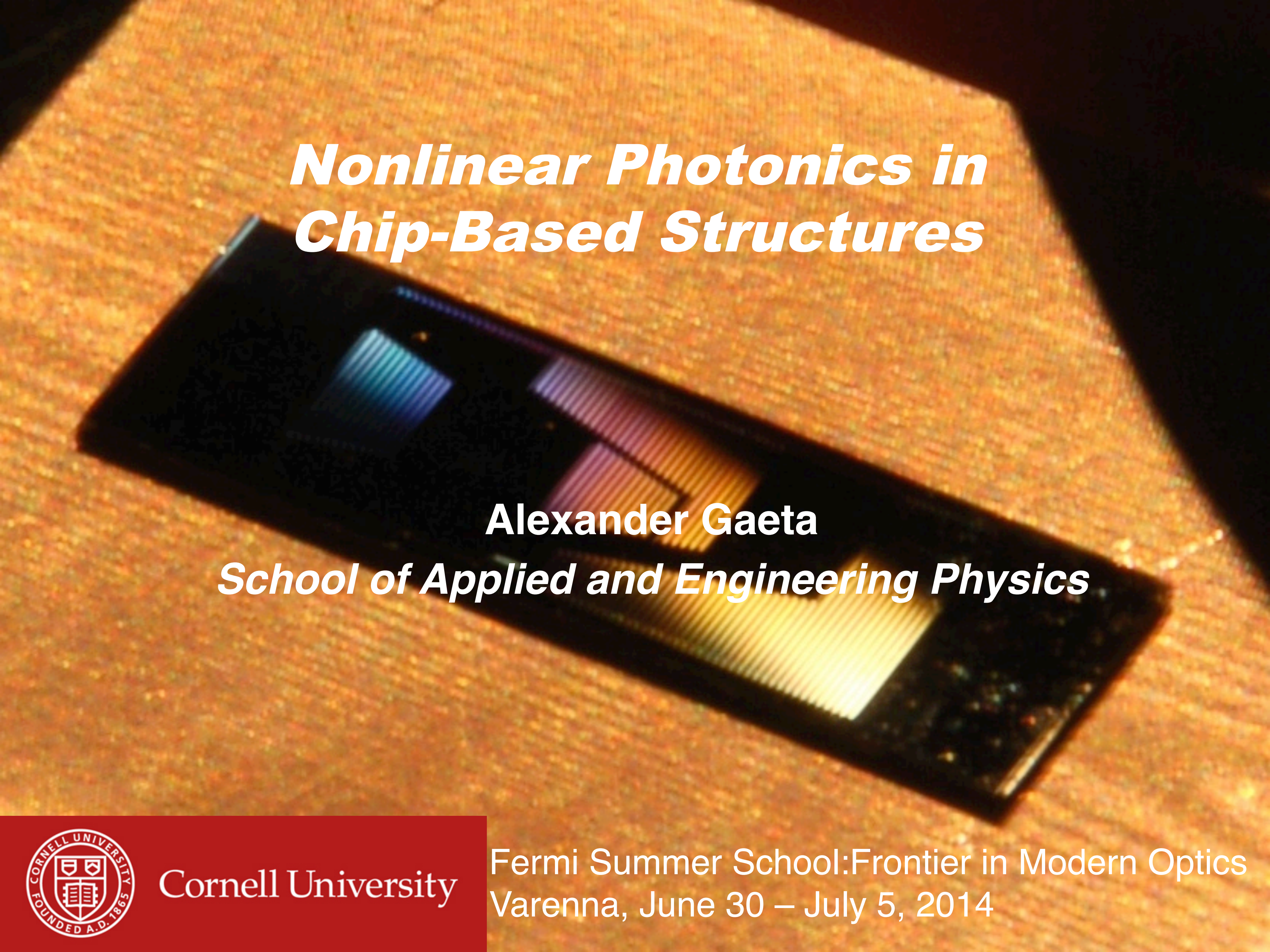


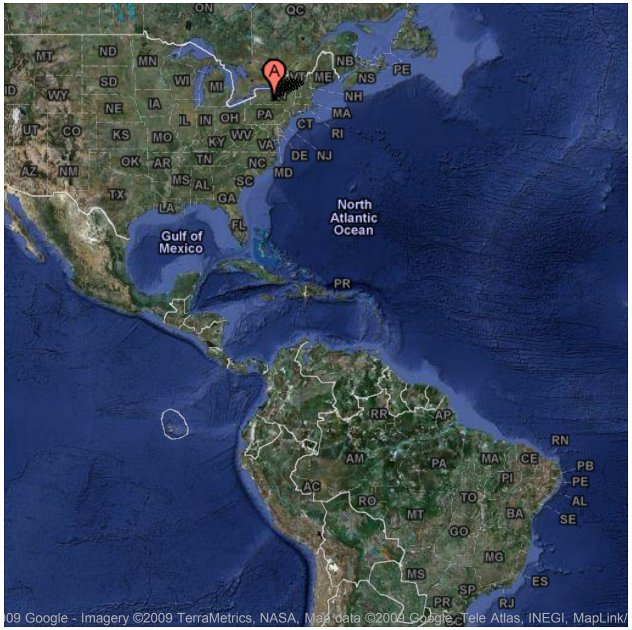
Nonlinear Photonics in Chip-Based Structures

Alexander Gaeta

School of Applied and Engineering Physics

Cornell University

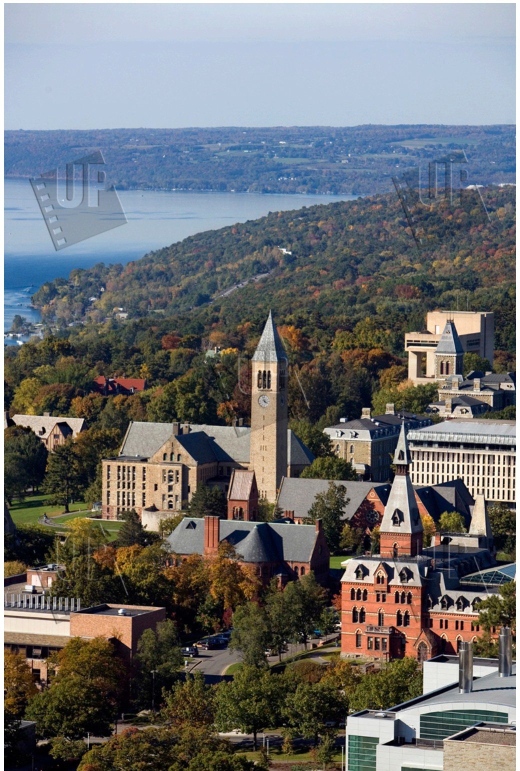
Fermi Summer School:Frontier in Modern Optics
Varenna, June 30 – July 5, 2014



Cornell University

Undergraduate: 14,000

Graduate: 6,000



Outline: Nonlinear Photonics in Chip-Based Structures

- Brief review of nonlinear optics
- Nonlinear processes in nanowaveguides
- Four-wave mixing (FWM) in Si nanowaveguides
    - ✧ Dispersion engineering
    - ✧ Ultra-broadband wavelength conversion
    - ✧ Application: correlated photons for quantum information
- Optical parametric oscillators
    - ✦ Broad-band frequency combs, ultrashort-pulse generation

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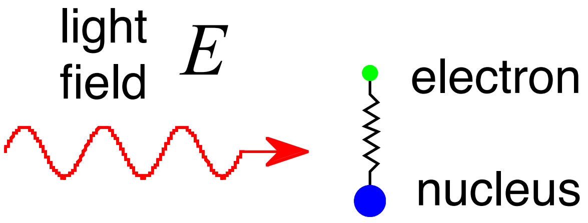
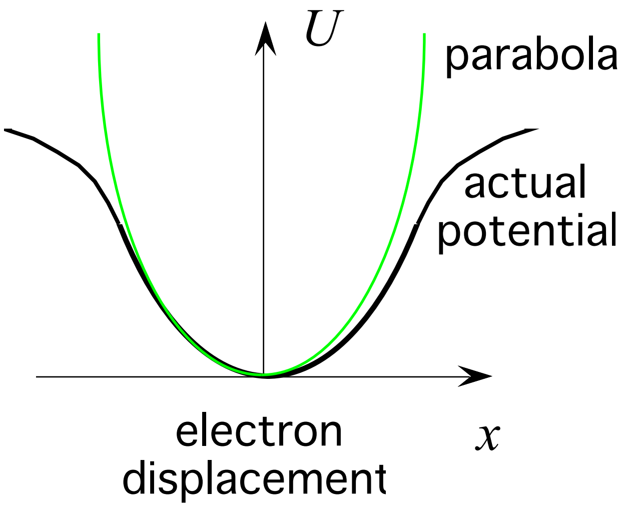
Nonlinear Optics: Interaction of Laser Light with Matter

- Microscopic picture: Lorentz-atom model.





restoring force: $F_{res} = -kx + ax^2 + bx^3 + \ldots$

- Macroscopic picture: nonlinear dependence on applied field.

polarization of the medium

$$P = \chi^{(1)}E + \chi^{\cancel{(2)}}E^2 + \chi^{(3)}E^3 + \ldots$$

linear susceptibility

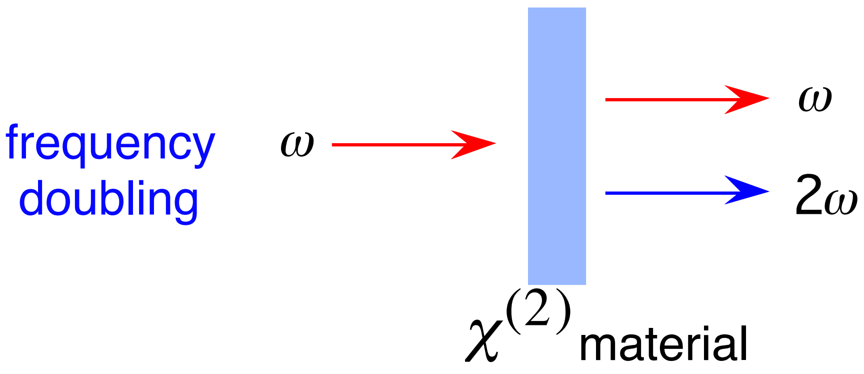
nonlinear susceptibilities

2nd-Order Nonlinear Processes

- Consider oscillating electric field: $E(t) = A\cos\omega t$

- $\chi^{(2)}$ effects: second-harmonic generation:

$$P^{(2)}(t) = \chi^{(2)}E^2(t) = \chi^{(2)}\frac{A^2}{2}(\underbrace{1}_{\text{dc}} + \underbrace{\cos 2\omega t}_{\text{second-harmonic}})$$



frequency doubling

ω

ω

2ω

$\chi^{(2)}$ material

- Other processes:
  - terahertz generation
  - sum- and difference-frequency generation
  - optical parametric amplification

- Only occurs in non-centrosymmetric crystals.
  - requires phase-matching (e.g., $n_\omega = n_{2\omega}$)

3rd-Order Nonlinear Processes

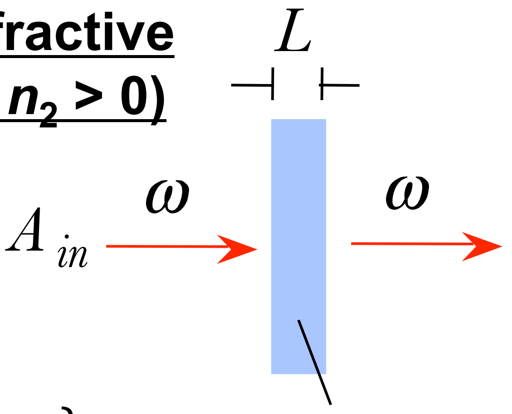
- $\chi^{(3)}$ effects: intensity-dependent refractive index

$$P^{(3)}(t) = \chi^{(3)} E^3(t) = \chi^{(3)} \frac{A^3}{4}(3\cos\omega t + \cos 3\omega t)$$

intensity-dependent refractive index

3rd-harmonic (usually not phase matched)

All materials exhibit a nonlinear refractive index (usually *n*₂ > 0)



$$A_{out} = A_{in} e^{i\phi_{nl}}$$

$$n_2^{glass} \sim 10^{-16} \text{ cm}^2/\text{W}$$

$$n_2^{Si} \sim 10^{-14} \text{ cm}^2/\text{W}$$

$$n_2^{air} \sim 10^{-20} \text{ cm}^2/\text{W}$$

$$n_2 \propto \text{Re}\left\{\chi^{(3)}\right\}$$

$$n = n_0 + n_2 I$$

intensity

- Nonlinear phase shift is important when: $\phi_{nl} = \frac{2\pi}{\lambda} n_2 I L \geq \pi$

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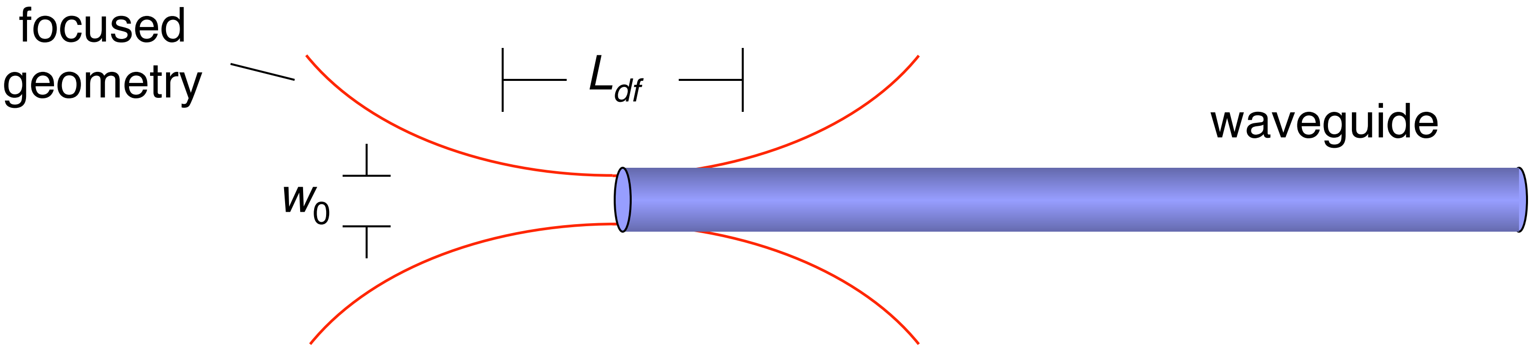
Nonlinear Interactions: Why Waveguides?



nonlinear parameter

$$\phi_{NL} = \kappa_{NL} \frac{P}{w_0^2} L$$

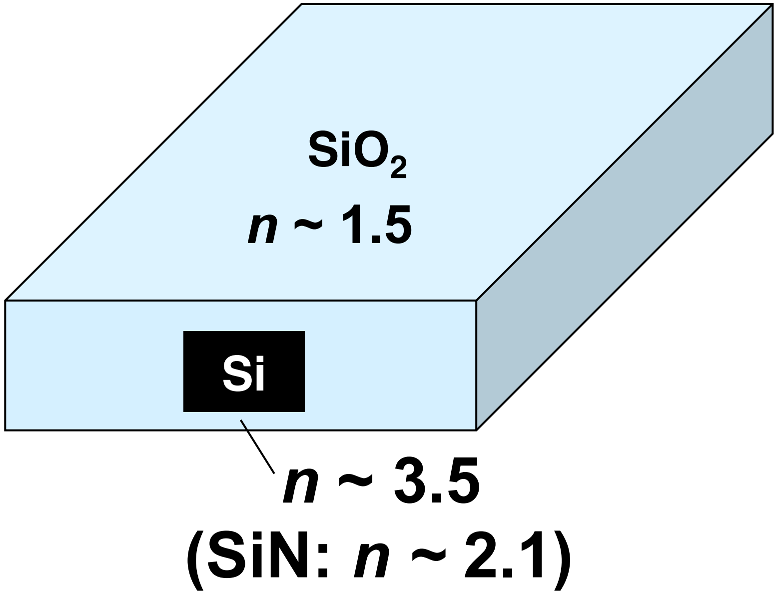
power

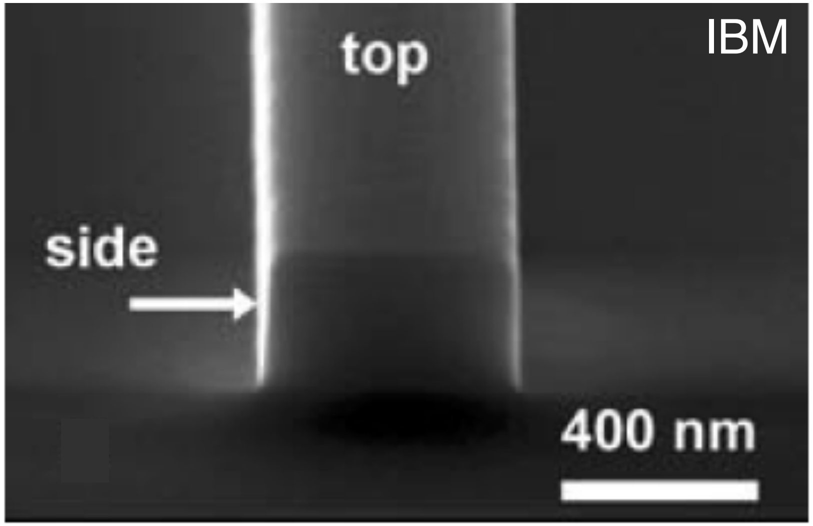
area

interaction length

- **Interaction length can be >> the diffraction length.**
- **Dispersion can be engineered.**

Nonlinear Optics in Silicon-Based Nanowaveguides

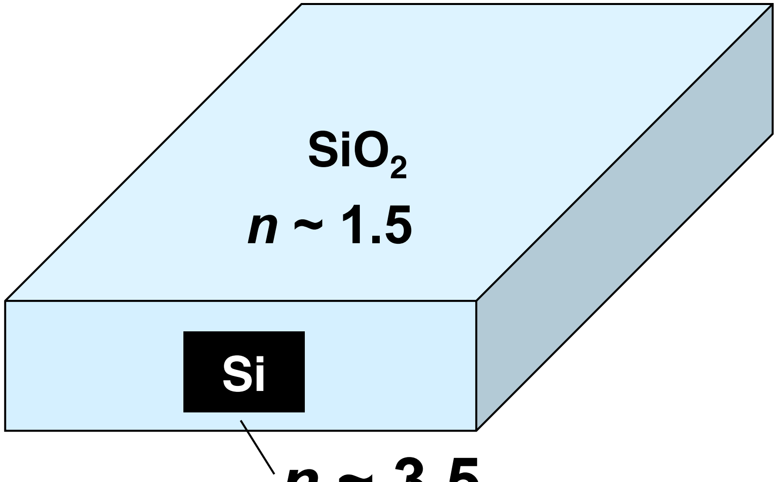




Absorption edge: Silicon => ~ 1.1 μm Si_3N_4 => ~ 400 nm

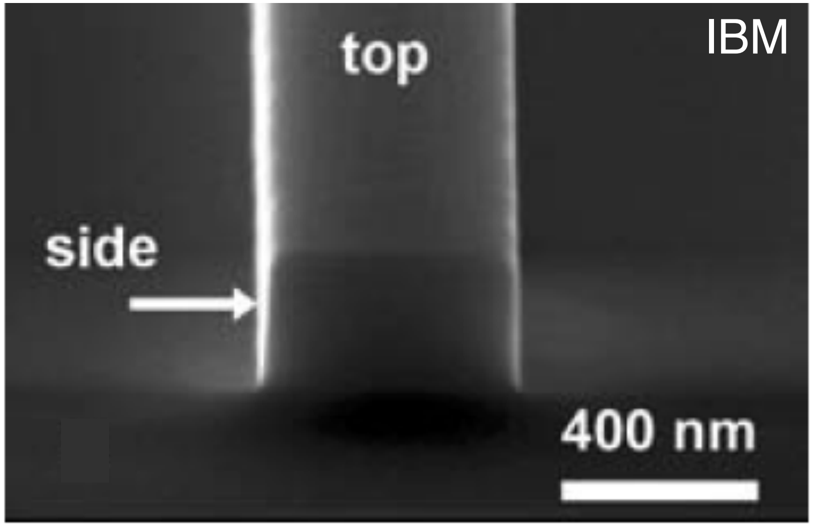
- Nonlinearity of Silicon 100X (Si_3N_4: 10X) silica
- Losses: Silicon – 2 dB/cm (Si_3N_4 – 0.2 dB/cm)
- Light confined to a region < than a wavelength.

Nonlinear Optics in Silicon-Based Nanowaveguides



$n \sim 3.5$

(SiN: $n \sim 2.1$)



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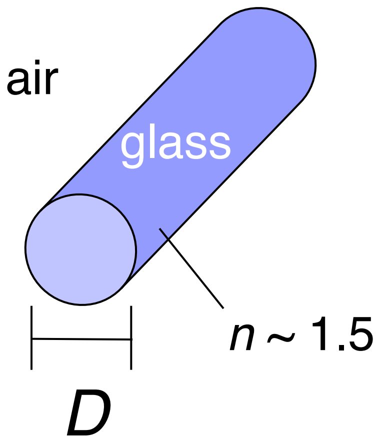
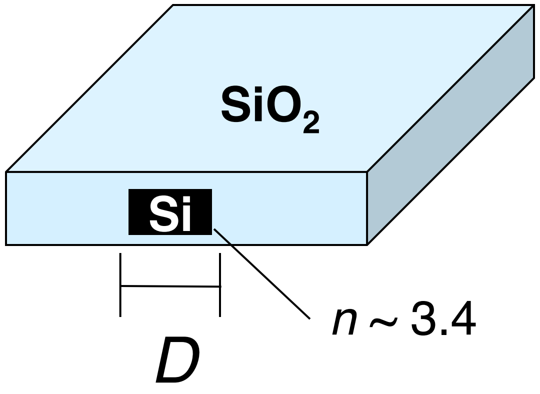
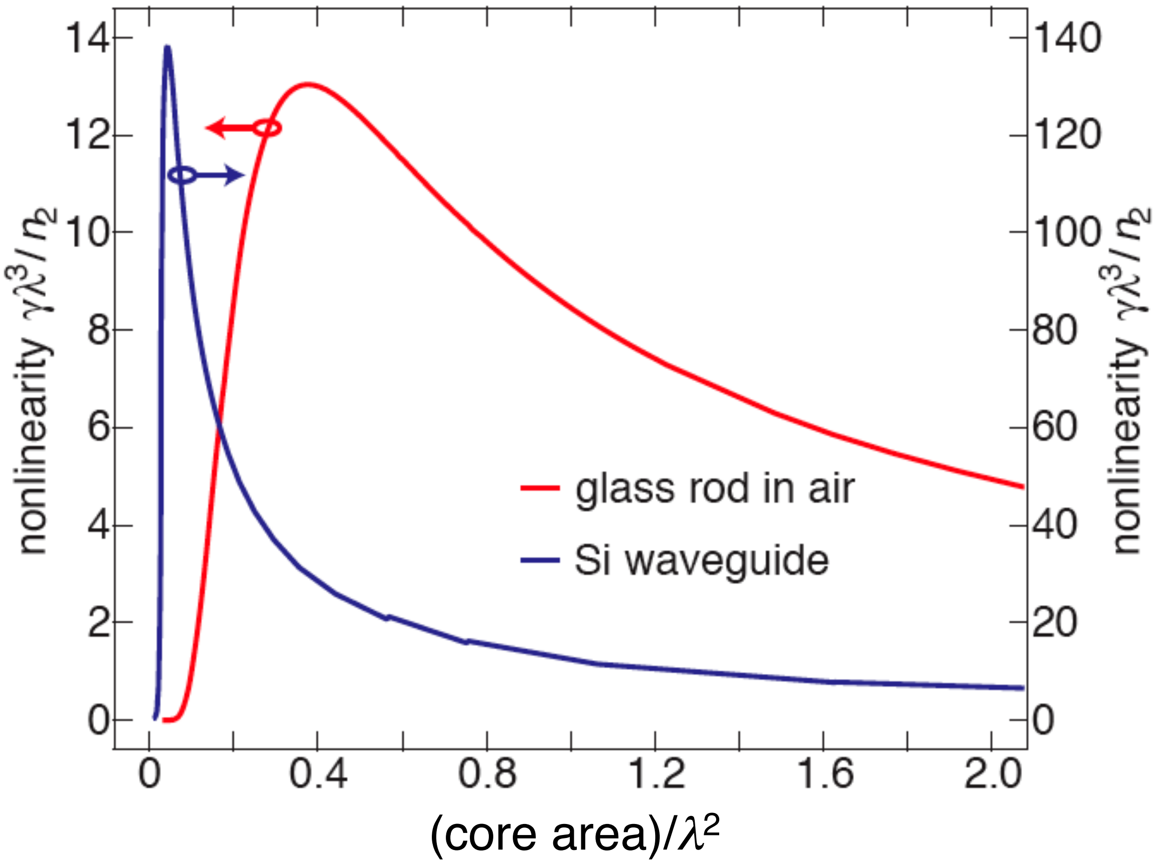
- **Dispersion can be engineered.**

Confinement Properties of Ultra-Small-Core Waveguides





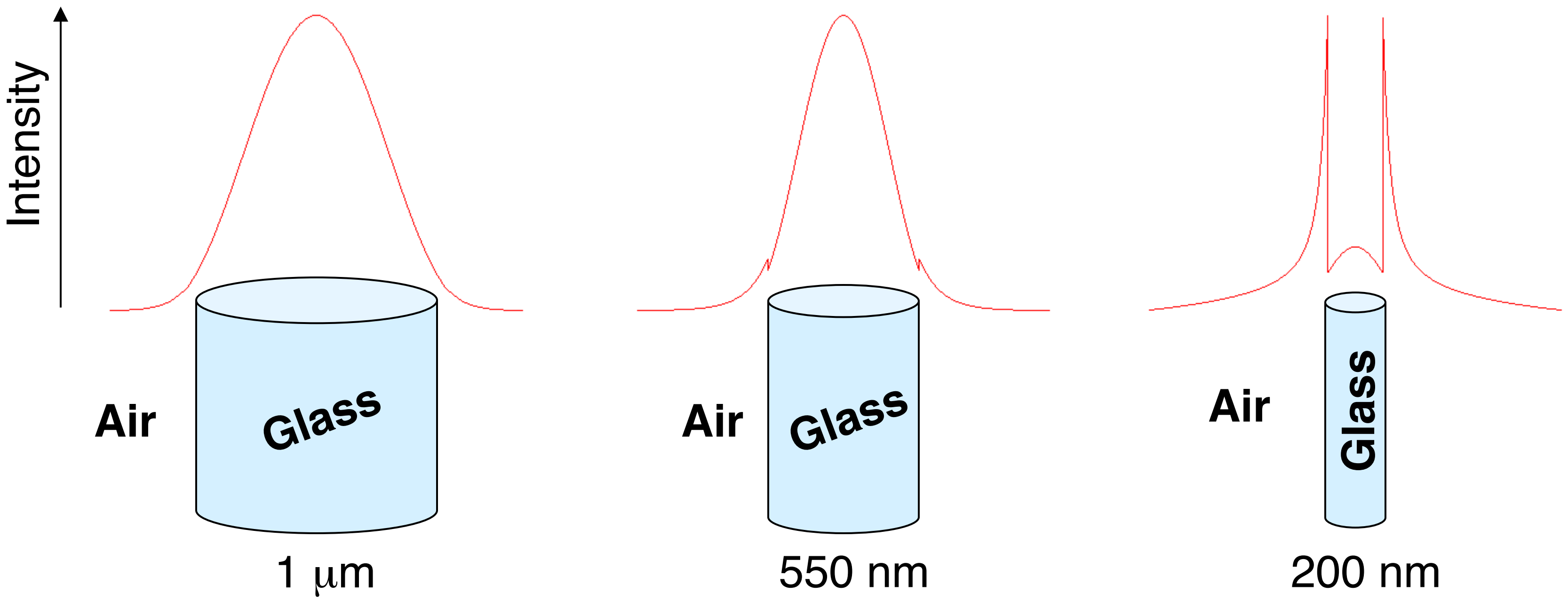


$$\boxed{D_{optimal} < \lambda}$$

Foster, Moll, and Gaeta, *Opt. Express* **12**, 2880 (2004)

Confinement Properties

λ = 800 nm



NLO in Silicon-Based Waveguides

- Raman scattering
  - Raman gain & oscillation [Claps et al 2003; Rong et al 2004; Espinola et al 2004; Xu, et al. 2005; Rong et al 2004; Boyraz et al 2004]
  - Raman-induced slow light [Okawachi et al 2006]
  - Zeno-switching [Wen et al 2011]
- Instantaneous Kerr nonlinearity
  - phase modulation & continuum generation [Tsang et al 2002; Boyraz et al 2004; Dulkeith et al, 2006; Hsieh et al, 2006; Hsieh, et al 2007; Koonath, et al 2007; Kuyken, et al. 2011; Halir, et al 2012]
  - harmonic generation [Corcoran et al. 2009; Levy et al. 2011]
- Four-wave mixing [Dimitropoulos et al 2004; Fukuda et al 2005; Espinola et al 2005; Yamada et al 2006; Rong et al 2006; Foster et al. 2006; Koos et al 2009); McMillan et al 2010; Zlatanovic et al. 2010; Xiaoping et al. 2010; Kuyken et al. 2011; Hu, et al. 2011]
  - generation of correlated photons [Sharping, et al. 2006; Harada et al 2007; Clemmen et al 2008]
  - signal regeneration [Salem, et al 2007, 2008]
  - parametric oscillation & comb generation (Levy, et al 2010; Foster et al 2011; Okawachi et al 2011; Ferdous et al. 2011); Herr, et al 2012]
  - ultrafast processing [Foster, et al 2008; Salem et al 2008; Corcoran et al 2010; Christian, et al. 2011]

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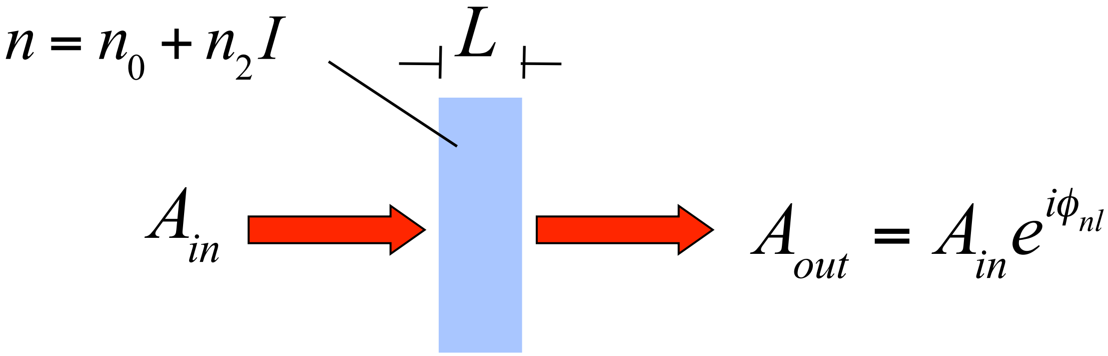
Reviews: Foster, et al. *Opt. Express* **16**, 1300 (2008)

Osgood, et al., *Adv. Opt. Phot.* **1**, 162 (2009)

Leuthold, et al., *Nat. Phot.* **4** 535 (2010).

Self- and Cross-Phase Modulation

- Self-phase modulation (SPM)



$n = n_0 + n_2 I$

L

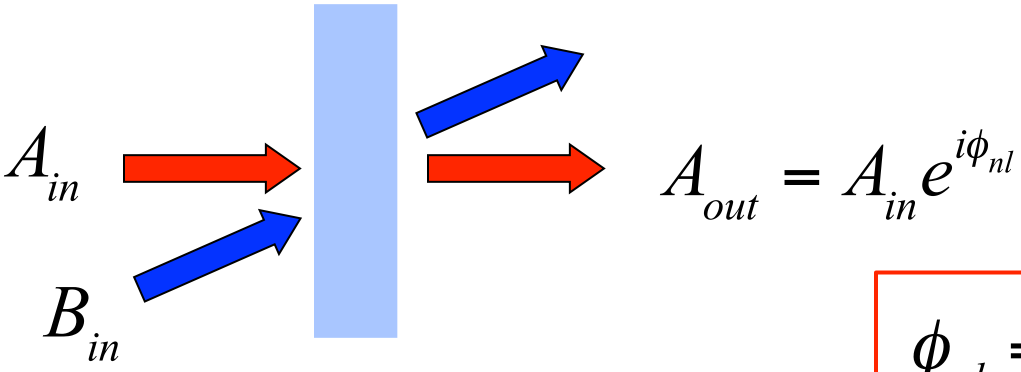
A_{in}

$A_{out} = A_{in} e^{i\phi_{nl}}$

SPM

$$\boxed{\phi_{nl} = \gamma\, I\, L}$$

- Cross-phase modulation (XPM)



A_{in}

B_{in}

$A_{out} = A_{in} e^{i\phi_{nl}}$

XPM

$$\boxed{\phi_{nl} = \gamma (I_A + 2 I_B) L}$$

Self-Phase Modulation w/ Pulses

- Neglect dispersion:

output field

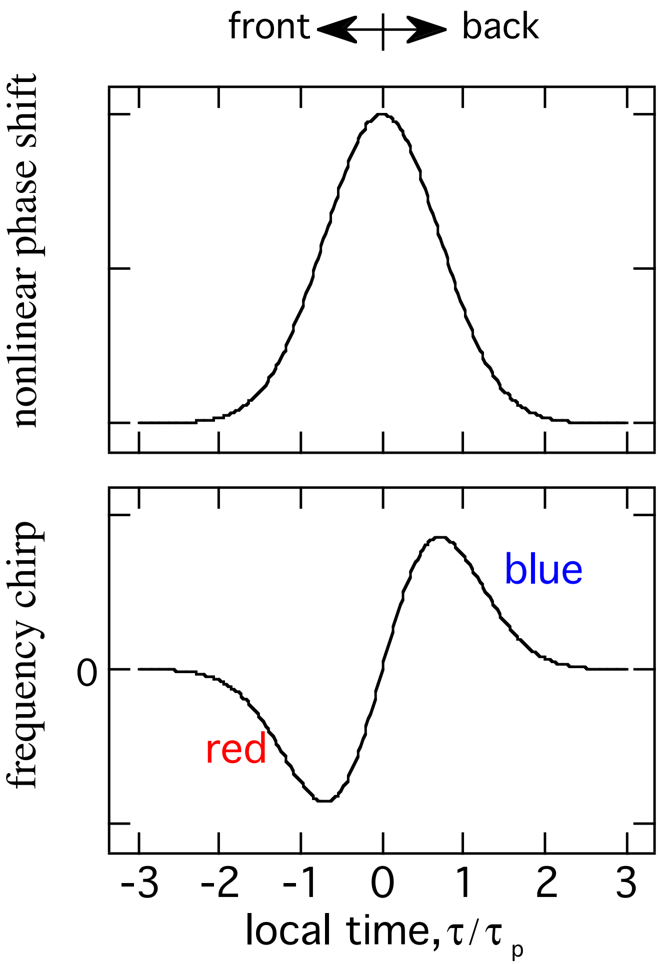
$$A_{out}(\tau) = A_{in}(\tau)e^{i\phi_{nl}(\tau)}$$

nonlinear phase shift

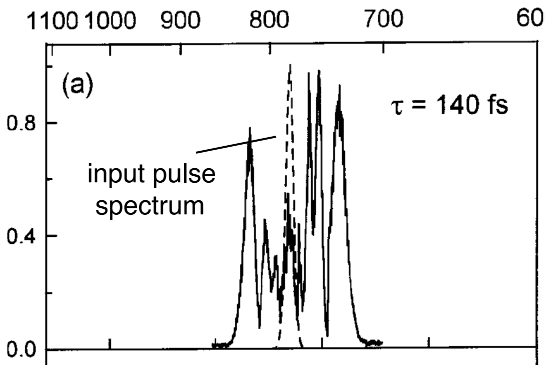
$$\phi_{nl}(\tau) = \frac{2\pi}{\lambda} n_2 I_{in}(\tau) L$$

frequency chirp

$$\delta\omega(\tau) = -\frac{\partial\phi_{nl}}{\partial\tau} \propto \frac{\phi_{nl}^{\max}}{\tau_p}$$



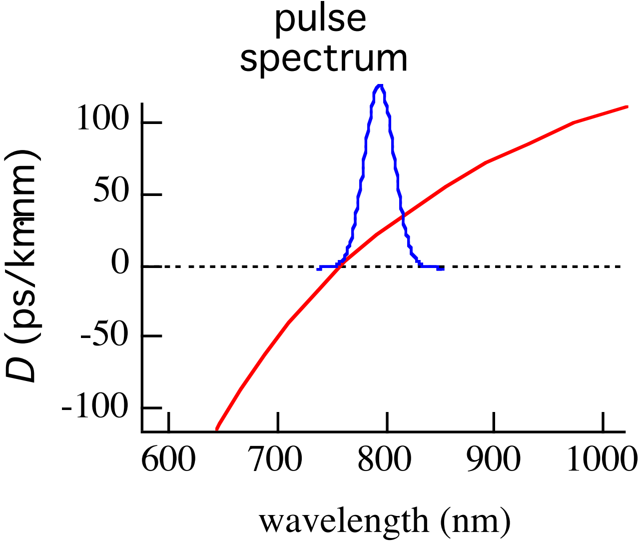
- Pulse duration is unchanged, but spectrum is broadened.

