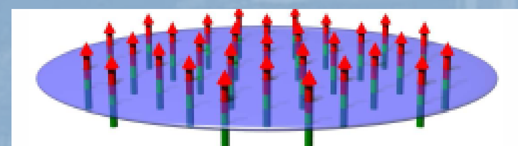
Leibniz Universität Hannover

Theory of dipolar gases (I)



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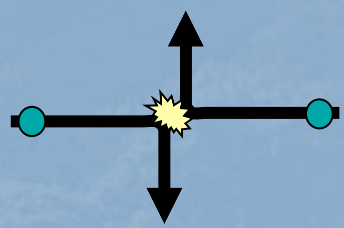
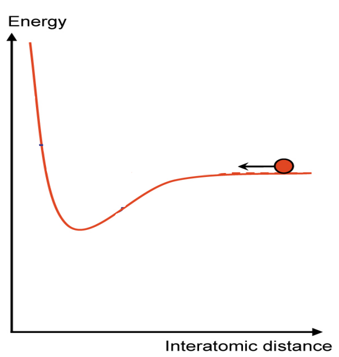
Varenna, July, 2014

Contact interaction



In typical experiments up to now the atoms interact via <u>short-range isotropic interactions</u>

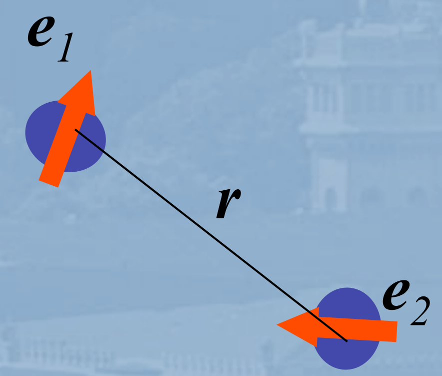
The interaction is given by **the s-wave scattering length "a"**



$$V\left(\vec{r}-\vec{r}\,'\right)\approx\frac{4\pi\hbar^{2}a}{m}\delta(\vec{r}-\vec{r}\,')\equiv g\delta(\vec{r}-\vec{r}\,')$$

Dipole-dipole interaction

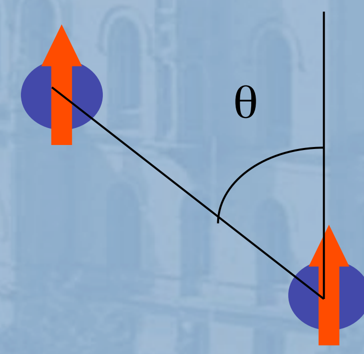
$$U_{\mathrm{dd}}(\boldsymbol{r}) = \frac{C_{\mathrm{dd}}}{4\pi} \frac{(\boldsymbol{e}_1 \cdot \boldsymbol{e}_2)\, r^2 - 3\, (\boldsymbol{e}_1 \cdot \boldsymbol{r})\, (\boldsymbol{e}_2 \cdot \boldsymbol{r})}{r^5}$$



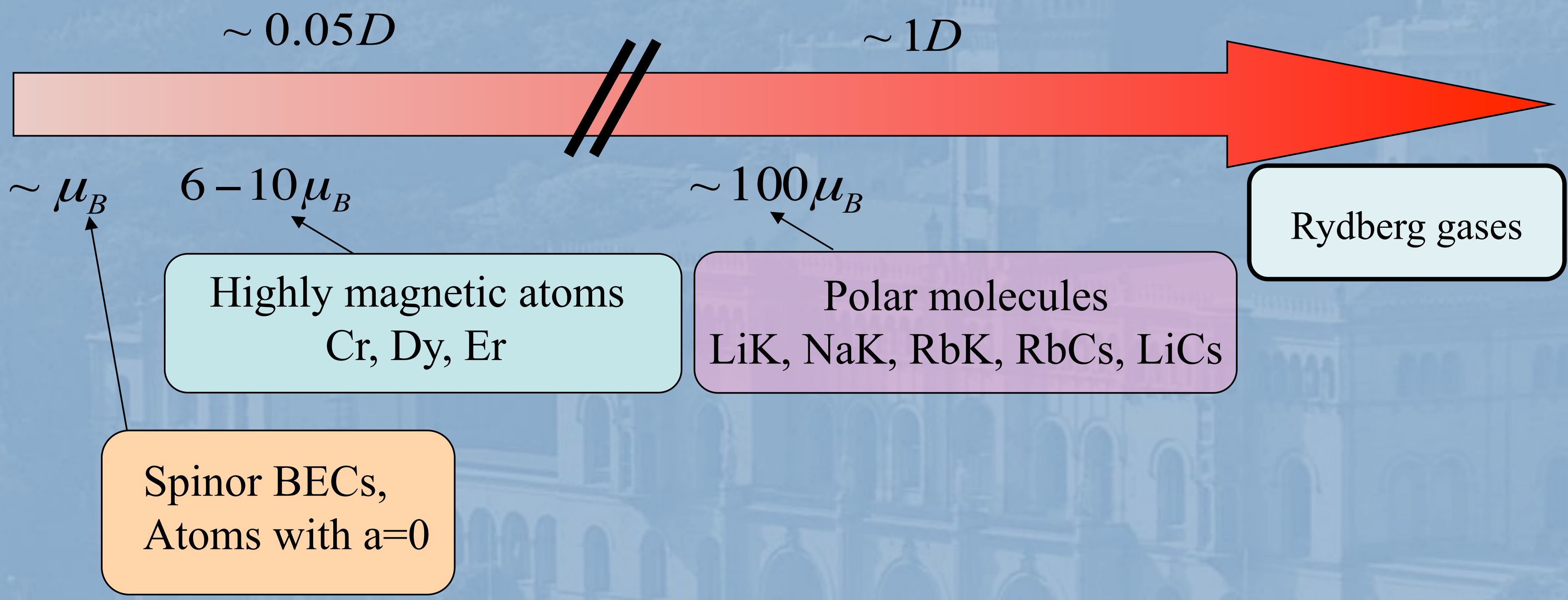
C_{dd} → $\mu_0 \mu^2$ (magnetic dipoles)

