Clean and dirty 1D quantum systems

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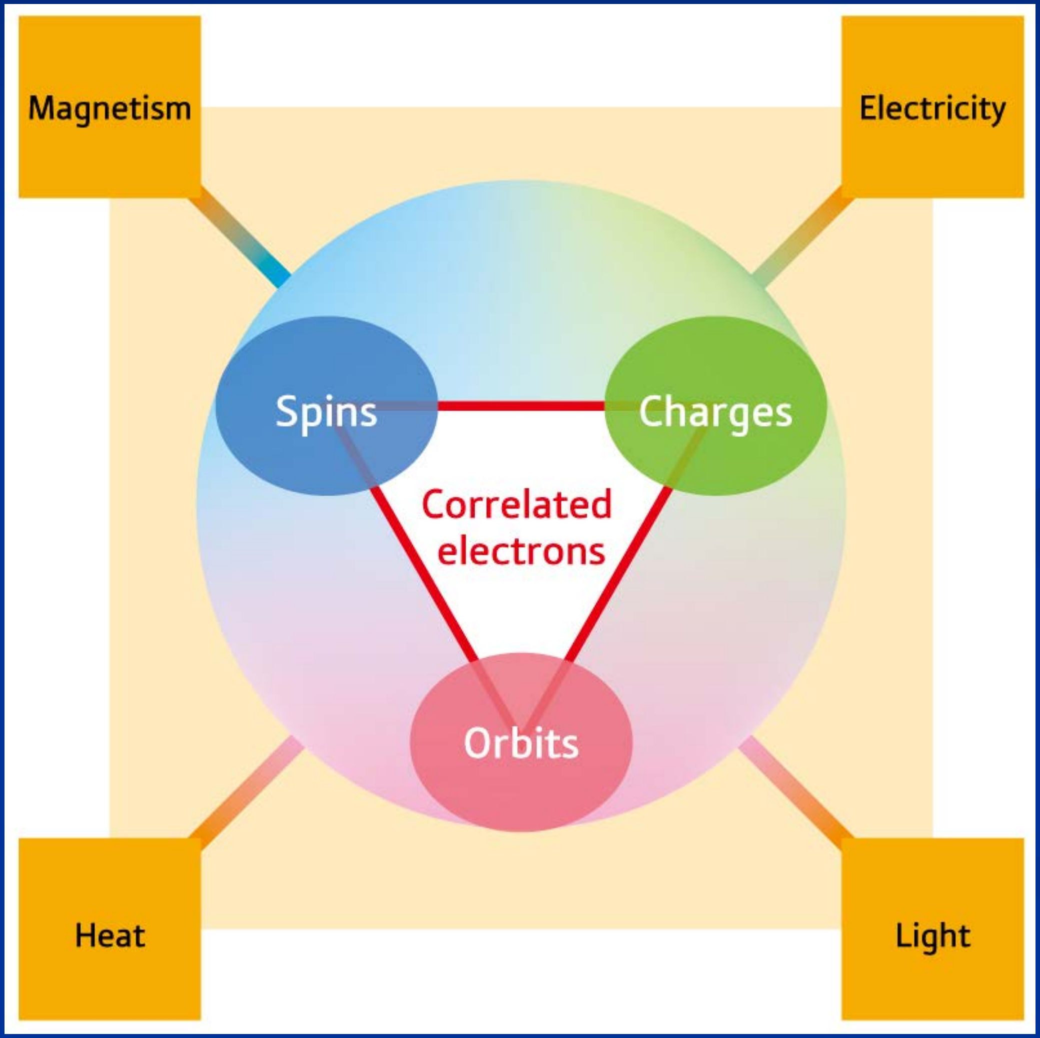
Why one dimension ?

Three urban legends about 1D

- It is a toy model to understand higher dimensional systems.
- It does not exist in nature ! This is only for theorists !
- Everything is understood there anyway !

So, why one dimension ?

Finding new functionalities /physics

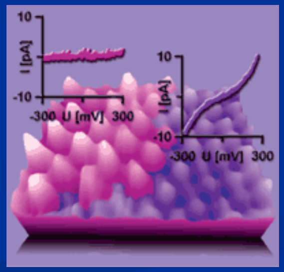


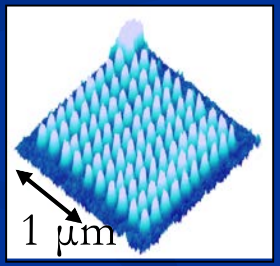
(Y. Tokura, Japan)

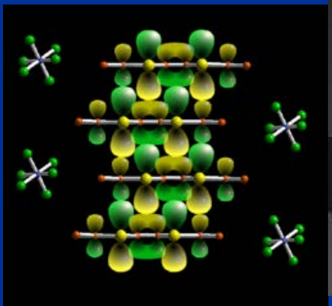
Need to understand interactions !

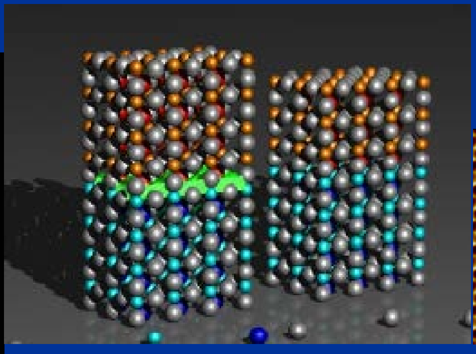
10^{23} particules + Interactions: Crucial fundamental problem ``non Fermi liquids” tomorrow’s materials

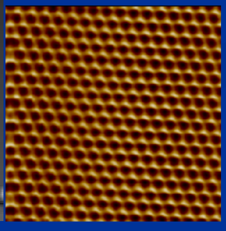
Ferroelectrics, High Tc, Manganites,
Oxydes, Organics,, nanomaterials….

