Spinor Bose gases

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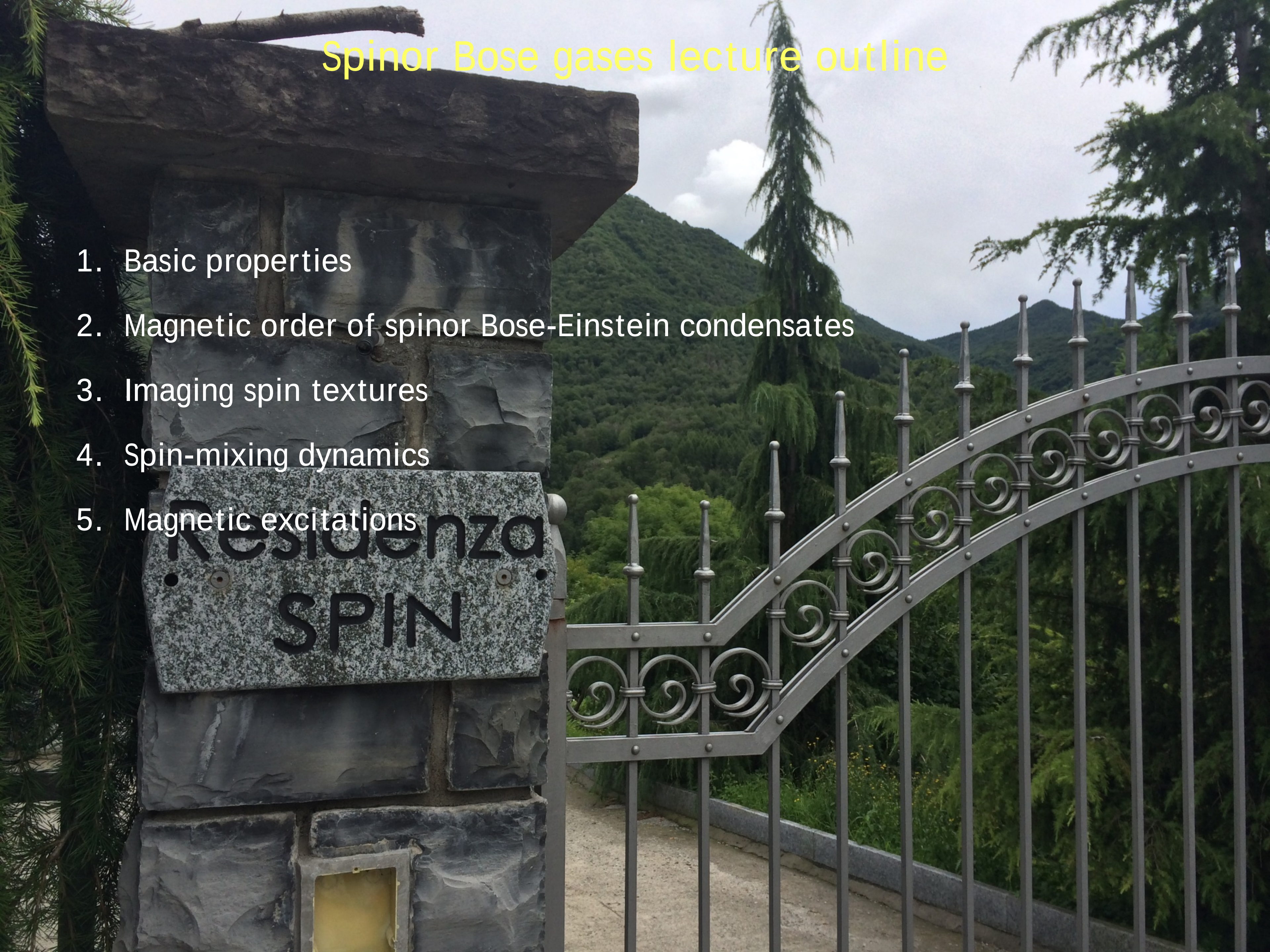
Spinor Bose gases lecture outline

1. Basic properties
2. Magnetic order of spinor Bose-Einstein condensates
3. Imaging spin textures
4. Spin-mixing dynamics
5. Magnetic excitations



1. Basic properties

a. atomic species

b. rotationally symmetric interactions

c. stability against dipolar relaxation

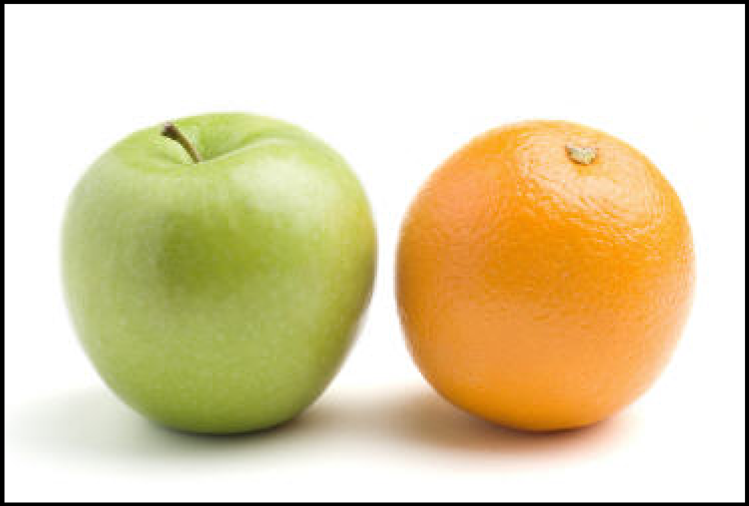
d. magnetic fields

The quantum fluids landscape

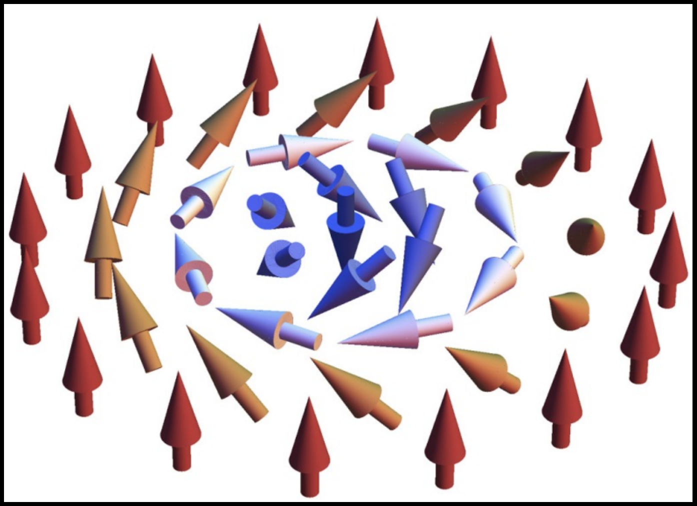
- pre-1995: a few quantum fluids.
  - ^{4}He: A scalar superfluid, incompressible, strongly interacting
  - Superconductors: Also scalar (mostly), charged (long-range interactions)
    - s-wave, d-wave, p-wave
  - 3He: Neutral BCS superfluid. Very interesting

- since 1995: A bonanza of quantum fluids!
  - atoms, molecules
  - bosons, fermions
  - resonant and tunable pairing
  - lots of "stable" internal states

→ Multistate quantum fluids with multicomponent order parameter



vs.



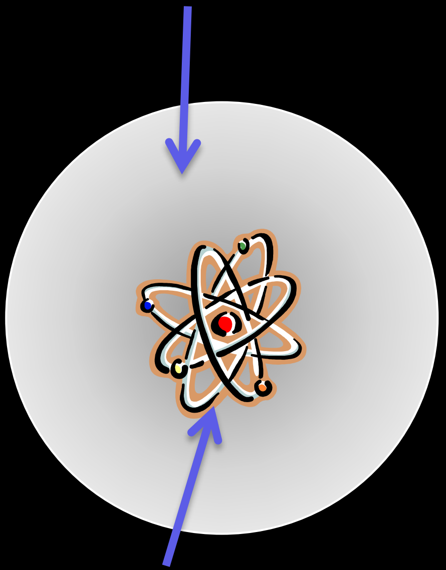
- States related by some accessible transition
- "Non-trivial" → (near) degeneracy of low-energy states
  - fine tuning or high degree of symmetry ← spatial and/or spin rotation
- Allowable dynamics

