Probing Many Body Localization with synthetic matter

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Harvard-MIT

$$ NSF, AFOSR NEW QUANTUM PHASES
MURI, DARPA OLE, ARO MURI
ATOMTRONICS, ARO MURI QUISM

Outline

Introduction to Many Body localized phases using real space RG. Loss of ergodicity

Interferometric probe of MBL

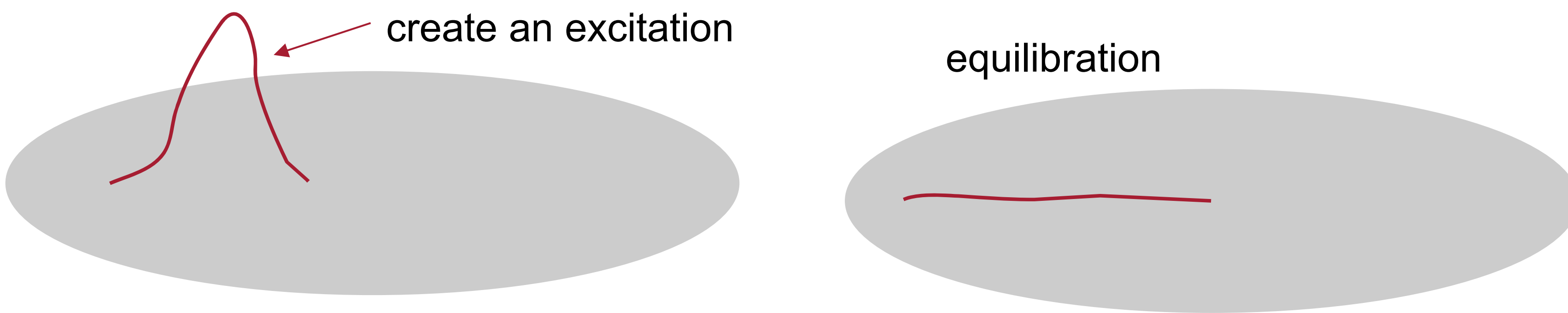
Refs: D. Pekker, et al., PRX (2014)
M. Knap, et al., PRL (2013)
M. Knap, et al., arXiv:1403.06
N. Yao, et al. arXiv:1311.7151

Many-Body Localization

Ergodicity: equivalence of temporal and ensemble averaging

Equilibration is exchange of particles, energy, ...

Thermalization means system acts as it's own bath



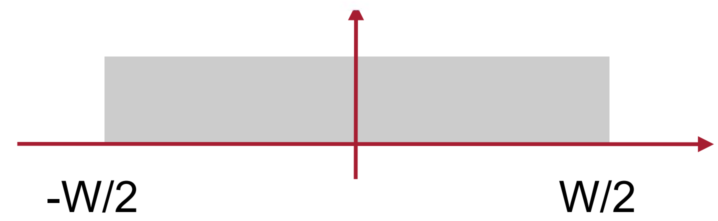
Many-Body Localized states: phases of interacting many-body systems, which do not exhibit ergodicity

Single-particle localization

Non-interacting particles in quenched disorder

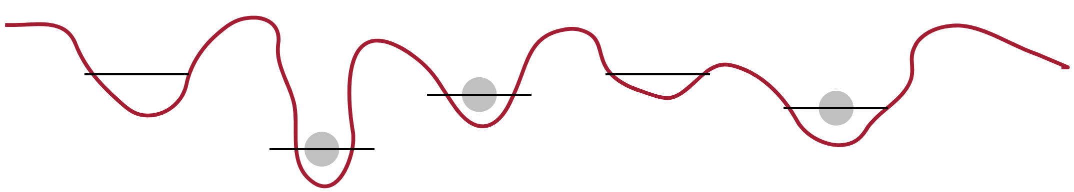
$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \sum_i V_i n_i$$

$$V_i \in [-\mathrm{W}/2, \mathrm{W}/2]$$



hopping cannot overcome disorder

P. W. Anderson, Phys. Rev. (1958)



zero conductivity

(critical strength of disorder depends on dimension)

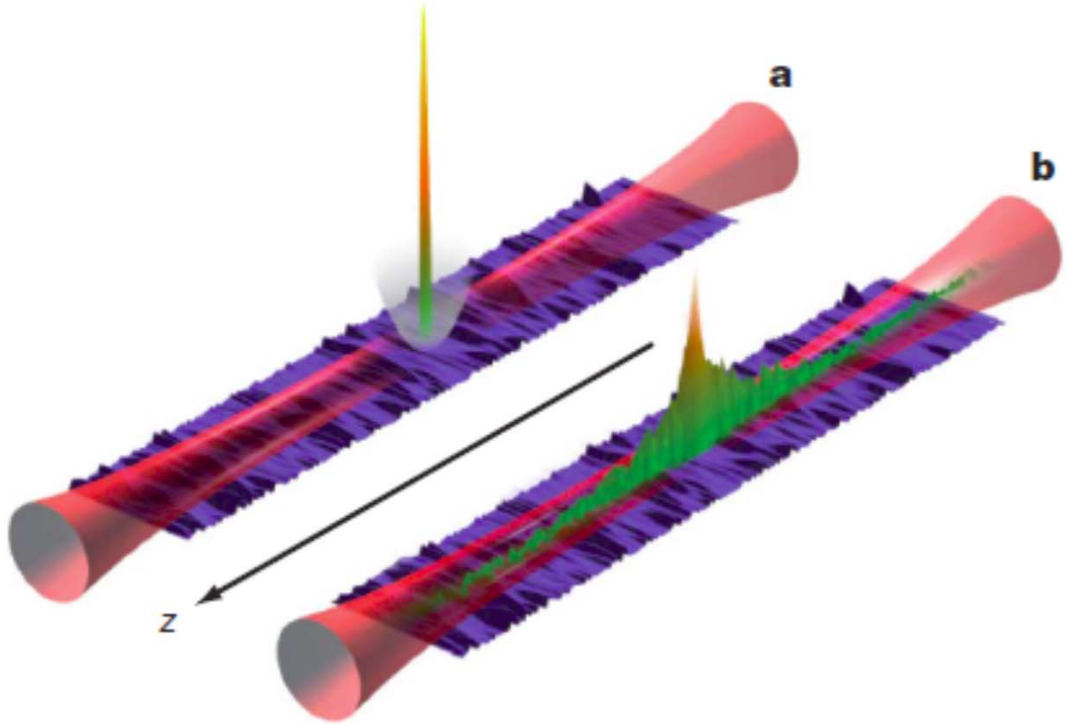
wave-functions are exponentially localized

$$\psi(r) \sim e^{-r/\xi}$$

localization length

Direct observation of Anderson localization of matter waves in a controlled disorder

Juliette Billy[1], Vincent Josse[1], Zhanchun Zuo[1], Alain Bernard[1], Ben Hambrecht[1], Pierre Lugan[1], David Clément[1], Laurent Sanchez-Palencia[1], Philippe Bouyer[1] & Alain Aspect[1]



Anderson localization of a non-interacting Bose–Einstein condensate

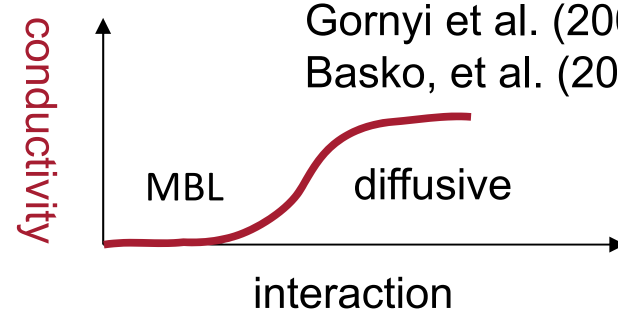
Giacomo Roati[1,2], Chiara D'Errico[1,2], Leonardo Fallani[1,2], Marco Fattori[1,2,3], Chiara Fort[1,2], Matteo Zaccanti Giovanni Modugno[1,2], Michele Modugno[1,4,5] & Massimo Inguscio[1,2]

Many-body localization (MBL)

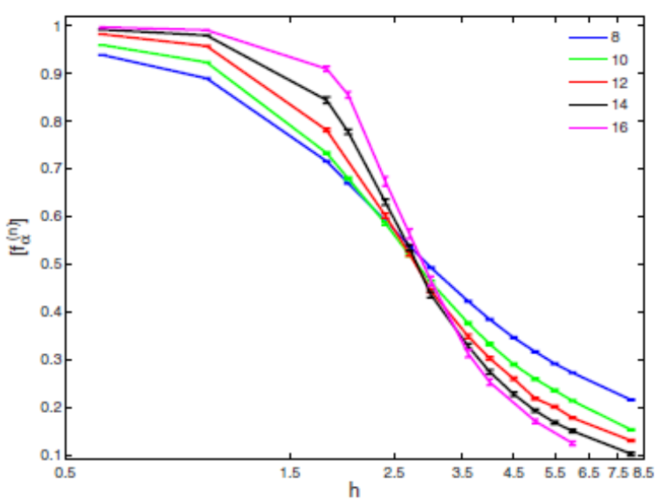
add interactions V: system can still be localized

Gornyi et al. (2005)
Basko, et al. (2006)

system does not act as its own bath (discrete local spectrum)
→ fails to thermalize