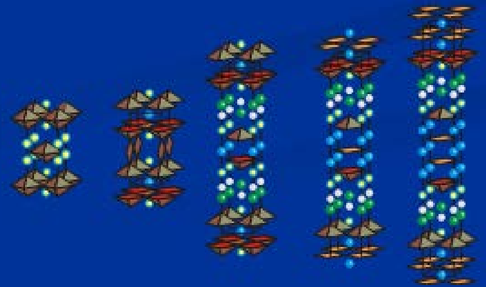


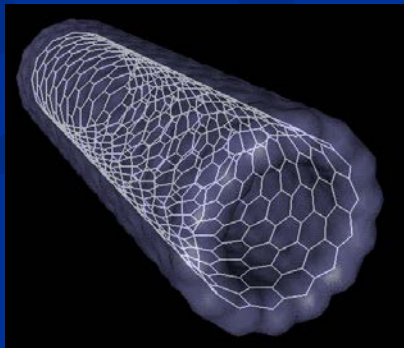






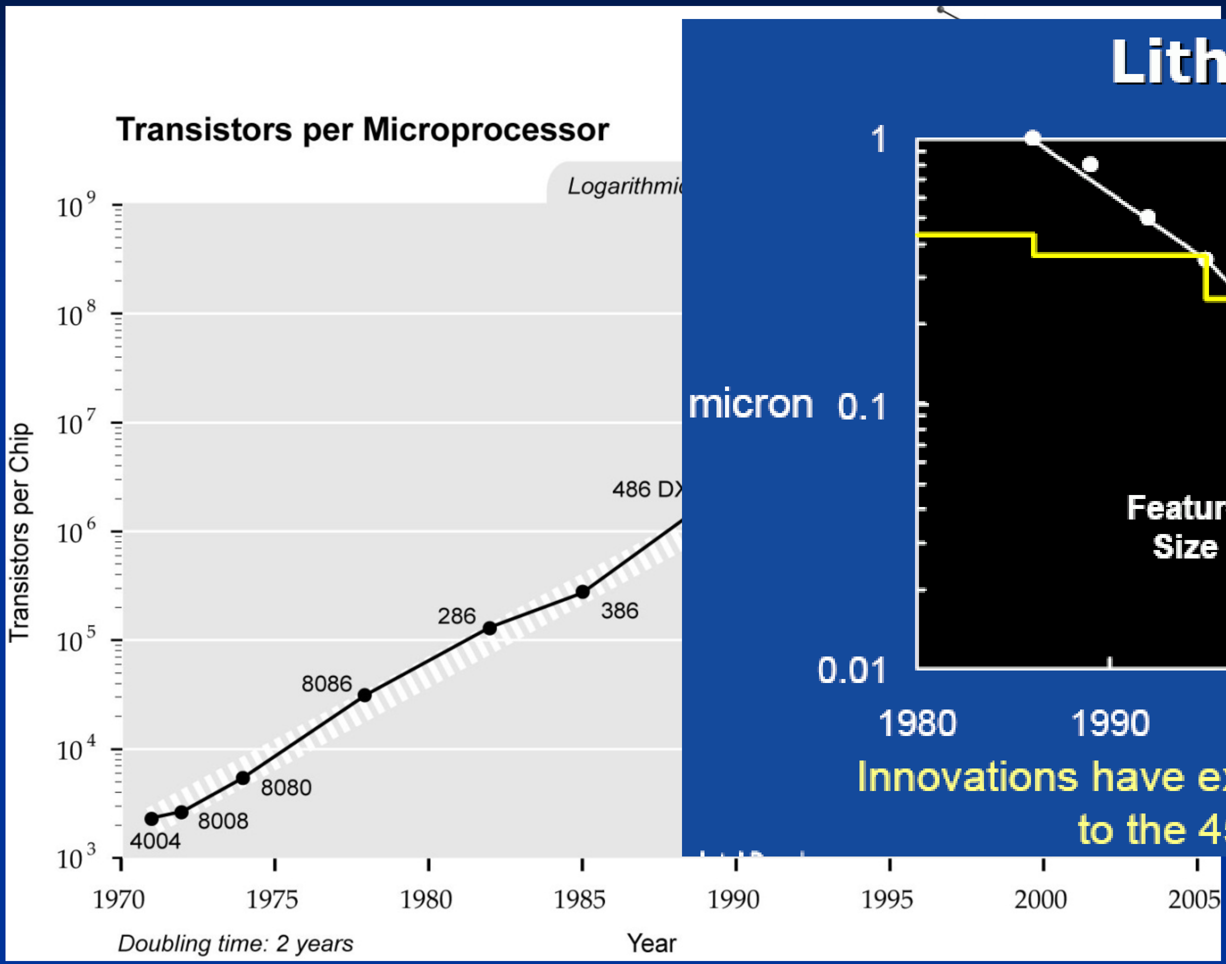


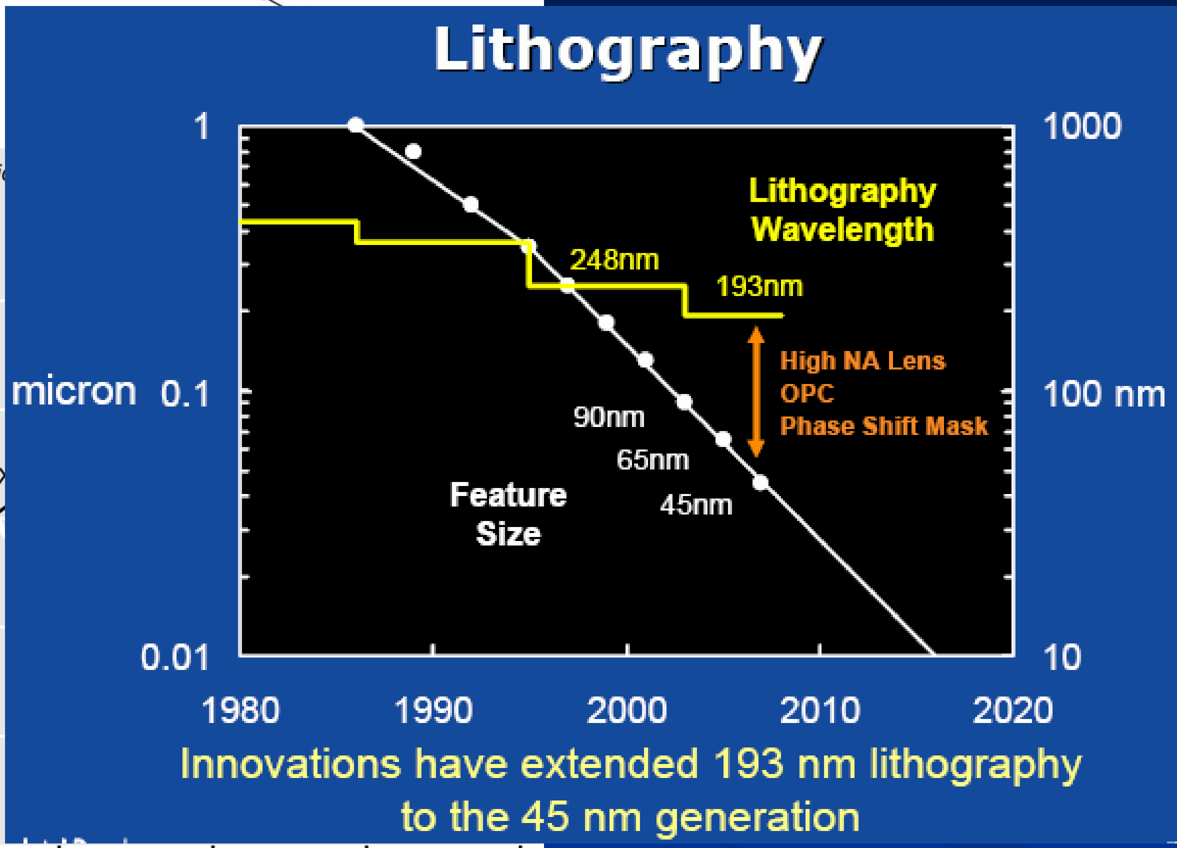




Reduced dimensionality

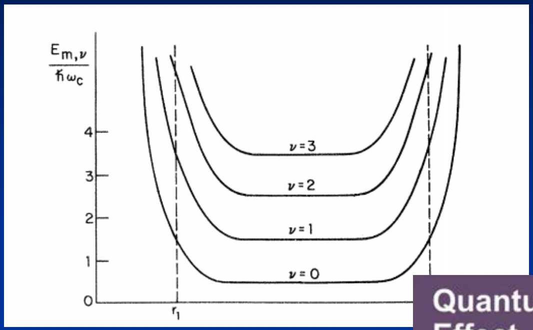
Future electronic

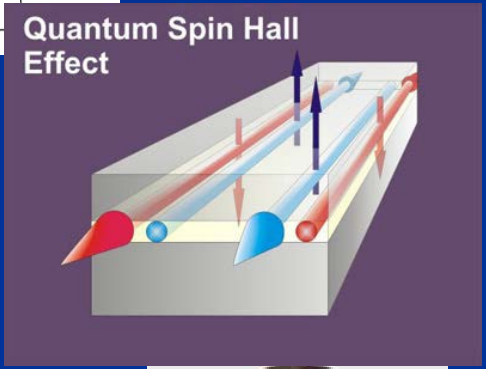


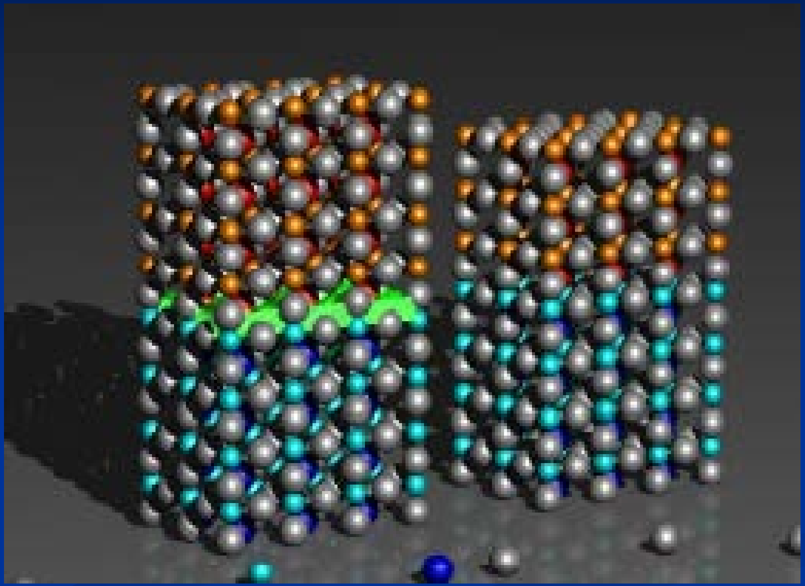


Need to worry about reduced dimensionality

Physics at the edge







Presence of edge
(B. I. Halperin)

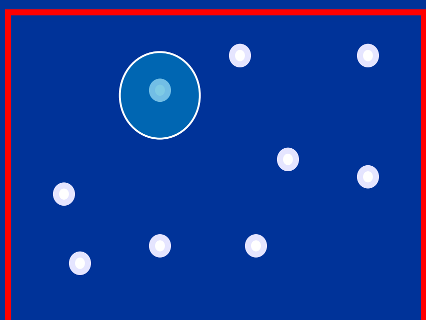
LaO/StO interface
(JM Triscone et al.)

Quantum hall effect
Topological insulators….

Superconductivity
between insulators…

One dimension is specially interesting

- No individual excitation can exist (only collective ones)



- Strong quantum fluctuations

$$\psi = |\psi| e^{i\theta}$$

Difficult to order

A good reason to work on 1D

However, my personal reason for working on one-dimensional problems is merely that they are fun. A man grows stale if he works all the time on the insoluble and a trip to the beautiful work of one dimension will refresh his imagination better than a dose of LSD.

Freeman Dyson (1967)

