SOCIETÀ ITALIANA DI FISICA

INTERNATIONAL SCHOOL OF PHYSICS "ENRICO FERMI"

UNDER THE SPONSORSHIP OF CAMERA DI COMMERCIO DI LECCO - ISTITUTO NAZIONALE DI FISICA NUCLEARE
CONSIGLIO NAZIONALE DELLE RICERCHE - ISTITUTO NAZIONALE DI RICERCA METROLOGICA
MUSEO STORICO DELLA FISICA E CENTRO STUDI E RICERCHE "ENRICO FERMI"

SUMMER COURSES 2014

VILLA MONASTERO – VARENNA, LAKE COMO

Course 191

QUANTUM MATTER AT ULTRALOW TEMPERATURES

7 – 15 July

Impurities and disorder in systems of ultracold atoms

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$$ NSF, AFOSR NEW QUANTUM PHASES MURI,
DARPA OLE, ARO MURI ATOMTRONICS,
ARO MURI QUISM

Harvard-MIT CUA

Outline of lectures

Lecture I. Bose polarons

Mean field, Renormalization Group, variational approach.
Equilibrium: binding energy and dispersion
Non-equilibrium: RF spectroscopy, Bloch oscillations of polarons

Lecture II. Systems with disorder

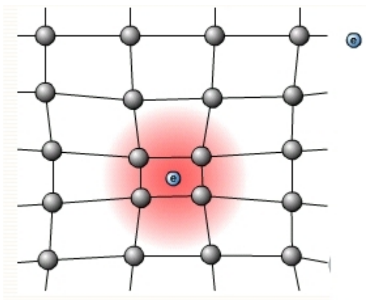
(complimentary to T. Giamarchi's lecture)
Many-body localization: real space RG perspective, loss or ergodicicty.
Probing MBL experimentally with interferometric probes.

Lecture III. Fermi polarons

Orthogonality catastrophe. Interferometric probe of orthogonality catastrophe in cold gases.
Rabi oscillations and Spin-bath model. Quantum flutter and Bloch oscillations in 1d.
Exotic Shiba molecules in Fermi superfluids

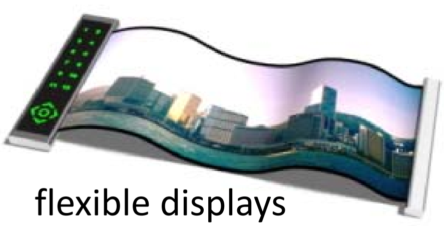
Polarons in condensed matter
and ensembles of ultracold atoms

Polarons in condensed matter physics and ensembles of ultracold atoms

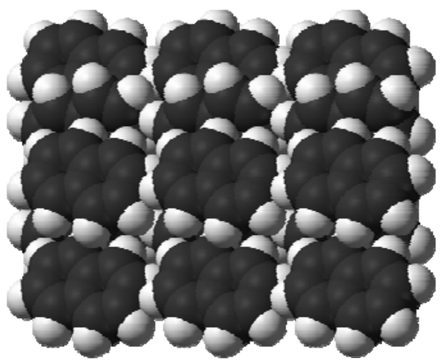


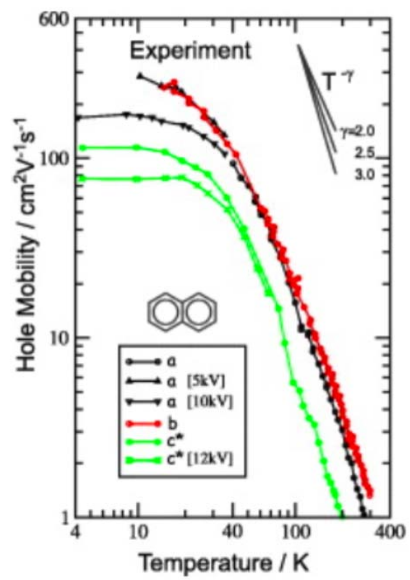
Lattice polarons:
Electrons dressed in phonons

Polaron transport in organic molecular crystal
(phonon dressing)



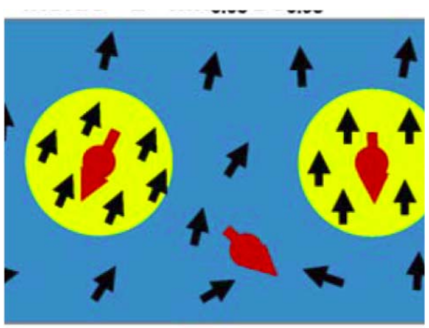
flexible displays
based on organic LEDs





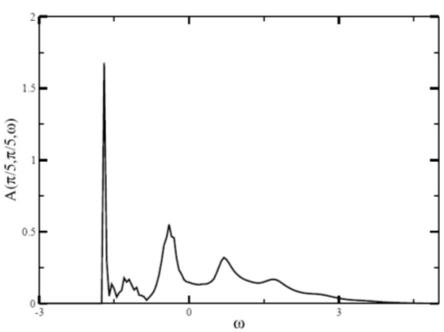
Naphthalene
hole mobility

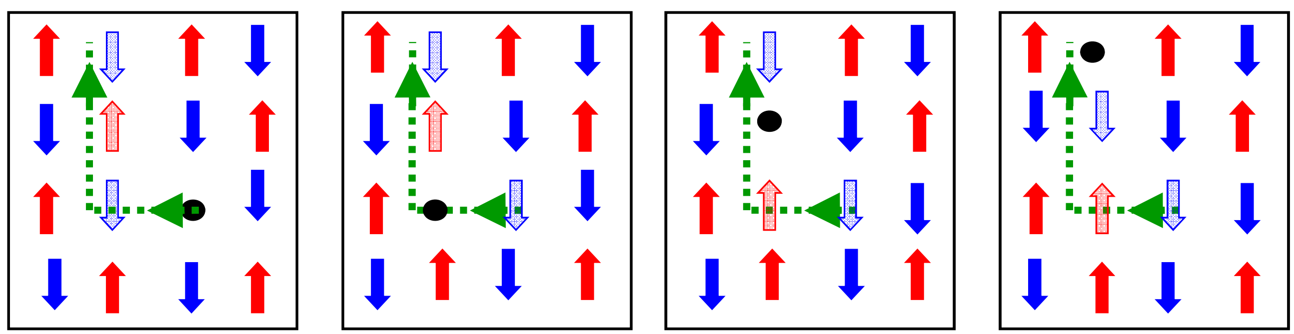
Polarons in condensed matter physics



Magnetic polarons:
Electrons dressed in magnetic polarization

Spectral function of a hole in an antiferromagnetic Mott insulator (spin wave dressing)

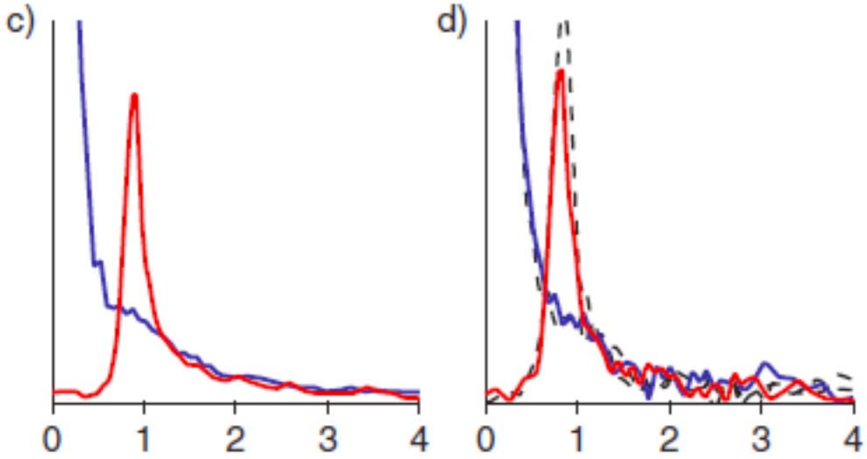




Polarons in ultracold Fermi gases

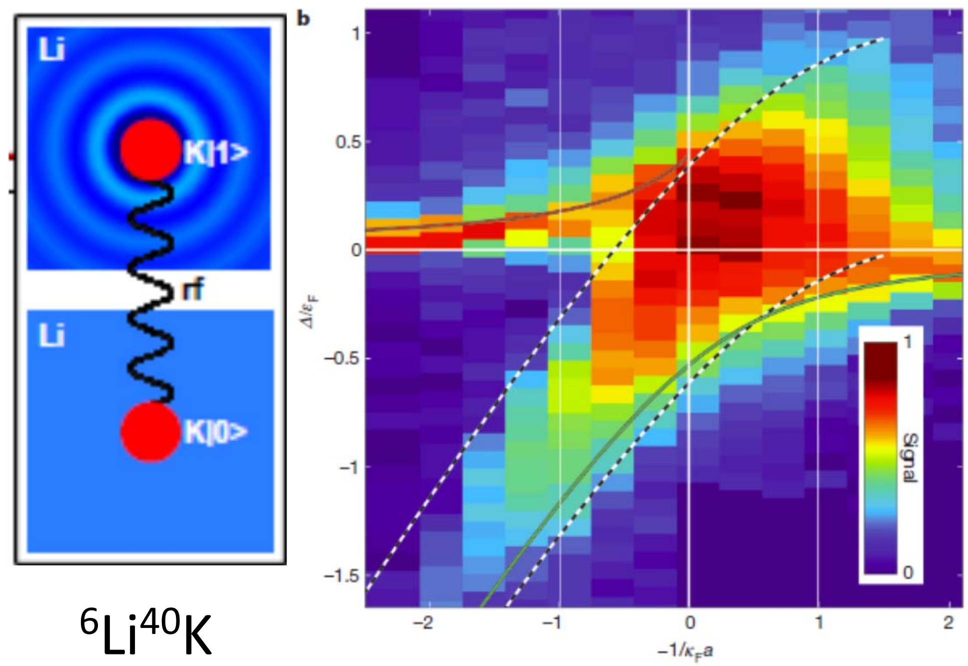
Zwierlein et al., PRL (2009)

Grimm et al., Nature (2012)



^{6}Li

also Kohl et al. (2011)