Many-body localization in spin systems in 1d



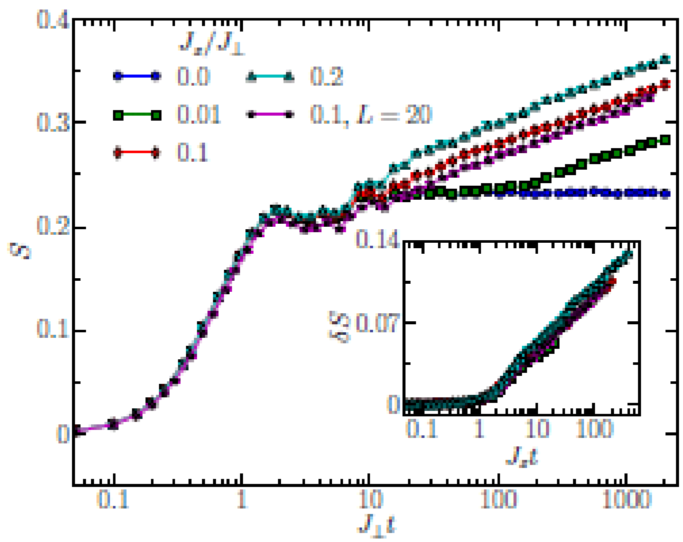
A. Pal, D. Huse (2006)

$$H = \sum_{i=1}^{L} [h_i \hat{S}_i^z + J \hat{\vec{S}}_i \cdot \hat{\vec{S}}_{i+1}]$$

The fraction of the initial spin polarization that is dynamic

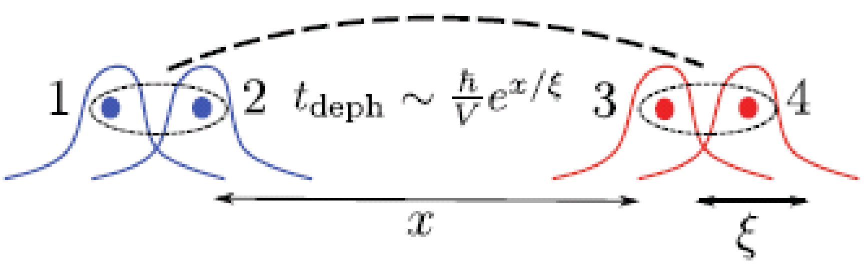
Entanglment growth in quenches with random spin XXZ model

Bardarson, Pollman, Moore, PRL 2012



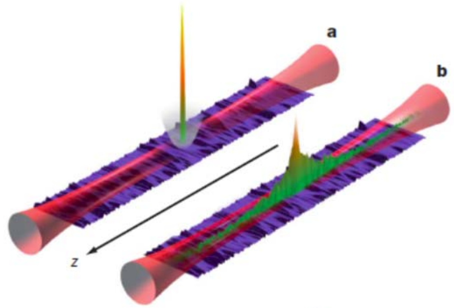
Exponentially small interaction induced corrections to energies

Vosk, Altman, PRL 2013
Serbin, Papic, Abanin, PRL 2013



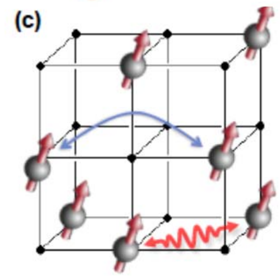
Systems with interactions and disorder

Ultracold atoms
- Aspect et al. (2008)
- Roati et al. (2008)
- DeMarco et al. (2011)



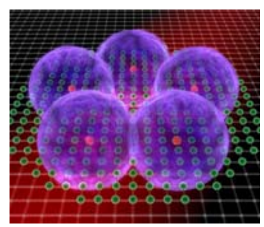
Disorder created by laser speckle

Ultracold atoms and
Polar molecules in optical lattices
- Ye et al. (2013)



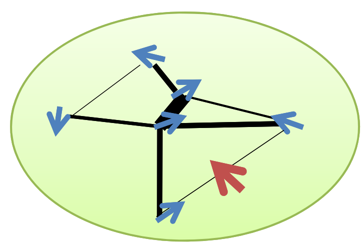
Angular momentum as spin degree of freedom

Rydberg atoms
- Pfau et al. (2008)
- Ryabtsev et al. (2010)
- Bloch et al. (2012)



Strong interactions due to large electric dipole moment

Central spin problem in q-dots
NV centers in diamond
- Marcus et al. (2004)
- Lukin et al. (2006)
- Jelezko et a. (2007)
- Awschalom et al. (2007)



Nuclear spin interactions mediated by electron spin

Many Body localized phases:

How to understand them

How to probe them in experiments

The Hilbert-glass transition: new universality of temperature-tuned many-body dynamical quantum criticality

Vosk and Altman, PRL (2012)
Pekker, Refael, Altman, Demler, Oganesyan, PRX (2014)

Contrast to weak disorder + interaction in 1d:

Giamarchi, Shulz, PRB (1988)

Hierarchical structure of excited many-body states in disordered systems: MBL and integrability

Vosk and Altman, PRL (2012)
Pekker, Refael, Altman, Demler, Oganesyan, PRX (2014)

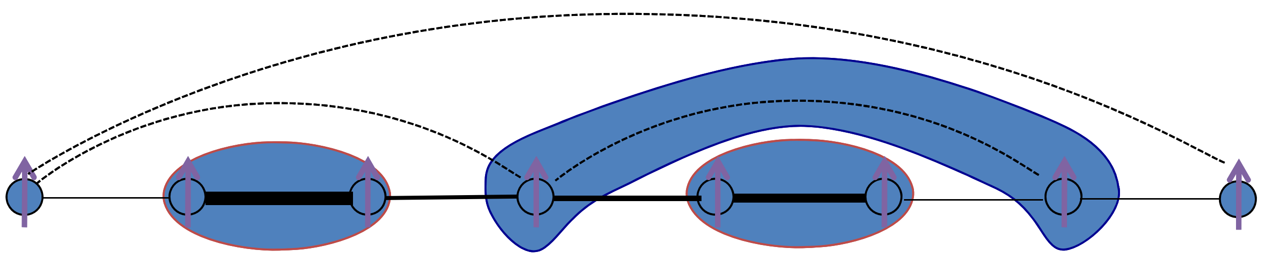
Conjecture

MBL states have hierarchical structure described by power law distributions of couplings and gaps. They are essentially integrable

Implications of the conjecture

Strongly coupled spins precess fast around each other.

They mediate coupling between outer spins

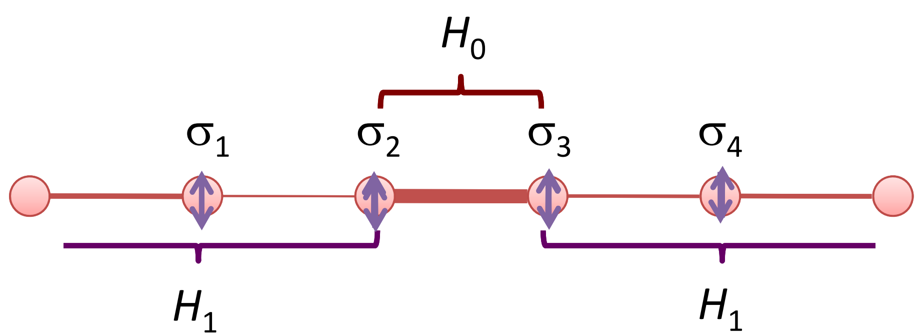


XY model: Beyond the ground state

- Real space RG
- Consider the 1D XY chain (free fermions with random hopping)

$$H = \sum_i J_i (\sigma_i^+ \sigma_{i+1}^- + \sigma_i^- \sigma_{i+1}^+)$$

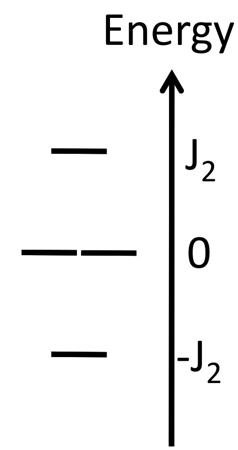
- RG decimation step: perturbation theory



$c_{23} = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

$|\uparrow\uparrow\rangle \qquad |\downarrow\downarrow\rangle$

$d_{23} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

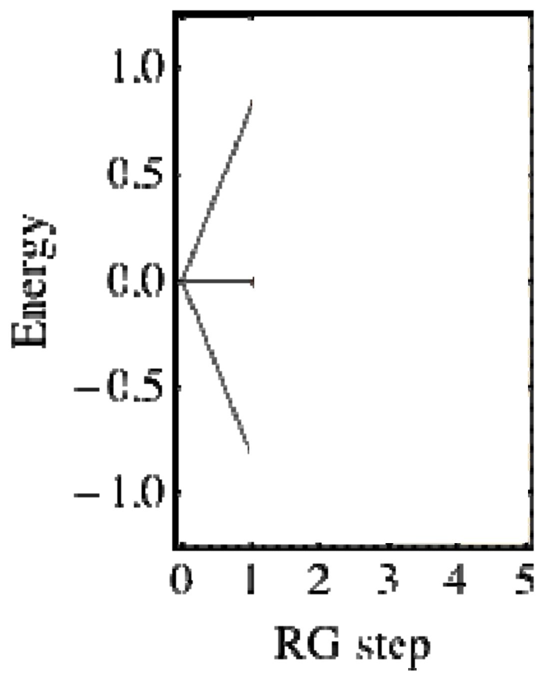


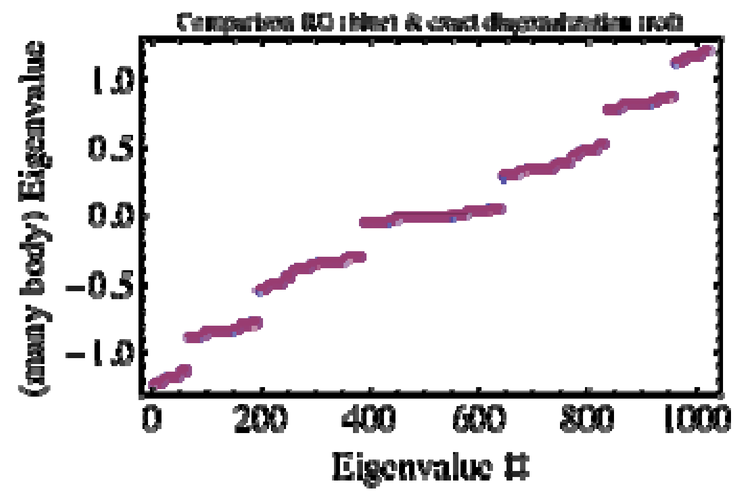
Effective coupling across cluster:

$$H^{(2)} = \frac{J_1 J_3}{J_2}(\sigma_1^+ \sigma_4^- + \sigma_1^- \sigma_4^+)\sigma_2^z \sigma_3^z + \frac{J_1^2 + J_3^2}{2 J_2}(P|d_{23}\rangle - P|c_{23}\rangle)$$