C_{dd} → d^2 / ε_0 (electric dipoles)

$$U_{\mathrm{dd}}(\boldsymbol{r}) = \frac{C_{\mathrm{dd}}}{4\pi} \frac{1 - 3\cos^2\theta}{r^3}$$



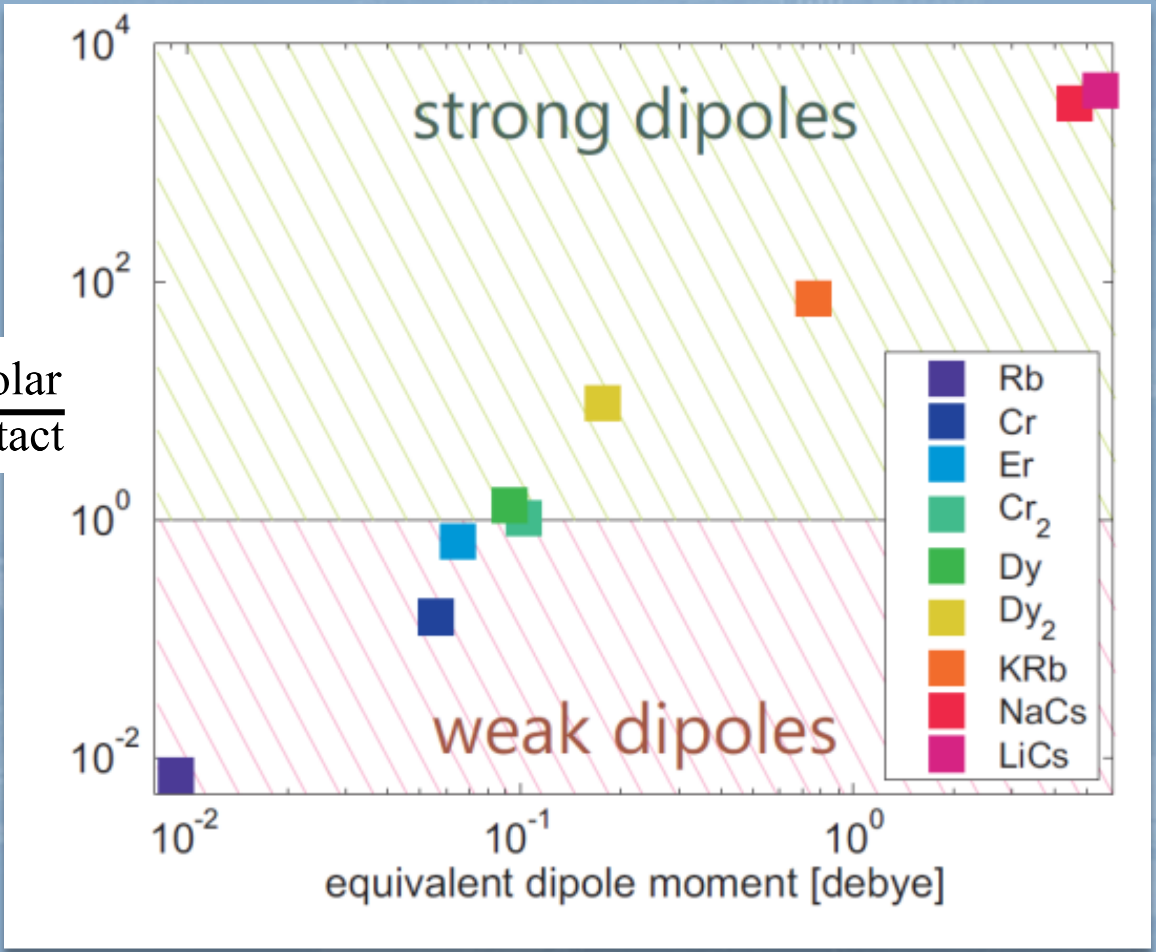
Dipolar gases: all the way from very weak to huge



