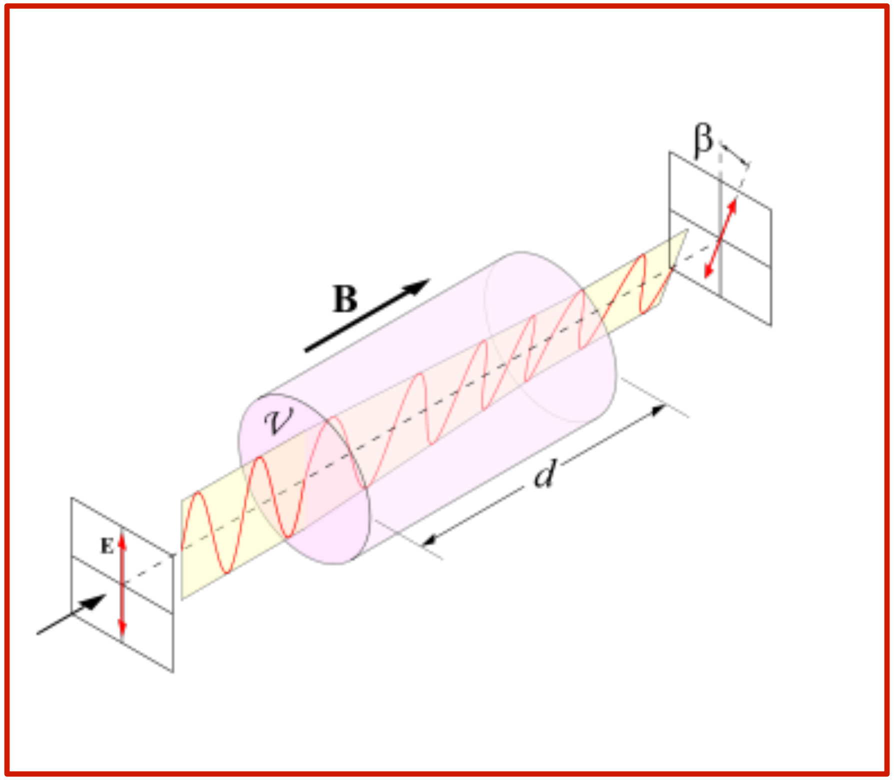


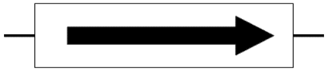
High-speed, integrated photonic platforms will play a crucial role in a broad range of applications

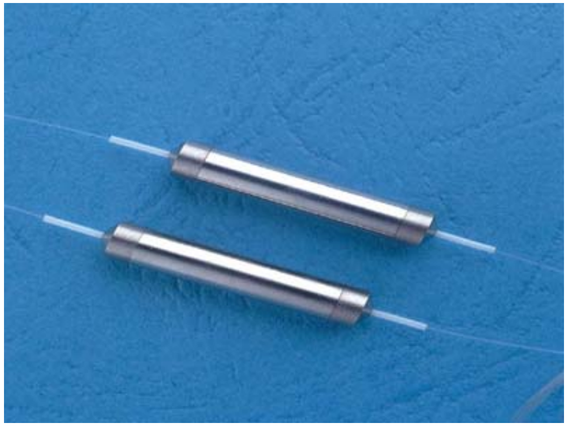
For this to materialize, all important components for light generation, switching, processing, and detection must be integrated on the *same wafer*.

Optical isolators are such indispensable components !

Optical isolators

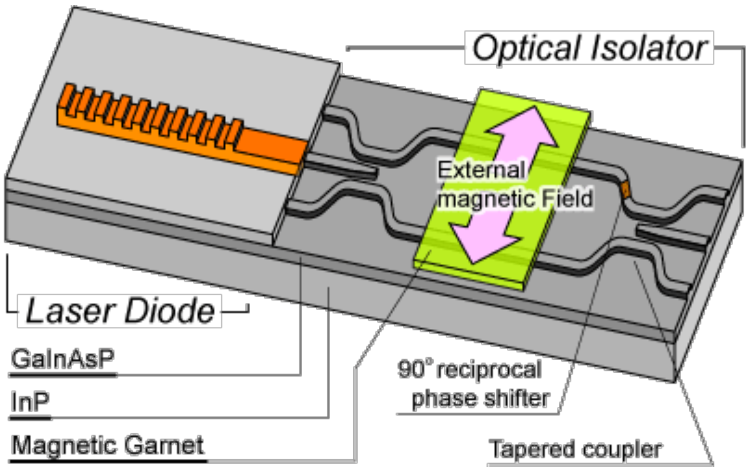




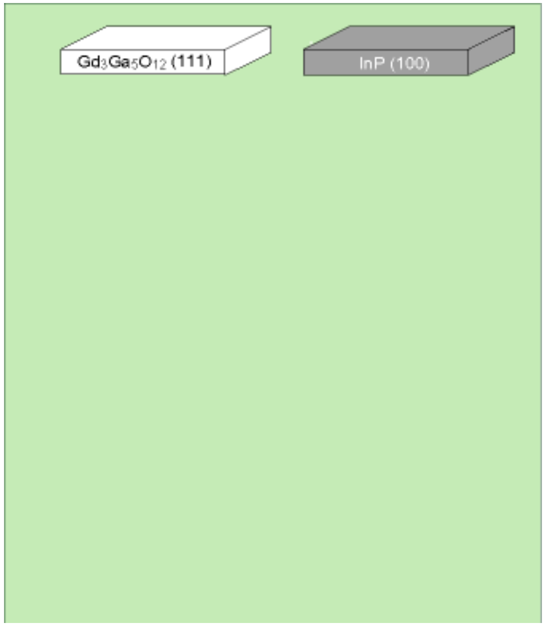


Optical isolators involve magneto-optic materials between polarizers or birefringent plates. Typically garnets are used with high Verdet constants.

Existing technologies



Mizumoto et al.

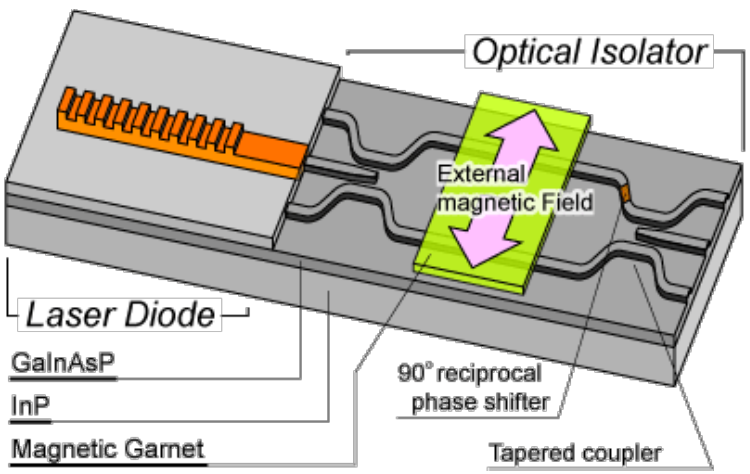


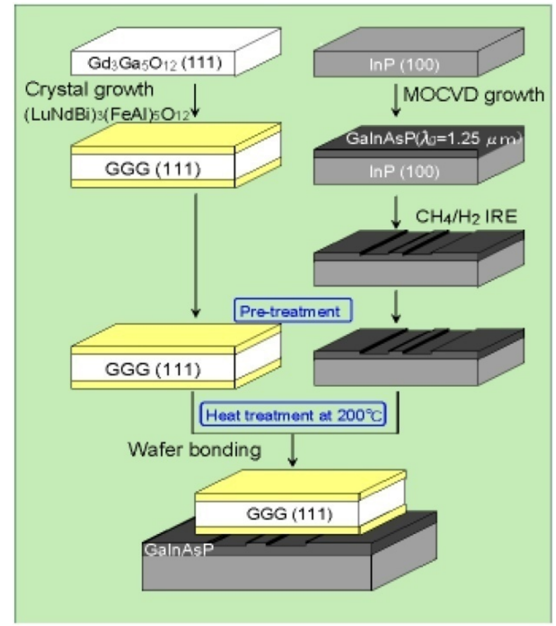
wafer direct bonding

magnetooptic garnets on
III-V compound semiconductors

Existing technologies

Non-Reciprocal Phase Shift in Magneto-Optic Thin Films



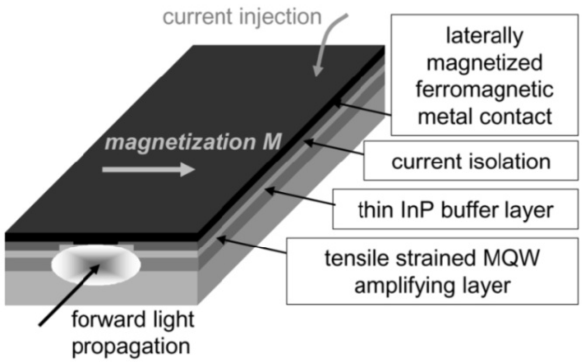


wafer direct bonding

Mizumoto et al.

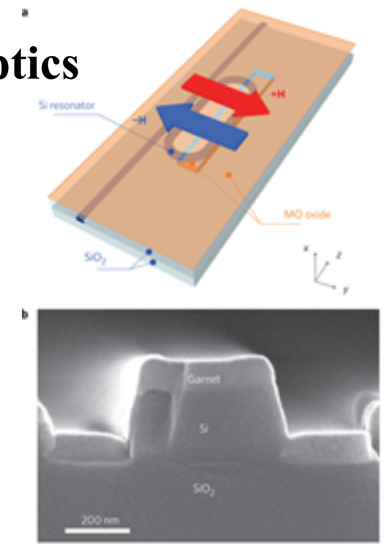
magnetooptic garnets on III-V compound semiconductors

Existing approaches for optical isolators



W. Van Parys et al, IEEE PTL, 19, pp 659, 2007

Si+magnetooptics

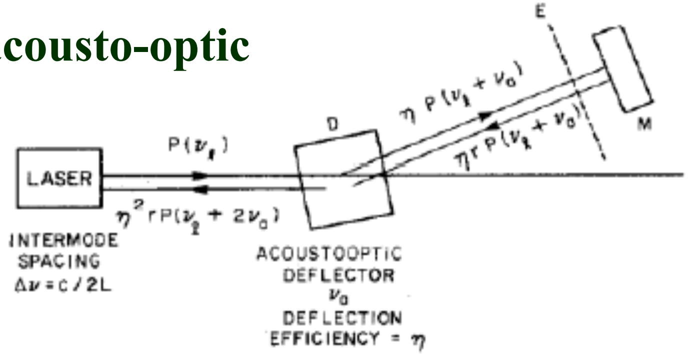


Kimerling's group, MIT, Nature Photonics 5, 758 (2011)

Are magneto-optic approaches necessary for optical isolation ?

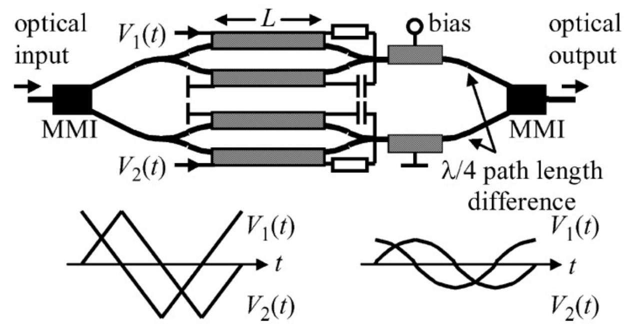
Alternative routes for non-reciprocity

acousto-optic

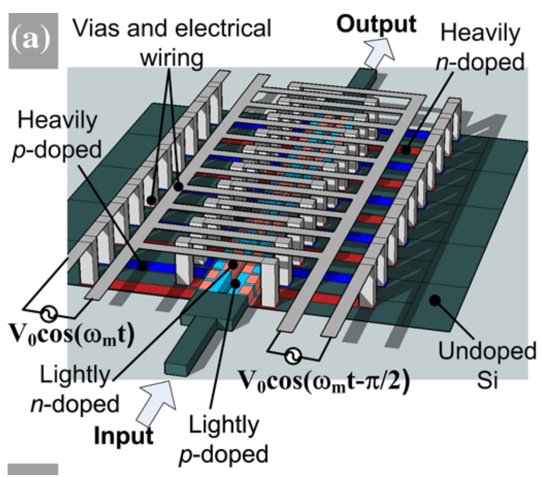


R. G. Smith, JQE 9, 545 (1973)

electro-optic



Ibrahim, S.K. et al, Electronics Letters, 40, 1293 (2004)
S. Bhandare et al, IEEE Sp. Top. Quant. Electron. (2005)
30 dB on III-V, 4.0 GHz



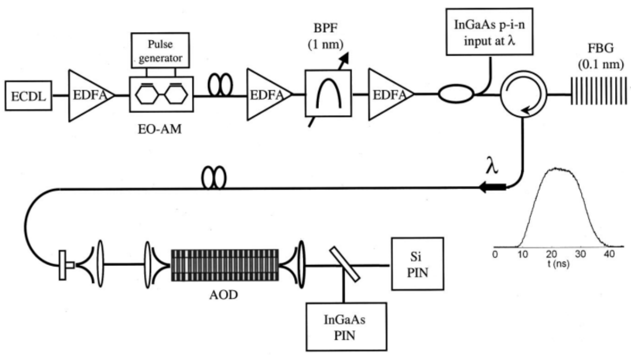
inter-band photonic transitions on a silicon chip

Yu & Fan, NP 3, 91 (2009)
Lira, Yu, Fan, Lipson, PRL 109, 033901 (2012)

Alternative routes for non-reciprocity

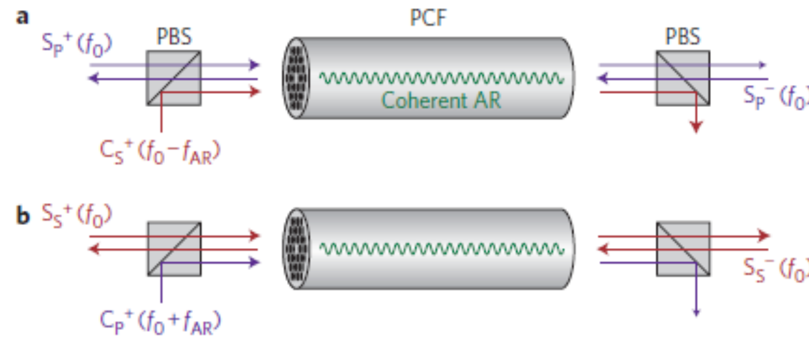
Nonlinear approaches

PPLN

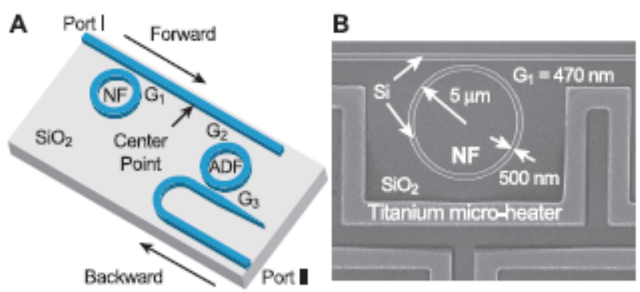


1-3 Watts
K. Gallo, G. Assanto, M. Fejer, APL 79, 314 (2001)

SBS



Kang, Butsch , Russell, NP 5, 549 (2011)



Minghao Qi , Weiner et al.,
Science 335, 447 (2012)

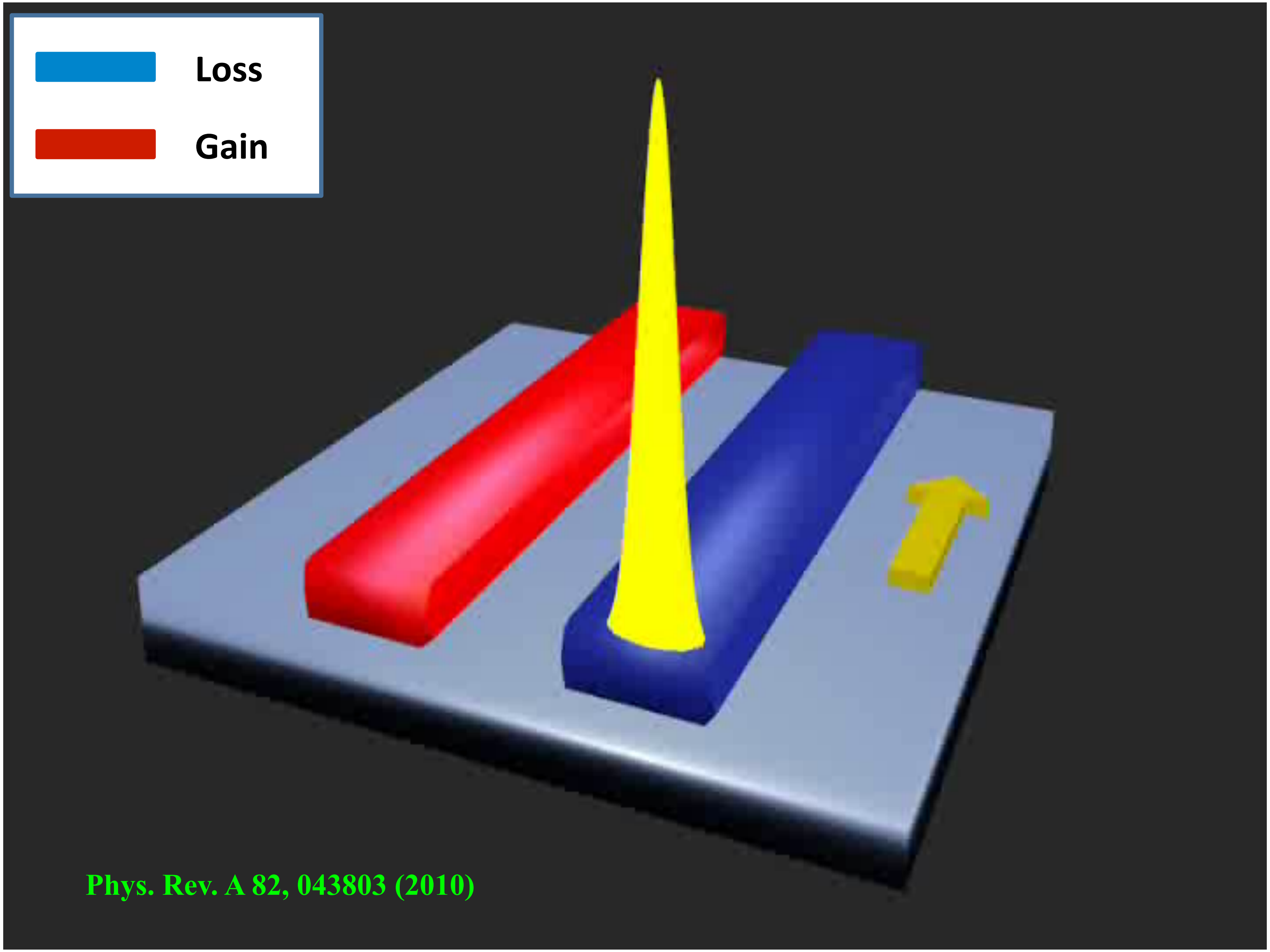
Requirements for on chip isolators and circulators

- Must be compact
- Broad-bandwidth
- Retain the color of the signal
- Polarization insensitive
- Use processes "indigenous" to the wafer itself

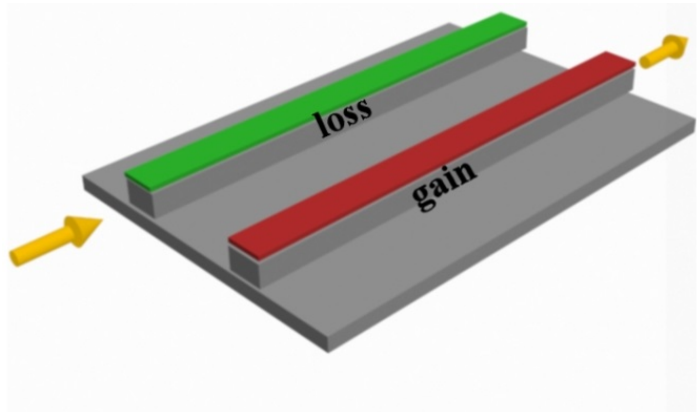
Nonlinear non-Hermitian unidirectional optical structures

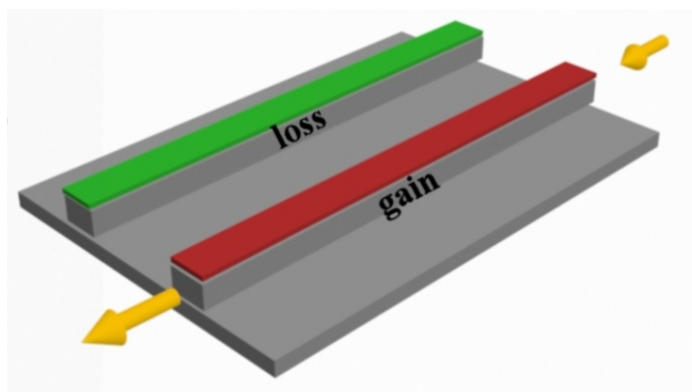
To achieve a high degree isolation nonlinearity is used in conjunction with gain/loss

The power in the system is forced to follow a non-linear roller-coaster


Loss
Gain
Phys. Rev. A 82, 043803 (2010)

Non-Hermitian nonlinear isolators



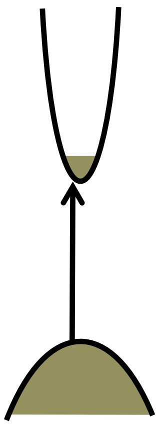


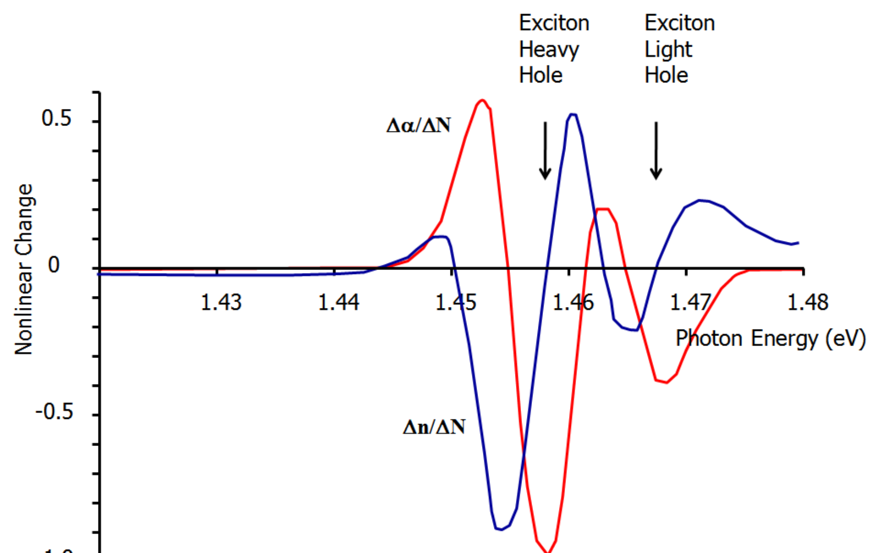
Semiconductor nonlinearities

Giant bandgap resonant nonlinearities resulting from

- **Exciton saturation**
- **Band filling effects**
- **Plasma effects**

$$n_2 = 1.4\times10^{-7}\ cm^2/W$$

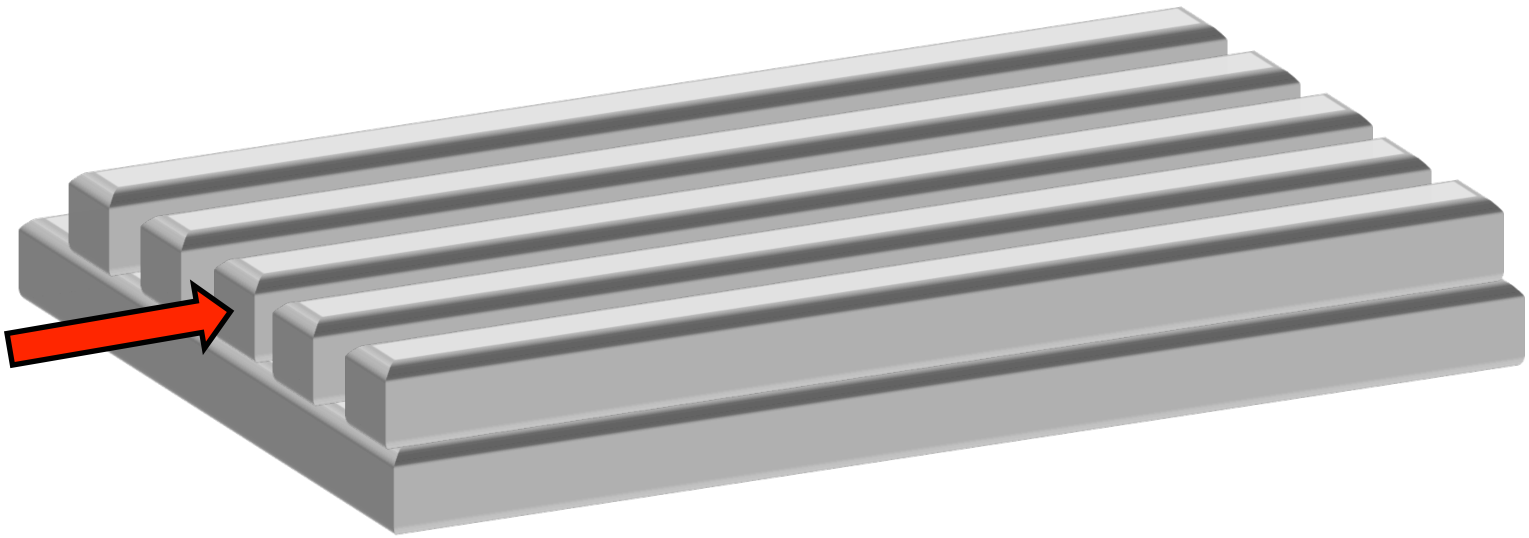




The large absorption associated with the nonlinearity is in this case <u>advantageous !</u>