Plan of the lectures (1)

- Lecture 1: 1D basics
  - What are one dimensional systems
  - Universal physics in one dimension (Luttinger liquid)
  - Some realizations with cold atoms or CM
  - Effect of a lattice: Mott transition

Plan of the lectures (2)

- Lecture 2: 1D and beyond
  - More on the Mott transition (string order)
  - Fermions and Spins
  - Systems with internal degrees of freedom (spin)
  - Impurities in Luttinger liquids; Non Luttinger liquids
  - Between 1D and 2D : ladders
  - Some open problems for pure systems

Plan of the lectures (3)

- Lecture 3: Disorder
  - Disorder and noninteracting quantum systems (Anderson localization)
  - Disorder and interactions in quantum systems (dirty bosons): Bose glass
  - Disorder and quasiperiodicity
  - Loose ends and open questions

References

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TG, Quantum physics in one dimension, Oxford (2004)

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Rev. Mod. Phys.83 1405 (2011)

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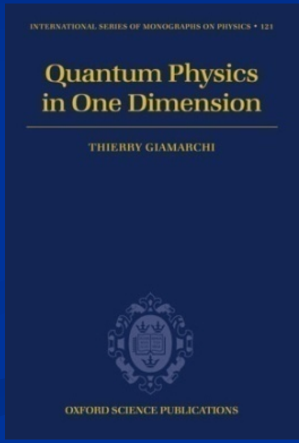
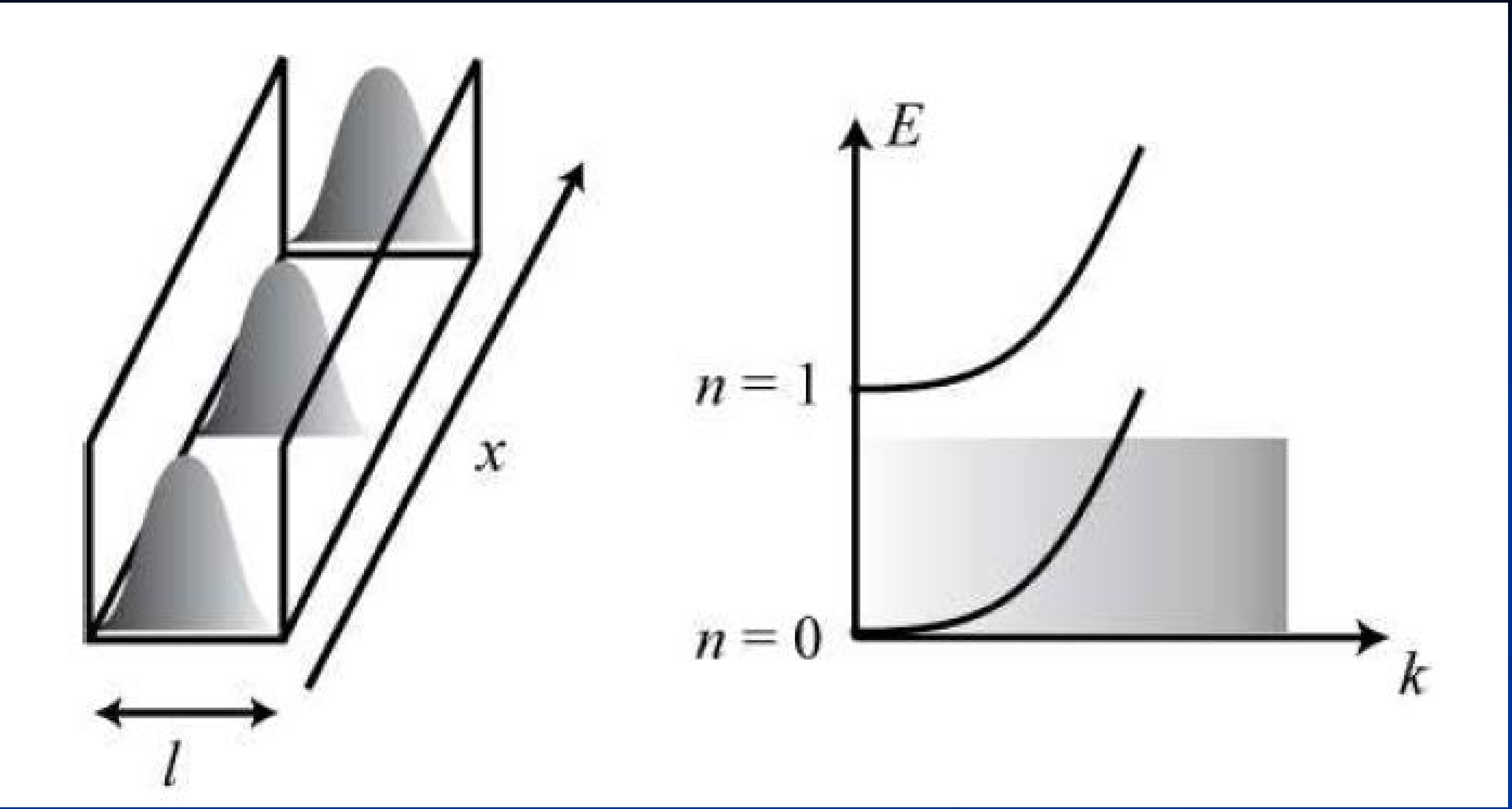