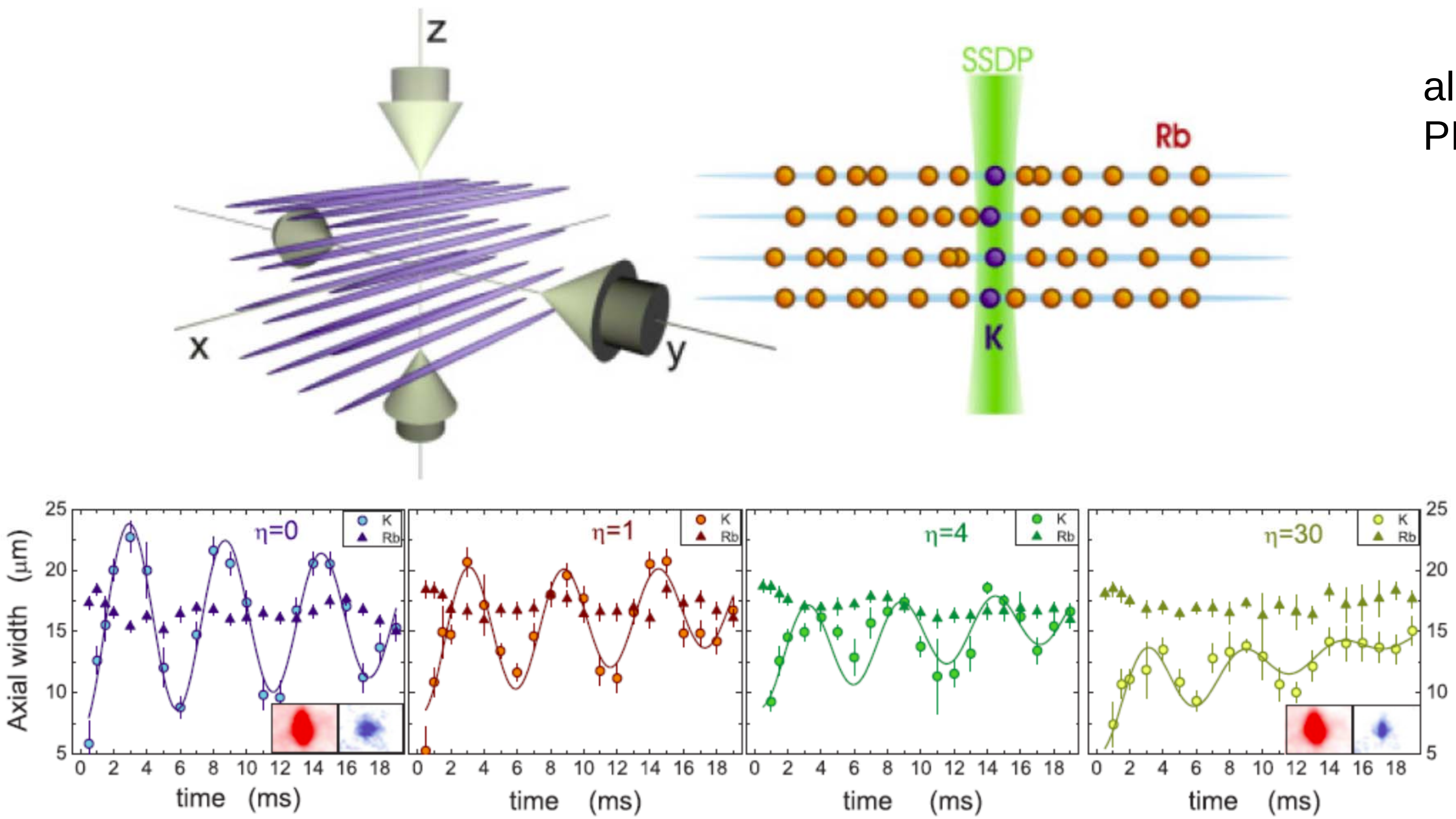


$^{6}Li^{40}K$

PHYSICAL REVIEW A 85, 023623 (2012)

Quantum dynamics of impurities in a one-dimensional Bose gas

J. Catani,[1,2] G. Lamporesi,[1,2] D. Naik,[1] M. Gring,[3] M. Inguscio,[1,2] F. Minardi,[1,2,*] A. Kantian,[4] and T. Giamarchi[4]



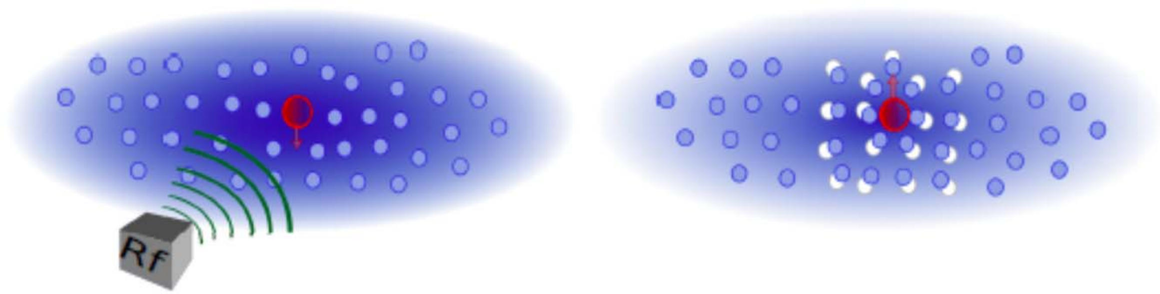
also M. Kohl et al., PRL (2009)

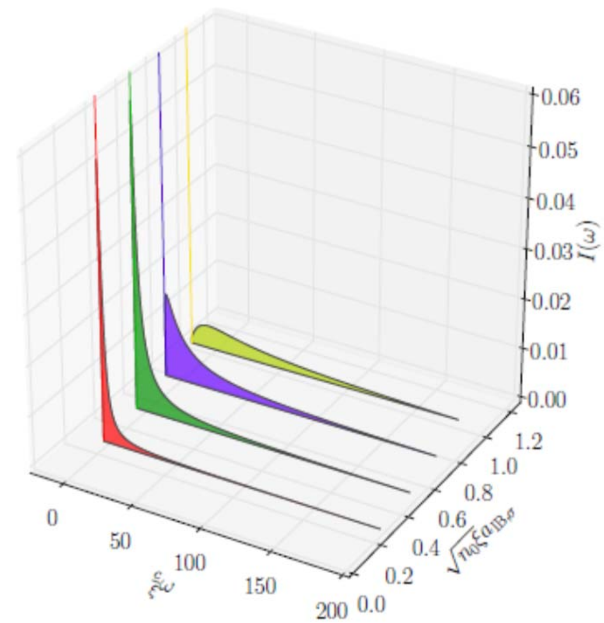
Polarons in BEC

Earlier theoretical work: Fisher, Zwerger (1986), Timmermans et al.(2006), Jaksch et al. (2008), Hofstetter et al. (2010), Devresee et al. (2011), Giamarchi et al. (2012), Rath, Schmidt (2013), Vlietnck et al. (2014)

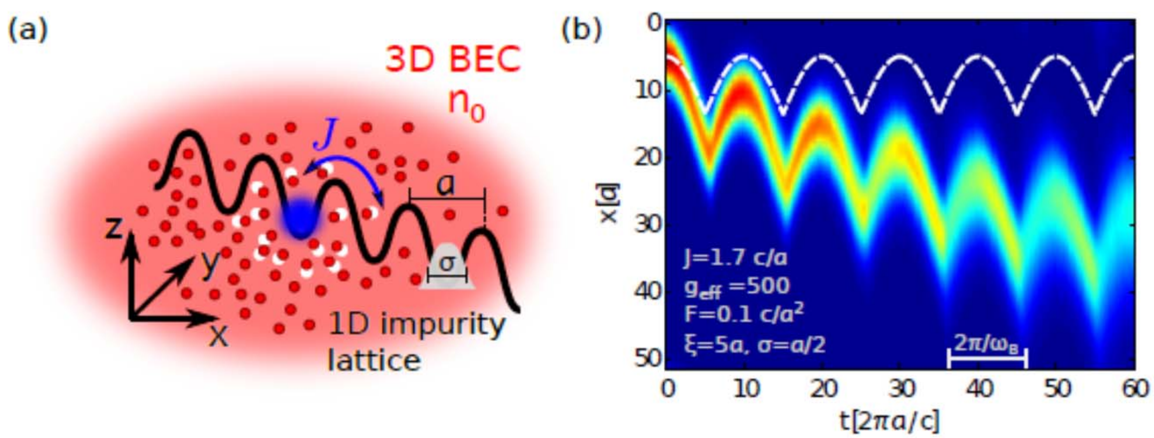
This work: A. Shashi, F. Grusdt, D. Abanin, E. Demler, Physical Review A, 89:053617 (2014)
+ F. Grusdt, Y. Schadilova, A. Rubtsov, D. Abanin, E. Demler, unpublished

RF spectroscopy of impurities in BEC

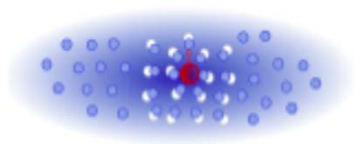




Polaron dynamics in optical lattices.
Bloch oscillations



Polarons in BEC. Froelich Hamiltonian



$$\mathcal{H} = \mathcal{H}_{\rm BEC} + \mathcal{H}_{\rm IMP} + \mathcal{H}_{\rm INT}$$

Bogoliubov approximation to BEC $\boxed{\mathcal{H}_{\rm BEC} = \sum_k \omega_k b_k^\dagger b_k}$ $\omega_{\mathbf{k}} = ck\sqrt{1 + \frac{(k\xi)^2}{2}}$

Number of condensate atoms N_0 . Density operator

$$\rho_k = \sum_p c_p^\dagger c_{p+k} = \sqrt{N_0}(c_k + c^\dagger_{-k}) = \sqrt{N_0}\,(u_k - v_k)\,(\,b_k + b^\dagger_{-k}\,)$$

Impurity: particle without the external potential $\boxed{\mathcal{H}_{\rm IMP} = \frac{P^2}{2M}}$