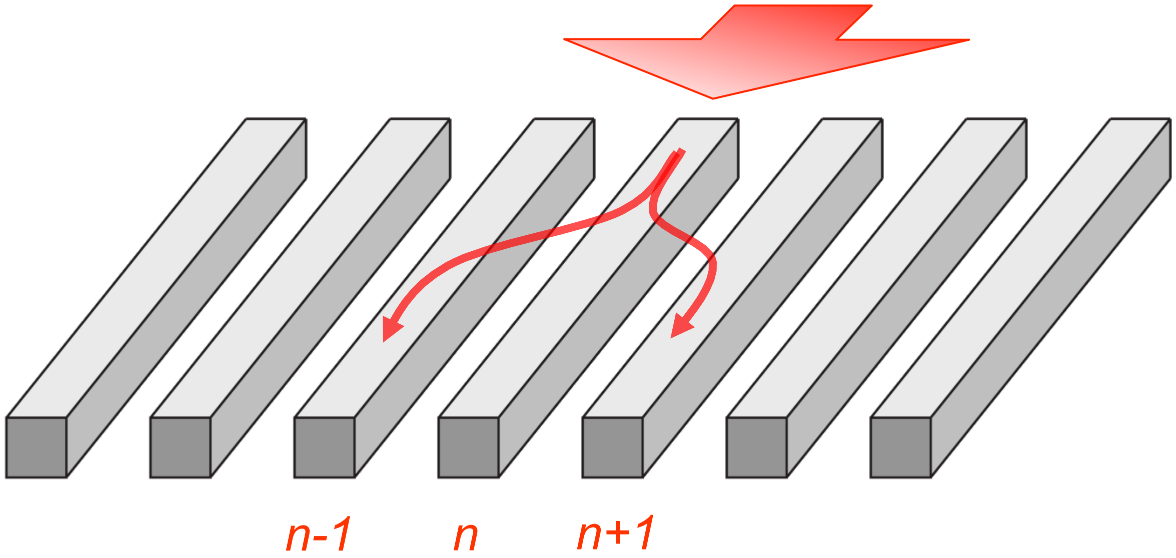
Silica glass: $n_2 = 2\times10^{-16}\ cm^2/W$

Structures & Parameters



- InGaAs quantum wells on InGaAsP
  Wavelength: 1.55μm
  Typical Losses: 20 cm^{-1}
  L ~ 1mm
  Power Response: ~50 μW
  Coupling lengths: ½ mm

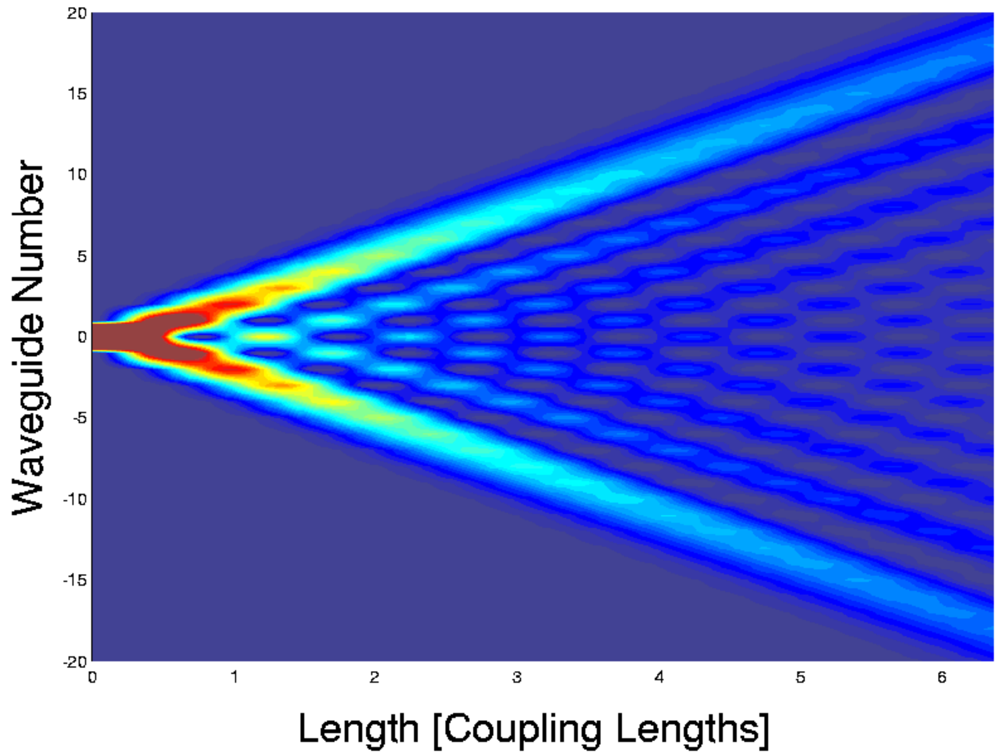
Waveguide arrays



$$i\frac{dE_n}{dz} + c(E_{n+1} + E_{n-1}) = 0$$

Jones, J. Opt. Soc. Am. 55, 261 (1965).
Somekh, Garmire, Yariv, Garvin, Hunsperger, APL 22, 46 (1973).
Christodoulides, Lederer, Silberberg, Nature 424, 817 (2003)

Linear operation of the array: discrete diffraction

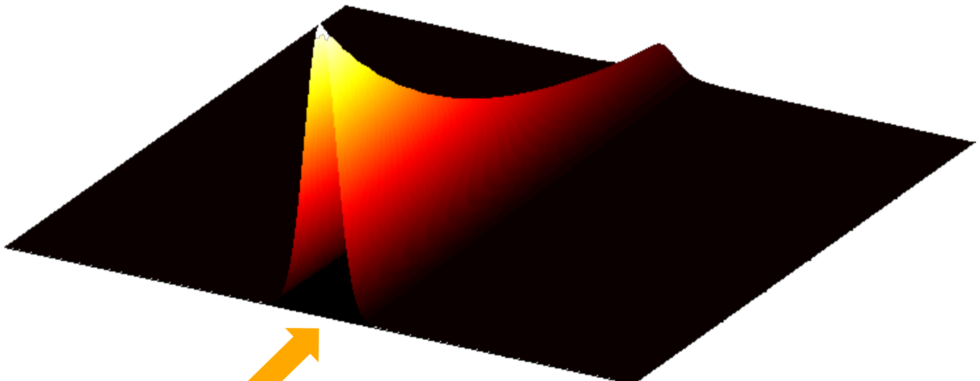


$$J_0(2cz) = 0$$

$$2cz = 2.4, 5.52, \ldots$$

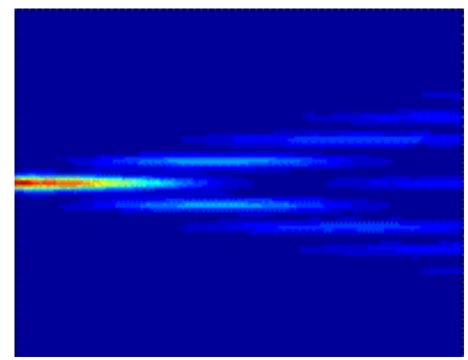
$$E_n = A_0 (i)^n J_n(2cz) \exp(i\beta z)$$

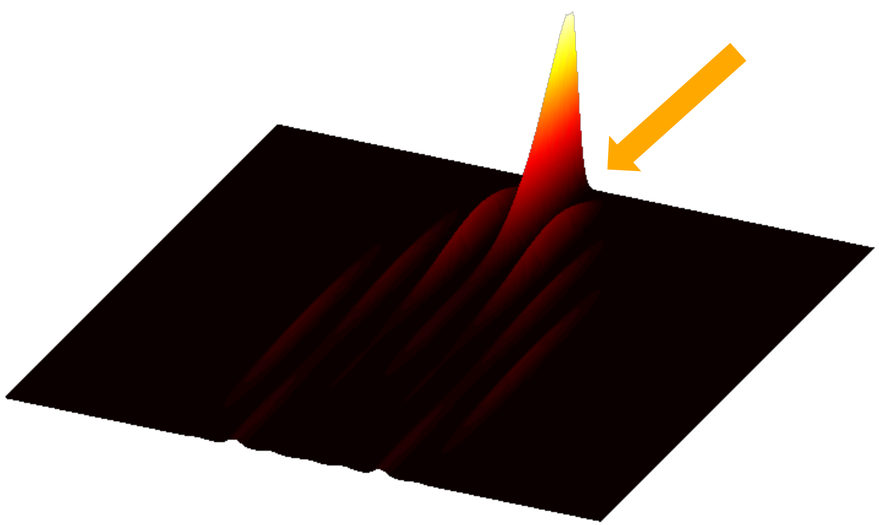
Non-Hermitian optical isolator



Forward

Isolation ratio: ~25 dB



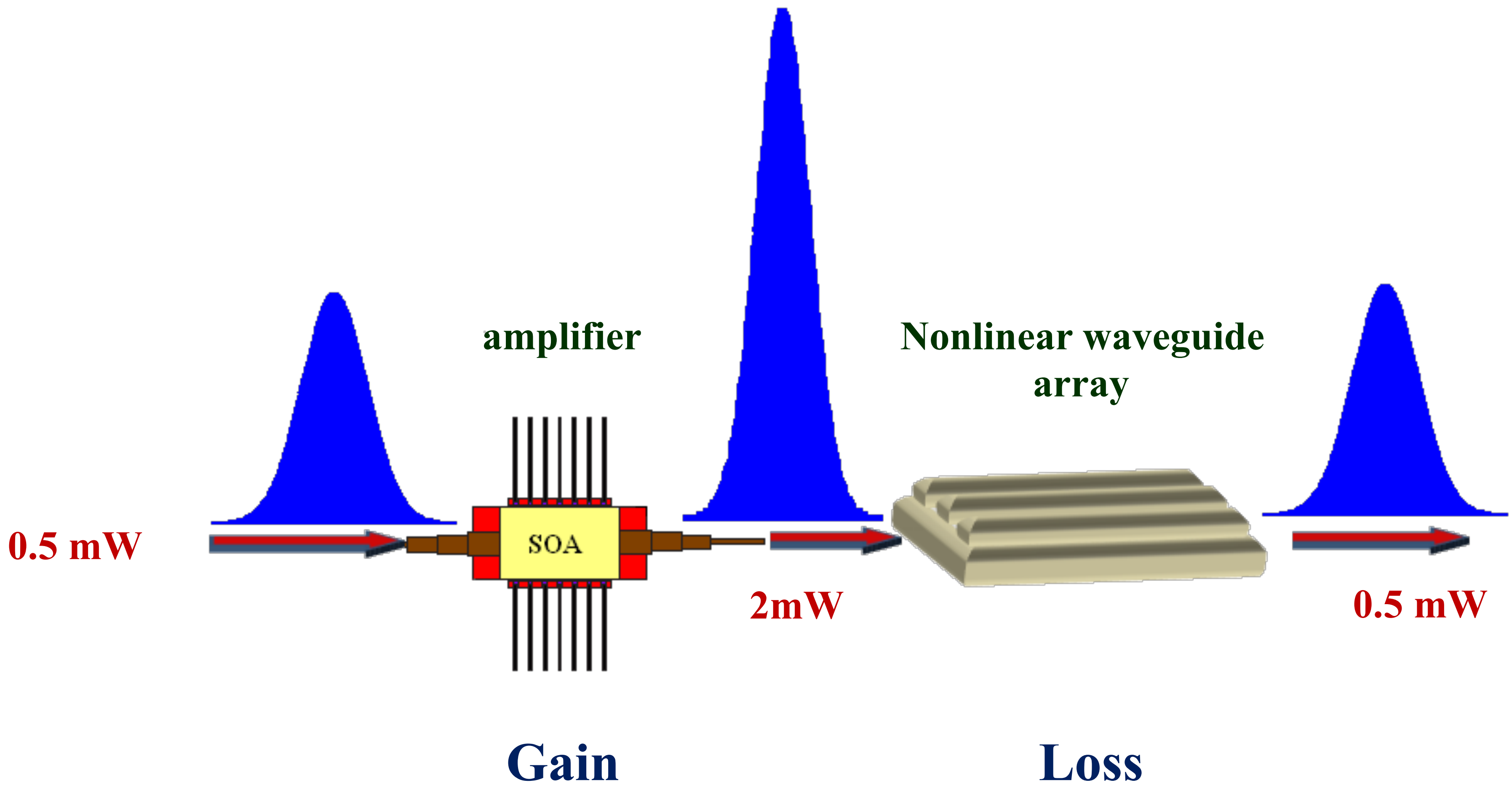


Backward mode

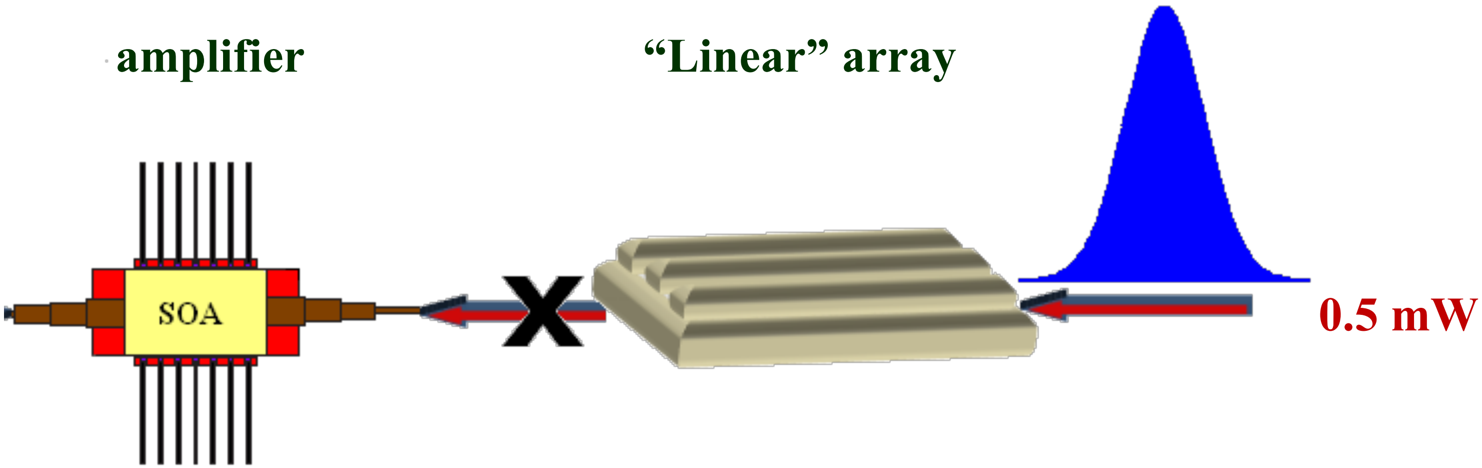
FORWARD:
L= 1.0 mm
Pin = 2 mW
Pout = 0.5 mW

BACKWARD:
Pin = 0.5 mW
Pout = 1 μW

Principle of operation

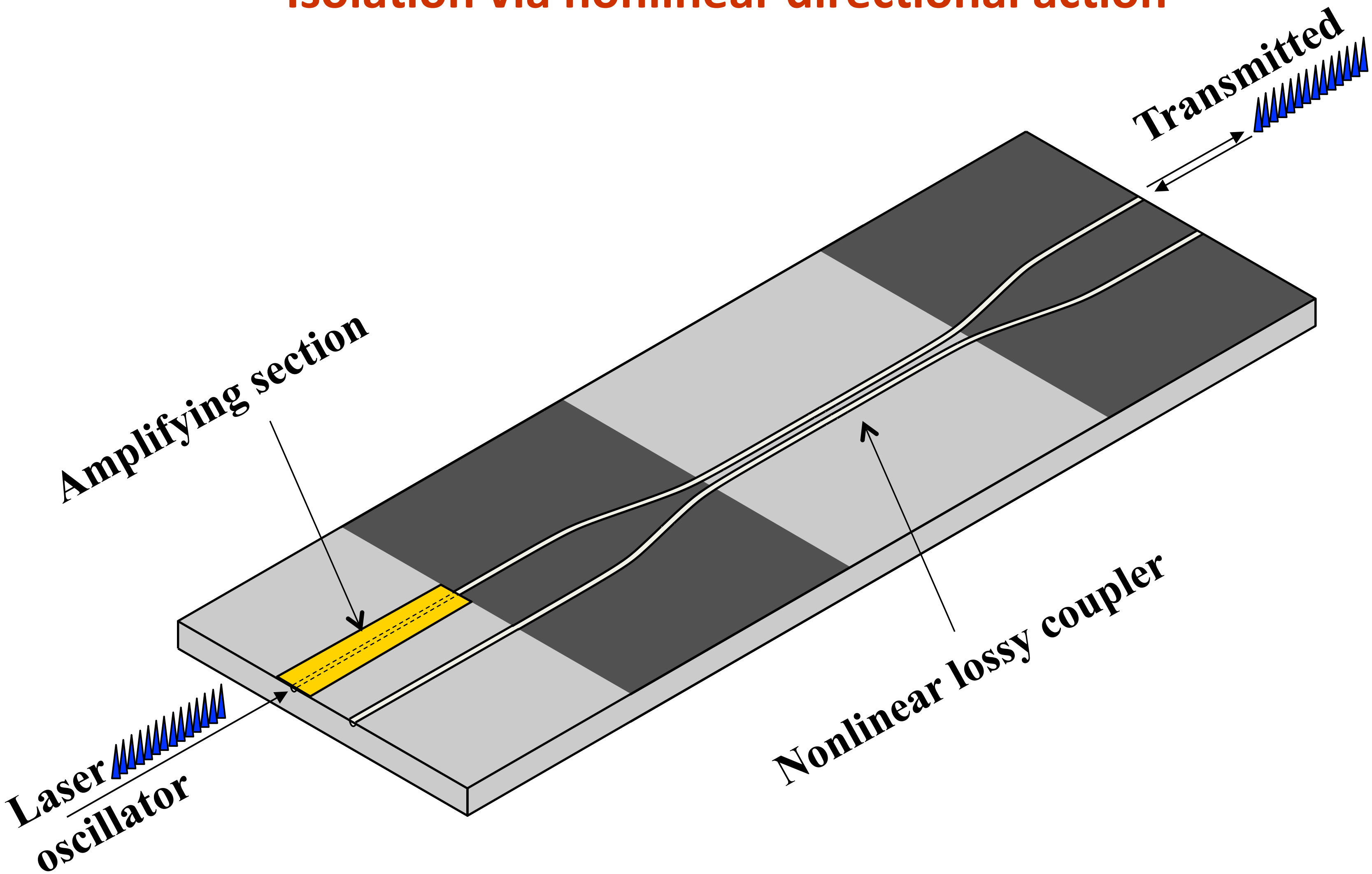


Principle of operation

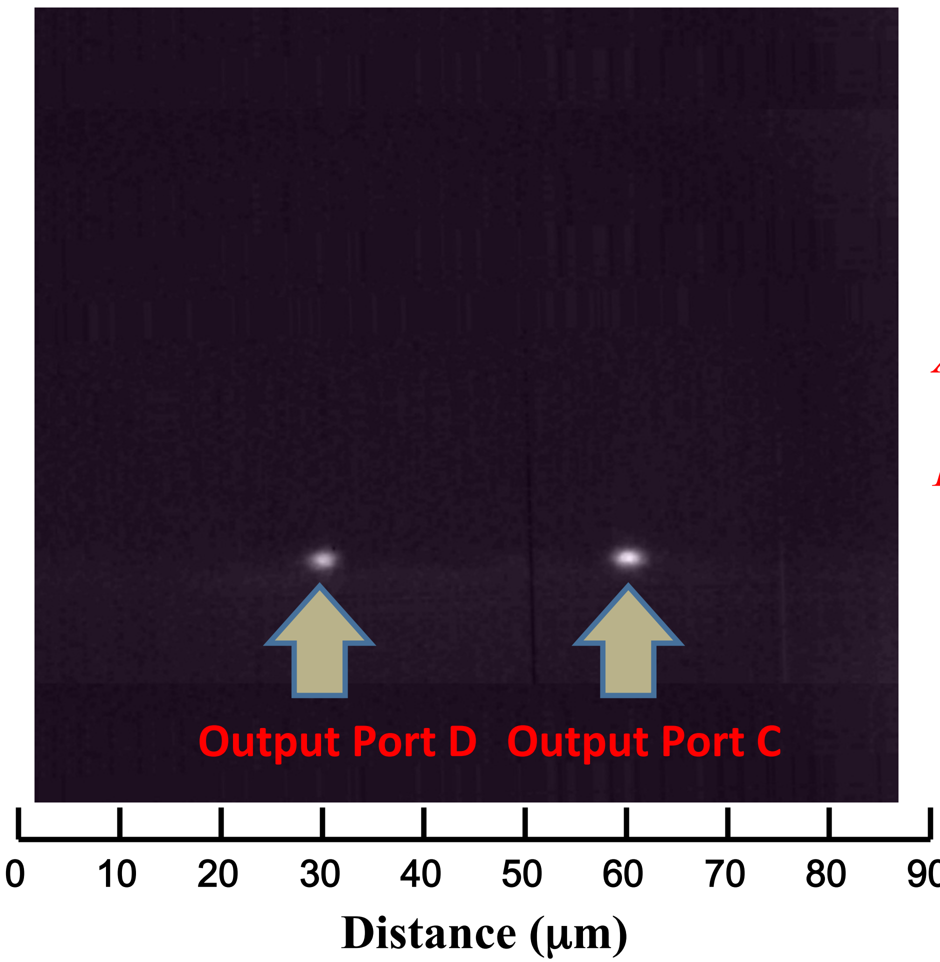


Light is blocked when launched from the other side

Isolation via nonlinear directional action



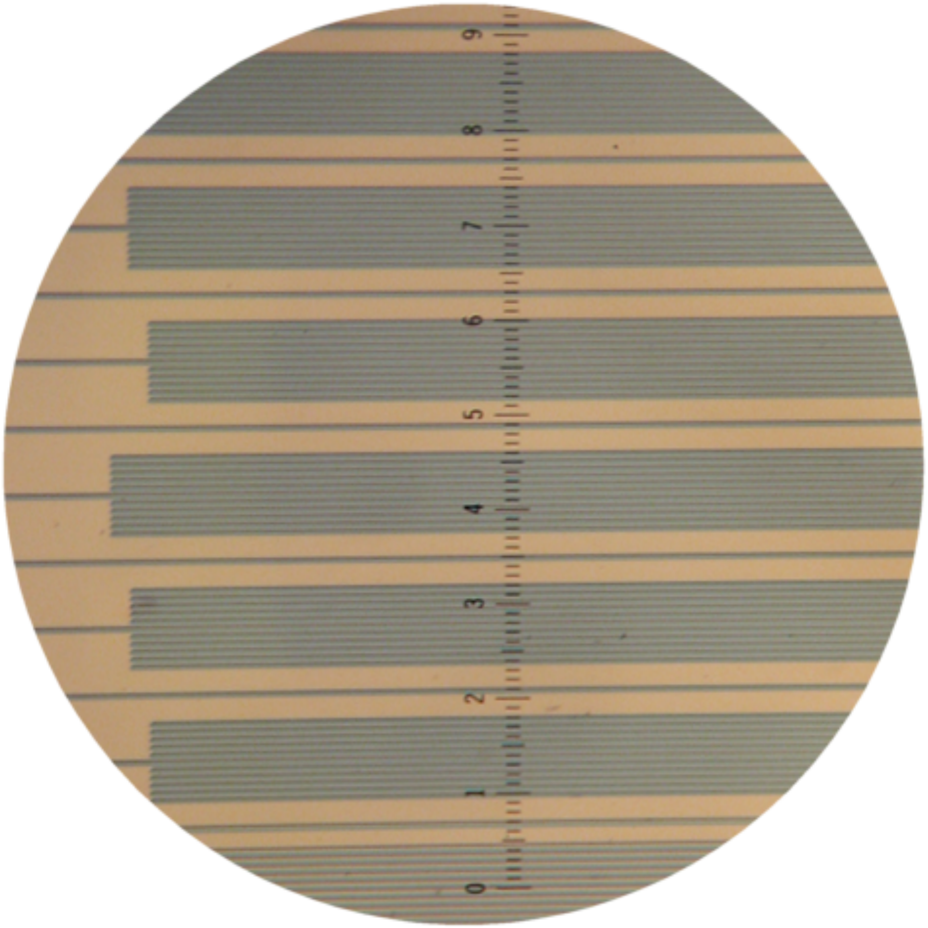
Photograph of Output Facet of the Directional Coupler

