Probing and Controlling Quantum Matter at the Single Atom Level

Ahmed Omran - Fermion Quantum Gas Microscope

Frauke Seesselberg - Ultracold Polar Molecules

Karen Wintersperger - Artificial Gauge Fields in Hexagonal Lattices

Max-Planck-Institut für Quantenoptik
Ludwig-Maximilians Universität

funding by
€ MPG, European Union, DFG
$ DARPA (OLE)

erc synergy

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Course Outline

LECTURE 1

Introduction

- Brief Review Lattice Basics
- Detection Methods
- Hubbard models

Single Atom Imaging/Control

- Single Atom Imaging Bosons/Fermions
- Probing Thermal and Quantum Fluctuations
- Single Spin Manipulation
- String Order - a Hidden Order Parameter
- Higgs Amplitude Mode

LECTURE 2 - Quantum Magnetism

Superexchange - from double wells to RVB/d-wave states on plaquettes

Probing Spin Correlations

Single Spin Impurity

Bound Magnons

AFM Order in the Fermi Hubbard Model

Quantum Magnetism with Rydberg atoms

LECTURE 3 - Artificial Gauge Fields

SSH model - the simplest Topological Insulator

Probing the Zak Phase in the SSH model
- Bulk-Edge correspondence in 1d -

'Aharonov Bohm' Interferometry for Measuring Band Geometry

- Berry connection/Berry curvature
- pi-flux Singularity in Graphene
- Stückelberg Interferometry

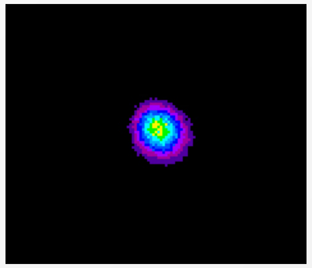
(non-Abelian Berry connection, Wilson loops)

Realizing Staggered Flux, Hofstadter & QSH Hamiltonian

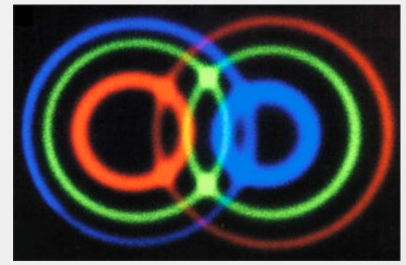
Hall Response and Chern Number in Hofstadter Bands

The Challenge of Many-Body Quantum Systems

Control of single and few particles



Single Atoms and Ions

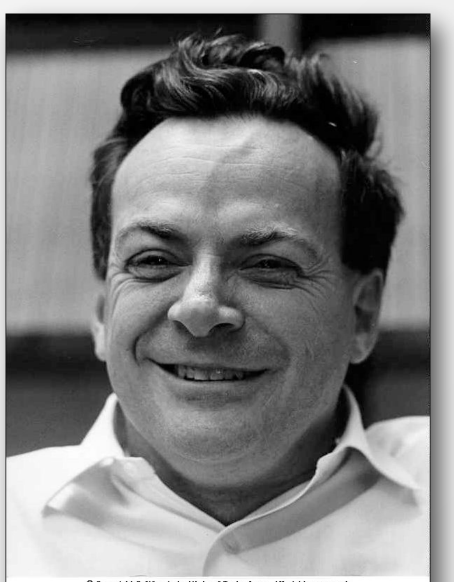


Photons

D. Wineland

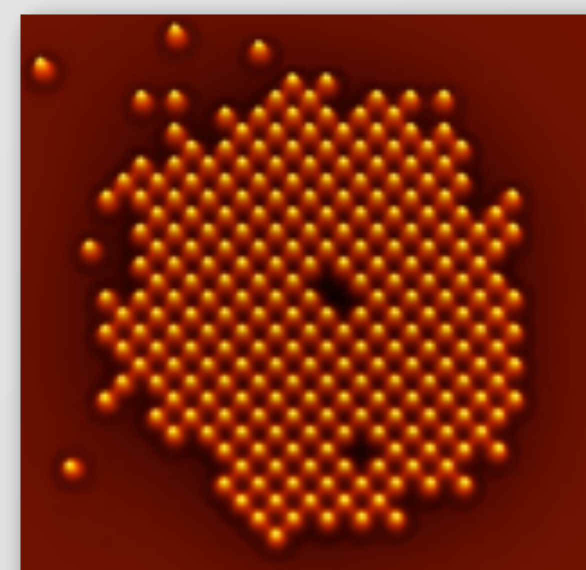
S. Haroche

Challenge: ... towards ultimate control of many-body quantum systems



R. P. Feynman's Vision

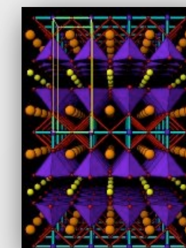
A Quantum Simulator to study the dynamics of another quantum system.

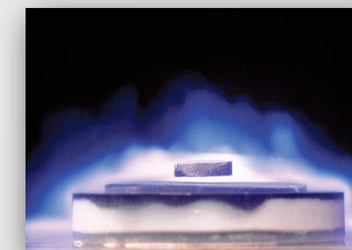


Crystal of Atoms Bound by Light

The Challenge of Many-Body Quantum Systems

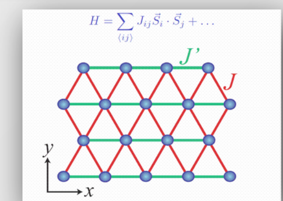
- **Understand and Design Quantum Materials** -
  one of the biggest challenge of Quantum Physics in the 21st Century





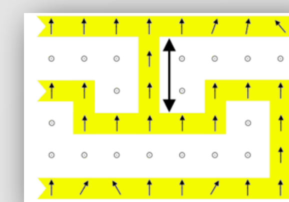
- **Technological Relevance**

  High-Tc Superconductivity (Power Delivery)

  Magnetism (Storage, Spintronics...)

  Novel Quantum Sensors (Precision Detectors)

  Quantum Technologies
  (Quantum Computing, Metrology, Quantum Sensors,...)

Many cases: lack of basic understanding of underlying processes

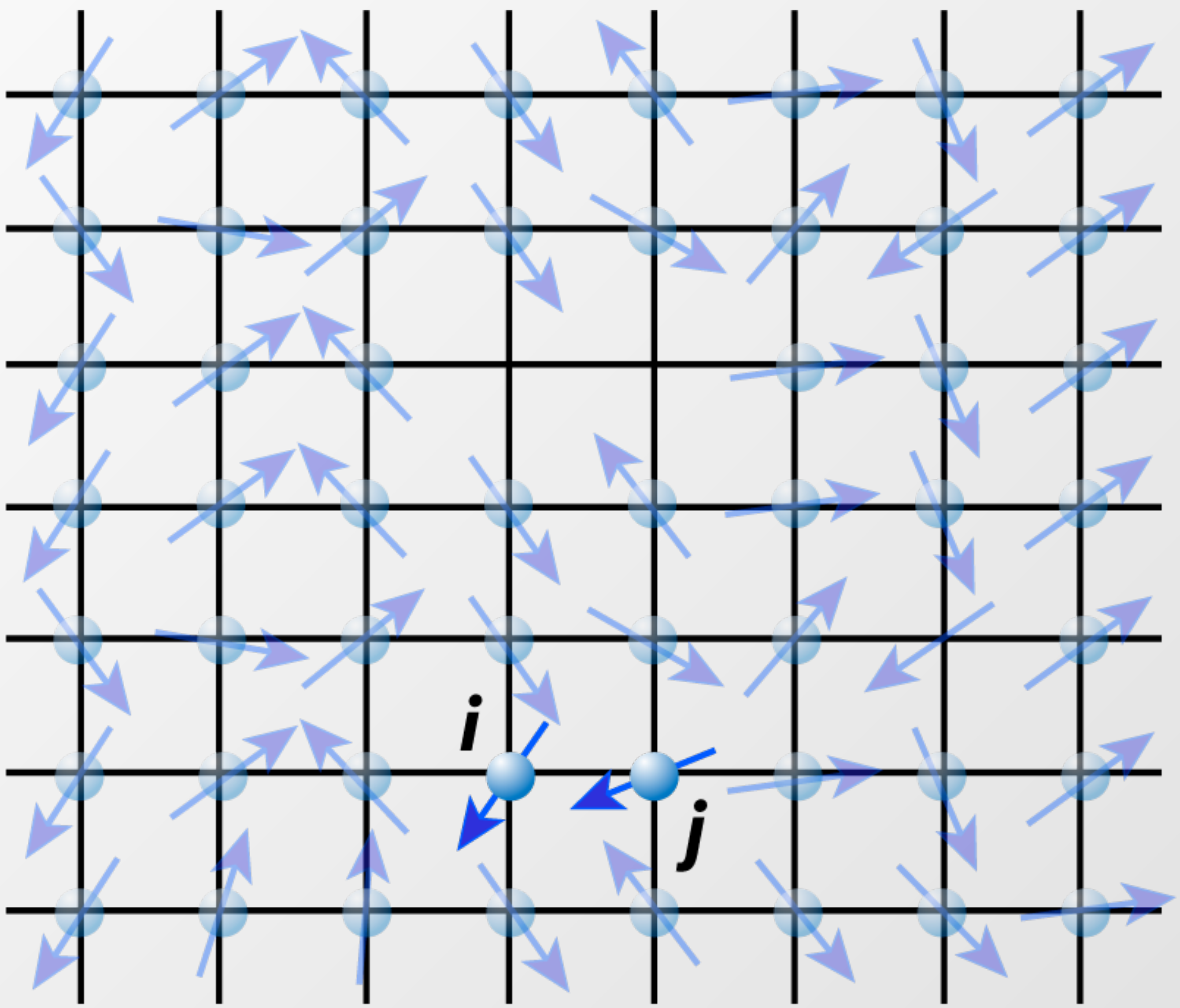
Difficulty to separate effects: probe impurities, complex interplay, masking of effects...

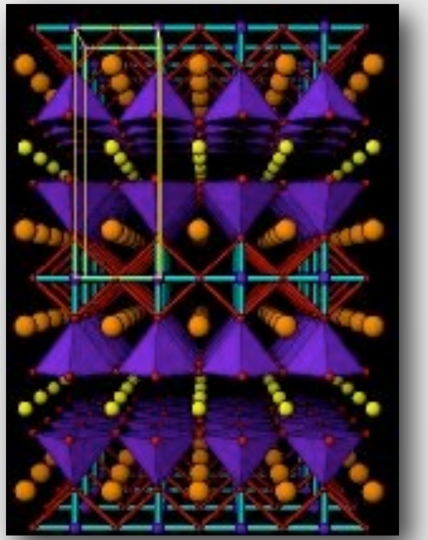
Many cases: even simple models "not solvable"

Need to synthesize new material to analyze effect of parameter change

Strongly Correlated Electronic Systems

$$H = -J \sum_{\langle i,j \rangle,\sigma} \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} + V_0 \sum_{i,\sigma} R_i^2 \hat{n}_{i,\sigma}$$





In strongly correlated electron system spin-spin interactions exist.

$$-J_{ex}\vec{S}_i \cdot \vec{S}_j$$

Underlying many solid state & material science problems:
Magnets, High-Tc Superconductors, Spintronics ....
see A. Georges (CdF)

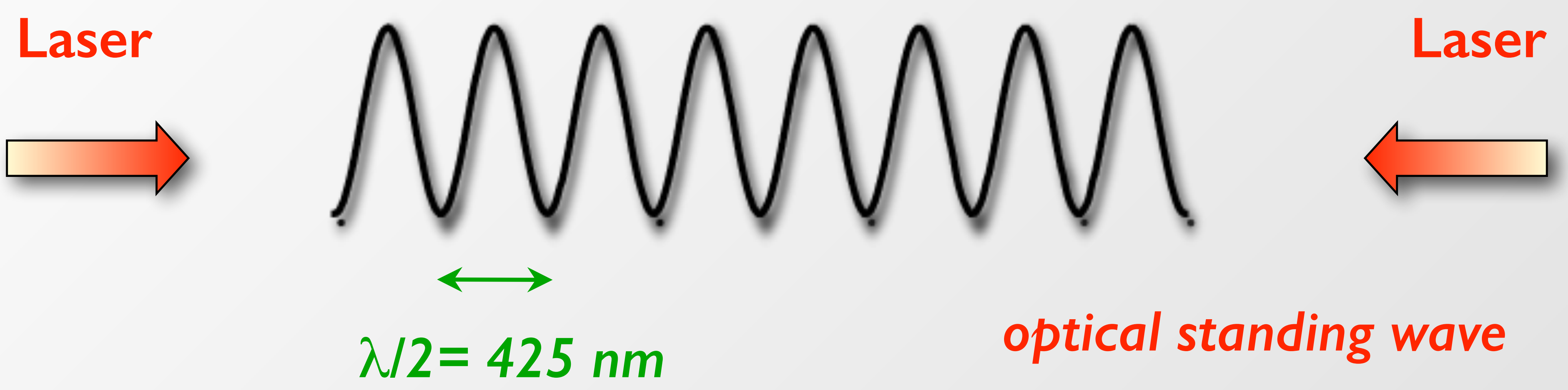
Three Central Goals

New probes & analysis techniques
- new light on known phenomena -

Quantitative predictions
- e.g. equation of state BEC-BCS crossover -

New phenomena / phases of matter
in accessible regimes

Optical Lattice Potential – Perfect Artificial Crystals

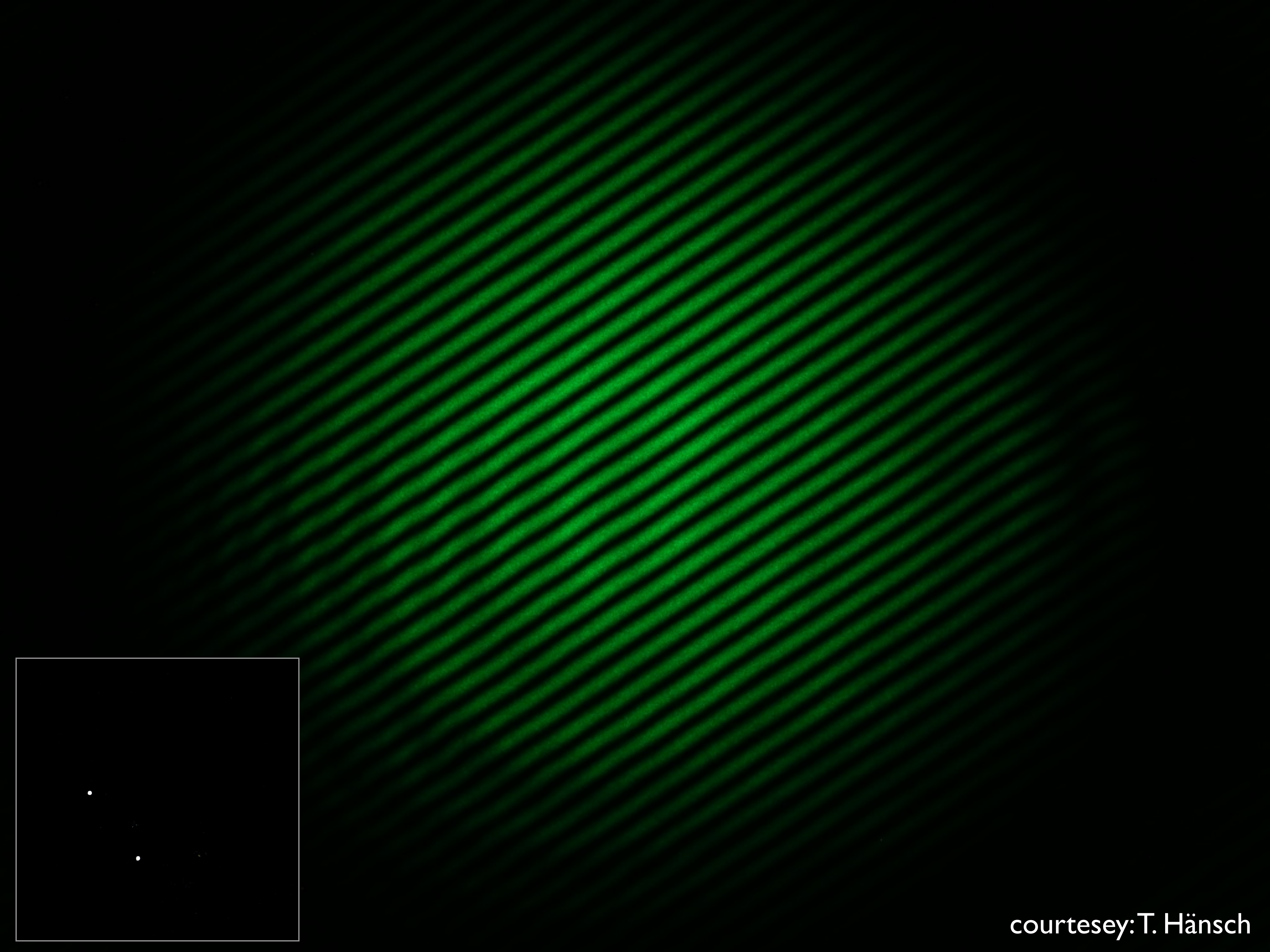


Fourier synthesize aribtrary lattices:

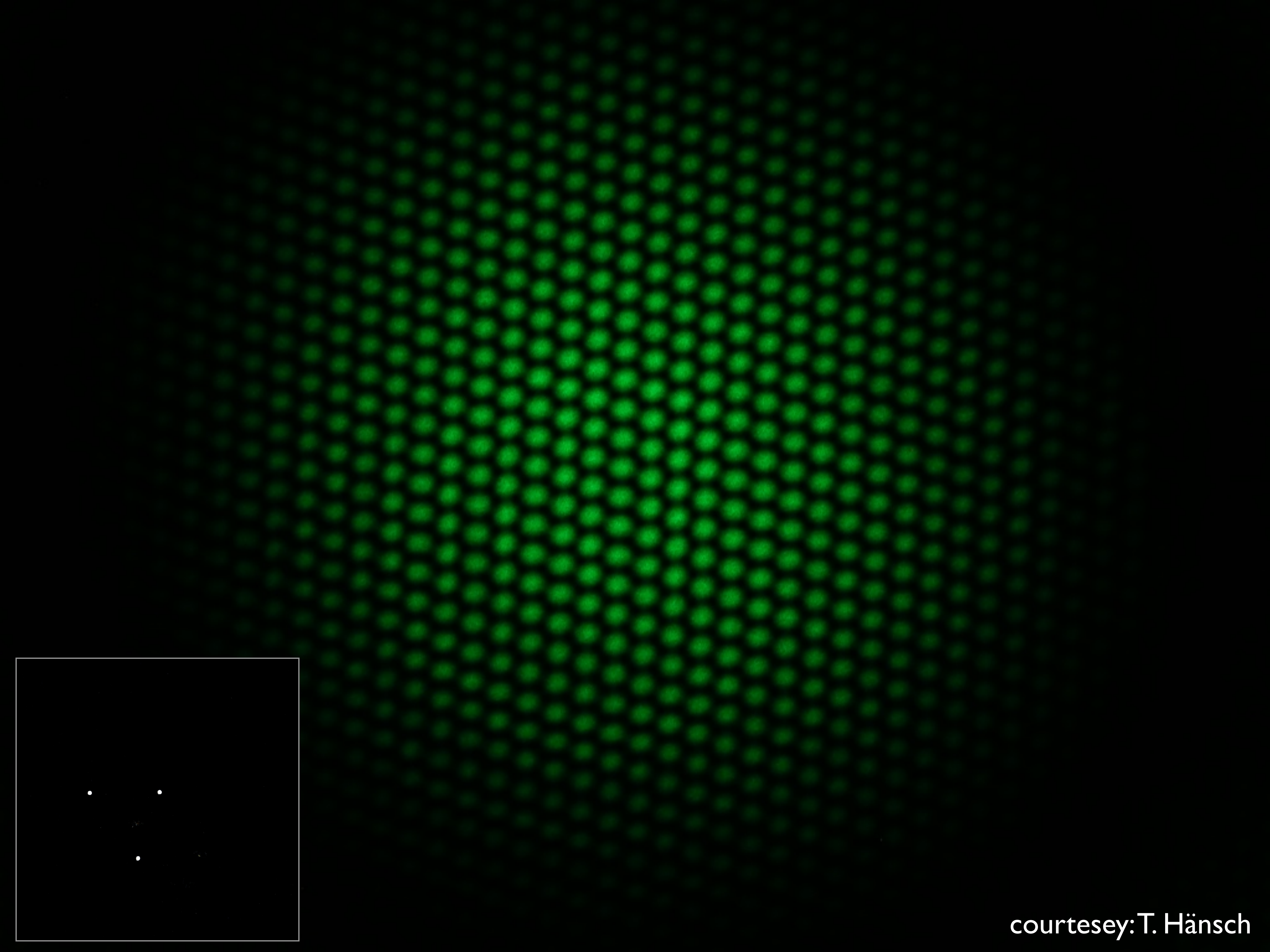
- Square
- Hexagonal/Triangular/Brick Wall
- Kagomé
- Superlattices
- *Spin dependent lattices*
- ...

Special case:
flux lattices...

Full **dynamical** control over **lattice depth, geometry, dimensionality!**



courtesey: T. Hänsch



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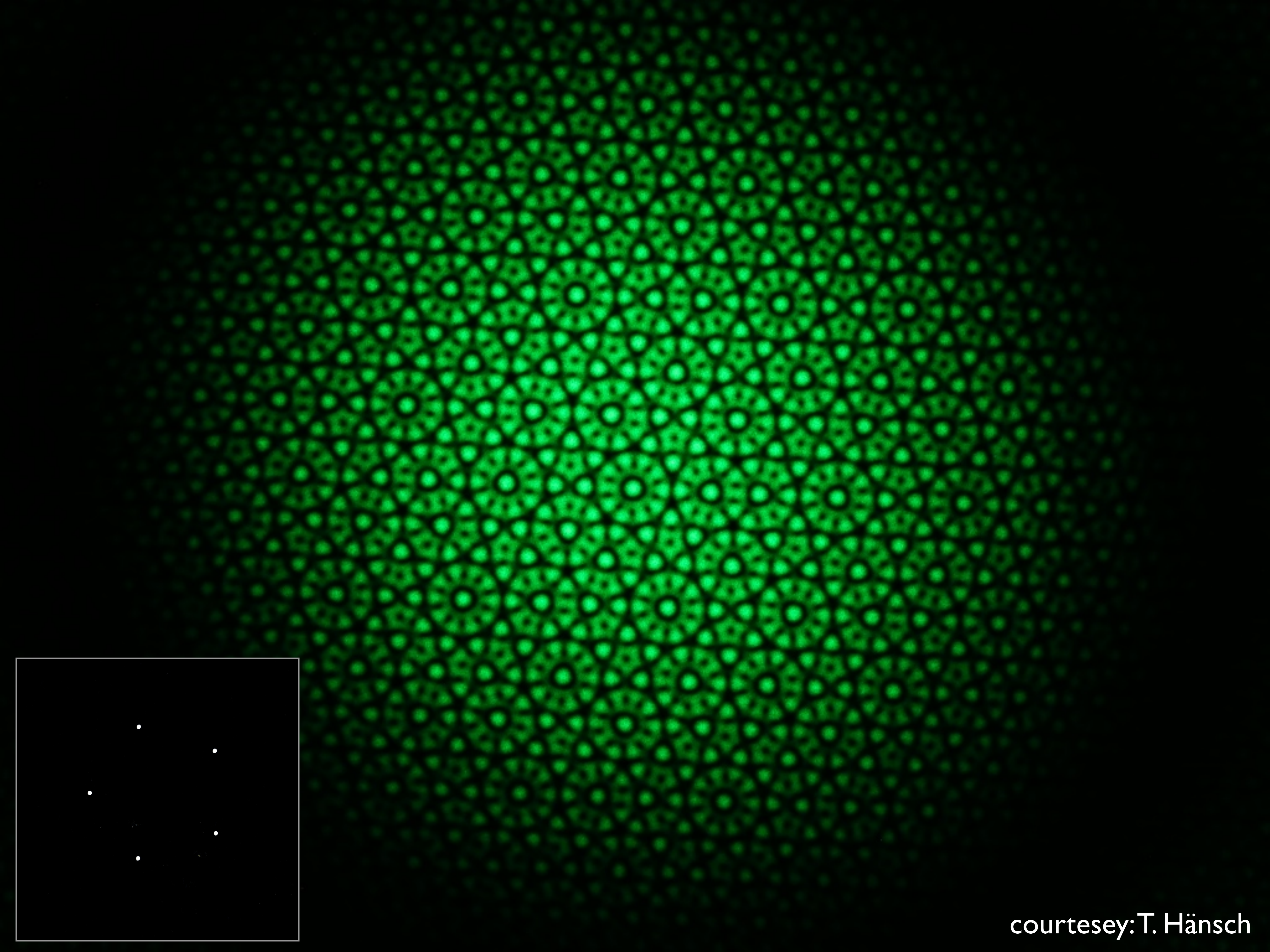