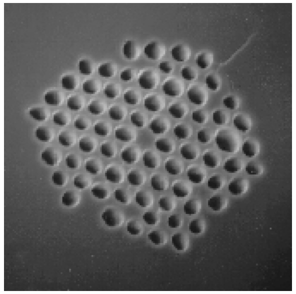


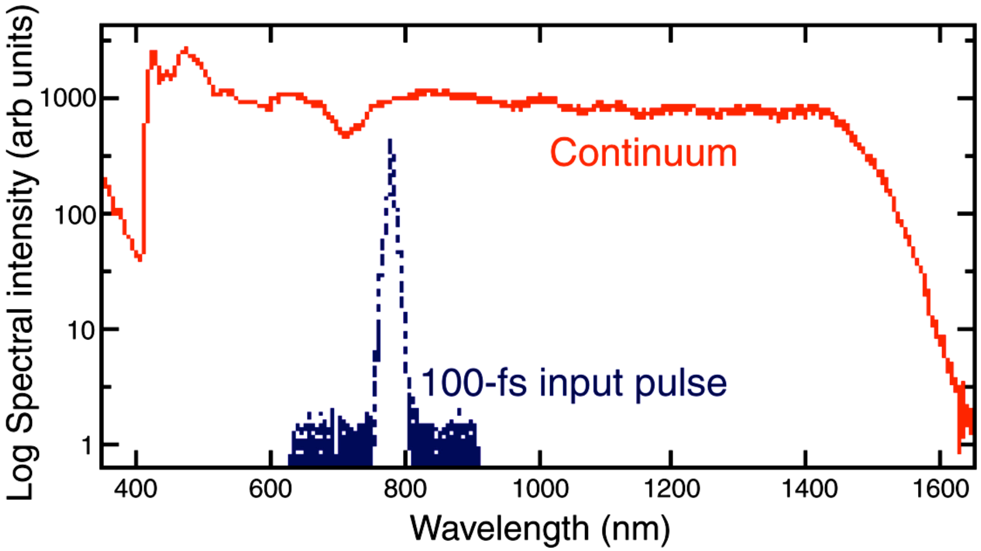
Nisoli, et al. Appl. Phys. B (1997)

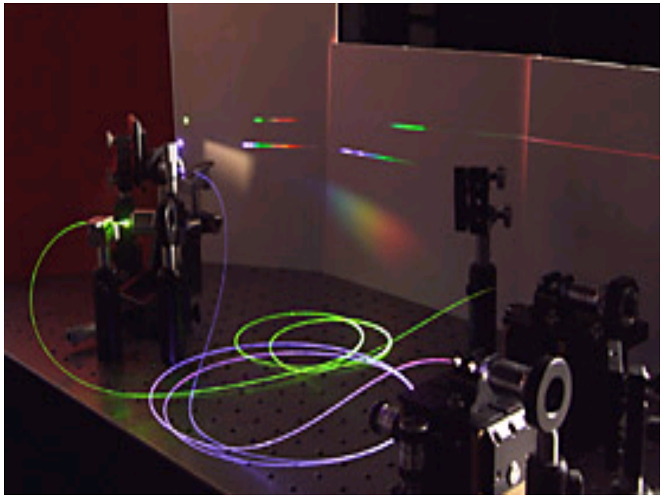
Supercontinuum Generation in Photonic Crystal Waveguides

- Initial observation: Inject < 100 fs pulses directly from Ti:sapphire modelocked (80 MHz) oscillator.
- Combination of small core and zero group-velocity dispersion allow for broad supercontinuum spanning > octave.









Ranka et al. 2001

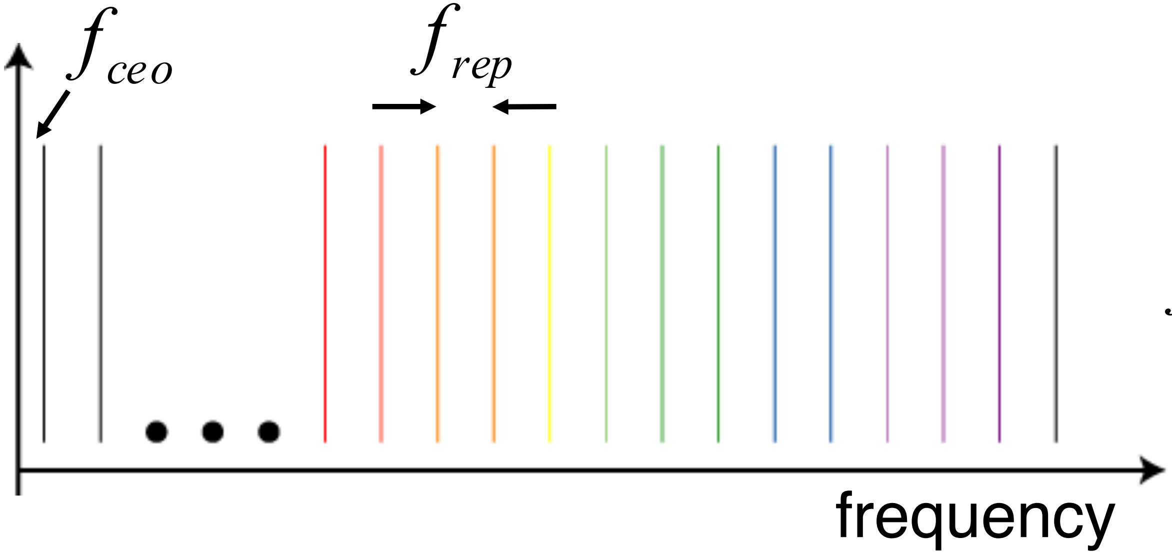
Optical Frequency Combs

100’s of THz span with mHz precision

Direct link between optical and microwave frequencies

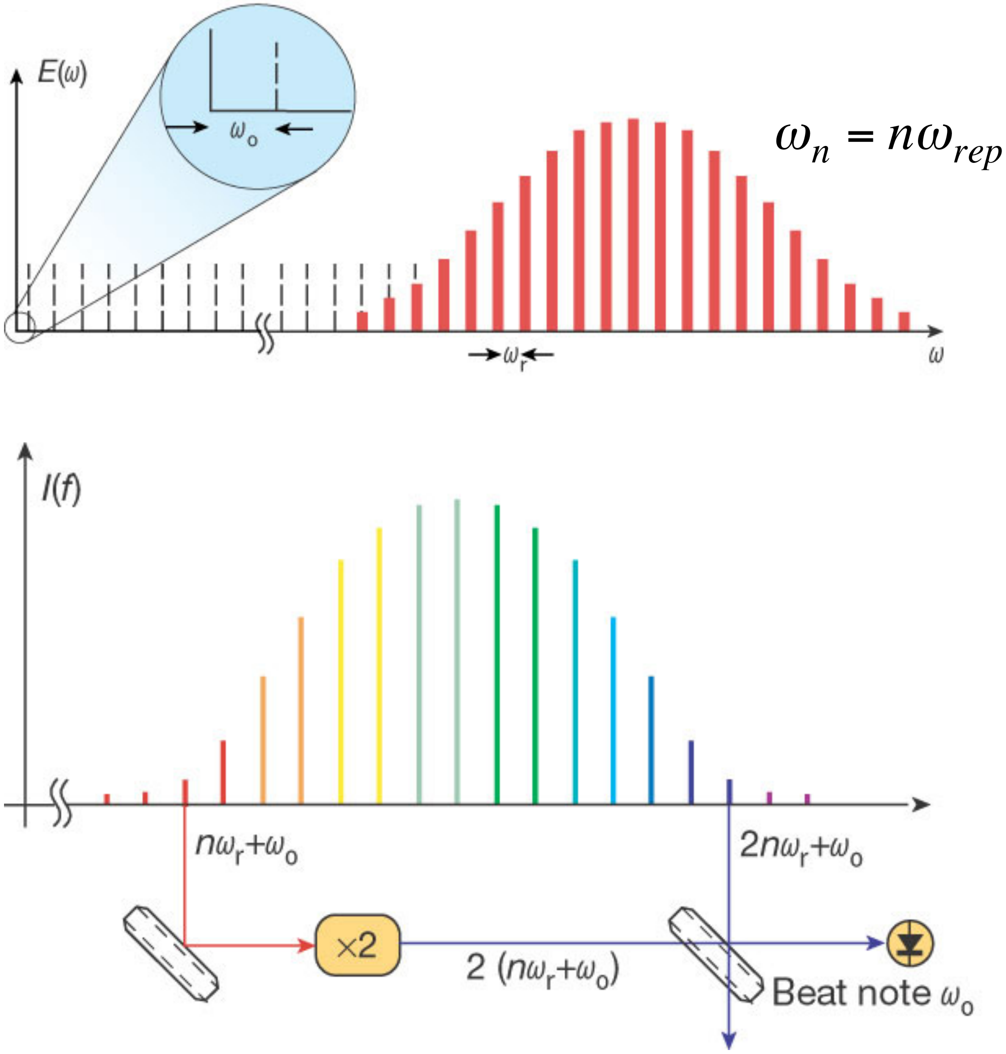
Telle, et al., Appl. Phys. B (1999).

Diddams, et al., Phys. Rev. Lett. (2000).



$$f_m = mf_{rep} + f_{ceo}$$

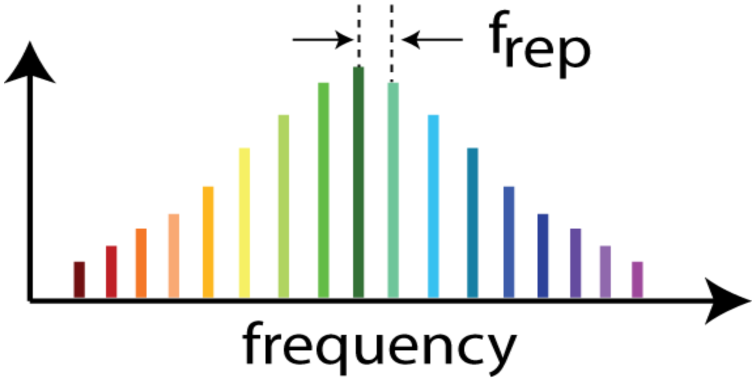
Why an Octave-Spanning Comb?



- Control of the position in the comb frequencies can be achieved.
- Link between microwave and optical frequencies.
- All-optical clock can be implemented.

Udem et al. *Nature* (2002)

Applications of Frequency-Comb Technology



- Optical clockwork
- Astronomical spectral calibration
- Chemical/biological sensing
- Optical communications & interconnects
- Tests of fundamental laws and constants (*R*, Lamb shift, fine-structure constant)
- Navigation (GPS)
- Very-long baseline interferometry
- Arbitrary-waveform generation
- Coherent control of molecules and reactions

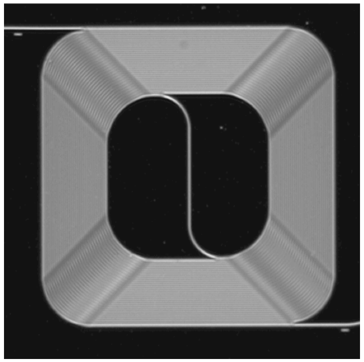
Experimental Setup for Supercontinuum Generation

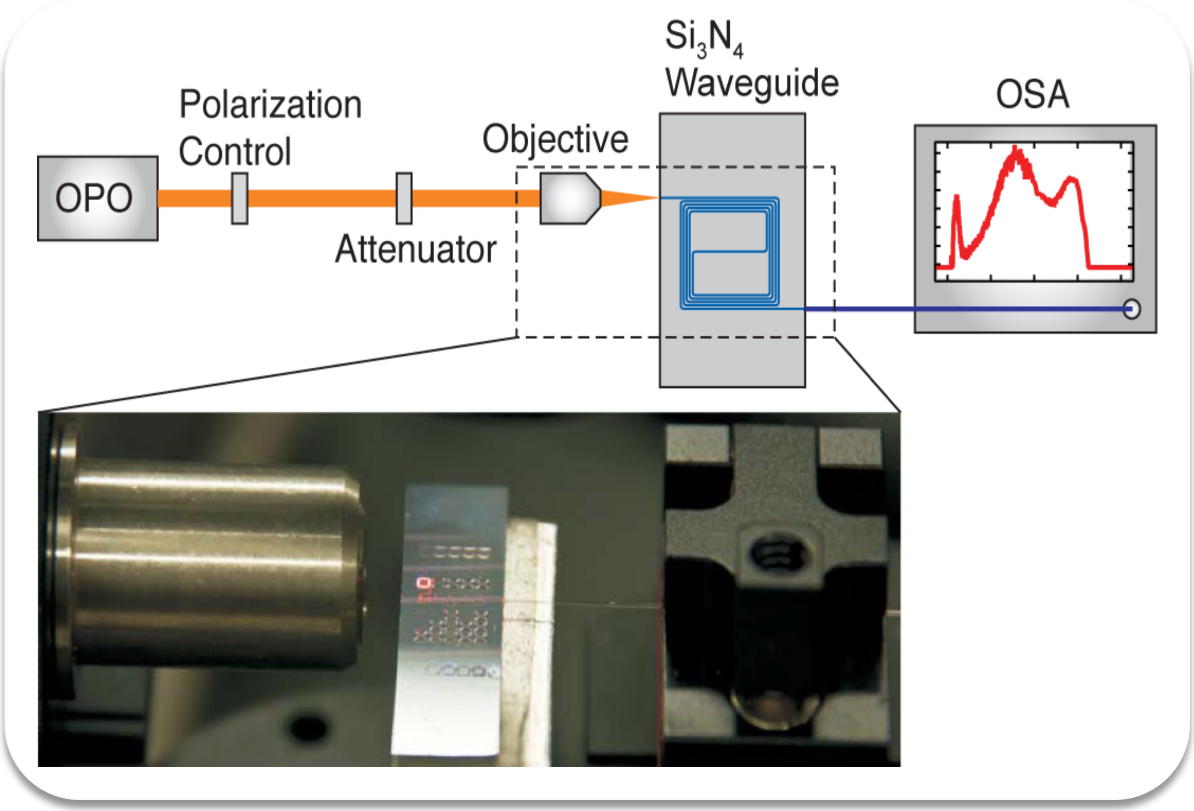
- Si_3N_4 spiral waveguide with 4.3 cm length, 715 x 1100 nm cross section

- 0.8 dB/cm propagation loss

- 80-MHz repetition rate, 200-fs pulsewidth OPO centered at 1335 nm

- Quasi-TE polarization





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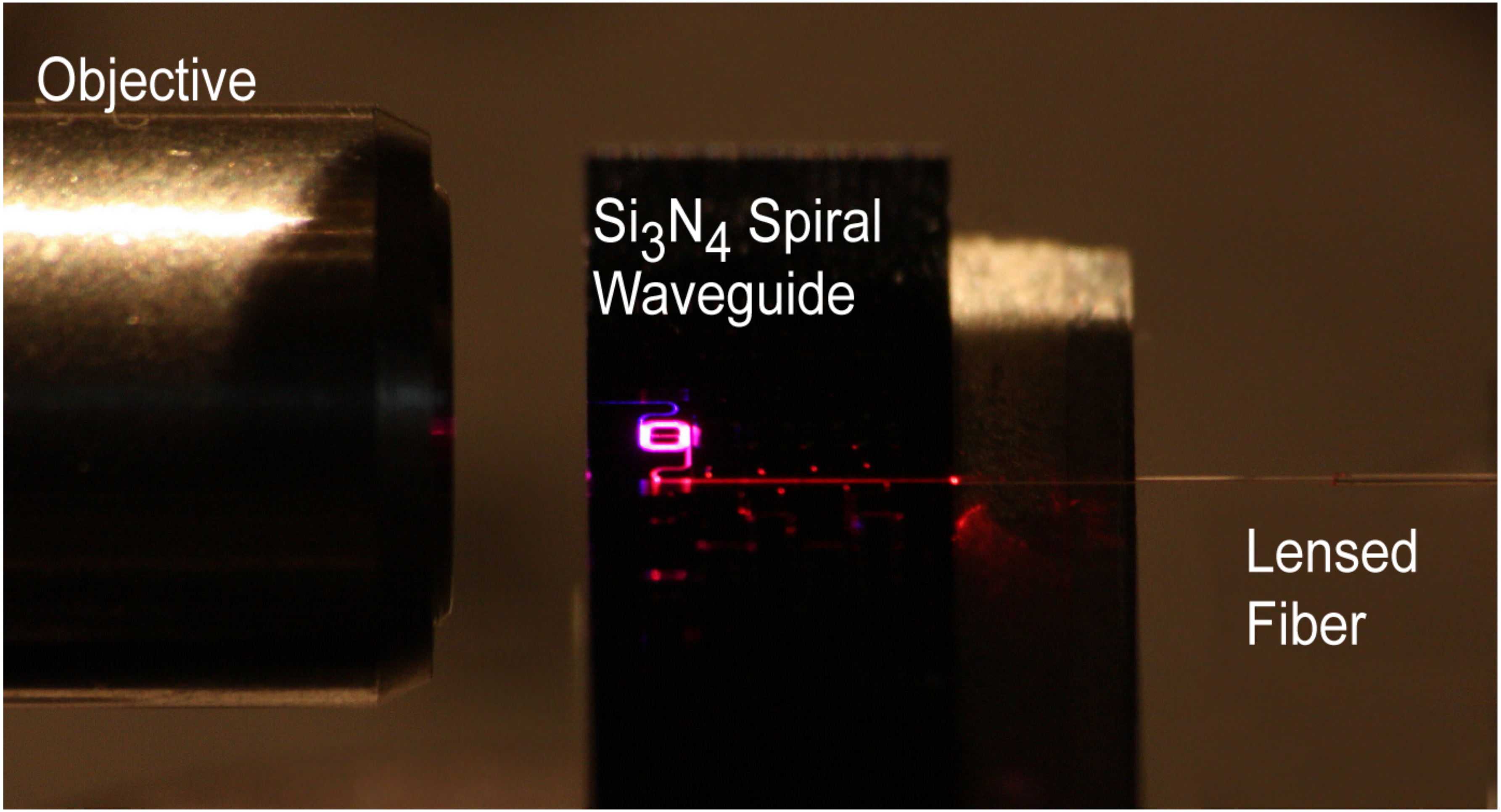
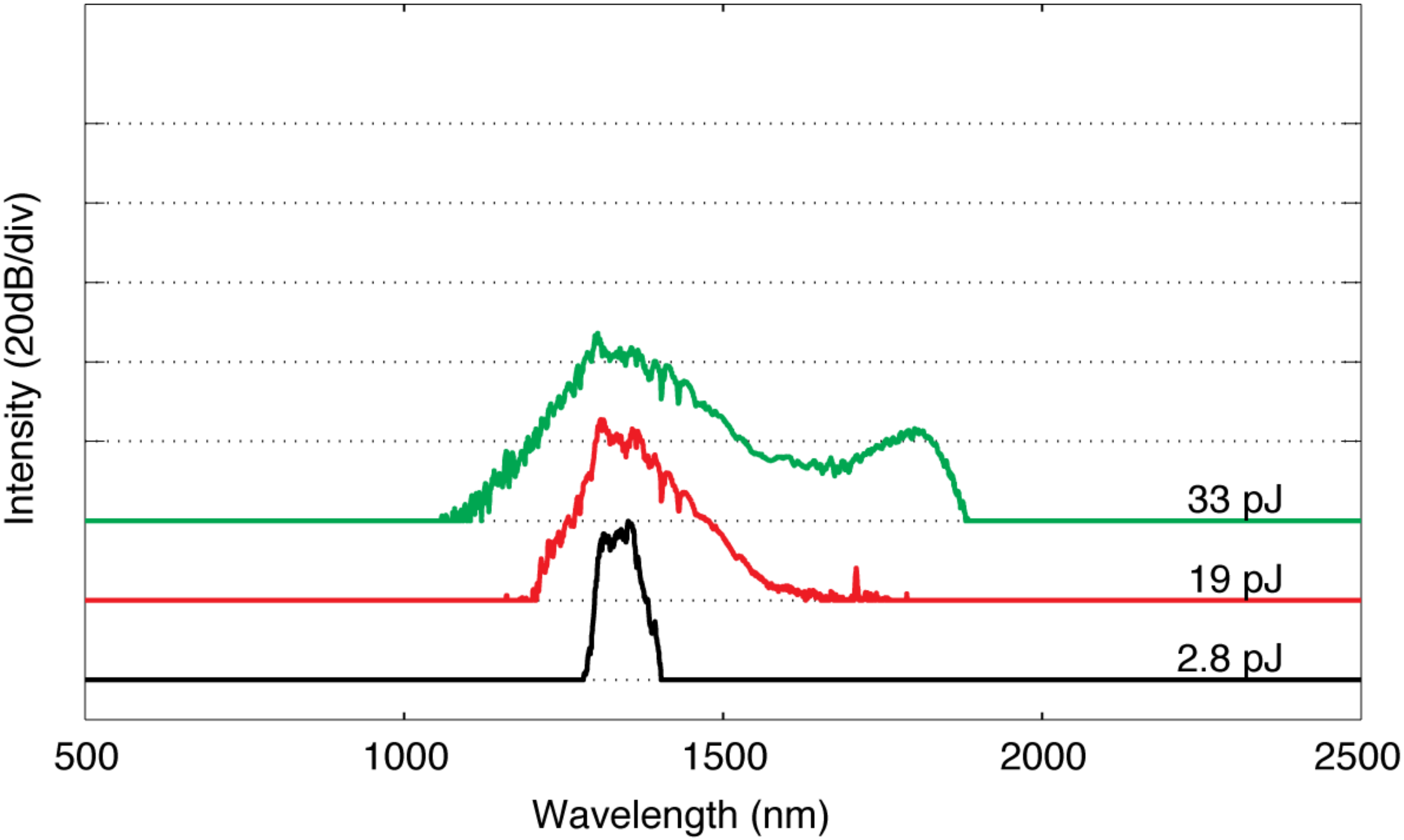


Supercontinuum Generation in Si_3N_4 Waveguide

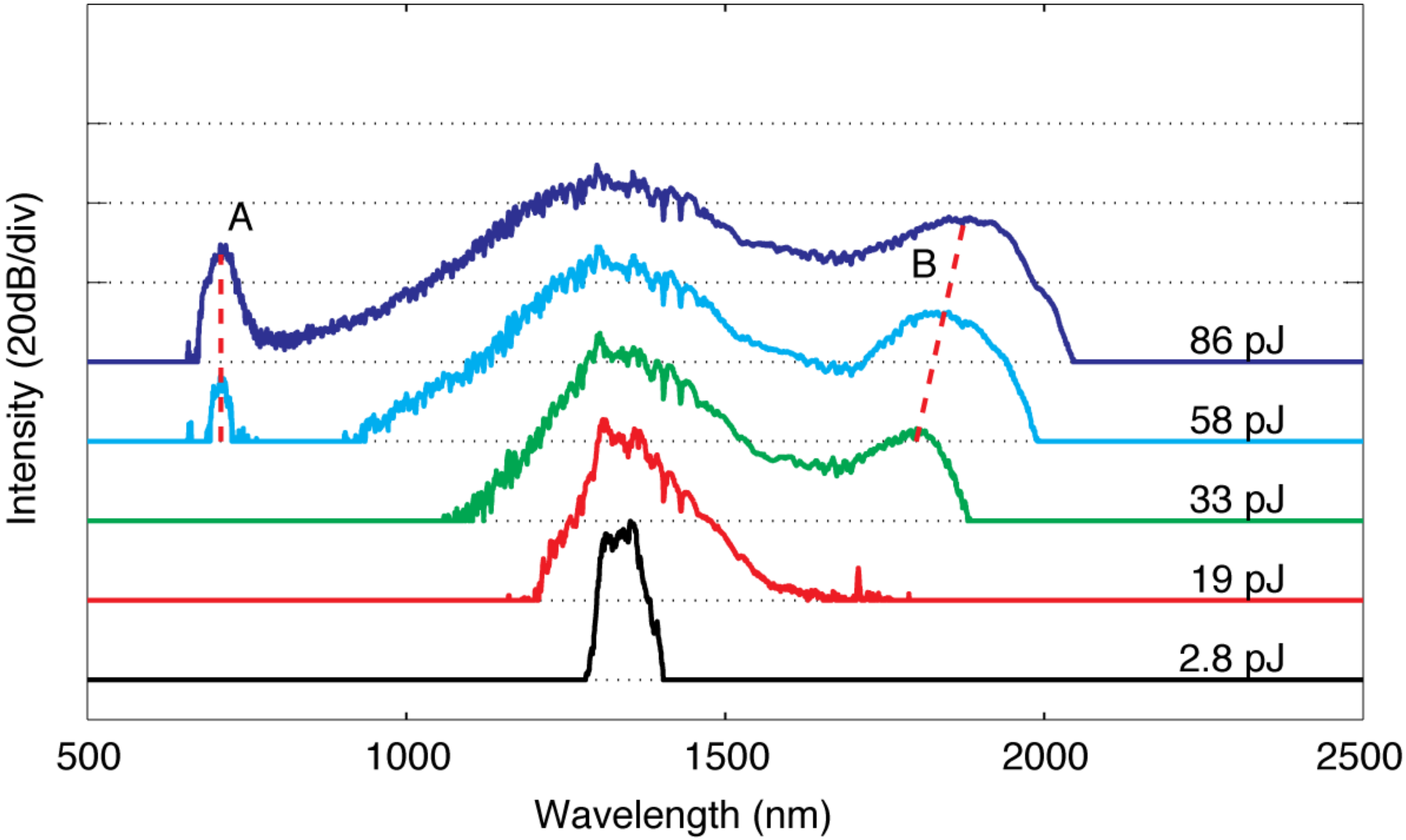


- Peak appears at 1800 nm

  → onset of soliton fission

Halir, Okawachi, Levy, Foster, Lipson, and Gaeta , *Opt. Lett.* (2012).

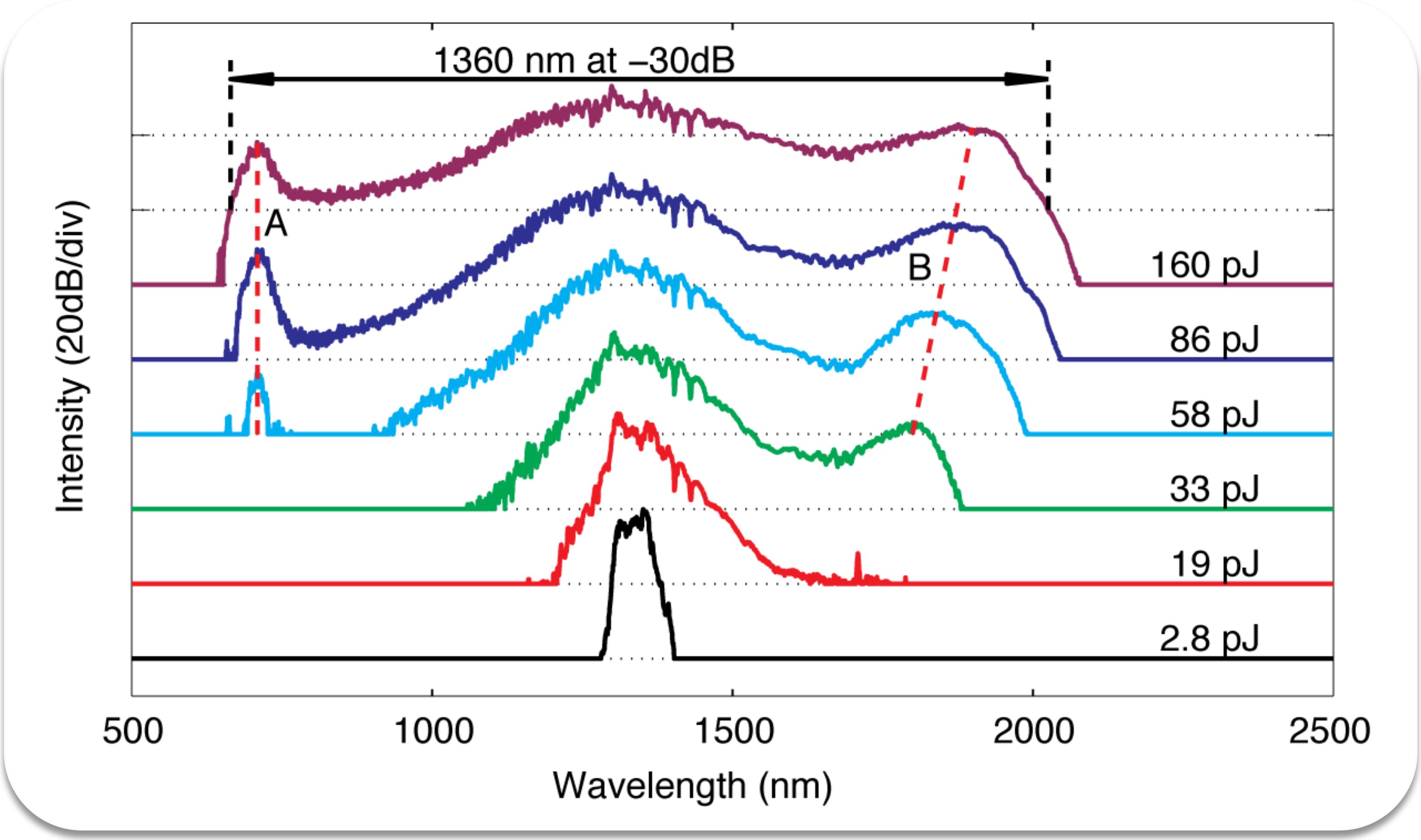
Supercontinuum Generation in Si_3N_4 Waveguide



- Self-frequency shift → 1800 nm peak to higher wavelengths
- Dispersive wave generation at 710 nm seeded by soliton fission

Halir, Okawachi, Levy, Foster, Lipson, and Gaeta , *Opt. Lett.* (2012).

Supercontinuum Generation in Si_3N_4 Waveguide



- Supercontinuum generation spans from 665 nm to 2025 nm

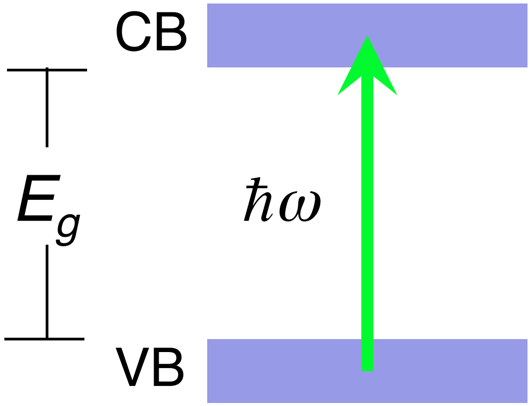
  → 1.6 octave span

Halir, Okawachi, Levy, Foster, Lipson, and Gaeta, *Opt. Lett.* (2012).