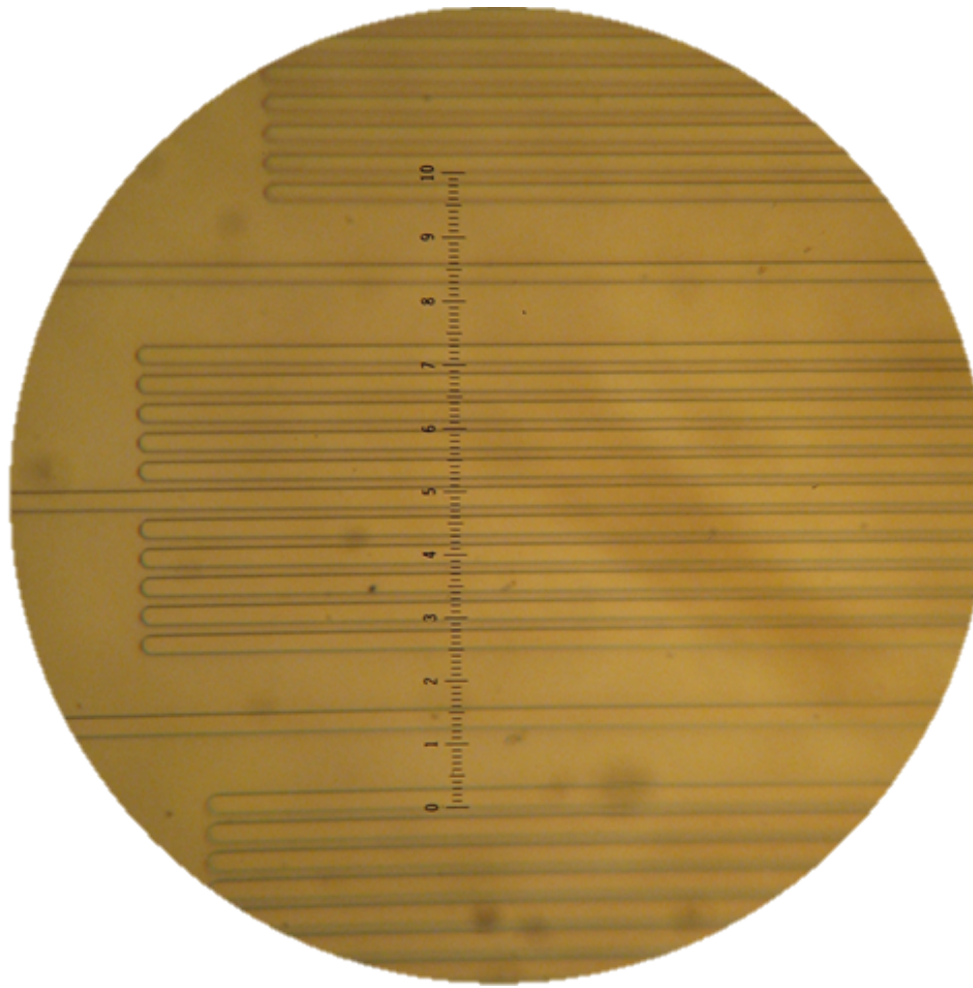


mode-locked
Er-fiber laser

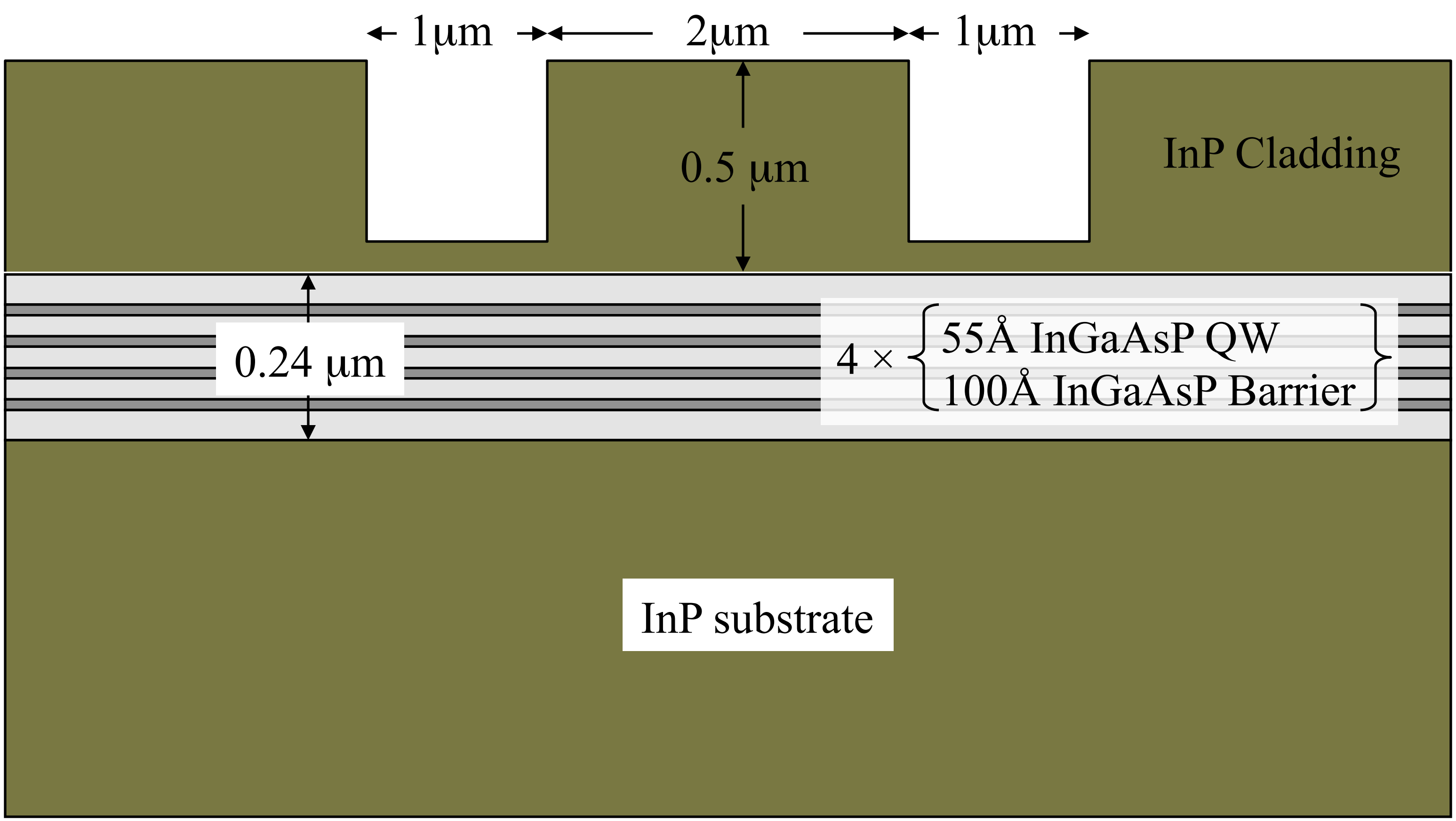
Device Under Test

Coupled Arrayed Waveguides Fabricated on InGaAsP MQW Structure





InGaAsP multiple quantum well structures



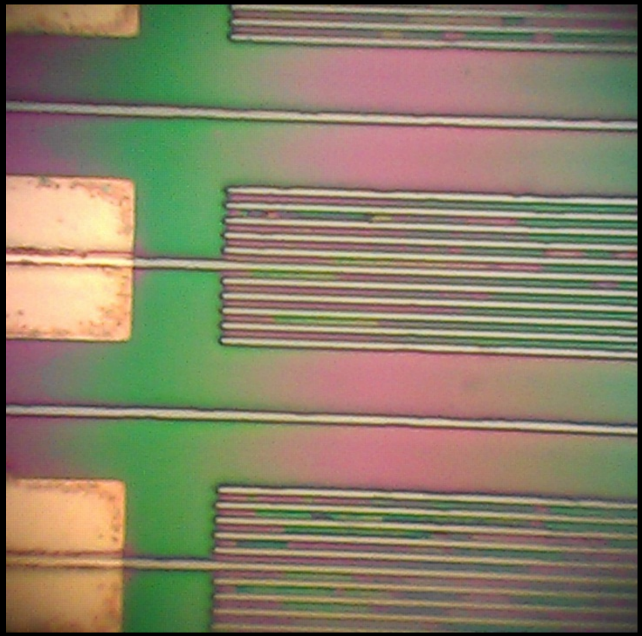
Non-Hermitian optical isolator waveguide array

Array: 11 waveguides, 2.5 μm wide with 2 μm separation. The array is 950 μm long.

The gain section (6-7 dBs) is 910 μm long and it is separated from the front of the array by 20 μm.

- **Broadband: ~20 nm**
- **Negligible ASE**
- **Both InGaAsP and AlGaAs systems tested**
- **Pulsed 1-10 ps**

Top view



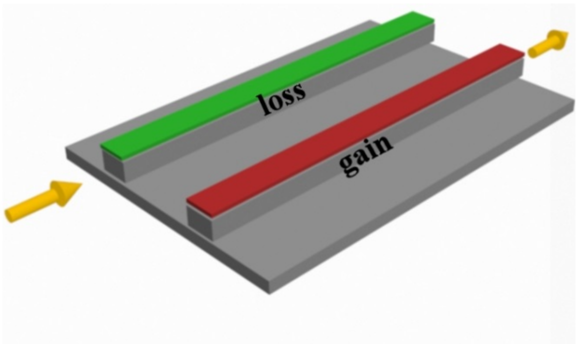
Isolation ratios obtained so far:

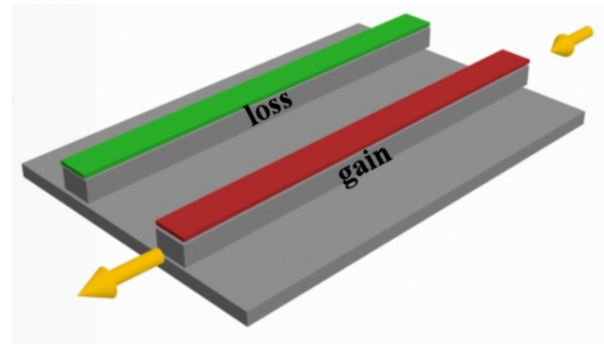
InGaAsP: 14 dBs

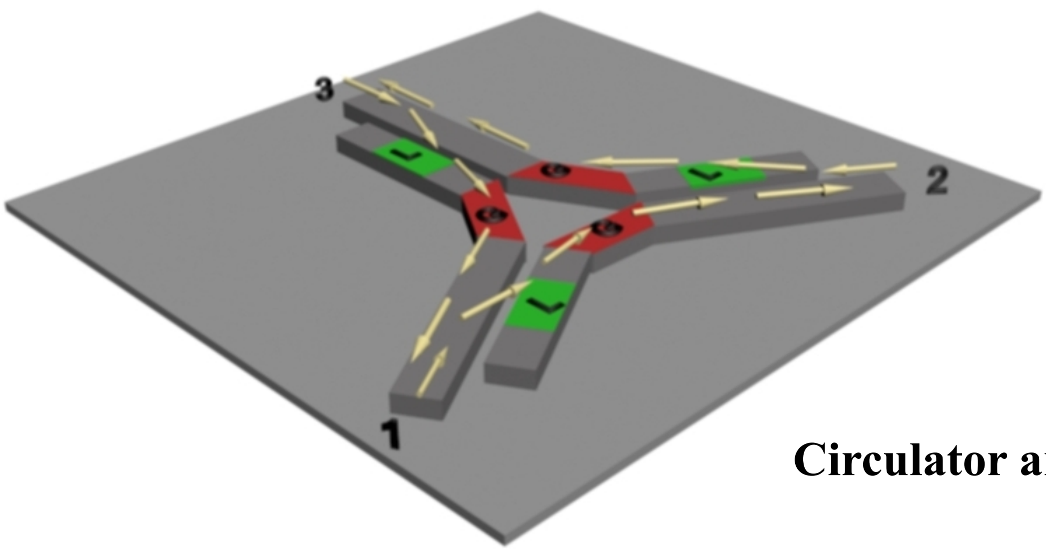
AlGaAs: 11 dBs

In principle 20-30 dBs should be possible by optimizing device fabrication tolerances and characterization

Non-Hermitian circulators



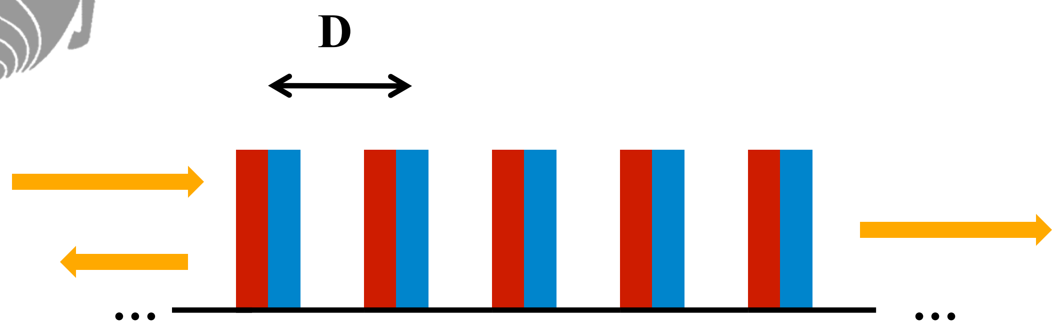




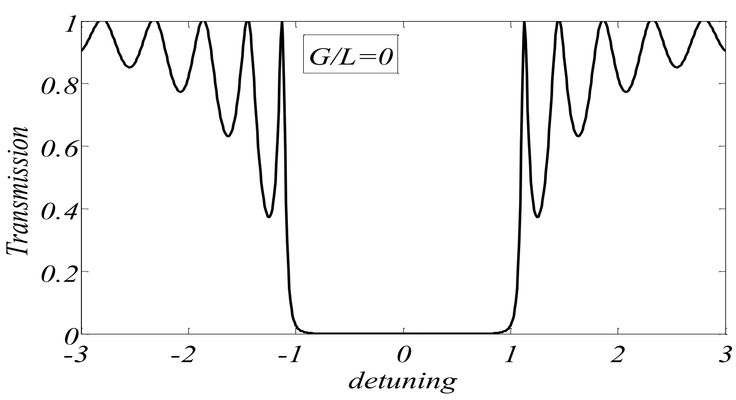
Circulator arrangement

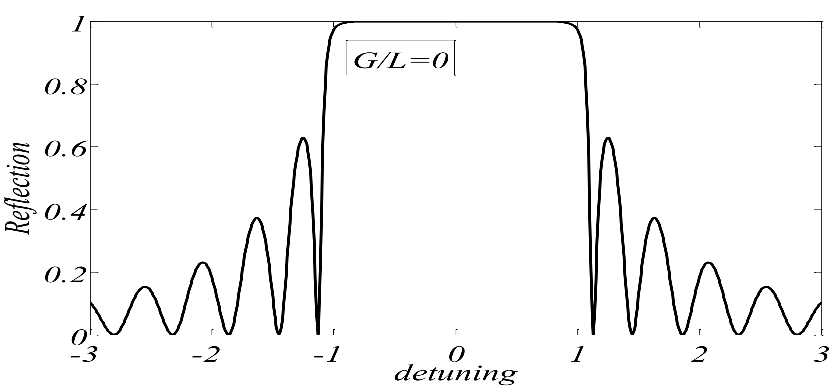
$\mathcal{PT}$ periodic tructures

$\mathcal{PT}$ cavities and scatterers

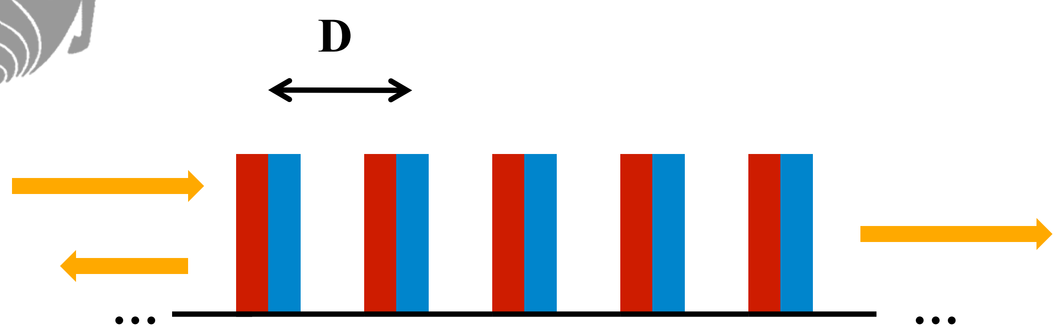


Passive cavity



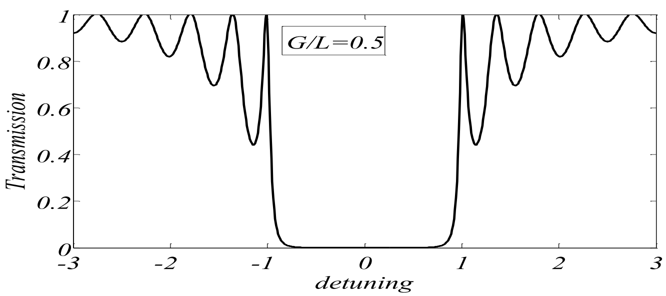


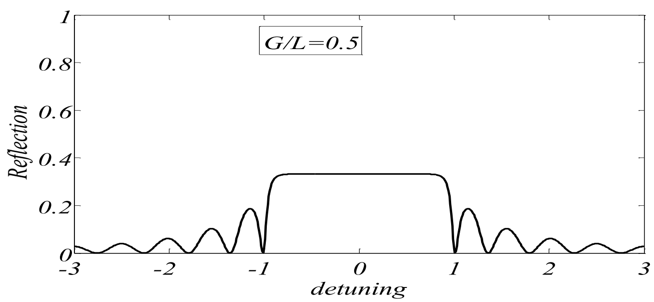
$\mathcal{PT}$ cavities and scatterers

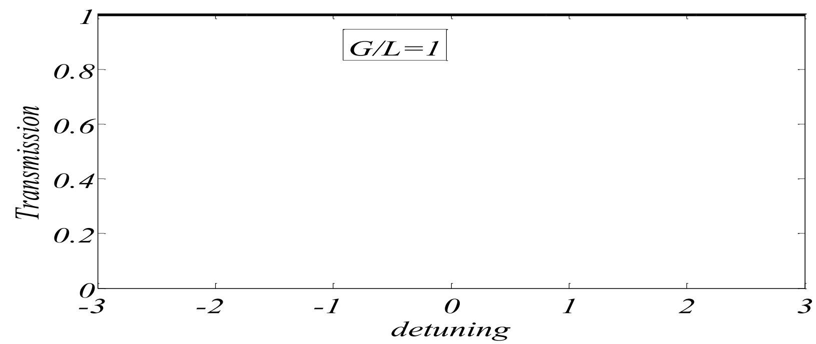


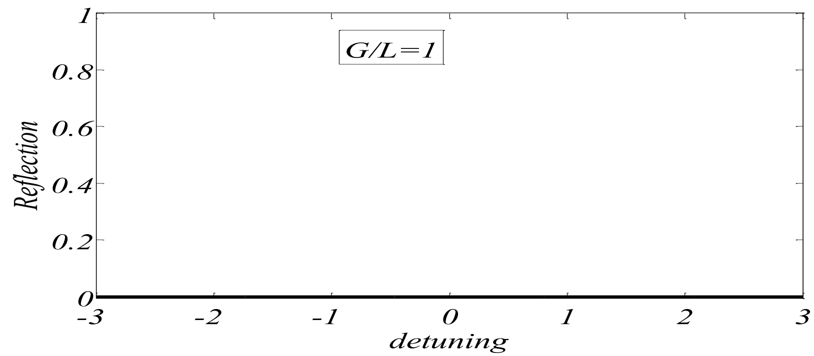
On PT threshold the grating becomes transparent and reflectionless

PT cavity





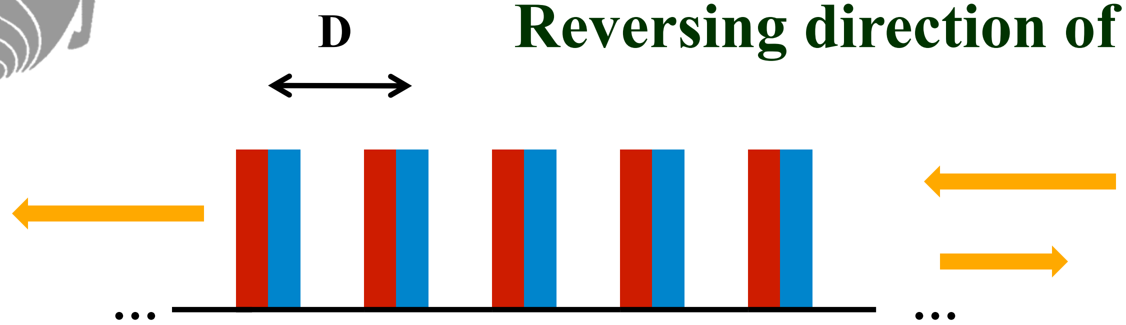


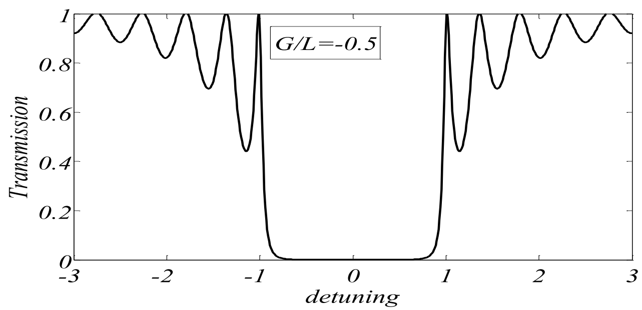


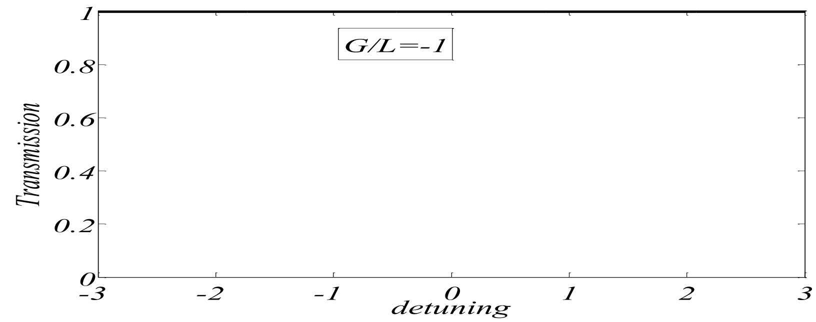
Lin, Eichelkraut, Ramezani, Kottos, Cao, and Christodoulides, PRL (2011)

