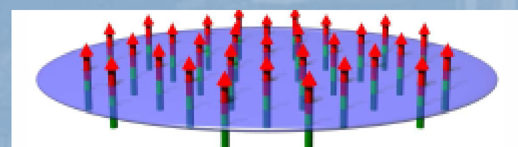
Leibniz Universität Hannover

Theory of dipolar gases (II)



Luis Santos

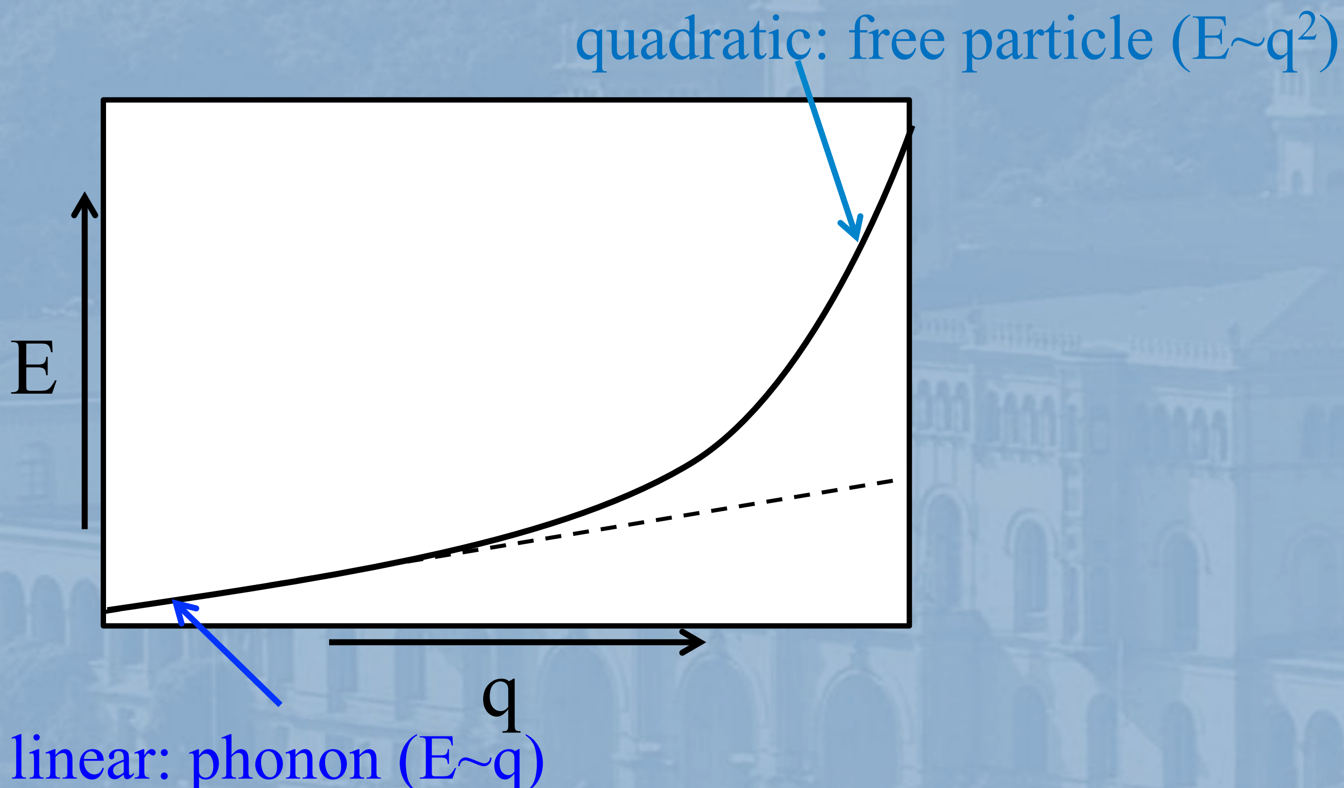
Institute of Theoretical Physics and

Center of Excellence QUEST

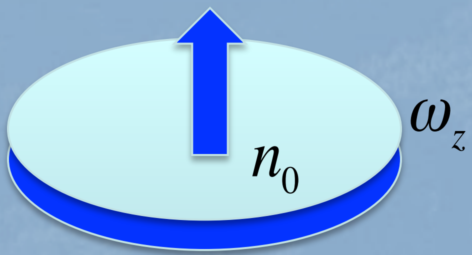
Leibniz Universität Hannover

Varenna, July, 2014

Bogoliubov spectrum



Two-dimensional dipolar condensates



$$\Psi(\vec{r}) = \phi_0(z)\psi(x,y) \qquad \phi_0(z) = \frac{1}{\pi^{1/4} l_z^{1/2}} e^{-z^2/2l_z^2}$$

$$\mu = \frac{(g+g_d)n_0}{\sqrt{2\pi}l_z} << \hbar\omega_z \qquad g_d = \frac{2}{3}C_{dd}$$

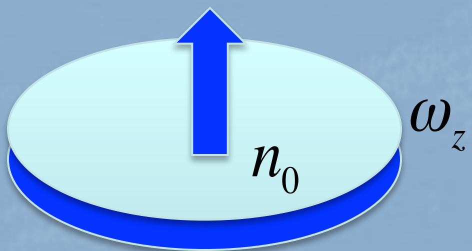
$$q \equiv q l_z \qquad g \equiv \frac{g n_0}{\sqrt{2\pi}l_z}\frac{2}{\hbar\omega_z}$$

$$E^2(q) = q^2\left[q^2 + 2\left(g + g_d G(q)\right)\right]$$

$$E \equiv \frac{2E}{\hbar\omega_z}$$

Recall that in 3D: $\varepsilon(\vec{p}) = \sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m} + 2gn_0\left(1 + \frac{C_{dd}}{3g}\left(3\cos^2\theta_p - 1\right)\right)\right]}$

Two-dimensional dipolar condensates



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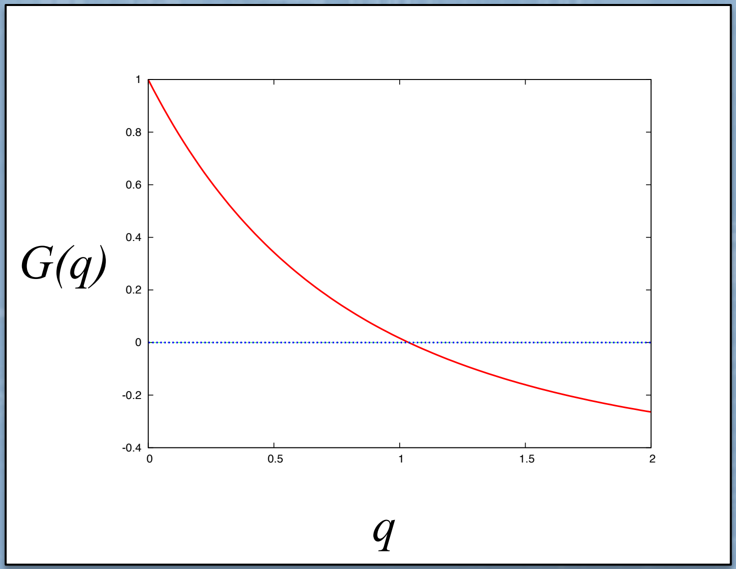
$$E^2(q) = q^2\left[q^2 + 2\left(g + g_d G(q)\right)\right]$$

$$E \equiv \frac{2E}{\hbar\omega_z}$$

$$G(q) = 1 - \frac{3\sqrt{\pi}}{2}\left(\frac{q}{\sqrt{2}}\right) erfc\left(\frac{q}{\sqrt{2}}\right) e^{q^2/2}$$



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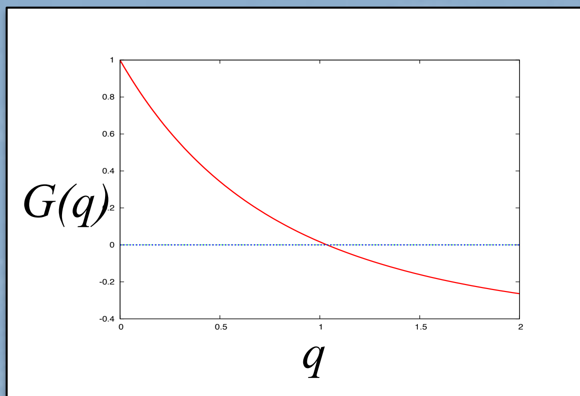
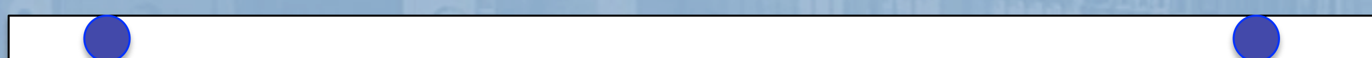
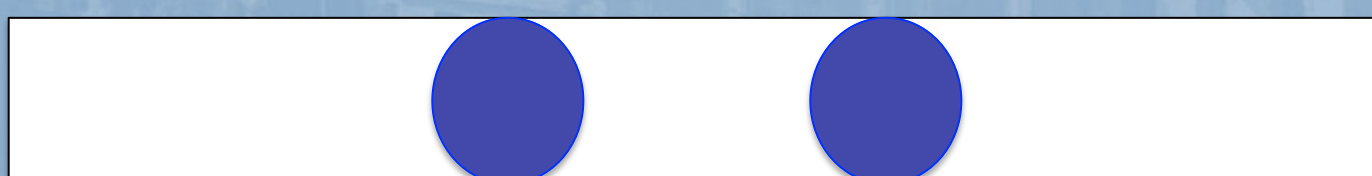


