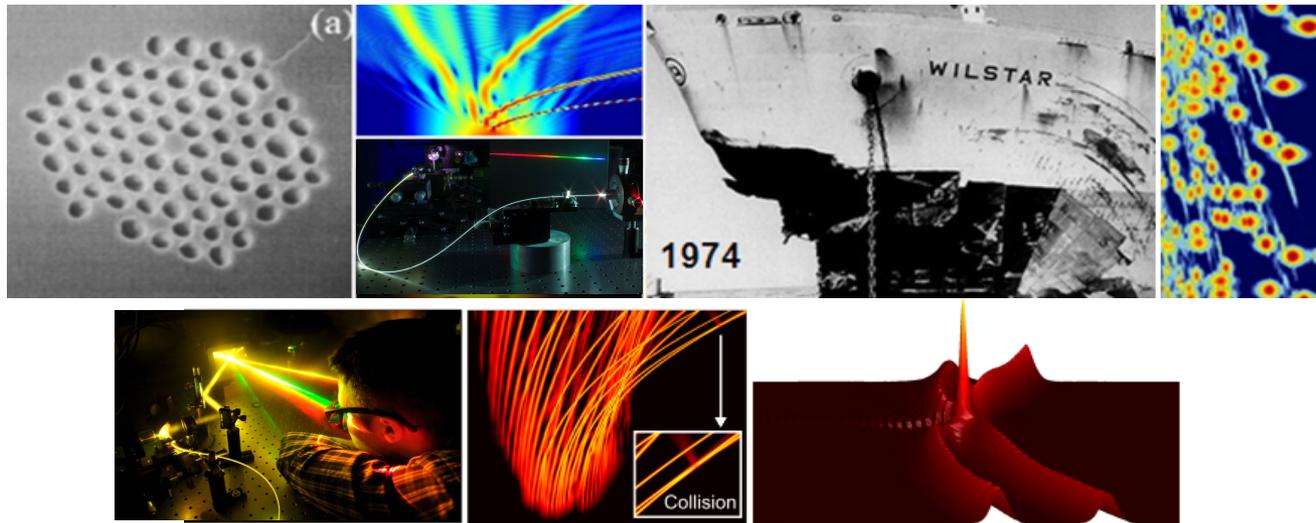


Supercontinuum



Goëry Genty

Optics Laboratory, Tampere University of Technology, Finland



Special thanks

John Dudley
FEMTO-ST-UFC
Besançon, France



Miro Erkintalo
University of Auckland,
New Zealand



Frederic Dias
UCD Dublin,
Ireland



Minna Surakka, Ari Friberg, Jari Turunen
University of eastern Finland, Finland

Bertrand Kibler, Christophe Finot,
Julien Fatome. Guy Millot
Université de Bourgogne,
France

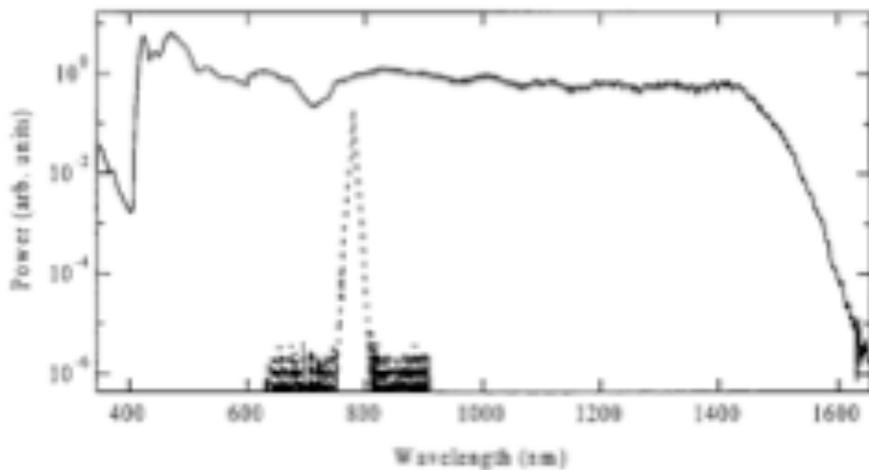
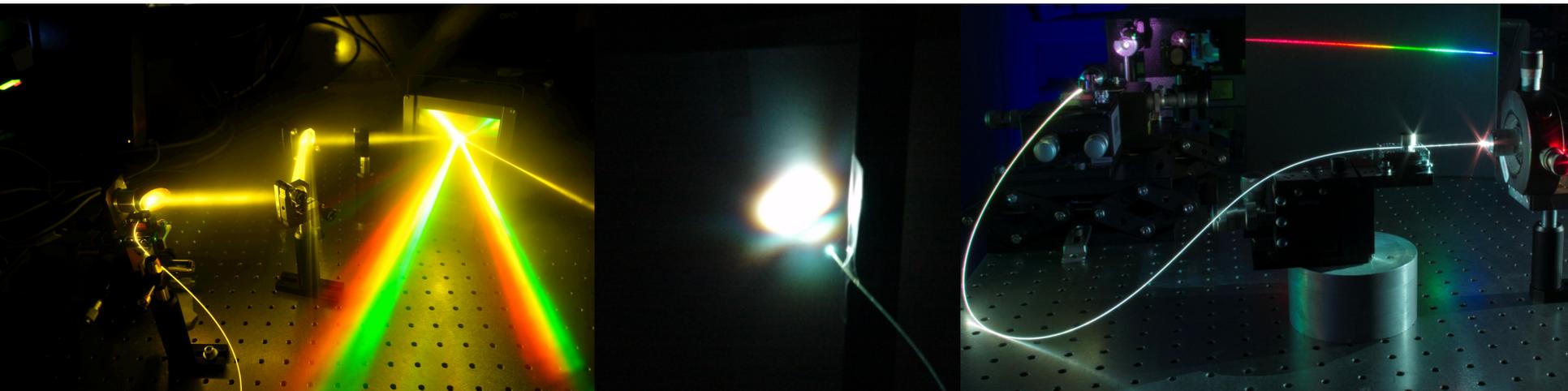


Nail Akhmediev
Research School of Physics
& Engineering
ANU ,Canberra, Australia

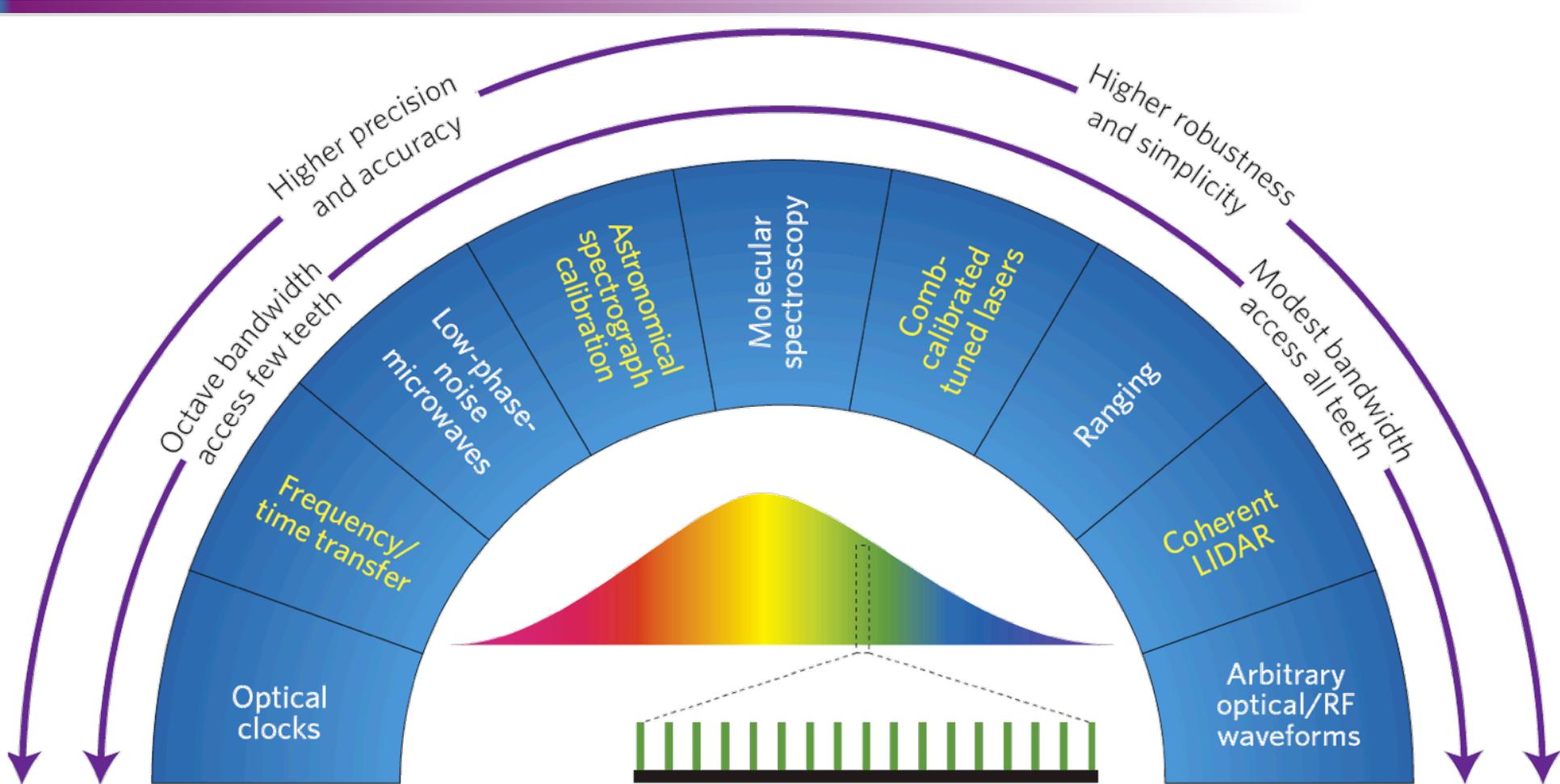


What is a supercontinuum?

Narrowband field experiences massive continuous spectral broadening in a nonlinear medium



Supercontinuum sources are actually useful!



Outline

- Basics of pulses propagation in nonlinear fibers
 - nonlinear phenomena, numerical modelling
- Regimes of supercontinuum generation
 - Deconstructing the dynamics
- Latest developments
 - Emerging structures
- Noise amplification dynamics
 - Modulation instability and extreme events

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- **Noise amplification dynamics**
 - Modulation instability and extreme localization

Supercontinuum is 45 years old

Special anniversary at SPIE Photonics West 2014, celebrating the discovery of supercontinuum 45 years ago

- Bob Alfano
- Alex Gaeta
- Roy Taylor
- Steve Cundiff
- Govind Agrawal
- ...

Supercontinuum is 45 years old

VOLUME 24, NUMBER 11

PHYSICAL REVIEW LETTERS

16 MARCH 1970

OBSERVATION OF SELF-PHASE MODULATION AND SMALL-SCALE FILAMENTS IN CRYSTALS AND GLASSES

R. R. Alfano* and S. L. Shapiro

Bayside Research Center of General Telephone & Electronics Laboratories Incorporated,
Bayside, New York 11360

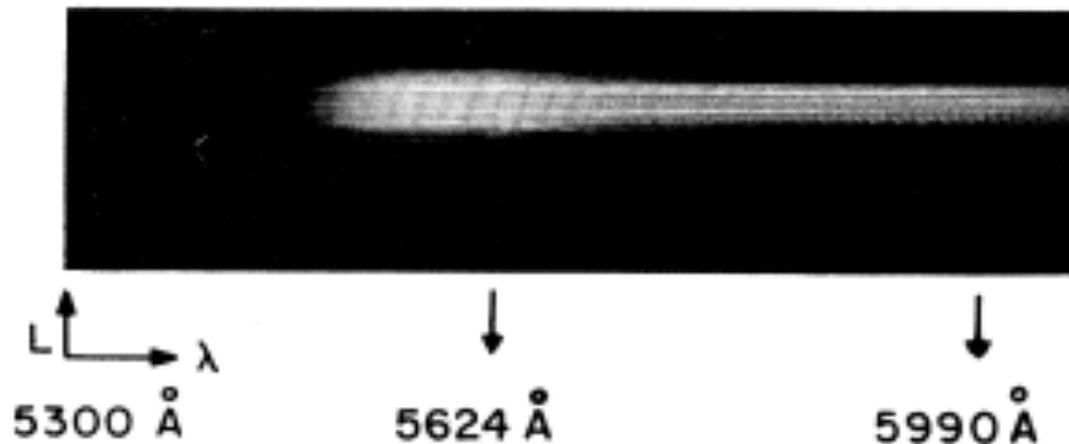
(Received 10 December 1969)

EMISSION IN THE REGION 4000 TO 7000 Å VIA FOUR-PHOTON COUPLING IN GLASS

R. R. Alfano and S. L. Shapiro

Bayside Research Center of General Telephone & Electronics Laboratories Incorporated,
Bayside, New York 11360

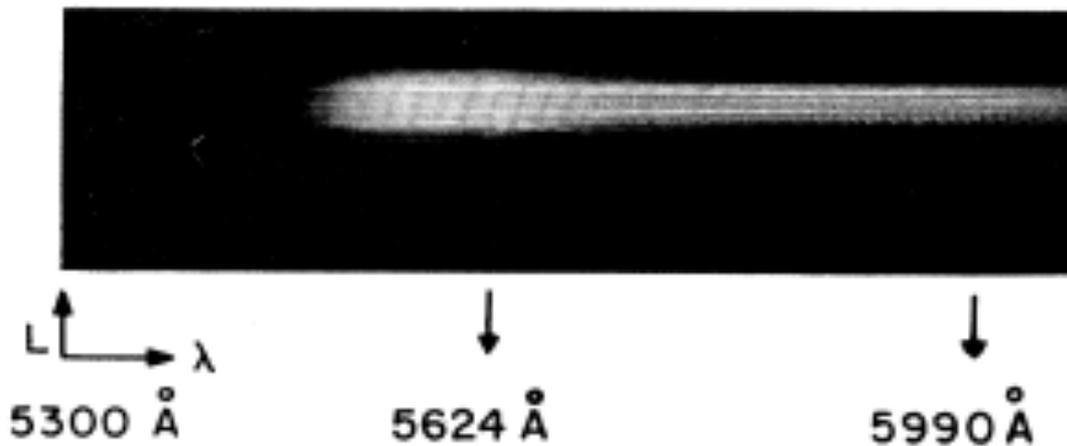
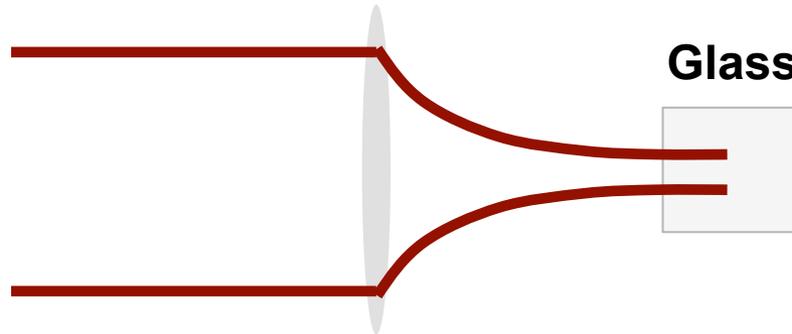
(Received 9 January 1970)



In bulk

Supercontinuum is 45 years old

6 ps pulses
5 mJ
 $\lambda = 532 \text{ nm}$



In bulk



There are some issues though

- Limitations
 - Walk off
 - Diffraction
 - Strong dispersion, limited broadening

- Need very high energy
 - Damage

- Fiber is the way to go!

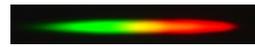
Supercontinuum timeline

Supercontinuum generation
in bulk silica
(R. Alfano)

Experimental
Fiber solitons
(L. Mollenauer)

Femtosecond
Ti:Sapphire
(Sibbett)

Supercontinuum
generation
in photonic crystal
fiber
(J. Ranka et al.)


Varenna

1970

1980

1990

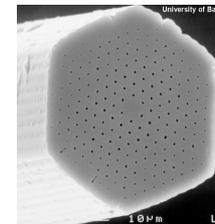
2000

2005

Today

Nonlinearity
in silica fiber
(R. Stolen & Lin)

Supercontinuum generation
in silica fiber
(R. Stolen & Lin)

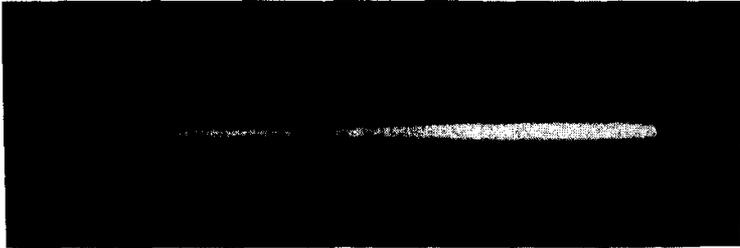


Solid core
photonic crystal fiber
(P. Russell)

Nobel prize
Hall & Hänsch

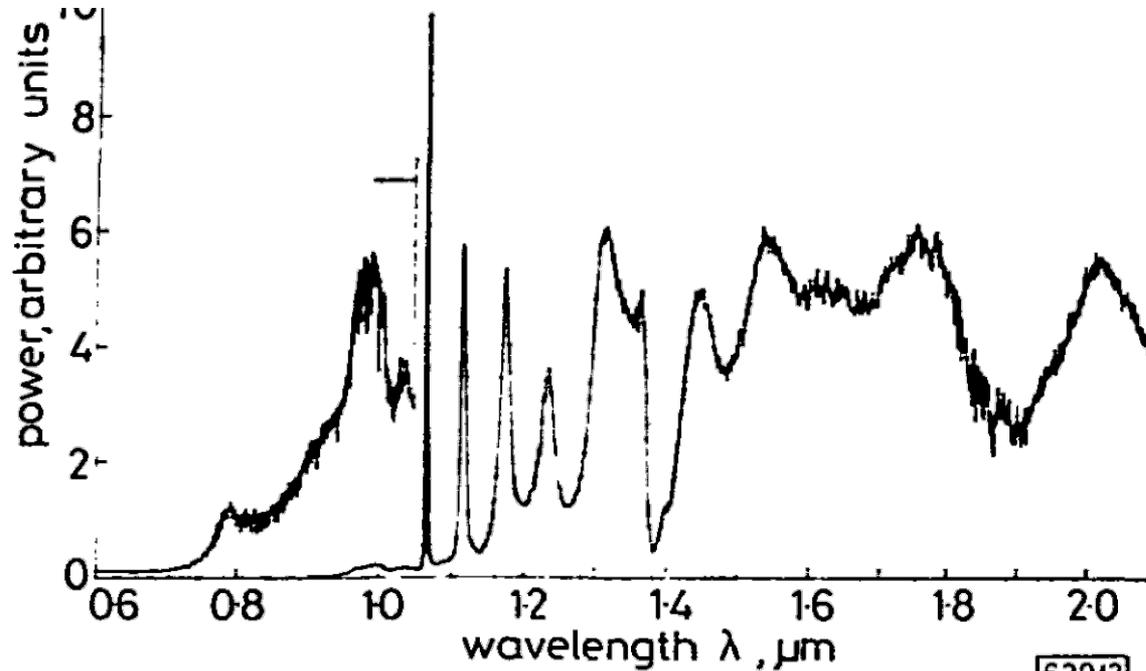
But “only” 36 years old in fiber

Lin *et al.*, Electron. Lett. 14, 822 (1978)



4416 Å

6328 Å



620/2

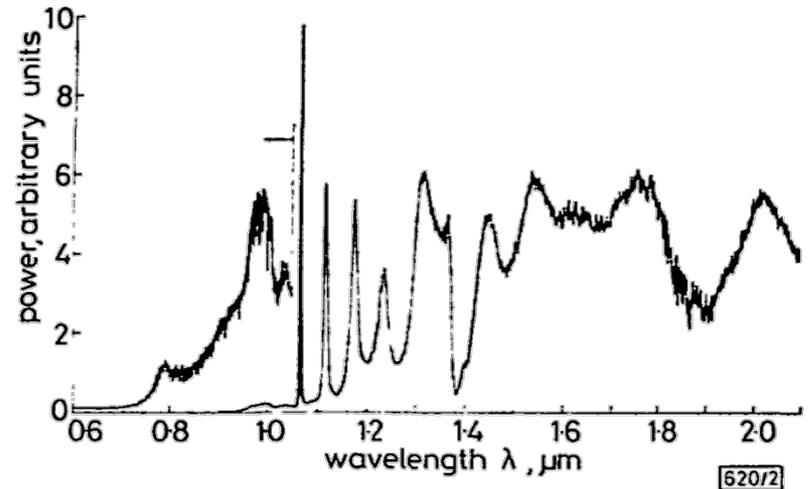
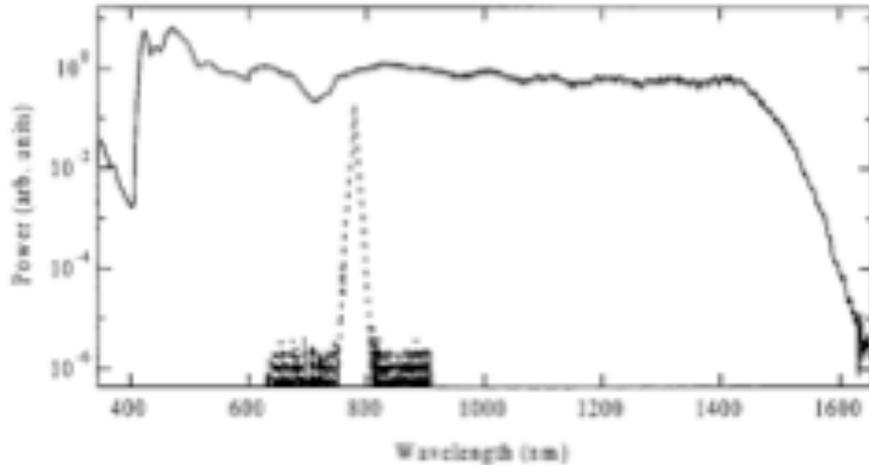
Truth be told: this is an amazing result!

But “only” 36 years old in fiber

Visible continuum generation in air-silica microstructure optical fibers with anomalous dispersion at 800 nm

Jinendra K. Ranka, Robert S. Windeler, and Andrew J. Stentz

Bell Laboratories, Lucent Technologies, 750 Mountain Avenue, Murray Hill, New Jersey 07974



Truth be told: this is an amazing result!

A historical note

PHYSICAL REVIEW A

VOLUME 21, NUMBER 4

APRIL 1980

Combined stimulated Raman scattering and continuum self-phase modulations

Joel I. Gersten*

Institute for Advanced Studies, Hebrew University, Mount Scopus Campus, Jerusalem, Israel

R. R. Alfano and Milivoj Belic

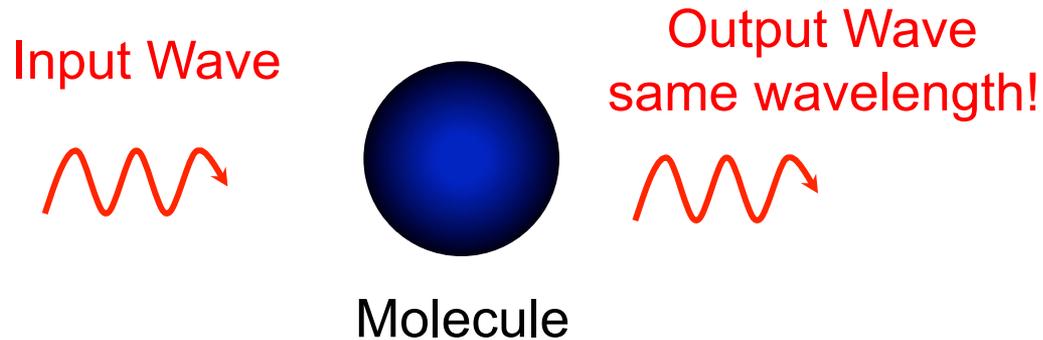
Department of Physics, The City College of New York, New York, N. Y. 10031

(Received 8 January 1979)

A theory describing the combined effects of stimulated Raman scattering and continuum self-phase modulation is developed. As may be expected, the effects are not simply additive. Calculations are presented which determine the interaction of these effects in various limits.

In the case of self-phase modulation (SPM), however, the repopulation of the spectral intensity takes place in a more gradual manner. Owing to the nonlinearity of the medium, the pulse heterodynes against itself and gradually increases its spectral width. There is a continuum of frequencies produced in this process. Supercontinuum generation spanning the visible and infrared region was first observed by Alfano and Shapiro when intense picosecond laser pulses were passed through liquids and solids.³

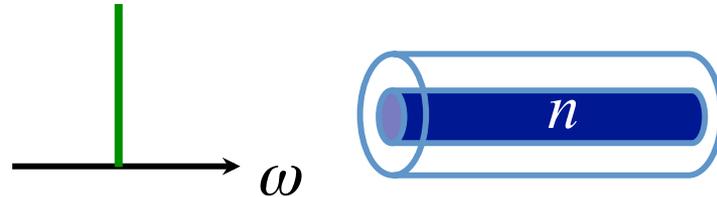
Linear Optics



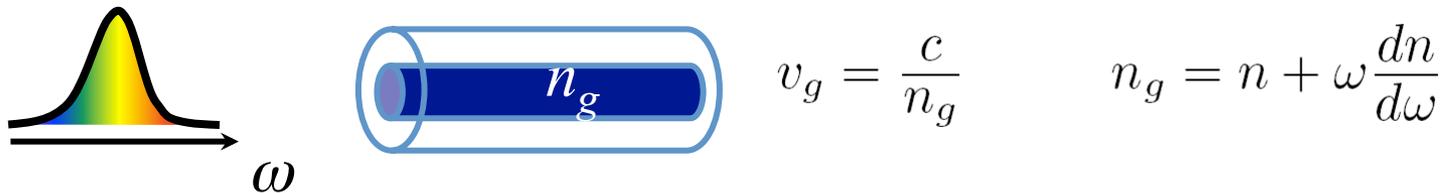
- Linear optics (attenuation and dispersion)
 - Optics of weak light (low intensity)
- Light is deflected or delayed
 - **FREQUENCY** unaffected

Dispersion

- Monochromatic light propagates with phase velocity c/n

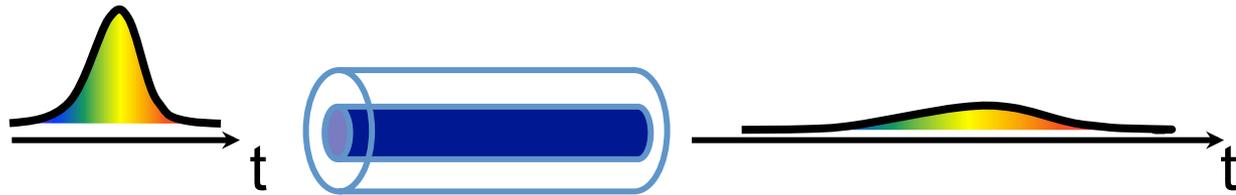


- Light pulse composed of multiple frequency components
 - travels at the group velocity



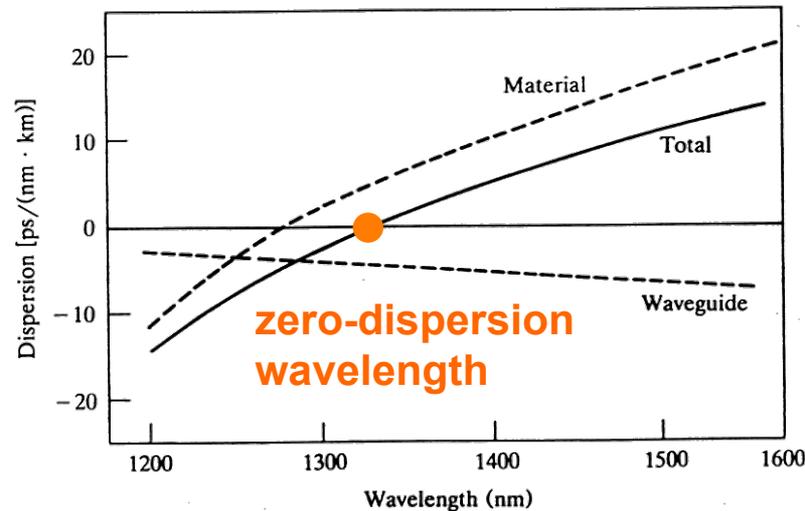
- Dispersion = frequency-dependence of v_g
 - frequency-dependence of refractive index of silica n (material dispersion)
 - frequency-dependence of the size of the mode (waveguide dispersion)

Dispersion



- Different spectral components of the pulse travel at different speeds
- Intensity of a pulse travelling through a fiber is dispersed in time \Rightarrow pulse spreads in time
- Spectrum unchanged

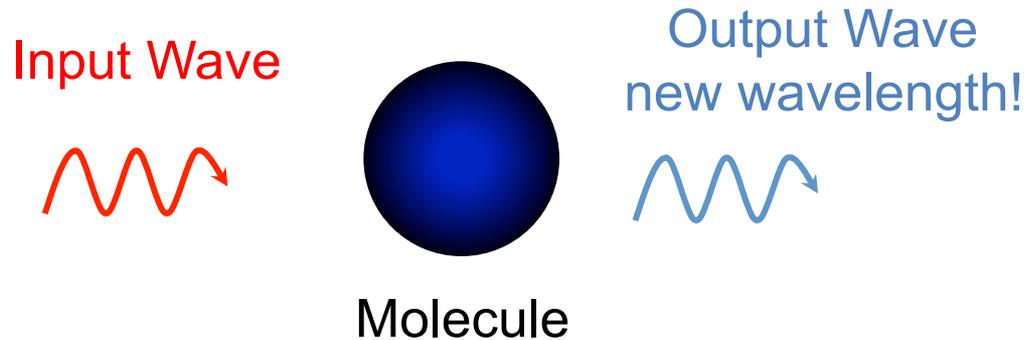
Dispersion



- $D < 0$: normal dispersion
 - Higher frequencies (shorter wavelengths) travel faster than lower frequencies (longer wavelengths)
- $D > 0$: anomalous dispersion
 - Higher frequencies (shorter wavelengths) travel slower than lower freq. (longer wavelengths)
- $D = 0$: zero-dispersion wavelength

- Dispersion leads to walk-off between spectral components of pulses
 - limits the efficiency of nonlinear effects

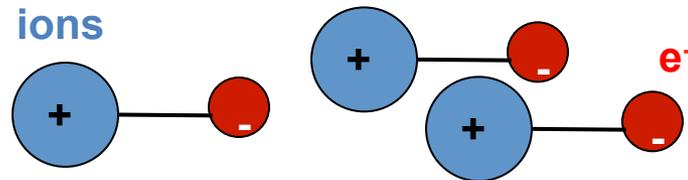
Nonlinear Optics



- Nonlinear optics (scattering and nonlinear refractive index)
 - Optics of intense light
- Light induces effects on its own **AMPLITUDE/PHASE**
 - Affects its **FREQUENCY**

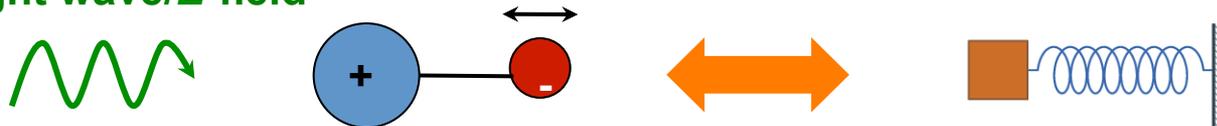
Origin of nonlinear effects

- Medium: collection of charged particles



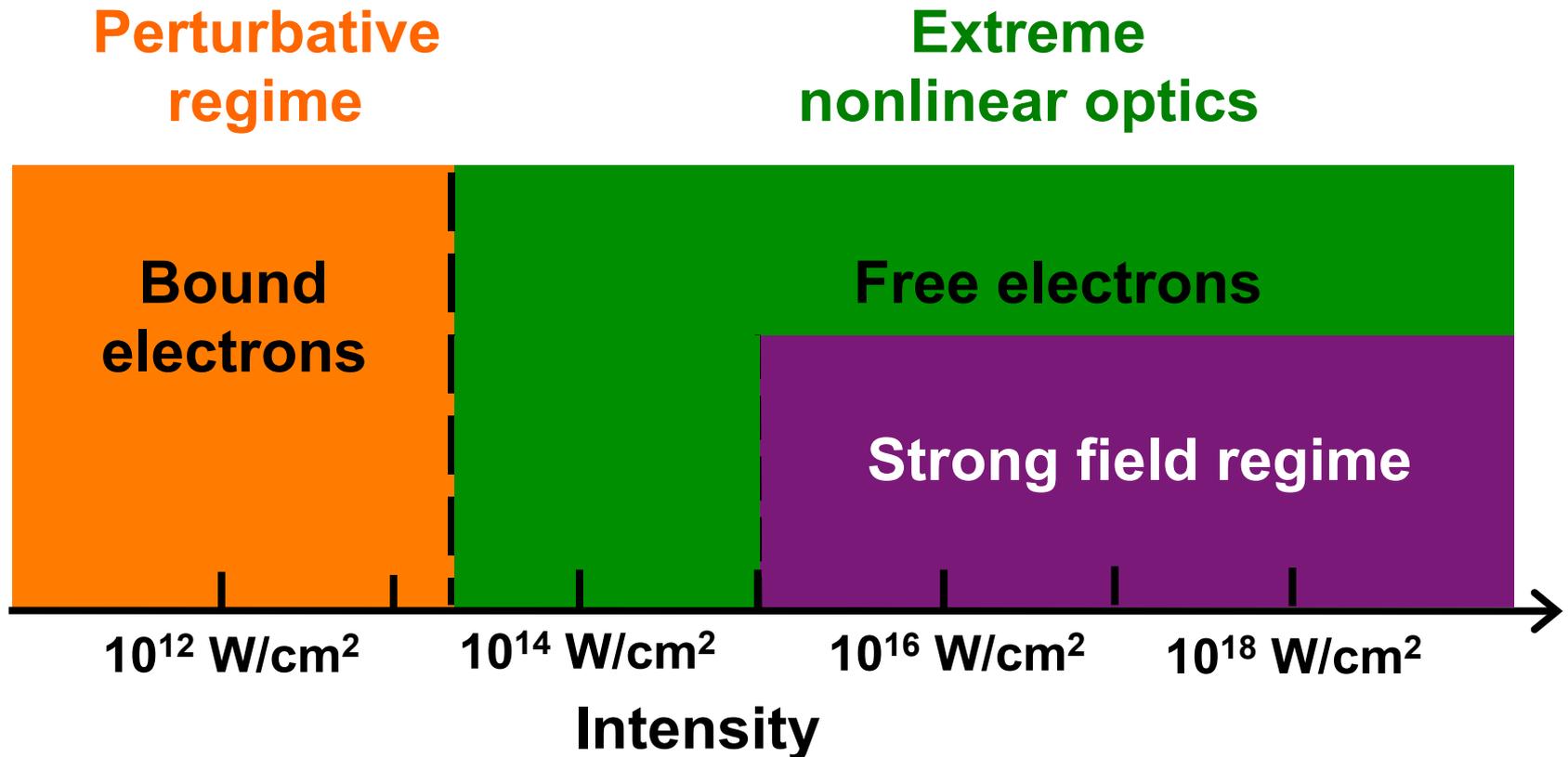
- Light wave travels in the material=oscillating electric field applied  charges moves, electric dipole
- Induced electric dipole moment (i.e. induced polarization)
 - Light radiated at same frequency

Light wave/E-field

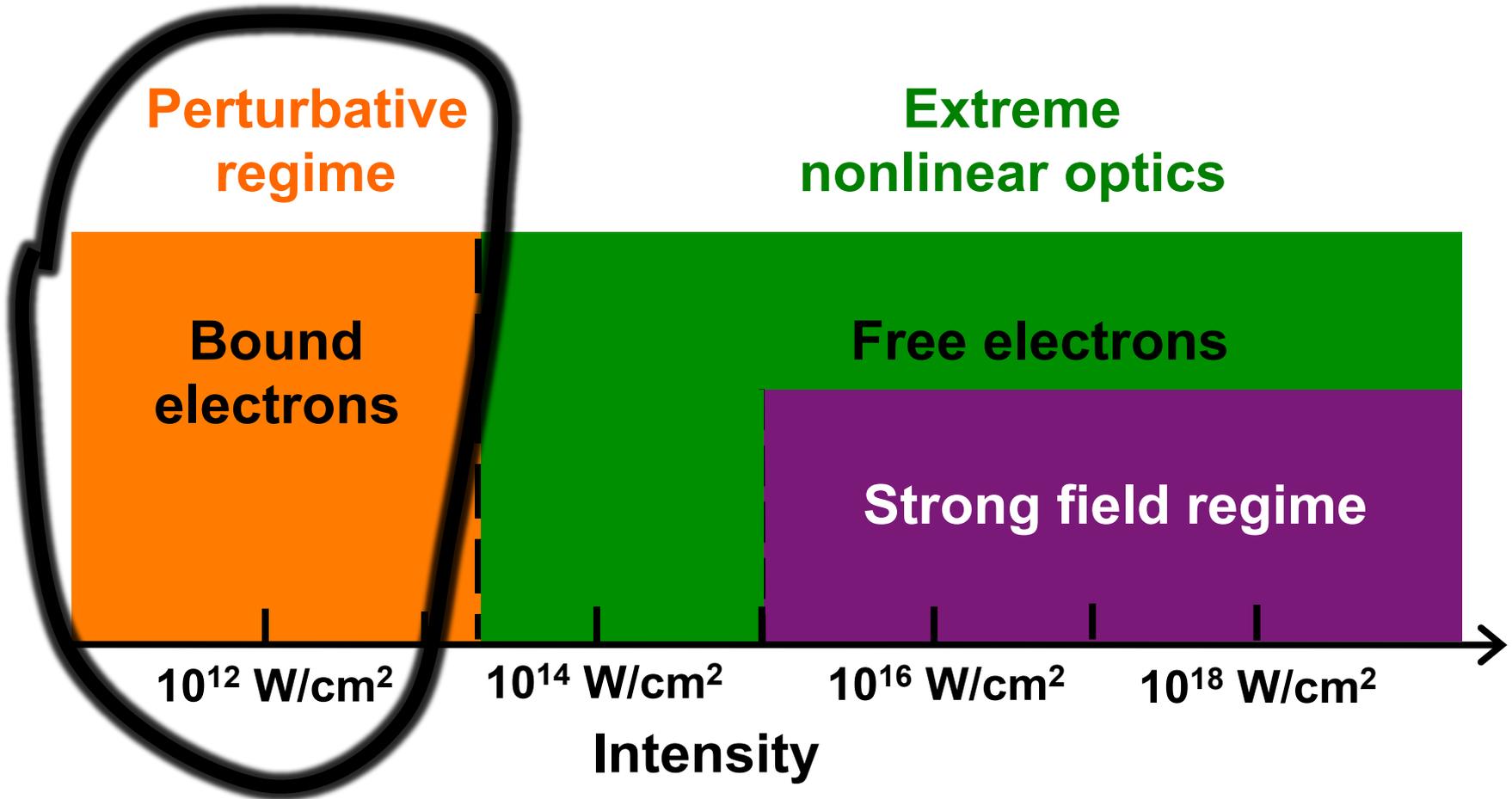


- If electric field large, motion of bound electrons is anharmonic = nonlinear (~spring distorted)
 - Light radiated at harmonic frequencies

Regimes of nonlinear optics



Regimes of nonlinear optics



Nonlinear pulse propagation in optical fibers

- Pulse propagation in optical fiber obeys :

$$\nabla \times \nabla \times E(r,t) + \frac{1}{c^2} \frac{\partial^2 E(r,t)}{\partial t^2} = -\mu_0 \frac{\partial^2 P(r,t)}{\partial t^2}$$

$$\text{with } P(r,t) = P_L(r,t) + P_{NL}(r,t)$$

$$P(r,t) = \underbrace{\varepsilon_0 \chi^{(0)} + \varepsilon_0 \chi^{(1)} E(r,t)}_{\text{Dispersion}} + \underbrace{\varepsilon_0 \chi^{(2)} E^2(r,t) + \varepsilon_0 \chi^{(3)} E^3(r,t) + \dots}_{\text{Nonlinearity}}$$

Dispersion

Nonlinearity

Nonlinear pulse propagation in optical fibers

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$$P(r,t) = \underbrace{\varepsilon_0 \chi^{(0)} + \varepsilon_0 \chi^{(1)} E(r,t)}_{\text{Dispersion}} + \underbrace{\cancel{\varepsilon_0 \chi^{(2)} E^2(r,t)} + \varepsilon_0 \chi^{(3)} E^3(r,t) + \dots}_{\text{Nonlinearity}}$$

- In silica: $\chi^{(2)}=0$ (centro-symmetric material)
- Only **THIRD-ORDER** nonlinear effects

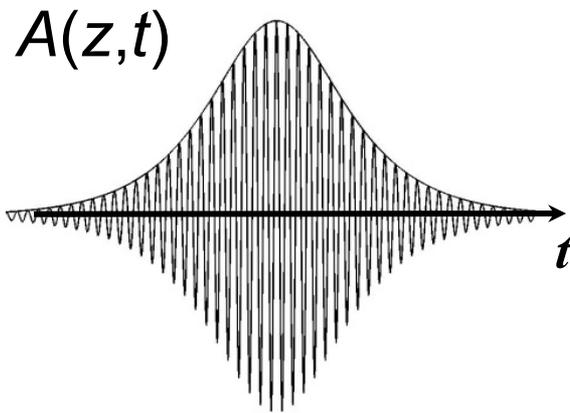
Representation of optical pulses

E-field associated with short laser pulses:

$$E(r, z, t) = F(r)A(z, t)e^{i\omega_0 t} \left\{ \begin{array}{l} F(r) \text{ modal distribution} \\ A(z, t) \text{ temporal envelope} \\ \omega_0 = \frac{2\pi c}{\lambda_0} \text{ carrier frequency} \end{array} \right.$$

Representation of optical pulses

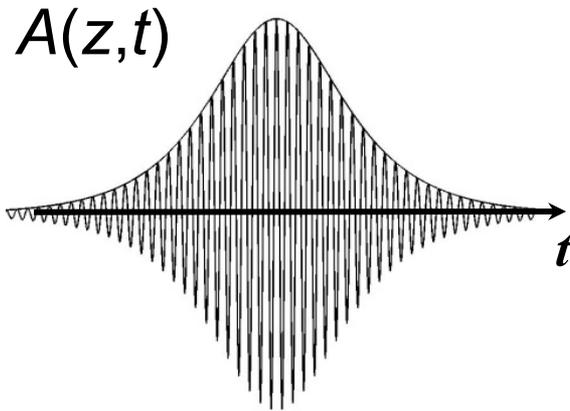
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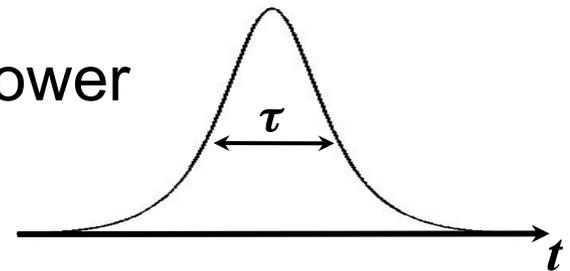
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What you measure in practice

$$P(z, t) = |A(z, t)|^2: \text{ power}$$



P_p : peak intensity (from 100 W to 10^5 W)

τ : pulse duration (from 10^{-9} s to 10^{-15} s)

Representation of optical pulses

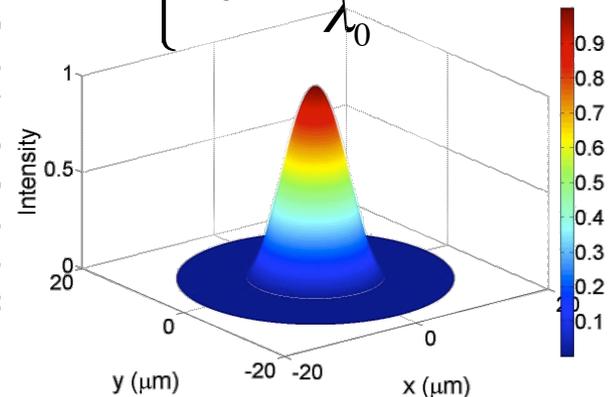
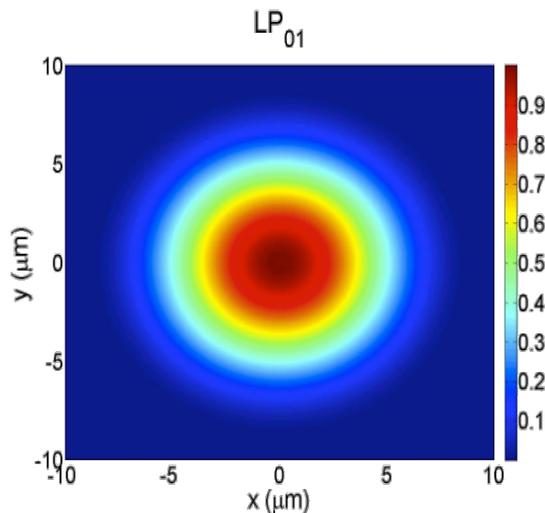
E-field associated with short laser pulses:

$$E(r, z, t) = F(r)A(z, t)e^{i\omega_0 t}$$

$F(r)$ modal distribution

$A(z, t)$ temporal envelope

$\omega_0 = \frac{2\pi c}{\lambda_0}$ carrier frequency



- Consider only fundamental mode (Gaussian)
- Does not vary with propagation
- Only temporal effects matters, no diffraction

Pulse propagation equation

- Generalized nonlinear Schrödinger Equation (GNLSE)

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left(1 + i\tau_{\text{shock}} \frac{\partial}{\partial T} \right) \left(A(z, t) \int_{-\infty}^{+\infty} R(T') |A(z, T - T')|^2 dT' \right)$$

Loss

Dispersion

Self-Steepening term

SPM, FWM, Raman

Intensity-dependent ref. index

$$\gamma = n_2 \omega_0 / c A_{eff}$$

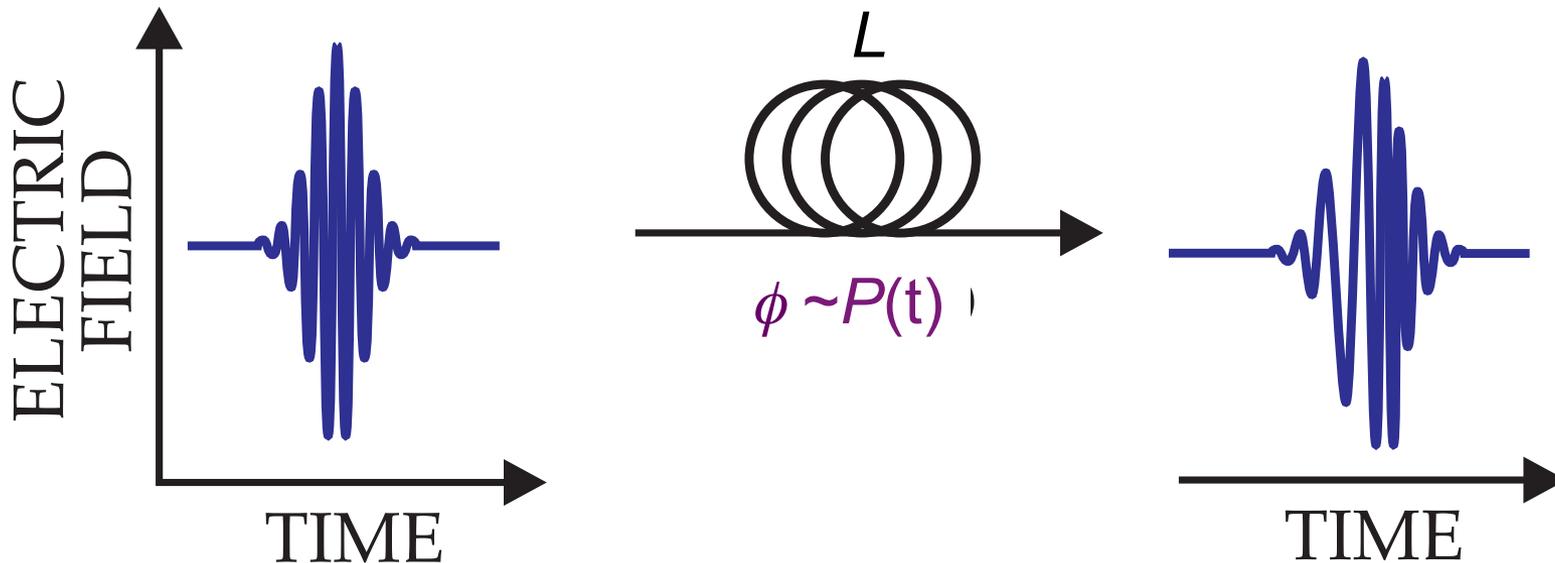
Raman response

$$R(T) = (1 - f_R)\delta(T) + f_R h_R(T)$$

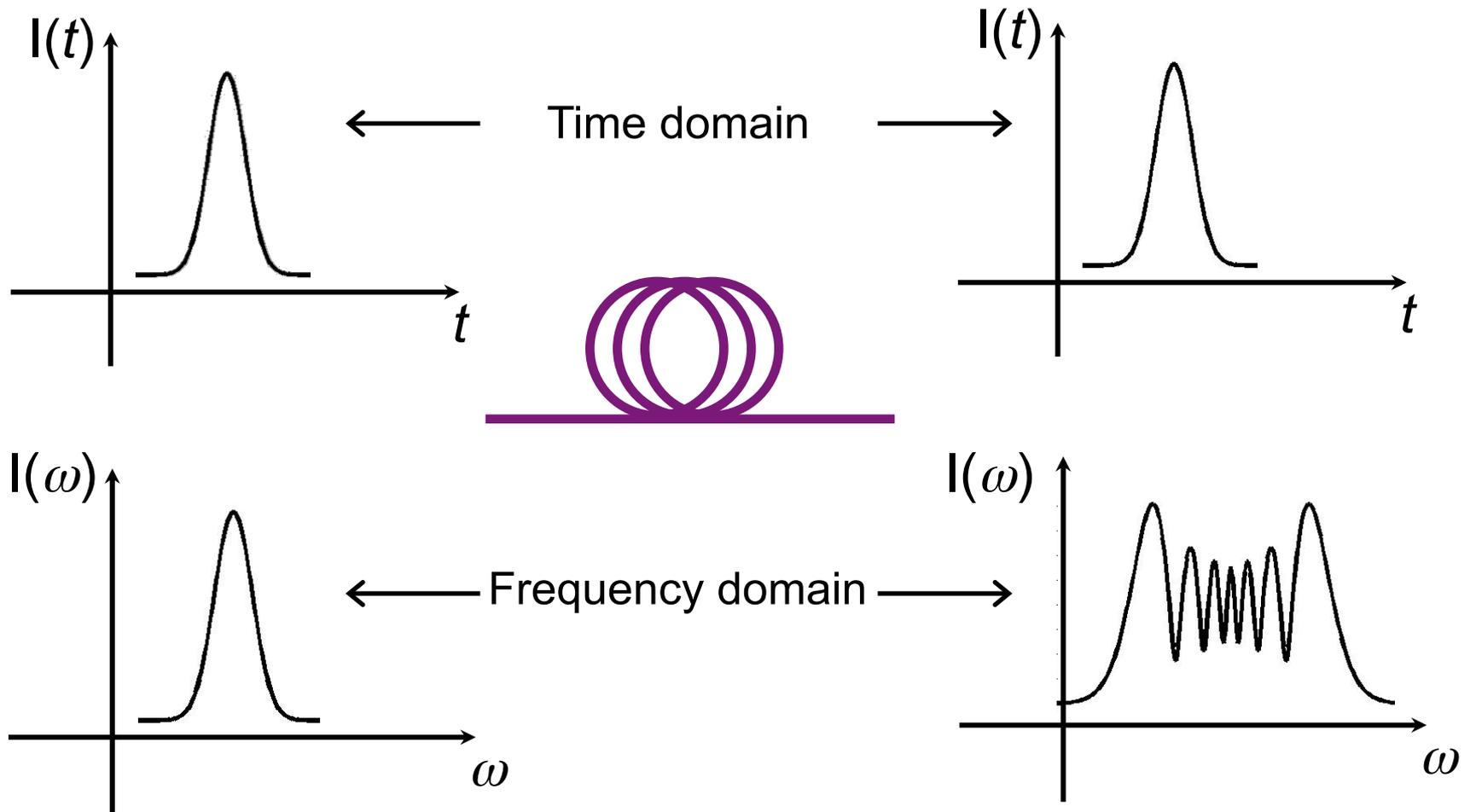
- Can include noise, frequency-dependent mode area, polarization...etc
- Validity: down to single cycle regime

Self-Phase Modulation

- Light modulates its own phase: $\phi_{NL}(t, L) = \gamma P(t) L$
- The frequency of the pulse is time-dependent: chirp



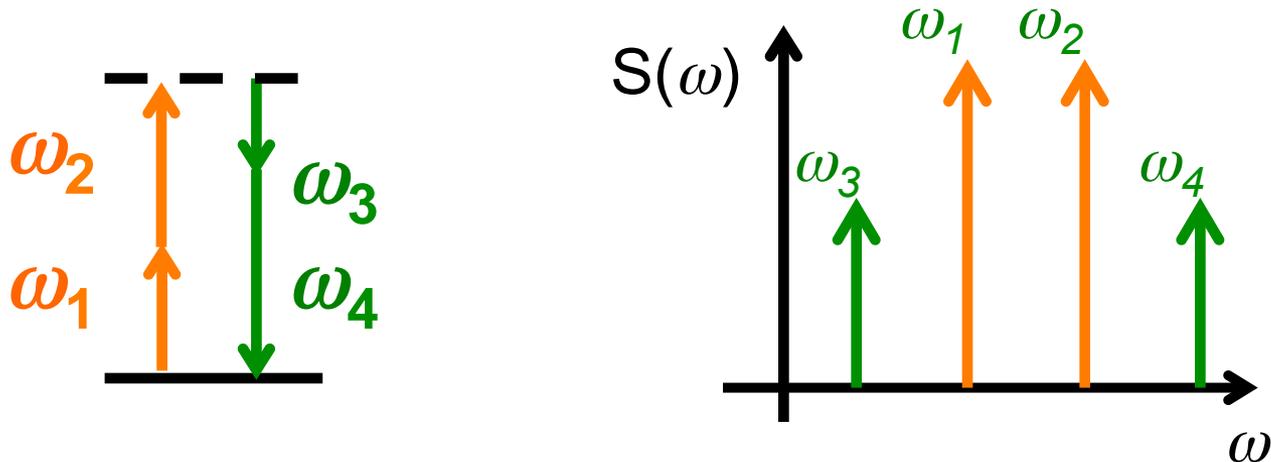
Self-Phase Modulation



- Spectrum broadens but temporal profile unchanged

Four-Wave Mixing

- Nonlinear mixing between two optical signals at different frequencies generates signals at the frequency difference