choice: components of quantum fluid come from an angular momentum manifold

Alkali spinor gases

e.g. 87Rb

37 electrons:
(36 core + 1 valence)
S = 1/2

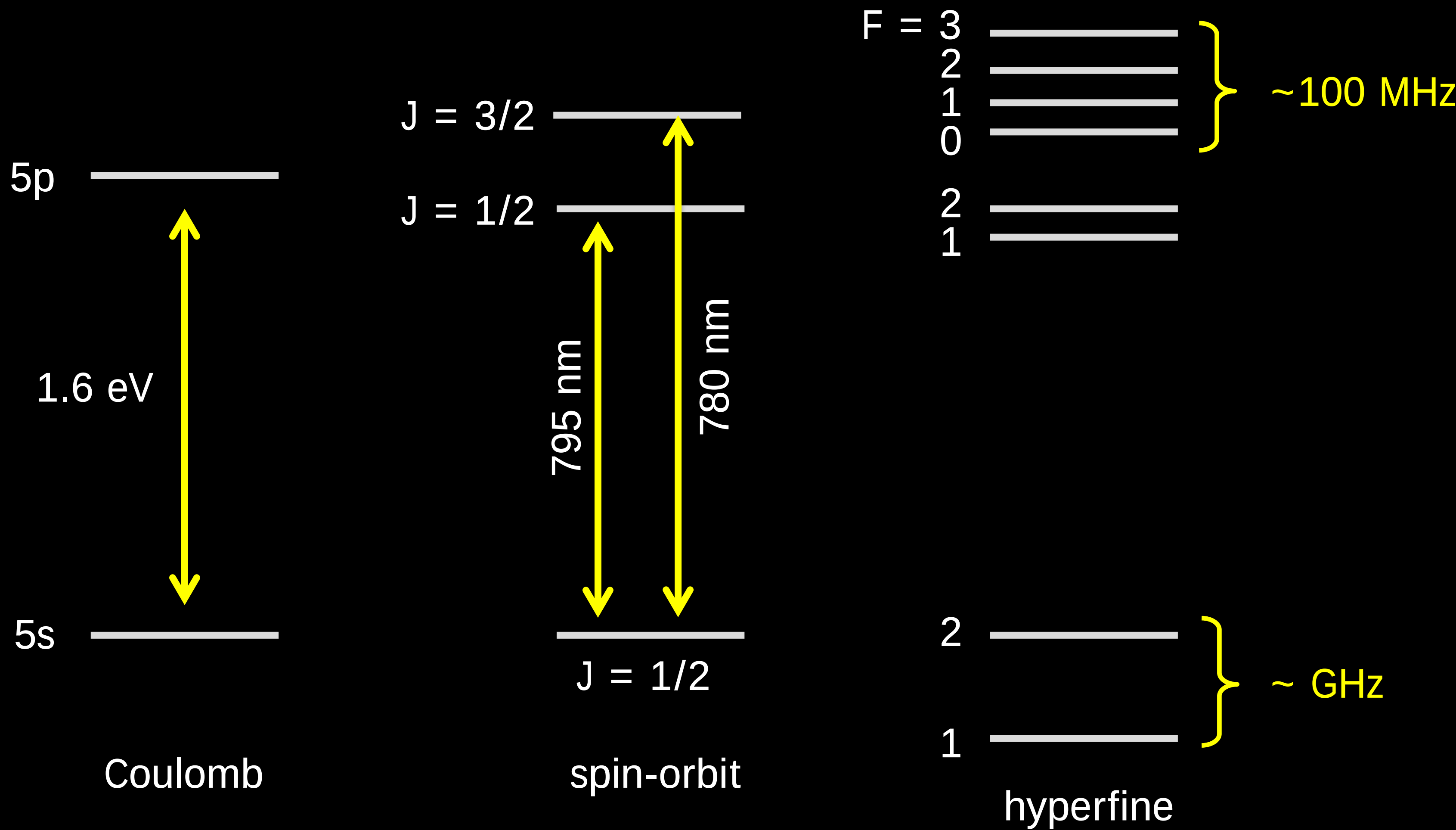


37 protons
50 neutrons
I = 3/2

$$\vec{F} = \vec{I} + \vec{J}$$

Alkali spinor gases

e.g. 87Rb

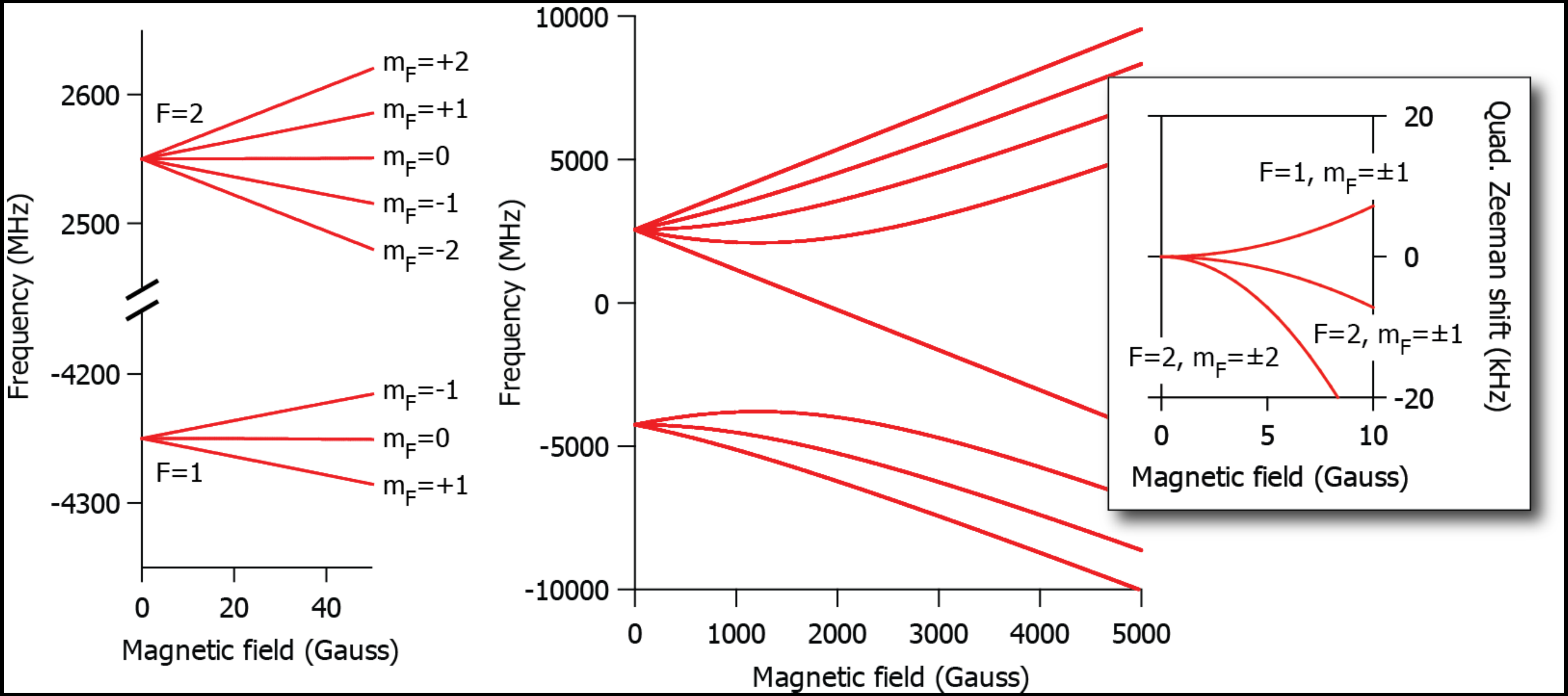


$$F = I + J$$

Breit-Rabi diagram

$$H_{hf} = ah\, I \cdot J \; - \mu \cdot B \qquad \mu = -g_J \mu_B\, J + g_I \mu_n\, I$$

note: here spin operators are dimensionless



$$g_F \simeq 2\, \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} = \frac{\pm 1}{I + 1/2}$$

TABLE I. Experimental candidates for the study of ultracold spinor Bose gases. Species are divided according to whether they are stable at zero magnetic field (information on thulium is lacking), and whether the dipolar relaxation rate is small enough to allow the longitudinal magnetization ($\langle F_z \rangle$) to be conserved in an experiment. The nature of the spin-dependent contact interactions is indicated in parentheses (f: ferromagnetic, af: antiferromagnetic, cyc: cyclic or tetrahedral, ?:,unknown). Stable pseudo-spin-1/2 gases of ^{87}Rb are indicated, with states labeled with quantum numbers $|F, m_F\rangle$ having the same low-field magnetic moment.

Stable: $\langle F_z \rangle$ conserved	Stable: $\langle F_z \rangle$ not conserved	Unstable
^{7}Li, $F = 1$ (f)	^{52}Cr, $F = 3$ (not f)	^{7}Li, $F = 2$
^{23}Na, $F = 1$ (af)	Dy, $F = 8$ (?)	^{23}Na, $F = 2$
^{41}K, $F = 1$ (f)	Er, $F = 6$ (?)	^{39}K
^{87}Rb, $F = 1$ (f)		^{85}Rb
^{87}Rb, $F = 2$ (af or cyc)		^{133}Cs
^{87}Rb pseudospin:	Tm, $F = 4$ (?)	
$\|1, 0\rangle$, $\|2, 0\rangle$		
$\|1, \pm 1\rangle$, $\|2, \mp 1\rangle$		

DMSK and M. Ueda, Rev. Mod. Phys. **85**, 1191 (2013)

High-spin atoms

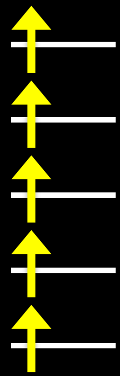
d-block

hydrogen 1 H 1.0079																		helium 2 He 4.0026
lithium 3 Li 6.941	beryllium 4 Be 9.0122												boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
sodium 11 Na 22.990	magnesium 12 Mg 24.305												aluminium 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
potassium 19 K 39.098	calcium 20 Ca 40.078		scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	zinc 30 Zn 65.39	gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80
rubidium 37 Rb 85.468	strontium 38 Sr 87.62		yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41	indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70 *	lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenium 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59	thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]
francium 87 Fr [223]	radium 88 Ra [226]	89-102 * *	lawrencium 103 Lr [262]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [266]	bohrium 107 Bh [264]	hassium 108 Hs [269]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]	unununium 111 Uuu [272]	ununbium 112 Uub [277]		ununquadium 114 Uuq [289]				

* Lanthanide series	lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
* * Actinide series	actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

f-block

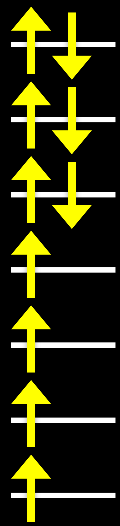
High-spin atoms



- Chromium

  configuration: $[Ar]4s^1\ 3d^5$ → S = 3; L = 0; I = 0; J = F = 3

  magnetic moment: $\mu = 6\ \mu_B$ (vs $\mu \leq 1\ \mu_B$ for alkalis)