$$E^2(q) = q^2\left[q^2 + 2\left(g + g_d G(q)\right)\right]$$

distances >> l_z $ql_z \ll 1$



distances ~ l_z $ql_z \sim 1$

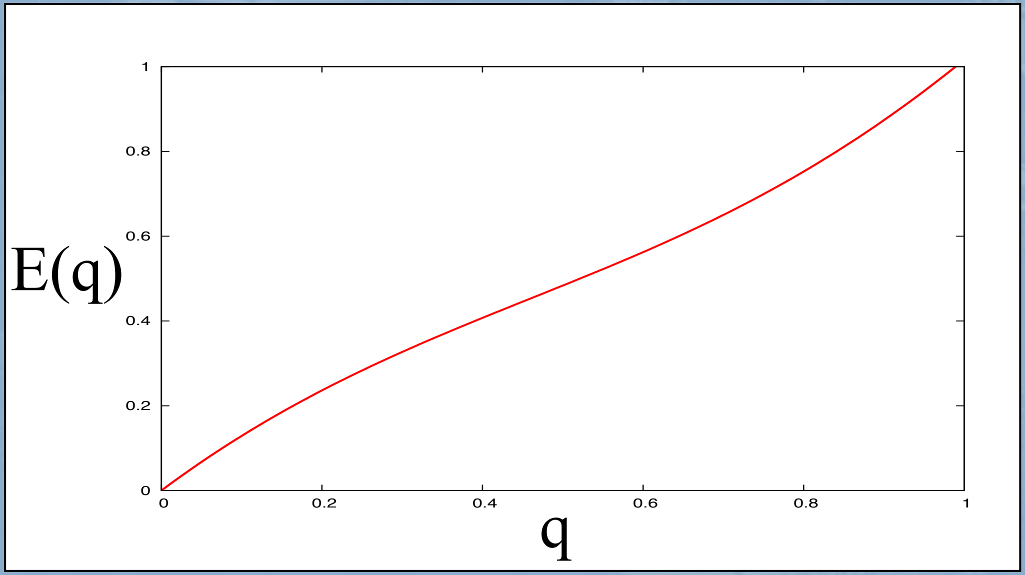


Two-dimensional dipolar condensates

$$E(q) = q\left[q^2 + 2\left(g + g_d G(q)\right)\right]^{1/2} \xrightarrow{q\approx 0} E(q) = q\left[2\left(g + g_d\right)\right]^{1/2}$$

As long as $g+g_d>0$ the sound velocity is real

If $g>0$, $E(q)$ always real and monotonously growing with q

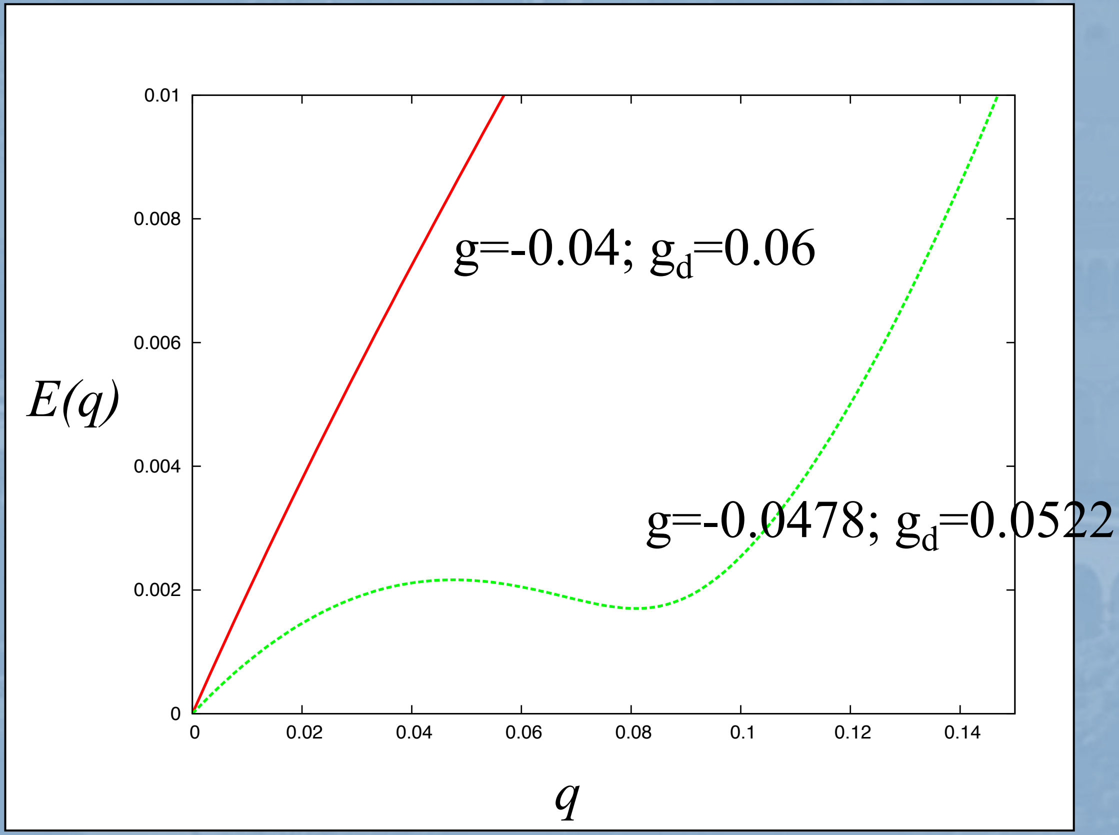


$$E^2(q) = q^2\left[q^2 + 2g_d G(q)\right]$$

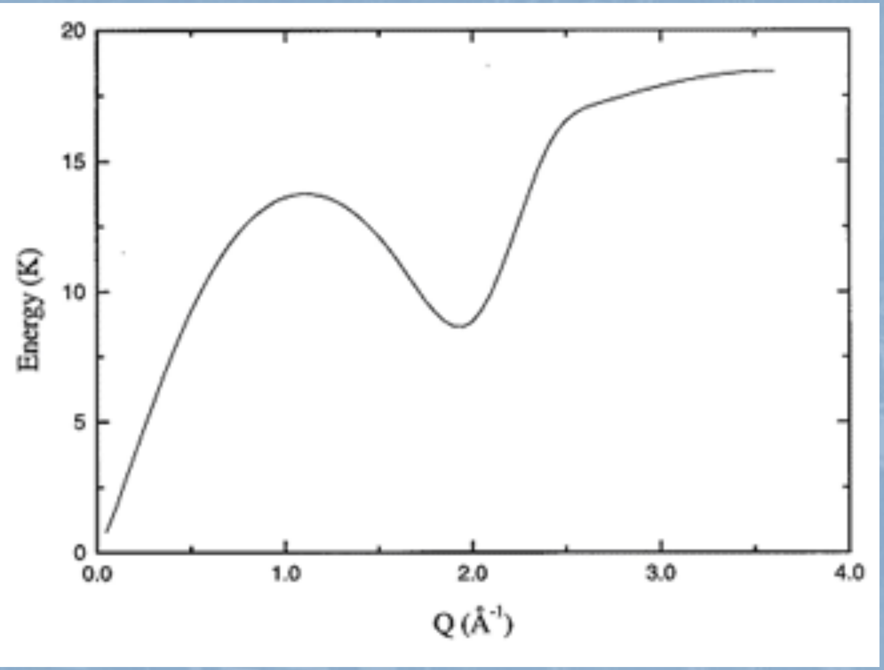
Roton-like minimum in 2D dipolar condensates

If g<0 $\Rightarrow$ $E^2(q) = q^2\left[q^2 + 2\left(-|g| + g_d G(q)\right)\right]$