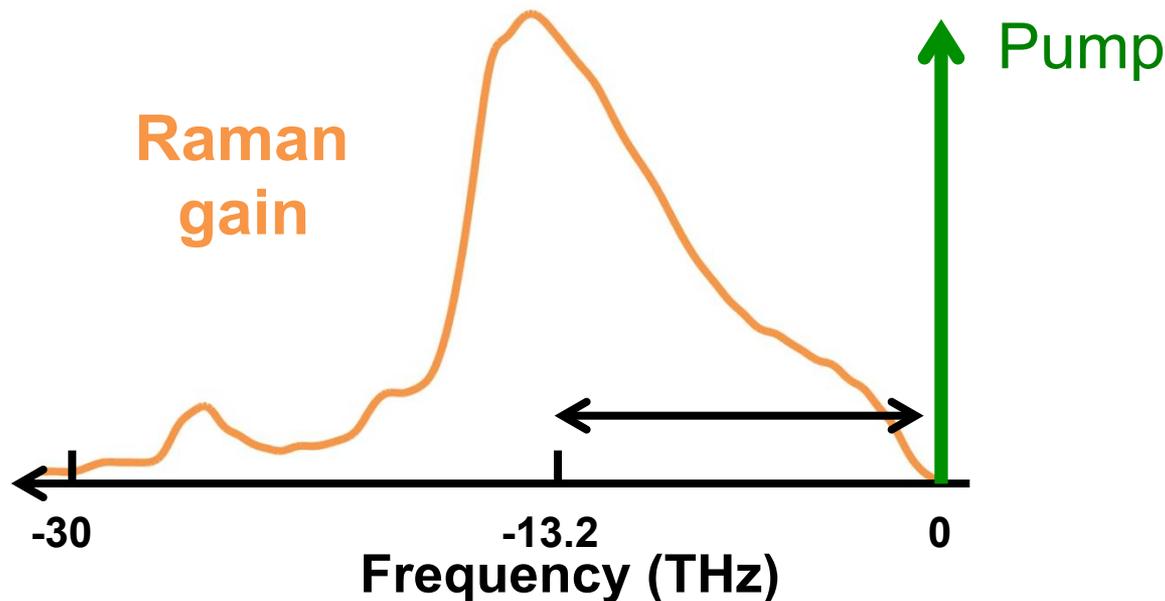
- Energy conservation + phase-matching condition

$$\omega_1 + \omega_2 \rightarrow \omega_3 + \omega_4$$

$$\beta(\omega_1) + \beta(\omega_2) \rightarrow \beta(\omega_3) + \beta(\omega_4)$$

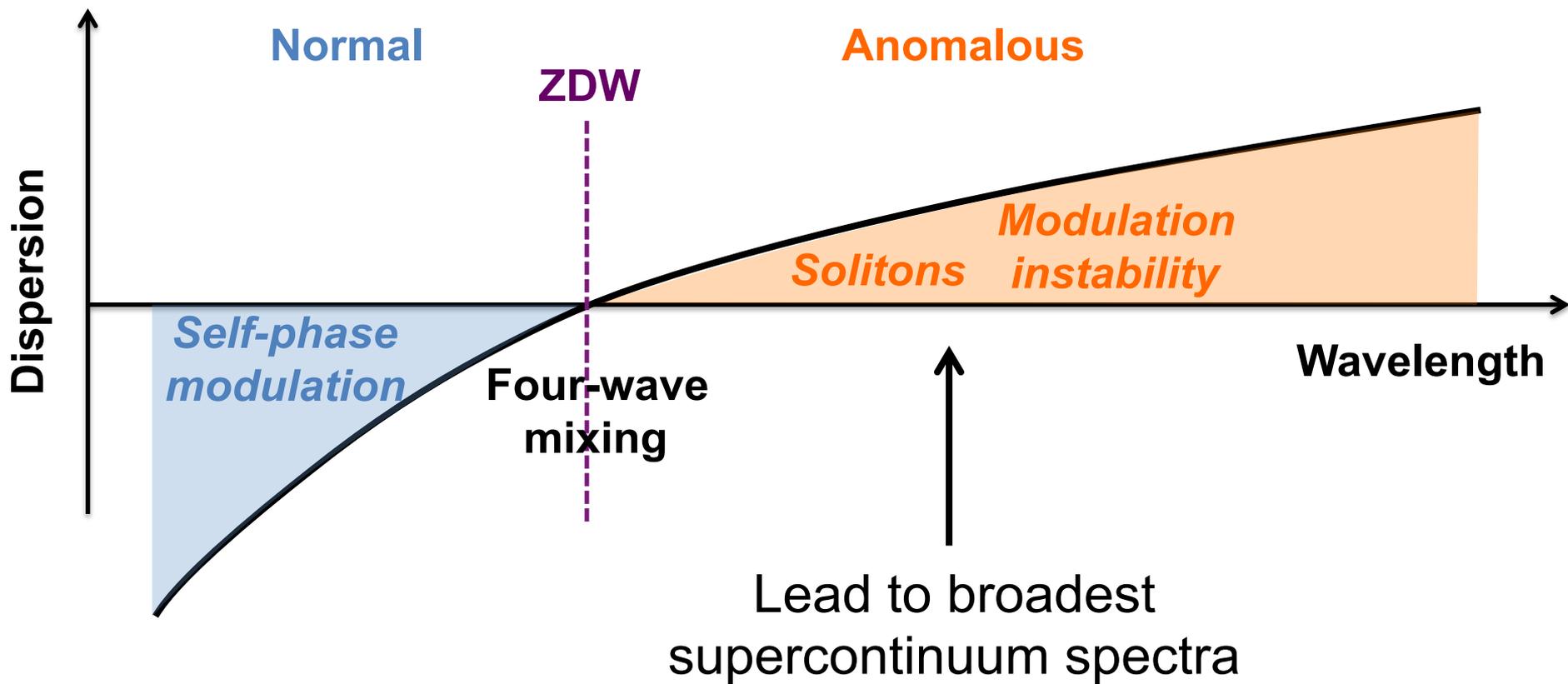
Stimulated Raman Scattering

- Interaction between light and vibrational modes of molecules
- High intensity pump induces gain for a wave with shorter frequency (longer wavelengths)



- Raman gain is broadband and depends on material

Pump wavelength is crucial



Fibers for nonlinear optics

- Pulse propagation in optical fibers
 - No diffraction, long interaction length



- Dispersion/nonlinearity can be controlled: crucial!
 - propagation dynamics depend on pump wavelength relative to fiber zero dispersion wavelength (ZDW)

“NEW” FIBERS

Small core, high doping
Photonic crystal fibers
Tapered fibers
Non-silica materials

PARAMETER CONTROL

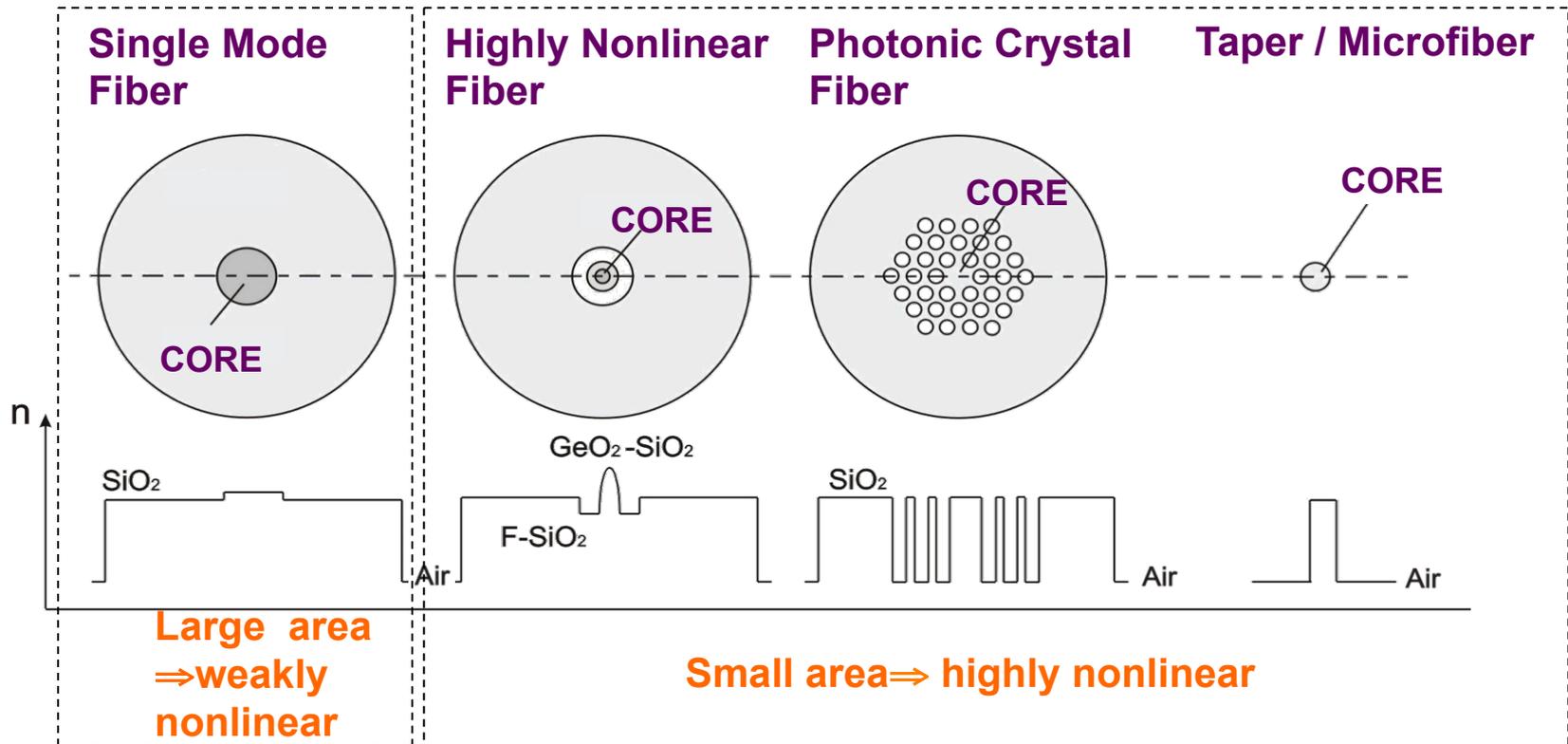
Dispersion
nonlinearity
Confinement

APPLICATIONS

Supercontinuum
Frequency conversion
Pulse compression
Amplification

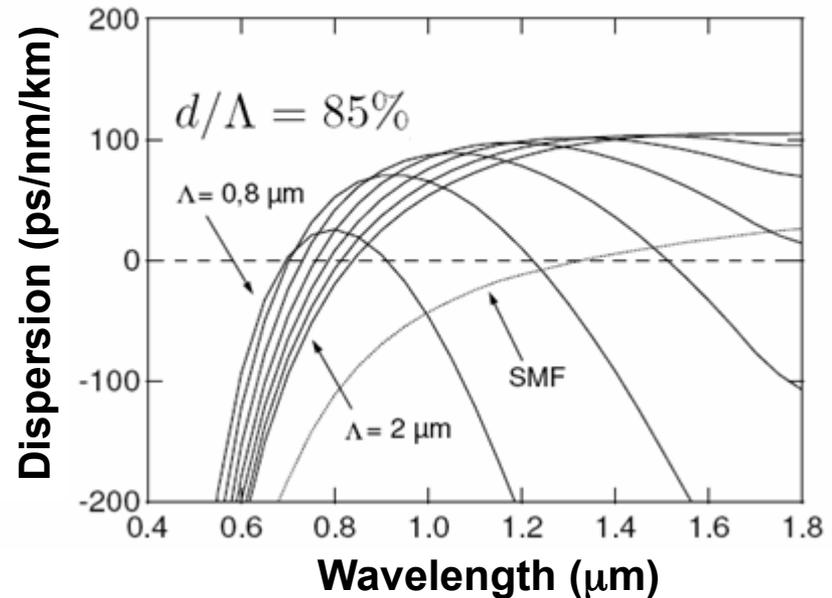
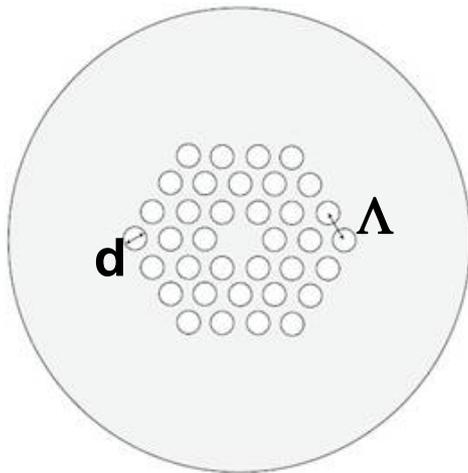
Fibers for nonlinear optics

- Both chemistry and geometry affects the refractive index profile
 - Determine modal confinement (nonlinearity) and dispersion characteristics
 - Wide range of possibilities



Photonic crystal fibers

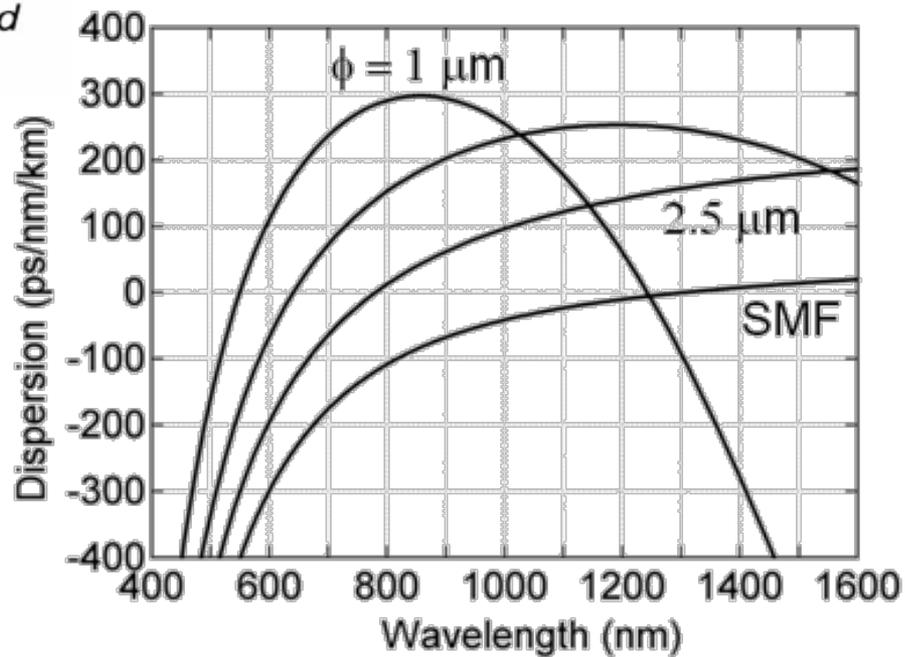
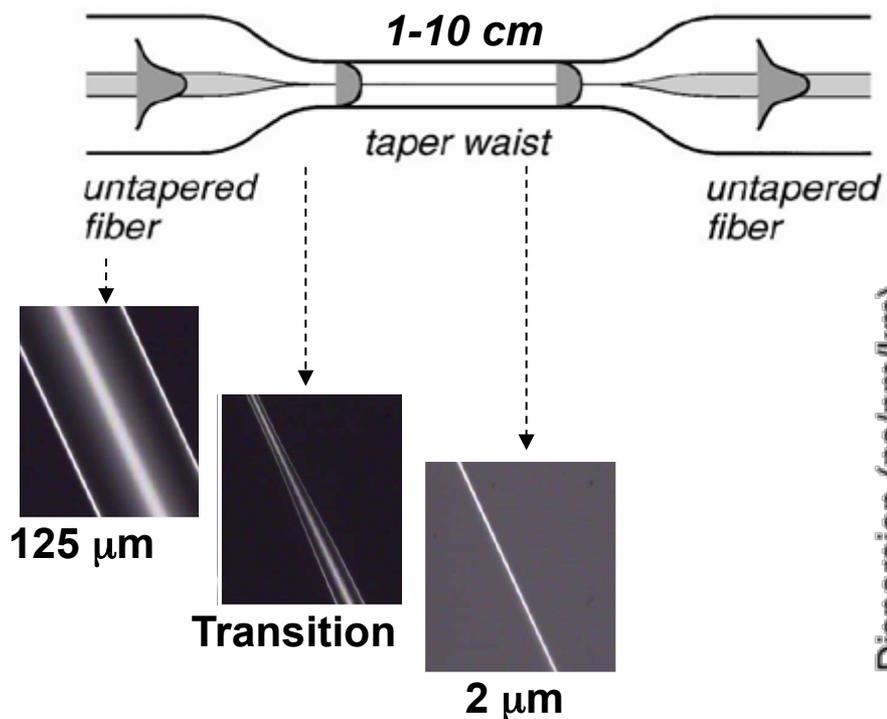
- Generally single material with a high air-fill fraction



- ZDW displaced to shorter wavelengths (match high-power short pulse sources)
- Large nonlinearity (x100 compared to standard fibers)
 \Rightarrow nonlinear effects dramatically enhanced

Taper/microfibers

- Tapering standard fibers: similar properties to photonic crystal fibers



Dispersion engineering

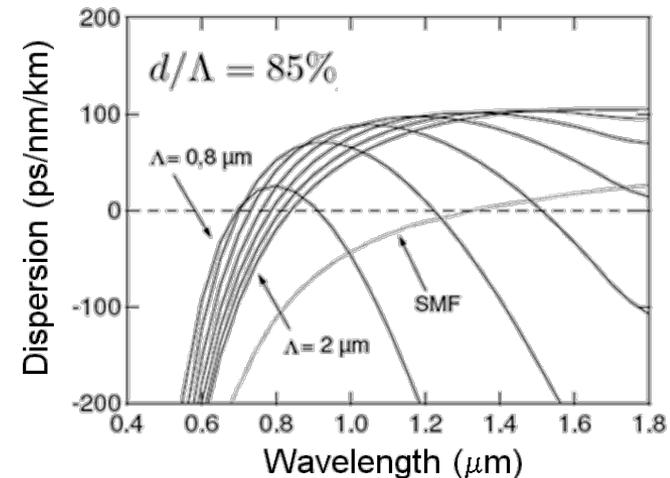
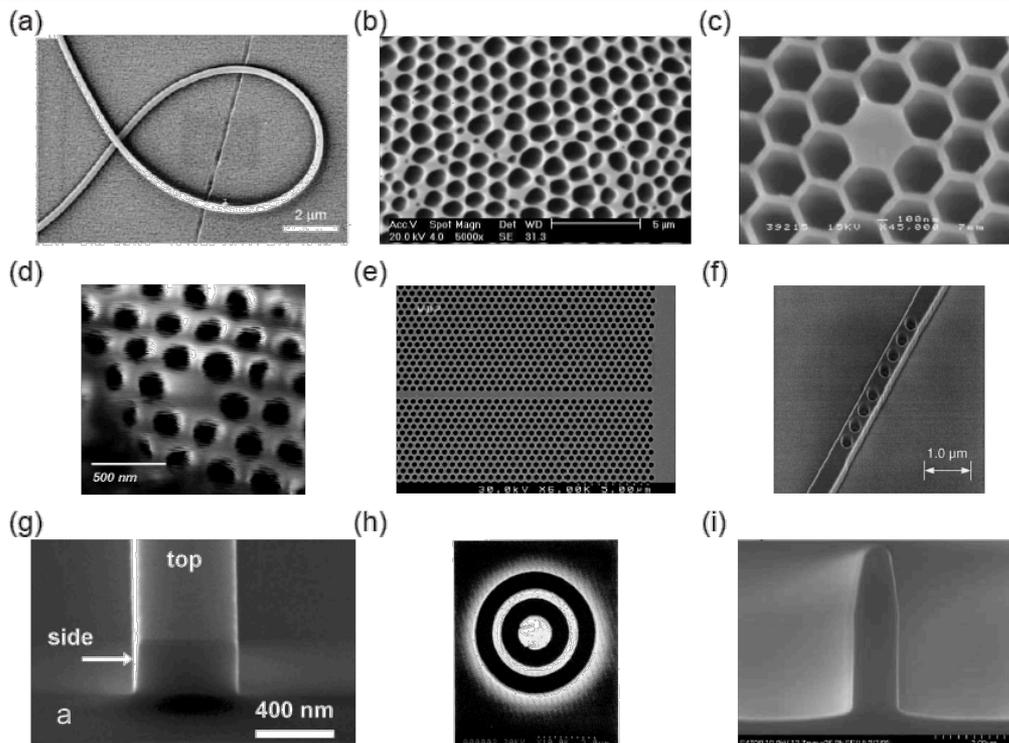
Micro/nano-structured waveguides: engineered nonlinearity and dispersion

(C) 2008 OSA

21 January 2008 / Vol. 16, No. 2 / OPTICS EXPRESS 1300

Nonlinear optics in photonic nanowires

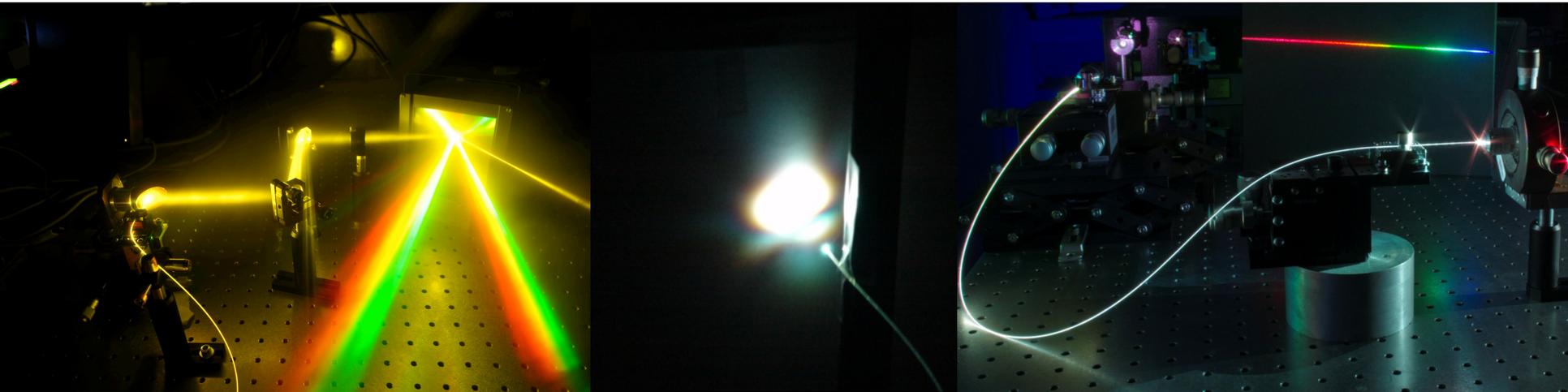
Mark A. Foster¹, Amy C. Turner², Michal Lipson², and Alexander L. Gaeta¹



- Tune dispersion vs. pump wavelength
- Light tightly confined → very large intensities!

Supercontinuum generation is easy

Pretty much anything works...

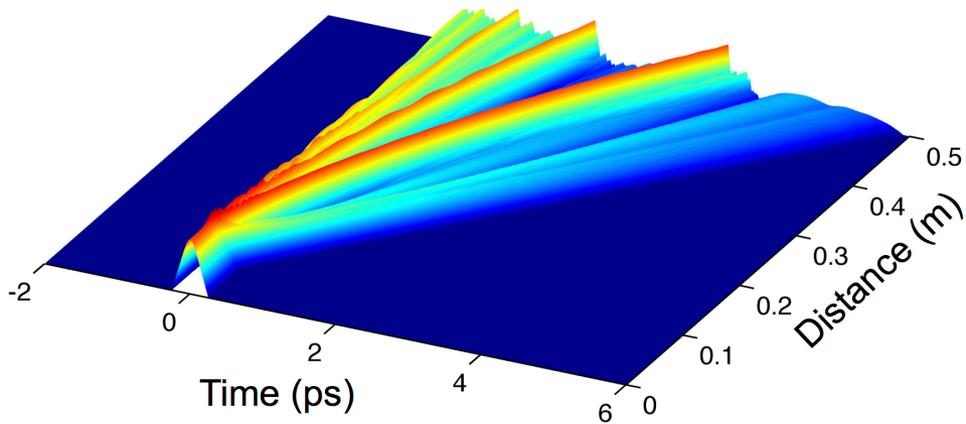


Source: fs, ps, ns pulses, CW lasers

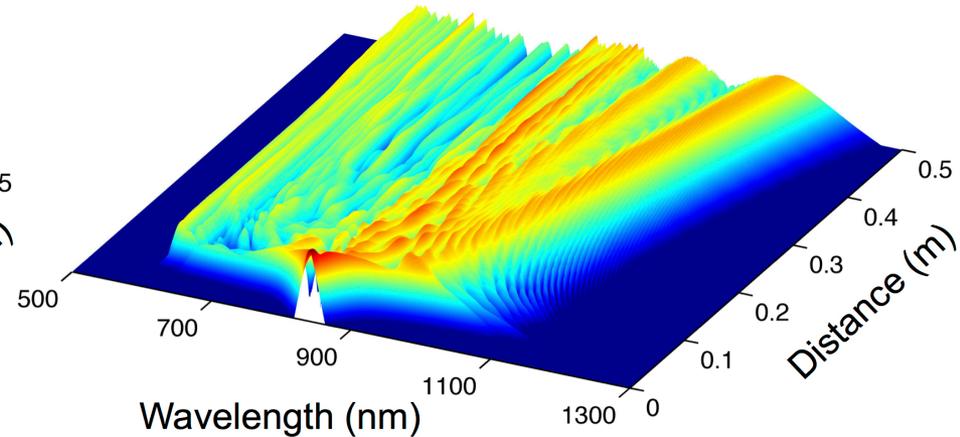
Fibers: PCFs, HNLf, DSF, SMF 28

Dynamics are rather complex...

Time evolution



Spectral evolution



Short pulses

Anomalous

- Soliton
- Dispersive waves

Long pulses

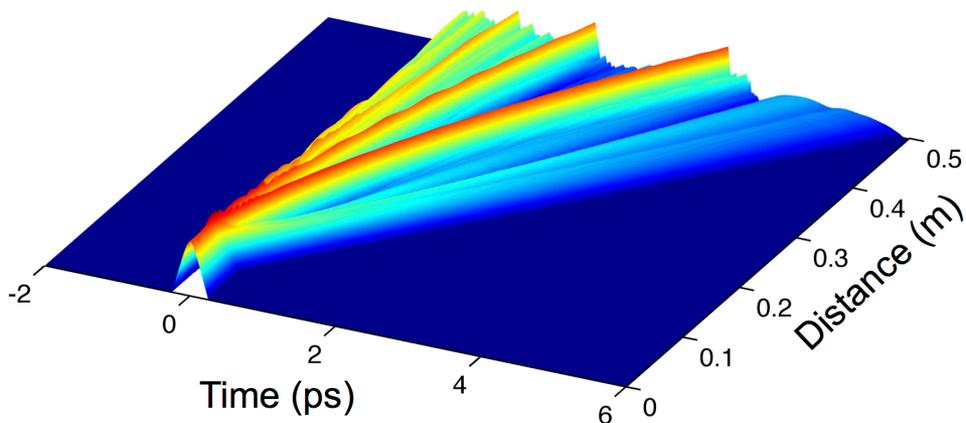
- Modulation instability
- Solitons dynamics

Normal

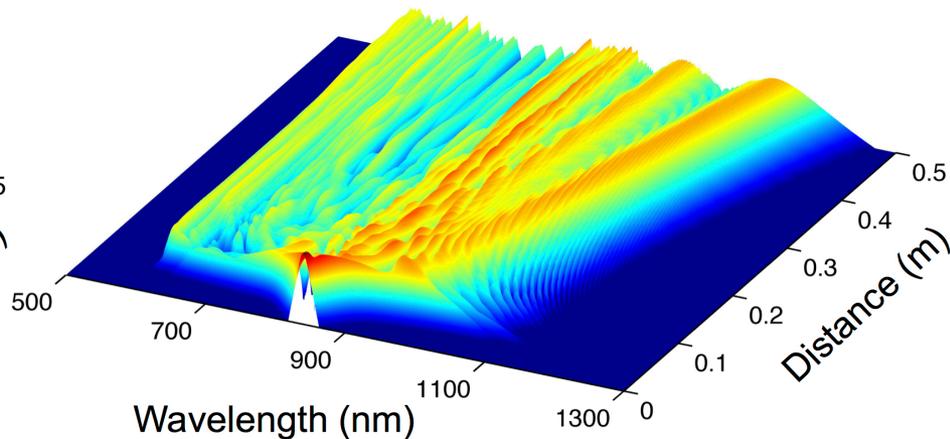
- Self-phase modulation
- Four-wave mixing
- Raman scattering
- Four-wave mixing

Dynamics are rather complex...

Time evolution



Spectral evolution



Short pulses

Long pulses

Anomalous

- Soliton
- Dispersive waves

- Modulation instability
- Solitons dynamics

Normal

- Self-phase modulation
- Four-wave mixing

- Raman scattering
- Four-wave mixing

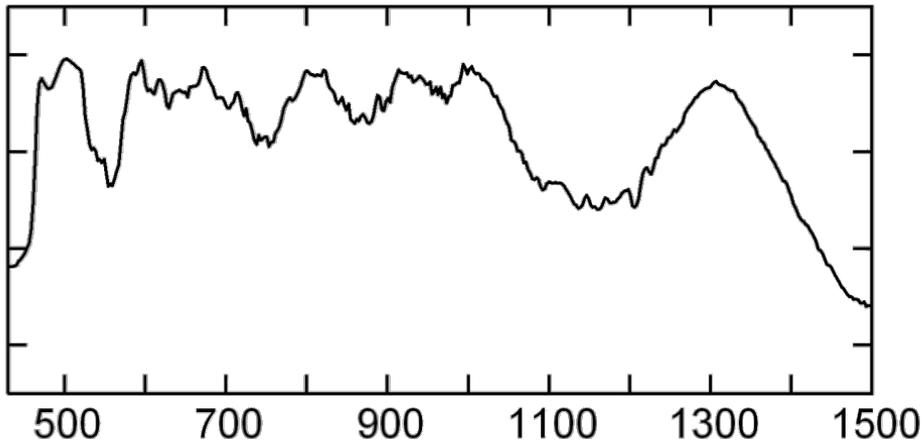
Coherent

Incoherent

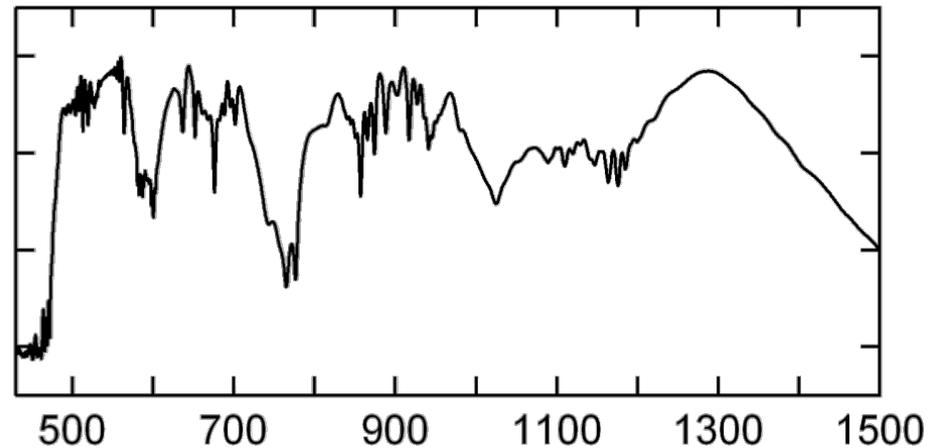
Modelling supercontinuum

- Good agreement with experimental results

Experiment



Simulation



- Success in modeling: physics well-understood

Understanding pulse propagation dynamics

- Numerical (analytical) modelling
- Visualization
- Need to be done properly!



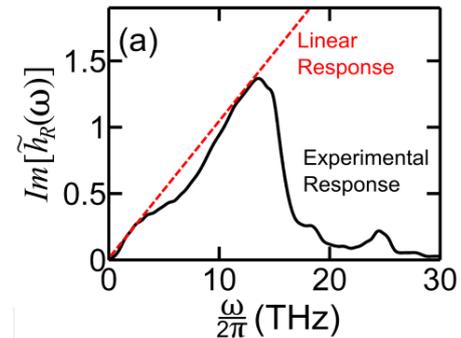
Modeling supercontinuum

- Generalized nonlinear Schrödinger Equation (GNLSE)

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left(1 + i\tau_{\text{shock}} \frac{\partial}{\partial T} \right) \left(A(z, t) \int_{-\infty}^{+\infty} R(T') |A(z, T - T')|^2 dT' \right)$$

- Important considerations

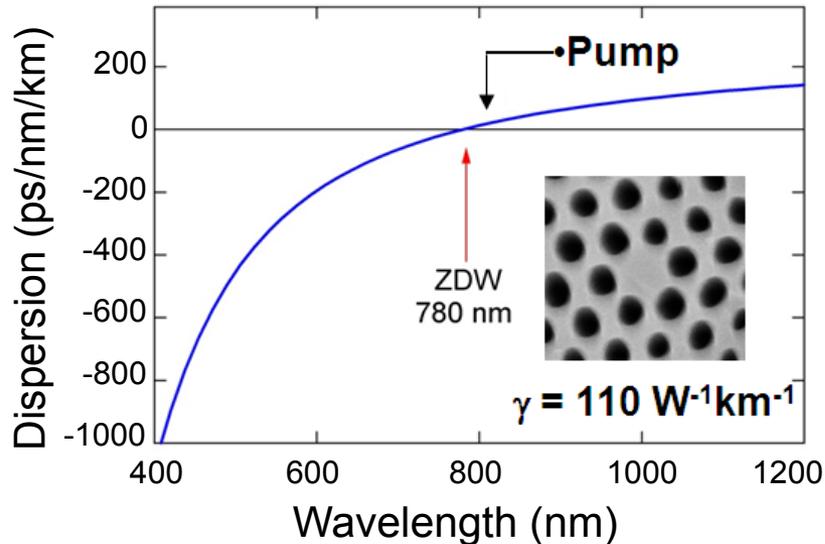
- window size (avoid wrapping around)
- temporal/spectral resolution
- step size (FWM artefacts)
- DO NOT use linear Raman model
- Disp. coeff. must be changed if you change the pump wavelength
- Better to use full β (and not 25th order...)



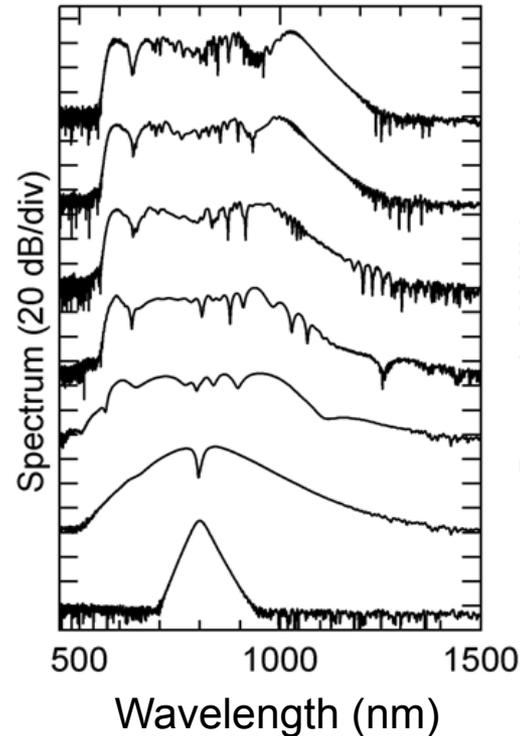
Short pulse regime and anomalous dispersion

- 30 fs, 10 kW input pulses

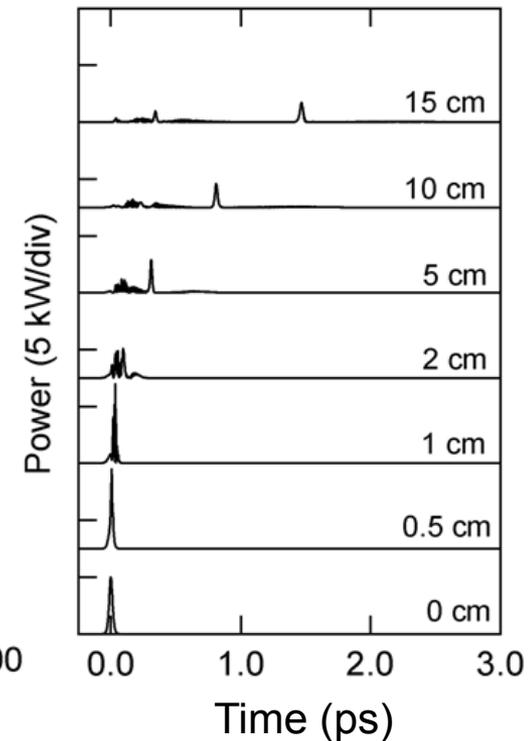
Anomalous pumping



(a) Spectral Evolution



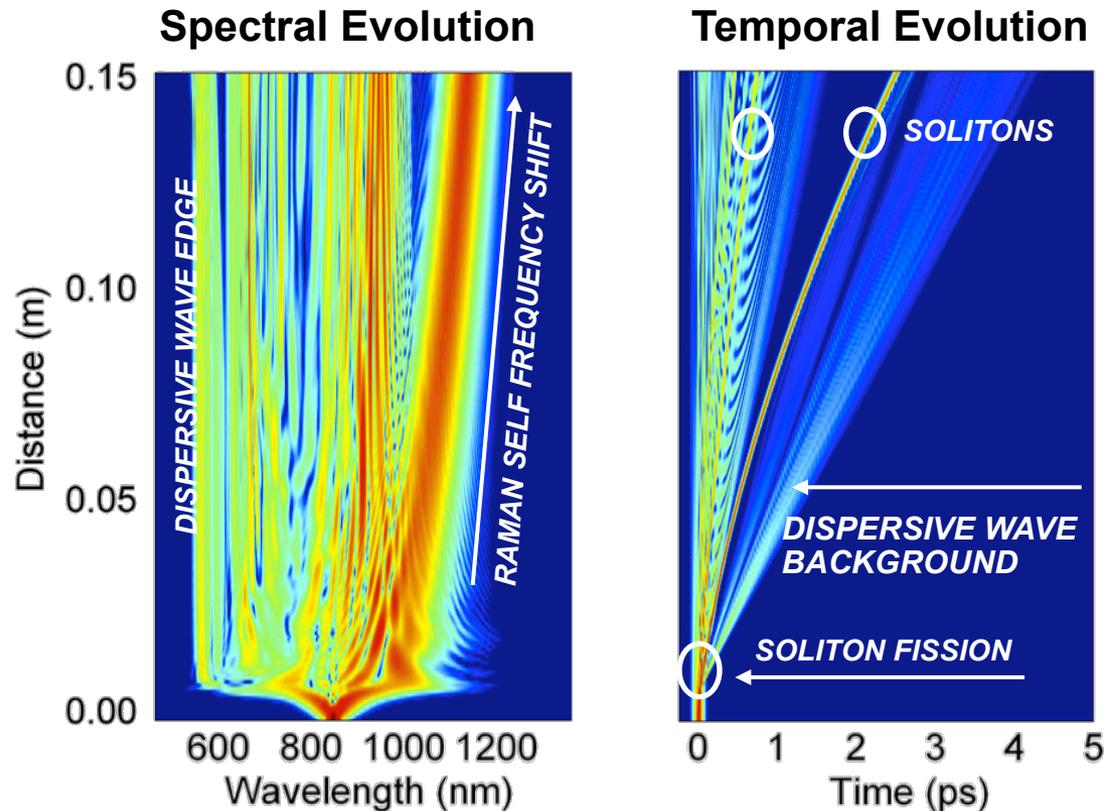
(b) Temporal Evolution



- Evolution can be divided in 3 stages

- Initial higher-order soliton compression (spectral broadening),
- soliton fission and dispersive wave generation
- Raman self-frequency shift

Better in color



- Evolution can be divided in 3 stages
 - Initial higher-order soliton compression (spectral broadening),
 - soliton fission and dispersive wave generation
 - Raman self-frequency shift

Deconstructing supercontinuum dynamics

- Understanding the complex dynamics: study separately the different stages

- Soliton propagation dynamics
 - Fundamental solitons
 - Higher-order solitons
 - Effect of perturbation: higher-order dispersion and Raman scattering

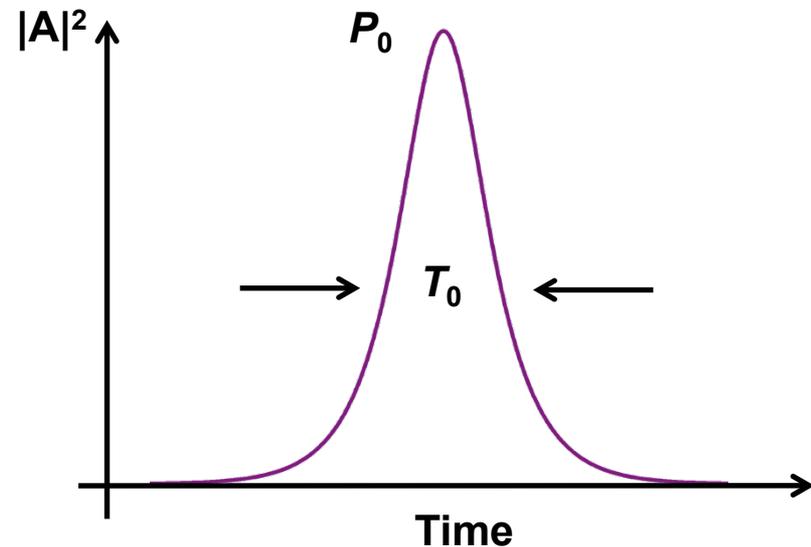
Fundamental soliton

- Fundamental soliton are invariant solution of the nonlinear Schrödinger equation (chirp from self-phase modulation balances chirp from ANOMALOUS dispersion)

$$i \frac{\partial A}{\partial z} + \frac{|\beta_2|}{2} \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0$$

GVD

Kerr effect



Requirements

$$\left[\begin{array}{l} A(z=0, T) = \sqrt{P_0} \operatorname{sech}(T/T_0) \quad A(z, T) = \sqrt{P_0} \operatorname{sech}(T/T_0) e^{jk_{sol}z} \\ N = \sqrt{L_d / L_{nl}} = \sqrt{\gamma P_0 T_0^2 / |\beta_2|} = 1 \end{array} \right. \text{Soliton number } \gamma P_0 / 2 \leftarrow$$

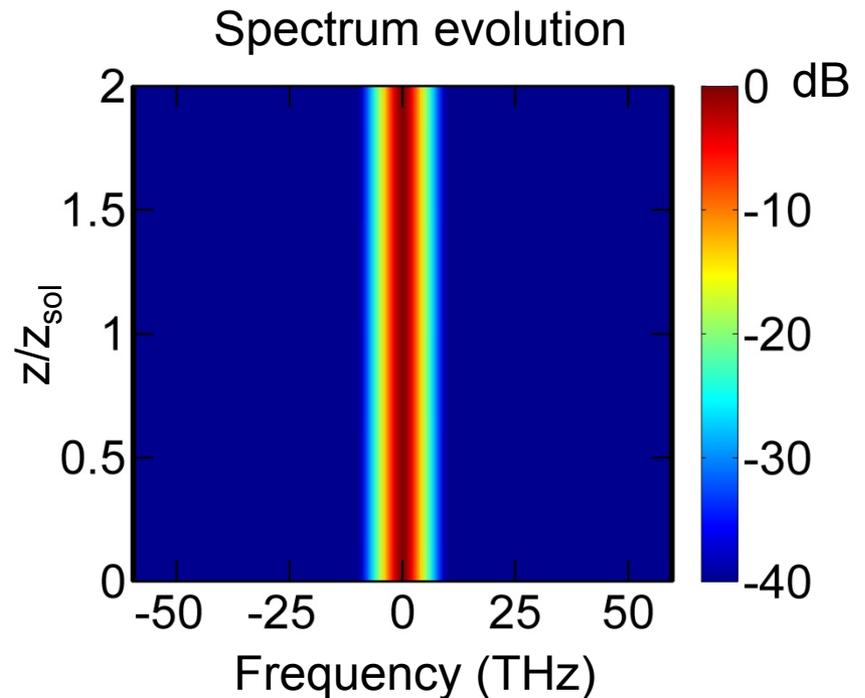
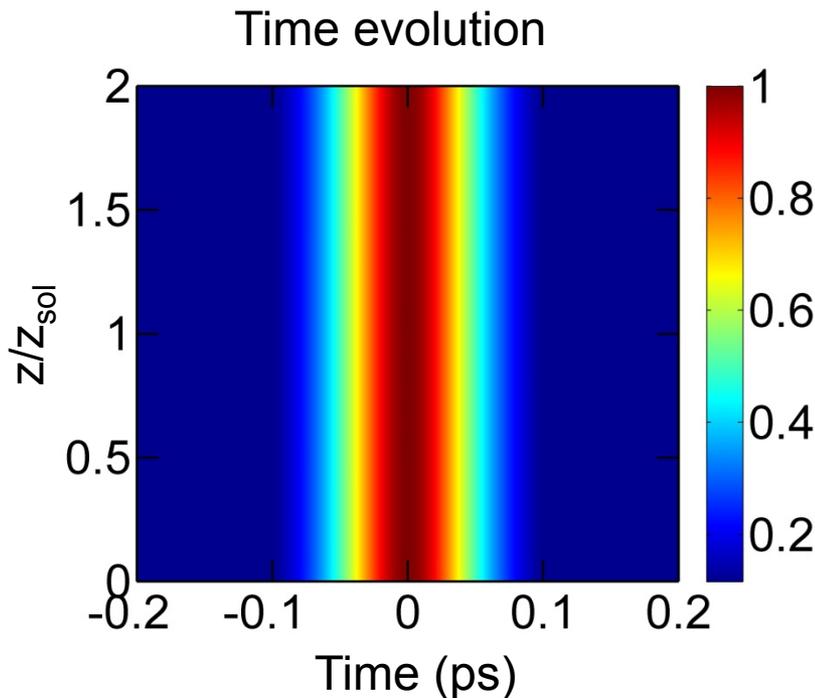
Fundamental soliton

- Fundamental soliton is invariant upon propagation (except for a constant nonlinear phase-shift)

Requirements

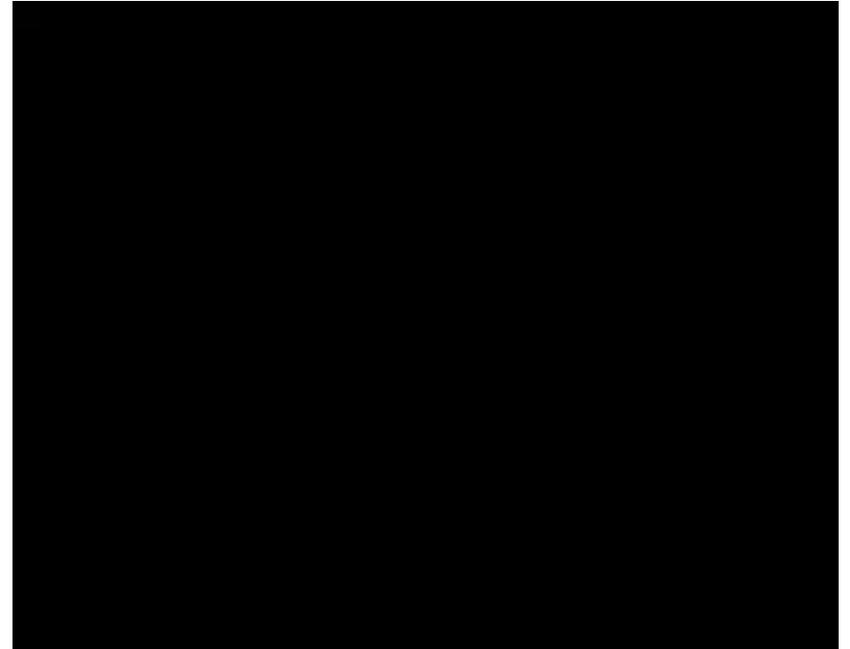
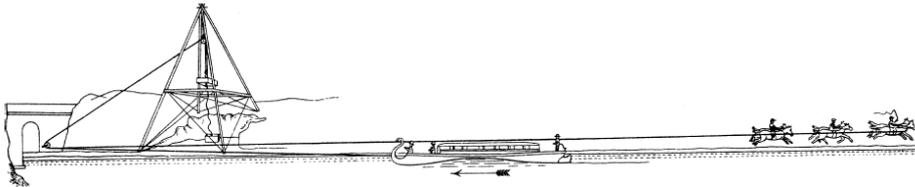
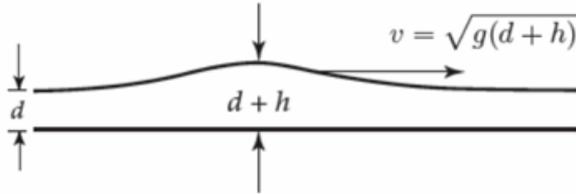
$$\begin{cases} A(z=0, T) = \sqrt{P_0} \operatorname{sech}(T/T_0) & A(z, T) = \sqrt{P_0} \operatorname{sech}(T/T_0) e^{jk_{sol}z} \\ N = \sqrt{L_d/L_{nl}} = \sqrt{\gamma P_0 T_0^2 / |\beta_2|} = 1 \end{cases}$$

Soliton number $\gamma P_0/2$ ←



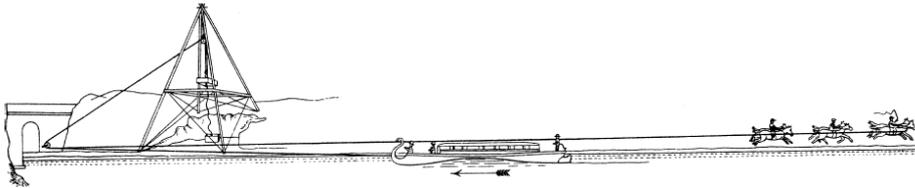
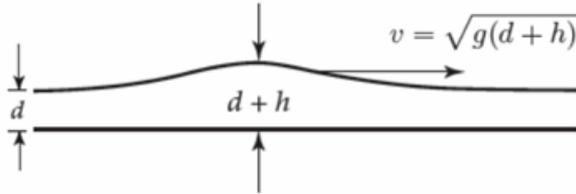
Solitons

- First soliton was observed as the “wave of translation” by Russell (1834)



Solitons

- First soliton was observed as the “wave of translation” by Russell (1834)



- Soliton experience elastic scattering



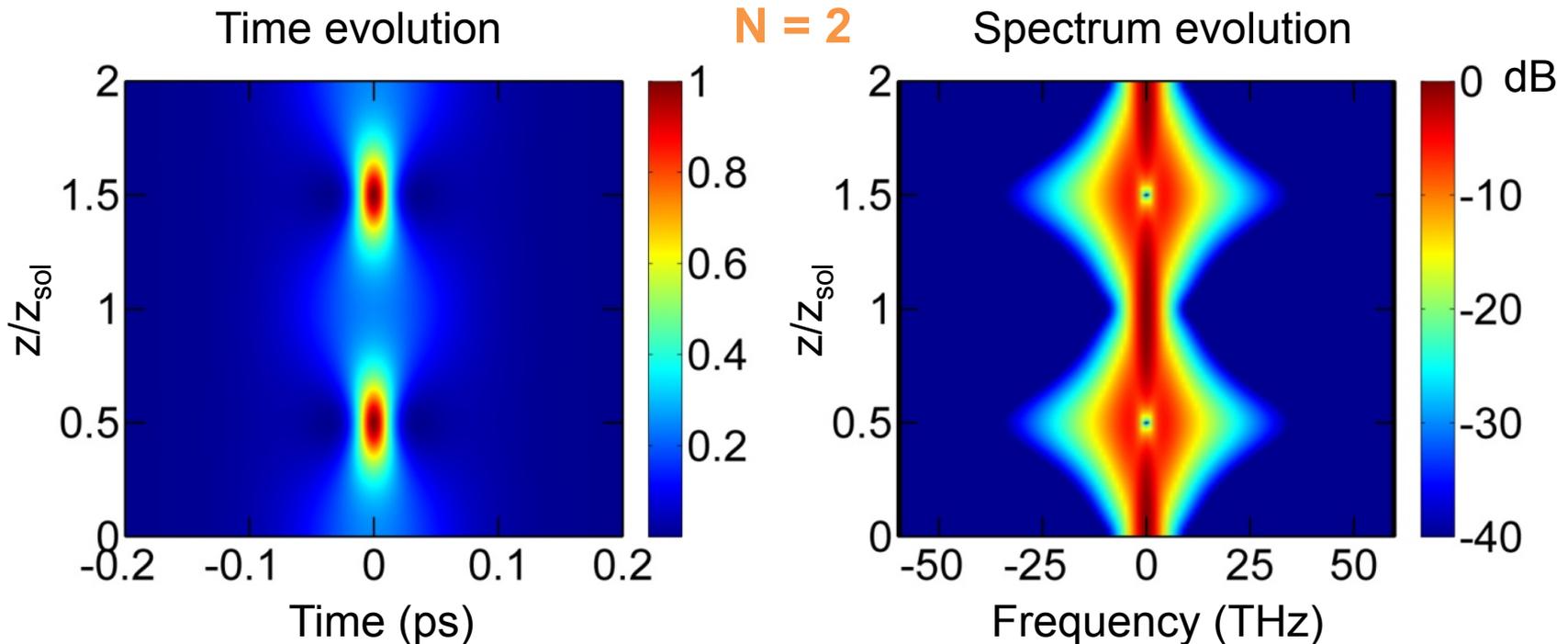
Higher-order solitons

- Higher-order soliton is periodic upon propagation $A(z+z_{\text{sol}}, T) = A(z, T)$

Requirements

$$\left[\begin{array}{l} A(z=0, T) = \sqrt{P_0} \operatorname{sech}(T/T_0) \\ N = \sqrt{L_d/L_{nl}} = \sqrt{\gamma P_0 T_0^2 / |\beta_2|} = 2, 3, 4, \dots \end{array} \right. \quad z_{\text{sol}} = \frac{\pi}{2} L_d = \frac{\pi}{2} T_0^2 / |\beta_2|$$

Quantized! → Soliton period



Higher-order solitons

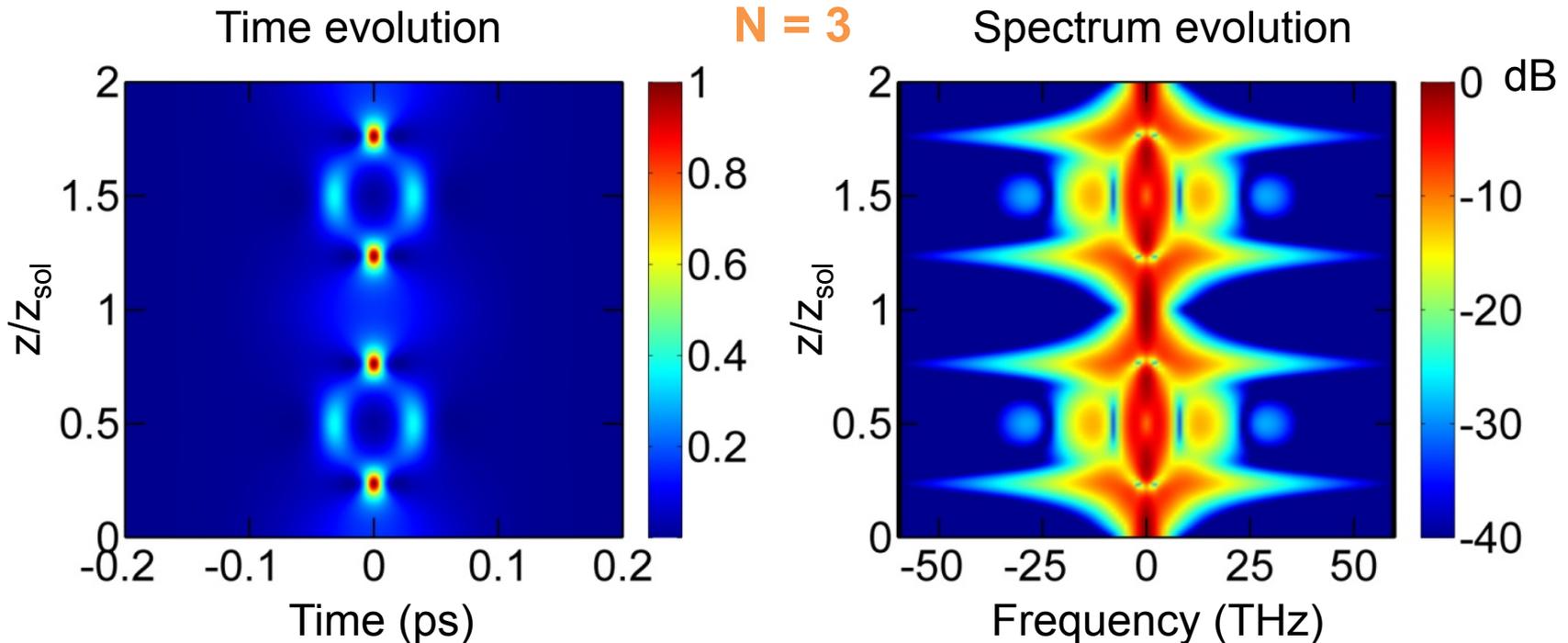
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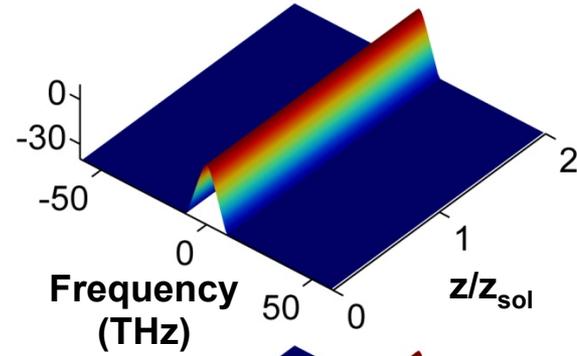
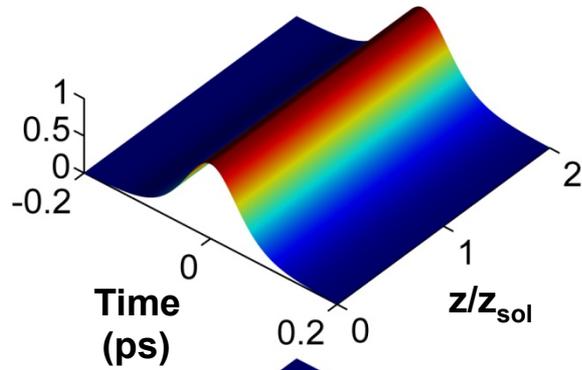
Soliton period

Quantized!