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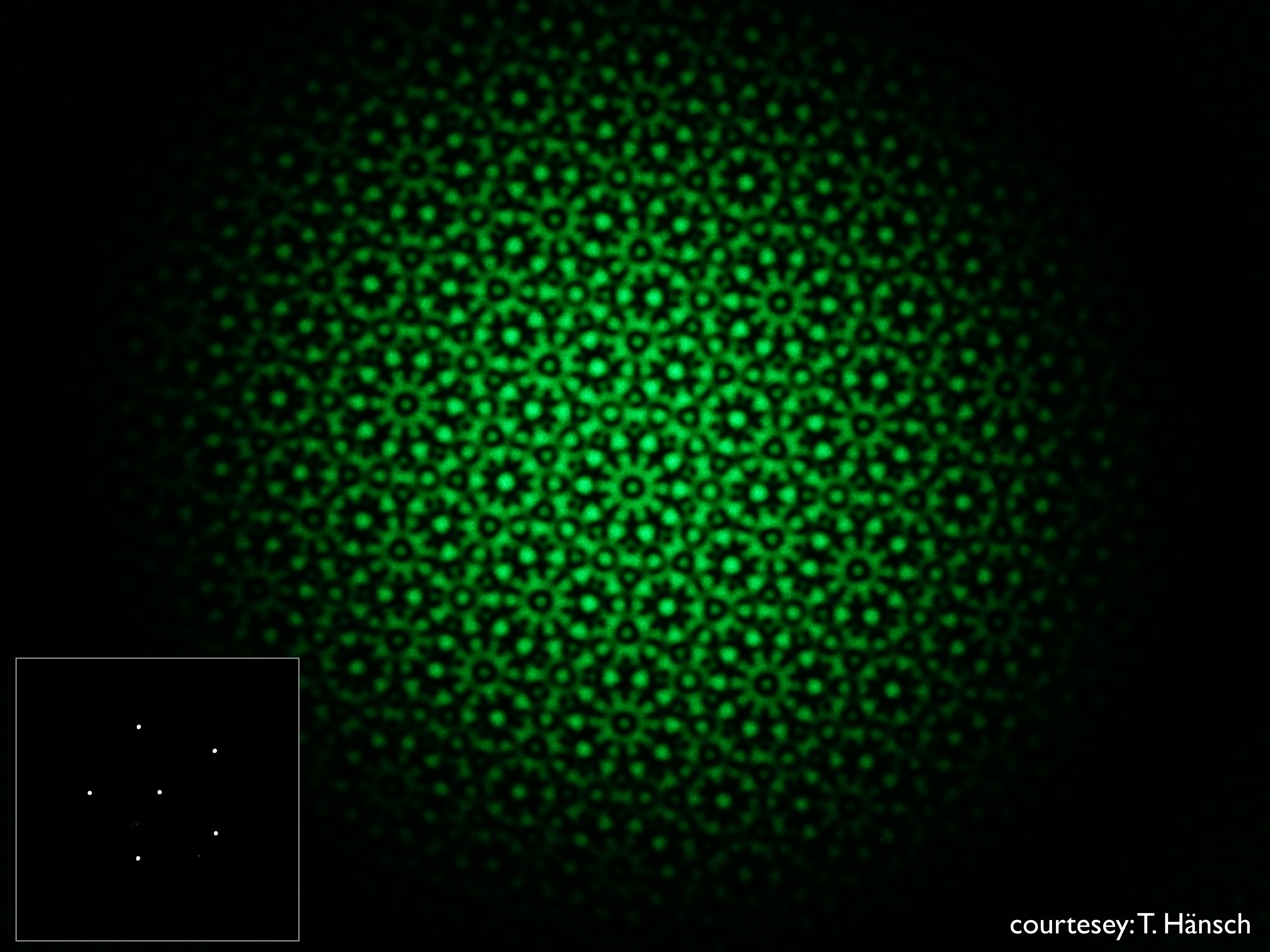
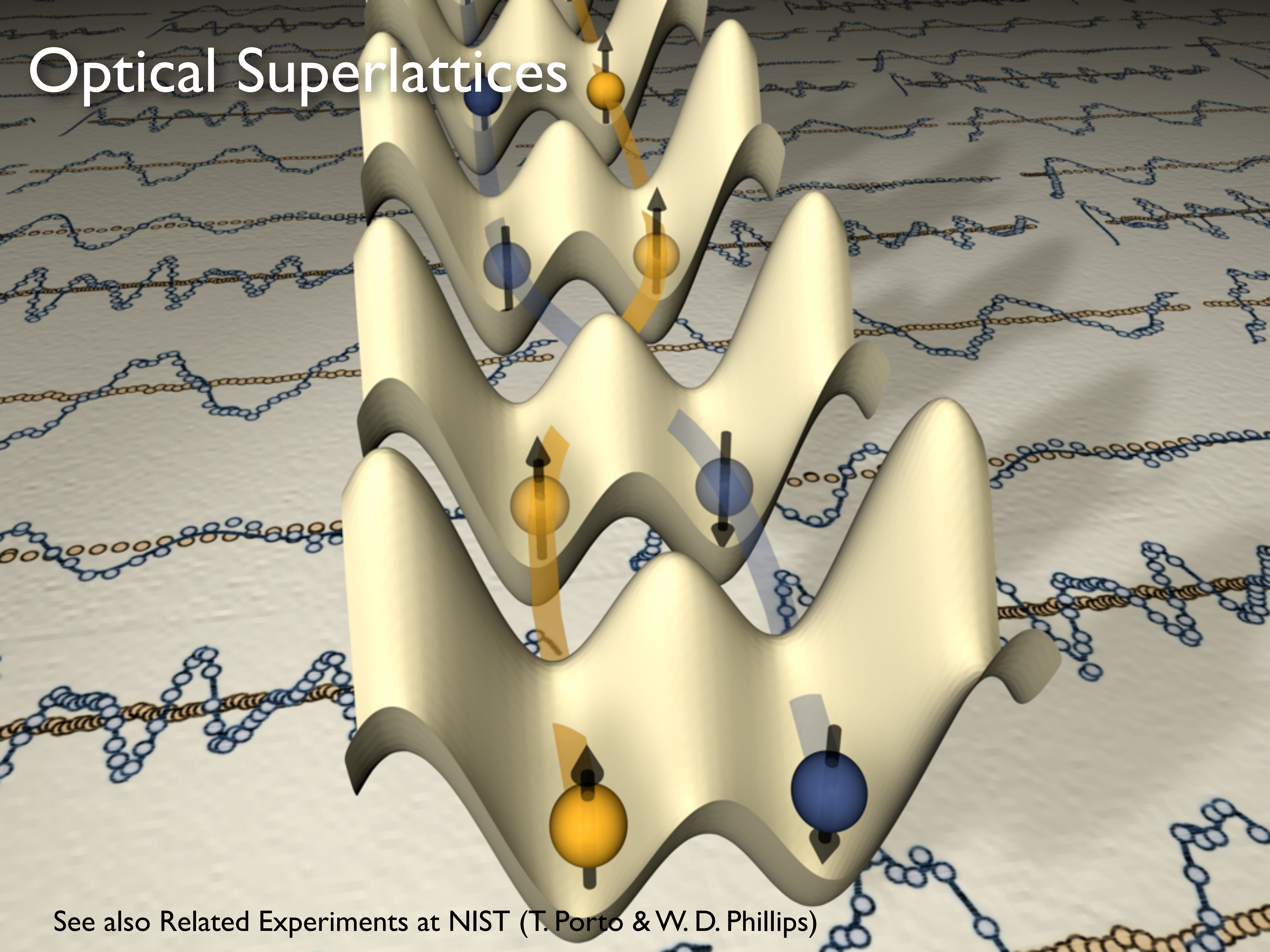


courtesey: T. Hänsch

Optical Superlattices

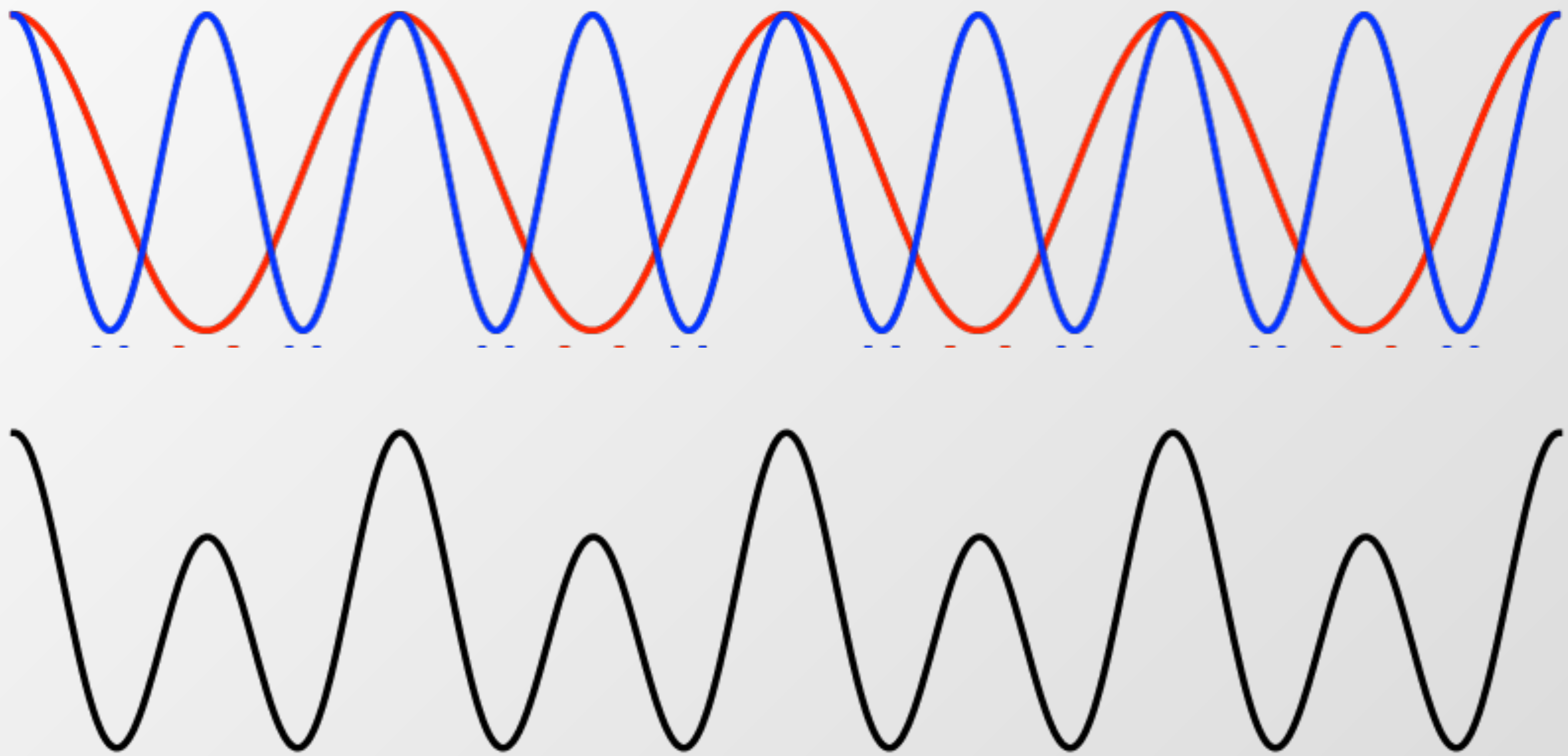


See also Related Experiments at NIST (T. Porto & W. D. Phillips)

Optical Superlattices

Superimpose two standing waves **with controllable phase & amplitude.**

1530 nm + 765 nm



Array of double wells

Note: two non-equivalent sites in unit cell!

Controllable Parameters

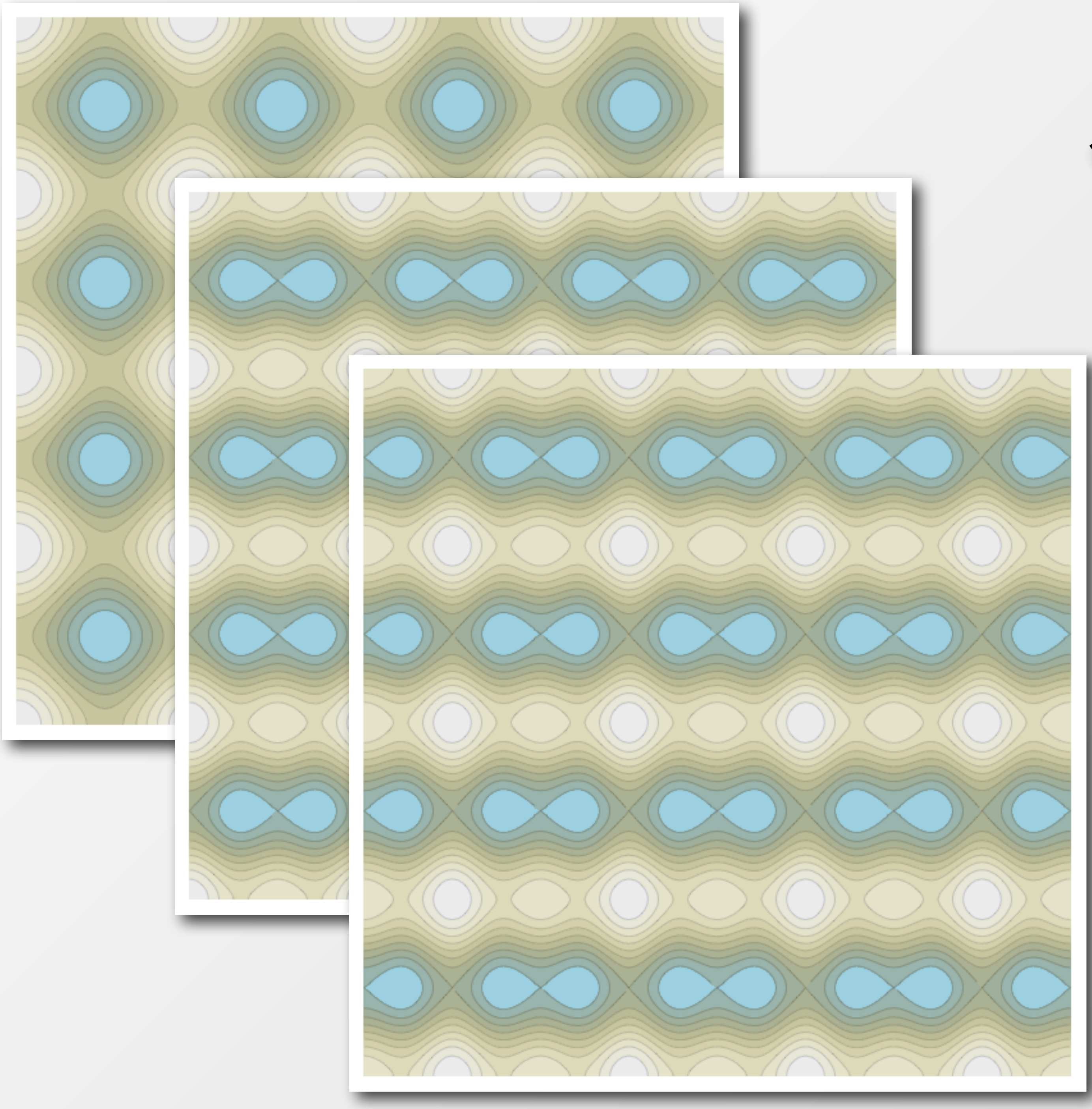
- **Original**

- **Intra & Interwell Barrier Depth**

- **Potential Bias**

All parameters can be changed dynamically & in-situ!

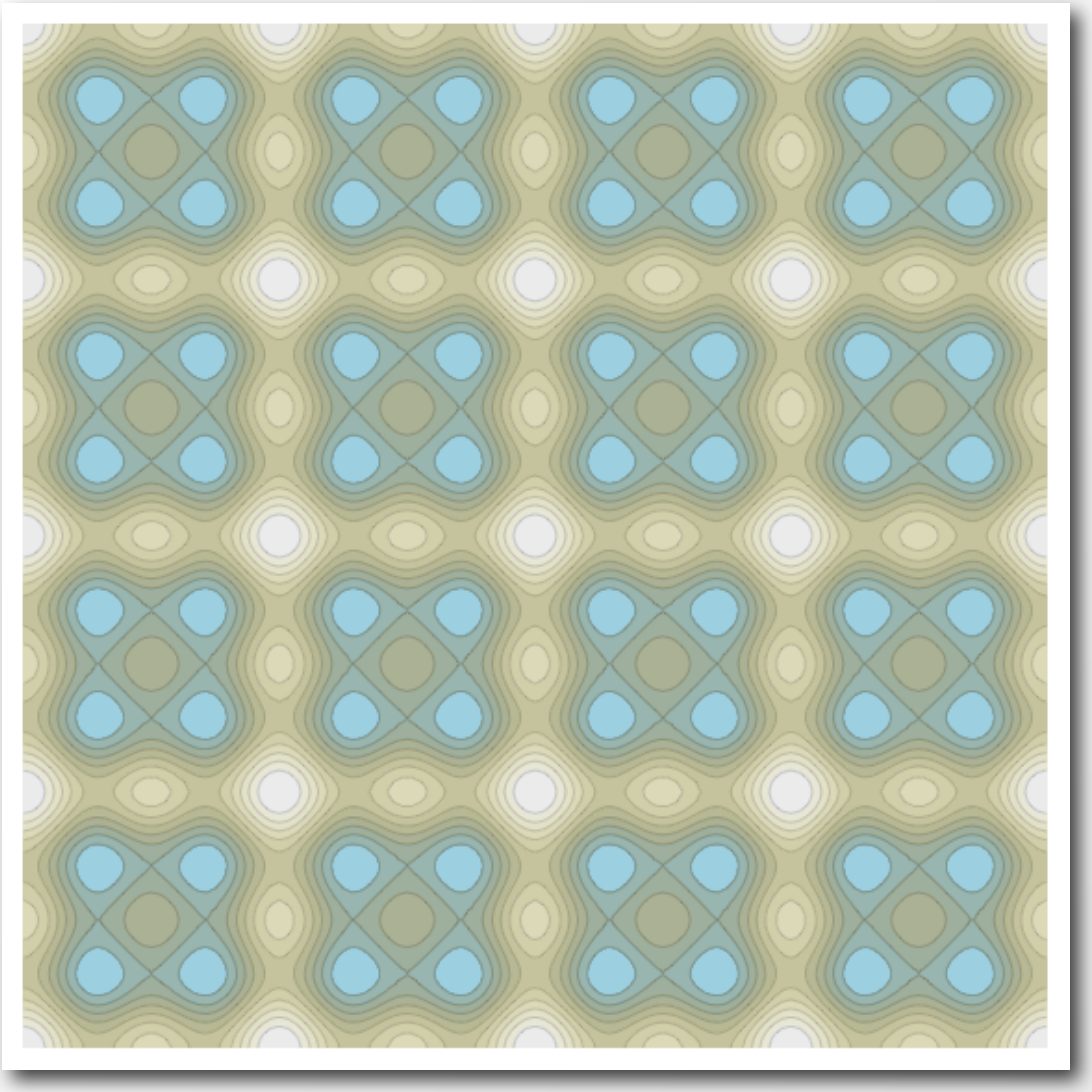
2D Superlattice Geometries (1 SL)



Controllable couplings and dynamics

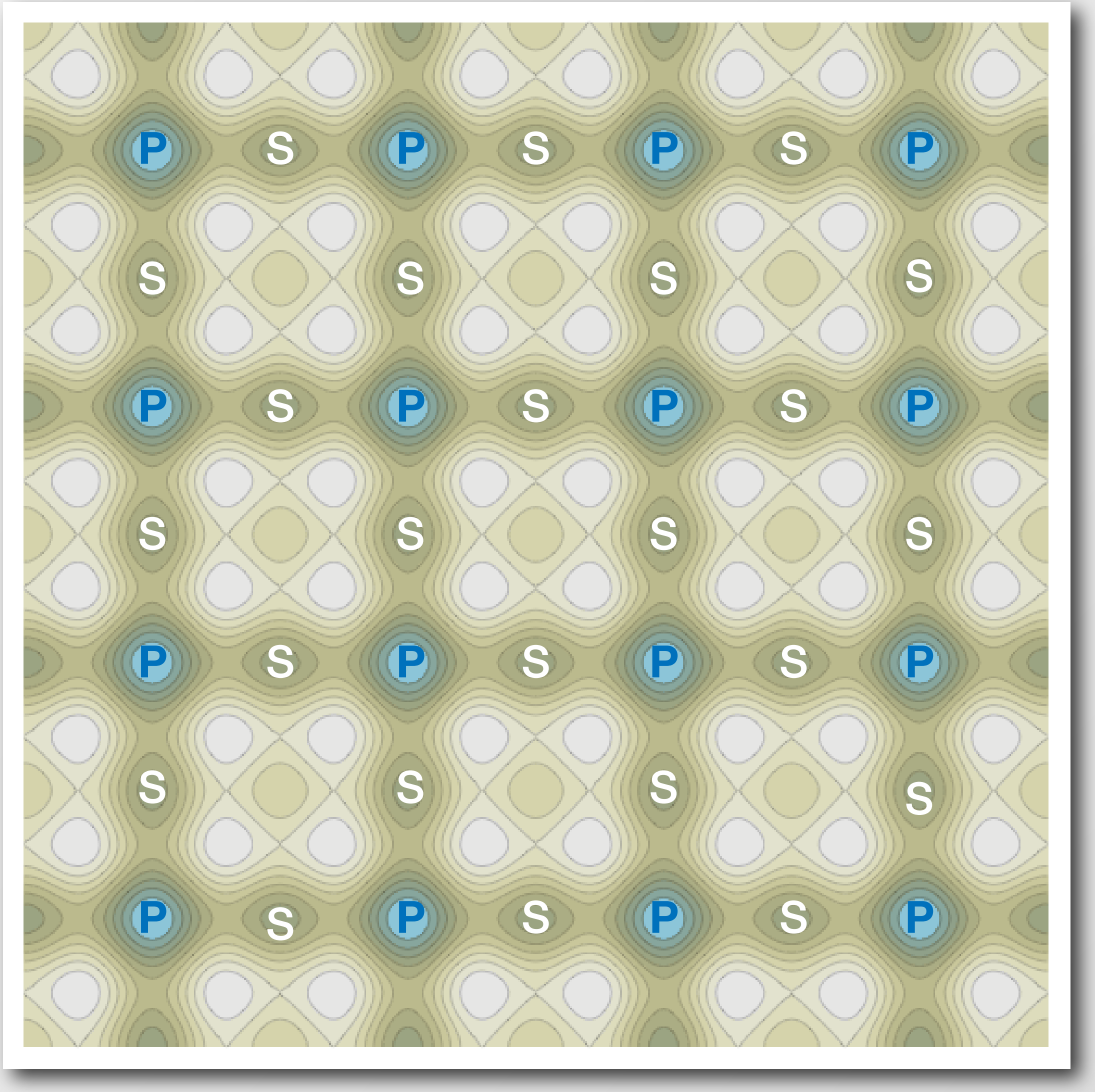
Superlattice useful for controlling and detecting spin correlations

2D Superlattice Geometries (2 SL)



Coupled Plaquette Systems

see B. Paredes & I. Bloch, PRA **77**, 23603 (2008)
S. Trebst et al., PRL **96**, 250402 (2006)



Higher Lattice Orbital Physics

see V. Liu, A. Ho, C. Wu and others work
exp: related to A. Hemmerich's exp.

Bose-Hubbard Hamiltonian

Expanding the field operator in the Wannier basis of localized wave functions on each lattice site, yields :

$$\hat{\psi}(\boldsymbol{x}) = \sum_i \hat{a}_i w(\boldsymbol{x} - \boldsymbol{x}_i)$$

Bose-Hubbard Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Tunnelmatrix element/Hopping element

$$J = -\int d^3x\, w(\boldsymbol{x} - \boldsymbol{x}_i) \left(-\frac{\hbar^2}{2m} \nabla^2 + V_{lat}(\boldsymbol{x}) \right) w(\boldsymbol{x} - \boldsymbol{x}_j)$$

Onsite interaction matrix element

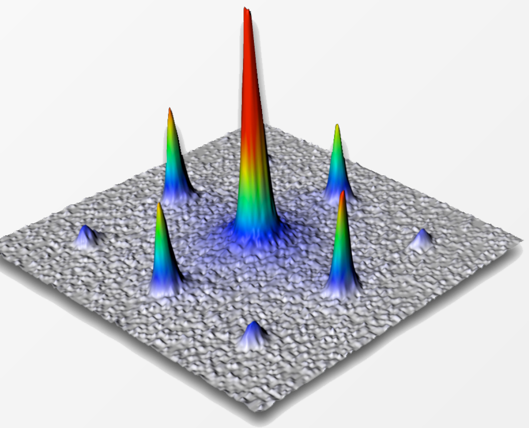
$$U = \frac{4\pi \hbar^2 a}{m} \int d^3x\, |w(\boldsymbol{x})|^4$$

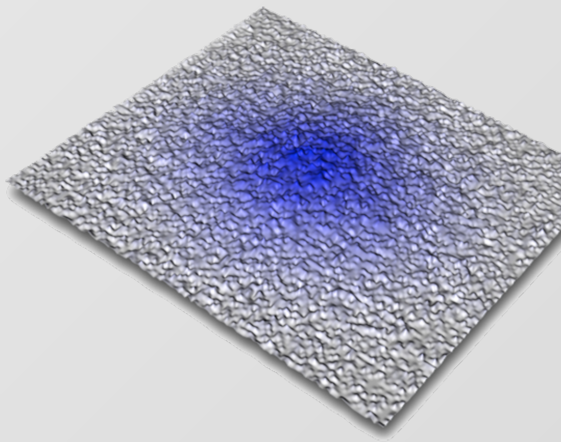
M.P.A. Fisher et al., PRB 40, 546 (1989); D. Jaksch et al., PRL 81, 3108 (1998)

Mott Insulators now at: Munich, Mainz, NIST, ETHZ, Texas, Innsbruck, MIT, Chicago, Florence,...
see also work on JJ arrays H. Mooij et al., E. Cornell,...

From Weak to Strong Interactions

$$\gamma = \frac{\text{Interaction Energy}}{\text{Kinetic Energy}} >> 1$$



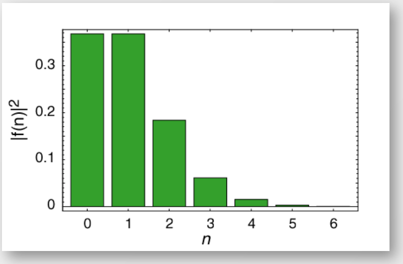


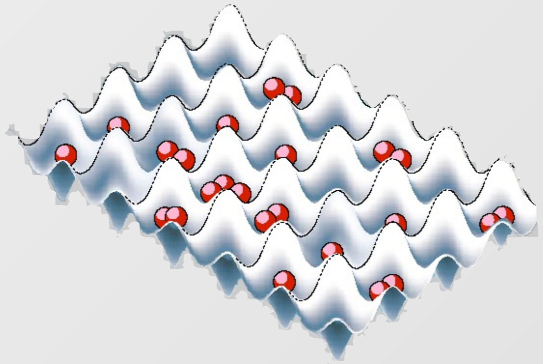
Quantum Phase Transition

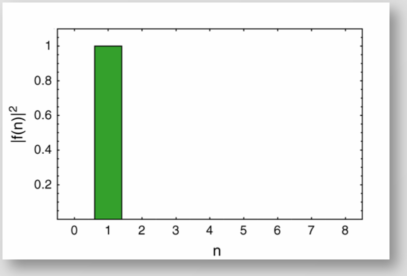
See S. Sachdev & B. Keimer Phys. Today 2011

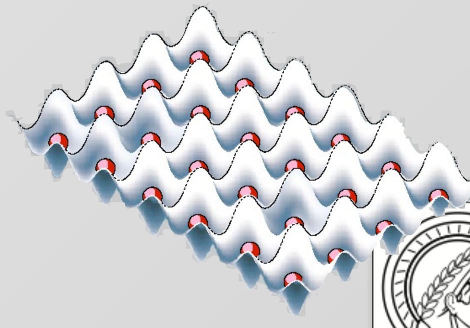
Weak Interactions

Strong Interactions





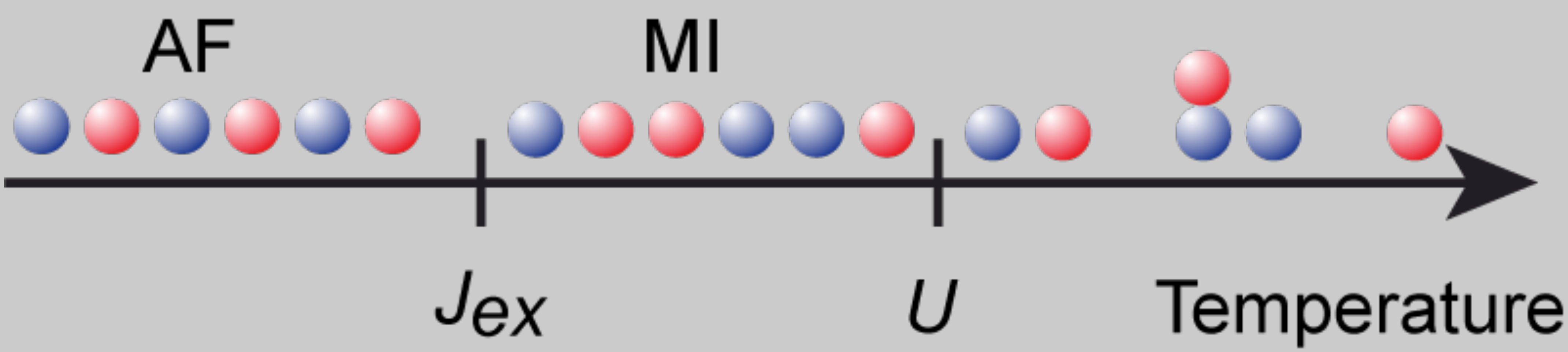




Strongly Interacting Fermions in Optical Lattices

$$\hat{H} = -J \sum_{\langle i,j \rangle,\sigma} \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\downarrow}\hat{n}_{i,\uparrow} + V_t \sum_{i,\sigma} i^2 \hat{n}_{i,\sigma}$$

Predicted phases at half filling for strong interactions $U/12J > 1$



max. Entropy
$S/N = k_B \, 2 \ln 2$

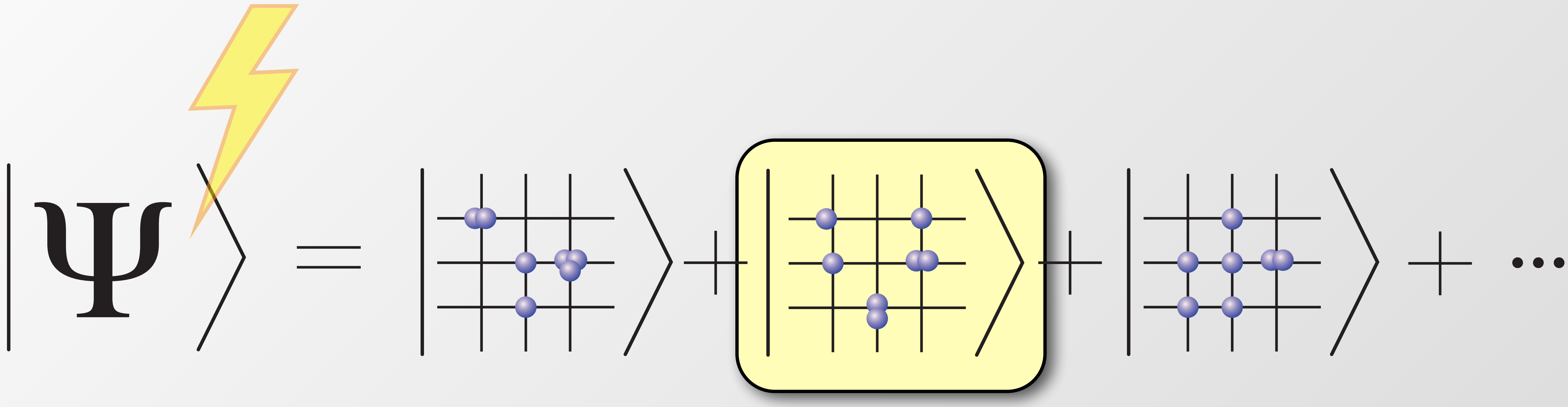
R. Jördens et al., Nature **455**, 204 (2008), U. Schneider et al., Science **322,** 1520 (2008),
D. Greif et al., Science **340**, 1307 (2013)