Results of the RG procedure

- Construct spectrum via choice of branch





Adding interactions: hJJ' model

- Twist on the random transverse field Ising model

$$H = \sum_i h_i \sigma_i^z - J_i \sigma_i^x \sigma_{i+1}^x - J_i' \sigma_i^z \sigma_{i+1}^z$$

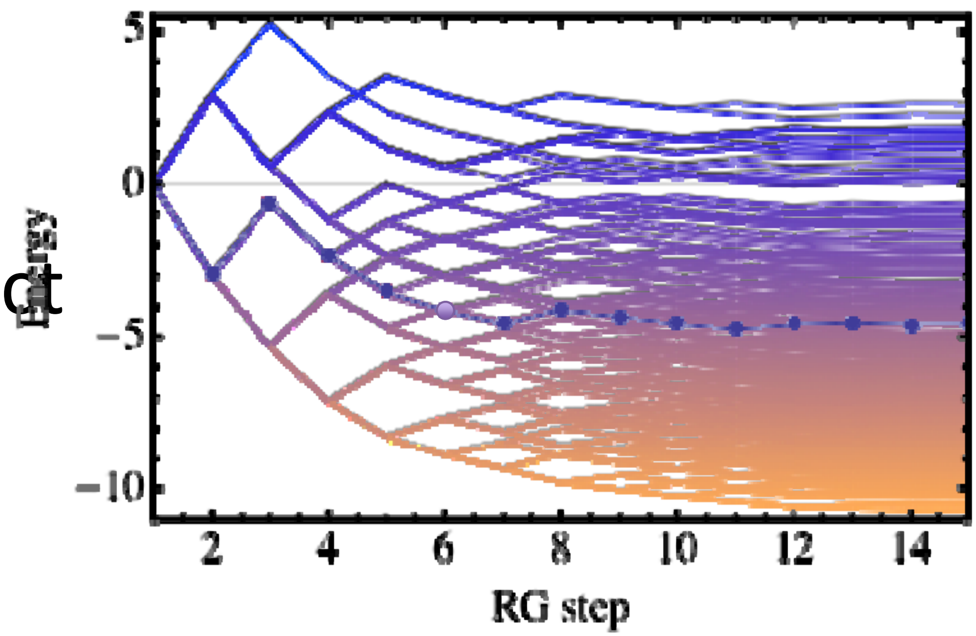
- Without *J*': solved by D. Fisher (equivalent to free fermions)
  - transition between *h*-dominated phase and a *J*-dominated phase
- With *J*': model becomes interacting
  - above transition becomes temperature tuned

Sampling the tree using Monte Carlo

- Start with a branch
- Propose a new branch
- Metropolis accept/reject

- Example of sampling:
  - finite freq. conductivity
  - run RG to ω scale

Tree for hJJ’ model

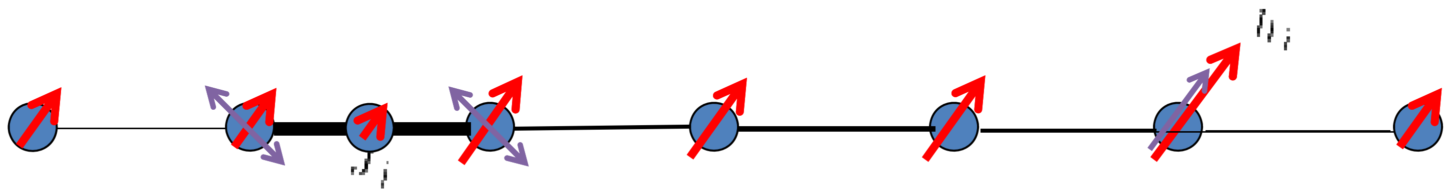


Random transverse field Ising model

- D. Fisher 1992

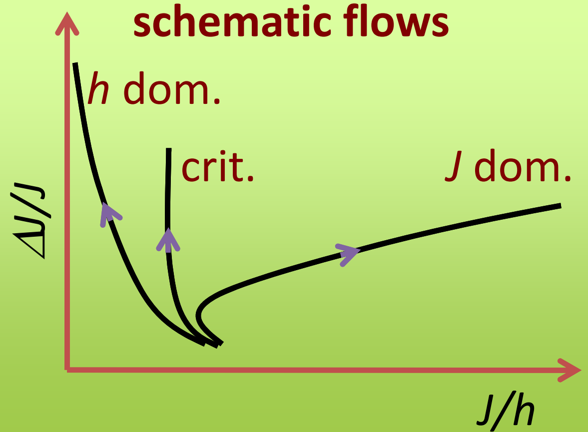
free fermions:
excited states identical except signs

$$H = \sum_i h_i \sigma_i^z - J_i \sigma_i^x \sigma_{i+1}^x$$

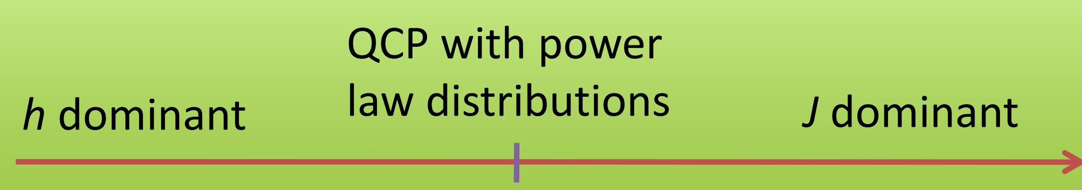


$$h_{\mathrm{eff}} = \frac{h_i h_{i+1}}{J_i} \qquad J_{\mathrm{eff}} = \frac{J_{i-1} J_i}{h_i}$$

schematic flows

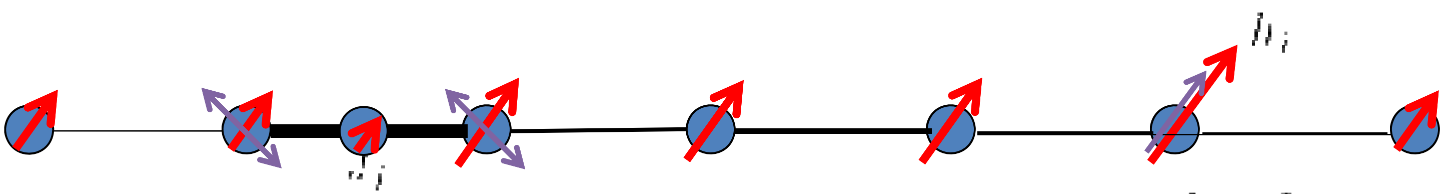


schematic phase diagram



Effect of J'

$$H = \sum_i h_i \sigma_i^z - J_i \sigma_i^x \sigma_{i+1}^x - J_i' \sigma_i^z \sigma_{i+1}^z$$



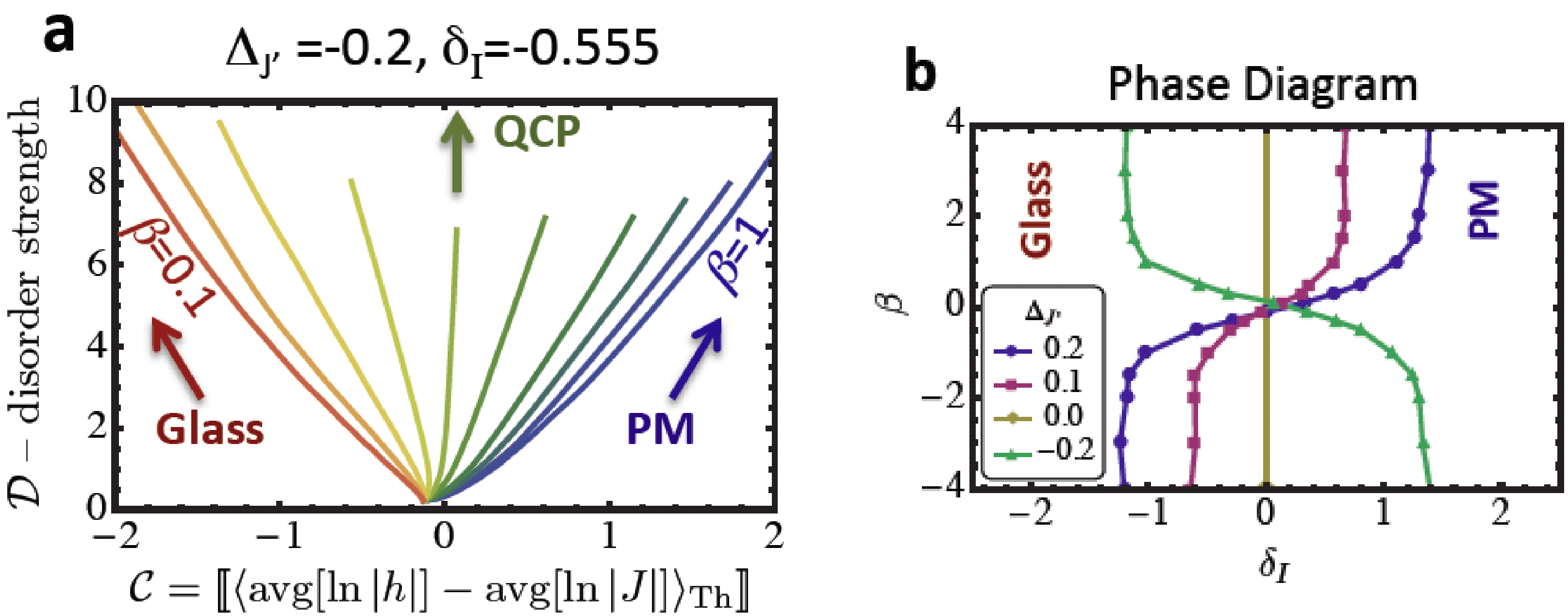
$$h_{\mathrm{eff}} = J_j' + \frac{h_j h_{j+1}}{J_j}$$