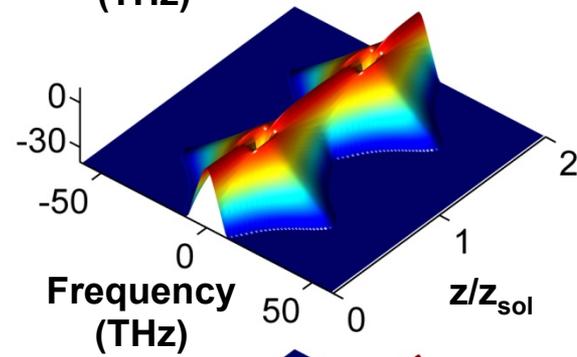
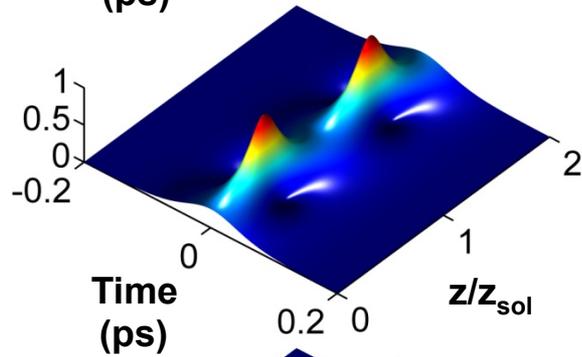


Nicer in 3D

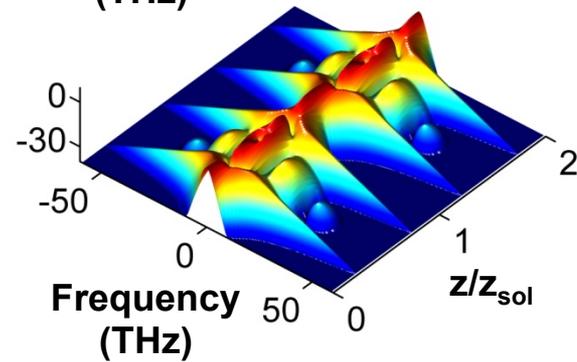
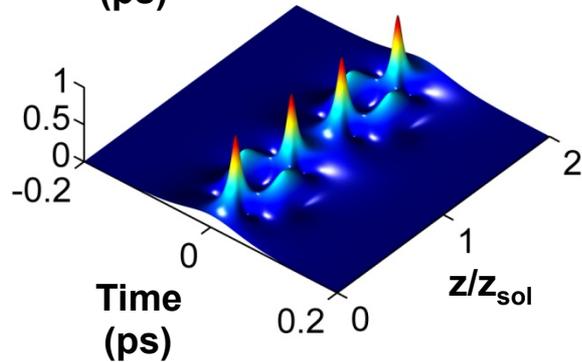
N=1



N=2

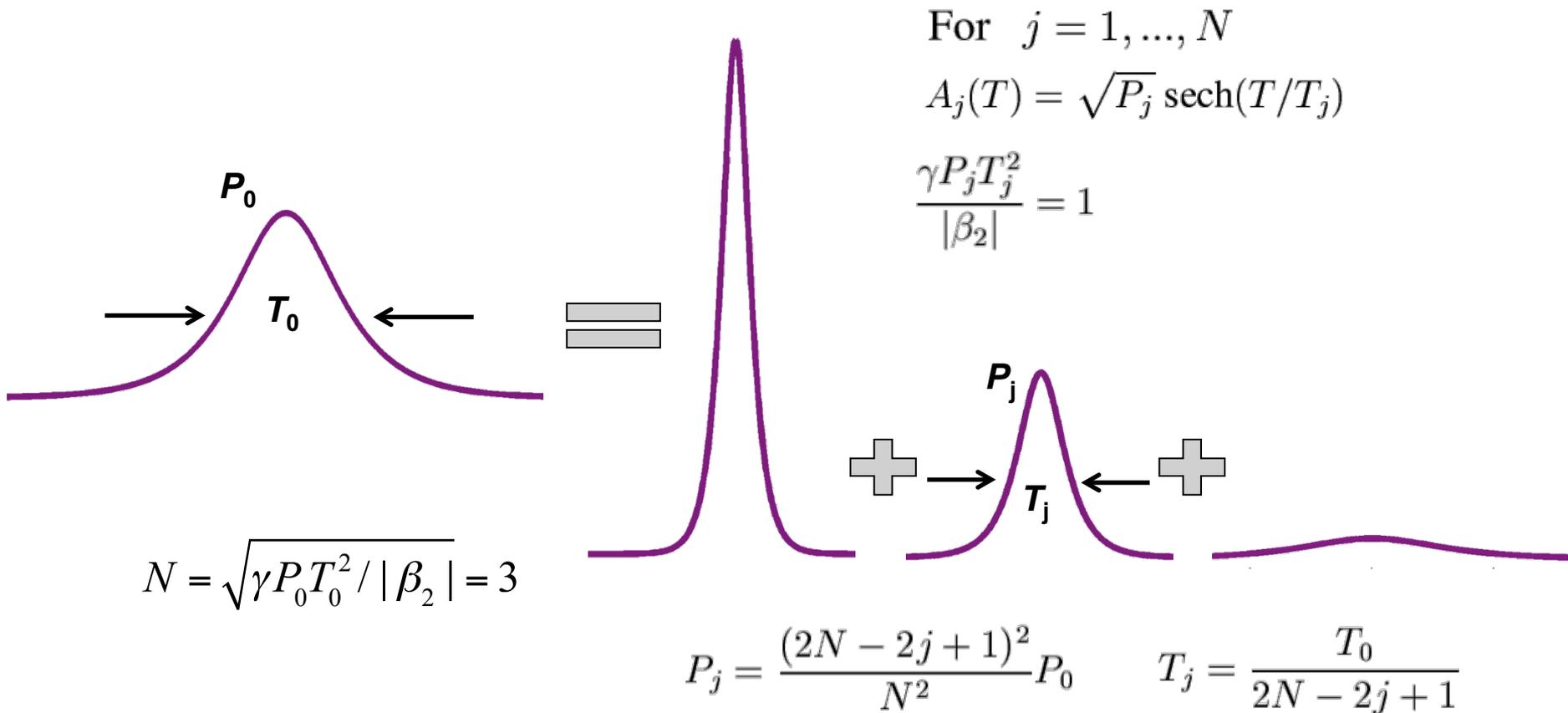


N=3



Higher-order solitons

- Higher-order soliton corresponds to the interference of fundamental solitons with different amplitudes (and phase)



Perturbations of solitons

- Solitons (fundamental/higher-order)
 - Solutions of pure NLS (only GVD and Kerr nonlinearity)

$$\frac{\partial A}{\partial z} + i \underbrace{\frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2}}_{\text{GVD}} = i \underbrace{\gamma |A|^2 A}_{\text{Kerr}}$$

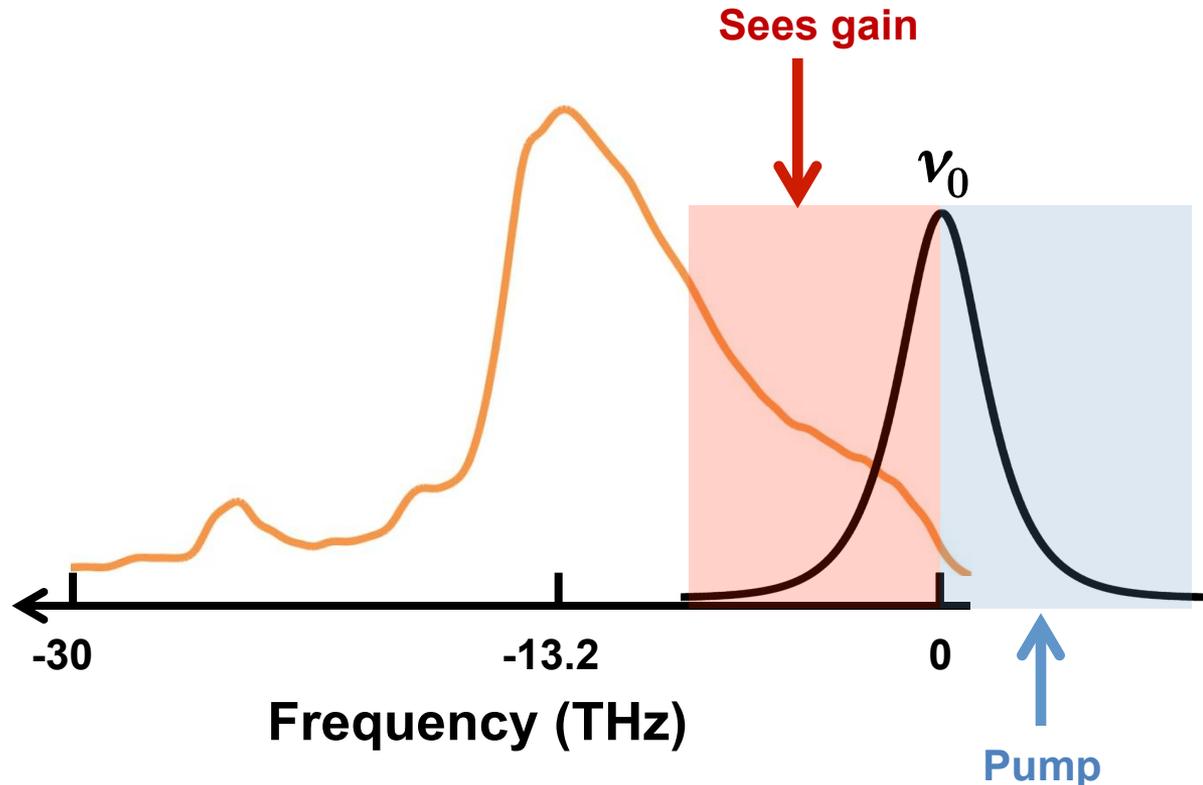
- Higher-order dispersion and Raman scattering perturb the evolution of solitons
 - Soliton self-frequency shift
 - Dispersive wave generation
 - Soliton fission

$$\frac{\partial A}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \underbrace{\frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} + \dots}_{\text{HOD}} = i \gamma (1 - f_R) |A|^2 A + \underbrace{f_R A (h_R * |A|^2)}_{\text{Raman}}$$

Raman perturbation of soliton

Mitschke and Mollenauer OL 11, 659 (1986)

Gordon OL 11, 662 (1986)

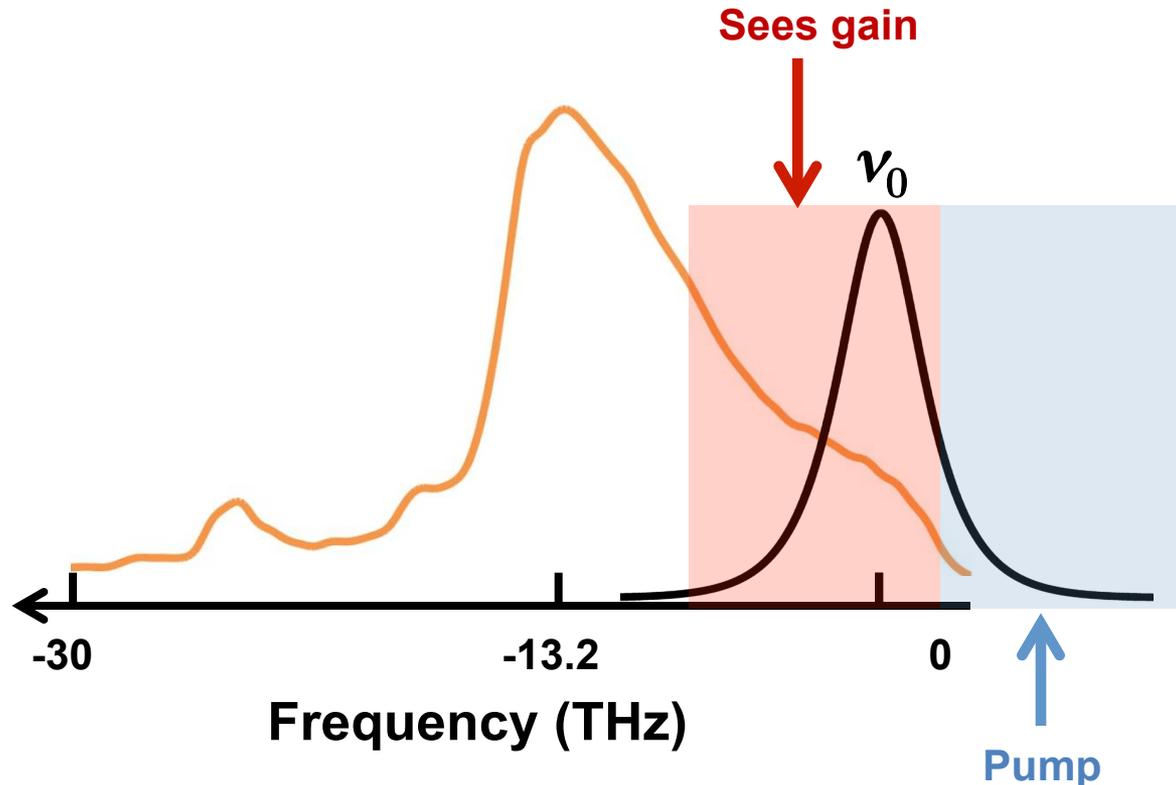


- Frequency of soliton shifts towards lower frequencies (longer wavelengths) with propagation

Raman perturbation of soliton

Mitschke and Mollenauer OL 11, 659 (1986)

Gordon OL 11, 662 (1986)

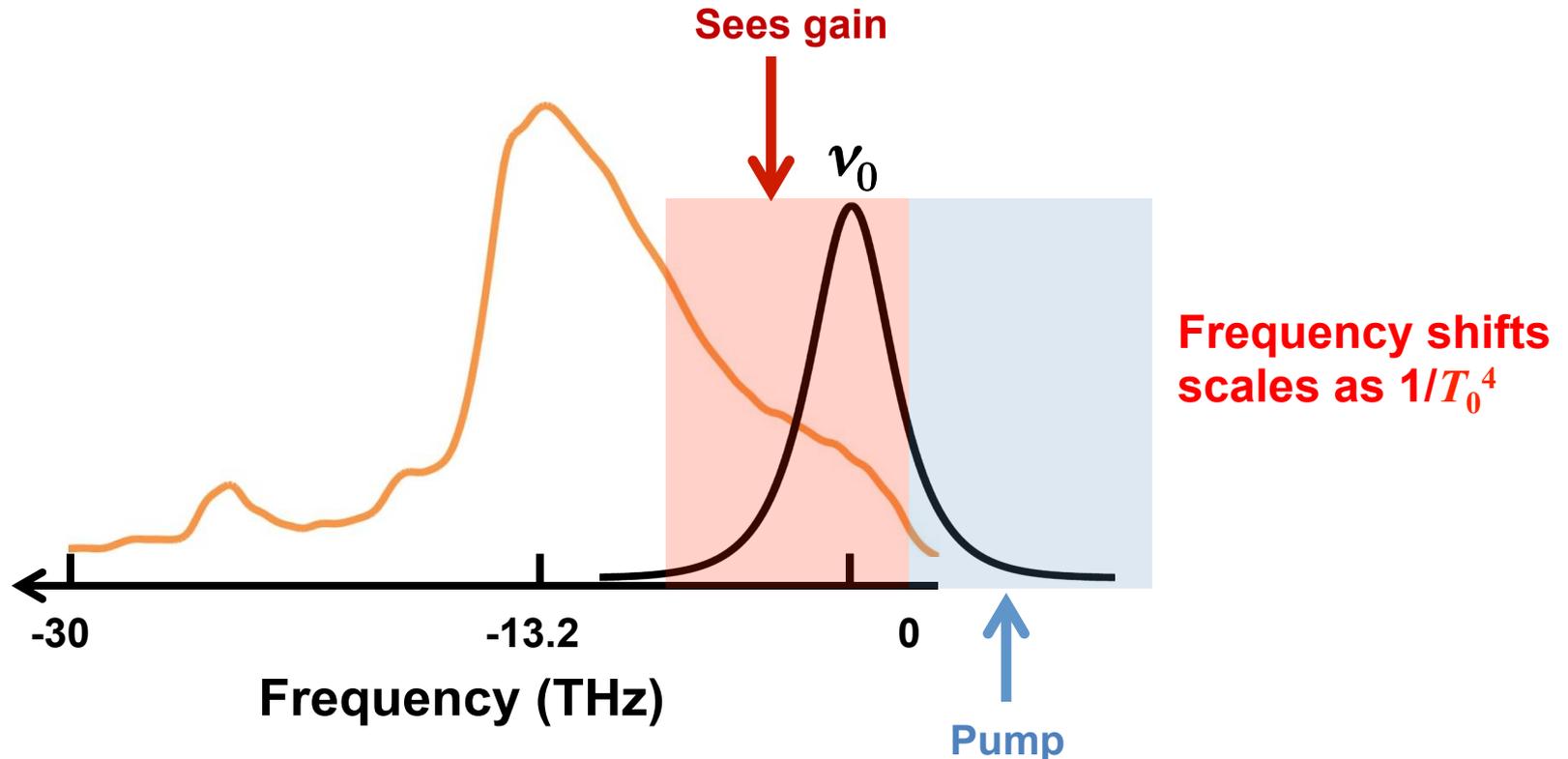


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Raman perturbation of soliton

Mitschke and Mollenauer OL 11, 659 (1986)

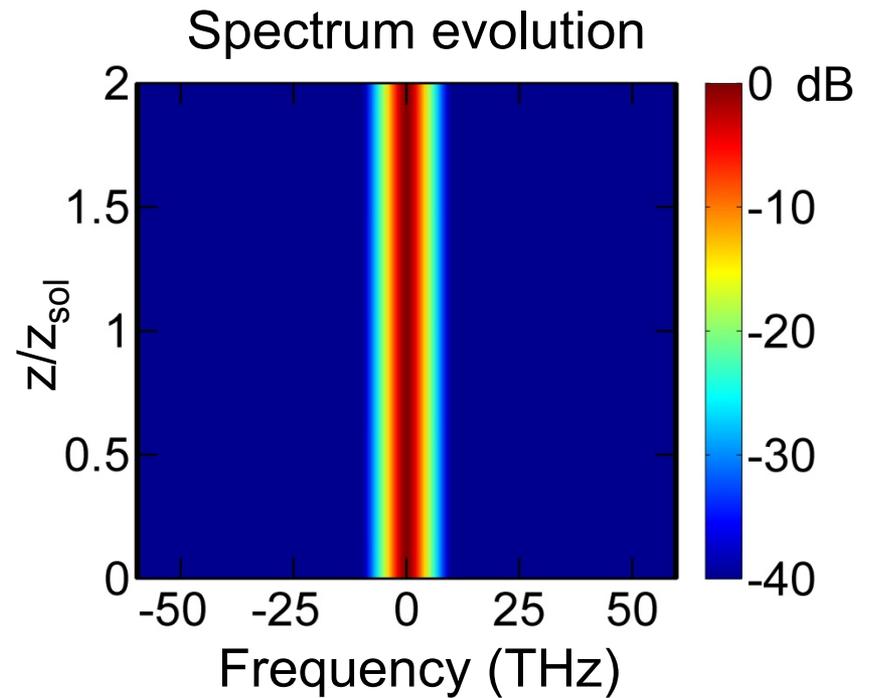
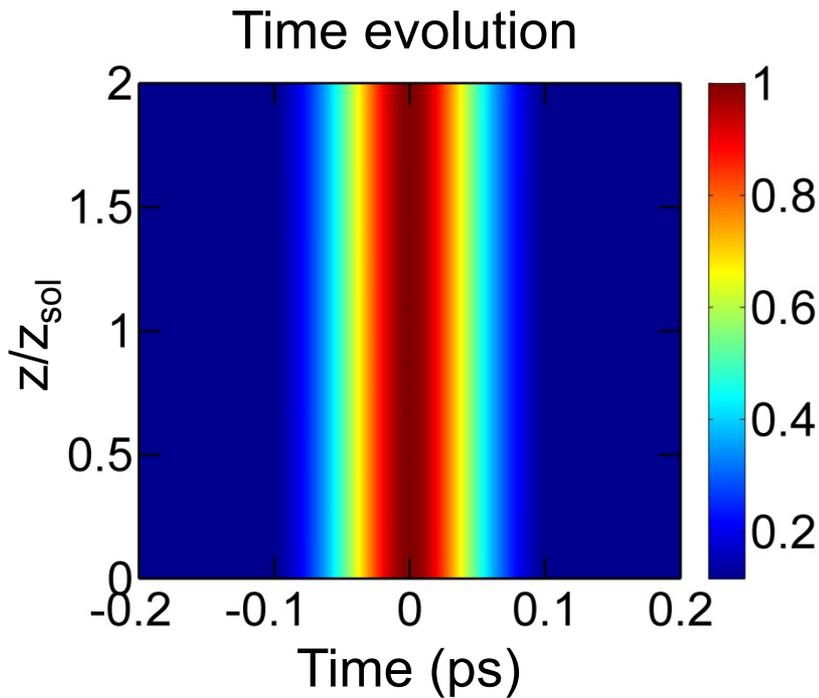
Gordon OL 11, 662 (1986)



- Frequency of soliton shifts towards lower frequencies (longer wavelengths) with propagation = **soliton SELF-frequency shift**

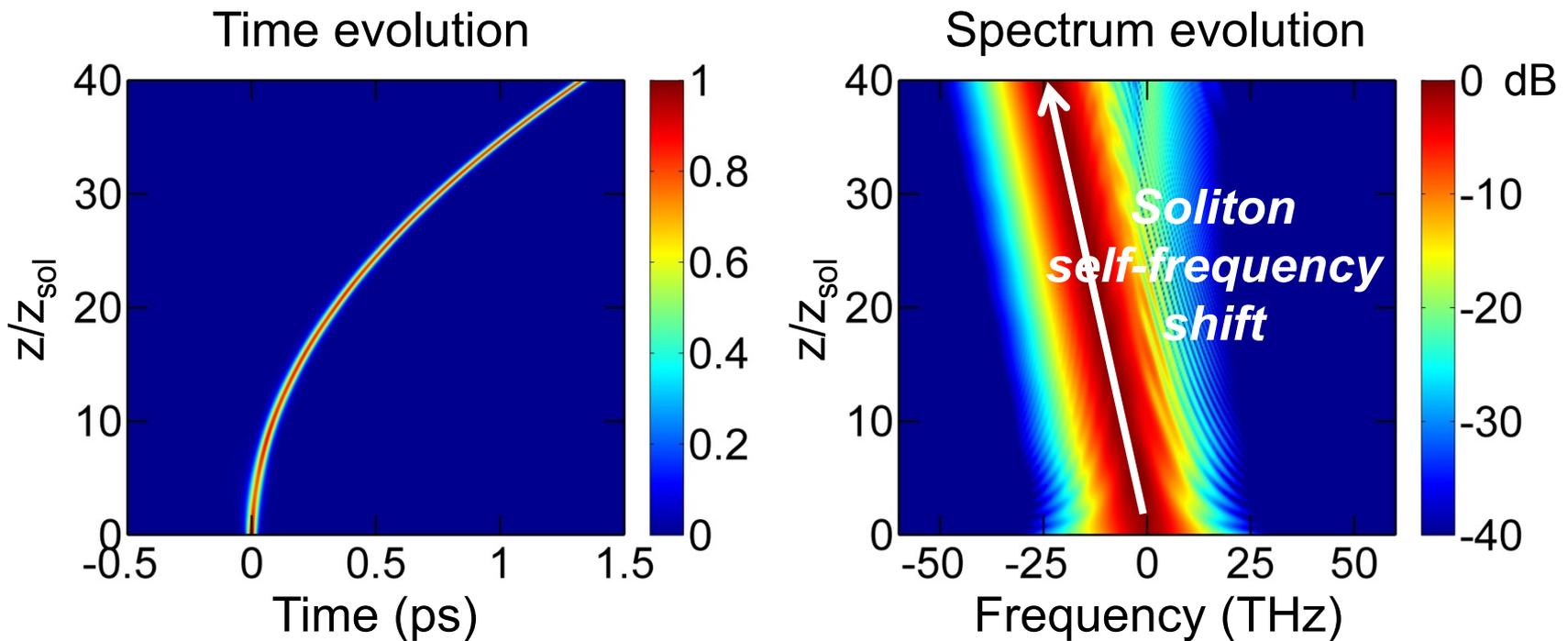
N=1 soliton

- Fundamental solitons are propagation-invariant



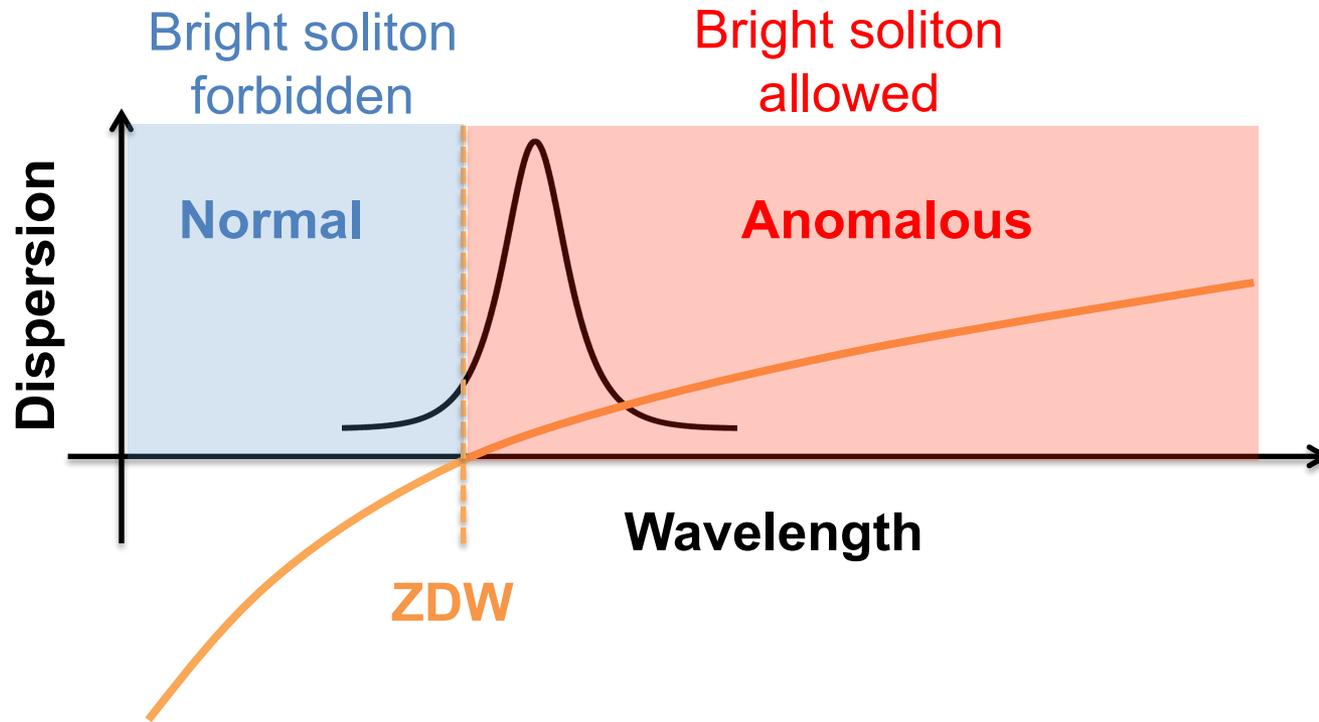
Soliton self-frequency shift

- Frequency of soliton shifts towards lower frequencies (longer wavelengths) with propagation = soliton SELF-frequency shift



- Parabolic/linear trajectory in the time/frequency domain

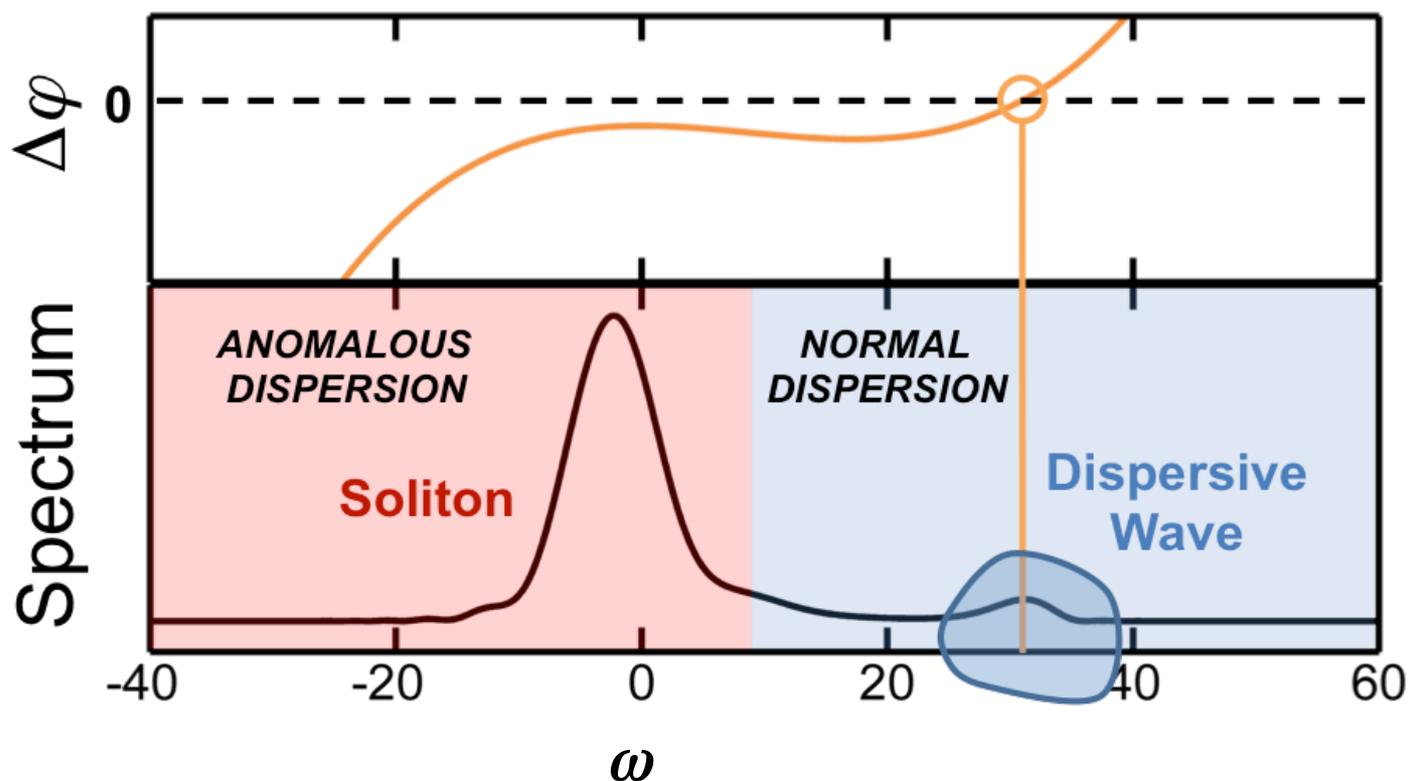
Higher-order dispersion perturbation



- Dispersion depends on wavelength/frequency
- Soliton near ZDW strongly perturbed: part of its spectrum extends in the normal dispersion regime!

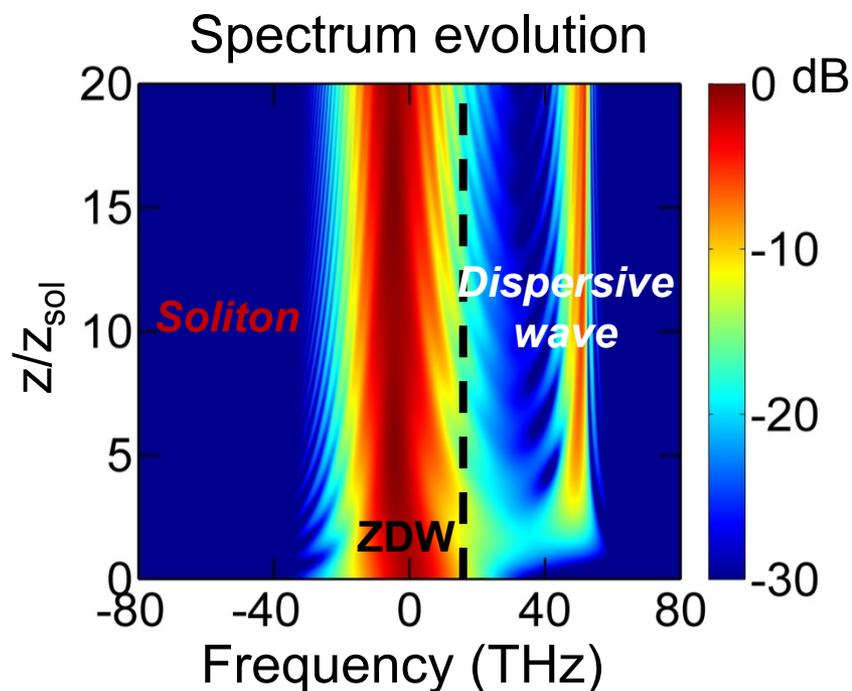
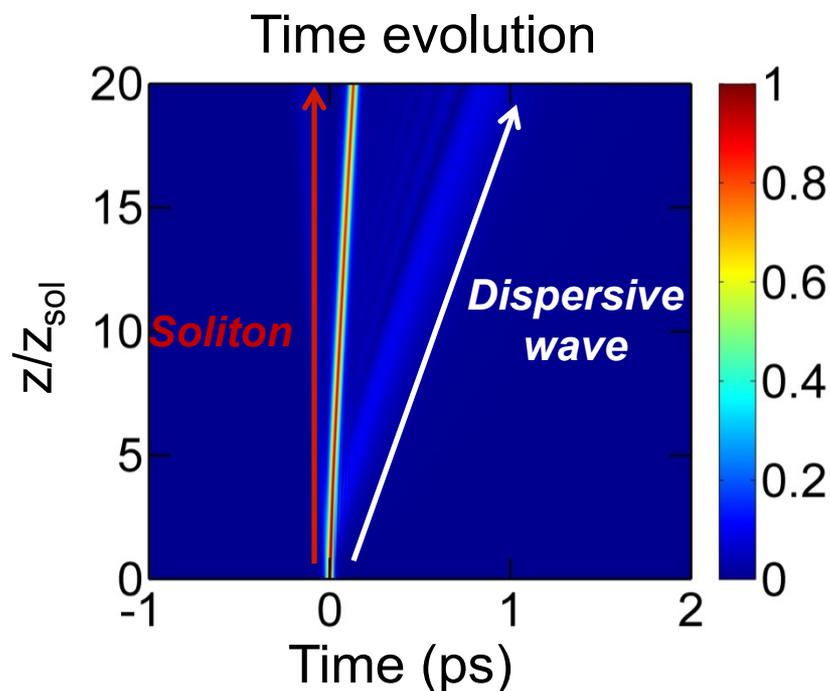
Dispersive wave generation

- Dispersive wave located in the normal dispersion regime
 - phase-matching condition: $\phi_{\text{soliton}} = \phi_{\text{disp. wave}}$



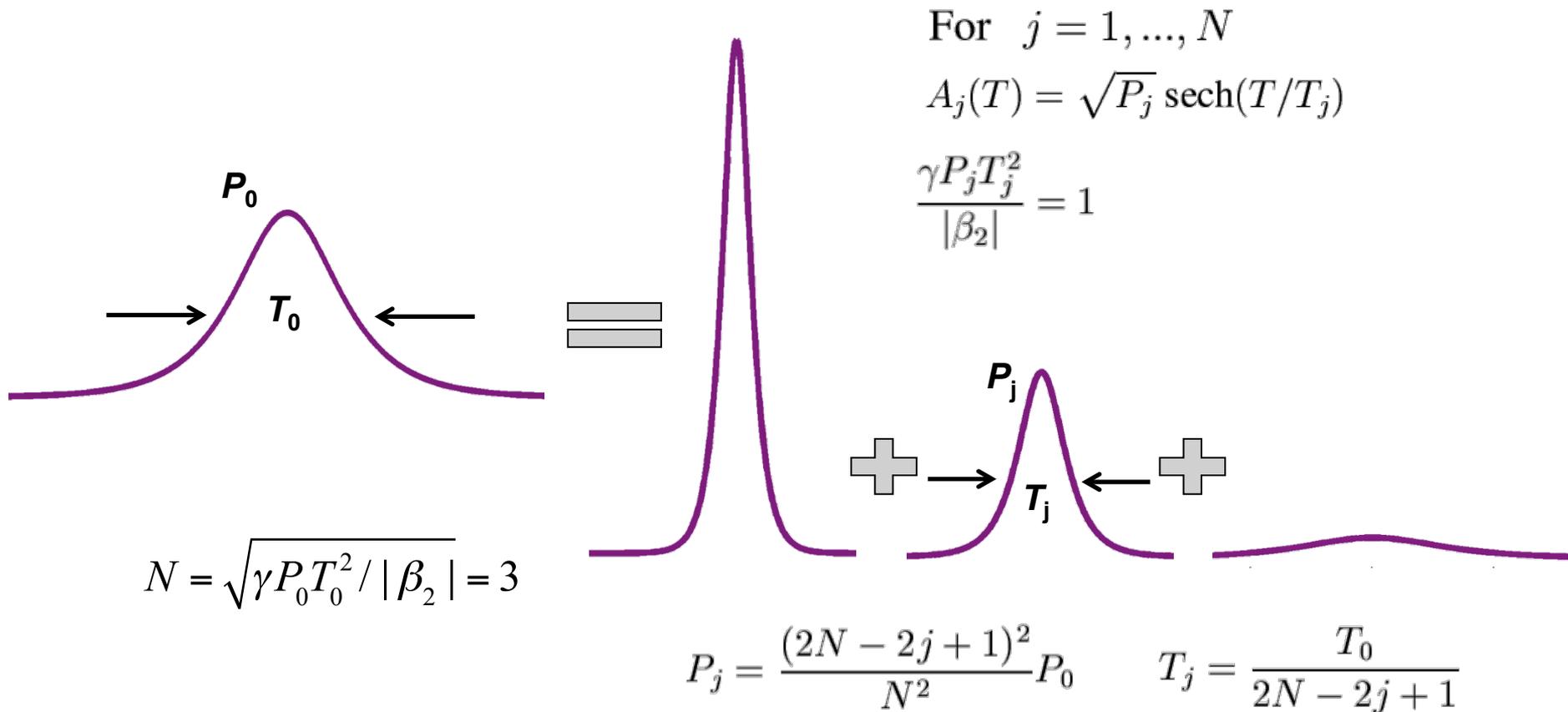
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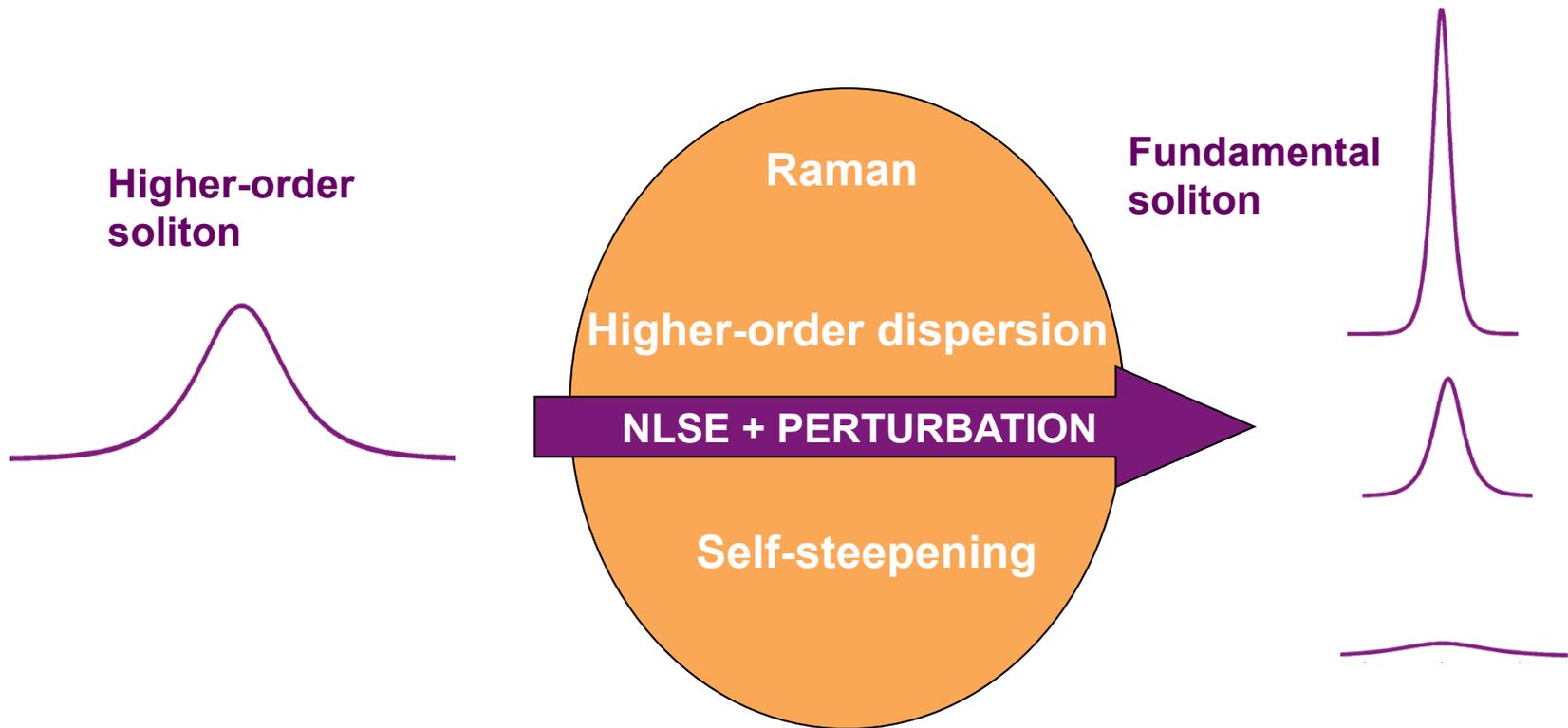
Higher-order solitons

- Higher-order soliton corresponds to the interference of fundamental solitons with different amplitudes (and phase)



Soliton fission

- Higher-order N -soliton is unstable, sensitive to perturbations
➔ N -soliton breaks up into N fundamental solitons



Soliton fission

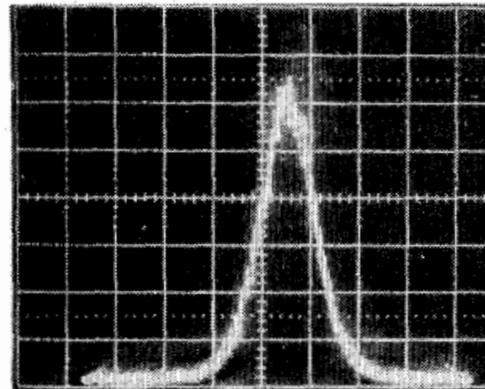
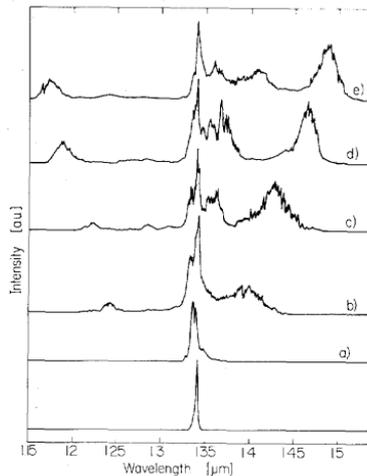
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1938

IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. QE-23, NO. 11, NOVEMBER 1987

Ultrashort Pulse Propagation, Pulse Breakup, and Fundamental Soliton Formation in a Single-Mode Optical Fiber

P. BEAUD, W. HODEL, B. ZYSSET, AND H. P. WEBER, SENIOR MEMBER, IEEE



Raman perturbation of N -Soliton

392 OPTICS LETTERS / Vol. 13, No. 5 / May 1988

Fission of optical solitons induced by stimulated Raman effect

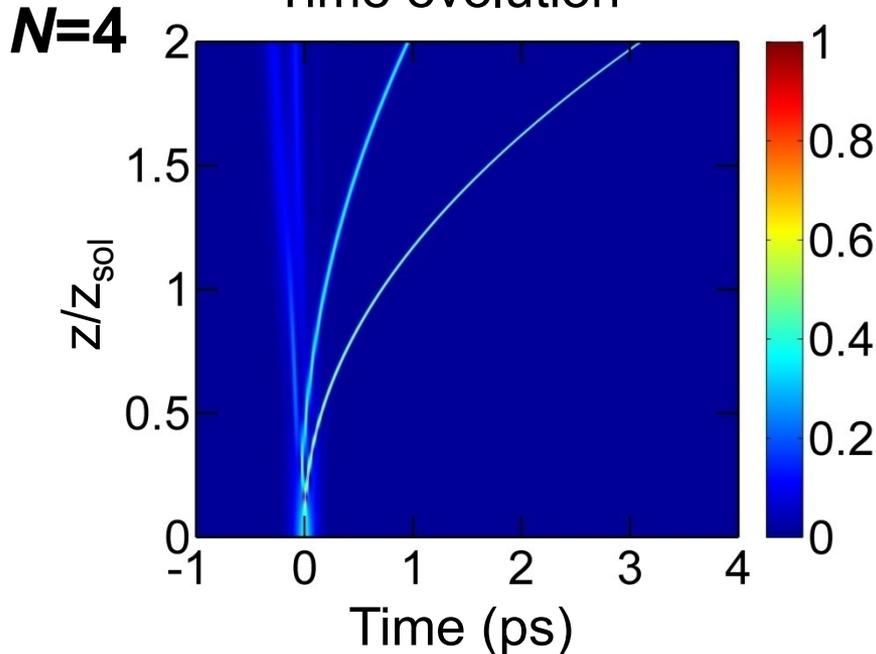
Kuo-chou Tai and Akira Hasegawa

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

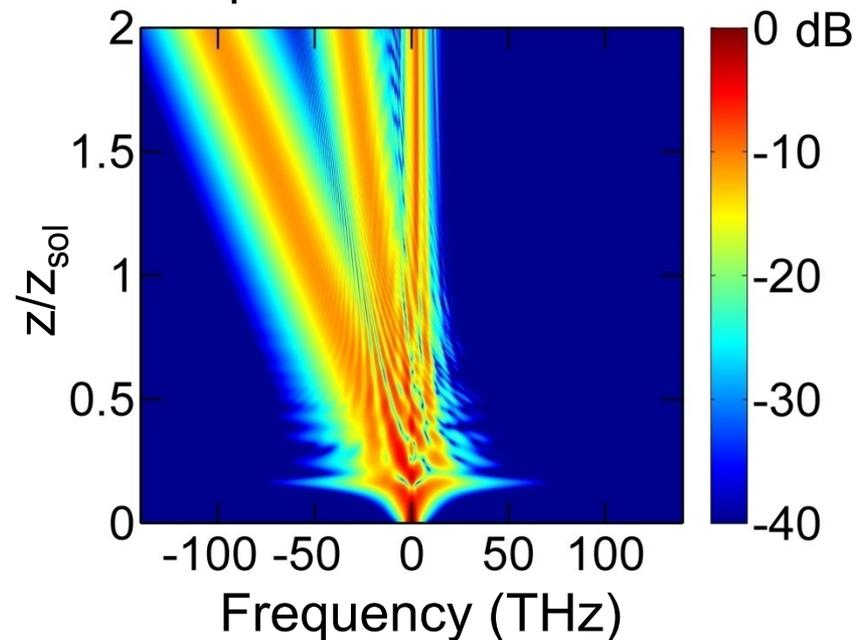
Naoaki Bekki

Institute for Fusion Studies, University of Texas, Austin, Texas 78712

Time evolution



Spectrum evolution

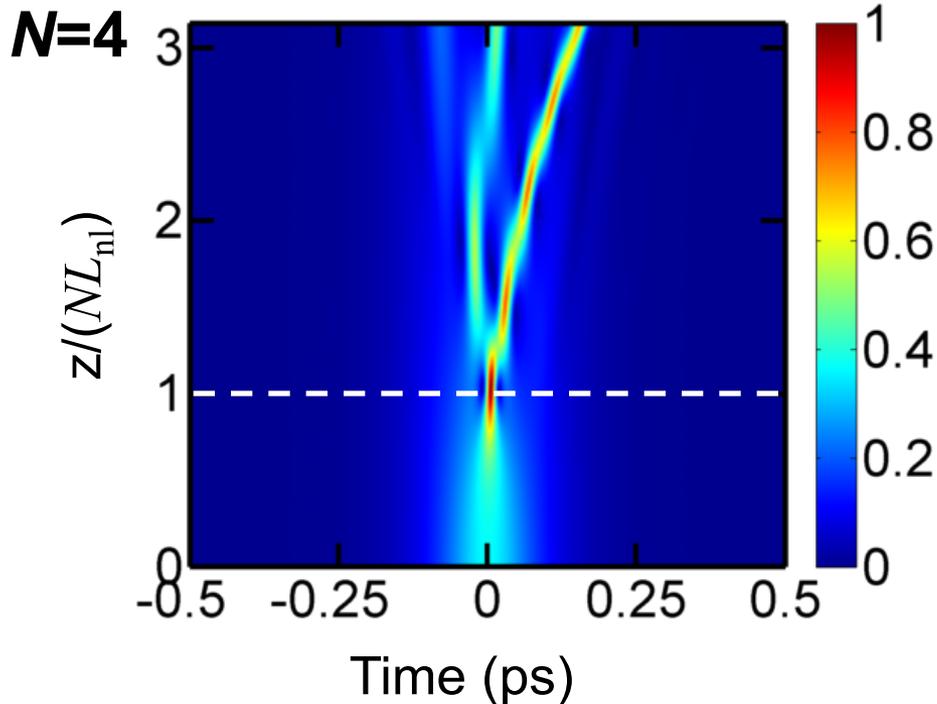


Fission of N -soliton

- Fission length: $L_{\text{fiss}} \approx L_d/N = N L_{\text{nl}}$

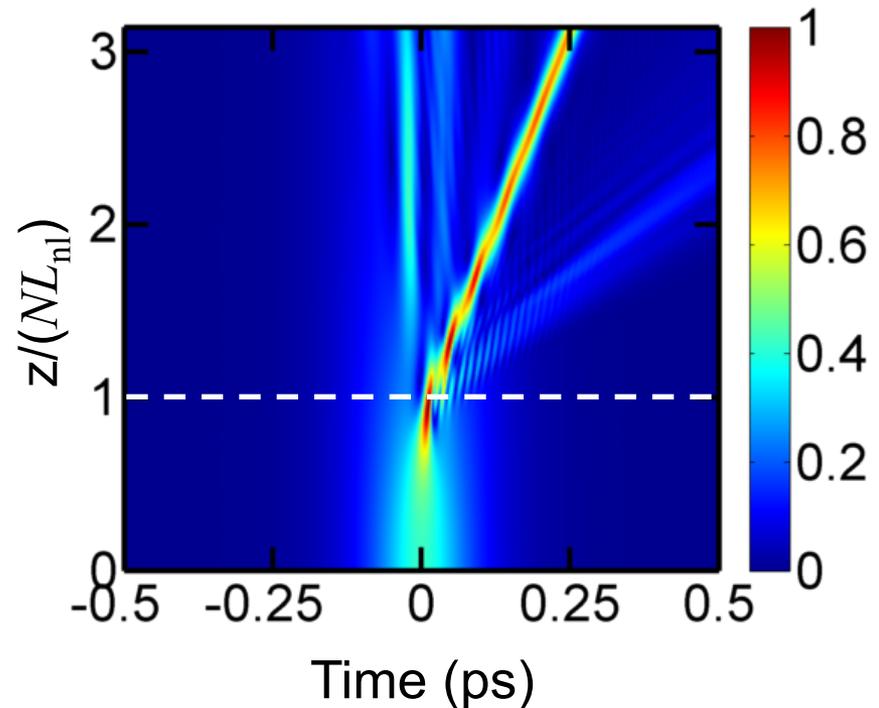
Raman
perturbation

Time evolution



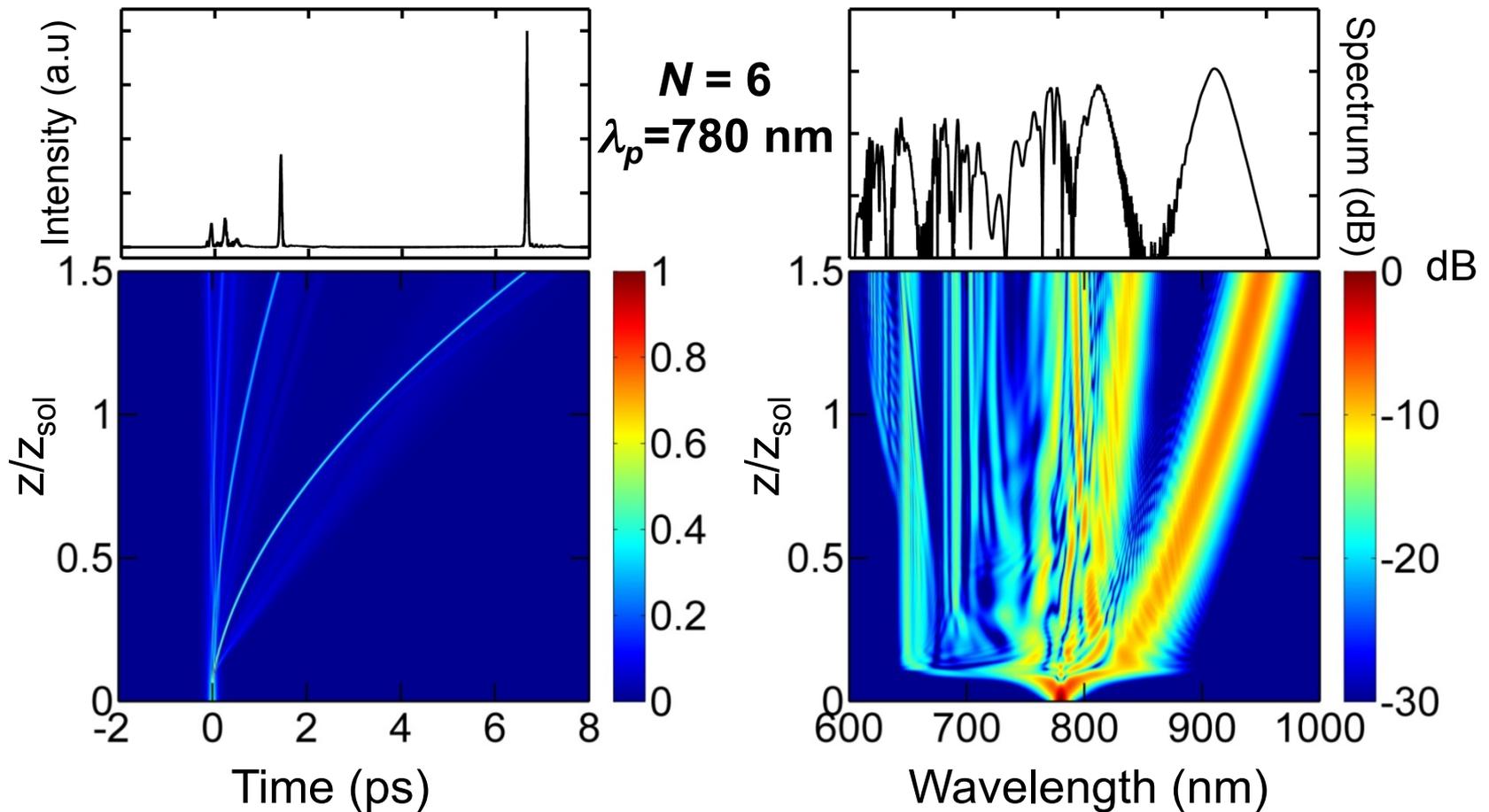
Higher-order dispersion
perturbation

Time evolution



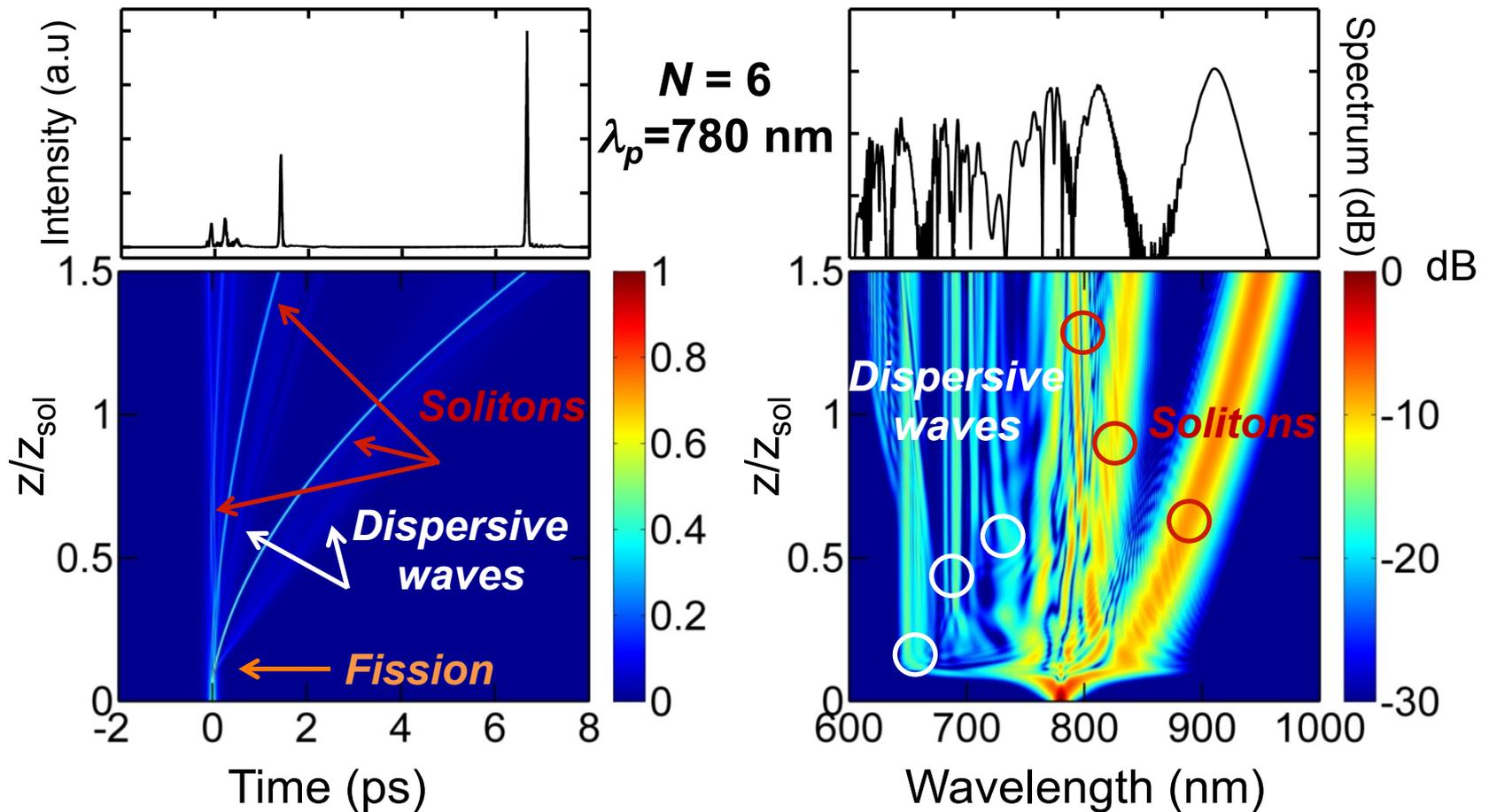
Let's put everything together

- Soliton fission + dispersive wave radiation + soliton self-frequency shift = supercontinuum generation