Dipolar gases: all the way from very weak to huge

$$\varepsilon_{dd} \equiv \frac{C_{dd}}{3g}$$

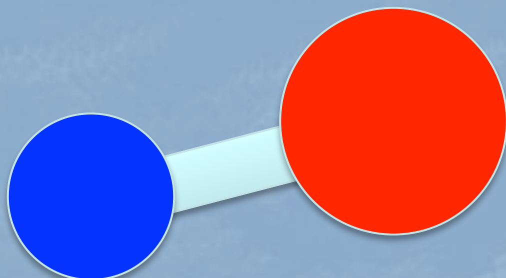
$\frac{\text{dipolar}}{\text{contact}}$



Rydberg

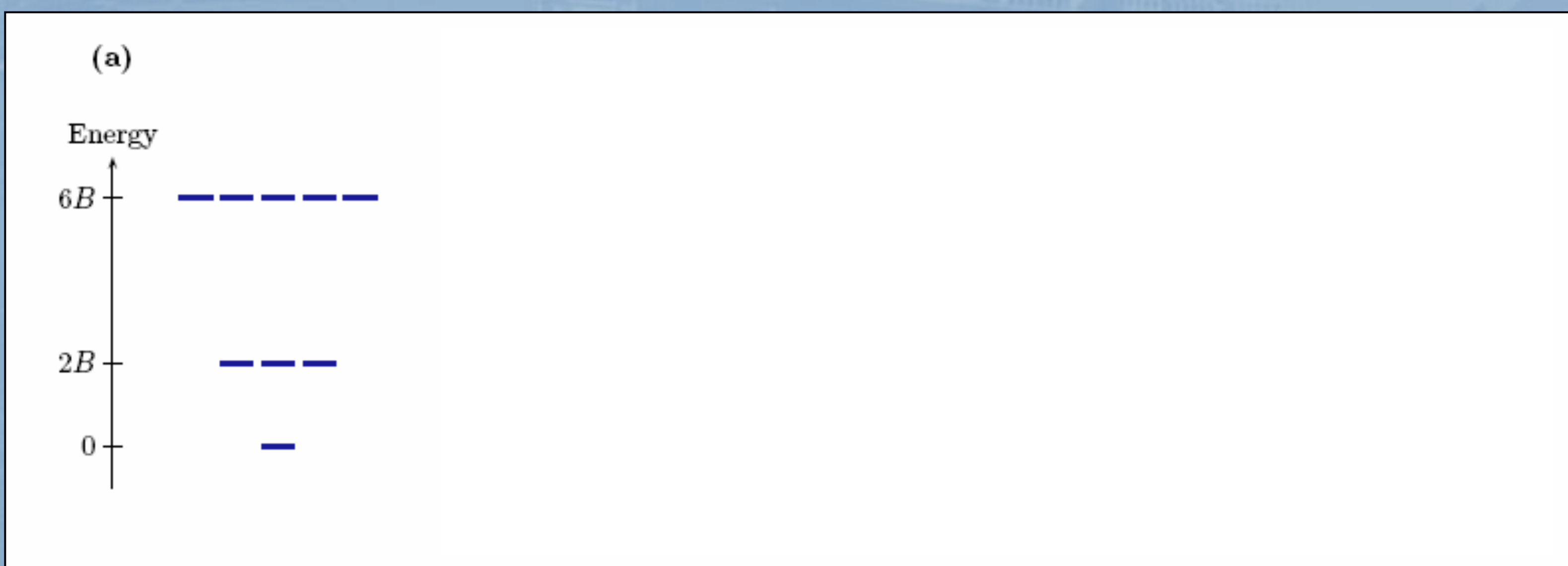
+ LiK, NaK, RbCs, SrRb...

Polar molecules

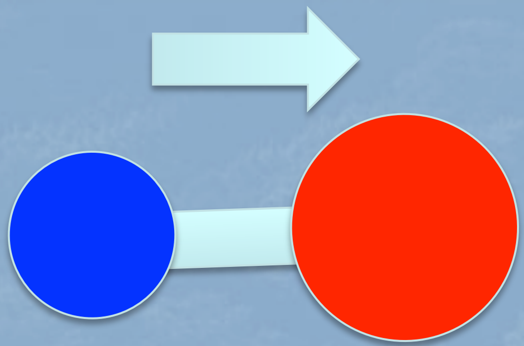


$$\hat{H} = B\hat{J}^2 \qquad \left|J, M\right\rangle \Rightarrow BJ(J+1)$$

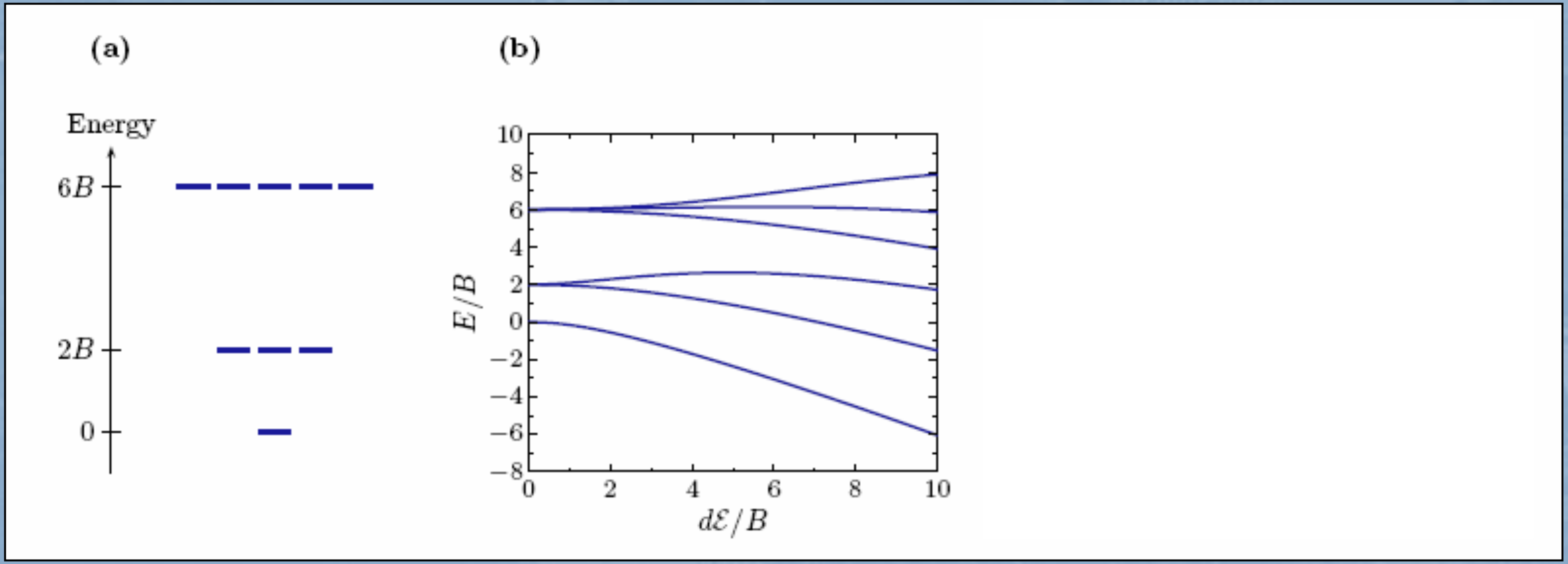
$$\left\langle 0,0\middle|d\middle|0,0\right\rangle = 0 \quad \left\langle 1,M\middle|d\middle|0,0\right\rangle \neq 0$$