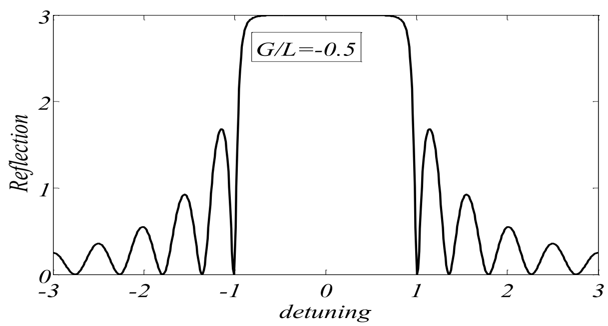
$\mathcal{PT}$ gratings

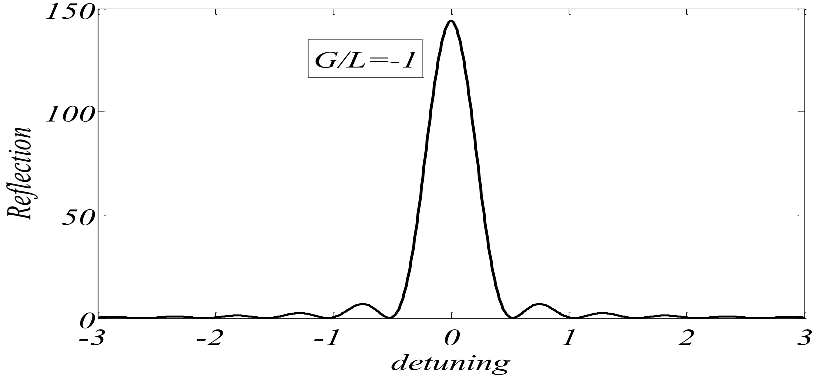
Reversing direction of entry





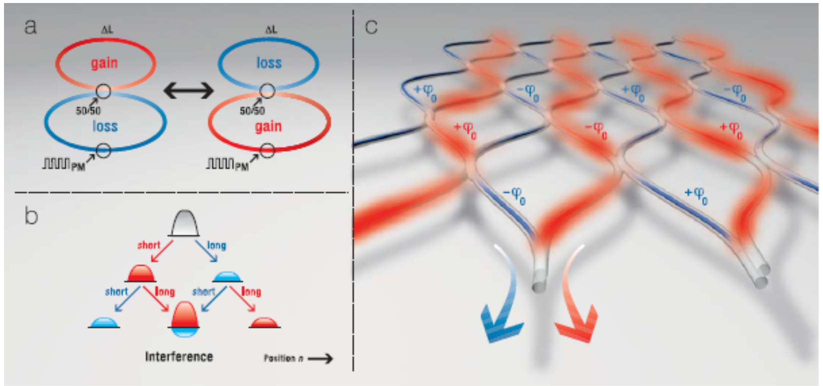


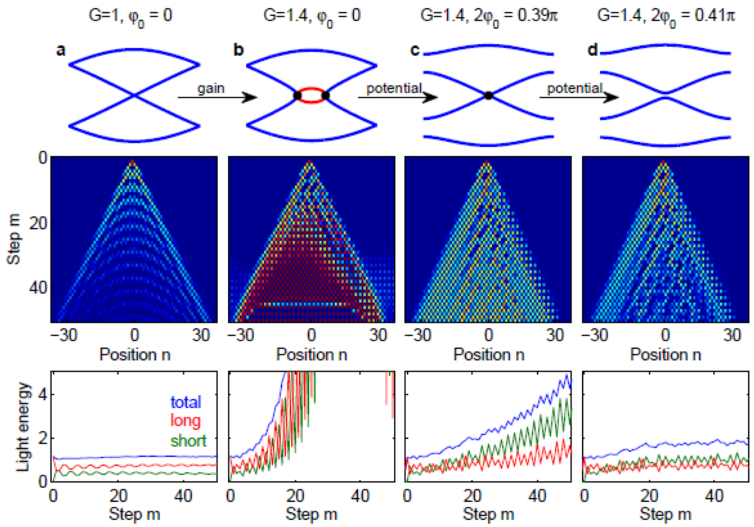


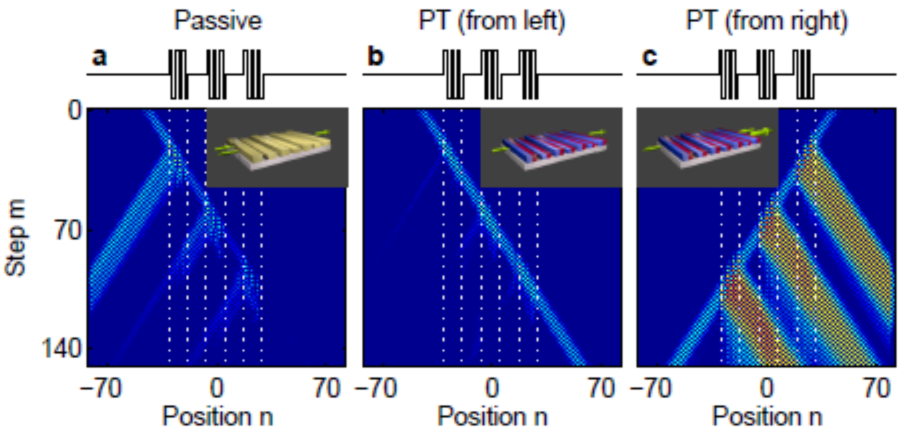


$\mathcal{PT}$ gratings

Experimental results



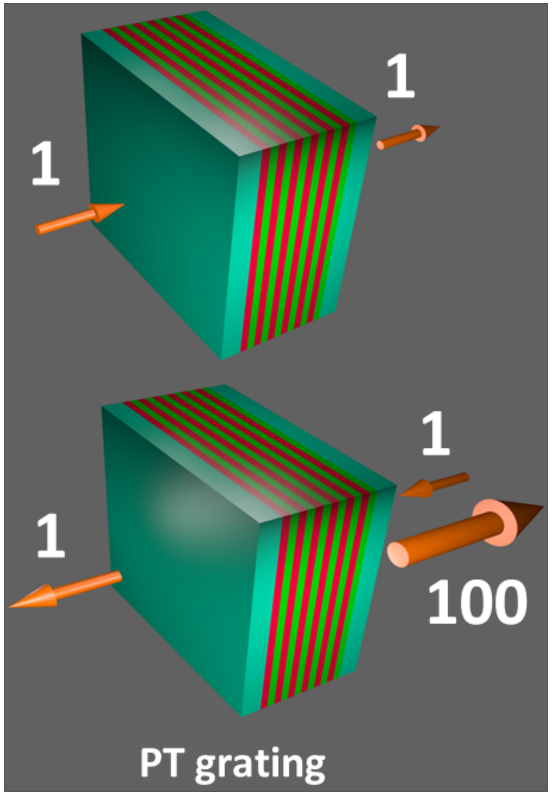




Nature 488, 167 (2012)

Max Planck Erlangen/CREOL

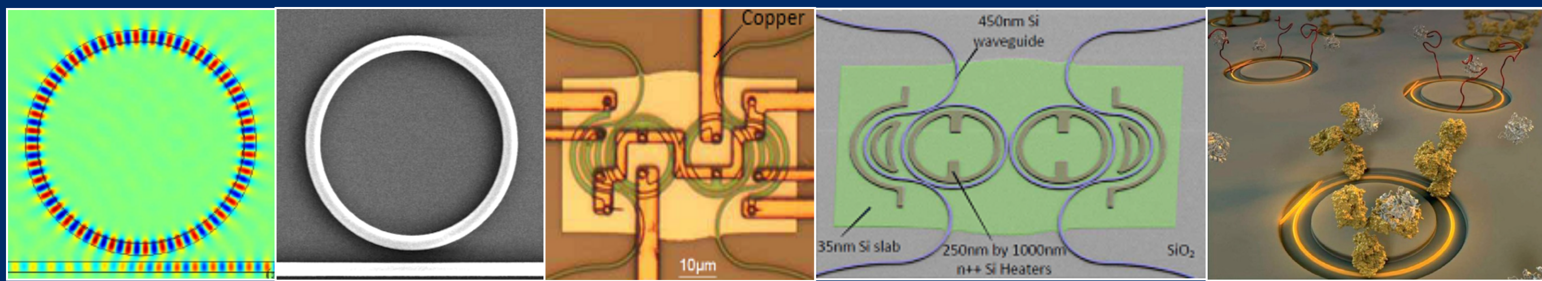
$\mathcal{PT}$-symmetric Optical Structures: Unidirectional Invisibility



Nature Materials, 12, 108 (2013)
(realization in silicon, Caltech)

PT-symmetric micro-ring lasers

Passive micro-ring resonators



H. L. R. Lira, C. B. Poitras, M. Lipson, Opt. Express **19**, (2011)

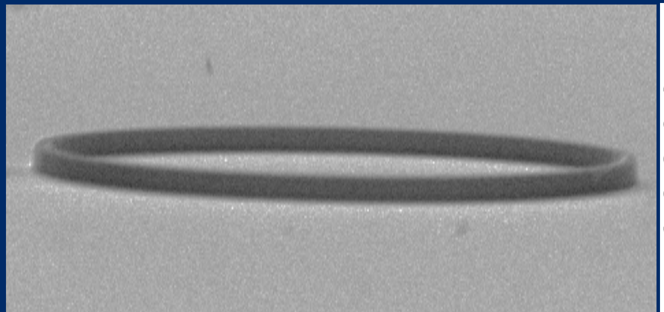
O. Scheler, et. al., *Biosensors & Bioelectronics*, 36, (**2012**)

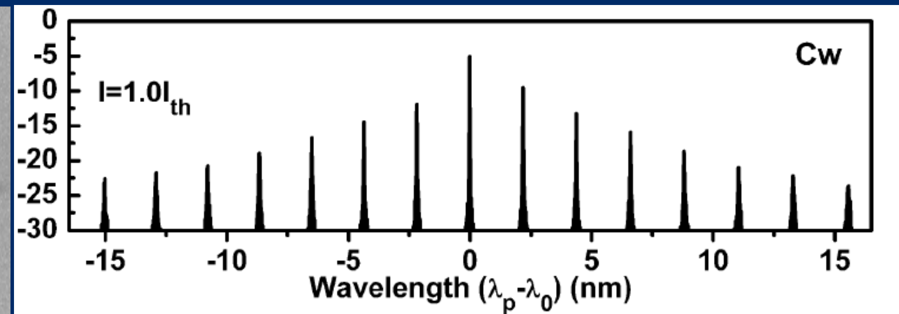
+ High quality factor
+ High confinement
+ Small footprint
+ Simple fabrication

ideal components for realization of **integrated photonic networks** and **sensing**

How about ring resonators as laser cavities?

Micro-ring lasers





Stamataki, I. et al., " *Quantum Electronics, IEEE Journal of* , vol.42, no.12, pp.1266,1273, 2006

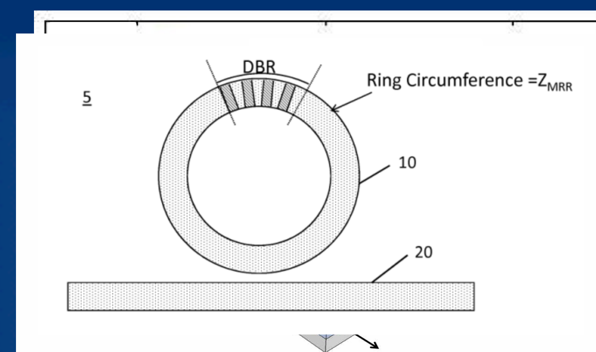
Drawback: Multi longitudinal mode operation within the broad gain bandwidth (over 300 nm for InGaAsP system)

Possible solutions:

1- spatially modulating the pump

2- external cavity tuning (like Vernier effect between coupled resonators)

3- DFB or DBR arrangements

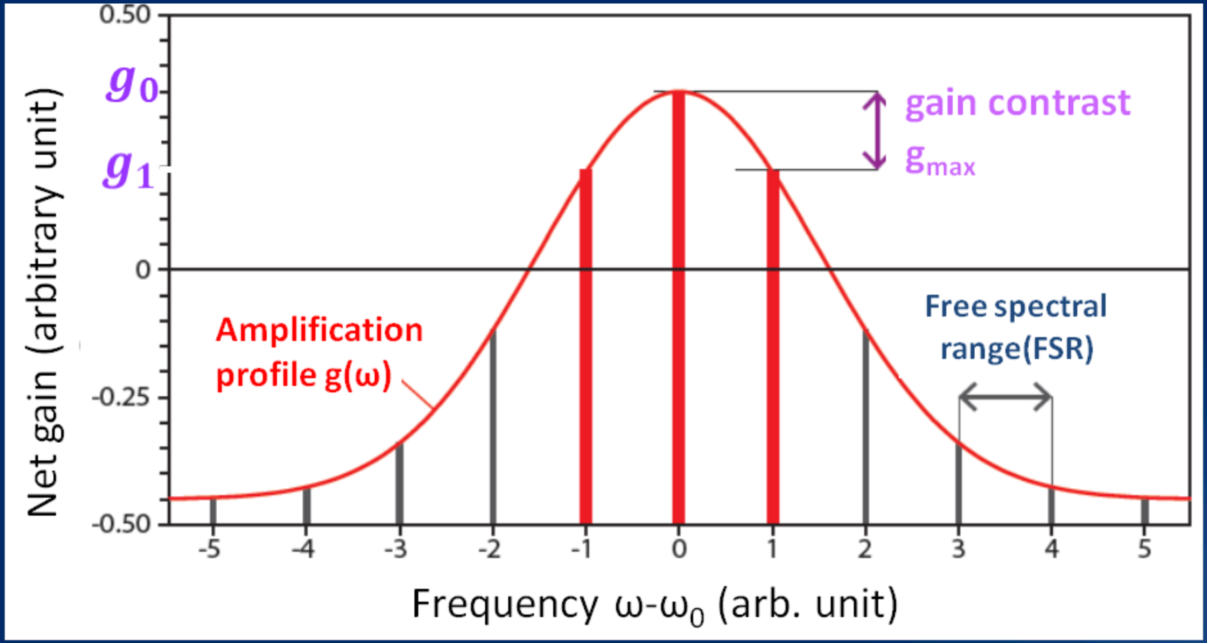


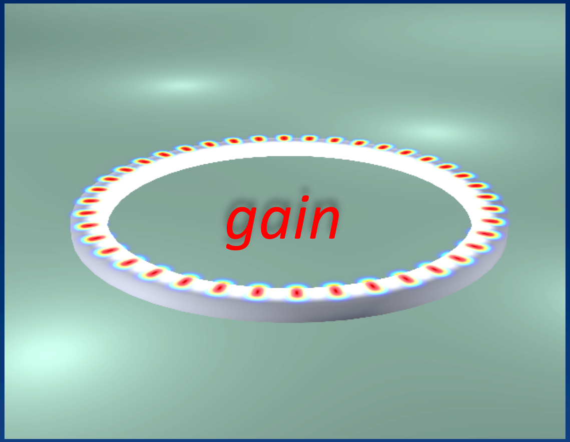
L. L. Goddard, Y. M. Kang, A. Arbabi, US Patent 20120063484 A1

T. Claes, W. Bogaerts, and P. Bienstman, Opt. Express **18**, 22747-22761 (2010)

Can we find a simple yet robust method for enforcing single mode operation in micro-ring lasers?

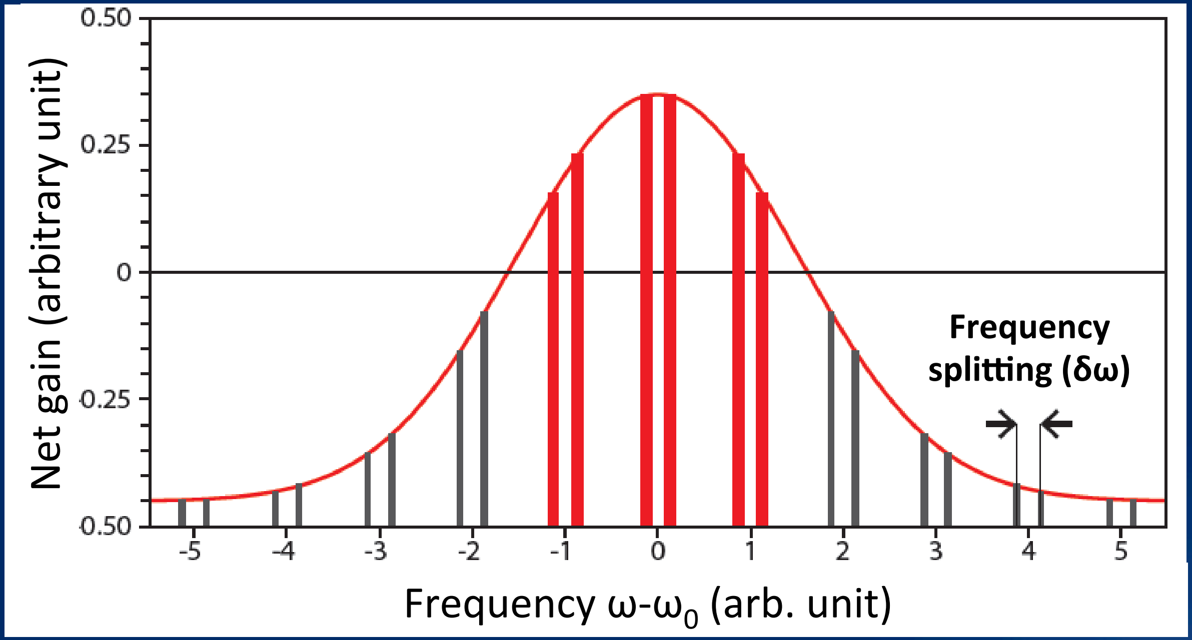
Single micro-ring laser

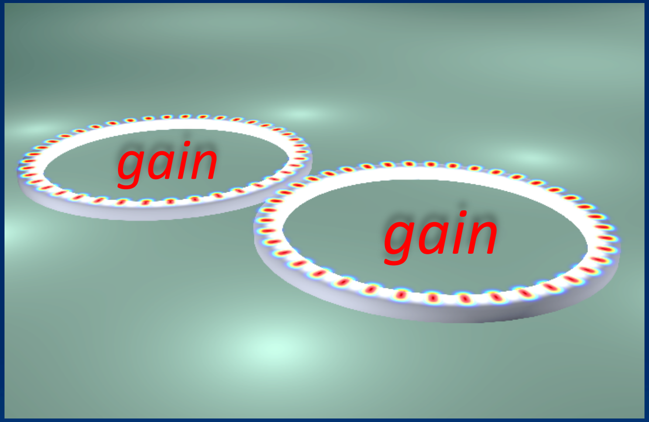




Maximum achievable gain maintaining single mode operation: $g_{max} = g_0 - g_1$

Two coupled micro-ring lasers





Longitudinal modes equation in time domain

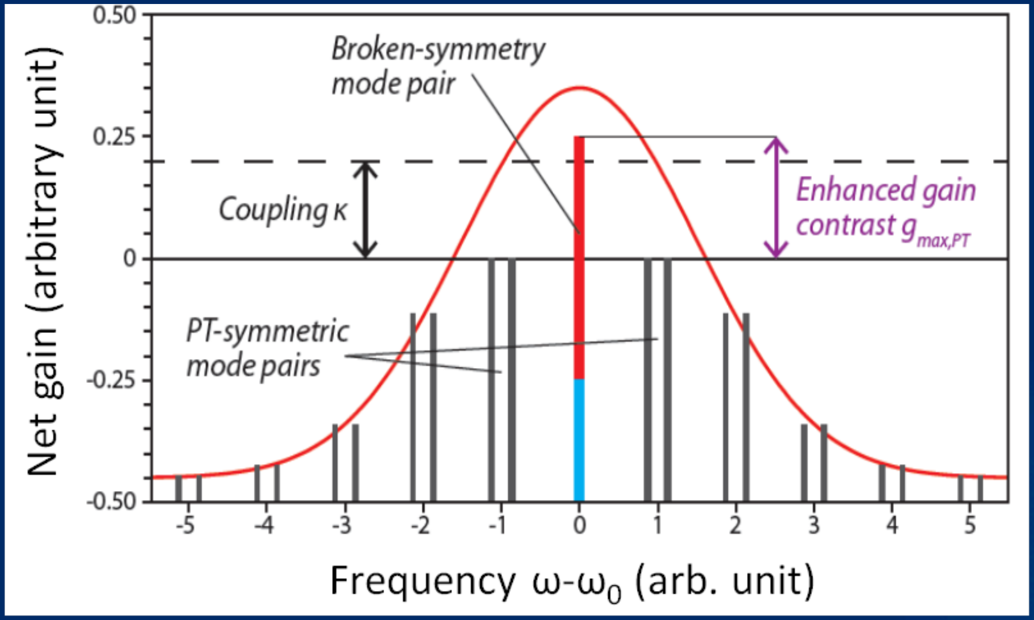
$$\begin{cases} \dfrac{da_n}{dt} = -i\omega_n a_n + i\kappa_n b_n + \gamma_{a_n} a_n \\ \dfrac{db_n}{dt} = -i\omega_n b_n + i\kappa_n a_n + \gamma_{b_n} b_n \end{cases}$$

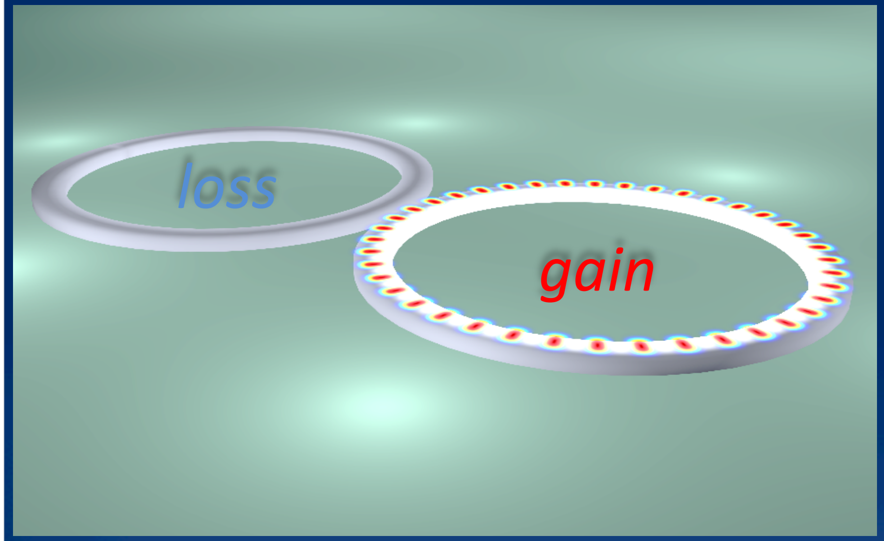
What if we use loss instead of gain in one of the rings?

Eigenfrequencies of the supermodes

$$\omega_n^{(1,2)} = \omega_n + i\frac{\gamma_{a_n} + \gamma_{b_n}}{2} \pm \sqrt{\kappa_n^2 - (\frac{\gamma_{a_n} - \gamma_{b_n}}{2})^2}$$

PT-symmetric micro-ring arrangement





Eigenfrequencies of the supermodes for generic two coupled resonators

$$\omega_n^{(1,2)} = \omega_n + i\frac{\gamma_{a_n} + \gamma_{b_n}}{2} \pm \sqrt{\kappa_n^2 - (\frac{\gamma_{a_n} - \gamma_{b_n}}{2})^2}$$

Eigenfrequencies of the super modes in PT-symmetric case

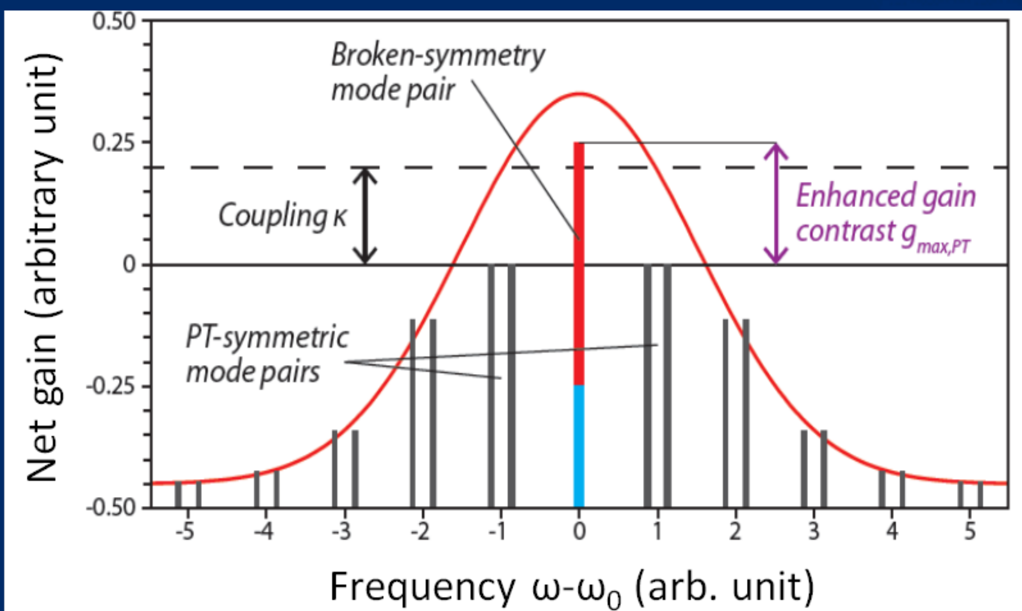
$$\omega_n^{(1,2)} = \omega_n \pm \sqrt{\kappa_n^2 - \gamma_n^2}$$

PT-symmetric micro-ring arrangement

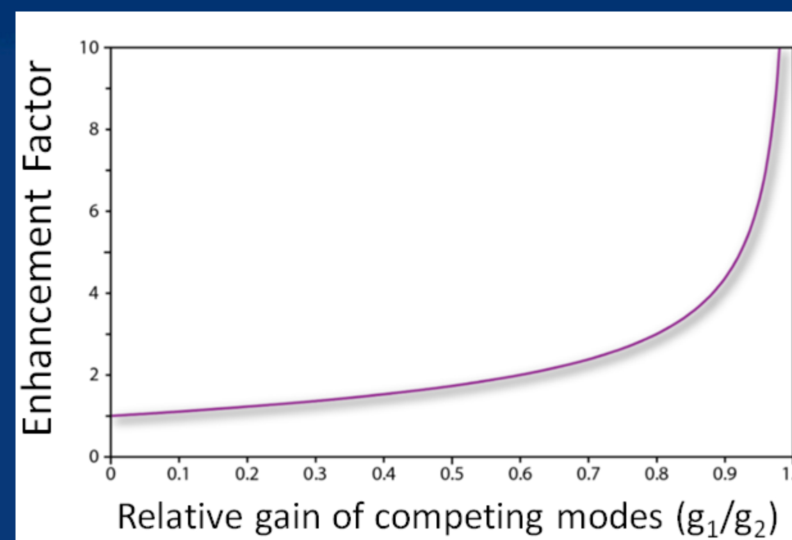
Maximum achievable gain while maintaining single mode operation in PT-symmetric arrangement

g_{ma}

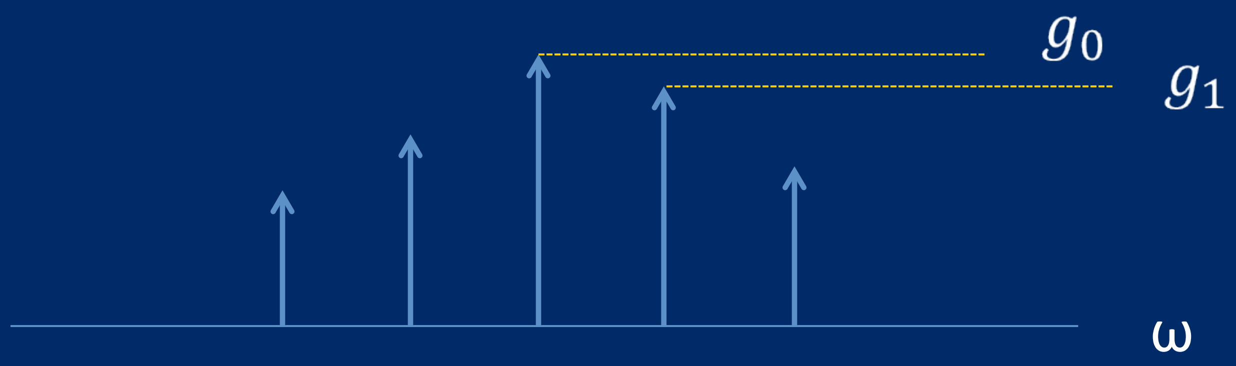
$$g_{max,PT} = \sqrt{g_0^2 - g_1^2} = (g_0 - g_1)\sqrt{\frac{g_0/g_1 + 1}{g_0/g_1 - 1}}$$