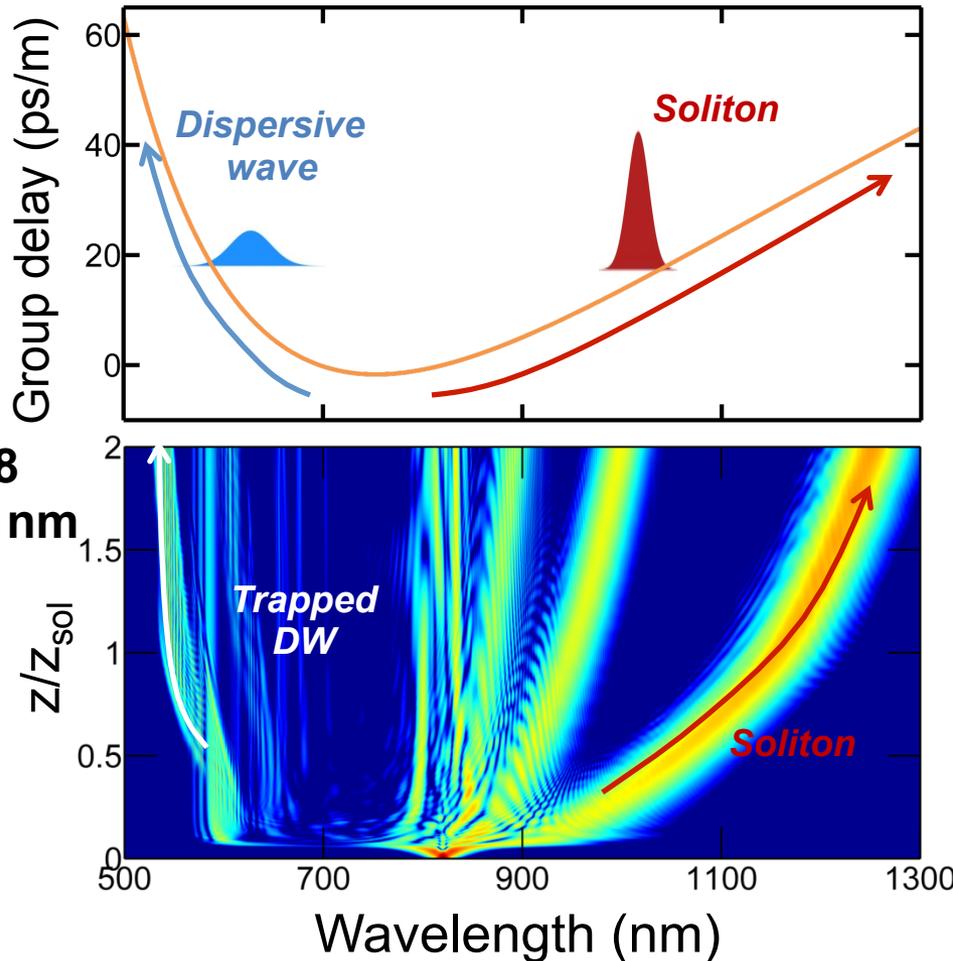
Let's put everything together

- Soliton fission + dispersive wave radiation + soliton self-frequency shift = supercontinuum generation



Dispersive wave trapping

- Continuously redshifting soliton induces a continuous blueshift of the dispersive wave



Soliton acts as a nonlinear potential barrier for the weaker dispersive wave

Effect of cross-phase modulation on supercontinuum generated in microstructured fibers with sub-30 fs pulses

G. Genty, M. Lehtonen, and H. Ludvigsen

Fiber-Optics Group, Department of Electrical and Communications Engineering, Helsinki University of Technology, P.O. Box 3500, FI-02015 HUT, Finland
goery.genty@hut.fi

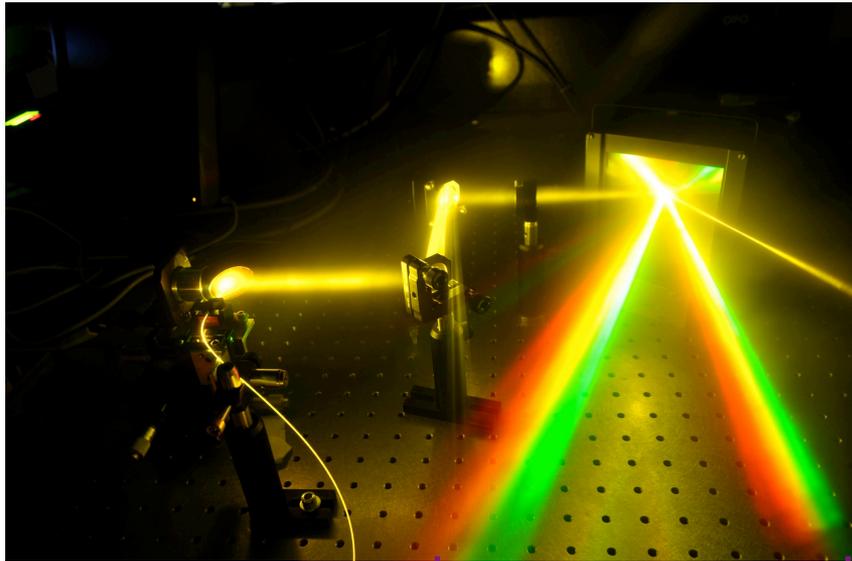
LETTERS

Light trapping in gravity-like potentials and expansion of supercontinuum spectra in photonic-crystal fibres

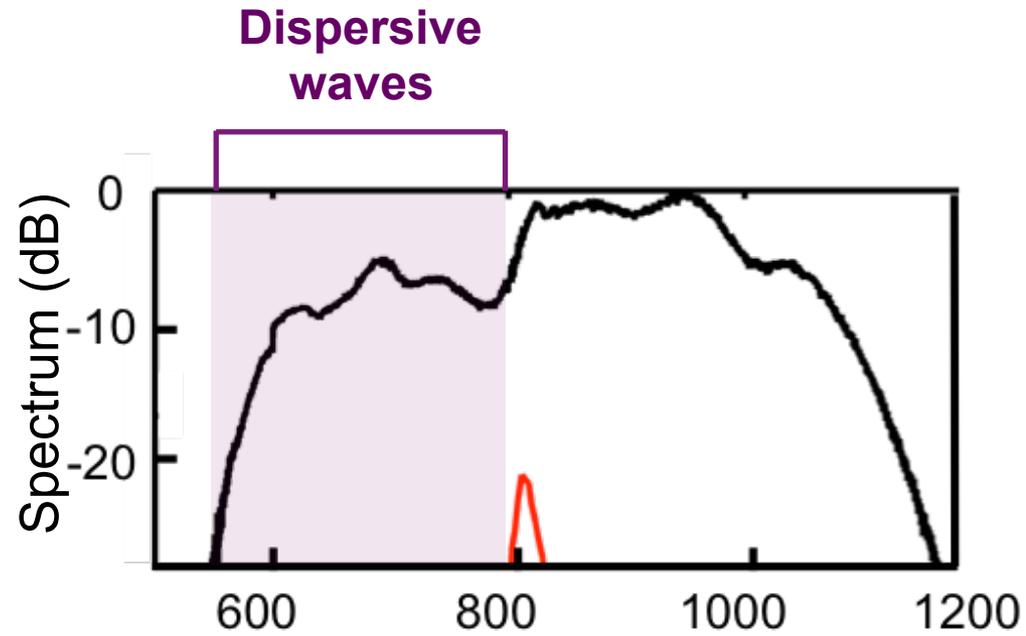
A. V. GORBACH AND D. V. SKRYABIN*

Centre for Photonics and Photonic Materials, Department of Physics, University of Bath, Bath BA2 7AY, UK
*e-mail: d.v.skryabin@bath.ac.uk

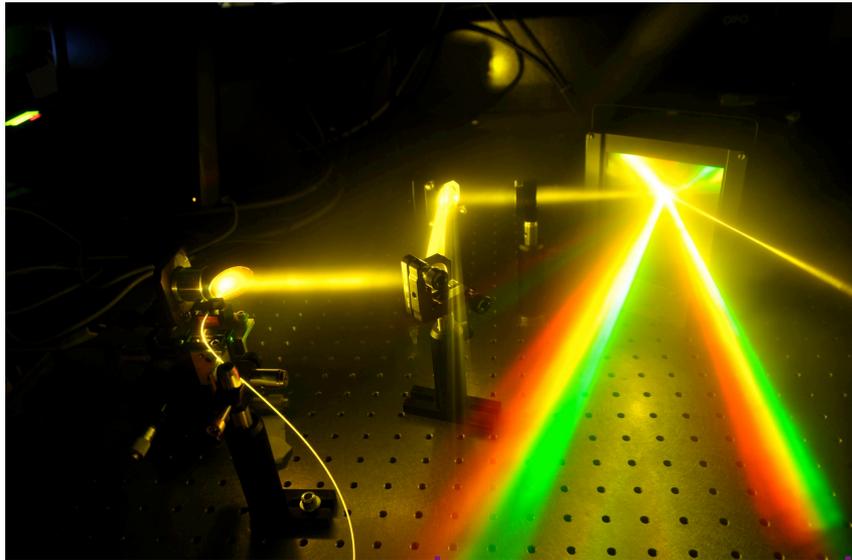
Dispersive waves corresponds to visible light



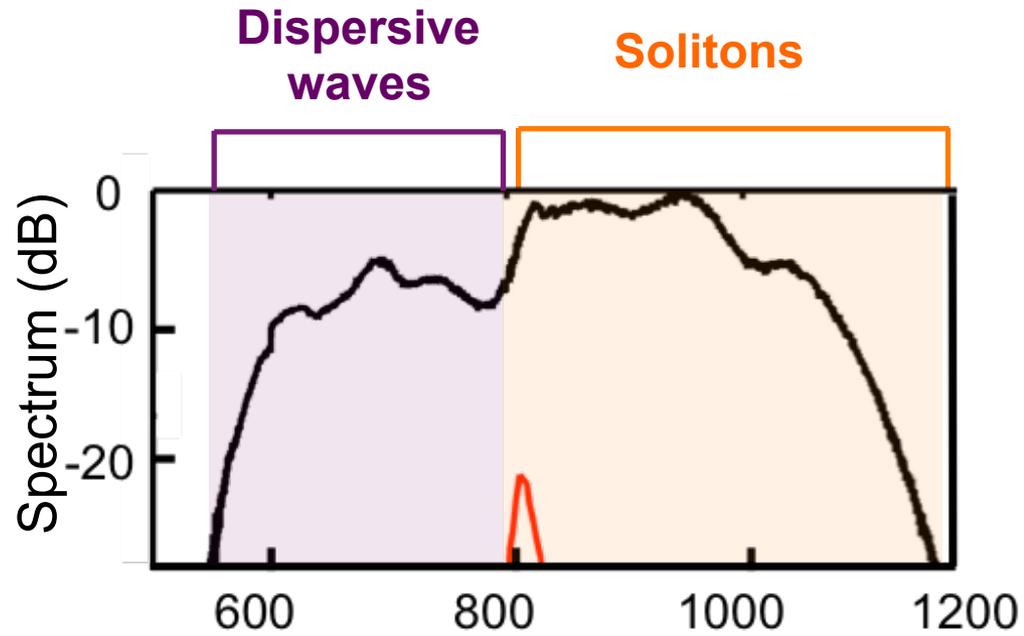
Dispersive waves



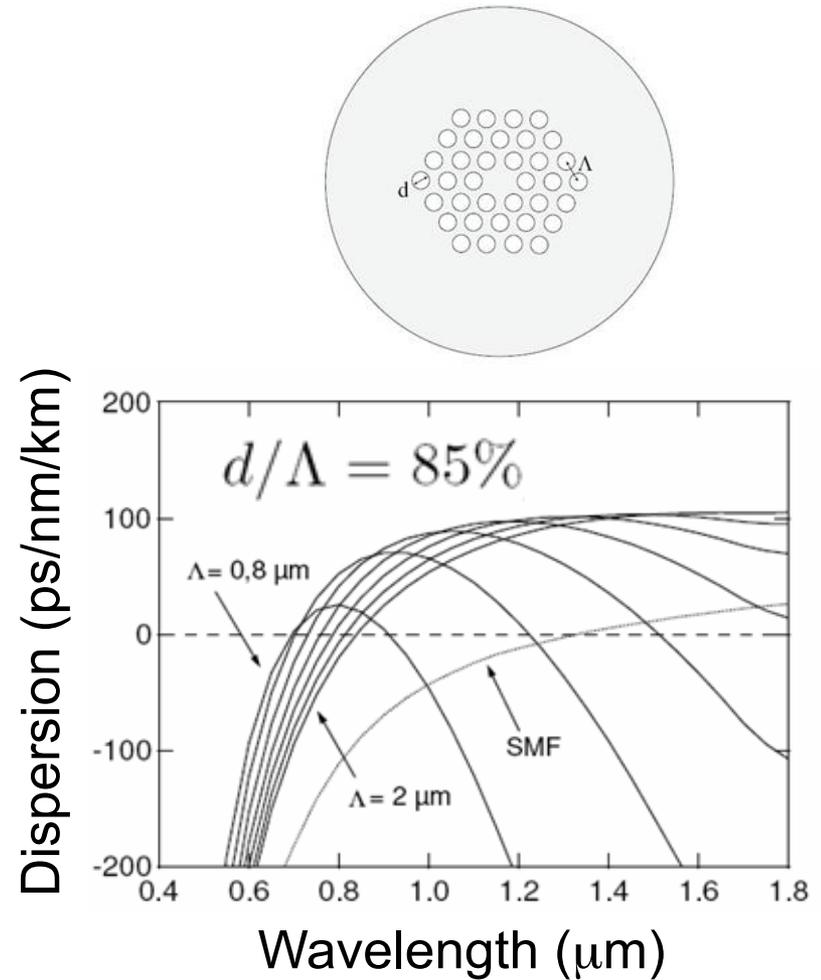
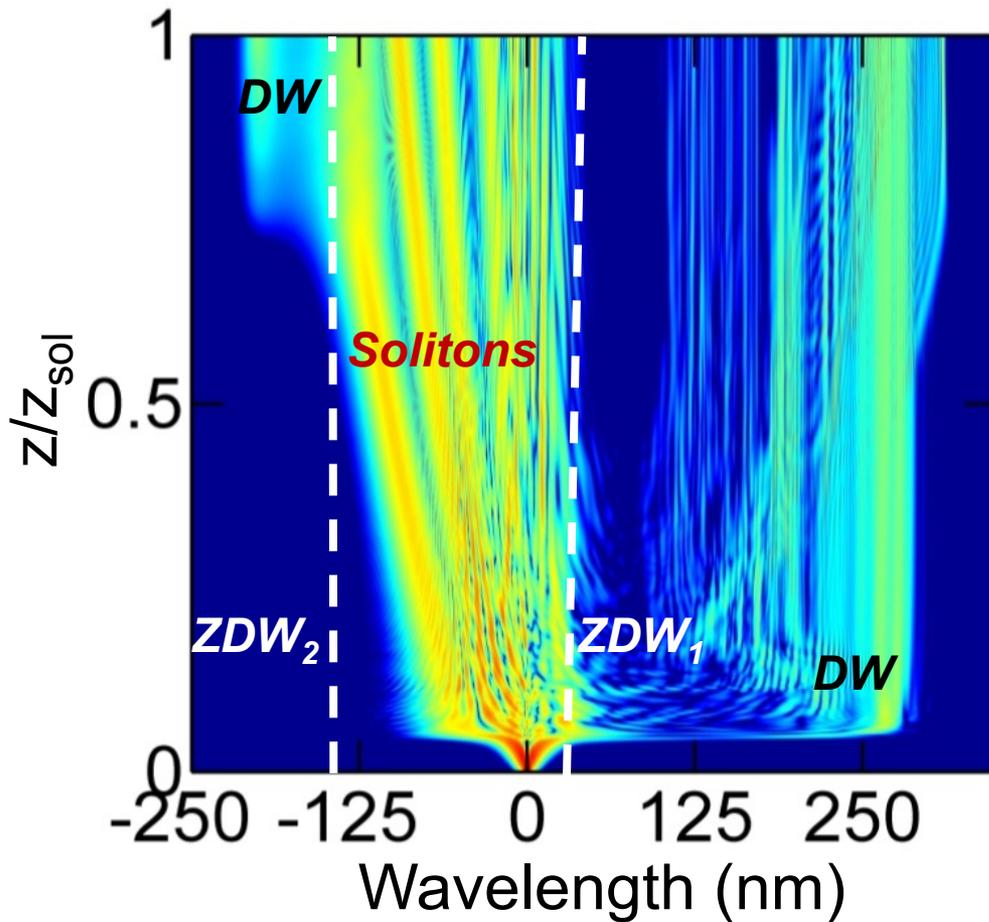
And solitons to infrared light



Dispersive waves

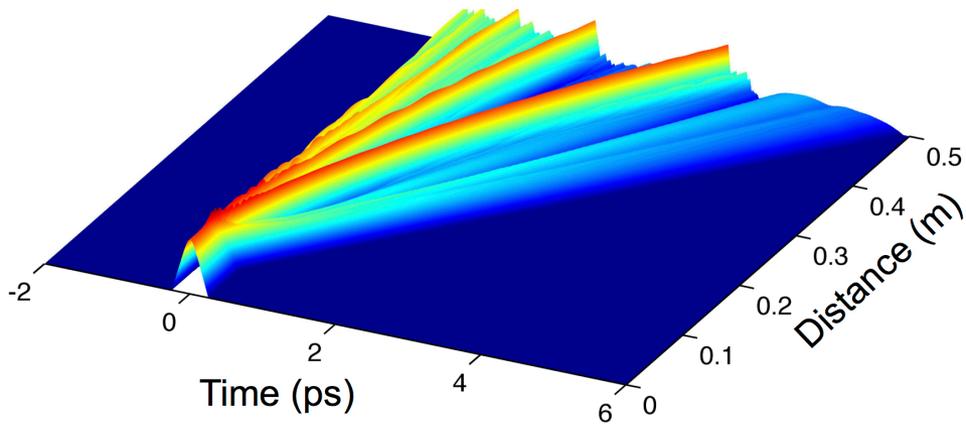


Fiber with two zero-dispersion wavelengths

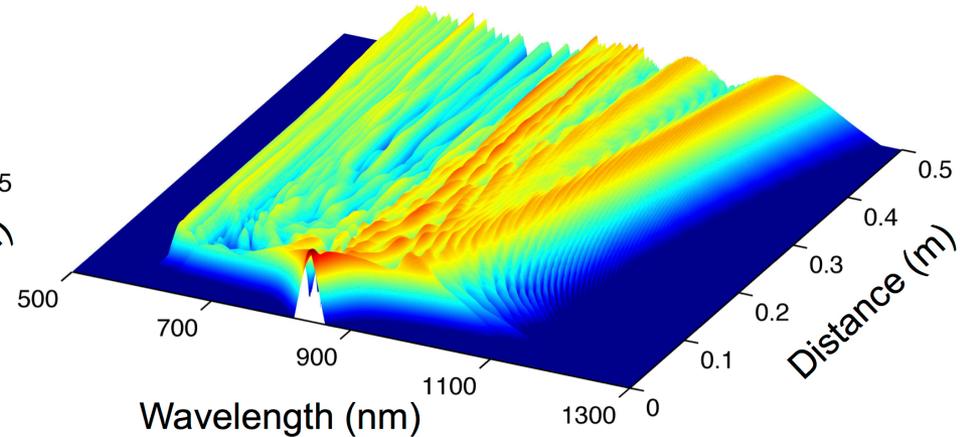


Dynamics are rather complex...

Time evolution



Spectral evolution



Short pulses

Anomalous

- Soliton
- Dispersive waves

Long pulses

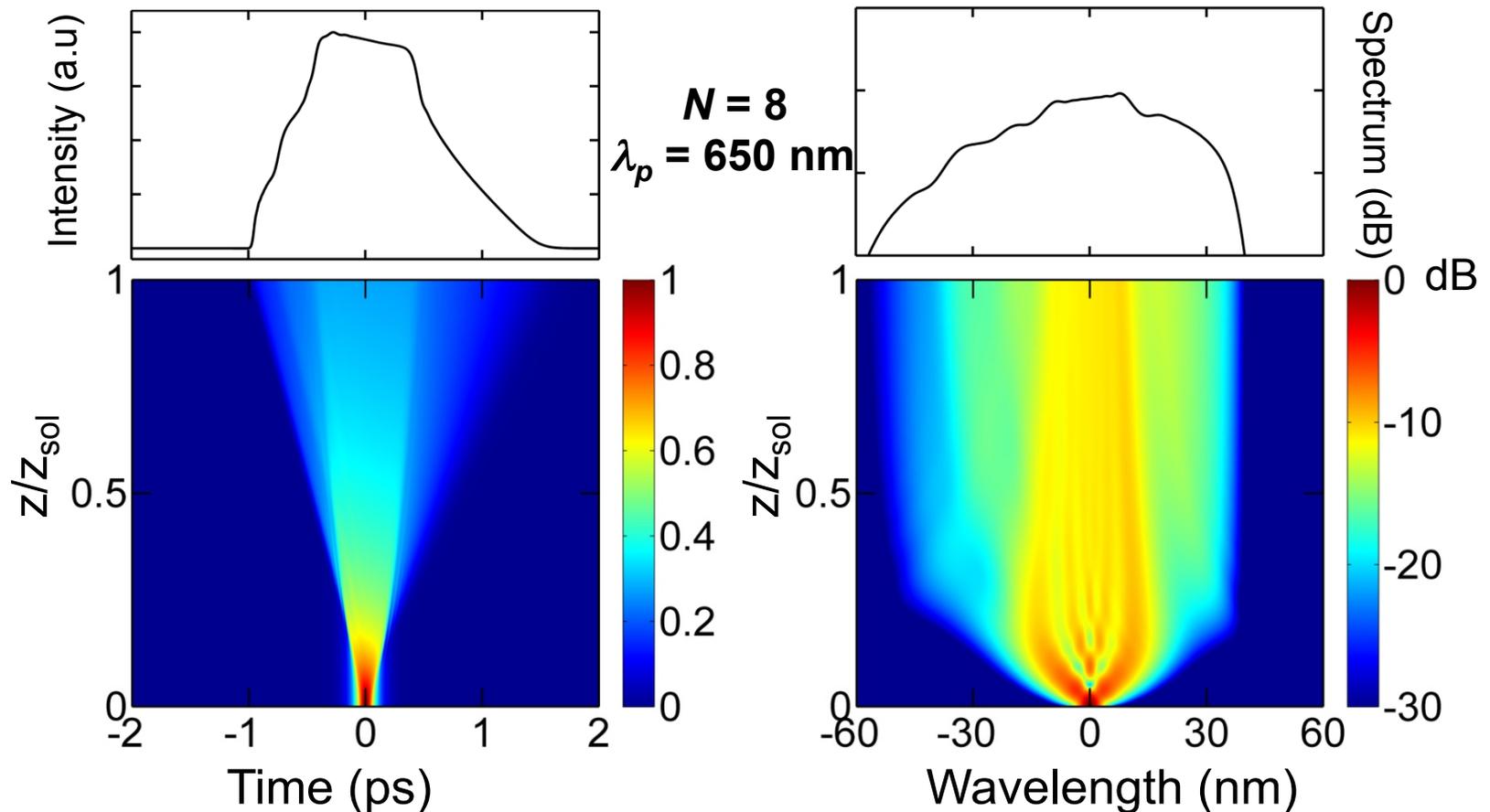
- Modulation instability
- Solitons dynamics

Normal

- Self-phase modulation
- Four-wave mixing
- Raman scattering
- Four-wave mixing

Short pulses and normal dispersion regime

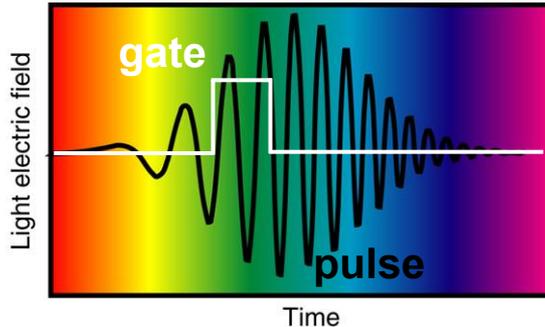
- No bright solitons in normal dispersion regime: different dynamics
- Self-phase modulation, four-wave mixing



Time-frequency analysis

- Ultrafast dynamics can be conveniently visualized in the time-frequency domain

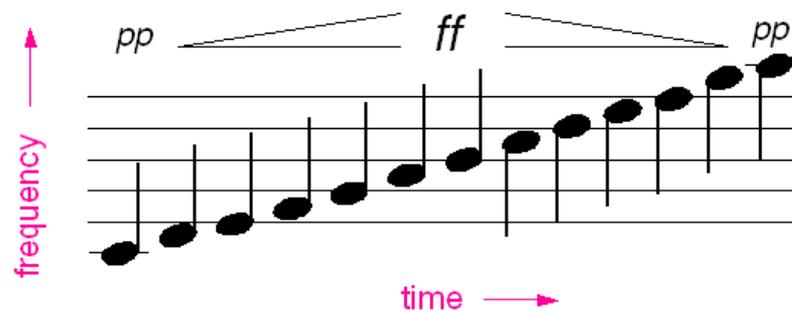
Spectrogram / short-time Fourier Transform



$$\Sigma_g^E(\omega, \tau) = \left| \int_{-\infty}^{\infty} E(t) g(t - \tau) \exp(i\omega t) dt \right|^2$$

Field to be measured *Gate*

Courtesy of R. Trebino



Time-frequency analysis

JOURNAL OF APPLIED PHYSICS

VOLUME 42, NUMBER 10

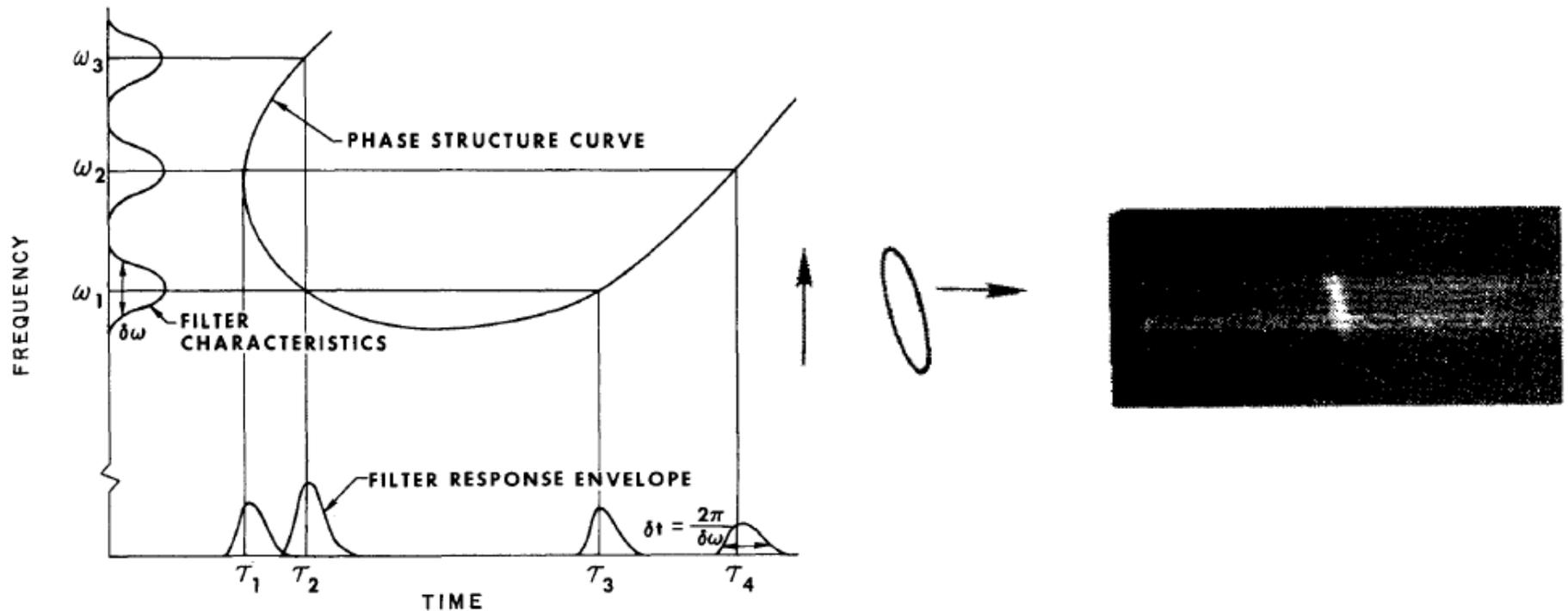
SEPTEMBER 1971

Measurement and Interpretation of Dynamic Spectrograms of Picosecond Light Pulses*

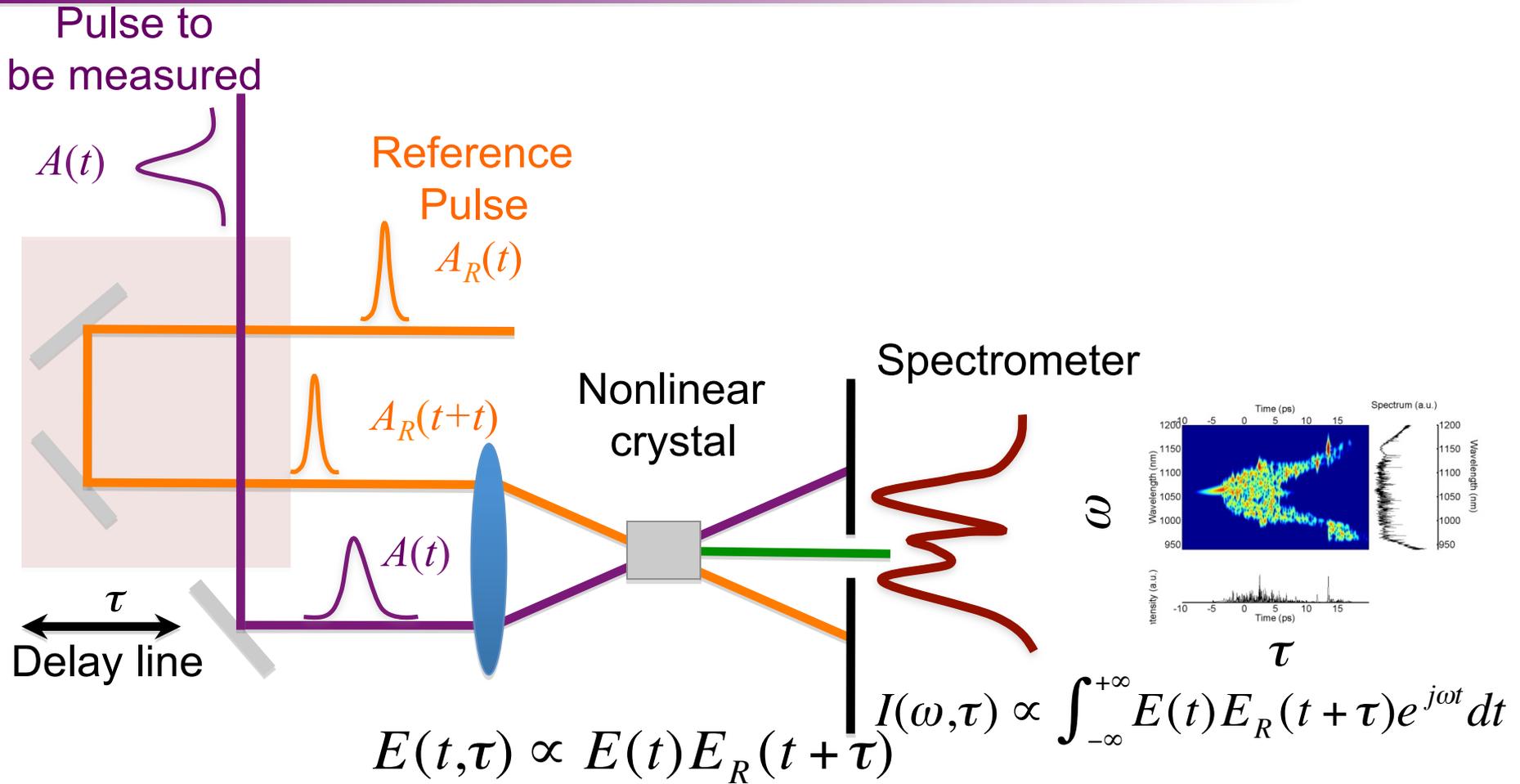
E. B. Treacy

United Aircraft Research Laboratories, East Hartford, Connecticut 06108

(Received 25 November 1970)



Experimental implementation



Cross-correlation Frequency Resolved Optical Gating

Short ref. pulse $\longrightarrow A(\omega, \tau)$

One more thing

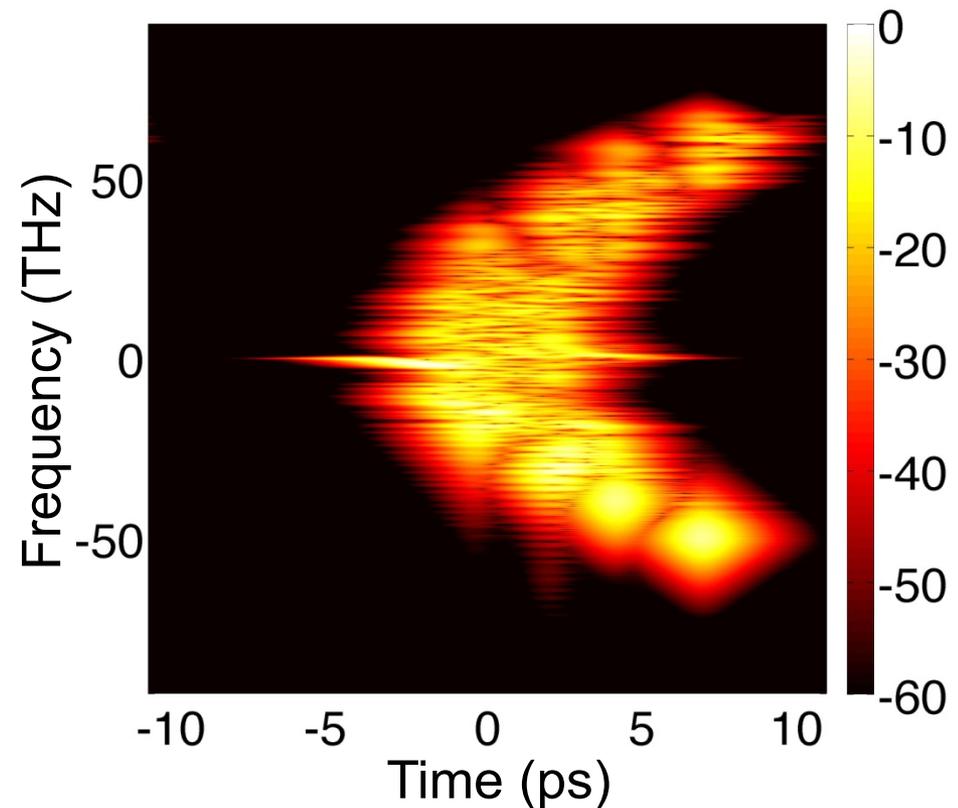
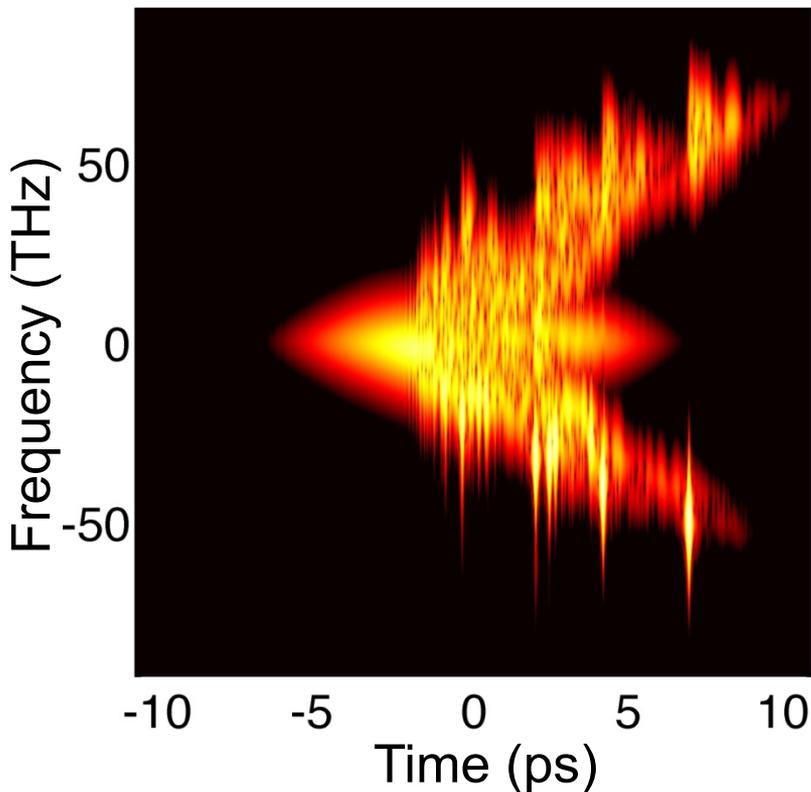
THEORY OF COMMUNICATION*

By D. GABOR, Dr. Ing., Associate Member.†

(The paper was first received 25th November, 1944, and in revised form 24th September, 1945.)

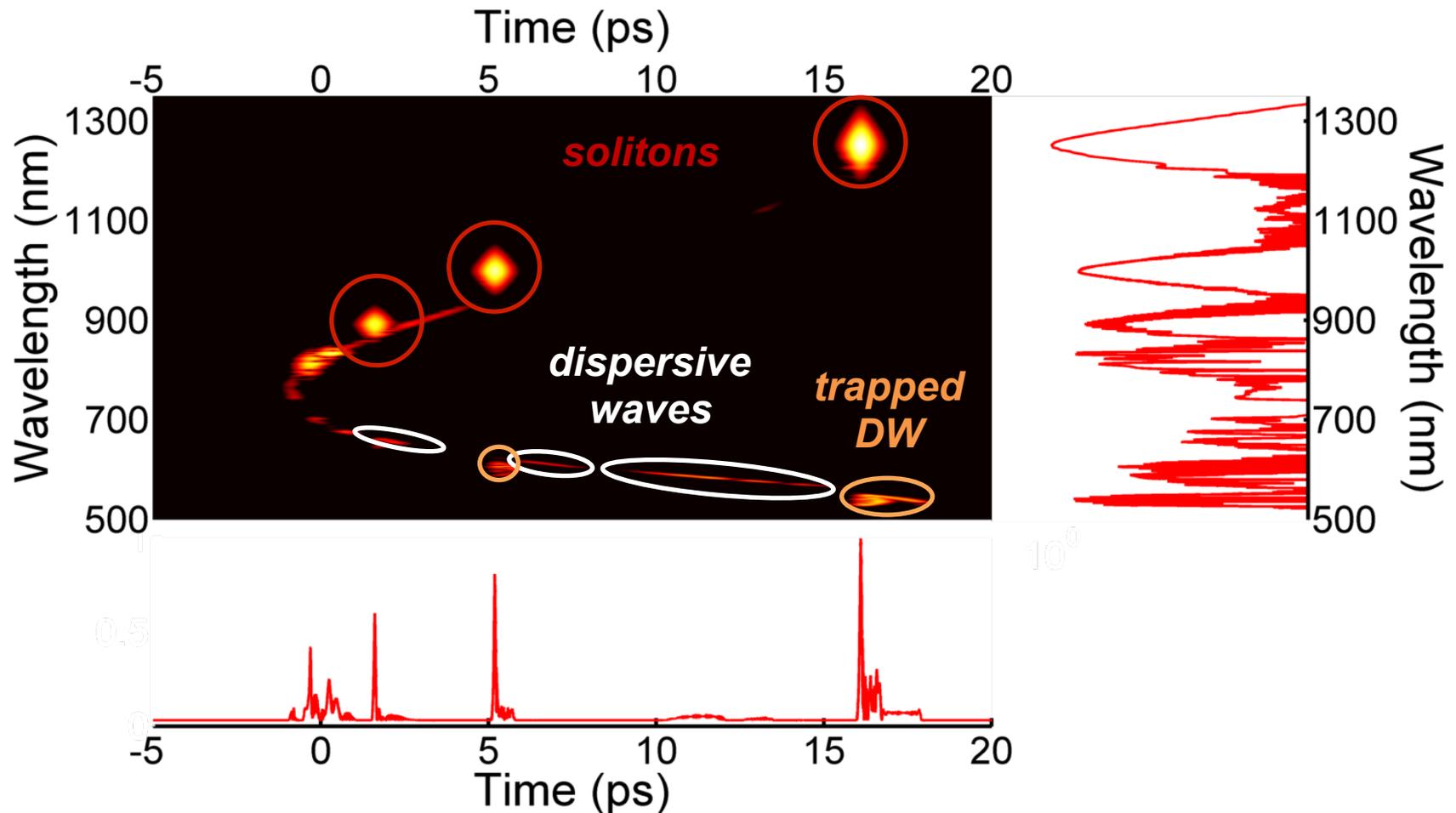
Reference pulse duration
determines the resolution!

$$\Delta t \Delta f \geq 1/2$$



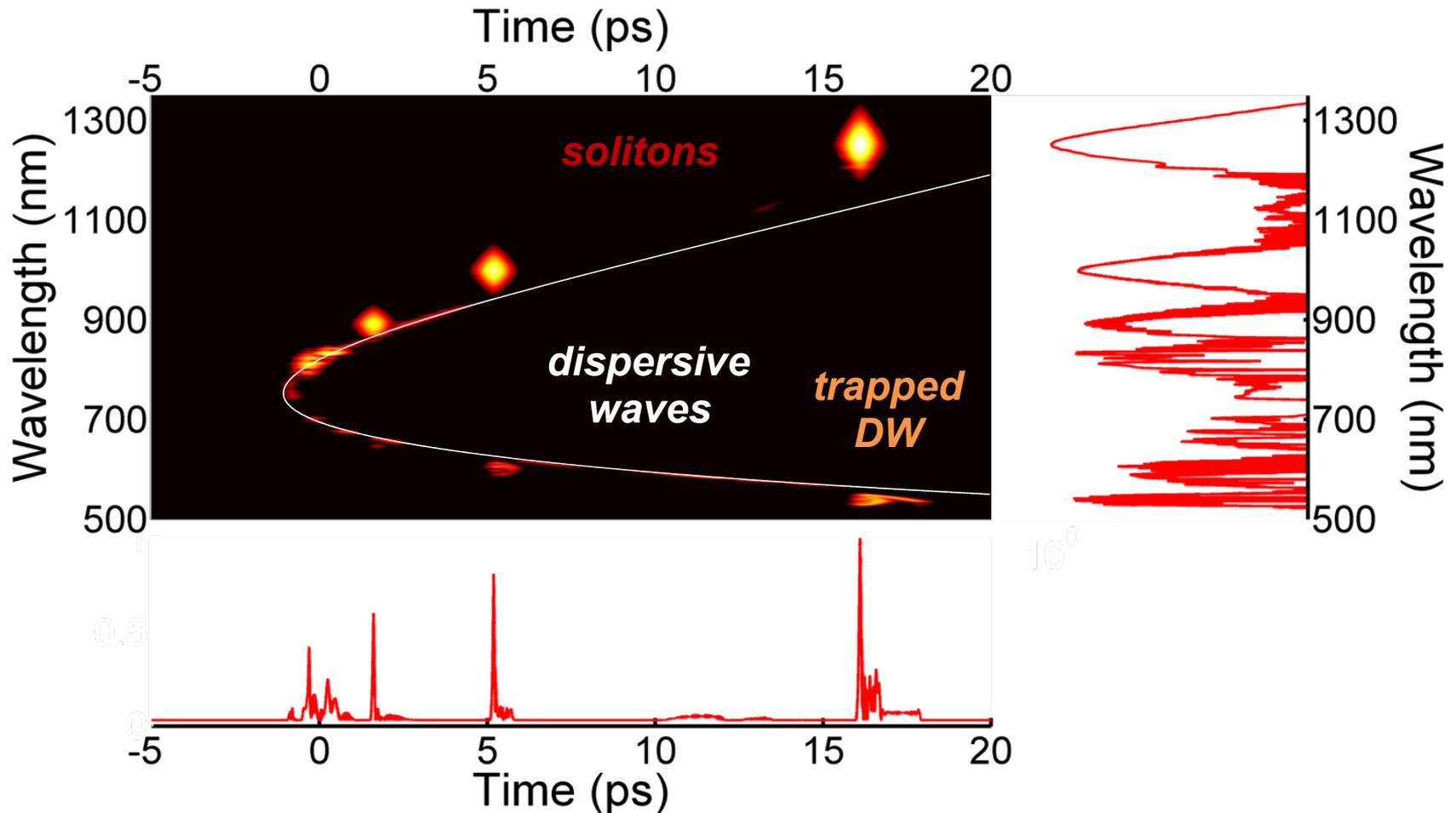
Spectrogram of supercontinuum

- Time-spectrum representation allows to conveniently identify dynamics



Spectrogram of supercontinuum

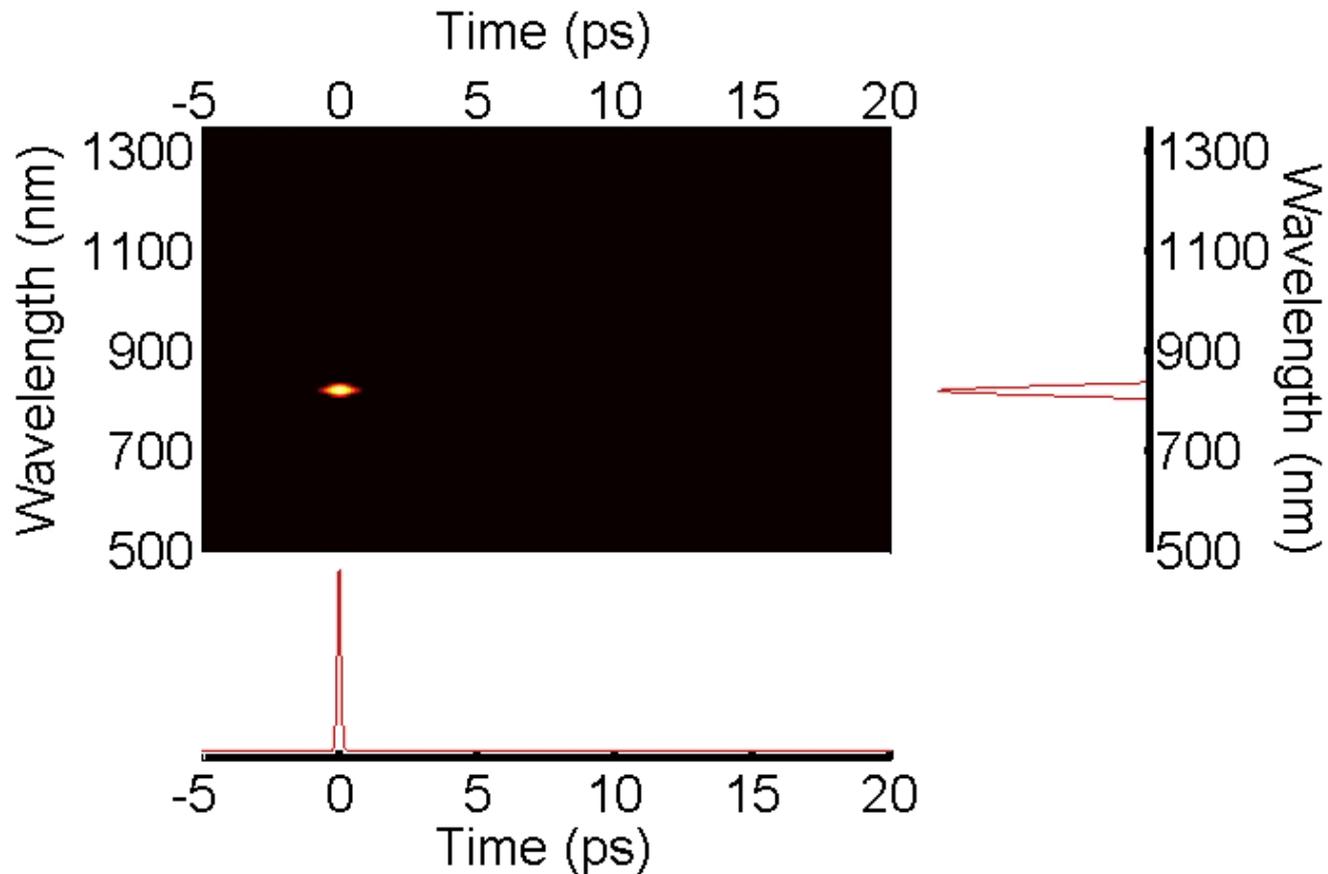
- Time-spectrum representation allows to conveniently identify dynamics



Visualizing dynamics

- Even better: spectrogram movies....