Contact interaction between impurity and host atoms. Scattering length $a_{\rm IB}$

$$\boxed{\mathcal{H}_{\rm INT} = \sum_k V_k e^{i\vec{k}\cdot\vec{R}}(b_{\vec{k}} + b^\dagger_{-\vec{k}})}$$

similar to electron-phonon
coupling in solids

Reduced mass $\frac{1}{\mu} = \frac{1}{M} + \frac{1}{m}$ $V_k = \frac{2\pi\, a_{\rm IB}\sqrt{N_0}}{\mu} \cdot \frac{\xi k}{\sqrt{2 + (\xi k)^2}}$

Polarons in BEC. Lee-Low-Pines transformation

Separate conserved total momentum of the system (transform to impurity frame)

$$\tilde{\mathcal{H}} = e^{iS}\mathcal{H}e^{-iS} \qquad S = \vec{R}\sum_k \vec{k}\, b_k^\dagger b_k \qquad e^{iS}\hat{\mathbf{p}}e^{-iS} = \hat{\mathbf{p}} - \sum_{\mathbf{k}} \mathbf{k}\hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}}$$

$$e^{iS} b_k e^{-iS} = b_k e^{-i\vec{k}\cdot\vec{R}}$$

Eliminate impurity degrees of freedom at the expense of introducing interactions between phonons

$$\tilde{\mathcal{H}} = \frac{1}{2M}\left(\mathbf{p} - \sum_{\mathbf{k}} \mathbf{k}\hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}}\right)^2 + \sum_{\mathbf{k}} V_{\mathbf{k}}(\hat{b}_{\mathbf{k}}^\dagger + \hat{b}_{-\mathbf{k}}) + \sum_{\mathbf{k}} \omega_{\mathbf{k}}\hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}}$$

p is now a parameter: total momentum of the system

Ground state. Mean field approximation

$$\tilde{\mathcal{H}} = \frac{1}{2M}\left(\mathbf{p} - \sum_{\mathbf{k}} \mathbf{k}\hat{b}^{\dagger}_{\mathbf{k}}\hat{b}_{\mathbf{k}}\right)^2 + \sum_{\mathbf{k}} V_{\mathbf{k}}(\hat{b}^{\dagger}_{\mathbf{k}} + \hat{b}_{-\mathbf{k}}) + \sum_{\mathbf{k}} \omega_{\mathbf{k}}\hat{b}^{\dagger}_{\mathbf{k}}\hat{b}_{\mathbf{k}}$$

Coherent state is exact solution when $M \to \infty$. Use it as a variational wavefunction.

$$|\,\Psi_{\mathrm{MF}}\,\rangle = \prod_{k} e^{(\,\alpha_k b^{\dagger}_k - \alpha^*_k b_k\,)}\,|\,0\,\rangle_{\mathrm{phon}}$$

Advantage: simple yet includes interaction between modes through self-consistency condition.
Disadvantage: factorizable wavefunction. No entanglement between modes.

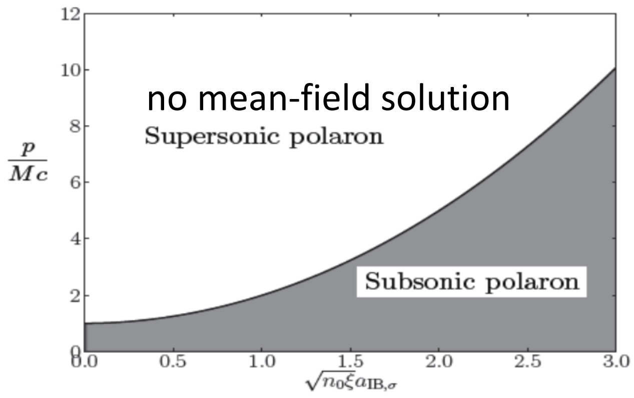
$$\alpha_k = \frac{V_k}{\omega_k + \frac{k^2}{2M} - \frac{k_{\|}}{M}(p - \Xi[\alpha_k])}$$

Mean-field self-consistency condition

$$\Xi = \sum_{\mathbf{k}} \frac{k_{\|} V^2_{\mathbf{k}}}{\left(\omega_{\mathbf{k}} + \frac{k^2}{2M} - \frac{k_{\|}}{M}(p - \Xi)\right)^2}$$

Polarons in BEC. Mean-field approximation

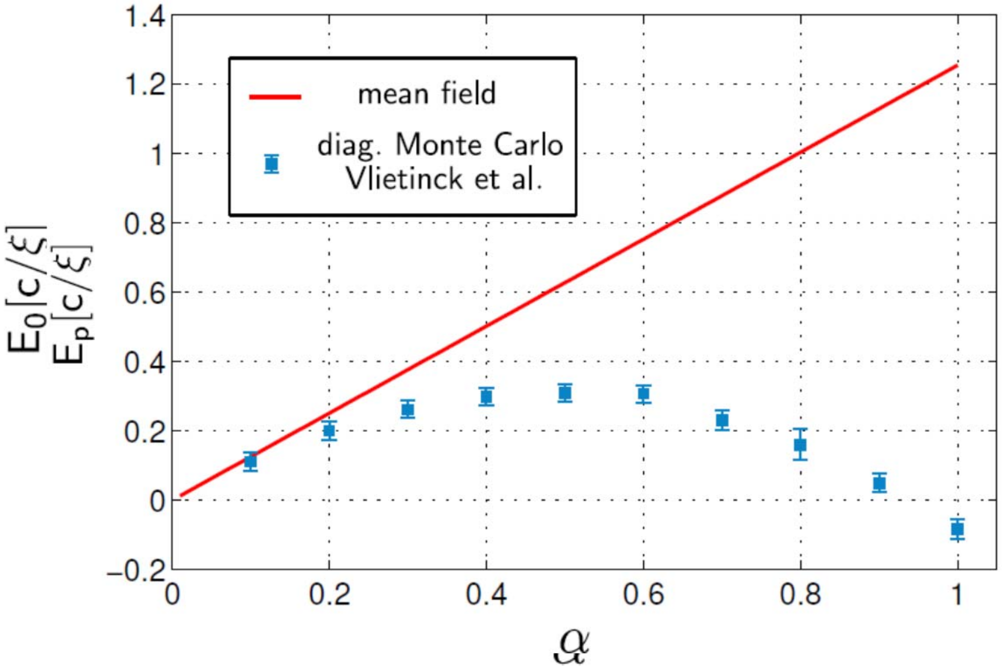
Where the mean-field solution exists



$$\boxed{a_{\mathrm{IB}}\sqrt{\rho_{\mathrm{BEC}}}\xi = \sqrt{\alpha/8\pi}}$$

Convenient definition because only $g_{\mathrm{IB}}\sqrt{\rho_{\mathrm{BEC}}}$ enters into analysis

How accurate is mean-field?
Comparison to Monte Carlo by Vlietnck et al.
arXiv:1406.6506



$$M/m_B = 0.263, q = 0, \Lambda_0 = 2000/\xi$$

Beyond mean-field.
Renormalization group analysis

Polarons in BEC. Equilibrium properties beyond mean-field

Expand around the mean-field solution $\hat{b}_k = \alpha_k + \hat{a}_k$

$$\mathcal{H} = E_{\rm MF} + \Delta E + \sum_k \Omega_k a_k^\dagger a_k + \frac{1}{2}\sum_{kk'} A_{kk'} \,:\, \Gamma_k \Gamma_{k'} \,:$$

Here $\Gamma_k = \alpha_k (a_k + a_k^\dagger) + a_k^\dagger a_k$ and $A_{kk'} = \frac{\vec{k}\vec{k}'}{M}$. Interaction proportional to 1/M.

Dimensions of operators

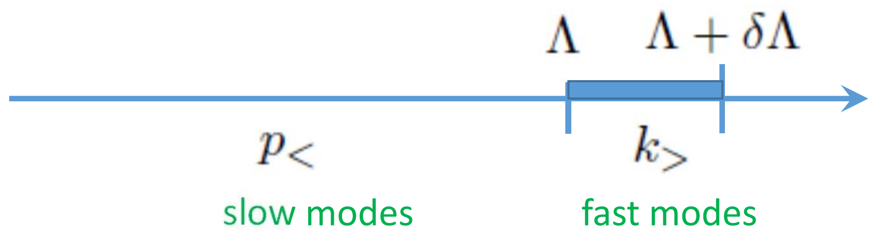
operator	$\Lambda \gg 1/\xi$	
$\hat{a}_k$	$\Lambda^{-d/2-1}$	
$\int d^dk\ d^dk'\ kk'\alpha_k\alpha_{k'}\ a_k a_{k'}$	Λ^{d-4}	irrelevant
$\int d^dk\ d^dk'\ kk'\alpha_k\ a_k a_{k'}^2$	$\Lambda^{d/2-3}$	irrelevant
$\int d^dk\ d^dk'\ kk' a_{k'}^2 a_k^2$	Λ^{-2}	marginal

In the long wavelength limit

Basic idea of RG analysis

$$\mathcal{H}_{\mathrm{fluct}} = \sum_k \Omega_k a_k^\dagger a_k + \frac{1}{2}\sum_{kk'} A_{kk'} :\Gamma_k \Gamma_{k'}:$$

Separate momentum components into slow and fast



$$\mathcal{H}_{\mathrm{S}} = \sum_{k_<} \Omega_k a_k^\dagger a_k + \frac{1}{2}\sum_{k_< k'_<} A_{kk'} \Gamma_k \Gamma_{k'}$$

$$\mathcal{H}_{\mathrm{F}} = \sum_{p_>} \Omega_p a_p^\dagger a_p + \mathrm{O}(\delta\Lambda^2)$$

$$\mathcal{H}_{\mathrm{MIX}} = \sum_{k_< p_>} A_{kp} \Gamma_k \Gamma_p \equiv \sum_{k_< p_>} A_{kp} \Gamma_k (\alpha_p (a_p + a_p^\dagger) + a_p^\dagger a_p)$$

Construct unitary transformation that decouples fast degrees of freedom.