Enhancement Factor



Differential gain contrast



Example

$g_0 = 4.1\ cm^{-1}$

$g_1 = 4\ cm^{-1}$

$g_0 - g_1 = 0.1\ cm^{-1}$

Traditional gain differential

PT gain differential

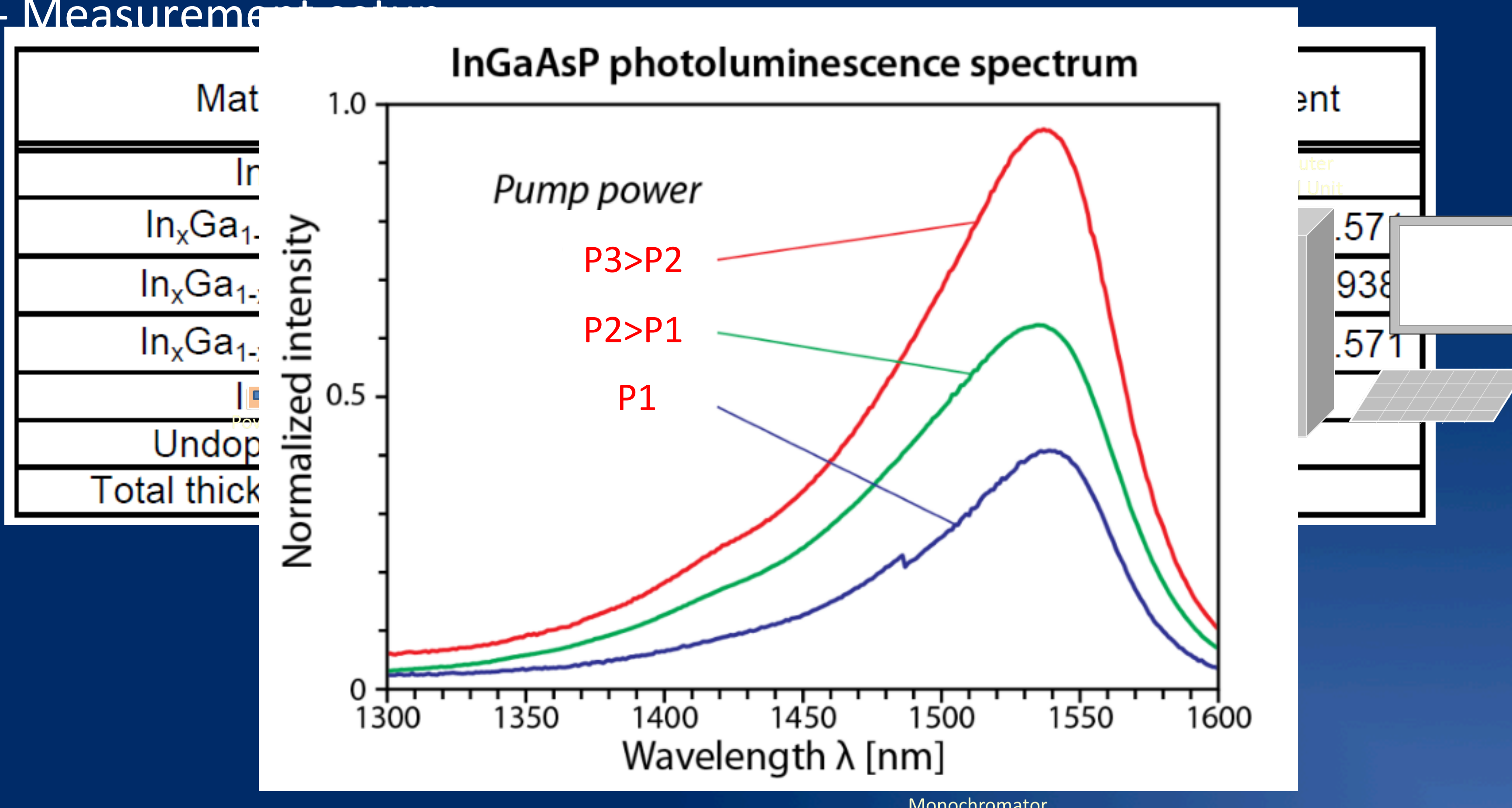
$$\sqrt{4.1^2 - 4^2} = 0.9\ cm^{-1}$$

Experimental platform

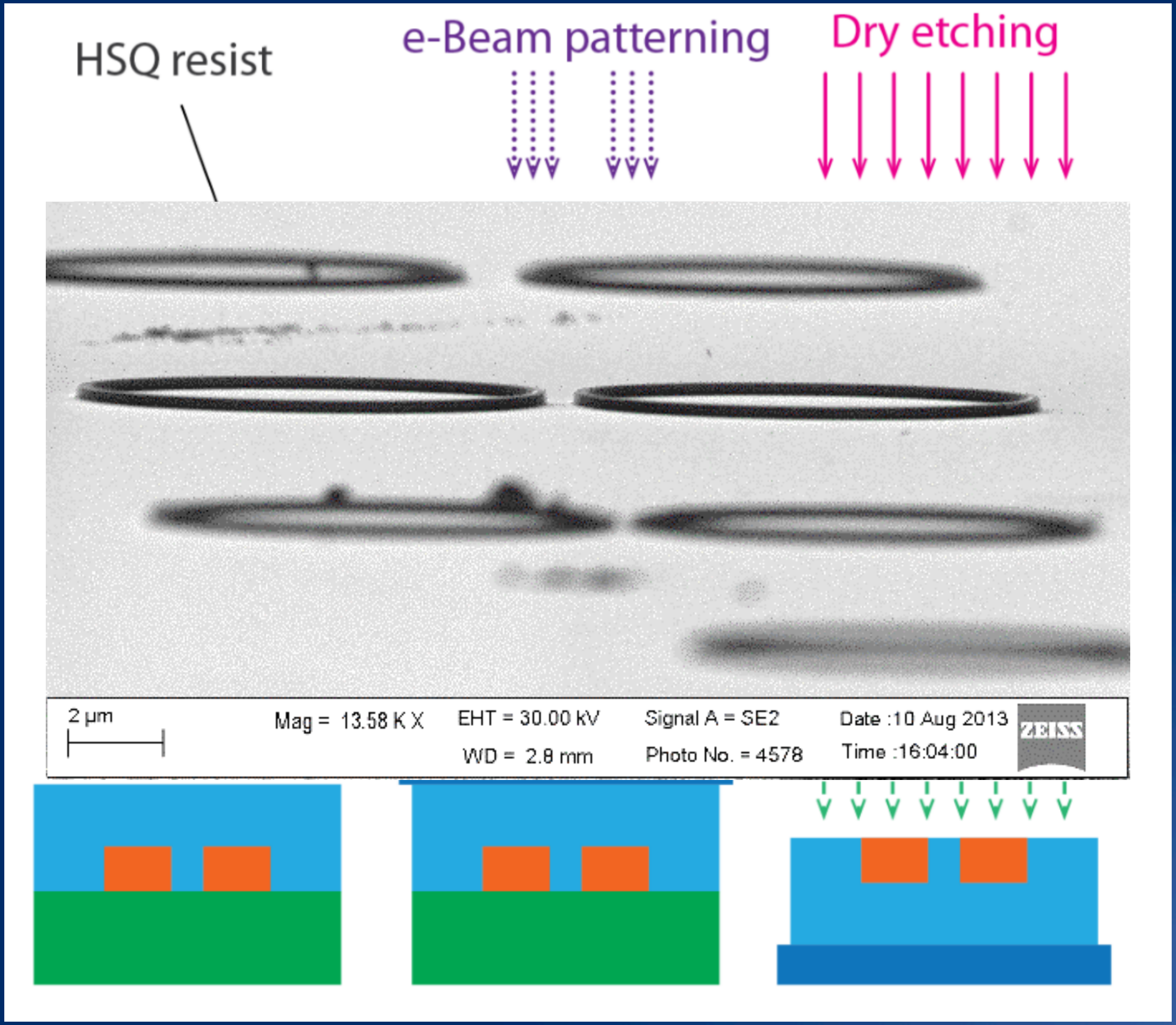
- Semiconductor quantum well system

- Gain material PL spectrum

- Measurement setup

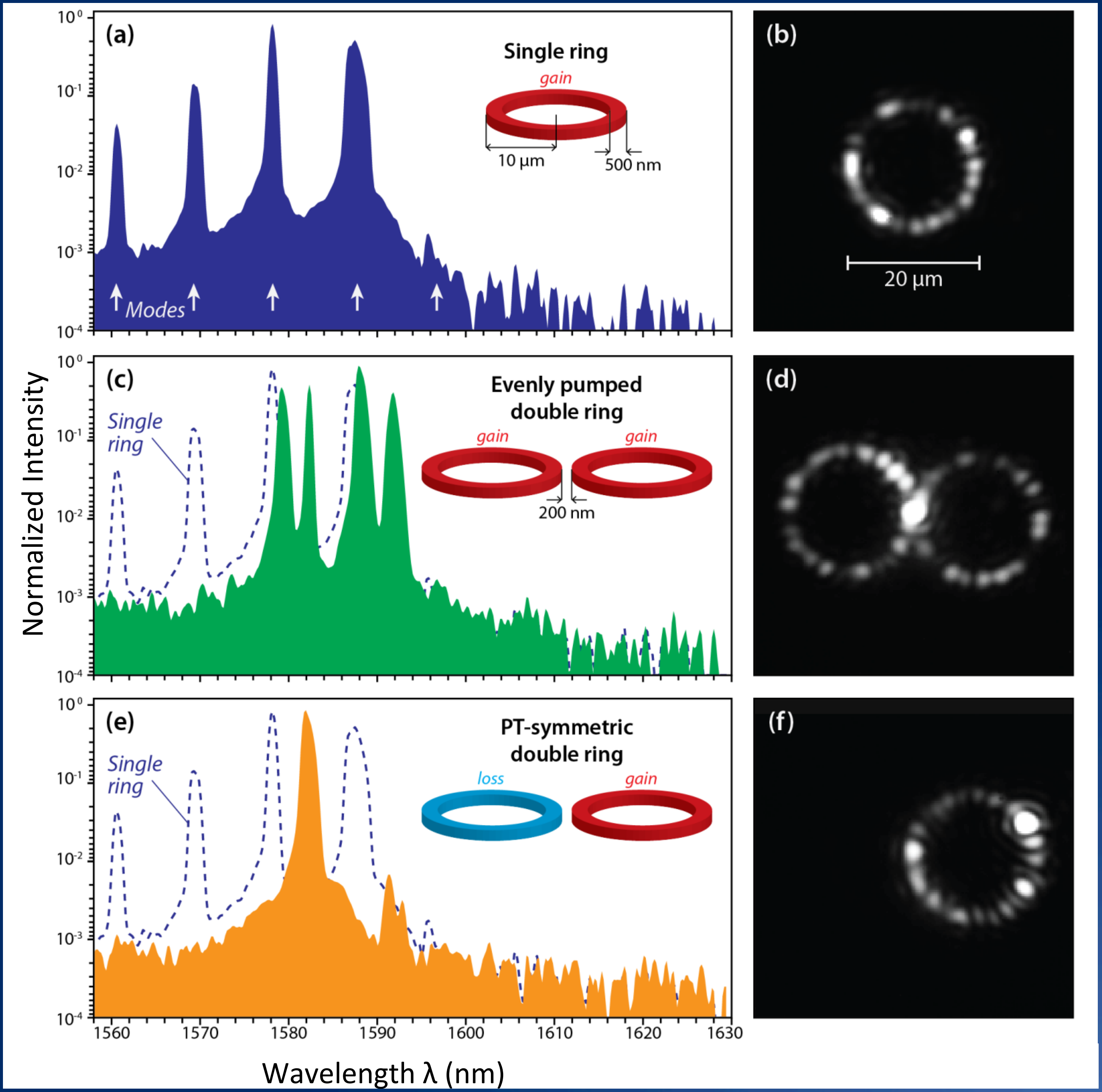


Sample preparation



Results

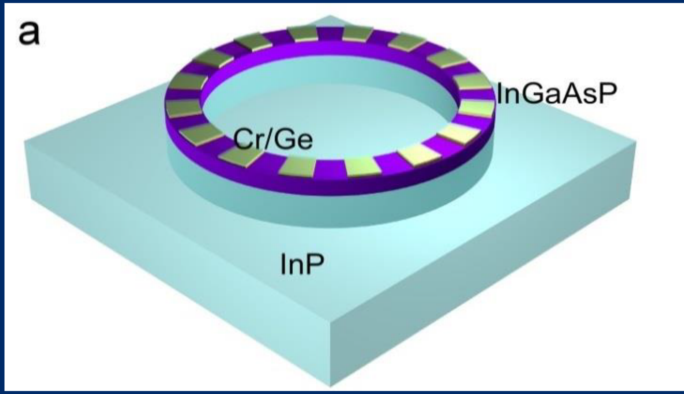
(Log Scale)

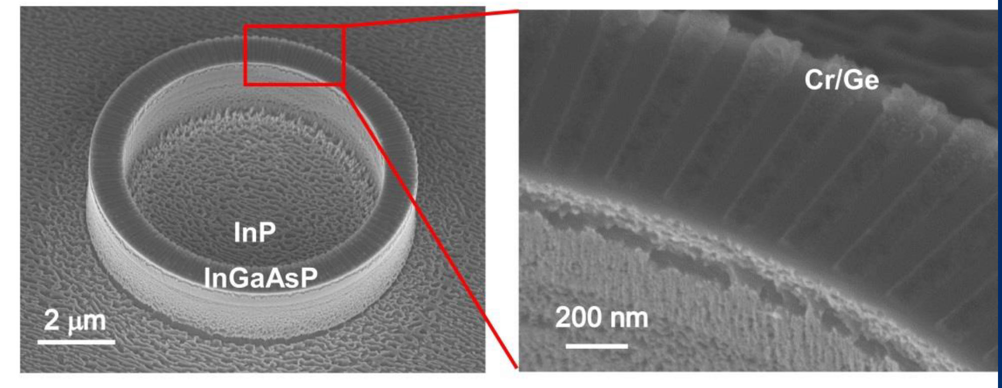


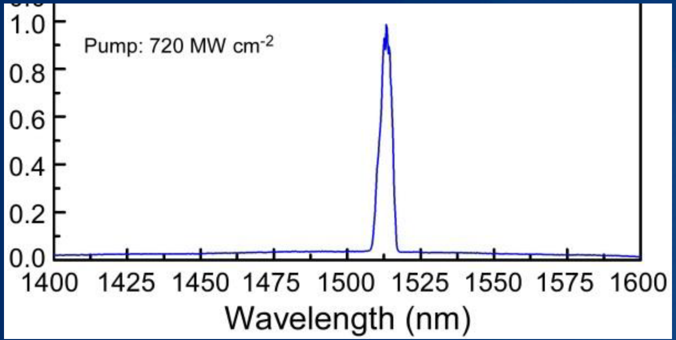
7.4 mW pump power at 1064 nm for each

The dead/lossy resonator plays the role of a keel in balancing a sailboat

Parity-Time Synthetic Laser





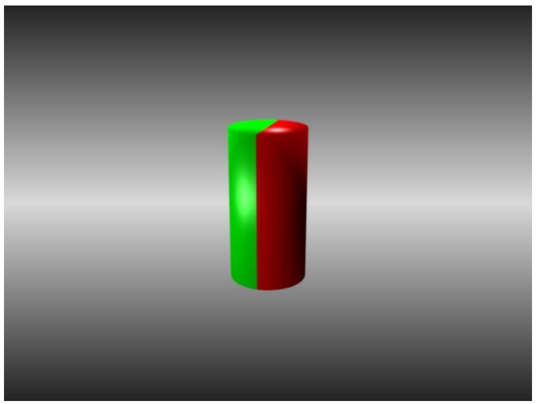


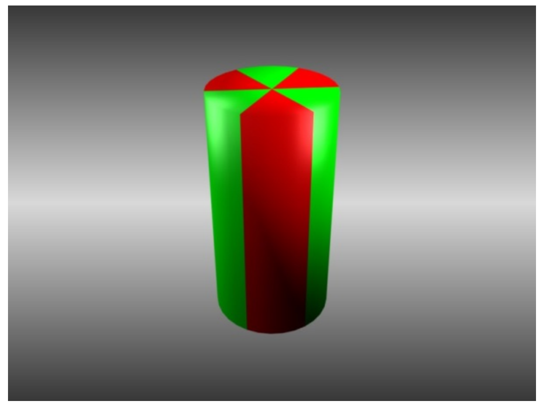
Zhang's group, Berkeley, arXiv:1405.2863.

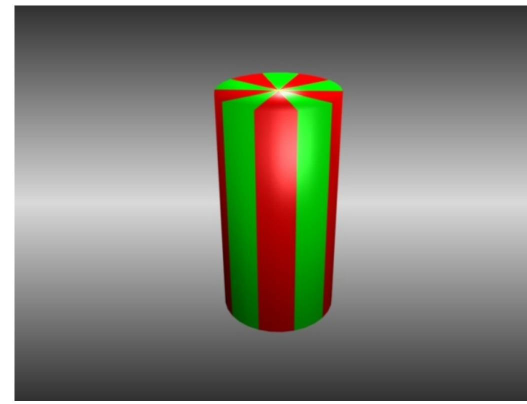
$\mathcal{PT}$ scatterers

Cylindrical scatterers

$N = 2$ $N = 6$ $N = 10$





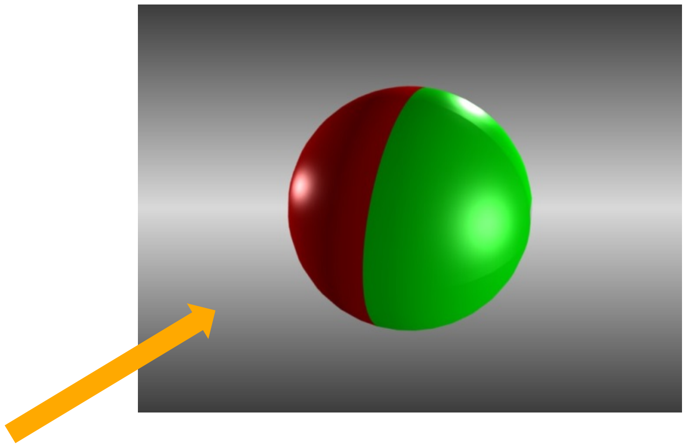


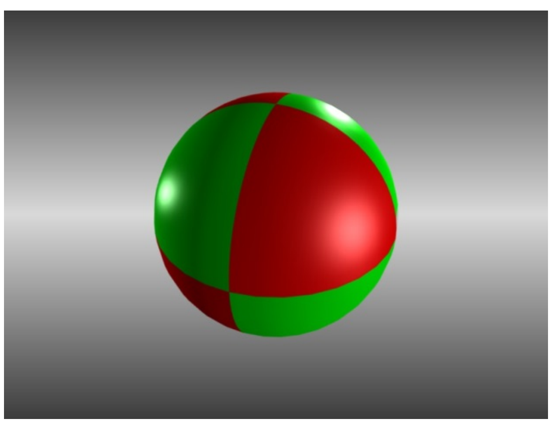
For PT symmetry

$$N = 4n + 2$$
$$n = 0, 1, 2, \ldots$$

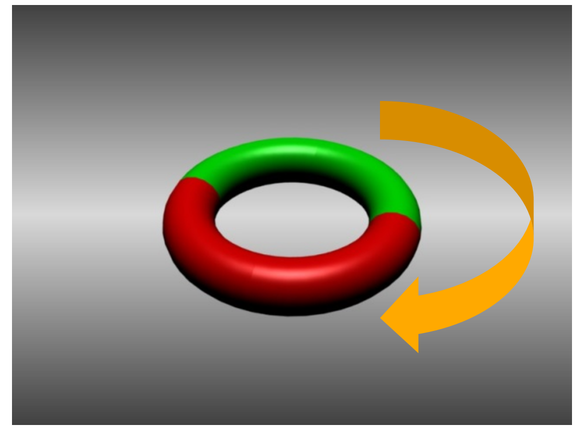
$\mathcal{PT}$ scatterers and cavities

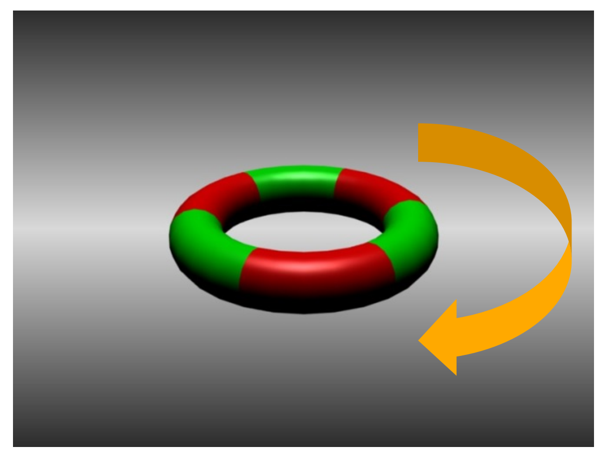
Spherical scatterers or galleries





Whispering galleries





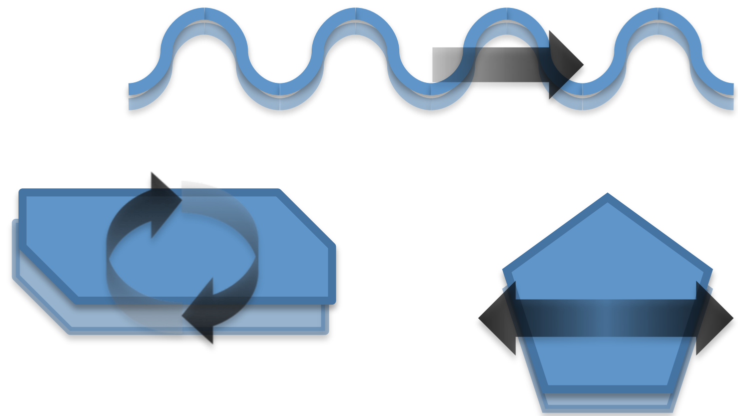
Supersymmetry (SUSY)

What is supersymmetry (SUSY) ?

Conventional symmetries:

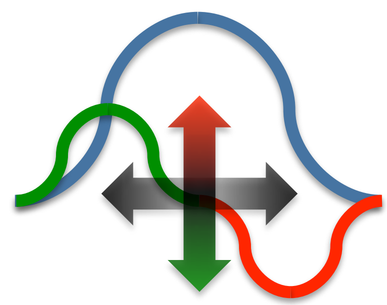
- Translation
- Rotation
- Parity

...