Lecture 1

What does “1D” means in the real (3D ?) world

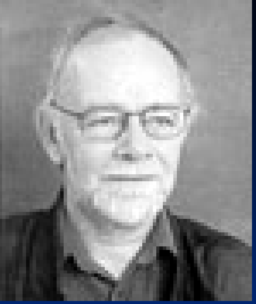


$$E = \frac{k_x^2}{2m} + \frac{k_y^2}{2m}$$

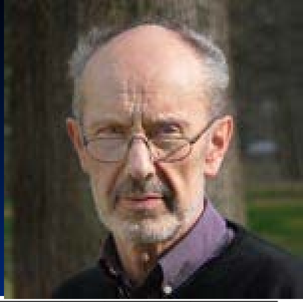
$$k_y = \frac{2\pi n}{L_y}$$

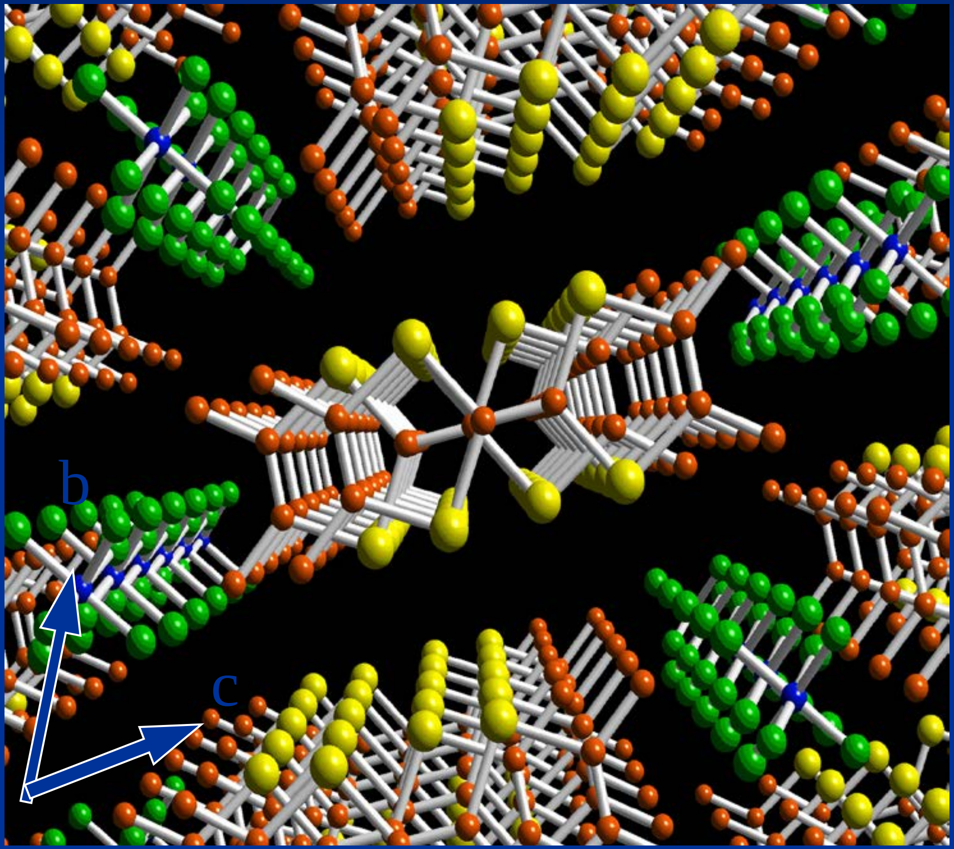
Fine …..
But does it exist ?

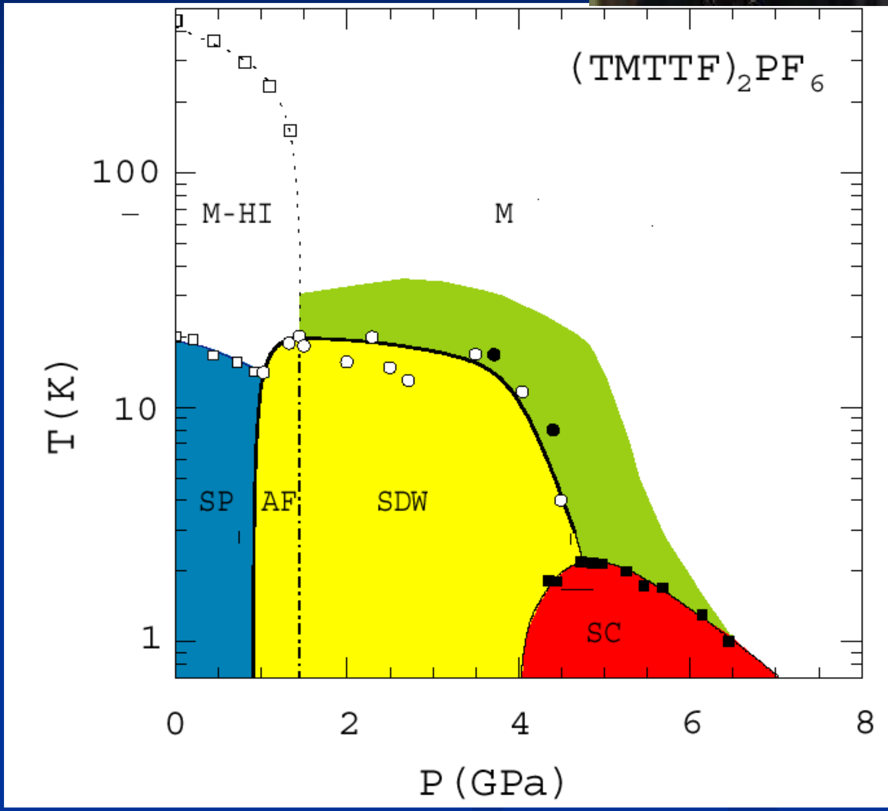
TG, Int J. Mod. Phys. B 26 1244004 (2012)