Small parameter $1/\Omega_{p_>}$

Compare to Born-Oppenheimer approximation for electron-ion system: for fixed coordinates of slow particles (ions) find the ground of fast particles (electrons), the energy of this ground state gives effective potential for slow particles (ions)

Basic idea of RG analysis

$$U = \exp\{\sum_{p_>} (F_p^\dagger a_p - F_p a_p^\dagger)\}$$

Transformation analogous to a “shift” operator for fast modes, that depends on slow modes

$$F_p = \frac{\alpha_p}{\Omega_p} \sum_{k_<} \frac{\vec{k}\,\vec{p}}{M} \Gamma_k - \frac{\alpha_p}{\Omega_p^2} \sum_{k_<} \Omega_k\,(\vec{k}\,\vec{p})\,(a_k^\dagger - a_k) - \frac{\alpha_p}{\Omega_p^2} \sum_{k_<\,k'_<} (\vec{k}\,\vec{p})\,(\vec{k}\,\vec{p}')\,\Gamma_k\,\Gamma_{k'}$$

Transformed Hamiltonian

$$U\,(\mathcal{H}_\mathrm{S} + \mathcal{H}_\mathrm{F} + \mathcal{H}_\mathrm{MIX})\,U^\dagger = \mathcal{H}_\mathrm{S} + \delta\mathcal{H}_\mathrm{S} + \sum_{p_>} (\Omega_p + \delta\Omega_p) a_p^\dagger a_p$$

$$\delta\Omega_p = \sum_{k_<} \frac{(\vec{k}\,\vec{p})}{M} \Gamma_k$$

$$\delta\mathcal{H}_\mathrm{S} = -\sum_{k_<k'_<} \Gamma_k\,\Gamma_{k'} \sum_{p_>} \frac{\alpha_p^2}{\Omega_p} \frac{(\vec{k}\,\vec{p})\,(\vec{k}'\,\vec{p})}{M^2} + \mathrm{O}(\delta\Lambda^2)$$

Renormalization of the Hamiltonian for slow modes

More accurate RG analysis requires keeping track of the generated anisotropy of 1/M, that serves as interactions constant.

RG flow equations

$$\delta M^{-1}_{\mu\nu} = -\frac{1}{2} M^{-1}_{\mu\lambda} \sum_{p_>} \frac{\alpha_p^2}{\Omega_p} k_\lambda k_\sigma M^{-1}_{\sigma\nu}$$

Anisotropy in M generates terms that couple linearly to Γ_k .
This leads to corrections in energy.

$$\frac{\partial M^{-1}_{\mu\nu}}{\partial\Lambda} = -2M^{-1}_{\mu\lambda} \int_{\mathrm{f}} d^2\boldsymbol{k}\, \frac{\alpha^2_{\boldsymbol{k}}}{\Omega^*_{\boldsymbol{k}}} k_\lambda k_\sigma\, M^{-1}_{\sigma\nu}$$

RG flow of interaction strength

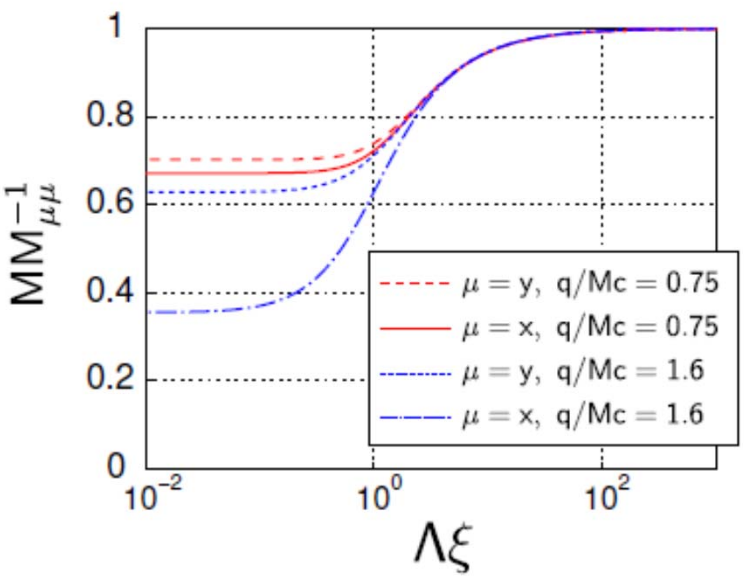
$$\frac{\partial Q_\mu}{\partial\Lambda} = 2M^{-1}_{\mu\nu} \int_{\mathrm{f}} d^2\boldsymbol{k}\, \frac{\alpha^2_{\boldsymbol{k}}}{\Omega^*_{\boldsymbol{k}}} k_\nu \left[-Q_\sigma k_\sigma + \frac{1}{2} k_\sigma \left(\delta_{\sigma\lambda} - M M^{-1}_{\sigma\lambda} \right) k_\lambda \right]$$

RG flow of momentum carried by phonons

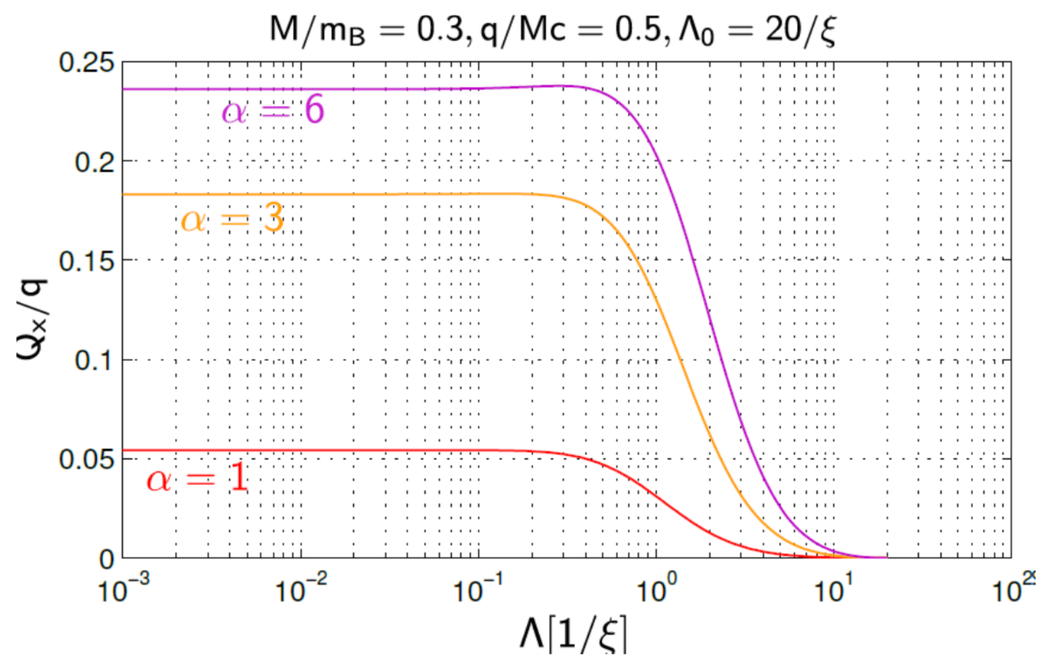
At the cut-off scale (before RG) we start with $Q = 0$ and $M^{-1}_{\mu\nu} = \frac{\delta_{\mu\nu}}{M}$. Correction to the energy

$$\Delta E = -\int^{\Lambda_0} d^3\boldsymbol{k} \left\{ \frac{|\alpha_{\boldsymbol{k}}|^2}{2M} k_\mu \left[\delta_{\mu\nu} - M M^{-1}_{\mu\nu}(k) \right] k_\nu + \frac{|\alpha_{\boldsymbol{k}}|^2}{\nu_{\boldsymbol{k}}} \left[\frac{Q_\mu(k)}{M} k_\mu + \frac{k_\mu k_\nu}{2} \left(M^{-1}_{\mu\nu}(k) - \frac{\delta_{\mu\nu}}{M} \right) \right]^2 \right\}$$

RG flow



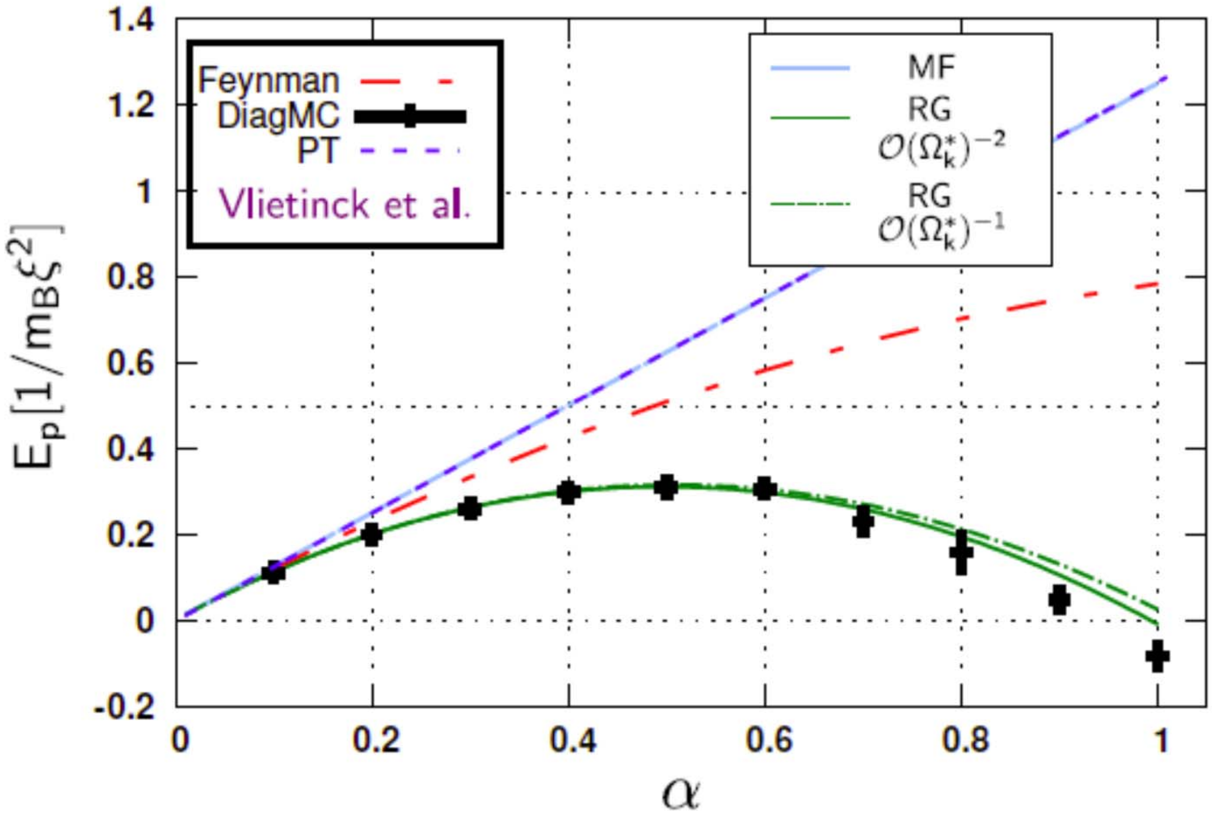
1
0.8
0.6
0.4
0.2
0
$MM^{-1}_{\mu\mu}$
10^{-2}
10^{0}
10^{2}
$\Lambda\xi$
$\mu = y$, $q/Mc = 0.75$
$\mu = x$, $q/Mc = 0.75$
$\mu = y$, $q/Mc = 1.6$
$\mu = x$, $q/Mc = 1.6$