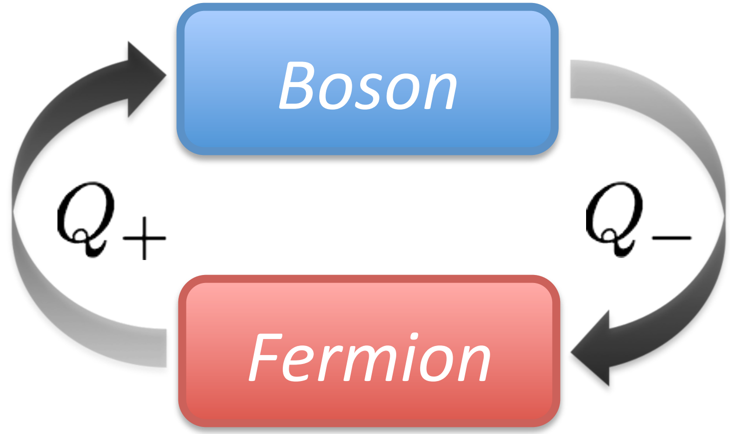
PT-symmetry

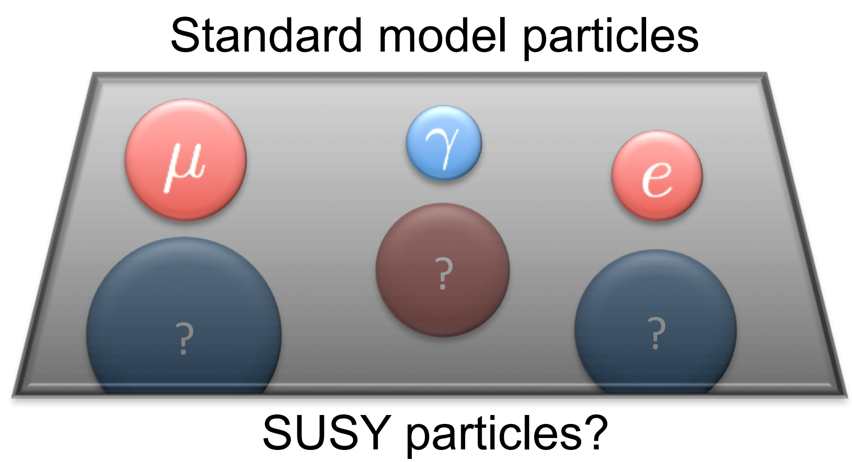
- Simultaneous reversal of *parity* and gain/loss (*time*)



Supersymmetry in quantum field theory

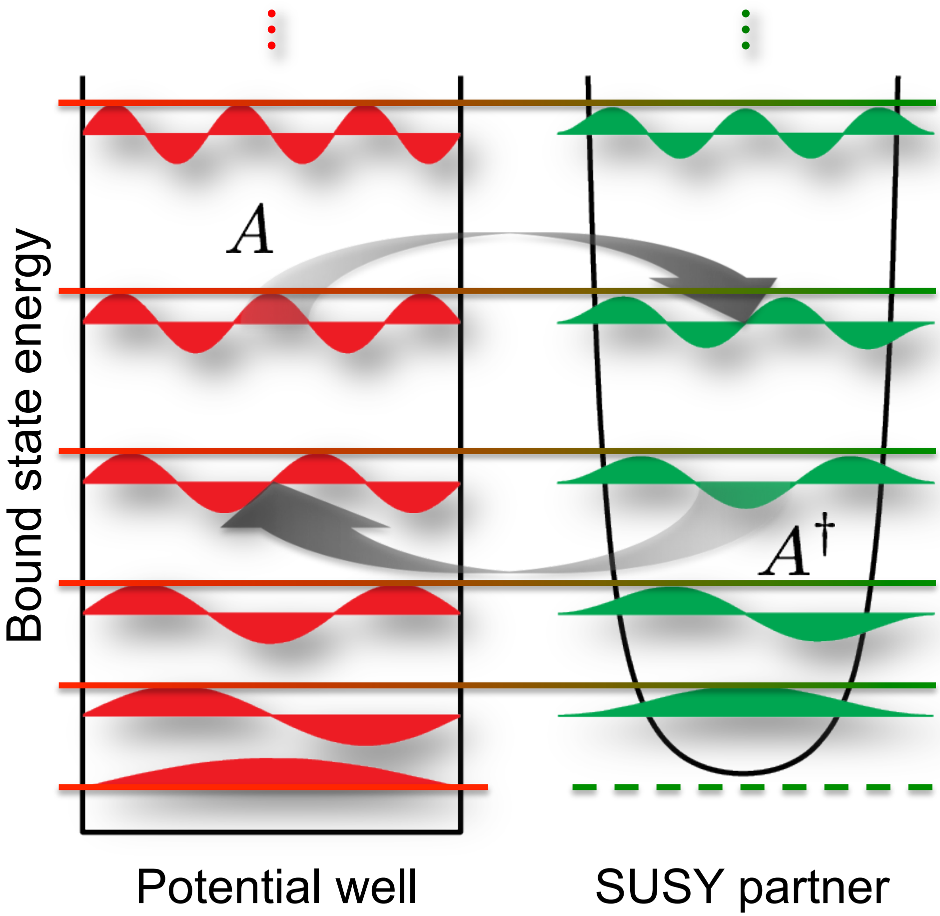
- Unified treatment of Bosons and Fermions
- SUSY particles as dark matter candidates?
- Key ingredient of a Grand Unified Theory?
- No experimental evidence to this day





Supersymmetry in quantum mechanics

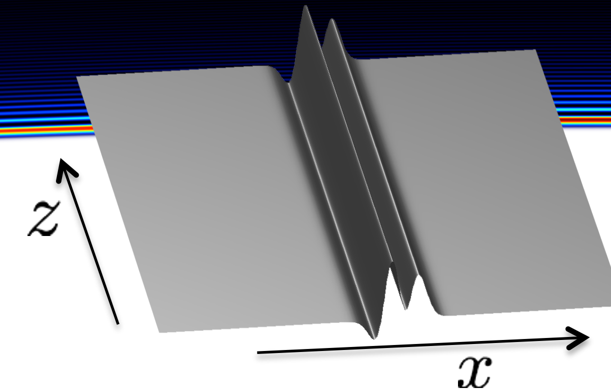
- Relation between two potentials and their sets of eigenstates
- Identical eigenvalue spectra
- Ground state energy missing in SUSY partner



Cooper *et al.*, "Supersymmetry in Quantum Mechanics," World Scientific (2001)

OPTICAL SUSY

SUSY optical structures



- Refractive index as "optical potential"
- TE waves in the Helmholtz regime:

$$\beta^2\psi = \underbrace{\left(\partial_x^2 + k_0^2 n^2(x)\right)}\psi$$

- Factorization of Hamiltonian H

$$H^{(1)} = A^\dagger A$$

$$H^{(2)} = AA^\dagger$$

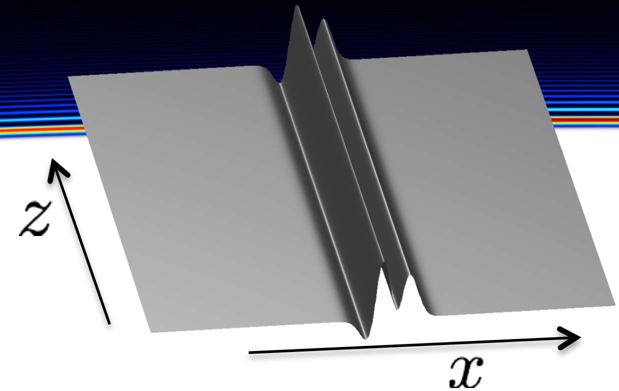
$$A = \partial_x - \partial_x \ln \psi_1^{(1)}$$

Fundamental mode

Propagation constant
of the fundamental mode

Miri et al., "Supersymmetric optical structures," Phys. Rev. Lett. **110**, 233902 (2013)

SUSY optical structures



- Refractive index as "optical potential"
- TE waves in the Helmholtz regime:

$$\beta^2\psi = \left(\partial_x^2 + \underbrace{k_0^2 n^2(x)}_{V}\right)\psi$$

- Optical potential

$$V^{(1)} = \left(\beta_1^{(1)}\right)^2 - W^2 + \partial_x W$$

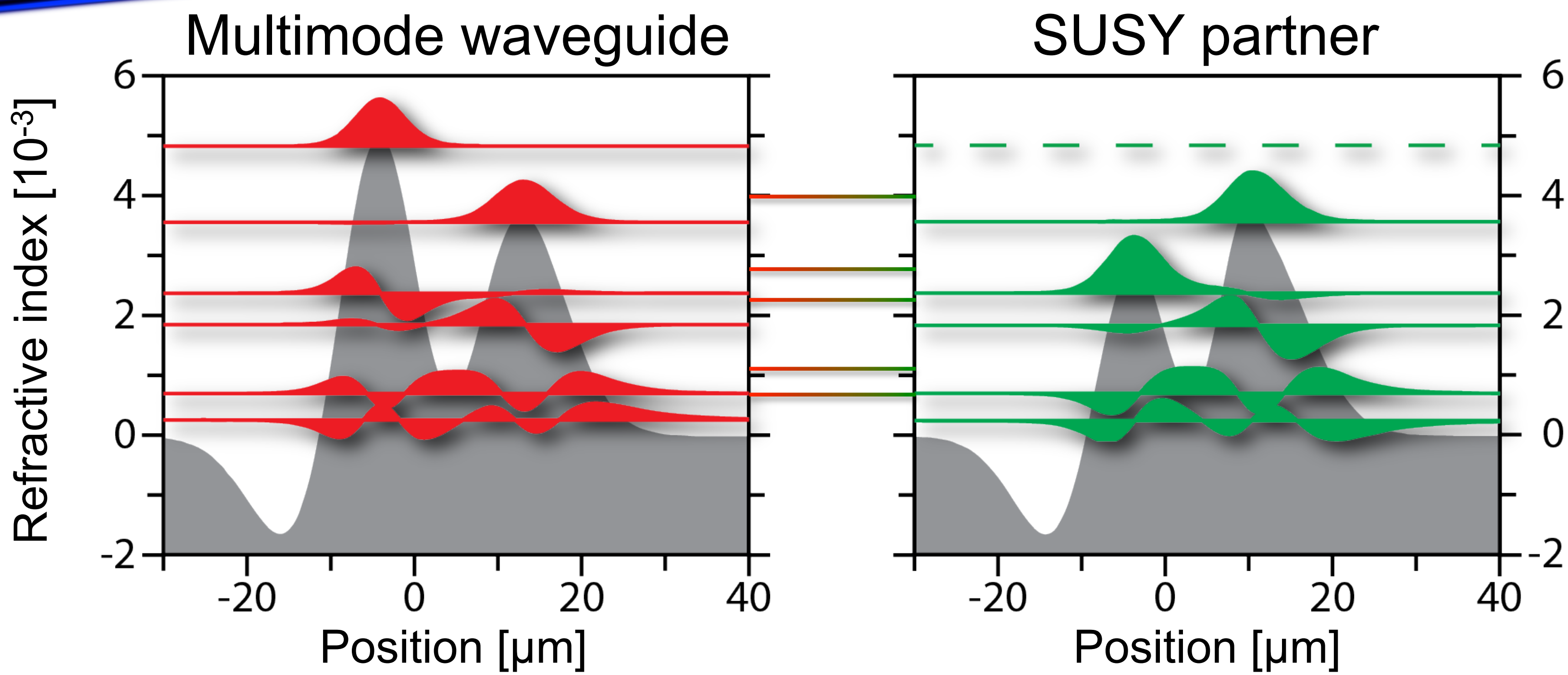
$$V^{(2)} = \left(\beta_1^{(1)}\right)^2 - W^2 - \partial_x W$$

$$W = -\partial_x \ln \psi_1^{(1)}$$

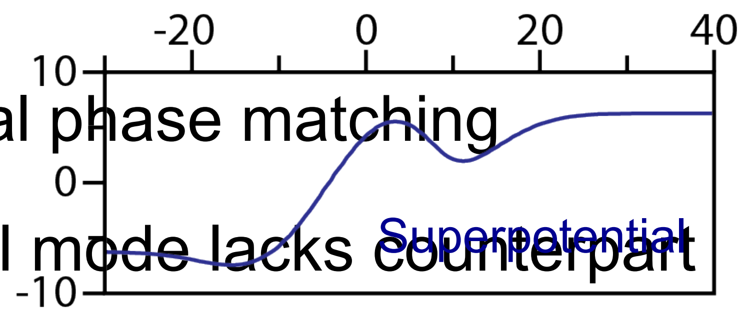
Superpotential

Miri et al., "Supersymmetric optical structures," Phys. Rev. Lett. **110**, 233902 (2013)

SUSY optical structures



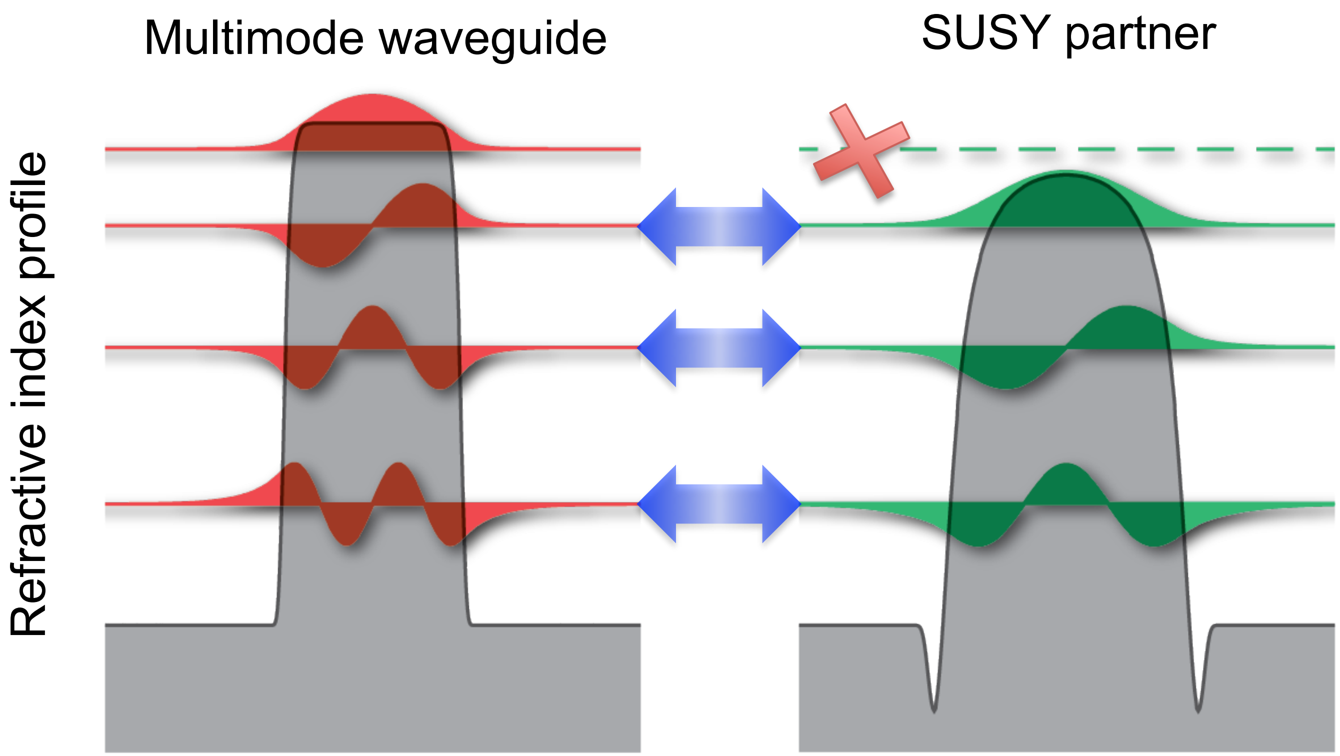
- Perfect global phase matching
- Fundamental mode lacks counterpart



$$W = -\partial_x \ln \psi_1^{(1)}$$

Miri et al., "Supersymmetric optical structures," Phys. Rev. Lett. **110**, 233902 (2013)

SUSY optical structures



- Elimination of fundamental mode
- Perfect global phase matching

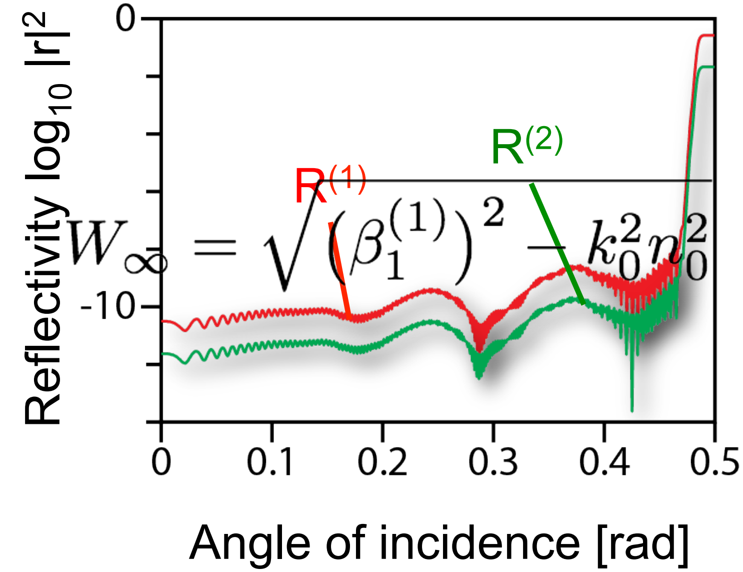
Scattering at SUSY structures

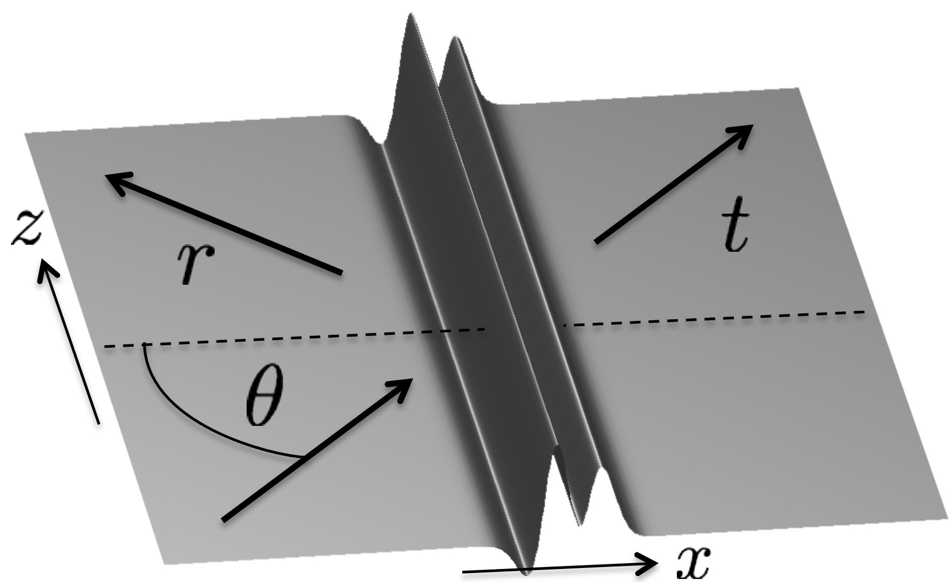
Reflectivity/transmission coefficients:

$$|r^{(2)}|^2 = \frac{W_\infty + ik_x}{W_\infty - ik_x} \cdot |r^{(1)}|^2$$

$$|t^{(2)}|^2 = \frac{W_\infty + ik_x}{W_\infty - ik_x} \cdot |t^{(1)}|^2$$

$$W_\infty = \sqrt{\left(\beta_1^{(1)}\right)^2 - k_0^2 n_0^2}$$

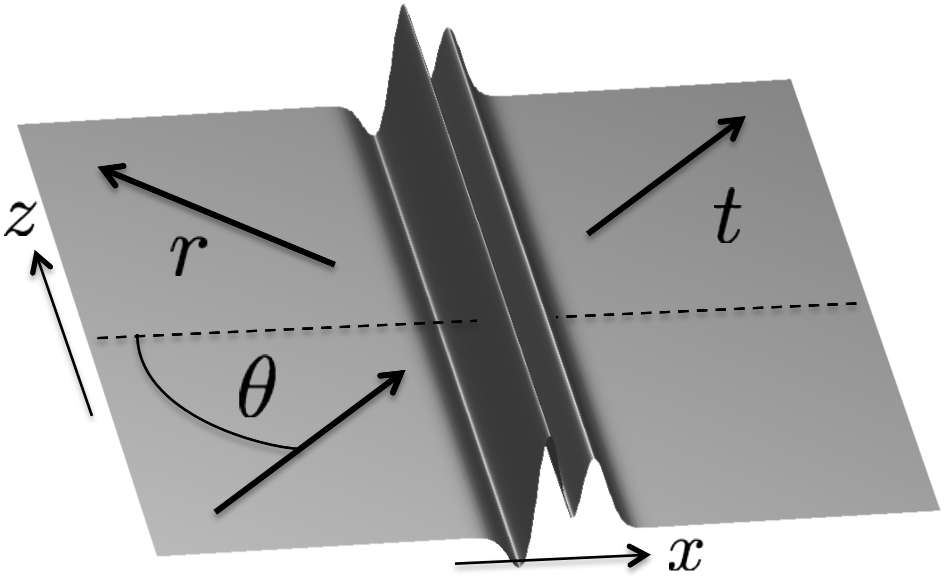


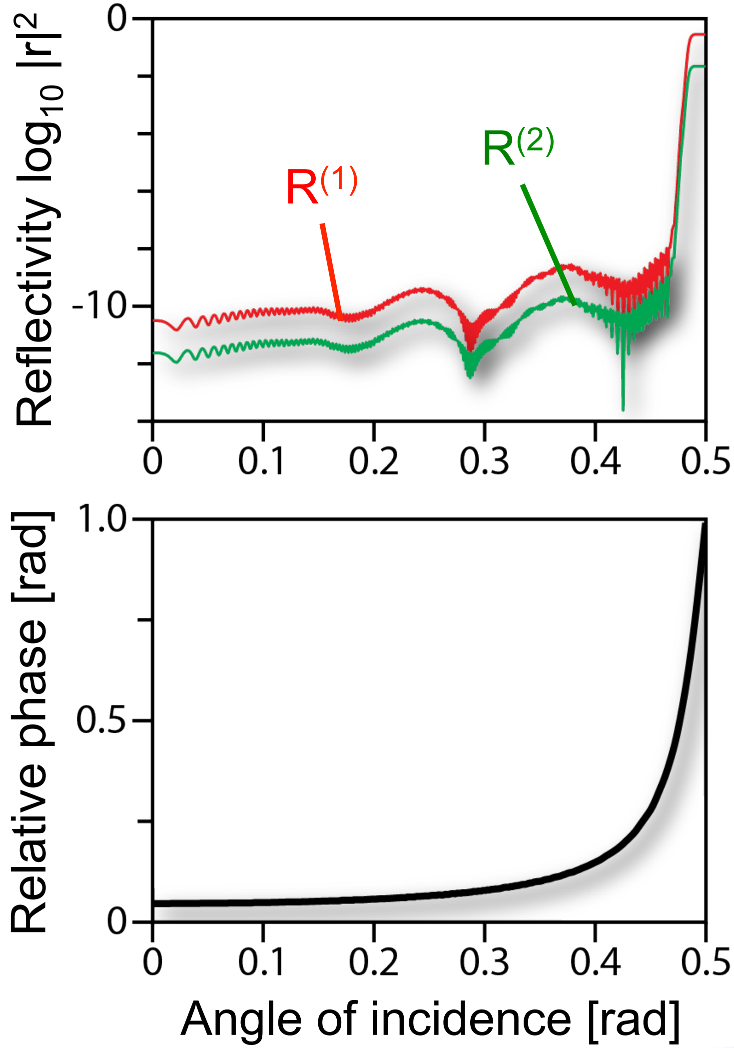


Miri et al., “Supersymmetric optical structures,” Phys. Rev. Lett. **110**, 233902 (2013)

Scattering at SUSY structures

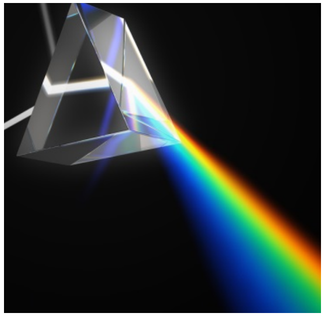
- Identical intensity scattering behavior
- Superpartners only distinguishable by direct interferometric measurements



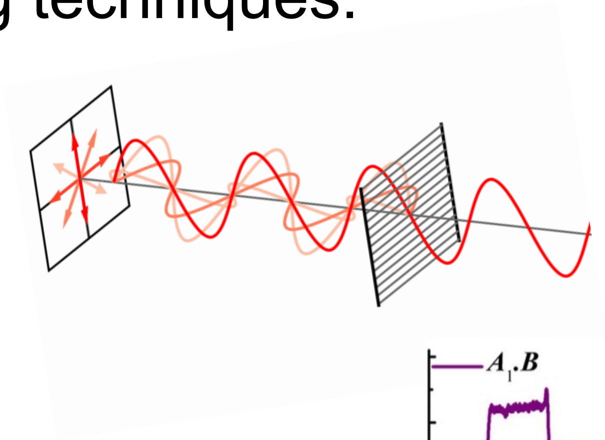


SUSY Mode Converters

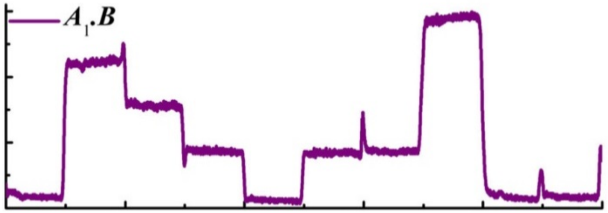
Maximizing the amount of data in an optical channel



Established multiplexing techniques:



- Wavelength
- Polarization
- Amplitude / phase

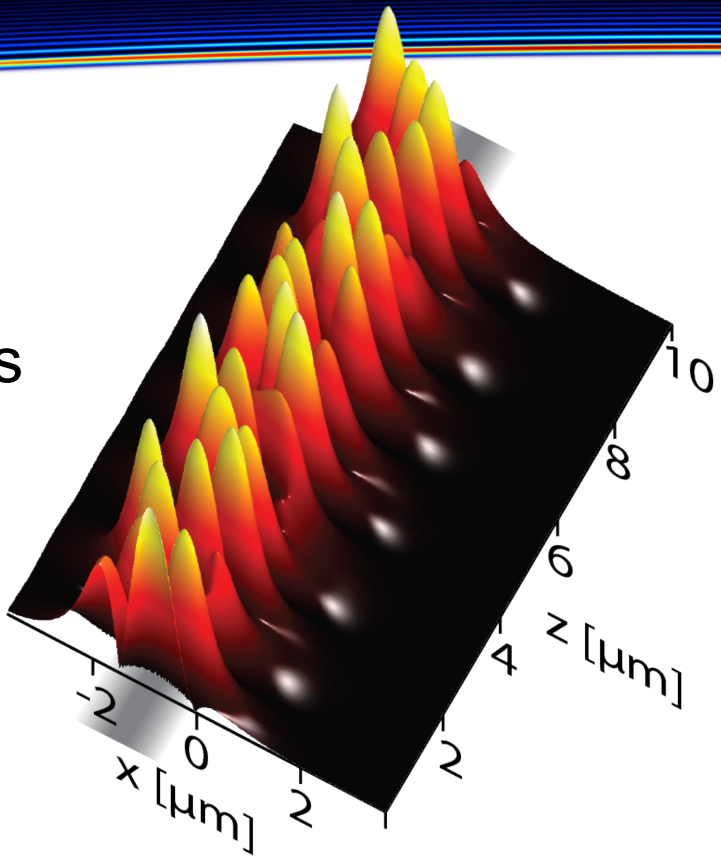


What about multimode structures?