Im{ $\chi^{(3)}$ } – Nonlinear Absorption

- 1- and 2-photon resonances lead to absorption

Saturated Absorption

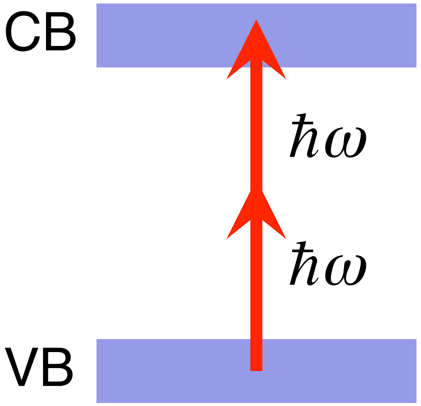


intensity

$$\frac{dI}{dz} = -\alpha_0 \left(1 - \frac{I}{I_s} \right) I$$

saturation intensity

Two-Photon Absorption



$$\frac{dI}{dz} = -\beta I^2$$

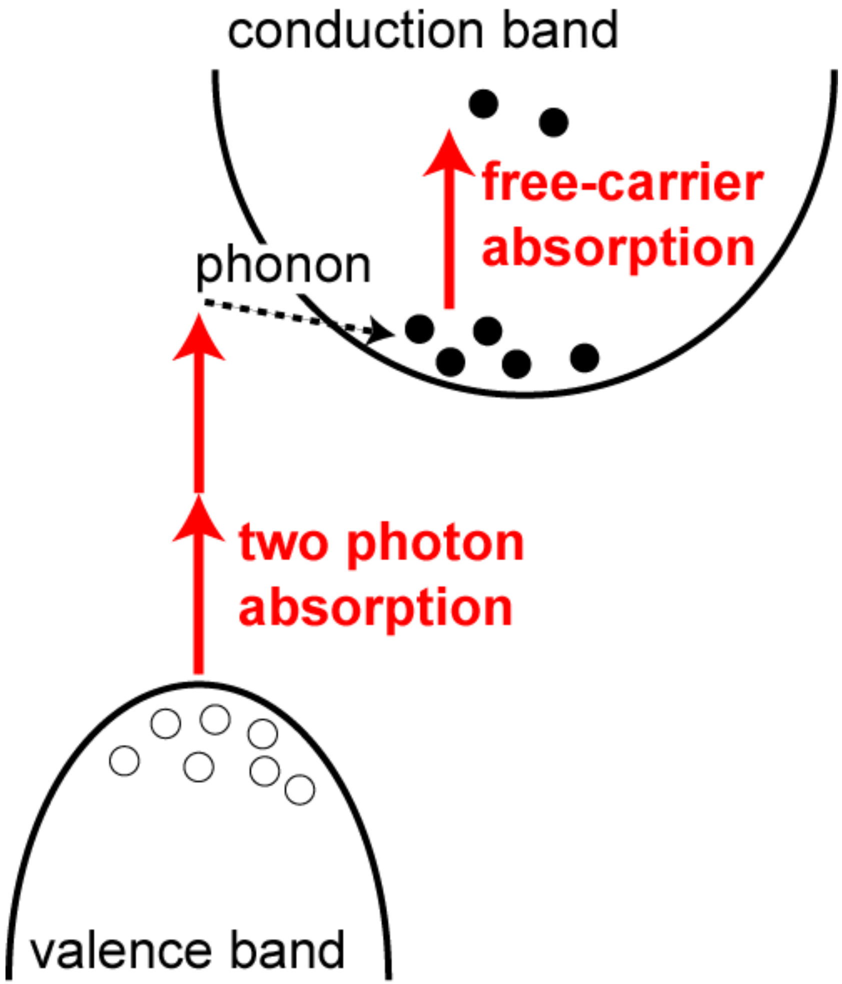
TPA coefficient

Issue for High-Power Operation: Nonlinear Absorption

- Two-photon absorption generates free carriers.
- Free carriers absorb incoming photons.
- Reduction of free-carrier lifetime can reduce loss.

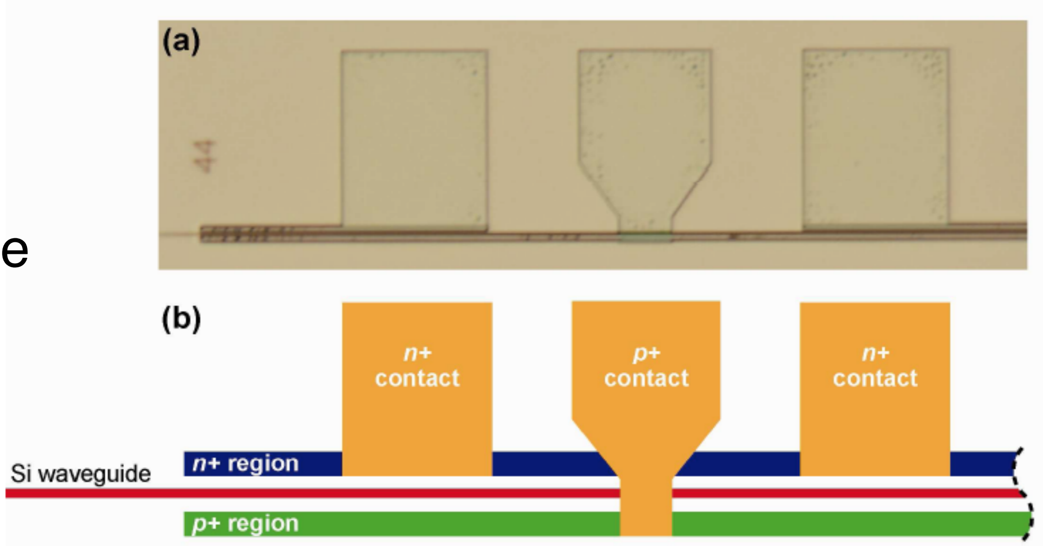
Solution:

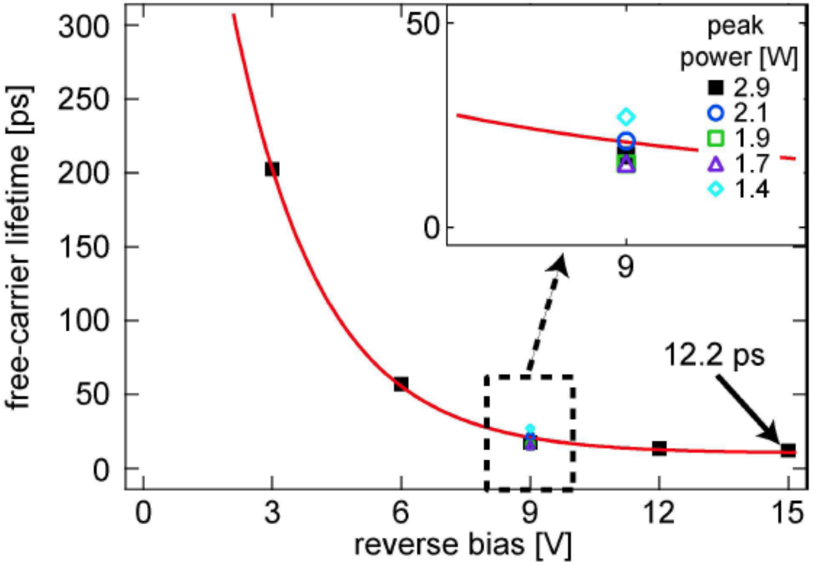
- Integrate PIN-diode structure into waveguides.
- Operate w/ pump > 2 μm
- Use SiN (broader band-gap).

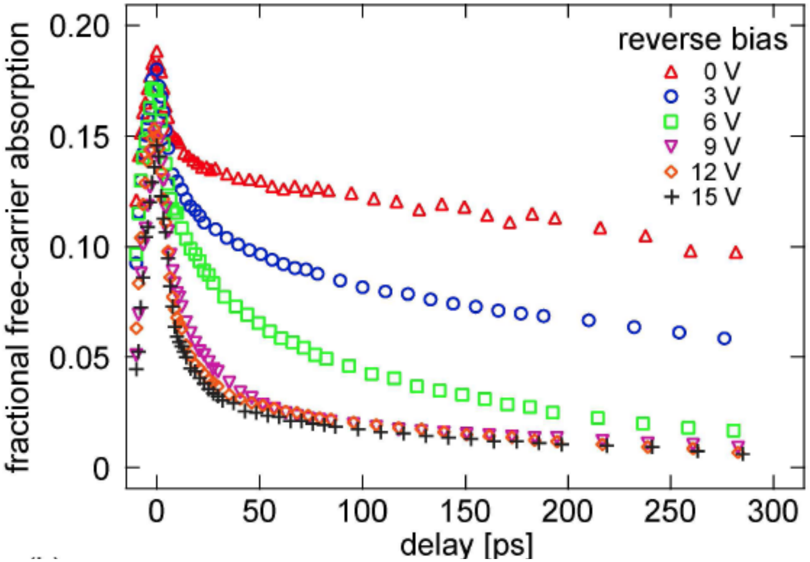


Reduction of Free-Carrier Lifetime

- Incorporate *p-i-n* structure

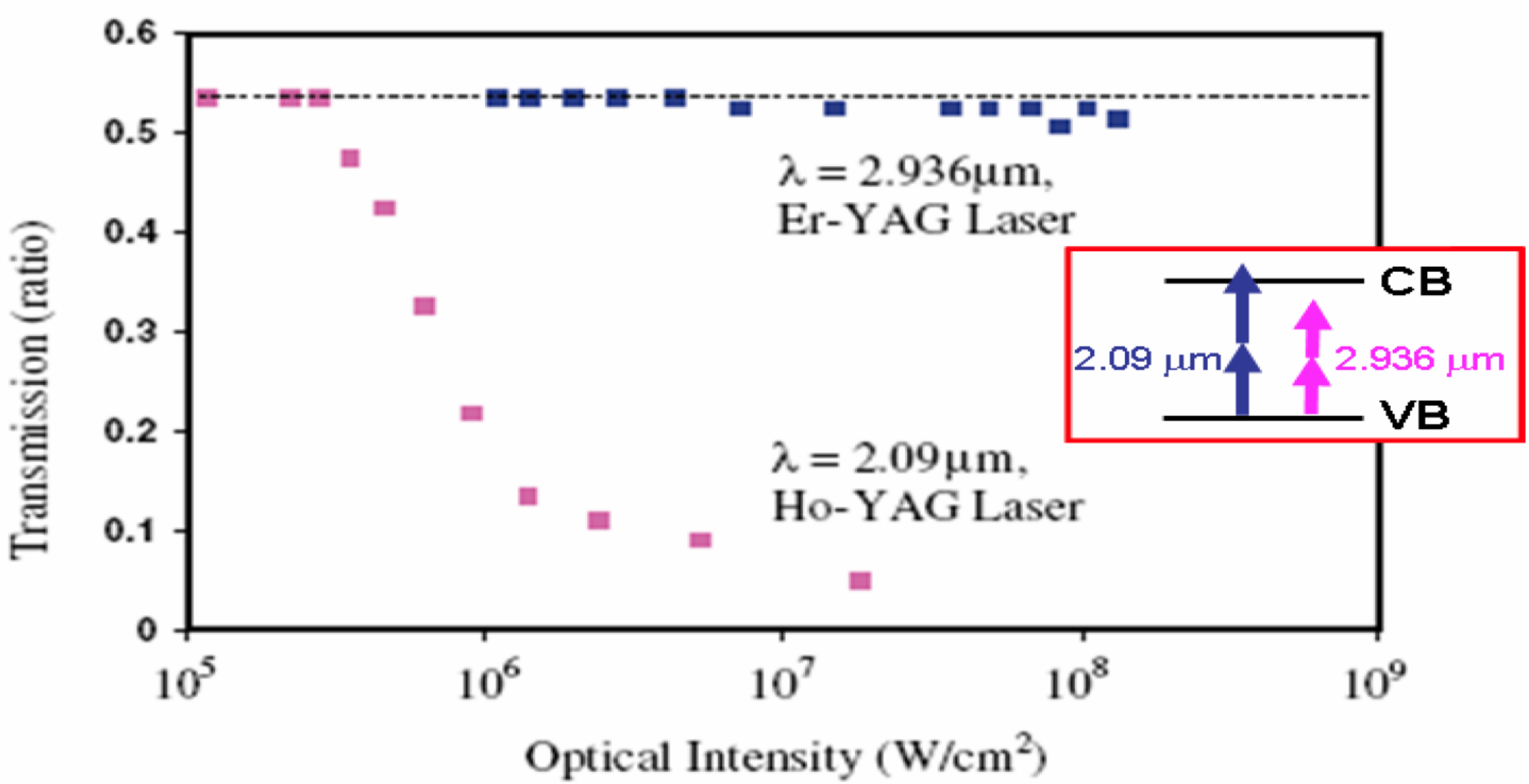






Effects of Two-Photon Absorption

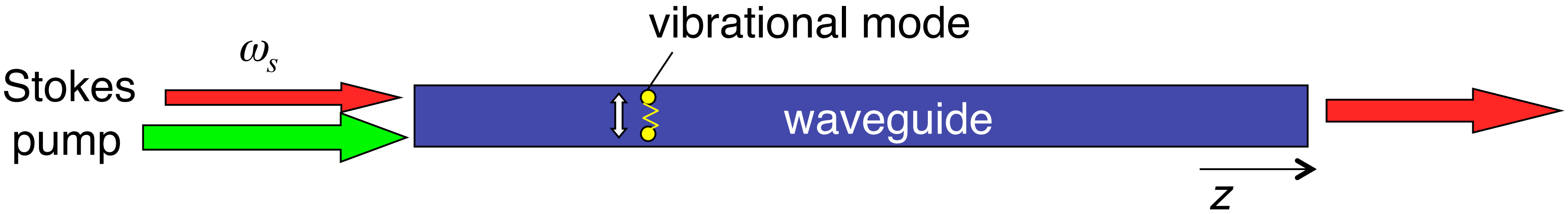
- 1- and 2-photon resonances lead to absorption



Raghunathan, et al. (2010).

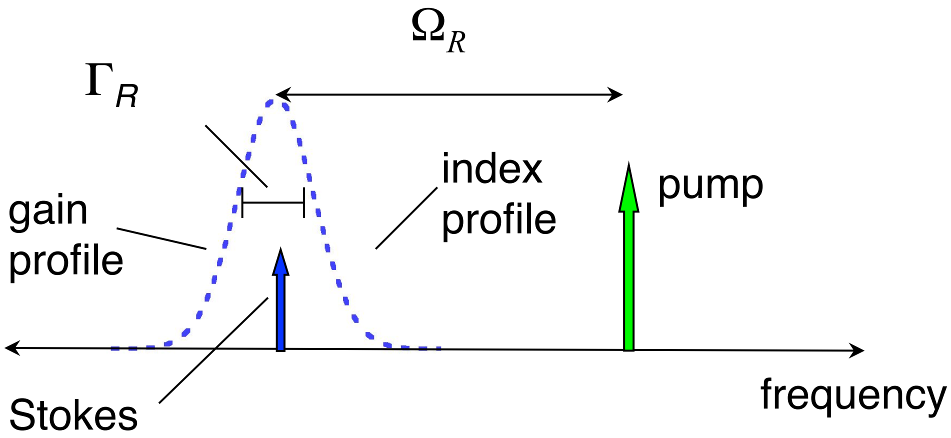
Im{ $\chi^{(3)}$ } – Raman Amplification

- Stimulated Raman scattering produces gain for Stokes wave.



Stokes intensity

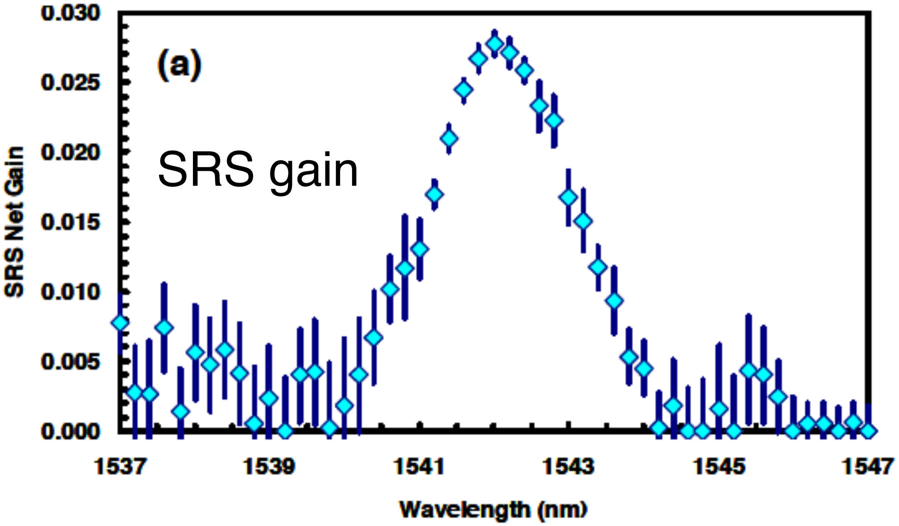
$$\frac{dI_s}{dz} = (g_R I_R) I_s \qquad g_R \propto \mathrm{Im}\left\{\chi_R^{(3)}\right\}$$

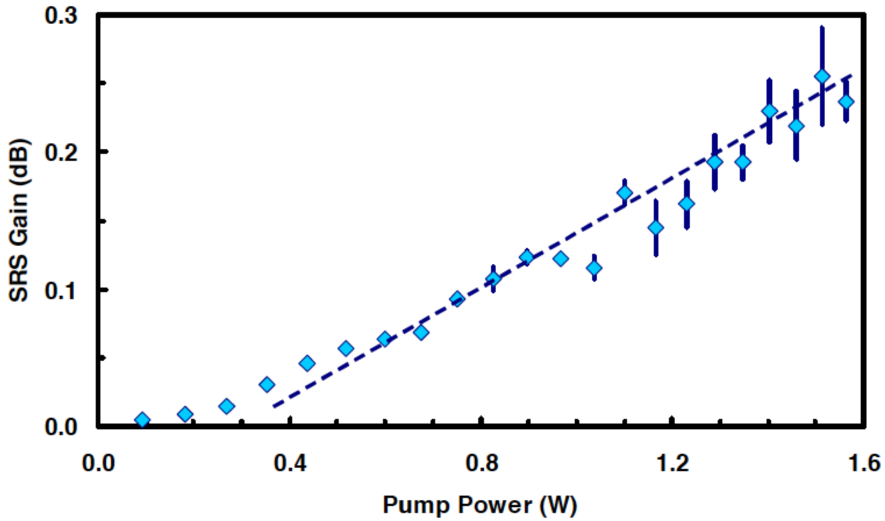


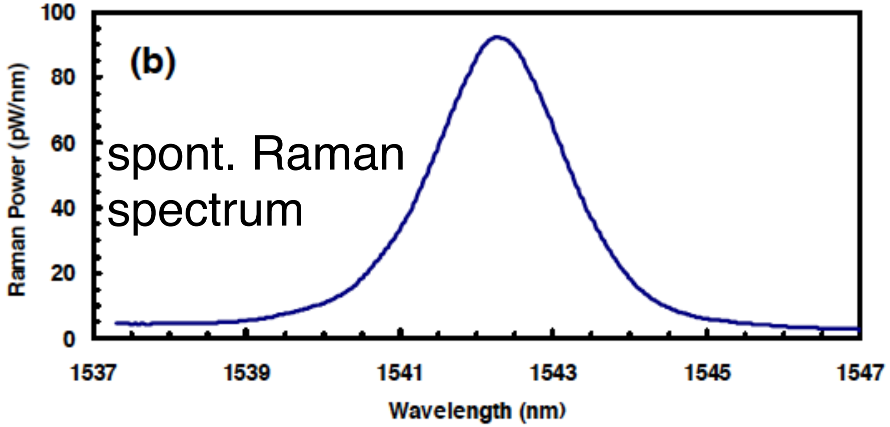
Typical Raman values

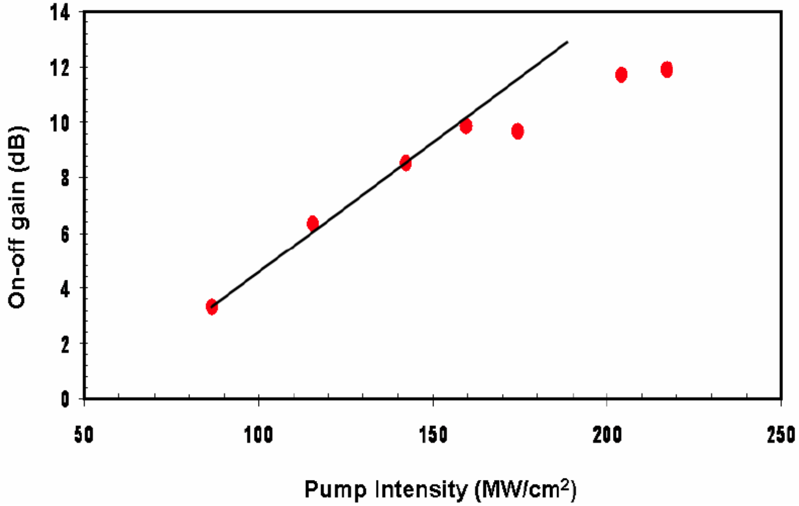
Silicon	**SiO$_2$**
Γ_R ~ 100 GHz	Γ_R ~ 3 THz
Ω_R ~ 13 THz	Ω_R ~ 12 THz

Raman Gain in Silicon-Based Nanowaveguides







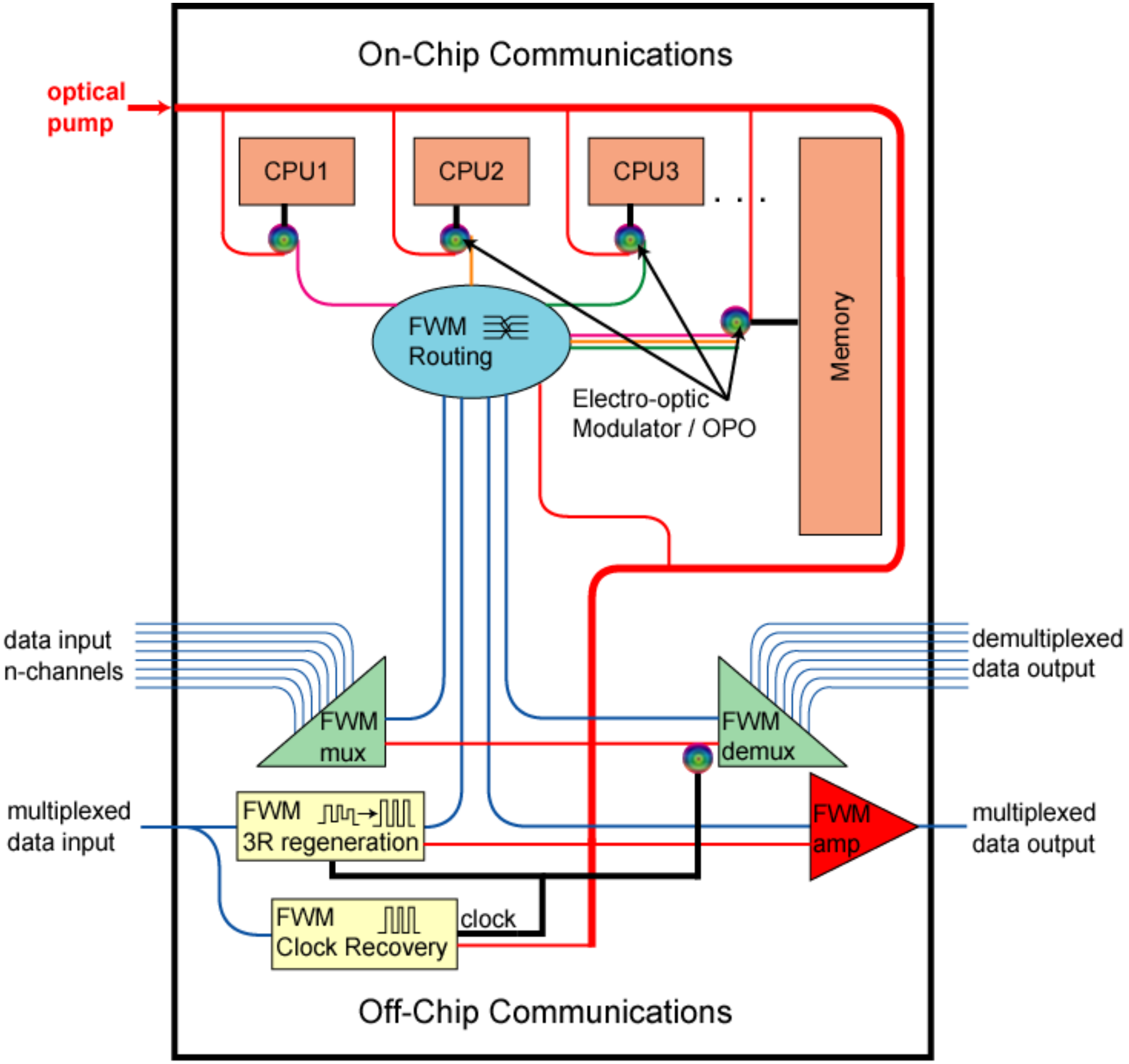


Claps, Dimitropoulos, Raghunathan, Han and Jalali, Opt. Express **11,** 1731 (2003).

Outline: Nonlinear Photonics in Chip-Based Structures

- Brief review of nonlinear optics
- Nonlinear processes in nanowaveguides
- Four-wave mixing (FWM) in Si nanowaveguides
  - ✧ Dispersion engineering
  - ✧ Ultra-broadband wavelength conversion
  - ✧ Application: correlated photons for quantum information
- Optical parametric oscillators
  - ✦ Broad-band frequency combs, ultrashort-pulse generation

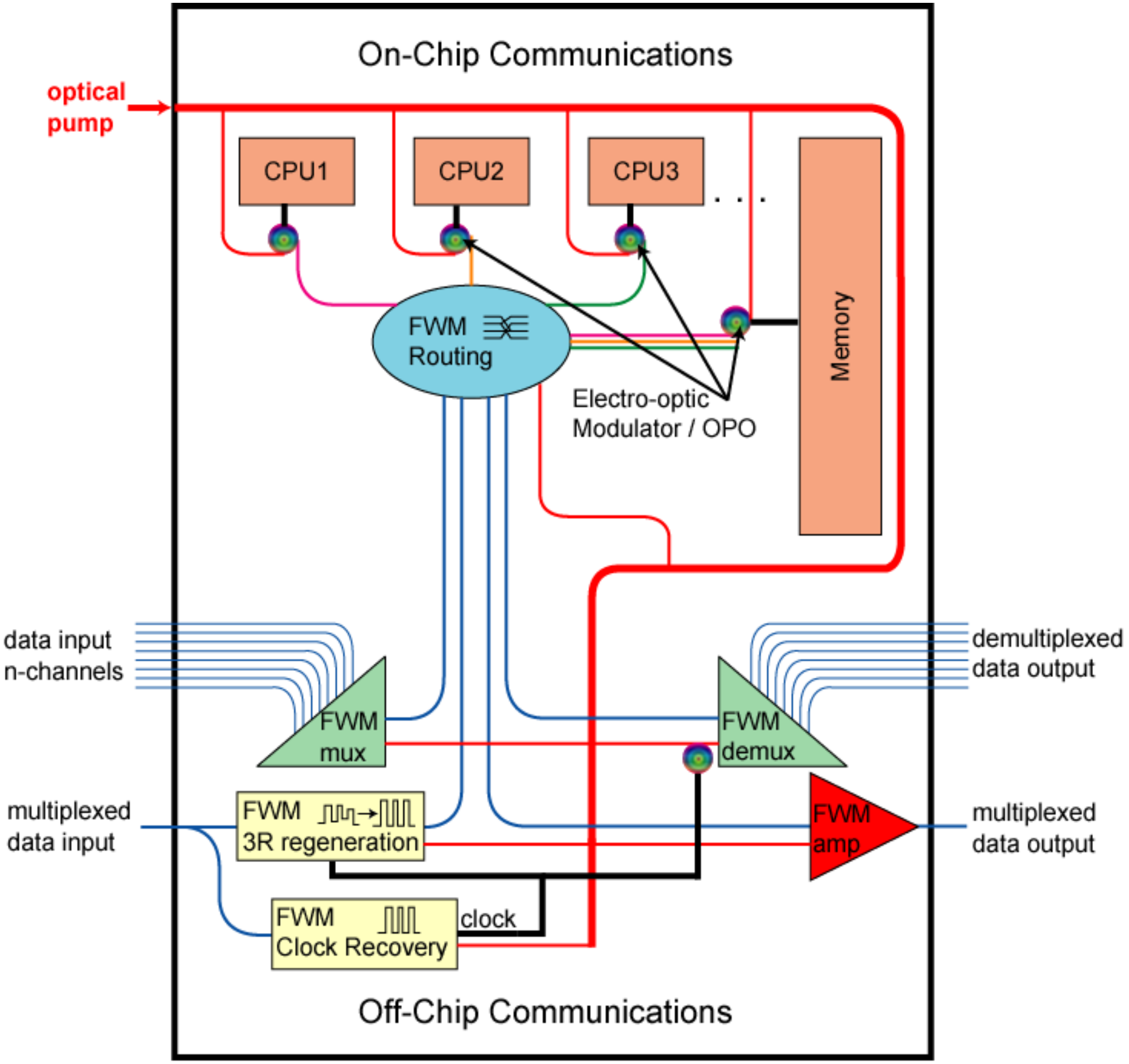
Nonlinear Photonics on Silicon Chip



On-Chip Processing

- Multiplexing/demux
- Regeneration
- Optical buffers
- Routing (switching / logic)
- Multicasting
- A-D conversion
- Wavelength conversion
- Amplification
- Oscillator/comb source

Nonlinear Photonics on Silicon Chip



On-Chip Processing

- Multiplexing/demux
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- Routing (switching / logic)
- Multicasting
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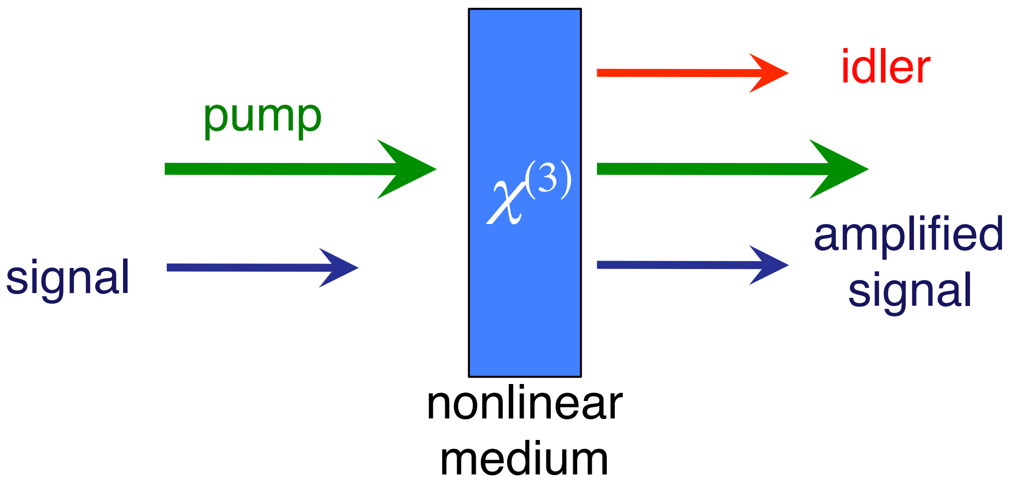
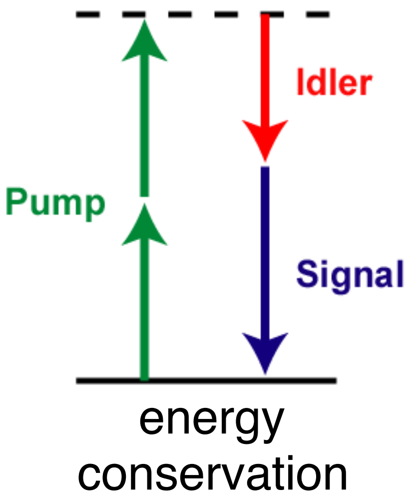
All performed via four-wave mixing.