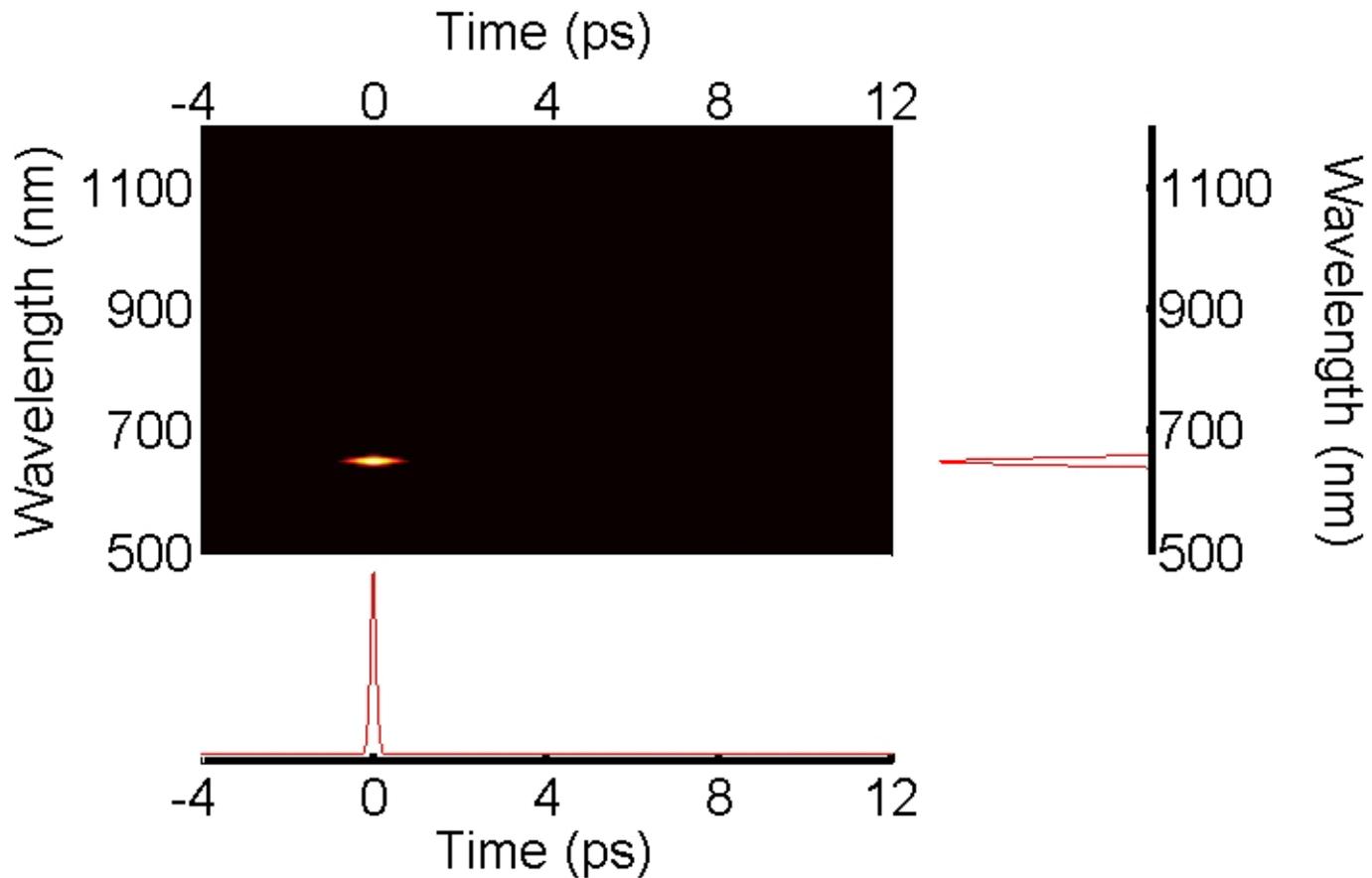
$\lambda_p = 820$ nm (anomalous)
 $N = 8$



Visualizing dynamics

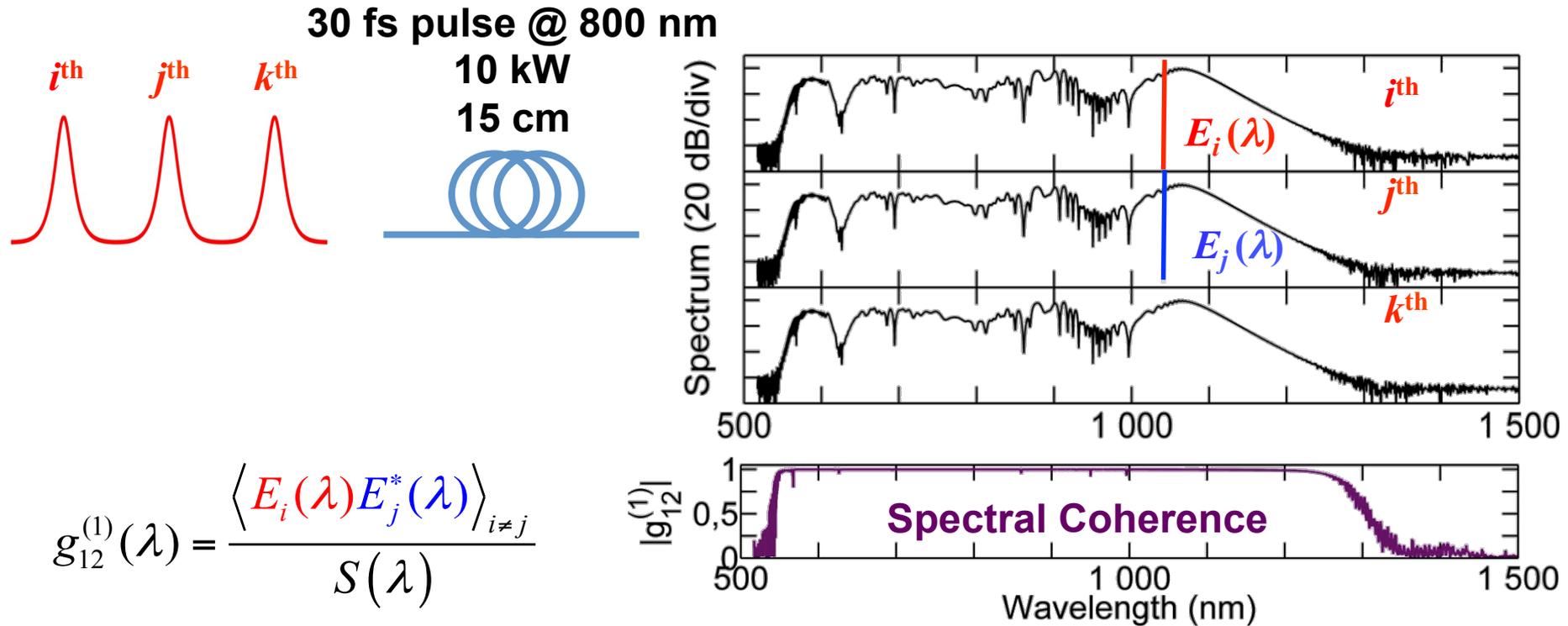
- Even better: spectrogram movies....

$\lambda_p = 650$ nm (normal)
 $N = 6$



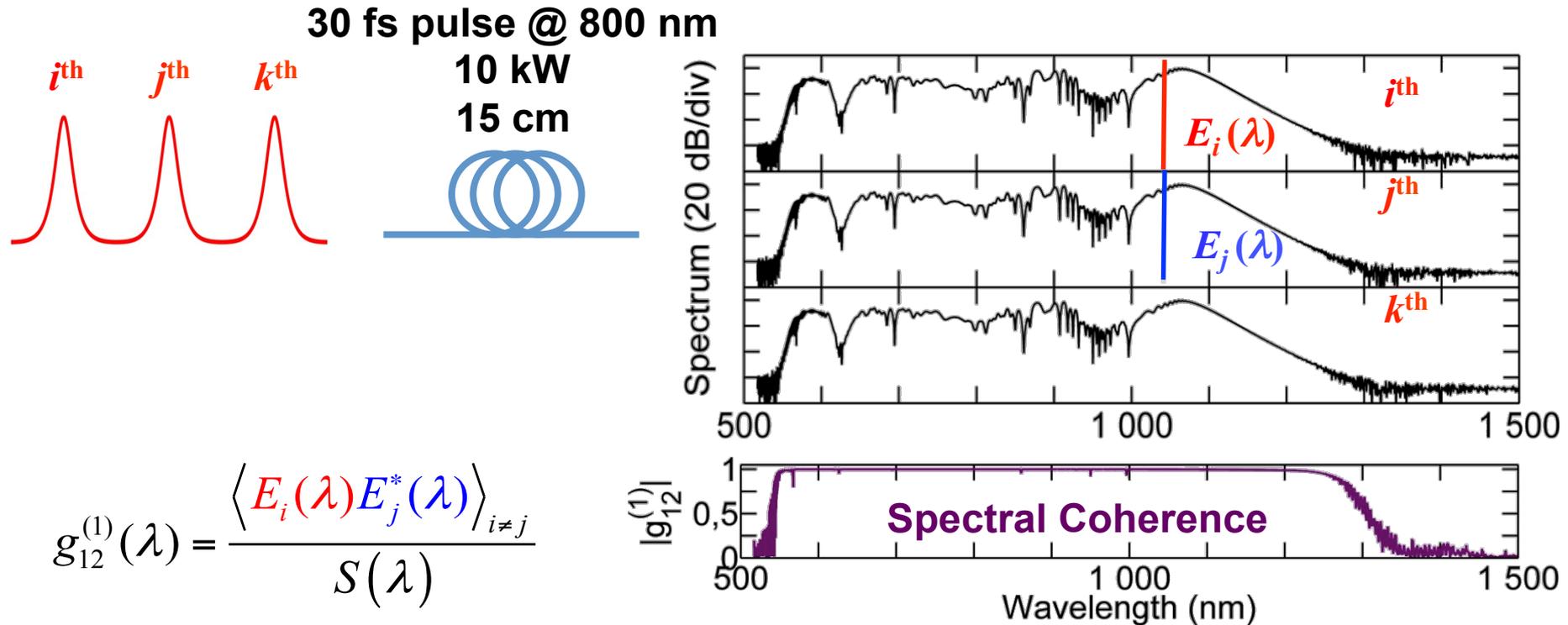
Coherence of supercontinuum

- Spectral coherence: “How spectra differs from shot to shot?”



Coherence of supercontinuum

- Spectral coherence: “How spectra differs from shot to shot?”

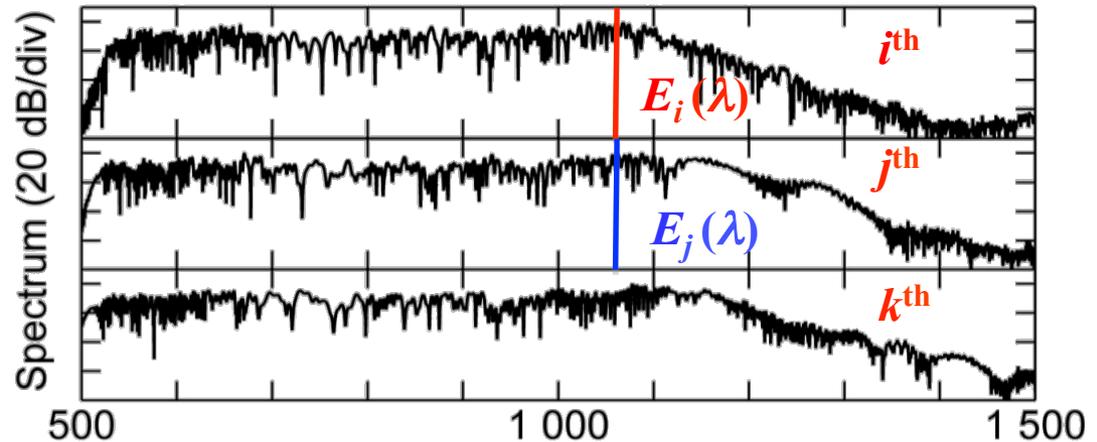
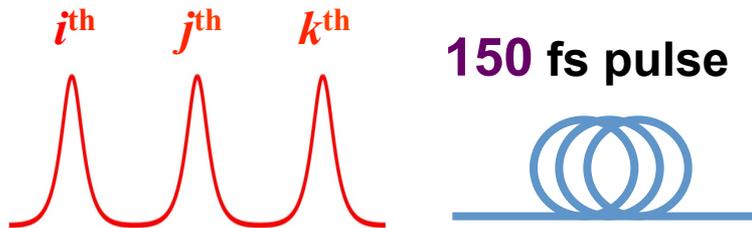


Dudley et al., OL 27, 1180 (2002)

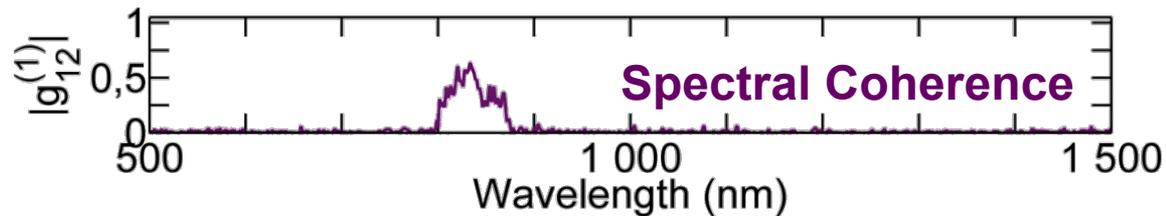
Frequency combs

Coherence of supercontinuum

- BUT supercontinuum is not necessarily coherent!



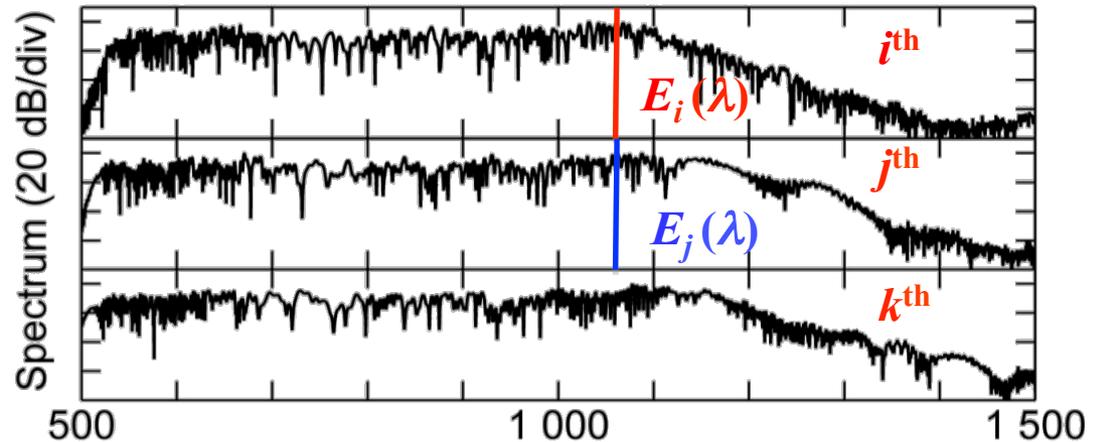
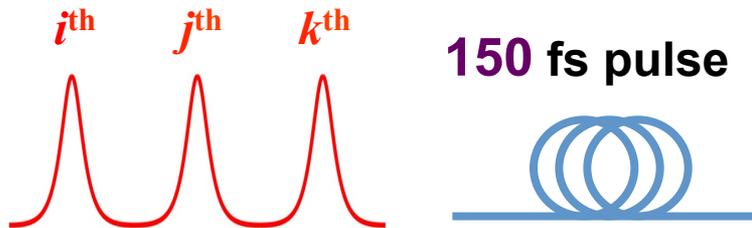
$$g_{12}^{(1)}(\lambda) = \frac{\langle E_i(\lambda) E_j^*(\lambda) \rangle_{i \neq j}}{S(\lambda)}$$



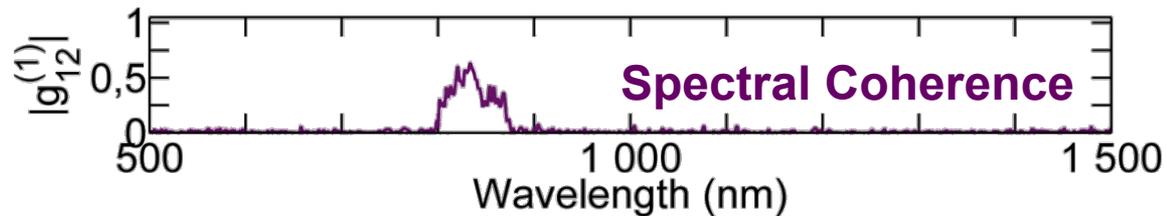
Dudley et al., OL 27, 1180 (2002)

Coherence of supercontinuum

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$$g_{12}^{(1)}(\lambda) = \frac{\langle E_i(\lambda) E_j^*(\lambda) \rangle_{i \neq j}}{S(\lambda)}$$



Dudley et al., OL 27, 1180 (2002)

Imaging, sensing

Supercontinuum coherence

$$N = \sqrt{\frac{\gamma P_p \tau^2}{|\beta_2|}} = \tau \sqrt{\frac{\gamma P_p}{|\beta_2|}} \approx 16$$

Short pulses

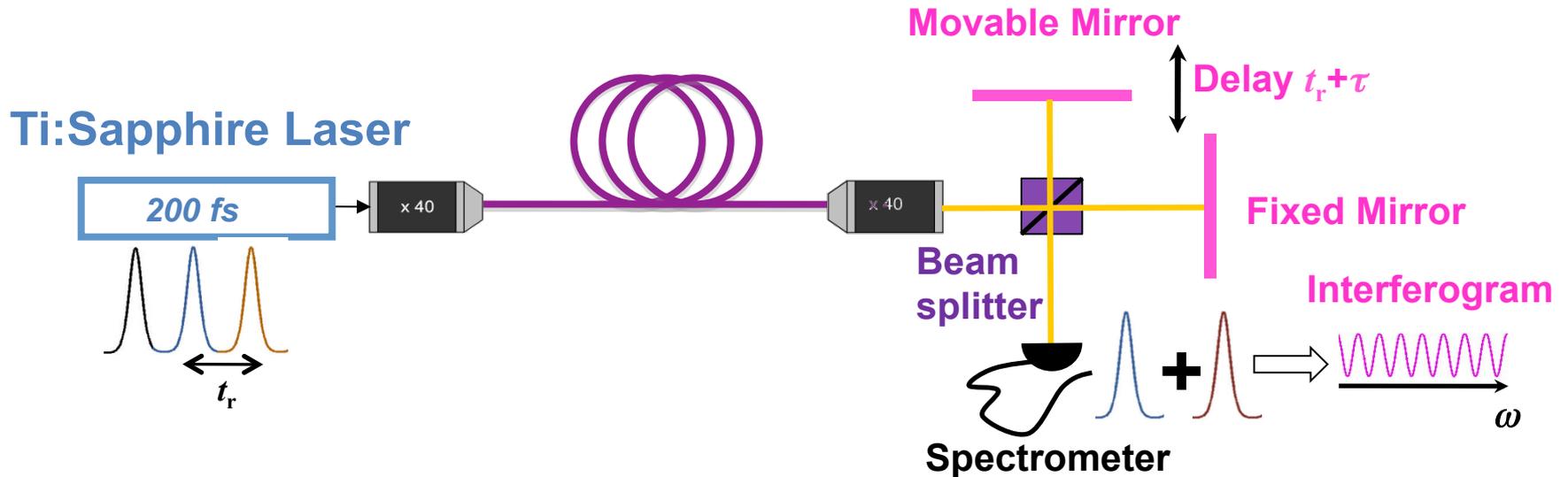
Long pulses

Anomalous	<ul style="list-style-type: none"> • Soliton • Dispersive waves 	<ul style="list-style-type: none"> • Modulation instability • Solitons dynamics
Normal	<ul style="list-style-type: none"> • Self-phase modulation • Four-wave mixing 	<ul style="list-style-type: none"> • Raman scattering • Four-wave mixing

Coherent

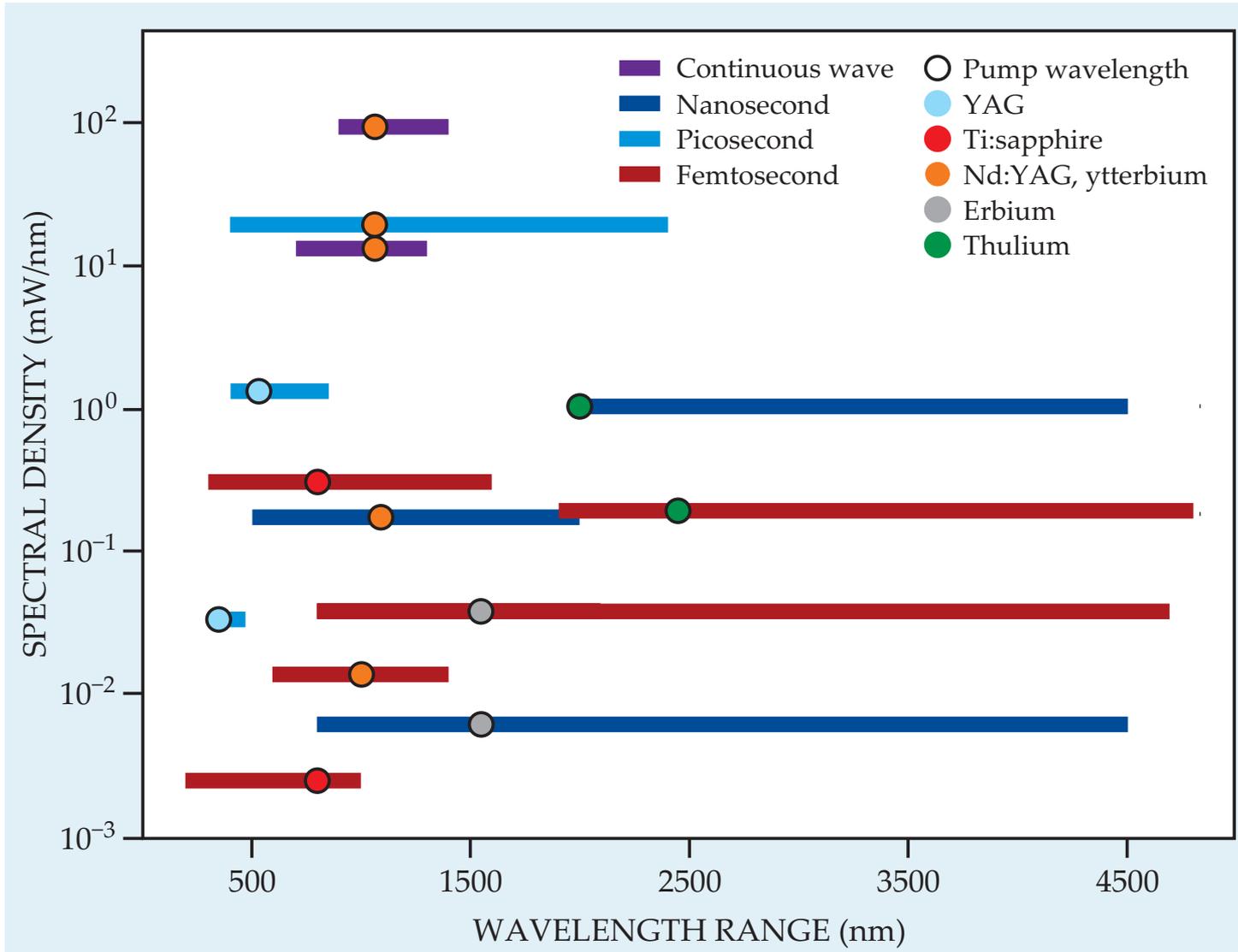
Incoherent

Measuring supercontinuum coherence

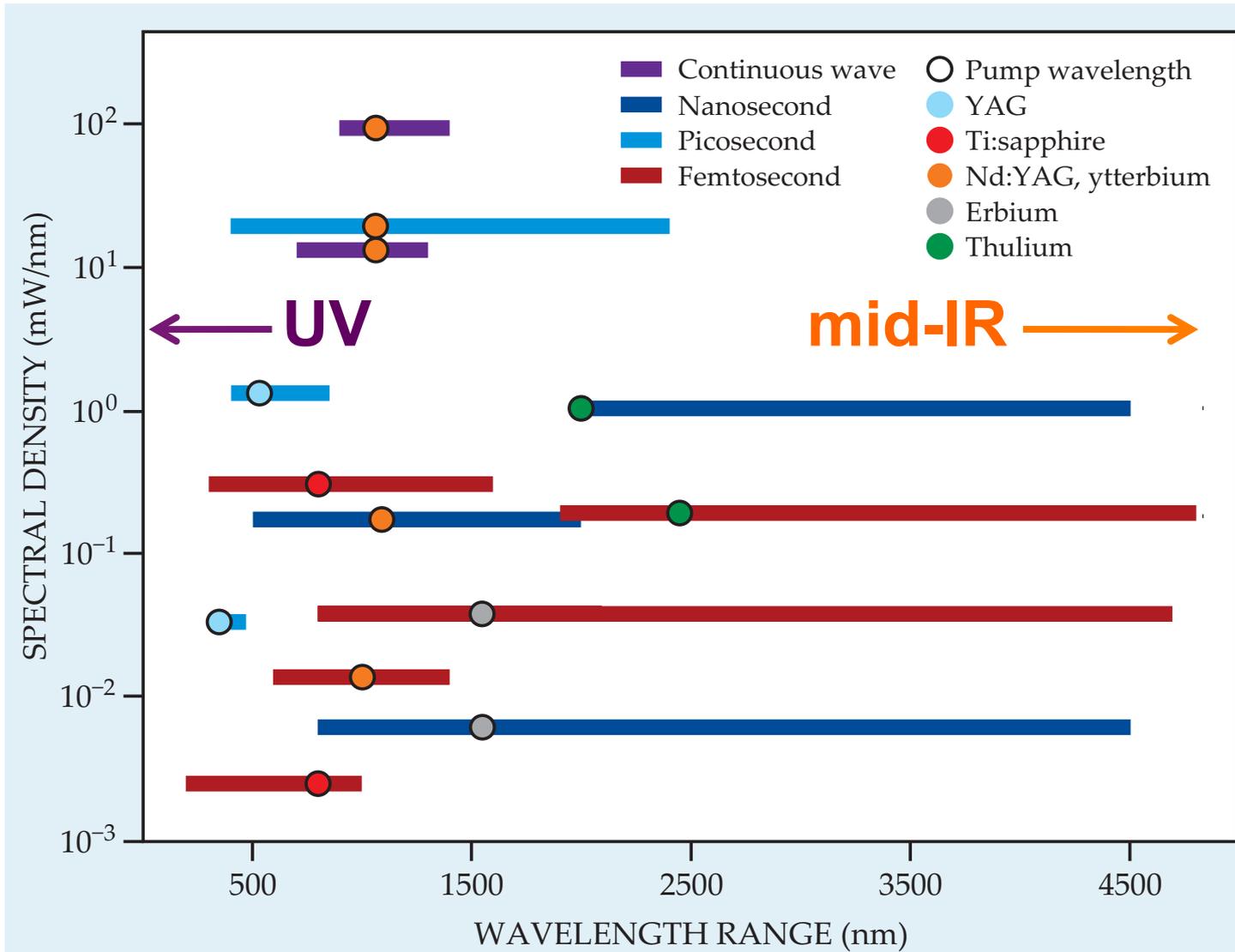


- Michelson interferometer
- Spectral interference
- Fringes visibility gives the coherence function

State-of-the-art



State-of-the-art



Extension to the mid-IR

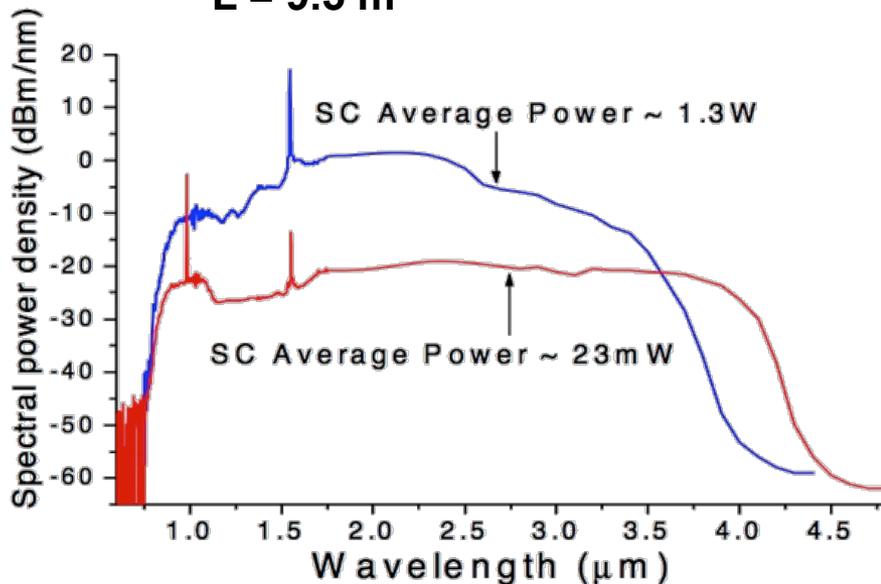
- Use of materials with low losses in the mid-IR

Fluoride (ZBLAN) fibers

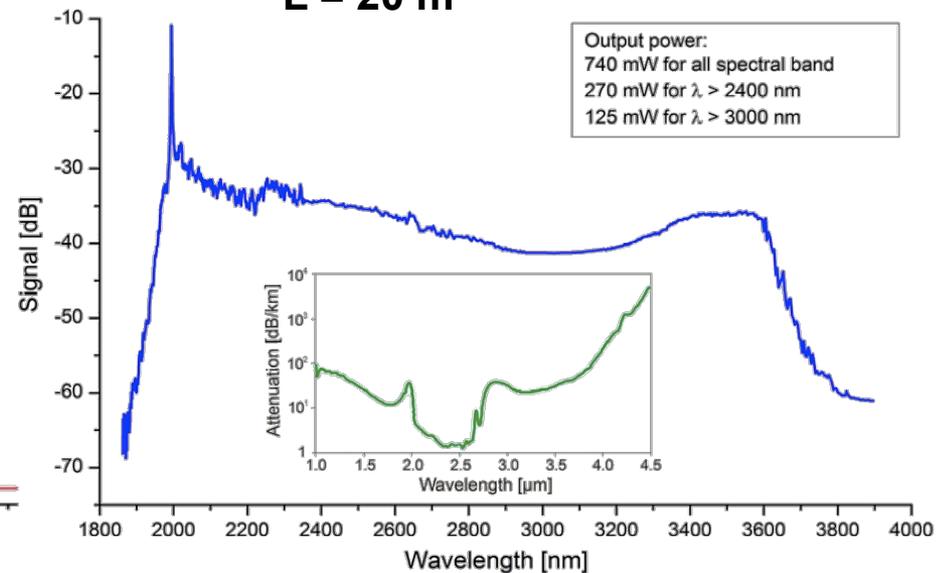
Material ZDW 1.6 microns

$$n_2 = n_2^{\text{silica}}$$

$P_p = 3 \text{ kW}$, $P_{av} = 1.3 \text{ W}$
 $L = 9.5 \text{ m}$



$P_p = 29 \text{ kW}$, $P_{av} = 0.4 \text{ W}$
 $L = 20 \text{ m}$



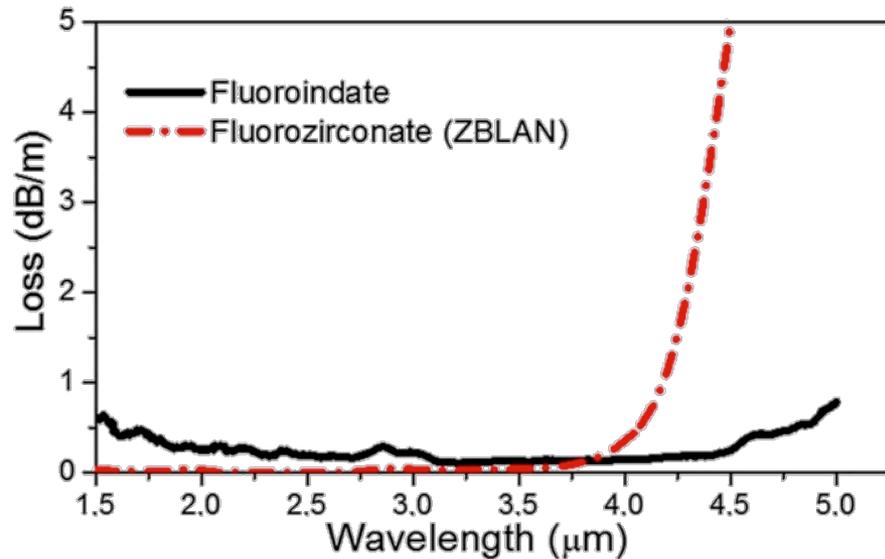
Xia et al. Opt. Express 15, 865 (2007)

Swiderski et al. Opt. Express 21, 7851 (2013)

Extension to the mid-IR

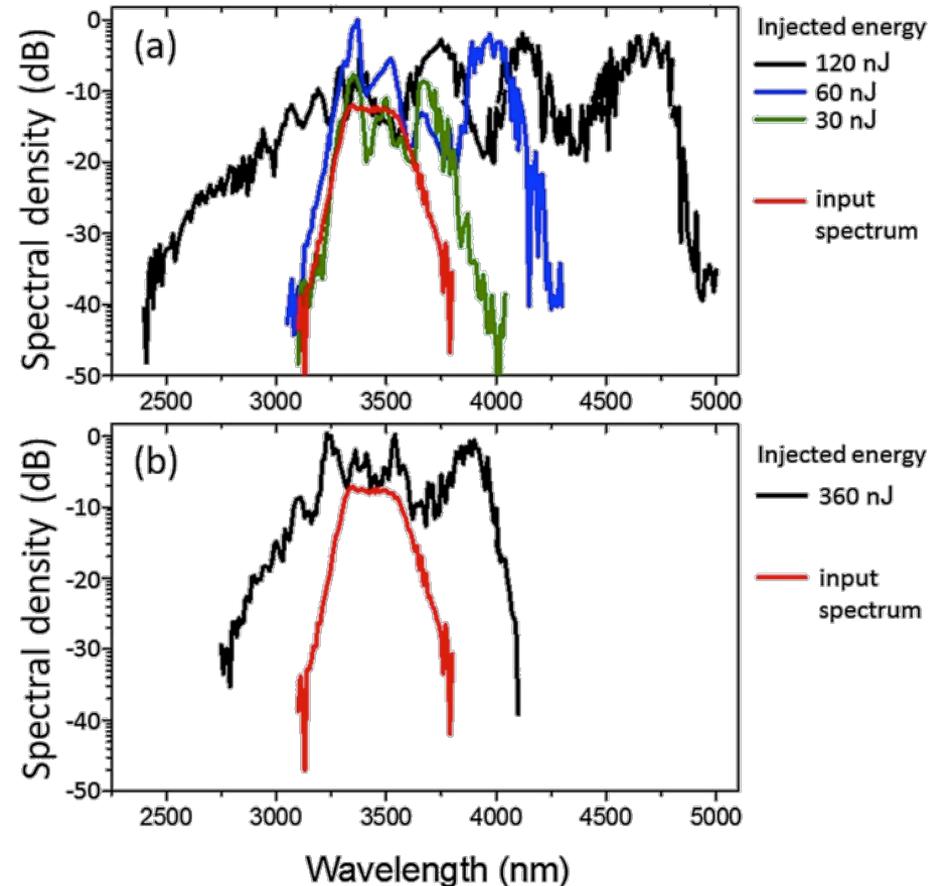
Fluoroindate fibers

Theberge et al., OL 38, 4683 (2013)



$P_p = 1.7 \text{ MW}$

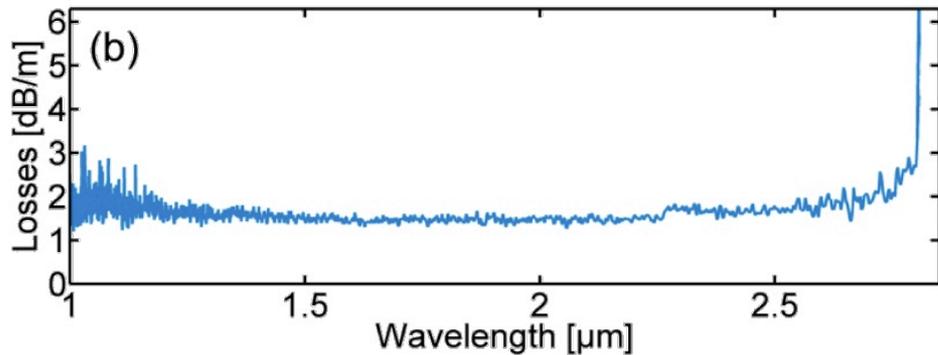
$L = 9.5 \text{ m}$



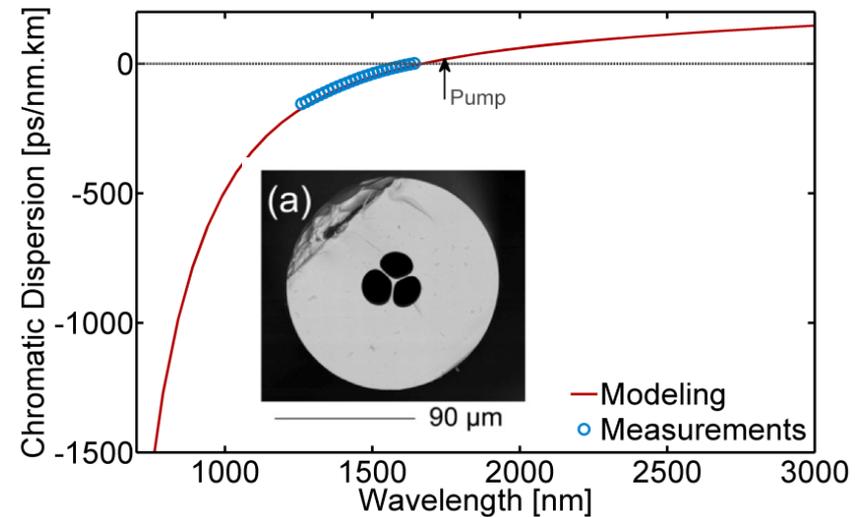
Extension to the mid-IR

- Enhanced nonlinearity

Tellurite fibers (PCFs)



Material ZDW 2.2 microns
 $n_2 = n_2^{\text{silica}} \times 10^{-20}$

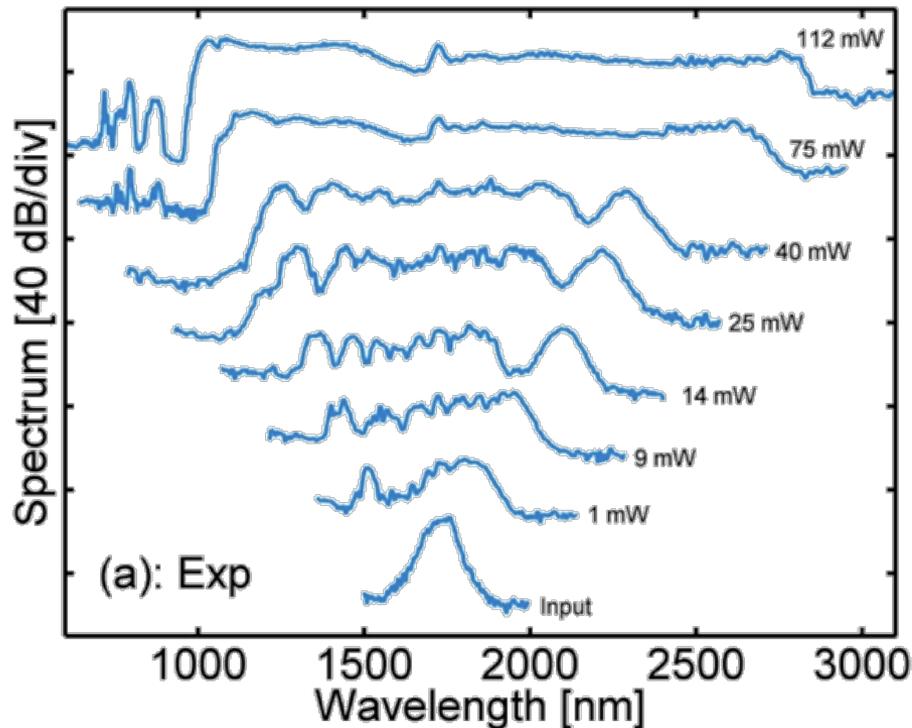


Extension to the mid-IR

- Enhanced nonlinearity

Tellurite fibers (PCFs)

Material ZDW 2.2 microns
 $n_2 = n_2^{\text{silica}} \times 10^{-20}$



$P_p = 7 \text{ kW}$, $P_{av} = 112 \text{ mW}$
 $L = 2 \text{ cm}$

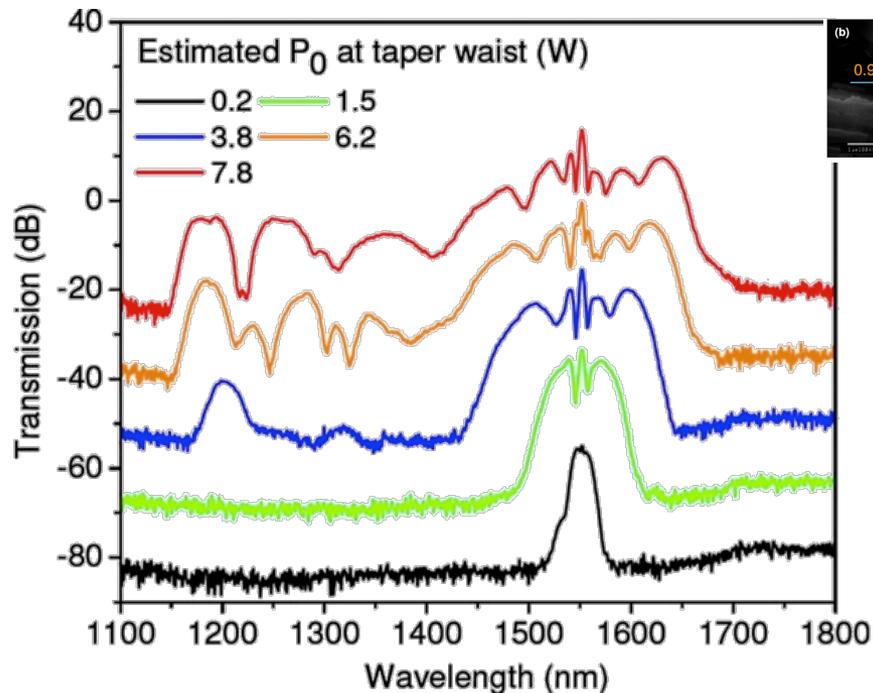
Savelii et al. Opt. Express 20, 27083
(2012)

Extension to the mid-IR

- Enhanced nonlinearity

Sulfide/Chalcogenide fibers

$P_p = 8 \text{ W}$, $P_{av} = 80 \text{ } \mu\text{W}$
 $L = 3 \text{ cm}$

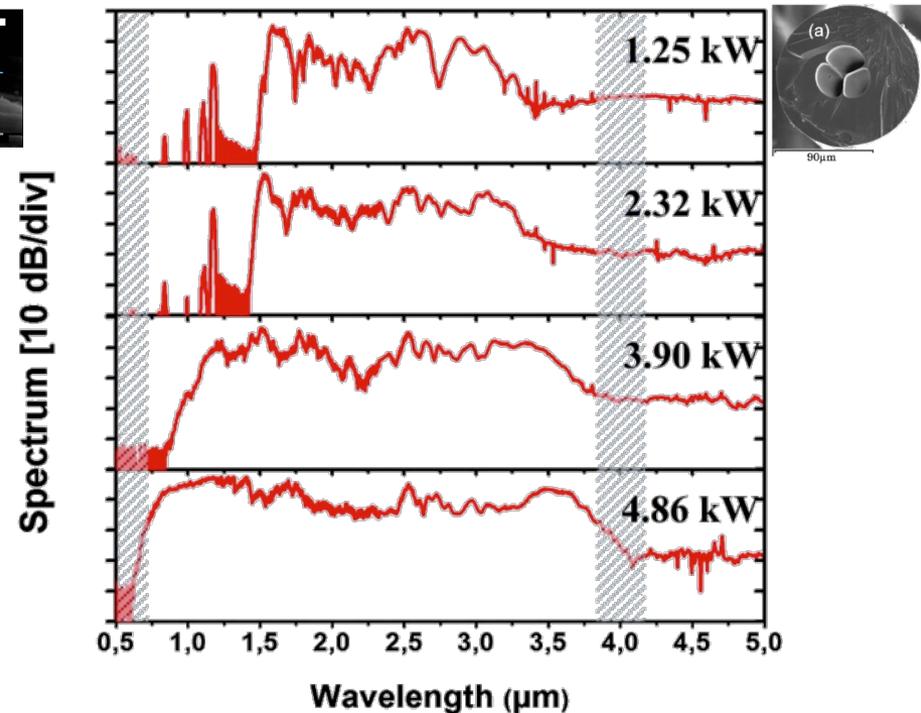


Yeom et al. OL 33, 660 (2008)

Material ZDW 5 microns

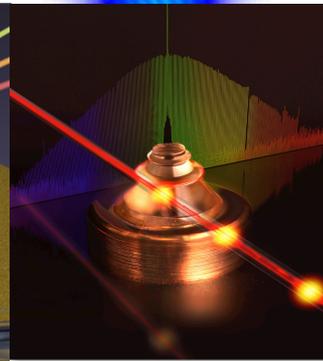
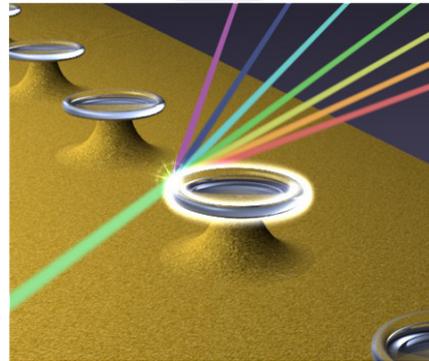
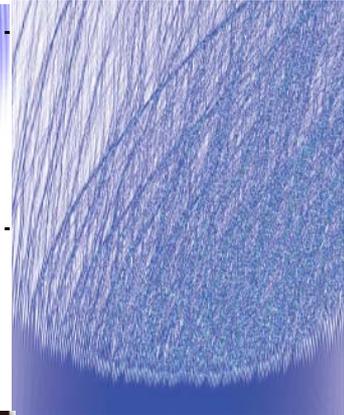
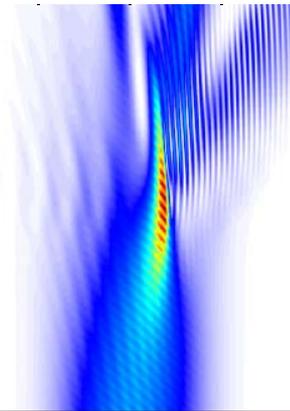
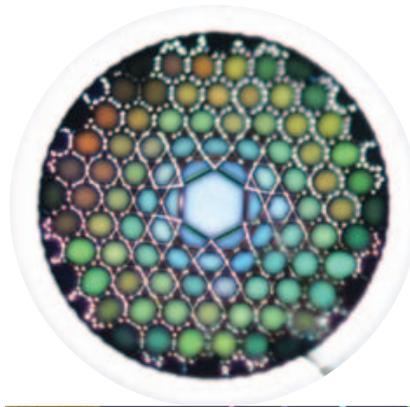
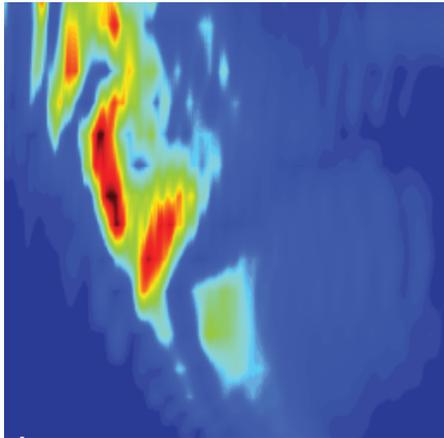
$$n_2 = n_2^{\text{silica}} \times 100$$

$P_p = 5 \text{ kW}$, $P_{av} = 80 \text{ mW}$
 $L = 2 \text{ cm}$



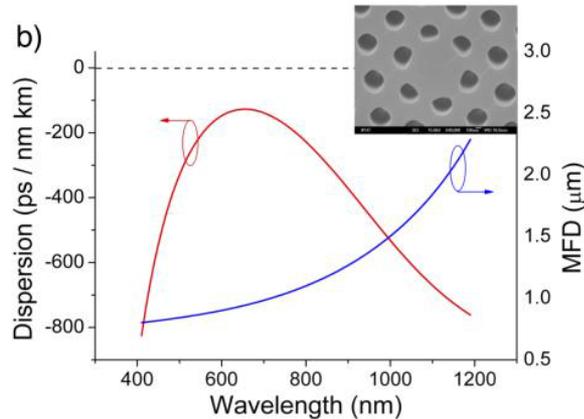
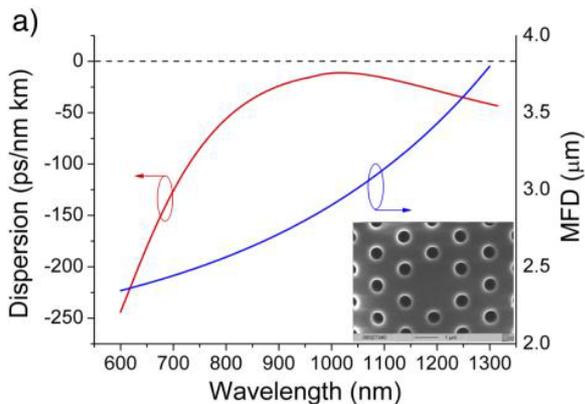
Mouawad et al. OL 39, 2685 (2014)

Emerging structures for supercontinuum generation



Fibers with all-normal dispersion

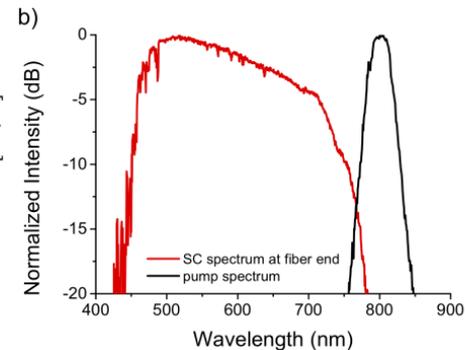
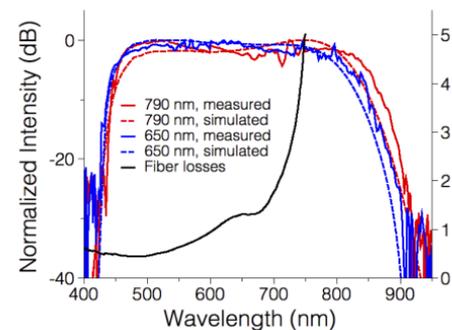
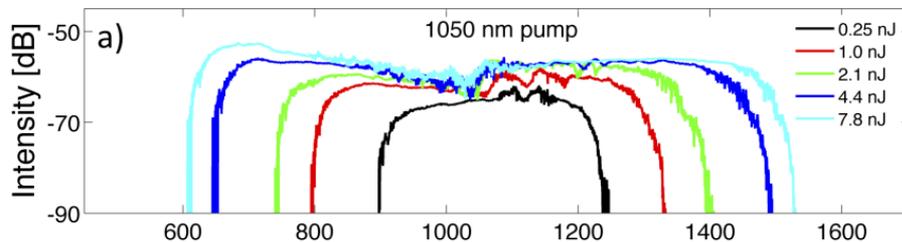
- Fibers with normal dispersion at all wavelengths: ANDI
- Allows for high coherence and stability, flat spectra
- Can reach octave-spanning



Coherent octave spanning near-infrared and visible supercontinuum generation in all-normal dispersion photonic crystal fibers

Alexander M. Heidt,^{1,2*} Alexander Hartung,² Gurthwin W. Bosman,¹ Patrizia Krok,¹ Erich G. Rohwer,¹ Heinrich Schwoerer,¹ and Hartmut Bartelt²

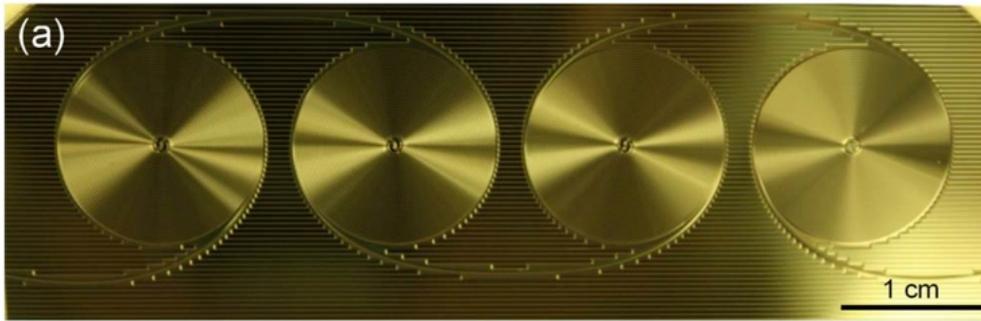
14 February 2011 / Vol. 19, No. 4 / OPTICS EXPRESS 3775



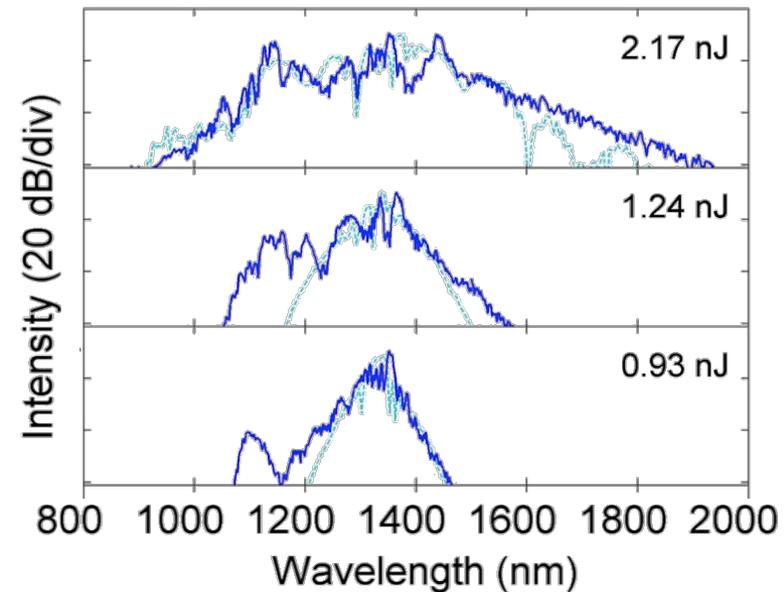
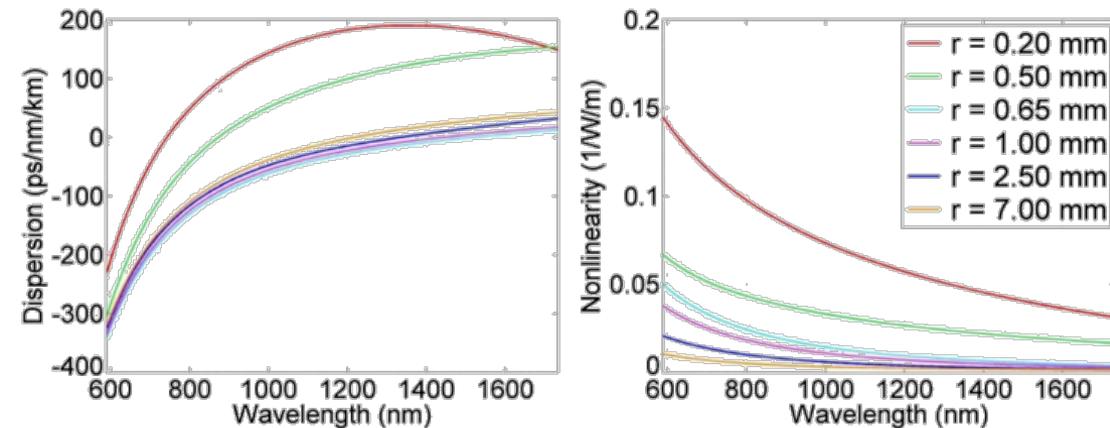
On-chip supercontinuum

Silica

Oh et al., OL 39, 1046 (2014)



