Introduction to the physics of artificial gauge fields

Lecture 2: Magnetism in a periodic lattice (continued)

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Outline

1. Gauge fields on a lattice

2. Hofstadter butterfly

3. Shaken lattices

4. Lattices combining several internal states

Laser assisted tunnelling

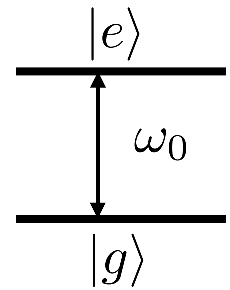
Flux lattices

Dalibard, Gerbier, Juzeliunas, Ohberg, Rev. Mod. Phys. 83, p.1523 (2011)

Goldman, Juzeliunas, Ohberg, Spielman, arXiv:1308.6533

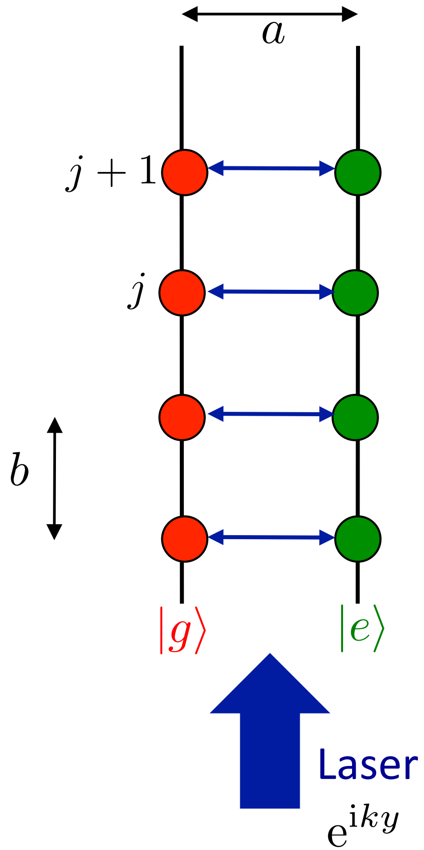
Ruostekoski et al., 2002

Laser assisted tunnelling



Atom with two internal states $|g\rangle, \ |e\rangle$

Ladder geometry with two parallel 1D lattices



The coupling with the laser induces a jump of the atom from one side of the ladder to the other one

$$|g, w_j\rangle \ \rightarrow \ |e, w_j\rangle$$

Tunnel matrix element along the line *j* :

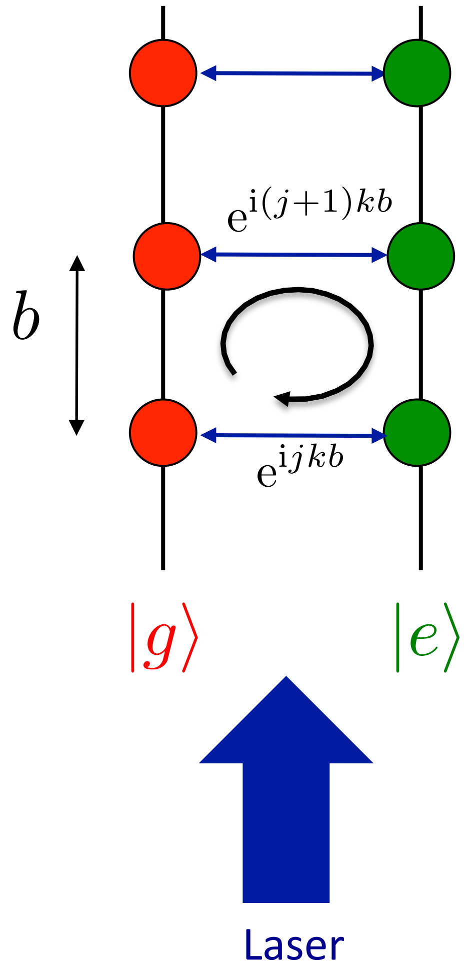
$$\underbrace{e^{ijkb}} \ \frac{\hbar\kappa}{2} \int \underbrace{w_0(x-a,y) \ \cos(ky) \ w_0(x,y) \ d^2r}$$

laser phase on line *j*

overlap of Wannier functions for $|g\rangle$ and $|e\rangle$

a can be “fictitious”: Celi et al. (2014)

Laser assisted tunnelling (continued)