E(q) may develop a „roton-like“ minimum



Roton minimum in Helium-II



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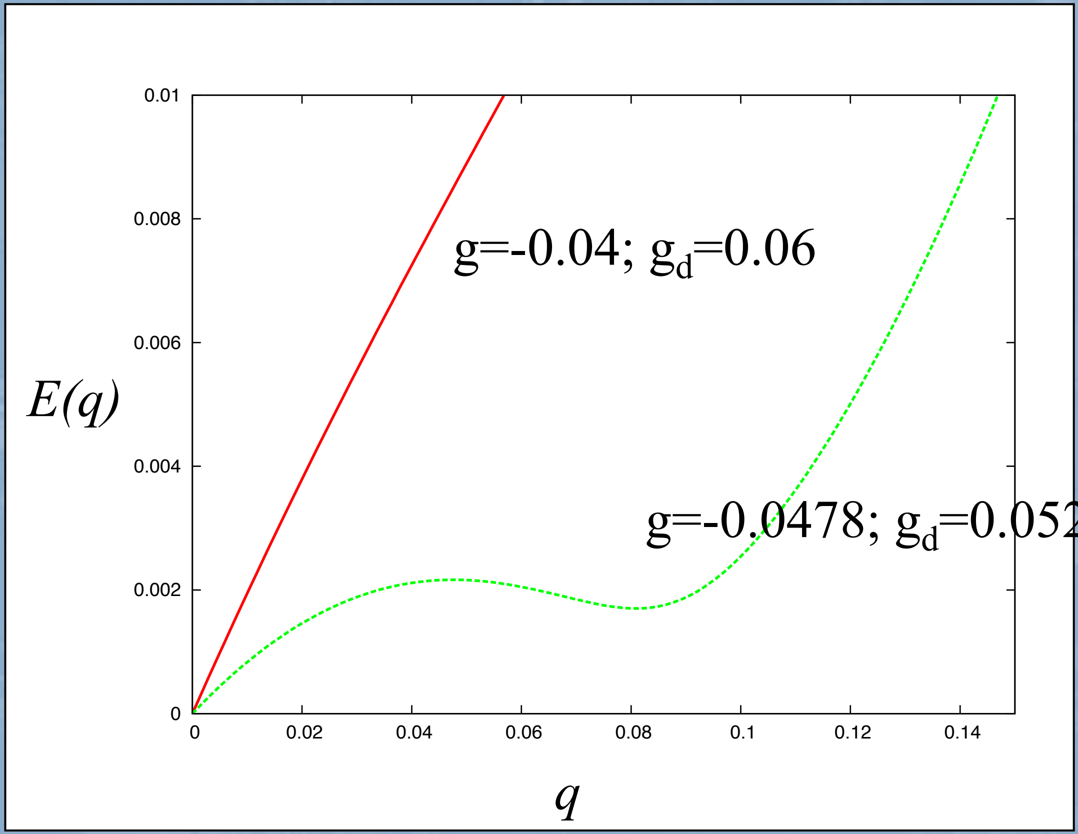
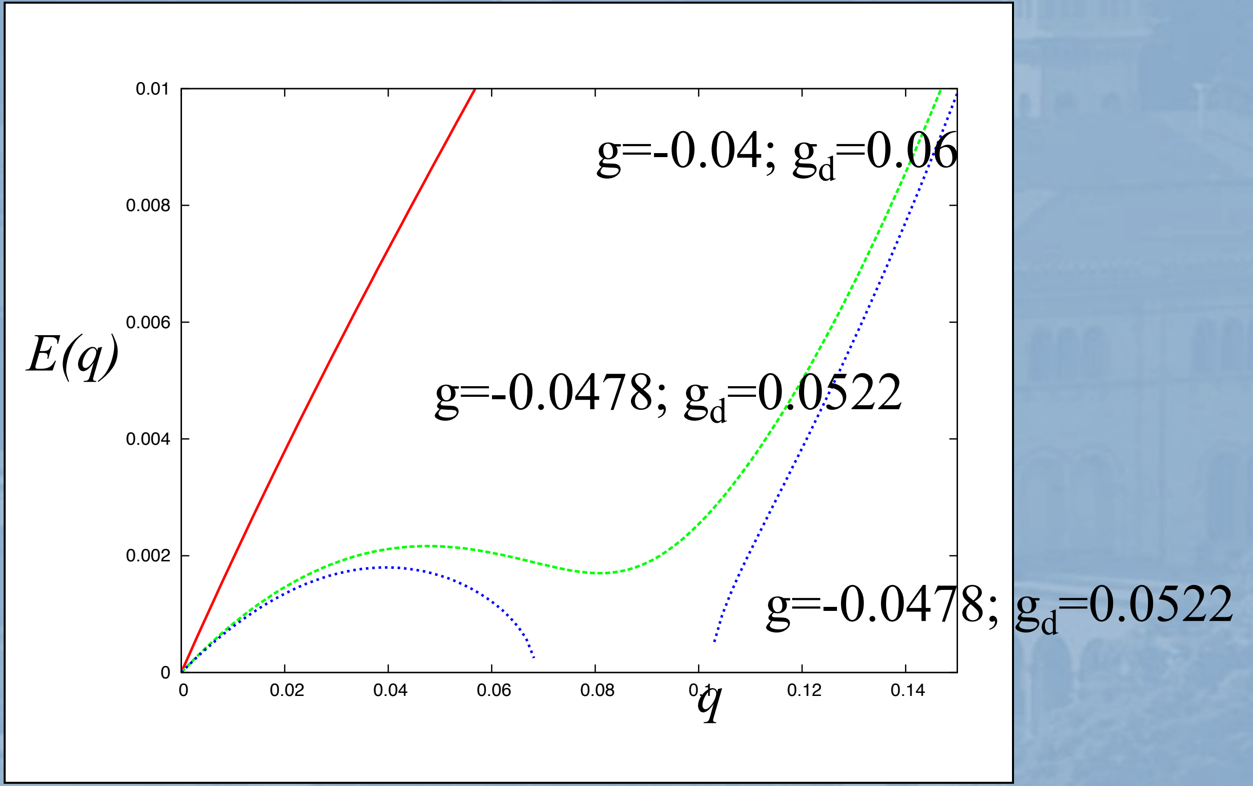


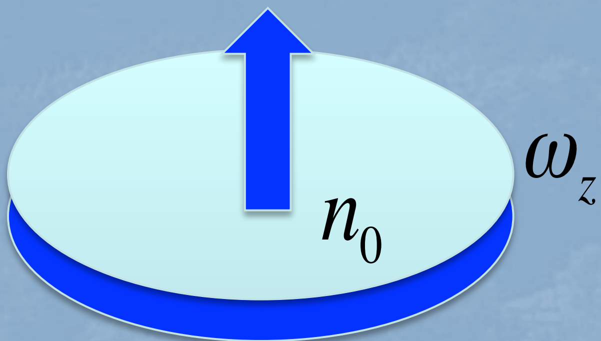
Roton instability

If gd/|g| is low enough the roton becomes unstable although low-q phonons are stable

Note that the dipole-dipole interaction stabilizes the system



Roton-like minimum in 3D (pancake) dipolar condensates

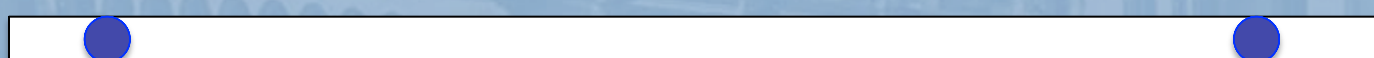


$$\mu > \hbar\omega_z \qquad g > 0$$

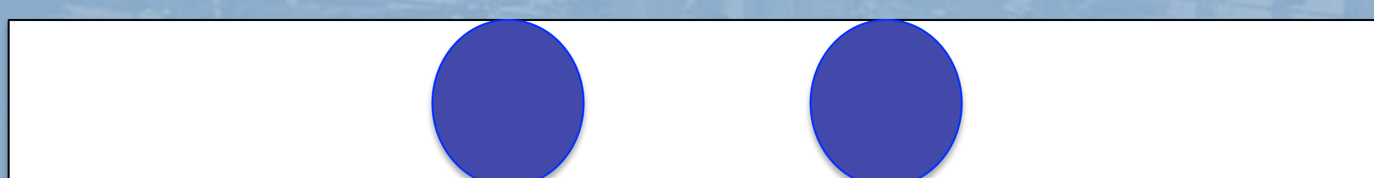
A roton-like minimum will appear now

Recall that in 2D: $E^2(q) = q^2\left[q^2 + 2\left(g + g_d G(q)\right)\right]$

distances >> l_z ql_z<<1

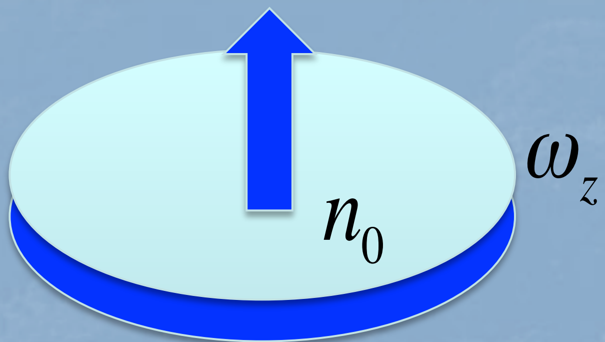


distances ~ l_z ql_z~1



Roton-like dispersion in 3D

[Santos et al., PRL **90**, 250403 (2003)]



$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + \frac{m\omega^2 z^2}{2} + g|\psi(\boldsymbol{r},t)|^2 + \int d\boldsymbol{r}' U_{dd}(\boldsymbol{r}-\boldsymbol{r}')|\psi(\boldsymbol{r}',t)|^2\right]\psi(\boldsymbol{r},t).$$

$$\psi(\vec{r},t) = \psi_0(z)\sqrt{n_0}\,e^{-i\mu t/\hbar}$$

$$\left[\frac{-\hbar^2}{2m}\frac{d^2}{dz^2} + \frac{m\omega^2 z^2}{2} + (g+g_d)|\psi_0|^2 - \mu\right]\psi_0(z) = 0$$

$$\psi(\boldsymbol{r},t) = \psi_0(z) + u(z)e^{i\boldsymbol{q}\cdot\boldsymbol{\rho}}e^{-i\omega t} + v^*(z)e^{-i\boldsymbol{q}\cdot\boldsymbol{\rho}}e^{-i\omega t}$$

$$\hbar\omega f_-(z) = \hat{H}_{kin} f_+(z),$$
$$\hbar\omega f_+(z) = \hat{H}_{kin} f_-(z) + \hat{H}_{int}[f_-(z)]$$