$M/m_B = 0.3, q/Mc = 0.5, \Lambda_0 = 20/\xi$
0.25
0.2
0.15
0.1
0.05
0
Q_x/q
$\alpha = 6$
$\alpha = 3$
$\alpha = 1$
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
$\Lambda[1/\xi]$

RG results

Comparison to Monte Carlo results by Vlietnck et al., arXiv:1406.6506



Beyond mean-field.
Variational approach

Variational wavefunction

Let's improve upon mean-field factorizable wavefunction

$$|\Psi_{\mathrm{MF}}\rangle = \prod_k e^{(\alpha_k b_k^\dagger - \alpha_k b_k)} |0\rangle_{\mathrm{phon}}$$

Introduce entanglement between modes via Gaussian wavefunction
(see also Kagan, Prokof'ev (1990) and Kraus, Cirac (2010))

$$\boxed{|\Psi_{GAUSS}\rangle = e^{\sum_k(\alpha_k b_k - \alpha_k^* b_k^\dagger)} e^{\sum_{kk'}(Q_{kk'} b_k^\dagger b_{k'}^\dagger - Q_{kk'}^* b_k b_{k'})} |0\rangle_{\mathrm{phon}}}$$

One can use $|\Psi_{GAUSS}\rangle$ to compute expectation values

$$\langle b_k \rangle = \alpha_k; \qquad \left\langle b_k b_{k'}^\dagger \right\rangle_c = I_{kk'} + \left[\sinh^2 Q\right]_{kk'} \qquad \langle b_k b_{k'} \rangle_c = \frac{1}{2}\left[\sinh 2Q\right]_{kk'}$$

where $\left\langle \hat{A}\hat{B} \right\rangle_c = \left\langle \hat{A}\hat{B} \right\rangle - \left\langle \hat{A} \right\rangle \left\langle \hat{B} \right\rangle$

higher order expectation values reduce to lower ones.

Expand energy up to second order in $Q_{kk'}$. Recall that each $Q_{kk'}$ is of order of 1/N.

Variational wavefunction

Saddle point equation on $Q_{kk'}$

$$\left(\Omega_q + \Omega_{q'} + \frac{\left(\vec{q}\cdot\vec{q}'\right)}{M}\right) Q_{qq'} = -\frac{\left(\vec{q}\cdot\vec{q}'\right)}{M}\alpha_q\alpha_{q'} - \sum_k \frac{\left(\vec{k}\cdot\vec{q}\right)}{M}\alpha_k\alpha_q Q_{q'k} - \sum_k \frac{\left(\vec{k}\cdot\vec{q}'\right)}{M}\alpha_k\alpha_{q'} Q_{qk}$$

Can be solved with ansatz (Kagan, Prokof'ev 1990)

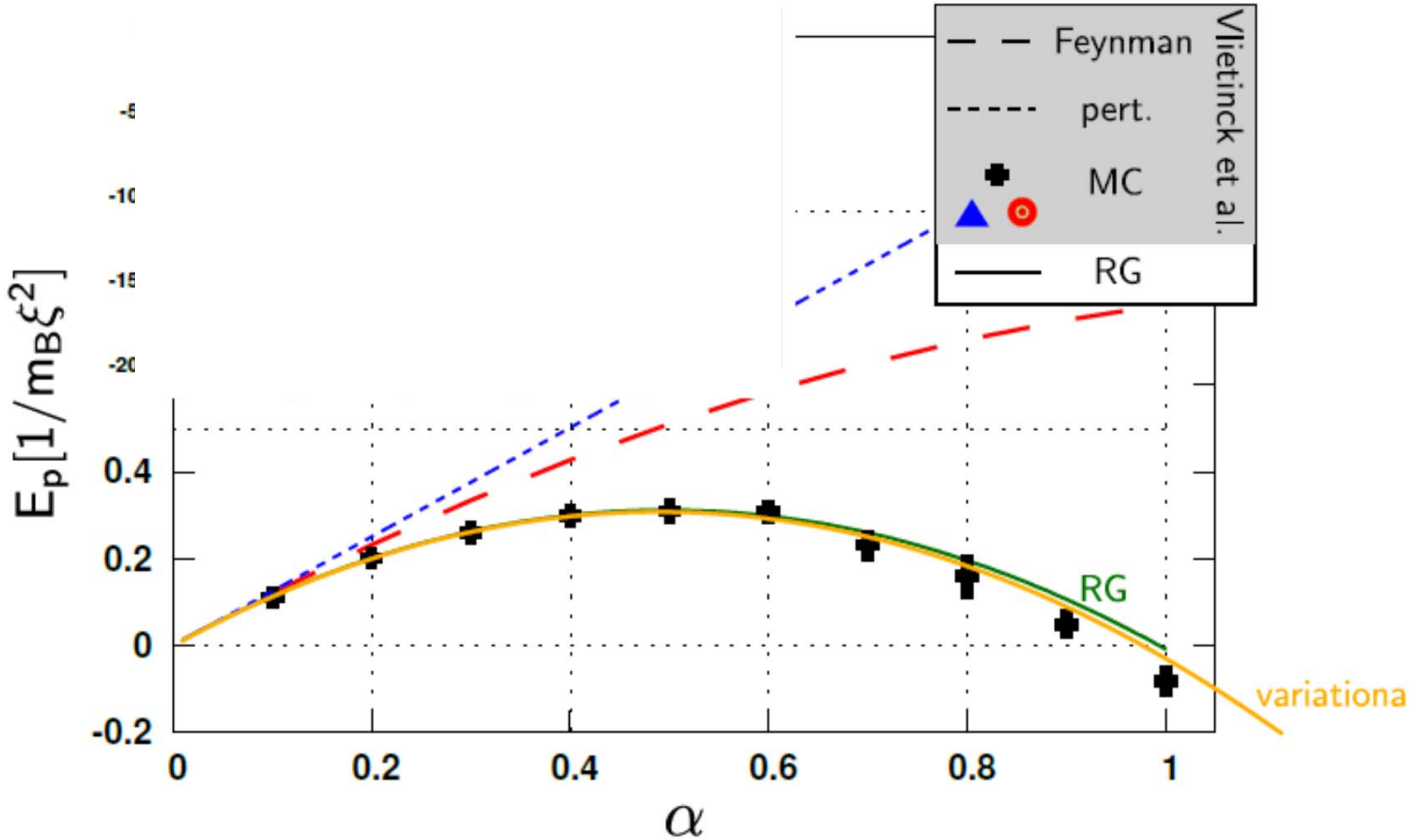
$$Q_{qq'} = -\frac{\left(\vec{q}\cdot\vec{q}'\right)}{M}\frac{\eta_q\alpha_q\alpha_{q'}\eta_{q'}}{\Omega_q + \frac{\left(\vec{q}\cdot\vec{q}'\right)}{M} + \Omega_{q'}}$$

Saddle point equations

$$\alpha_q = -\frac{V_q}{\omega_q + \frac{q^2}{2M}\eta - \frac{\vec{q}}{M}\left(\vec{P} - \sum \vec{k}\left|\alpha_q\right|^2\right)}$$

$$\eta_q = 1 - \eta_q \int d^3k \frac{\left(\vec{k}\cdot\vec{e_q}\right)^2}{M}\frac{\alpha_k^2\eta_k}{\Omega_q + \Omega_k + \frac{\left(\vec{q}\cdot\vec{k}\right)}{M}}$$

Variational wavefunction

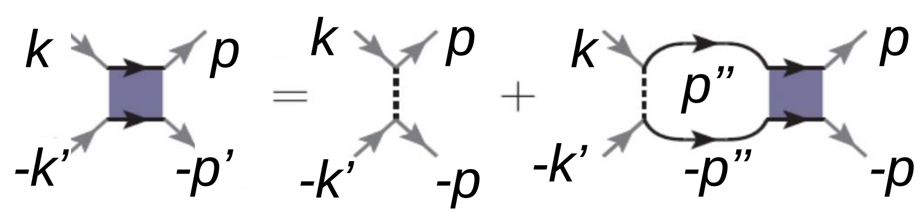


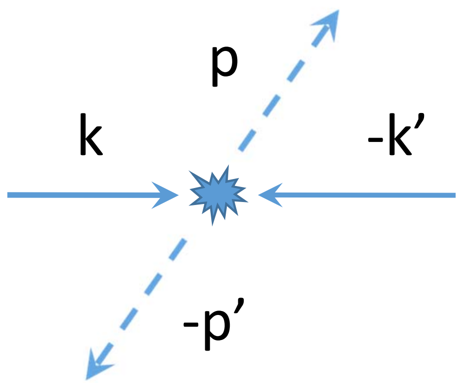
Subtlety: regularizing divergences

Power law divergence in the mean-field energy

$$E_0 = g_{IB} n_0 \qquad E_{\mathrm{MF}} = E_B - \left(\frac{2\pi g_{IB}}{\mu}\right)^2 n_0 \sum_k \frac{2\mu}{k^2}$$