Four-Wave Mixing





input field

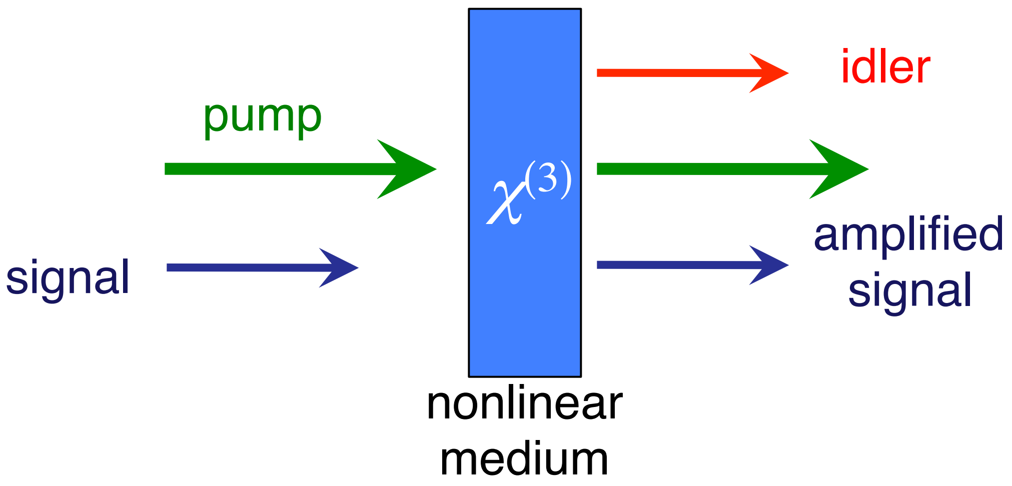
$$E_{input} = E_p + E_s = A_p e^{-i\omega_p t} + A_s e^{-i\omega_s t} + c.c.$$

nonlinear polarization

$$P^{(3)} = \chi^{(3)} E^3 = \underbrace{\chi^{(3)} A_p^2 A_s^* e^{-i(2\omega_p - \omega_s)t}}_{\text{idler driving term}} + \text{other terms}$$

$$\omega_i = 2\omega_p - \omega_s$$

Four-Wave Mixing



pump

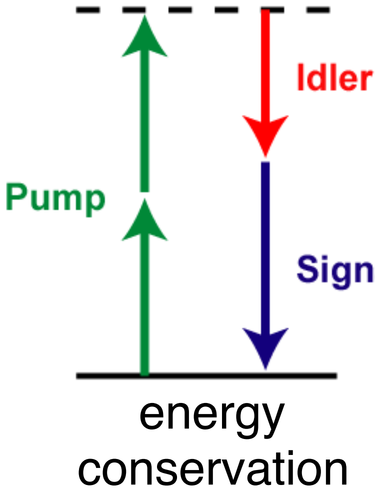
signal

$\chi^{(3)}$

idler

amplified signal

nonlinear medium



Pump

Idler

Signal

energy conservation

input field

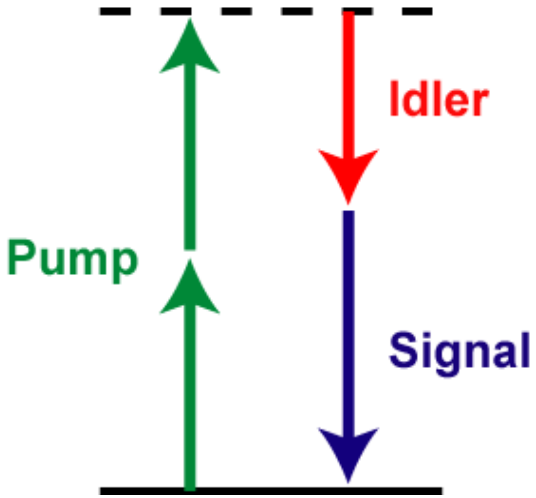
$$E_{input} = E_p + E_s = A_p e^{-i\omega_p t} + A_s e^{-i\omega_s t} + c.c.$$

nonlinear polarization

$$P^{(3)} = \chi^{(3)} E^3 = \chi^{(3)} \left[\underbrace{A_p^2 A_s^*}_{\text{idler driving term}} + \underbrace{2 | A_p |^2 A_i}_{\text{cross-phase modulation}} \right] e^{-i(2\omega_p - \omega_s)t}$$

- Efficient generation requires momentum conservation (i.e., phase matching)

Requirements for Efficient Four-Wave Mixing



- Energy conservation: $2\omega_p - (\omega_s + \omega_i) = 0$
- Momentum conservation: $\Delta k = 2k_p - (k_s + k_i) + \Delta k_{nl}$
    - Balance of GVD and effects of self-phase modulation & cross-phase modulation

group-velocity dispersion:

$$\boxed{\mathrm{GVD} \propto -\frac{d^2 n}{d\lambda^2} \geq 0}$$

Nonlinear Propagation in Waveguides

[*Nonlinear Fiber Optics*, Agrawal (2001)]

- Starting point: Maxwell's wave equation:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E = \frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2}$$

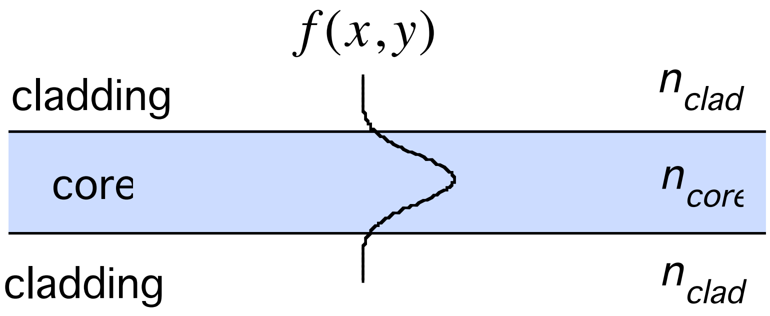
linear & nonlinear contributions

- Separate transverse-spatial parts of field:

$$E(\vec{r},t) = f(x,y)\,E(z,t)$$

mode profile

field amplitude



Inclusion of Dispersion

- Expand propagation constant in power series expansion

$$\beta(\omega) = \beta_0 + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2 + \ldots \qquad \beta_m = \left.\frac{d^m k}{d\omega^m}\right|_{\omega=\omega_0}$$

propagation constant

$$\beta_0 = \frac{n_{mode}(\omega_0)\,\omega_0}{c}$$

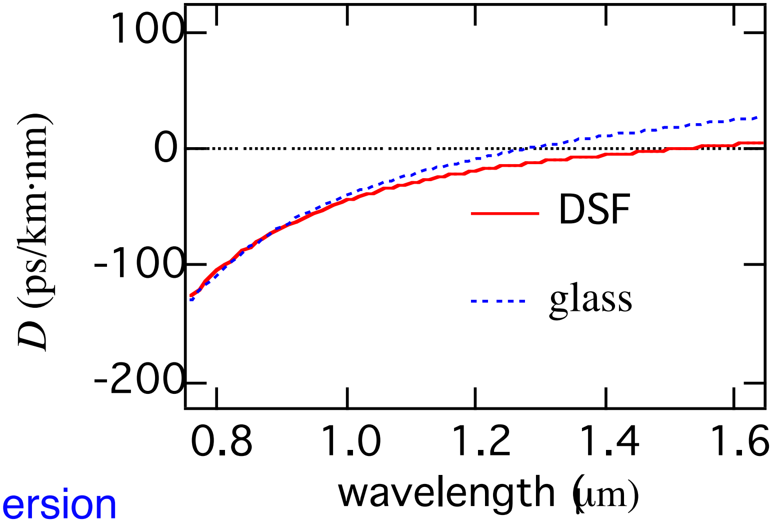
inverse of group velocity

$$\beta_1 = \left.\frac{dk}{d\omega}\right|_{\omega=\omega_0} = \frac{1}{v_{gr}}$$

group-velocity dispersion

$$\beta_2 = \left.\frac{d^2 k}{d\omega^2}\right|_{\omega=\omega_0} = -\frac{\lambda^2}{2\pi c} D$$

dispersion parameter



$D > 0$ anomalous dispersion

$D < 0$ normal dispersion

Nonlinear Schrödinger Equation

[*Nonlinear Optics*, Boyd (2009)]

$$z = z / L_{ds}$$

$$\tau = \frac{t - z / v_{gr}}{\tau_p}$$

local time

input pulse duration

$$\boxed{\frac{\partial u}{\partial z} = -\mathrm{sgn}(\beta_2)\frac{i}{2}\frac{\partial^2 u}{\partial \tau^2} + i\frac{L_{ds}}{L_{nl}}|u|^2 u}$$

dispersion

nonlinearity

$$u = \frac{A}{A_0}$$

input amplitude

dispersion length $L_{ds} = \dfrac{\tau_p^2}{|\beta_2|}$

nonlinear length $L_{nl} = \dfrac{cA_{eff}}{\omega_0 n_2 P_0}$

effective mode area

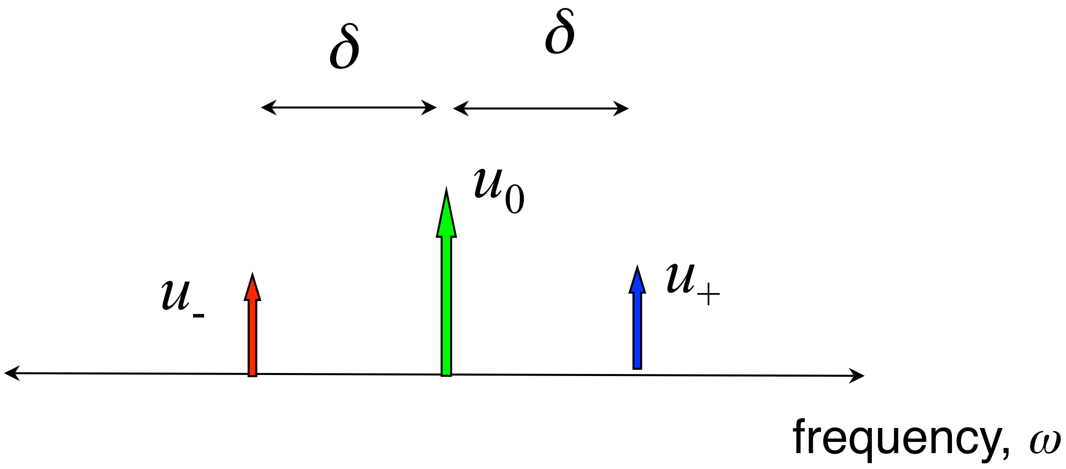
$P_0 \propto |A_0|^2$ input power

- Describes very well propagation of pulses in waveguides.
- Additional nonlinearities (e.g., Raman scattering) can be easily included.
- Addition of self-steepening allows for treatment of ~ single-cycle pulses.

Calculate Gain for Sidemodes: Modulational Instability

[*Nonlinear Optics*, Boyd (2009)]

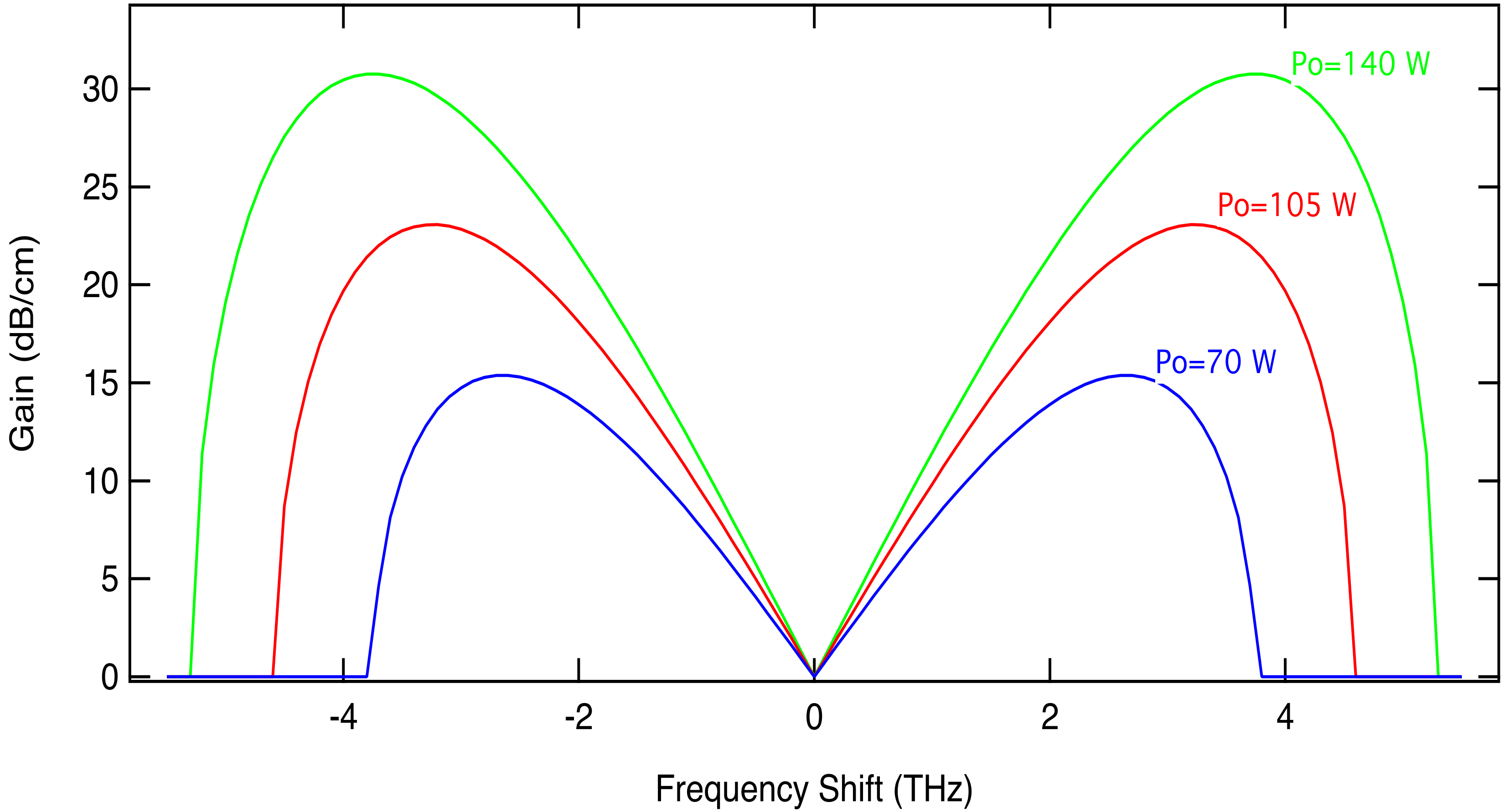
$$u(z,\tau) = u_0(z) + u_+(z)e^{-i\delta\tau} + u_-(z)e^{i\delta\tau}$$



- Derive gain (i.e., assume $u_\pm \sim e^{\lambda z}$) by looking for conditions under which Real(λ) > 0.

Phase-Matched Four-Wave Mixing

- Peak gain occurs where $\Delta k = 0$ ($D > 0$, i.e., anomalous GVD)

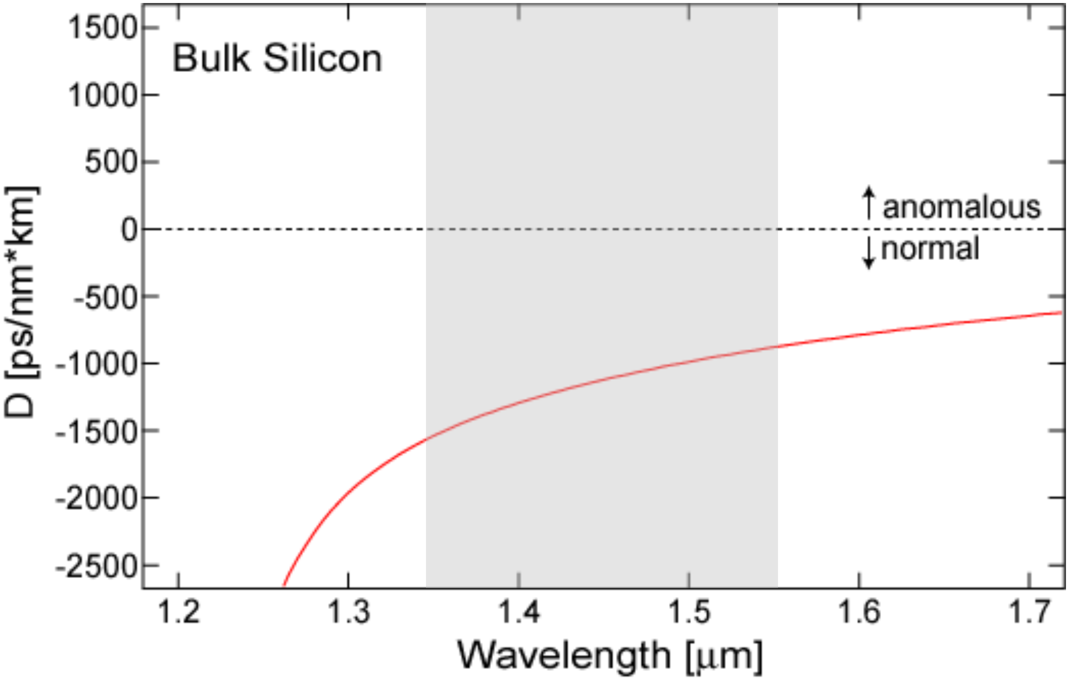


Challenge: Large Normal GVD of Si

- Bulk Silicon
  - absorption band edge @ 1.1 μm
  - Si @ 1.55 μm: ***D* ~ − 1000 ps/(nm*km)**

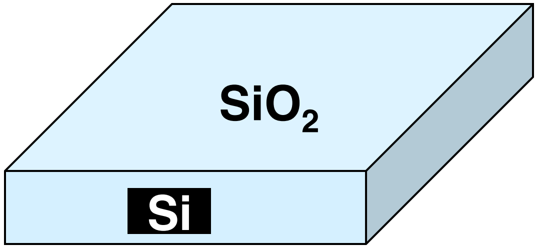
    [silica glass @ 1.5 μm: ***D* ~ 20 ps/(nm*km)**]

GVD



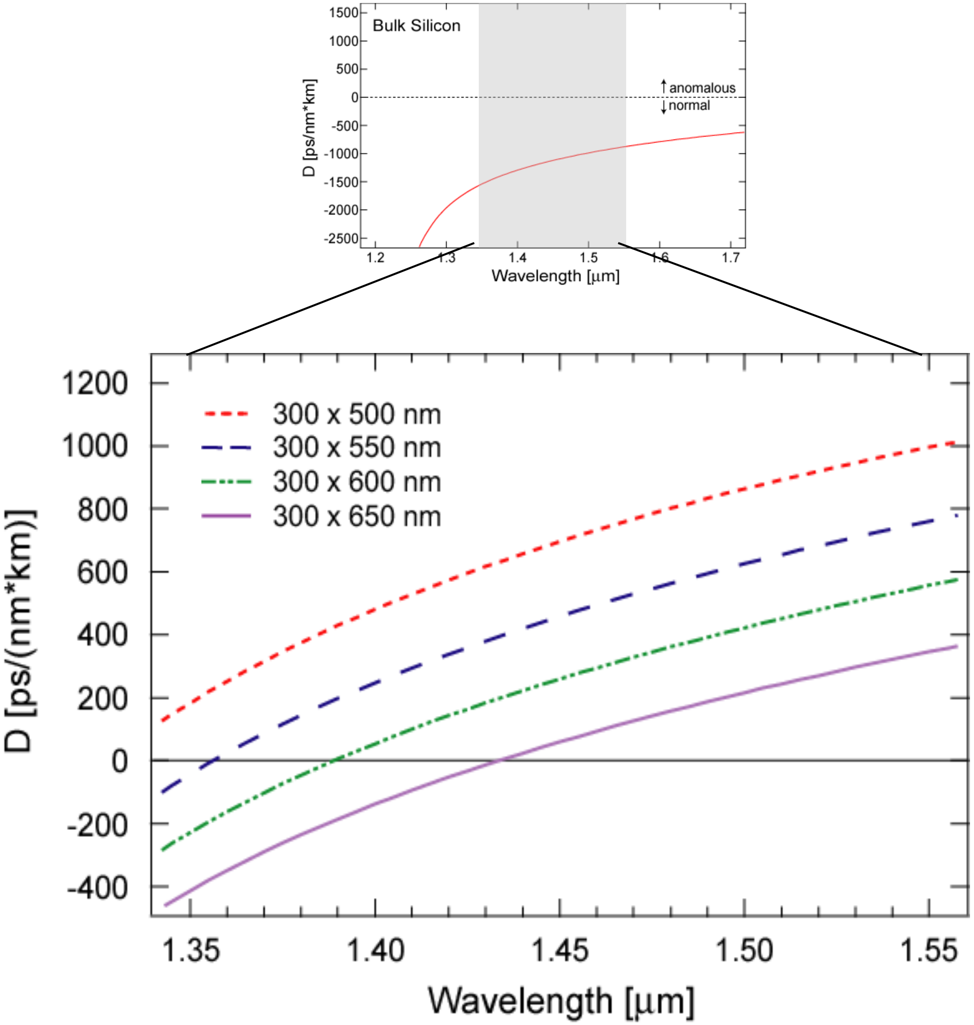
Tailoring of GVD in Si Waveguides

- Utilize waveguide dispersion to tune GVD.



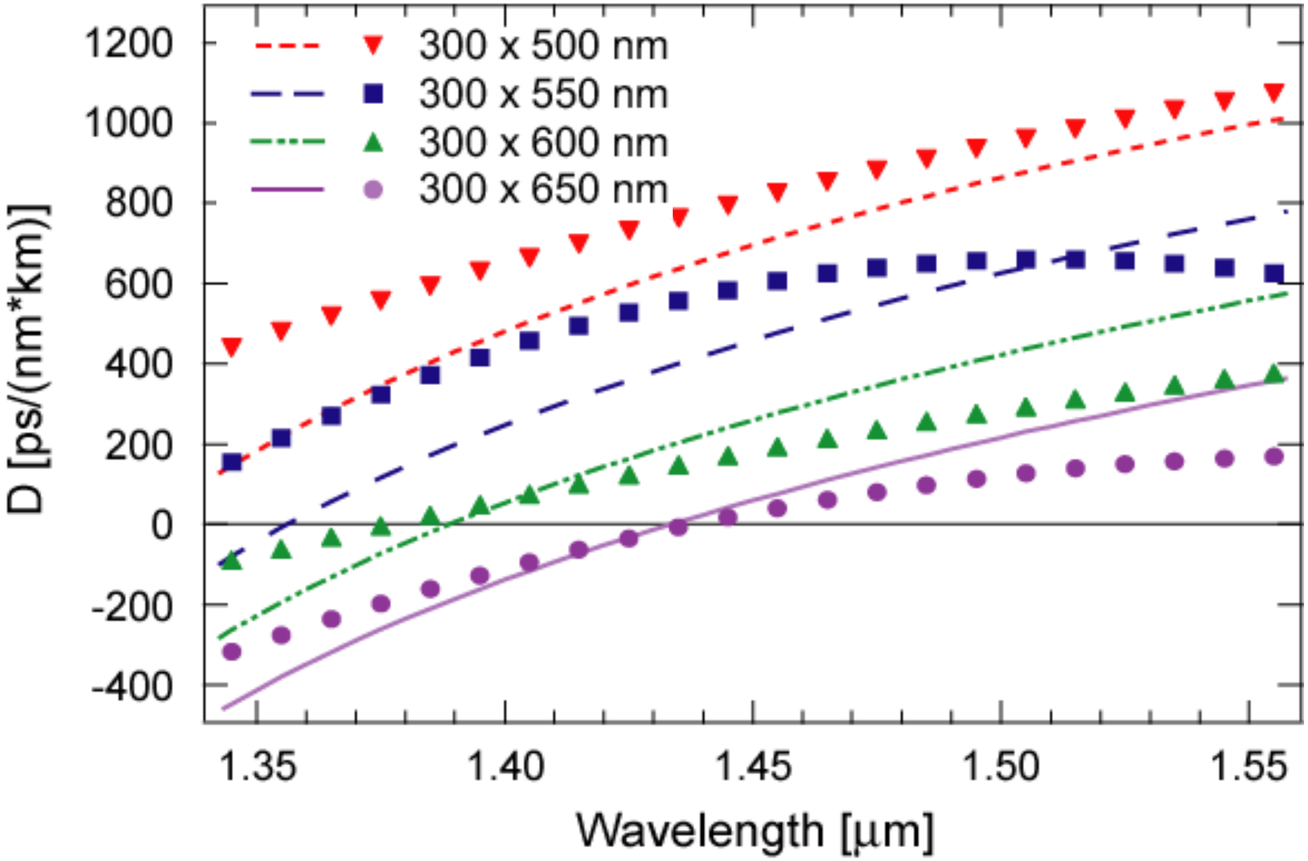
- GVD can be tuned by varying waveguide shape and size.

Turner et al. (2006)
Lin et al. (2006)



Predicted anomalous-GVD ~50X SMF-28 fiber [20 ps/(nm·km)].

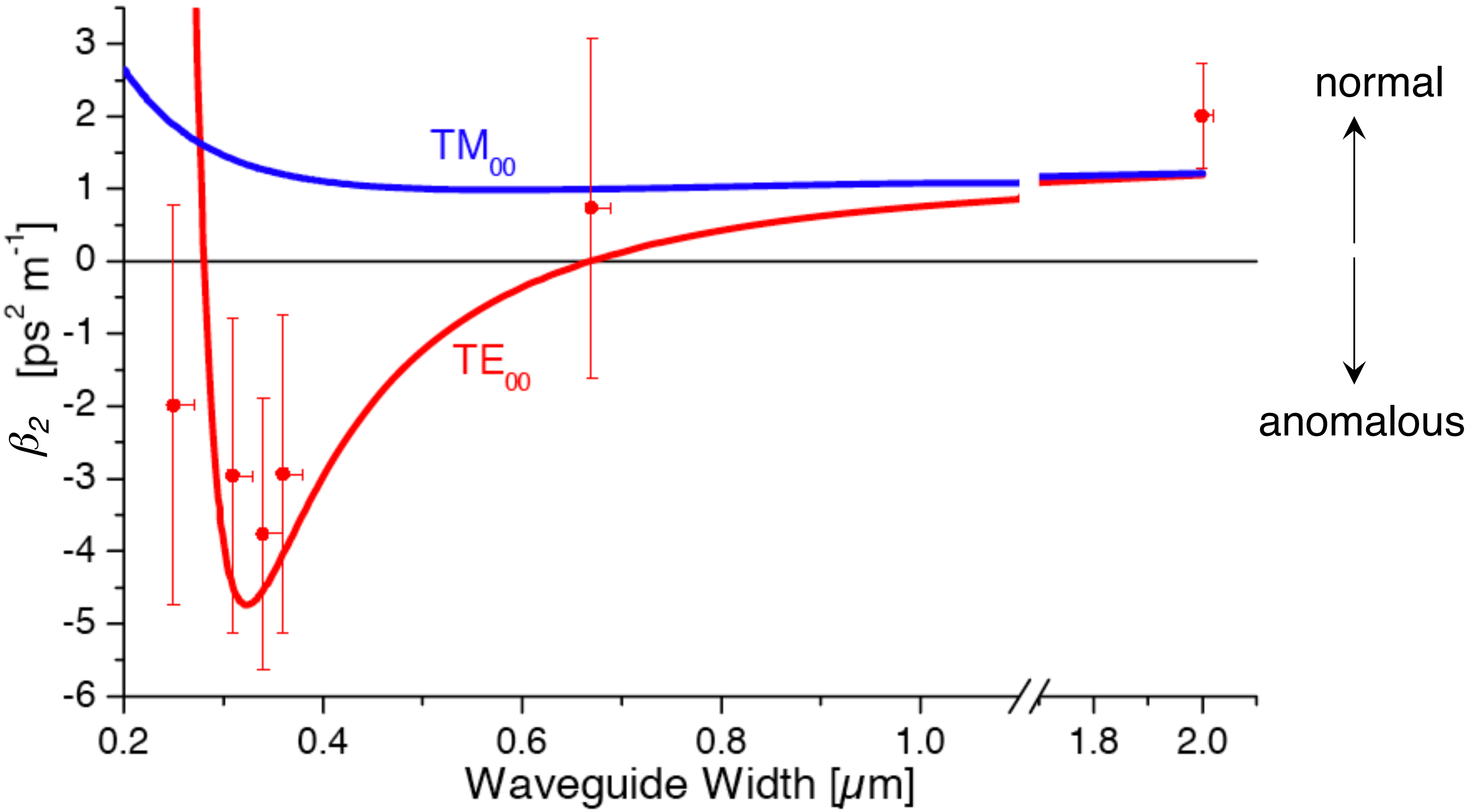
Measurement of Anomalous-GVD in Si Waveguides



Turner, Manolatou, Schmidt, Lipson, Foster, Sharping, and Gaeta, *Opt. Express* **14**, 4357 (2006).

Dulkeith, Xia, Schares, Green, and Vlasov, *Opt. Express* **14**, 3853 (2006).

Dispersion Control in AlGaAs Waveguides

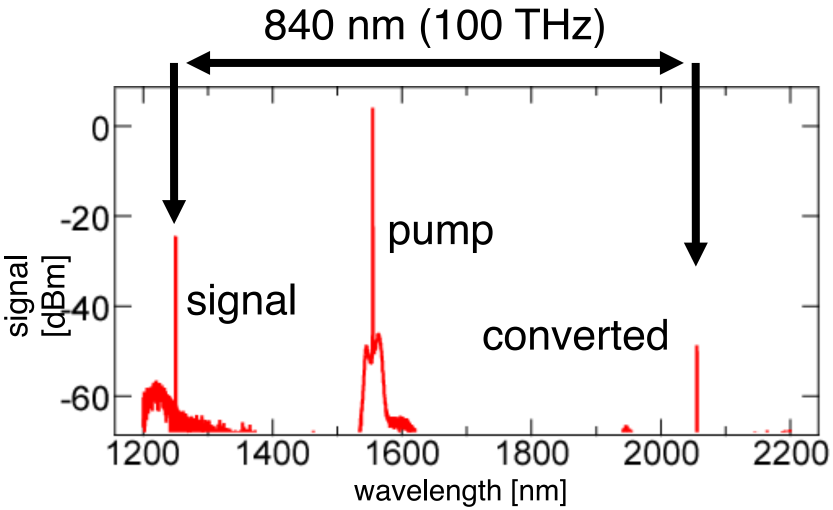


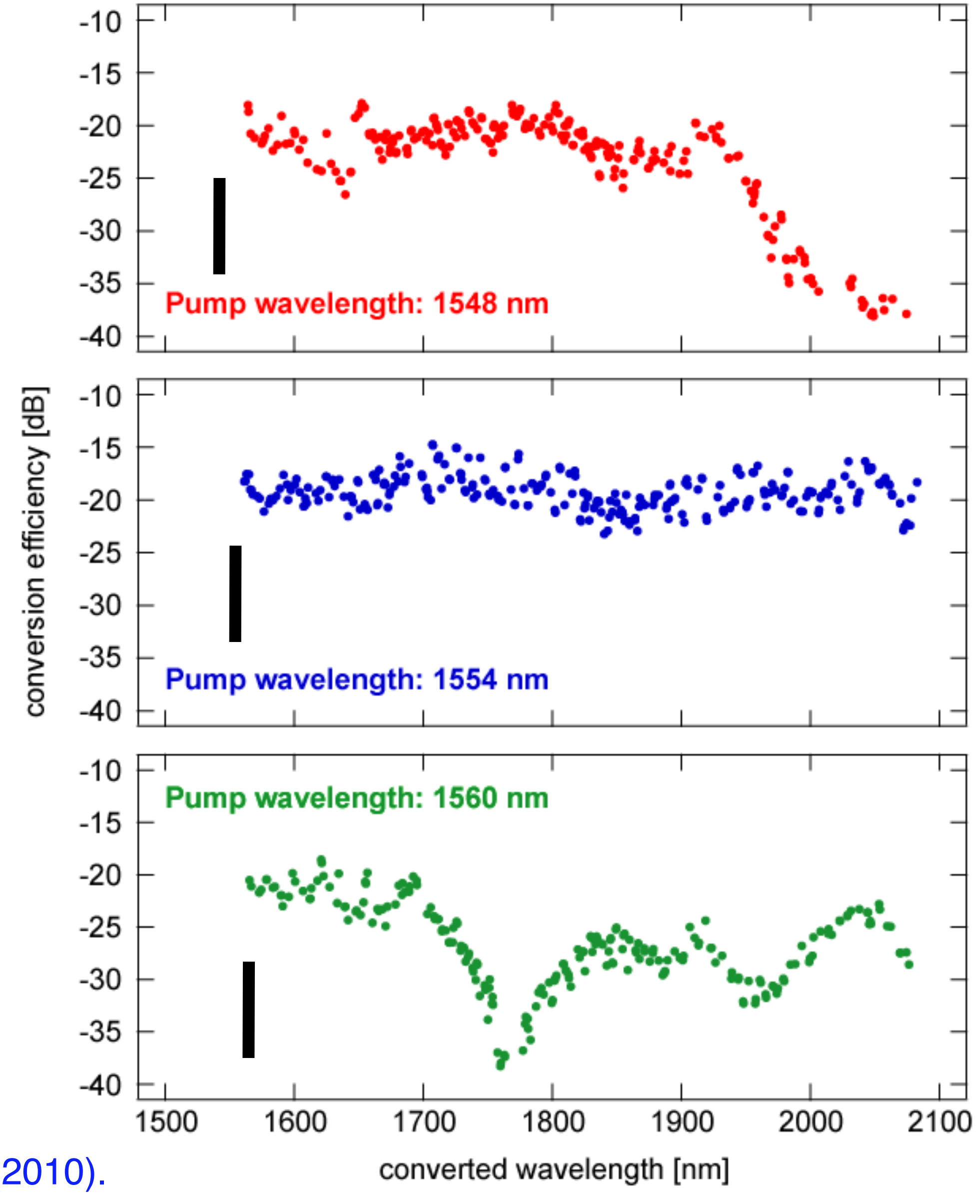
Meier, Mohammed, Jugessur, Qian, Mojahedi, and Aitchison, *Opt. Express* **15**, 12755 (2007).

CW Wavelength Conversion over 900-nm Bandwidth

- Pump: 50 mW

- **Conversion bandwidth > 2/3 of an octave!**

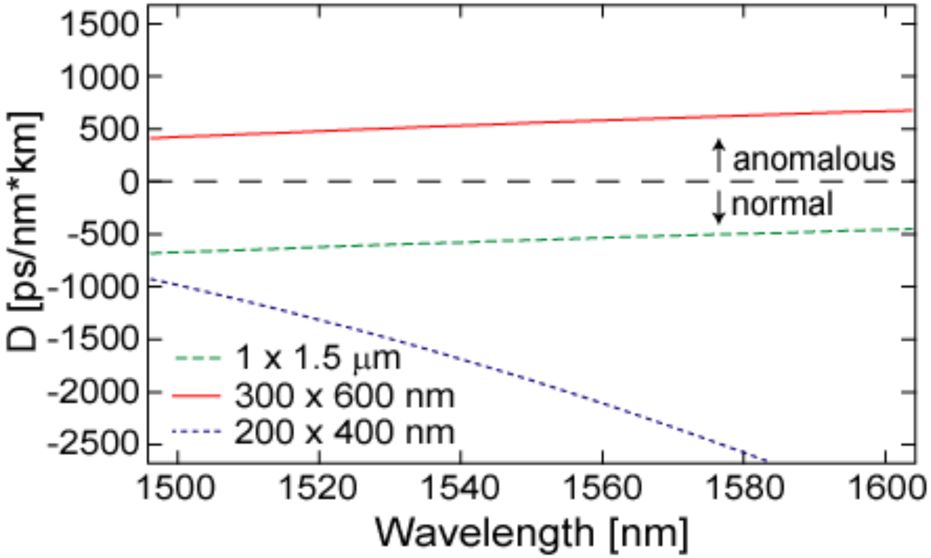


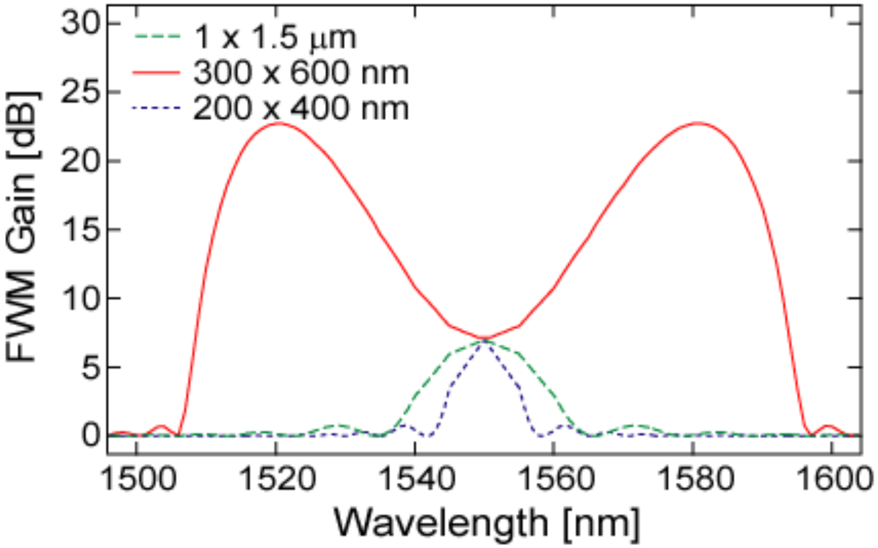


Turner, Lipson, Foster, and Gaeta, *Opt. Express* (2010).