- Dysprosium

  configuration: $[Xe]6s^2\ 4f^{10}$ → S = 2; L = 6; I = 0; J = F = 8

  magnetic moment: $\mu = 10\ \mu_B$

- Erbium

  configuration: $[Xe]6s^2\ 4f^{12}$ → S = 1; L = 5; I = 0; J = F = 6

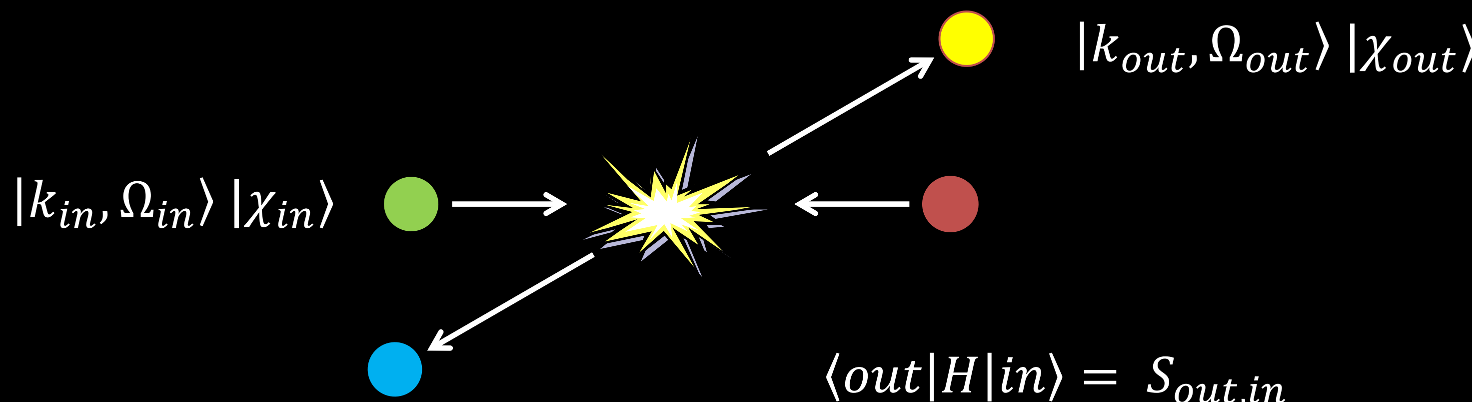
  magnetic moment: $\mu = 7\ \mu_B$

- Thulium

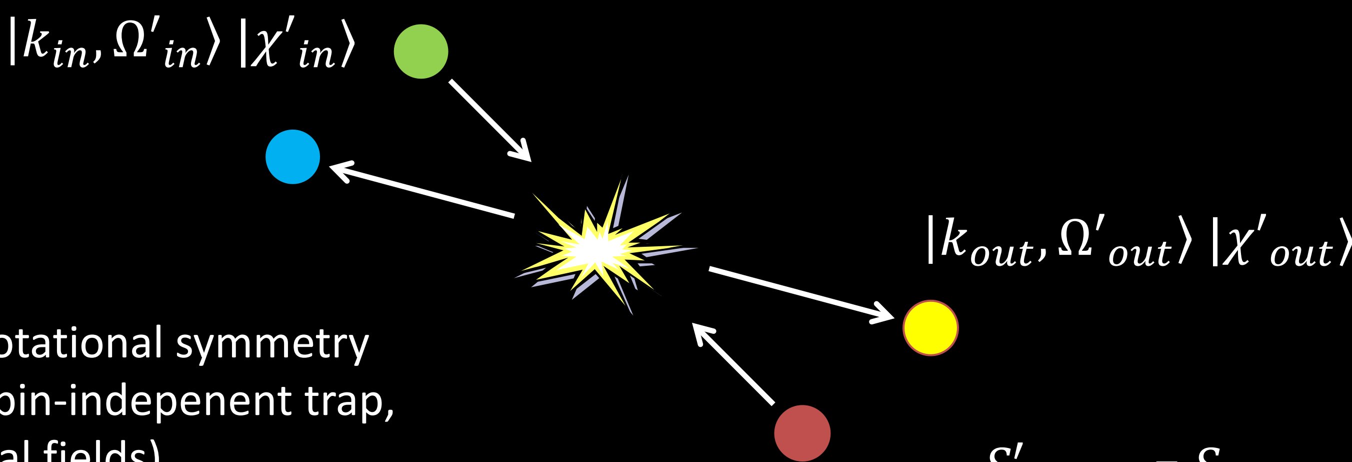
  configuration: $[Xe]6s^2\ 4f^{13}$ → S = 1/2; L = 3; I = 1/2; J = 7/2; F = 3

  magnetic moment: $\mu = 4\ \mu_B$

Rotationally symmetric interactions



$\langle out|H|in\rangle = S_{out,in}$

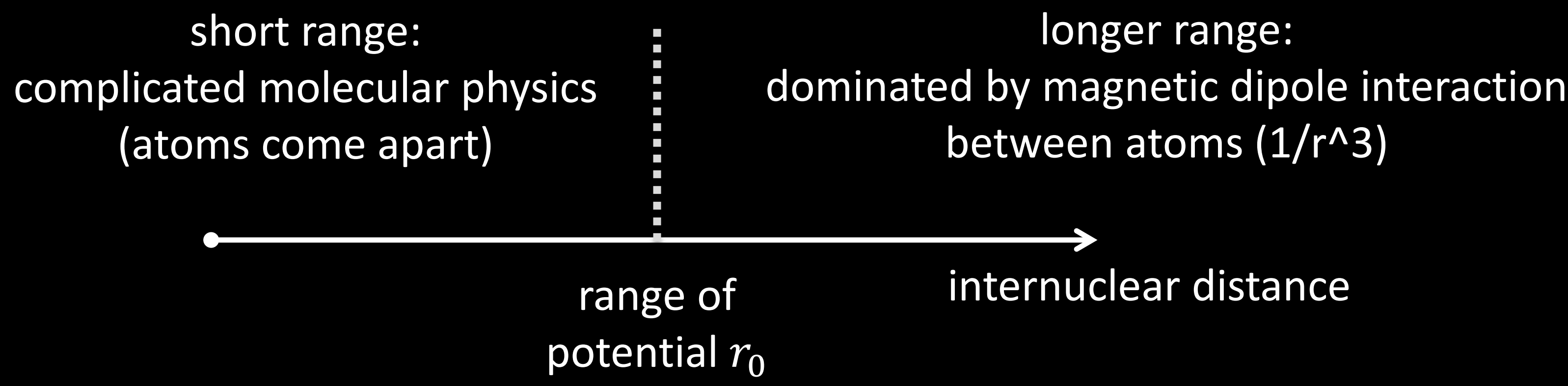


in case of rotational symmetry
(isotropic spin-indepenent trap,
zero external fields)

$S'_{out,in} = S_{out,in}$

Collisions: a series of approximations

- separate short-range and longer-range potential



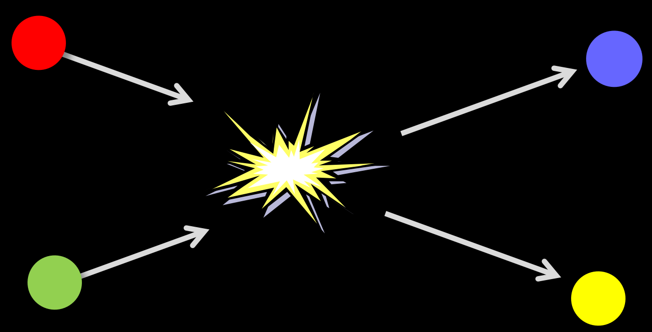
- ***incident*** low energy $\lambda_{dB} \ll r_0$
  - short-range potential gives s-wave scattering only
  - long-range has to be treated separately and carefully

- rotational symmetry
  - Total angular momentum (orbital + spin) of colliding pair is conserved

- weak dipolar interactions in short-range potential (not valid for all atoms!)
  - Spin angular momentum of colliding pair is separately conserved

Two spin-dependent interactions

contact interactions

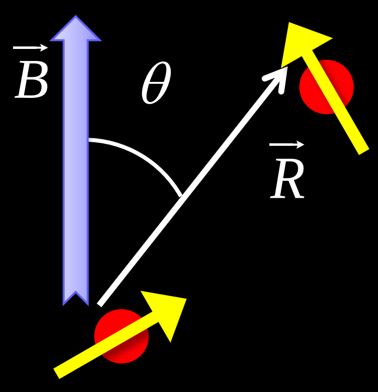


$$V = \sum_{pairs} \sum_{F_{tot}} \frac{4\,\pi\hbar^2 a_{Ftot}}{m}\ \delta(r)\ \hat{P}_{Ftot}$$

here, $F_{tot} \in \{0, 2, 4 \ldots 2\ F\}$

- symmetric under rotation in spin space
- valid in (low) magnetic field
  - Zeeman regime, away from F. resonance
  - spin conserved in Larmor precessing frame

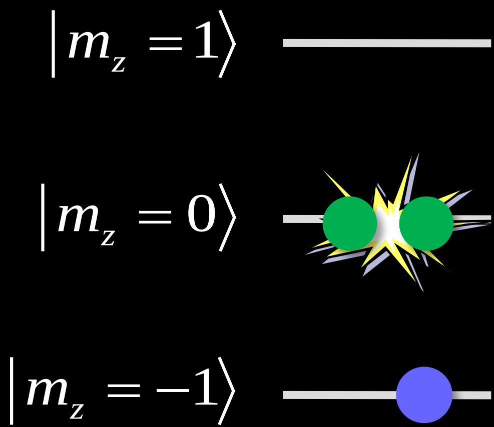
magnetic dipolar interactions



$$\sum_{pairs} \frac{\mu_0 \mu_B^2 g_F^2}{4\,\pi} \left\langle \frac{\boldsymbol{F}_i \cdot \boldsymbol{F}_j\ -\ 3\ \boldsymbol{F}_i \cdot \boldsymbol{e}_r\ \boldsymbol{F}_j \cdot \boldsymbol{e}_r}{r^3} \right\rangle_{time}$$

- symmetric under combined spin/position space rotations
- magnetic field breaks this symmetry