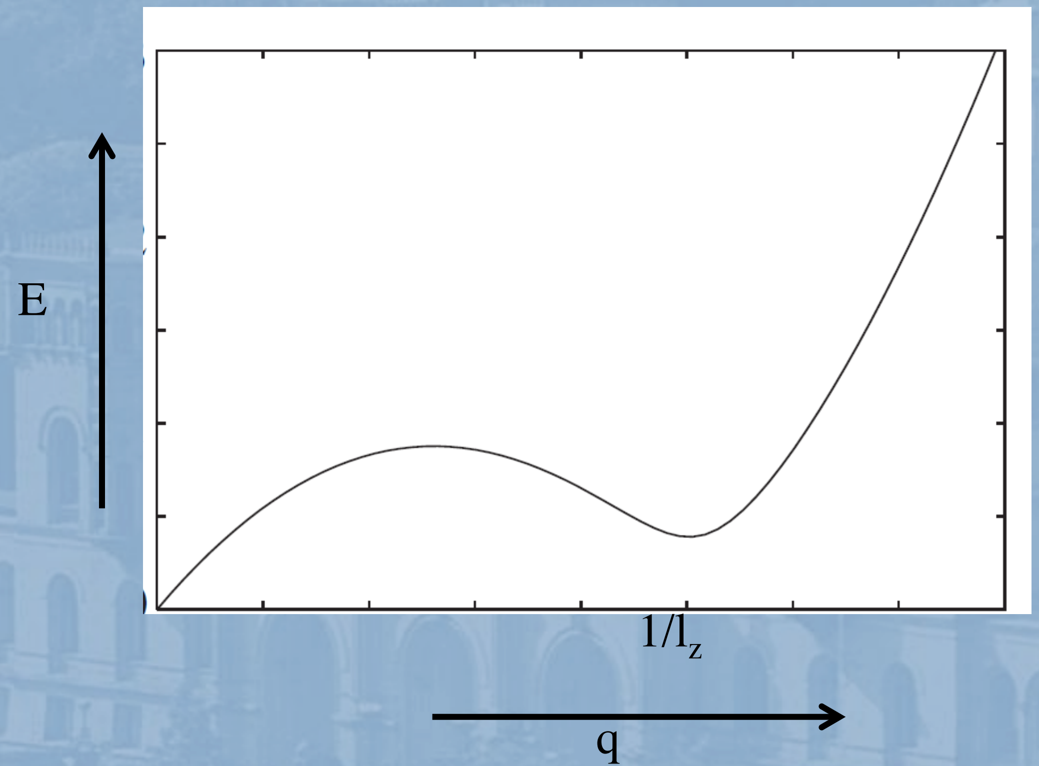
$$\hat{H}_{kin} = \frac{\hbar^2}{2m}\left[-\frac{d^2}{dz^2} + q^2 + \frac{\nabla^2\psi_0}{\psi_0}\right],$$

$$\hat{H}_{int}[f_-] = 2(g+g_d)\psi_0^2(z)f_-(z) - \frac{3}{2}qg_d\psi_0(z)\int_{-\infty}^{\infty} dz'\psi_0(z')\exp[-q|z-z'|]f_-(z')$$

Roton-like dispersion

A roton minimum appears in the vicinity of $ql_z \sim 1$

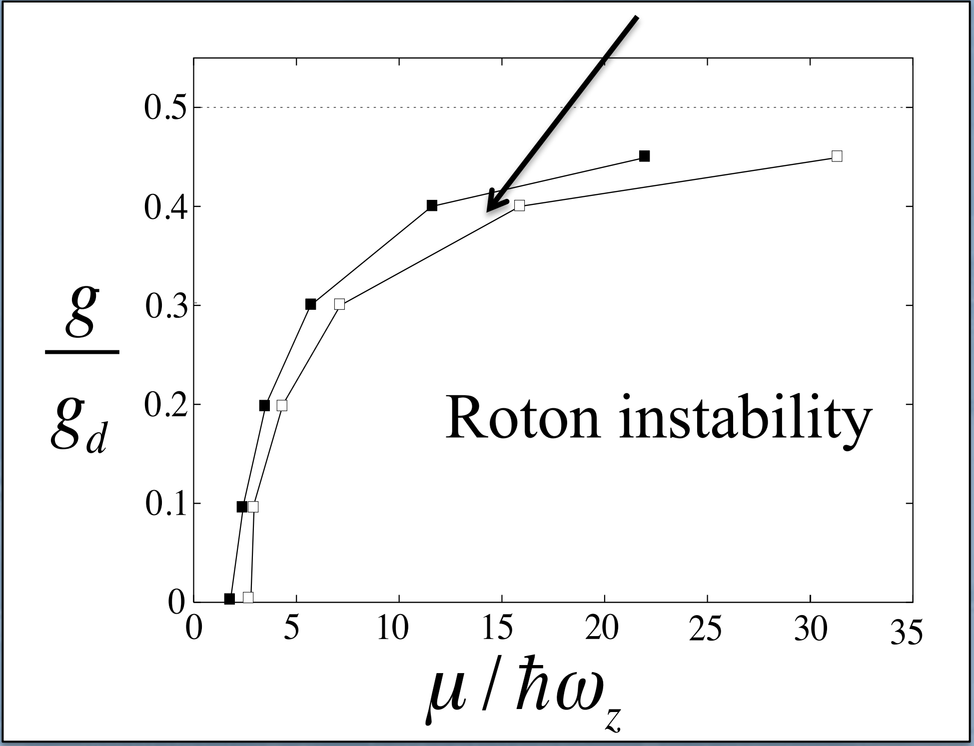
Roton instability may occur



Roton instability

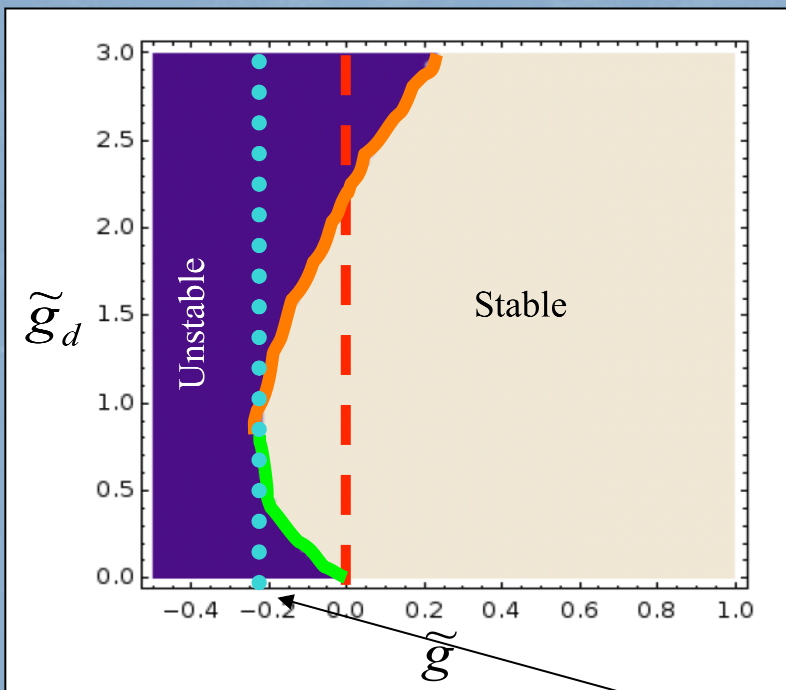
$$\varepsilon_{dd} = \frac{C_{dd}}{3g} > 1$$

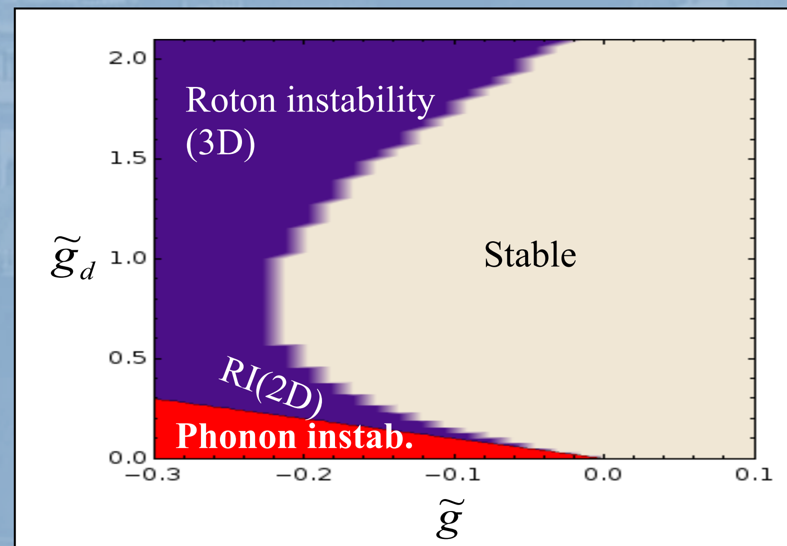
Rotonization



The dipole-dipole interaction destabilizes the system

Stability





Critical $\tilde{g} \equiv \dfrac{g n_0}{\sqrt{2\pi} l_z \hbar\omega_z} \cong -0.22$

Roton-like minimum (with g>0)

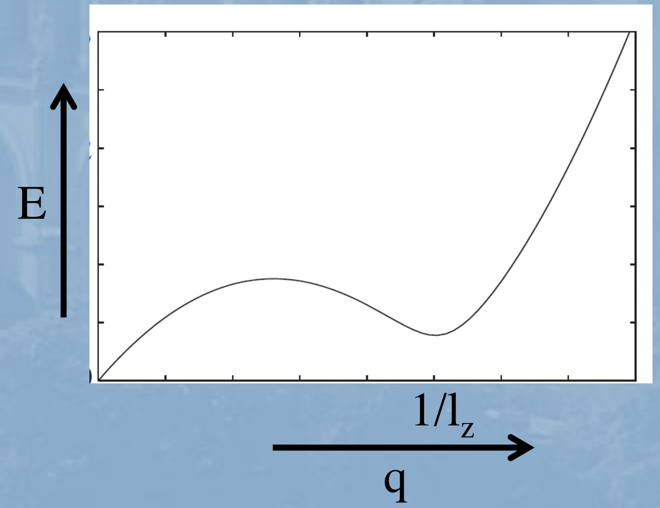
[Santos et al., PRL **90**, 250403 (2003)]

The roton-like minimum results from the q-dependence of the DDI

Conditions for a roton-like minimum (for g>0)

- Large-enough ratio DDI//contact
- Pancake traps (but in the 3D regime)