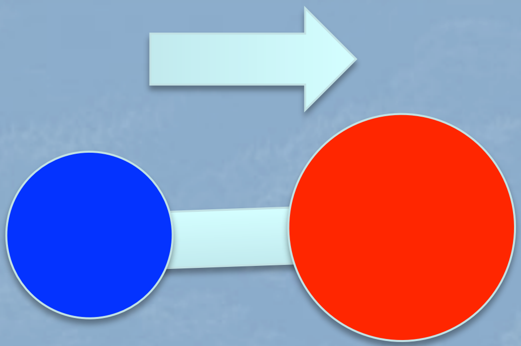
Polar molecules



$$\hat{H} = B\hat{J}^2 - \vec{d}\cdot\vec{E}$$

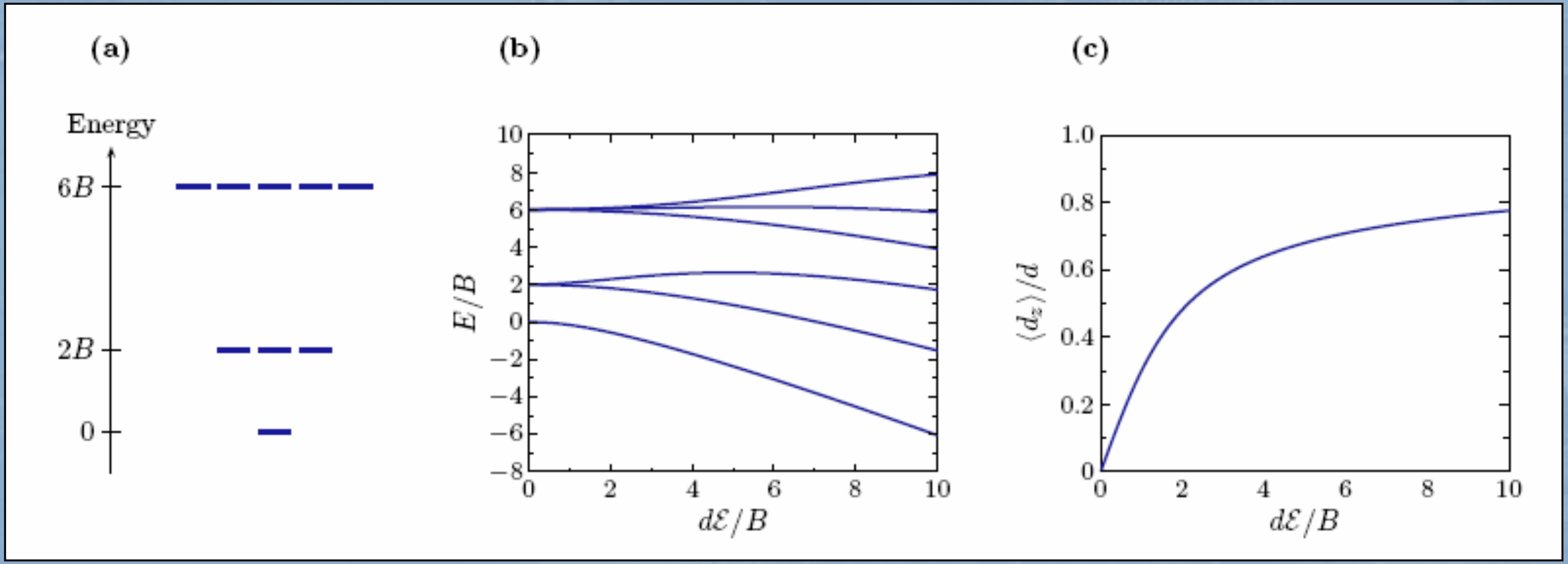


Polar molecules



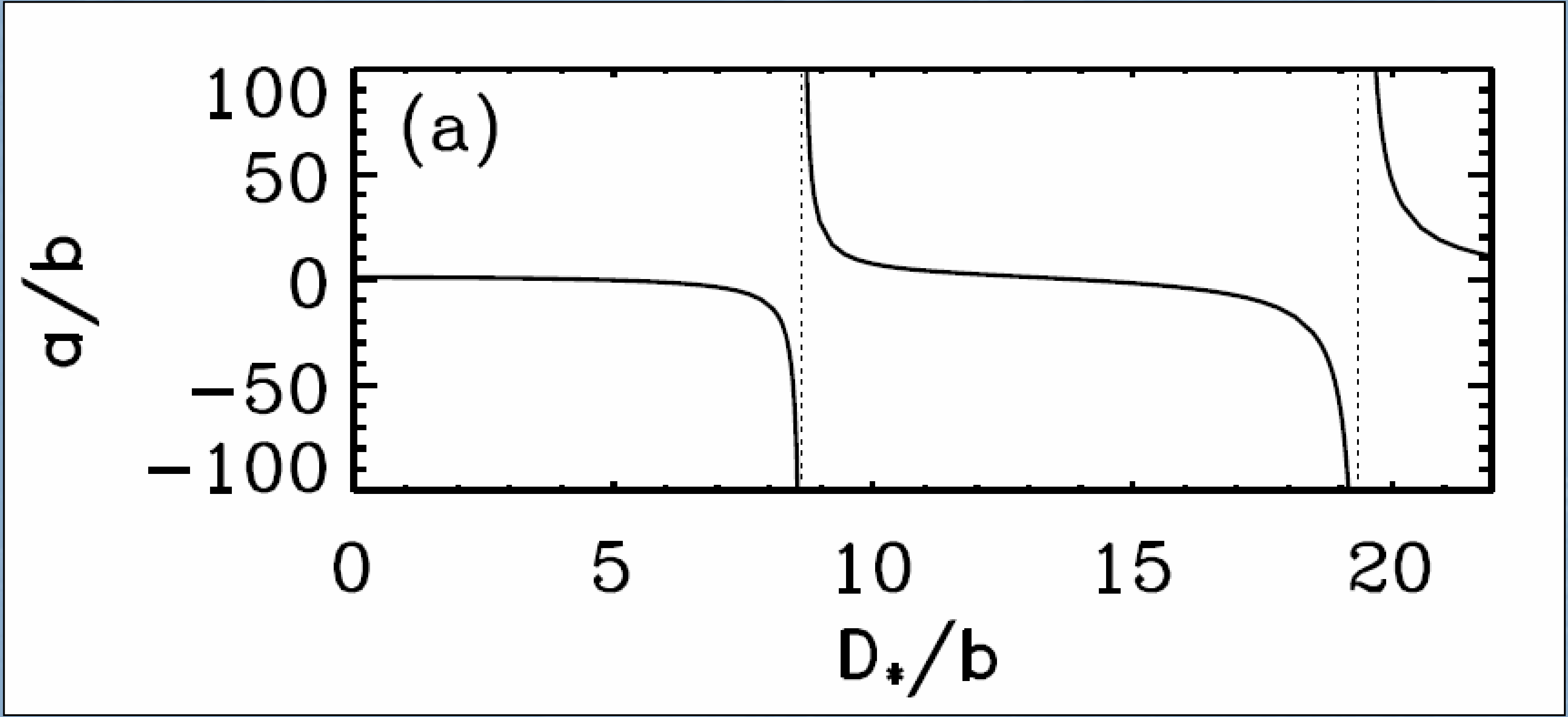
$$\hat{H} = B\hat{J}^2 - \vec{d}\cdot\vec{E}$$

$$\langle \phi_0 | d | \phi_0 \rangle \neq 0$$



Pseudopotential

$$U_{SR}(r) + U_{dd}(\vec{r}) \;\Rightarrow\; g(d)\delta(r) + U_{dd}(\vec{r})$$



[From Bortolotti et al., PRL **97**, 160402 (2006)]

Dipolar BECs: Nonlocal Gross-Pitaevskii equation

$$H = \int dr \hat{\psi}^{+}(\vec{r}) \left[-\frac{\hbar^2}{2m} \nabla^2 + V_T(r) - \mu \right] \hat{\psi}(\vec{r})$$

$$+ \frac{1}{2} \iint dr\, dr' \hat{\psi}^{+}(\vec{r}) \hat{\psi}^{+}(\vec{r}\,') U\left(\vec{r} - \vec{r}\,'\right) \hat{\psi}(\vec{r}\,') \hat{\psi}(\vec{r})$$

$$U(\vec{r}) = g\delta(r) + U_{dd}(\vec{r})$$

Dipolar BECs: Nonlocal Gross-Pitaevskii equation

$$\hat{H} = \int d\boldsymbol{r}\hat{\psi}^\dagger(\boldsymbol{r})\left[-\frac{\hbar^2}{2m}\nabla^2 + V_T(\boldsymbol{r}) - \mu + \frac{1}{2}g\hat{\psi}^\dagger(\boldsymbol{r})\hat{\psi}(\boldsymbol{r})\right]\hat{\psi}(\boldsymbol{r})$$