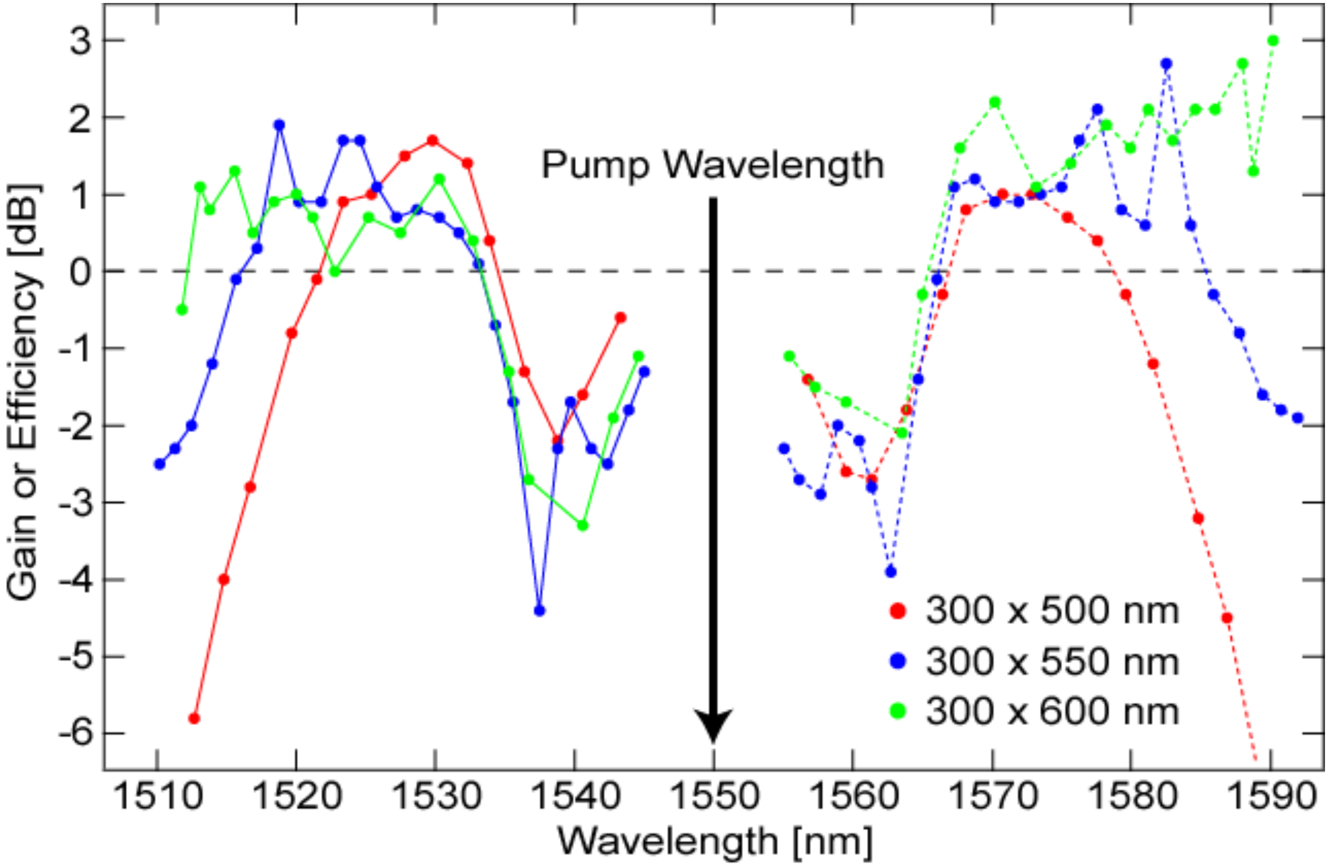
Phase-Matched Four-Wave Mixing in Si

- Broad regions of FWM gain predicted.





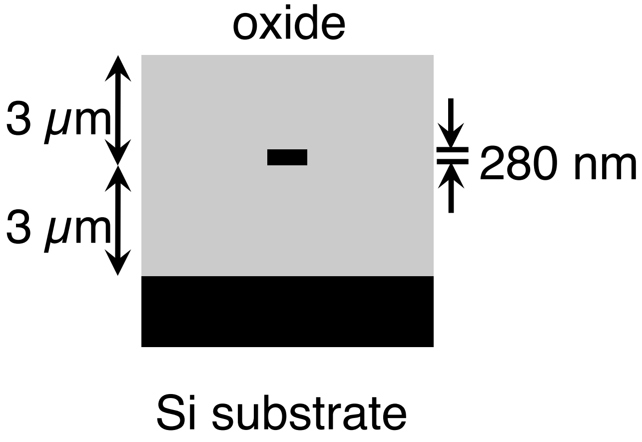
Four-Wave Mixing Amplification



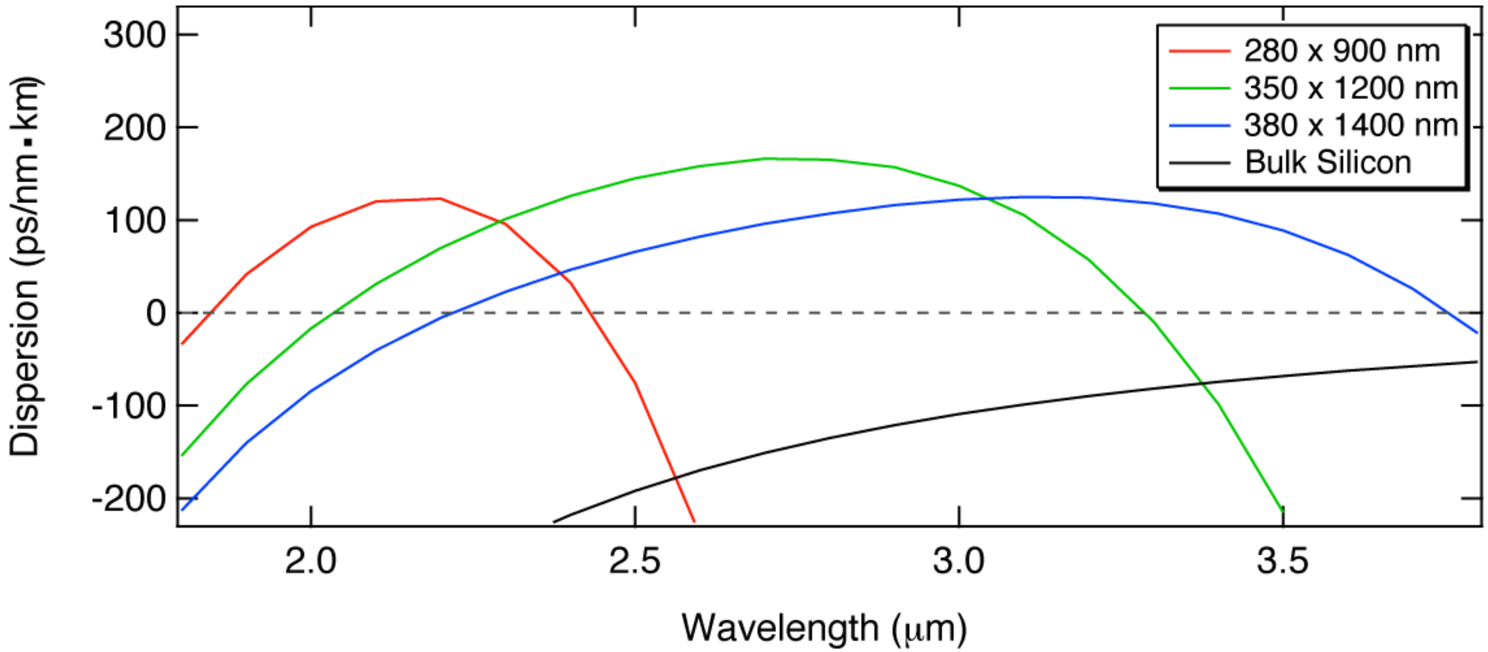
- First observation of broadband gain in Si. (Raman gain bandwidth ~ 1 nm)

Foster, Turner, Sharping, Schmidt, Lipson, and Gaeta, *Nature* **441**, 960 (2006).

Dispersion Engineering into Mid-IR

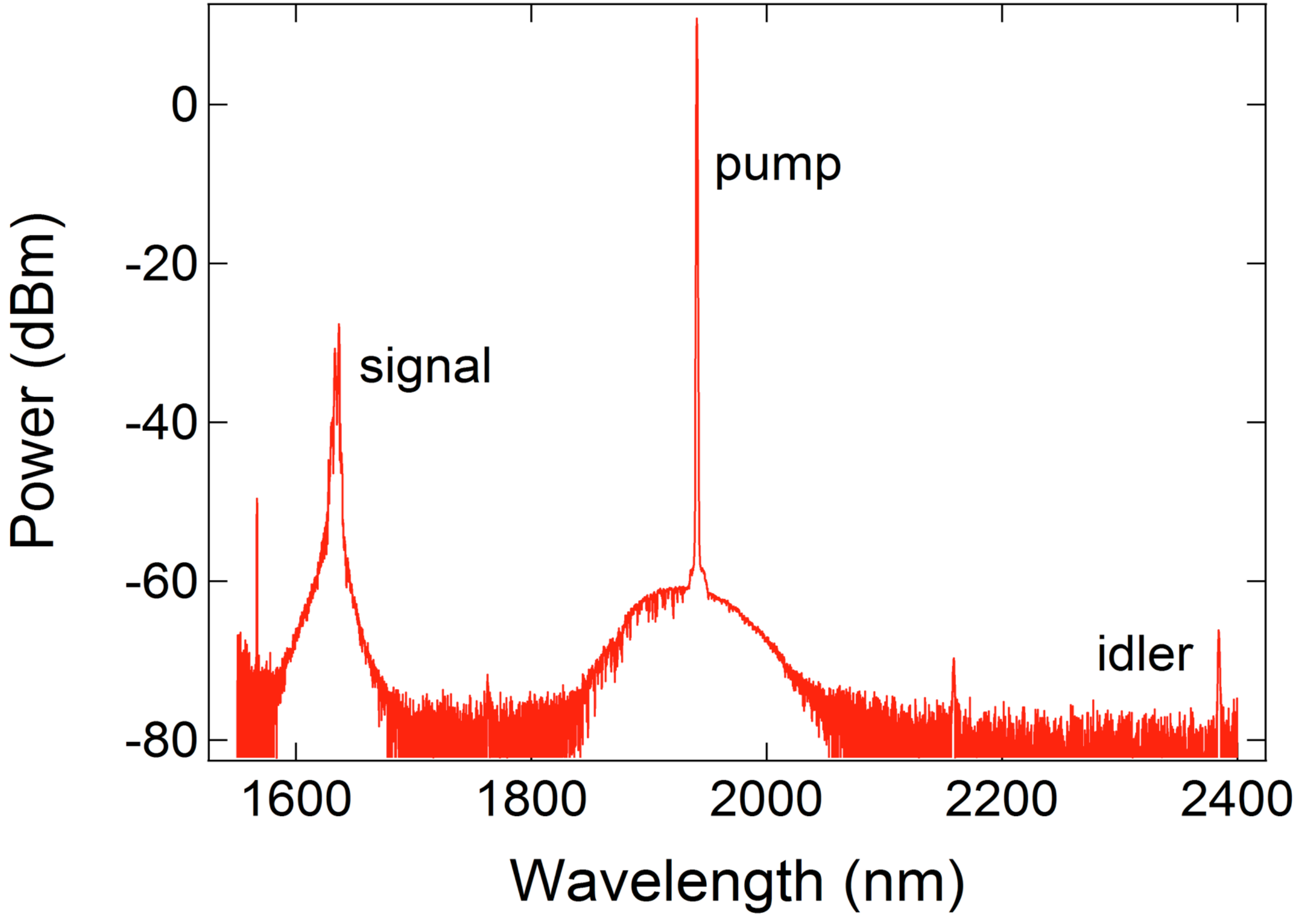


- Broadband anomalous GVD in MIR.



Mid-IR Frequency Conversion

Lau, et al., Lipson, and Gaeta, *Opt. Lett.* (2011).



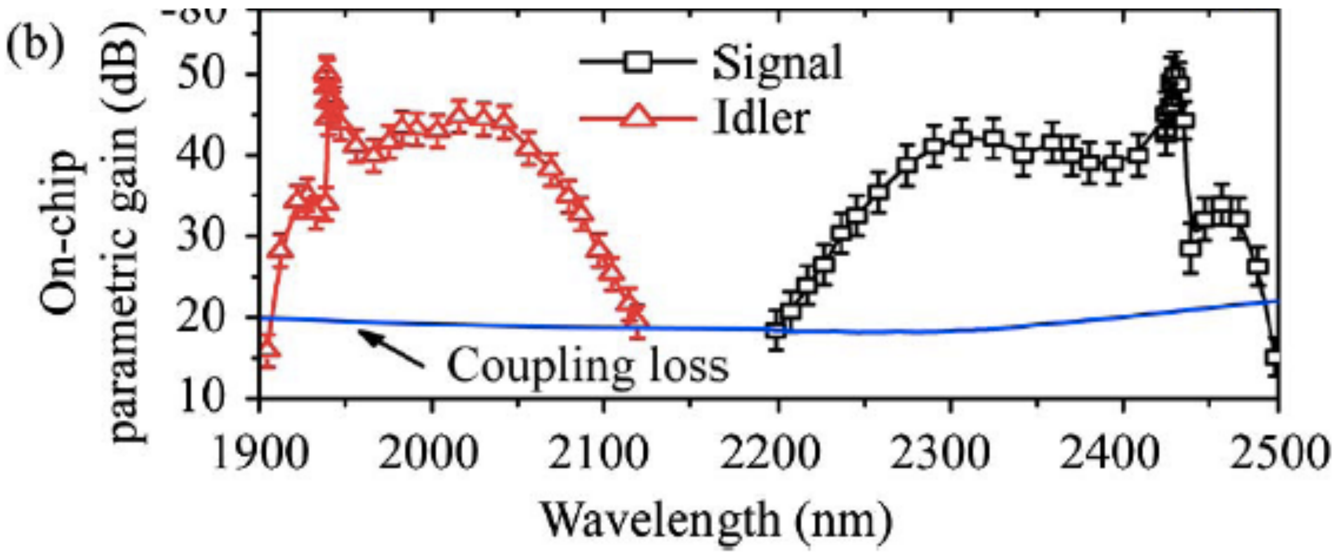
- Pulsed conversion (w/ gain) [Zlatanovic et al. (2010); Kuyken et al. (2010, 2011).]
- Need other cladding materials (e.g., sapphire, SiN) for longer MIR wavelengths [Baehr-Jones et al. (2010)]

Four-Wave Mixing Amplification in Mid-IR

November 15, 2011 / Vol. 36, No. 22 / OPTICS LETTERS 4401

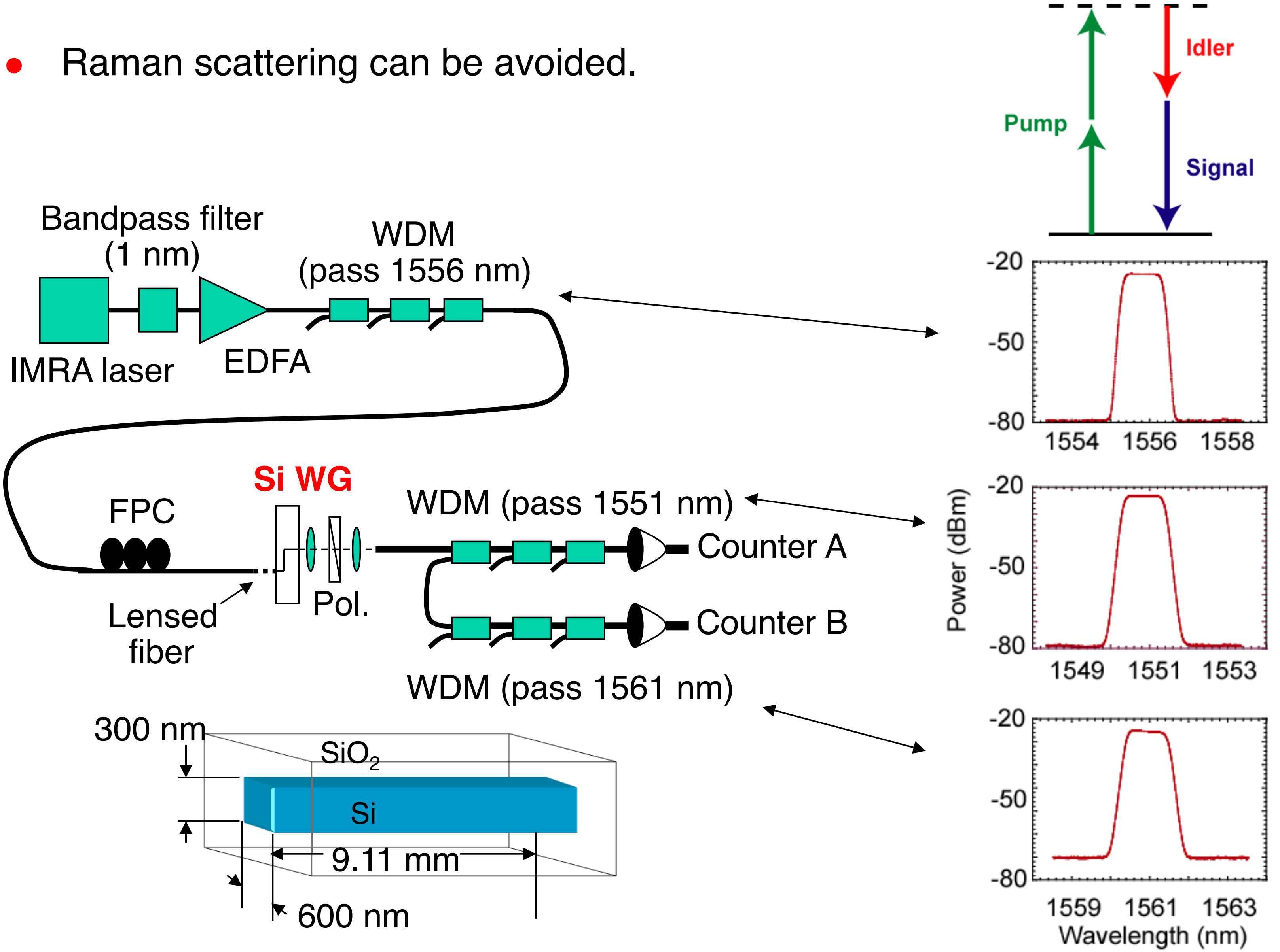
50 dB parametric on-chip gain in silicon photonic wires

Bart Kuyken,[1,†] Xiaoping Liu,[2,4,†] Günther Roelkens,[1] Roel Baets,[1]
Richard M. Osgood, Jr.,[2] and William M. J. Green[3,*]
[1]*Photonics Research Group, Department of Information Technology, Ghent University—imec, Ghent B-9000, Belgium*
[2]*Department of Electrical Engineering, Columbia University, 1300 S. W. Mudd Building, 500 W. 120th Street, New York, New York 10027, USA*
[3]*IBM Thomas J. Watson Research Center, 1101 Kitchawan Road, Yorktown Heights, New York 10598, USA*
[4]*Current address: OFS Labs, 19 Schoolhouse Road, Somerset, New Jersey 08873, USA*
**Corresponding author: wgreen@us.ibm.com*

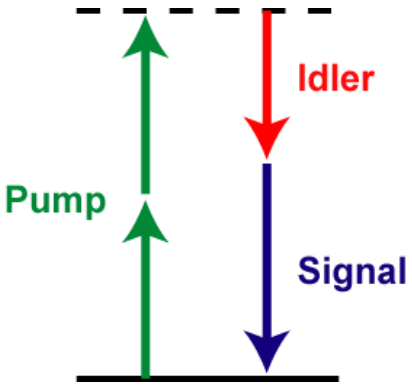


Application: Chip-Based Source for Correlated Photons

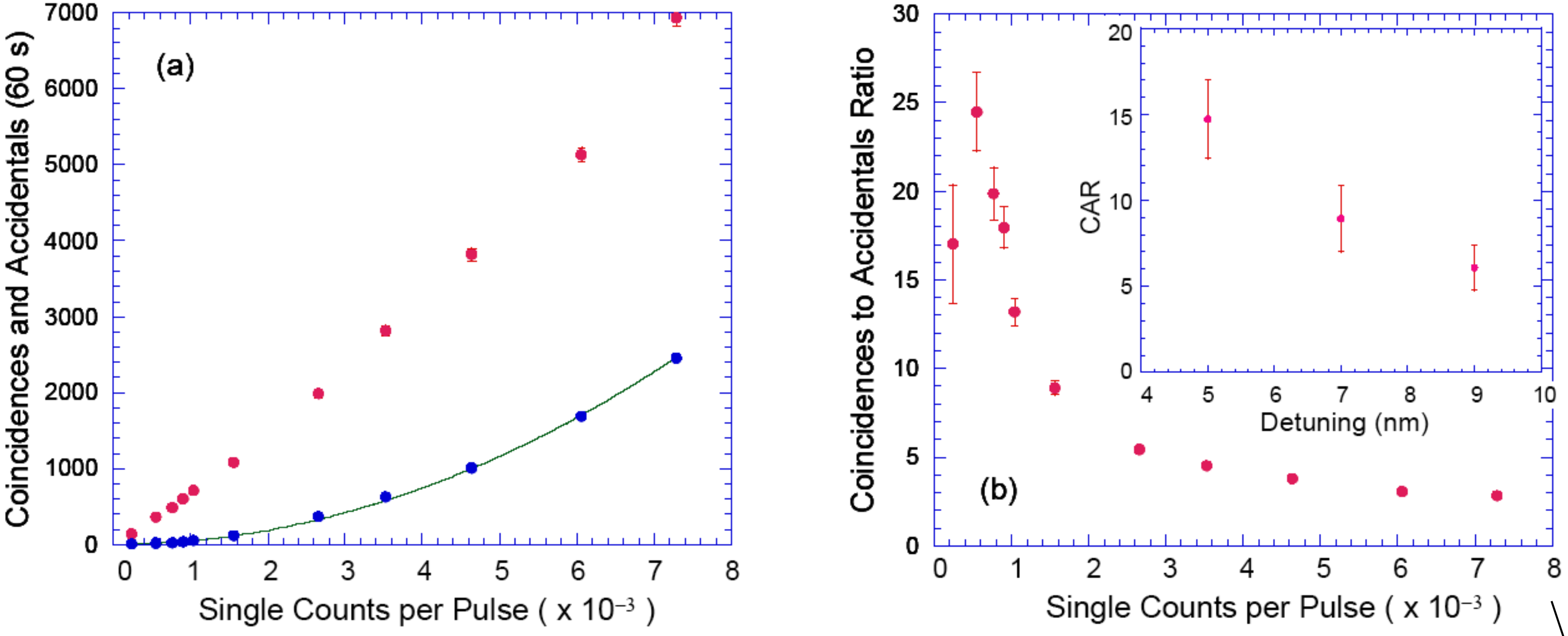
- Raman scattering can be avoided.



Generation of Correlated Photons in Si



- Results show good quantum characteristics.
- Raman scattering can be avoided.



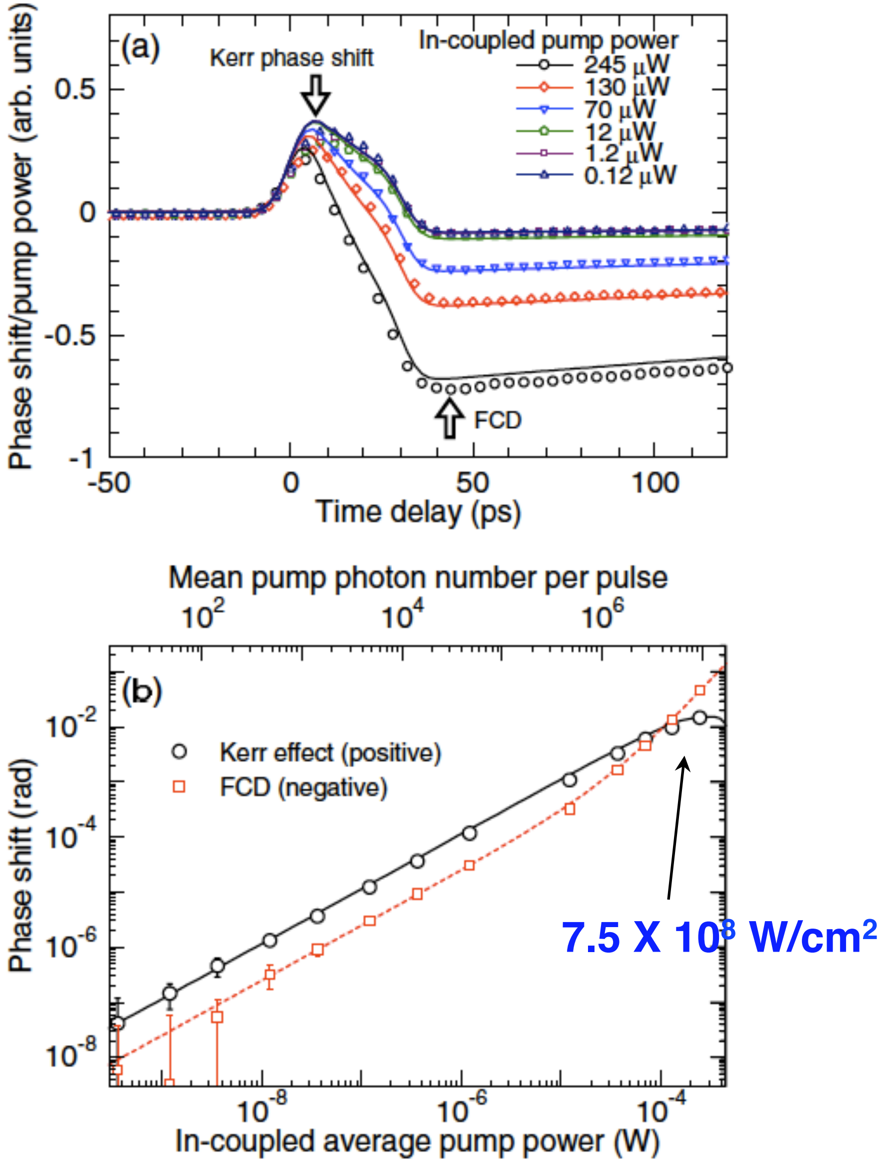
10^9 W/cm^2

Sharping, Lee, Foster, Turner, Lipson, Gaeta, and Kumar, *Opt. Express* **14**, 12388 (2007),

Contribution of Free-Carriers Spoils Correlations

- Plasma dispersion effect leads to generation of blue photons.
- $\chi^{(3)}$ due to free-carriers comparable to electronic for peak intensities ~ 7.5 X 10^8 W/cm^2
- Performance can be improved by incorporating p-i-n structure.

Engin, *et al.*, Opt. Express. **21**, 27826 (2013).



Matsuda, *et al.*, Appl. Phys. Lett. **95**, 171110 (2009).

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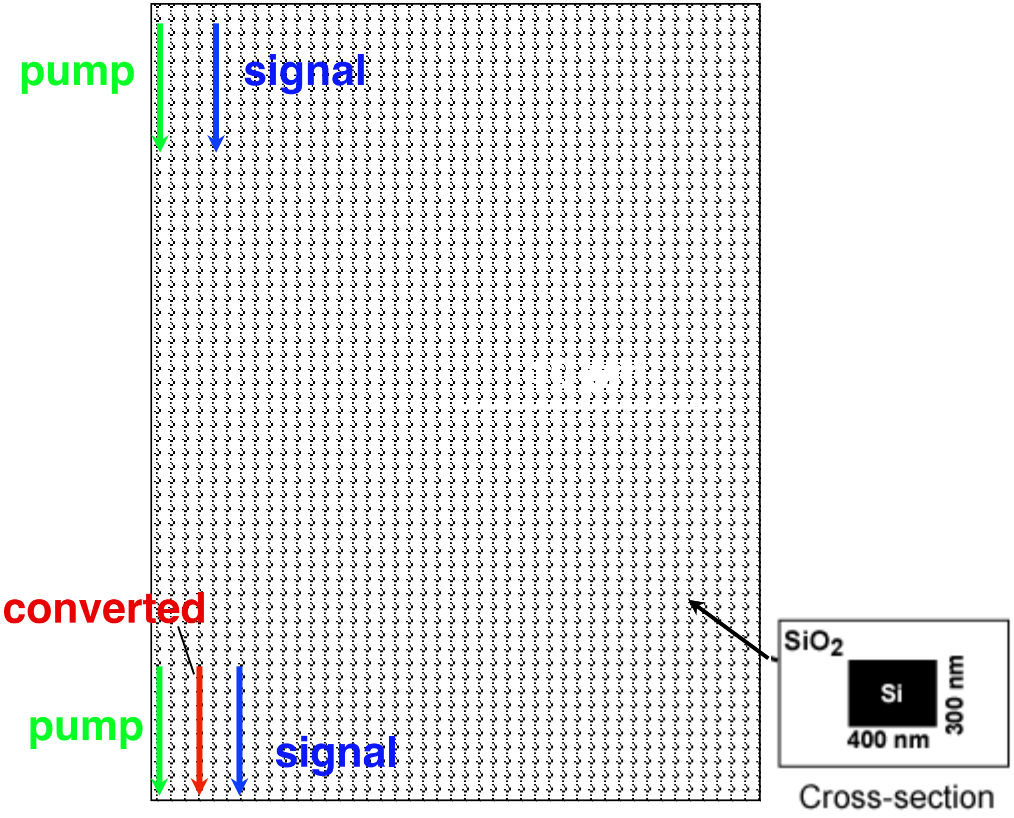
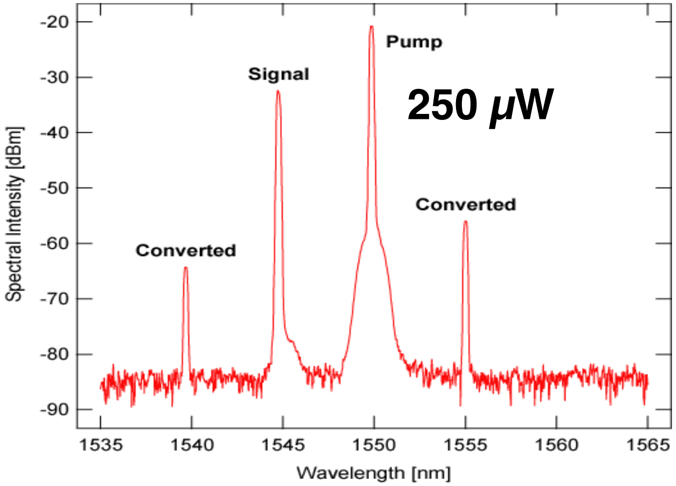
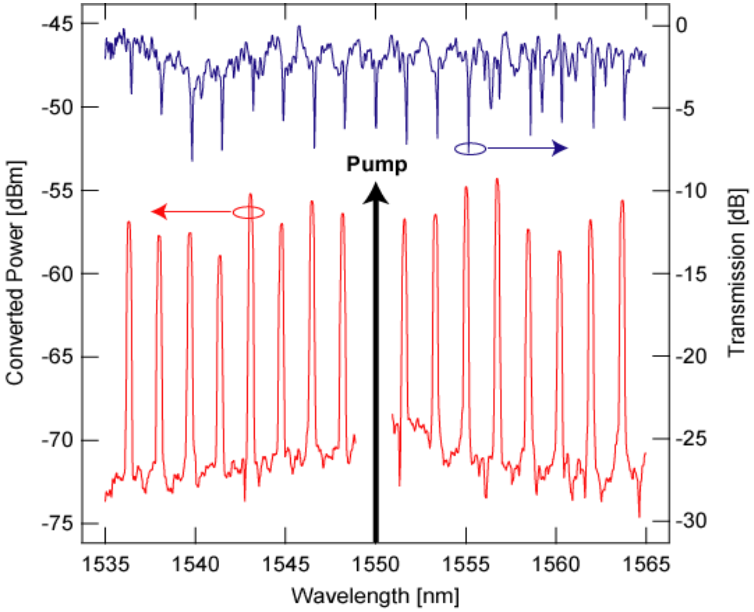
FWM in Silicon Microresonators: Ultralow Power Frequency Conversion

- Use ring resonator to enhance efficiency of FWM.
- Frequency conversion: < **mW** cw powers.

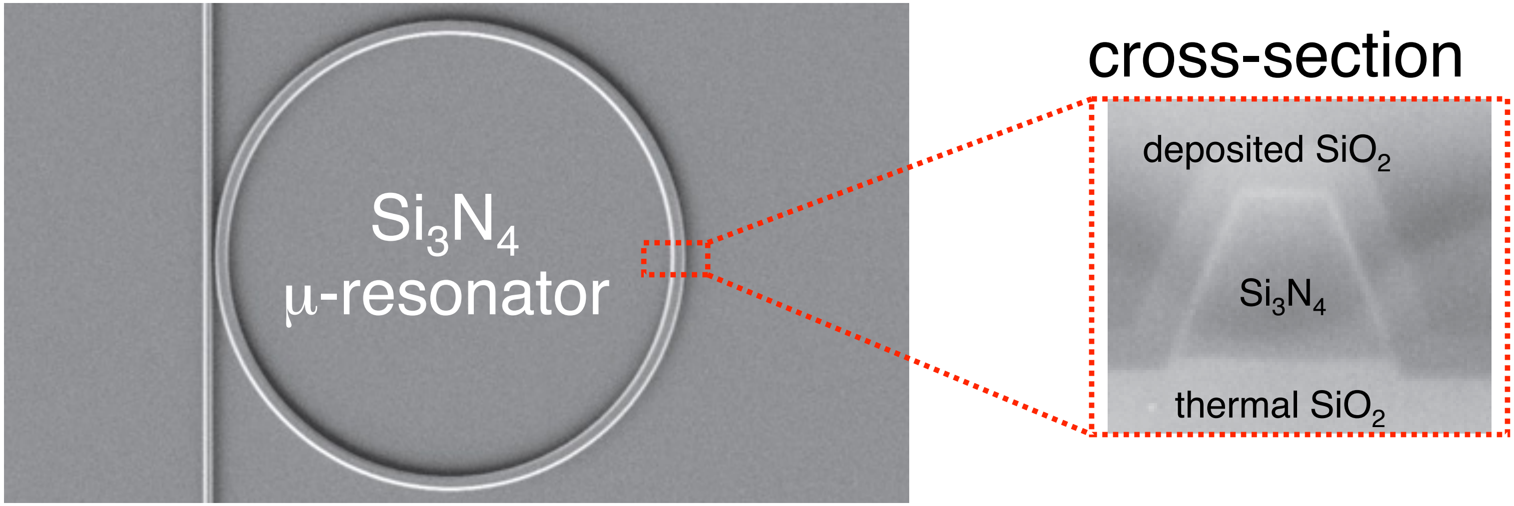






Turner, Foster, Gaeta, and Lipson, *Opt. Express* (2008).