Organic conductors



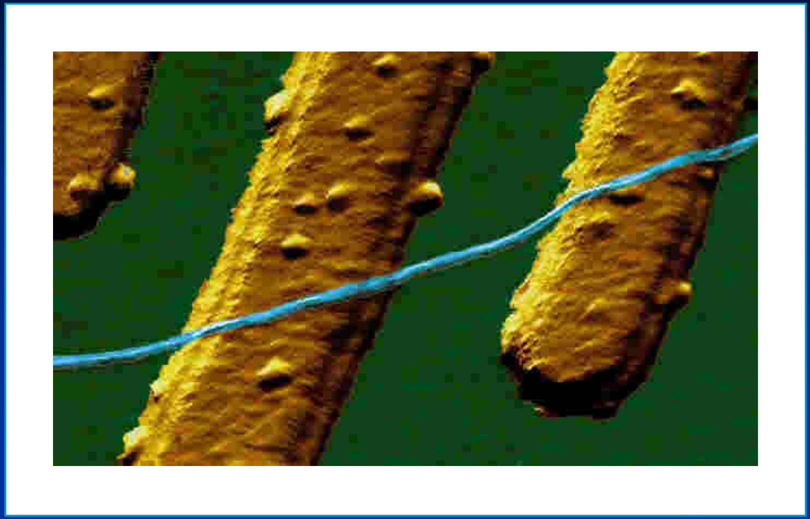


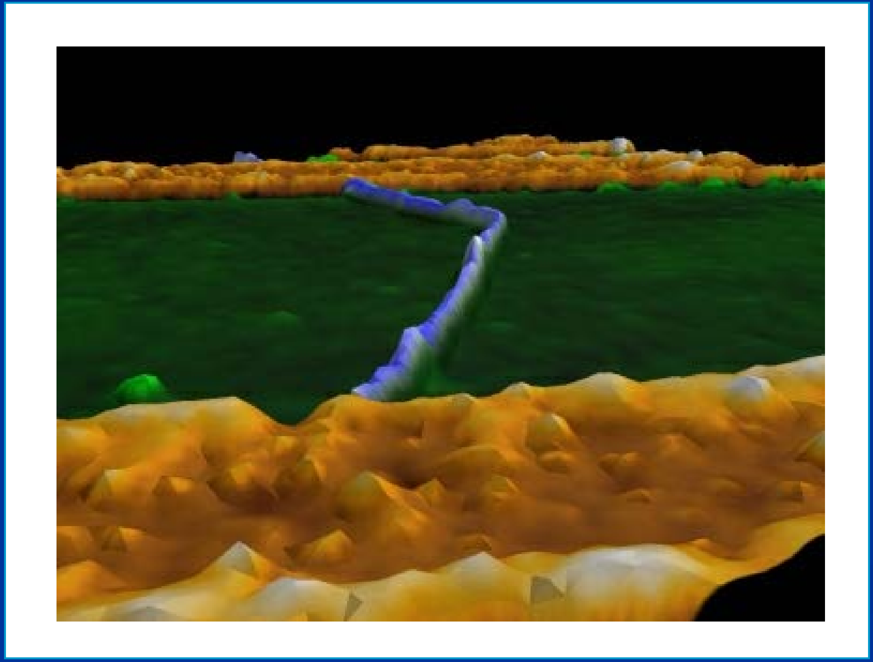


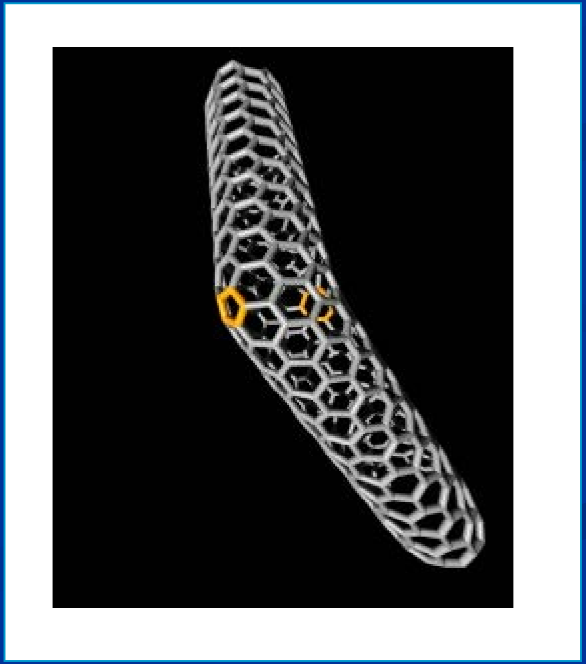
D. Jaccard et al., J. Phys. C, 13 L89 (2001)