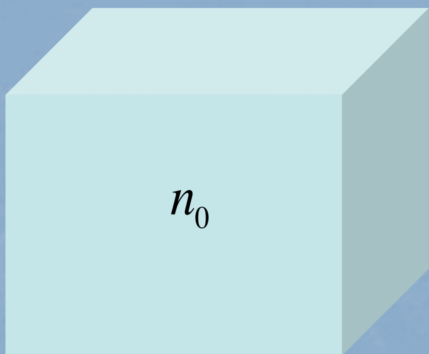
$$+\frac{1}{2}\int d^3r d^3r' \hat{\psi}^\dagger(\boldsymbol{r})\hat{\psi}^\dagger(\boldsymbol{r}')U_{\rm dd}(\boldsymbol{r}-\boldsymbol{r}')\hat{\psi}(\boldsymbol{r}')\hat{\psi}(\boldsymbol{r}),$$

$$\hat{\psi}(\vec{r}) \cong \psi(\vec{r})$$

Dipolar BECs: Nonlocal Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) = \Big[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - \mu + g|\psi(\boldsymbol{r},t)|^2 + \frac{C_{dd}}{4\pi}\int d\boldsymbol{r}'\frac{1-3\cos^2\theta}{|\boldsymbol{r}-\boldsymbol{r}'|^3}|\psi(\boldsymbol{r}',t)|^2\Big]\psi(\boldsymbol{r},t)$$

Stability: homogeneous space



$$\hat{\psi}(\boldsymbol{r}) = \sum_{\boldsymbol{p}} \hat{a}_{\boldsymbol{p}} \exp[i\boldsymbol{p}\cdot\boldsymbol{r}/\hbar]\sqrt{V}$$

$$\hat{H} = \sum_{\boldsymbol{p}} \frac{p^2}{2m}\hat{a}^\dagger_{\boldsymbol{p}}\hat{a}_{\boldsymbol{p}} + \frac{1}{2V}\sum_{\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{q}} (g + \tilde{U}_{\rm dd}(\boldsymbol{q}))\hat{a}^\dagger_{\boldsymbol{p}_1+\boldsymbol{q}}\hat{a}^\dagger_{\boldsymbol{p}_2-\boldsymbol{q}}\hat{a}_{\boldsymbol{p}_2}\hat{a}_{\boldsymbol{p}_1}$$

$$\tilde{U}_{\rm dd}(\boldsymbol{q}) = \frac{C_{\rm dd}}{3}(3\cos^2\theta_q - 1)$$

$$\hat{a}_0, \hat{a}^\dagger_0 \simeq \sqrt{N}$$

$$\hat{H} = \sum_{\boldsymbol{p}\neq 0} \frac{p^2}{2m}\hat{a}^\dagger_{\boldsymbol{p}}\hat{a}_{\boldsymbol{p}} + \frac{n_0}{2}\sum_{\boldsymbol{p}}(g + \tilde{U}_{\rm dd}(\boldsymbol{q}))(2\hat{a}^\dagger_{\boldsymbol{p}}\hat{a}_{\boldsymbol{p}} + \hat{a}^\dagger_{\boldsymbol{p}}\hat{a}^\dagger_{-\boldsymbol{p}} + \hat{a}_{\boldsymbol{p}}\hat{a}_{-\boldsymbol{p}})$$

$$\epsilon(\boldsymbol{p}) = \sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m} + 2n_0\left(g + \tilde{U}_{\rm dd}(\boldsymbol{p})\right)\right]}$$

Stability: homogeneous space

$$\varepsilon(\vec{p}) = \sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m} + 2gn_0\left(1 + \frac{C_{dd}}{3g}\left(3\cos^2\theta_p - 1\right)\right)\right]}$$

For a short-range interacting gas with a<0

$$\varepsilon(\vec{p}) = \sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m} - 2|g|n_0\right]} \approx p\sqrt{\frac{-|g|n_0}{m}} = ip|c_s|$$

Stability: homogeneous space

$$\varepsilon(\vec{p}) = \sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m} + 2gn_0\left(1 + \frac{C_{dd}}{3g}\left(3\cos^2\theta_p - 1\right)\right)\right]}$$

$$\varepsilon(\vec{p}) \cong p\sqrt{\frac{gn_0}{m}}\sqrt{1 + \frac{C_{dd}}{3g}\left(3\cos^2\theta_p - 1\right)}$$

If $\varepsilon_{dd}>1$ one has dynamical instability (phonon instability) but only in some directions

Stability: trapped case

$$\psi(\rho, z) = \frac{\sqrt{N}}{\pi^{3/4} l_\rho l_z^{1/2}} e^{-z^2/2l_z^2} e^{-\rho^2/2l_\rho^2}$$

$$E = \frac{N\hbar^2}{2m}\left\{\frac{1}{l_z^2} + \frac{2}{l_\rho^2}\right\} + \frac{Nm}{4}\left\{2\omega_\rho^2 l_\rho^2 + \omega_z^2 l_z^2\right\}$$

$$+ \frac{gN^2}{2(2\pi)^{3/2} l_z l_\rho^2} + \frac{C_{\mathrm{dd}} N^2}{3(2\pi)^{3/2} l_\rho^2 l_z} f(\kappa), \qquad \kappa = l_\rho / l_z$$

$$f(\kappa) \equiv \left\{\frac{2\kappa^2 + 1}{\kappa^2 - 1} - \frac{3\kappa^2}{(\kappa^2 - 1)^{3/2}} \arctan[\sqrt{\kappa^2 - 1}]\right\}$$

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$$+ \frac{gN^2}{2(2\pi)^{3/2} l_z l_\rho^2} + \frac{C_{\mathrm{dd}} N^2}{3(2\pi)^{3/2} l_\rho^2 l_z} f(\kappa), \qquad \kappa = l_\rho / l_z$$