Roton-like minimum at $q_r \sim 1/l_z$



Roton-like dispersion: approximate expression

[Santos et al., PRL **90**, 250403 (2003)]

$$\psi(\vec{r},t) = e^{-i\mu t/\hbar}\left[\psi_0(\vec{r}) + u(\vec{r})e^{-i\varepsilon t/\hbar} - v(\vec{r})^* e^{i\varepsilon^* t/\hbar}\right] \qquad \psi_0(\vec{r}) = \sqrt{n_0\left[1 - z^2/L^2\right]}$$

$$f_\pm(\vec{r},t) = \left[u(\vec{r}) \pm v(\vec{r})\right] = e^{i\vec{q}\cdot\vec{\rho}} f_\pm(z)$$

$$f_+(z) = W(z)\psi_0(z)$$

$$qL >> 1$$

$$x \equiv \frac{z}{L} \qquad \beta \equiv \frac{g}{g_d}$$

$$\left[\frac{1}{2}(1-x^2)\frac{d^2W}{dx^2} - \left(1 + \frac{3}{2(1+\beta)}\right)x\frac{dW}{dx}\right]\hbar^2\omega^2 +$$

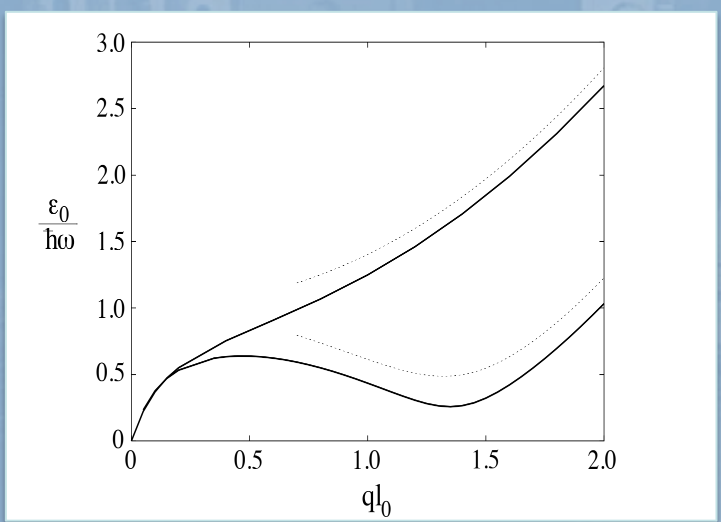
$$\left[\epsilon^2 - E_q^2 - \frac{2\beta-1}{1+\beta}\mu E_q(1-x^2) - \frac{3\hbar^2\omega^2}{2(1+\beta)}\right]W = 0$$

Roton-like dispersion: approximate expression

[Santos et al., PRL **90**, 250403 (2003)]

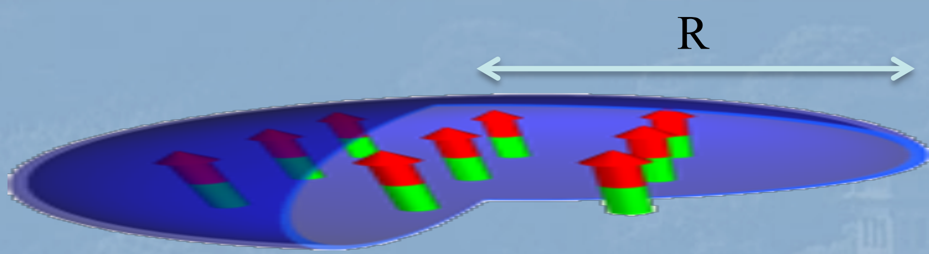
$$W(x) \approx 1 + \sum_{j>0} a_j C_j^{\lambda}(x) \qquad \lambda = (4+\beta)/2(1+\beta)$$

$$\varepsilon^2 W(x) = \left[\left(\frac{\hbar^2 q^2}{2m} \right)^2 - \frac{(1-2\beta)(5+2\beta)}{3(1+\beta)(2+\beta)} \left(\frac{\hbar^2 q^2}{2m} \right) \mu + \hbar^2 \omega^2 \right] W(x)$$



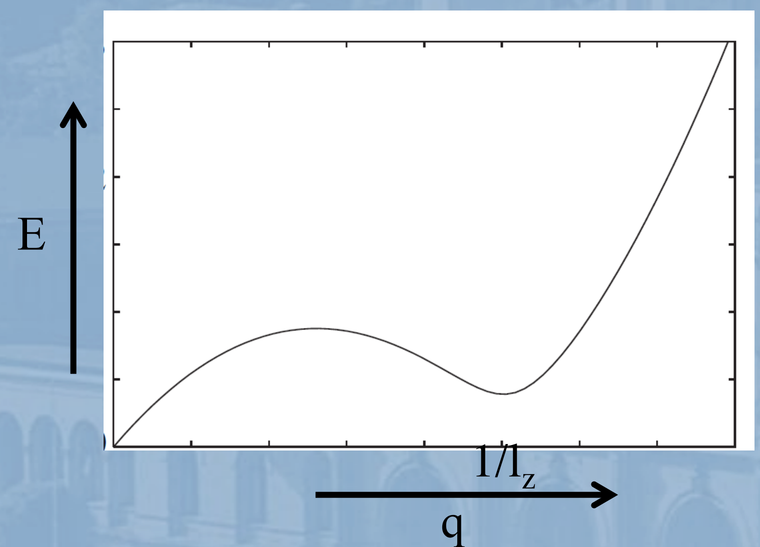
The roton properties depend on chemical potential and hence on the 2D density n_0

Pancake traps: local-density approximation



$$V(\vec{r}) = \frac{m}{2}\left(\omega^2\rho^2 + \omega_z^2 z^2\right)$$

$$\lambda \equiv \omega_z / \omega >> 1$$



$$q_r R \approx \frac{R}{l_z} >> 1$$

LDA is a good approximation
Although q_r is rather large the minimum remains very density-dependent

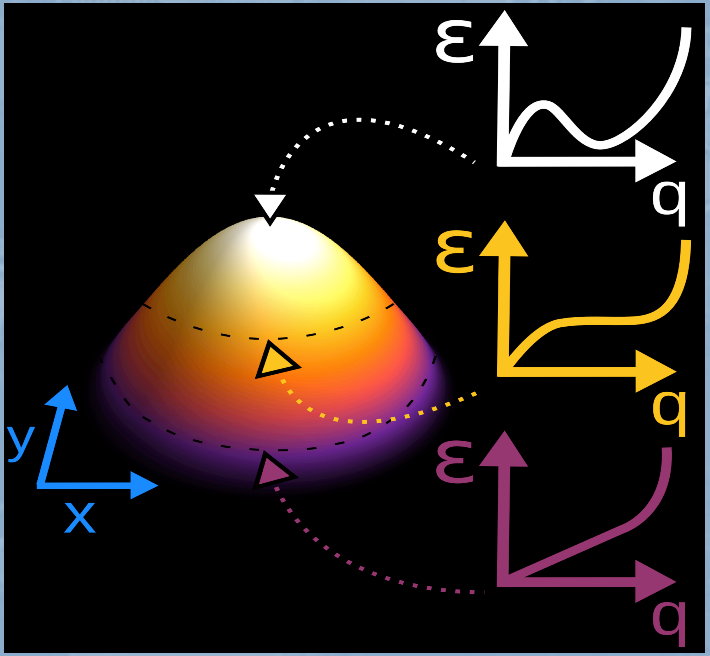
Local roton spectrum

[Jona-Lasinio, Lakomy and Santos, PRA **88**, 013619 (2013)]

The roton dispersion in trapped BECs has a local character for sufficiently pancake traps

$$\varepsilon(q,\rho)^2 \approx \left(\frac{\hbar^2 q^2}{2m}\right)^2 - F\left(\frac{g_d}{g}\right)\left(\frac{\hbar^2 q^2}{2m}\right)\mu(\rho) + (\hbar\omega)^2$$

$$\mu(\rho) = \mu_0\left(1 - \rho^2 / R^2\right)$$



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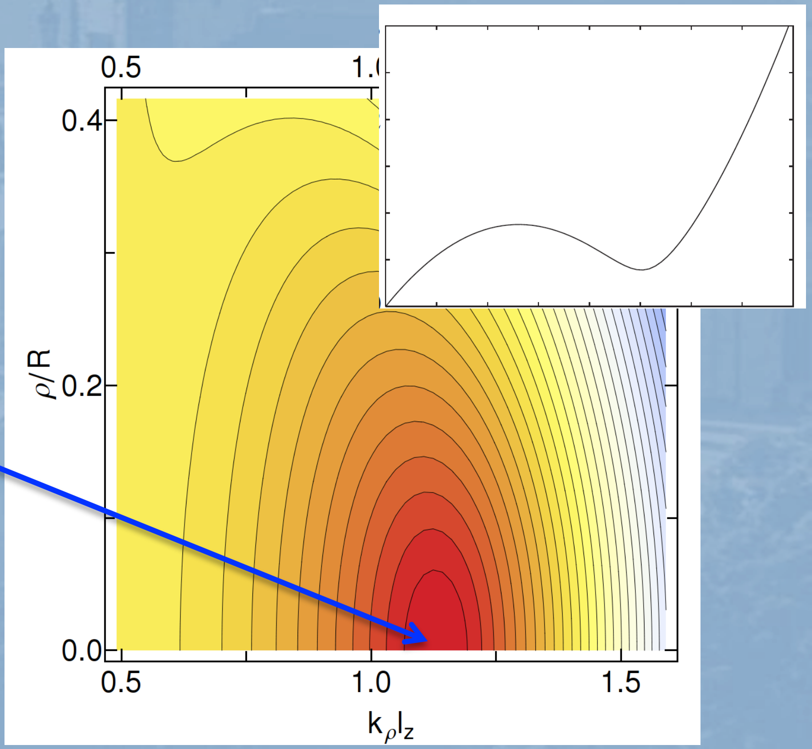
Minimum in both q and ρ

Roton confinement!