Single Atom Detection in a Lattice

Sherson et al. Nature 467, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

Measuring a Many-Body Quantum System

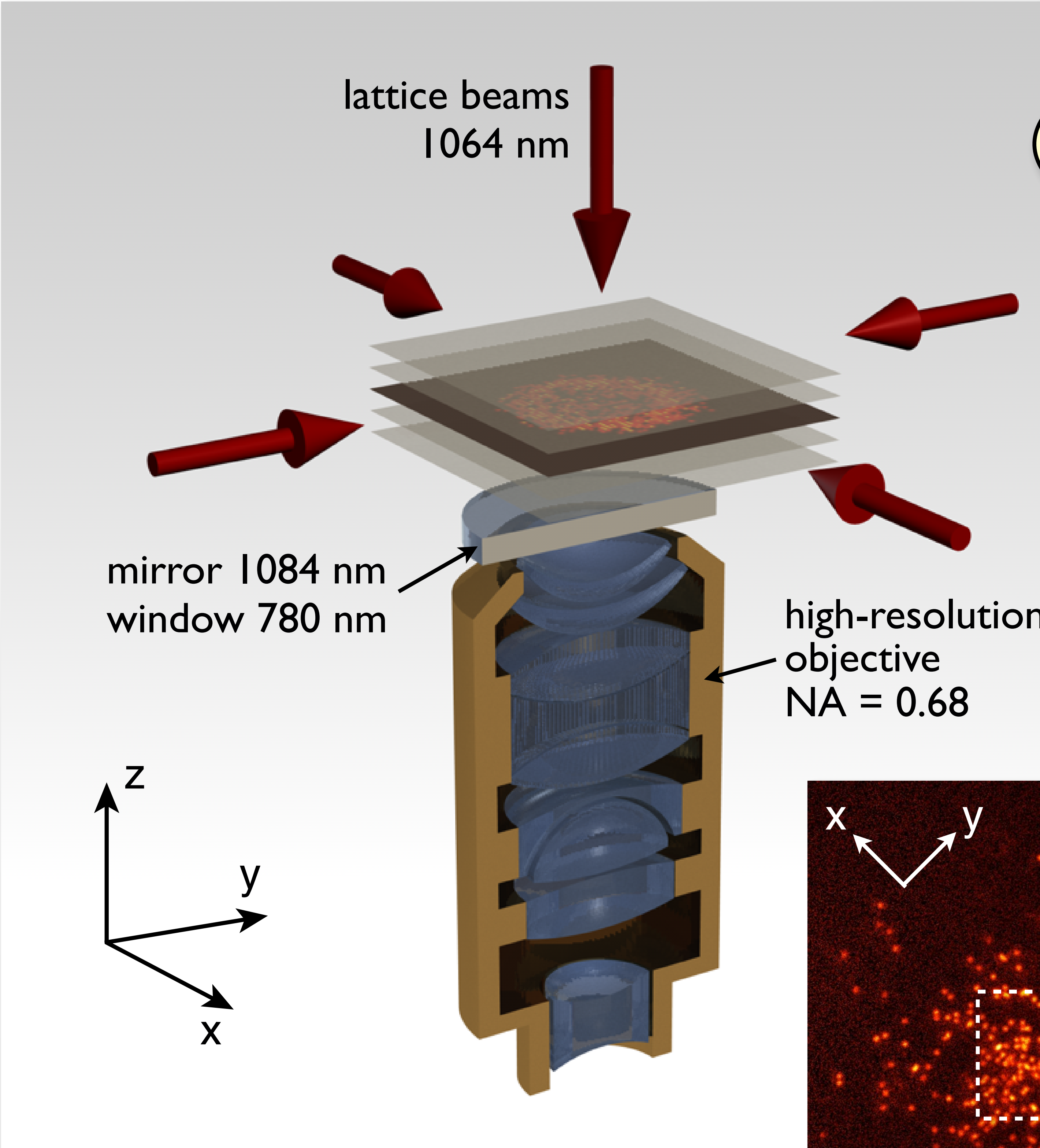
Local occupation measurement



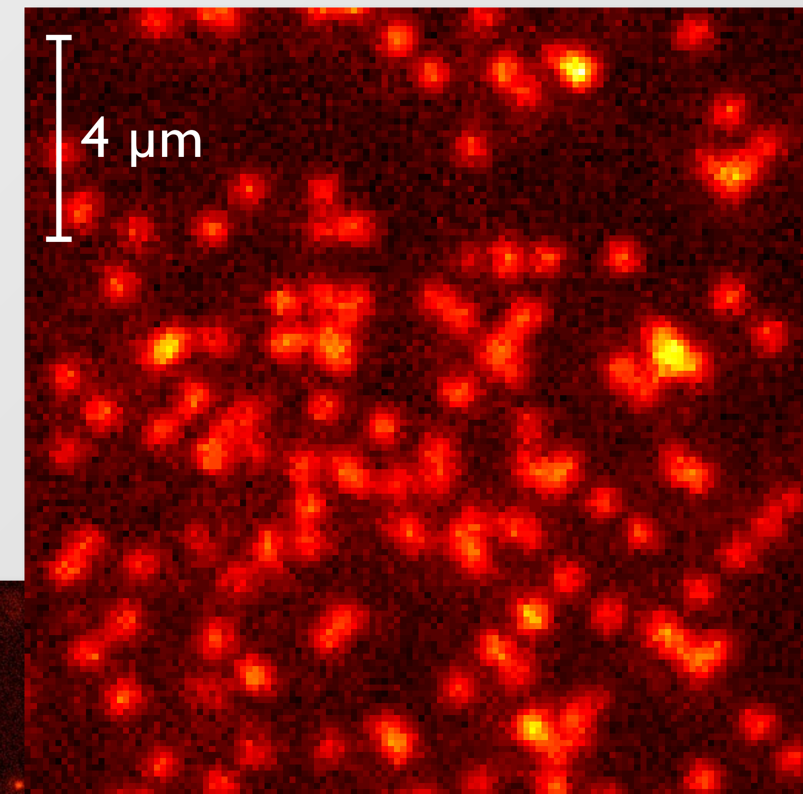
Enables access to all position correlation between particles!

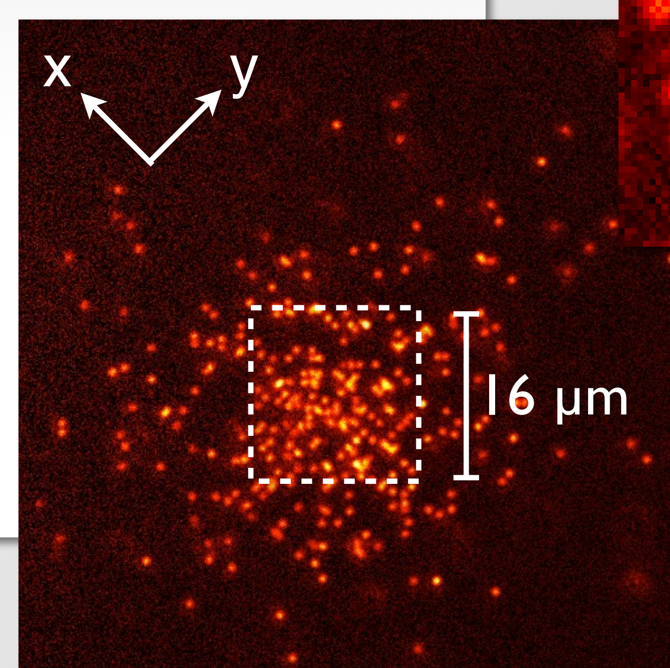
Extendable to other observables (e.g. local currents etc…)

Experimental Setup



single 2D degenerate gas
~ 1000 ^{87}Rb atoms (bosons)



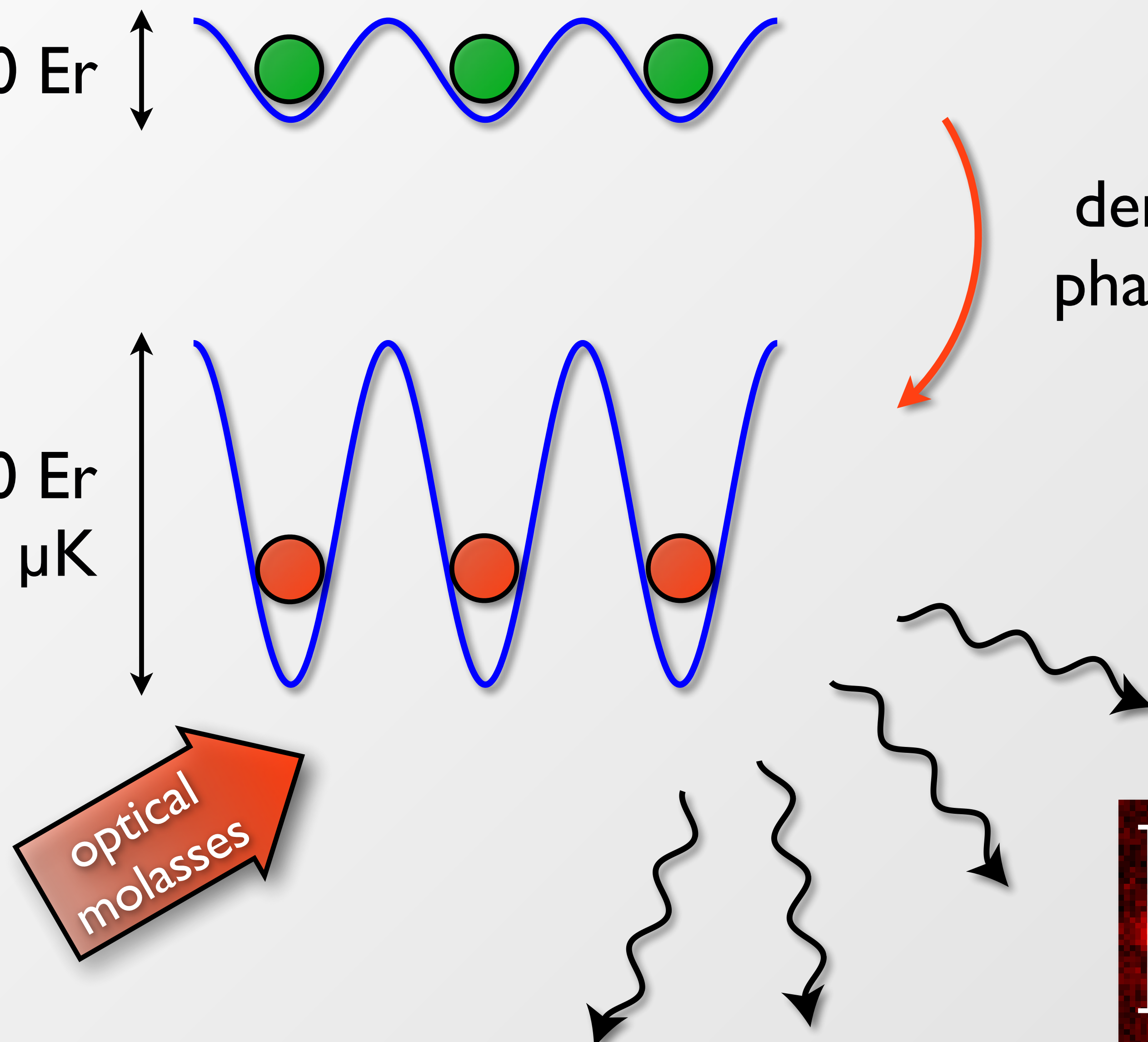


resolution of the imaging system: ~700 nm

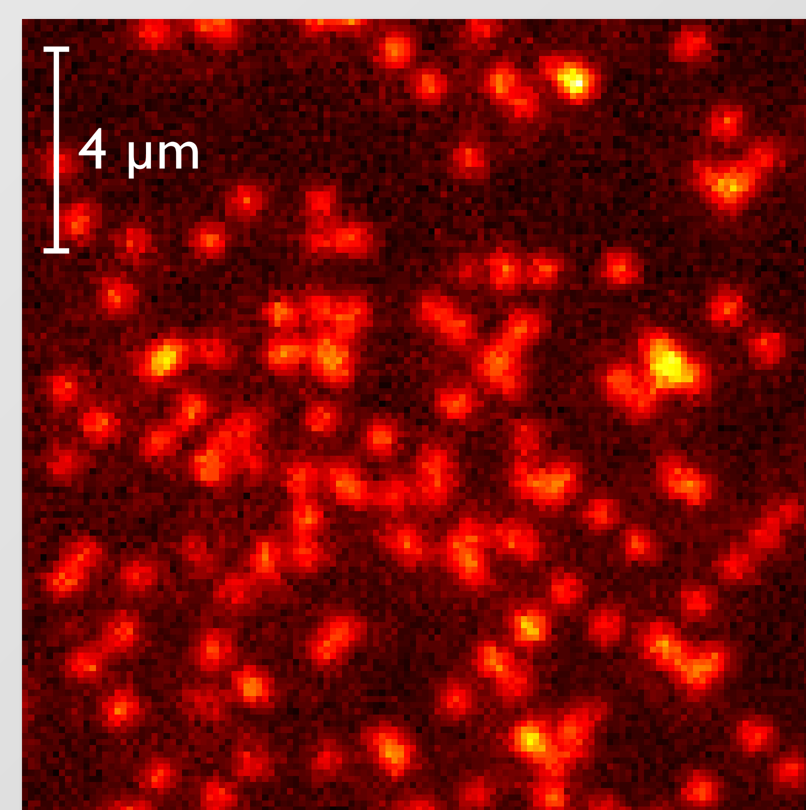
Fluorescence imaging

depth ~ 10 Er

depth ~ 4000 Er
300 μK



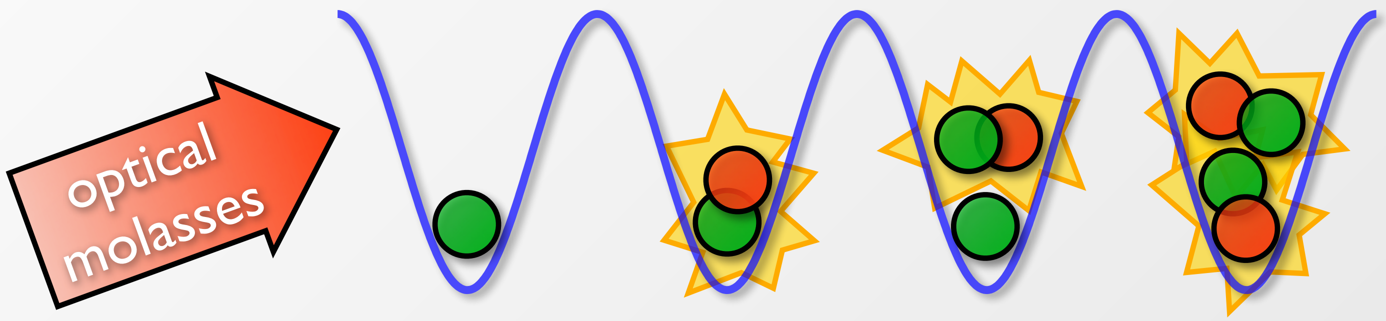
density distribution frozen
phase coherence lost



fluorescence rate / atom: 60 kHz
~ 5000 photons / atom collected in 900ms

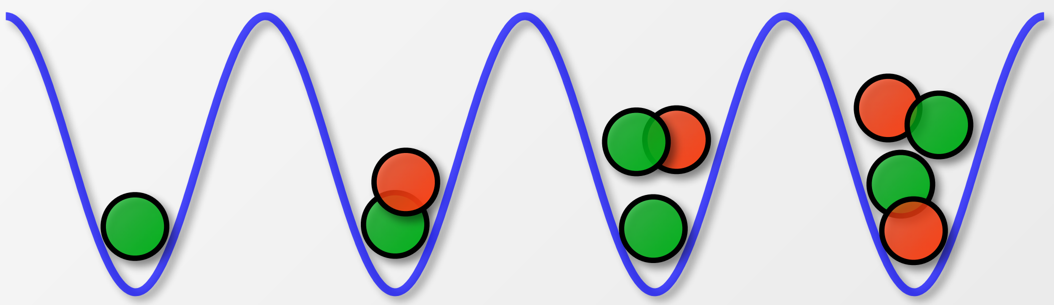
Parity projection

dePue *et al.*, PRL **82**, 2262 (1999)

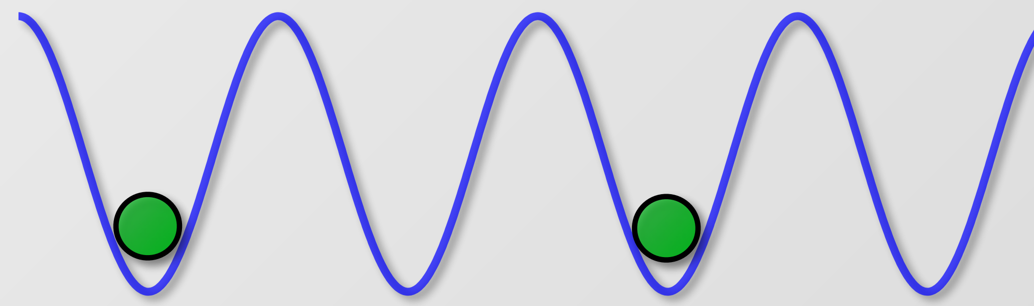


Light-induced collisions

initial density distribution



measured density distribution



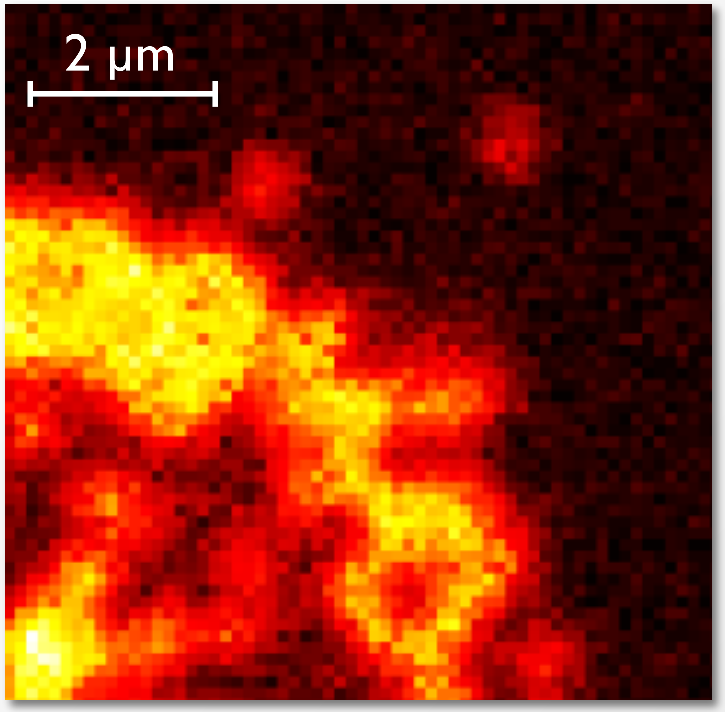
measured occupation: $n_{\text{det}} = \text{mod}_2 n$

measured variance: $\sigma^2_{\text{det}} = \langle n^2_{\text{det}} \rangle - \langle n_{\text{det}} \rangle^2$

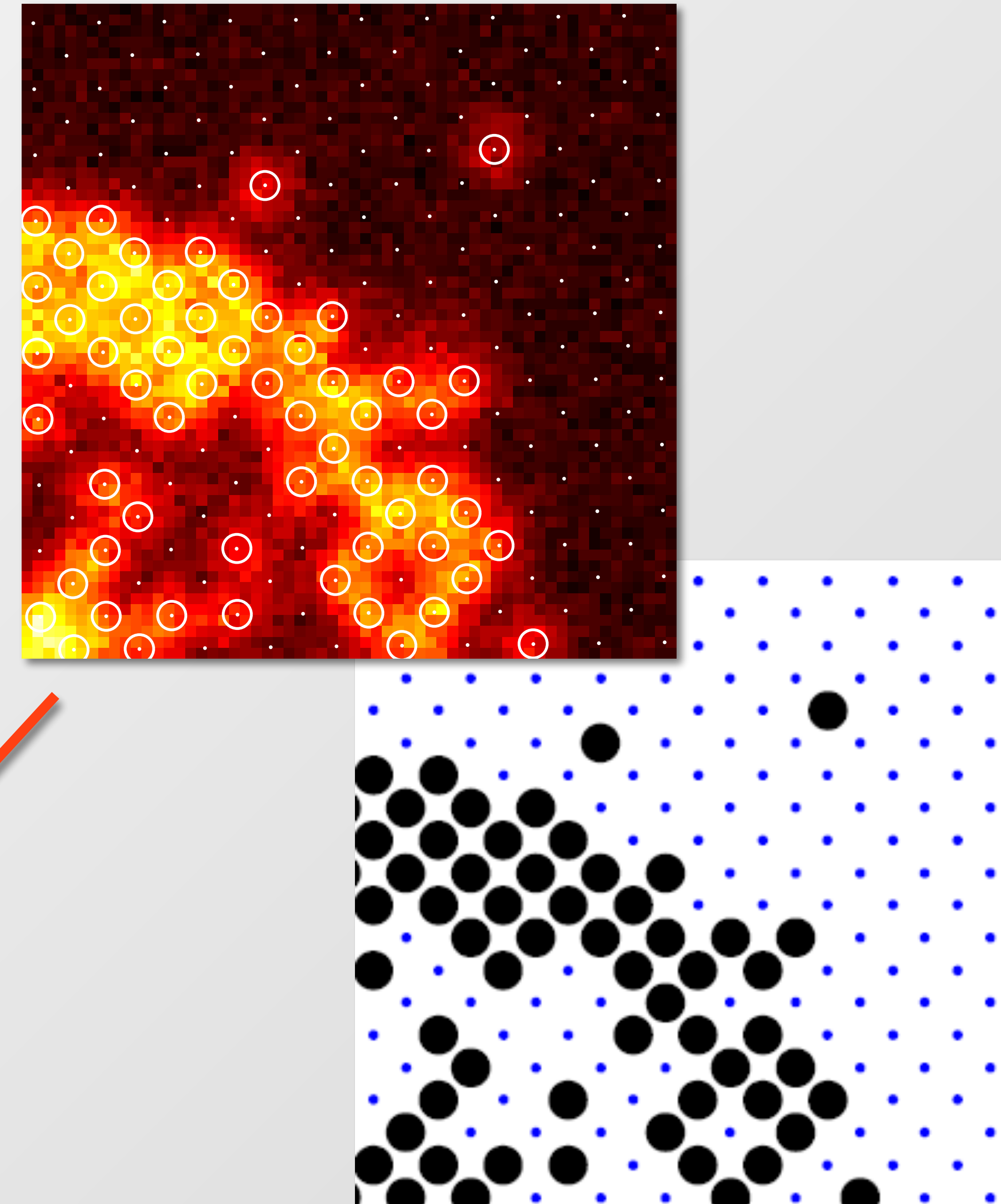
parity projection $\Rightarrow$ $\langle n^2_{\text{det}} \rangle = \langle n_{\text{det}} \rangle$

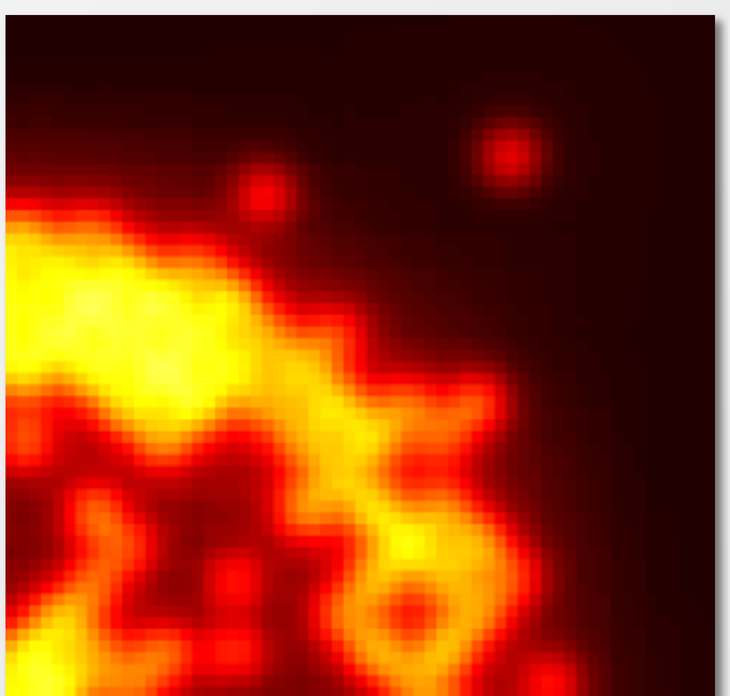
see also E. Kapit & E. Mueller, Phys. Rev. A **82**, 013644 (2010)