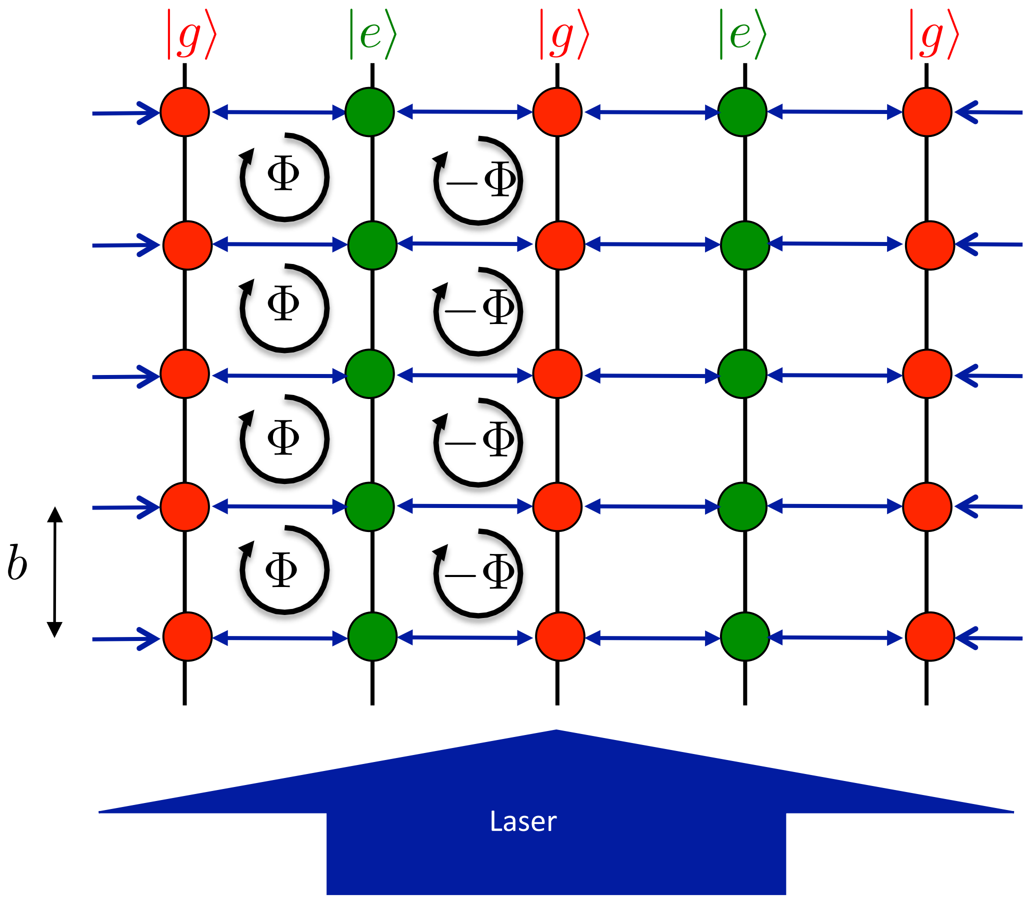
Phase accumulated on a unit cell

$$\Theta = 0 + (j+1)kb + 0 - jkb$$
$$= kb$$

$k = 2\pi/\lambda$ $\qquad$ $b \sim \lambda$

Θ can take adjustable values between 0 et 2π , by controlling the angle of the coupling laser with the axis y.

Extension in two dimensions



Staggered flux : on a given line, from left to right, the phase of the tunnel coefficient oscillates between

$$+j\,kb \quad \text{for} \quad |g\rangle \rightarrow |e\rangle$$

and

$$-j\,kb \quad \text{for} \quad |e\rangle \rightarrow |g\rangle$$

One can rectify this flux with slightly more complicated schemes

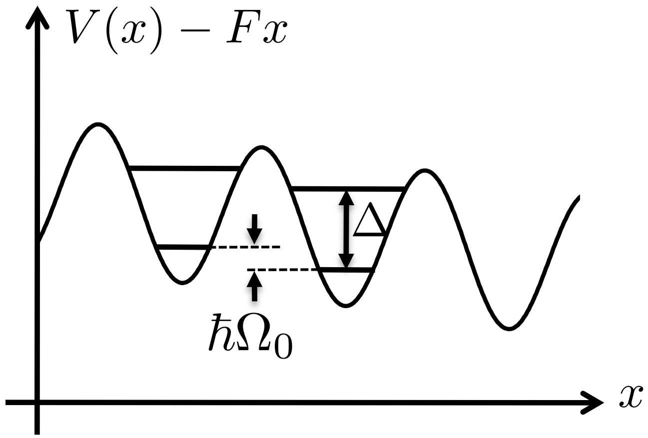
Jaksch & Zoller (2003), Gerbier & Dalibard (2010)

Is it worth using internal atomic states?

Difficulties associated to the simulation of lattice magnetism:

- transitions to higher bands (not taken into account here)
- heating due to the shaking

For a tilted+shaken lattice



- to avoid interband transitions: $\Omega_0 \ll \Delta$
- to avoid heating due to shaking, good hierachy in the time scales: $\bar{J} \sim J \ll \Omega$

The resonance condition $\Omega = \Omega_0$ then imposes

$\bar{J} \ll \Omega \ll \Delta$: small tunnel coefficients...

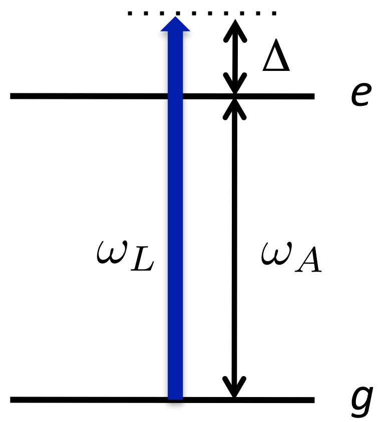
For tunnelling between different internal states:

Only one inequality to fullfill: $\bar{J} \sim \kappa \ll \Delta$

but inelastic collisions between various internal states can create other difficulties

A different approach to artificial magnetism: Flux lattices

Reminder: two-level atom in a light field, Rabi frequency κ, detuning Δ

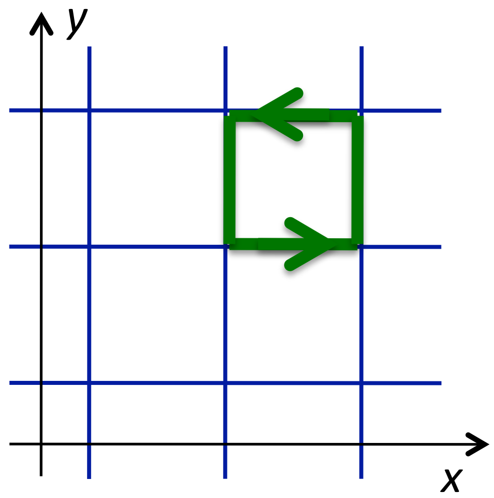


$$\hat{V}_{\mathrm{A-L}} = \frac{\hbar\Omega}{2}\begin{pmatrix} \cos\theta & \mathrm{e}^{-\mathrm{i}\phi}\sin\theta \\ \mathrm{e}^{\mathrm{i}\phi}\sin\theta & -\cos\theta \end{pmatrix}$$

$$\Omega = \left(\Delta^2 + |\kappa|^2\right)^{1/2} \qquad \tan\theta = |\kappa|/\Delta \qquad \kappa = |\kappa|\,\mathrm{e}^{\mathrm{i}\phi}$$

$$\vec{A} = \frac{\hbar}{2}(1-\cos\theta)\vec{\nabla}\phi$$

What happens in a static fully periodic lattice ?