$$J_{\mathrm{eff}} = \frac{J_{i-1} J_i}{h_i}$$

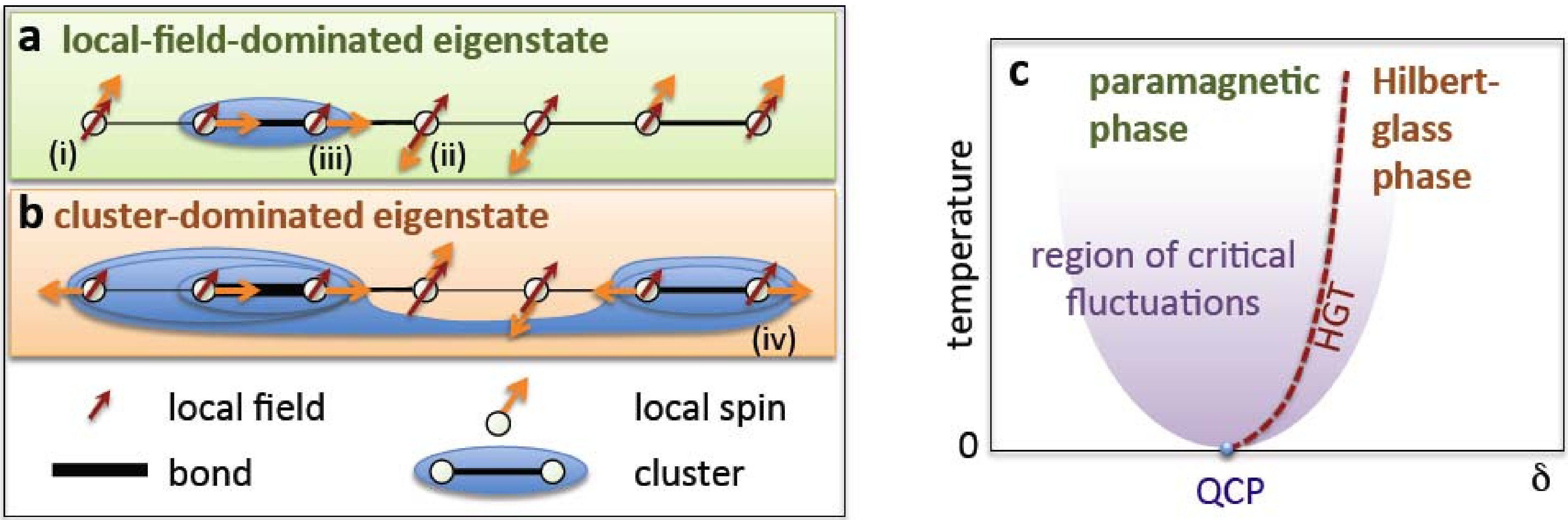
$$J_{\mathrm{eff}}' = 0$$

RG flows of JJ' model

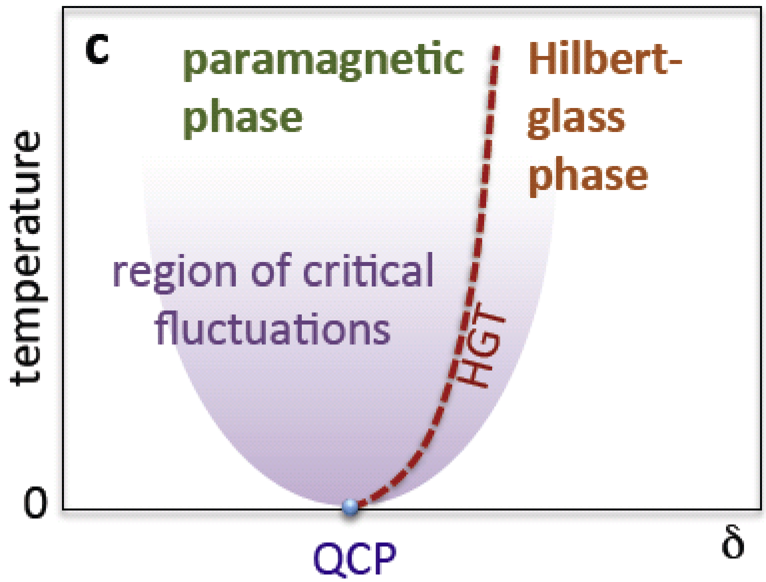
$$\delta = \mathcal{C}/\mathcal{D}$$

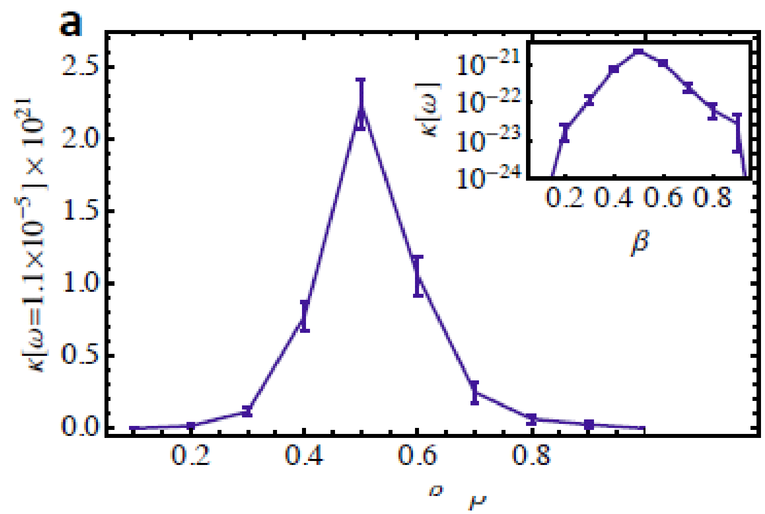


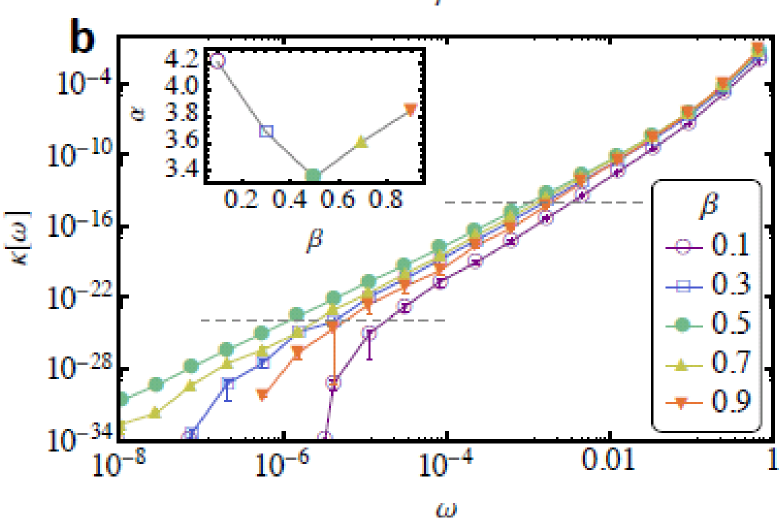
Phase diagram of JJ' model



Manifestations of Hilbert Glass transition







$$\kappa(\omega) \;\sim\; \omega^{\alpha}$$

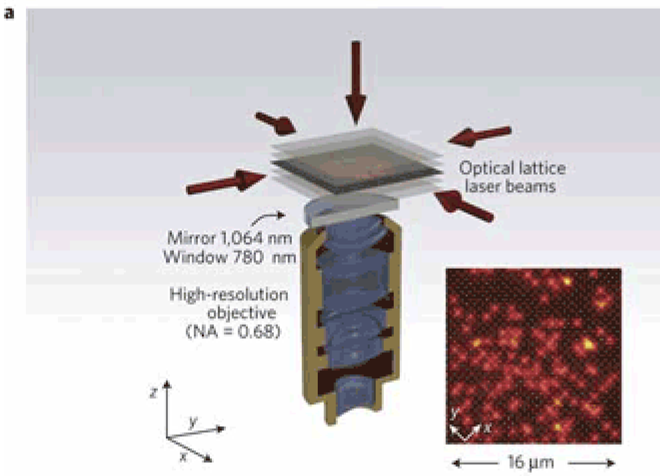
$$\alpha \;\sim\; 3 + \;\mathrm{const} \times |\beta - \beta_c|$$