Reconstruction of site occupation



Reconstruction algorithm





Digitized image convoluted with point-spread function

digitized image
no experimental noise

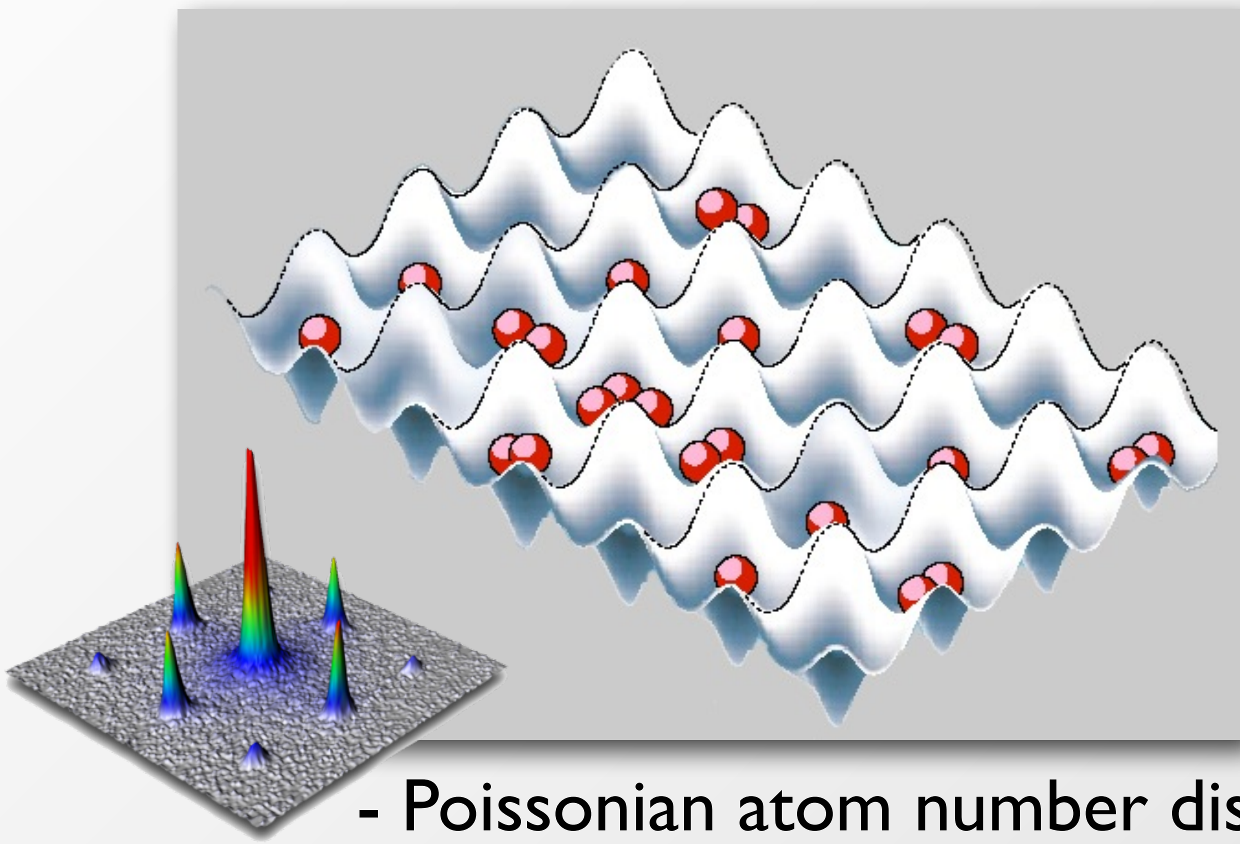
In-Situ Imaging of a Mott Insulator

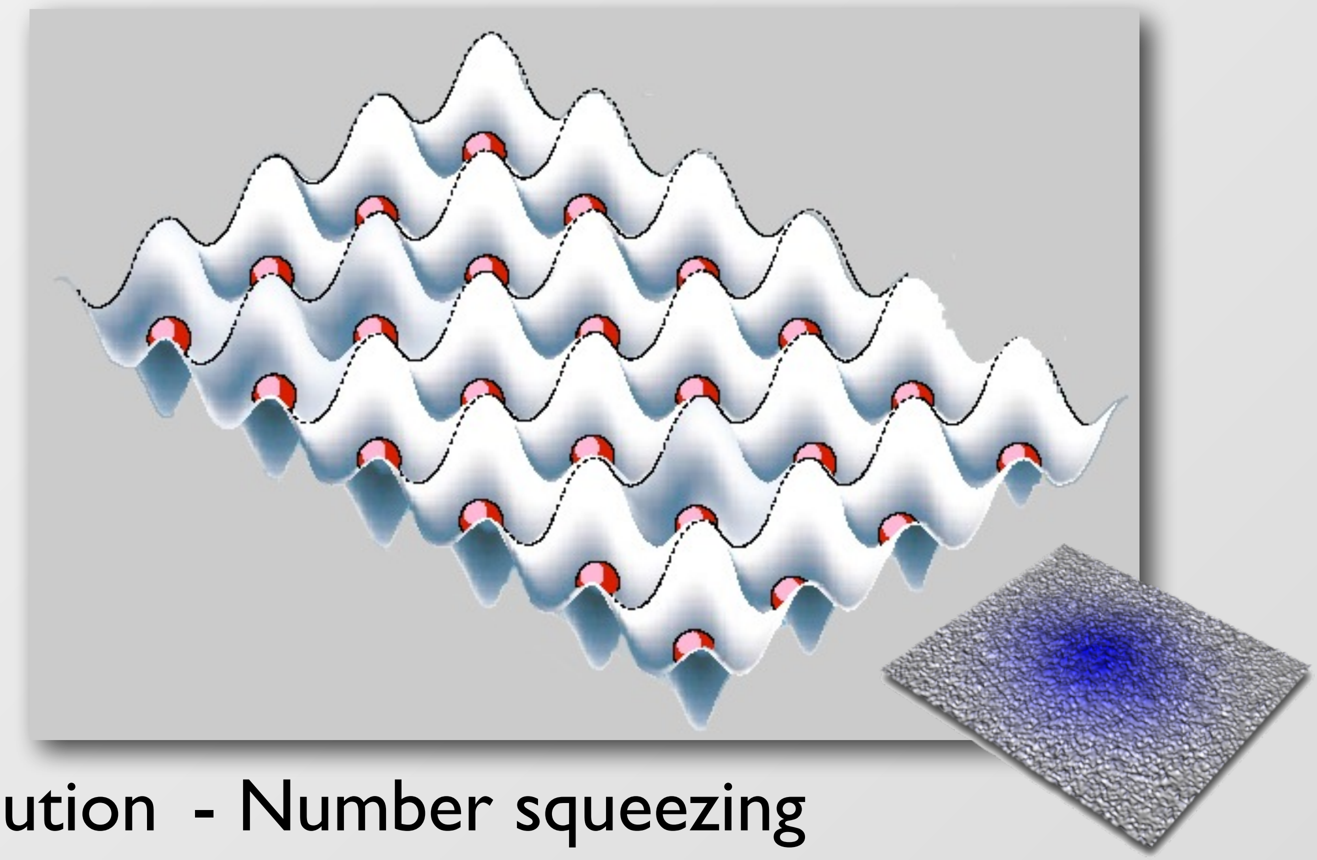
J. Sherson et al. Nature **467**, 68 (2010),
see also S. Fölling et al. Phys. Rev. Lett (2006), G.K. Campbell et al. Science (2006)
N. Gemelke et al. Nature (2009), W. Bakr et al. Science (2010)

Mott Insulators

Superfluid

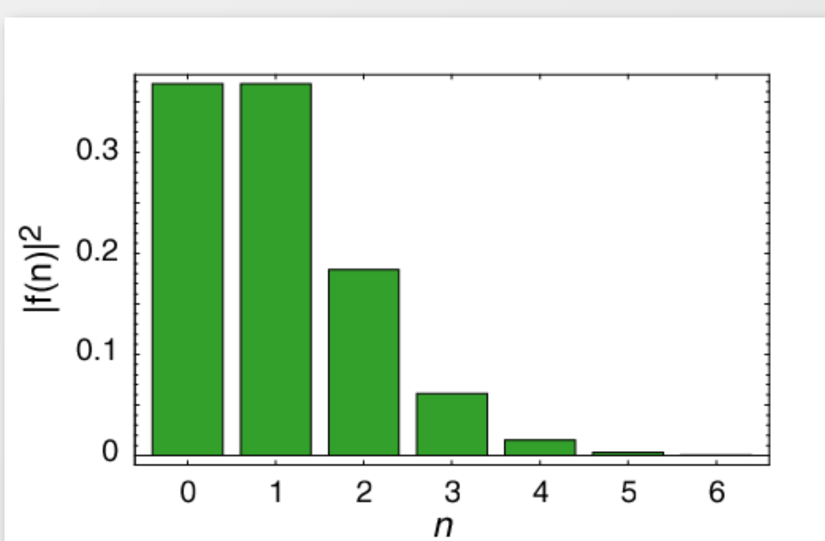
Mott-Insulator

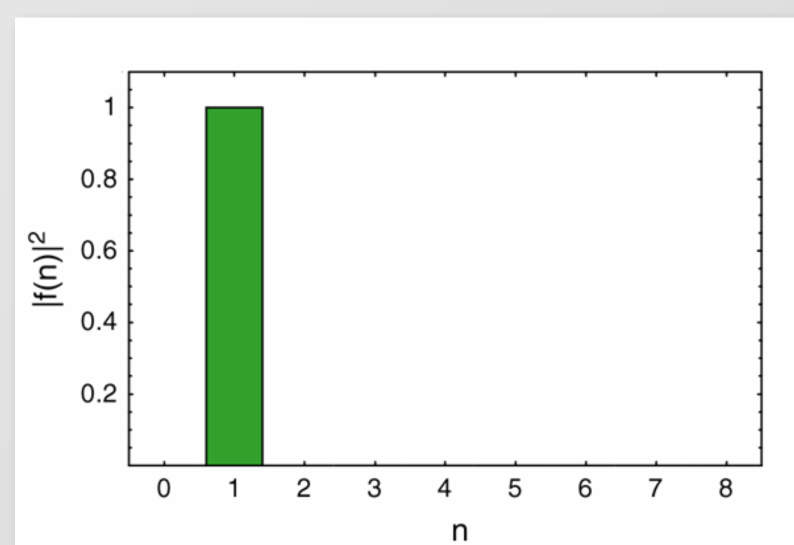




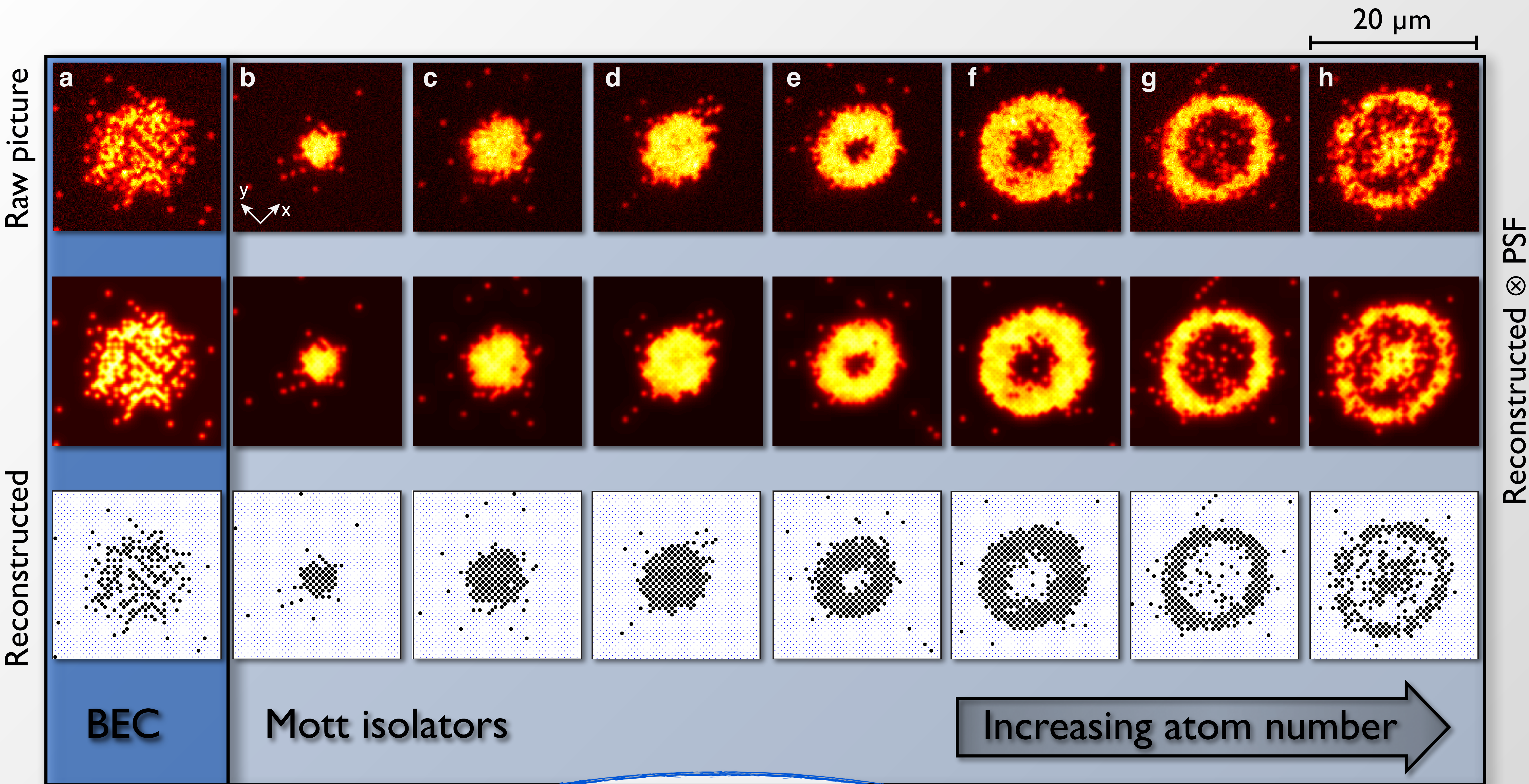
- Poissonian atom number distribution
- Long range phase coherence

- Number squeezing
- No phase coherence





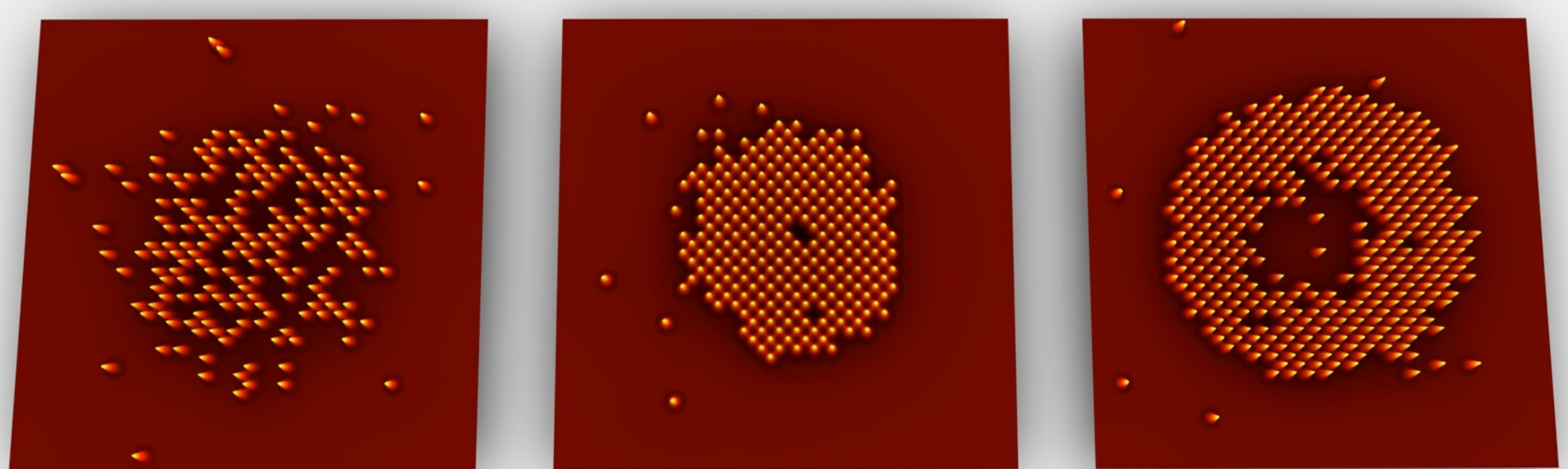
In-situ observation of a Mott insulator



for the Mott insulators: U/J ~ 300 (critical U/J ~ 16)

⇒ only thermal fluctuations

Snapshot of an Atomic Density Distribution



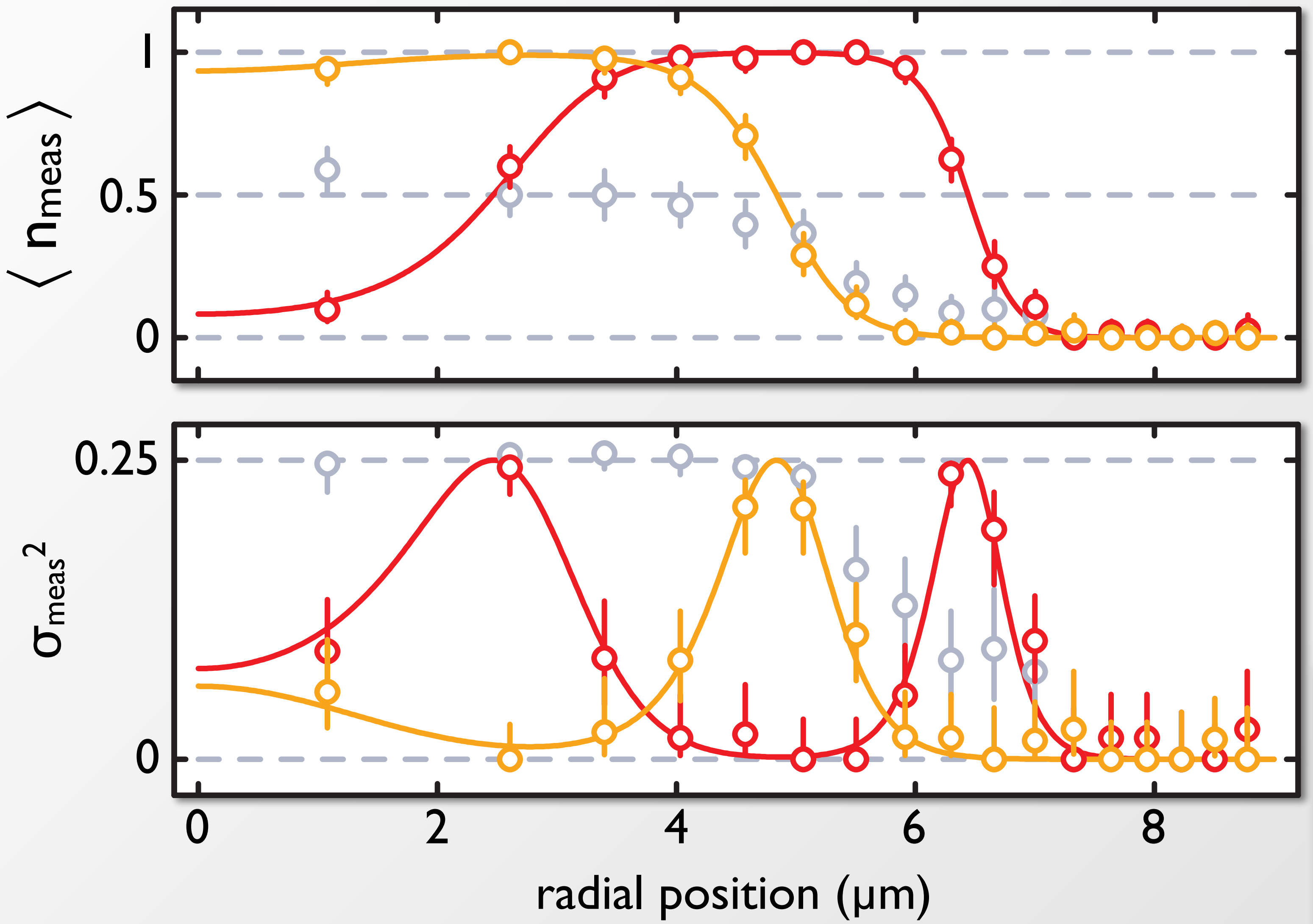
BEC

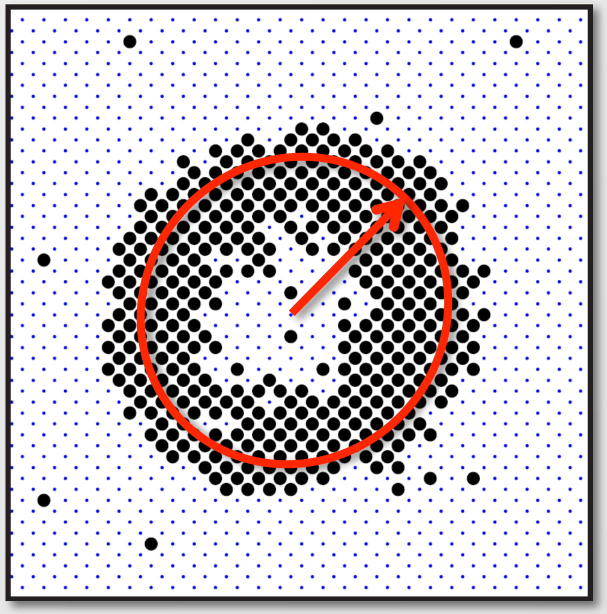
n=1
Mott Insulator

n=1 & n=2
Mott Insulator

J. Sherson *et al.* Nature **467**, 68 (2010)

Single Shot Thermodynamics



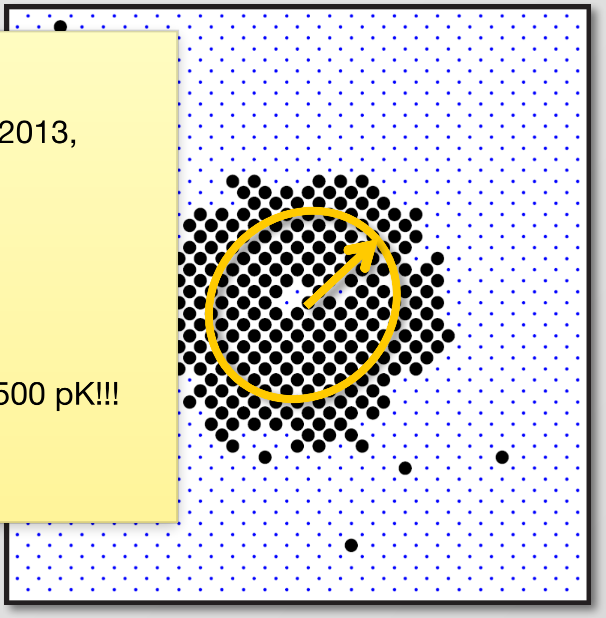


T = 0.074(5) U/k_B, μ = 1.17(1) U
N = 610(20)

Imported Author 23 Oct 2013, 7:24
2 kHz=100nK
1 kHz=50 nK

0.1 U approx 5 nK

measurement precision 500 pK!!!



T = 0.090(5) U/k_B, μ = 0.73(3) U
N = 300(20)

Simple Theory - Atomic Limit Mott Insulator

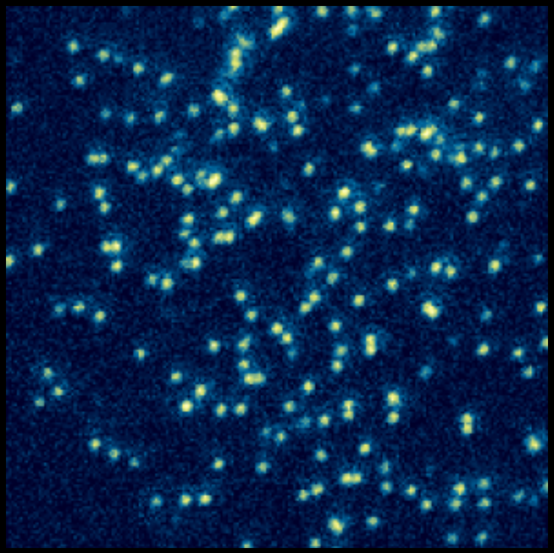
occupation probability: $$p_n(r) = \frac{e^{-\beta(E_n - \mu(r)n)}}{Z(r)}$$

interaction energy: $$E_n = \frac{1}{2}Un(n-1)$$

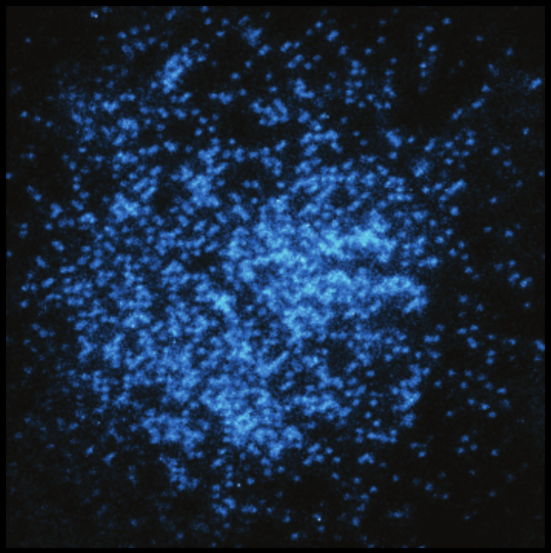
fit parameters: $$T/U, \mu/U, U/\omega^2$$

Fermionic Quantum Gas Microscopes

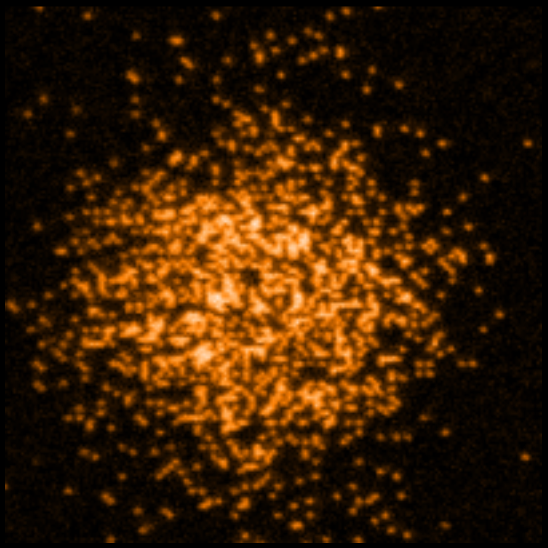
now also for fermions!



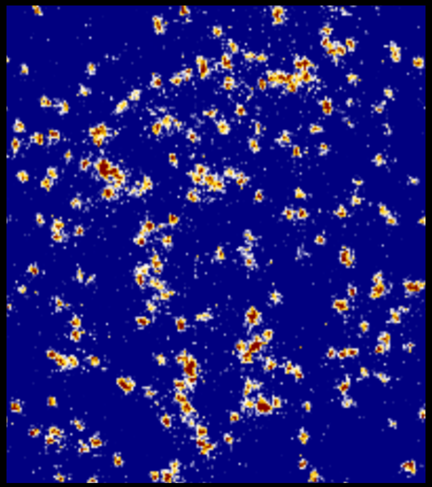
Strathclyde (^{40}K)



Harvard (^{6}Li)



MIT (^{40}K)

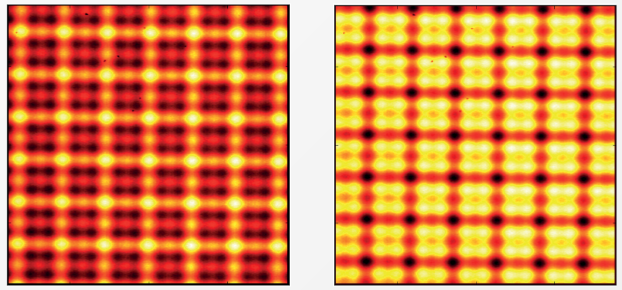


Toronto (^{40}K)