Mode division multiplexing

Benefits

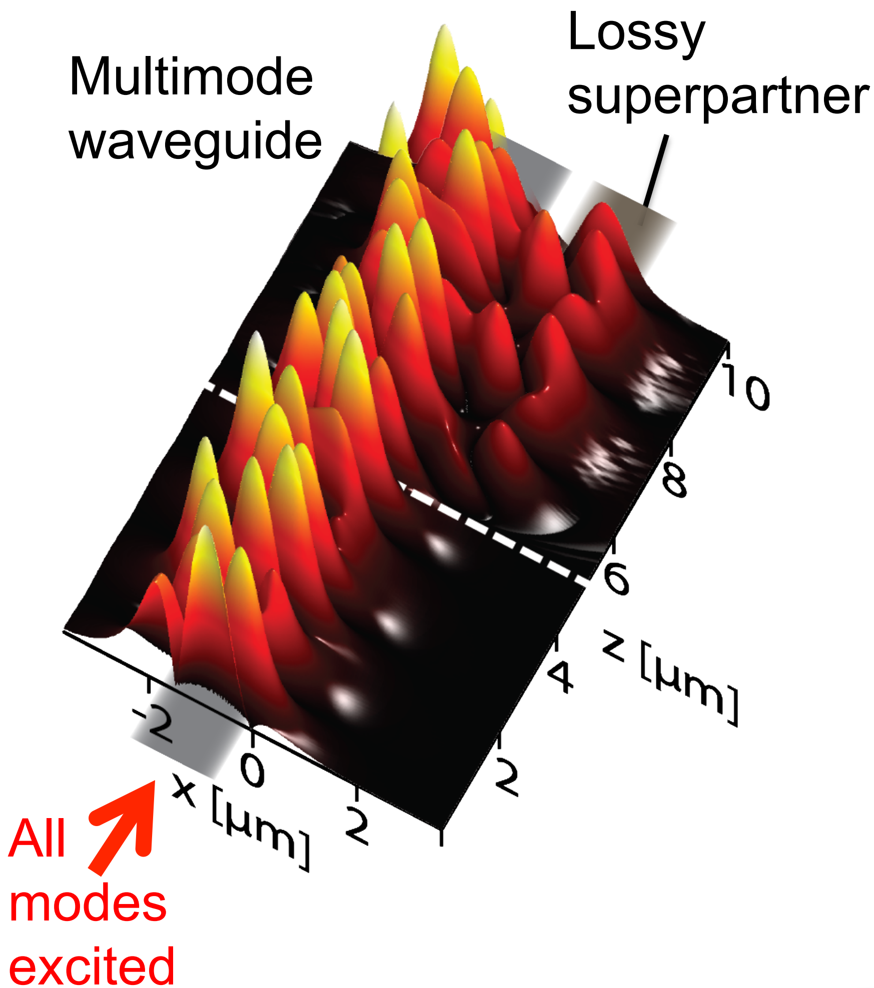
- Compatible to existing approaches
- No crosstalk between channels
- Larger mode area, reduced intensity

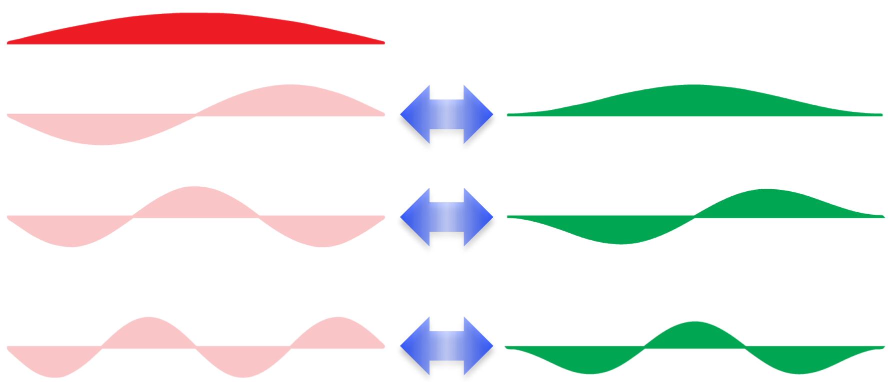


Challenges

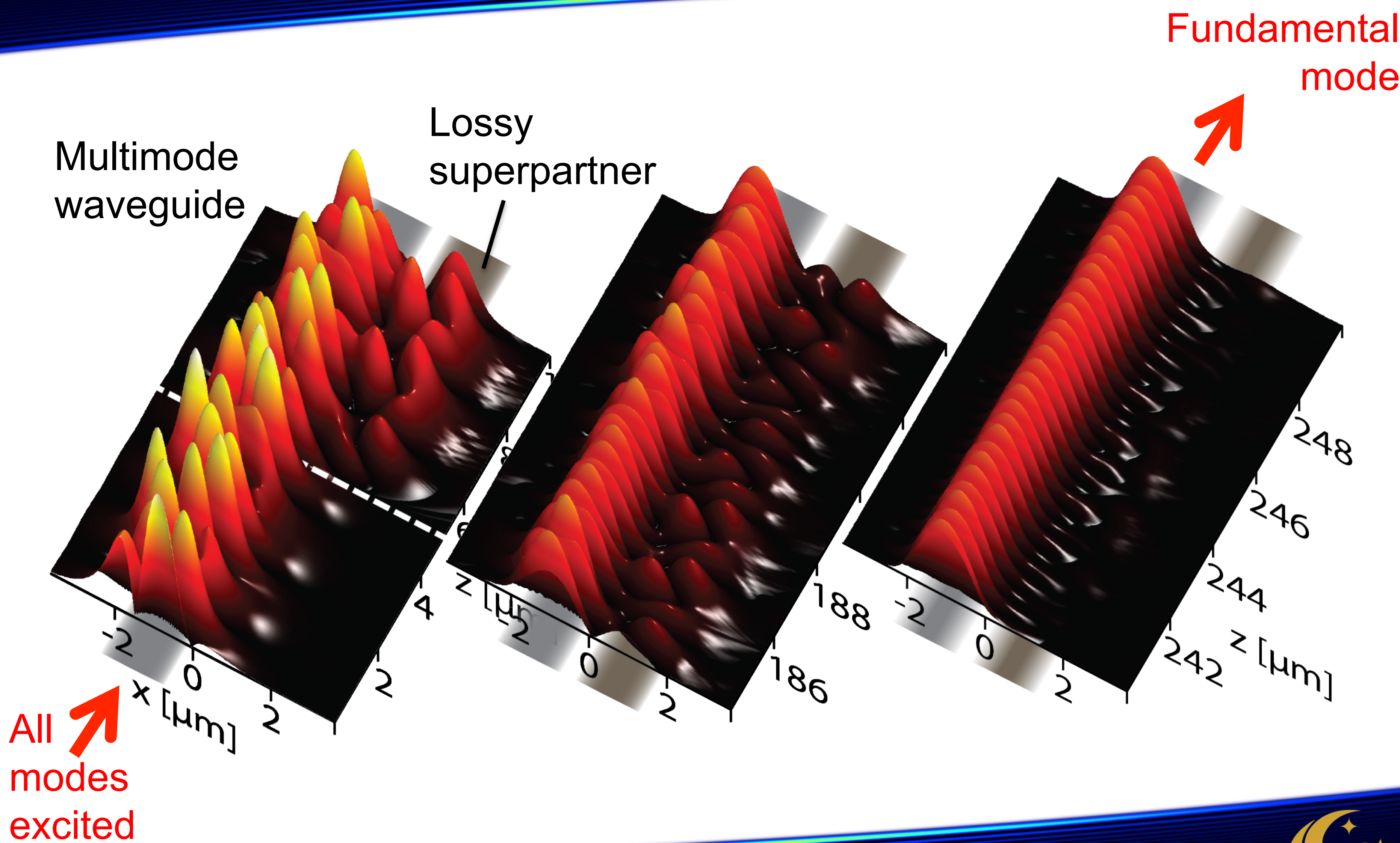
- Design: Structures allow only indirect control over modes
- Integration: Selective population/interrogation

Suppression of higher order modes

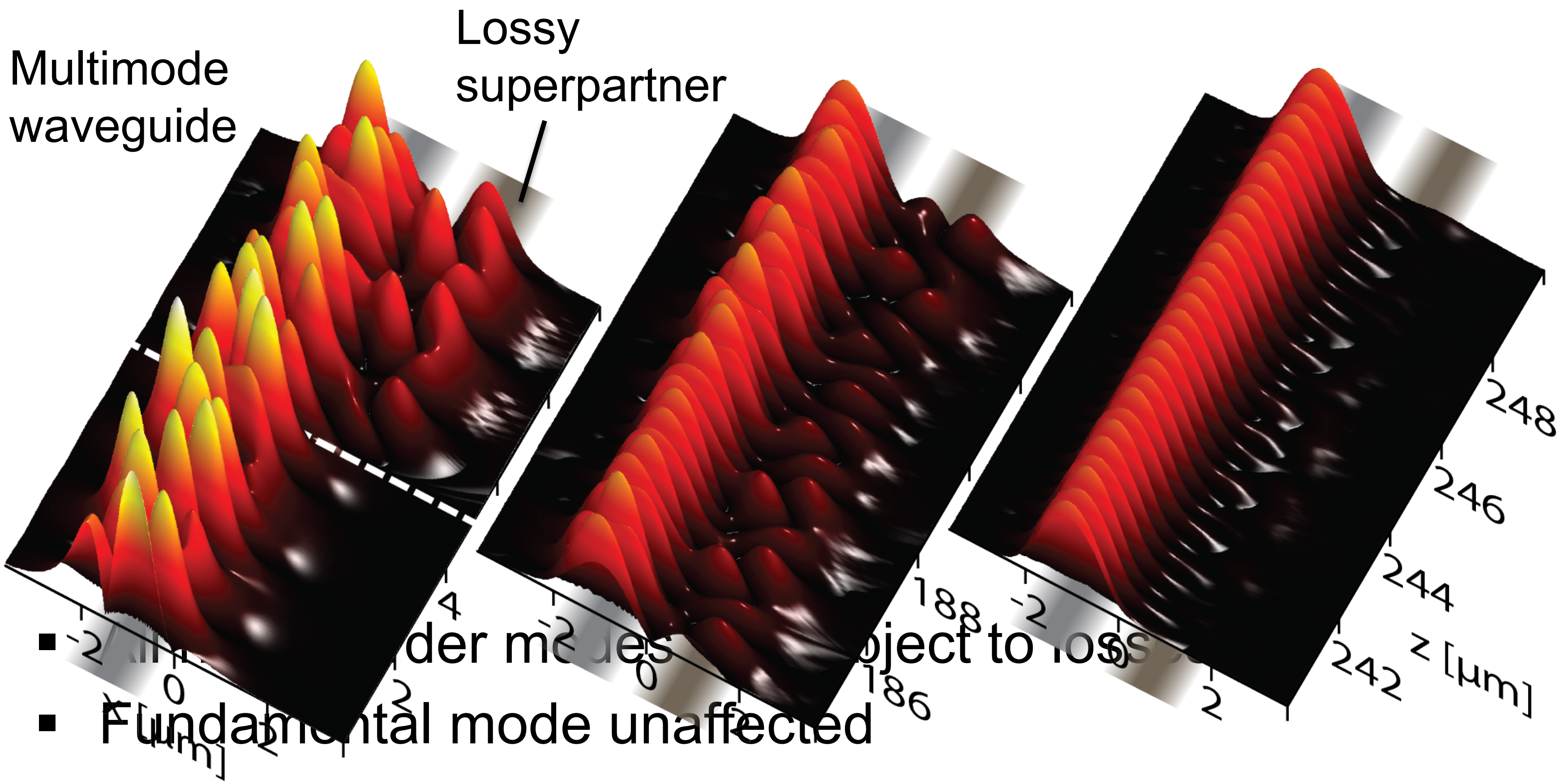




Suppression of higher order modes

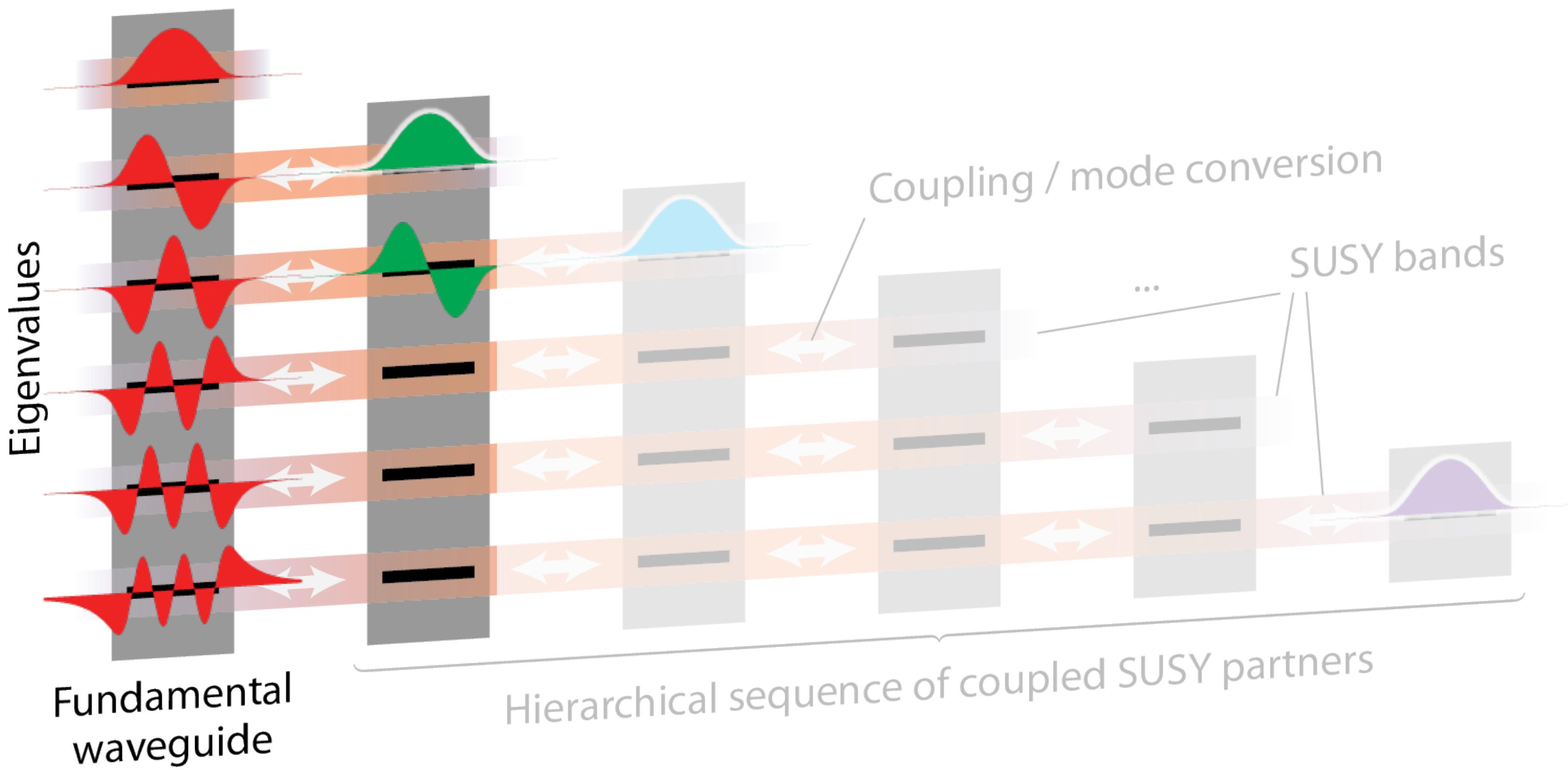


Suppression of higher order modes



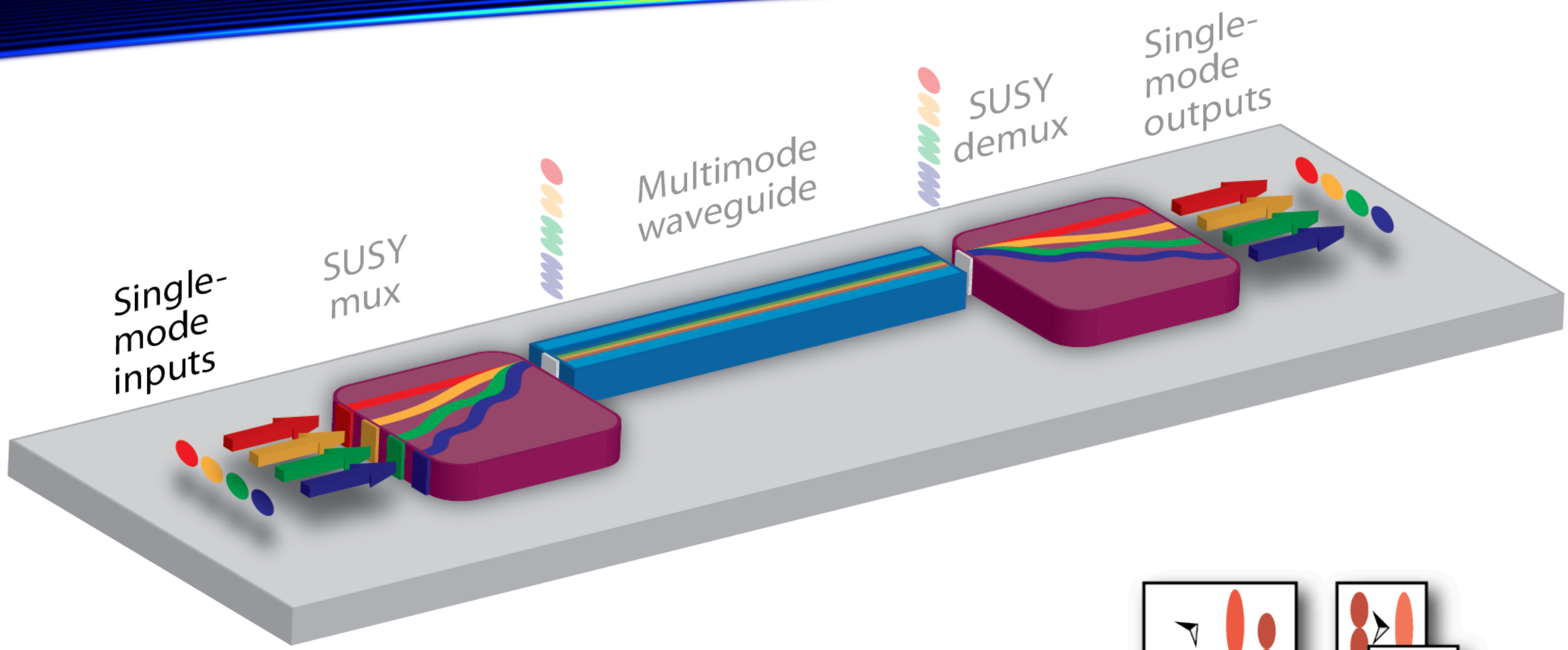
- A[illegible]der m[illegible]des [illegible]ject to los[illegible]
- Fundamental mode unaffected
- Selective lasing in large-area multimode systems

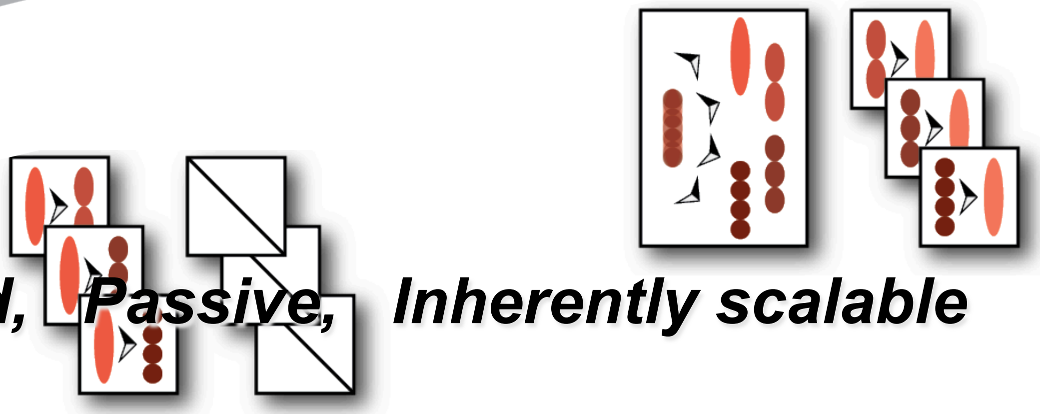
SUSY Ladder



Eigenvalues
Coupling / mode conversion
SUSY bands
...
Fundamental waveguide
Hierarchical sequence of coupled SUSY partners

SUSY mode division multiplexing



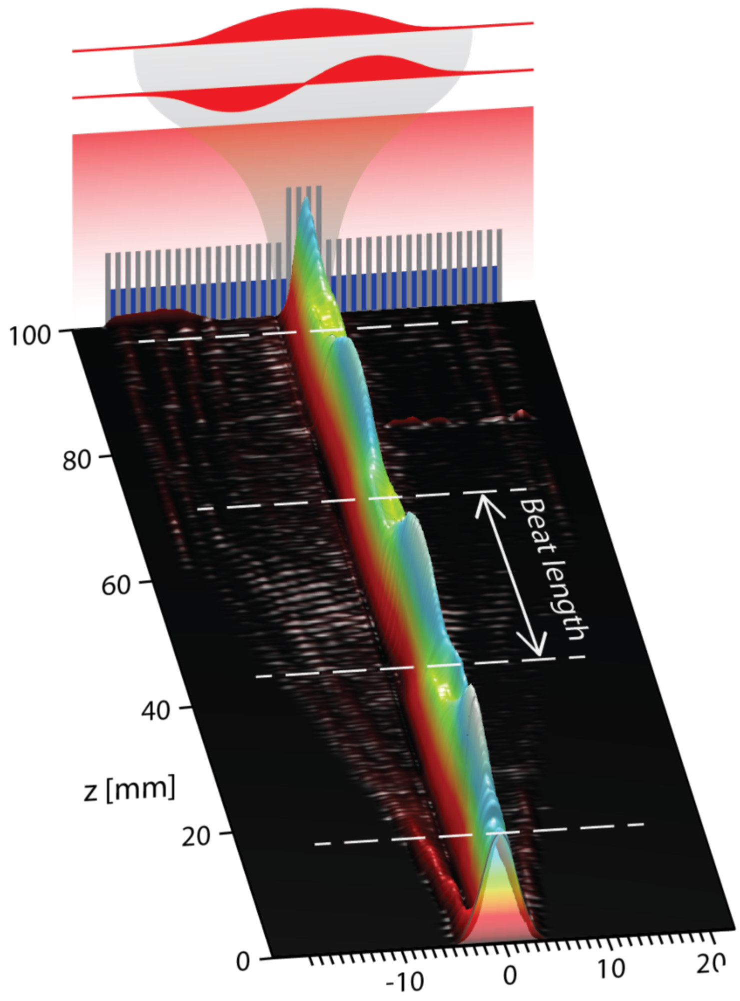


Fully integrated, Passive, Inherently scalable

Heinrich et al., "Supersymmetric Mode Converters," Nature Commun. **5**, 3698 (2014)