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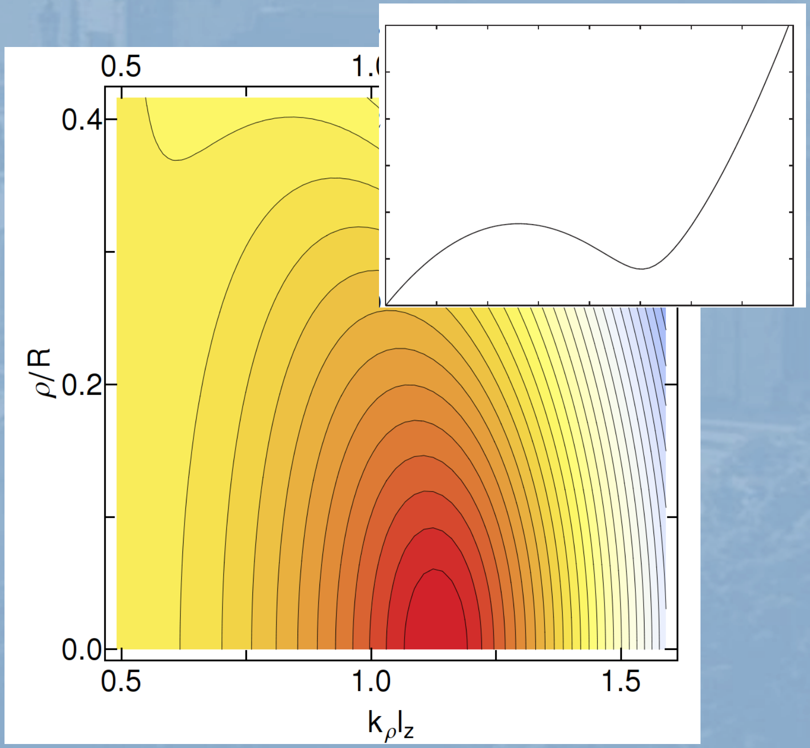
$$\mu(\rho) = \mu_0\left(1 - \rho^2 / R^2\right)$$

$$\varepsilon(q,\rho)^2 \approx \varepsilon_r^2 + \frac{\hbar^2}{2m_*}(q-q_r)^2 + \frac{1}{2}m_*\omega_*^2\rho^2$$

Roton localization length $l_* \equiv \sqrt{\frac{\hbar}{m_*\omega_*}} \cong 2^{1/4}\sqrt{\frac{R}{q_r}} << R$



Local roton spectrum

[Jona-Lasinio, Lakomy and Santos, arXiv:1301.4907]

$$\psi_0(\vec{r}) = \sqrt{n_0\left[1 - \rho^2/R^2 - z^2/L^2\right]}$$

$$f_\pm(\vec{r},t) \approx F(\vec{\rho})W(x)\psi_0(\vec{r})$$

$F(\vec{\rho})$ narrowly peaked around q_r

$$\varepsilon^2 W(x) \approx \left[\varepsilon_r^2 + \frac{\hbar^2}{2m_*}(q-q_r)^2 + \frac{1}{2}m_*\omega_*^2\rho^2\right]W(x)$$

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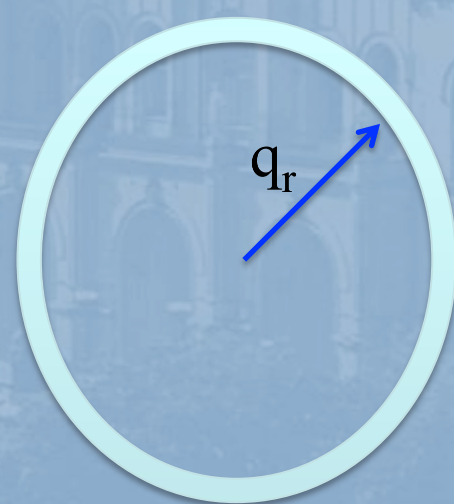
$$\varepsilon^2 F(\vec{\rho}) \approx \left[\varepsilon_r^2 + \frac{\hbar^2}{2m_*}(\hat{q} - q_r)^2 + \frac{1}{2}m_*\omega_*^2\rho^2\right]F(\vec{\rho})$$

Rashba-like dispersion like trapped BECs with spin-orbit coupling



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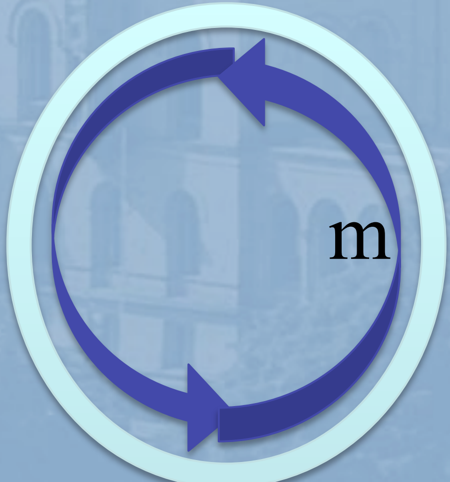
$$\varepsilon_{ns}^2 \approx \varepsilon_r^2 + \left(\frac{(m^2-1/4)}{2(q_r l_*)^2} + n + \frac{1}{2}\right)\hbar\omega_*$$

Lowest localized roton states

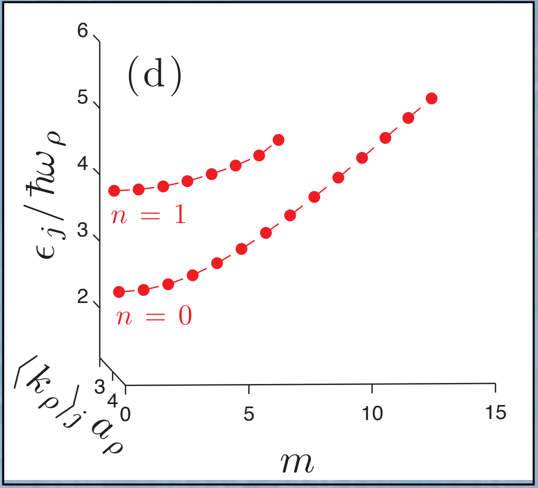
$$F(\vec{\rho}) = \Psi_{0m}(\rho,\phi) \propto e^{im\phi}J_m(q_r\rho)e^{-\rho^2/2l_*^2}$$

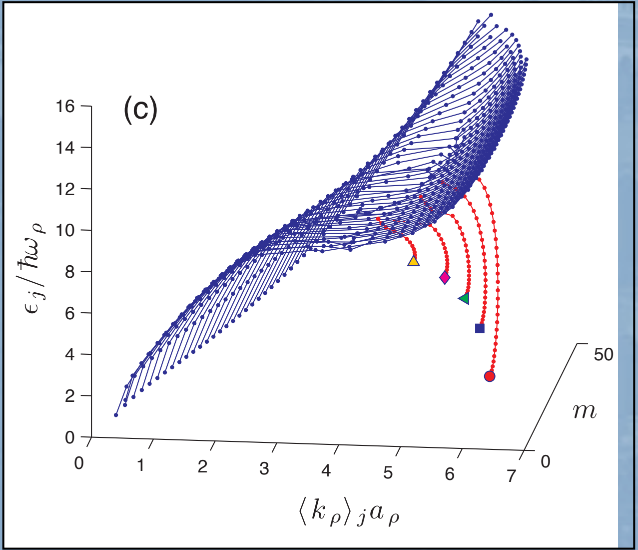


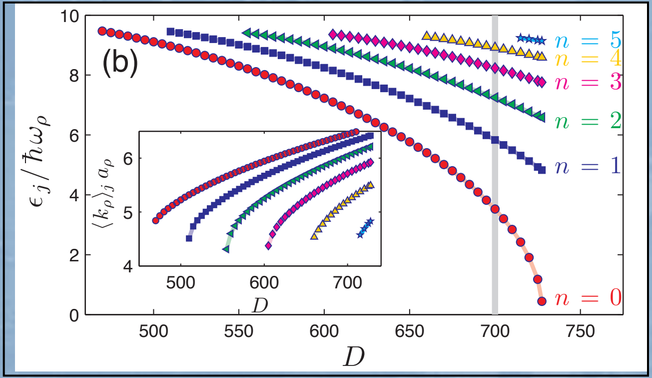
Roton spectrum of a trapped dBEC

[Bisset, Baillie, and Blakie, PRA **88**, 043606 (2013)]

Numerical spectra match very well with the behavior predicted by the local spectrum







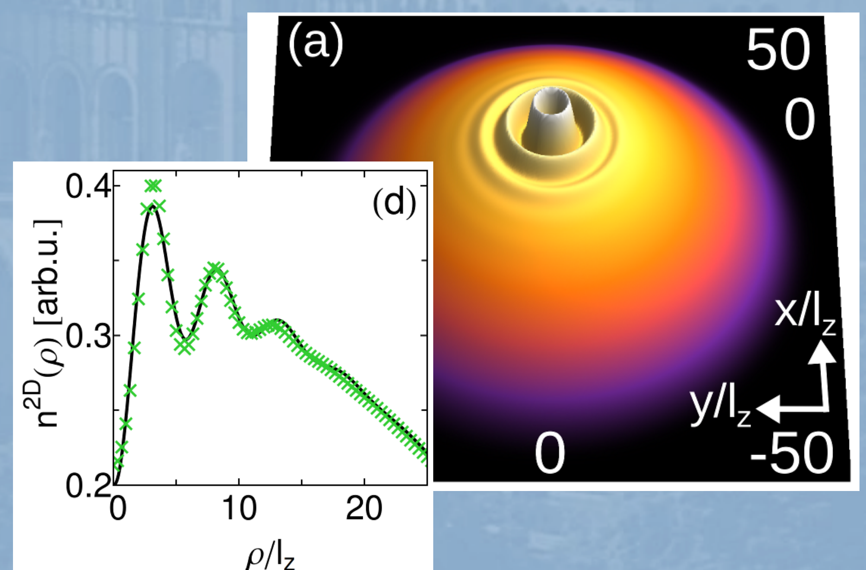
Roton modulational instability

[Jona-Lasinio, Lakomy and Santos, arXiv:1301.4907]

