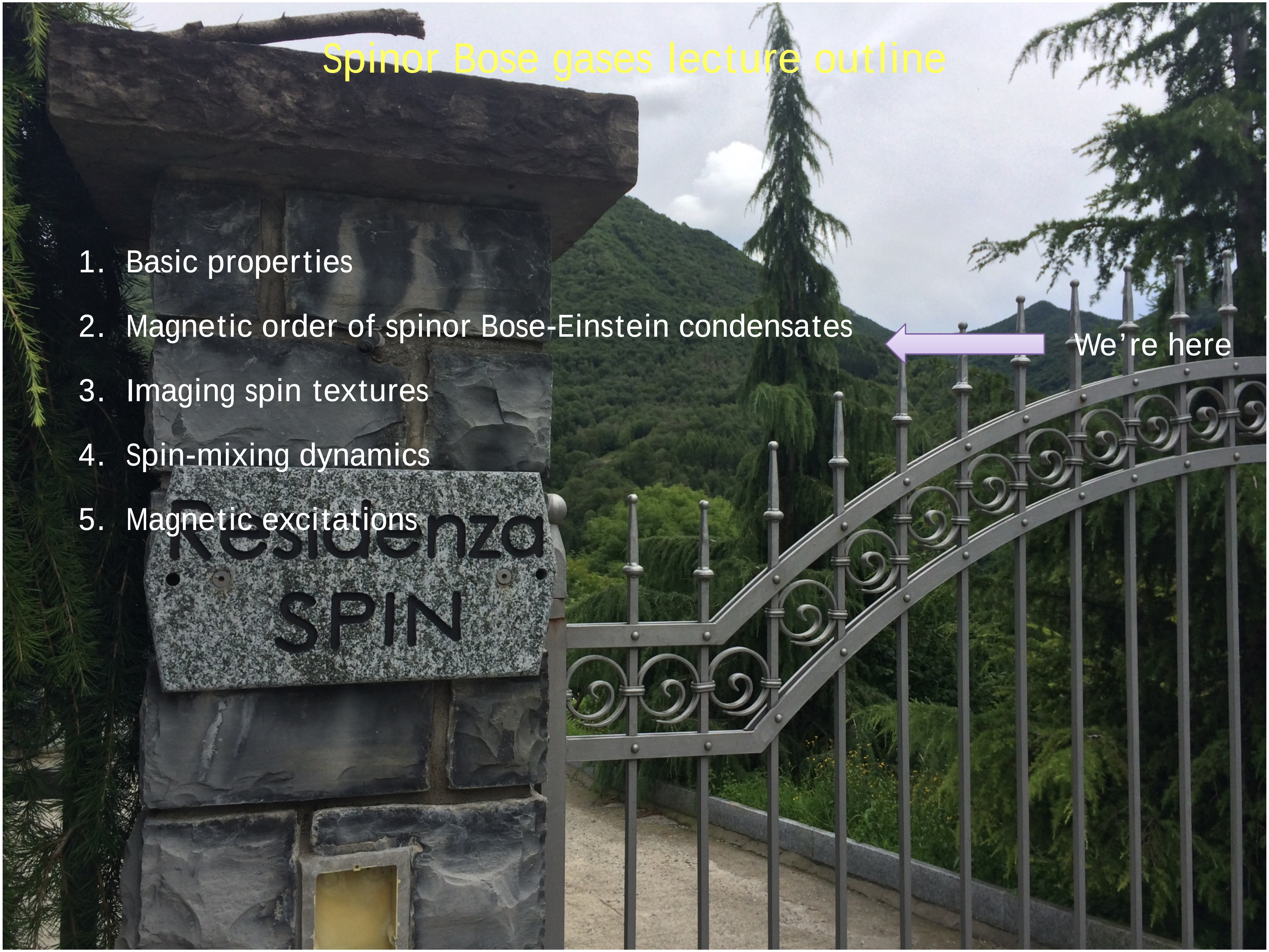
Spinor Bose gases lecture outline

1. Basic properties
2. Magnetic order of spinor Bose-Einstein condensates
3. Imaging spin textures
4. Spin-mixing dynamics
5. Magnetic excitations

We're here

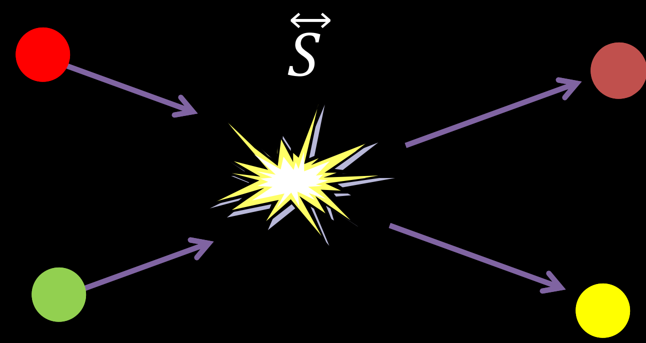


Coupling strengths

$|F=1,m_{F1}\rangle\ |F=1,m_{F2}\rangle$

or

$|F_{pair},m_{Fpair}\rangle$



$|F=1,m'_{F1}\rangle\ |F=1,m'_{F2}\rangle$

or

$|F_{pair},m_{Fpair}\rangle$

$$c_0^{(1)}\delta^3(r)\ +c_1^{(1)}\,\boldsymbol{F}_A\cdot\boldsymbol{F}_B\,\delta^3(r)$$

	$\|m_F=\ +1\rangle$	$\|m_F=0\rangle$	$\|m_F=-1\rangle$
$\|m_F=\ +1\rangle$	$c_0^{(1)}+c_1^{(1)}$	$c_0^{(1)}+c_1^{(1)}$	$c_0^{(1)}-c_1^{(1)}$
$\|m_F=0\rangle$	$c_0^{(1)}+c_1^{(1)}$	$c_0^{(1)}$	$c_0^{(1)}+c_1^{(1)}$
$\|m_F=-1\rangle$	$c_0^{(1)}-c_1^{(1)}$	$c_0^{(1)}+c_1^{(1)}$	$c_0^{(1)}+c_1^{(1)}$

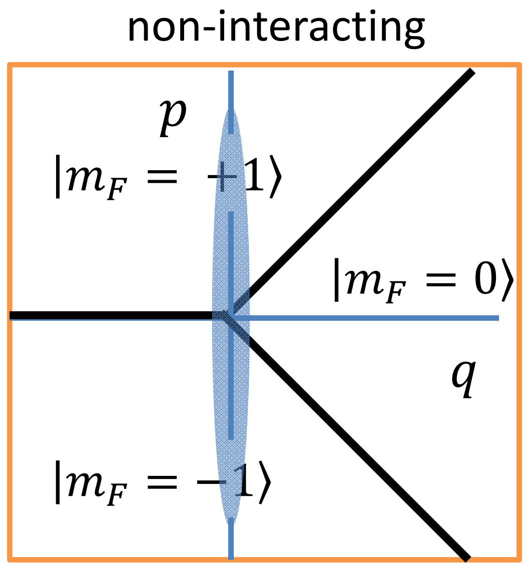
$$c_1^{(1)}$$

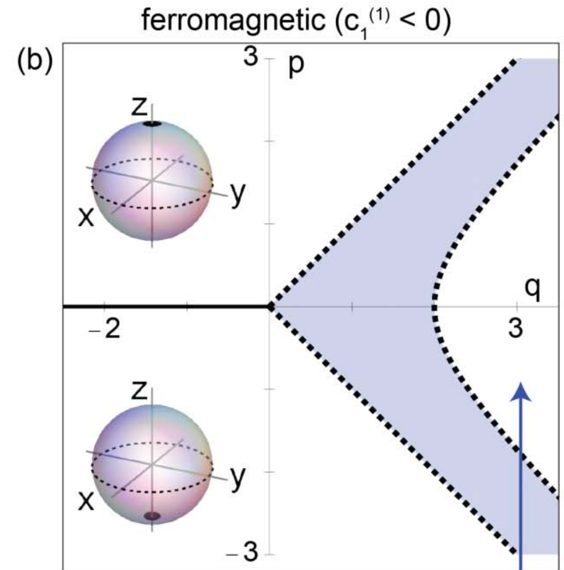
$$|F=1,m_{F1}=0\rangle\ |F=1,m_{F2}=0\rangle\leftrightarrow|F=1,m_{F1}=1\rangle\ |F=1,m_{F2}=-1\rangle$$

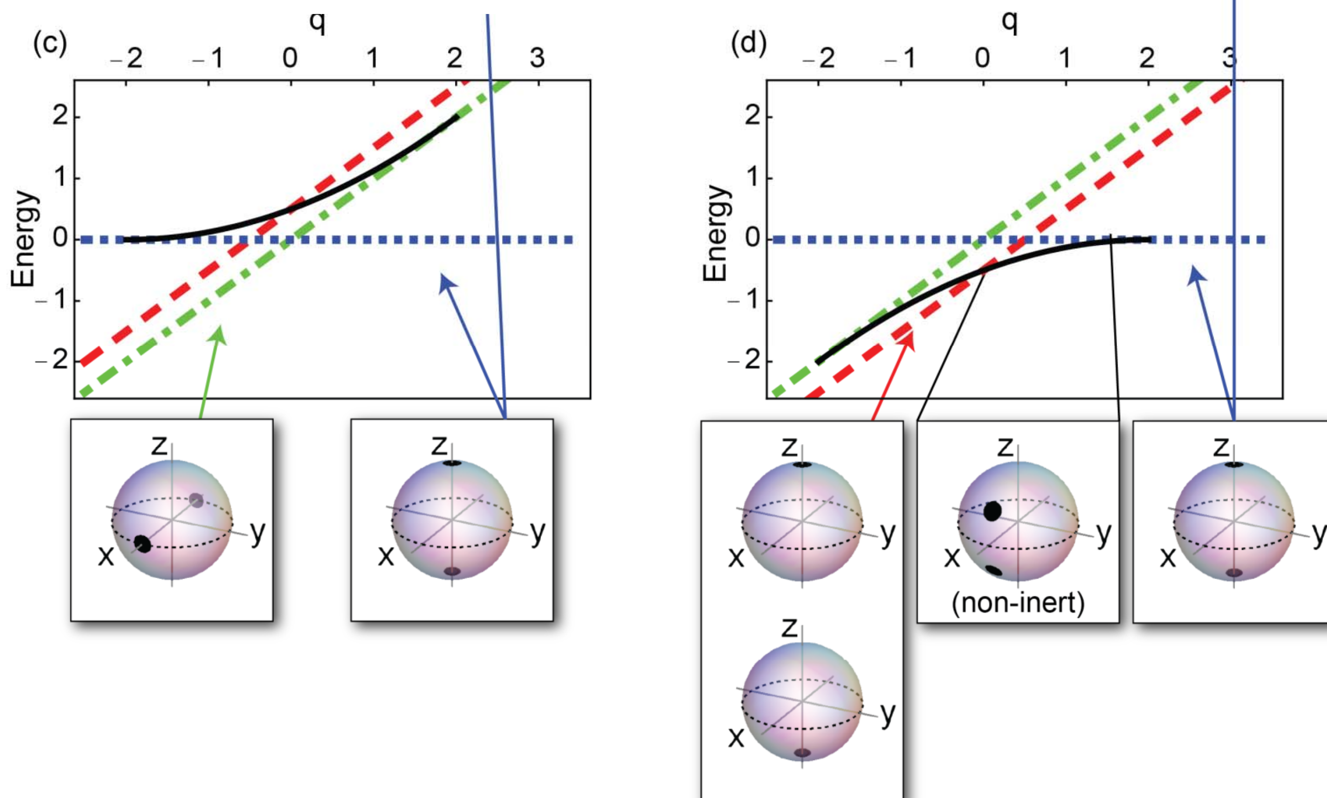
Two more ingredients:

fixed magnetization
$p\,\langle F_z \rangle$

quadratic Zeeman shift
$q\,\langle F_z^2 \rangle$







Stenger et al., Nature
396, 345 (1998)

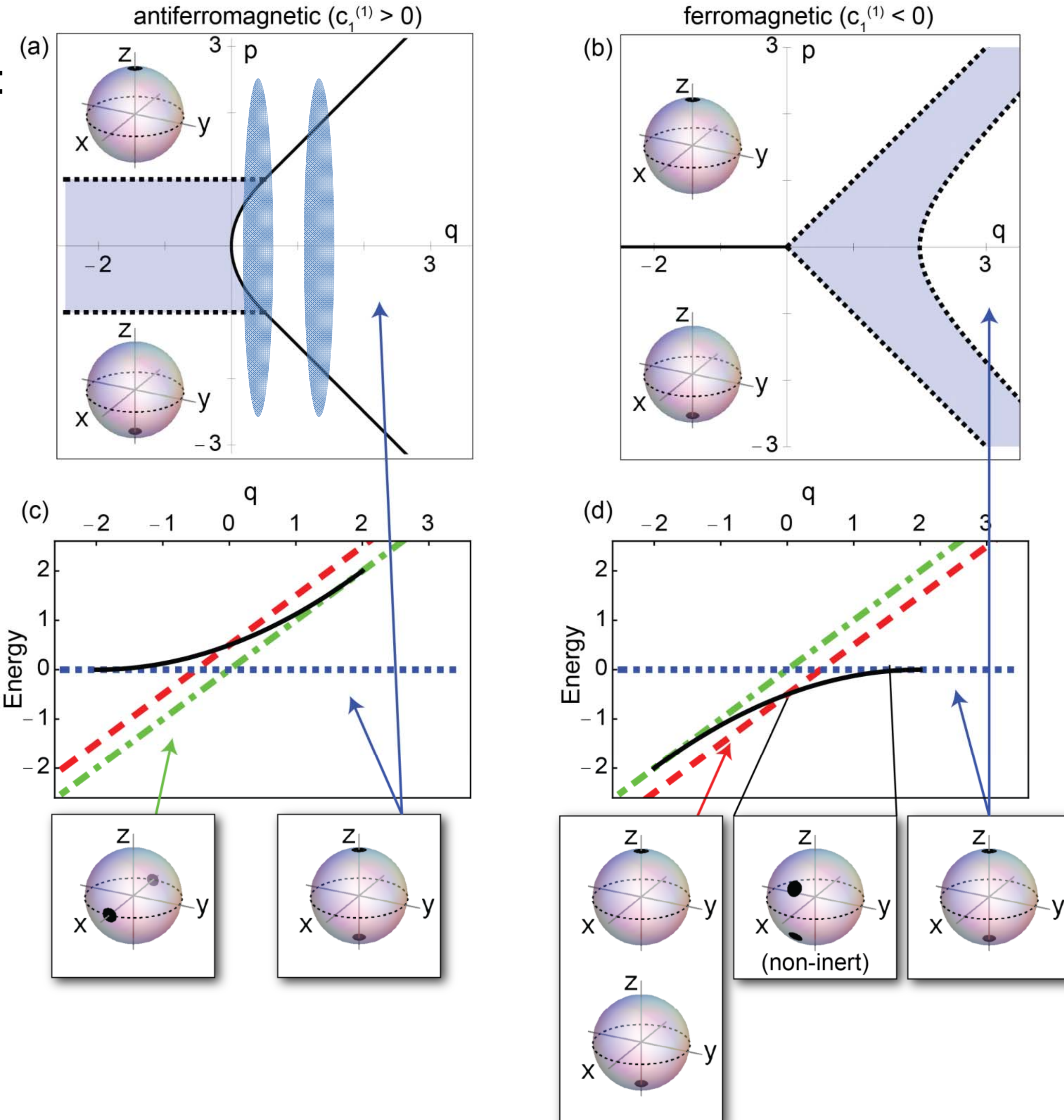
Two more ingredients:

fixed magnetization

$p\,\langle F_z \rangle$

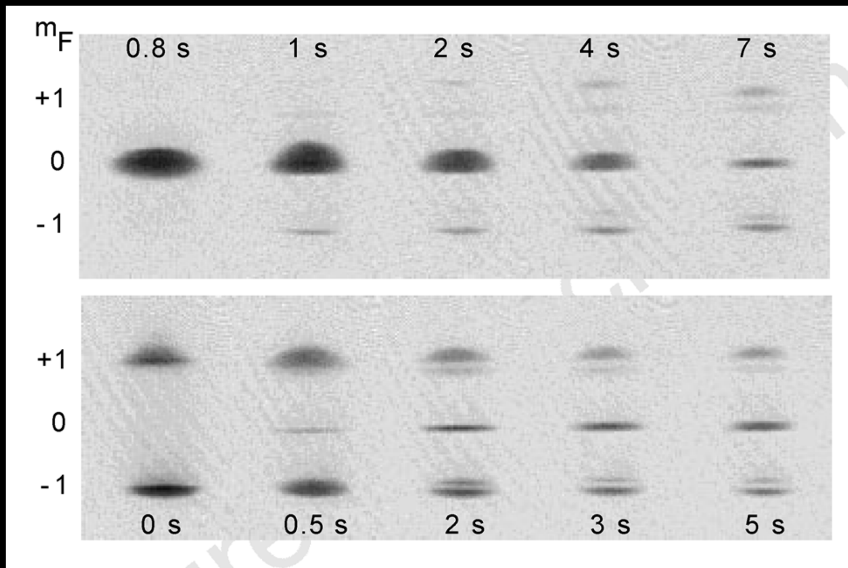
quadratic Zeeman shift

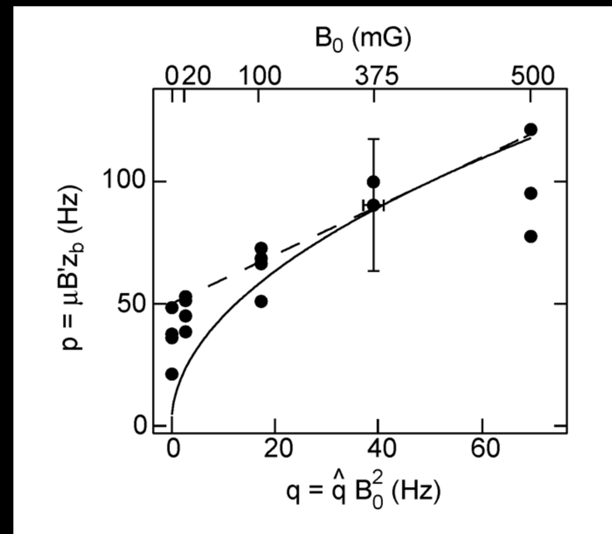
$q\,\langle F_z^2 \rangle$



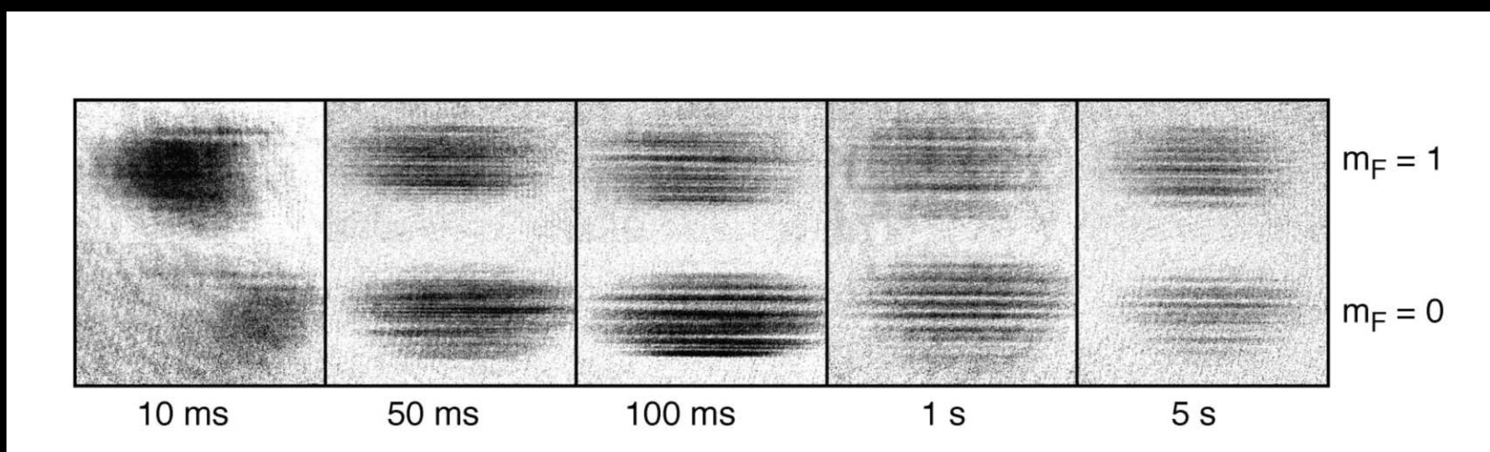
Stenger et al., Nature **396**, 345 (1998)

Evidence for antiferromagnetic interactions of F=1 Na





Stenger et al., Nature **396**, 345 (1998)

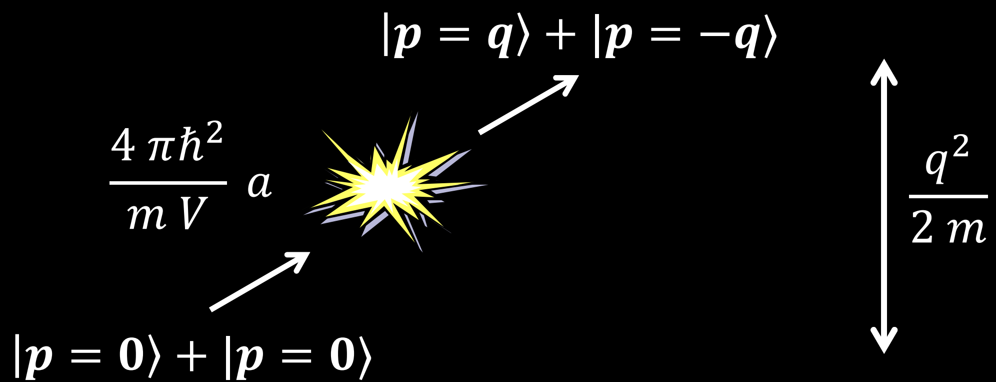


Miesner et al., PRL **82**, 2228 (1999).

Many body ground state

Is mean-field approximation good?

Scalar Bose gas (Bogoliubov, 1947): Quantify the quantum depletion



Quantum depletion = fraction of atoms outside condensate $\propto \sqrt{n\,a^3}$

Many body ground state

Is mean-field approximation good?

Spinor Bose gas [Law, Pu, and Bigelow, PRL **81**, 5727 (1998); others]

$$\frac{c_1^{(1)}}{V}$$

$$|m_F = 0\rangle + |m_F = 0\rangle \quad \longleftrightarrow \quad \longleftrightarrow |m_F = +1\rangle + |m_F = -1\rangle$$

No barrier to quantum depletion? Is there a Bose-Einstein condensate at all?

Many body ground state

Spin-1 gas: Law, Pu, and Bigelow, PRL **81**, 5727 (1998); others]

$$\sum_{pairs} c_1^{(1)}\, \boldsymbol{F}_A \cdot \boldsymbol{F}_B\, \delta^3(r) \;\longrightarrow\; \frac{c_1^{(1)} n}{2}\left(\frac{\boldsymbol{F}_{coll}^2}{N} - 2\right) \qquad F_{coll} = \sum_{atoms} \boldsymbol{F}_i$$

$c_1^{(1)} < 0$ (ferromagnetic, ^{87}Rb)

$$|F_{coll} = N, m_{Fcoll}\rangle$$

- N-fold degenerate ground state
- broken-symmetry mean-field state = coherent superposition of these degenerate states
- some subtlety if magnetization is exactly conserved

$c_1^{(1)} > 0$ (antiferromagnetic, ^{23}Na)

$$|F_{coll} = 0, m_{Fcoll} = 0\rangle$$

- unique ground state
- unbroken rotational symmetry
- not BEC: fractionated condensate
- observable by correlations: $m_{Fcoll} = 0$ along any axis

3. Imaging spin textures

a. methods

   i. dispersive imaging

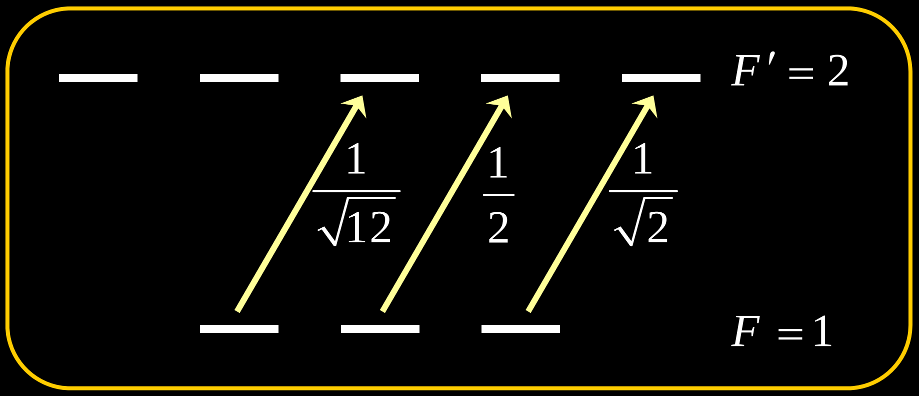
   ii. absorptive spin-sensitive in-situ imaging