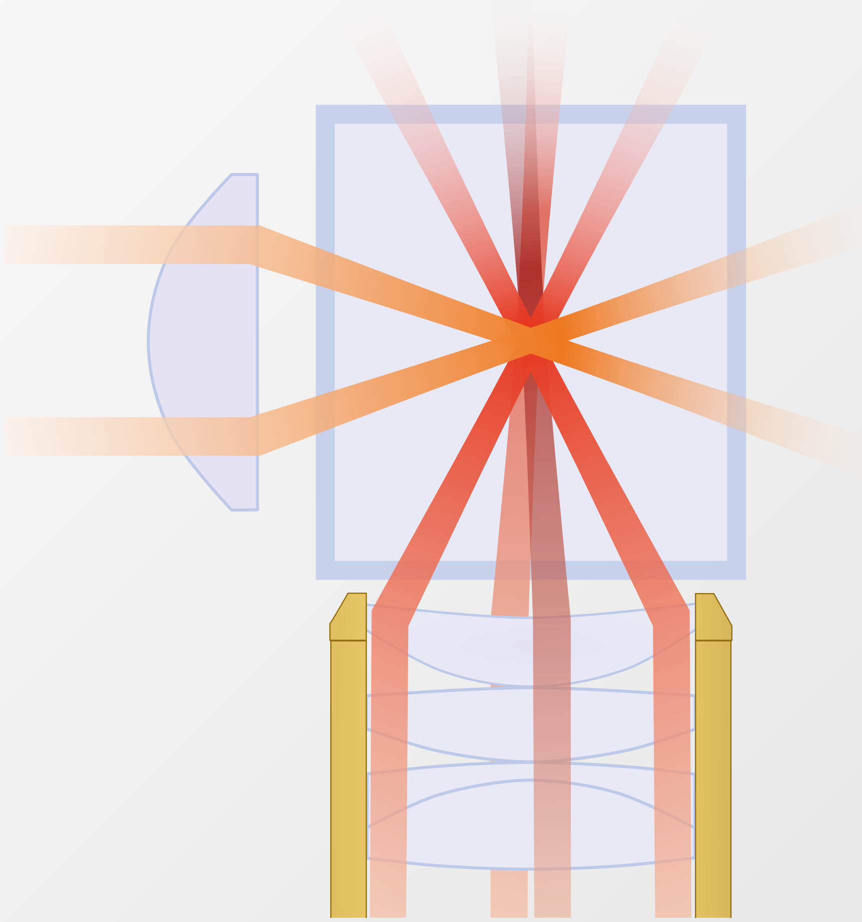
Fermionic Quantum Gas Microscopes

Experimental Setup

xy-lattice

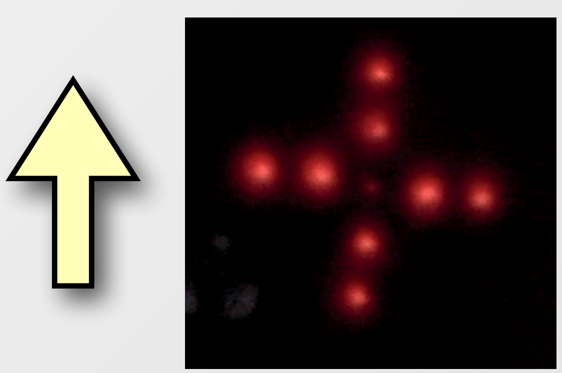


xy-lattice short lattice spacing 1.2 μm



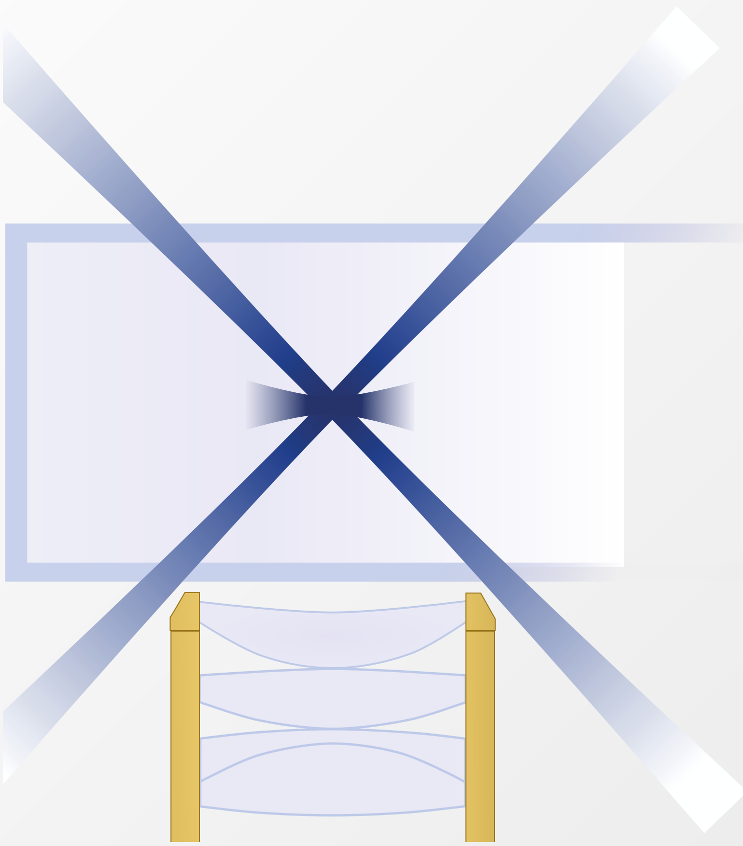
z-lattice

Physics lattice



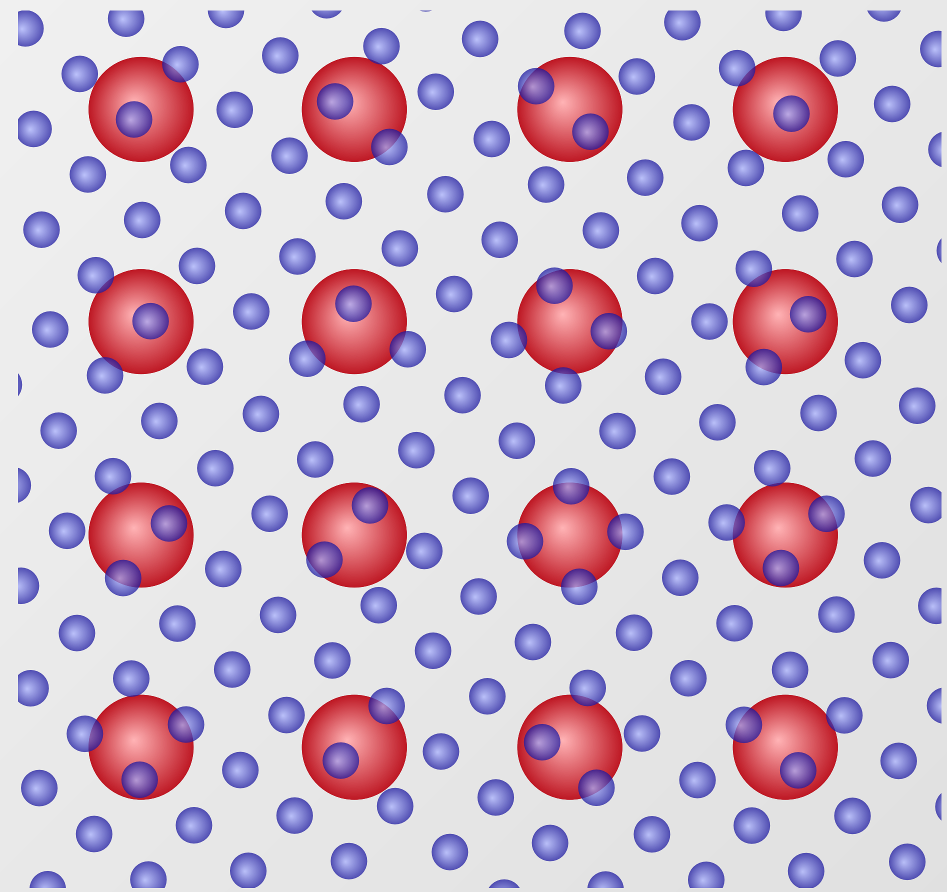
Detection 'Pinning' Lattice

Pinning lattice 1064 nm



Physics Lattice

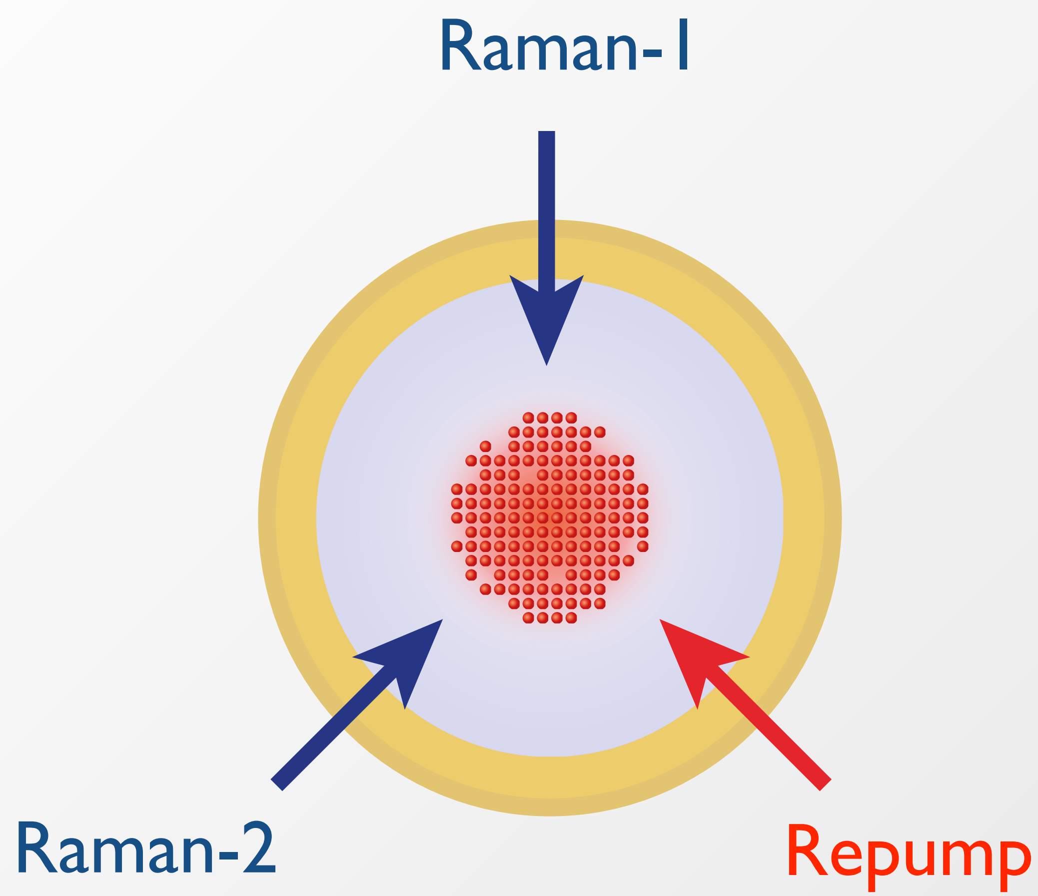
Physics+Pinning Lattice

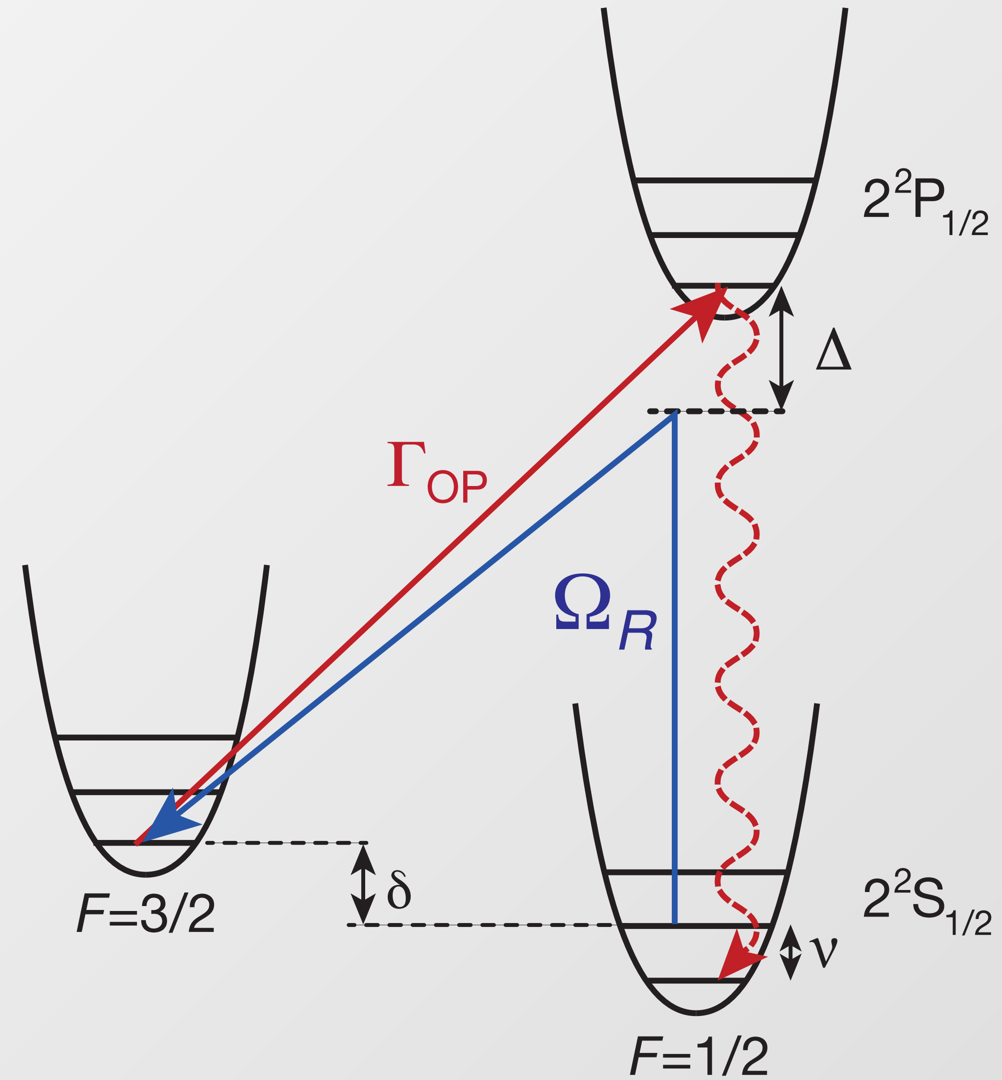


Pinning Spacing 532 nm

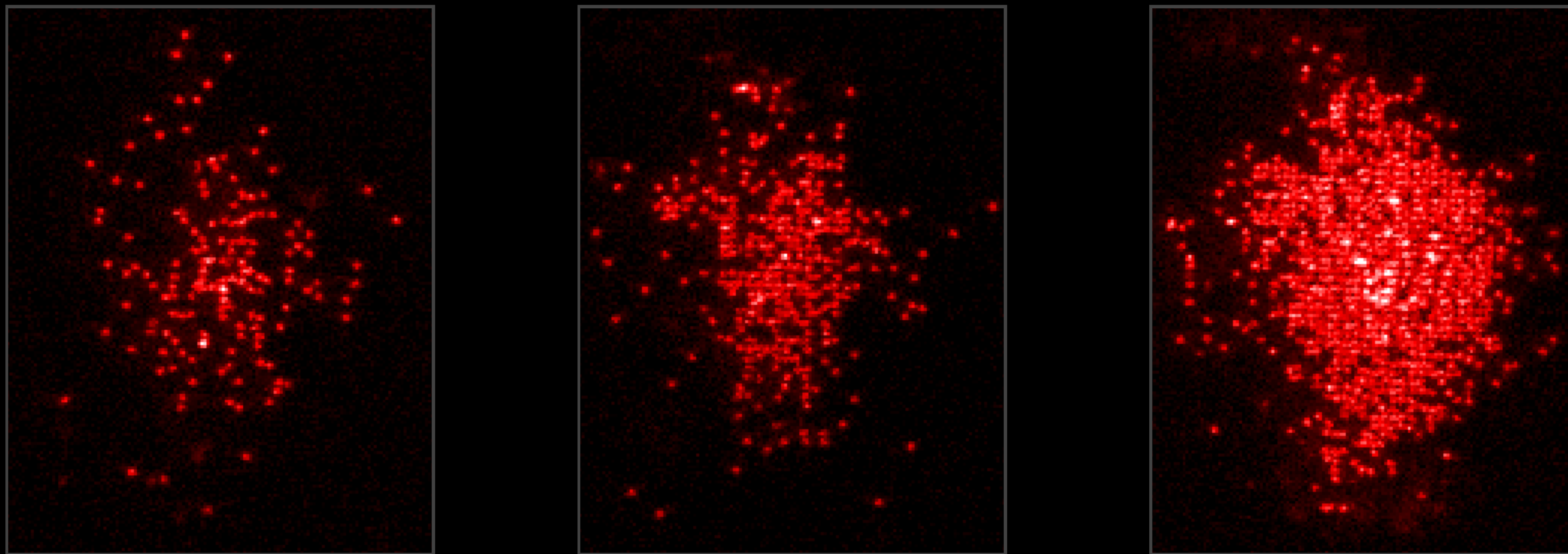
Onsite Trap Freq. 1.4 MHz

Raman Cooling in Pinning Lattice





7 kHz Photon Scattering Rate!

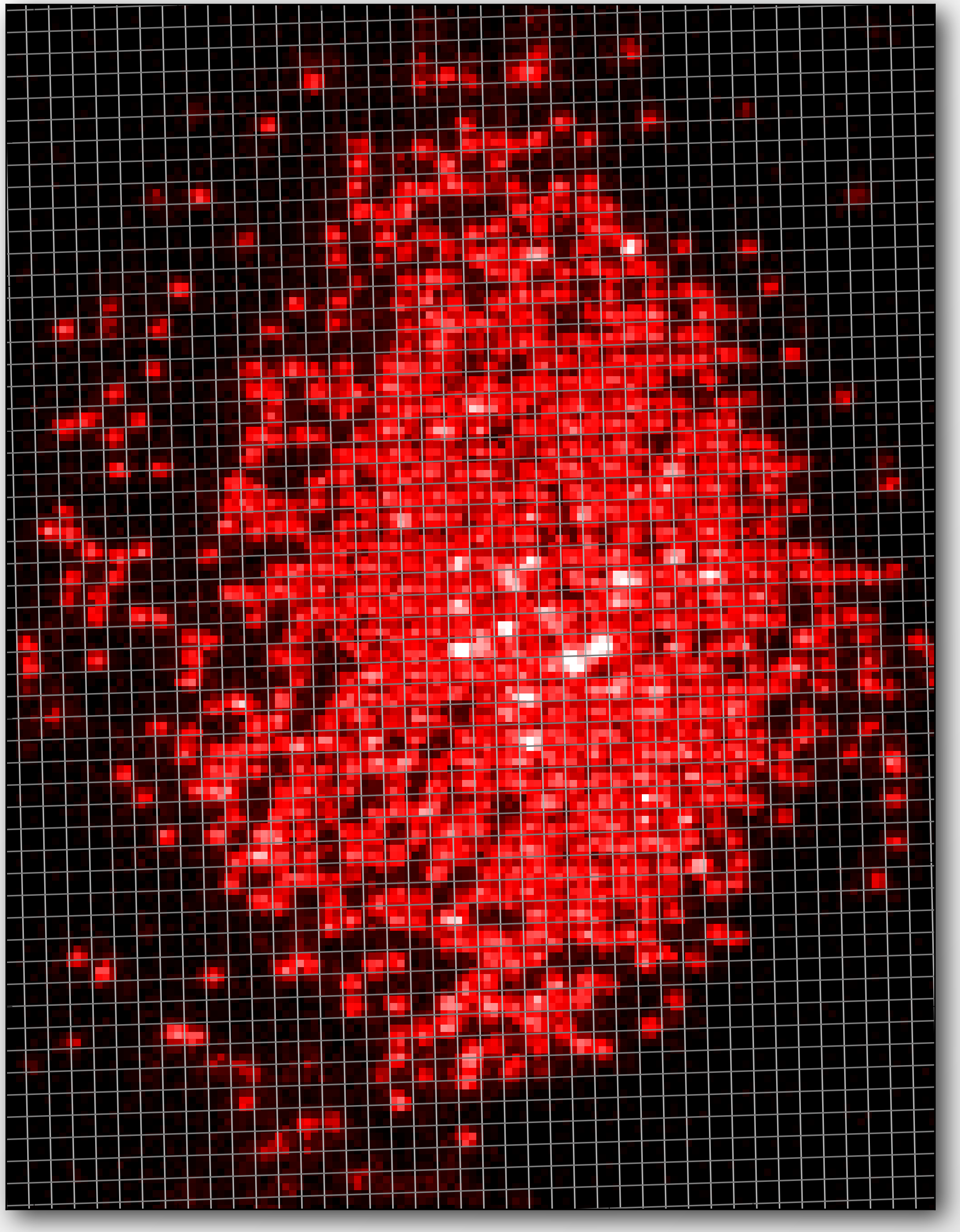


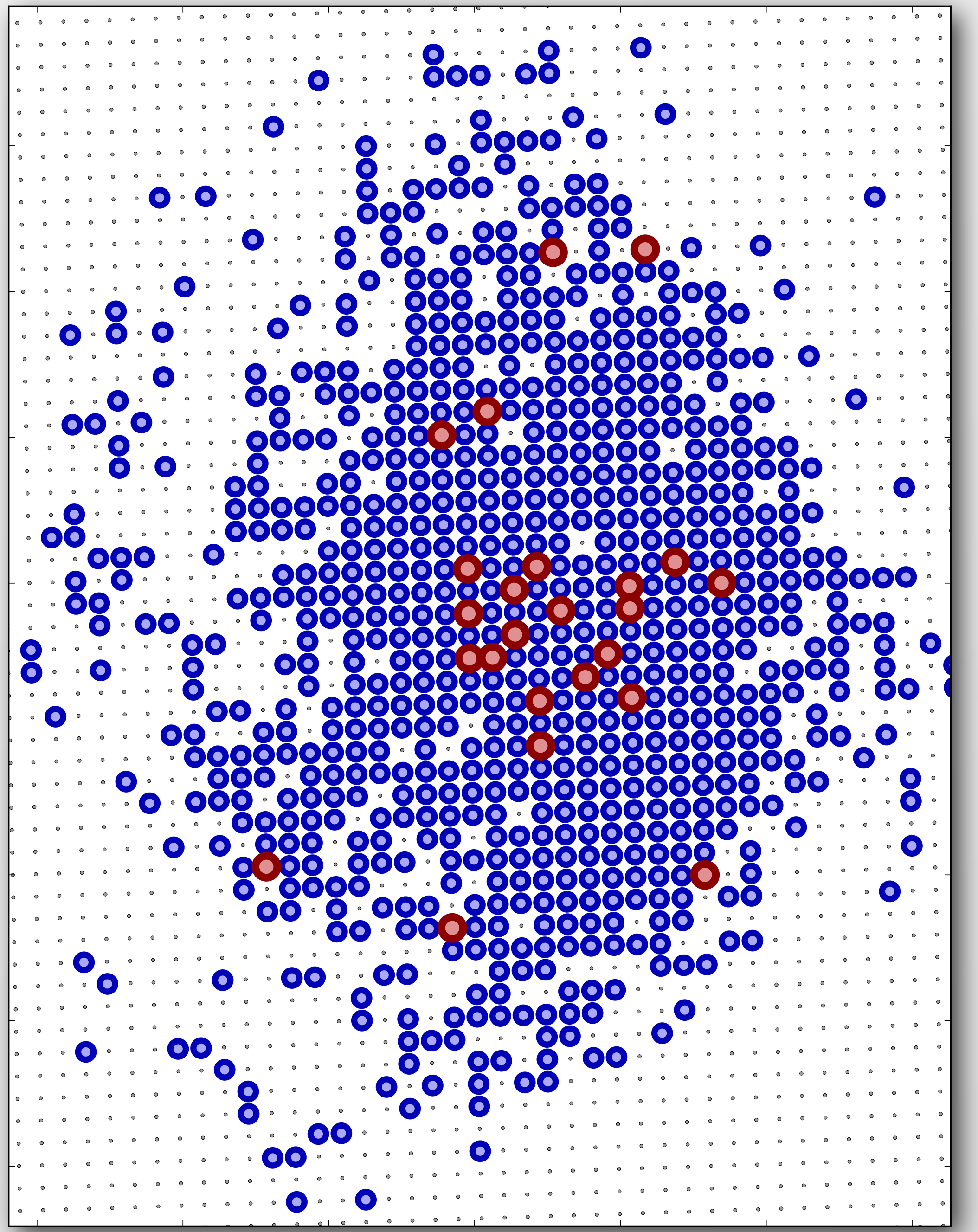
dilute medium dense - Band Insulator

Single Atom Fluorescence Imaging 6-Li

A. Omran et al. PRL **115**, 263001 (2015)

Reconstruction of BI

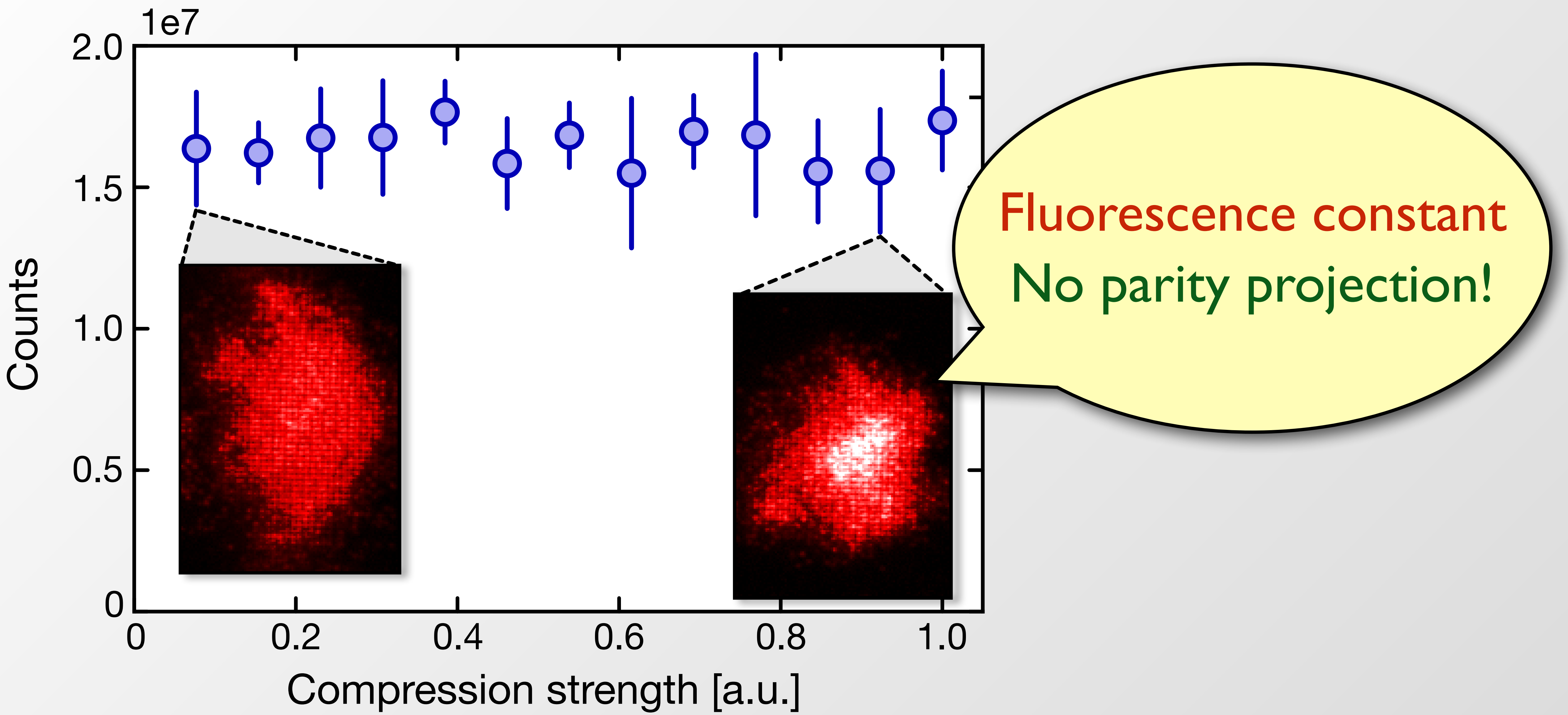




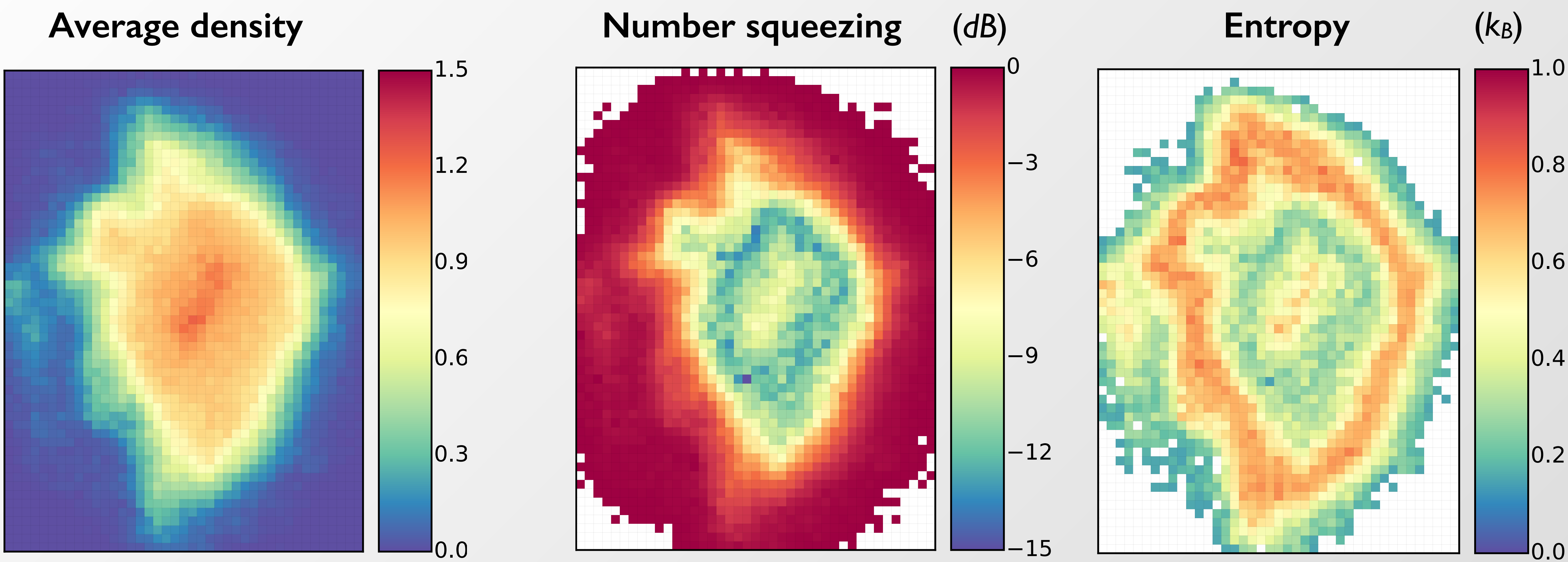
~800 atoms in image

field of view ~2000 lattice sites

Avoiding Parity Projection



Site Resolved Many-Body State Analysis



$\bar{n}_i$

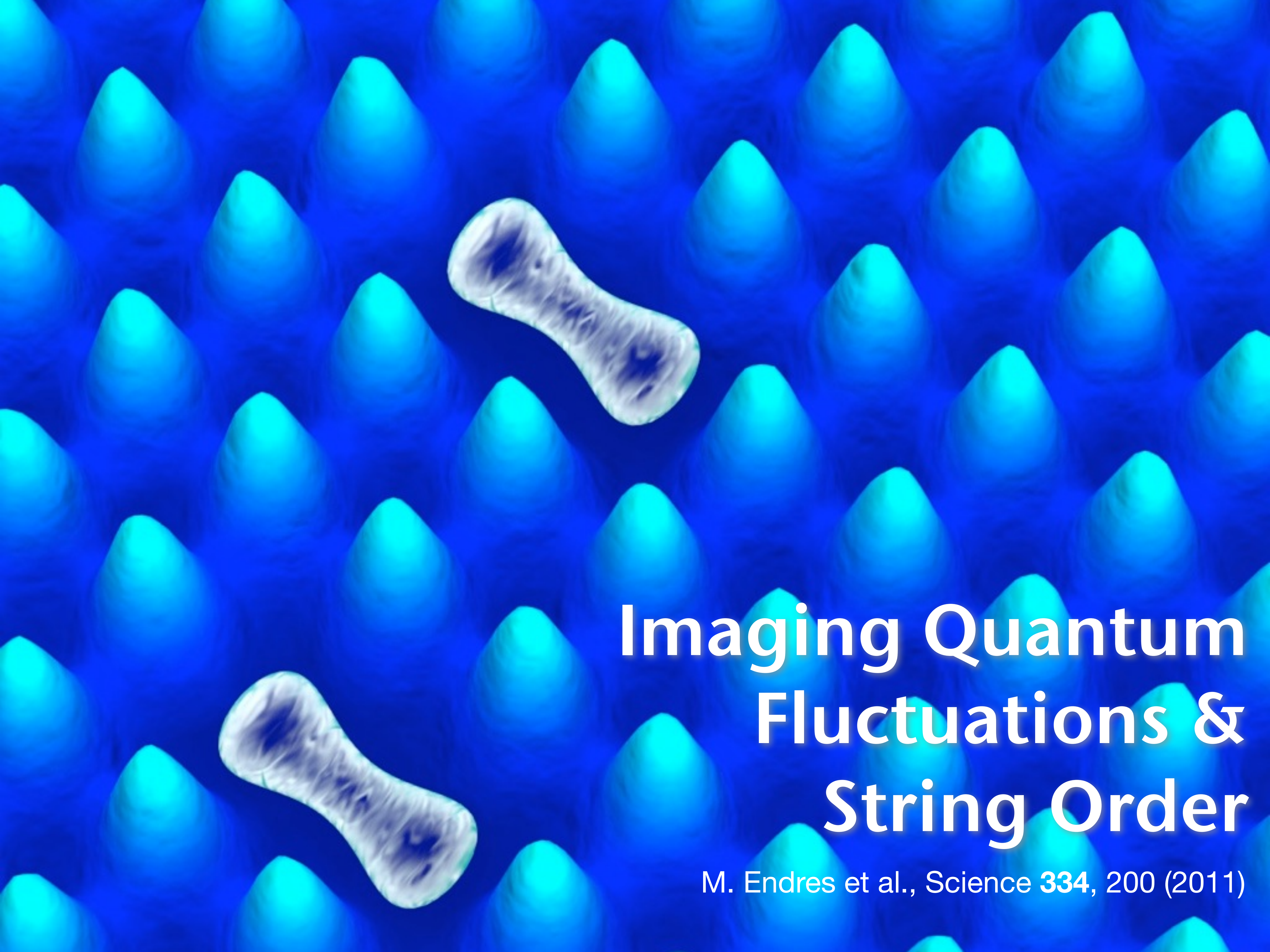
$\frac{\sigma^2_{n,i}}{\bar{n}_i}$

$S_i = -k_B \sum_{j\in 0,1,2} p_{i,j} \log p_{i,j}$

Analysis from ~500 single shot images!