The localized roton states $\Psi_{0s}(\rho,\phi) \propto e^{is\phi} J_s(q_r\rho) e^{-\rho^2/2l_*^2}$ play a crucial role in the roton instability

$a_i > a_{cr}$ (Roton-stable) → $a_f < a_{cr}$ (Roton-unstable)

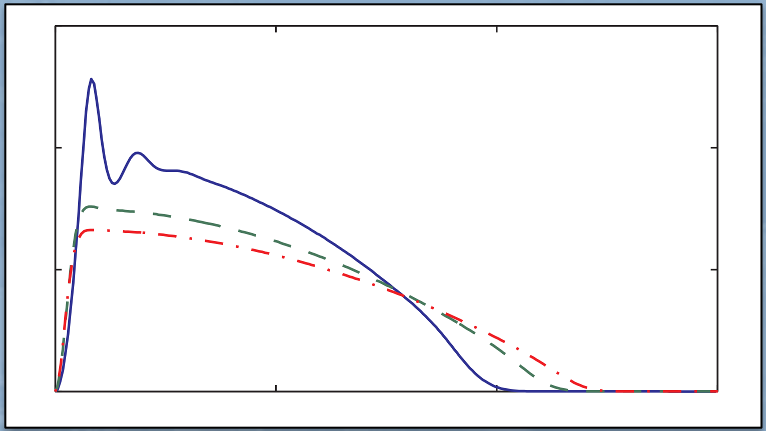
$$\frac{n(\vec{r},t) - n_0(\vec{r})}{n_0(\vec{r})} \approx e^{|\varepsilon| t/\hbar} \mathrm{Re}\left[\psi_{0m}(\vec{\rho})\right]$$



Consequences of the roton spectrum

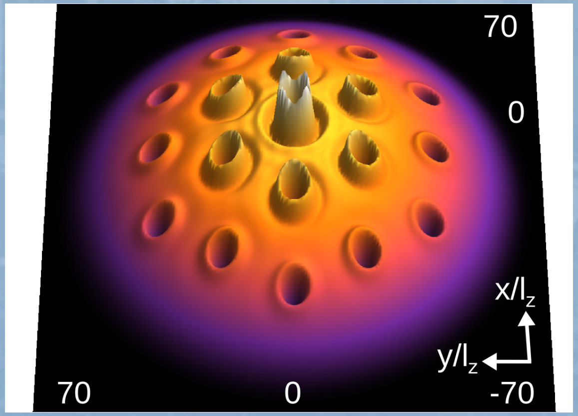
A deep roton leads to an enhanced susceptibility against the creation of density modulations with the roton wavelength, when imposing a perturbation (e.g. a vortex)

[Yi and Pu, PRA **73**, 061602(R) (2006).
Wilson, Ronen, Bohn, and Pu, PRL **100**, 245302 (2008)]



From [Yi and Pu, PRA **73**, 061602(R) (2006)]

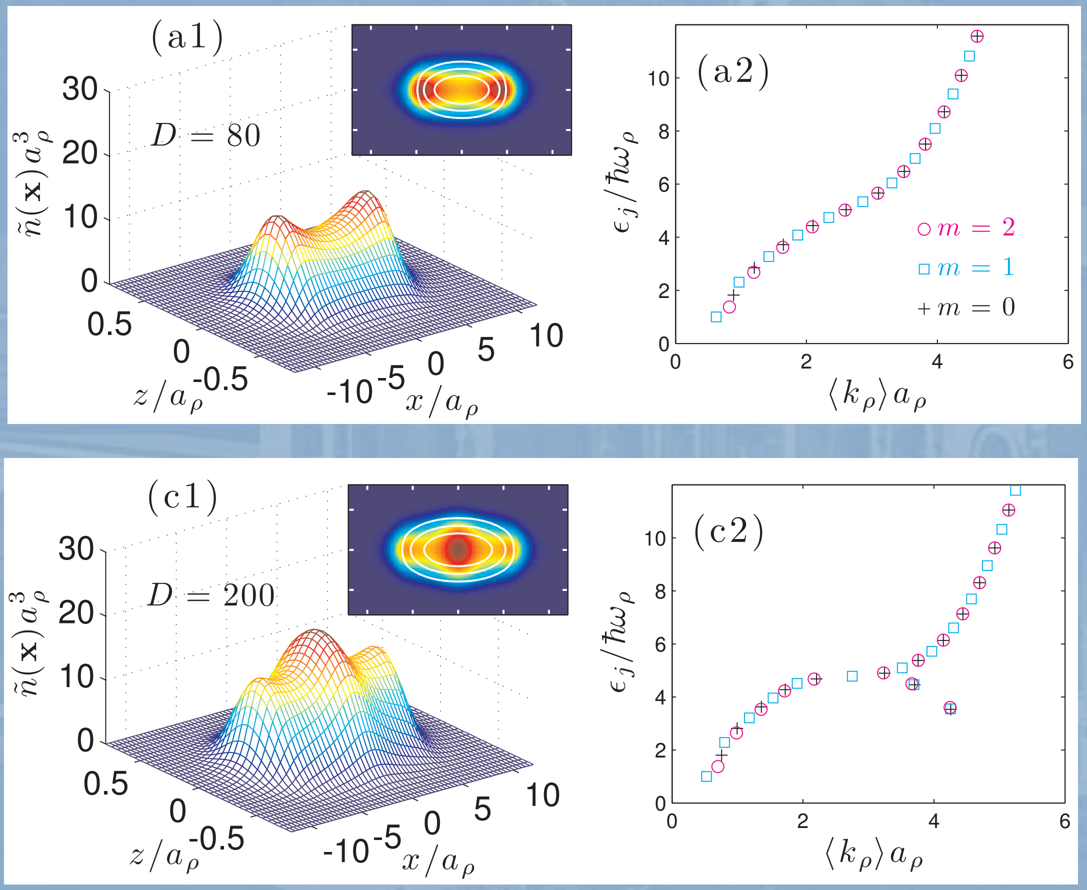
Since the roton depth is local the susceptibility is also local. This may be seen e.g. in vortex lattices.



Local depletion

[Blakie, Baillie, and Bisset, PRA **88**, 013638 (2013)]

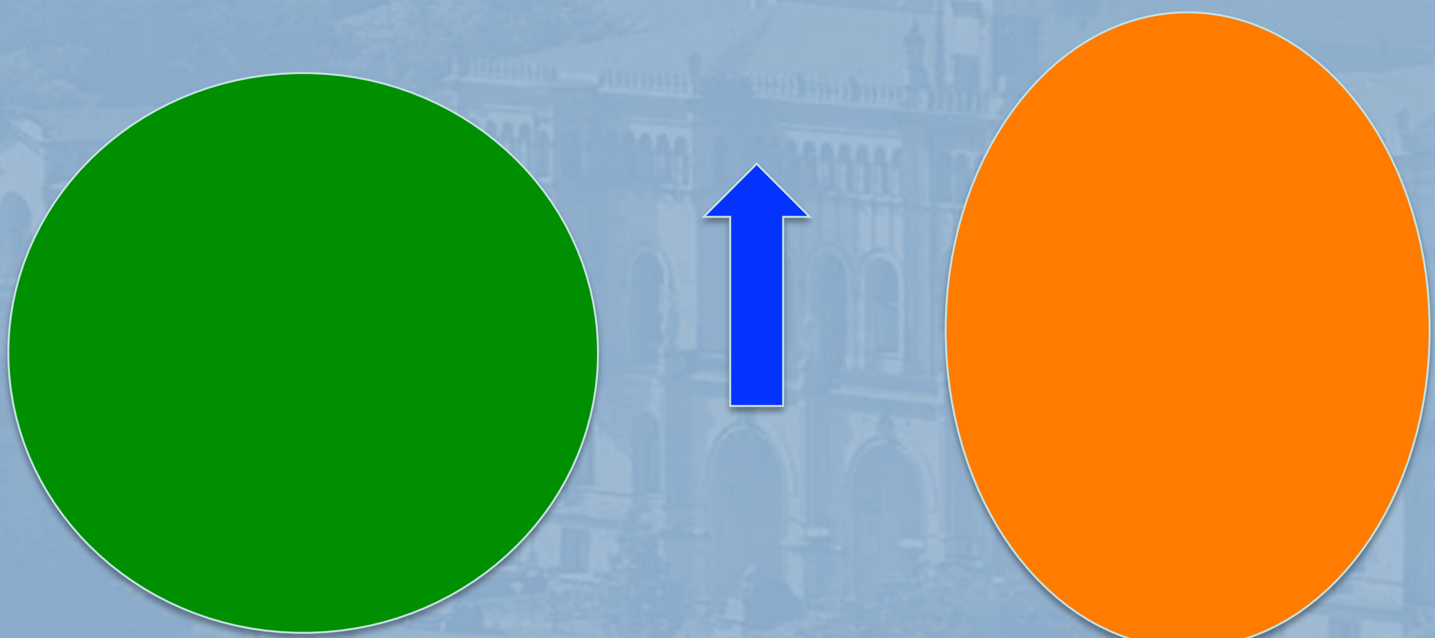
$$\sum_j \left[\bar{n}_j |u_j(\mathbf{x})|^2 + (\bar{n}_j + 1)|v_j(\mathbf{x})|^2\right]$$



Dipolar Fermi gases: Deformation of the Fermi surface

For short-range interacting Fermi gases the Fermi surface is isotropic

What happens in a dipolar Fermi gas?