Probing Many-Body Localization with many-body Ramsey interferometry

M. Knap, et al., PRL (2013)
M. Knap, M. Serbyn, et al., arXiv:1403.0693

Probing spin dynamics in synthetic matter

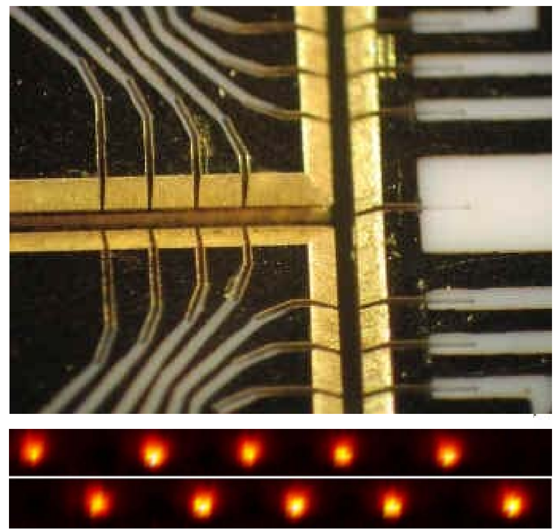
Ultracold atoms



Harvard, MPQ, …

- **Bosons in random potential, spin models**

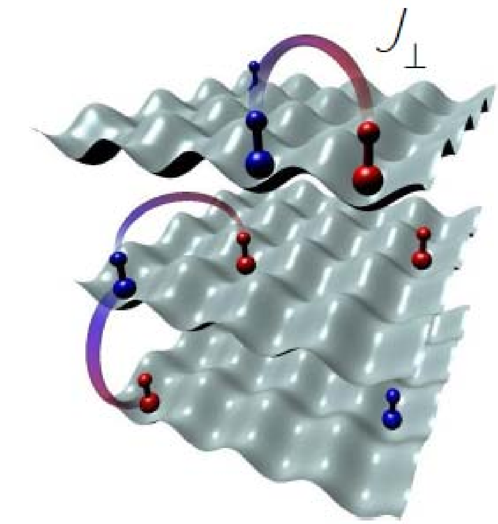
Trapped ions



JQI group

- **LR transverse field Ising model**
- interactions mediated by phonons
- e.g. ^{171}Yb

Polar molecules



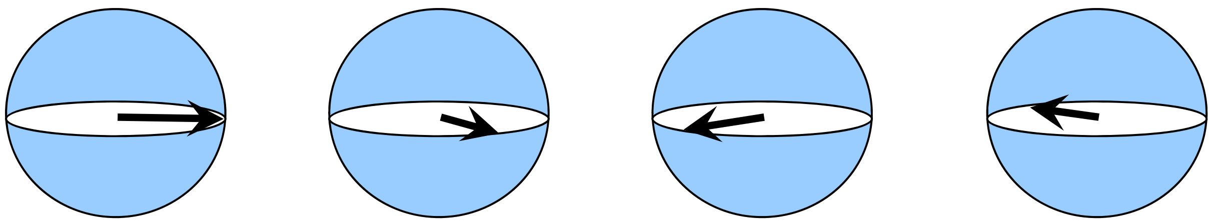
JILA group

- **LR XX model**
- dipolar interactions
- e.g. KRb

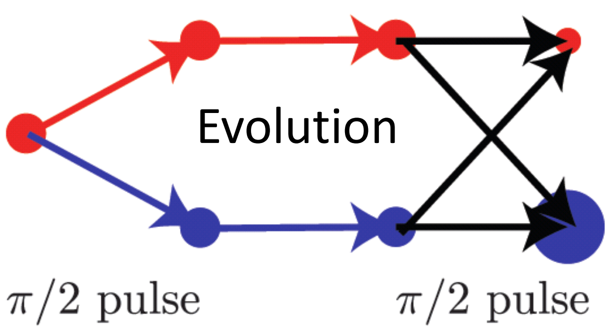
Tools of atomic physics: Ramsey interference

$\frac{\pi}{2}$ pulse $\quad |\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}|\downarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle$

Evolution $\quad |\Psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_\downarrow t} |\downarrow\rangle + \frac{1}{\sqrt{2}} e^{-i\mathcal{H}_\downarrow t} |\uparrow\rangle$



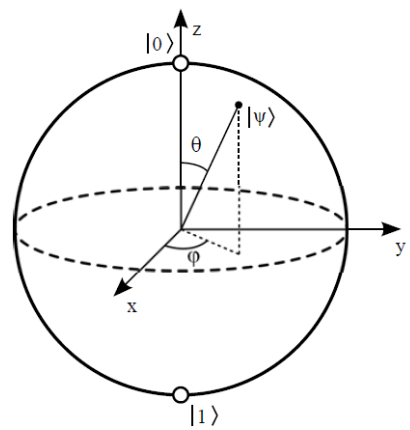
$\frac{\pi}{2}$ pulse + measurement ot S_z gives relative phase accumulated by the two spin components



Used for atomic clocks, gravitometers, accelerometers, magnetic field measurements

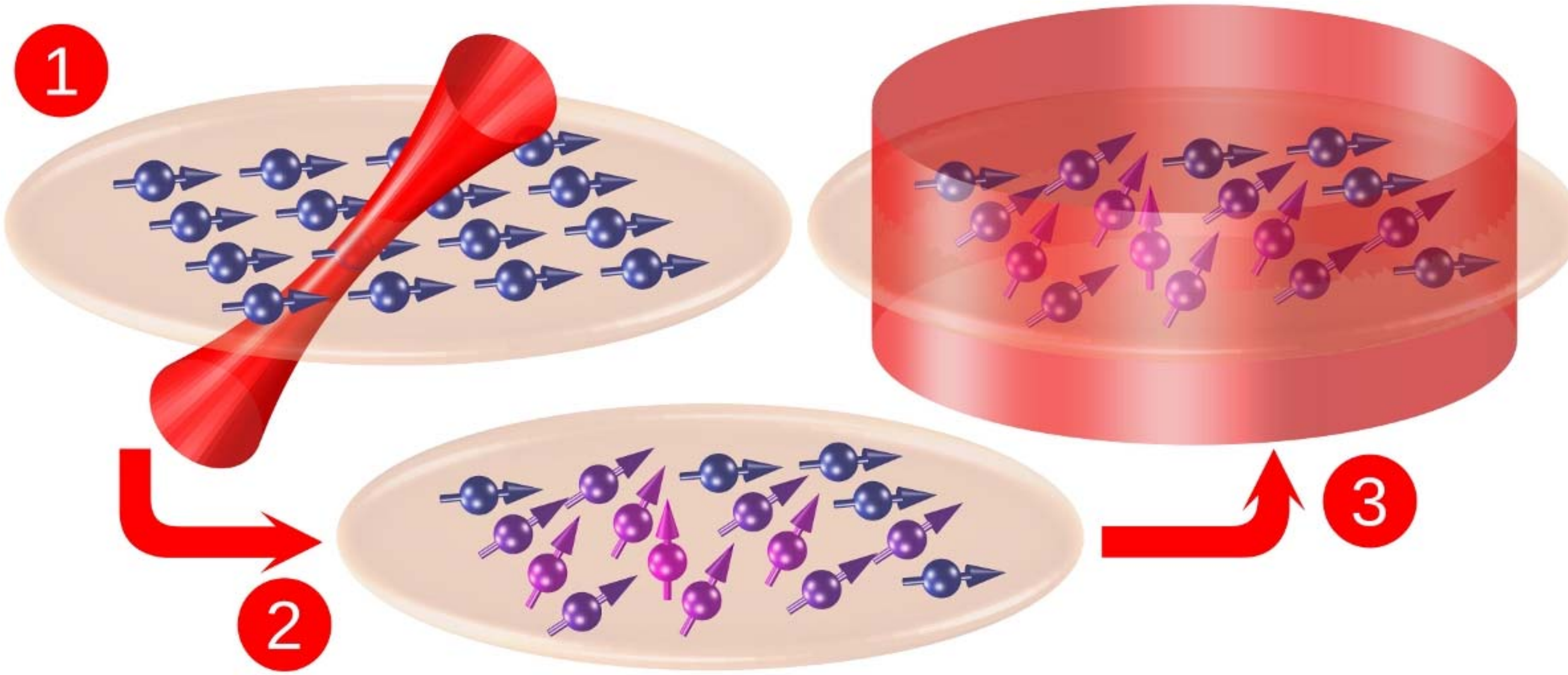
Spin rotations

$$R_j(\theta, \phi) = \hat{1} \cos \frac{\theta}{2} + i(\sigma_j^x \cos \phi - \sigma_j^y \sin \phi) \sin \frac{\theta}{2}$$



π/2 pulse: $R_j(\frac{\pi}{2}, \phi) = \frac{1}{\sqrt{2}} (1 + e^{i\phi} \sigma_j^+ + e^{-i\phi} \sigma_j^-)$

Many-body spin Ramsey protocol



Measures the usual retarded spin correlation function

$$\frac{\theta(t)}{Z} \sum_n e^{-\beta E_n} \langle n \,|\, S_i^x(0)\, S_i^x(t) \,-\, S_i^x(0)\, S_i^x(t) \,|\, n \rangle$$

Spin correlation function as quantum quench

$$\langle n \,|\, S_i^x(t)\, S_i^x(0) \,|\, n \rangle = \langle n \,|\, e^{i\mathcal{H}t}\, S_i^x(0)\, e^{-i\mathcal{H}t}\, S_i^x(0) \,|\, n \rangle$$

$$= \langle n \,|\, e^{i\mathcal{H}t}\, e^{-i\tilde{\mathcal{H}}_i t} \,|\, n \rangle$$