$P_p = 12 \text{ kW}$, $P_{av} = 175 \text{ mW}$
 $L = 3.5 \text{ cm}$

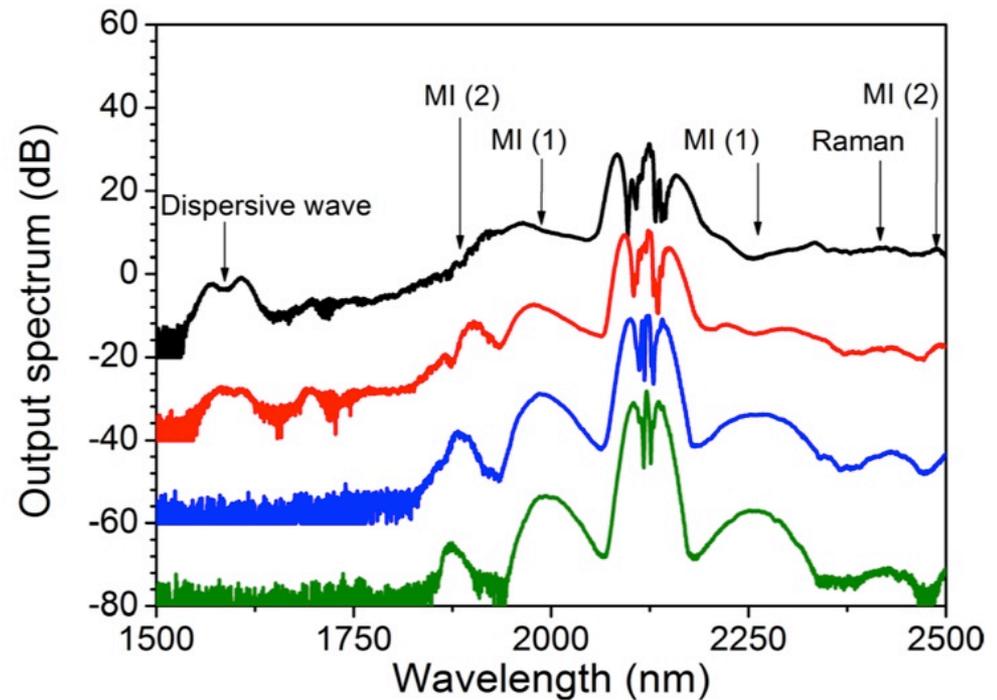
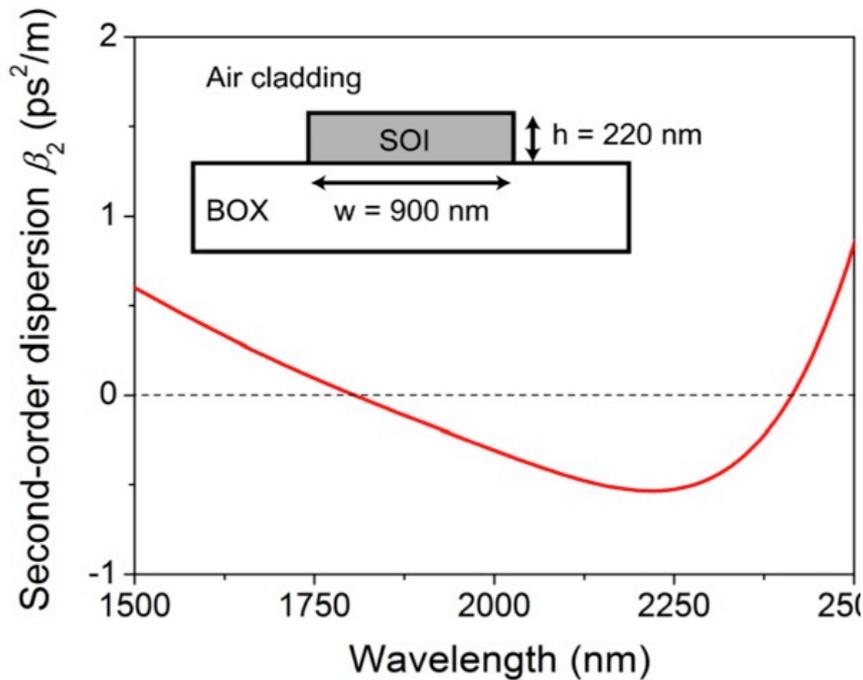


On-chip supercontinuum

Silicon

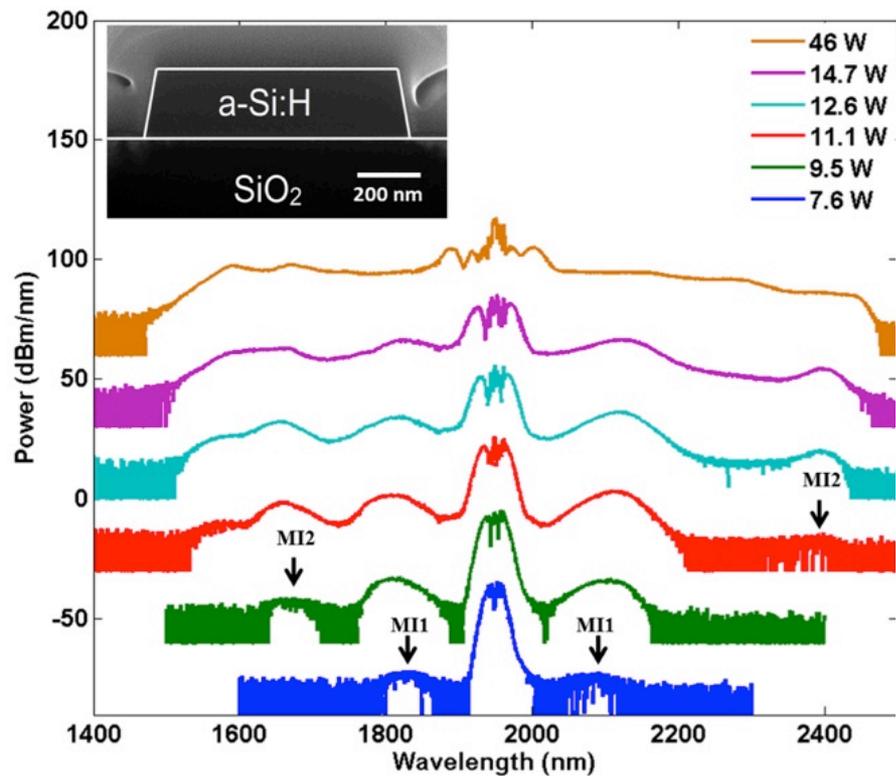
Kuyken et al., Opt. Express 19, 20172 (2011)

$P_p = 12 \text{ W}$
 $L = 1 \text{ cm}$



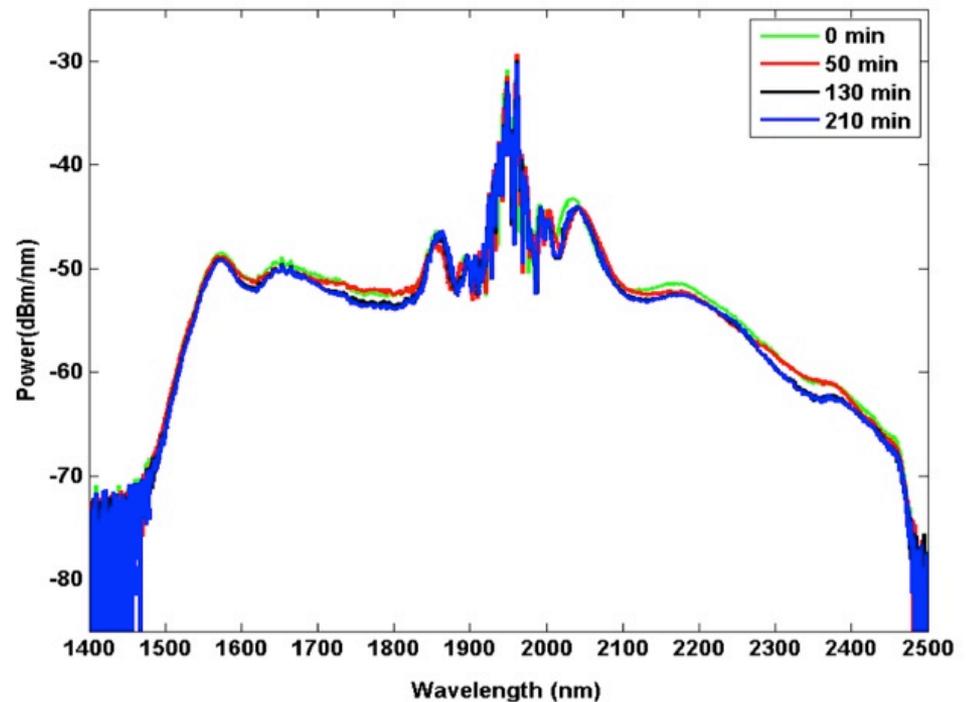
On-chip supercontinuum

Hydrogenated Silicon



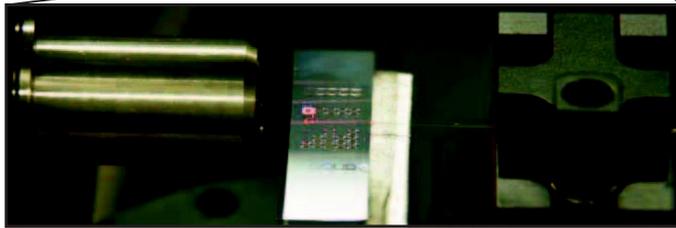
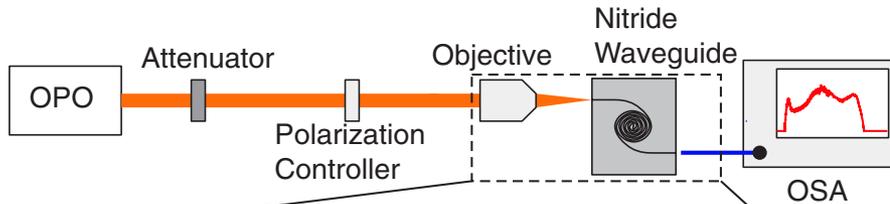
Dave et al., Opt. Express 21, 32034 (2013)

$P_p = 16.5 \text{ W}$, $P_{av} = .5 \text{ mW}$
 $L = 1 \text{ cm}$



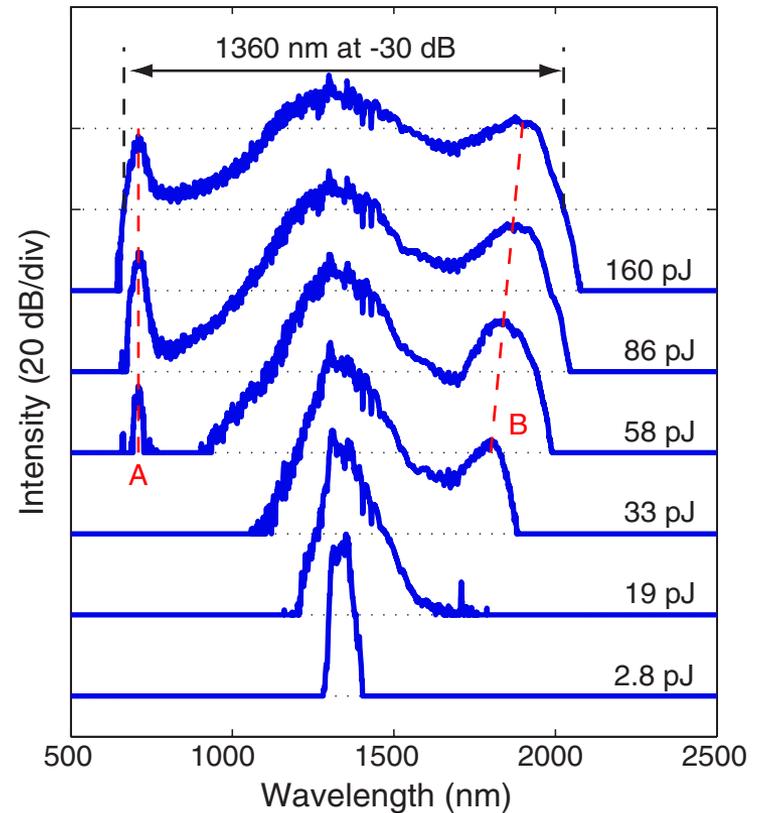
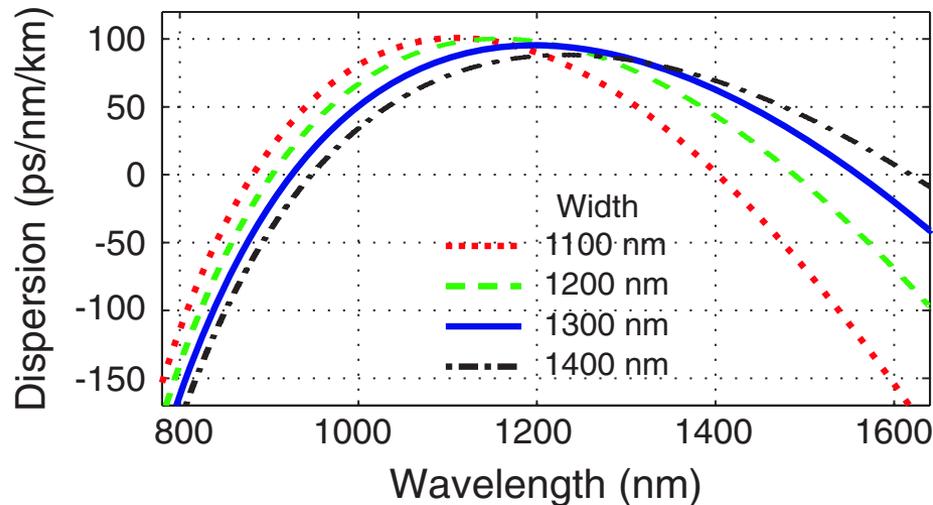
On-chip supercontinuum

Silicon nitride

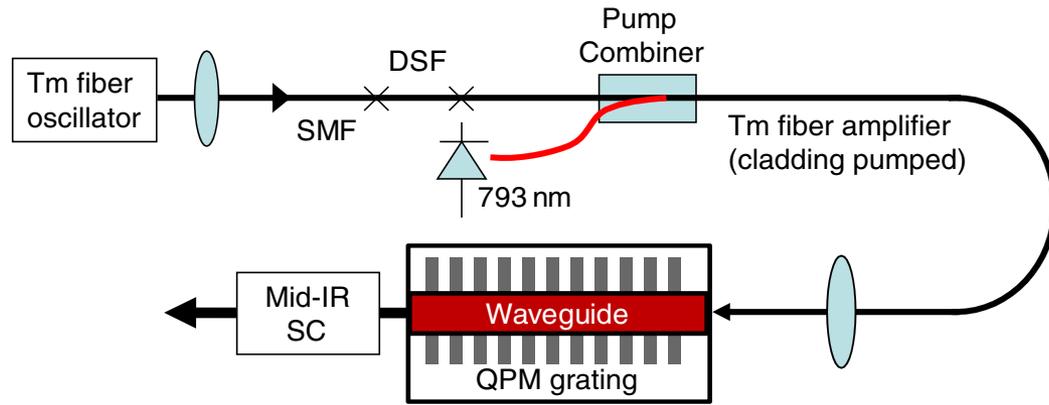


Halir et al. Opt. Lett 37, 1685 (2012)

$P_p = 800 \text{ W}$, $P_{av} = 13 \text{ mW}$
 $L = 4.3 \text{ cm}$

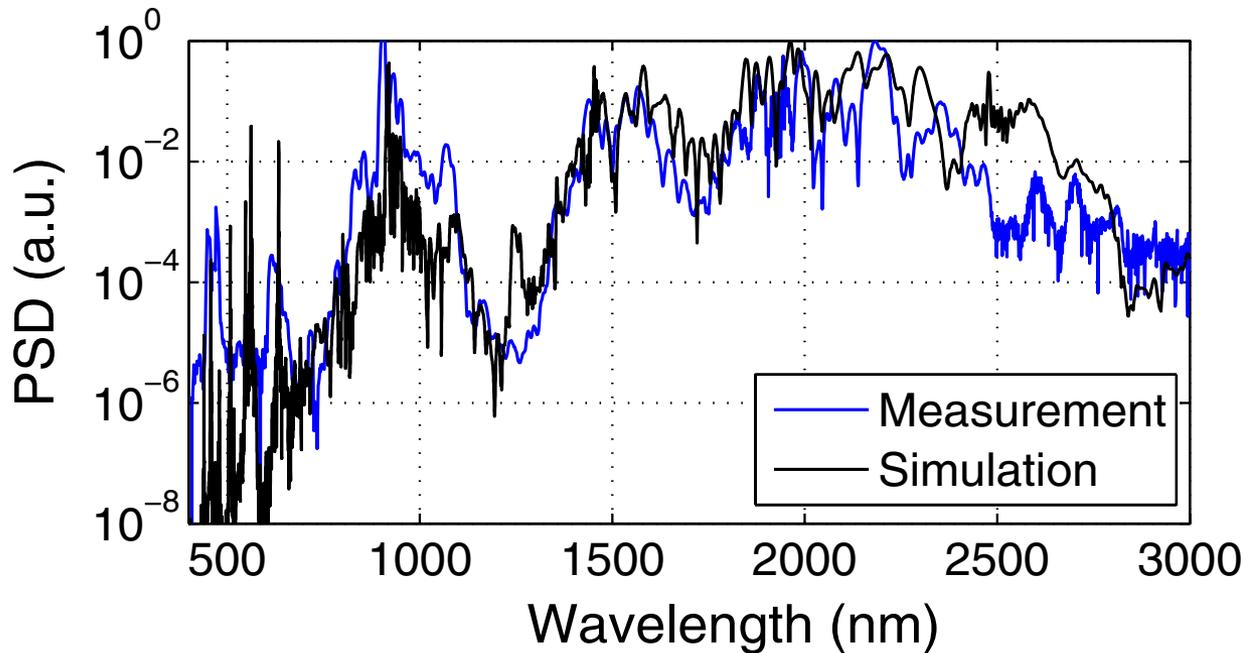


On-chip supercontinuum



Lithium Niobate

Phillips et al. Opt. Lett 36, 3912-3914 (2011)

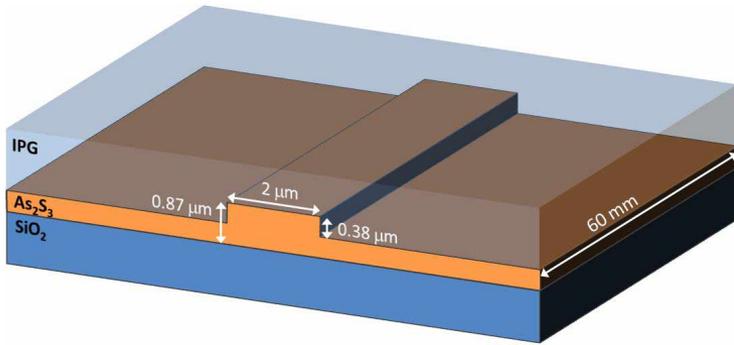


$P_p = 72 \text{ kW}$, $P_{av} = 500 \text{ mW}$
 $L = 1.85 \text{ cm}$

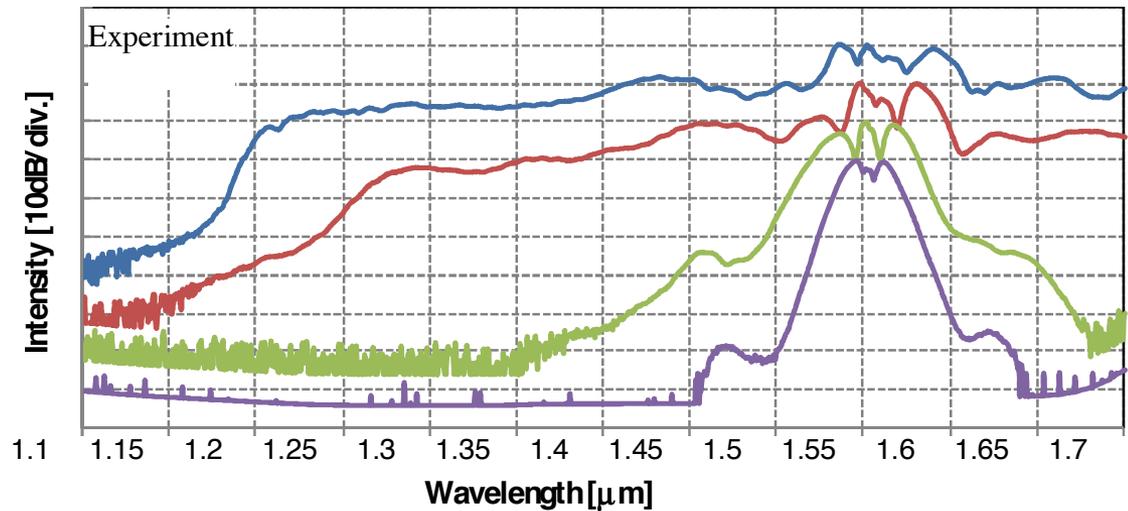
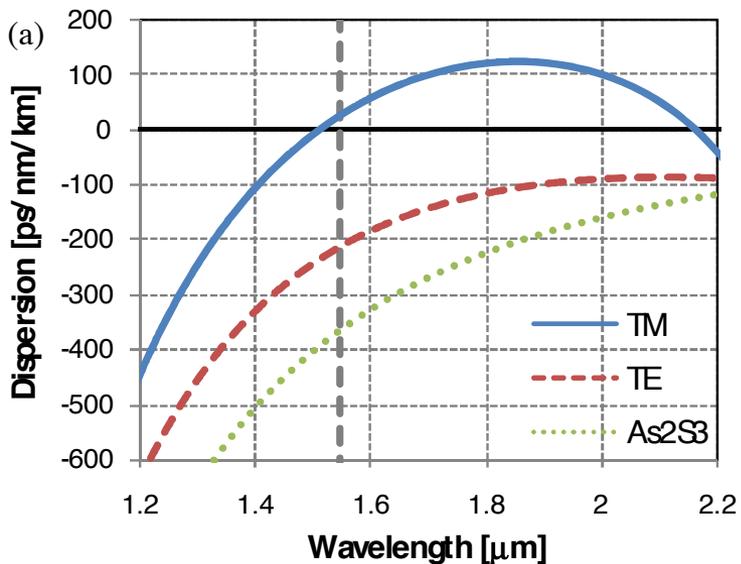
On-chip supercontinuum

Chalcogenide

Lamont et al. Opt. Express 16, 14938 (2008)



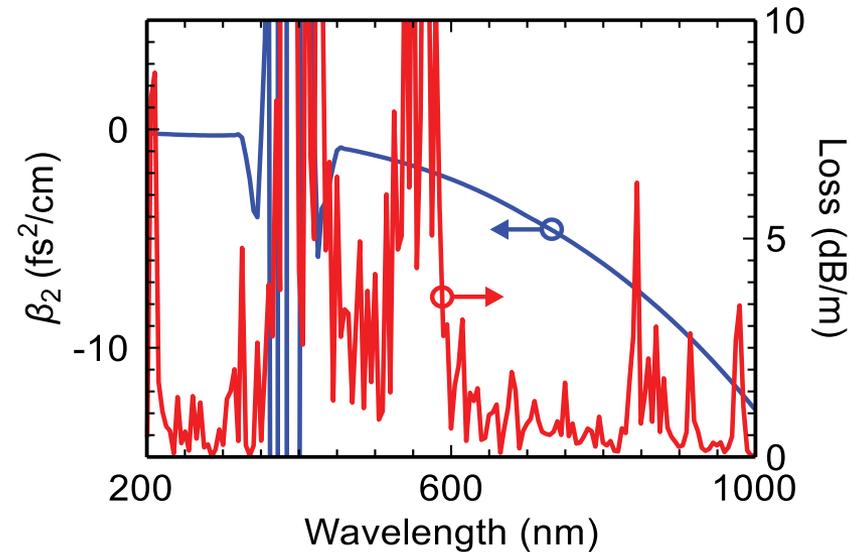
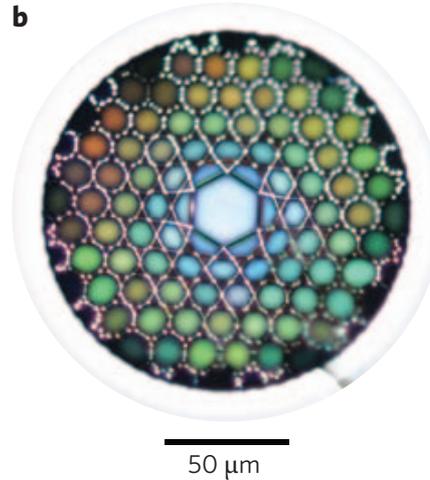
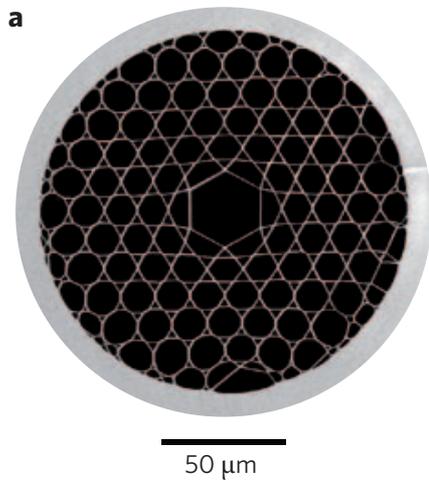
$P_p = 68\ \text{W}$, $P_{av} = .5\ \text{mW}$
 $L = 6\ \text{cm}$



Hollow core photonic crystal fibers

- Kagome fibers
 - Broad guidance

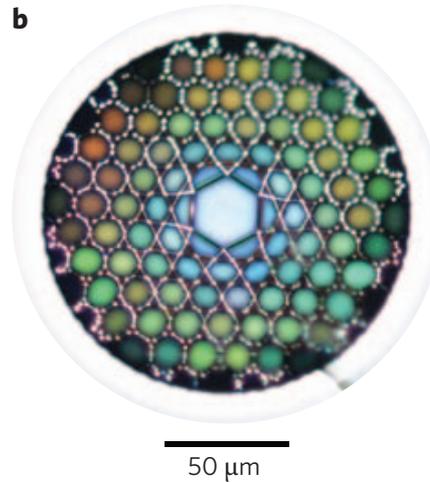
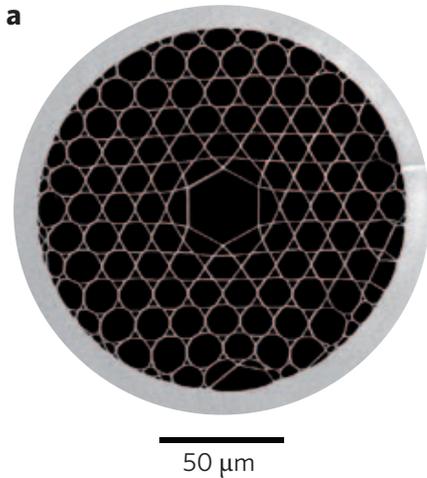
Travers et al., JOSAB 28, 11 (2011)



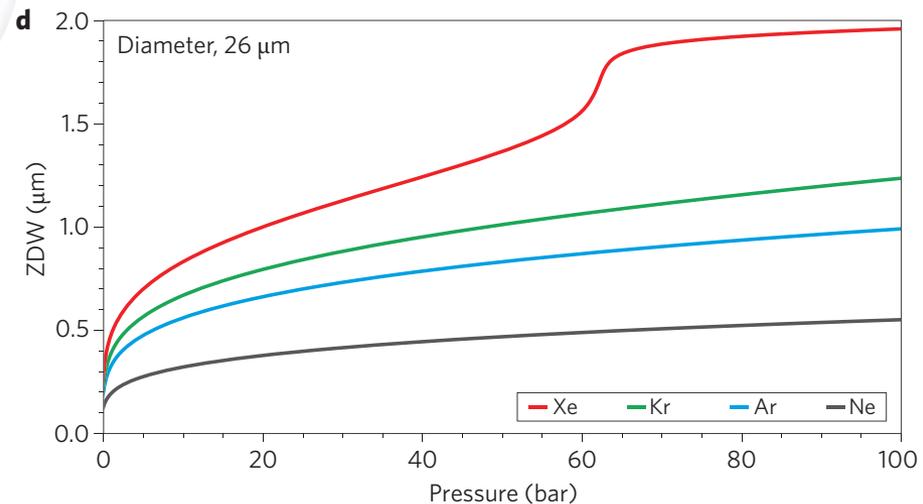
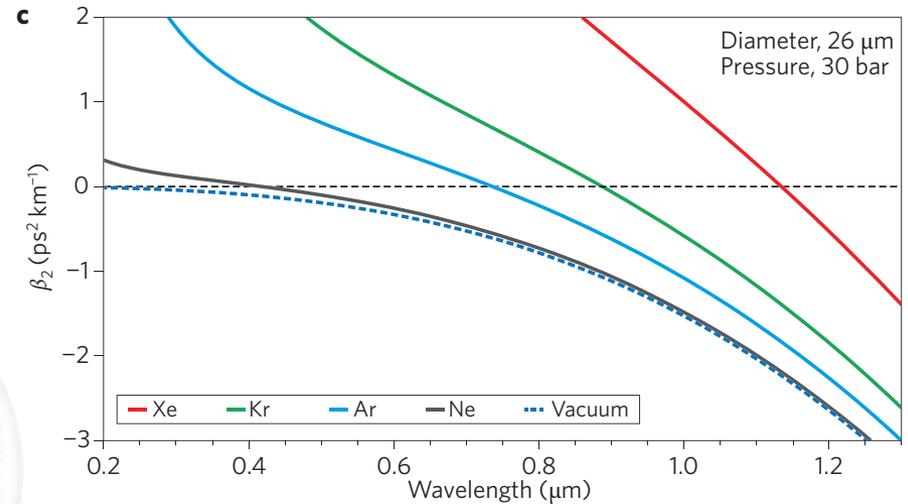
Hollow core photonic crystal fibers

- Kagome fibers

- Broad guidance
- Dispersion-control

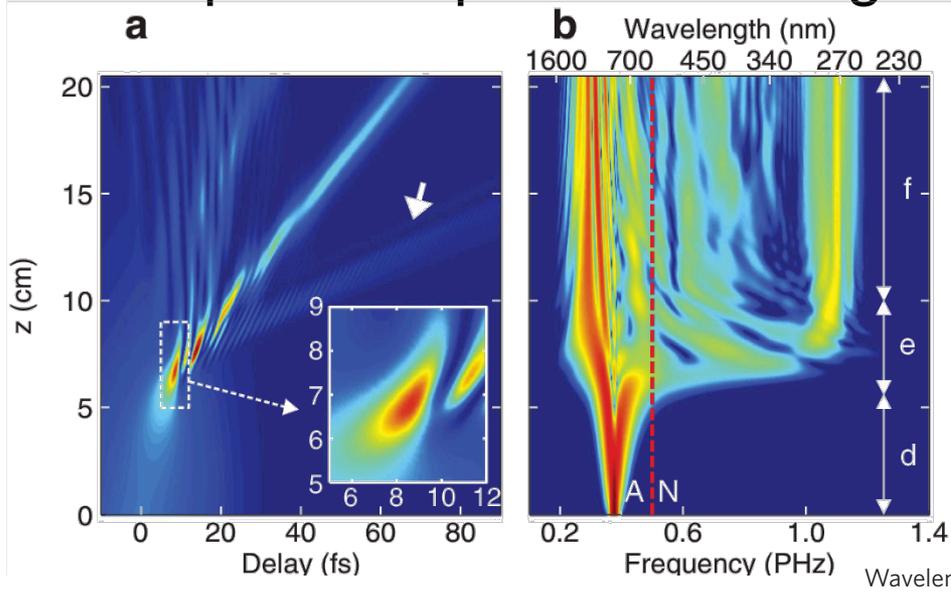


Russell et al., Nat. Photon. 8, 278 (2014)

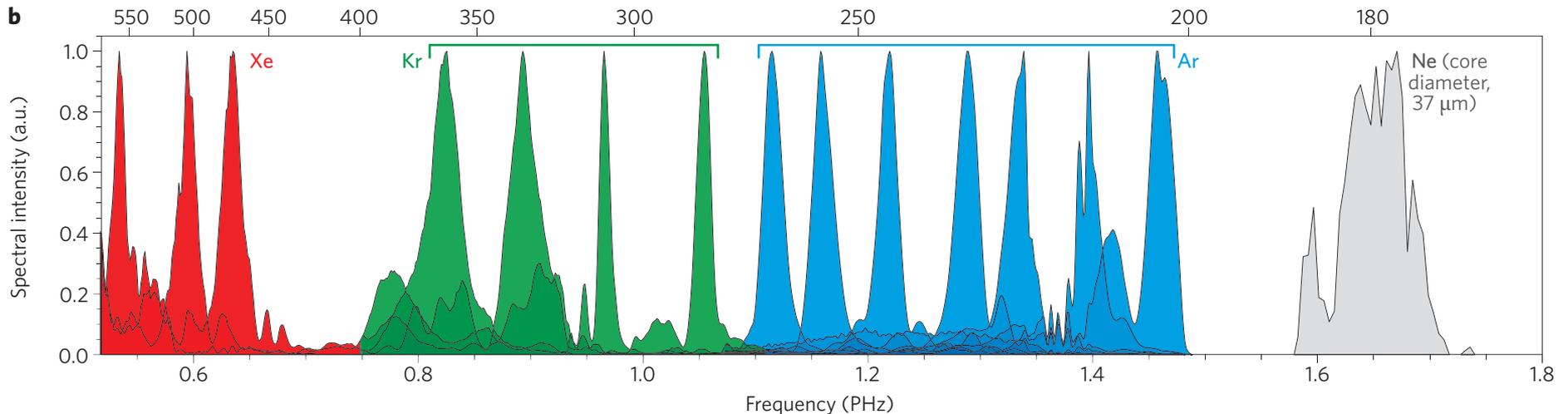


Deep UV generation

- Exploits dispersive wave generation by solitons



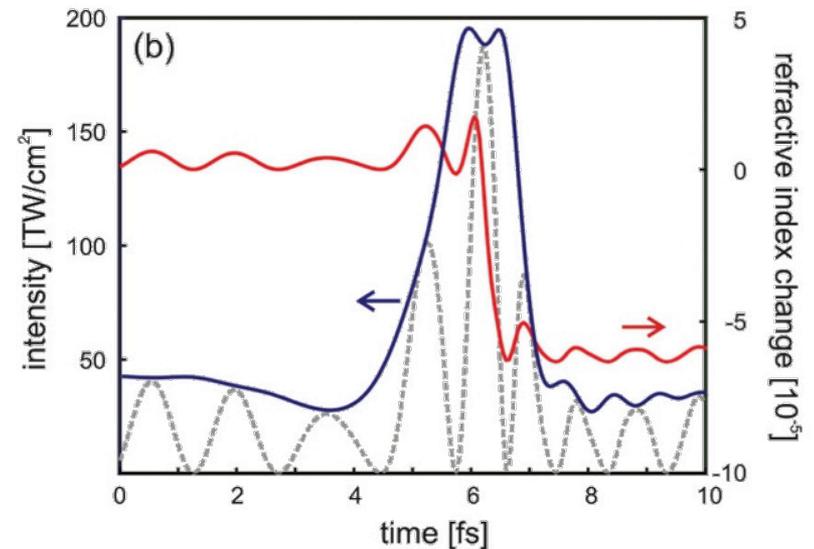
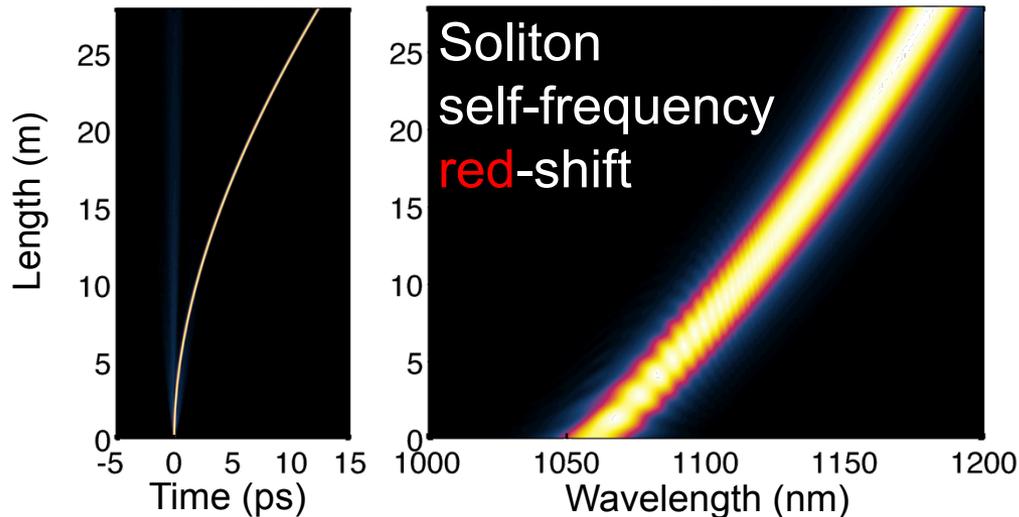
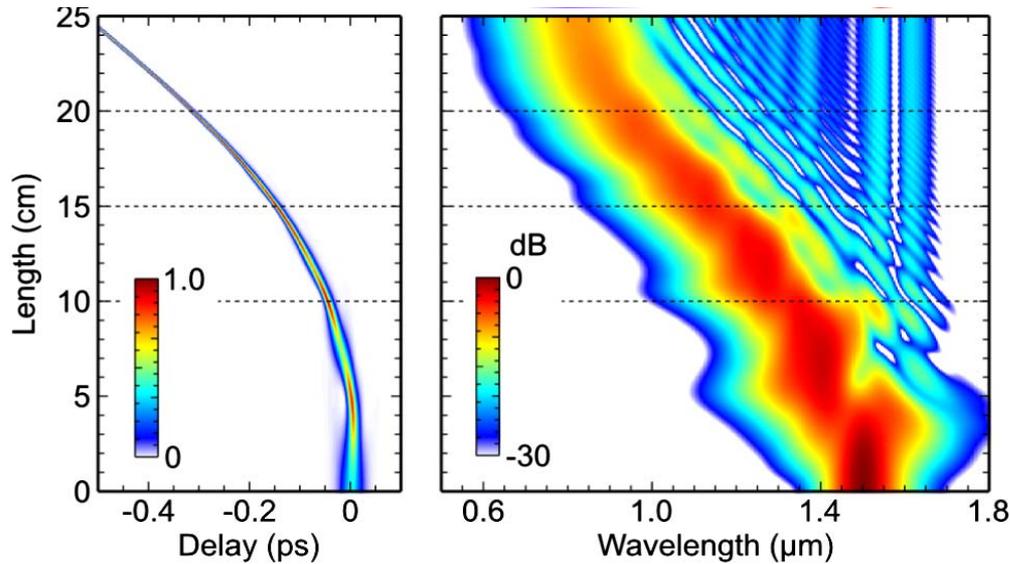
Russell et al., Nat. Photon. 8, 278 (2014)



Novel dynamics in the ionization regime

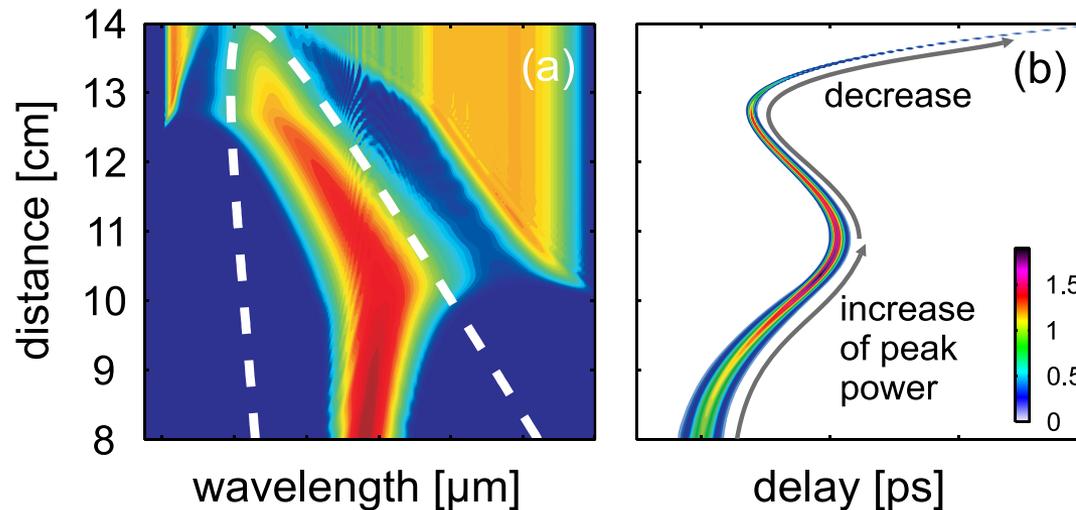
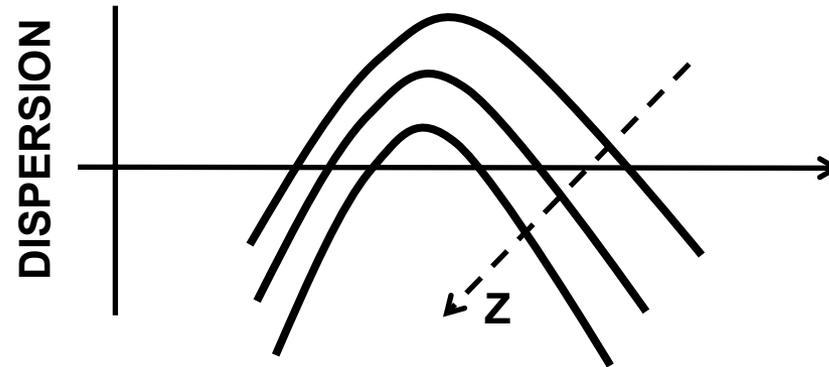
- Solitons self-frequency **blue-shift**

Chang et al., OL 38, 29884 (2013)



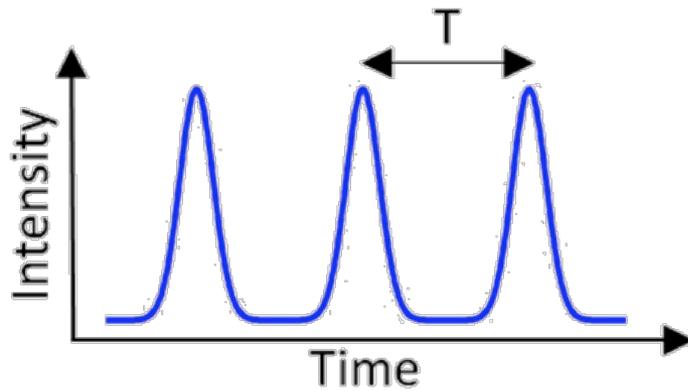
BTW, also possible in tapered fibers

- Solitons self-frequency **blue-shift** Stark et al., PRL 106, 083903 (2011)

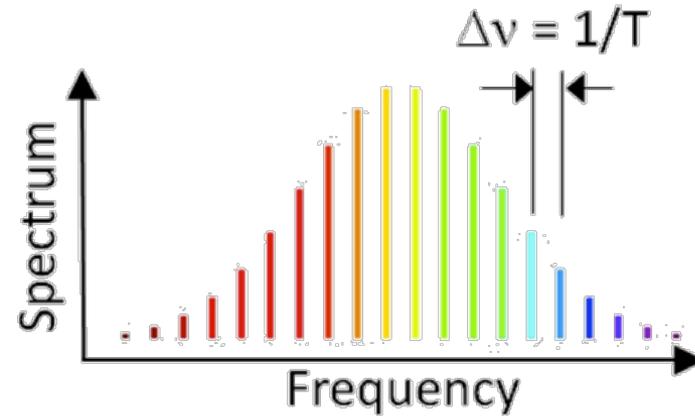


Soliton and frequency combs

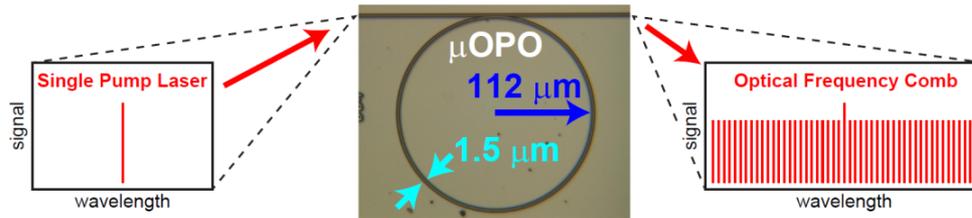
A pulse train in time



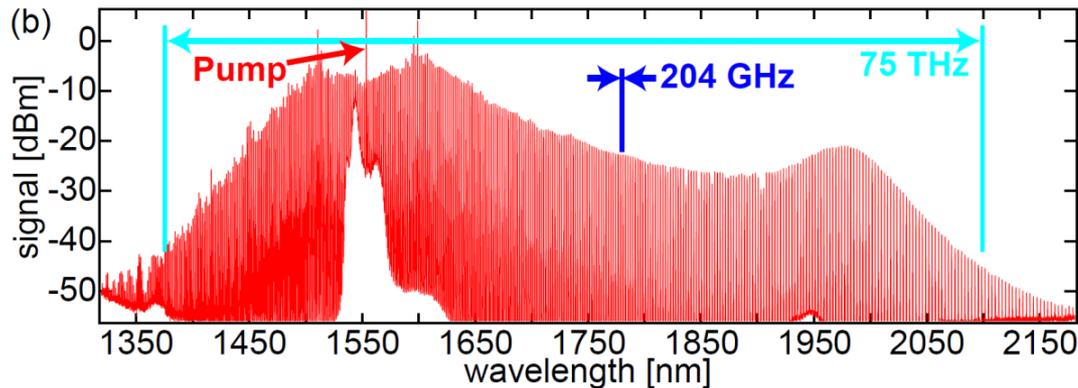
A frequency comb



(a)



(b)



Foster et al., Optics Express
19, 14233 (2011)

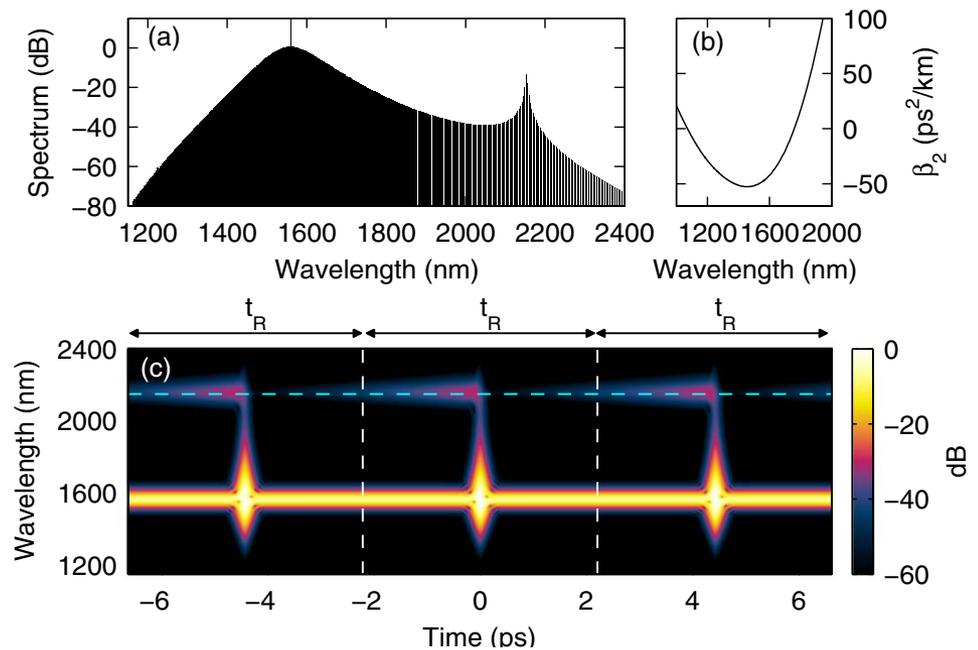
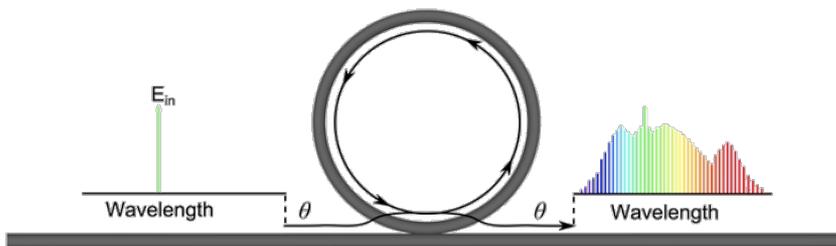
Soliton dynamics at the micron scale

- Soliton dynamics also in microresonator frequency combs

Coen et al., OL 38, 37 (2013)

$$t_R \frac{\partial E(t, \tau)}{\partial t} = \left[-\alpha - i\delta_0 + iL \sum_{k \geq 2} \frac{\beta_k}{k!} \left(i \frac{\partial}{\partial \tau} \right)^k + i\gamma L |E|^2 \right] E + \sqrt{\theta} E_{\text{in}},$$

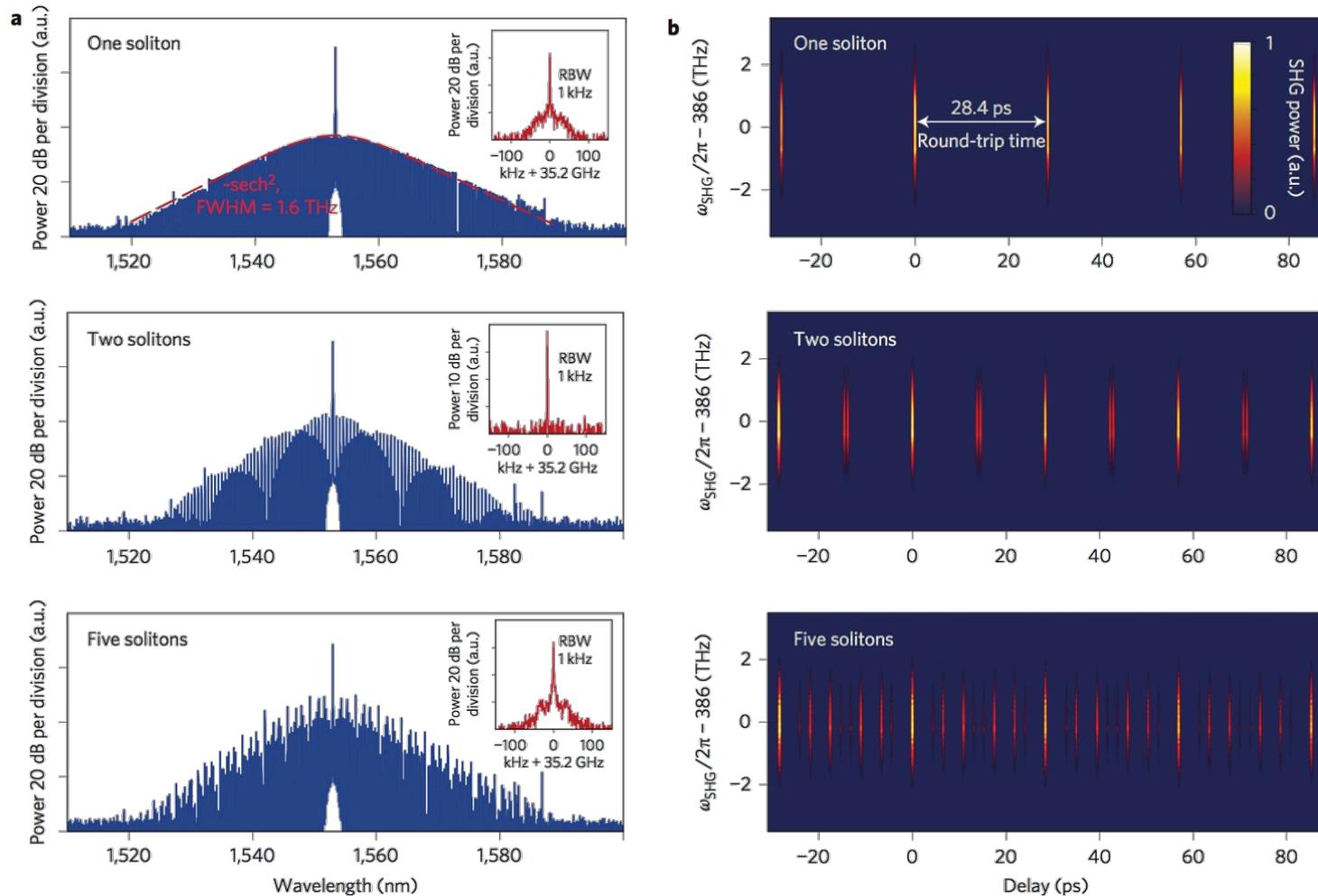
Lugiato-Lefever model



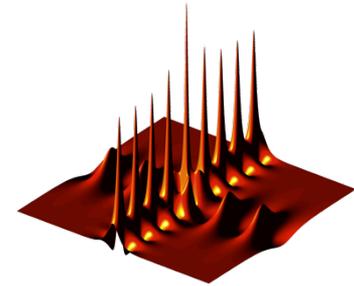
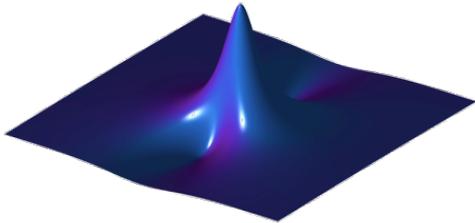
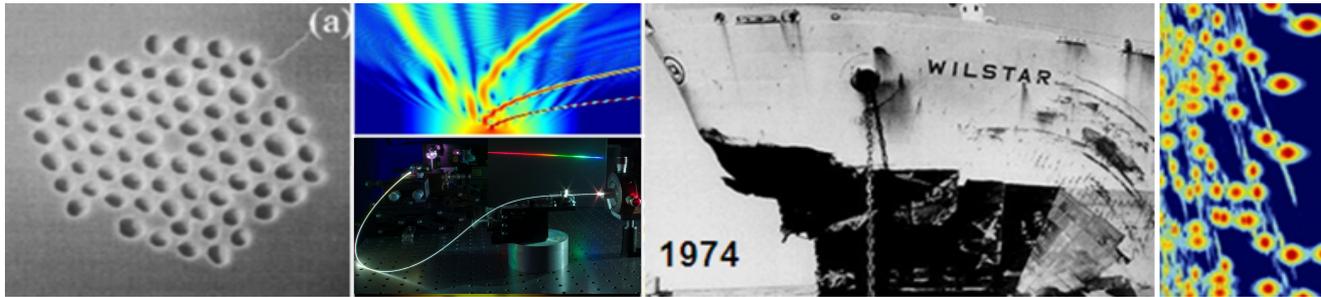
Soliton dynamics at the micron scale

- Soliton dynamics also in microresonator frequency combs

Herr et al., Nat. Photon. 8, 145–152 (2014)

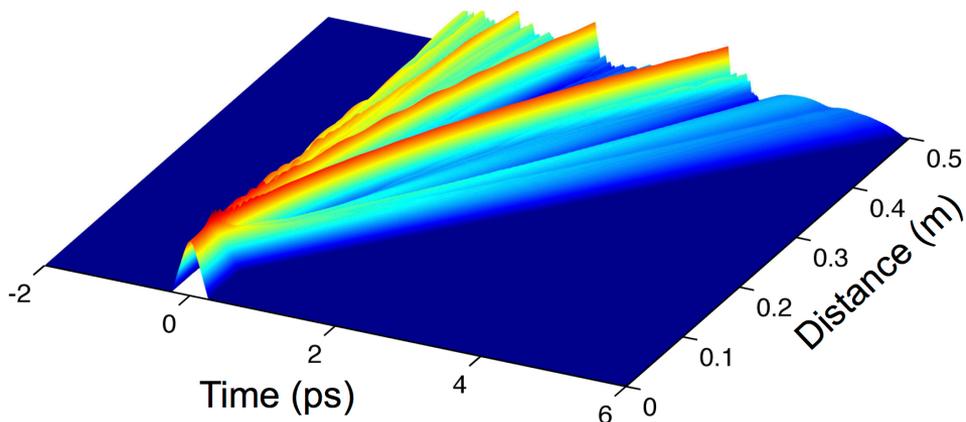


The long pulse regime: nonlinear instabilities et extreme events

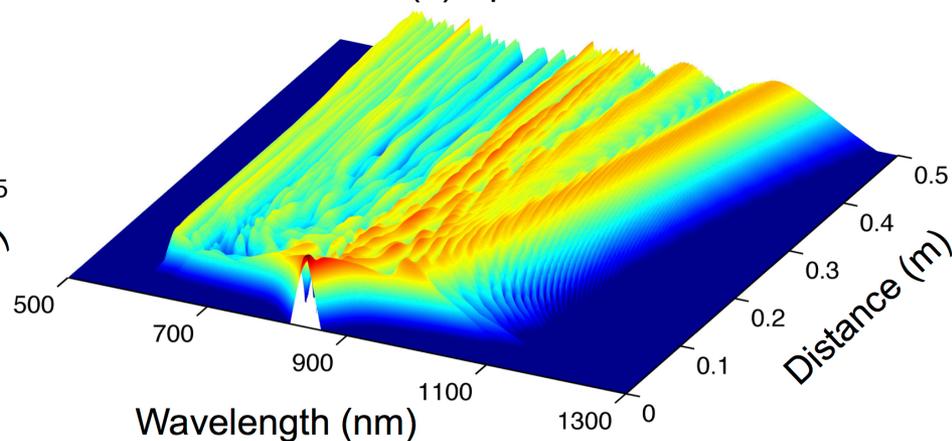


Dynamics are rather complex...