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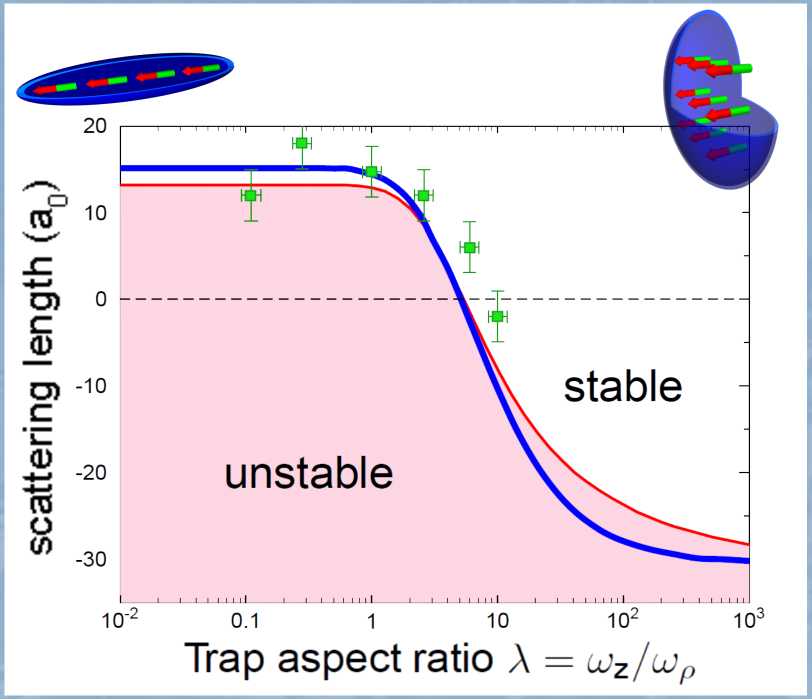
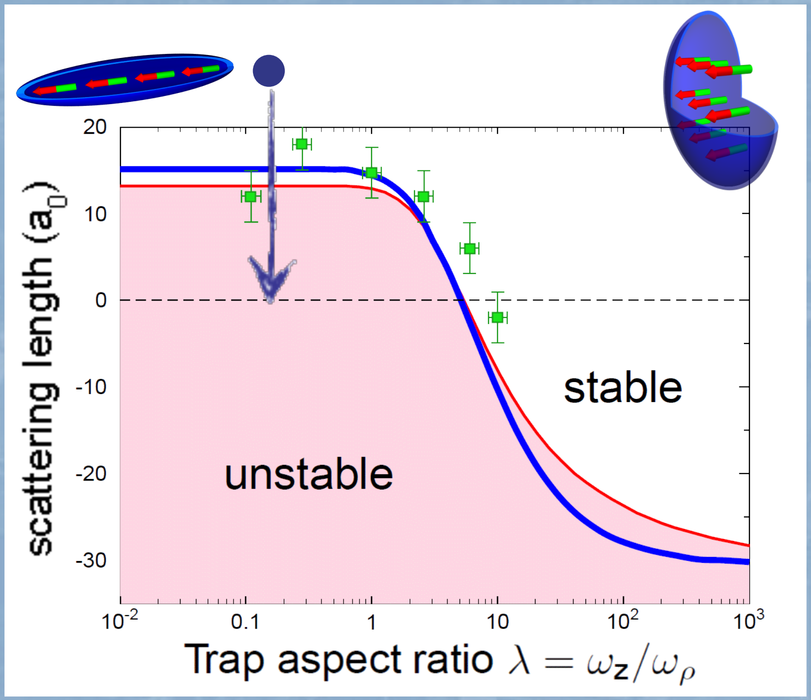
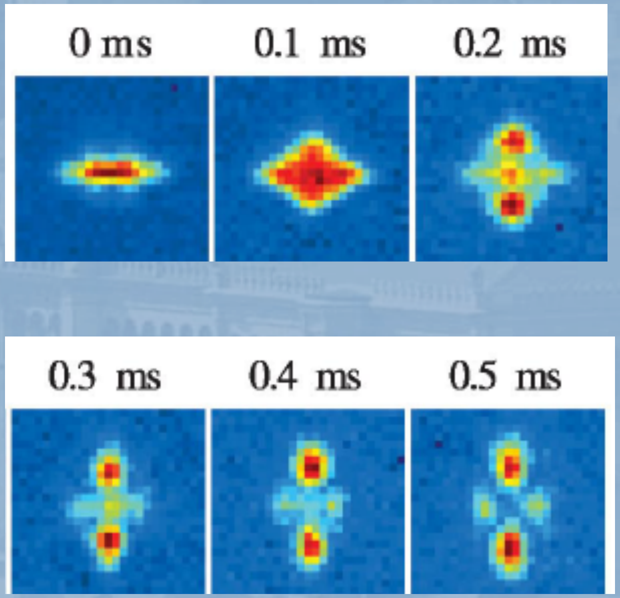
Geometry-dependent stability



[T. Koch et al., Nat. Phys. 4, 218 (2008);
J. L. Bohn, R. M. Wilson and S. Ronen, Laser Physics 19, 547 (2008)]

Trap-dependent stability and d-wave collapse





[Lahaye et al., PRL **101**, 080401 (2008)]

[T. Koch et al., Nat. Phys. 4, 218 (2008);
J. L. Bohn, R. M. Wilson and S. Ronen, Laser Physics 19, 547 (2008)]