$\tilde{\mathcal{H}}_i$ differs from $\mathcal{H}$ by

$$S_i^y \to -S_i^y$$
$$S_i^z \to -S_i^z$$

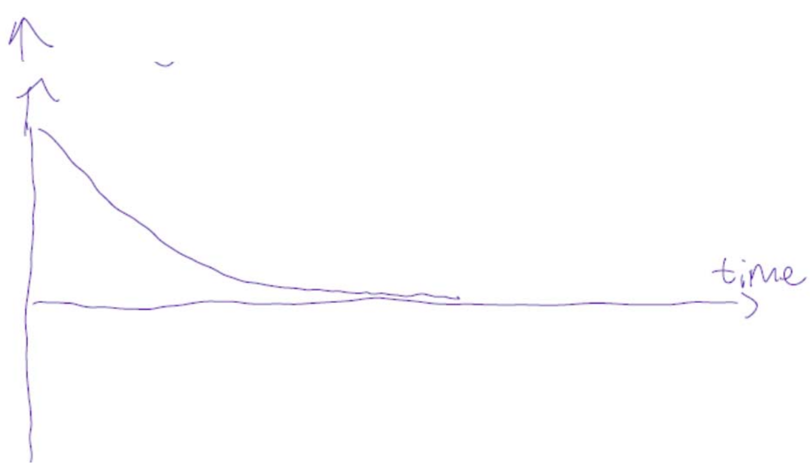
Spin correlation function as quantum quench

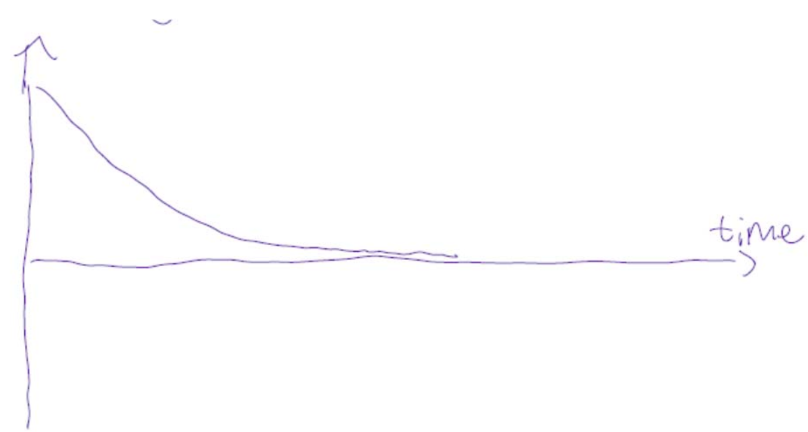
$$\langle n | e^{i\mathcal{H}t} e^{-i\tilde{\mathcal{H}}_i t} | n \rangle$$

In a localized phase, local quench affects only a few excitations. For each eigenstate expect non-decaying oscillations

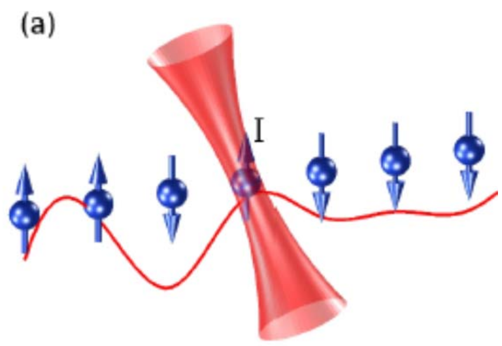
After averaging over thermal ensemble (and/or disorder realizization) find decay

In a delocalized phase (diffusive regime), local quench affects all excitations. Expect decay akin orthogonality catastrophe

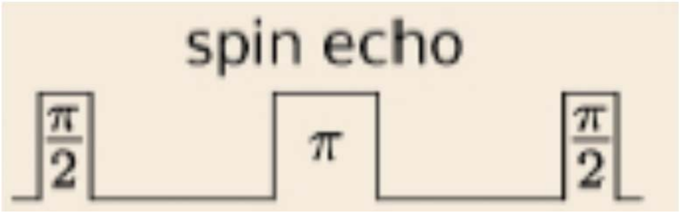




Ramsey + spin echo



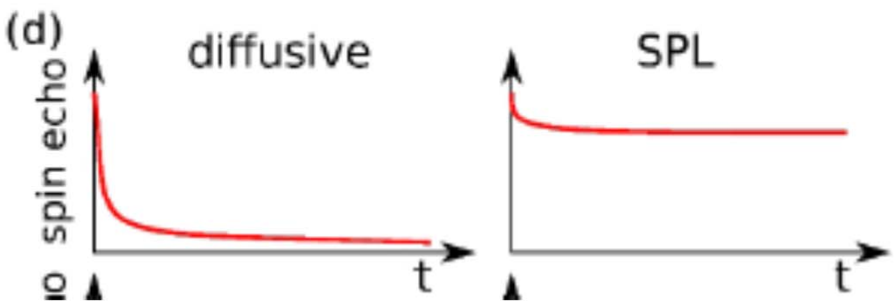
M. Knap, S. Gopalakrishnan, M. Serbyn, et al.



“Cartoon” model of the localized phase $\mathcal{H}_{\mathrm{SPL}} = \sum_i h_i^z S_i^z$

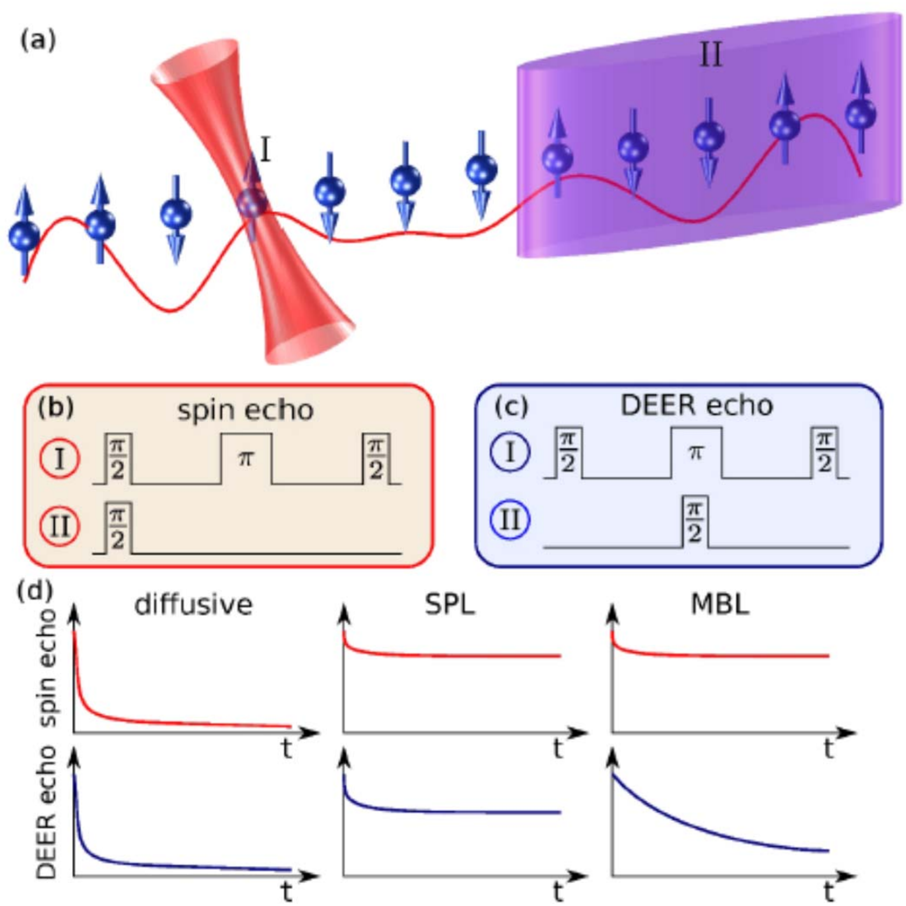
Spin echo $h_i^z \to -h_i^z$

$$|\Psi_{zi}(t)\rangle = e^{i\mathcal{H}_{zi}\frac{t}{2}} e^{-i\mathcal{H}_{zi}\frac{t}{2}} |\Psi_{zi}(0)\rangle$$



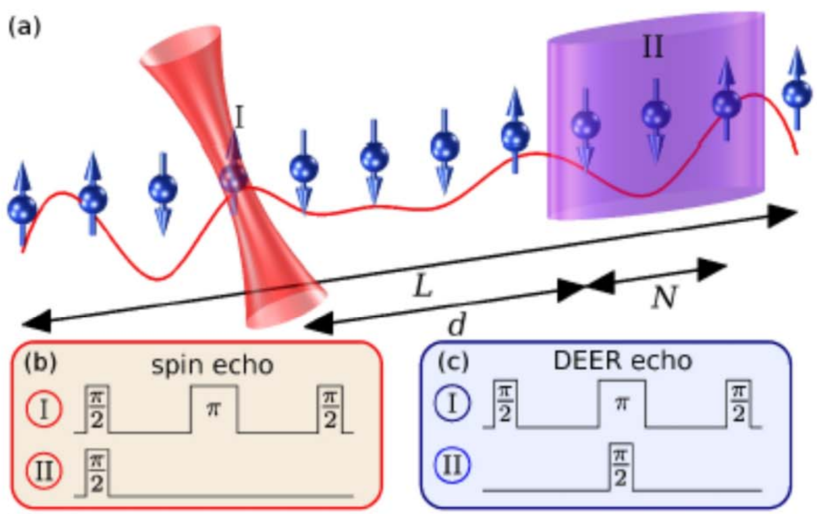
Double Electron-Electron Resonance Ramsey sequence