Ω, θ, ϕ : periodic functions of *x,y*

$\vec{A} = \frac{\hbar}{2}(1-\cos\theta)\vec{\nabla}\phi$ is also periodic, hence one expects naively

$$\oint_C \vec{A}\cdot d\vec{l} = 0 \qquad \iint B_z(x,y)\,dx\,dy = 0$$

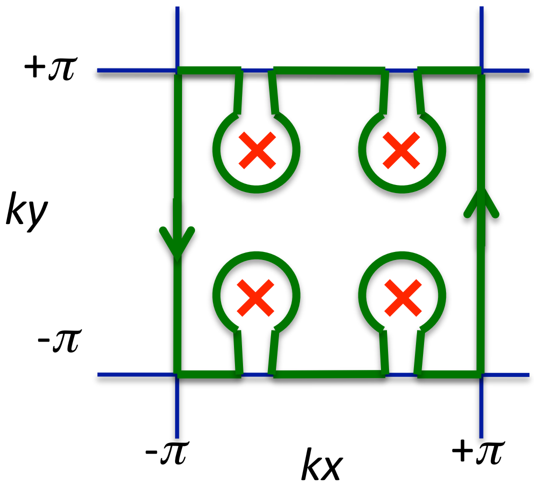
Correct only if $\vec{A}$ has no singularity in the unit cell

Singularities of $\vec{A}$ can occur at any point where $\sin\theta$ vanishes: ϕ undefined

An example of flux lattice (Cooper 2011)

$$\hat{V}_{\mathrm{A-L}} = \frac{\hbar\Omega}{2}\begin{pmatrix} \cos\theta & \mathrm{e}^{-\mathrm{i}\phi}\sin\theta \\ \mathrm{e}^{\mathrm{i}\phi}\sin\theta & -\cos\theta \end{pmatrix}$$ with $$\begin{cases} \cos\theta \propto \sin(kx)\sin(ky) \\ \sin\theta e^{i\phi} \propto \cos(kx) + i\cos(ky) \end{cases}$$

Singularities for $\boldsymbol{A}$ in points where $\cos(kx) = 0$ and $\cos(ky) = 0$



$$\iint B_z(x,y)\,dx\,dy = -\pi\hbar \sum_{\text{singular } j} \mathrm{sign}(j)$$

$$= 4\pi\hbar$$

$$B_z = \frac{\hbar k^2}{2}\frac{1 - C_x^2 C_y^2}{(1 + C_x^2 C_y^2)^{3/2}}\,, \quad C_x = \cos kx,\; C_y = \cos ky$$

How to operate a flux lattice?

Tight binding limit ? Because of the relation $\iint B_z(x,y)\, dx\, dy = -\pi\hbar \sum_{\text{singular } j} \text{sign}(j)$

there is an integer number of flux quanta per plaquette: not so interesting

Leaving the tight binding limit $\hbar\Omega \sim E_{\text{recoil}}$

The adiabatic approximation becomes questionable and one has to

calculate the band structure of $\frac{\hat{p}^2}{2M} + \frac{\hbar\Omega}{2} \begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix}$

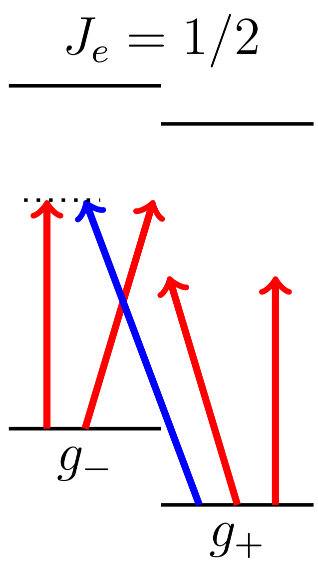
Look e.g. at the lowest band $n = 0$:

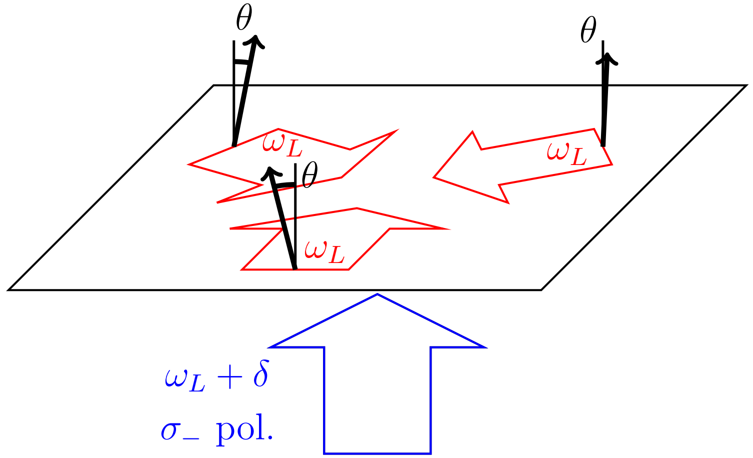
- should be as flat as possible:
  Mimic the degeneracy of Landau levels, increase the role of interactions
- should possess “magnetic properties”, characterized by its Chern index C