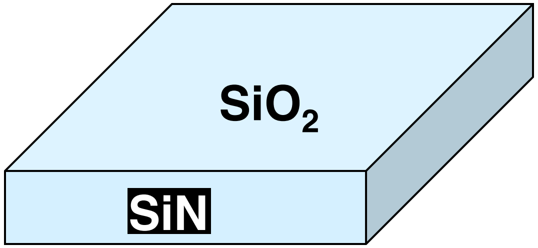
Chip-Based Silicon Nitride Microrings for Parametric Oscillators

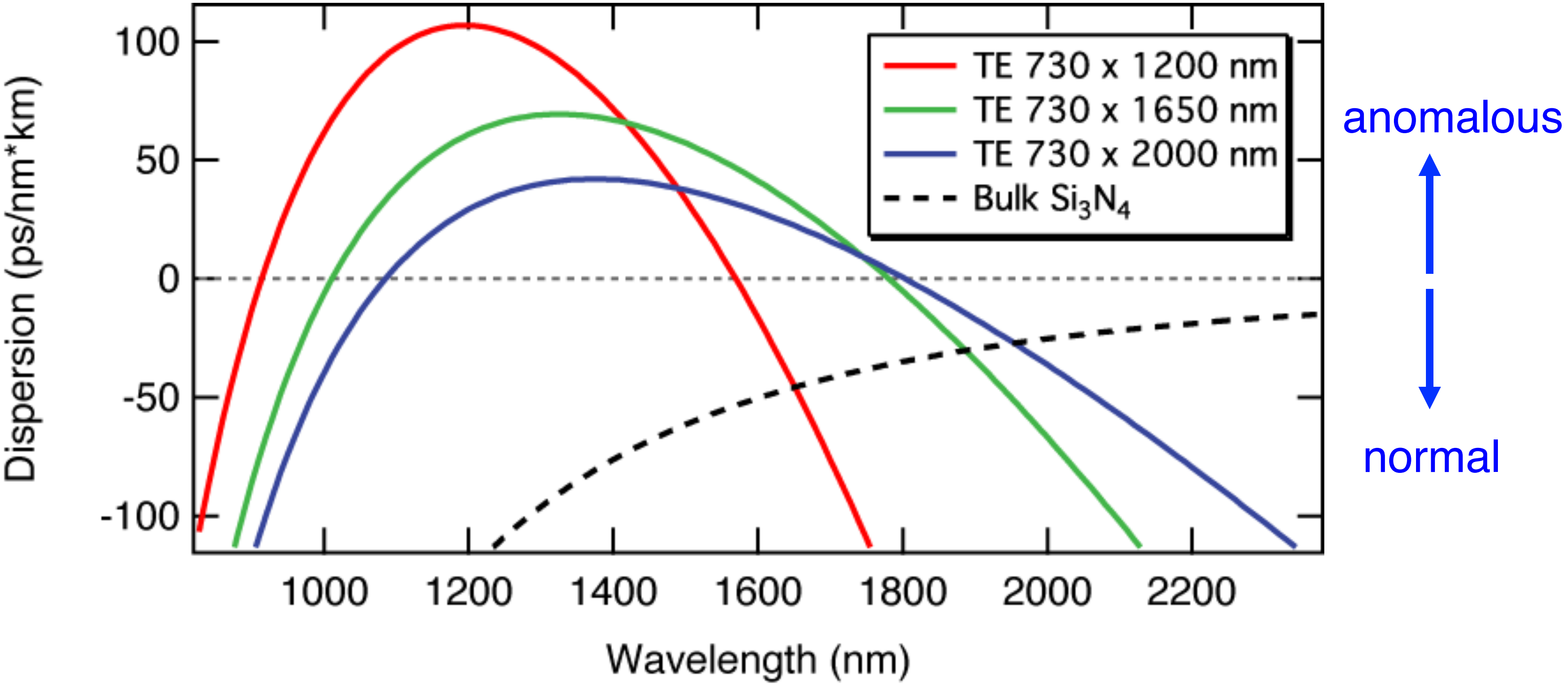


- CMOS-compatible material
- Fully monolithic and sealed structures and couplers
- No 2-photon absorption
- High-Q resonators → $Q = 3\times10^6$ [Gondarenko, et al., *Opt. Express* (2009).]
- High nonlinearity → $n_2 \sim 10\times$ silica [Ikeda, et al., Opt. Express (2008).]
- Waveguide dispersion can be engineered [Turner-Foster, et al., *Opt. Express* (2006); Tan, Ikeda, Sun, and Fainman, Appl. Phys. Lett. (2010).]

Tailoring of GVD in Silicon-Nitride Waveguides

- GVD can be tuned by varying waveguide shape and size.
- Same chip can operate w/ different pump wavelengths.

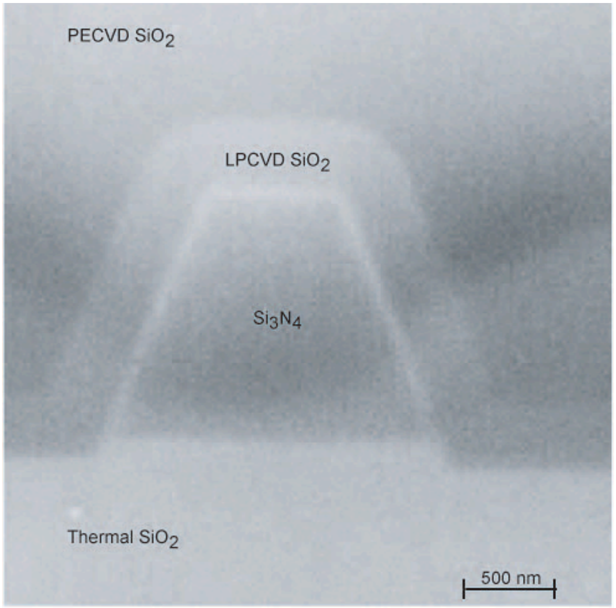


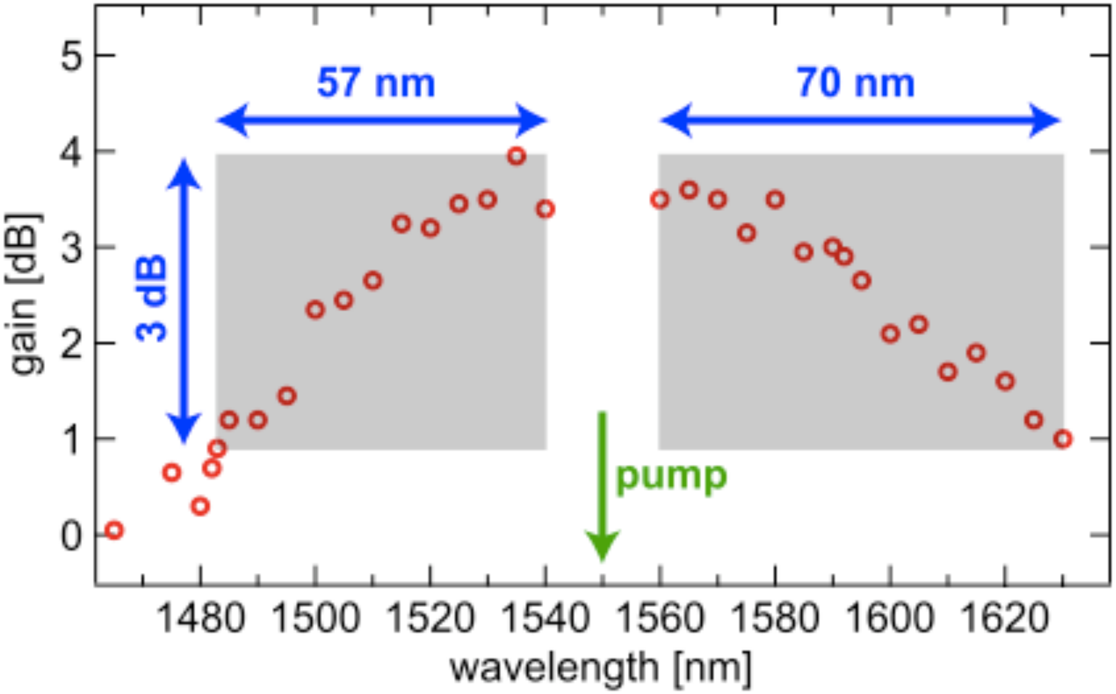


Foster, Turner, Sharping, Schmidt, Lipson, and Gaeta, *Nature* **441**, 960 (2006).
Turner-Foster, Foster, Salem, Gaeta, and Lipson, *Opt. Express* **18**, 1904 (2010).

FWM Gain in Si Nitride

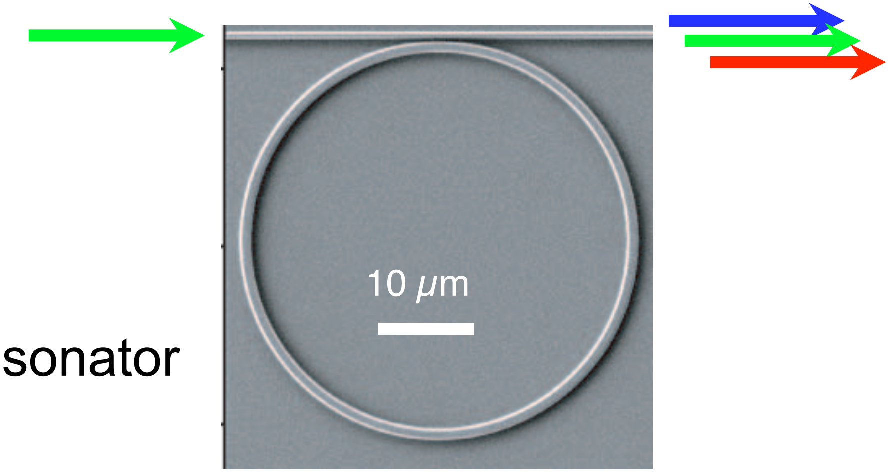
- Dispersion engineered for anomalous GVD.
- 6-cm-long waveguide.



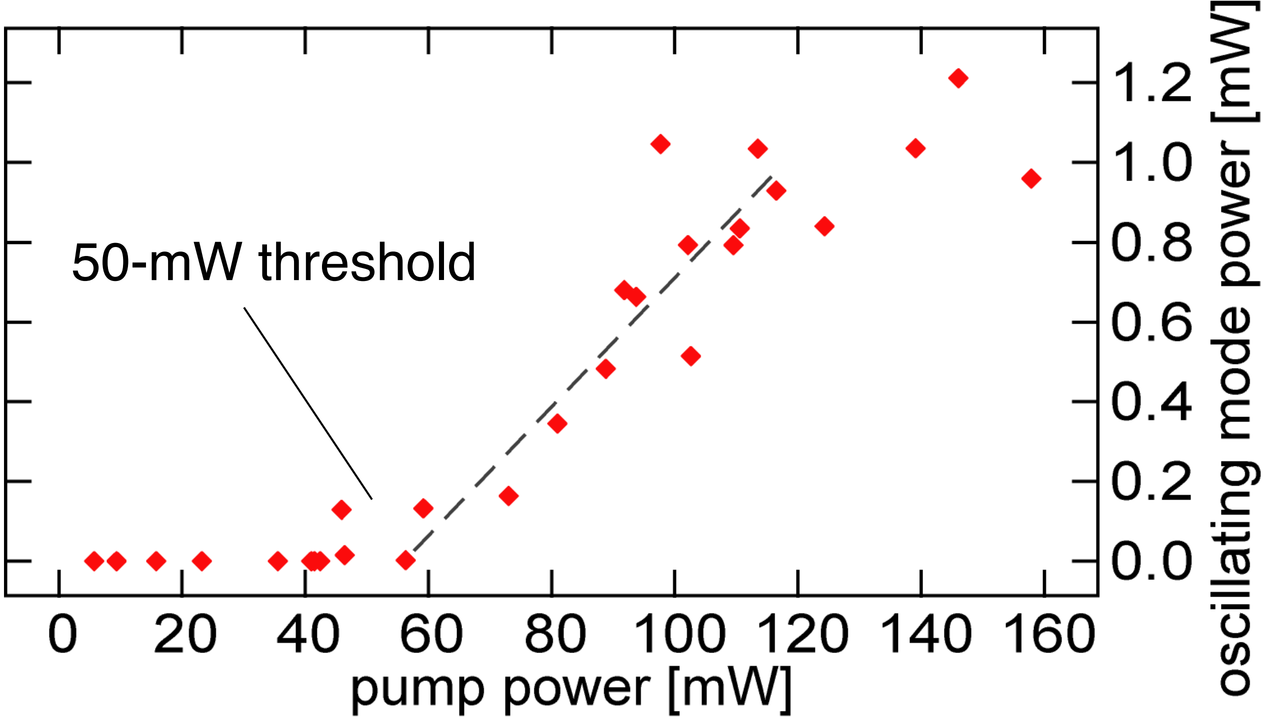


Threshold for Oscillation in SiN Microring

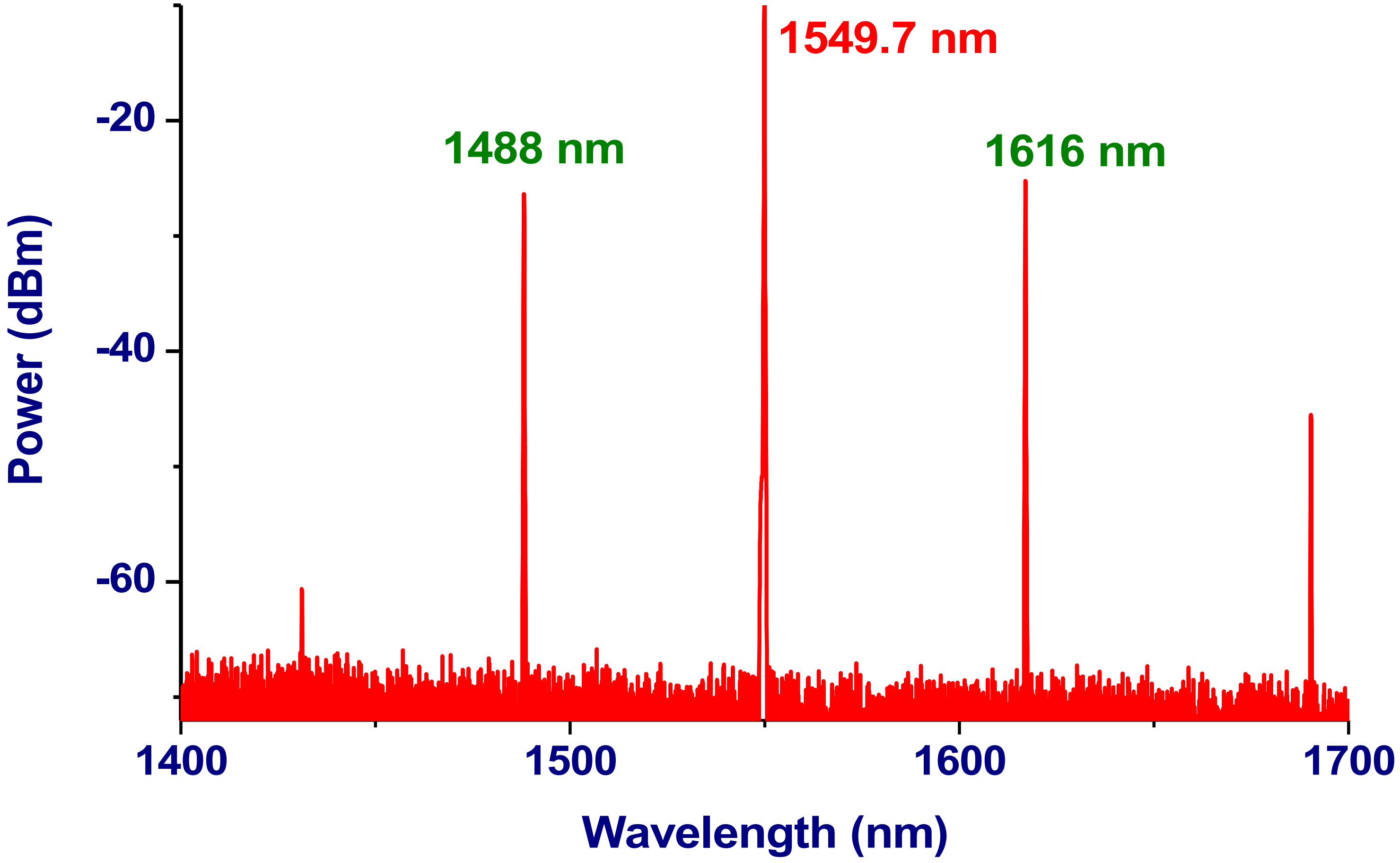
continuous-wave
pump



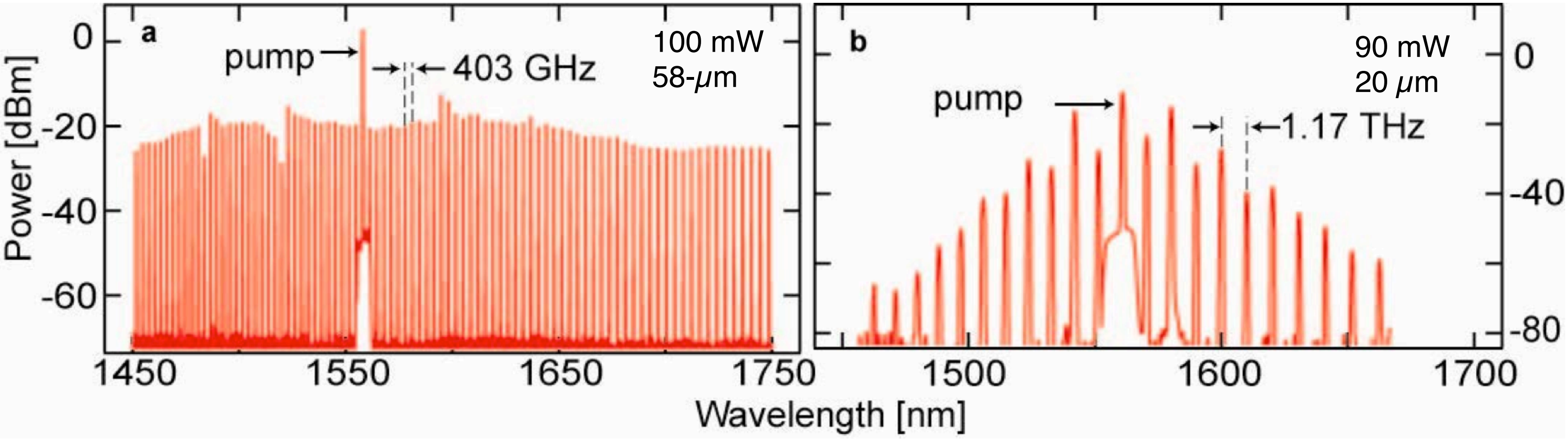
microresonator

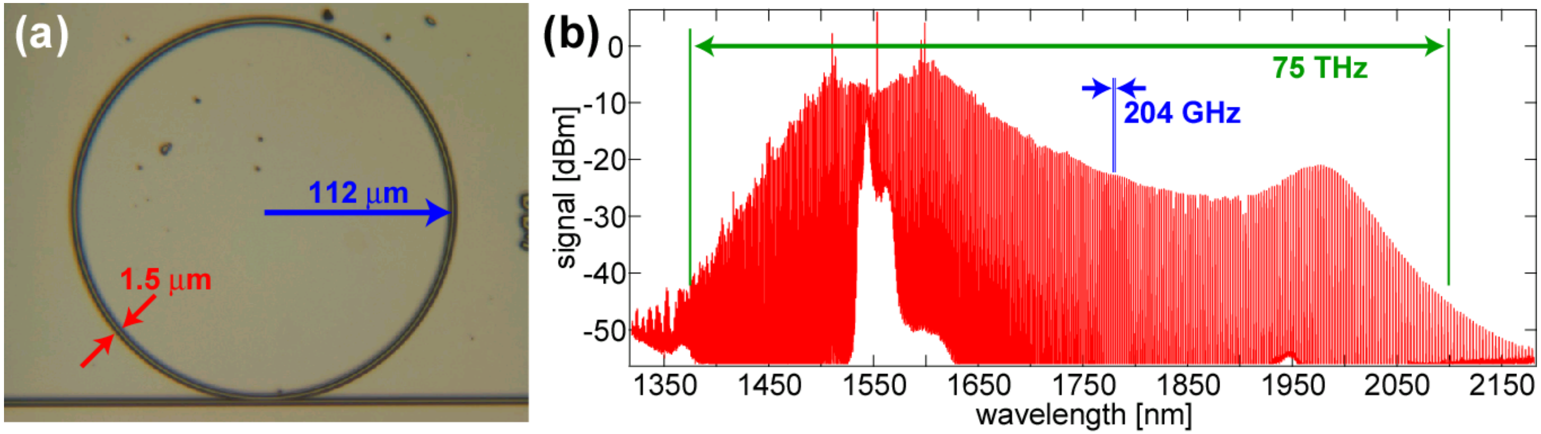


Triply-Resonant OPO – Near Threshold



Chip-Based FWM Frequency Comb





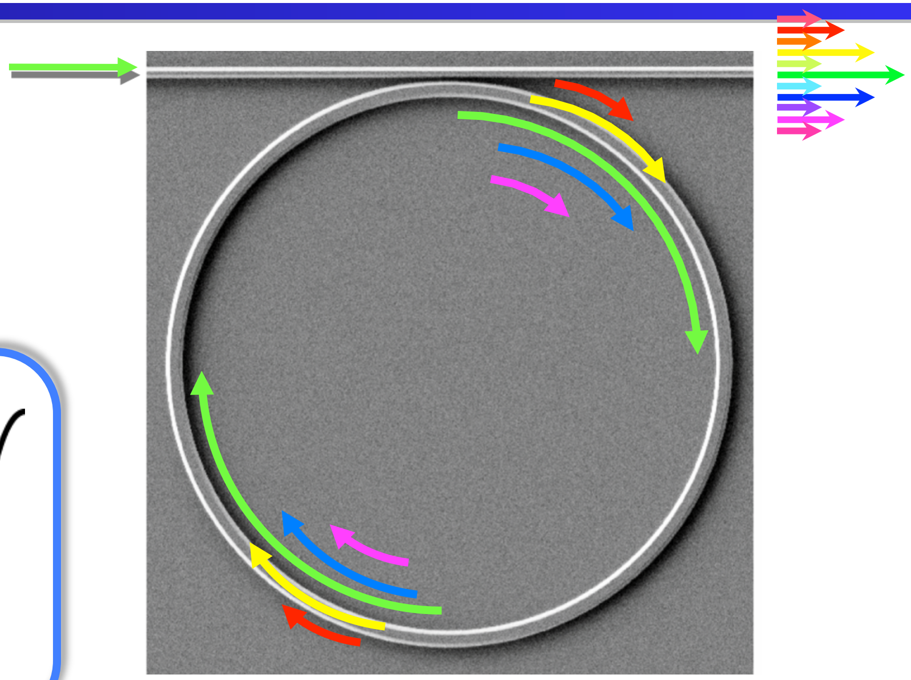
- Octave-spanning comb possible with suitable waveguide design and sufficiently high powers (~ 500 mW).

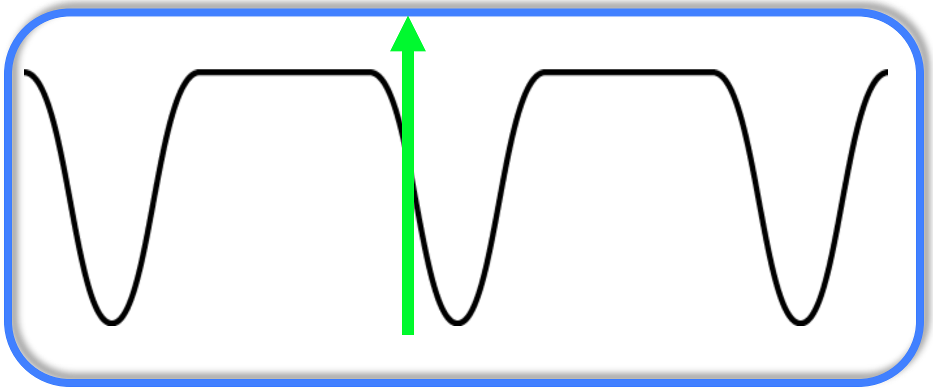
Levy, Gondarenko, Foster, Turner-Foster, Gaeta, and Lipson, *Nature Photon.* **4**, 37 (2010).

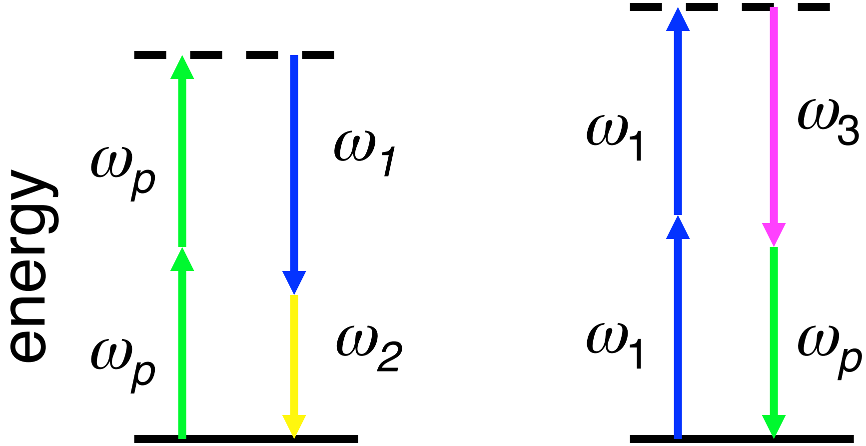
Frequency Comb Generation

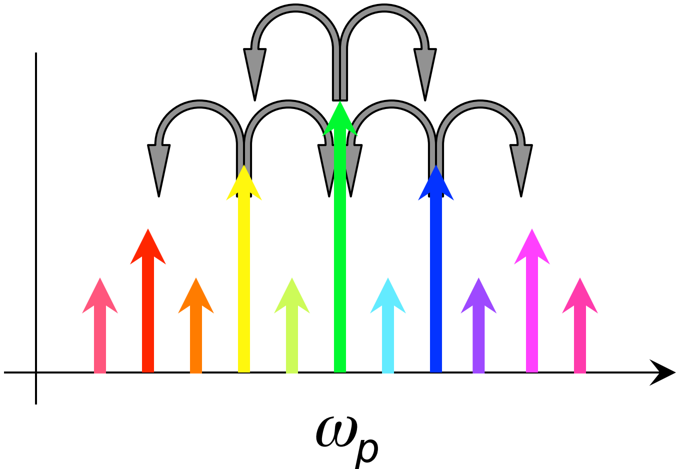
- Single input wavelength
- Parametric gain > loss

→ parametric oscillation

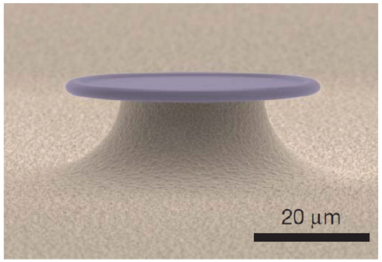






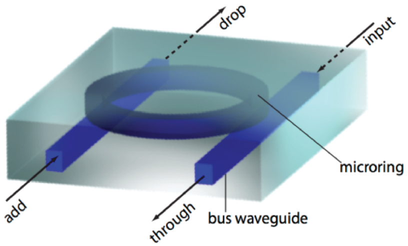


Microresonator-Based Parametric Combs



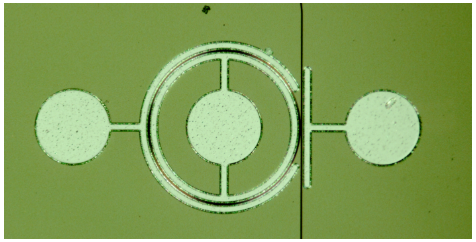
silica μ-toroids

Del' Haye *et al.*, Nature (2007).
Del' Haye *et al.*, PRL (2008).



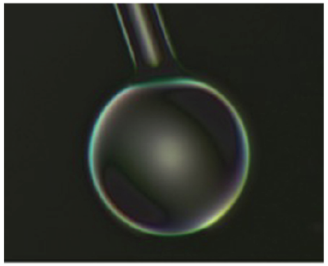
high-index glass μ-rings

Razzari *et al.*, Nature Photon. (2010).
Pasquazi *et al.*, Opt. Express (2013).



Silicon

Griffith *et al.*, (2014).

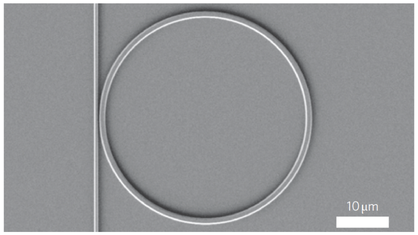


silica μ-spheres

Agha *et al.*, PRA (2007).
Agha *et al.*, Opt. Express (2009).

Silica disks

Li *et al.*, PRL (2012).

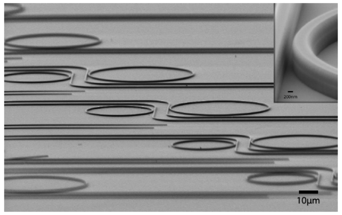


Si nitride

Levy *et al.*, Nat. Photon. (2010).
Okawachi *et al.*, Opt. Lett. (2011).
Ferdous *et al.*, Nat Photon. (2012).
Herr *et al.*, Nat. Photon. (2012).

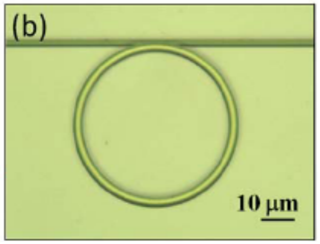
CaF_2, MgF_2, & quartz

Savchenkov *et al.*, PRL (2008).
Liang *et al.*, Opt. Lett. (2011).
Papp & Diddams, PRA (2011).
Herr *et. al.*, Nat. Phot. (2012).



diamond

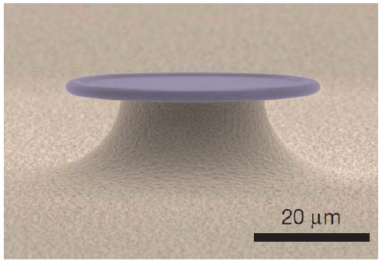
Hausmann *et al.*, Nature Photon. (2013).



Al nitride

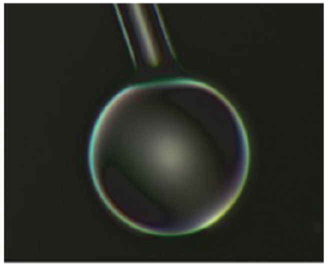
Jung *et al.*, Opt. Lett. (2013).

Microresonator-Based Parametric Combs



silica μ-toroids

Del' Haye *et al.*, Nature (2007).
Del' Haye *et al.*, PRL (2008).



silica μ-spheres

Agha *et al.*, PRA (2007).
Agha *et al.*, Opt. Express (2009).

CaF$_2$, MgF$_2$, & quartz

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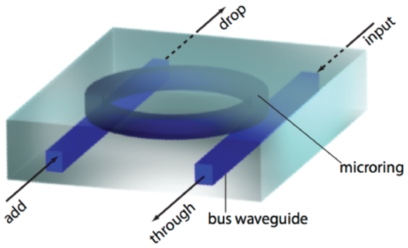
Liang *et al.*, Opt. Lett. (2011).
Papp & Diddams, PRA (2011).
Herr *et. al.*, Nat. Phot. (2012).

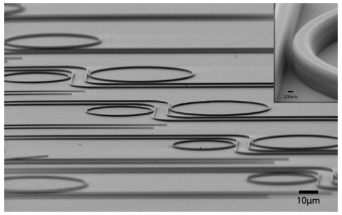


high-index glass μ-rings

Razzari *et al.*, Nature Phot. (2010).
Pasquazi *et al.*, Opt. Express (2013).

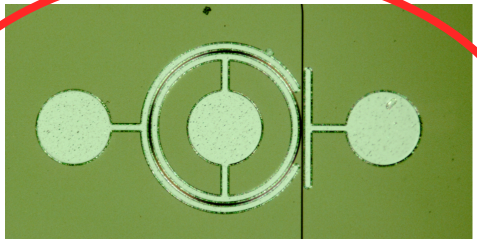
Silica disks

Li *et al.*, PRL (2012).



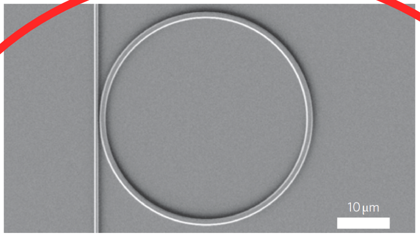
diamond

Hausmann *et al.*, Nature Phot. (2013).



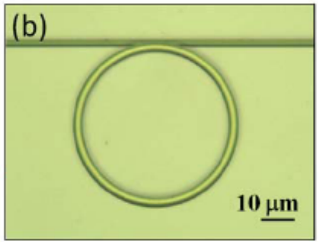
Silicon

Griffith *et al.*, (2014).



Si nitride

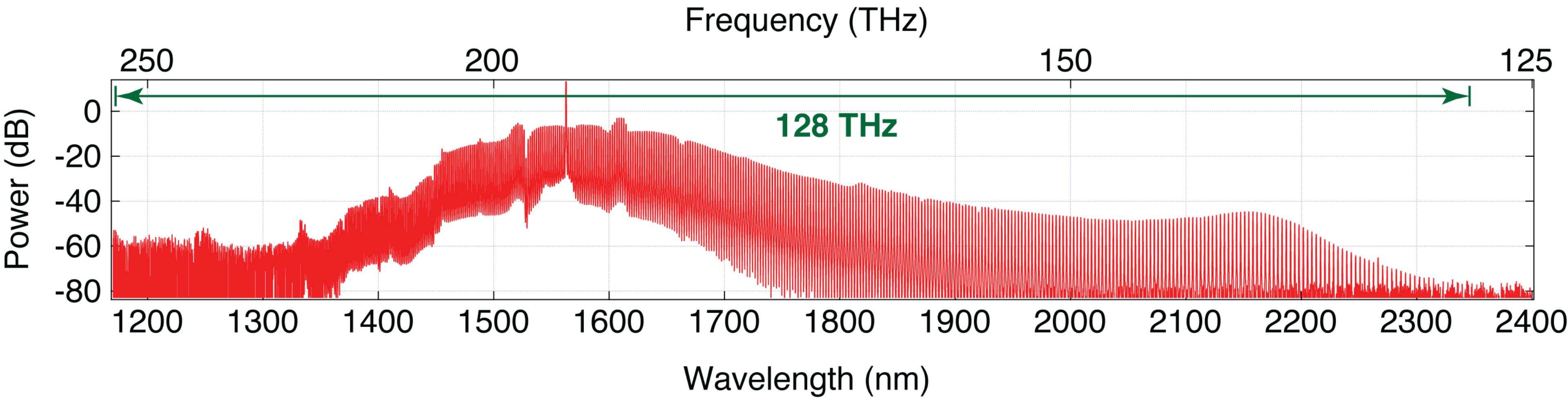
Levy *et al.*, Nat. Photon. (2010).
Okawachi *et al.*, Opt. Lett. (2011).
Ferdous *et al.*, Nat Phot. (2012).
Herr *et al.*, Nat. Phot. (2012).



Al nitride

Jung *et al.*, Opt. Lett. (2013).