Assume Grand Canonical also allows to obtain μ,T,k…

A. Omran et al. PRL **115**, 263001 (2015)



Imaging Quantum Fluctuations & String Order

M. Endres et al., Science **334**, 200 (2011)

Probing Hidden Non-Local String Order

M. Endres, M. Cheneau, T. Fukuhara, Ch. Weitenberg, P. Schauss,
L. Mazza, M.C. Bañuls, L. Pollet, I. Bloch, S. Kuhr

discussions: Emanuele Dala Torre, Ehud Altman

E. G. Dalla Torre et al. Phys. Rev. Lett. **97**, 260401 (2006),
E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008).

Order Parameters

Typical Order Parameter in Landau Paradigm of Phase Transition

$$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \langle \hat{A}(\mathbf{x})\hat{A}(\mathbf{y})\rangle = c$$

Order Parameter:

Examples: gnetism, AFM,...)

Function)

or

General classification scheme for all phases of matter ???

Order Parameter Chara Ground State Correlations

Local ordering!

String Order in 1D Systems

E.g. in 1D gapped systems where $\langle \hat{A}(\mathbf{x}) \hat{A}(\mathbf{y}) \rangle$ decays exponentially with distance

However, they can show hidden non-local order:

$$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \langle \hat{A}(\mathbf{x}) \left(\prod_{\mathbf{z}\in S(\mathbf{x},\mathbf{y})} \hat{B}(\mathbf{z}) \right) \hat{A}(\mathbf{y}) \rangle = c$$

We say the order is hidden, because a "global view" of the underlying state is required. (**Topological Order:** X.-G. Wen)

Allows us to characterize state only via its ground state correlations!

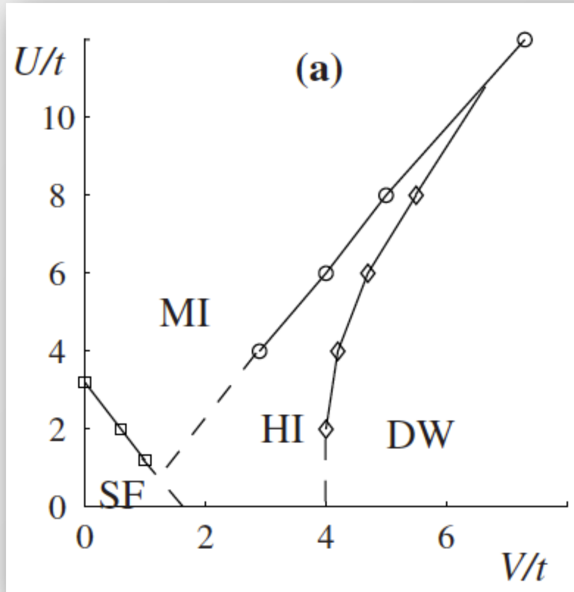
M. den Nijs, K. Rommelse, Phys. Rev. B 40, 4709 (1989).
E. Kim, G. Fa´th, J. So´lyom, D. Scalapino, Phys. Rev. B 62, 14965 (2000)
E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
F. Anfuso, A. Rosch, Phys. Rev. B 75, 144420 (2007)
E. Berg, 1 E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

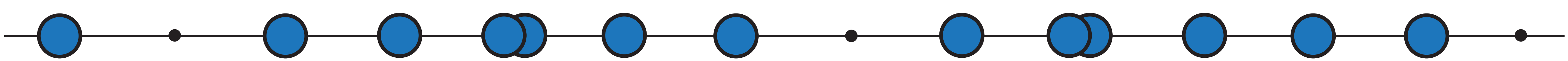
An Example: Haldane Insulator in 1D

E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
E. Berg, I E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

$$H = -J\sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1) + V\sum \hat{n}_i\hat{n}_{i+1}$$

Bose-Hubbard with next-neighour interaction





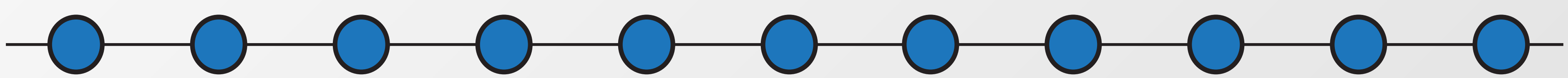
Hidden Non-local Order Captured by String Correlator

$$\mathcal{O}_S^2 = -\lim_{|i-j|\to\infty} \left\langle \delta\hat{n}_i \left(\prod_{i<k<j} e^{i\pi\delta\hat{n}_k} \right) \delta\hat{n}_j \right\rangle$$

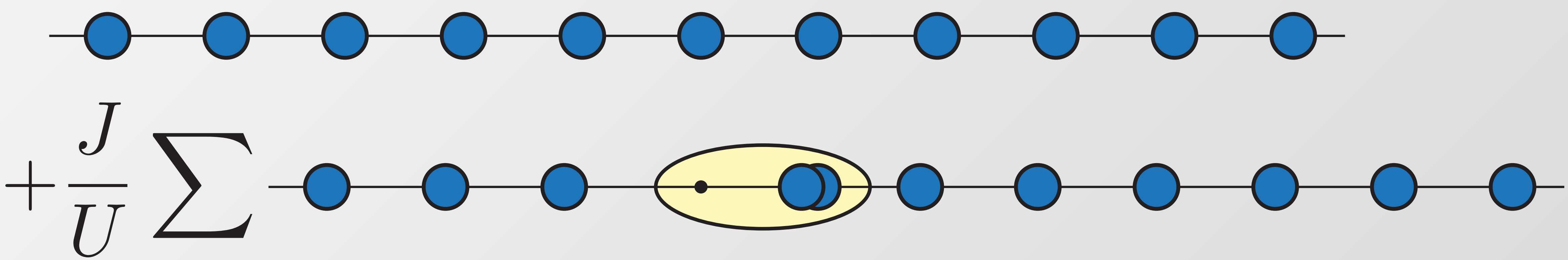
Ground State of a MI

$$H = -J\sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Starting Point: MI in Atomic Limit (*J*=0) ***No fluctuations!***



Small Tunneling (First order perturbation)

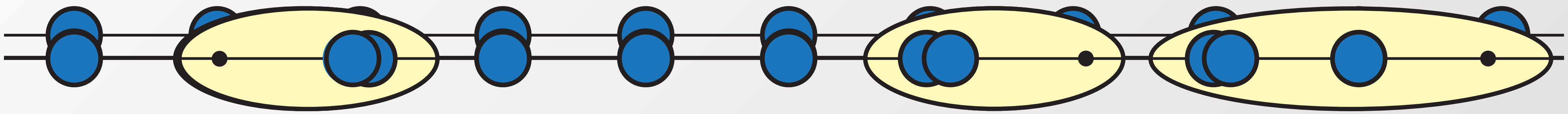


Quantum Fluctuations appear in form of
Quantum Correlated Particle Hole Pairs

In constrast: *thermal fluctuations appear as uncorrelated fluctuations!*

Towards Larger J/U

Increasing *J/U*



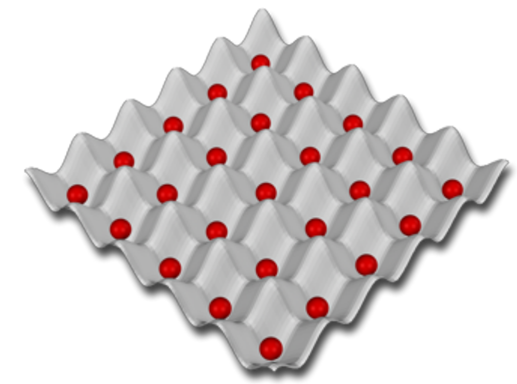
- Particle-Hole Pairs **Proliferate**
- Particle-Hole Pairs **Extend in Size**
  (leading to Deconfinement at Transition Point)

Deviations from the Atomic Mott Insulator

Ground state for J=0 :

``atomic´´ Mott insulator

$$|\Psi_0\rangle = \prod_i |n_0\rangle_i$$

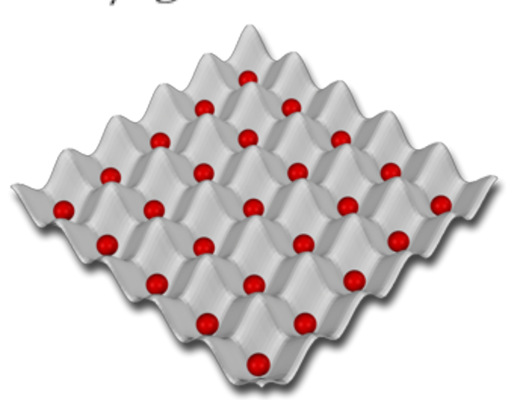


Ground state for finite J<<U :

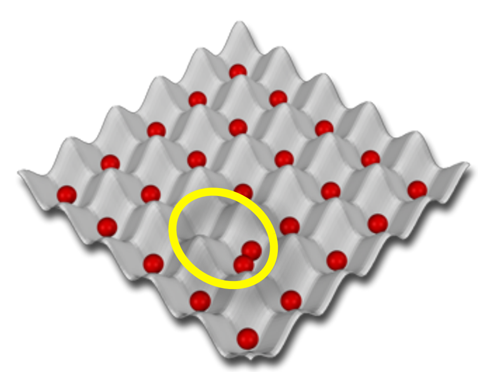
treat the hopping term H_{hop} in 1st order perturbation

$$|\Psi_1\rangle = -\sum_{n\neq g} \frac{H_{\mathrm{hop}}}{E_g^{(0)} - E_n^{(0)}} |\Psi_0\rangle$$

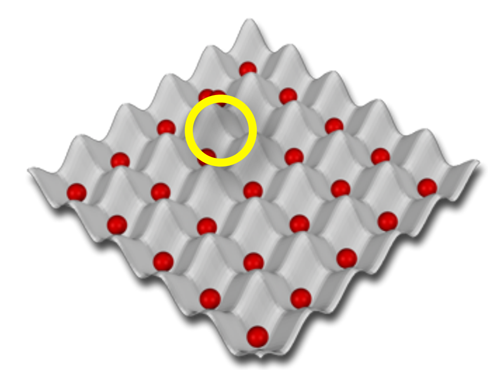
=



$+ \frac{J}{U}$



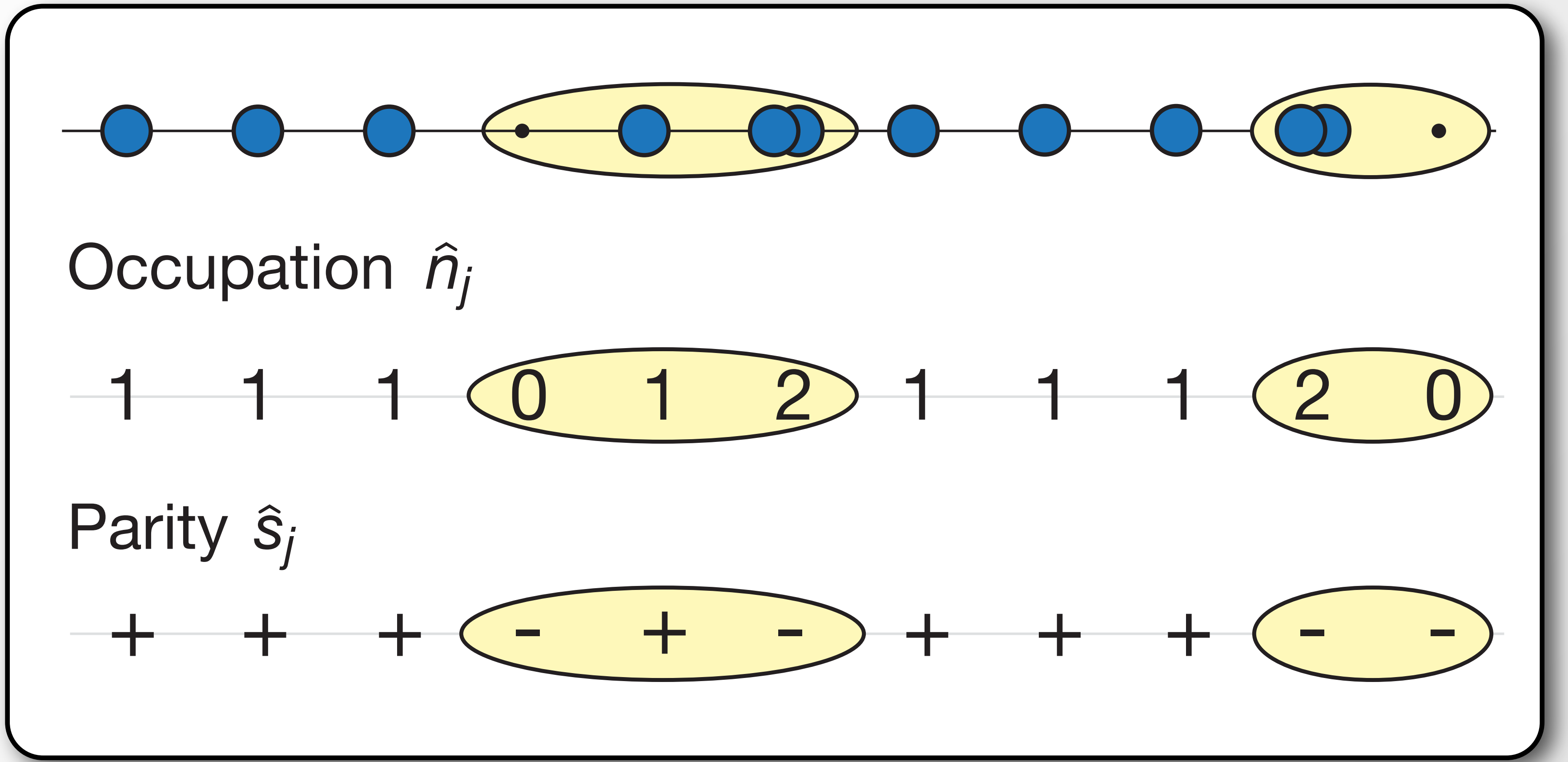
$+ \frac{J}{U}$



$+ \ldots$

Coherent admixture of particle/holes at finite J/U

String Order in a 1D Mott Insulator

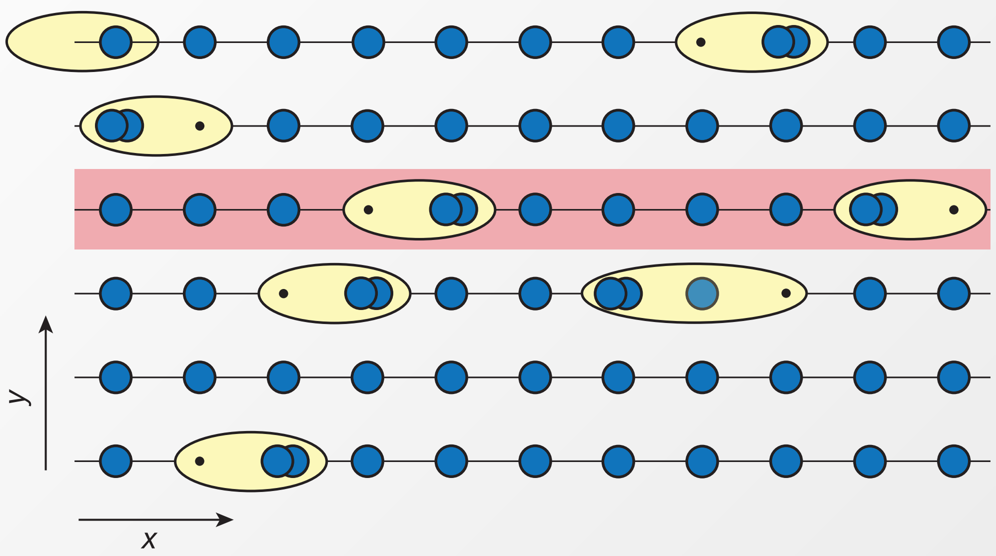


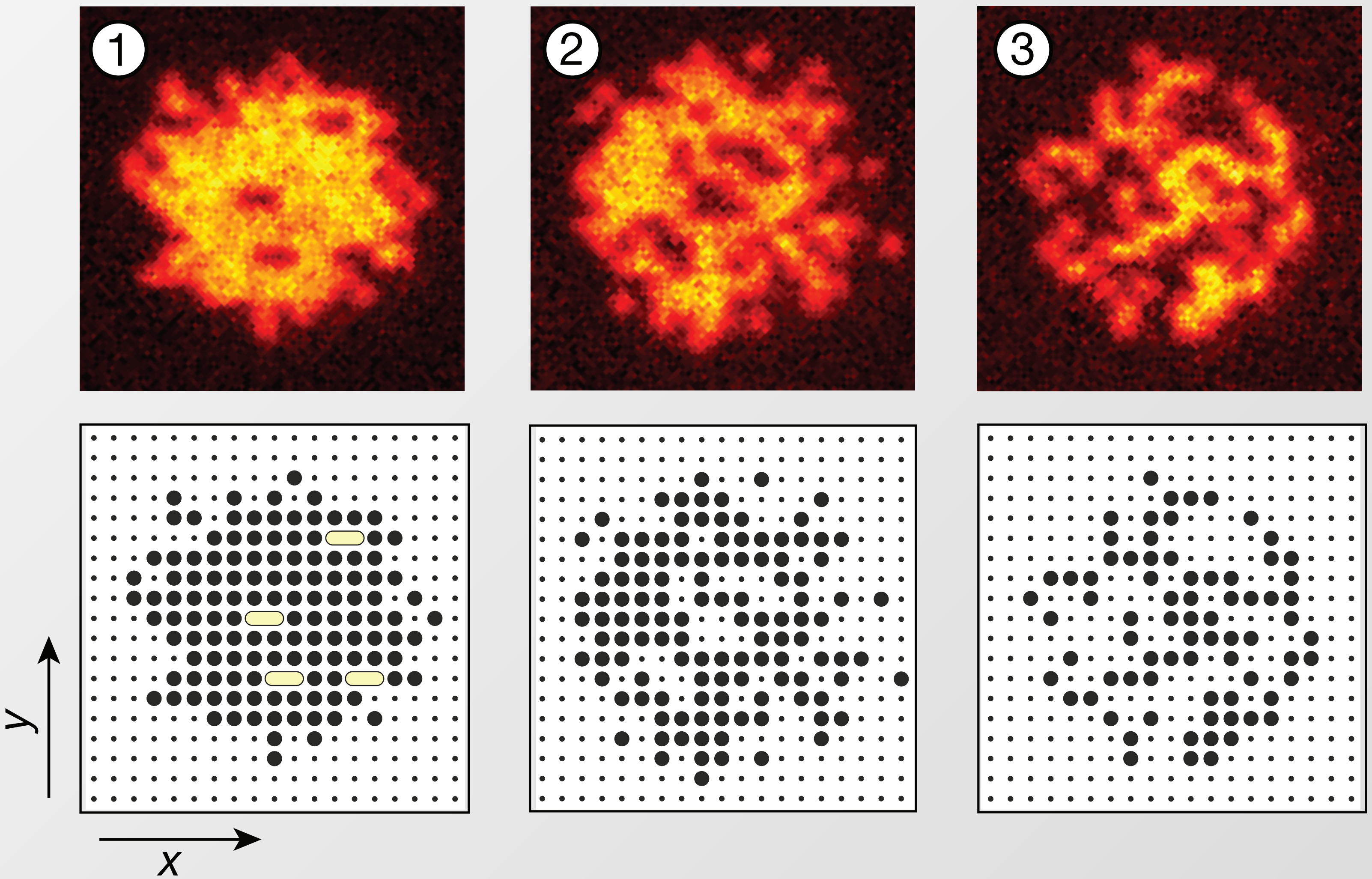
Hidden Non-Local Order Parameter of MI

$$\mathcal{O}_P^2 = \lim_{|i-j|\to\infty} \left\langle \prod_{i\le k\le j} e^{i\pi\delta\hat{n}_j} \right\rangle$$

E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
E. Berg, I E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

Quantum Correlated Particle Hole Correlations



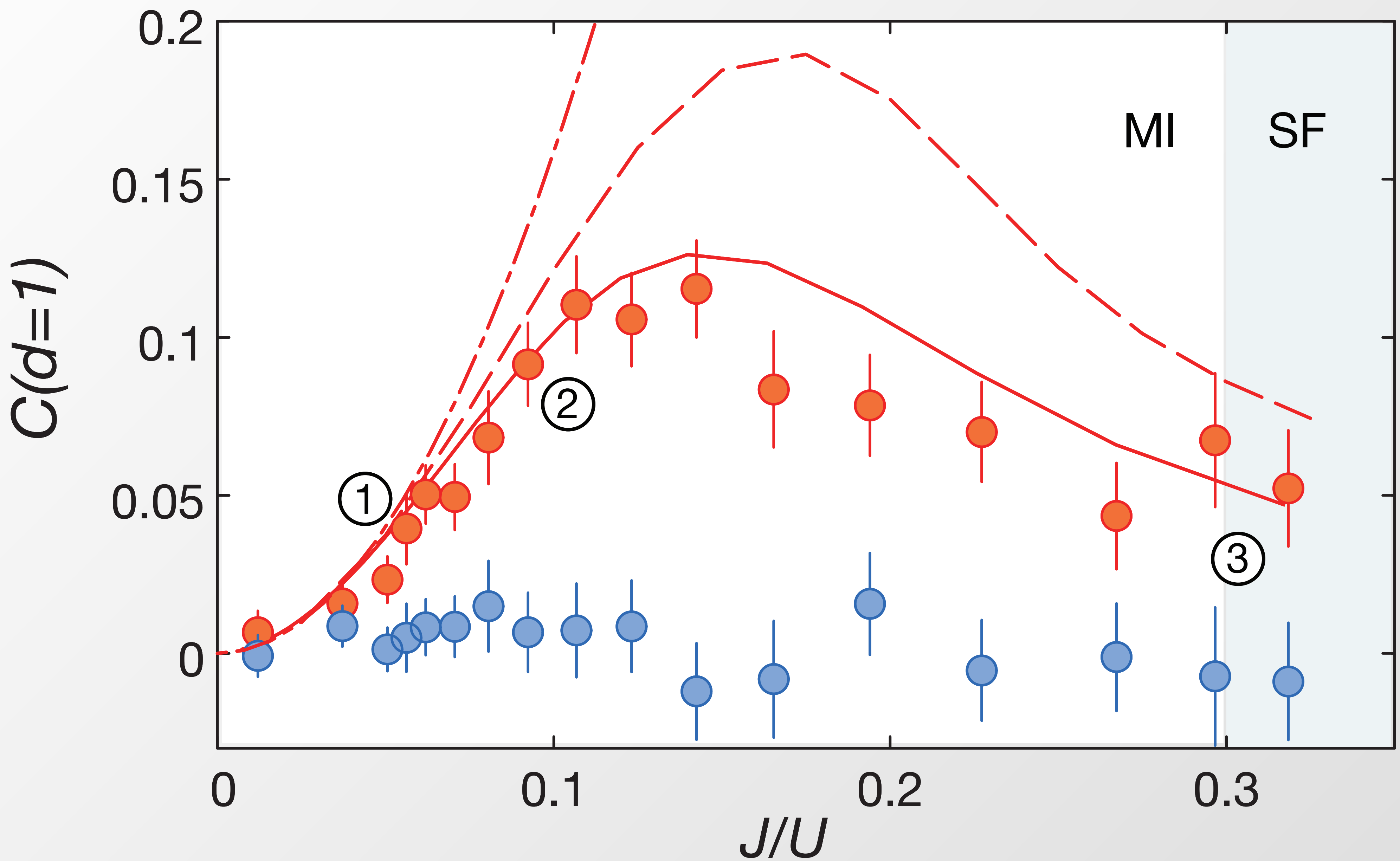


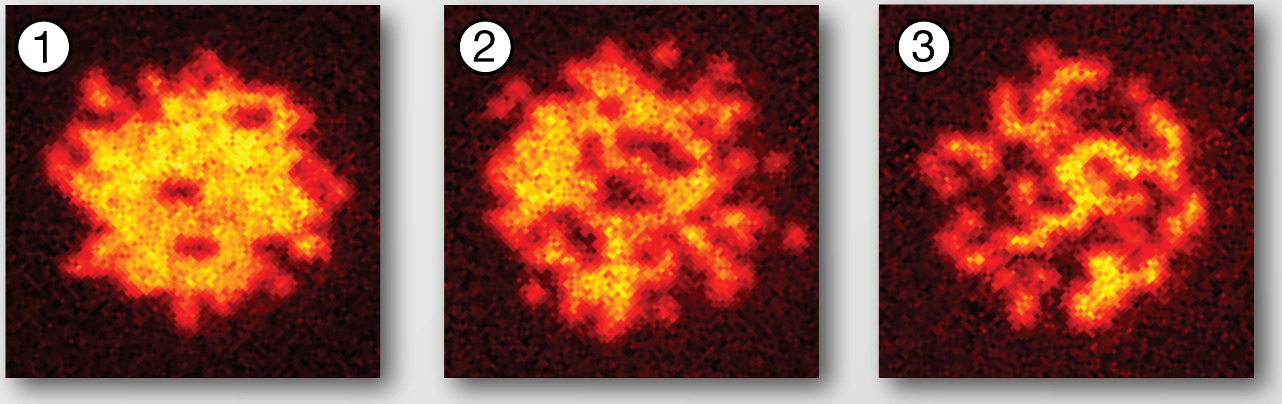
J/U=0.06 *J/U*=0.11 *J/U*=0.3

$$C(d) = \langle \hat{s}_k \hat{s}_{k+d} \rangle - \langle \hat{s}_k \rangle \langle \hat{s}_{k+d} \rangle$$