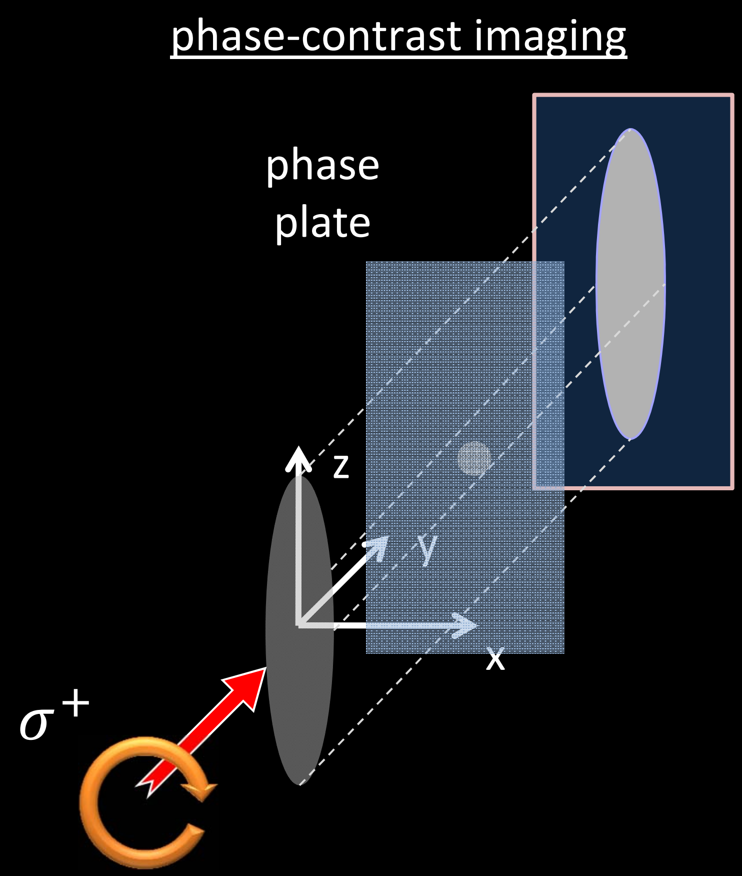
b. equilibration toward ground-state

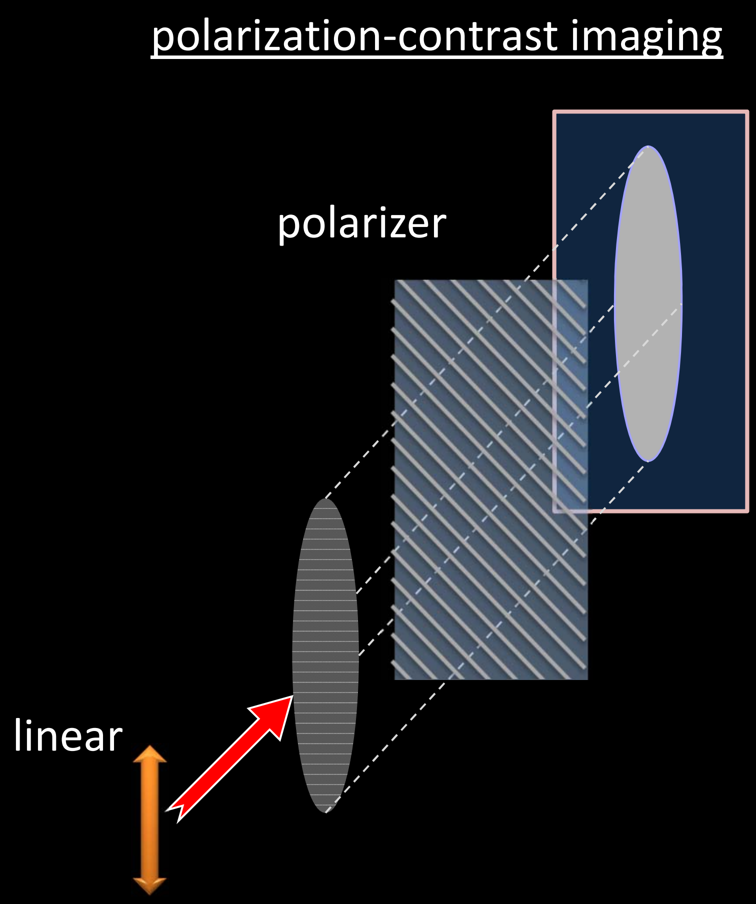
c. topological structures

d. magnetization curvature

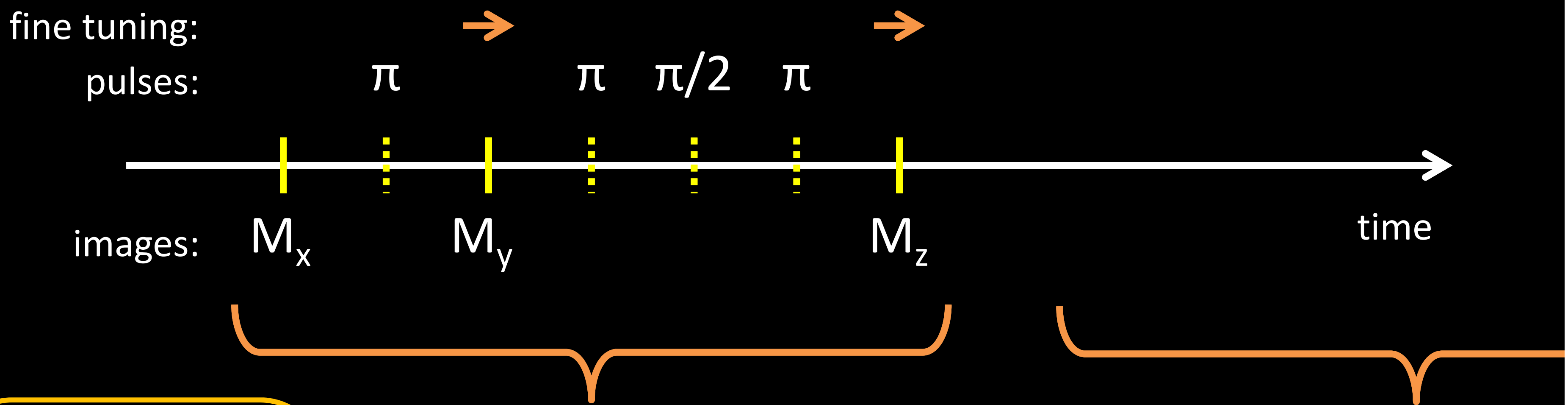
Dispersive birefringent imaging

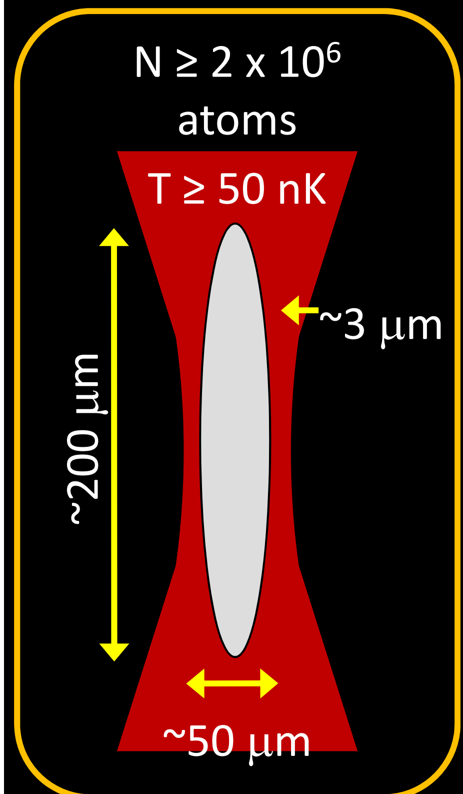




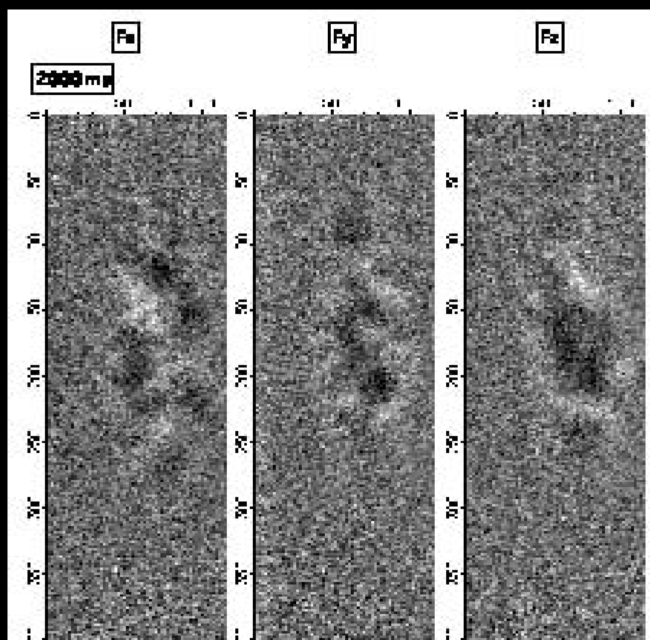


Spin echo imaging





vector imaging sequence



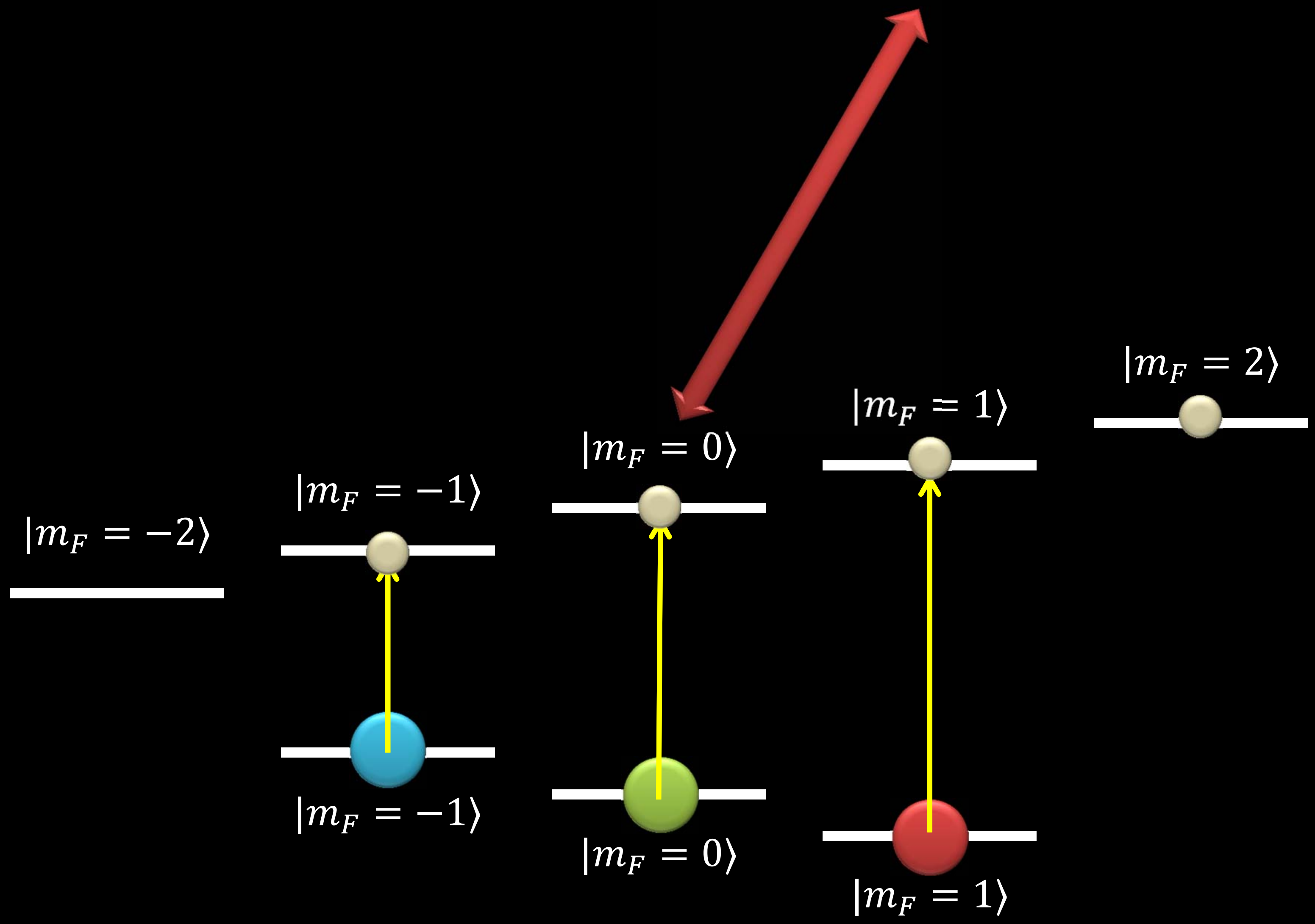
repeat?

geometry
≈ surfboard

Absorptive spin-sensitive in-situ imaging

"ASSISI"

Absorptive spin-sensitive in-situ imaging



Expected ground states

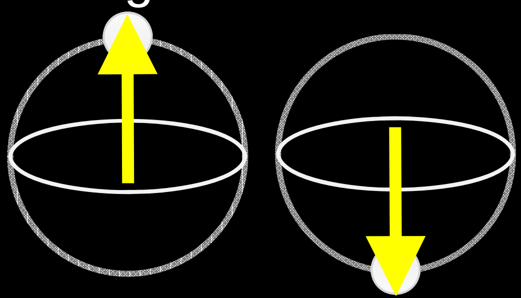
zero magnetization
$p = 0$

$$E = -\left|c_1^{(1)}\right| n \langle \vec{F} \rangle^2 + q \langle F_z^2 \rangle$$

<u>ferromagnetic states</u>

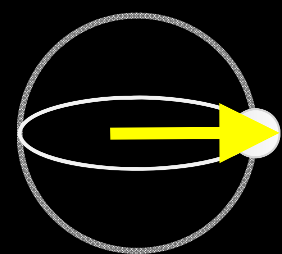
<u>unmagnetized state</u>

longitudinal axis

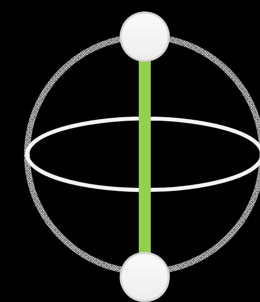


$Z_2 \times U(1)$

transverse plane



$SO(2) \times U(1)$



$U(1)$

$|m_z = 0\rangle$

"Majorana representation," Majorana, Nuovo Cimento **9**, 43 (1932)

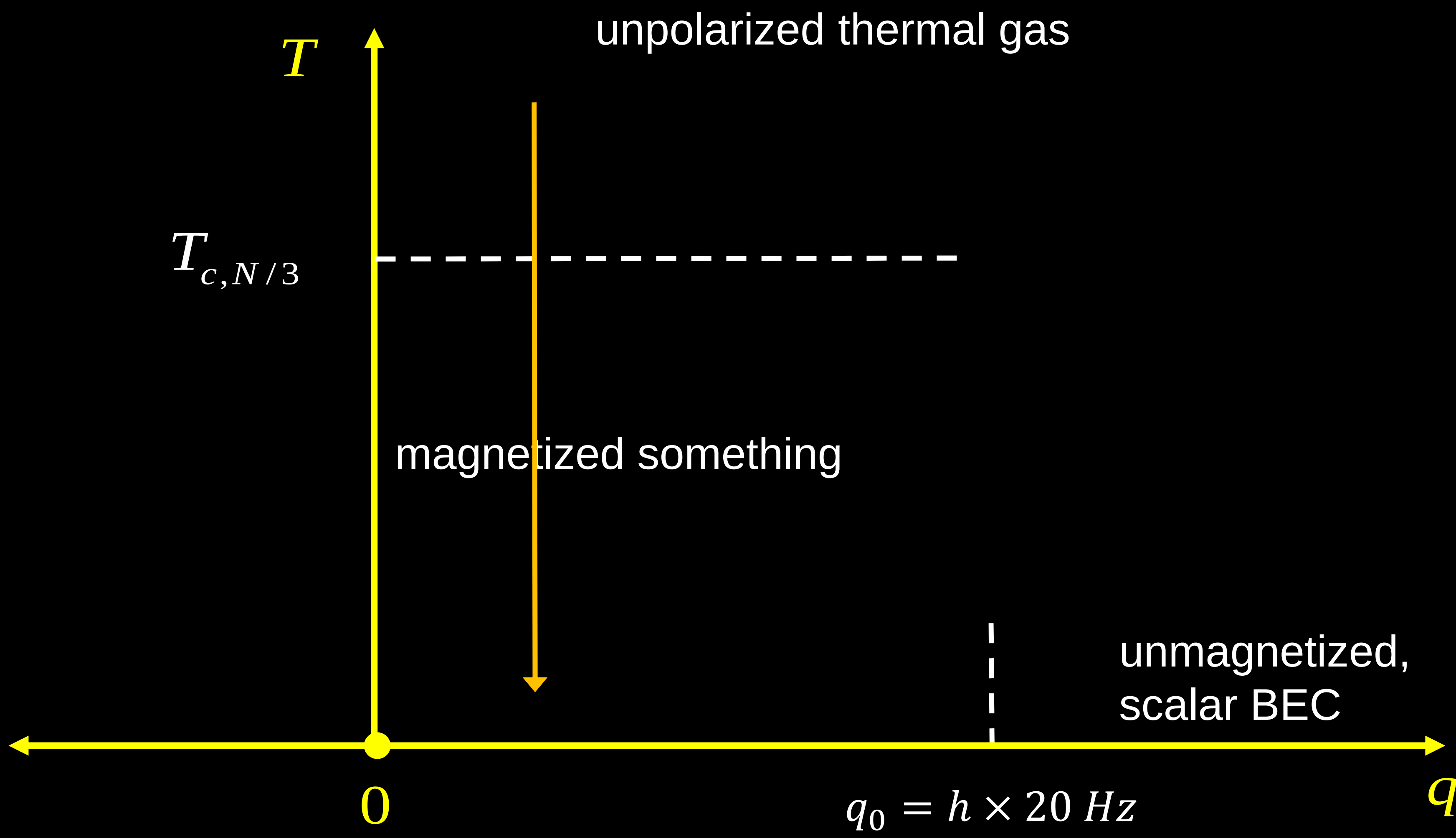
0

$q_0 = 2 \left|c_1^{(1)}\right|$ n

q

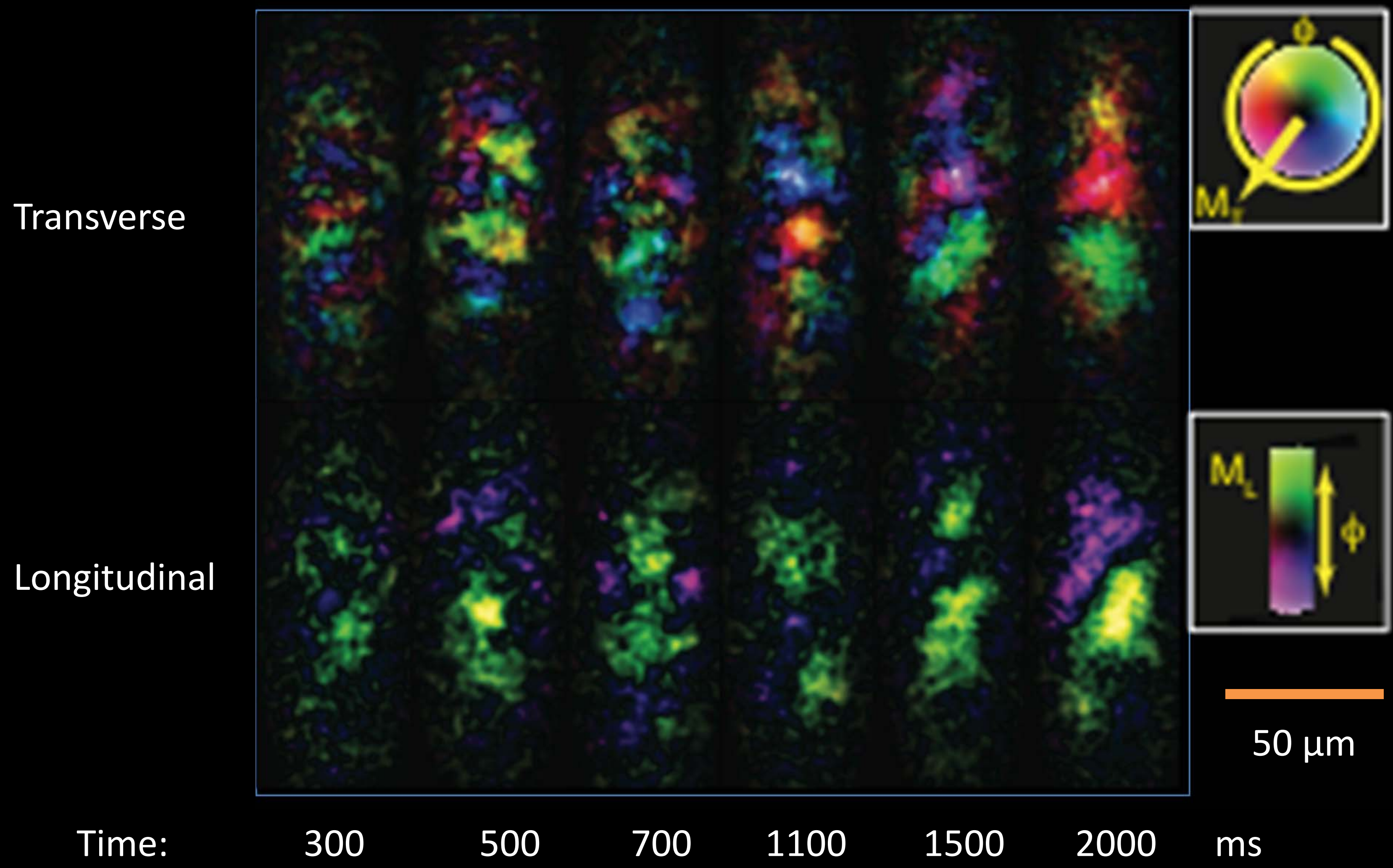
F=1 ^{87}Rb gas at thermal equilibrium?

- prepare fully depolarized thermal gas in uniform magnetic field
- lower temperature
- what happens?