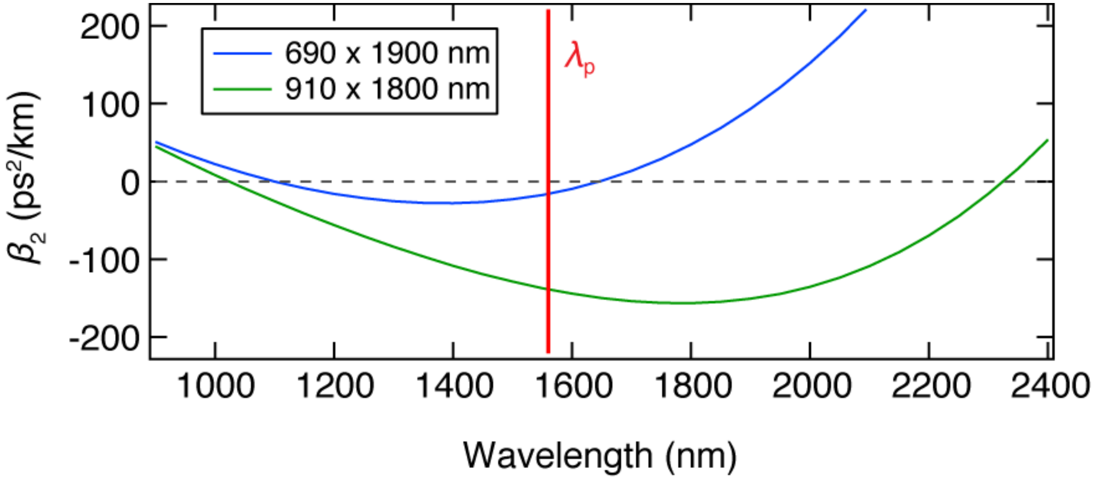
Octave-Spanning Comb in SiN

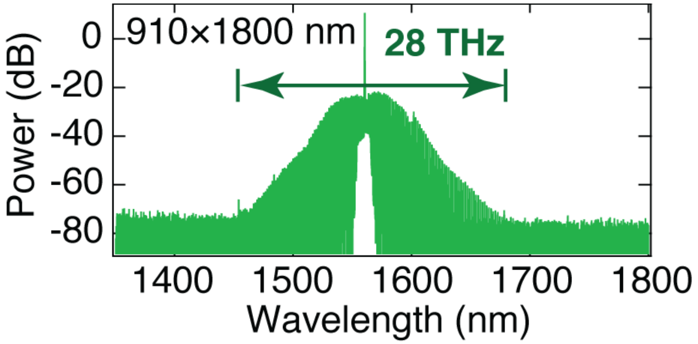


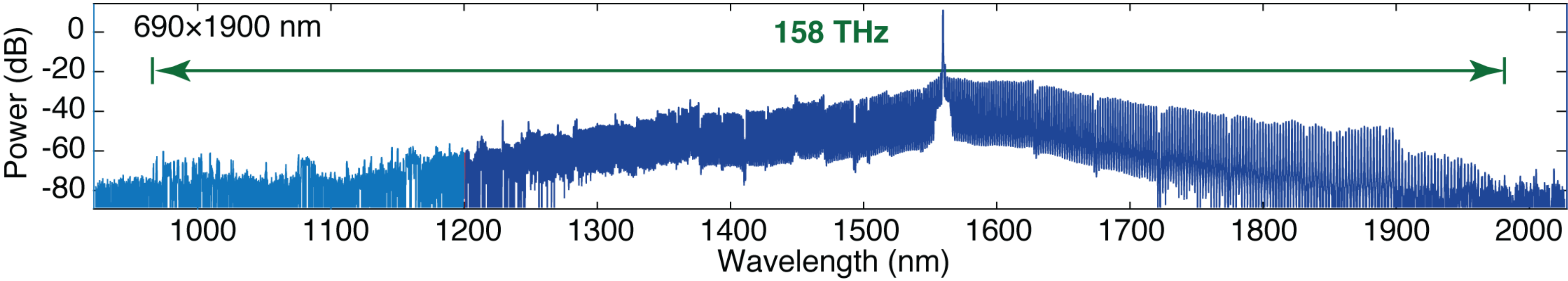
- 128 THz bandwidth with 230 GHz spacing
- Stable, robust, highly compact comb source for clock applications
- Modest power requirements (100’s of mW)

Okawachi, et al., Lipson, and Gaeta, *Opt. Lett.* (2011).

Comb Bandwidth Tailoring with Dispersion Engineering



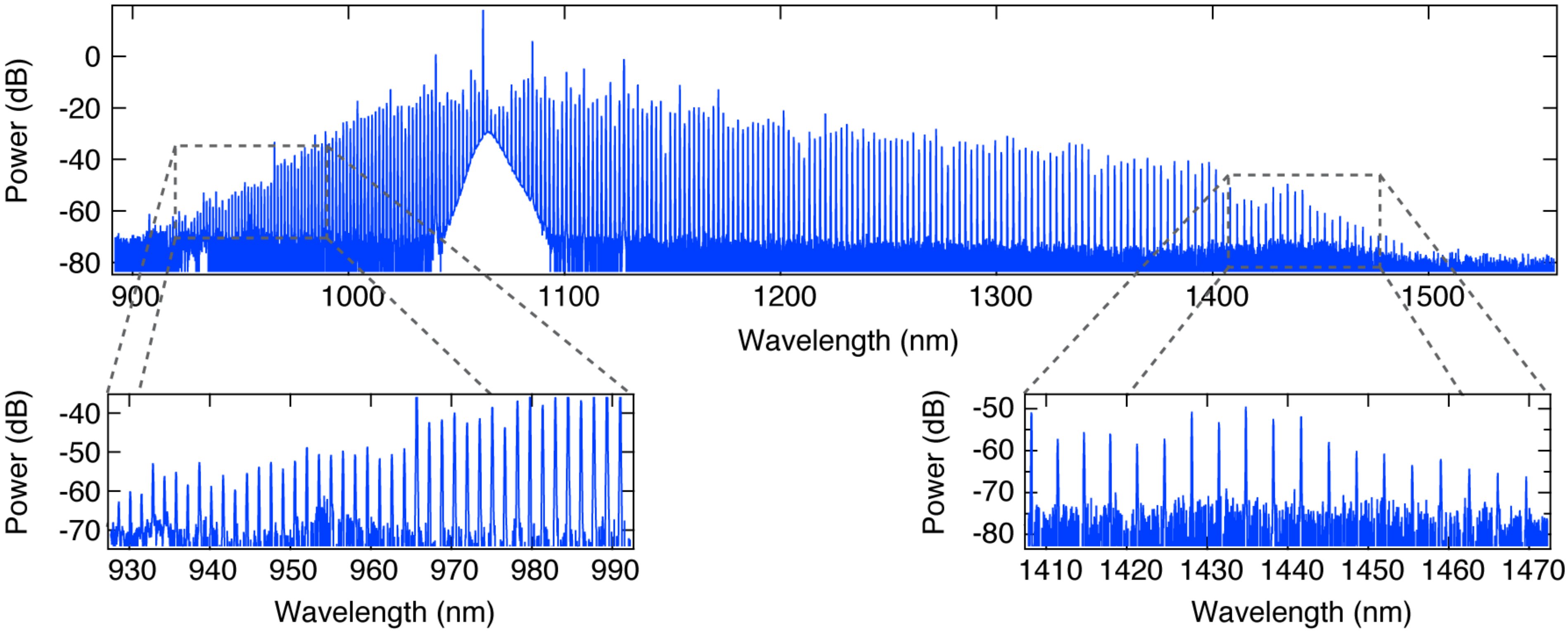




- Experimentally measured spectra shows good agreement with model
- Broadest comb in Si_3N_4 microresonators to date

Okawachi, *et al.*, Opt. Lett. 2014.

Broadband Combs with 1-μm Pump

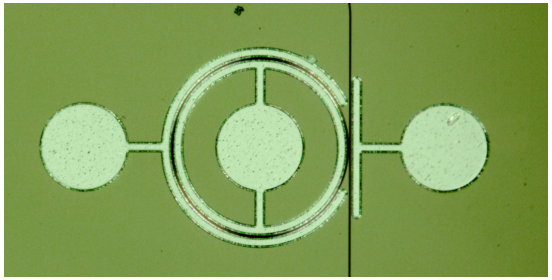


- 690 x 1400 nm cross section, 46-μm resonator radius (500 GHz FSR)
- >2/3 octave of continuous comb bandwidth

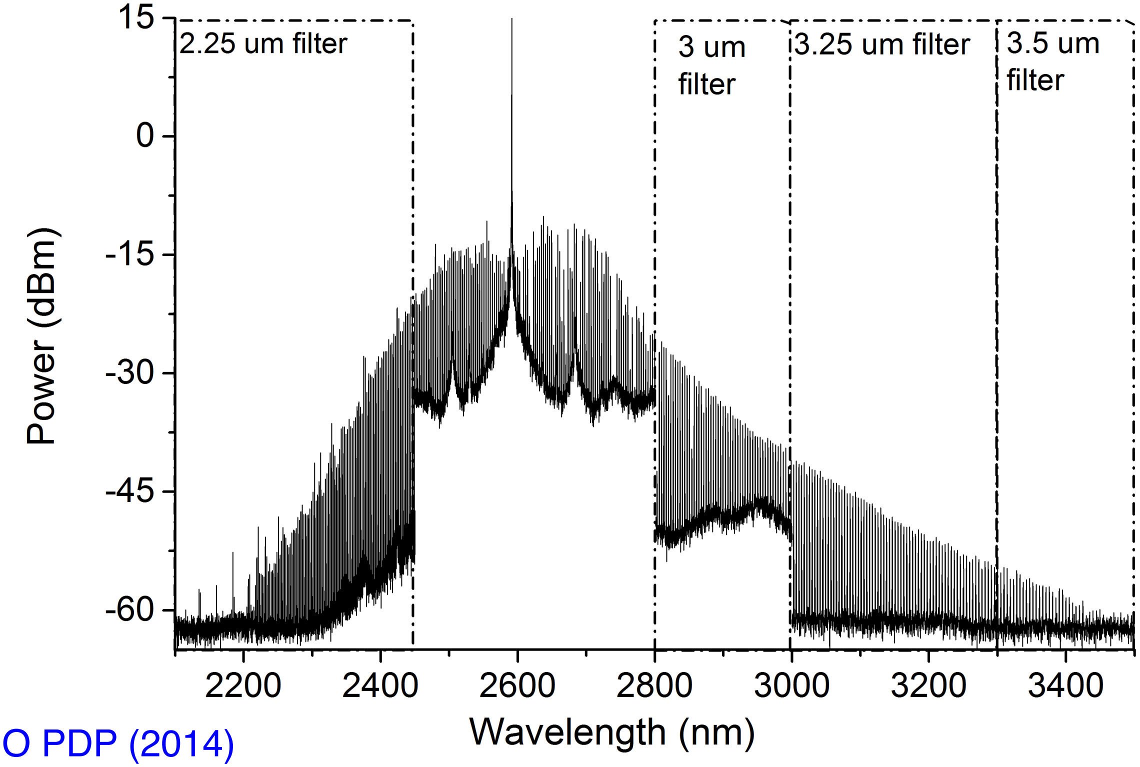
Saha, et al., Lipson, and Gaeta, Opt. Express (2012)
Luke et al. Lipson, Gaeta, to be published (2014).

Silicon Microresonator-Based Mid-IR Parametric Frequency Comb

- 500×1400 nm **etchless** silicon microresonator with ***p-i-n* structure**
- Q-factor ~10^6
- Measurement with FTIR OSA
  ➔Bandwidth limited by dynamic range of OSA

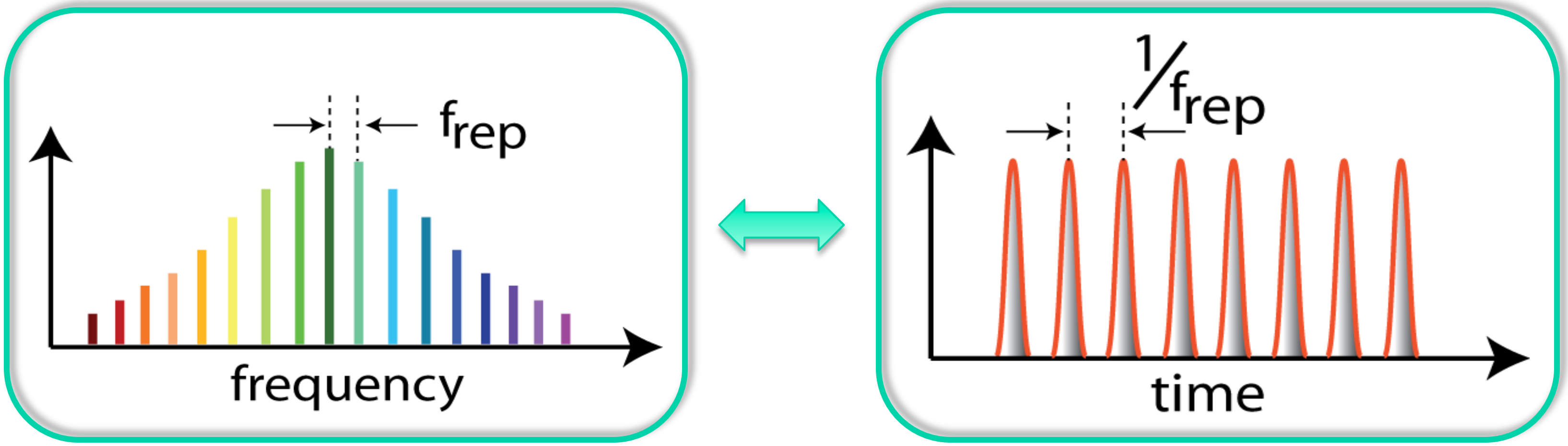


- 2608-nm pump
- 750-nm bandwidth
- 125-GHz FSR
  (100 μm radius)

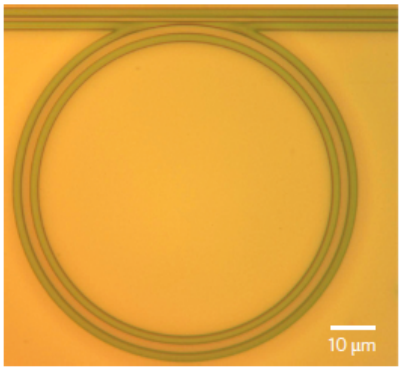


Griffith, et al., Lipson, and Gaeta,CLEO PDP (2014)

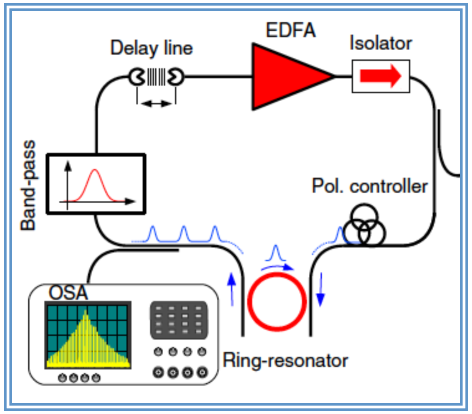
Comb Generation Dynamics



Few ps pulses
Papps & Diddams
Phys. Rev. A (2011).

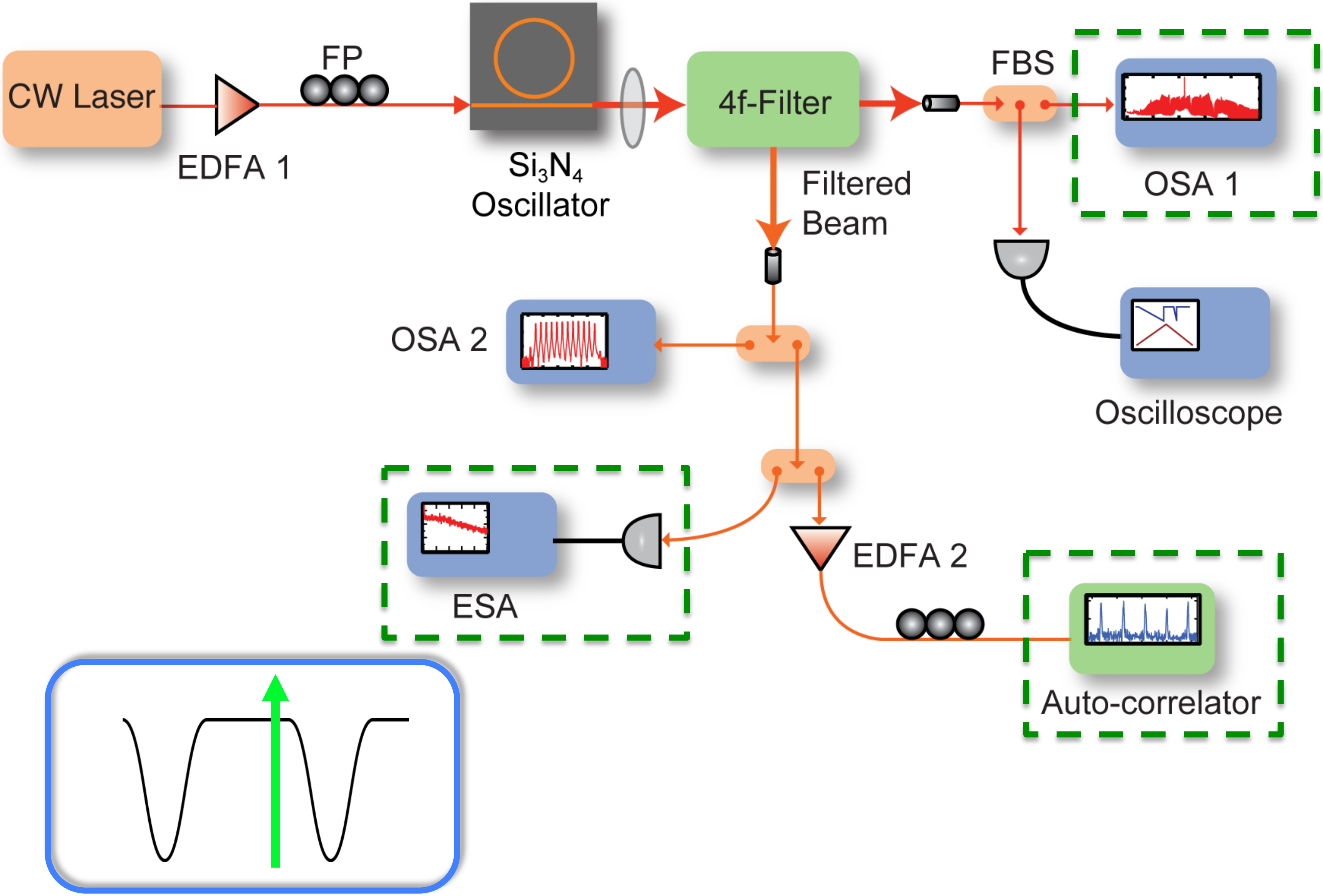


430 fs pulses,
External modulation
Ferdous et al.,
Nature Photon. (2011).

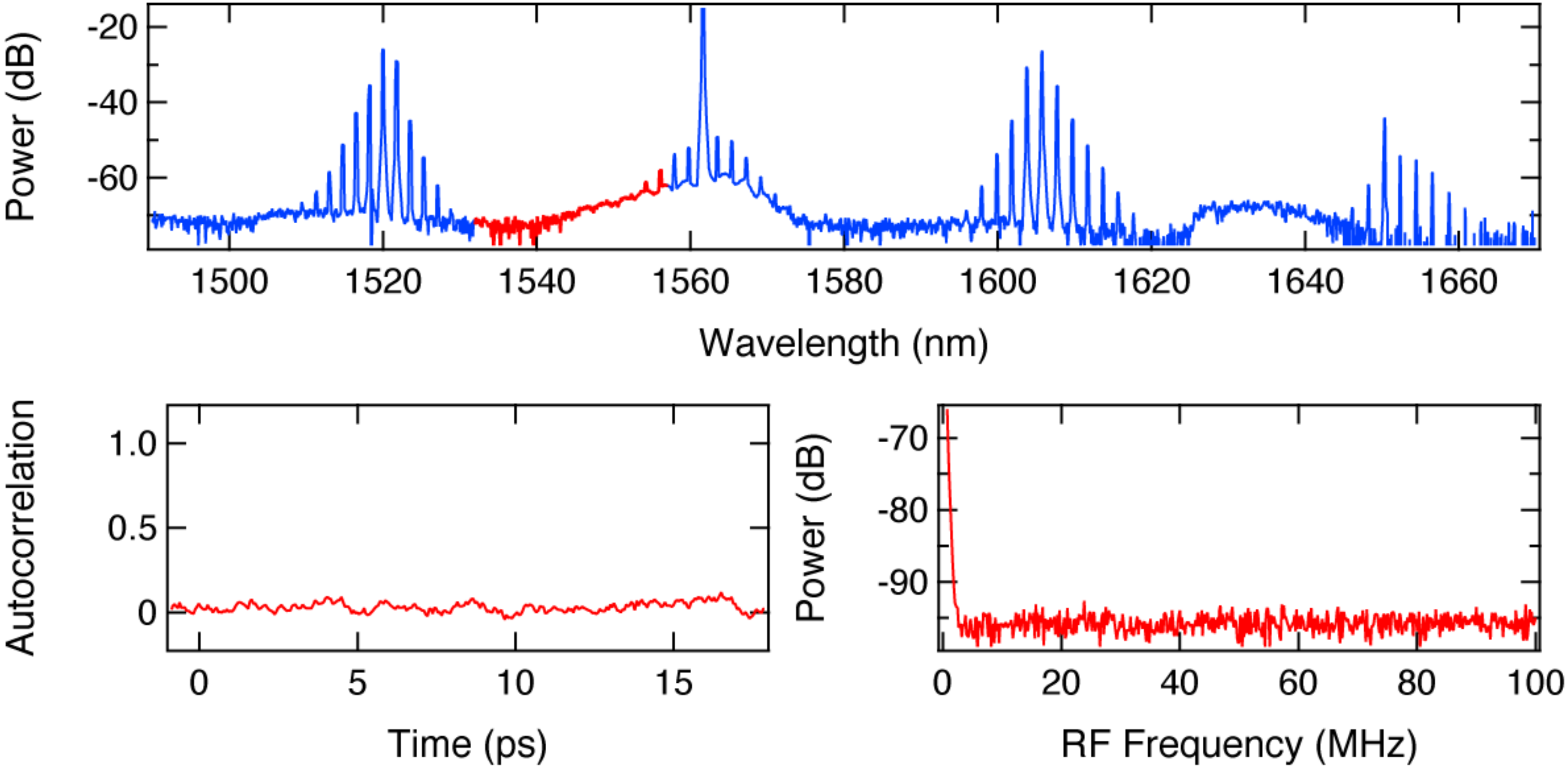


Few ps pulses,
External cavity
Peccianti et al.,
Nature Comm. (2012).

Characterization of Comb Dynamics

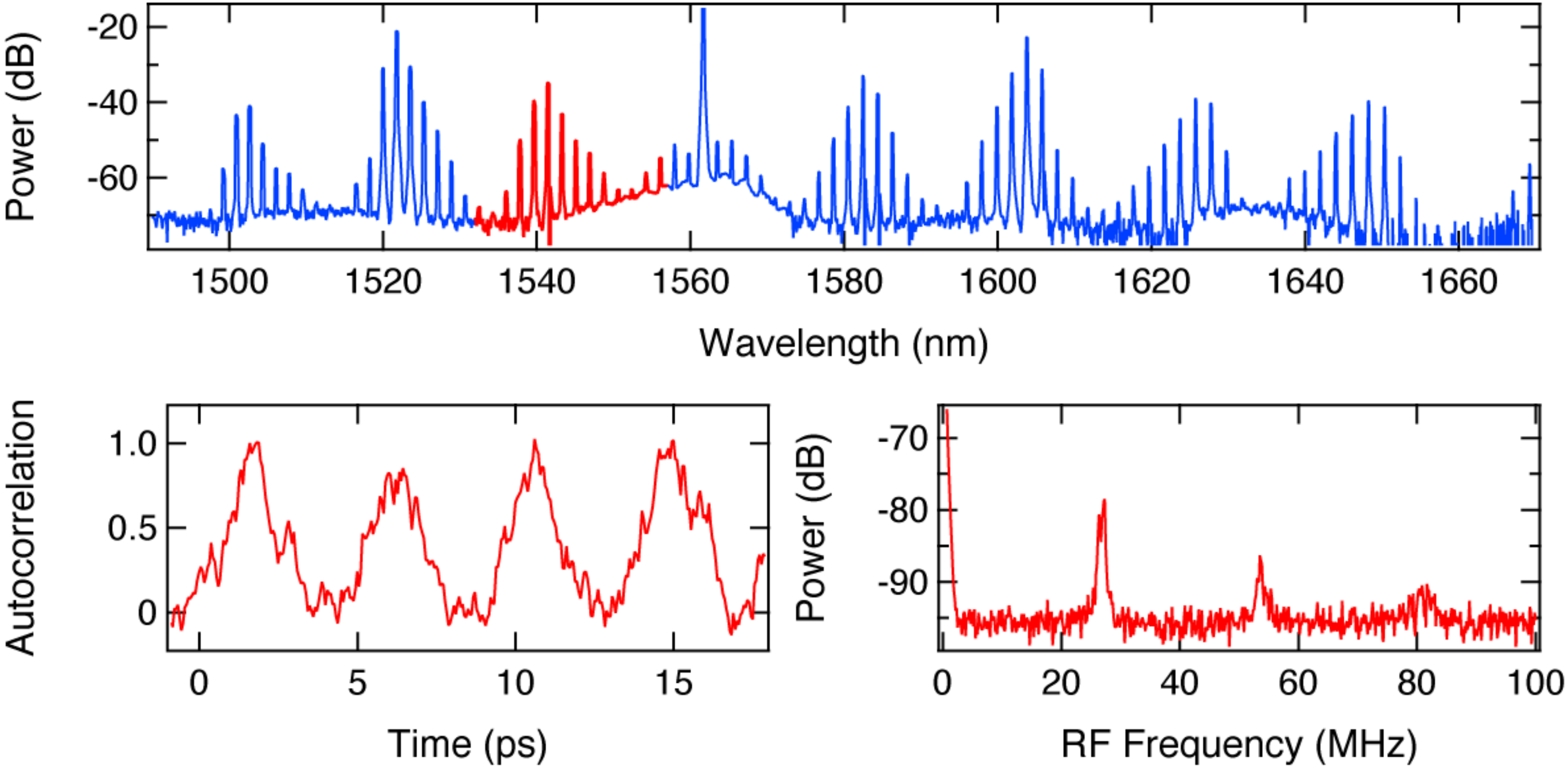


Temporal and Spectral Comb Generation Dynamics



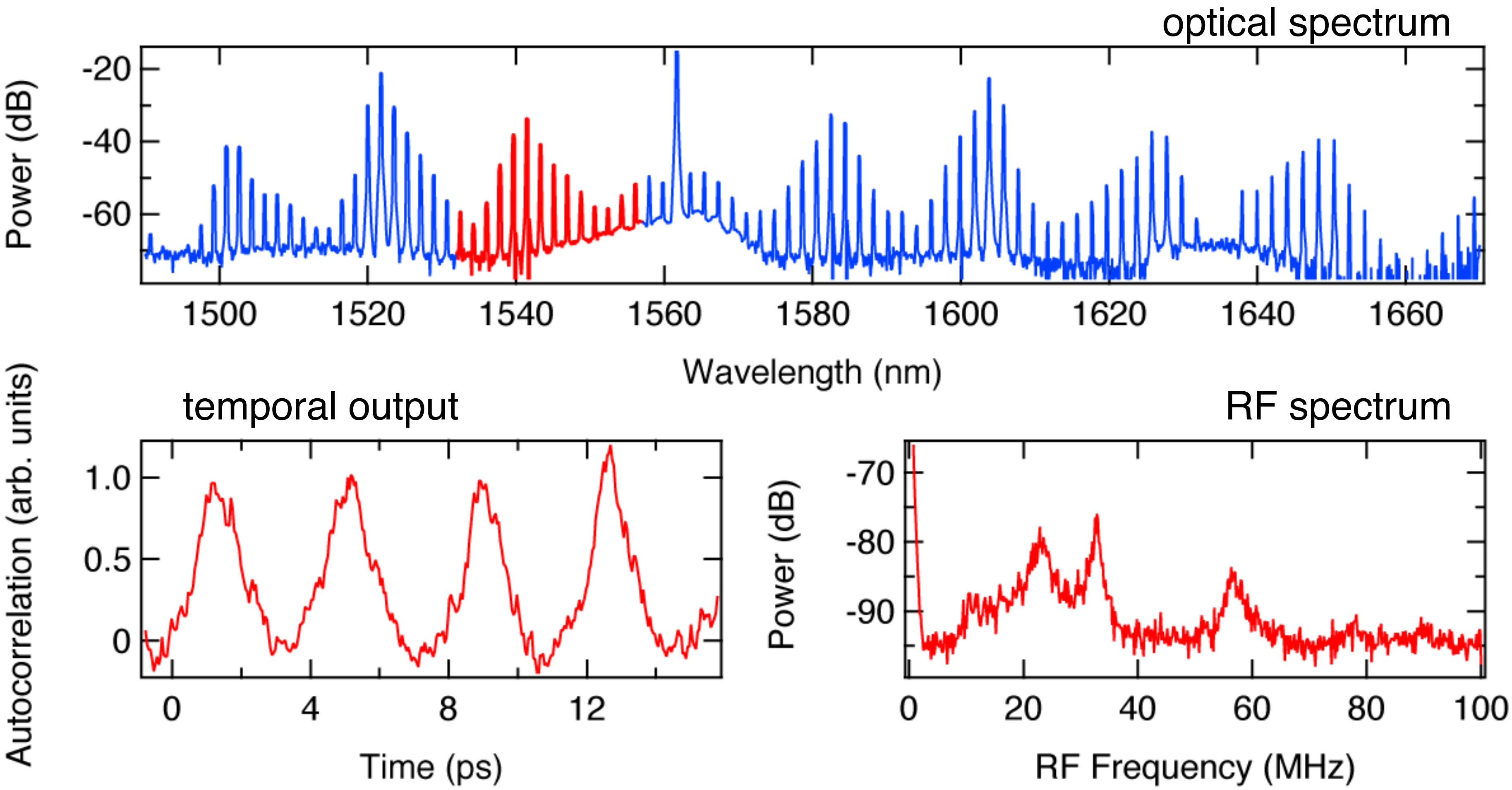
- Filter 25-nm section of comb centered at 1545 nm (red)

Comb Generation Dynamics



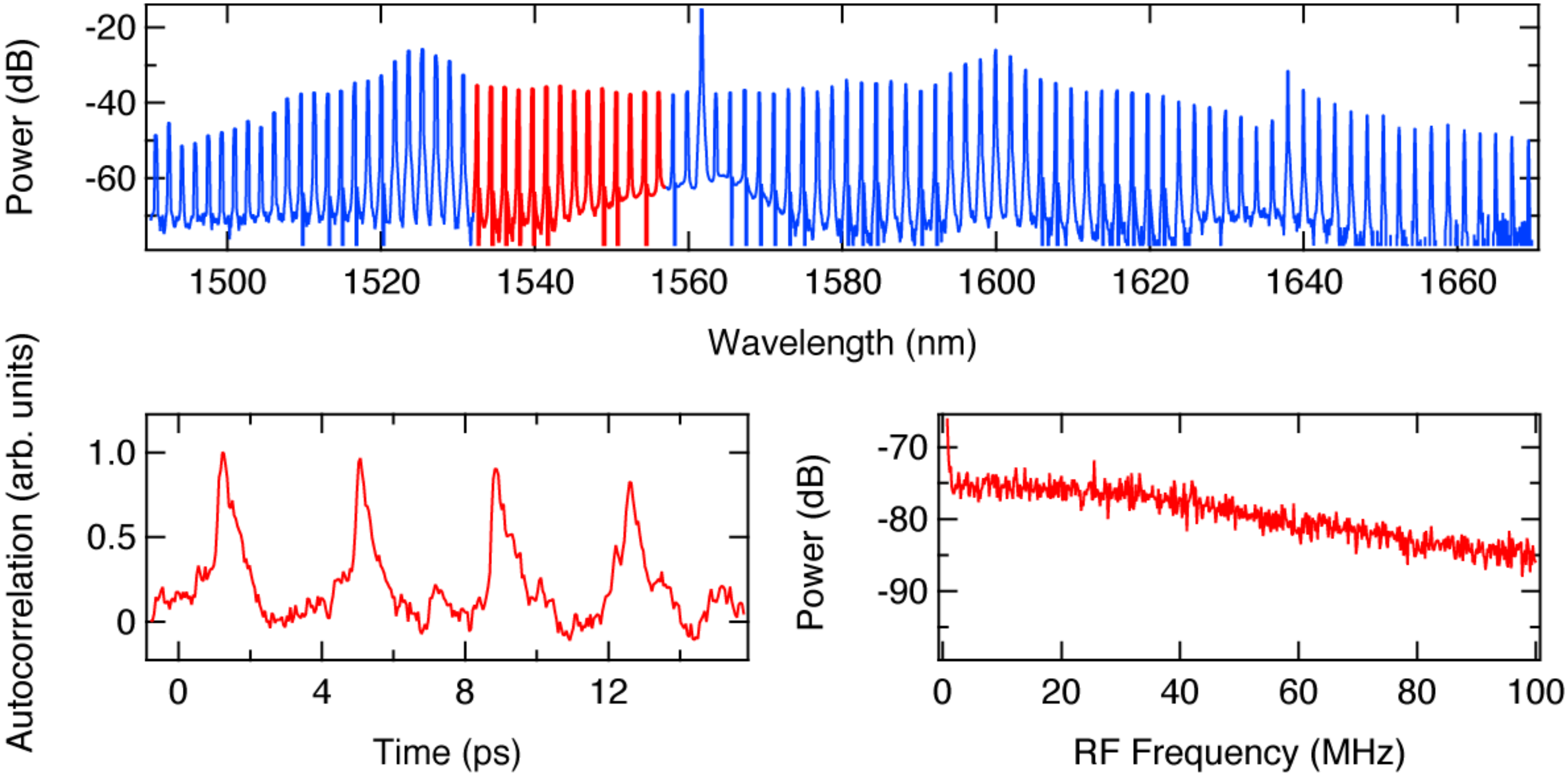
Saha, et al., Lipson, and Gaeta, *Opt. Express* (2013).
Herr, et al., Kippenberg, *Nature Phot.* (2012).