Linear Zeeman shift in a uniform magnetic field



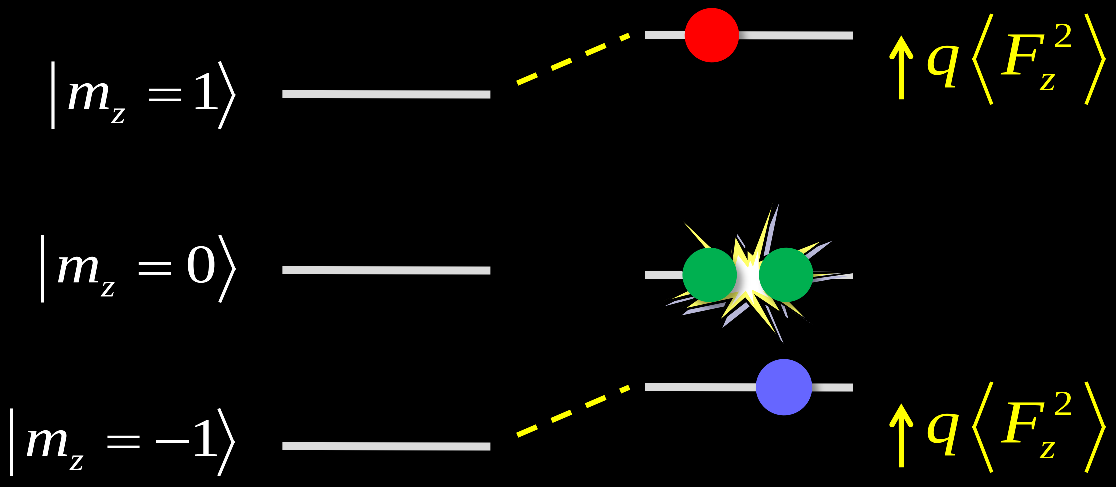
However, dipolar relaxation is extremely rare (for alkali atoms)
$\rightarrow$ linear Zeeman shift is irrelevant!

in other words, including the constraint of constant (longitudinal) magnetization

$$H(B=0) - g_F \mu_B\, F_z B_z \rightarrow H(B=0) \underbrace{- g_F \mu_B\, F_z B_z \;-\lambda\, F_z}_{p\, F_z}$$

presto… magnetic field is gone!

Quadratic Zeeman shifts



spin-mixing collisions are allowed

q = quadratic Zeeman shift

2. Magnetic order of spinor Bose-Einstein condensates

a. Bose-Einstein ferromagnet

b. entropy, energy

c. double condensation

non-interacting

d. mean-field, single mode, spin-1 gases

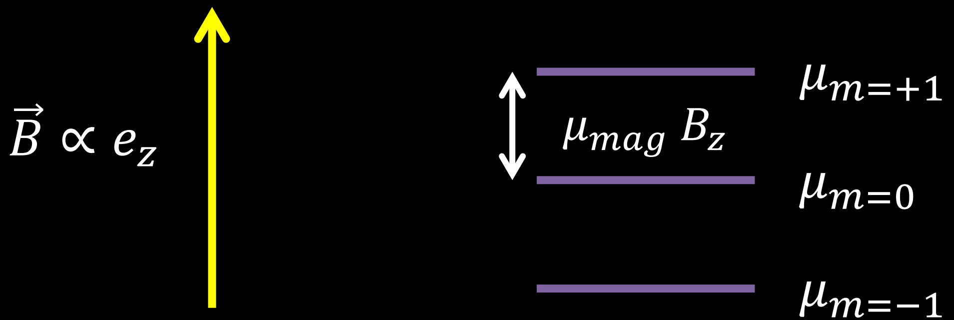
e. many-body ground state

interacting

Non-interacting spin-1 Bose gas

Yamada, “Thermal properties of the system of magnetic Bosons,”
Prog. Theo. Phys. 67, 443 (1982)

- non-interacting spin-1 Bose gas, no magnetization constraint, in B field
  - e.g. Cr: Pasquiou et al, “Thermodynamics of a Bose-Einstein condensate with free magnetization,” PRL **108**, 045307 (2012)



- or, consider constant longitudinal magnetization and allow spin mixing collisions

chemical equilibrium: $2\,\mu_{m=0} = \mu_{m=+1} + \mu_{m=-1}$

$$\mu_{m=+1} - \mu_{m=0} = \mu_{m=0} - \mu_{m=-1}$$

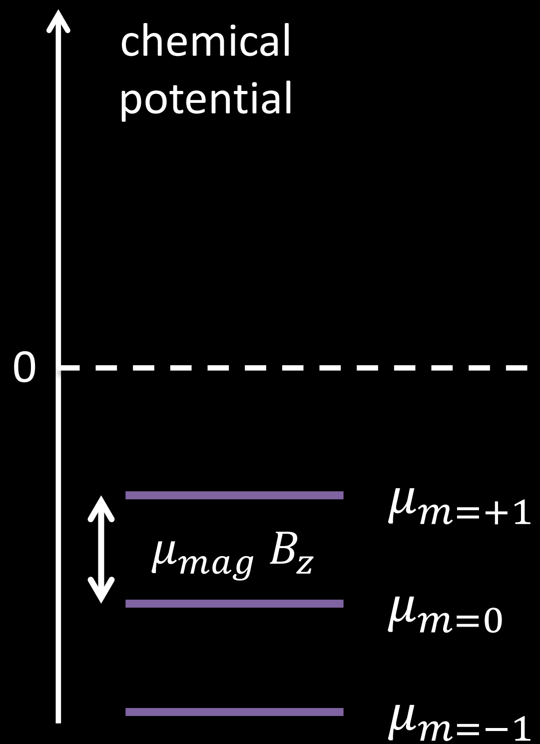
$$g_{3/2}(z) = \sum_{j=1}^{\infty} \frac{z^j}{j^{3/2}} \qquad \lambda_T = \sqrt{\frac{h^2}{2\pi\, m\, k_B T}}$$

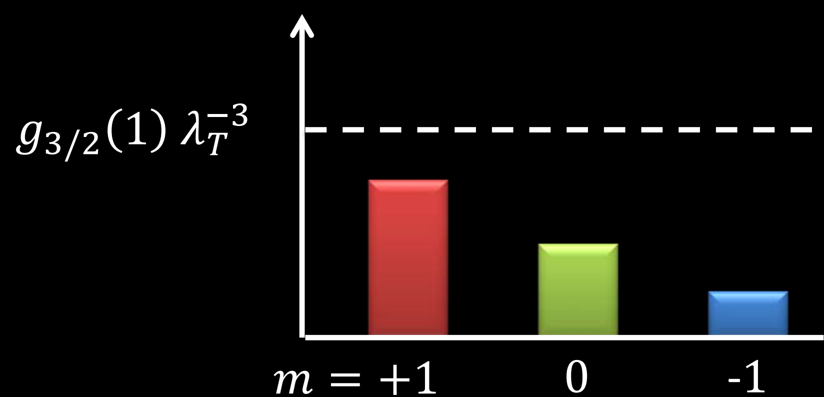
two possibilities:

1. $\mu_{m=+1} < 0$: non-degenerate gas

$$n_{th,mF} = g_{3/2}\left(\exp\left[\frac{\mu_{m=0} + m_F \mu_{mag} B_z}{k_B T}\right]\right) \lambda_T^{-3}$$

$$n_{tot} = n_{th,tot} = n_{th,+1} + n_{th,0} + n_{th,-1}$$





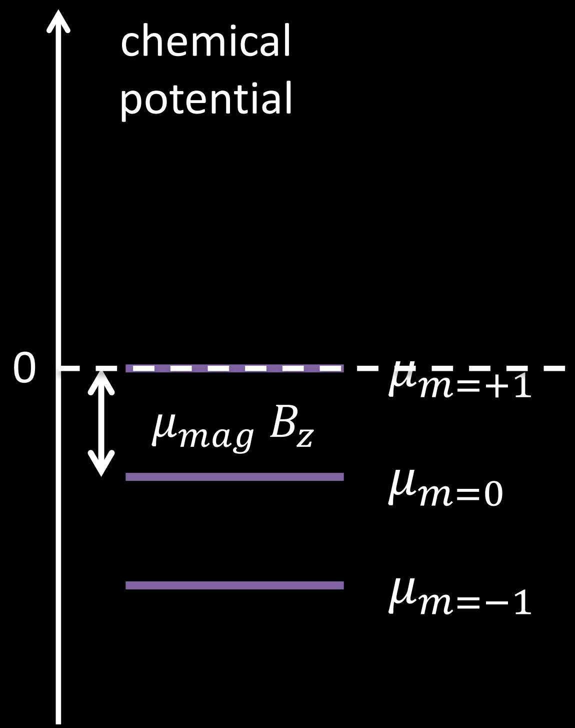
thermal paramagnet

fully magnetized in infinite field limit

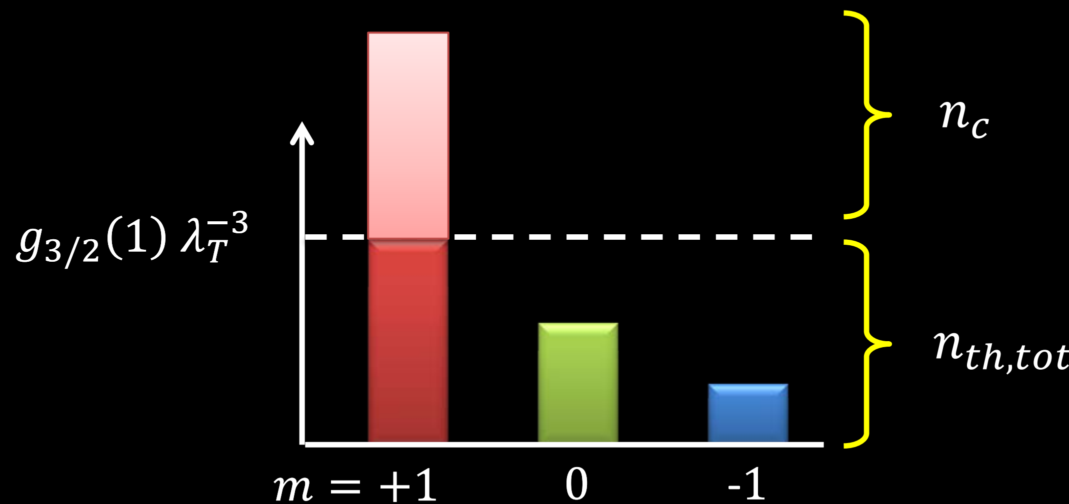
$$g_{3/2}(z) = \sum_{j=1}^{\infty} \frac{z^j}{j^{3/2}} \qquad \lambda_T = \sqrt{\frac{h^2}{2\pi\, m\, k_B T}}$$

two possibilities:

2. $\mu_{m=+1} = 0$: degenerate gas



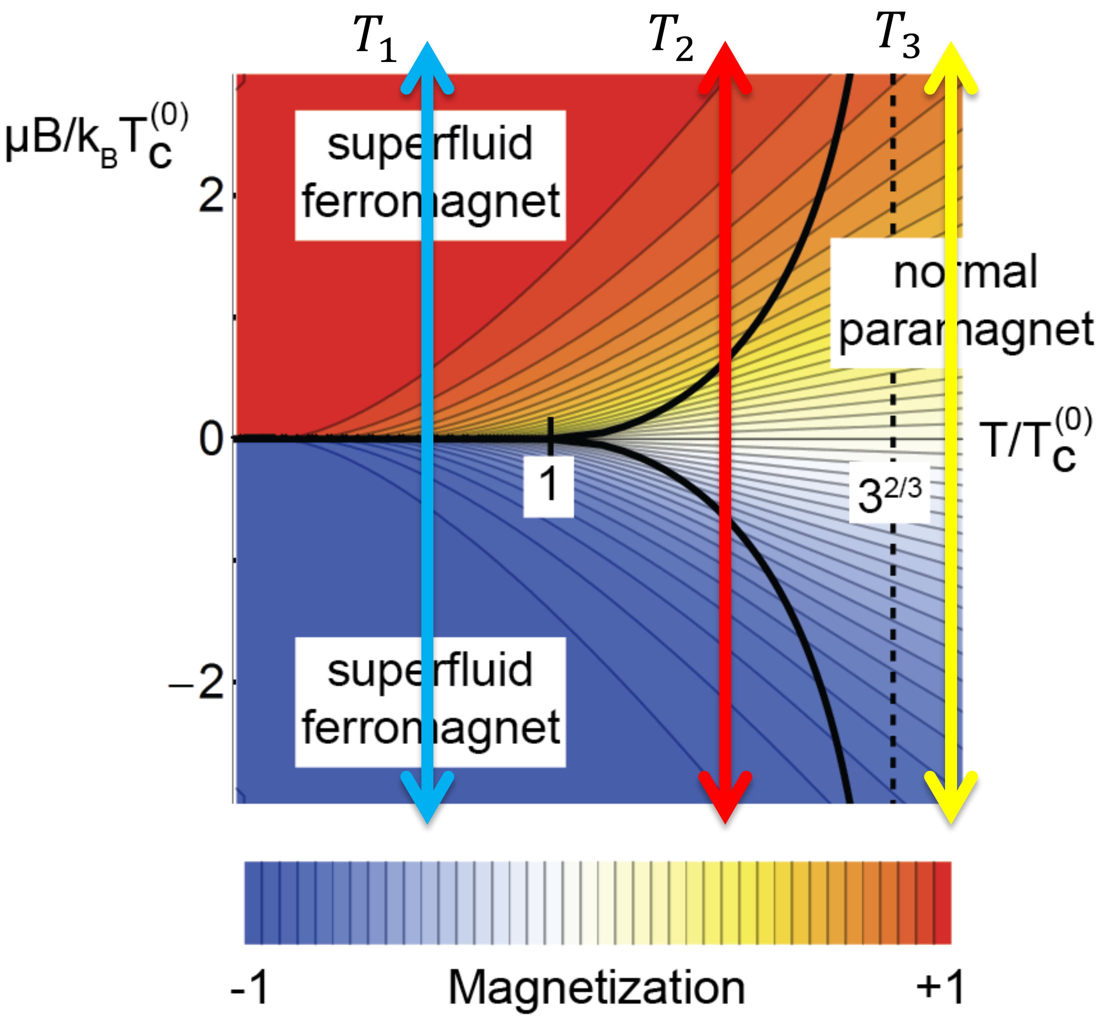
$$n_{tot} = n_{th,tot} + n_c$$



condensate is fully magnetized,
thermal gas is partly magnetized

Bose-Einstein magnetism

magnetization of a non-interacting, spin-1 Bose gas in a magnetic field:

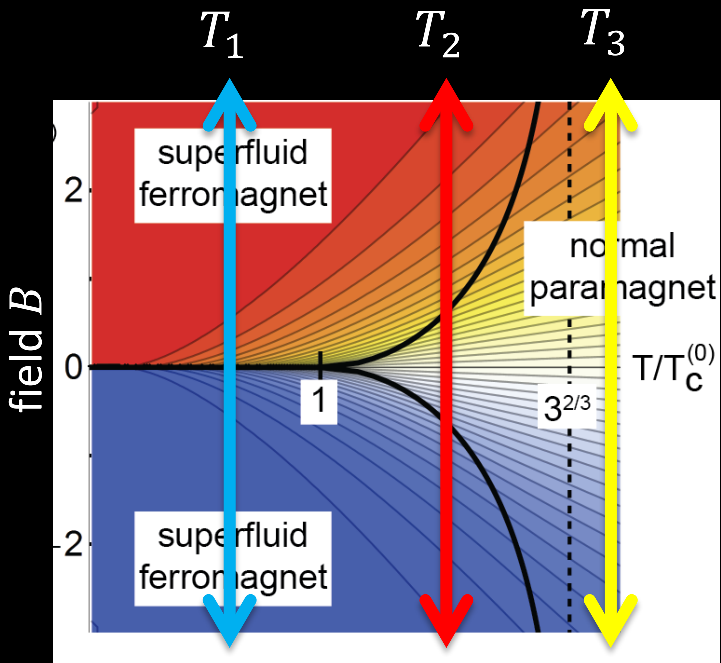


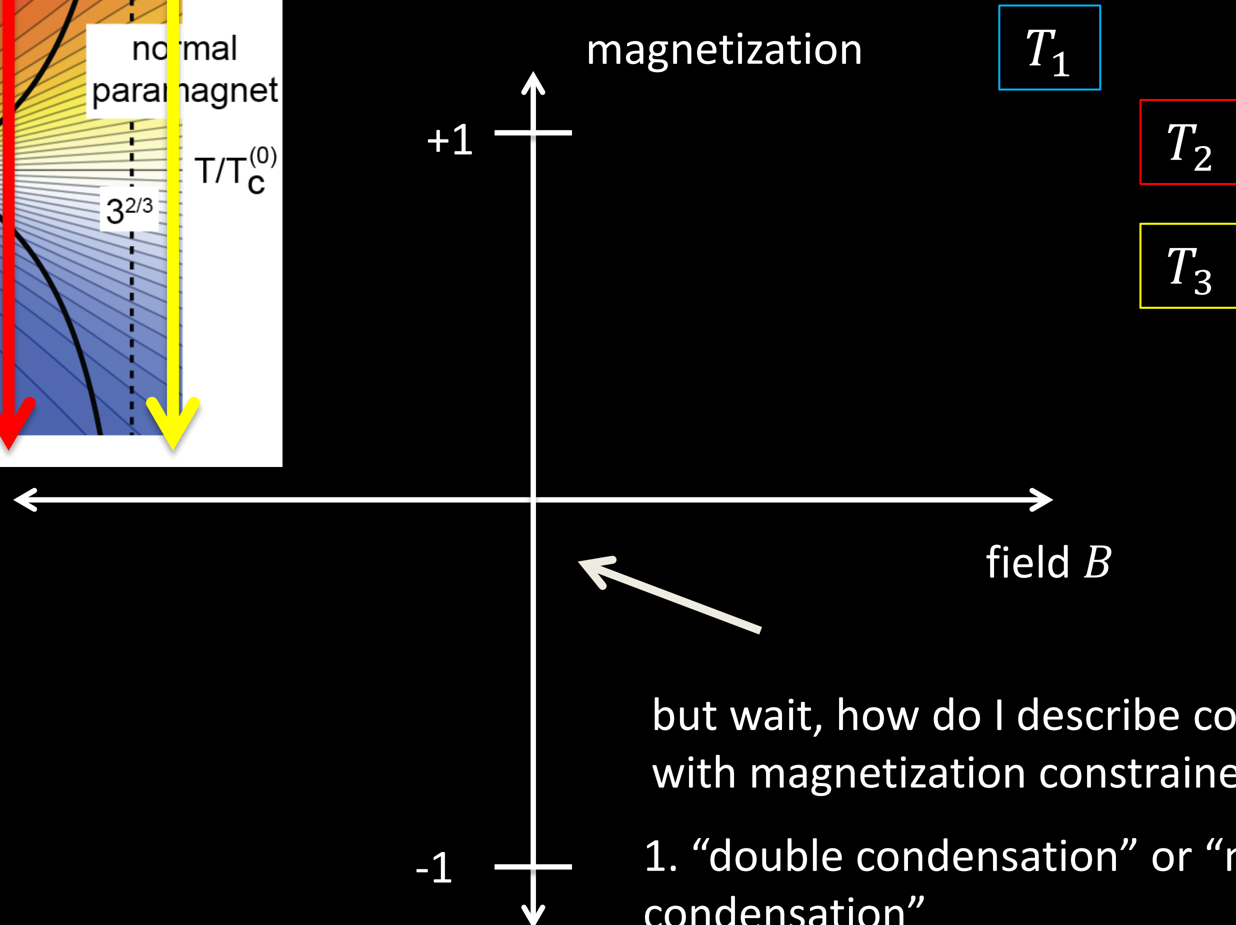
- Bose-Einstein condensation occurs at lower temperature at lower field (opening up spin states adds entropy)
- Magnetization jump at zero-field below Bose-Einstein condensation transition

Yamada, "Thermal Properties of the System of Magnetic Bosons," Prog. Theo. Phys. 67, 443 (1982)

Expt. with chromium:
Pasquiou, Laburthe-Tolra et al., PRL **106**, 255303 (2011).

magnetic ordering is "parasitic"





but wait, how do I describe cold system with magnetization constrained in here?