Dipolar Fermi gases: Deformation of the Fermi surface

[Miyakawa, Sogo, and Pu, PRA **77**, 061603 (2008)]

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \int \frac{d^3k}{(2\pi)^3} f\left(\frac{\boldsymbol{r}+\boldsymbol{r}'}{2},\boldsymbol{k}\right) \mathrm{e}^{i\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}')}$$

$$\begin{aligned} n(\boldsymbol{r}) &= \rho(\boldsymbol{r},\boldsymbol{r}) = (2\pi)^{-3}\int d^3k\, f(\boldsymbol{r},\boldsymbol{k}) \\ \tilde{n}(\boldsymbol{k}) &= (2\pi)^{-3}\int d^3r\, f(\boldsymbol{r},\boldsymbol{k})\ . \end{aligned}$$

$$E_{kin} = \int d^3k\ \tilde{n}(\vec{k})\frac{\hbar^2 k^2}{2m}$$

$$E_{trap} = \int d^3r\ n(\vec{r})U(\vec{r})$$

$$E_{DDI;direct} = \frac{1}{2}\iint d^3r d^3r'\ n(\vec{r})n(\vec{r}\,')V_{ddi}(\vec{r}-\vec{r}\,')$$

$$E_{DDI;exchange} = -\frac{1}{2}\iint d^3r d^3r' \left|\rho(\vec{r},\vec{r}\,')\right|^2 V_{ddi}(\vec{r}-\vec{r}\,')$$

Deformation of the Fermi surface (homogeneous system)

[Miyakawa, Sogo, and Pu, PRA **77**, 061603 (2008)]

$$f(\vec{r},\vec{k}) = f(\vec{k})$$

$$E_{kin} = V\int \frac{d^3k}{(2\pi)^3} f(\vec{k}) \frac{\hbar^2 k^2}{2m}$$

$$n(\vec{r}) = n_f$$

$$E_{DDI;exchange} = -\frac{V}{2}\iint \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3} f(\vec{k}) f(\vec{k}') V_{ddi}(\vec{k}-\vec{k}')$$

$$E_{DDI;exchange} = -\frac{V}{2}\iint \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3} f(\vec{k}) f(\vec{k}') \frac{C_{dd}}{3}\left(3\cos^2\theta_k - 1\right)$$

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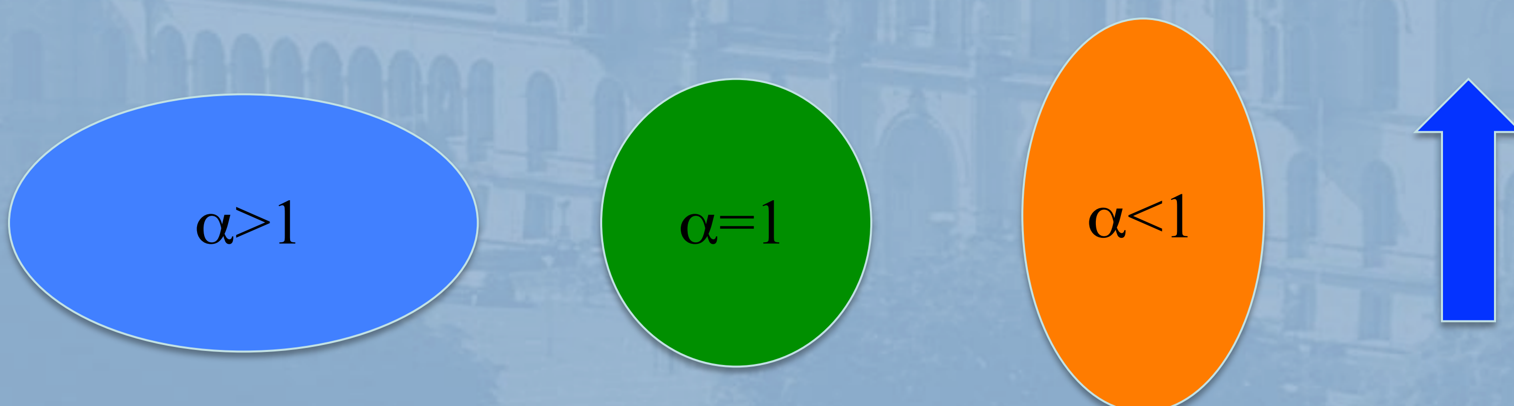
$$f(\boldsymbol{k}) = \Theta\left(k_F^2 - \frac{1}{\alpha}(k_x^2 + k_y^2) - \alpha^2 k_z^2\right)$$



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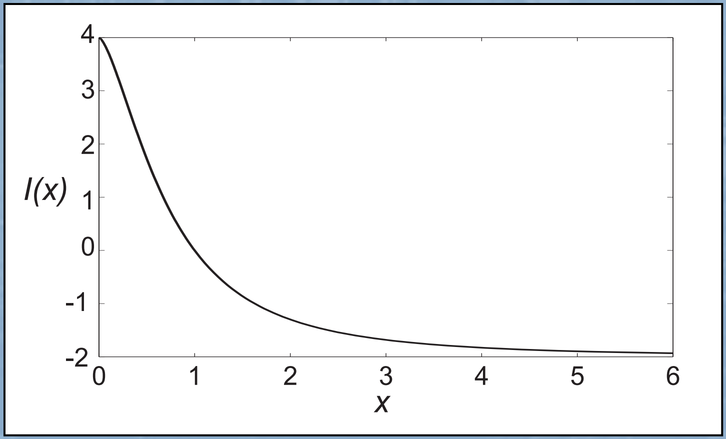
$$E_{kin} = \frac{V}{5}\frac{\hbar^2 k_F^2}{2m} n_f \left(\frac{1}{\alpha^2} + 2\alpha\right)$$

$$E_{DDI;exchange} = -V\frac{C_{dd}}{12} n_f^2 I(\alpha)$$



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$$E_{kin} = \frac{V}{5}\frac{\hbar^2 k_F^2}{2m} n_f \left(\frac{1}{\alpha^2} + 2\alpha\right)$$

prefers α=1

$$E_{DDI;exchange} = -V\frac{C_{dd}}{12} n_f^2 I(\alpha)$$

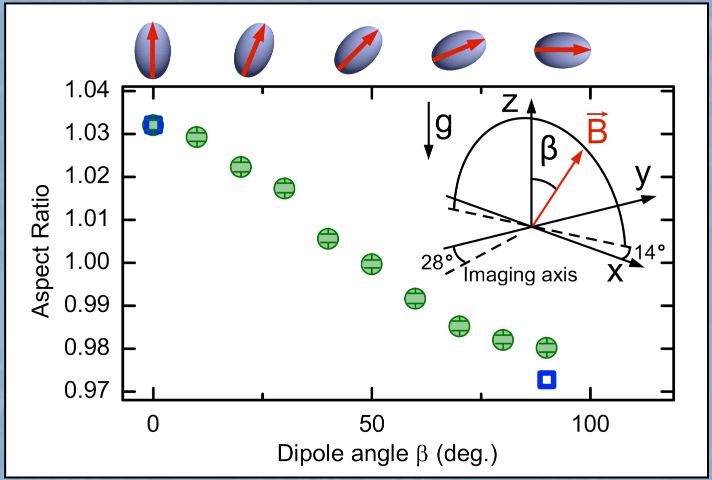
prefers α=0

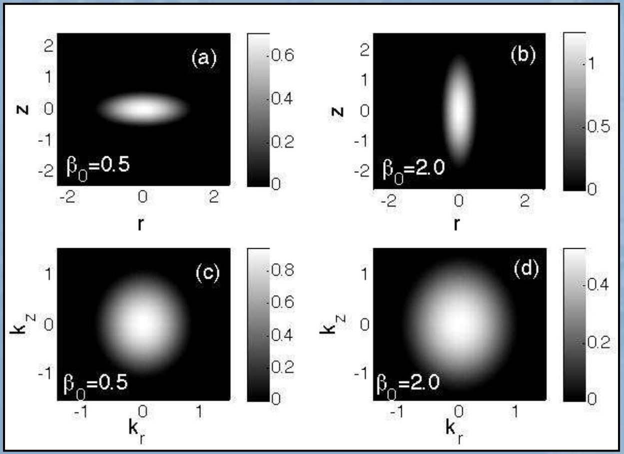
Deformation of the Fermi surface (trapped case)

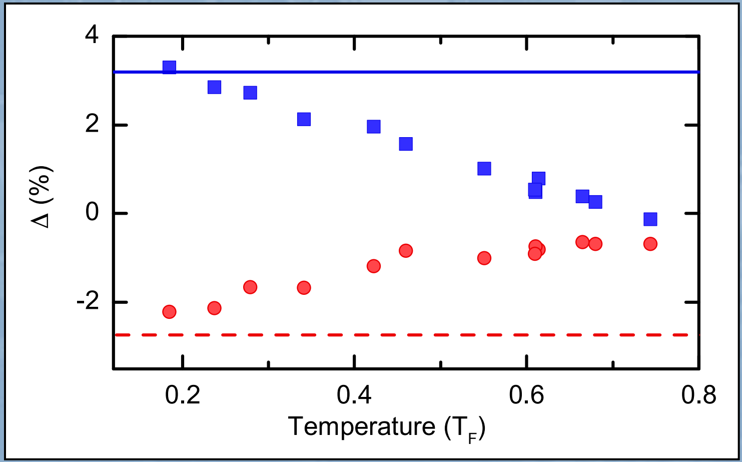
[Aikawa et al., arXiv:1405.2154]

Deformation $\sim k_F r_*$

$$r_* \equiv \frac{m}{\hbar^2}\left(\frac{C_{dd}}{4\pi}\right)$$







[Miyakawa, Sogo, and Pu, PRA **77**, 061603 (2008)]