Physical scattering length determined
from the Lippman-Schwinger equation



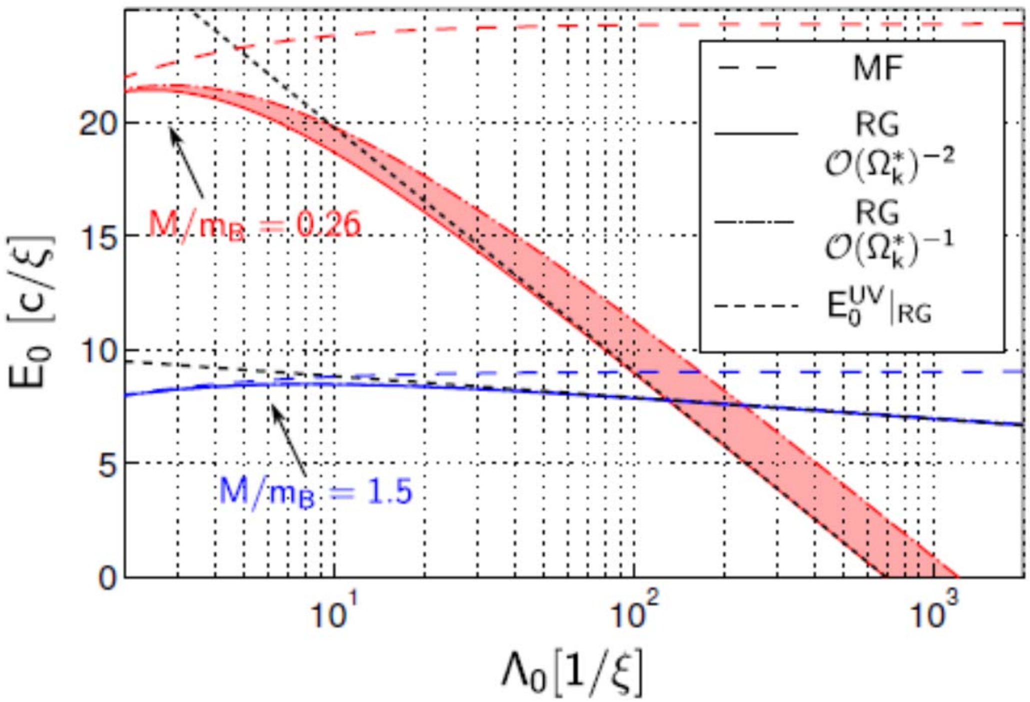


$$\frac{1}{g_{IB}} = \frac{\mu}{2\pi a_{IB}} - \sum_k \frac{2\mu}{k^2}$$

$$\boxed{E_0 + E_{\mathrm{MF}} = \frac{2\pi a_{IB} n_0}{\mu} + E_B}$$

Subtlety: regularizing divergencies

Log divergence of the fluctuation part of energy



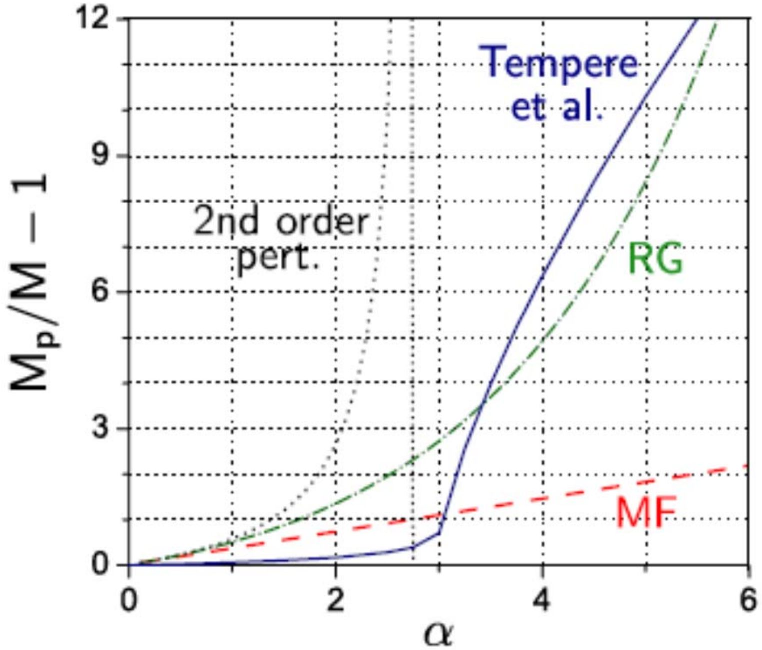
Physical cut-off should be taken as distance between Bose atoms

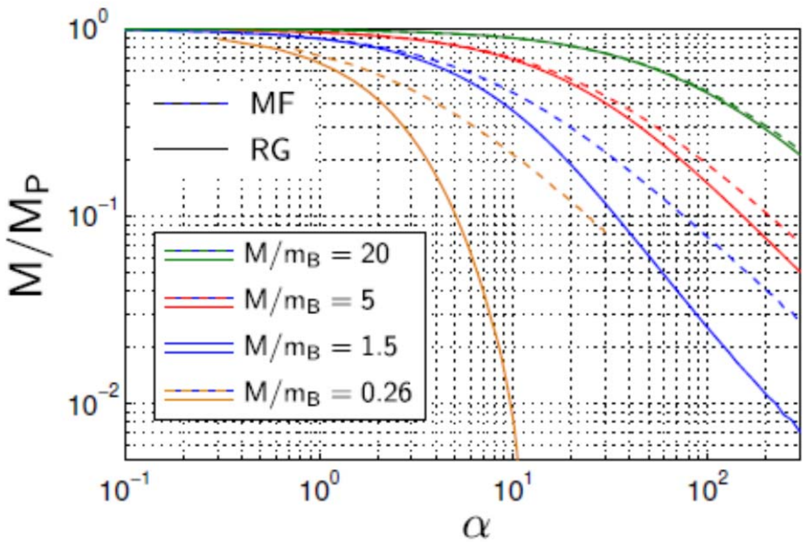
Why not just use Monte-Carlo?
Why do we need RG and variational approaches?

RG results for the effective mass

Effective mass

$$\frac{\partial \epsilon_p}{\partial p} = v_p \equiv \frac{p}{M_p}$$

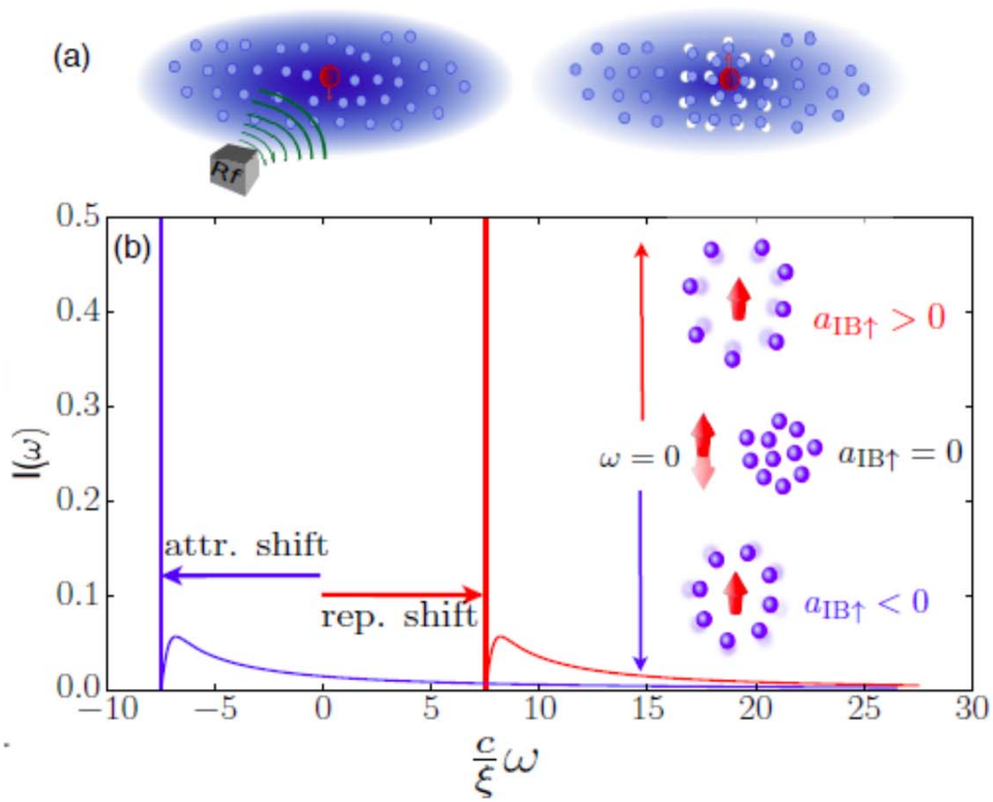




Mean-field vs RG

RF spectroscopy
(mostly mean-field)

RF spectroscopy of impurities in BEC



$$I(\omega) = \sum_n |\langle n \uparrow | \mathcal{H}_{\mathrm{RF}} | 0 \downarrow \rangle|^2 \delta(\omega - \omega_{0n})$$

Summation goes over states with 0,1,2,… excitations

So far we only discussed the position of the quasiparticle peak (ground state to ground state transition)

RF spectroscopy of polarons in BEC

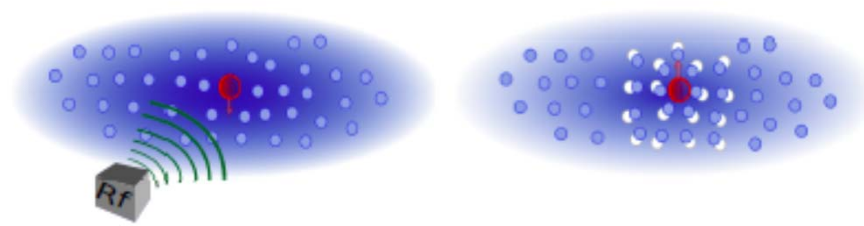
$$\mathcal{H} = \mathcal{H}_{\mathrm{BEC}} + \mathcal{H}_{\mathrm{IMP}} + \mathcal{H}_{\mathrm{INT}}$$

Bogoliubov approximation to BEC $\quad \mathcal{H}_{\mathrm{BEC}} = \sum_k \omega_k b_k^\dagger b_k \qquad \omega_{\mathbf{k}} = ck\sqrt{1 + \frac{(k\xi)^2}{2}}$

Contact interaction between impurity and host atoms.

$$\mathcal{H}_{\mathrm{IMP}} = \frac{P^2}{2M} + g_\sigma P_\sigma \sum_k V_k e^{i\vec{k}\cdot\vec{r}}(b_{\vec{k}} + b^\dagger_{-\vec{k}})$$