1. “double condensation” or “magnon condensation”
2. interactions now certainly play a role

Ground states

s-wave interactions: $$\frac{4\,\pi\,\hbar^2}{m}\delta^3(r)\left(a_0\hat{P}_0 + a_2\hat{P}_2 + a_4\hat{P}_4 + \ \dots\right)$$

more familiar form (e.g. spin-1): use two identities

1. Identity operator $$I_A \otimes I_B = \hat{P}_0 + \hat{P}_1 + \hat{P}_2$$

$$(I_A \otimes I_B)_S = \hat{P}_0 + \hat{P}_2$$

restricted to symmetric states

2. Spin dot product (Heisenberg interaction)

$$\boldsymbol{F_A}\cdot\boldsymbol{F_B} = \sum_{Fpair}\left(F_{pair}\left(F_{pair}+1\right) - 2\,F(F+1)\right)\hat{P}_{Fpair}$$

$$(\boldsymbol{F_A}\cdot\boldsymbol{F_B})_{\boldsymbol{S}} = 2\,\hat{P}_2 - \hat{P}_0$$

---

spin-1: $$\frac{4\,\pi\,\hbar^2}{m}\delta^3(r)\left[\frac{2\,a_2+a_0}{3}\,I_A\otimes I_B + \frac{a_2-a_0}{3}\,\boldsymbol{F}_A\cdot\boldsymbol{F}_B\right]$$

$$= \ c_0^{(1)}\delta^3(r) \ + c_1^{(1)}\,\boldsymbol{F}_A\cdot\boldsymbol{F}_B\,\delta_3(r)$$

Mean-field ground states

$$E = c_1^{(1)} n \langle \vec{F} \rangle^2$$

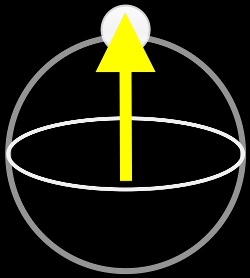
rotational symmetry: look for "most symmetric states" (inert states)

3He: Barton and Moore, J. Phys. C Solid State **7**, 4220 (1974); **8**, 970 (1975)

Spinor gas: Makela and Suominen, PRL **99**, 190408 (2007); Yip, PRA **75**, 023625 (2007)

"magnetic"
"oriented"

$$\Psi = \hat{R}\left| m_z = 1 \right\rangle$$

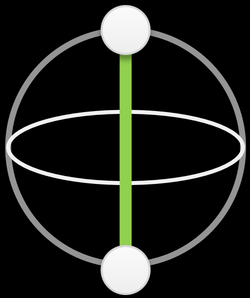


favored for ^{87}Rb

disfavored for ^{23}Na

"non-magnetic"
"nematic"
"aligned"

$$\Psi = \hat{R}\left| m_z = 0 \right\rangle$$



disfavored for ^{87}Rb

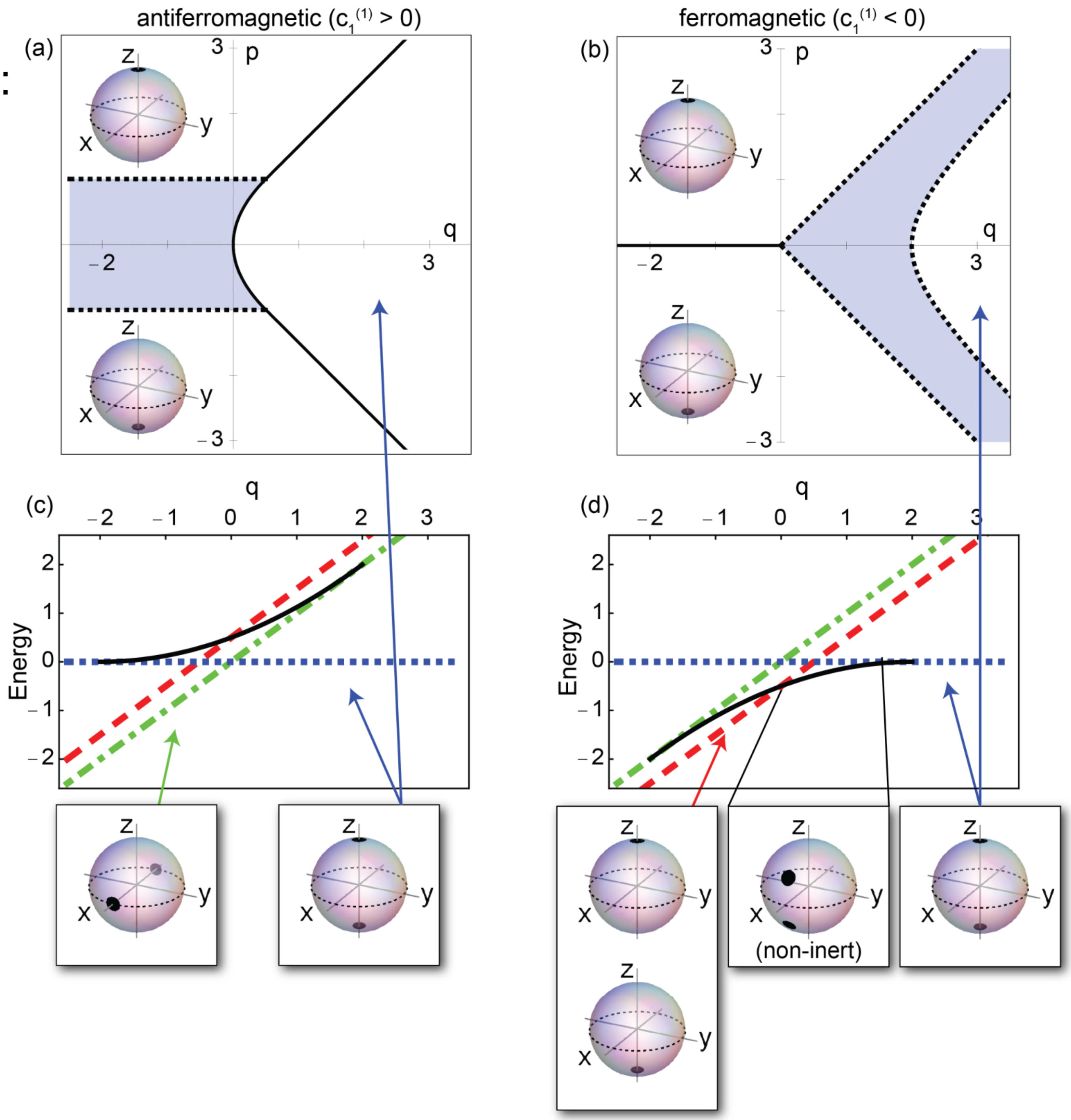
favored for ^{23}Na

Majorana, Nuovo Cimento **9**, 43 (1932)

Two more ingredients:

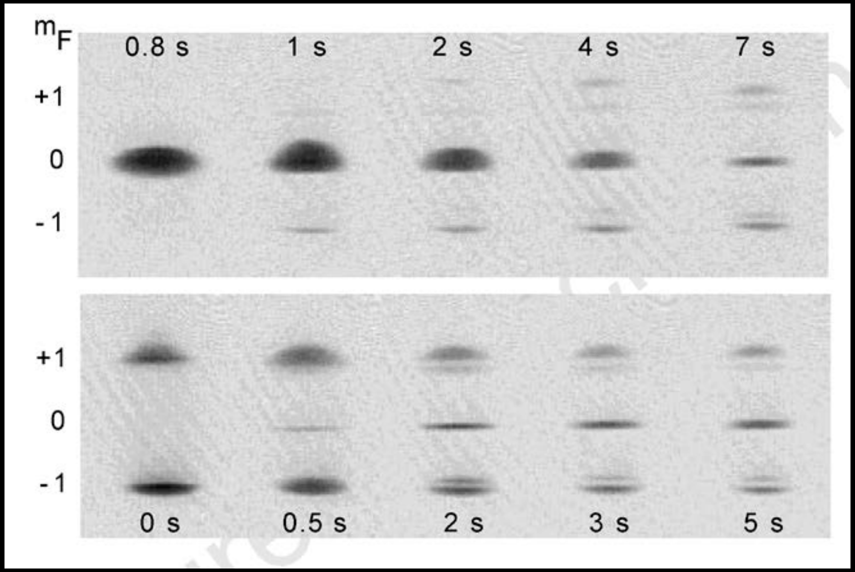
fixed magnetization
$p \langle F_z \rangle$

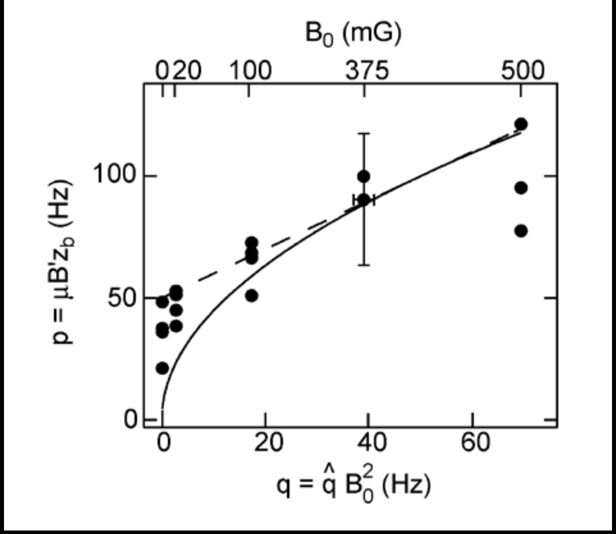
quadratic Zeeman shift
$q \langle F_z^2 \rangle$