CARBON NANOTUBES

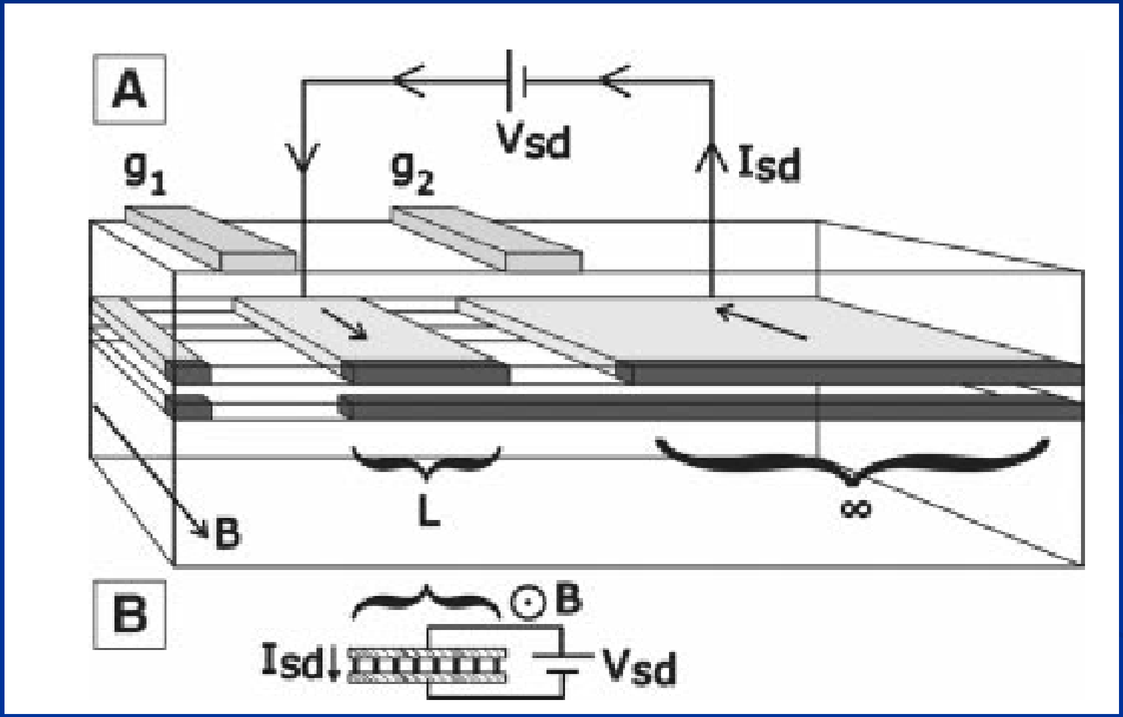
Cees Dekker







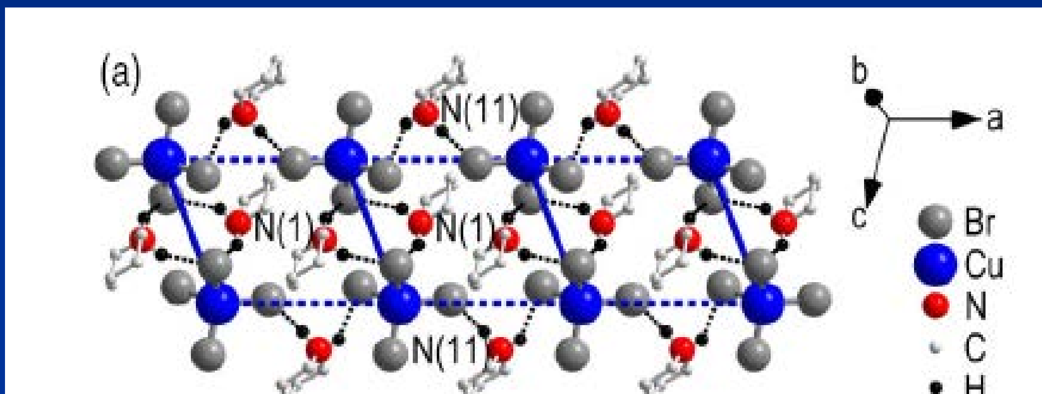
Quantum Wires



O.M Ausslander et al., Science **298** 1354 (2001)

Spin chains and ladders

B. C. Watson et al., PRL 86 5168 (2001)



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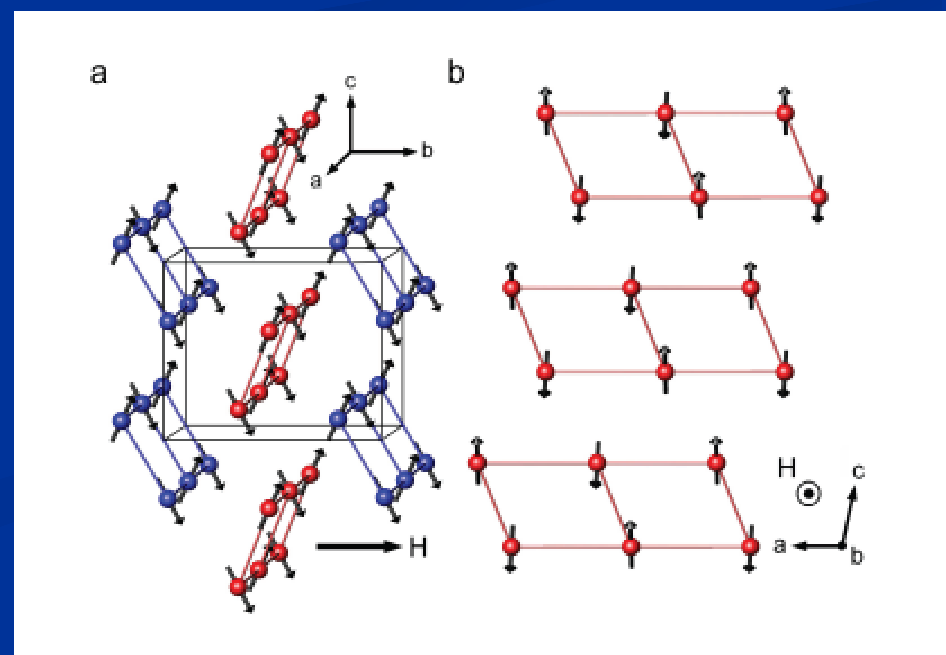
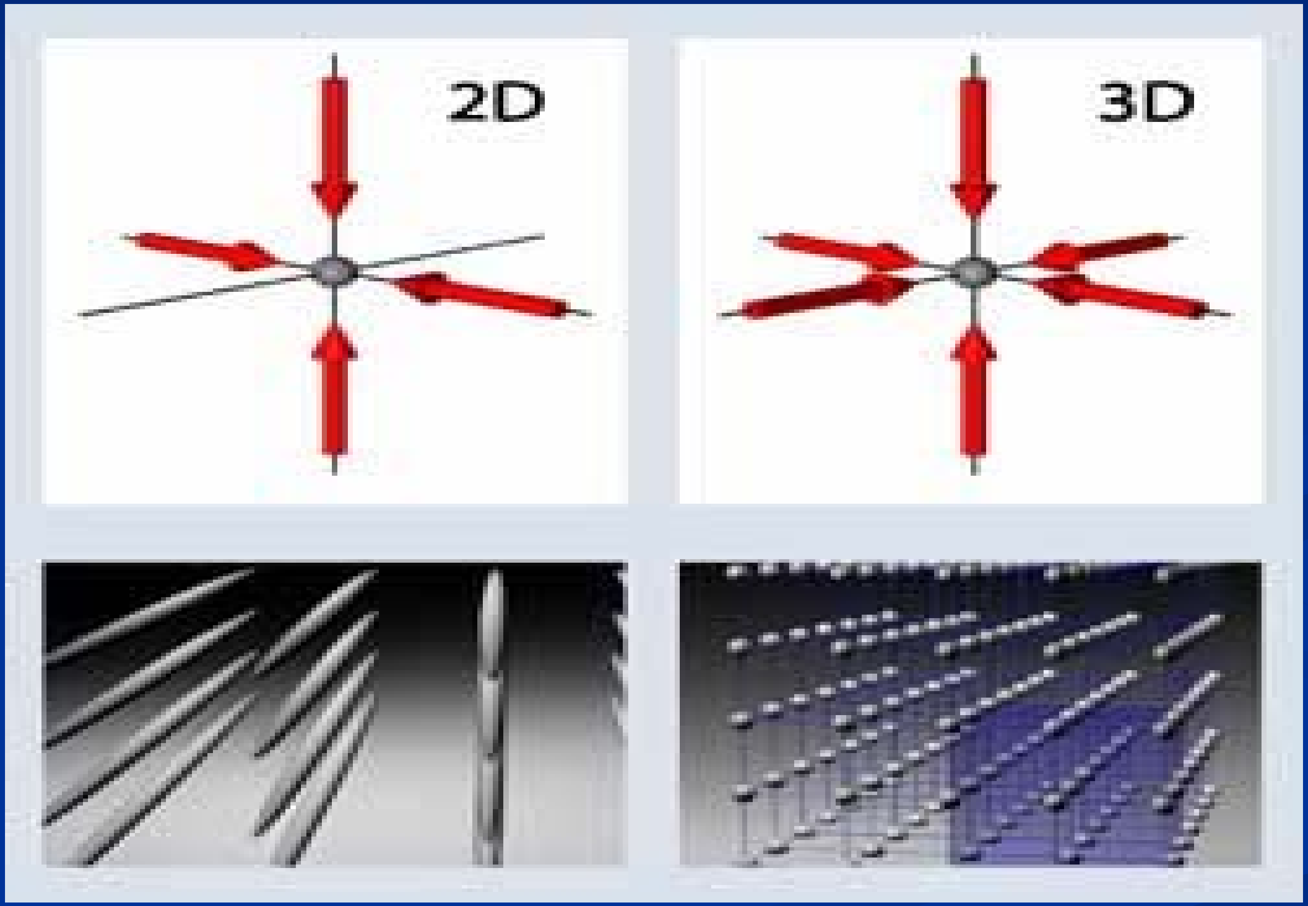