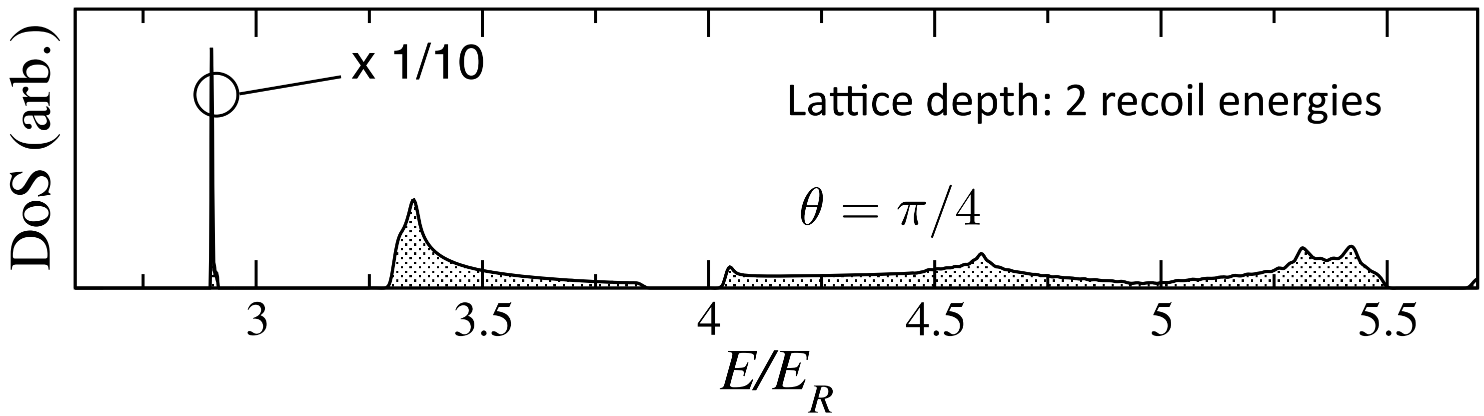
Realistic example for alkali atoms

Cooper & Dalibard 2011

Raman coupling between
two ground states







Lowest band : Chern index=+1, width=0.01 recoil energy

Gap above the lowest band: 0.4 recoil energy

Summary of some methods for simulating orbital magnetism

Goal : Reach a given cyclotron frequency ω_c or spin-orbit coupling

	Time-independent Hamiltonian	Time-dependent Hamiltonian frequency Ω
No use of internal states	Rotation with conserved angular momentum	Rotation with a stirrer Shaken lattices
Use of internal states	Berry's phase based schemes Flux lattices Laser assisted tunnelling Spin-orbit coupling	Spin-orbit coupling

Lecture 3 :
Artificial magnetism and interactions

Discussion for the case of a rotating Bose gas, with the single-particle Hamiltonian

$$\hat{H} = \frac{\hat{\boldsymbol{p}}^2}{2M} + \frac{1}{2}M\omega^2\hat{\boldsymbol{r}}^2 - \Omega\hat{L}_z$$

in the rotating frame

Can be extended to any other simulation of uniform magnetism, taking

Cyclotron frequency ω_c	⟷	*Rotation frequency* $\Omega = \omega_c / 2$

Outline of this part

1. From the standard vortex lattice to the mean-field lowest Landau level (LLL)

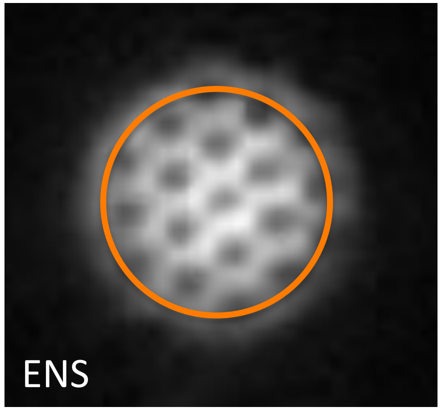
2. Beyond mean-field: quantum-Hall like states

Fetter, Rev. Mod. Phys. 81, p. 647 (2009)

Cooper, Advances in Physics, 57, p. 539 (2008)

Bloch, Dalibard, Zwerger, Rev. Mod. Phys. 80, p.885 (2008)

Rotation and "standard" vortex lattices



Standard response of a superfluid to rotation

- 2π phase winding around each vortex
- Vortex core size: healing length ξ with $\mu = \dfrac{\hbar^2}{2M\xi^2}$

Baym-Pethick
Dalfovo-Stringari

Number of vortices N_v ?
Answer from Feynman: compare the quantum velocity field $\boldsymbol{v} = \dfrac{\hbar}{M}\boldsymbol{\nabla}[\text{phase}]$
and the expected one for a classical fluid $\boldsymbol{v} = \boldsymbol{\Omega} \times \boldsymbol{r}$

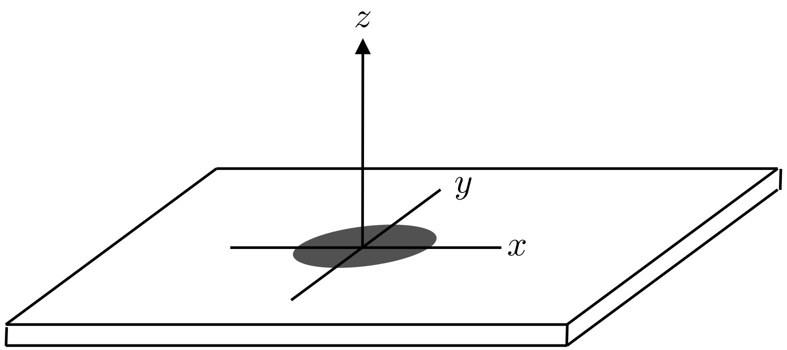
Along a circle of radius R: $\displaystyle\oint \boldsymbol{v}\cdot \mathbf{d}\boldsymbol{r} = \frac{\hbar}{M} 2\pi N_v \longleftrightarrow 2\pi\Omega R^2$