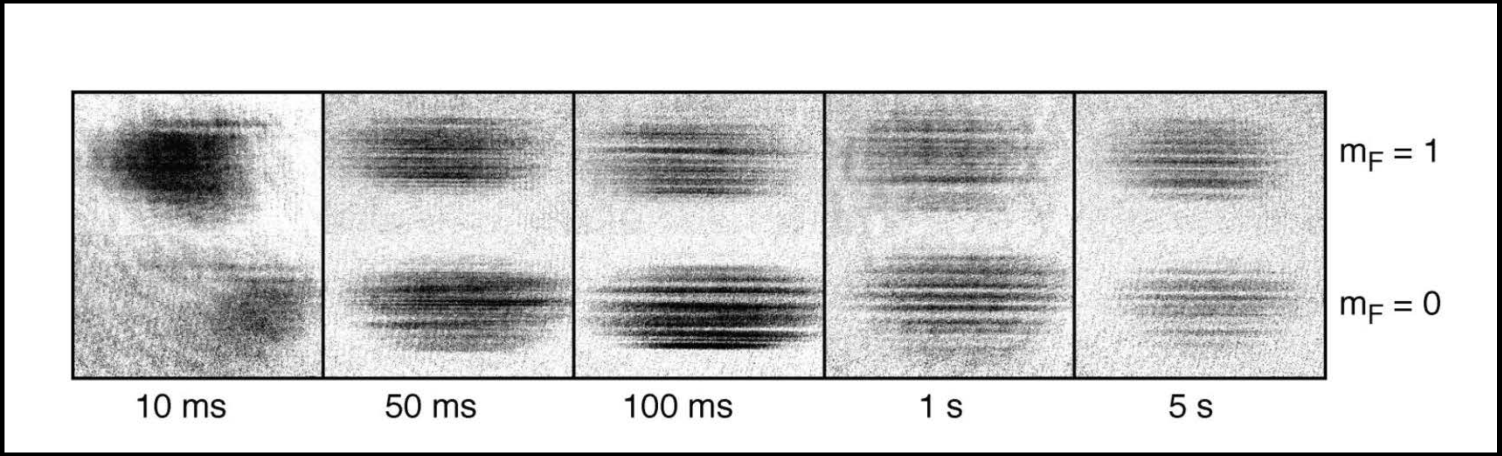
Stenger et al., Nature
396, 345 (1998)

Evidence for antiferromagnetic interactions of F=1 Na





Stenger et al., Nature **396**, 345 (1998)

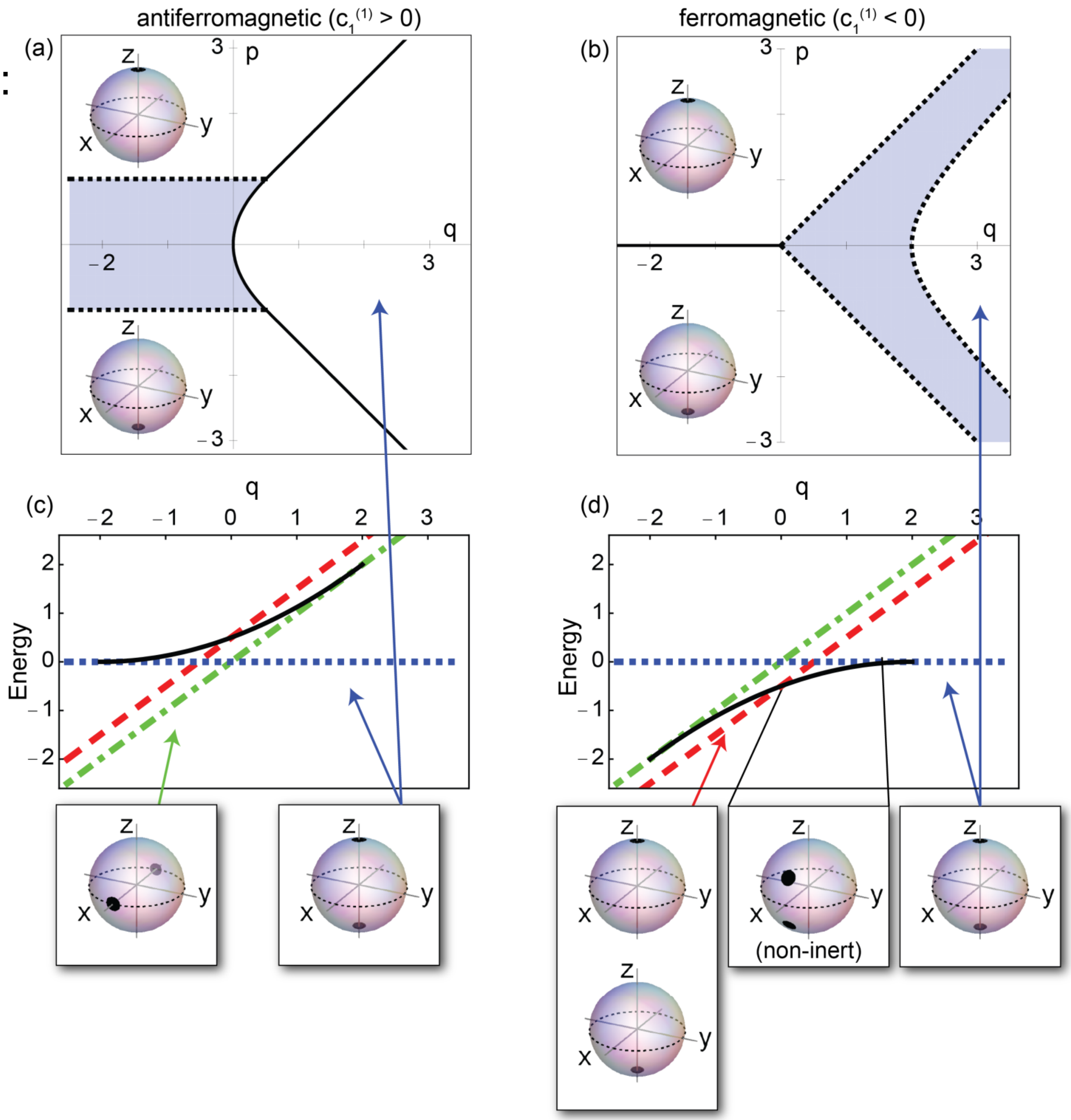


Miesner et al., PRL **82**, 2228 (1999).

Two more ingredients:

fixed magnetization
$p \langle F_z \rangle$

quadratic Zeeman shift
$q \langle F_z^2 \rangle$



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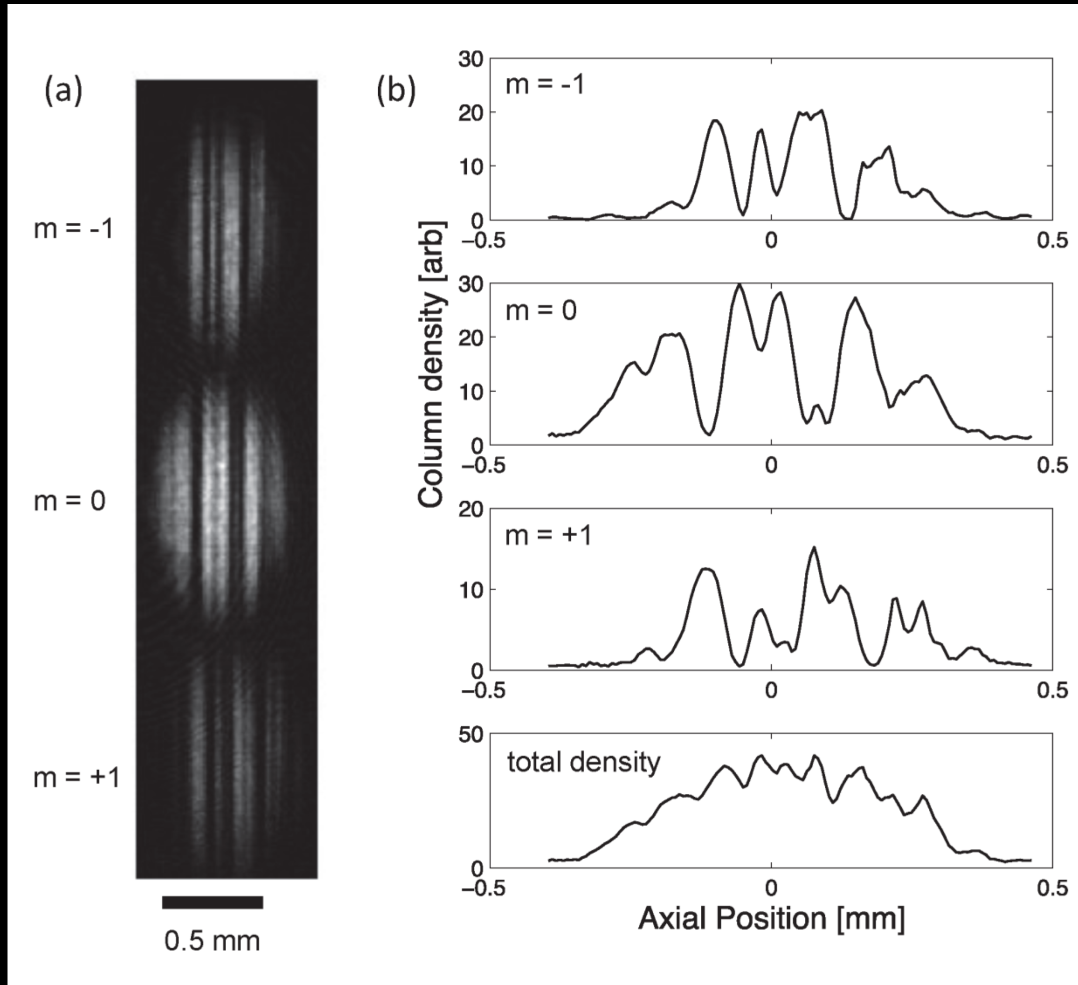

Bookjans, E.M., A. Vinit, and C. Raman, Quantum Phase Transition in an Antiferromagnetic Spinor Bose-Einstein Condensate. Physical Review Letters 107, 195306 (2011).