B. Thielemann et al.,

PRB **79**, 020408® 2009

Cold atoms

Control on the dimension



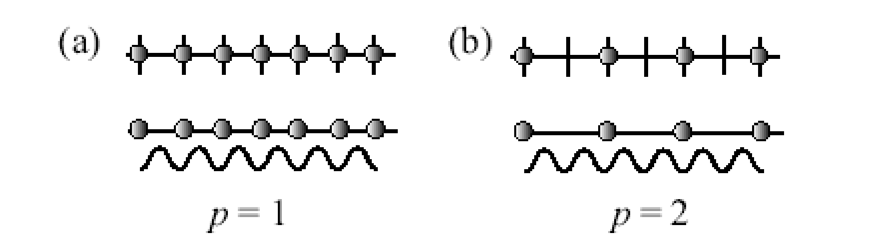
I. Bloch, Nat. Phys 1, 23 (2005)

Typical problem (e.g. Bosons)

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu \int dx\, \rho(x)$$

- Lattice:



$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

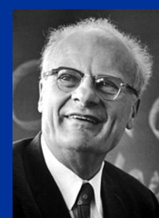
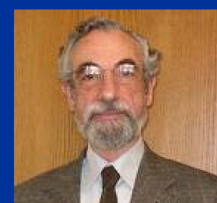
How to treat ?

- ``Standard” many body theory

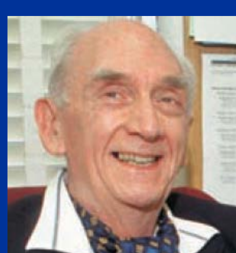




- Exact Solutions (Bethe ansatz)

- Field theories (bosonization, CFT)

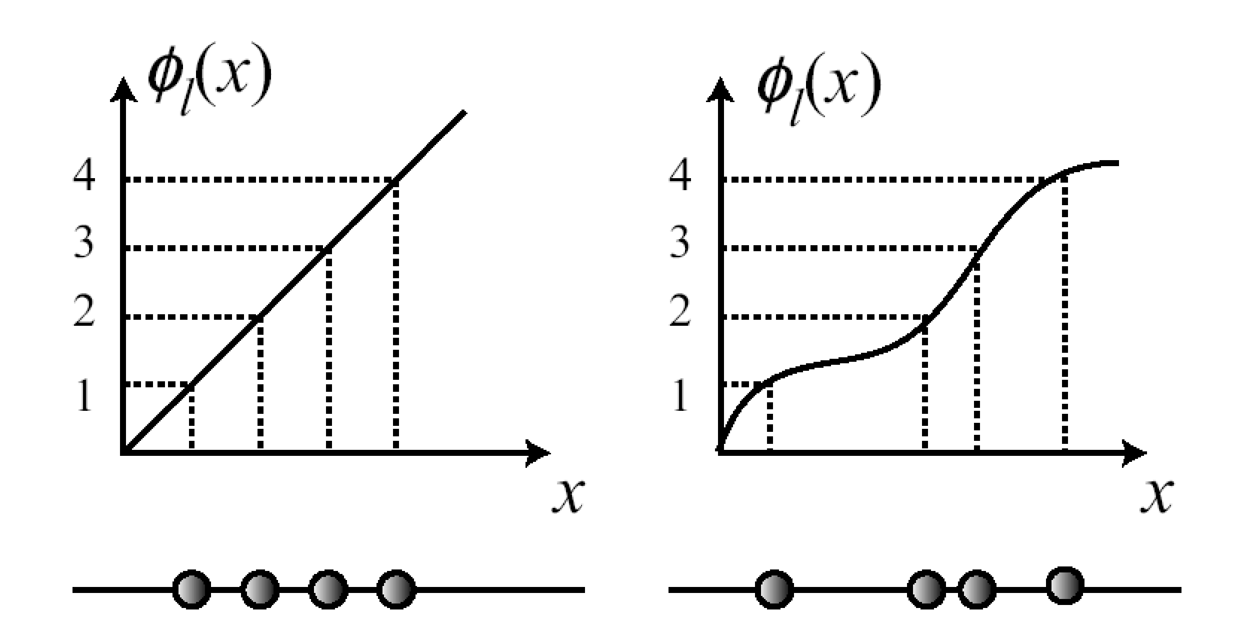

- Numerics (DMRG, MC, etc.)