$P_\sigma = |\,\sigma\,\rangle\,\langle\,\sigma\,|$ projection operator



RF coupling $\quad \mathcal{H}_{\mathrm{RF}} = \Omega\,(\,\sigma_+\,e^{-i\omega t} + \mathrm{c.c.})$

Fermi's golden rule for the probability of transferring impurity atoms into the other spin state

$$I(\omega) = \sum_n |\langle\, n\uparrow\, |\mathcal{H}_{\mathrm{RF}}|\, 0\downarrow\rangle|^2 \delta(\omega - \omega_{0n})$$

After Lee-Low-Pines transformation

$$\tilde{\mathcal{H}}_\sigma = \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2M}(\vec{p} - \sum_k \vec{k}\, b_k^\dagger\, b_k\,)^2 + g_\sigma P_\sigma \sum_k V_k(\, b_k + b_{-k}^\dagger\,)$$

RF spectroscopy as quench dynamics $\quad I(\omega) = \sum_n |\langle\, n \uparrow\, |\mathcal{H}_{\rm RF}|\, 0 \downarrow \rangle|^2 \delta(\omega - \omega_{0n})$

$$I(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\, e^{iE_{o\downarrow}t}\, \langle\, \tilde{0}_{\rm ph\downarrow}\,|\, e^{i\tilde{\mathcal{H}}_\uparrow t}|\, \tilde{0}_{\rm ph\downarrow}\,\rangle$$

Propagation amplitude $\quad A_p(t) \,=\, \langle\, \tilde{0}_{\rm ph\downarrow}\,|\, e^{i\mathcal{H}_\uparrow t}|\, \tilde{0}_{\rm ph\downarrow}\,\rangle$

When $M \to \infty$ time dependent coherent states give exact solution of time dependent wavefunctions. We consider mean-field coherent states as a variational ansatz for dynamics.

$$|\phi(t)\rangle = e^{-i\chi(t)} e^{\sum_{\mathbf{k}} \alpha_{\mathbf{k}}(t)\hat{b}_{\mathbf{k}}^\dagger - \frac{1}{2}|\alpha_{\mathbf{k}}(t)|^2} |0\rangle$$

RF spectroscopy of polarons. Mean-field approach

$$|\phi(t)\rangle = e^{-i\chi(t)} e^{\sum_{\mathbf{k}} \alpha_{\mathbf{k}}(t)\hat{b}^{\dagger}_{\mathbf{k}} - \frac{1}{2}|\alpha_{\mathbf{k}}(t)|^2} |0\rangle$$

Use variational dynamics (Jackiw, Kerman (1979)). Construct “classical-like” Lagrangian

$$L[\chi(t), \alpha_k(t), t] = i\langle\, \Psi_{\mathrm{var}}(t)\,|\, \partial_t \,|\Psi_{\mathrm{var}}(t)\,\rangle - \langle\, \Psi_{\mathrm{var}}(t)\,|\, \tilde{\mathcal{H}} \,|\Psi_{\mathrm{var}}(t)\,\rangle$$

and construct equations of motion

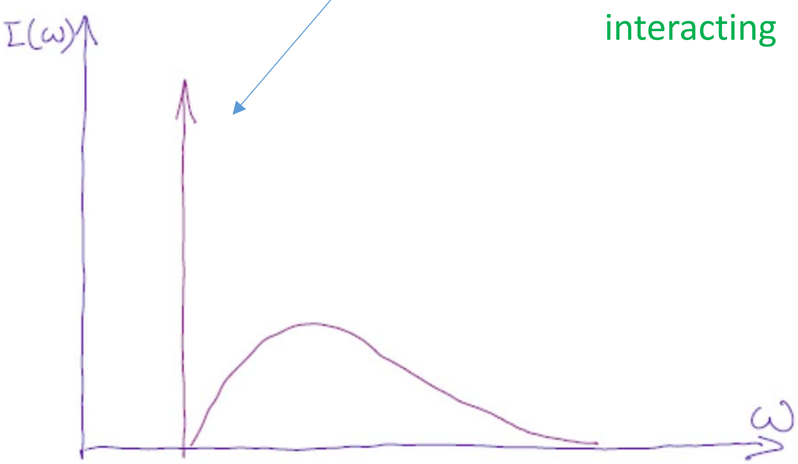
$$\dot{\chi}(t) = \frac{p^2}{2M} - \sum_{\mathbf{k},\mathbf{k}'} \frac{\mathbf{k}.\mathbf{k}'}{2M} |\alpha_{\mathbf{k}}|^2 |\alpha_{\mathbf{k}'}|^2 + \frac{1}{2}\sum_{\mathbf{k}} V_{\mathbf{k}}(\alpha_{\mathbf{k}} + \alpha^*_{\mathbf{k}})$$

$$i\dot{\alpha}_{\mathbf{k}}(t) = \left(\Omega_{\mathbf{k}} - \frac{\mathbf{p}.\mathbf{k}}{M} + \frac{\mathbf{k}}{M}.\sum_{\mathbf{k}'} \mathbf{k}'|\alpha_{\mathbf{k}'}(t)|^2\right)\alpha_{\mathbf{k}}(t) + V_{\mathbf{k}}$$

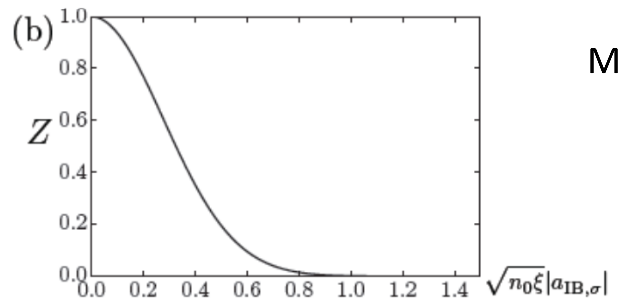
RF spectroscopy of polarons. Quasiparticle residue

Quasiparticle residue

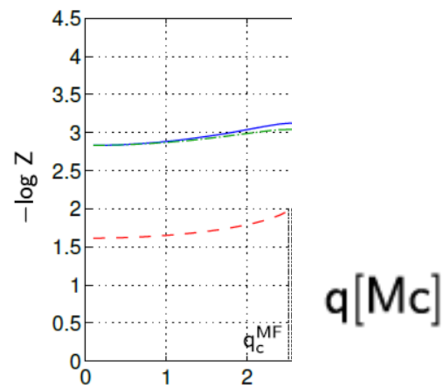
$$Z = |\langle 0 | 0_{\uparrow p} \rangle|^2 = \exp\left[-\sum_{\mathbf{k}} \frac{V_{\mathbf{k}}^2}{\left(\omega_{\mathbf{k}} + \frac{k^2}{2M} - \frac{k_{\parallel}}{M} \left(p - \Xi[\alpha_{\mathbf{k}}^{\mathrm{MF}}]\right)\right)^2} \right]$$



non-interacting
to
interacting



Mean-field

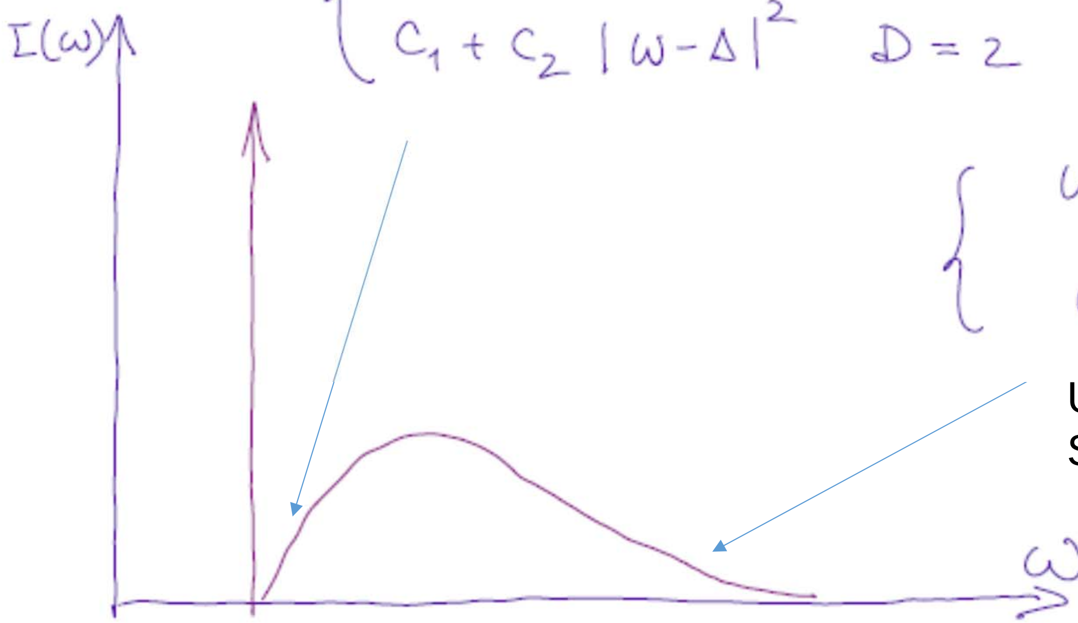


Residue vs momentum
RG vs mean-field.
α=4

Polarons in BEC. Mean-field approach to dynamics

$$\begin{cases} \omega - \Delta & D = 3 \\ c_1 + c_2 \, |\omega - \Delta|^2 & D = 2 \end{cases}$$

Long times asymptote



$$\begin{cases} \omega^{-3/2} & D = 3 \\ \omega^{-2} & D = 2 \end{cases}$$

Universal high frequency tail
S. Tan (2008)