(a) Time Evolution



(b) Spectral Evolution



Short pulses

Long pulses

Anomalous

- Soliton
- Dispersive waves

- Modulation instability
- Solitons dynamics

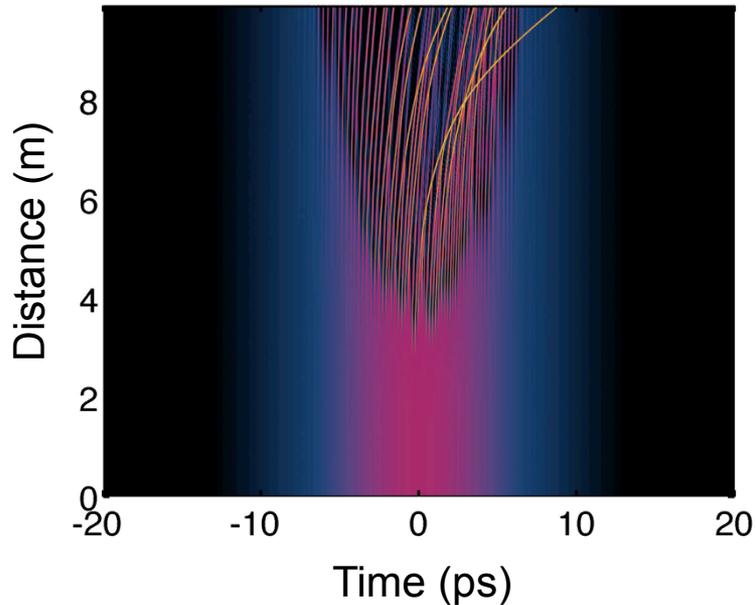
Normal

- Self-phase modulation
- Four-wave mixing

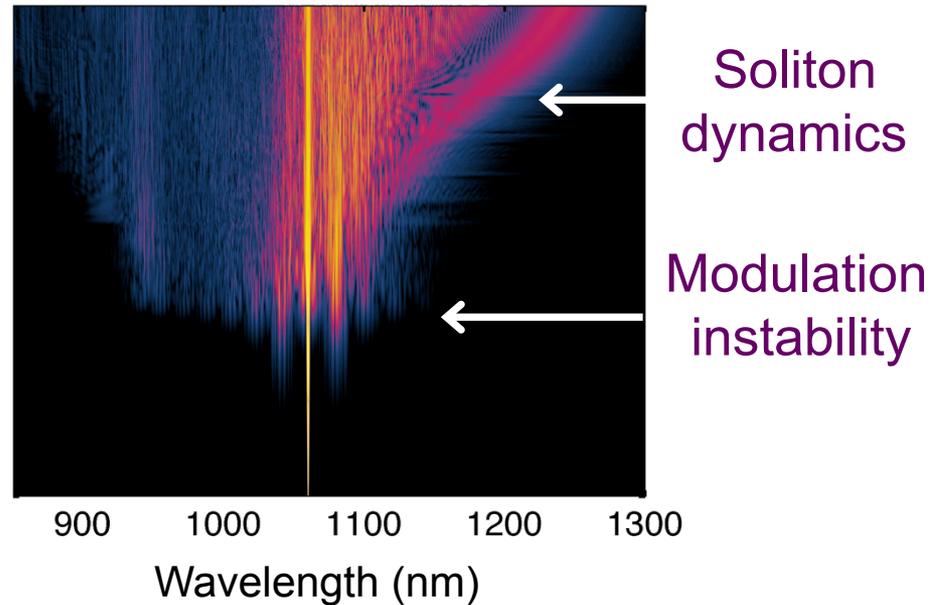
- Raman scattering
- Four-wave mixing

SC generation with long pulses

Time Evolution



Spectral Evolution

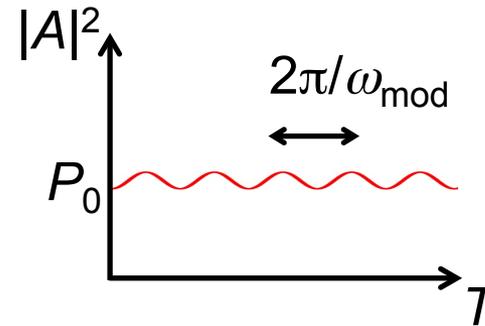


- Modulation instability is seeded by noise (ASE, intensity-noise...etc)

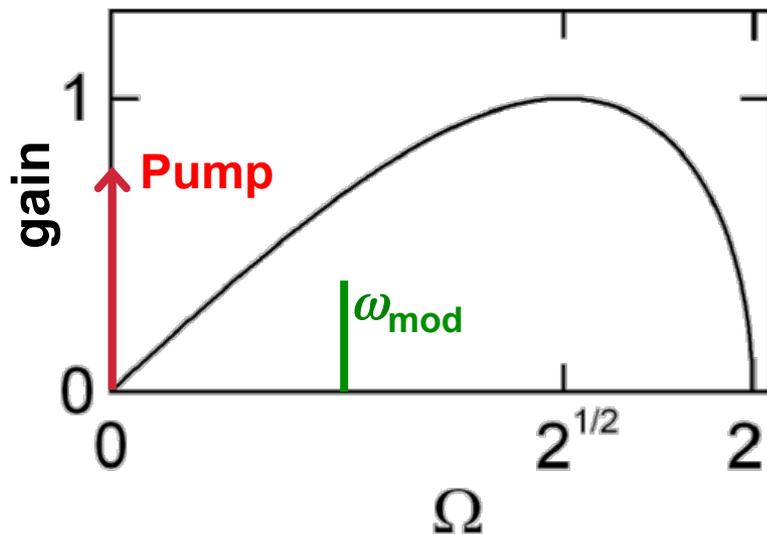
Modulation instability

- Single frequency excitation

$$A(T) = \sqrt{P_0} [1 + \delta \cos(\omega T)]$$



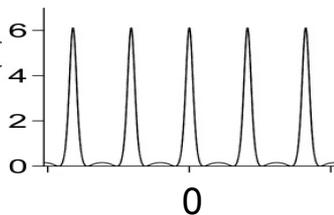
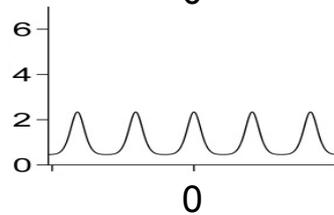
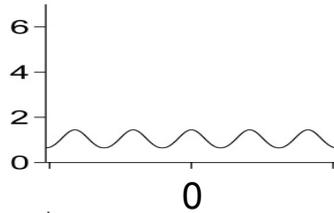
- Gain for a range of frequencies



Modulation instability

0
z ↓

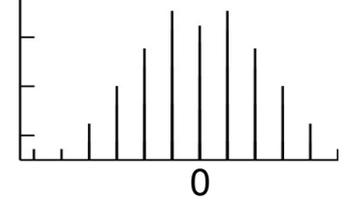
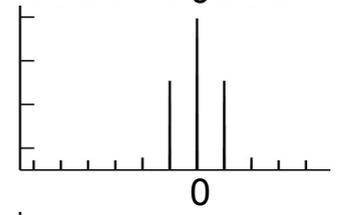
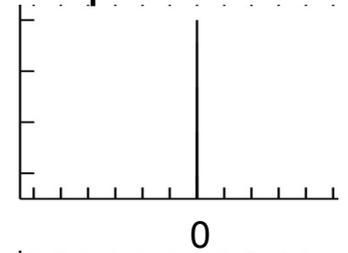
Time



t

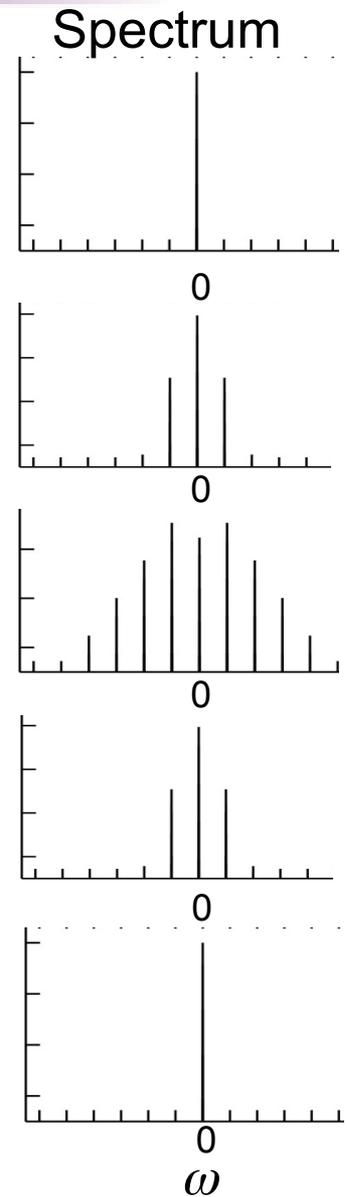
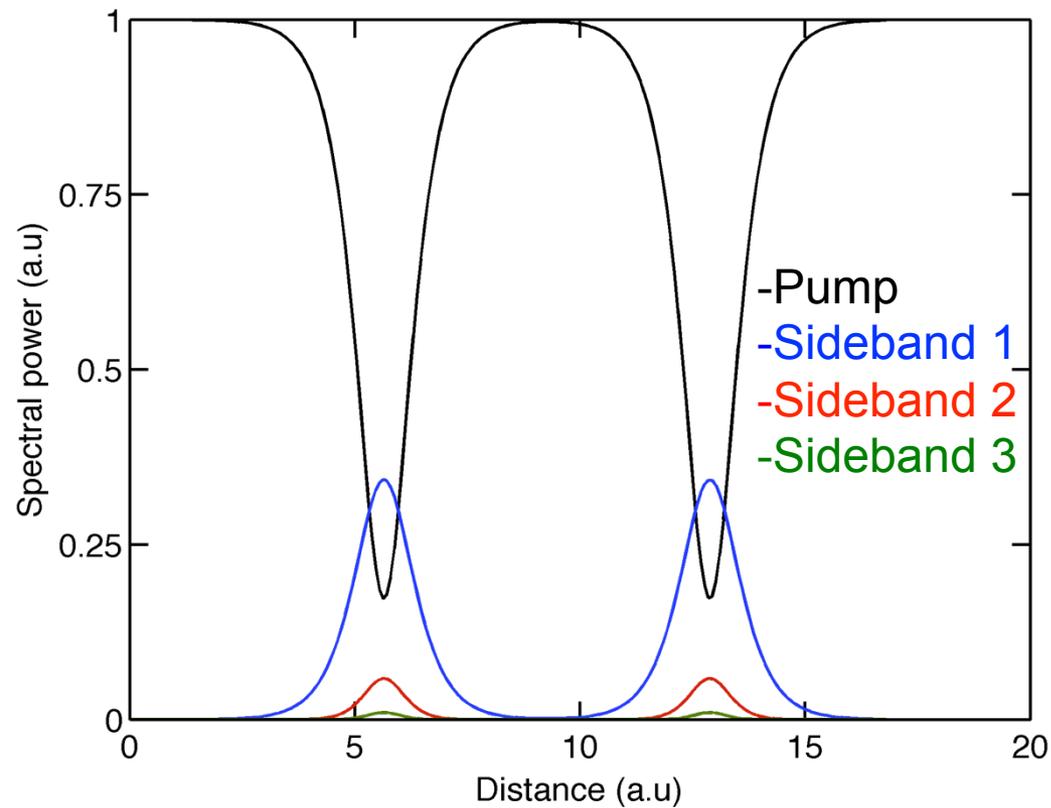
Modulation grows exponentially

Spectrum



ω

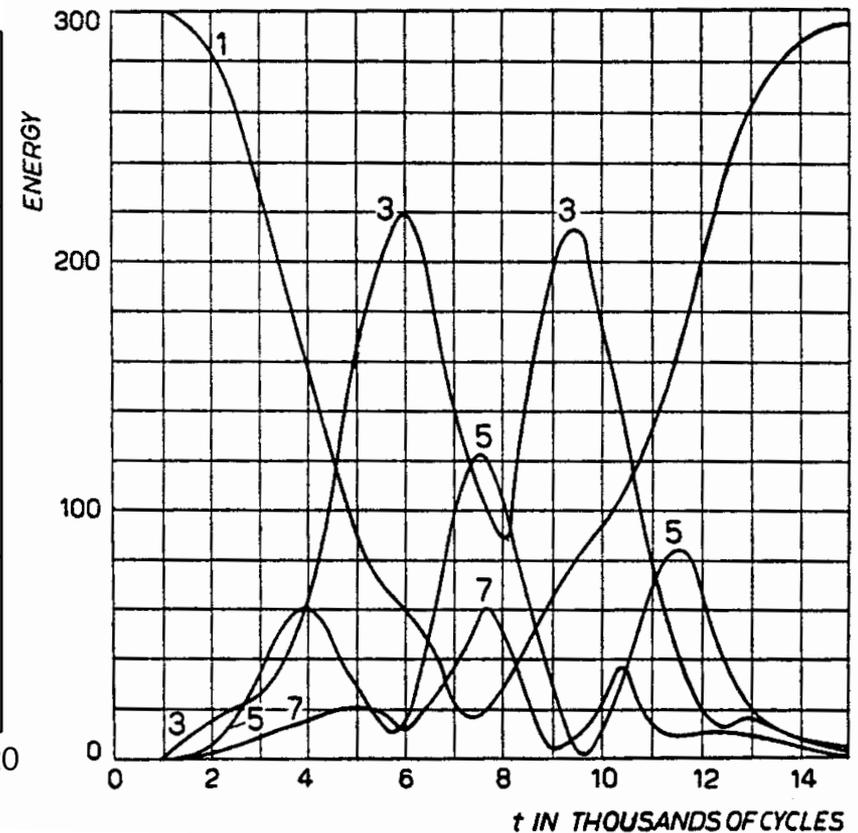
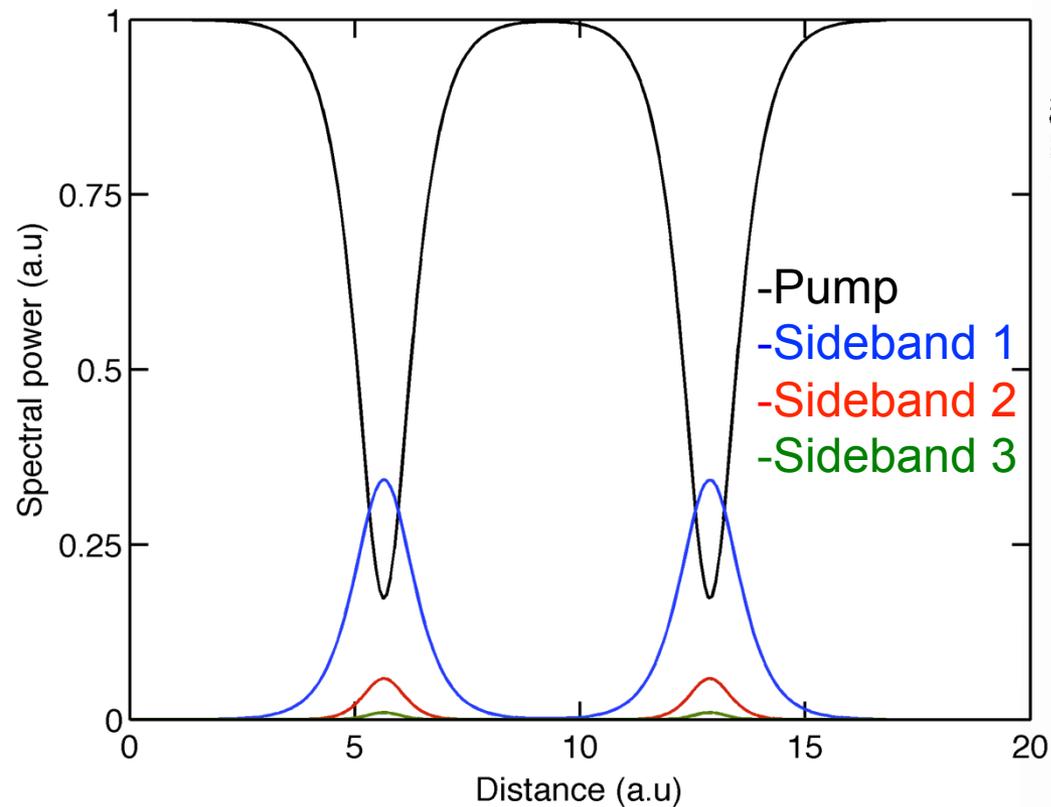
FPU recurrence



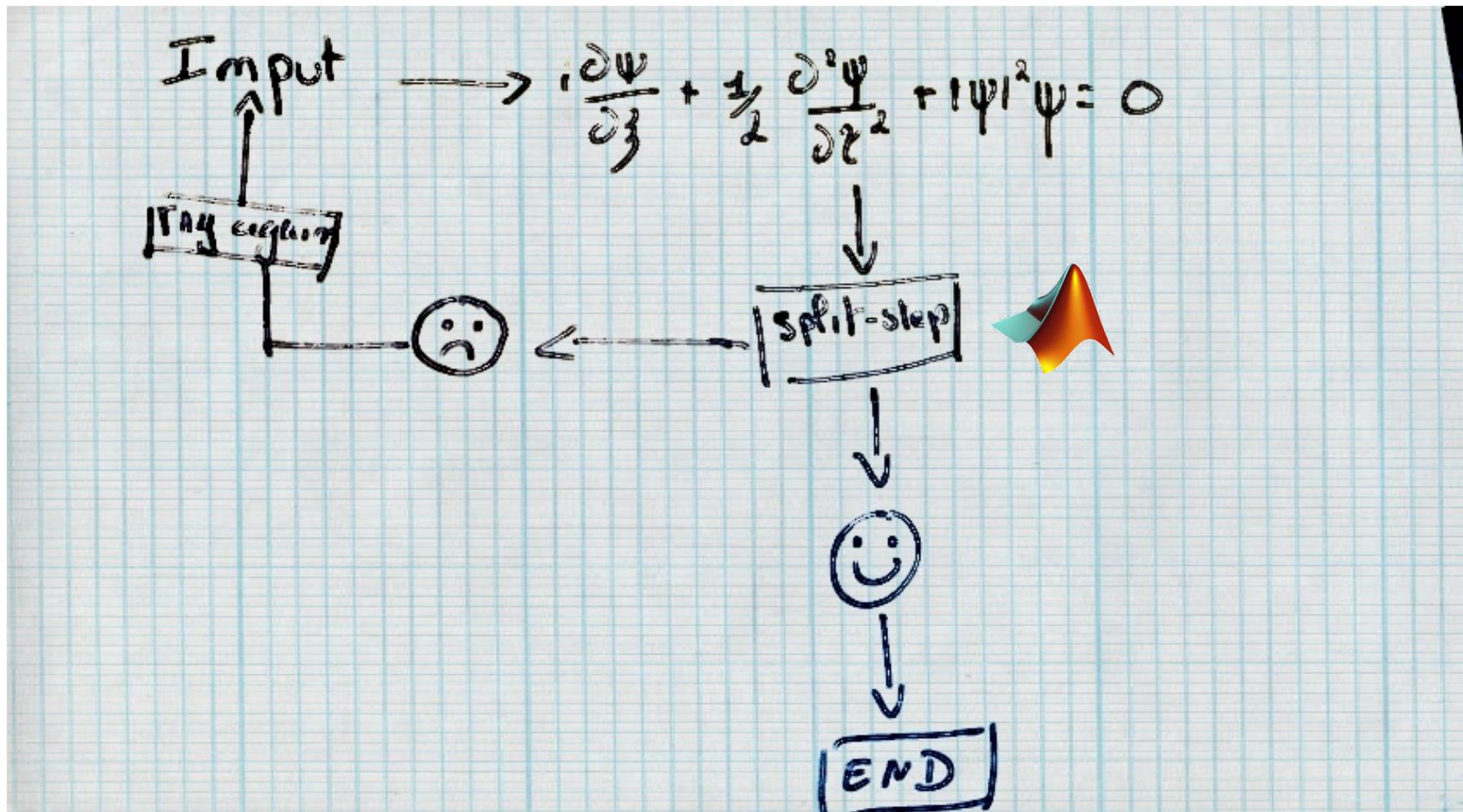
FPU recurrence

STUDIES OF NON LINEAR PROBLEMS

E. FERMI, J. PASTA, and S. ULAM
Document LA-1940 (May 1955).

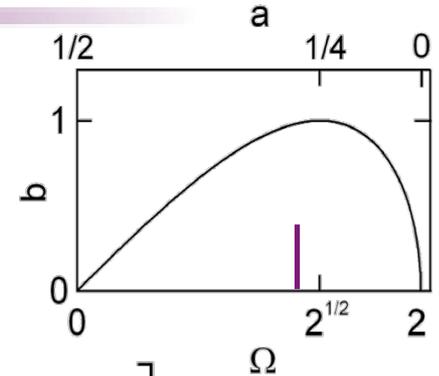


Luckily, it's more simple these days...



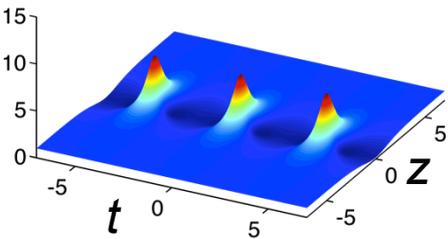
Solitons on finite background (SFBs)

- **Highly localized** solutions of the NLS
- **Single-parameter** family of solutions



$$\psi(\xi, \tau) = e^{i\xi} \left[\frac{(1 - 4a) \cosh(b\xi) + ib \sinh(b\xi) + \sqrt{2a} \cos(\Omega\tau)}{\sqrt{2a} \cos(\Omega\tau) - \cosh(b\xi)} \right]$$

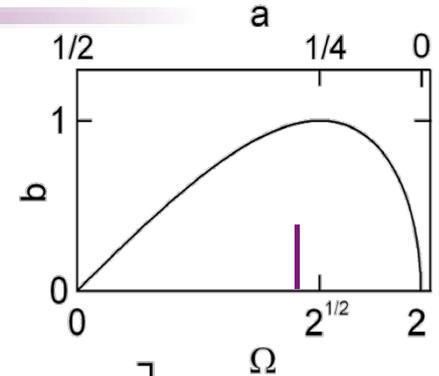
$$0 < a < 1/2$$



Akhmediev
breather (AB)

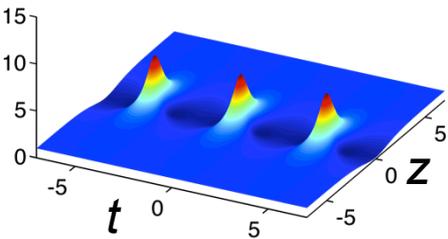
Solitons on finite background (SFBs)

- **Highly localized** solutions of the NLS
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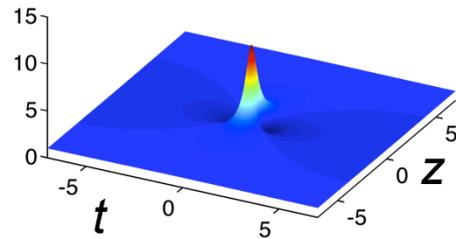
$$\psi(\xi, \tau) = e^{i\xi} \left[\frac{(1 - 4a) \cosh(b\xi) + ib \sinh(b\xi) + \sqrt{2a} \cos(\Omega\tau)}{\sqrt{2a} \cos(\Omega\tau) - \cosh(b\xi)} \right]$$

$0 < a < 1/2$



Akhmediev
breather (AB)

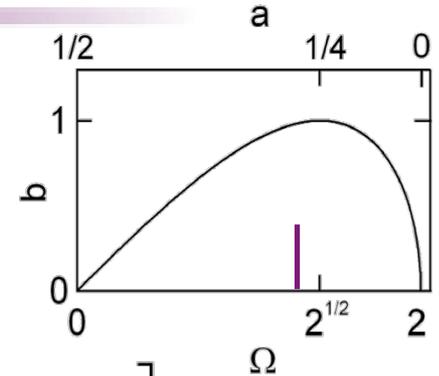
$a = 1/2$



Peregrine
soliton (PS)

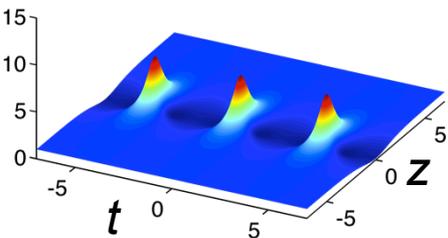
Solitons on finite background (SFBs)

- **Highly localized** solutions of the NLS
- **Single-parameter** family of solutions



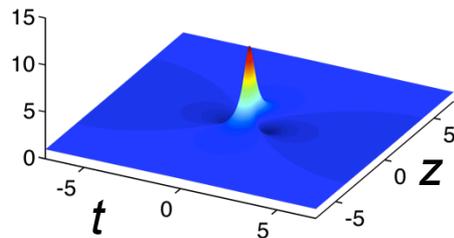
$$\psi(\xi, \tau) = e^{i\xi} \left[\frac{(1 - 4a) \cosh(b\xi) + ib \sinh(b\xi) + \sqrt{2a} \cos(\Omega\tau)}{\sqrt{2a} \cos(\Omega\tau) - \cosh(b\xi)} \right]$$

$0 < a < 1/2$



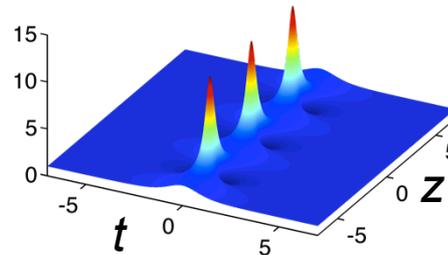
Akhmediev
breather (AB)

$a = 1/2$



Peregrine
soliton (PS)

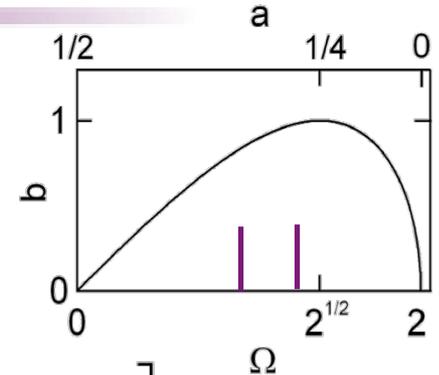
$1/2 < a$



Kuznetsov-Ma
soliton (KM)

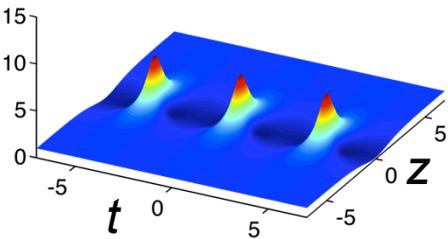
Solitons on finite background (SFBs)

- **Highly localized** solutions of the NLS
- **Single-parameter** family of solutions



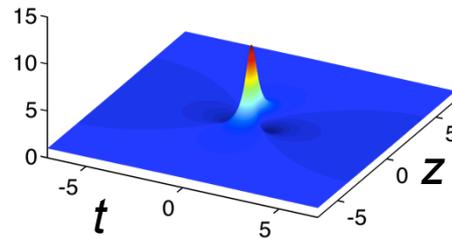
$$\psi(\xi, \tau) = e^{i\xi} \left[\frac{(1 - 4a) \cosh(b\xi) + ib \sinh(b\xi) + \sqrt{2a} \cos(\Omega\tau)}{\sqrt{2a} \cos(\Omega\tau) - \cosh(b\xi)} \right]$$

$0 < a < 1/2$



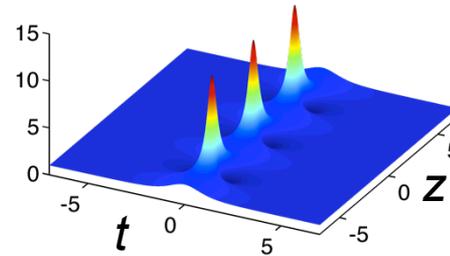
Akhmediev
breather (AB)

$a = 1/2$



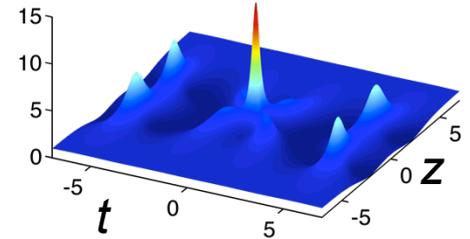
Peregrine
soliton (PS)

$1/2 < a$



Kuznetsov-Ma
soliton (KM)

$0 < a < 1/2$
 $0 < a' < 1/2$

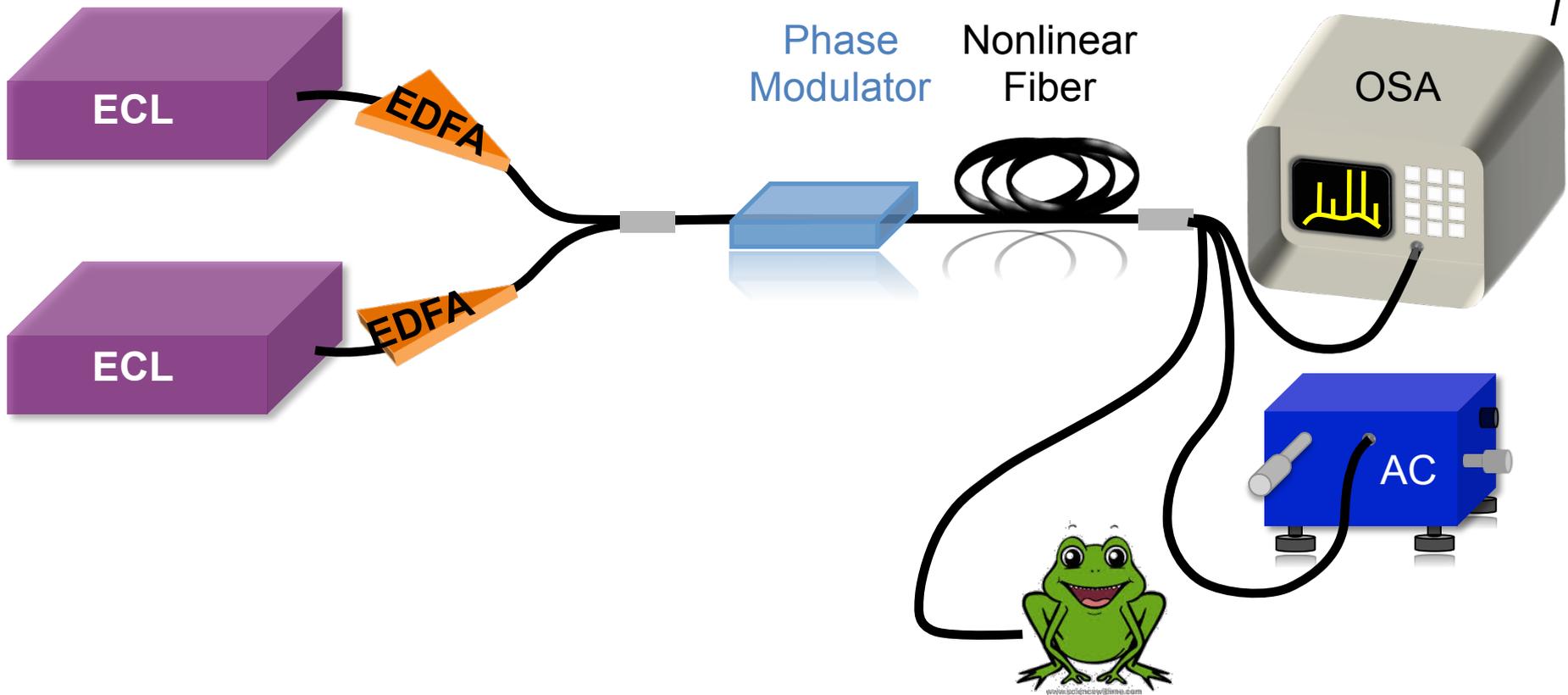
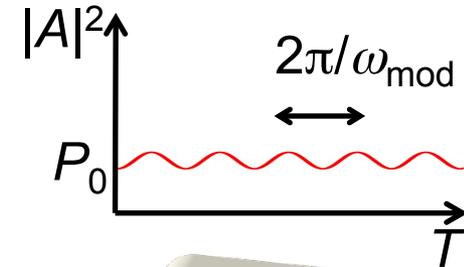


Higher-order
AB

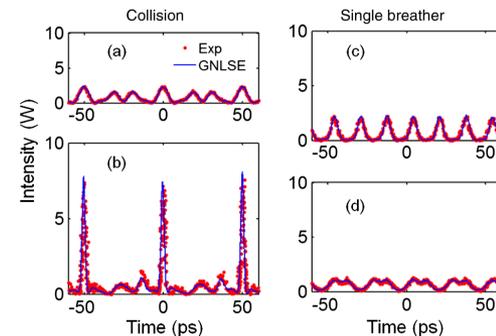
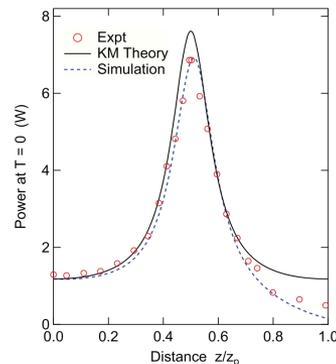
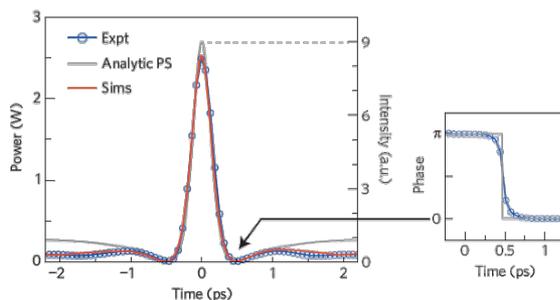
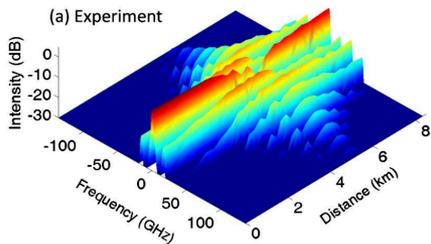
Observing experimentally SFBs

- Use of multi-frequency CW fields to generate a modulated signal

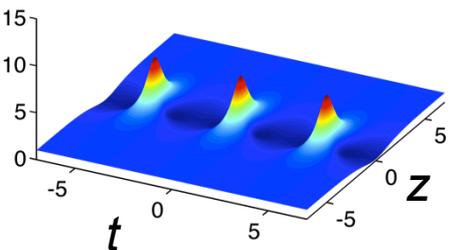
$$A(z = z_0, T) = \sqrt{P_0} [1 + \alpha_{\text{mod}} \exp(i\omega_{\text{mod}} T)]$$



Observing experimentally SFBs



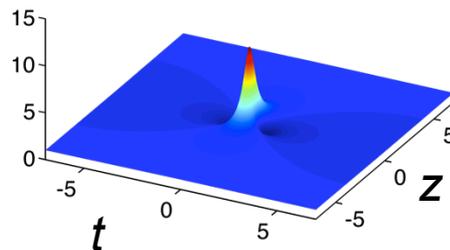
$$0 < a < 1/2$$



Akhmediev breather (AB)

Hammani *et al.*,
OL (2011)

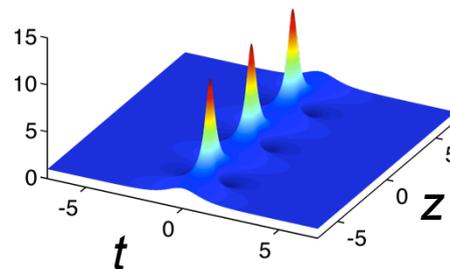
$$a = 1/2$$



Peregrine soliton (PS)

Kibler *et al.*,
Nat. Phys. (2010)

$$1/2 < a$$

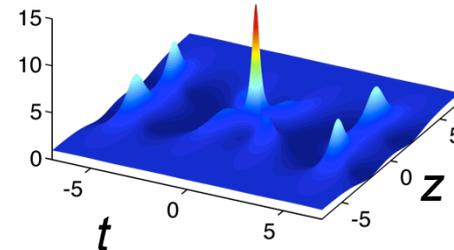


Kuznetsov-Ma soliton (KM)

Kibler *et al.*,
Sci. Rep. (2012)

$$0 < a < 1/2$$

$$0 < a' < 1/2$$



Higher-order breathers

Frisquet *et al.*,
PRX (2013)

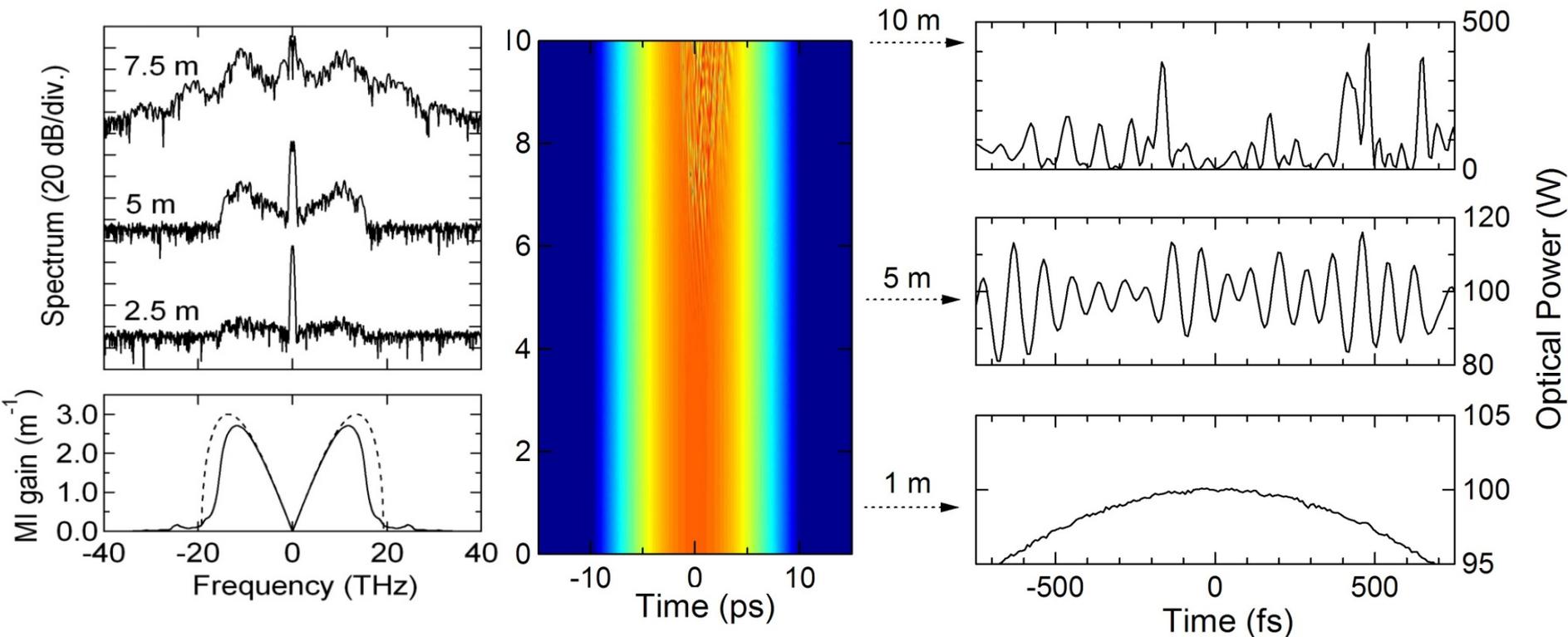
Initial stage of supercontinuum with long pulses

- Modulation instability seeded by noise

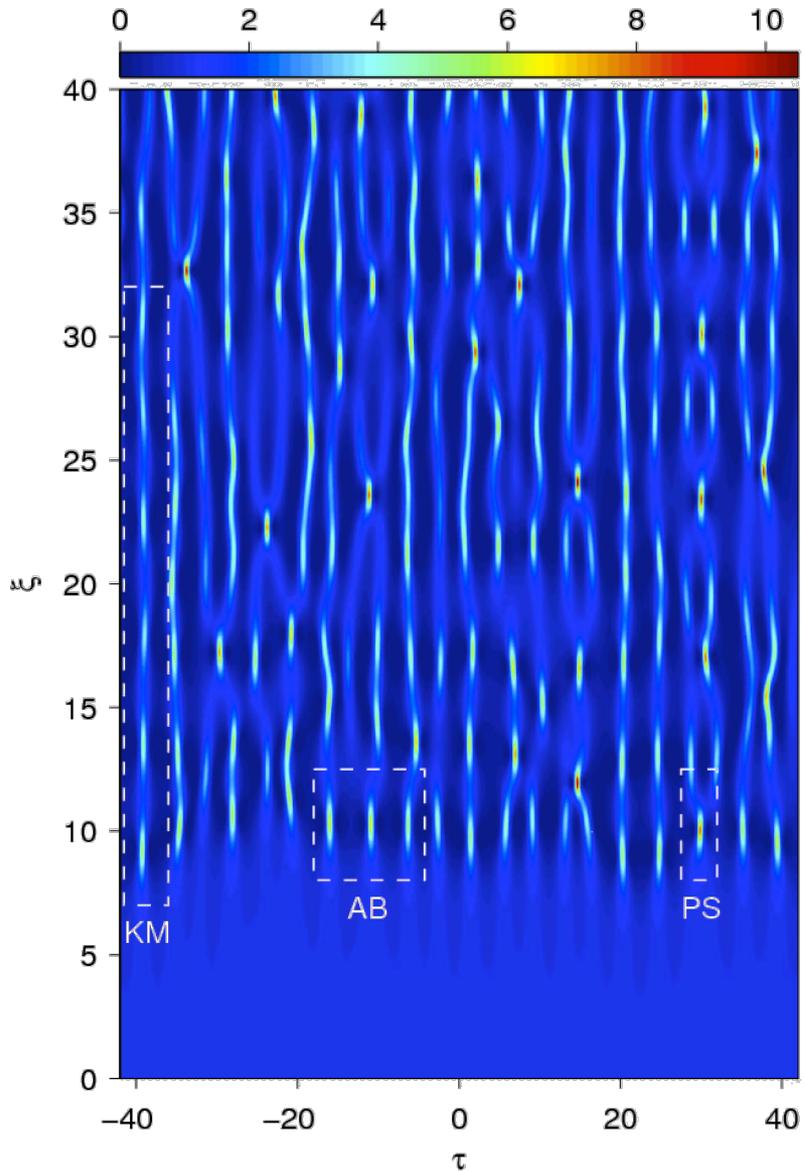
$$i \frac{\partial A}{\partial z} + \frac{|\beta_2|}{2} \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0$$

GVD

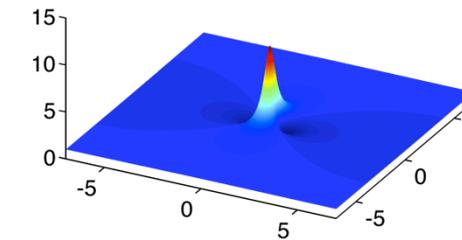
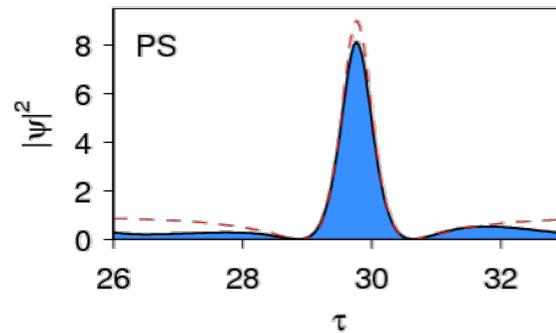
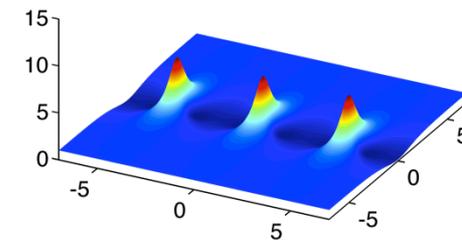
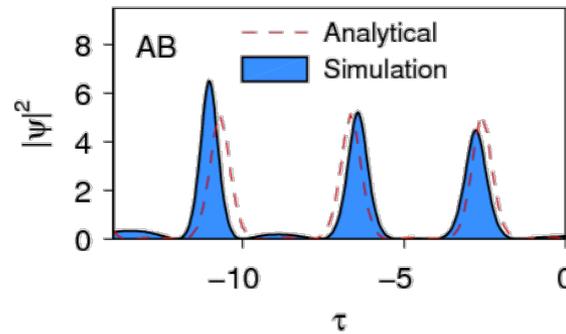
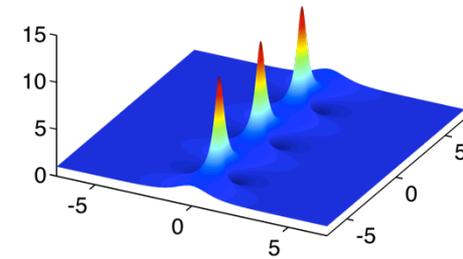
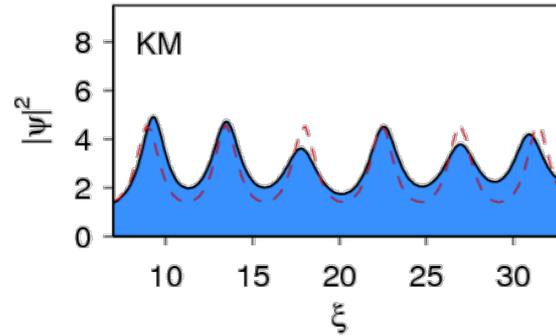
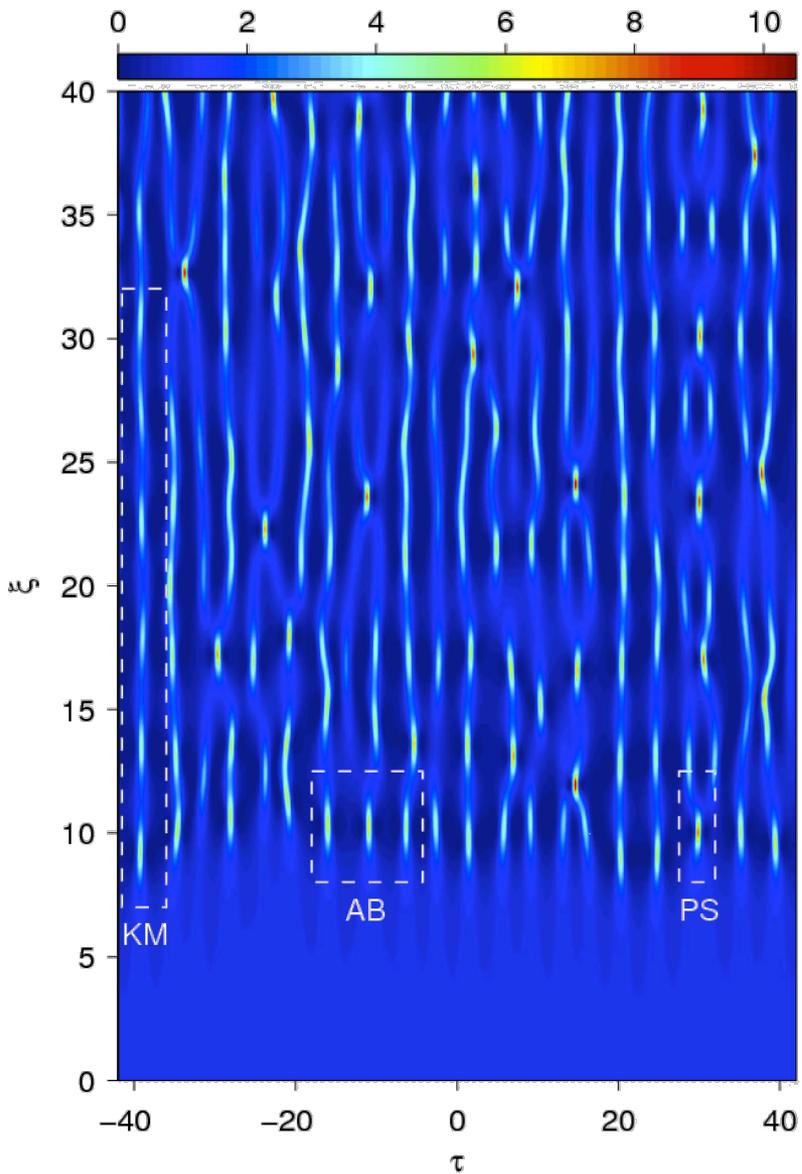
Kerr effect



Signatures of SFBs in noise-seeded MI



Signatures of SFBs in noise-seeded MI



Ocean rogue waves

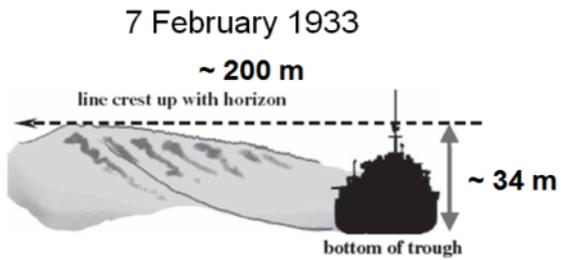
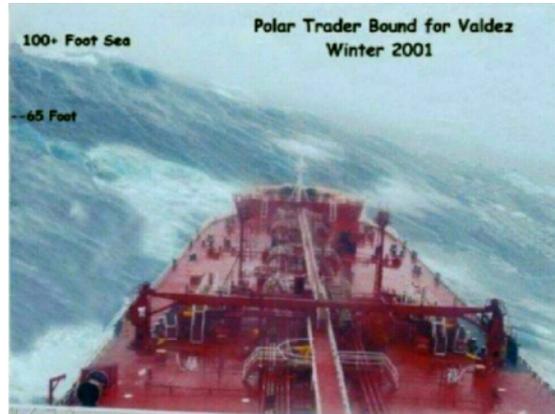
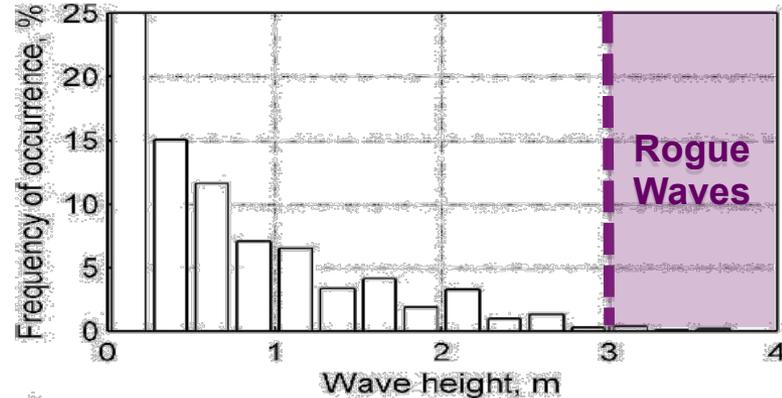
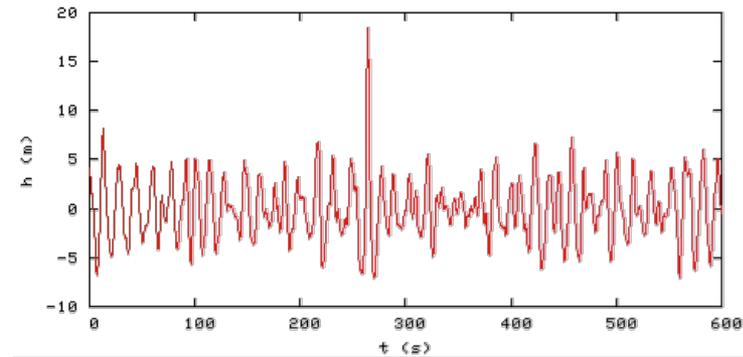


Fig. I.2 Observation of the highest reported wave by the crew members of "Ramapo" (Dennis and Wolff 1996)



Ocean rogue waves

- Rogue waves: statistically-rare wave height



- Possible mechanisms

- Directional focusing
- Random superposition of independent wave trains

Linear

- Amplification of surface noise
- Localized wave structures

Nonlinear

Linear vs. nonlinear waves

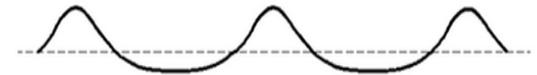
Linear waves

speed is independent of the amplitude



Nonlinear waves

speed depends on the amplitude



Linear



Nonlinear

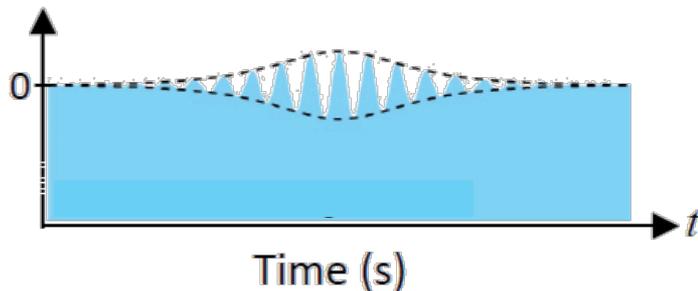
Where is the analogy with hydrodynamics?