Luttinger liquid physics

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

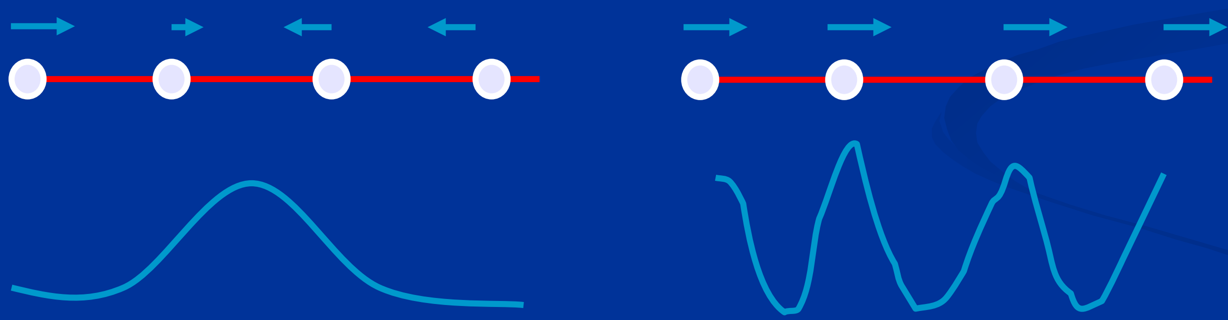
1D: unique way of labelling



$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly



CDW

$q \sim 0$ $\qquad$ $q \sim 2\pi\rho_0$

$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

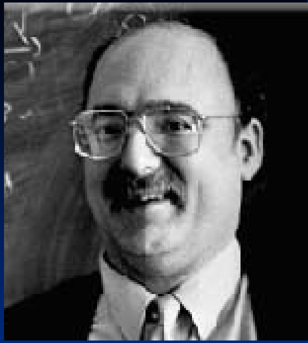
θ: superfluid phase

$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

Luttinger liquid concept

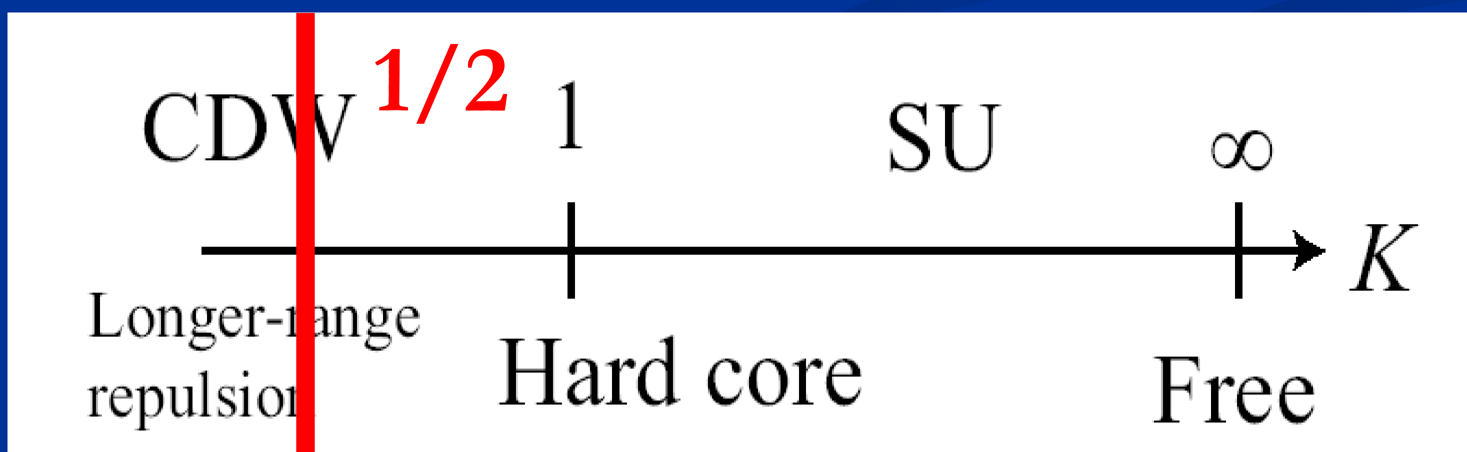


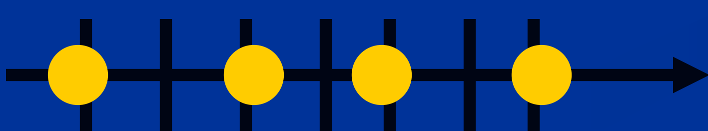
- How much is perturbative ?
- Nothing (Haldane): provided the correct u,K are used
- Low energy properties: Luttinger liquid (fermions, bosons, spins…)

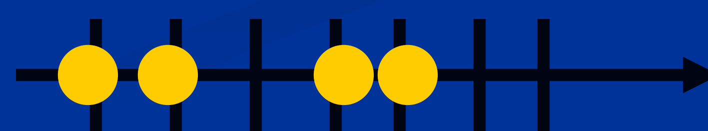
Correlations

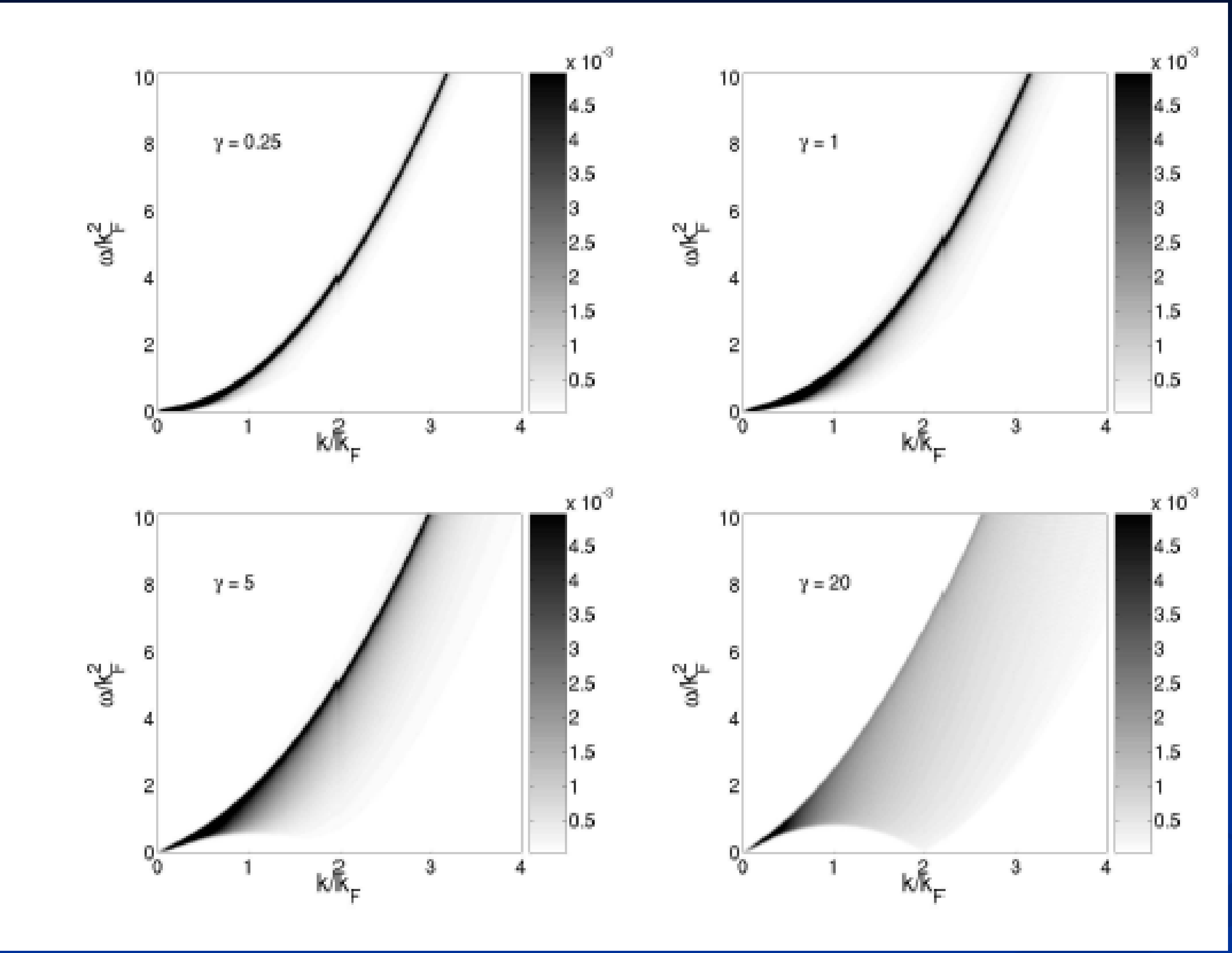
$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$









S(q,!) J.S. Caux et al PRA 74 031605 (2006)

Finite temperature

Conformal theory