Polarons in BEC. Mean-field approach to dynamics

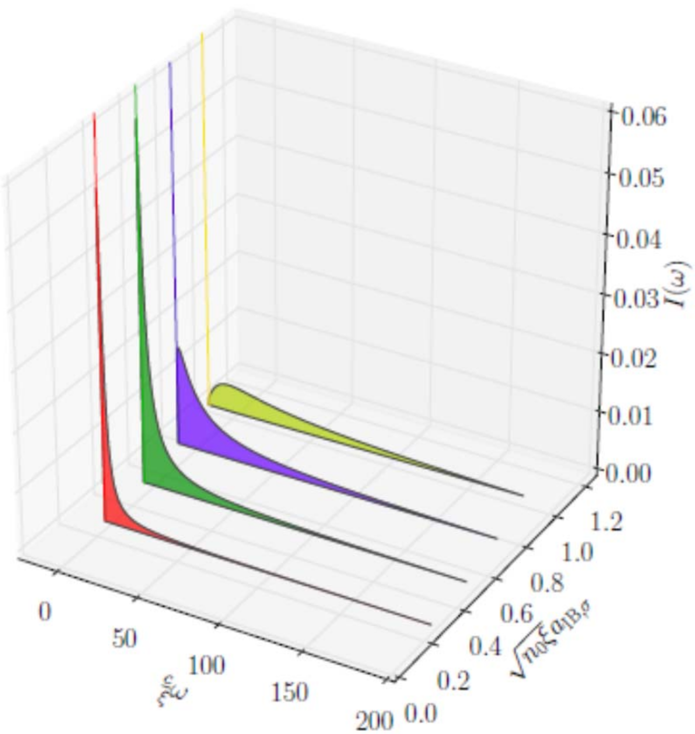
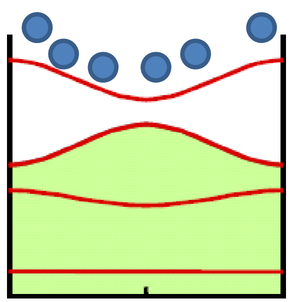


FIG. 4. RF spectra for different initial impurity interaction strengths. The quantity $a_{IB,\sigma}\sqrt{n_0\xi}$ is a dimensionless ratio between the mean free path of the impurity and the length scale over which bosons are localized (a non-interacting BEC has completely delocalized bosons). We observe that the spectral weight starts almost entirely in the coherent part of the spectrum, corresponding to a nearly free impurity, and gradually shifts to higher energies as more excitations of the BEC are generated by increasing impurity-bose interactions. The spectra presented above were obtained for an experimentally relevant mass ratio M/m of 2.5; there is a weak dependence of the spectra on mass ratio, and is not observable on the scale shown here.

Polaron dynamics in optical lattices. Bloch oscillations

Tools of atomic physics: Bloch oscillations

$$\boxed{\frac{dk}{dt} = F}$$



C. Salomon et al., PRL (1996)

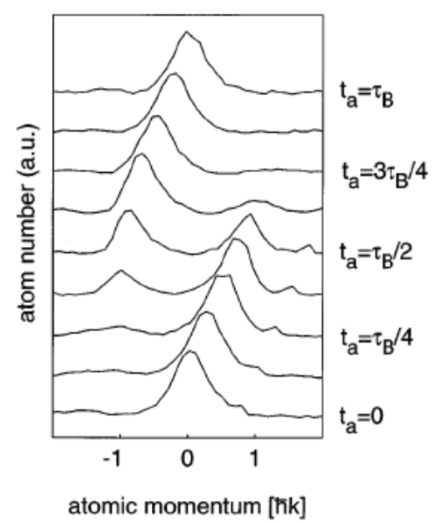
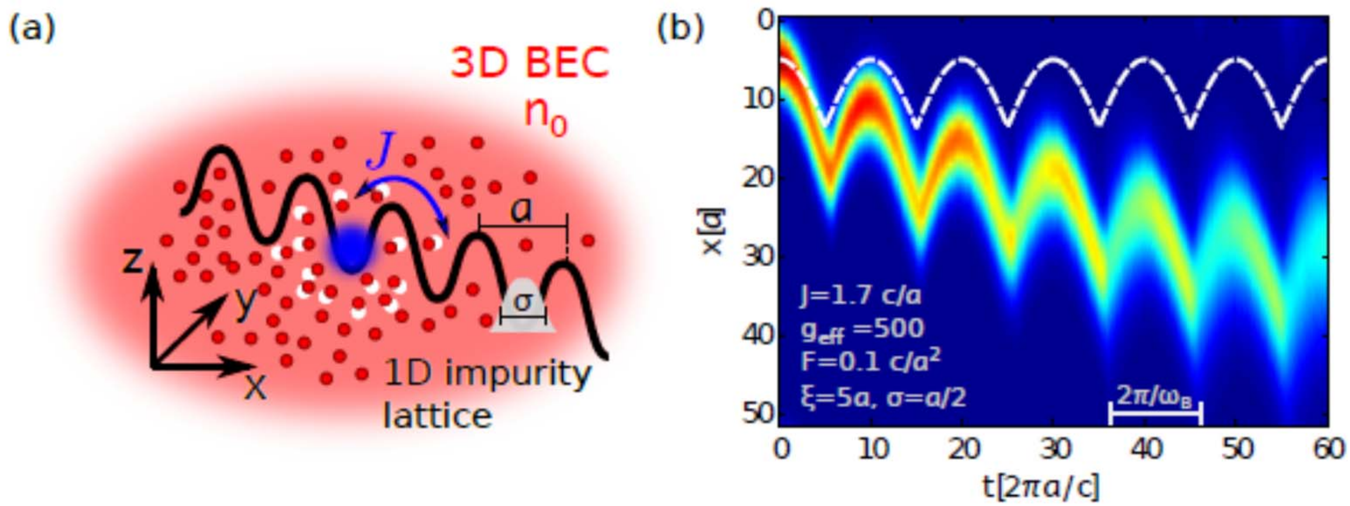


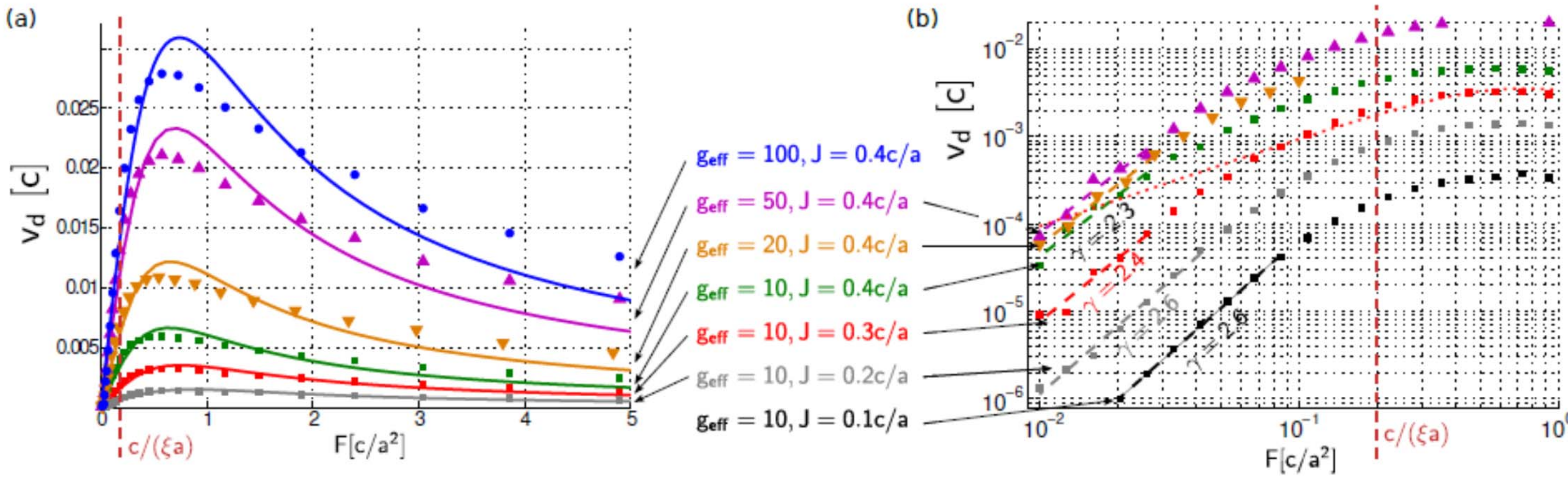
FIG. 2. Bloch oscillations of atoms: momentum distributions in the accelerated frame for equidistant values of the acceleration time t_a between $t_a = 0$ and $t_a = \tau_B = 8.2$ ms. The light potential depth is $U_0 = 2.3E_R$ and the acceleration is $a = -0.85$ m/s^2. The small peak in the right wing of the first five spectra is an artifact.

More than 30,000 oscillations in expts of Inguscio et al (2011), Nagerl et al. (2011)

Polaron dynamics in optical lattices. Bloch oscillations



BO of polarons in optical lattices. Drift = dissipation



Solid lines: fit to Esaki-Tsu relation

$$v_\mathrm{d} = 2J^*_\mathrm{eff}a\frac{\omega_\mathrm{B}\tau}{1+(\omega_\mathrm{B}\tau)^2}$$

Ohm's law
for small F

Fermi's golden rule analysis for small F

BO of polarons in optical lattices. Drift = dissipation

Fermi's golden rule for emitted energy

$$\omega \,|\, M(\omega) \,|^2 \, \nu(\omega)$$

This energy should be supplied by particle drifting downstream

$$E_{\text{Force}} \sim F \; v_{\text{drift}}$$

Drift velocity

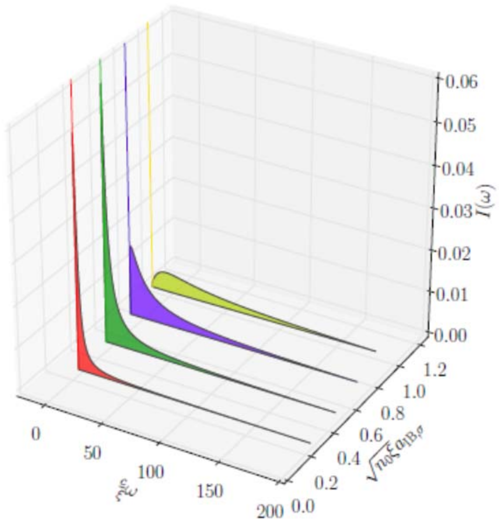
$$\boxed{v_{\text{drift}} \sim F^d}$$

No Ohm's law in d=2,3

Summary

Polarons in BEC

Froelich type effective model
equilibrium solution with LLP transformation + mean-field
RG analysis beyond mean-field, Variational approach
RF spectra



Polarons in BEC in optical lattices

renormalized dispersion
damped Bloch oscillations beyond Esaki-Tsu formula