$$N_v = \frac{M\Omega R^2}{\hbar}$$

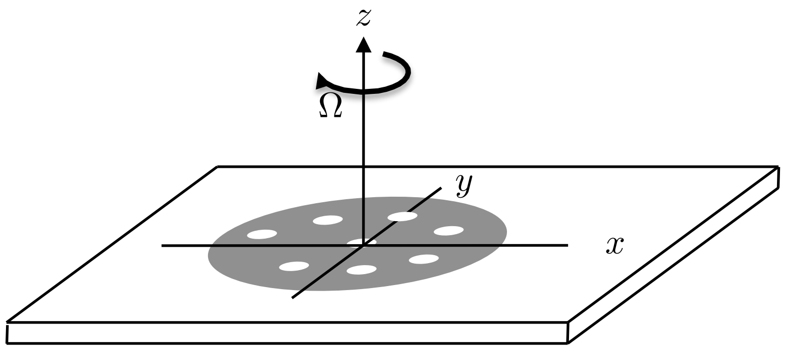
vortex density: $\rho_v = \dfrac{N_v}{\pi R^2} = \dfrac{M\Omega}{\pi\hbar}$

Checked experimentally at MIT

The limit of fast rotation



Condensate at rest



Rotating condensate

Rotation frequency Ω ⟶ Trap frequency ω

The effective trapping potential is reduced by the centrifugal potential: $\frac{1}{2}M\left(\omega^2-\Omega^2\right)r^2$

- The cloud density goes down.
- The healing length ξ (core size) goes up.
- The vortex density tends to a constant (Feynman): $\rho_v = \frac{M\Omega}{\pi\hbar} \approx \frac{M\omega}{\pi\hbar}$

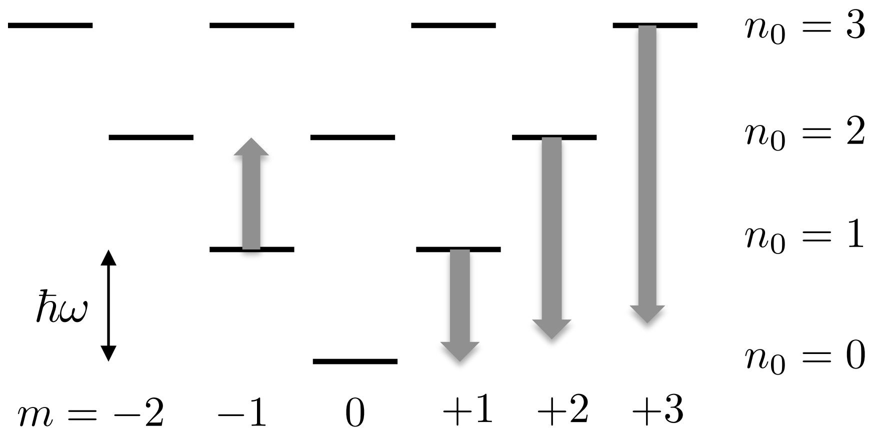
The limit $\rho_v\xi^2 \gtrsim 1$ or equivalently $\mu \lesssim \hbar\omega$ corresponds to the entrance in the LLL regime

Ho, 2001
Cooper, Komineas, Read, 2004
Fischer, Watanabe, Baym, Pethick, 2004

Aftalion, Blanc, Dalibard, 2005
Matveenko, Kovrizhin, Ouvry, Shlyapnikov, 2009

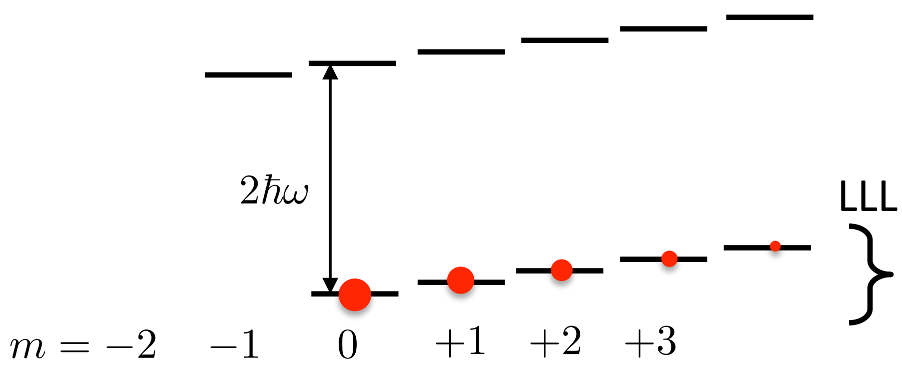
Single particle states for $\Omega \to \omega$

Harmonic trap at $\Omega = 0$



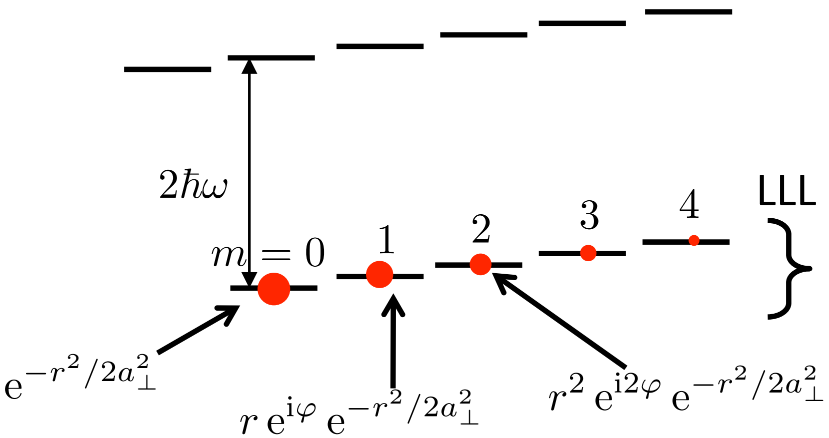
Add $-\Omega L_z = -m\hbar\Omega$
to go in the rotating frame

Harmonic trap for $\Omega \approx \omega$



The limit $\mu < \hbar\omega$ corresponds to
a restriction to the lowest Landau level

LLL states



Eigenstates : $\phi_m(r,\varphi) \propto r^m\, e^{im\varphi}\, e^{-r^2/2a_\perp^2}$

$$= u^m\, e^{-r^2/2a_\perp^2}$$

with : $u = x + iy$

$$a_\perp = \sqrt{\hbar/M\omega}$$

General LLL wave function : $\phi(\boldsymbol{r}) = \sum_m \alpha_m \phi_m(\boldsymbol{r}) \;= P(u)\, e^{-r^2/2a_\perp^2}$

polynomial : $P(u) = \sum_m \alpha_m u^m = \prod_{m=1}^{m_{\max}} (u - u_m)$

Around the root u_m of the polynomial, the phase rotates by $+2\pi$: vortex !