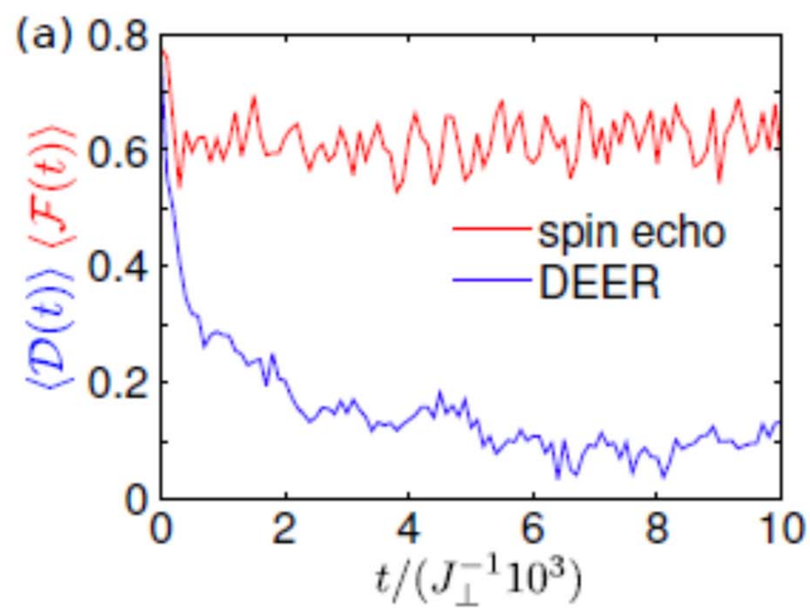
M. Knap, S. Gopalakrishnan, M. Serbyn, et al.



Double Electron-Electron Resonance
Ramsey sequence

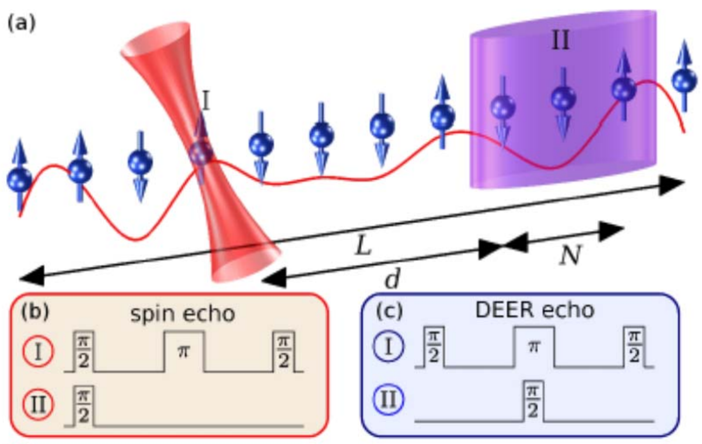
$$\hat{H} = \frac{J_\perp}{2}\sum_{\langle ij\rangle}(\hat{S}_i^+\hat{S}_j^- + \hat{S}_j^+\hat{S}_i^-) + J_z\sum_{\langle ij\rangle}\hat{S}_i^z\hat{S}_j^z + \sum_i h_i\hat{S}_i^z$$

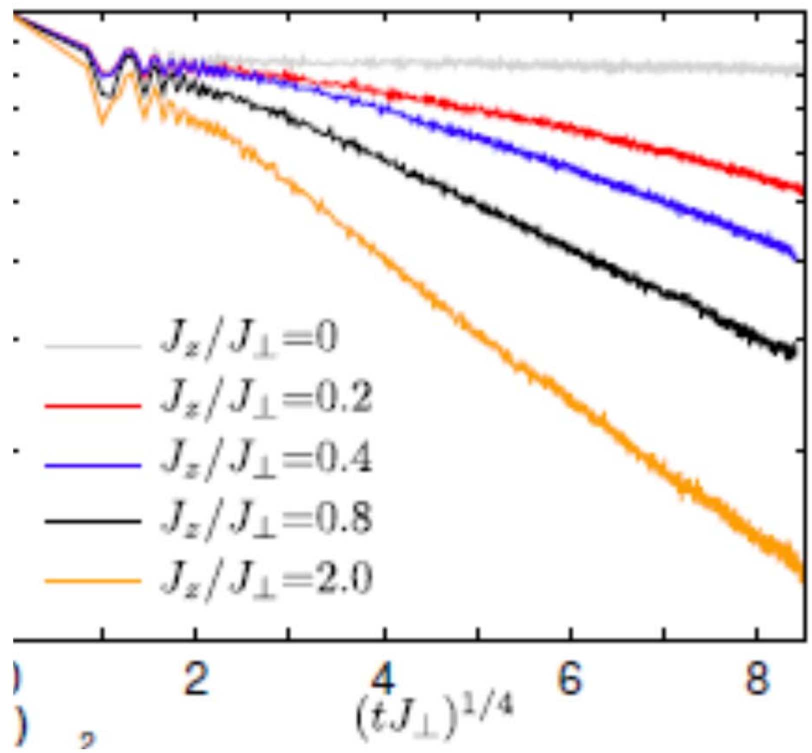




single realization
thermal averaging over 50 eigenstates

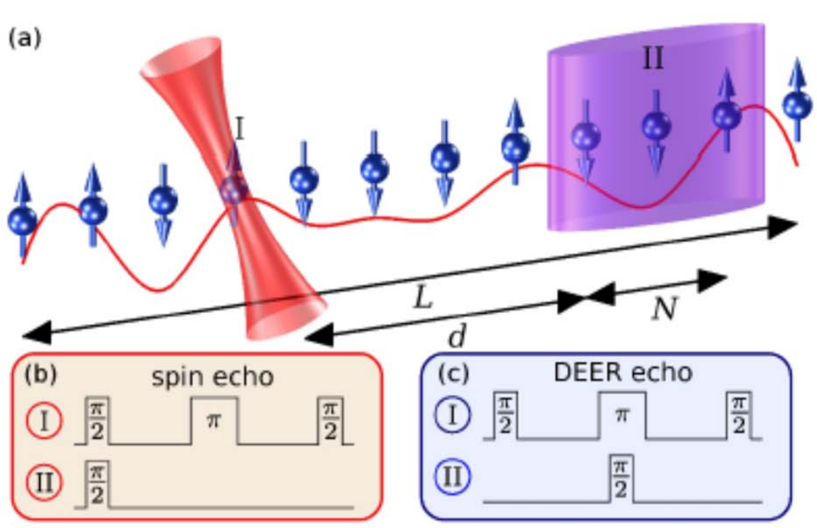
Double Electron-Electron Resonance Ramsey sequence

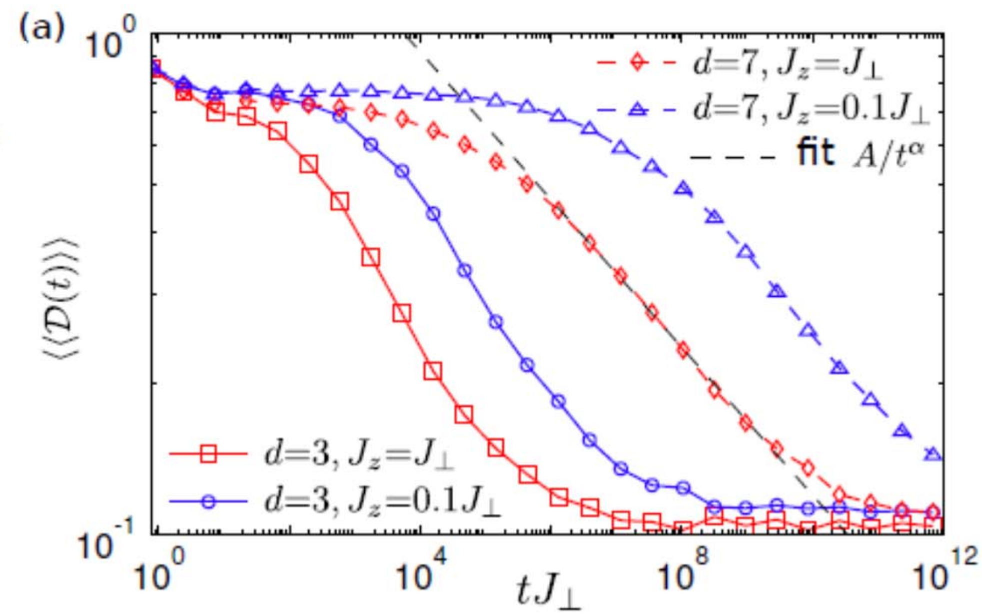




Double Electron-Electron Resonance Ramsey sequence

Power law decay of DEER signal with time





thermal and ensemble averaging

MBL as integrable model

$$\hat{H} = \sum_i \tilde{h}_i \tau_i^z + \sum_{ij} J_{ij} \tau_i^z \tau_j^z$$

"Cartoon" model of MBL phase

Interaction strength decays as $J_{\mathrm{I}j} \propto \exp(-|j-\mathrm{I}|/\xi)$

$$\mathcal{D}(t) \equiv \langle \psi(t) | \hat{\tau}_{\mathrm{I}}^z | \psi(t) \rangle = \frac{1}{2^N} \mathrm{Re} \prod_{j \in \mathrm{II}} \{1 + e^{2iJ_{\mathrm{I}j}\tau_j t}\}$$