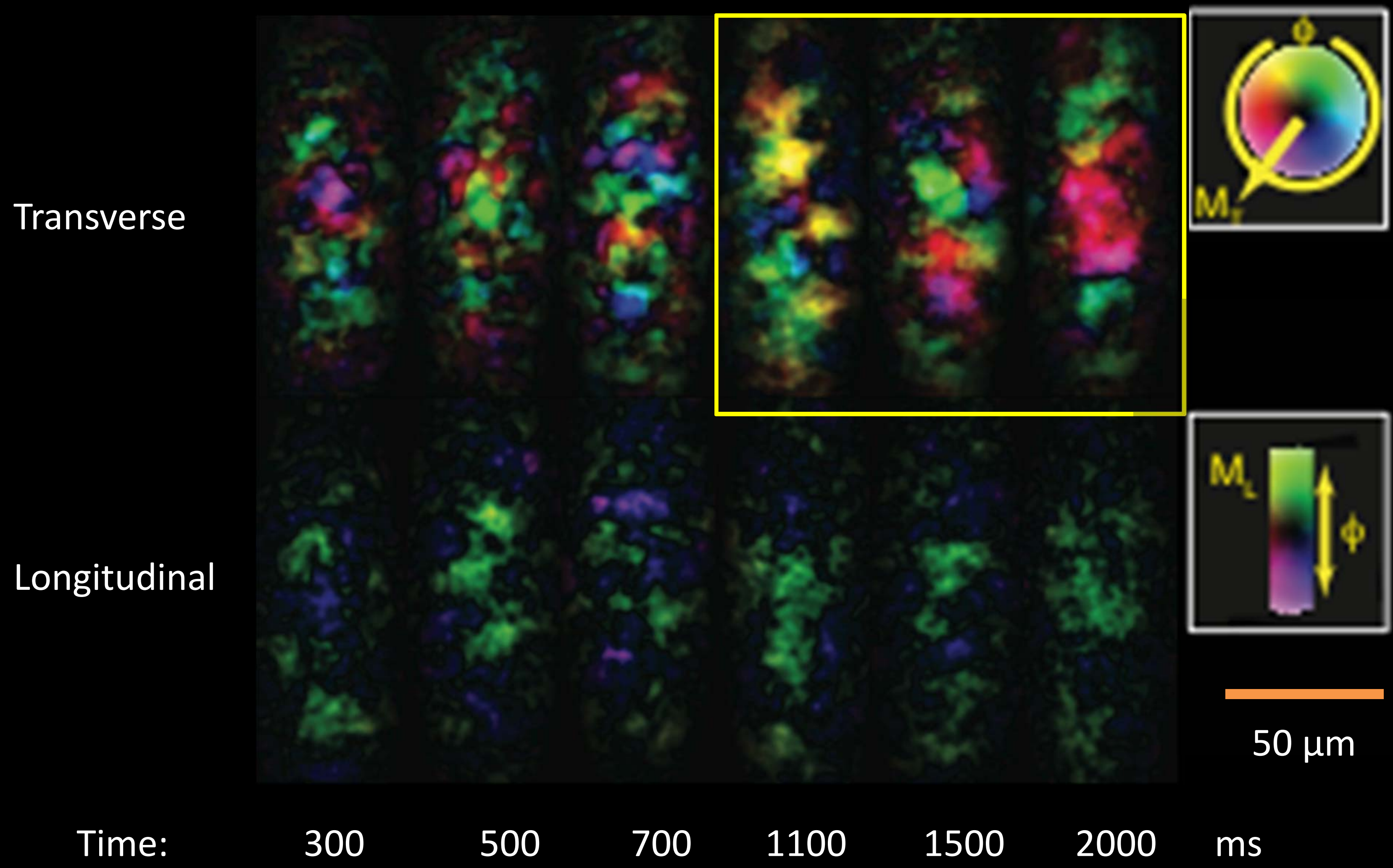
Development of spin texture

q/h = 0



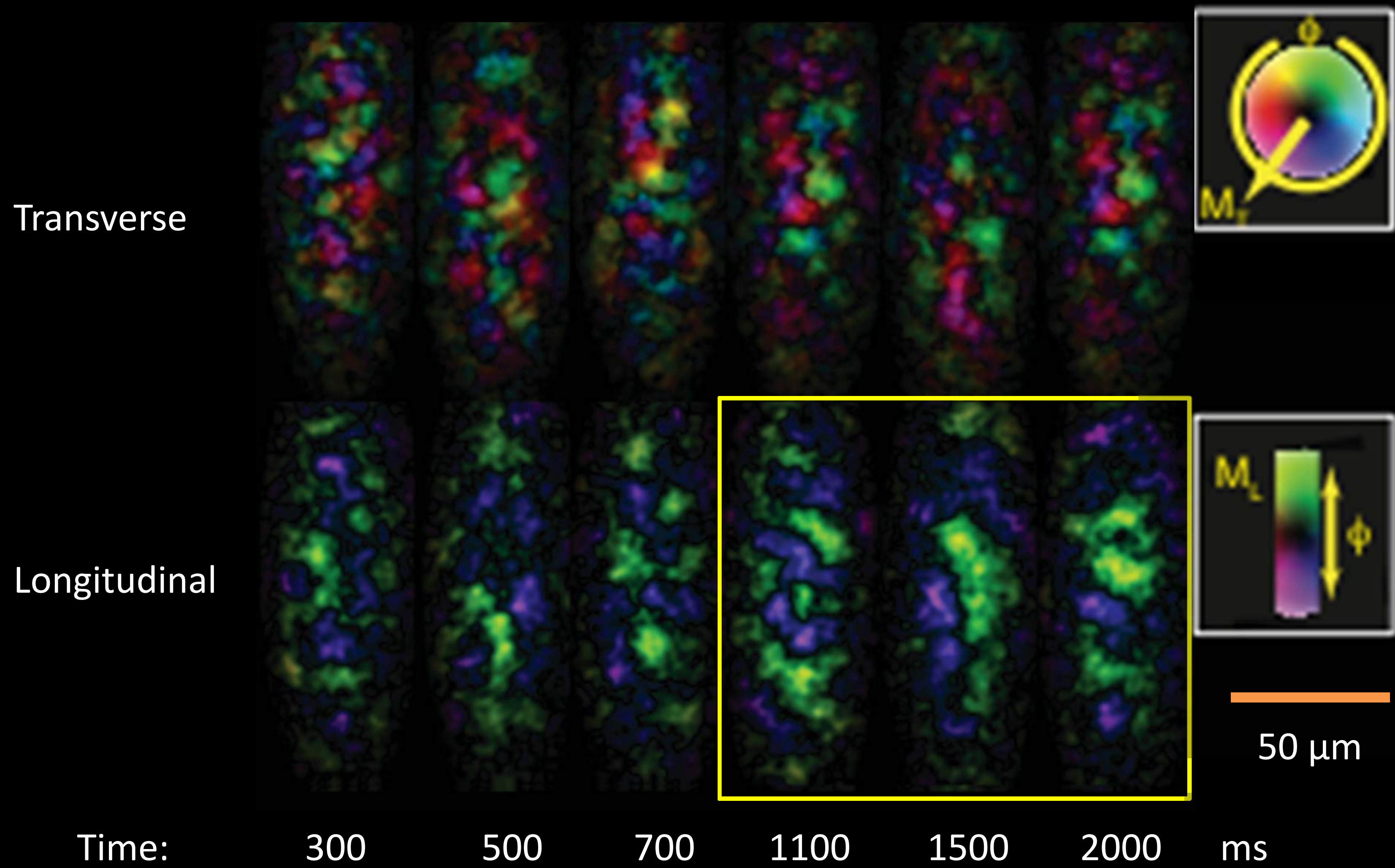
Development of spin texture

q/h = + 5 Hz



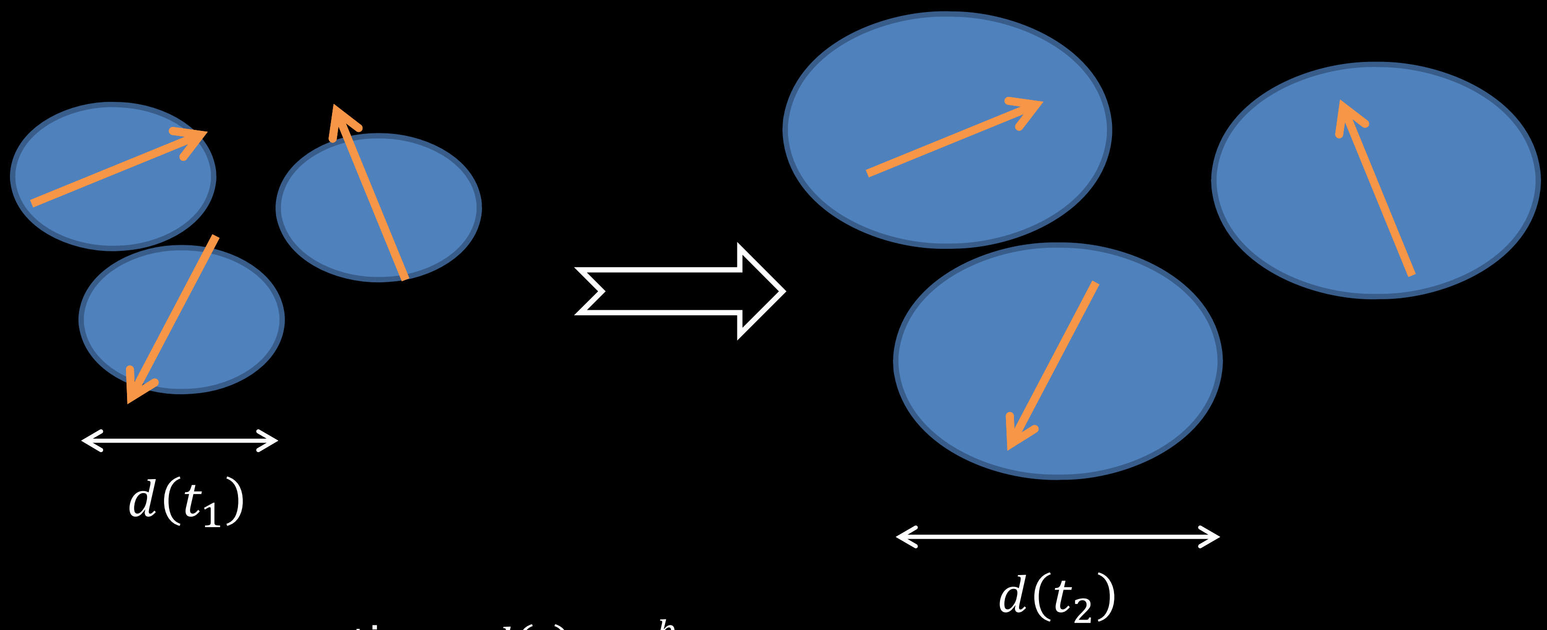
Development of spin texture

q/h = - 5 Hz



Questions and opportunities

- Why is equilibration so slow?
  - Partial answer: coarsening dynamics
    [A. J. Bray, Adv. Phys. **51**, 481 (2002)]



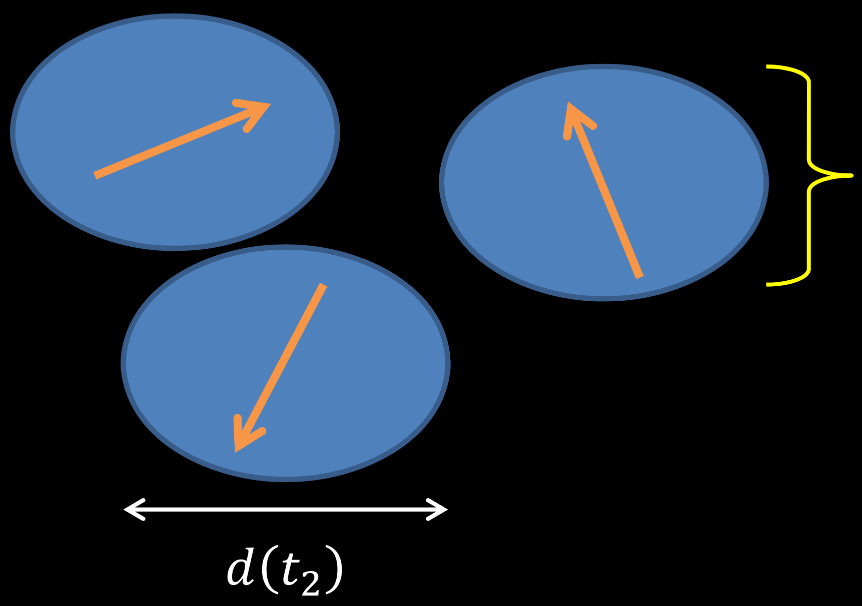
sometimes: $d(t) \propto t^b$

does scaling hold? how to determine this from small, short-lived samples?

what are underlying mechanisms?

Questions and opportunities

- Spin correlation functions: Fluctuations and spin susceptilibity
  - Equal time (one image): static spin structure
  - Unequal time (repeated image): dynamical spin structure factor



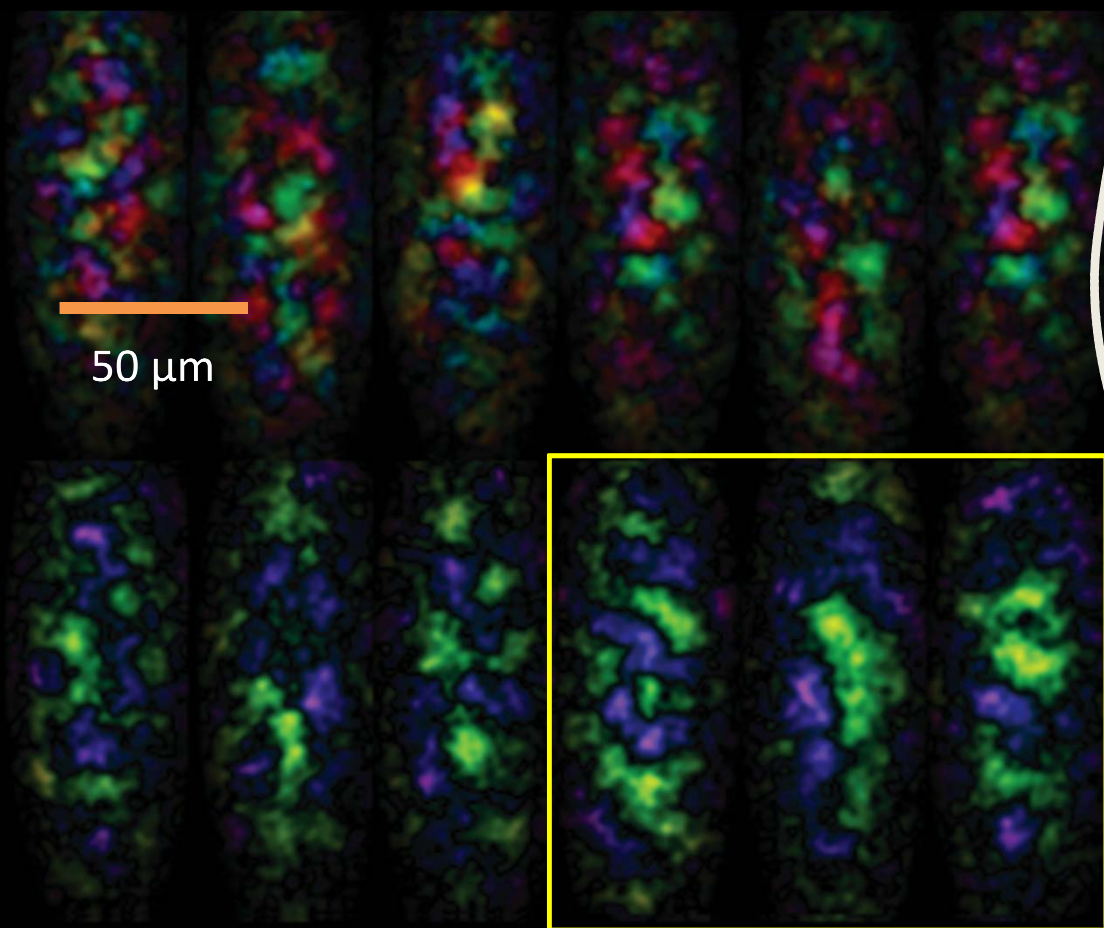
what can we learn from examining within one coarsened region, and from variation between regions?

What do such measurements tell us when we're unsure of equilibrium?

Precedent and guide from studies of density fluctuations:

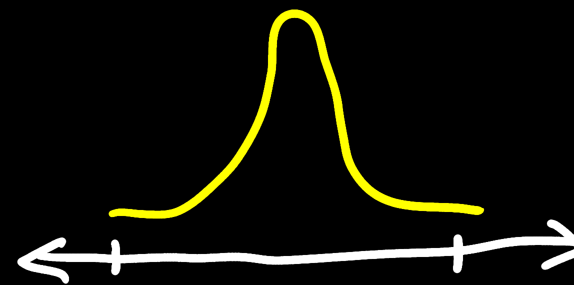
Hung et al., "Extracting density–density correlations from *in situ* images of atomic quantum gases," NJP **13**, 075019 (2011).

q/h = - 5 Hz
(easy axis feromagnet)



300 500 700 1100 1500 2000 ms

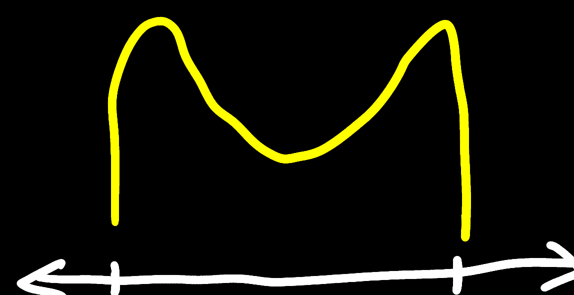
e.g. examining M_z



above T_c

~ Gaussian fluctuations

$$(\Delta M_z)^2 = k_B T \, \chi_M(\lim \omega \to 0)$$



below T_c

from data: $\chi_M(\lim \omega \to 0)$ has diverged

Topological structures

- Symmetry breaking: Hamiltonian of system has symmetry that state of system does not
  - there are symmetry operations that do change the state, but do not change its energy
  - continuous manifold of degenerate (equilibrium, ground, metastable) states

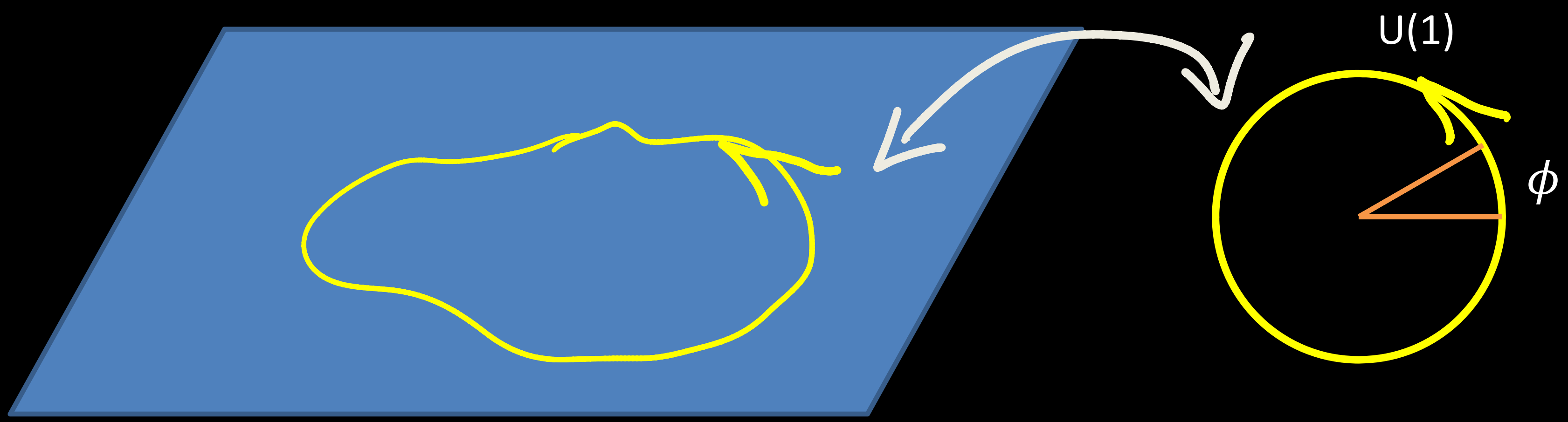
Examples of continuous symmetries:

Transformation	Group	Symmetry breaking system
"multiply wavefunction by complex phase"	$U(1)$	scalar superfluid
"rotations in 3D"	$SO(3)$	ferromagnetic superfluid