- Initial stage: field is narrowband \longrightarrow NLS regime

(a) Deep water wave group envelope

$$i \frac{\partial u}{\partial z} + \frac{1}{g} \frac{\partial^2 u}{\partial t^2} + \frac{\omega_0^6}{g} \frac{\partial^2 u}{\partial t^2} = 0$$

GVD Nonlinearity

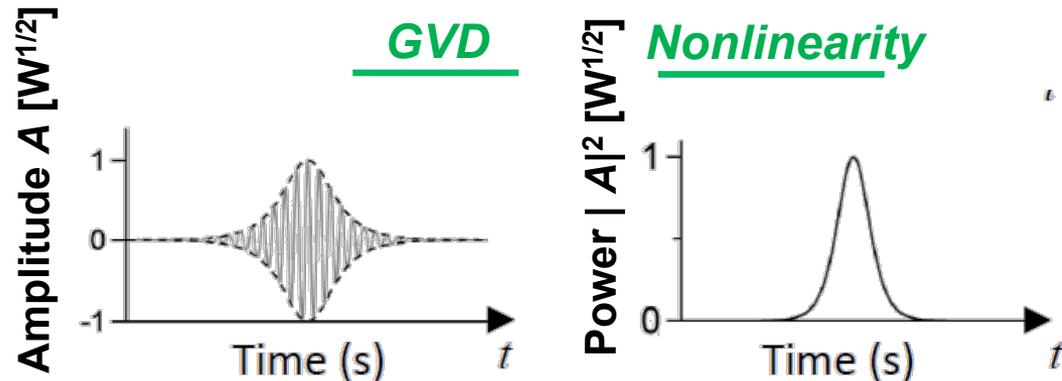


- Envelope modulating the amplitude of a group of free surface waves

(b) Light pulse envelope in fibre

$$i \frac{\partial A}{\partial z} + \frac{1}{2} |\beta_2| \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0$$

GVD Nonlinearity

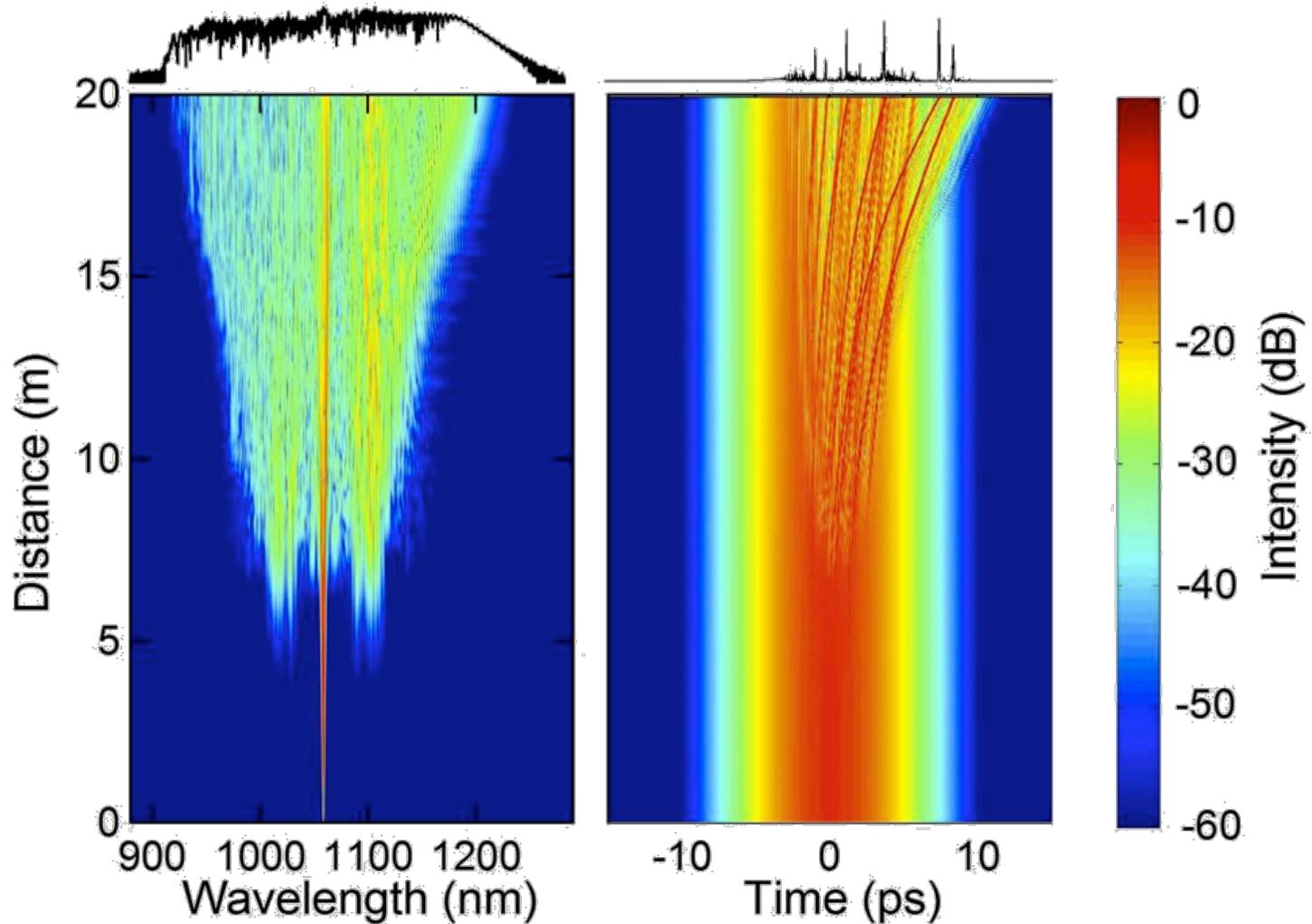


- Envelope modulating the amplitude of a large number of cycles of the electric field

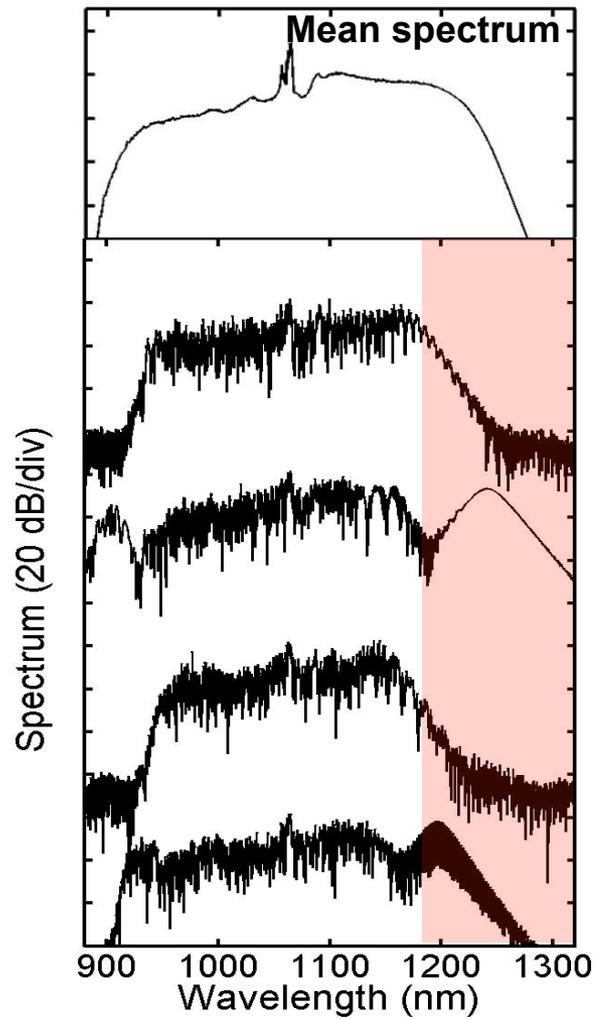
Both cases: anomalous dispersion and self-focussing

Long term soliton dynamics

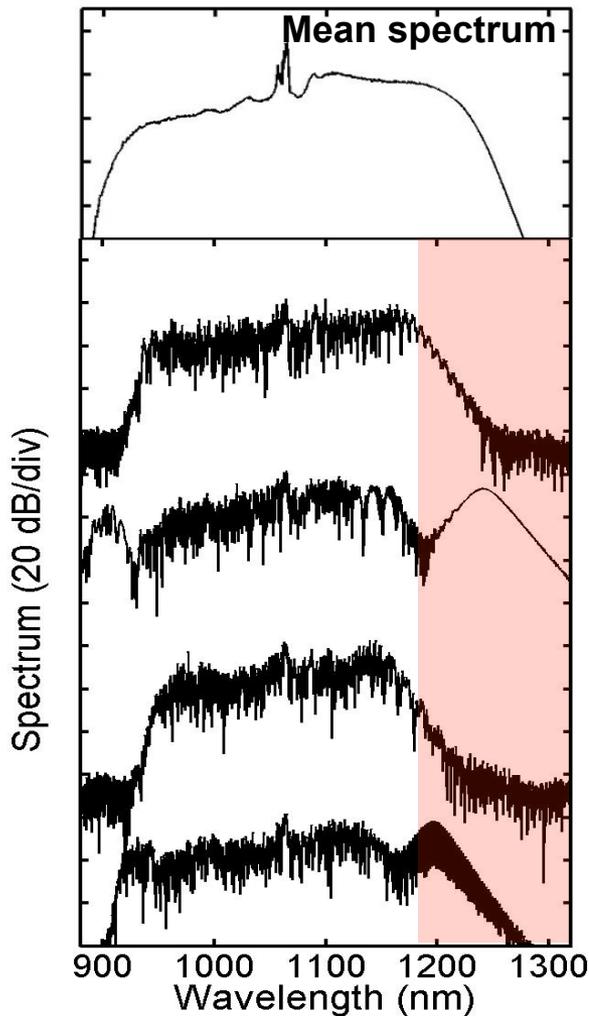
- “turbulent solitons gas”



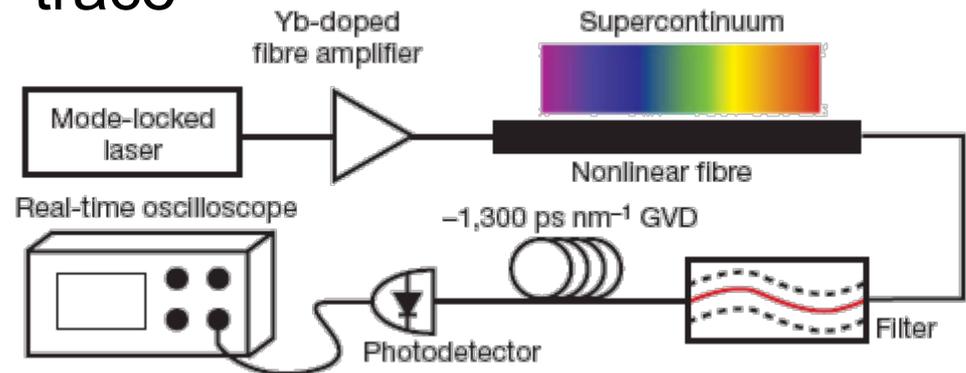
Shot-to-shot fluctuations



Shot-to-shot fluctuations



- Filtering selects long wavelength edge
- Dispersive fiber stretches the time trace



nature

Vol 450 | 13 December 2007 | doi:10.1038/nature06402

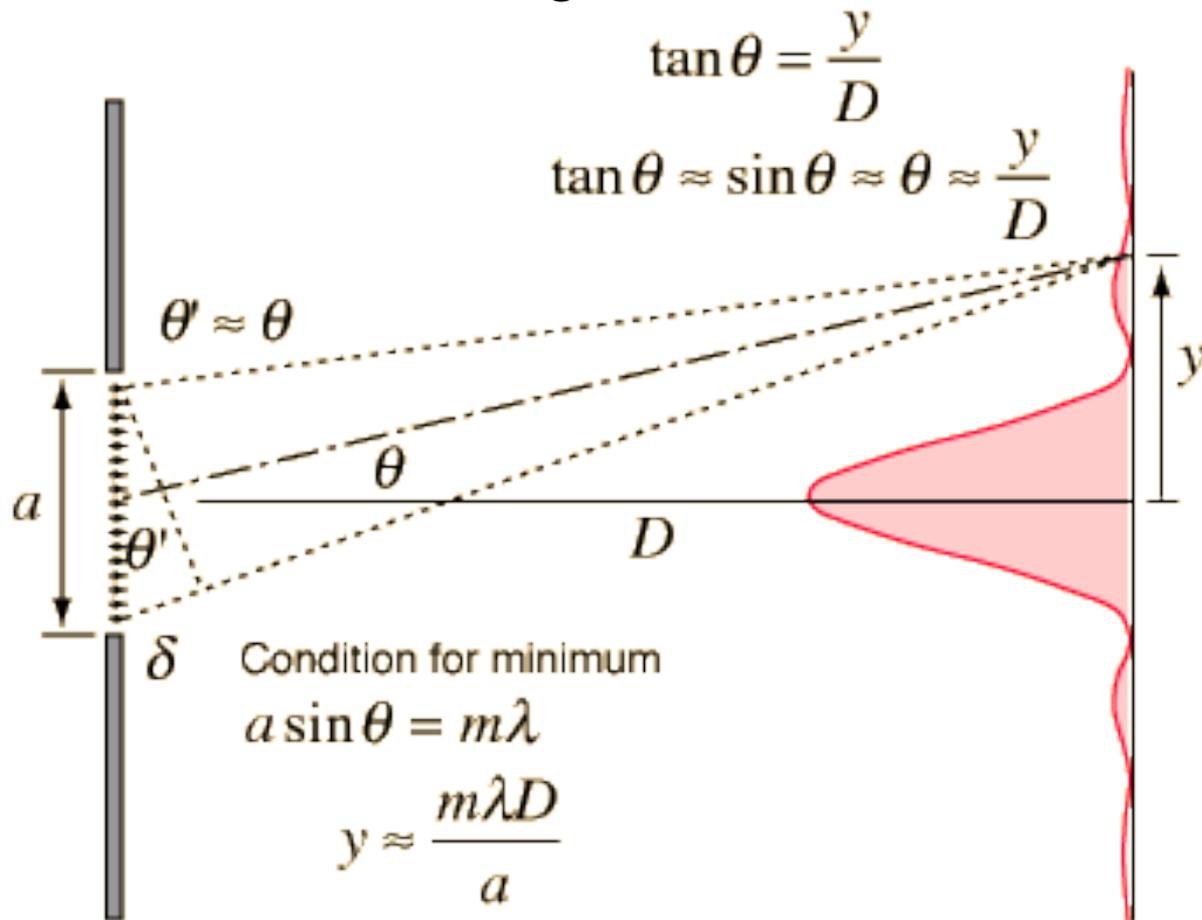
LETTERS

Optical rogue waves

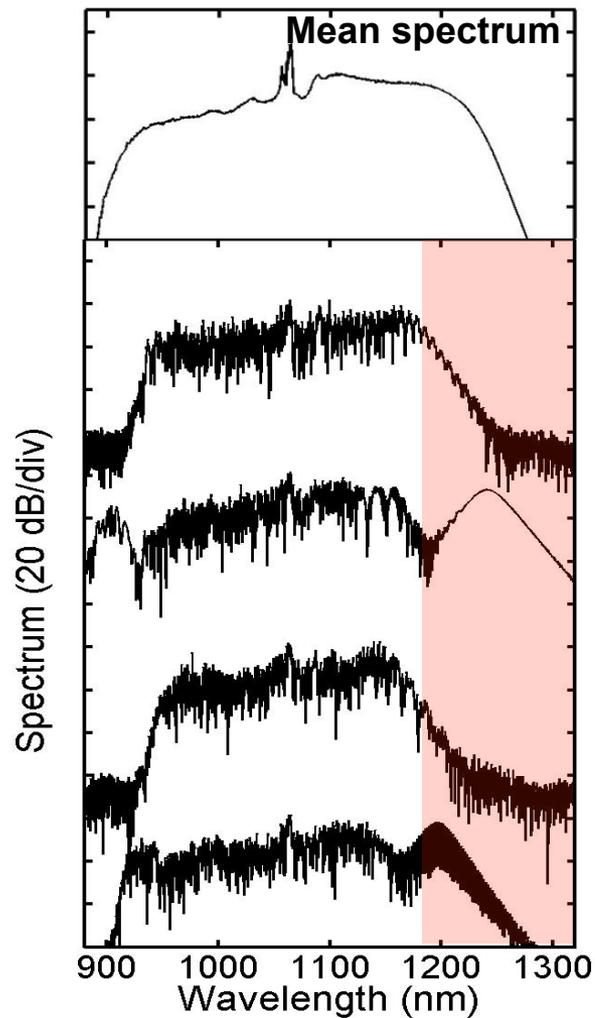
D. R. Solli¹, C. Ropers^{1,2}, P. Koonath¹ & B. Jalali¹

Space-time duality

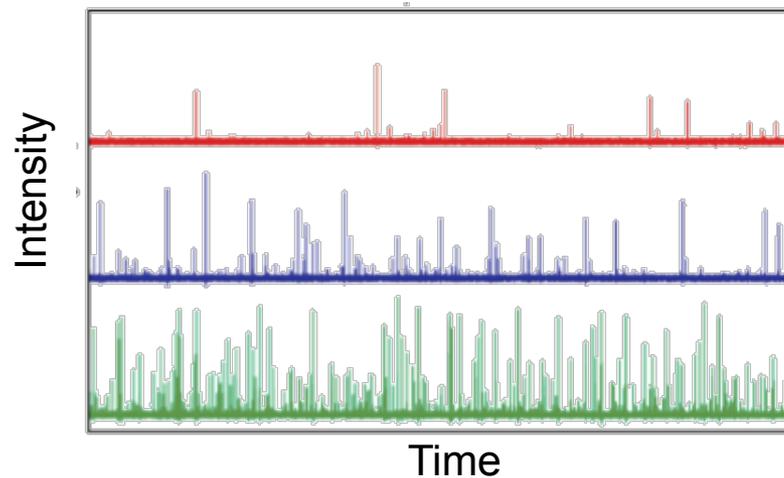
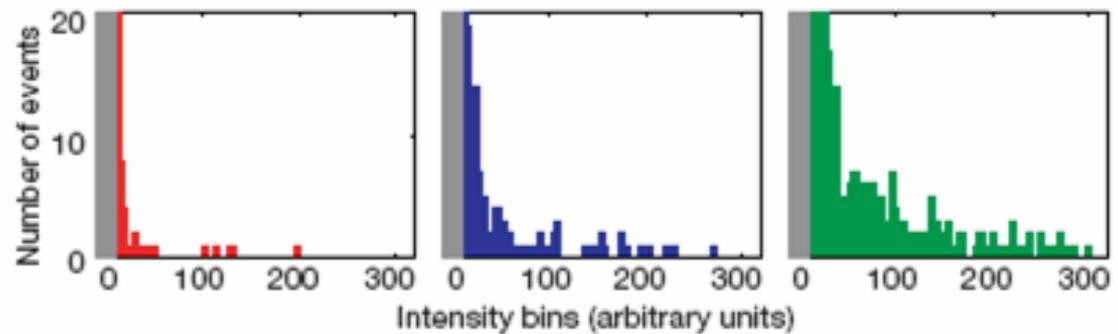
- Field diffraction pattern of an aperture is the Fourier transform of the diffracting mask



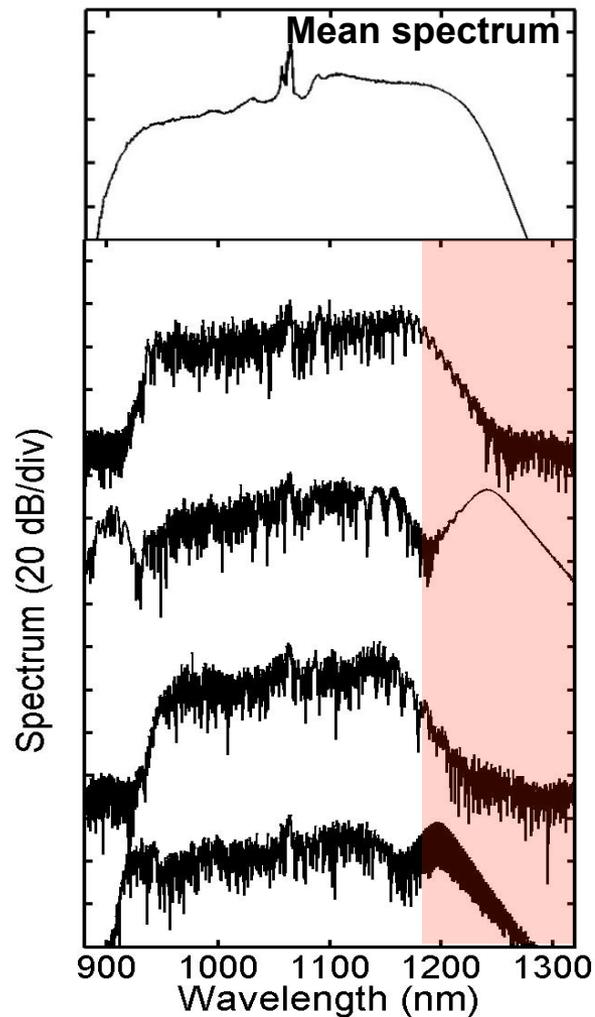
Shot-to-shot fluctuations statistics



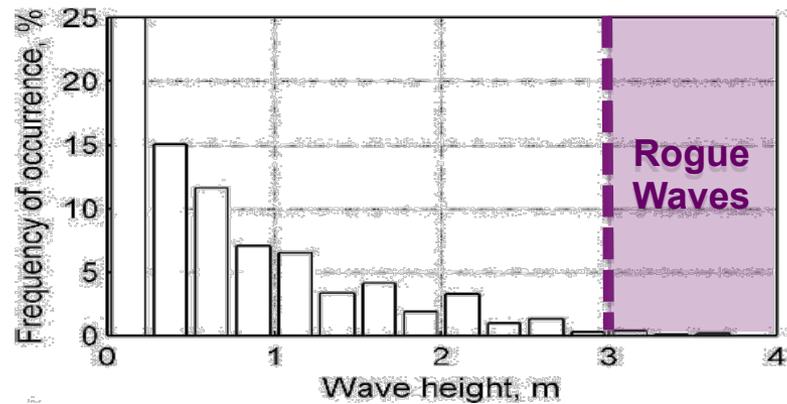
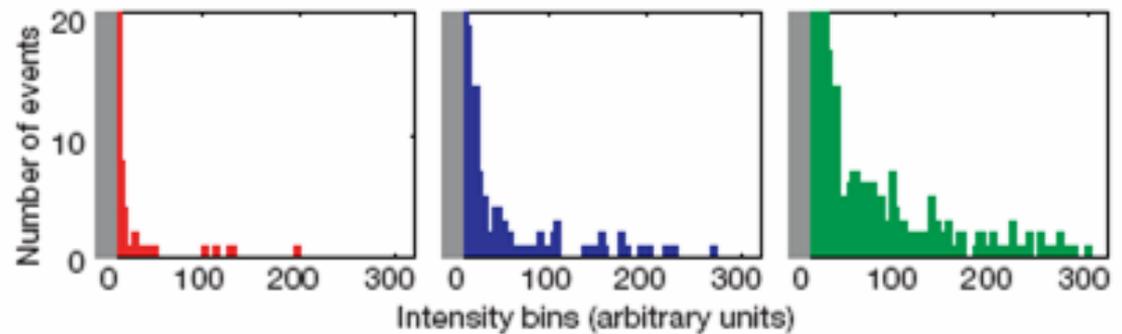
- Long tail statistics at the SC edge



Shot-to-shot fluctuations statistics

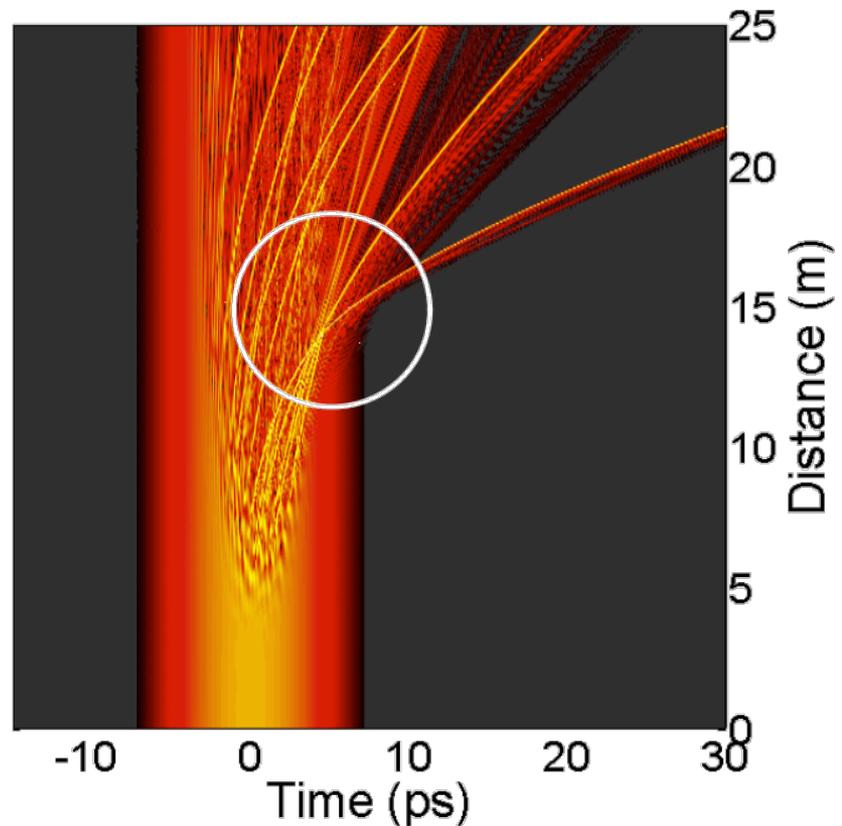
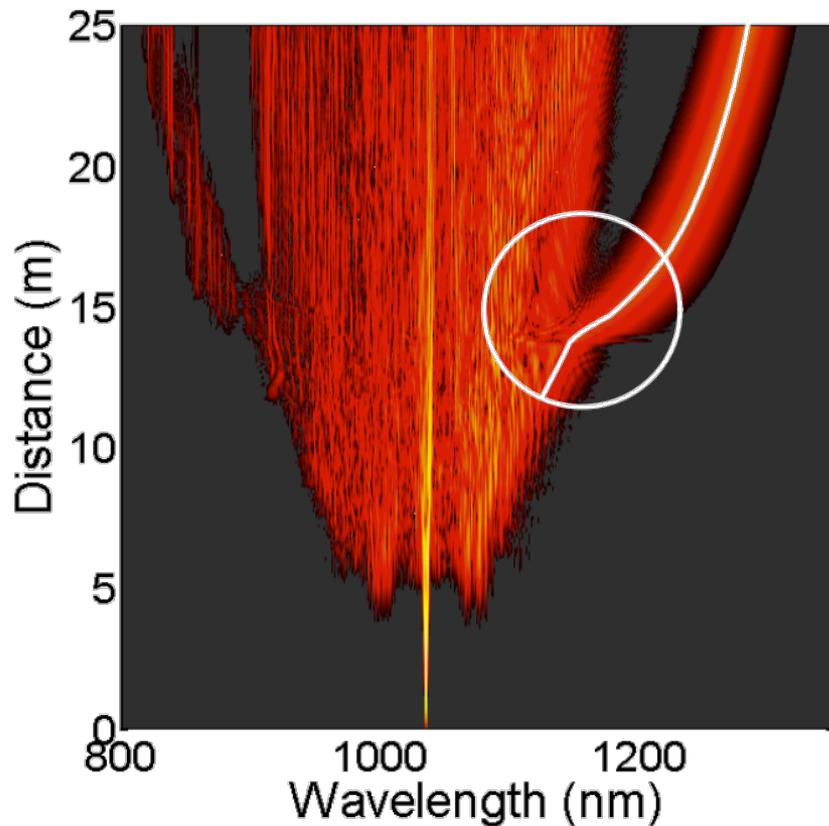


- Analogy with rogue waves?



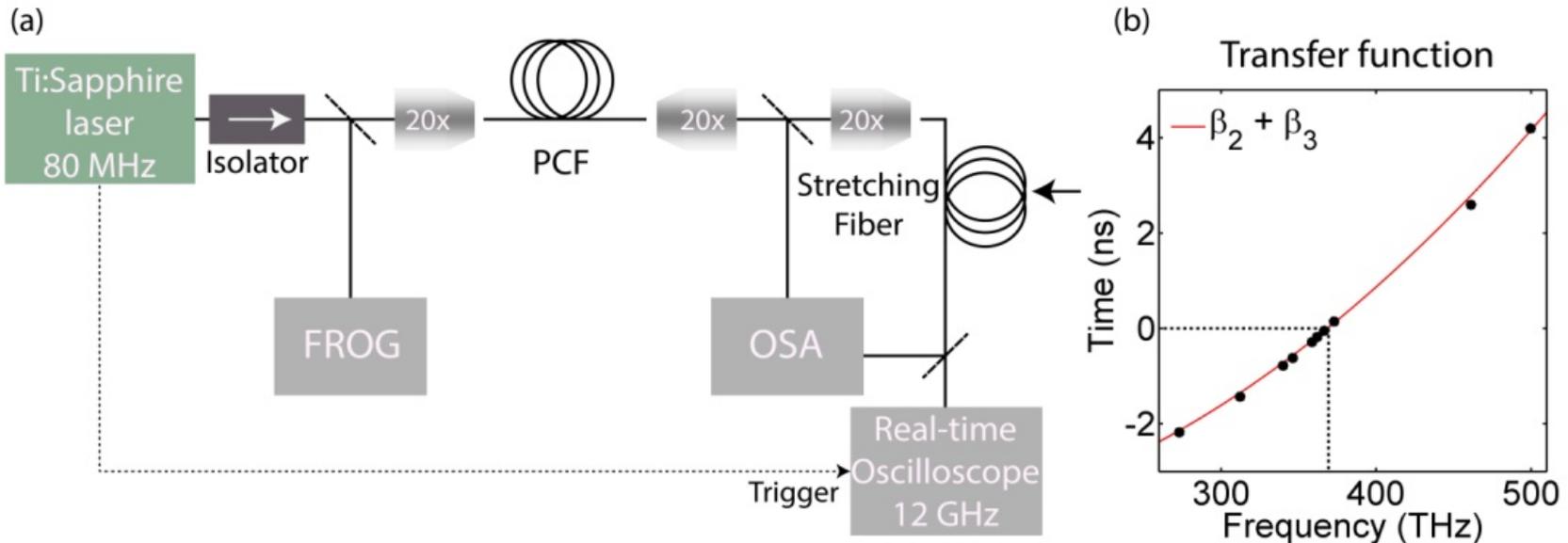
What causes rogue solitons?

- Collisions between solitons lead to energy exchange and enhanced redshift



Capturing single shot spectra

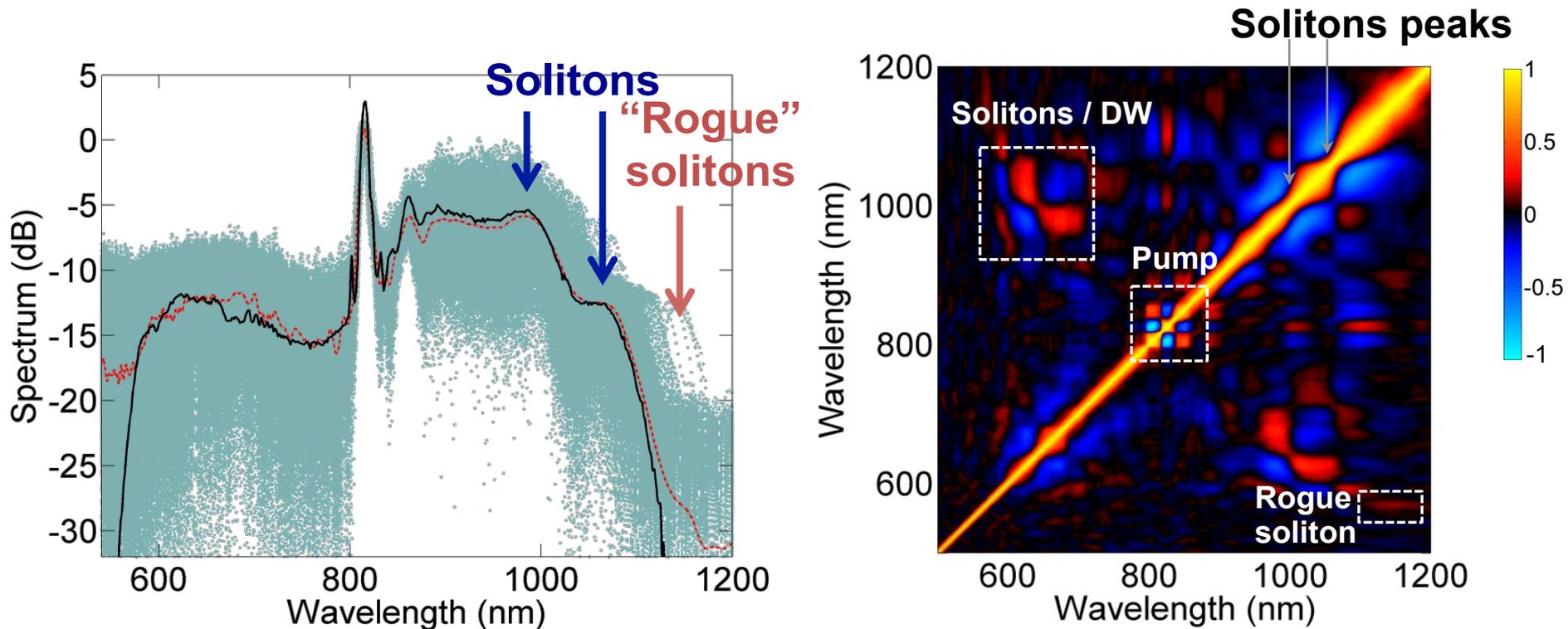
- Real-time measurement of single shot spectra is possible
- Use dispersive time-to-frequency transformation



- At large distance in the dispersive fiber, the stretched temporal trace corresponds to the spectrum: analog to far-field diffraction

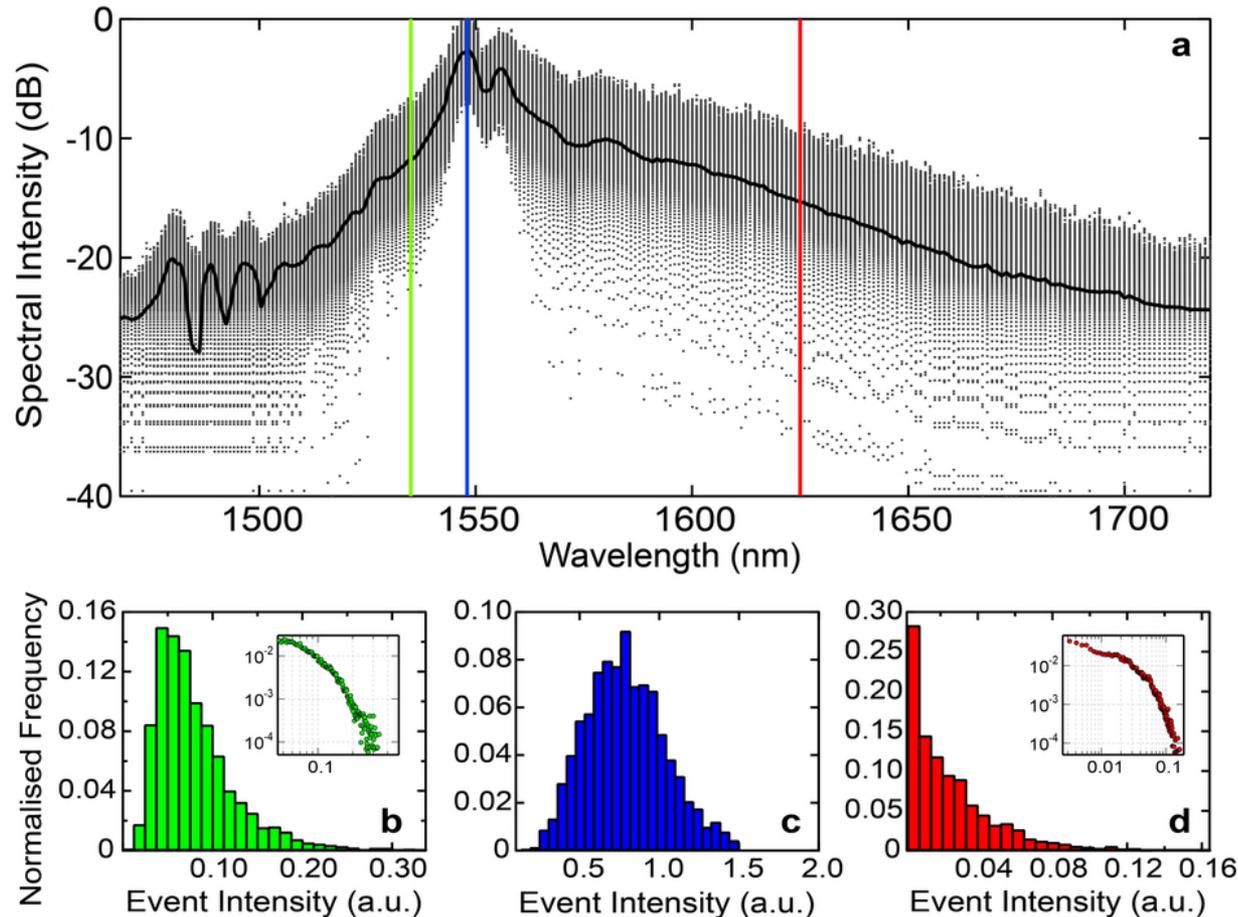
Spectral fluctuations

- Octave-spanning fluctuations can be measured



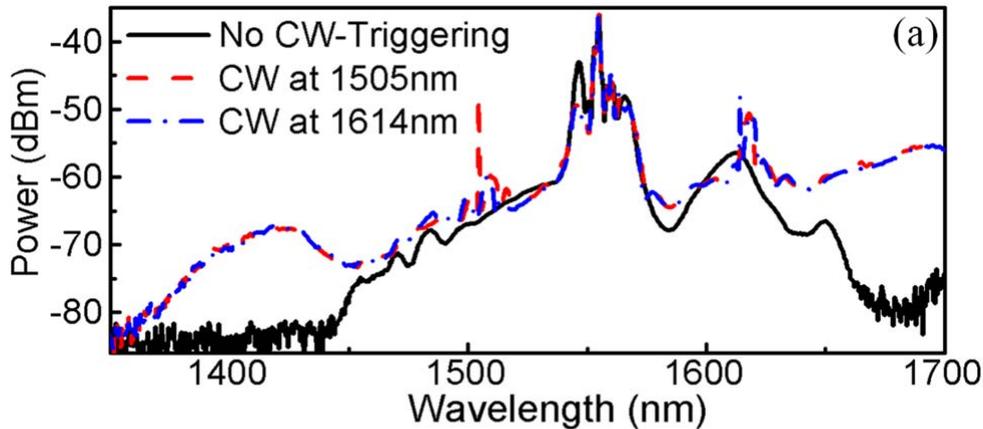
Capturing single shot spectra

- Direct access to spectral fluctuations at ALL wavelengths

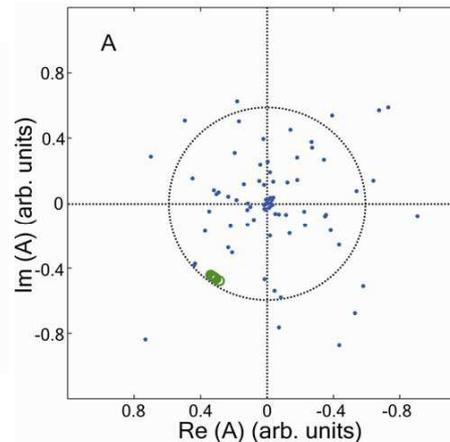
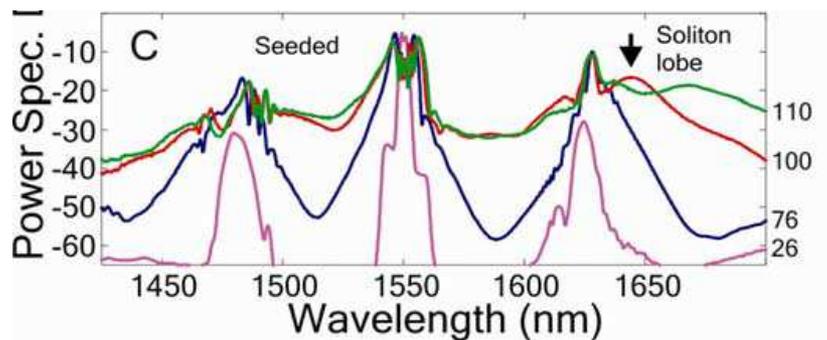


Controlling SC fluctuations and rogue solitons

- Stable SC from long pump pulses
 - Seeding modulation instability



Cheung et al., OL 36,
160-162 (2011)

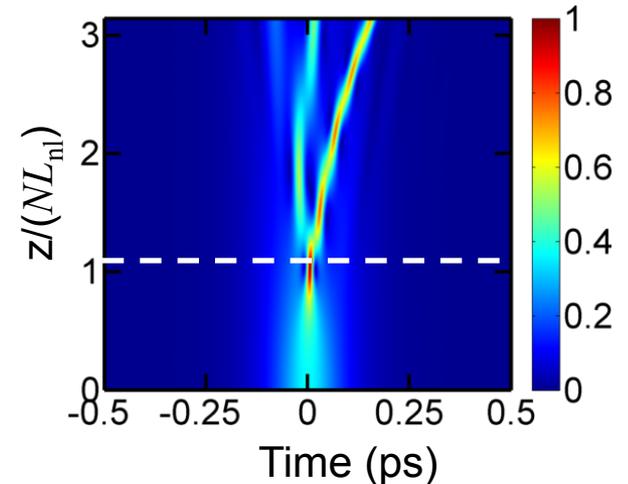
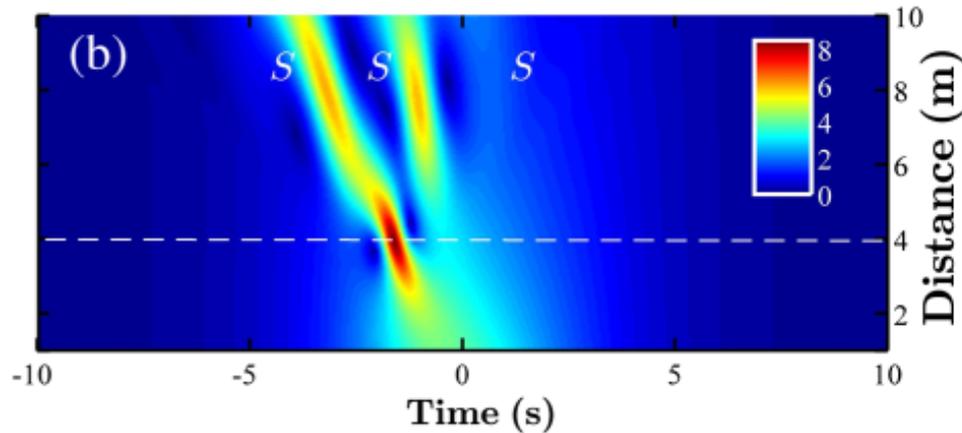


Solli et al., PRL 101, 233902 (2008)

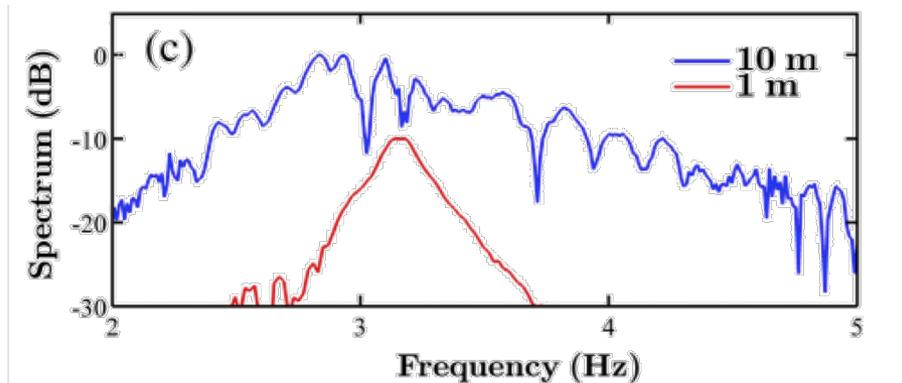
That's all folks!

Even SC in water waves!

- Perturbations can also lead to soliton fission, just as in optics...



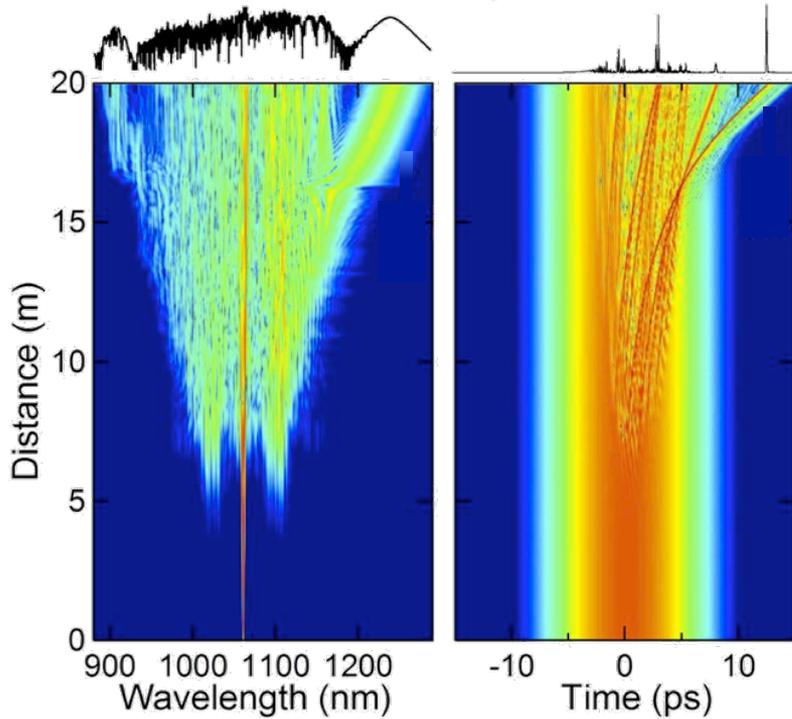
- Fission length $L_{fiss} \approx L_d/N = N L_{nl}$ is also the same



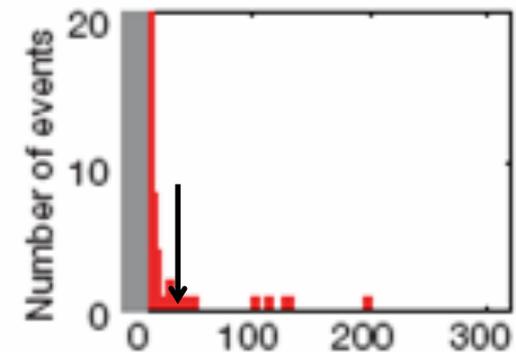
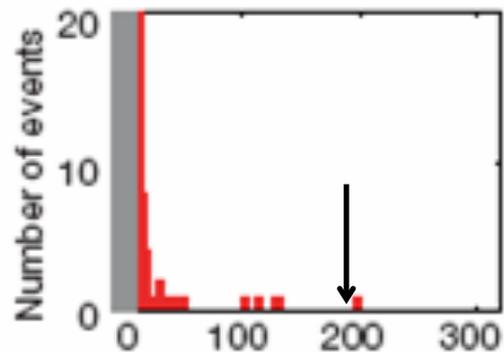
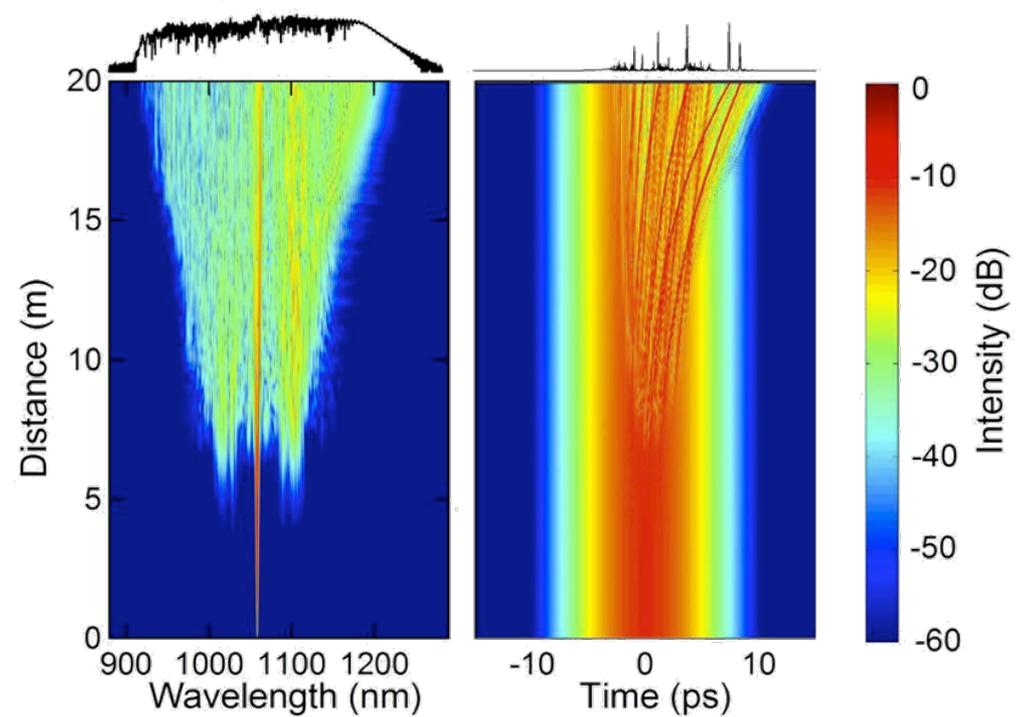
Hydrodynamic
supercontinuum

Rogue solitons

(a) Evolution of "rogue" event

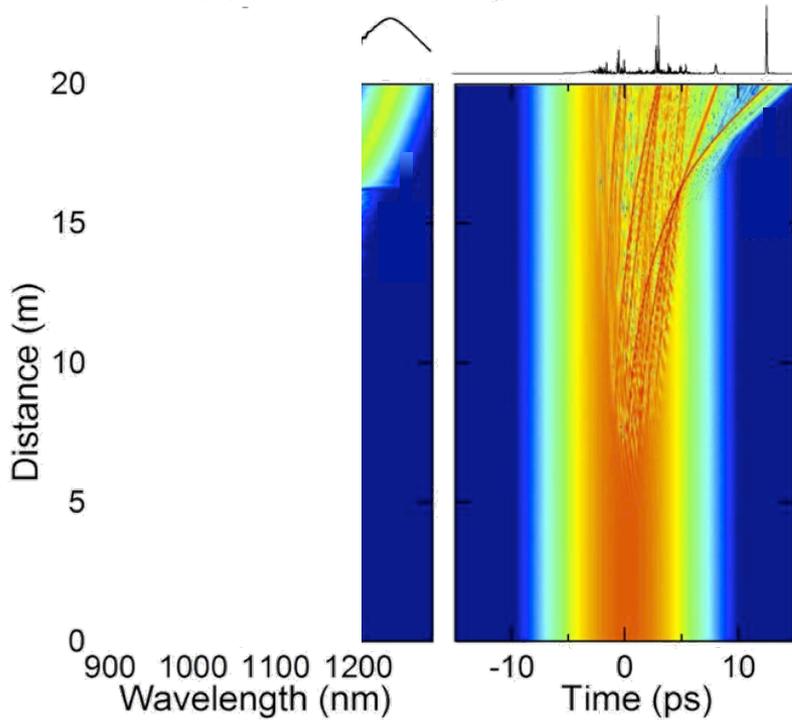


(b) Evolution of "median" event

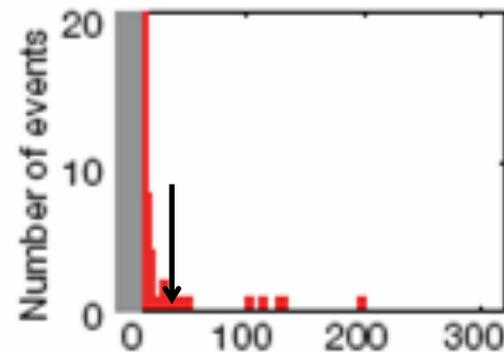
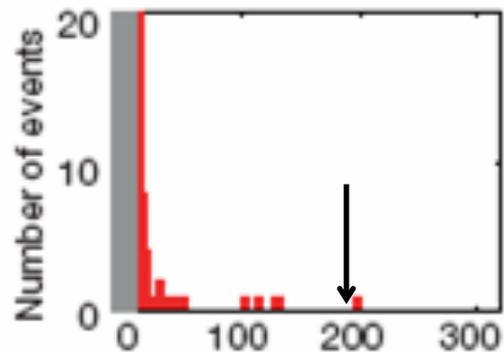
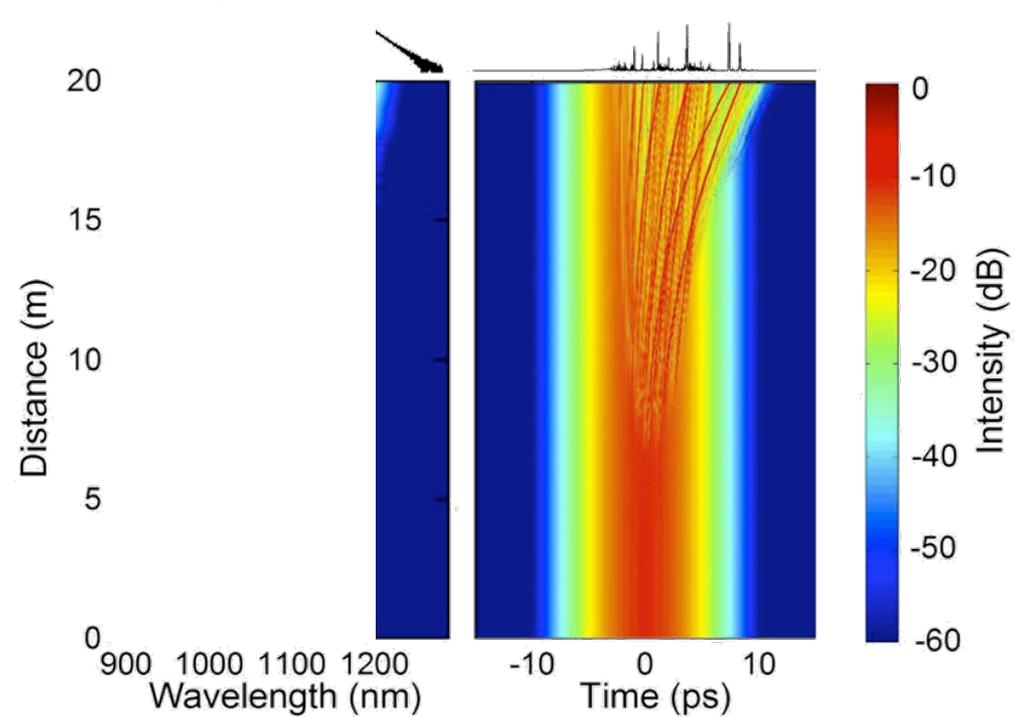


Rogue solitons

(a) Evolution of "rogue" event

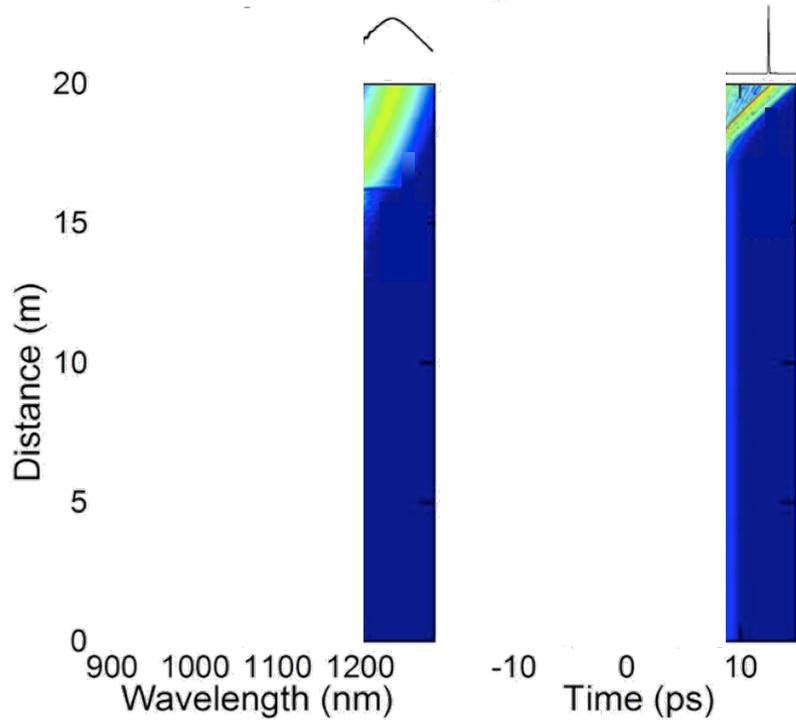


(b) Evolution of "median" event



Rogue solitons

(a) Evolution of "rogue" event



(b) Evolution of "median" event