For a given time *t* we can separate fast modes $J_{\mathrm{I}j} t \gg 1$

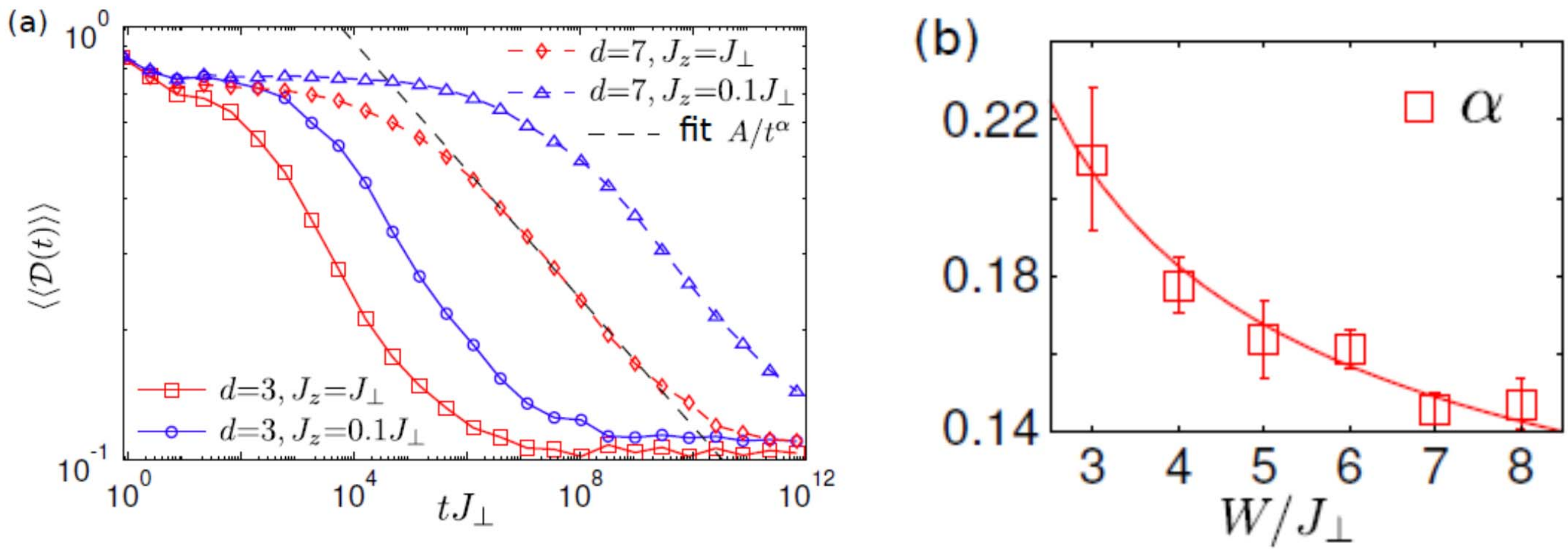
and slow modes $J_{\mathrm{I}j} t \ll 1$

$$\mathcal{D}(t) = \bar{\mathcal{D}}(t) + \mathcal{D}_{\mathrm{osc}}(t), \qquad \bar{\mathcal{D}}(t) = 1/2^{N_{\mathrm{fast}}(t)}$$

$$N_{\mathrm{fast}}(t) \sim \xi \log t$$

$$\boxed{\bar{\mathcal{D}}(t) \sim \frac{1}{(1 + t/t_0)^{\alpha}} \qquad \alpha = \xi \ln 2}$$

Double Electron-Electron Resonance Ramsey sequence



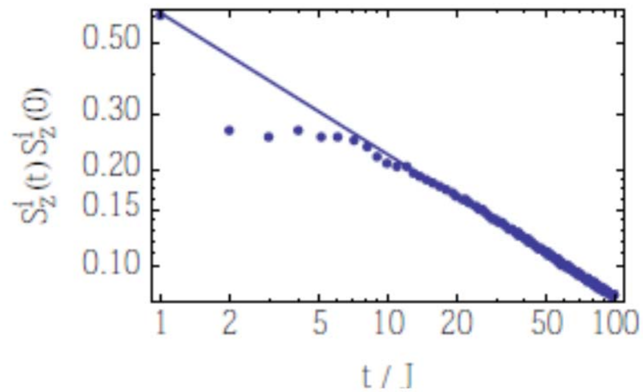
Anomalous diffusion/Griffiths phase before MBL

K. Agarwal, unpublished

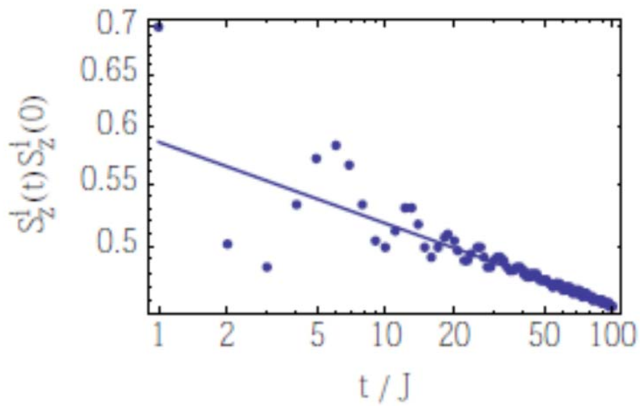
Return probability

$$\langle S_z^i(t) S_z^i(0) \rangle$$

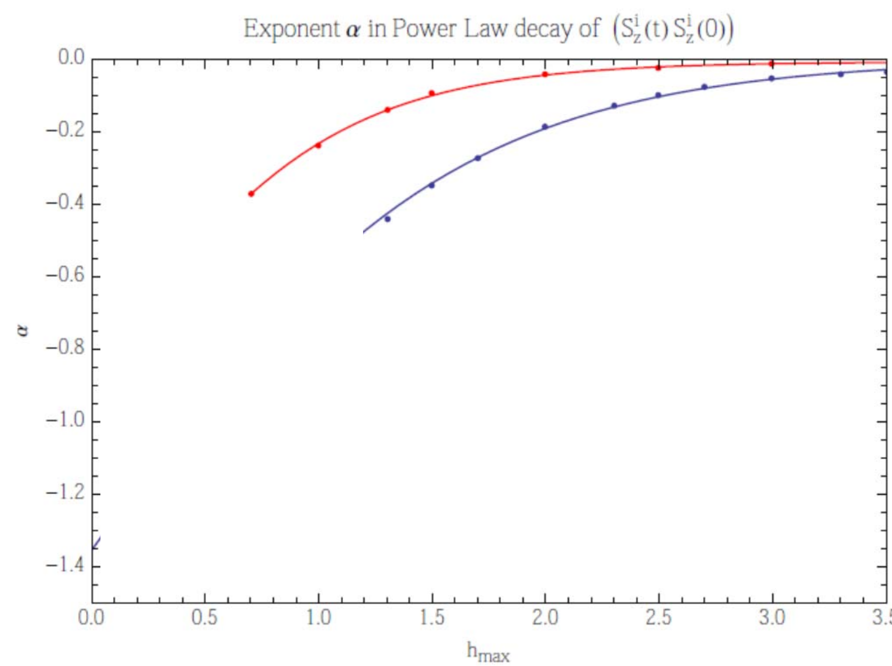
$h_{max} = 1.3$



$h_{max} = 3.$



$$H = \sum_{i=1}^{L} [h_i \hat{S}_i^z + J \hat{\vec{S}}_i \cdot \hat{\vec{S}}_{i+1}]$$

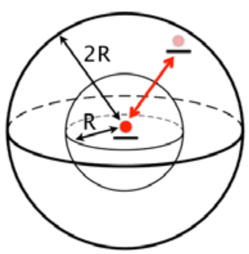


Power laws in optical conductivity, long tails in resistivity, …

Outlook

Localization with Long-Range Hops/Interactions Burin pairs

$$H = \sum_i \mu_i n_i + \sum_{ij} \frac{t_{ij}}{R_{ij}^{\alpha}} a_i^{\dagger} a_j$$

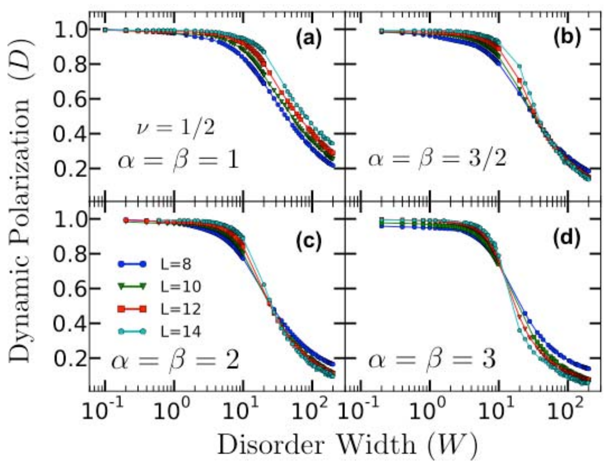


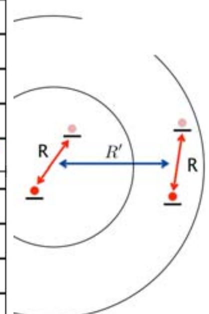
d-dimension, a- exponent of hopping

For d ≥ α, there is no localization

P. W. Anderson, Phys. Rev. (1958)

$$H = \sum_i \mu_i n_i + \sum_{ij} \frac{t_{ij}}{R_{ij}^{\alpha}} a_i^{\dagger} a_j + \sum_{ij} \frac{V_{ij}}{R_{ij}^{\beta}} n_i n_j$$





With interactions localization disappears for 2d ≥ α

Consequencies

N. Yao, C. Laumann, S. Gopalakrishnan, M. Knap et al. arXiv:1311.7151

1) Dipoles in 2D are delocalized
2) Experimental – tunable power-laws

Summary

- Interesting realizations of Many Body Localized phases can be probed with "synthetic" matter (cold atoms, polar molecules, Rydberg atoms, NV centers)

- "Smoking gun" signature of MBL can be obtained from spine echo/DEER experiments