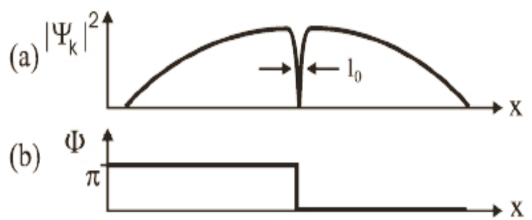
2D solitons

Gross-Pitaevskii equation

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = \left\{ \frac{-\hbar^2}{2m}\nabla^2 + V(\vec{r},t) + \frac{4\pi\hbar^2 a}{m} N|\psi(\vec{r},t)|^2 \right\}\psi(\vec{r},t)$$

Dark Solitons (a>0)

[Burger et al, PRL **83**, 5198 (1999)]
[Denschlag et al., Science **287**, 97 (2000)]

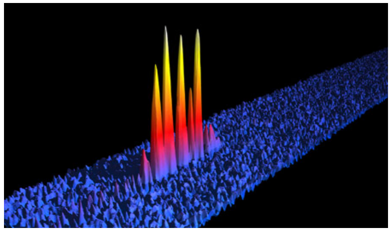


1D NLSE

[Zakharov & Shabat., JETP **34**, 62 (1972)]

Bright solitons (a<0)

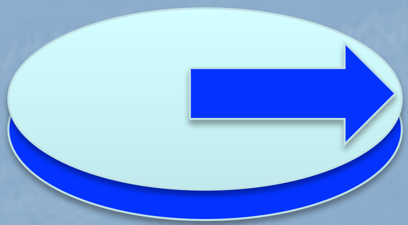
[Strecker et al., Nature **417**, 150 (2002)]
[Khaykovich et al., Science **296**, 1290 (2002)]



Continuous solitons become unstable in 2D and 3D

2D solitons

[Pedri and Santos, PRL **95**, 200404 (2005);
Tikhonenkov, Malomed, and Vardi, PRL **100**, 090406 (2008)]



$$\psi(\boldsymbol{r}) = \frac{1}{l_y^{3/2}} \frac{1}{\pi^{3/4}\Lambda_x\Lambda_z} e^{\frac{1}{2l_y^2}\left(\frac{x^2}{\Lambda_x^2}+\frac{z^2}{\Lambda_z^2}+y^2\right)}$$

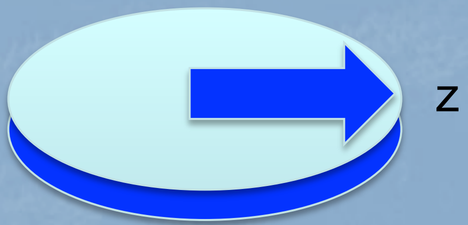
$$\epsilon \equiv \frac{E}{N\hbar\omega_y} = \frac{1}{4(\Lambda_x^2+\Lambda_z^2)} + \frac{\tilde{g}}{4\pi\Lambda_x\Lambda_z}\left[1+\epsilon_{dd}h\left(\frac{\Lambda_x}{\Lambda_z},\frac{1}{\Lambda_z}\right)\right]$$

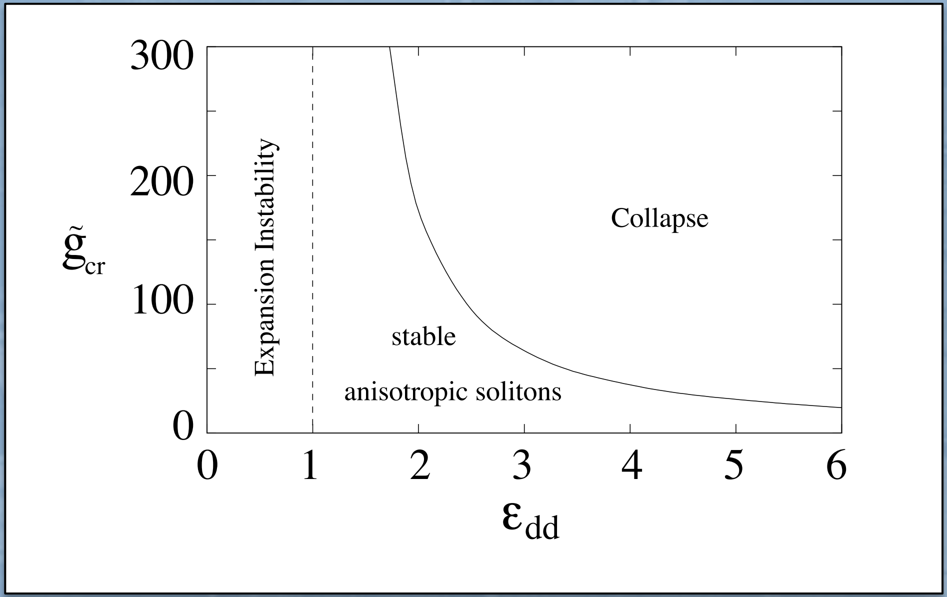
$$\tilde{g} = \frac{m}{\hbar^2}\frac{Ng}{\sqrt{2\pi}l_y}$$

$$h(\alpha,\beta) = -1 + 3\int_0^1 ds \frac{3\alpha\beta s^2}{[1+\alpha^2-1)s^2]^{1/2}\,[1+\beta^2-1)s^2]^{1/2}}$$

Without dipole $\Rightarrow$ $\epsilon(\Lambda = \Lambda_x = \Lambda_z) = \dfrac{(1+\tilde{g}/2\pi)}{2\Lambda^2}$

2D solitons


z


300
200
100
0
0
1
2
3
4
5
6
$\tilde{g}_{cr}$
ε_{dd}
Expansion Instability
Collapse
stable
anisotropic solitons

2D solitons