Two point correlator

Particle-Hole Correlations 1D



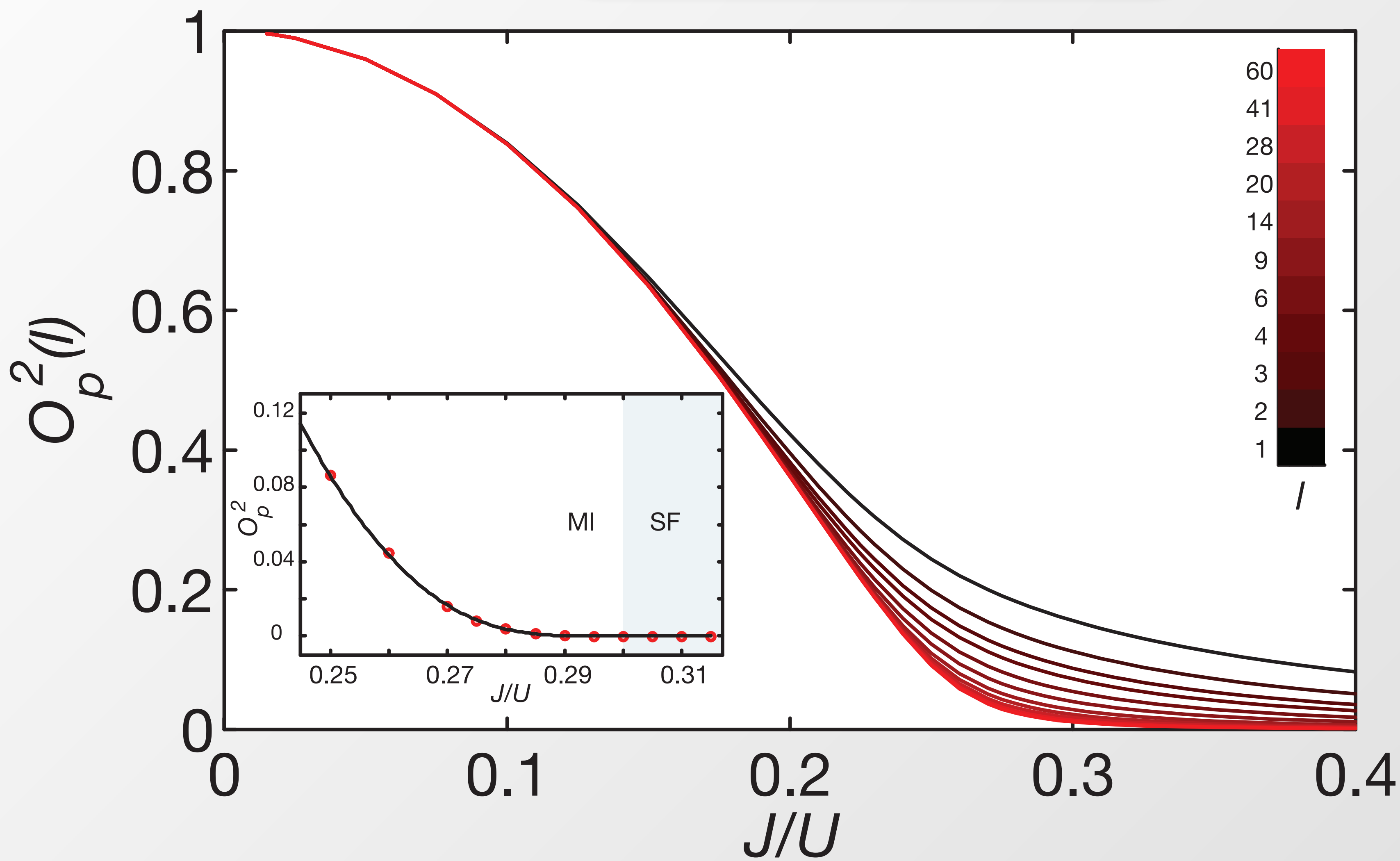


Single Shot Images

Theory: (dashed and solid line) DMRG & MPS by L. Mazza & M.C. Bañuls

String Order - an Order Parameter for the Mott Insulator?

$$\mathcal{O}_P^2(l) = \left\langle \prod_{j=k}^{k+l} \hat{s}_j \right\rangle$$



DMRG T=0 Simulations, chain length 216, nbar=1

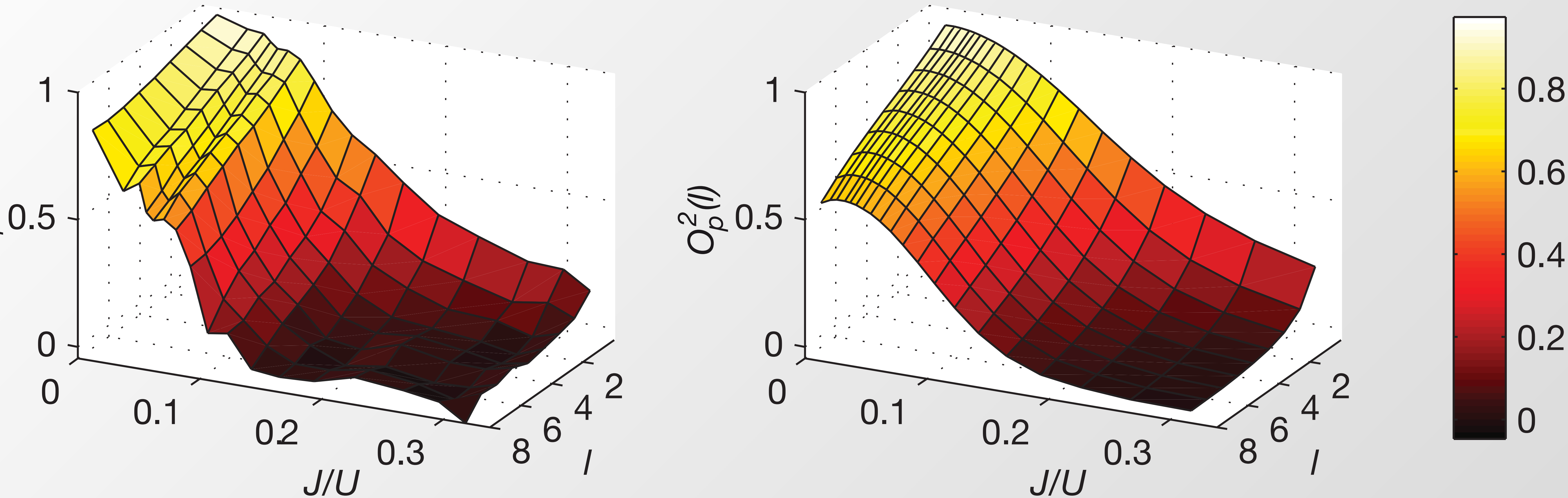
Inset: Finite size scaling of string correlator

Fit to $e^{-\frac{a}{\sqrt{(J/U)_c-(J/U)}}}$ Berezinskii-Kosterlitz-Thouless

$$(J/U)_c = 0.295 - 0.32$$

String Order

$$\mathcal{O}_P^2(l) = \left\langle \prod_{j=k}^{k+l} \hat{s}_j \right\rangle$$



Experiment

Theory (MPS *T*=0.09*U*)

Note:

- decay for larger string lengths due to thermal excitations
- shift of transition point due to inhomogenous trapping (pointy Mott lobes in 1D)

Conclusion

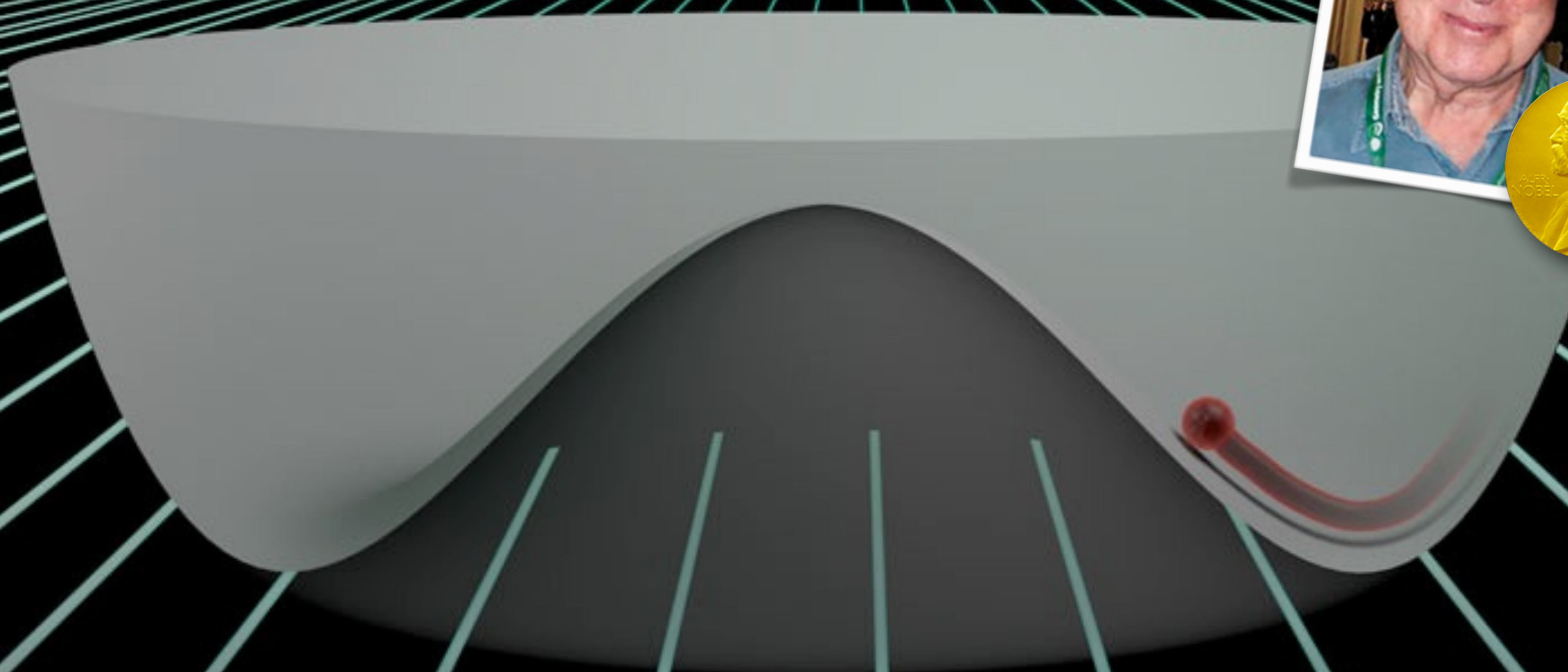
- Mott Insulator contains many *quantum correlated particle-hole pairs*, induced by quantum fluctuations.
- Particle-hole pairs *deconfine* at Mott-Superfluid transition
- Deconfinement is captured by *hidden non-local order parameter*
- String Order useful concept for finite lengths
- Another Deconfinement Transition from *Mott Insulator to Haldane Phase* for next neighbour interactions
- **First Measurement of a Non-Local Order Parameter**

Extension to 2D:

S. P. Rath, W. Simeth, M. Endres, W. Zwerger, Annals of Physics, 334, p. 256-271

Dynamics: L. Mazza, D. Rossini, M. Endres & R. Fazio Phys. Rev. B 90, 020301(R) (2014)

M. Strinati, L. Mazza, M. Endres, D. Rossini & R. Fazio Phys. Rev. B 94, 024302 (2016)



‘Higgs’ Amplitude Mode in Flatland

M. Endres, T. Fukuhara, M. Cheneau, P. Schauss, D. Pekker, E. Demler, S. Kuhr & I.B.

M. Endres et al. Nature (2012)
Chubukov & Sachdev, PRB 1993; Sachdev, PRB 1999; Zwerger, PRL 2004; Altman, Blatter, Huber, PRB 2007, PRL 2008; U. Bissbort et al. Phys. Rev. Lett. (2011); D. Podolsky, A. Auerbach, D. Arovas, PRB 2011

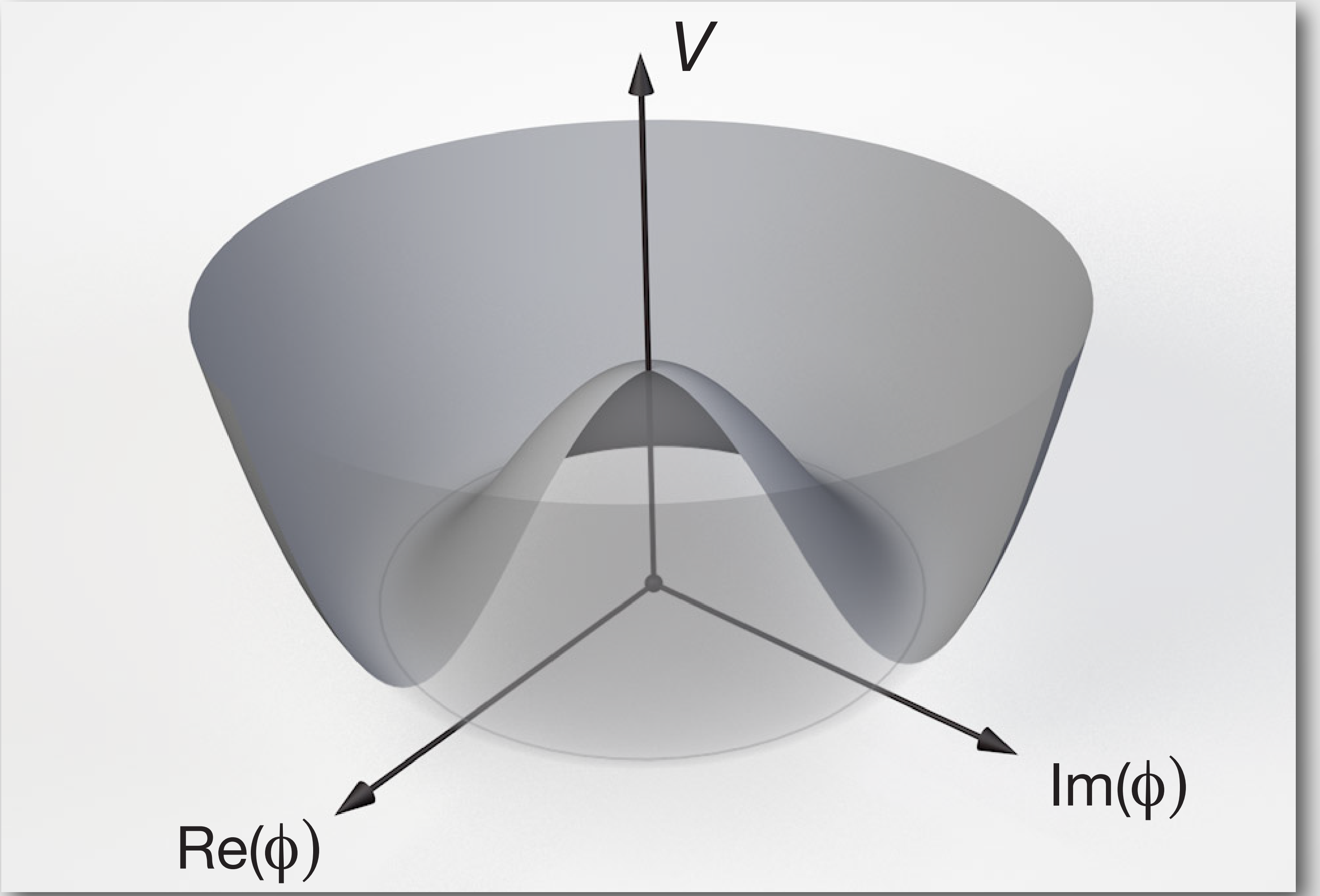
Spotaneous Symmetry Breaking

$$L = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{1}{2}\lambda(\phi^*\phi)^2$$

Relativistic Quantum Field-Theory of complex field ϕ with mass m.

$$L = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi - \frac{1}{2}\lambda(\phi^*\phi)^2$$

Imagine negative mass term.



$$L = \partial_\mu \phi^* \partial^\mu \phi - V(\phi)$$

$$\phi(x) \rightarrow \phi(x) e^{i\theta}$$

Lagrangian is U(1) symmetric

Spontaneous Symmetry Breaking - Modes

$$V(\phi) = -\frac{1}{2}\lambda v^2 \phi^* \phi + \frac{1}{2}\lambda(\phi^*\phi)^2 \qquad v^2 = -2m^2/\lambda$$

Minimum of Mexican Hat at: $|\phi|^2 = \frac{v^2}{2}$

Pick one vacuum state! Expand field around: $\phi = \frac{1}{\sqrt{2}}(v + \varphi_1 + i\varphi_2)$

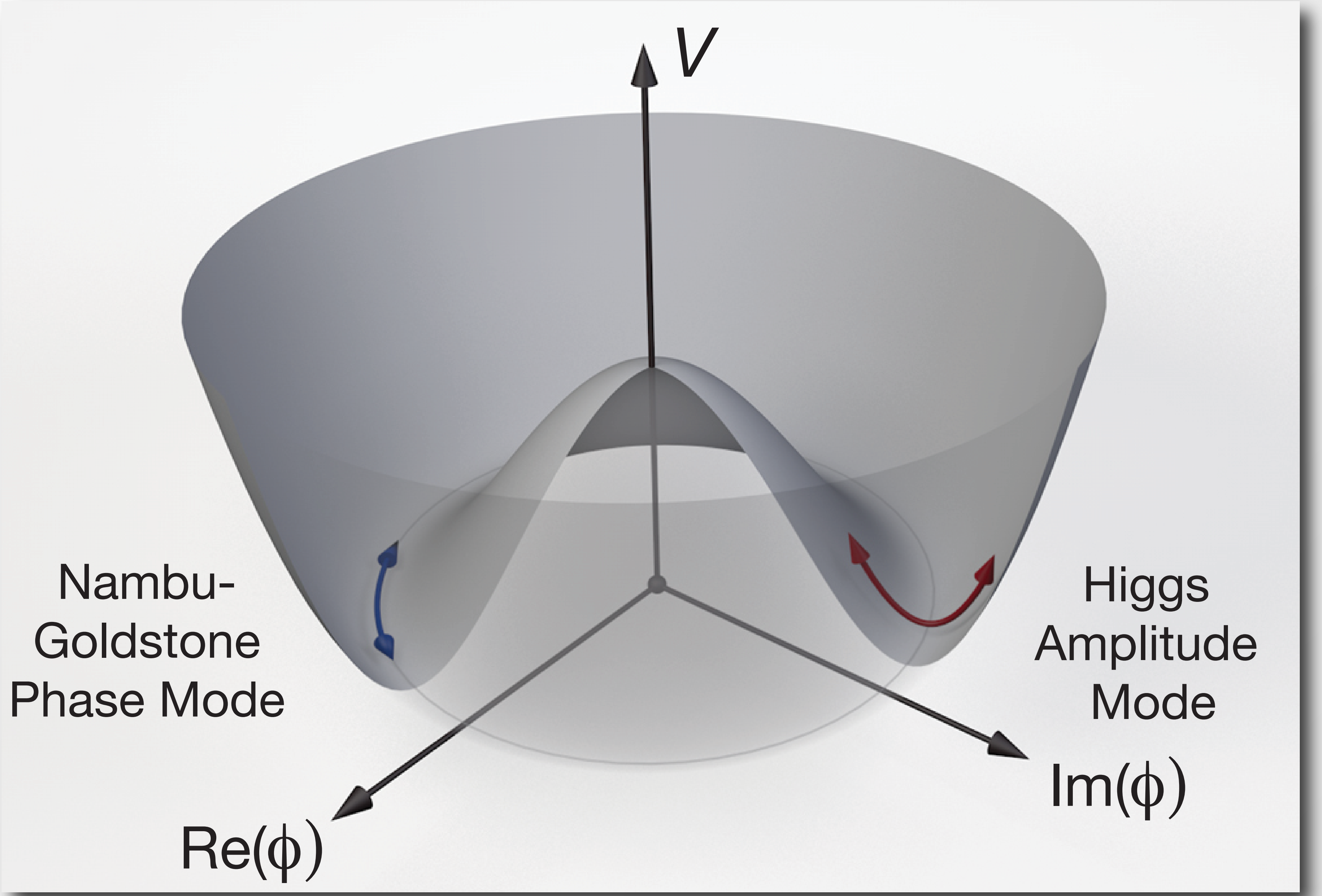
$$L = \frac{1}{2}\left[(\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2)^2\right] - \frac{1}{2}\lambda v^2 \varphi_1^2 + \dots$$

Massless Nambu-Goldstone mode

Massive Higgs mode

φ_1, φ_2 real scalar fields

Spontaneous Symmetry Breaking - Modes



Excitations in radial direction are gapped due to ‘Higgs mass’!

Anderson-Higgs Mechanism

$\theta \rightarrow \theta(x)$ Extend to local U(1) gauge symmetry.

$A_\mu \rightarrow A_\mu(x) - \frac{1}{e}\partial_\mu \theta(x)$ introduces vector potential

$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi$ minimal coupling

$$L = D_\mu \phi^* D^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi)$$

Breaking symmetry leads to:

$$L = \frac{1}{2}(\partial \varphi_1)^2 + \frac{1}{2}(\partial_\mu \varphi_2 + evA_\mu)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}\lambda v^2 \varphi_1^2 + \ldots$$

Photons have become massive (m^2=ev)! ⇒ Meissner effect Anderson 1963

Similiar for non-Abelian gauge theory U(1)xSU(2) ⇒ W,Z bosons acquire mass

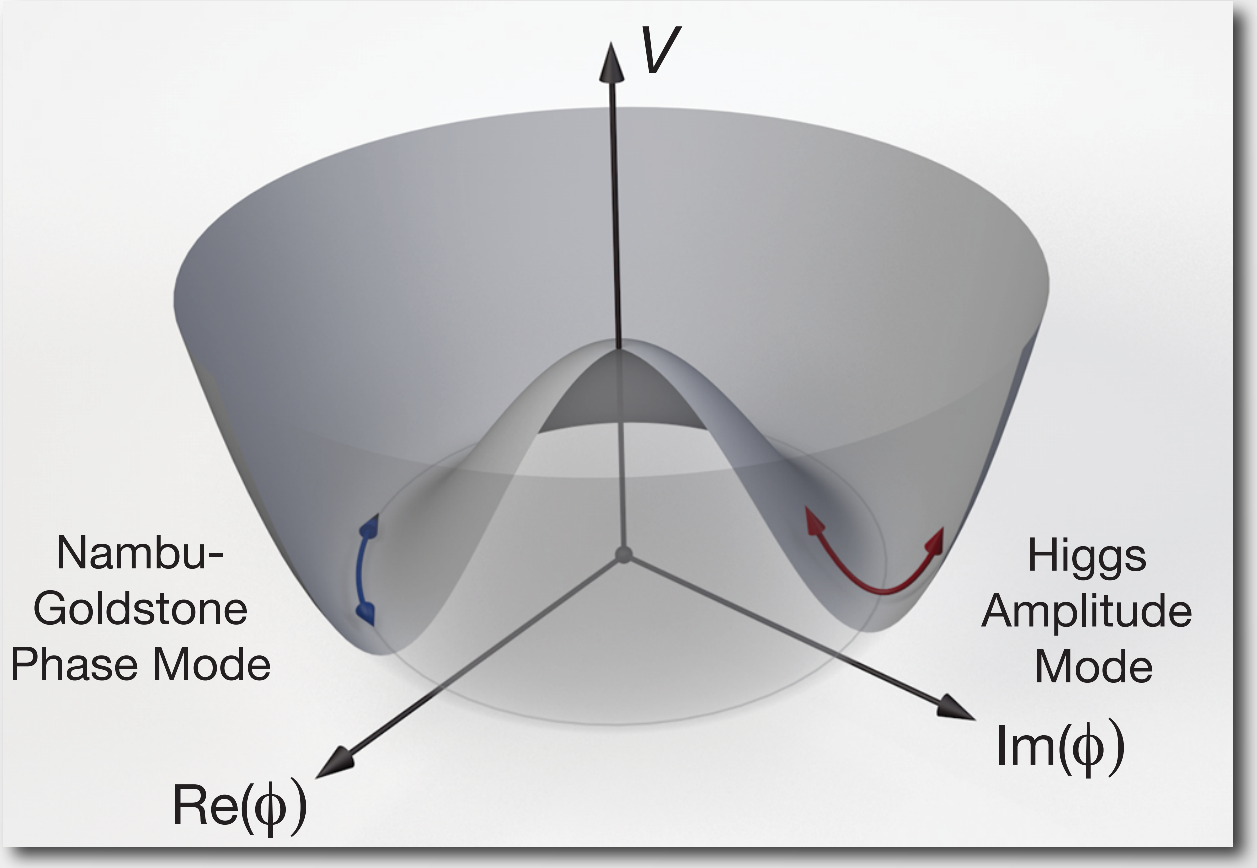
Englert, Brout, Higgs, Guralnik, Hagen, Kibble, Weinberg ~1964

LMU

Higgs Mode in CM Physics

Close to a quantum critical point, effectively relativistic field theory!
see e.g.: Subir Sachdev, Quantum Phase Transitions

Here: SF-MI transition for n=1, O(2) field theory in 2+1 dimension



Fundamental question in 2D:
is mode observable or overdamped?

Chubukov & Sachdev, PRB 1993
Sachdev, PRB 1999; Zwerger, PRL 2004; Altman, Blatter, Huber, PRB 2007, PRL 2008; Menotti & Trivedi, PRB 2008; Podolsky, Auerbach, Arovas, PRB 2011; Pollet & Prokof'ev PRL 2012; Sachdev & Podolsky, PRB 2012; ...

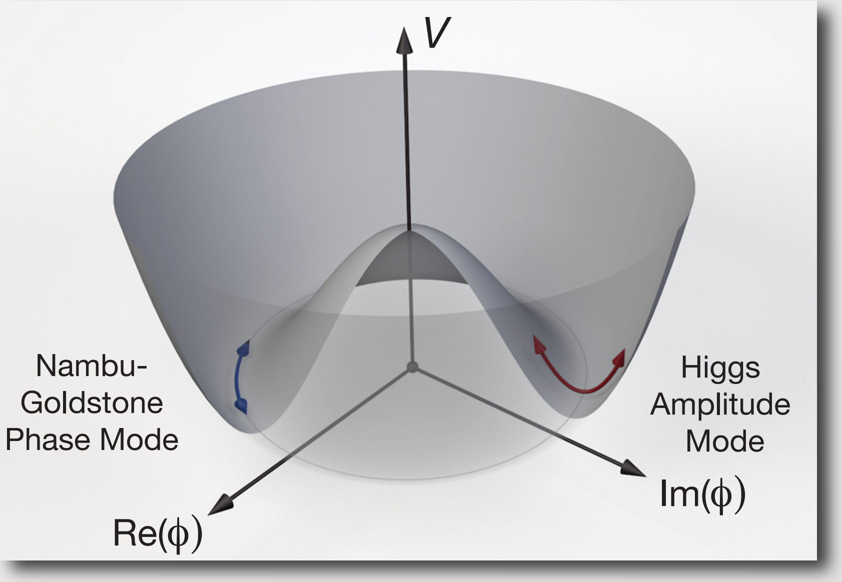
Other systems: Quantum spin systems O(3) in 3+1 dimensions
Ch. Rüegg et al. Physical Review Letters (2008)

in superconductors coupled to CDW:
C. Varma & P. Littlewood PRL, PRB (1981,1982)

Higgs Mode in CM Physics

Theoretically difficult/debated problem!

Sachdev, QPT book, 2011:
„..., for N>1, d<3, there **is no Higgs particle**...“
Bose-Hubbard in 2d?



Podolsky, 2011: **It‘s there!**
- use „correct“ response function
- Lowest-order analysis
- Some weird properties

Experiment, 2012: **It‘s there!**
- Measure „correct“ response function
- Finite temp.
- Trap

Pollet, 2012: **It‘s there!**
- Quantum Monte Carlo
- Reliable?
- Calculation in im. time

Podolsky, Sachdev, 2012:
It‘s there!
Higher order analysis
Very cumbersome
Not reliable for N=2

Dynamics in the Superfluid Phase

Far from the Mott lobe, SF described by Gross-Pitaevskii action:

$$S = \int d^3r dt \left(-i\psi^* \partial_t \psi - \frac{1}{2m^*} |\nabla \psi|^2 + \mu |\psi|^2 - g |\psi|^4 \right)$$

Galilean invariant. Predicts massless Goldstone mode, but no Higgs mode.