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$p\,\langle F_z \rangle$

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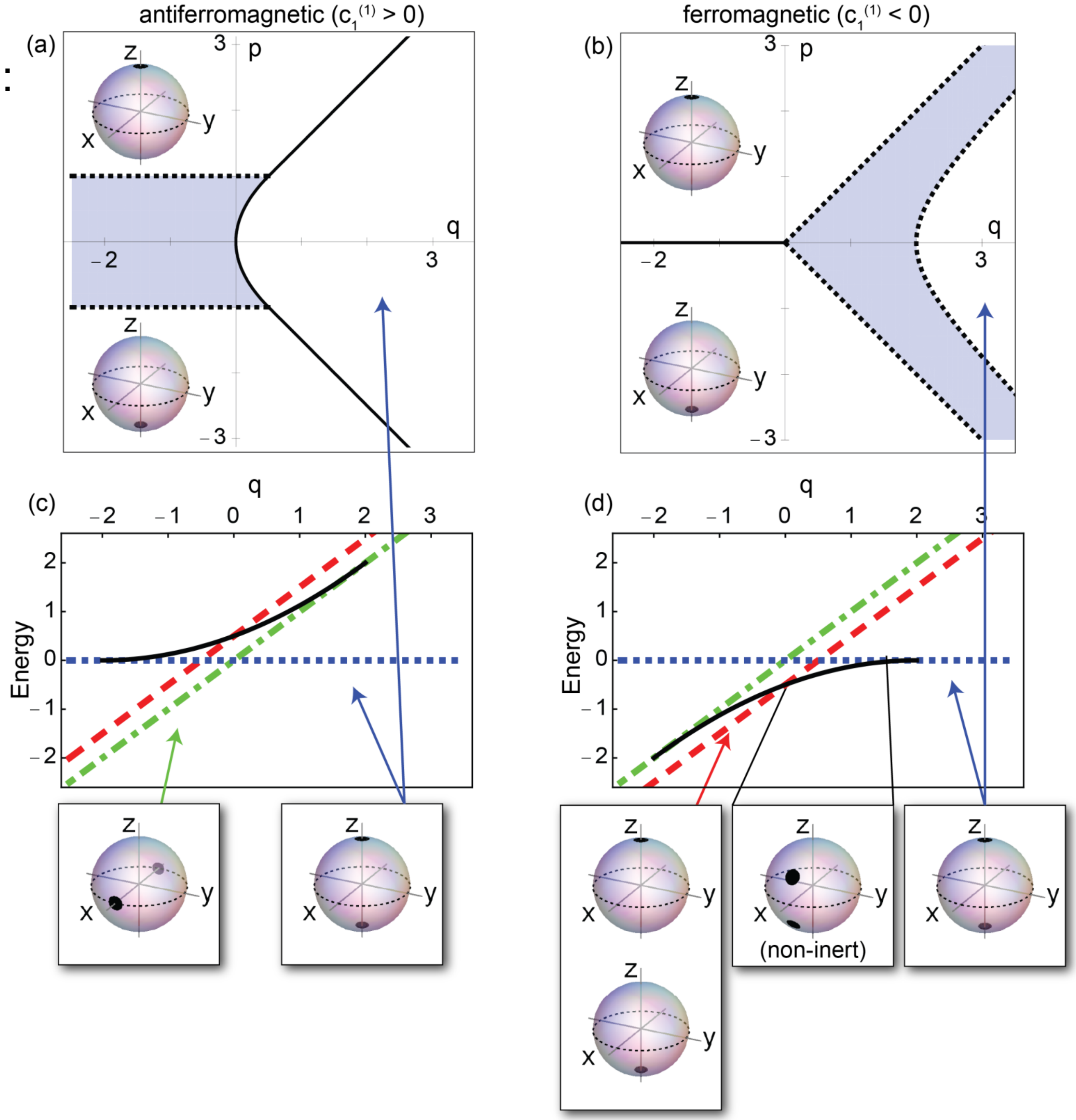
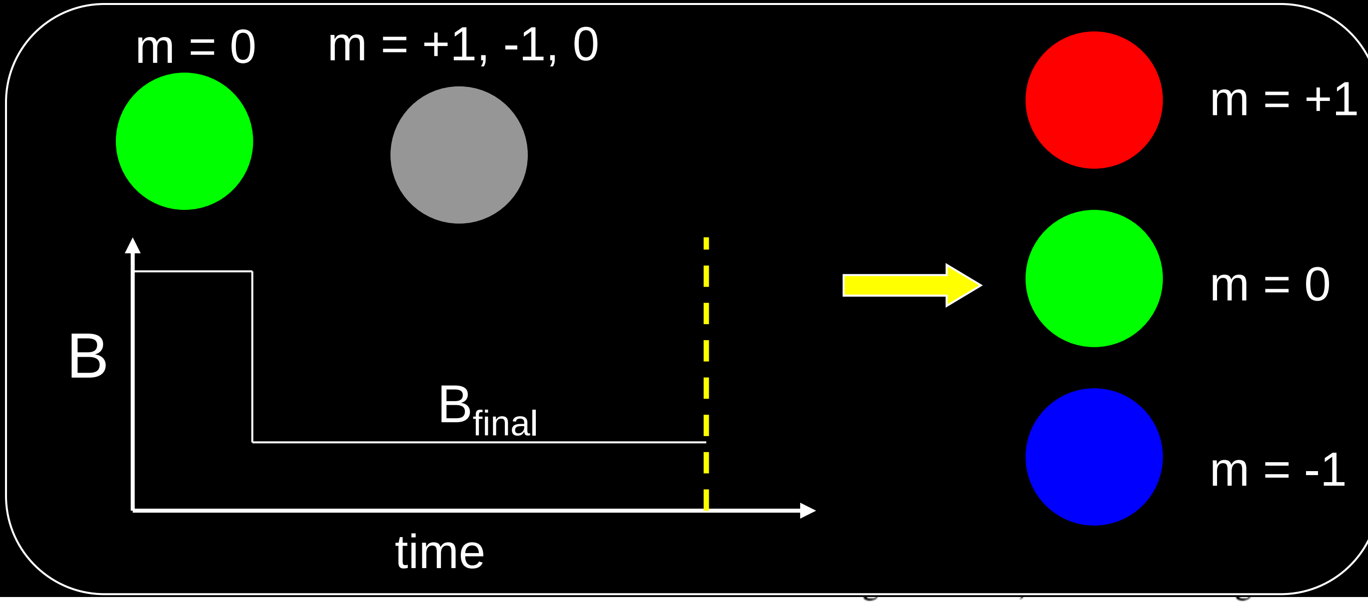
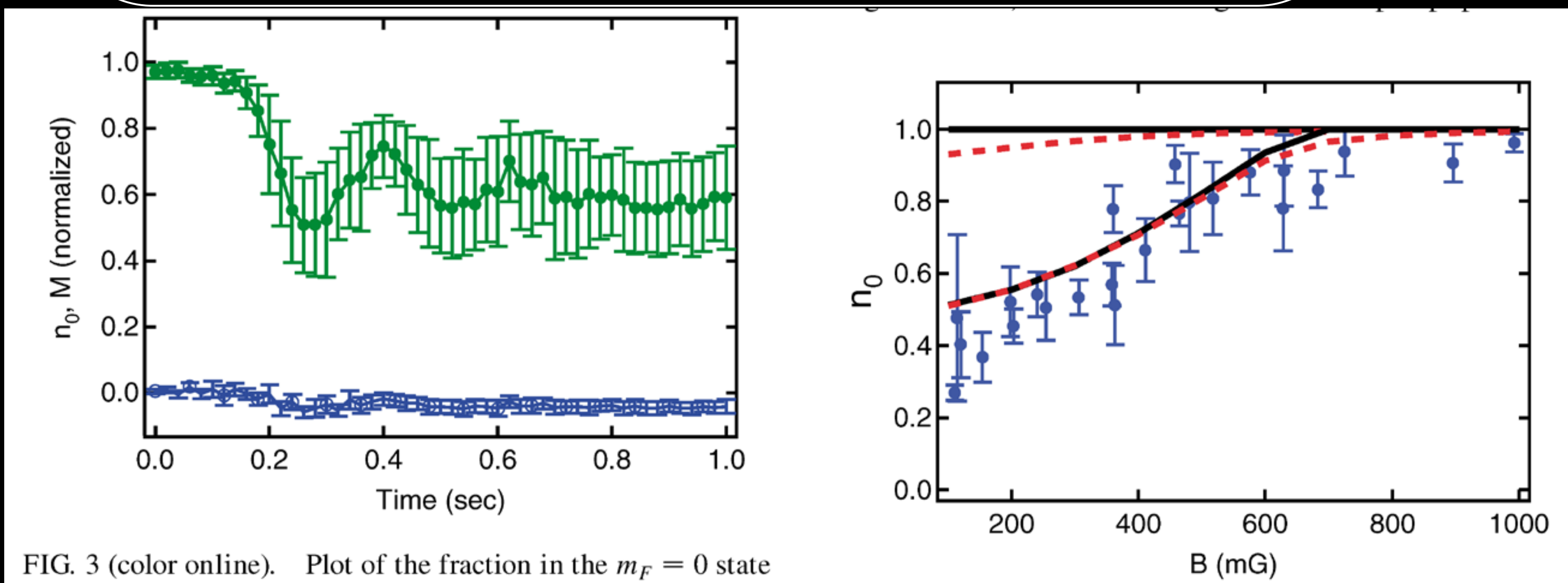
Stenger et al., Nature **396**, 345 (1998)







Chang, M.-S., et al., *Observation of spinor dynamics in optically trapped Rb Bose-Einstein condensates. PRL* ***92,*** *140403 (2004)*

$$\vec{\Psi} = \begin{pmatrix} \frac{1}{2} e^{i\phi_1} \\ \frac{1}{\sqrt{2}} e^{i\phi_0} \\ \frac{1}{2} e^{i\phi_{-1}} \end{pmatrix}$$

?

?

$$\vec{\Psi} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

Ferromagnetic state

$$\left\langle \vec{F} \right\rangle = 1$$

points in x-direction

$$\vec{\Psi} = \begin{pmatrix} \frac{1}{2} \\ i\frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

Polar state !!!

$$\left\langle \vec{F} \right\rangle = 0$$

“points nowhere”
along the y+z axis