Comb Generation Dynamics

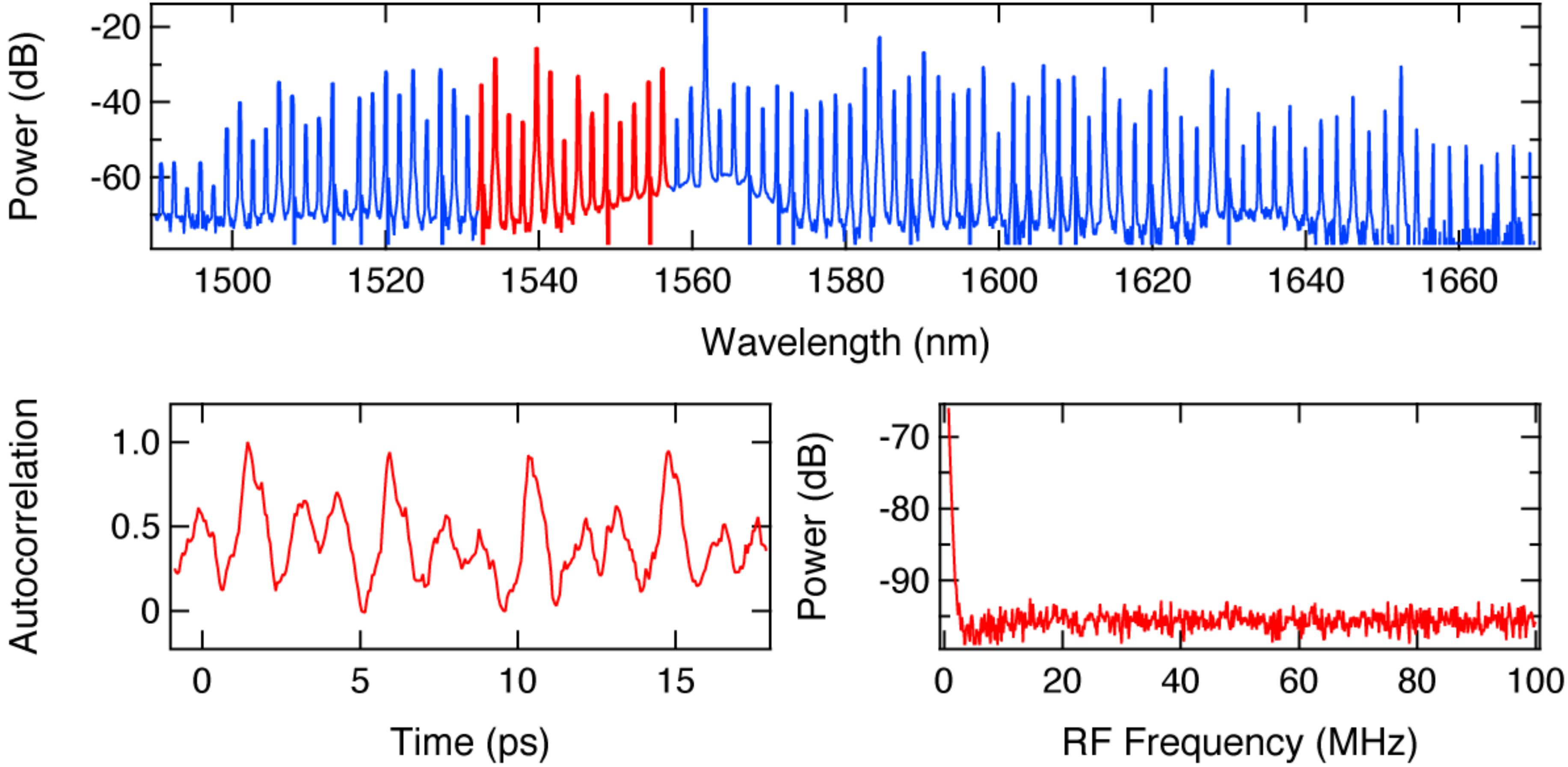


Saha, et al., Lipson, and Gaeta, *Opt. Express* (2012).
Herr, et al., Kippenberg, *Nature Phot.* (2012).

Temporal and Spectral Comb Generation Dynamics

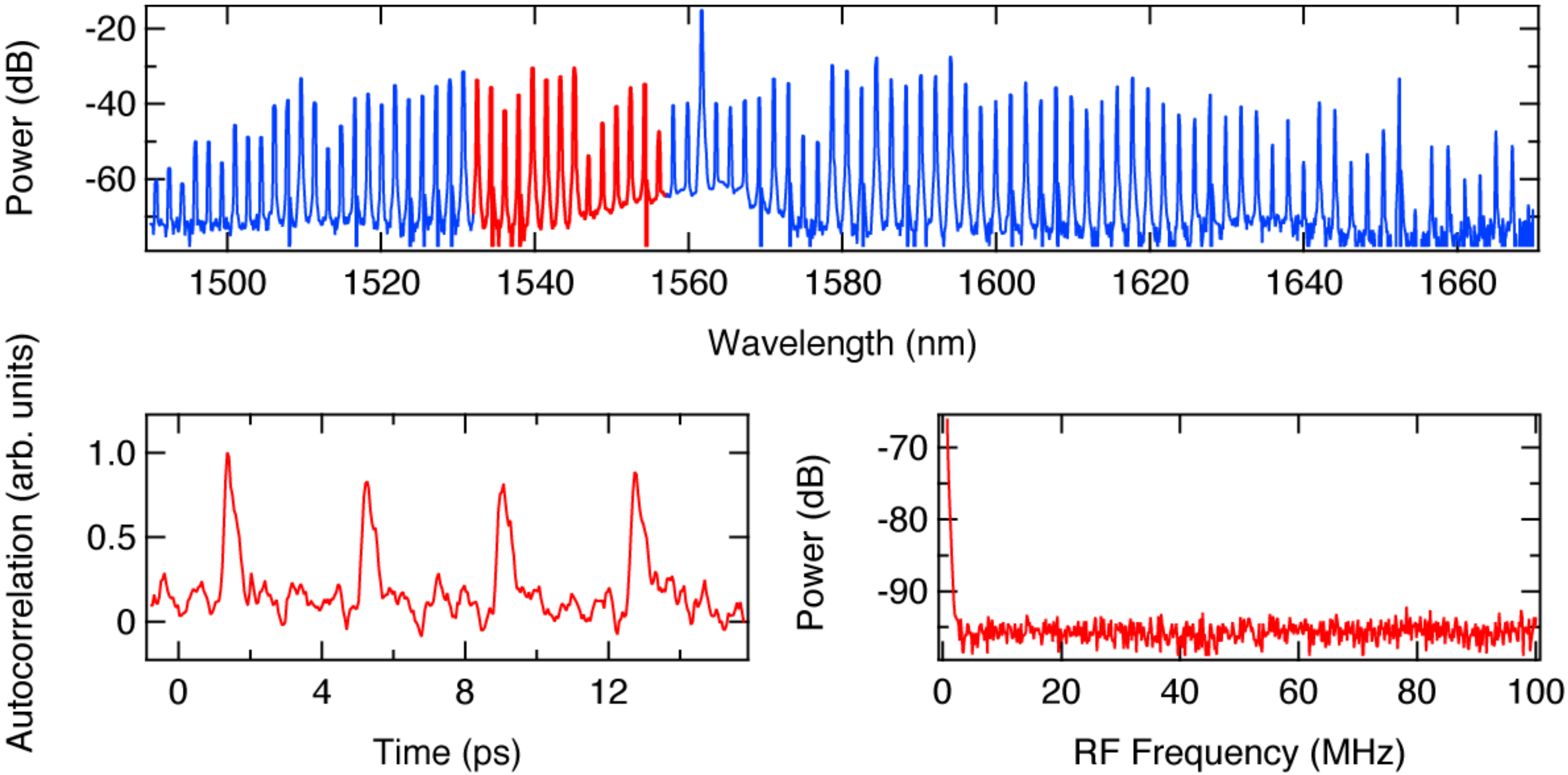


Temporal and Spectral Comb Generation Dynamics



Transition to multi-pulse modelocking.

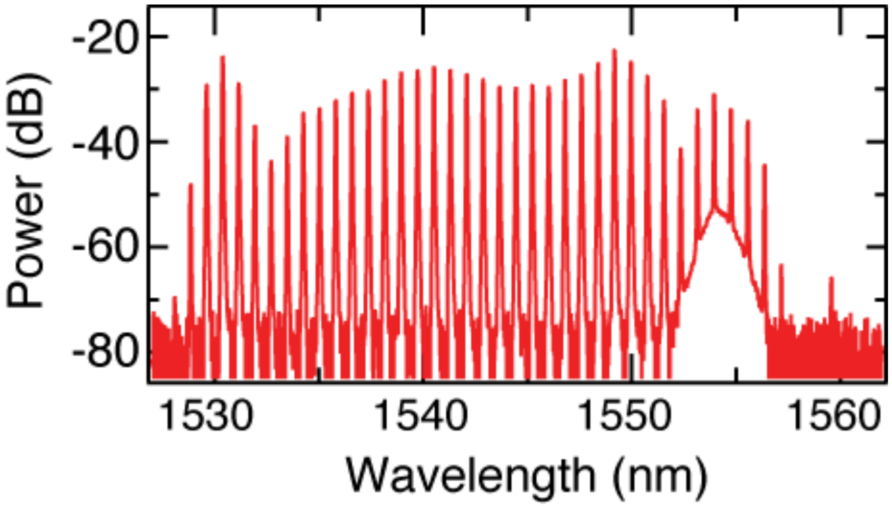
Temporal and Spectral Comb Generation Dynamics

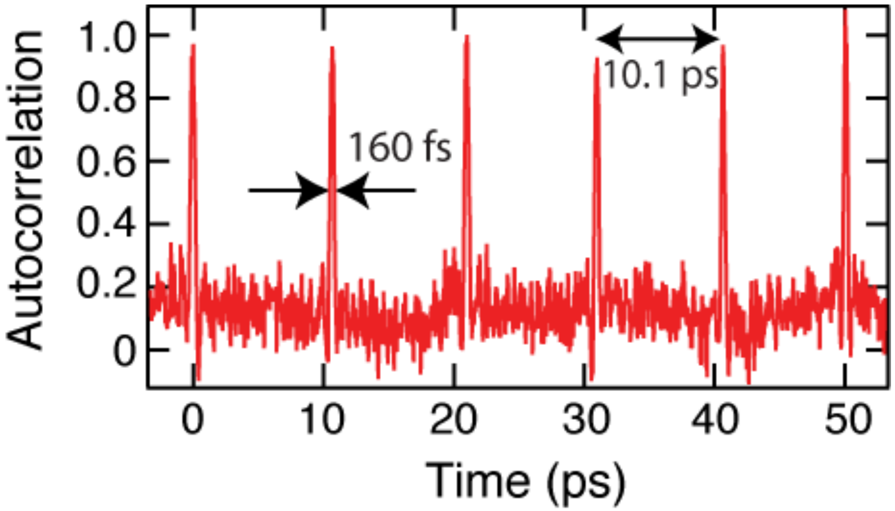


Transition to single-pulse modelocking.

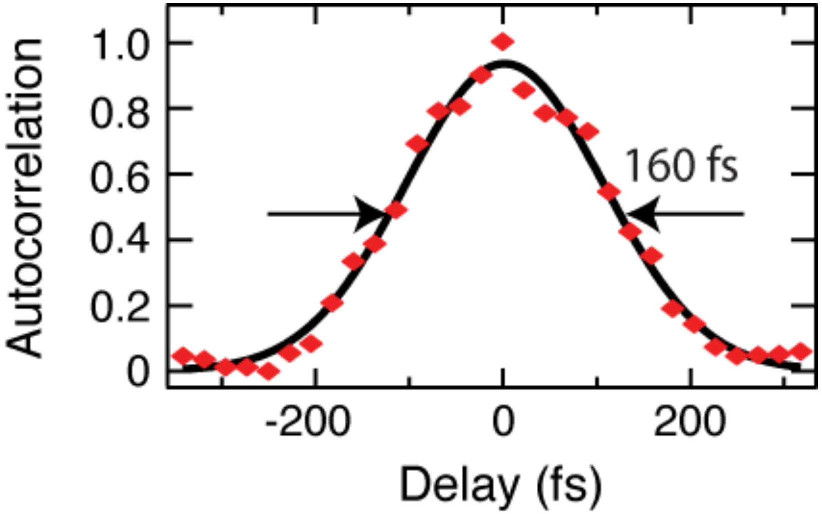
Saha, et al., *Opt. Express* (2013).

Ultrashort Pulses at 99 GHz





- 99-GHz repetition rate
- 160-fs pulses



Saha, Okawachi, Shim, Levy, Foster, Salem, Johnson, Lipson, and Gaeta, *Opt. Express* (2013).

Theoretical Model for Comb Generation

- Lugiato-Lefever model: NLSE with ring-resonator B.C.

Lugiato and Lefever, (1987); Haelterman, et al.(1992); Leo, et al. (2010); Matsko, et al.(2011); Coen, et al. (2013) Chembo and Menyuk (2013); Lamont et al. (2013).

$$\frac{\partial}{\partial t}A = -(\alpha + i\delta)A - i\frac{L}{L_{DS}}\frac{\partial^2 A}{\partial \tau^2} + i\frac{L}{L_{NL}}|A|^2 A + \eta E_{in}$$

Cavity loss & detuning

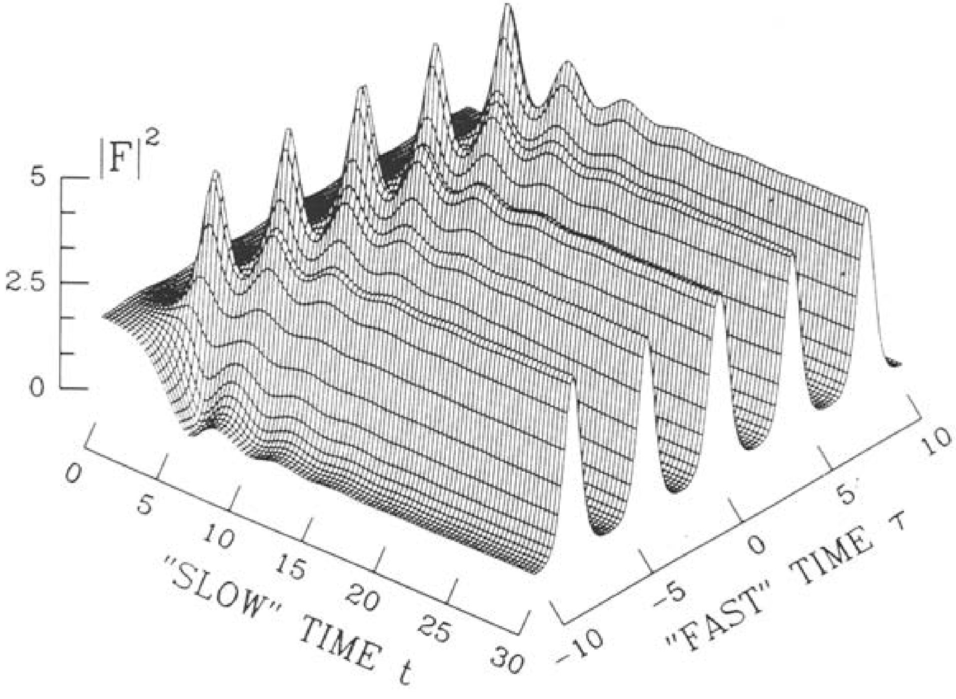
Dispersion

Nonlinearity

Pump

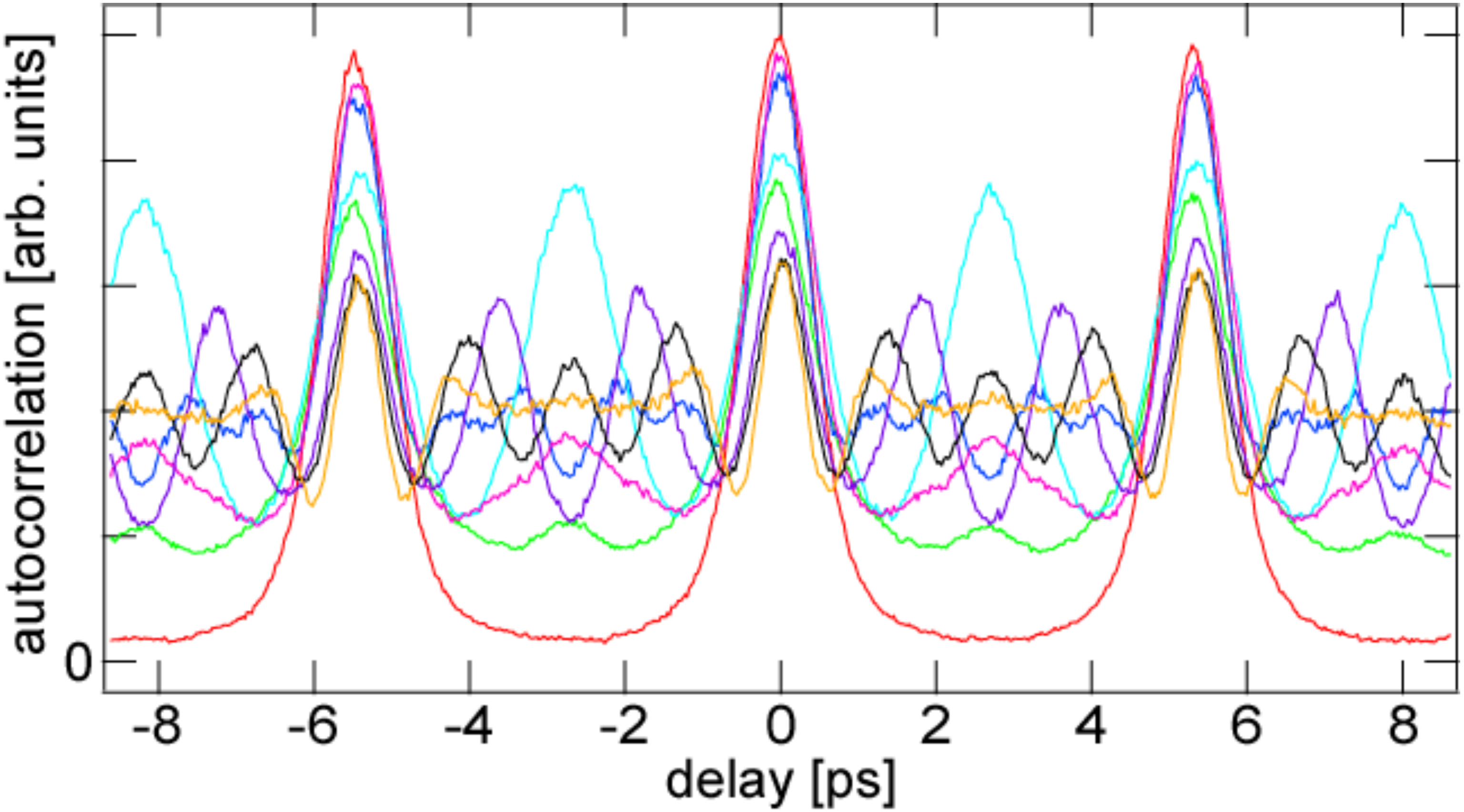
Multiple soliton solutions exist

$$A(\tau) \sim C_1 + C_2 \sum_{j=1}^{N} \text{sech}\left[(\tau - \tau_j)/\tau_0\right]$$



Haelterman, Trillo, and Wabnitz, *Opt. Commun.* (1992).
Wabnitz, *Electron. Lett.* (1993).
Nakazawa, et al. (1988).
Herr, et al., Nature Phot. (2014).

Multiple Cavity-Soliton Formation



- Temporal output varies with tuning of pump laser.

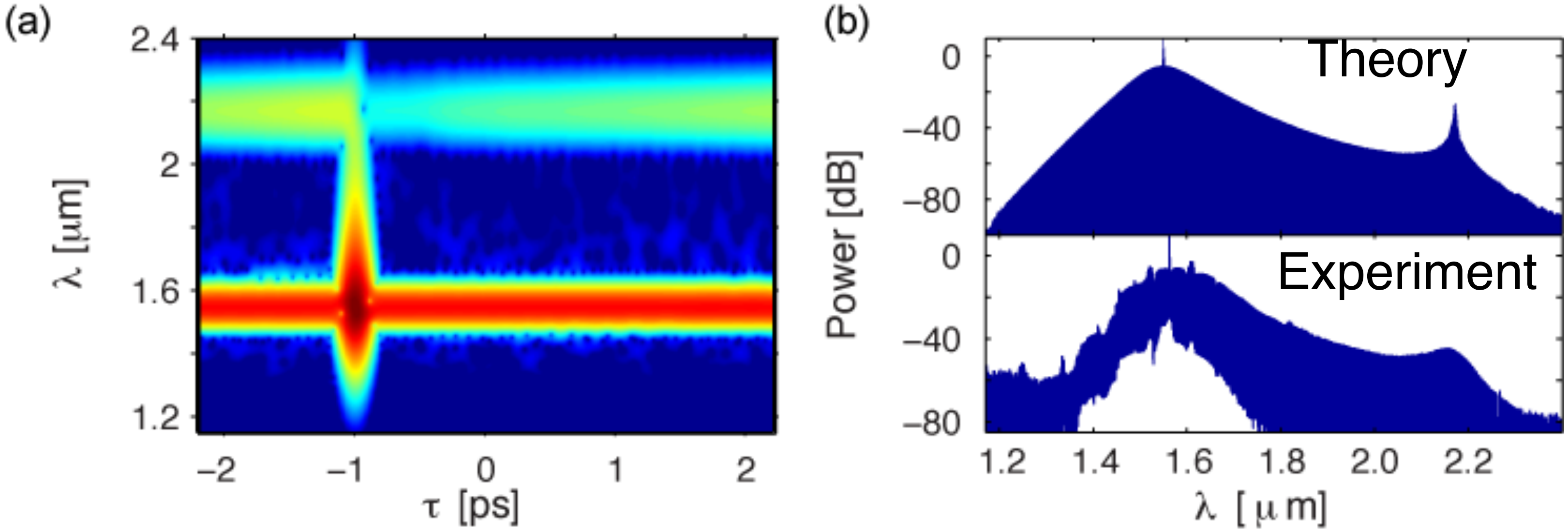
Foster, et al. Lipson and Gaeta, *arXiv* (2008); Saha, et al. Lipson and Gaeta, *Opt. Express* (2013).
Herr, Brasch, Gorodetsky, and Kippenberg, *Nature Phot.* (2014).

Modeling of Octave-Spanning Combs

- Includes higher-order dispersion and self-steepening for octave combs

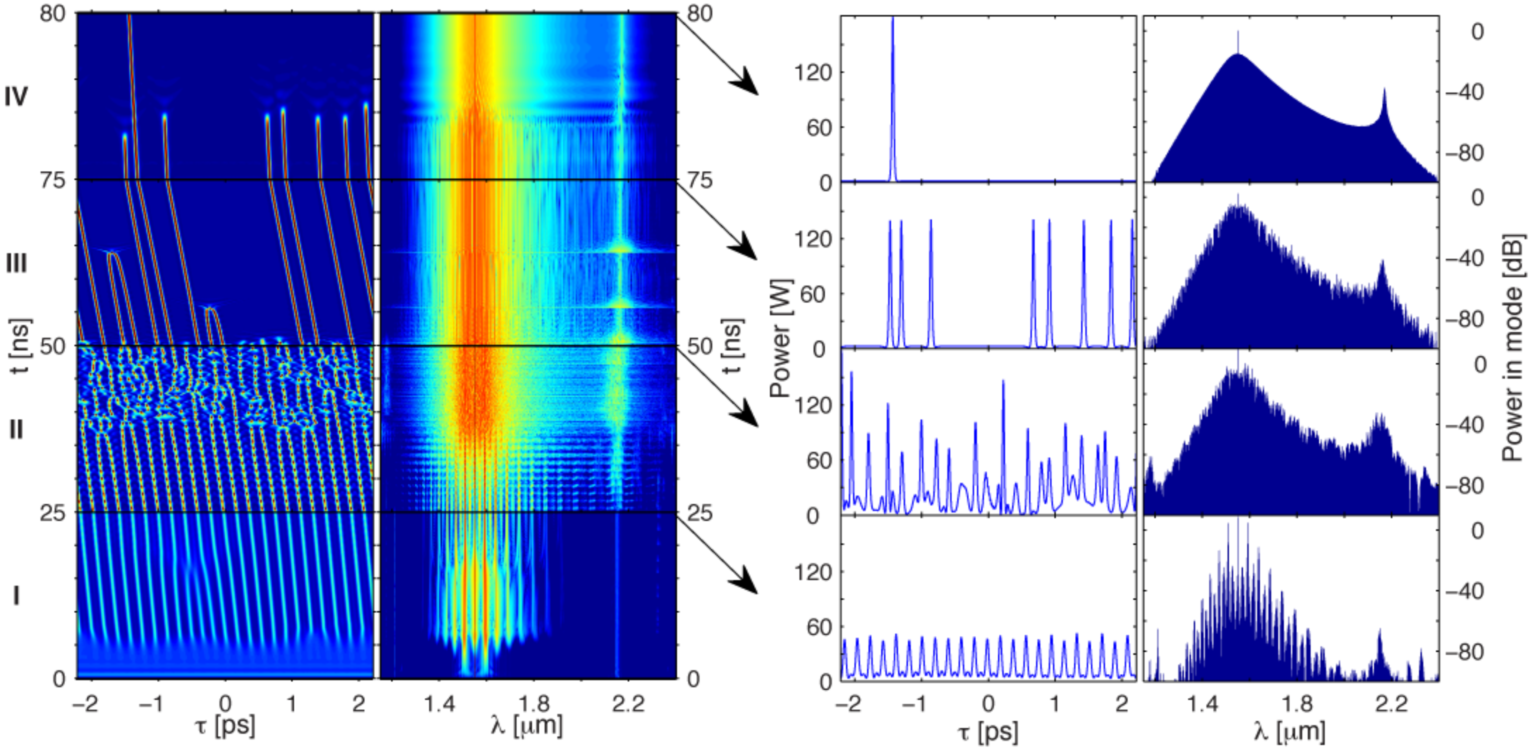
$$T_R \frac{\partial E(t,\tau)}{\partial t} = \sqrt{\kappa} E_{\text{in}} + \left[-\frac{\alpha}{2} - \frac{\kappa}{2} - i\delta_0 + iL \sum_{k\geq 2} \frac{\beta_k}{k!}\left(i\frac{\partial}{\partial\tau}\right)^k + i\gamma L\left(1+\frac{i}{\omega_0}\frac{\partial}{\partial\tau}\right)\left|E(t,\tau)\right|^2 \right] E(t,\tau)$$

- Consistent with experimental data from Okawachi et al. (2011).



Coen, et al. *Opt. Lett.* (2013); Lamont, Okawachi, and Gaeta, *Opt. Lett.* (2013).

Evolution to Modelocked State



- Evolution to stabilized single-pulse modelocking
- Each stage represents increase in pump detuning from on-resonance to δ_0 = 0.02, 0.04, and 0.05642

Lamont, Okawachi, and Gaeta, *Opt. Lett.* (2013);
Erkintalo and Coen, *Opt. Lett.*. **39**, 283 (2014).

Synchronization: A universal model for self-organization dynamics

The Kuramoto Model

$$\dot{\phi}_i = \omega_i + \sum_{j=1}^{N} \Gamma_{ij}(\phi_i - \phi_j)$$

$$\Gamma_{ij}(\phi_i - \phi_j) = Sin(\phi_i - \phi_j)$$

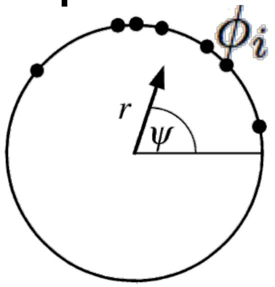
Order Parameter

$$re^{i\psi} = \frac{1}{N} \sum_{j=1}^{N} e^{i\phi_j}$$

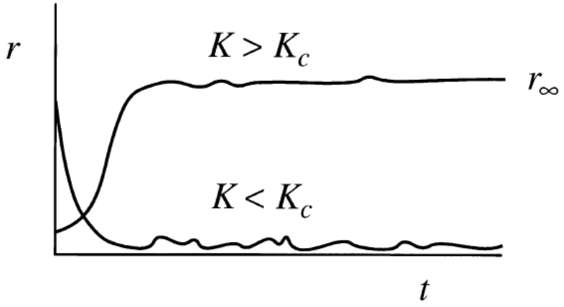
$$\dot{\phi}_i = \omega_i + KrSin(\psi - \phi_i)$$

Acebron, et al. Rev. Mod. Phys. (2005)
Strogatz, Physica D (2000)

The phase picture



Transition behavior



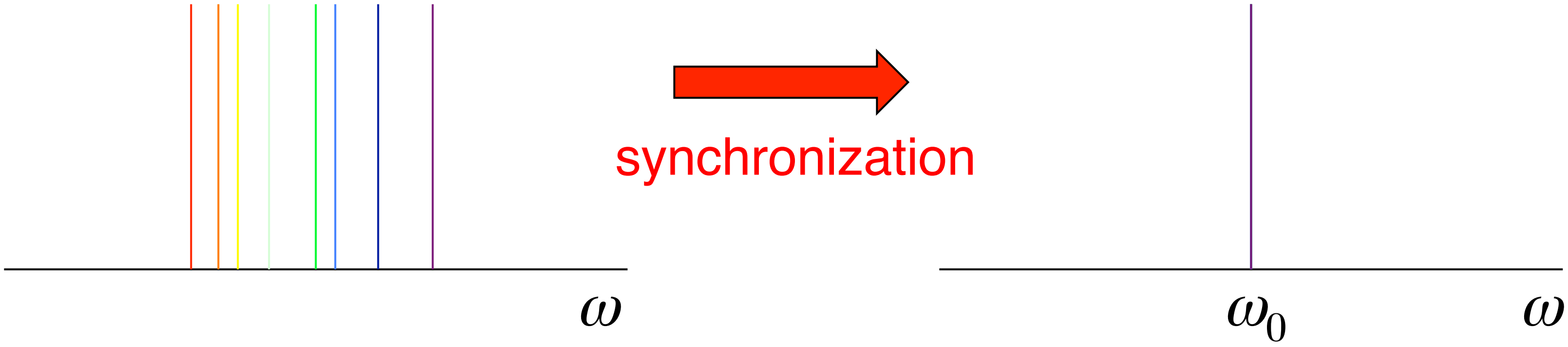
Synchronized fireflies

Maffit, YouTube (2009)

Synchronization

frequency ensemble

identical frequencies



synchronization

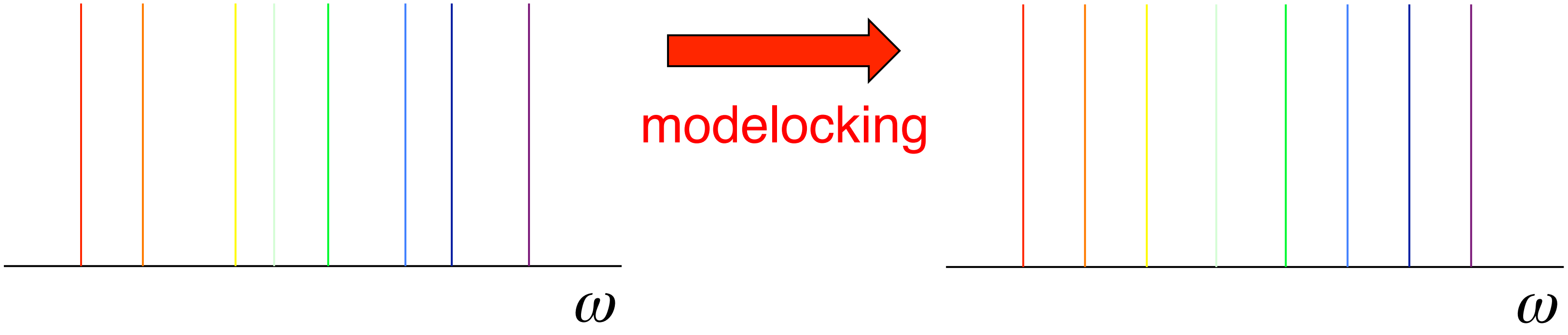
ω

ω_0

ω

mode distribution

identical mode spacings

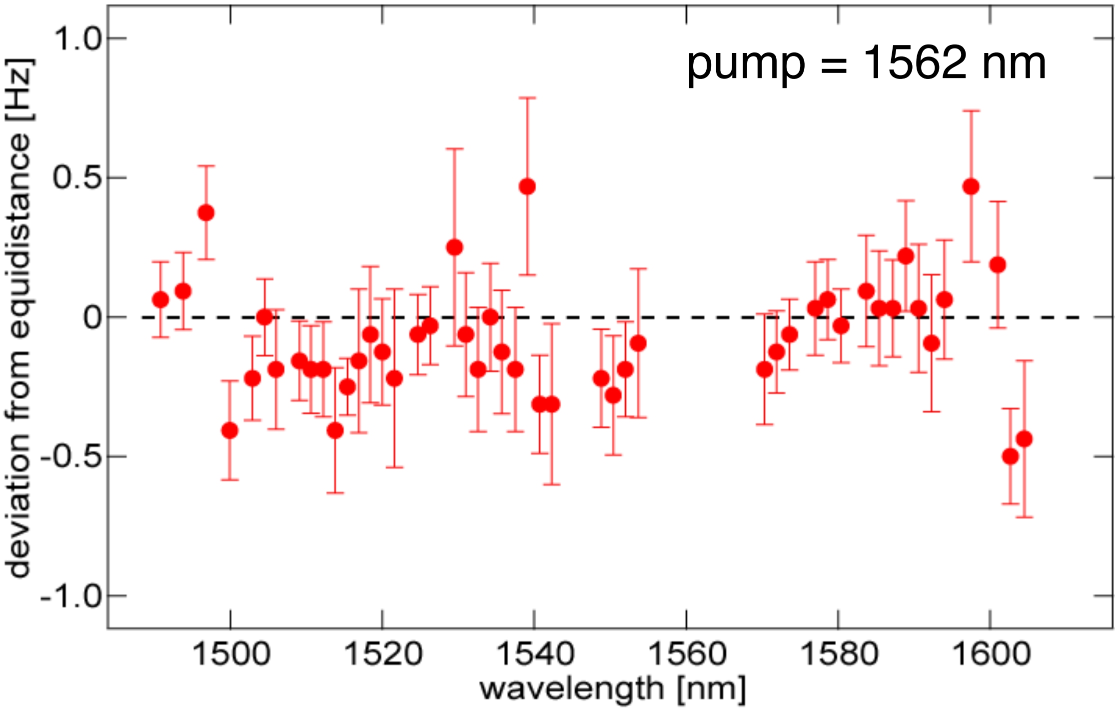


modelocking

ω

ω

Sub-Hz Equidistance for Modelocked Comb



< 0.5 Hz equidistance over 115 nm (14.5 THz)

- 3×10^{-14} × measurement bandwidth
- 3×10^{-15} × optical frequency

Foster, et al., Lipson and Gaeta, *Opt. Express* (2011).

Future: Silicon-Based Combs

- Ultralow power nonlinear optical devices
- Highly flexible platform from visible to mid-IR
- Highly compact, robust chip-based optical clock.
- Multiple-wavelength WDM CMOS-compatible source.
- Highly flexible, chip-based ultrashort pulse source.
- Microwave photonics

Acknowledgements

M. Foster
A. Johnson
B. M. Lamont
Y. Okawachi
K. Saha
H. Wen

<u>M. Lipson</u> (Electrical & Computer Engineering)
J. Levy
K. Luke
A. Turner-Foster
A. Griffith

Cornell University

Funding: DARPA, AFOSR