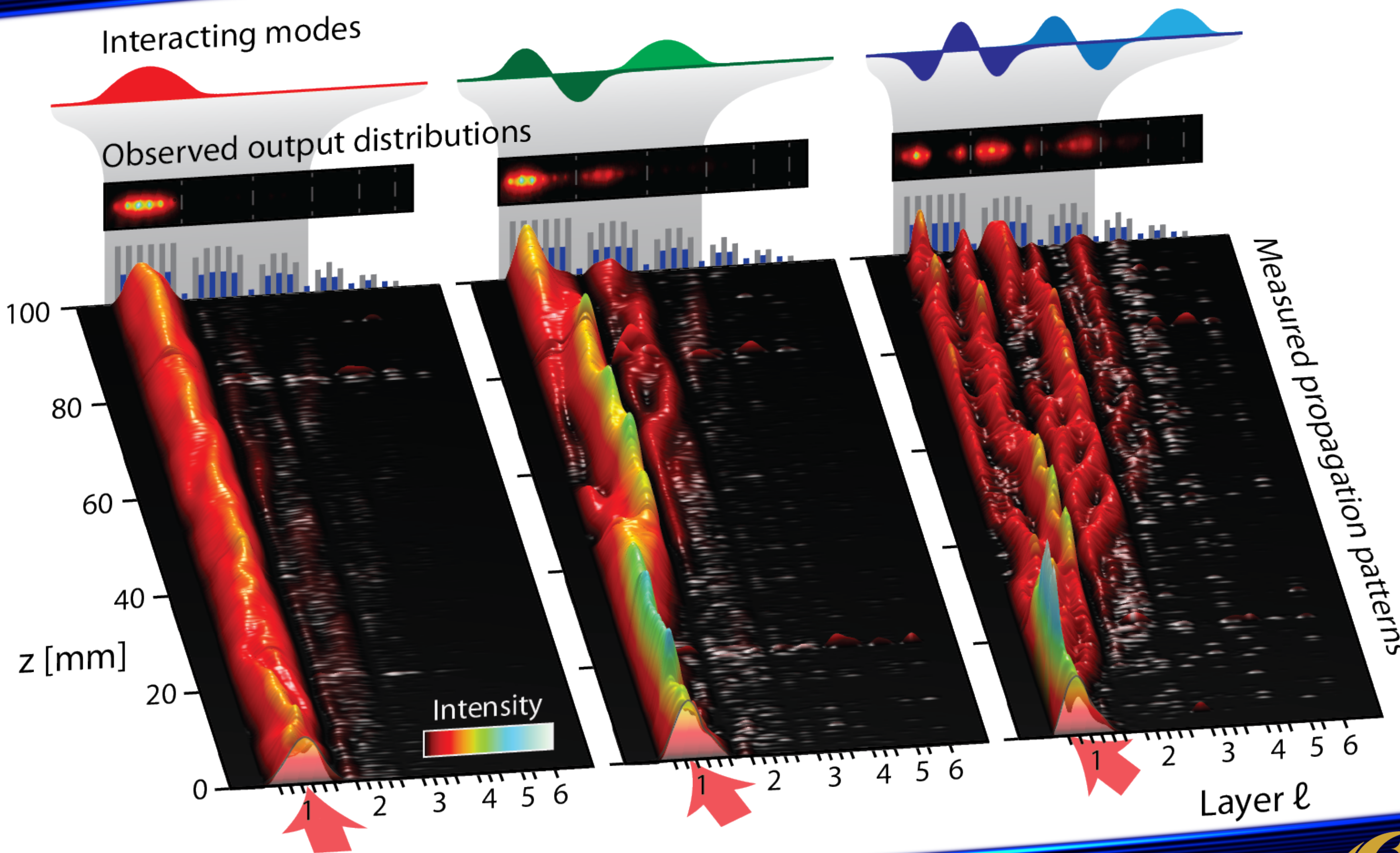
Example: State removal

Fundamental lattice



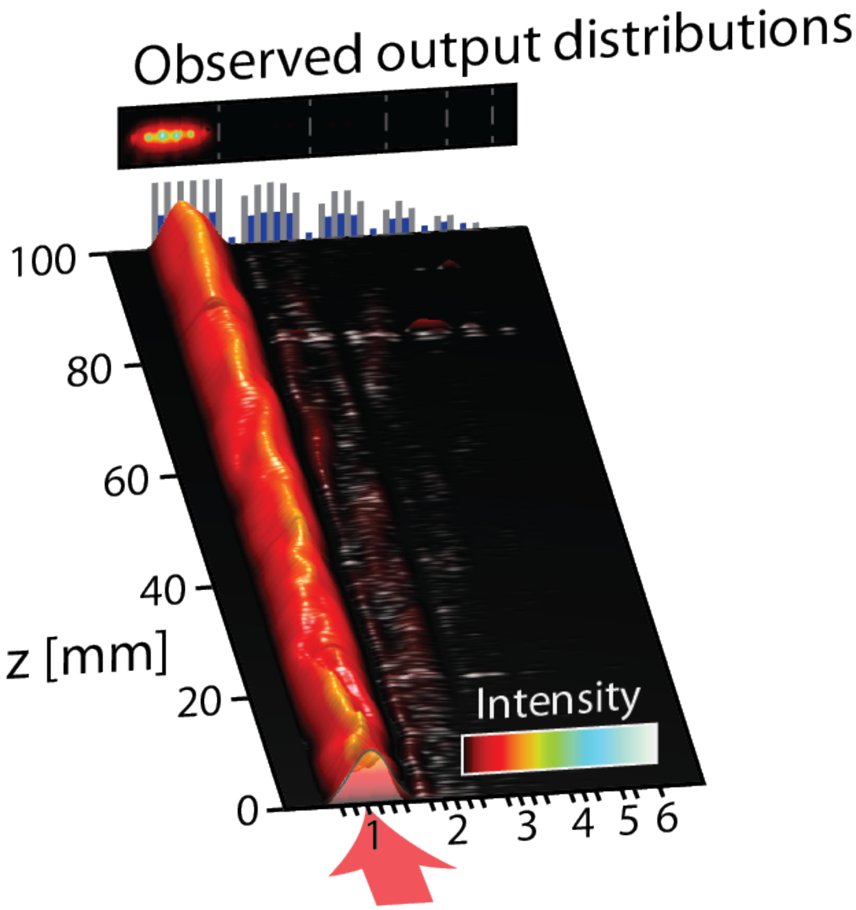
Superpartner lattice

SUSY Ladder: Mode conversion



Heinrich et al., “Supersymmetric Mode Converters,” Nature Commun. **5**, 3698 (2014)

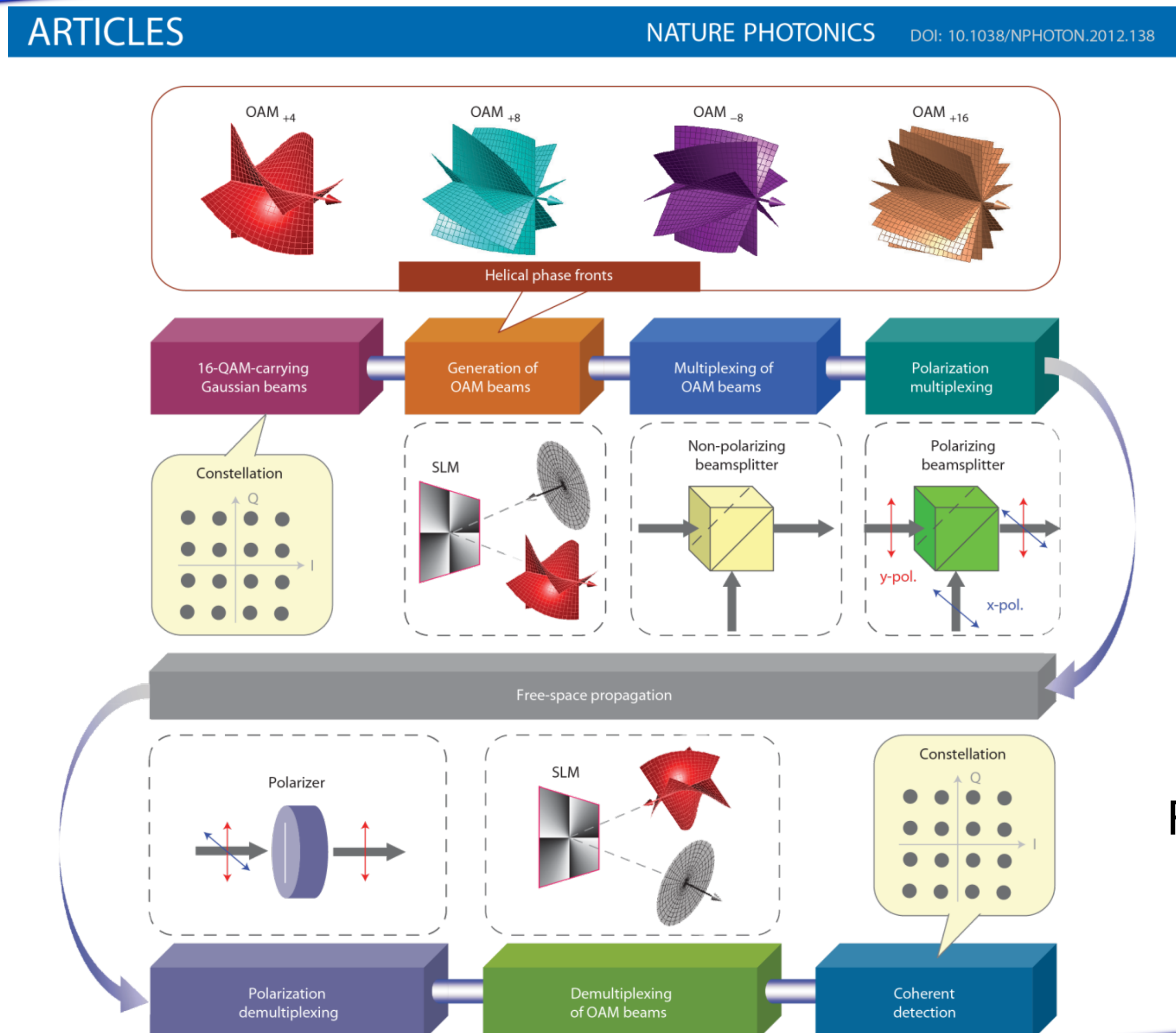
SUSY Ladder: Mode isolation



Heinrich et al., "Supersymmetric Mode Converters," Nature Commun. **5**, 3698 (2014)

SUSY FIBERS

Digression: OAM multiplexing



Record data rate of 2.5 Tbit/s

Wang *et al.*, Nature Photon. **6**, 488 (2012)

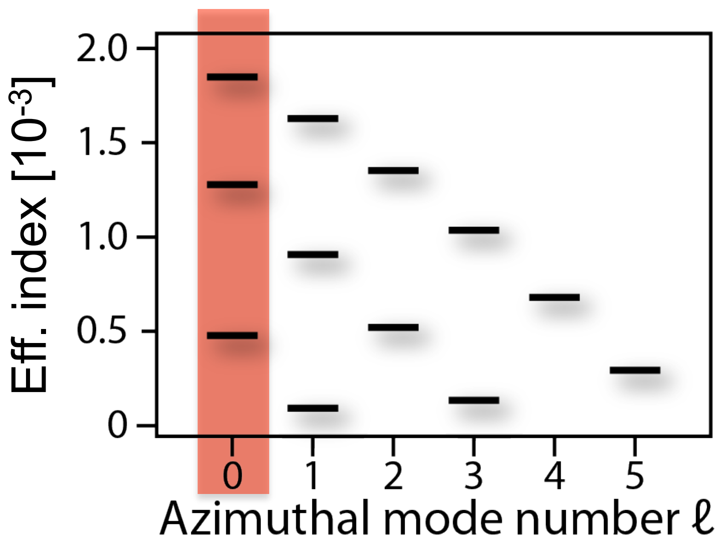
An integrated solution?

Multimode fibers:

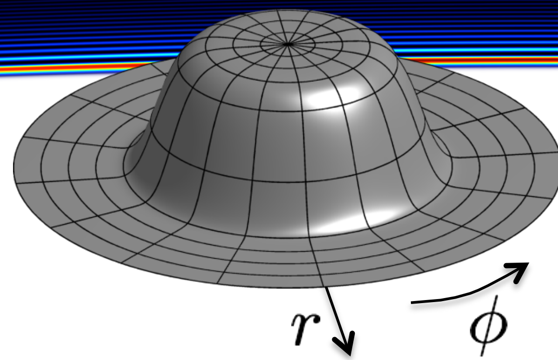
- High order modes carry angular momentum

$$U = e^{i\mu z}\boxed{e^{i\ell\phi}}R_{\ell,m}(r)$$

- Is there a systematic way to populate and interrogate them individually?



Extension of SUSY to fibers



- Factorization requires 1D problem
- 2D system with cylindrical symmetry:

$$\left(-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} - 2n_0 k_0^2 \Delta n(r)\right) U = i\frac{\partial}{\partial z} U$$

- Effective potential:

$$V_{\text{eff}}(r) = 2n_0 k_0^2 \Delta n(r) + \frac{1/4 - \ell^2}{r^2}$$

for the modes

$$U = e^{i\mu z} e^{i\ell\phi} R_{\ell,m}(r)$$

Azimuthal mode number

Radial mode number

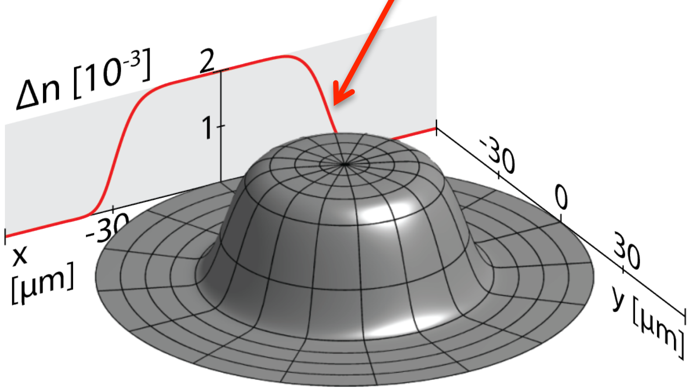
Extension to fibers

- 1D effective eigenvalue equation: $\left(-\frac{d^2}{dr^2} - V_{\text{eff}}(r)\right) u = -\mu u$

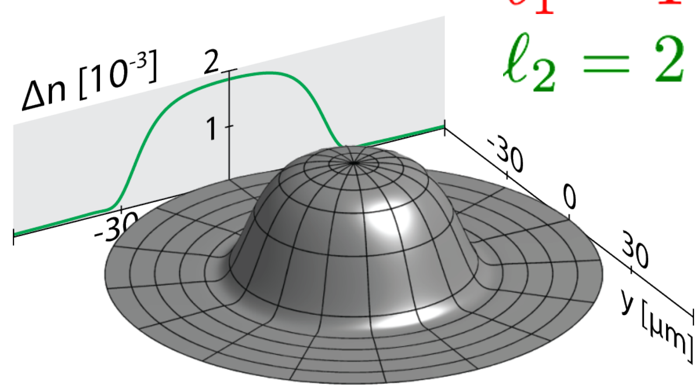
- SUSY partner fibers:

$$\Delta n^{(2)}(r) = \Delta n^{(1)}(r) + \frac{1}{n_0 k_0^2} \cdot \frac{d^2}{dr^2} \ln\left(r^{\frac{\ell_1^2 - \ell_2^2 + 1}{2}} \cdot R^{(1)}_{\ell_1 1} \right)$$

$$\ell_1 = 1$$
$$\ell_2 = 2$$

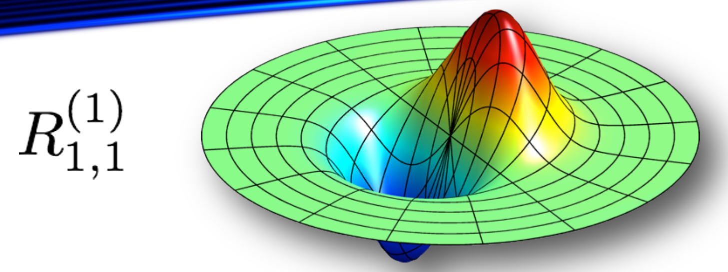


Original fiber

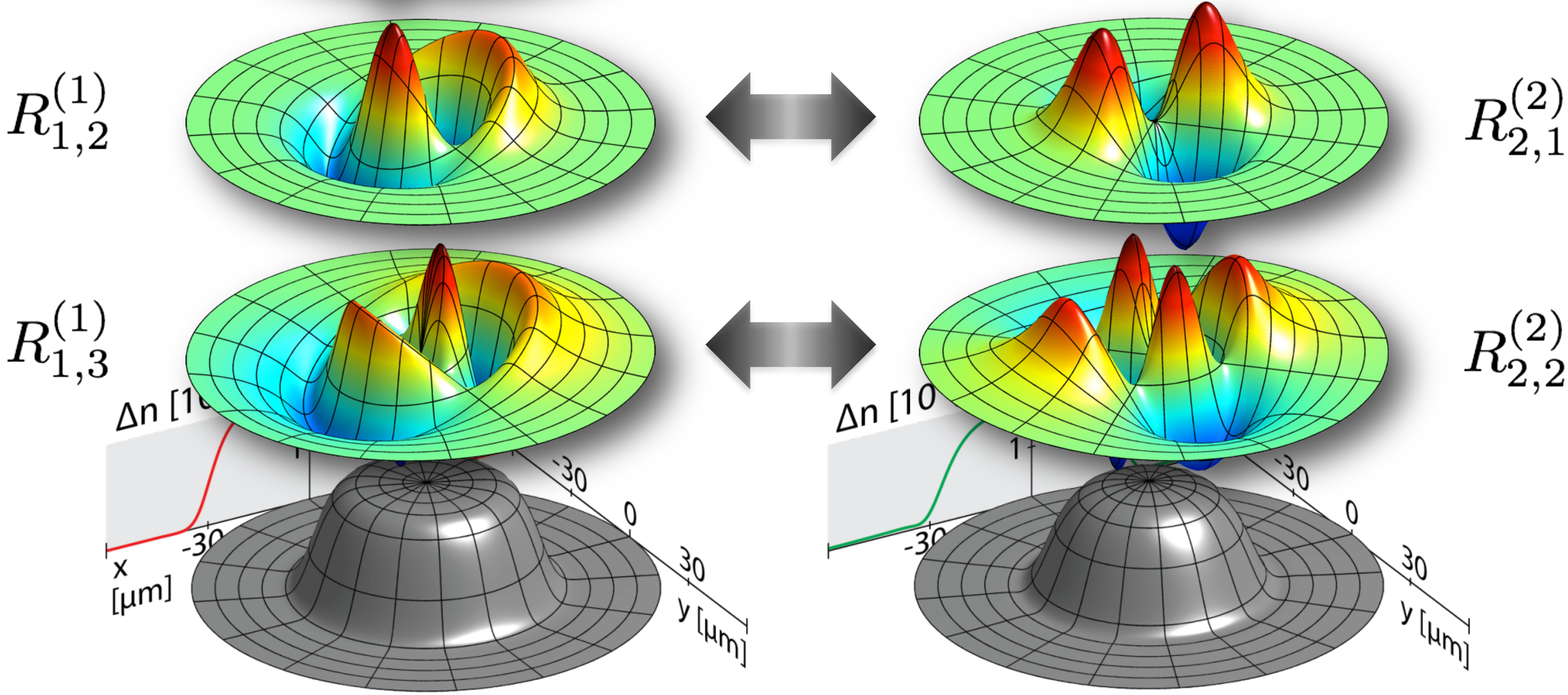


SUSY partner fiber

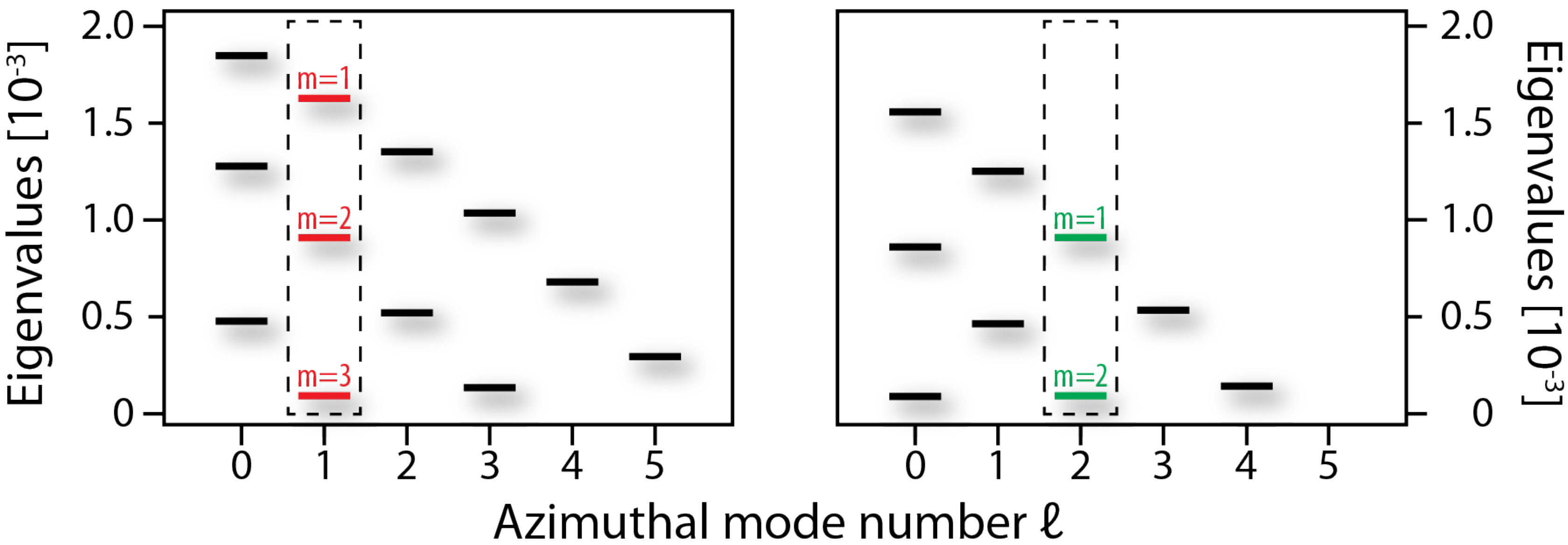
Link between azimuthal mode subsets



Mode orthogonality irrelevant for evanescent coupling

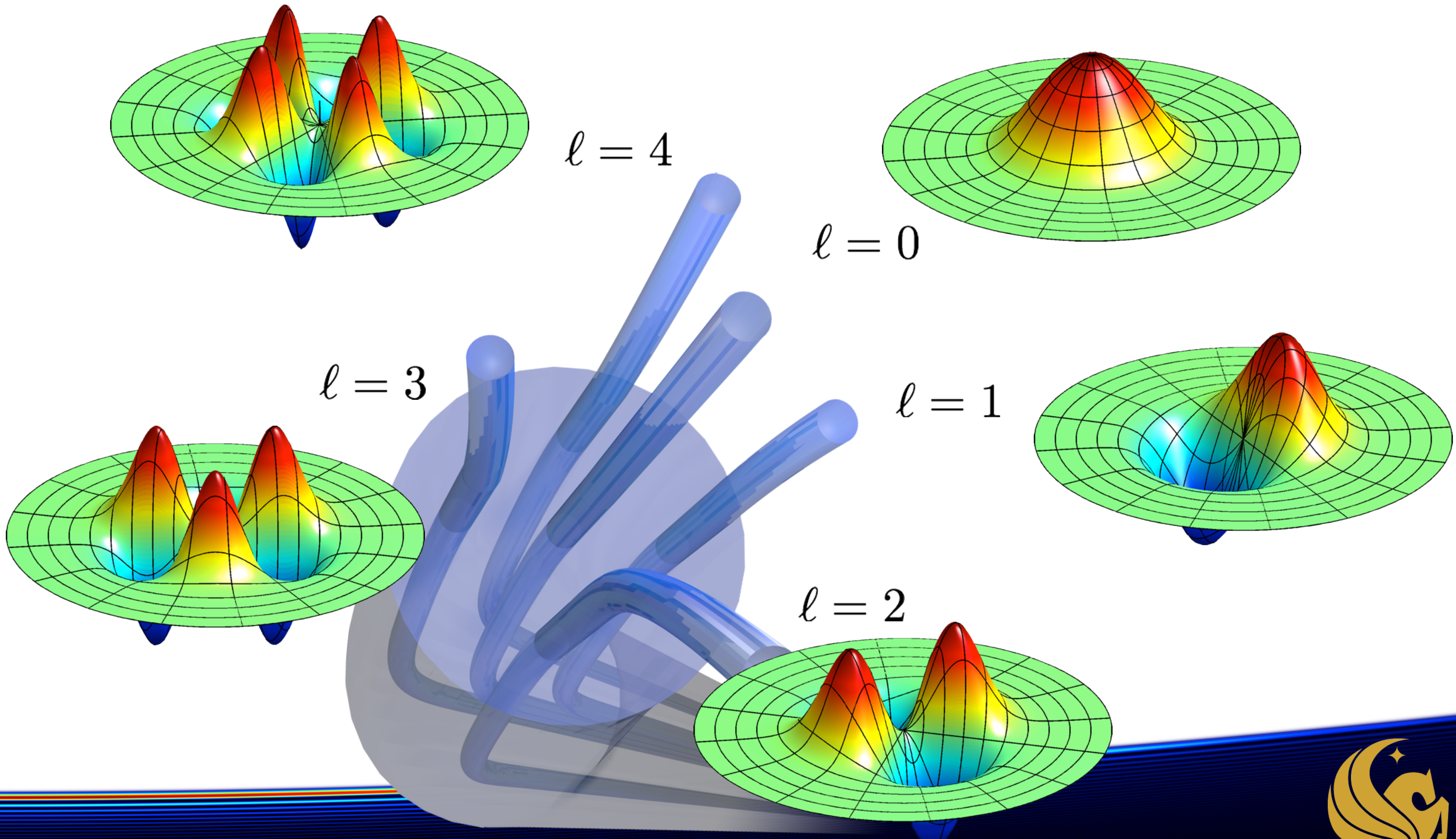


Link between azimuthal mode subsets



- Perfect SUSY phase matching for $|\ell^{(2)}| = |\ell^{(1)}| + 1$
- Eigenvalues of other modes remain disjoint

Integrated angular momentum multiplexing



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