Topological structures

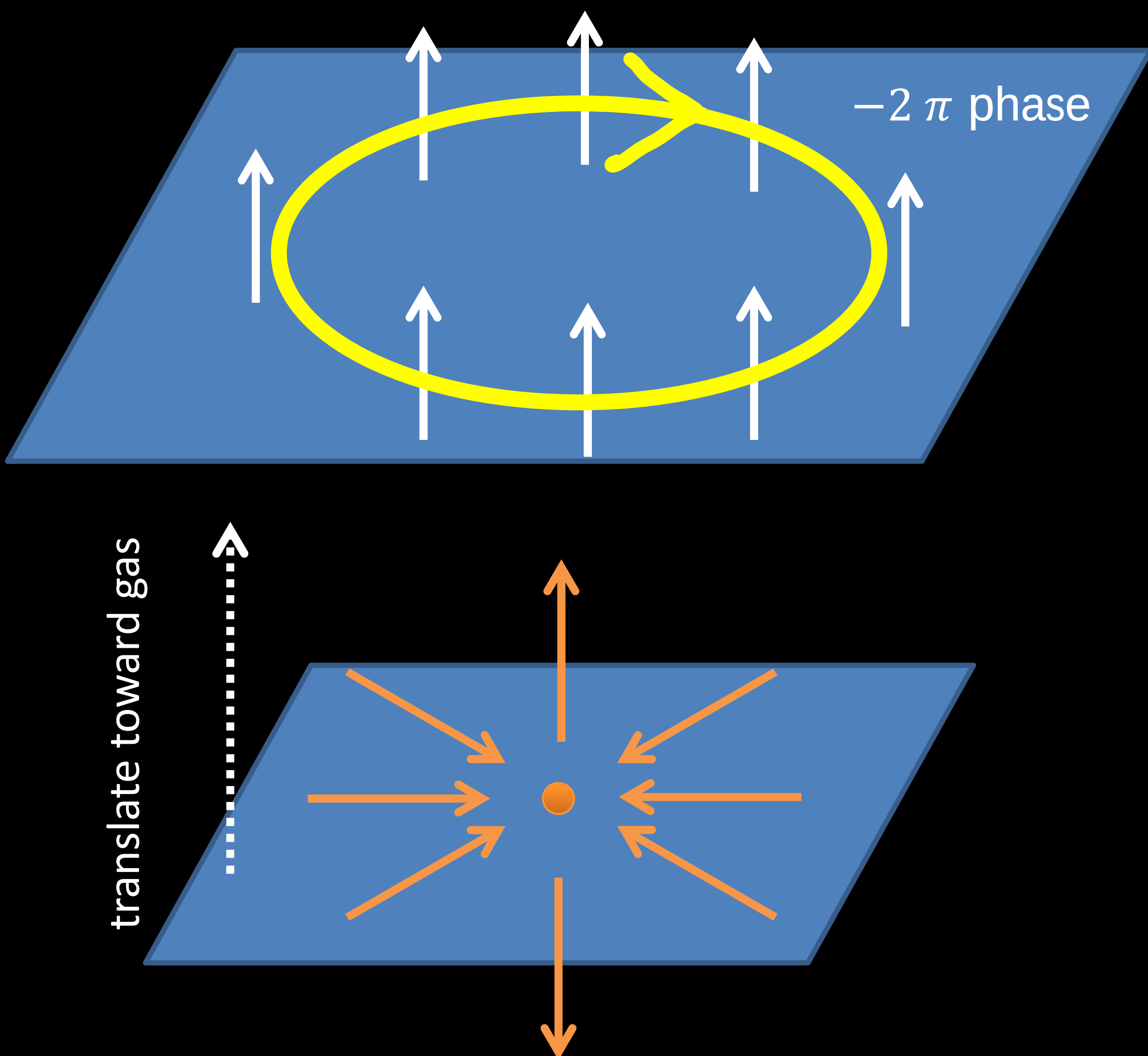
- mapping between order parameter space and a d-dimensional contour that cannot be smoothly undone

e.g. 1d contour, U(1) vortices



n windings possible (homotopy group is Z)

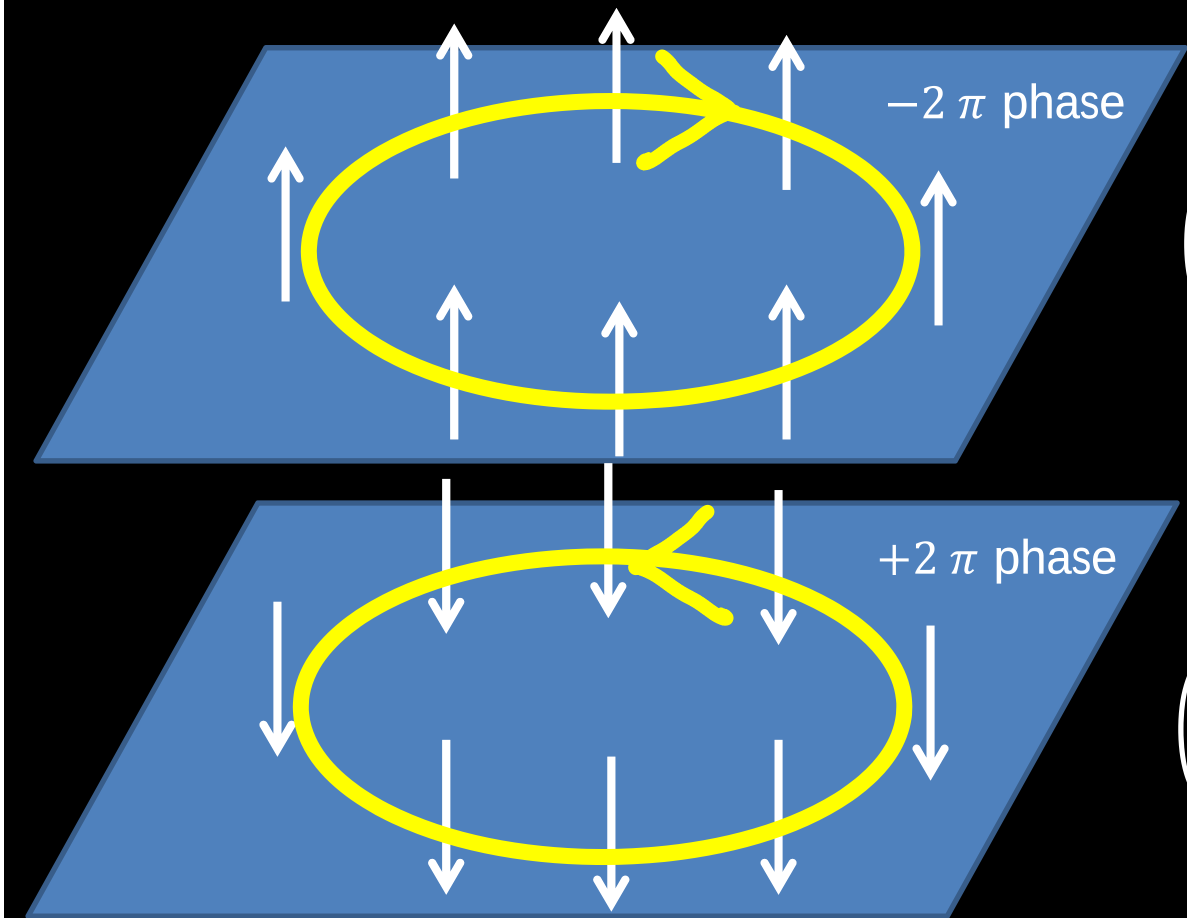
Vortices in a spin-1 ferromagnetic superfluid



spherical quadrupole magnetic field

$$\begin{pmatrix} a(r)e^{-i\phi} \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a(r)e^{-i\phi} \\ b(r) \\ c(r)e^{+i\phi} \end{pmatrix}$$

Vortices in a spin-1 ferromagnetic superfluid

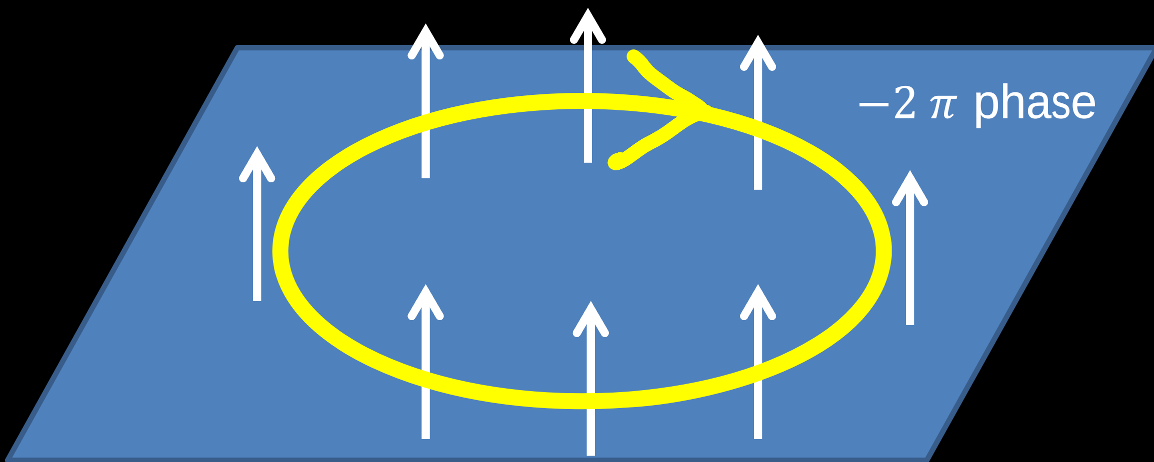


$$\begin{pmatrix} a(r)e^{-i\phi} \\ 0 \\ 0 \end{pmatrix}$$

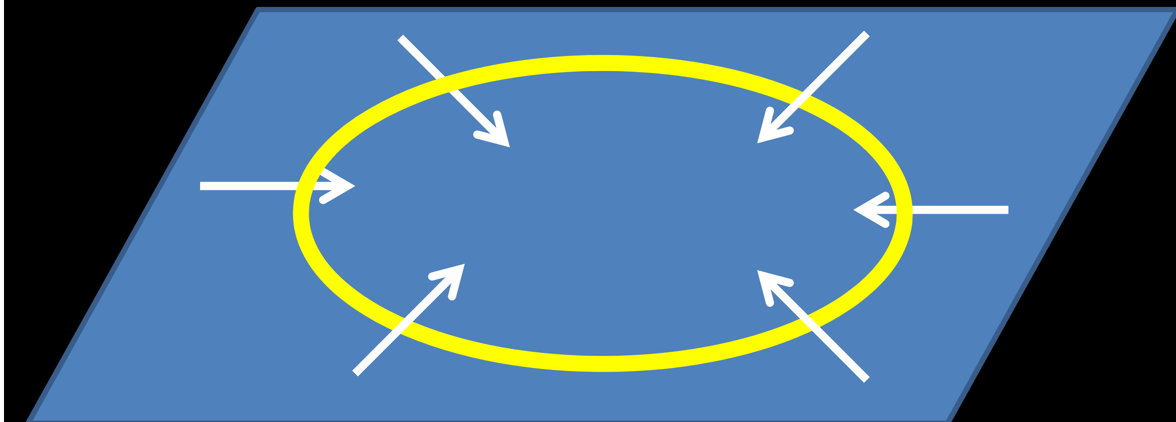
$$\begin{pmatrix} 0 \\ 0 \\ c(r)e^{+i\phi} \end{pmatrix}$$

- +1, -1 vortices are topologically equivalent. Also +2 vortex is equivalent to no structure at all! (homotopy group is Z_2)

Vortices in a spin-1 ferromagnetic superfluid



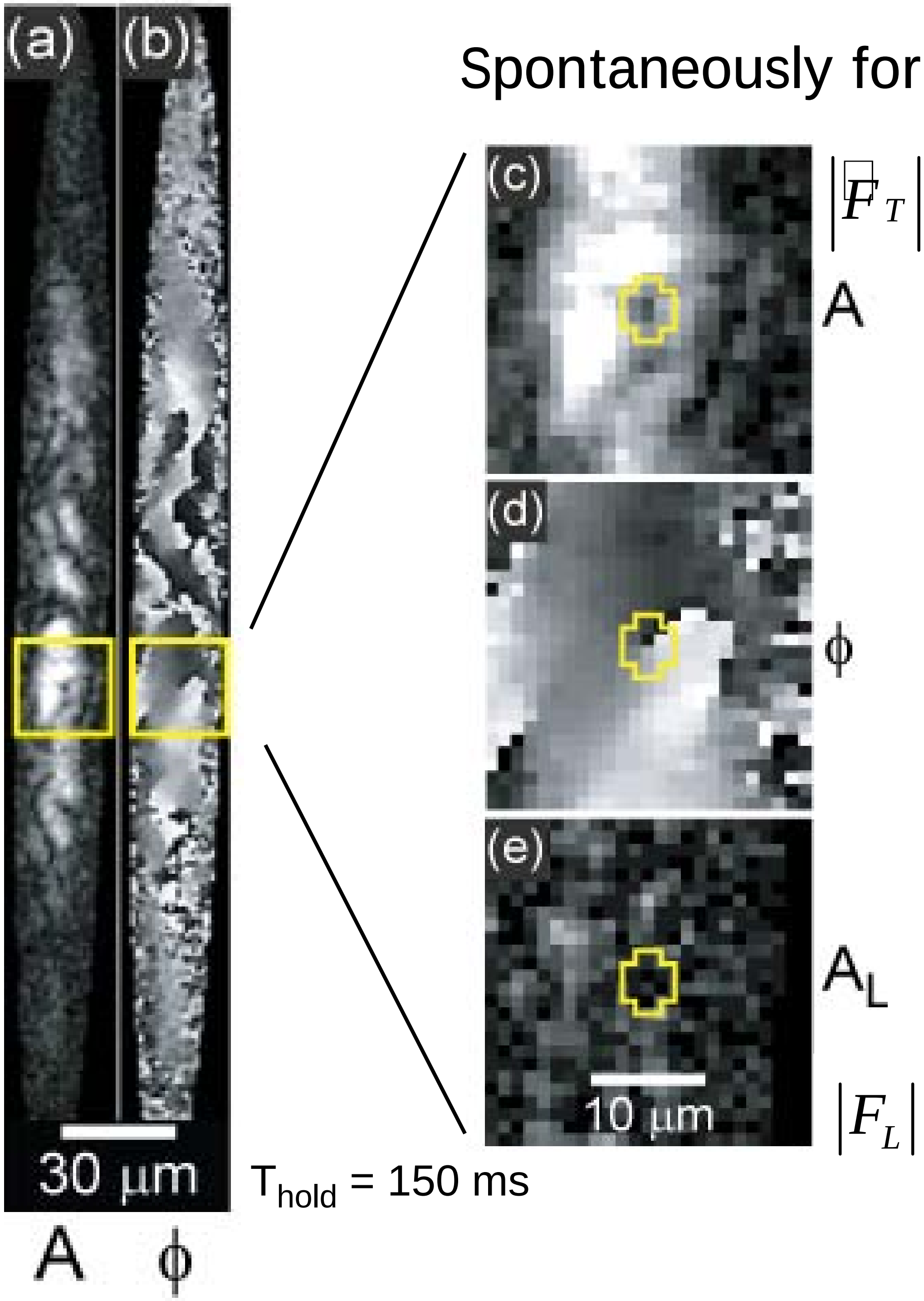
$$\begin{pmatrix} a(r)e^{-i\,\phi} \\ 0 \\ 0 \end{pmatrix}$$



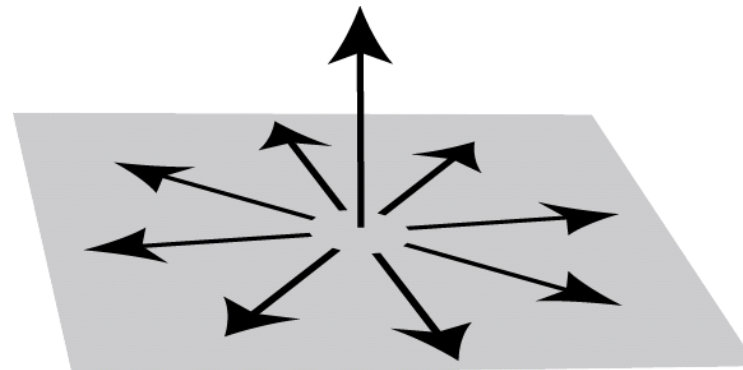
$$\begin{pmatrix} a(r)e^{-i\,\phi} \\ b(r) \\ c(r)e^{+i\,\phi} \end{pmatrix}$$

- +1, -1 vortices are topologically equivalent. Also +2 vortex is equivalent to no structure at all! (homotopy group is Z_2)
- polar-core spin vortex

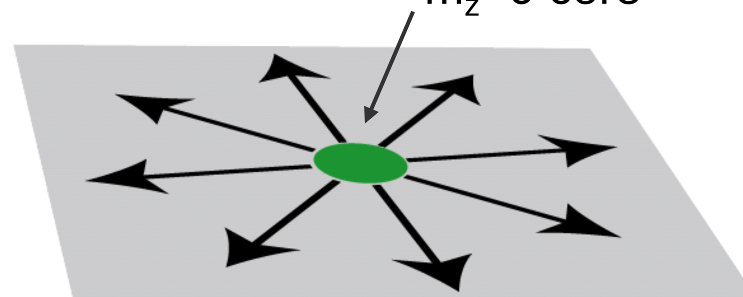
Spontaneously formed spin vortices



candidates:

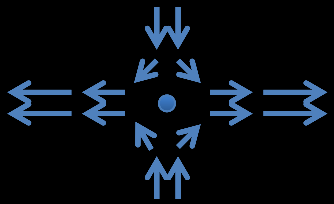


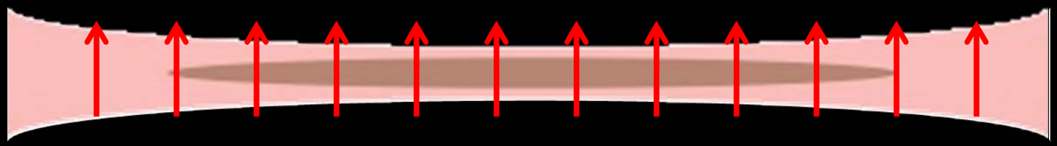
Mermin-Ho vortex (meron)



“Polar core” spin vortex

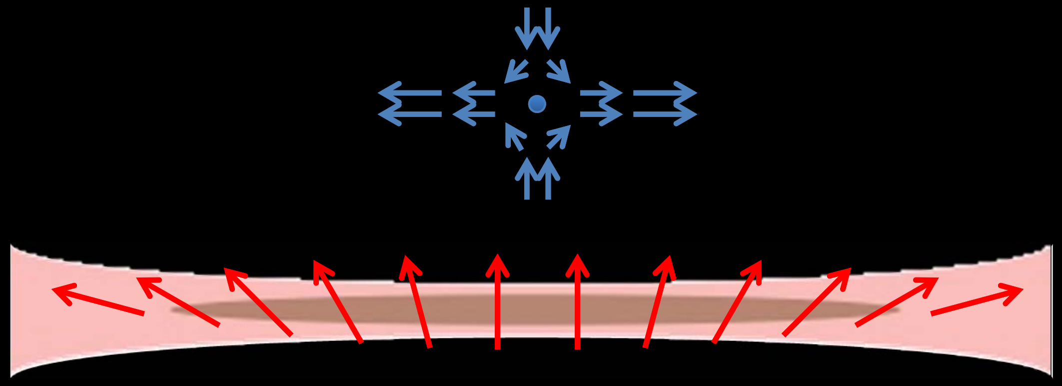
Making a spin texture





A. Leanhardt, *et. al.* PRL 90.140403 (2003)

Making a spin texture



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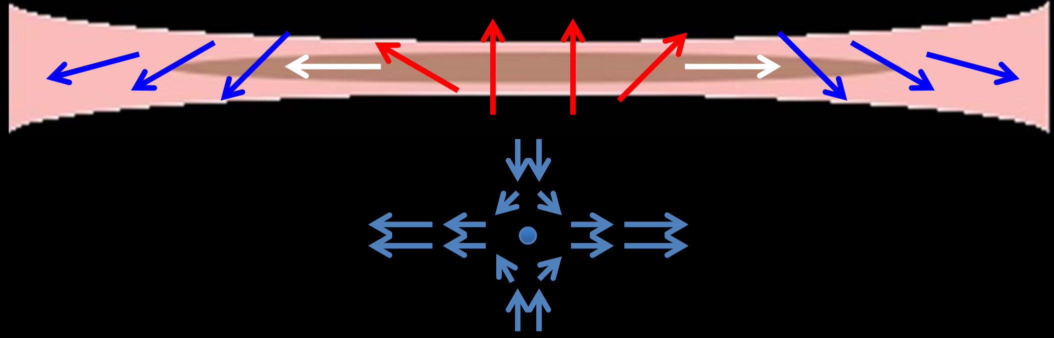
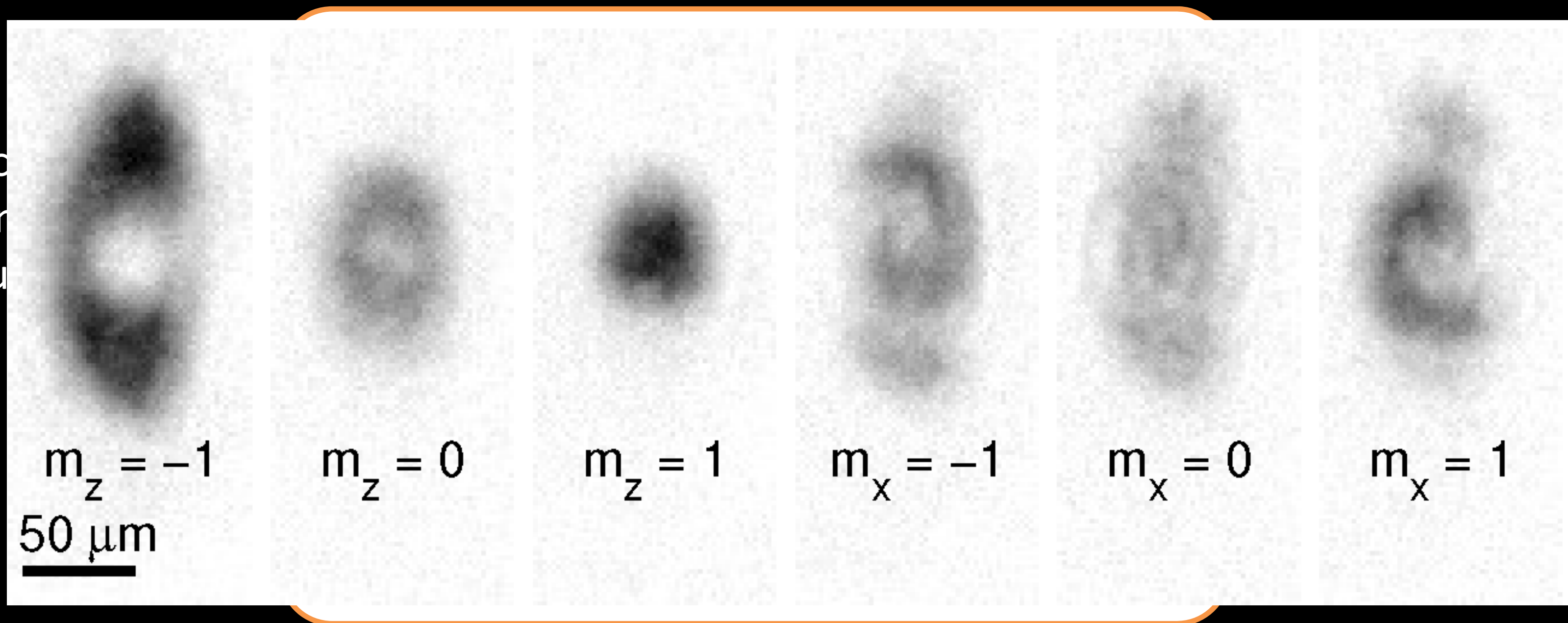
Making a spin texture

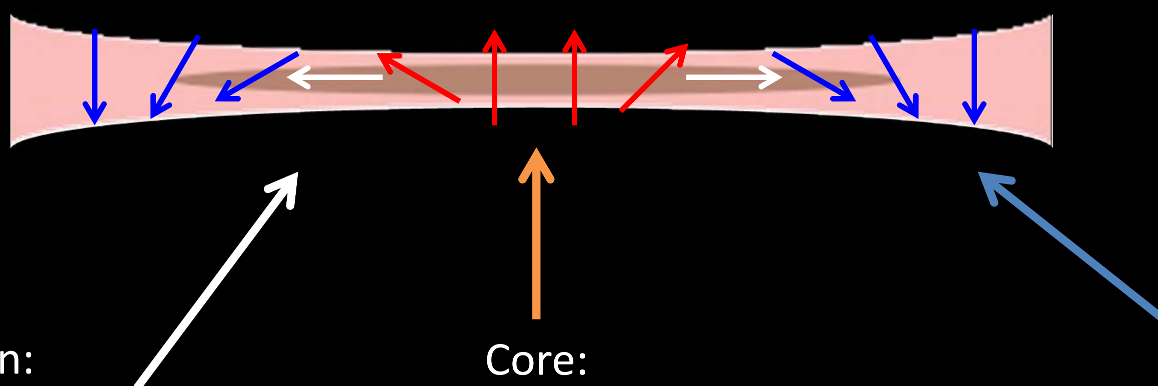


A. Leanhardt, *et. al.* PRL 90.140403 (2003)

Making a spin texture

Direc
magn
textu



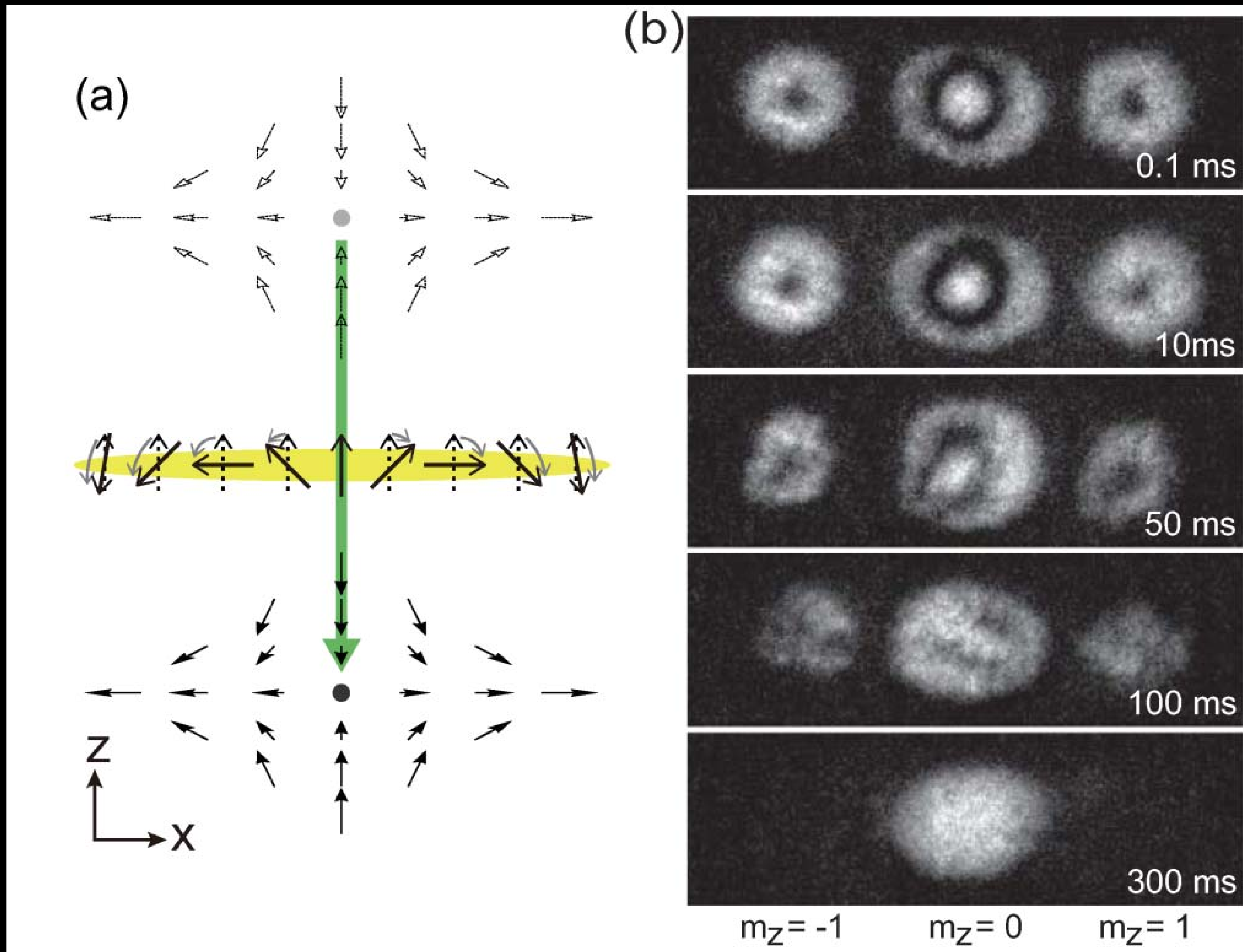


Between:
magnetized
radially (sideways)

Core:
magnetized up

Periphery:
magnetized
down

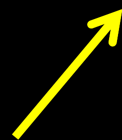
A. Leanhardt, *et. al.* PRL 90.140403 (2003)

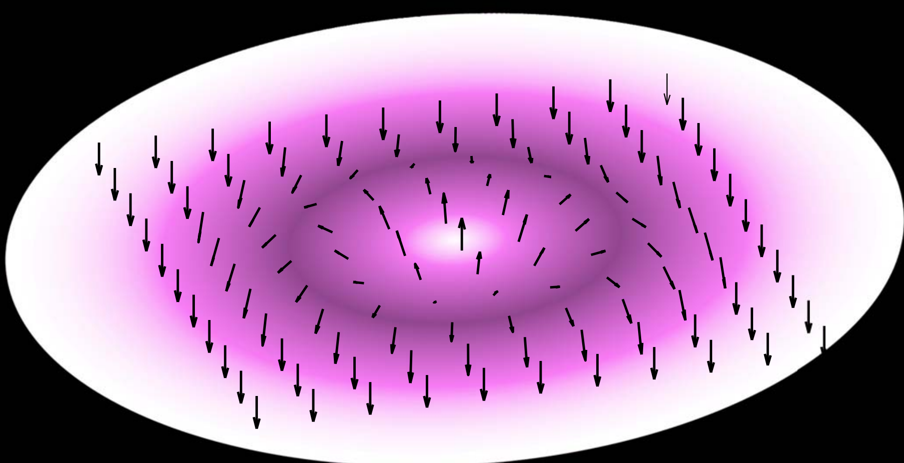


Choi, J.Y., W.J. Kwon, and Y.I. Shin, Observation of Topologically Stable 2D Skyrmions in an Antiferromagnetic Spinor Bose-Einstein Condensate. PRL **108**, 035301 (2012)

skyrmion, or not skyrmion?

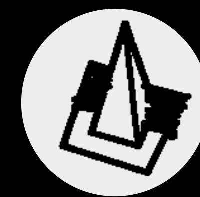
Ferromagnet or polar spinor condensate:

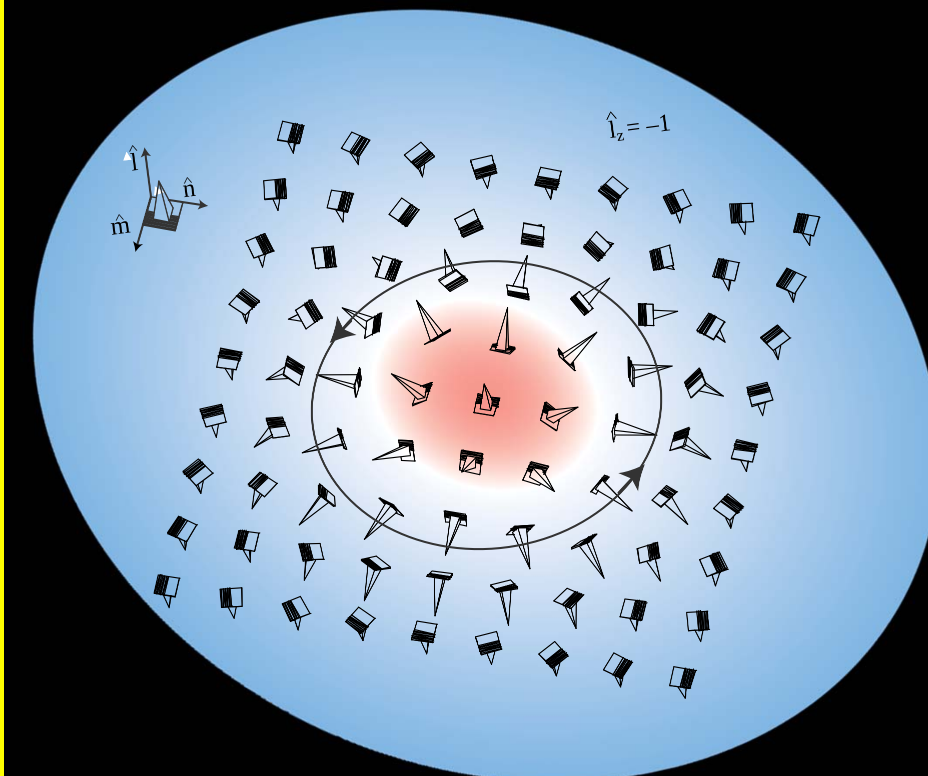
Order parameter = 



= skyrmion (topological)