In the LLL, it is equivalent to specify the wave function
(coefficients α_m) or the vortex positions (roots u_m)

The vortex core size is similar to vortex separation

Aftalion, Blanc, Dalibard

Ground state in the LLL (mean field)

Two-dimensional problem: interactions described by a contact potential $\frac{\hbar^2}{M} g\, \delta^{(2)}(\boldsymbol{r})$

Dimensionless constant $g = \sqrt{8\pi}\, \frac{a}{a_z}$

a : 3D scattering length

a_z : "thickness" along the z direction $= \sqrt{\frac{\hbar}{M\omega_z}}$

Typical values: *g* between 0.01 to 1

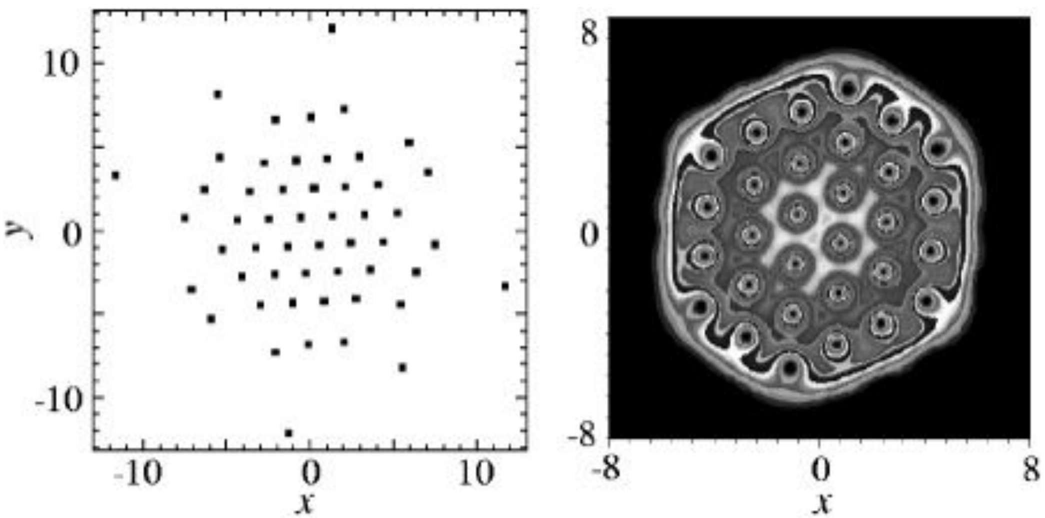
Three dimensionless parameters:

$$N,\ g,\ \Omega/\omega$$

but only one relevant parameter:

$$\Lambda = \frac{Ng}{1-\Omega/\omega}$$

Example for $\Lambda = 3000$ →



Regular lattice at center

Inverted parabola shape typical of Thomas-Fermi regime with $R_{\mathrm{TF}} \propto \Lambda^{1/4}$

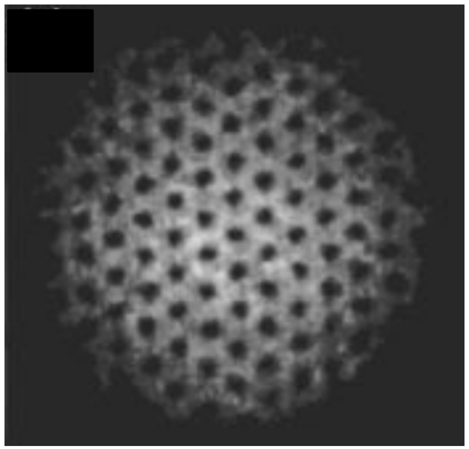
Vortex number: $N_v \propto \Lambda^{1/2}$

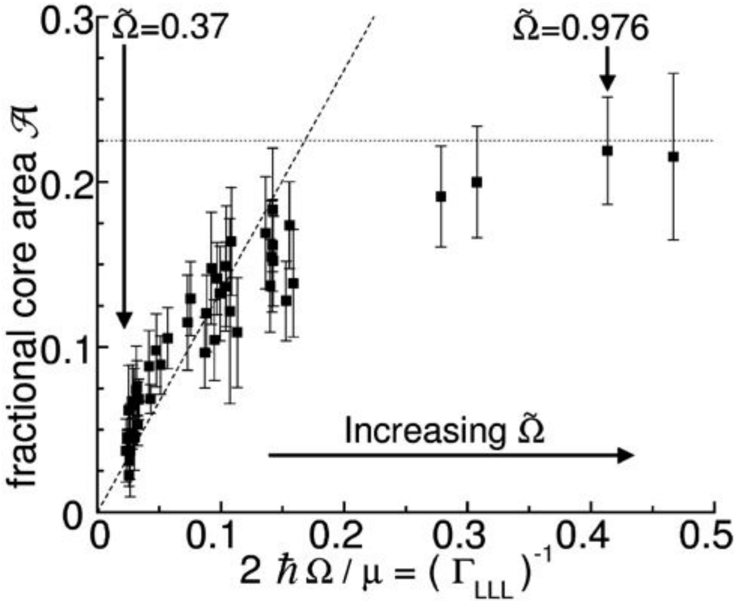
Experiments in the LLL

Boulder : evaporative spin-up method, which allowed to reach

$$\Omega = 0.993\,\omega$$

Fractional core area of the vortices Ω : good agreement with the predictions





100 000 Rb atoms, the trap is effectively 2D once rotating

Outline of this part

1. From the standard vortex lattice to the mean-field LLL

2. Beyond mean-field: quantum-Hall like states

Melting of the vortex lattice, Laughlin state, ...