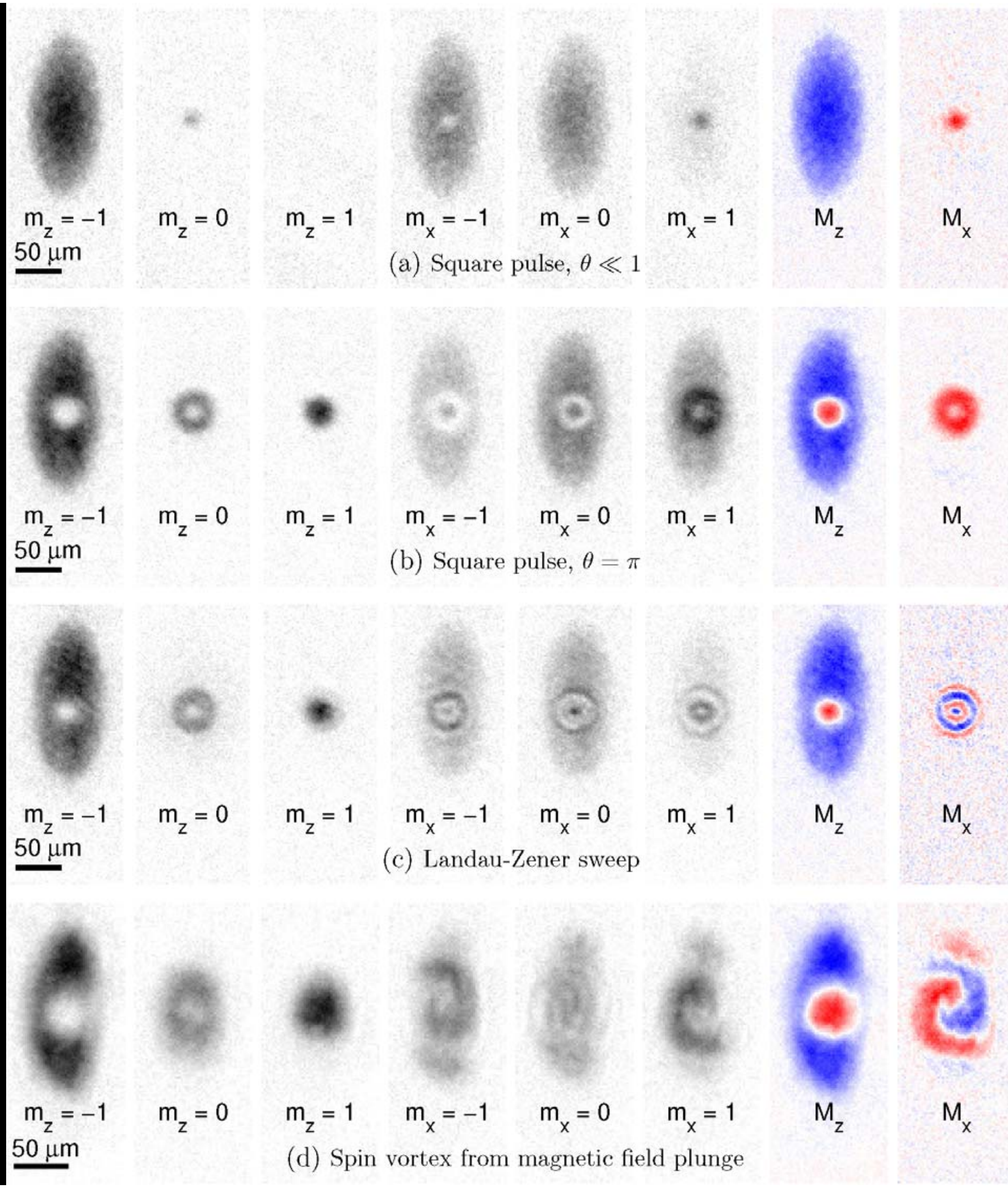
Ferromagnetic spinor condensate:

Order parameter = 



≠ skyrmion (not topological)

Other localized spin structures



Spinor superfluid hydrodynamics

Lamacraft PRA **77**, 063622 (2008)

Barnett, Podolsky and Refael, PRB **80**, 024420 (2009)

- Modification of irrotational flow condition:

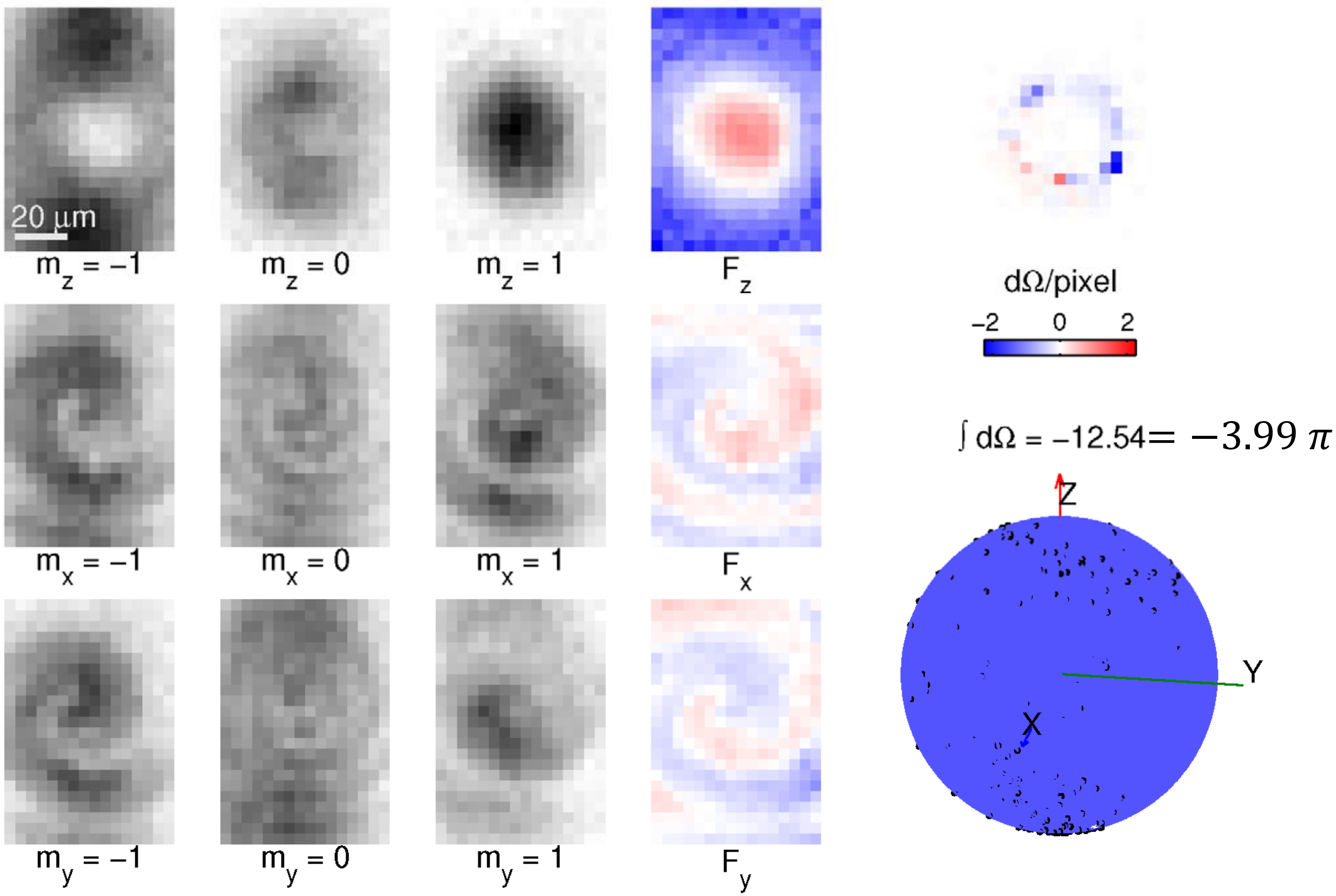
$$(\nabla \times \vec{v})_i = \frac{1}{2}\epsilon_{ijk}\vec{n} \cdot \partial_j \vec{n} \times \partial_k \vec{n}$$

$\vec{n}$ = direction of magnetization

magnetization curvature (topological density) creates Lorentz-like force
[similar to “topological Hall effect” in solid-state; see PRL 102, 186602 (2009)]

In-situ measurement of magnetization curvature

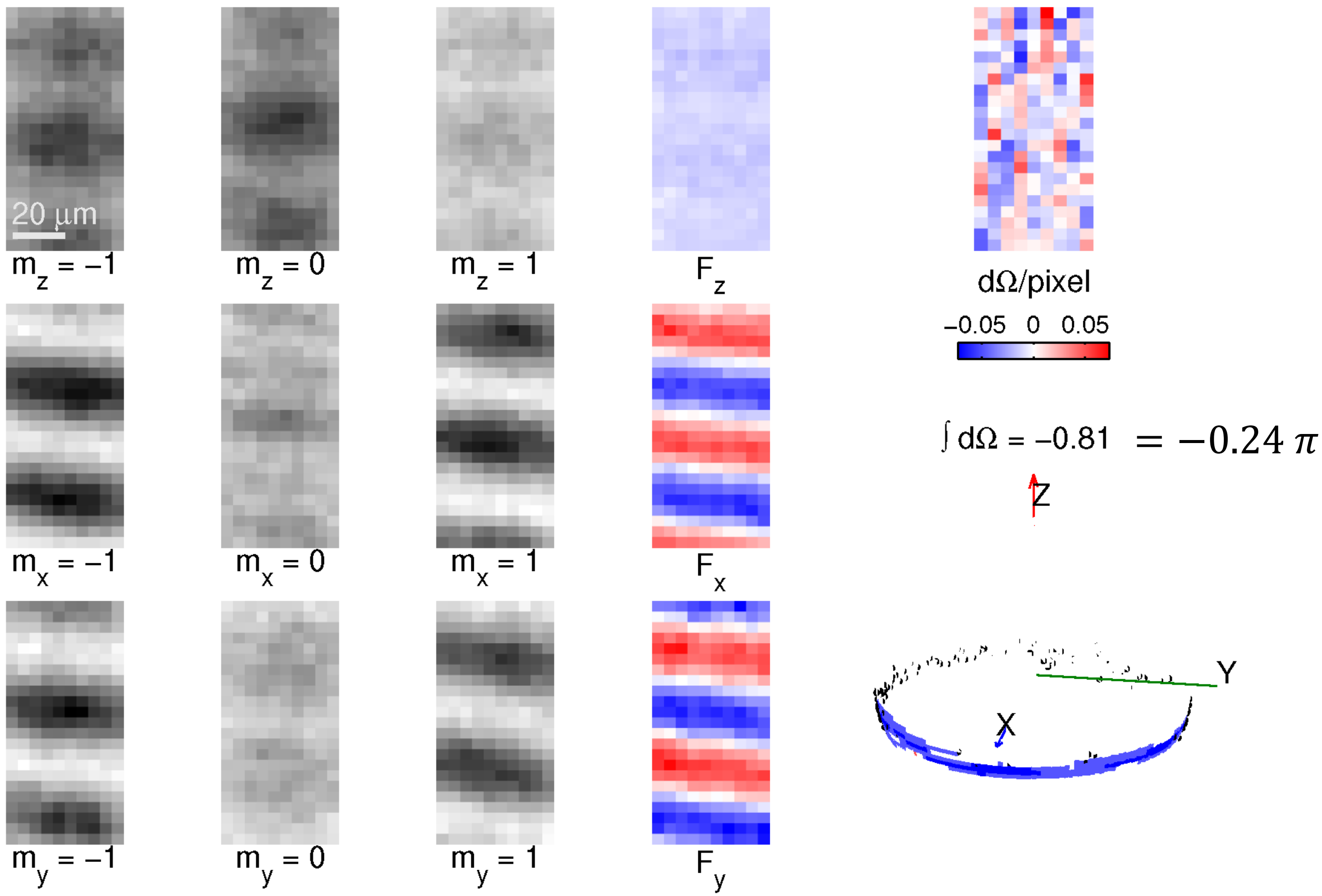
skyrmion-like spin texture



$\int d\Omega = -12.54 = -3.99\,\pi$

In-situ measurement of magnetization curvature

spin helix spin texture



4. Spin mixing dynamics

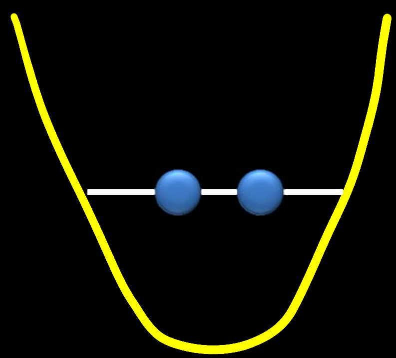
a. microscopic spin mixing oscillations

b. SMA, mean-field dynamics

c. spin-mixing instability

Microscopic spin mixing

Consider just two particles in a tight trap



$$H = const. + \frac{4\,\pi\hbar^2\langle n\rangle}{m}\left(a_0\hat{P}_0 + a_2\,\hat{P}_2\right)$$

say initially both atoms are in $|m_F = 0\rangle$

$$|\Psi(0)\rangle = \ |0,2,0\rangle$$

superposition of states with two different total spin:

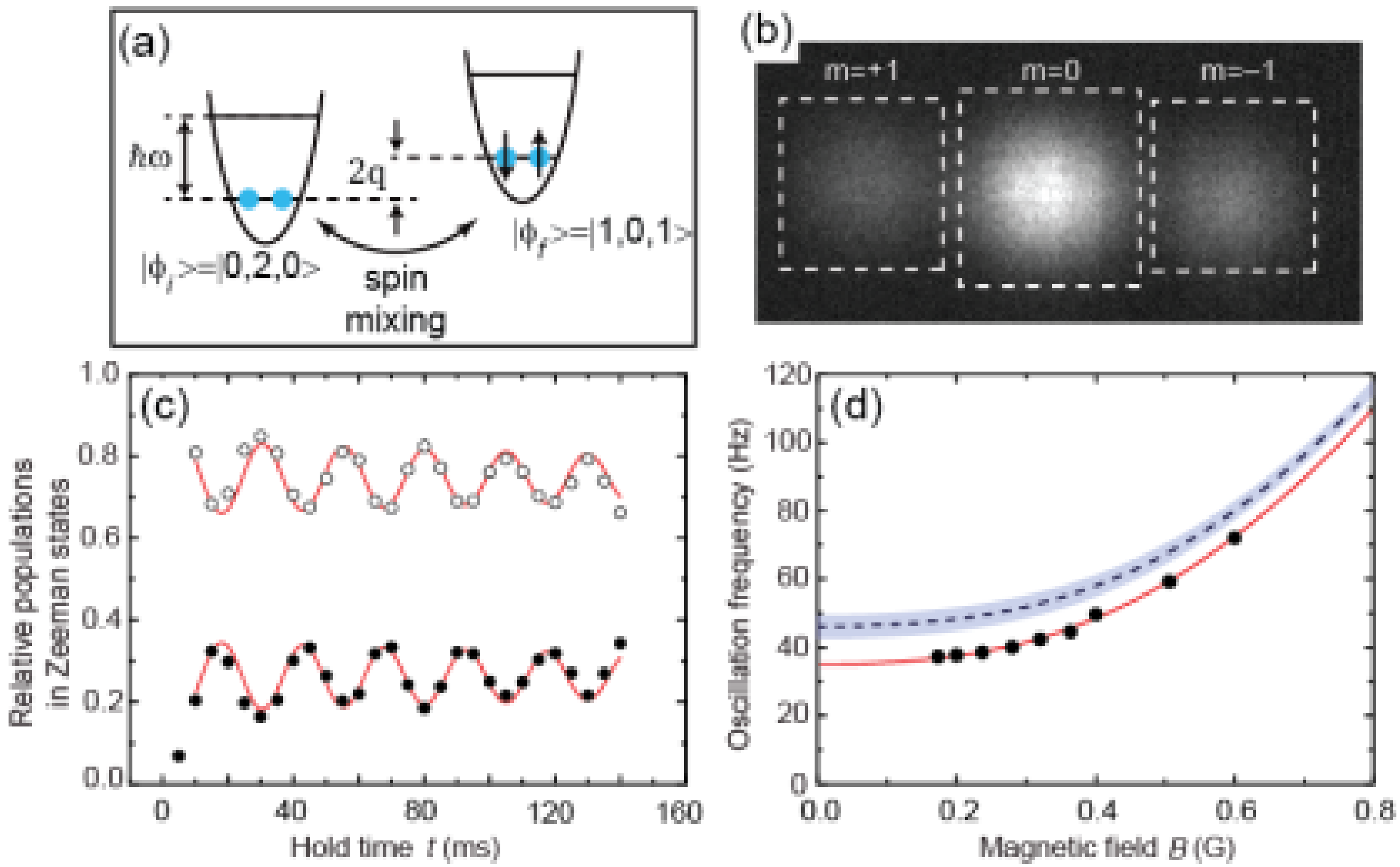
$$\left|\mathrm{F_{pair}} = 2,0\right\rangle = \sqrt{{}^{2}\!/_{3}}\ |0,2,0\rangle + \sqrt{{}^{1}\!/_{3}}\ |1,0,1\rangle$$ evolves at ω_2

$$\left|\mathrm{F_{pair}} = 0,0\right\rangle = -\sqrt{{}^{1}\!/_{3}}\ |0,2,0\rangle + \sqrt{{}^{2}\!/_{3}}\ |1,0,1\rangle$$ evolves at ω_0

$$|\Psi(t)\rangle = \left(\frac{2}{3}e^{-i\,\omega_2 t} + \frac{1}{3}e^{-i\,\omega_1 t}\right)|0,2,0\rangle + \left(\frac{\sqrt{2}}{3}e^{-i\omega_2 t} - \frac{\sqrt{2}}{3}e^{-i\omega_0 t}\right)\,|1,0,1\rangle$$

spin mixing of many atom pairs

Widera et al., PRL **95**, 190405 (2005)



Mean-field macroscopic spin mixing

Zhang et al., PRA 72, 013602 (2005)

- derive spinor Gross-Pitaevskii equation (lots of papers, looks complicated)
- Identify energy landscape and dynamical variables:

$$E = \frac{c_1^{(1)} n}{2} \langle \boldsymbol{F} \rangle^2 + p \langle F_z \rangle + q \langle F_z^2 \rangle$$

$$\psi_{mF} = \sqrt{\rho_{m_F}} \exp(-i\, \theta_{m_F})$$

$$\rho_{+1} + \rho_0 + \rho_{-1} = 1$$

$$\rho_{+1} - \rho_{-1} = M$$

$$\theta_{+1} + \theta_0 + \theta_{-1} = 3\,\bar{\theta}$$ unimportant

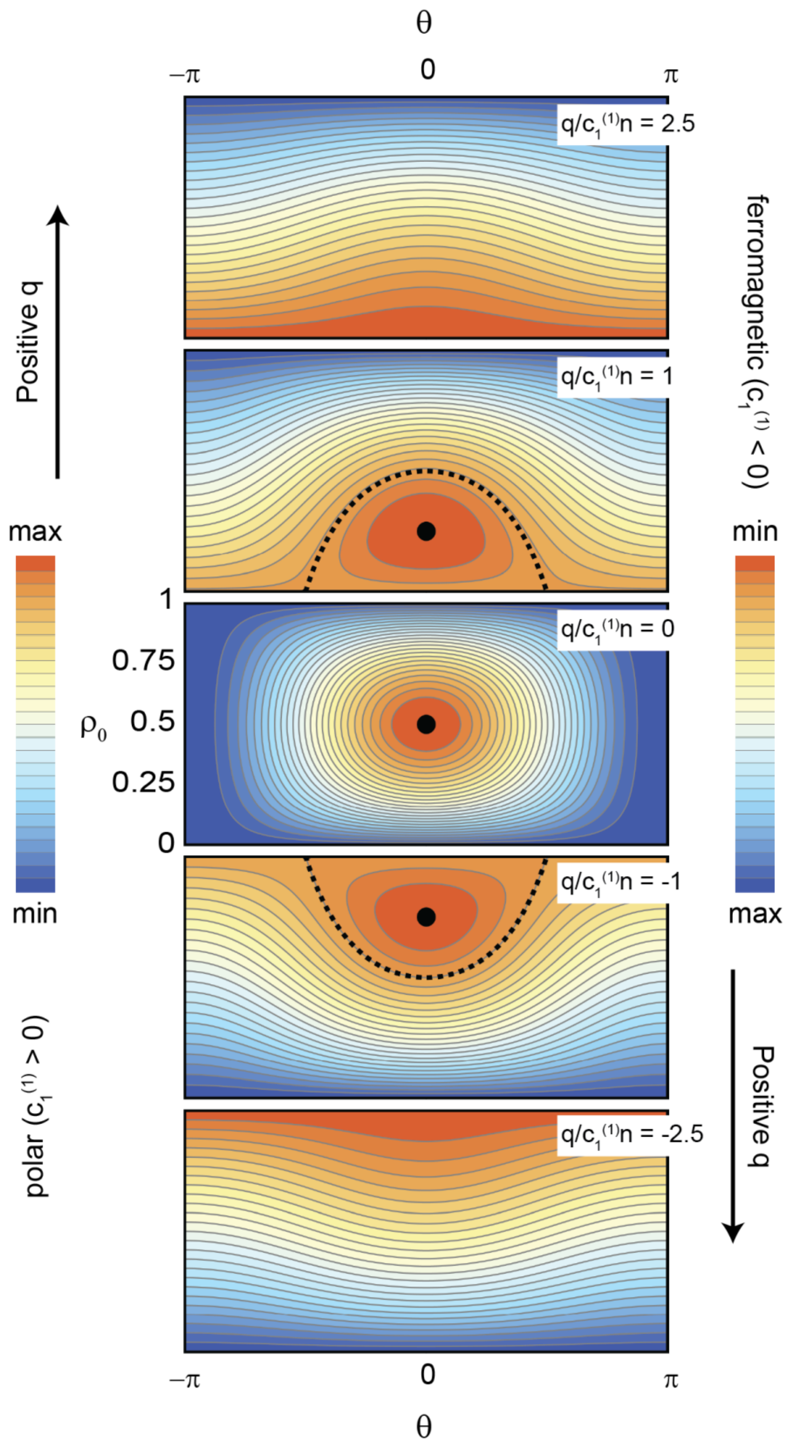
$$\theta_{+1} - \theta_{-1}$$ unimportant

only one density to keep track of: ρ_0

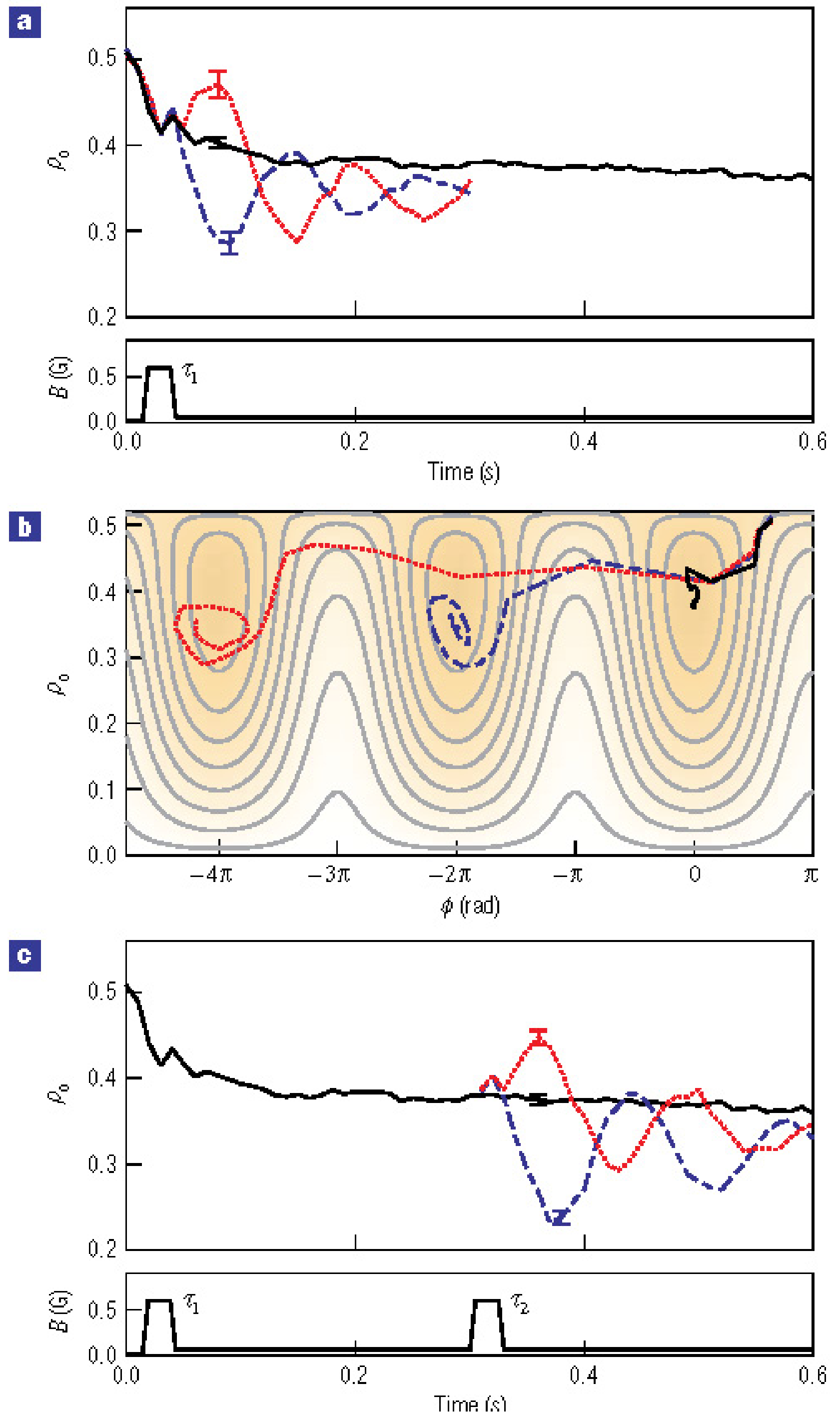
only one phase to keep track of:

$$\theta = \theta_{+1} + \theta_{-1} - 2\,\theta_0$$

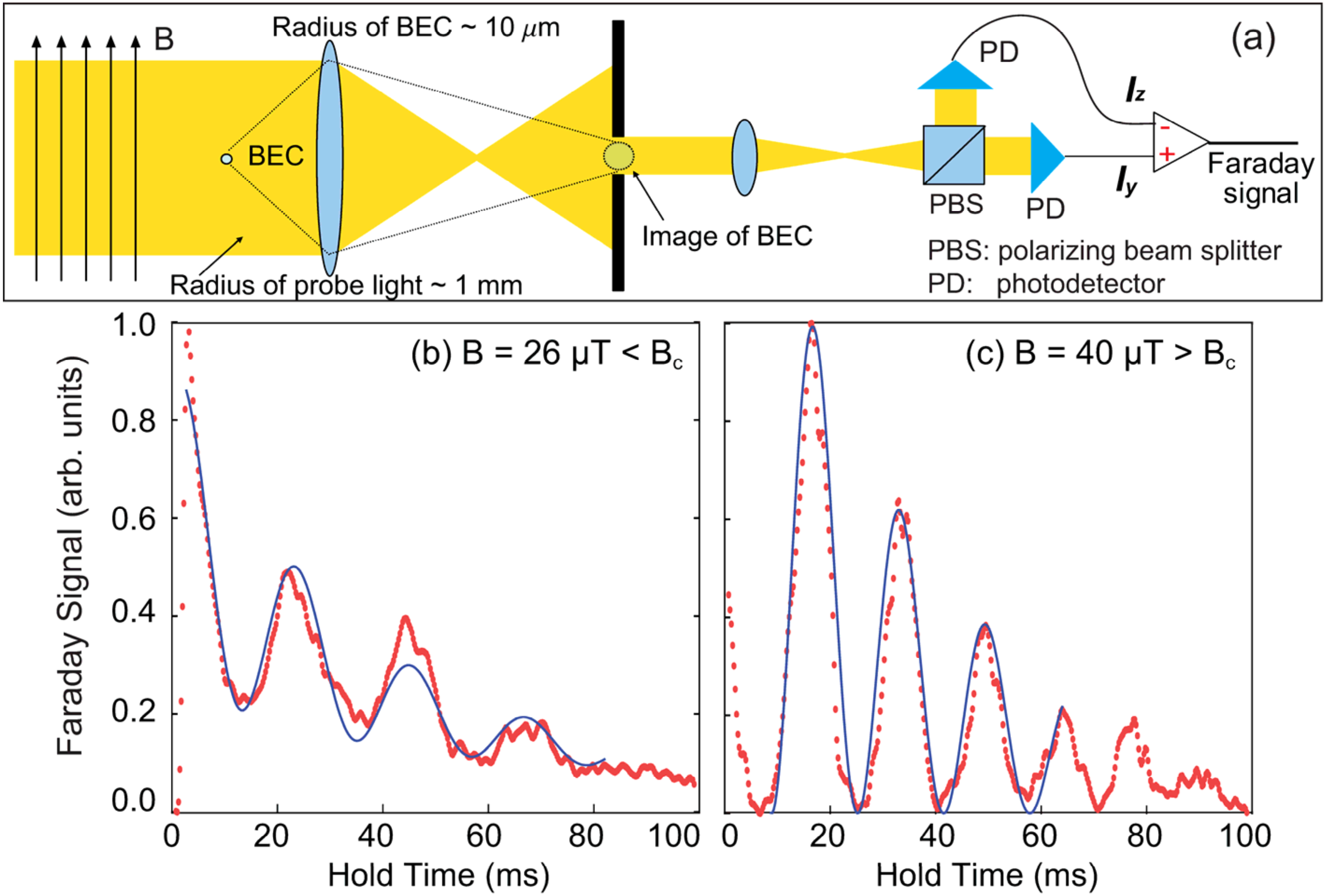
$$\frac{d\rho_0}{dt} = -\frac{2}{\hbar} \frac{\partial E}{\partial \theta} \qquad \frac{d\theta}{dt} = +\frac{2}{\hbar} \frac{\partial E}{\partial \rho_0}$$



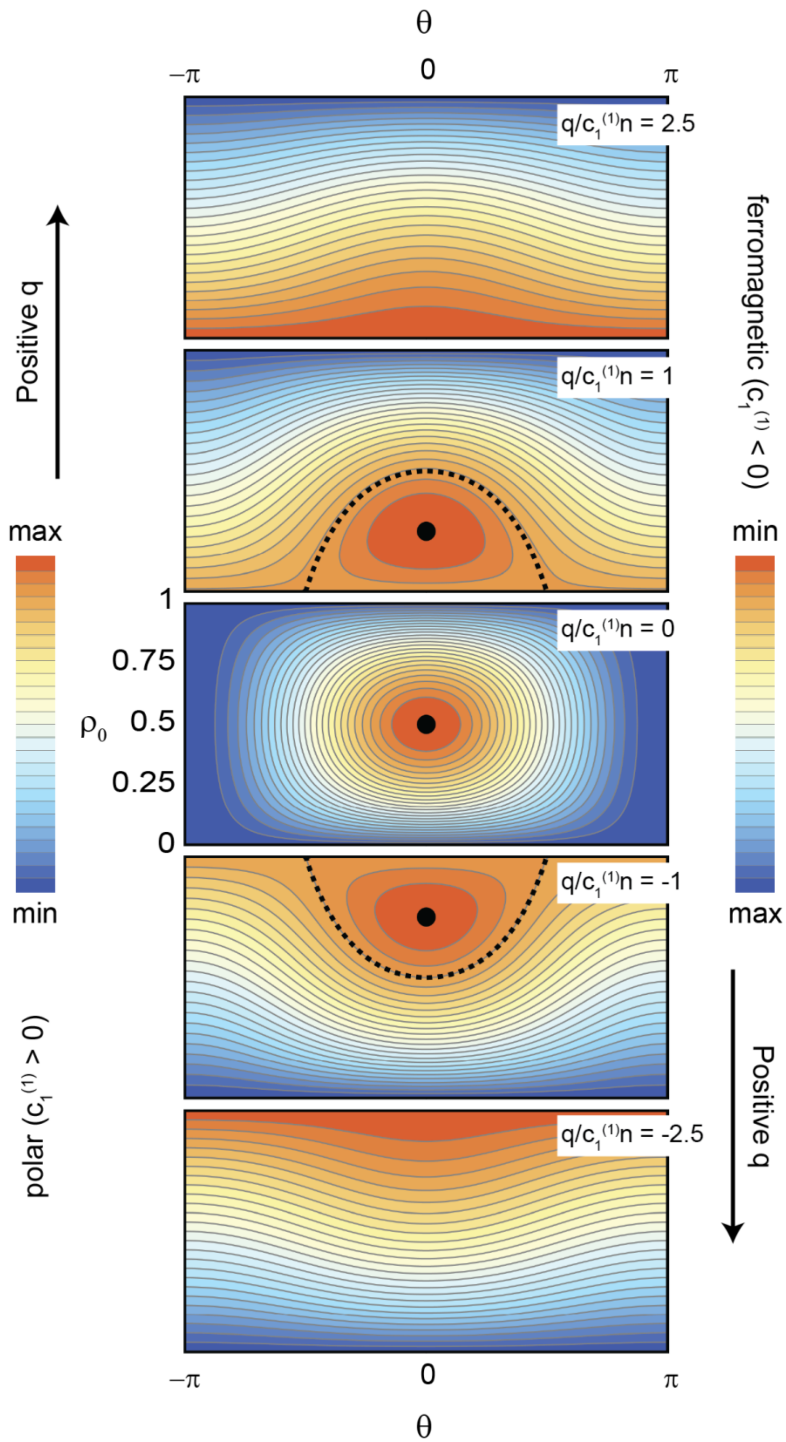
θ
−π
0
π
q/c1(1)n = 2.5
q/c1(1)n = 1
q/c1(1)n = 0
q/c1(1)n = -1
q/c1(1)n = -2.5
ferromagnetic (c1(1) < 0)
Positive q
max
min
ρ0
1
0.75
0.5
0.25
0
polar (c1(1) > 0)



M. S. Chang et al, Nature Physics **1**, 111 (2005)



Liu, Y., S. Jung, S.E. Maxwell, L.D. Turner, E. Tiesinga, and P.D. Lett, Quantum Phase Transitions and Continuous Observation of Spinor Dynamics in an Antiferromagnetic Condensate. PRL **102**, 125301 (2009.


θ
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Positive q
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ρ₀ 0.5
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polar (c₁⁽¹⁾ > 0)
Positive q
−π
0
π
θ