Fetter, Rev. Mod. Phys. 81, p. 647 (2009)

Cooper, Advances in Physics, 57, p. 539 (2008)

Bloch, Dalibard, Zwerger, Rev. Mod. Phys. 80, p.885 (2008)

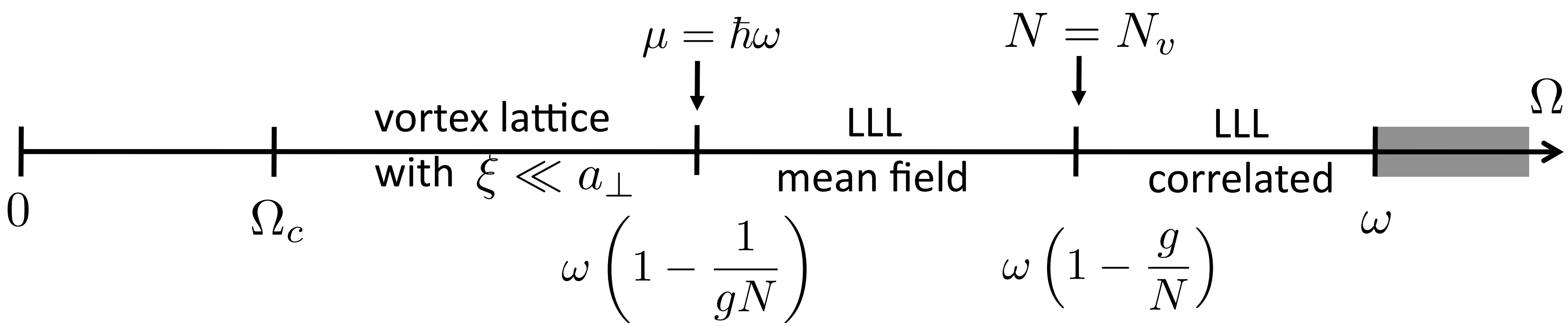
Limits of the LLL mean-field theory

Number of particles : N

Number of single-particle states that are effectively occupied = number of vortices

$$N_v \approx \Lambda^{1/2} \approx \left(\frac{Ng}{1-\Omega/\omega}\right)^{1/2}$$

When $N_v \sim N$, one can significantly lower the ground state energy by considering correlated states $\Psi(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) \neq \psi(\boldsymbol{r}_1) \ldots \psi(\boldsymbol{r}_N)$



valid for a weak-enough interaction: $g < 1$

How to find these correlated states?

Ground state of $\hat{H} = \sum_i \left(\frac{\hat{\boldsymbol{p}}_i^2}{2M} + \frac{1}{2} M\omega^2 \hat{\boldsymbol{r}}_i^2 \right) + \frac{\hbar^2}{M} g \sum_{i<j} \delta^{(2)}(\boldsymbol{r}_i - \boldsymbol{r}_j)$

for a given total angular momentum $\mathcal{L}$

LLL : $\Phi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) = \mathcal{P}(u_1, u_2, \ldots, u_N) \exp(-\sum_i r_i^2 / 2a_\perp^2)$

where $\mathcal{P}$ is a symmetric polynomial of the variables $u_1, u_2, \ldots, u_N$ $\quad u_j = x_j + \mathrm{i}y_j$

For example $N = 2,\ \mathcal{L} = 2 :\quad \mathcal{P}(u_1, u_2) = \alpha(u_1^2 + u_2^2) + \beta u_1 u_2$

Generally, sum of $u_1^{\alpha_1} \ldots u_N^{\alpha_N} \quad \sum_i \alpha_i = \mathcal{L}$

A few remarkable configurations

Cooper, Wilkin, Gunn
Sinova, MacDonald, et al
Lewenstein, Barberan, et al

➡ When $N_v \sim \frac{N}{10}$, the vortex lattice melts because of quantum fluctuations

➡ When $N_v = 2N$ or more precisely $m_{\max} = 2N$ (filling factor ½), Laughlin state

$$\mathcal{P}_{\text{Lau.}}(u_1, u_2, \ldots, u_N) = \prod_{i<j} (u_i - u_j)^2$$

total polynomial degree:
$\mathcal{L} = N(N-1)$

Never two particles at the same place: strong correlations!

Zero interaction energy for a contact potential: $\frac{\hbar^2}{M}\, g \sum_{i<j} \delta^{(2)}(\boldsymbol{r}_i - \boldsymbol{r}_j)$

Separated by a gap from all other states with the same angular momentum

$$E_{\text{gap}} \approx 0.1\, g\, \hbar\omega$$

Regnault-Jolicoeur

➡ When $N_v = N$ or more precisely $m_{\max} = N$ (filling factor 1),
Moore-Read (Pfaffian) state (never 3 particles at the same location)

$$E_{\text{gap}} \sim 0.05\, g\, \hbar\omega$$

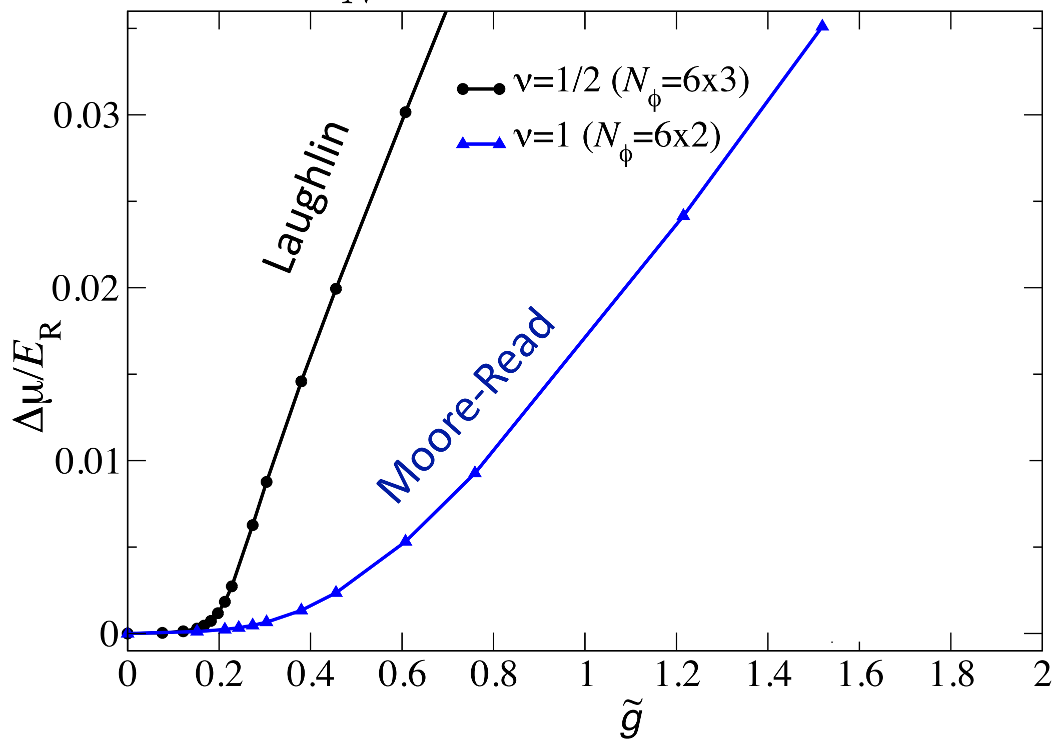
Chang et al.

Similar study for atoms in a flux lattice with a very narrow lowest band

Band width: $0.015\ E_R$ Gap: $0.7\ E_R$

Calculation of the gap as function of the interaction strength g

$$\Delta\mu = \frac{1}{N}\left[E(N+1) + E(N-1) - 2E(N)\right]$$



Filling factors ½ and 1

Incompressible states even for moderate interaction strength

Cooper & Dalibard
PRL **110**, 185301 (2013)

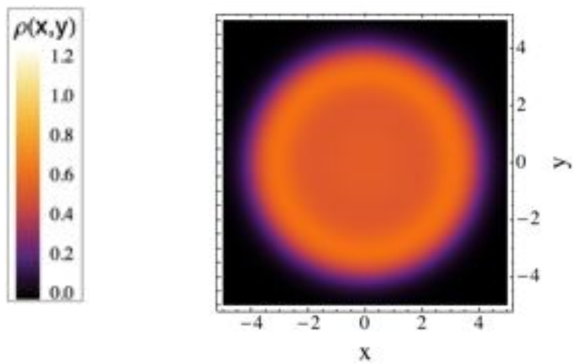
How can one detect these correlated states?

→ Reduction of inelastic losses

Laughlin state: never two particles at the same location

→ Gap between the ground state and all excited states

Flat density profile for a Laughlin state in a harmonic potential

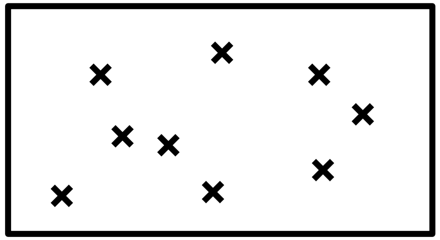


Roncaglia, Rizzi, Dalibard, calculation for 9 particles

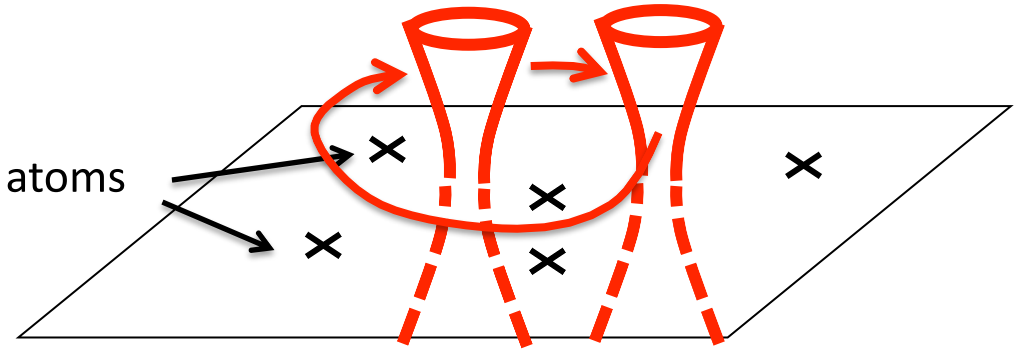
How can one detect these correlated states (continued)?

→ Detection of individual atoms after a time-of-flight

Reconstruction of spatial correlation functions



→ Quest for non conventional statistics [anyons, (Wilczek, 1982)] for the excited states of these fluids



Paredes *et al.*

→ create two excitations (holes) with two laser beams

→ rotate one excitation around the other one

→ Detect the accumulated phase

Conclusions

We now have a vast range of tools to simulate a one-body Hamiltonian with magnetic properties

Subtle topological states can be evidenced in this way

A (very) big challenge is to produce strongly correlated states with these one-body Hamiltonians + interactions

Would allow one to address important open questions on topological quantum matter