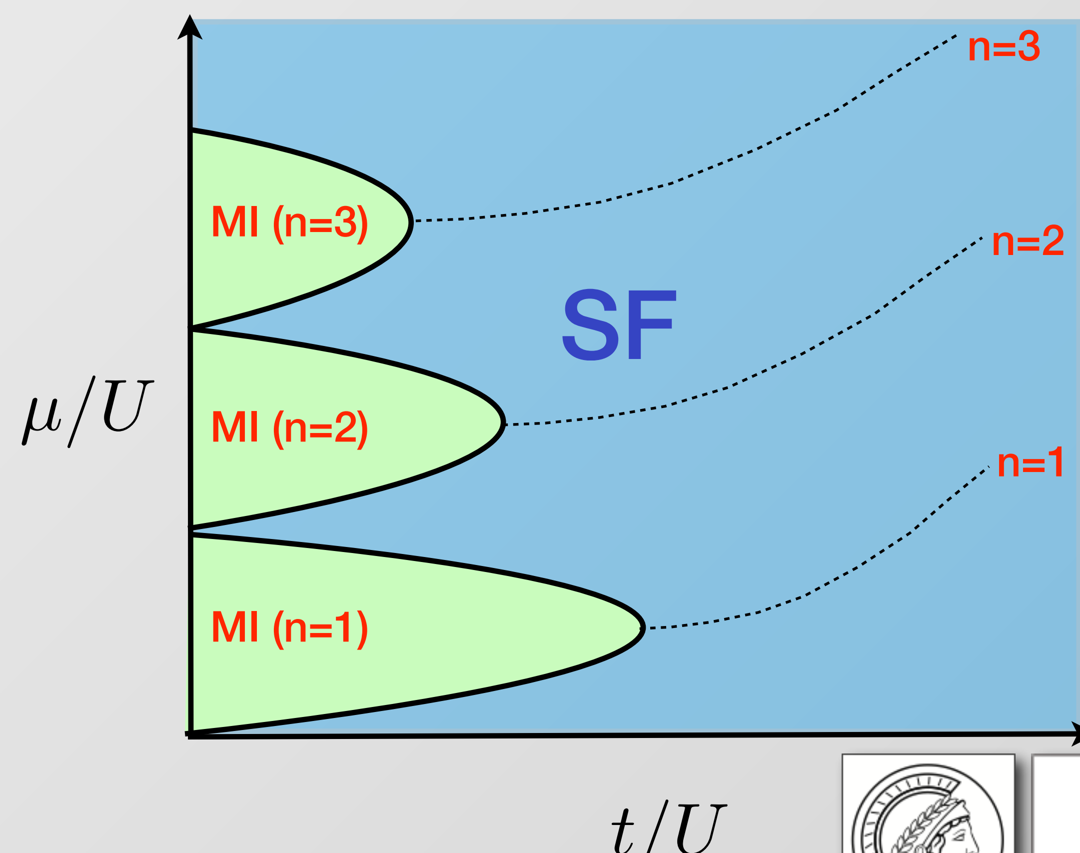
Imported Author 23 Oct 2013, 7:24
GPE: Phase and amplitude mode are c.c. variables! Therefore only one mode!

Close to QCP: Phase and ampltiude of order parameter

Near the Mott lobe at integer filling, particle-hole symmetry leads to relativistic dynamics:

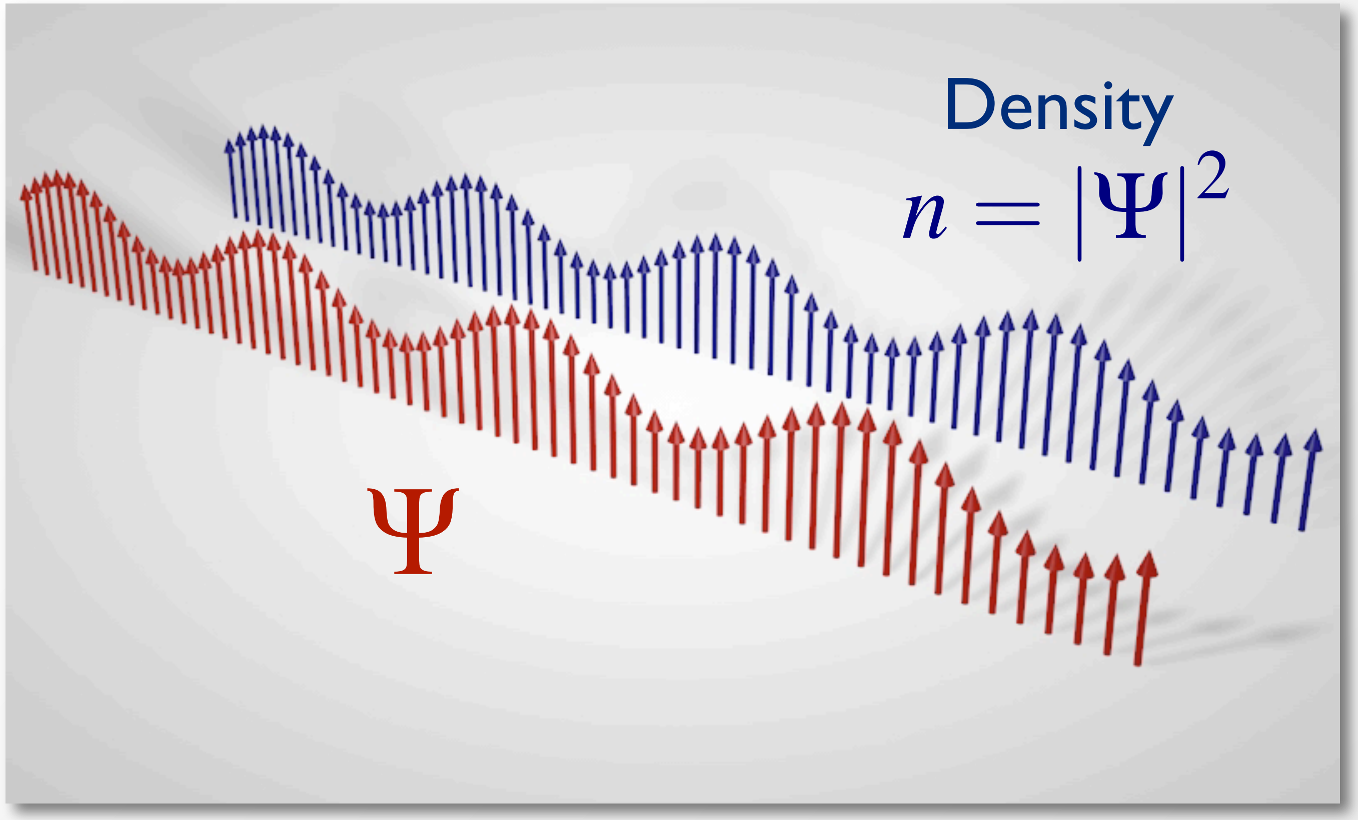
$$S = \int d^3r\, dt \left(|\partial_t \psi|^2 - c^2 |\nabla \psi|^2 + r|\psi|^2 - u|\psi|^4 \right)$$

*Lorentz invariant. Predicts Goldstone mode **and** Higgs mode.*



Courtesey: Danny Podolsky (Technion)

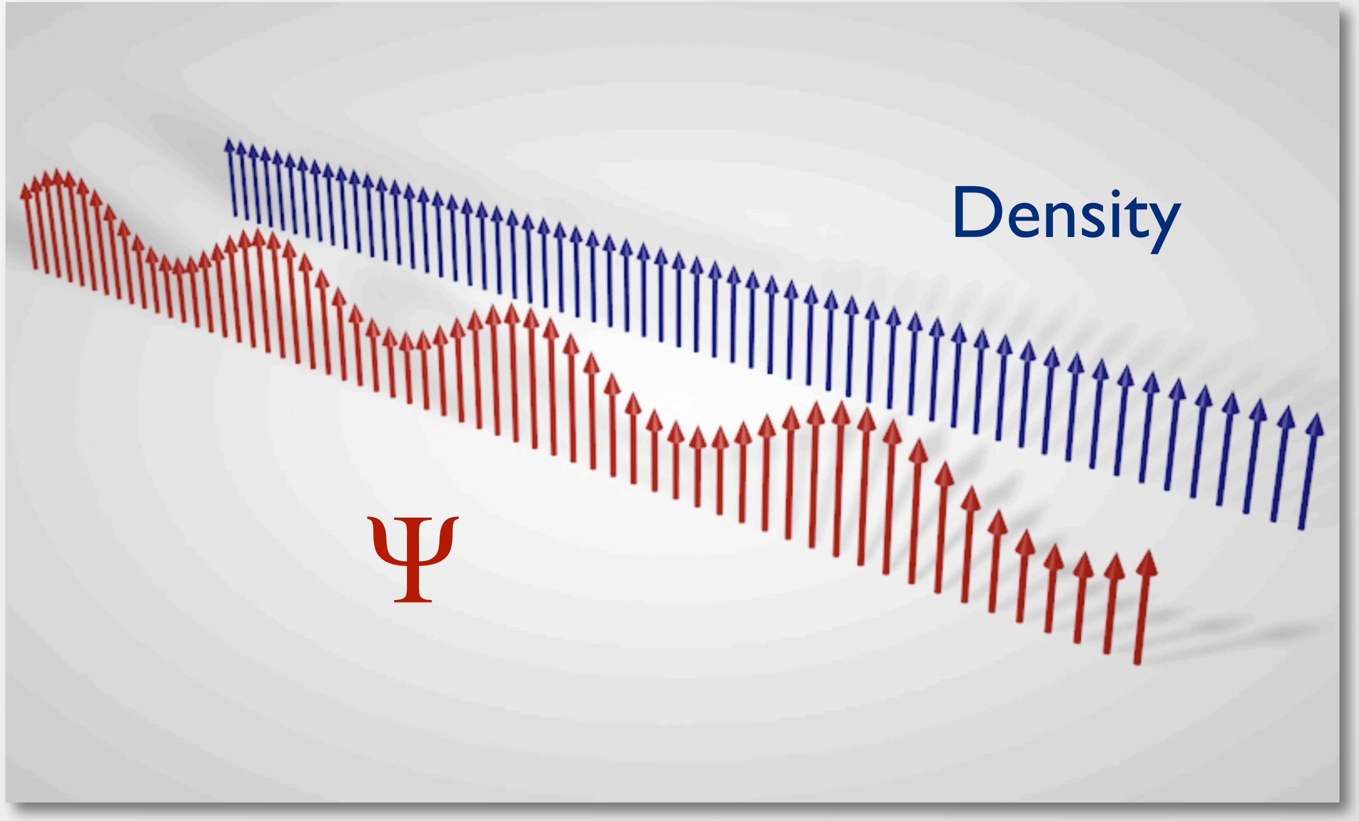
Relativistic vs Non-Relativistic Dynamics



Weakly Interacting BEC

(non-relativistic)

$$\omega(\tilde{k}) = \sqrt{\mu^2 \tilde{k}^2 (\tilde{k}^2 + 2)}$$



SF @ Quantum Critical Point

(relativistic)

$$\omega_1(k) = \sqrt{\Delta_0^2 + c_s^2 k_1^2}$$

$$\omega_2(k) = c_s k$$

Relativistic vs Gross-Pitaevskii Dynamics

From Euler-Lagrange equation, we obtain:

Lorentz invariant action

$$\ddot{\varphi}_1 = c_s^2 \nabla^2 \varphi_1 - \Delta_0^2 \varphi_1$$
$$\ddot{\varphi}_2 = c_s^2 \nabla^2 \varphi_2$$

$$\omega_1(k) = \sqrt{\Delta_0^2 + c_s^2 k_1^2}$$ Relativistic Mode — Amplitude!

$$\omega_2(k) = c_s k$$ Sound Mode — Density!

Galilean invariant action

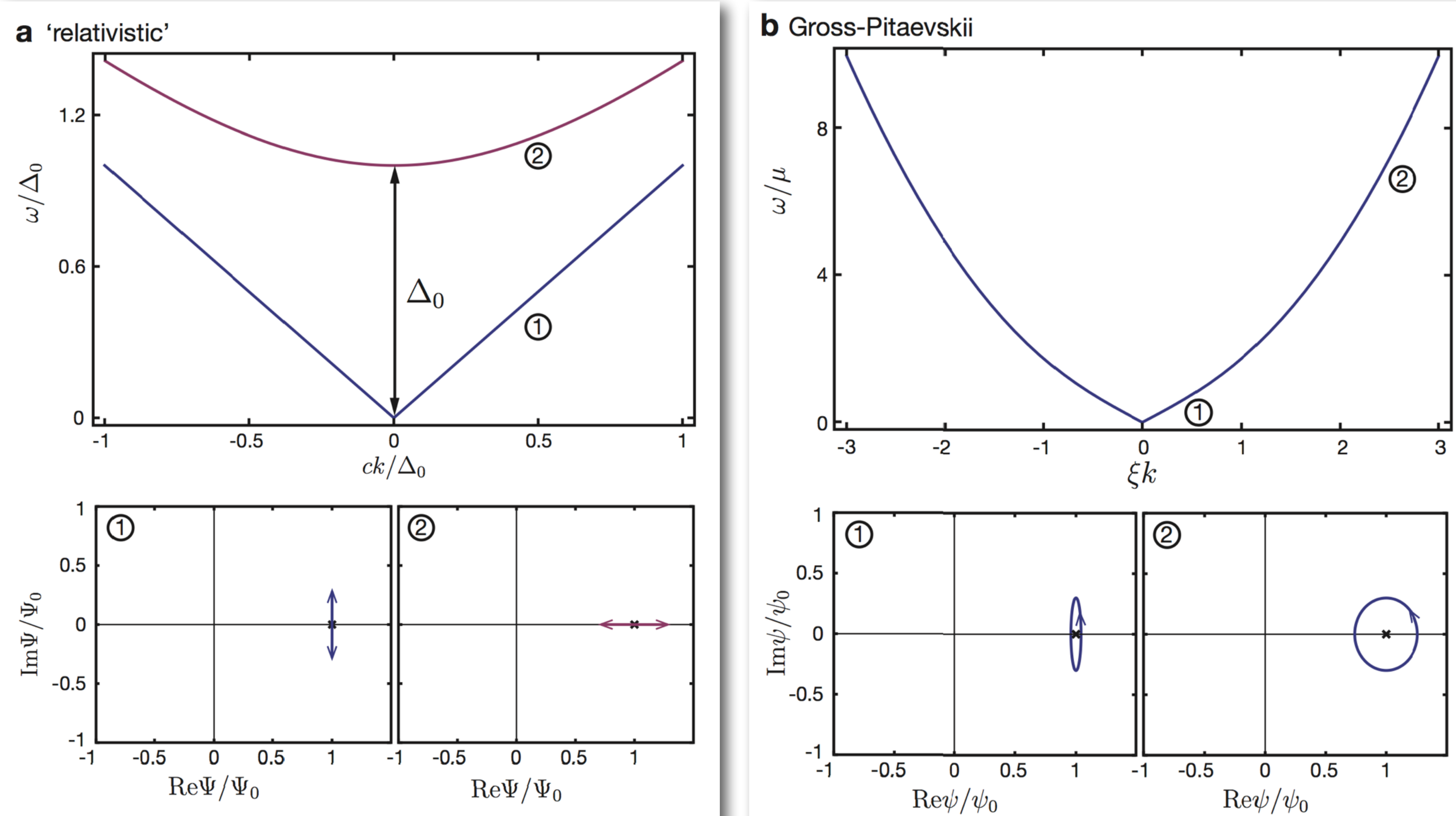
$$-\dot{\varphi}_1 = \frac{\hbar^2}{2m} \nabla^2 \varphi_2$$
$$\dot{\varphi}_2 = \frac{\hbar^2}{2m} \nabla^2 \varphi_1 - 2\mu \varphi_1$$

$$\omega(\tilde{k}) = \sqrt{\mu^2 \tilde{k}^2 (\tilde{k}^2 + 2)}$$

Bogoliubov Mode

Amplitude-Density Coupled!

Relativistic vs Gross-Pitaevskii Dynamics



'Relativistic'
Lorentz Invariant

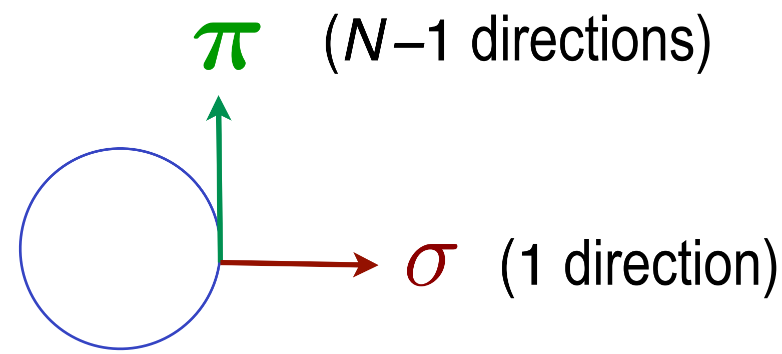
'Classical
Galilean Invariant

Broken Symmetry and Collective Modes

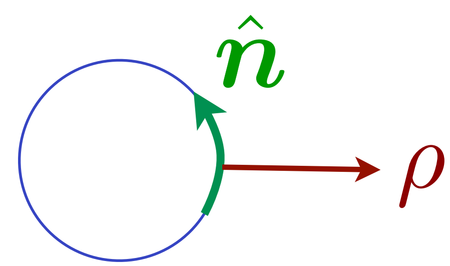
Two ways to parameterize deviations from the ordered state :

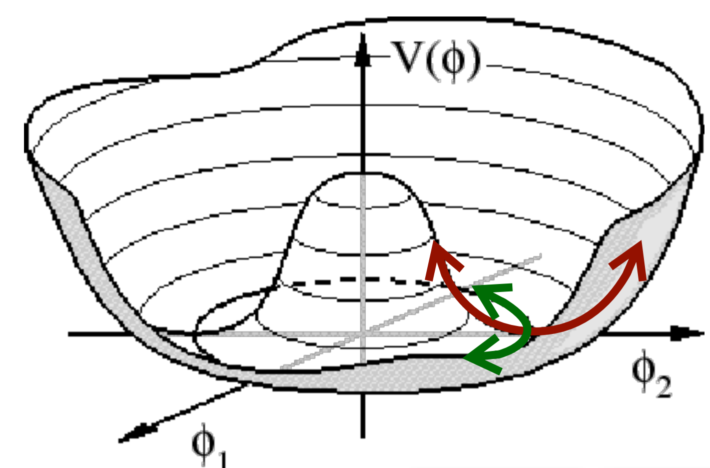
1) Cartesian : $\phi = (\sqrt{N} + \sigma, \boldsymbol{\pi})$

$$\mathcal{L}_0 = \frac{1}{2g}\left[(\partial_\mu \sigma)^2 - m^2\sigma^2 + (\partial_\mu \boldsymbol{\pi})^2\right]$$



2) Polar : $\phi = \sqrt{N}\,(1+\rho)^{1/2}\,\hat{\boldsymbol{n}}$



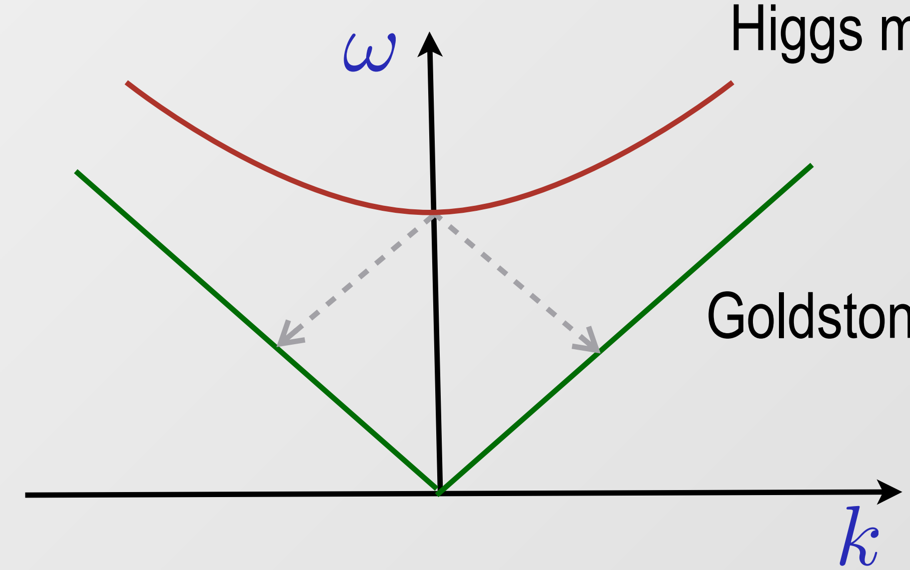


Courtesey: Danny Podolsky (Technion)

Lifetime of Higgs Excitation

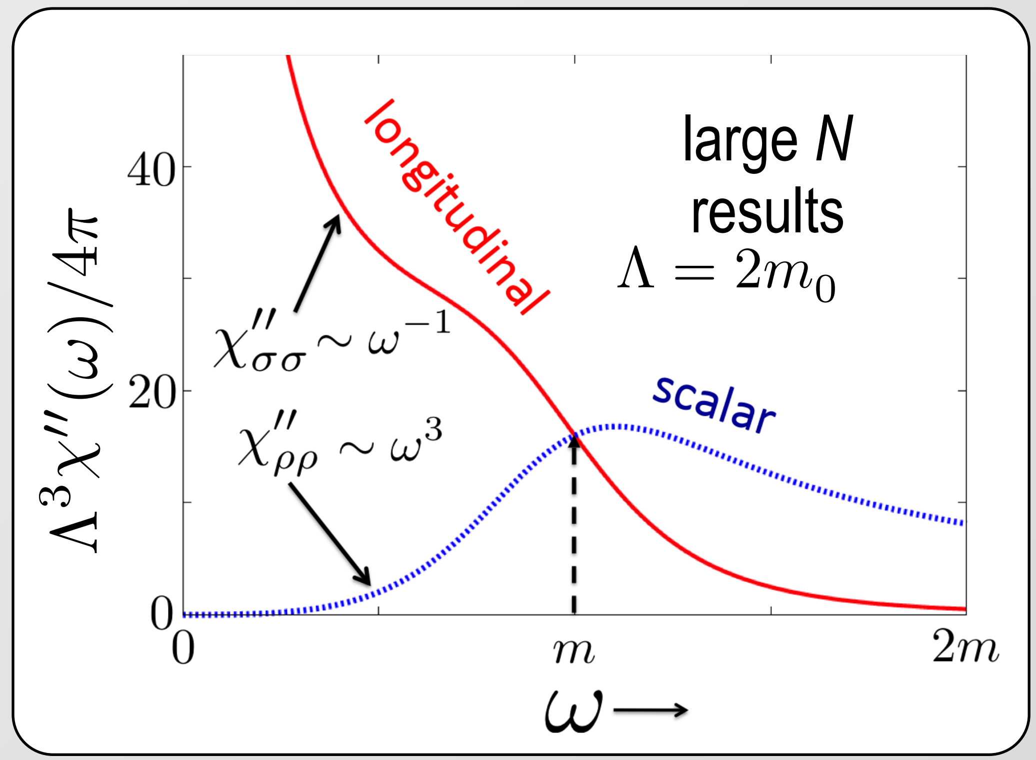
It can decay into a pair of Goldstone bosons :

$$\mathcal{L}_{\text{int}} \propto \begin{cases} \sigma \boldsymbol{\pi}^2 & \text{(Cartesian)} \\ \rho \left(\partial_\mu \hat{\boldsymbol{n}}\right)^2 & \text{(polar)} \end{cases}$$



Cartesian and polar calculations correspond to *different correlation functions.*

Depends on the type of experiment performed.





Courtesey: Danny Podolsky (Technion)

D. Podolsky, D., Auerbach, A. & Arovas, Phys. Rev. B 84, 174522 (2011)
L. Pollet, N. Prokof'ev, Phys. Rev. Lett. 109, 010401 (2012)
S. Gazit, D. Podolsky, A. Auerbach, Phys. Rev. Lett. 110, 140401 (2013)
D. Podolsky and S. Sachdev, Phys. Rev. B 86, 054508 (2012)

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Higgs Response Function

The longitudinal response function is measured by an experiment where the probe couples directly to the order parameter field:

$$S_{\mathrm{probe}} = \int d^d x \int dt \, \boldsymbol{h}(\boldsymbol{x},t) \cdot \boldsymbol{\phi}(\boldsymbol{x},t)$$

Example : neutron scattering in an antiferromagnet.

The scalar response function is measured by an experiment where the probe couples directly to the magnitude of the order parameter field:

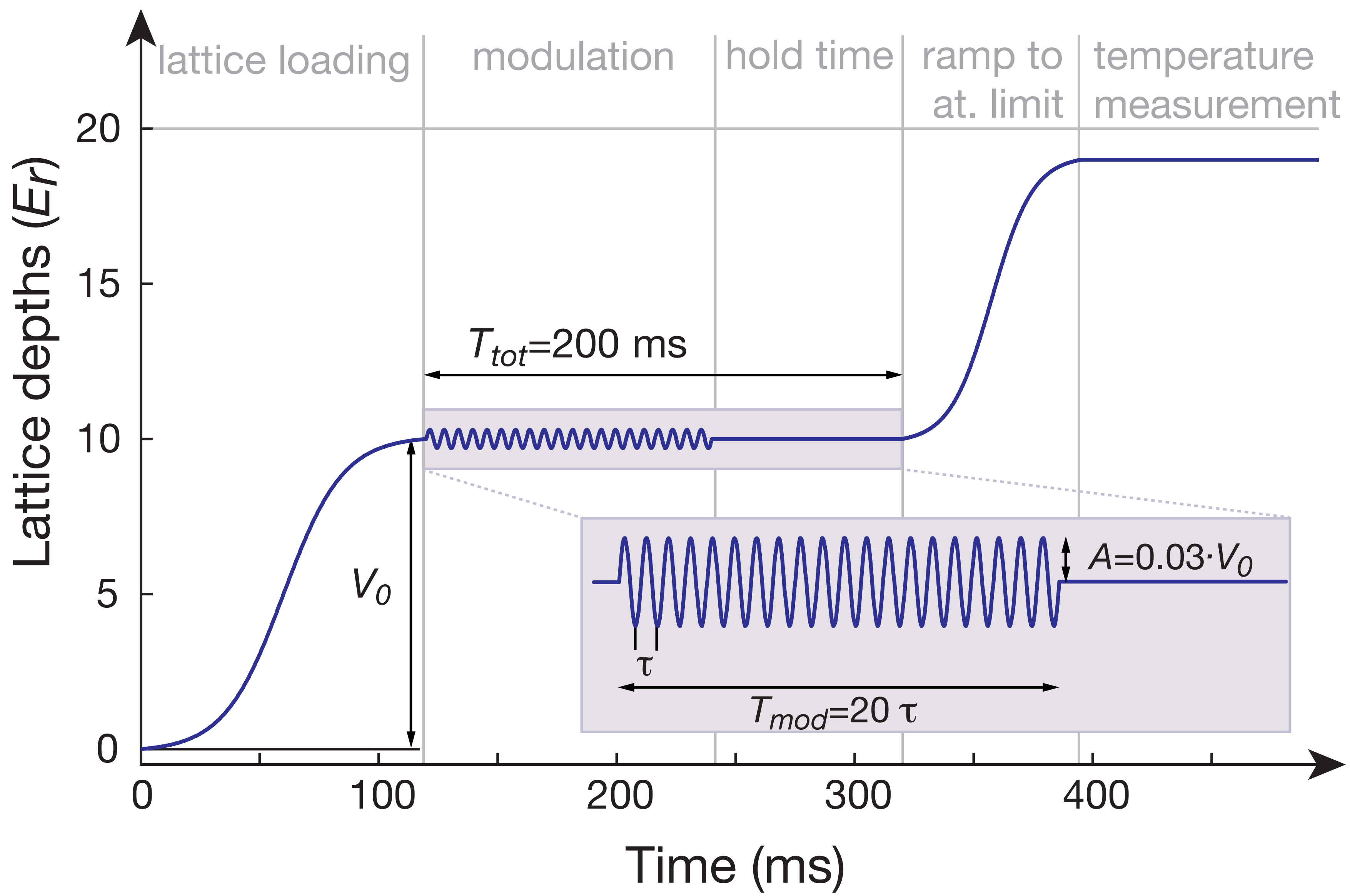
$$S_{\mathrm{probe}} = \int d^d x \int dt \, u(\boldsymbol{x},t) \left| \boldsymbol{\phi}(\boldsymbol{x},t) \right|^2$$

$$|\phi|^2 = N(1+\rho)$$

Examples : lattice modulation spectroscopy

Courtesey: Danny Podolsky (Technion)

Exciting the Amplitude Mode



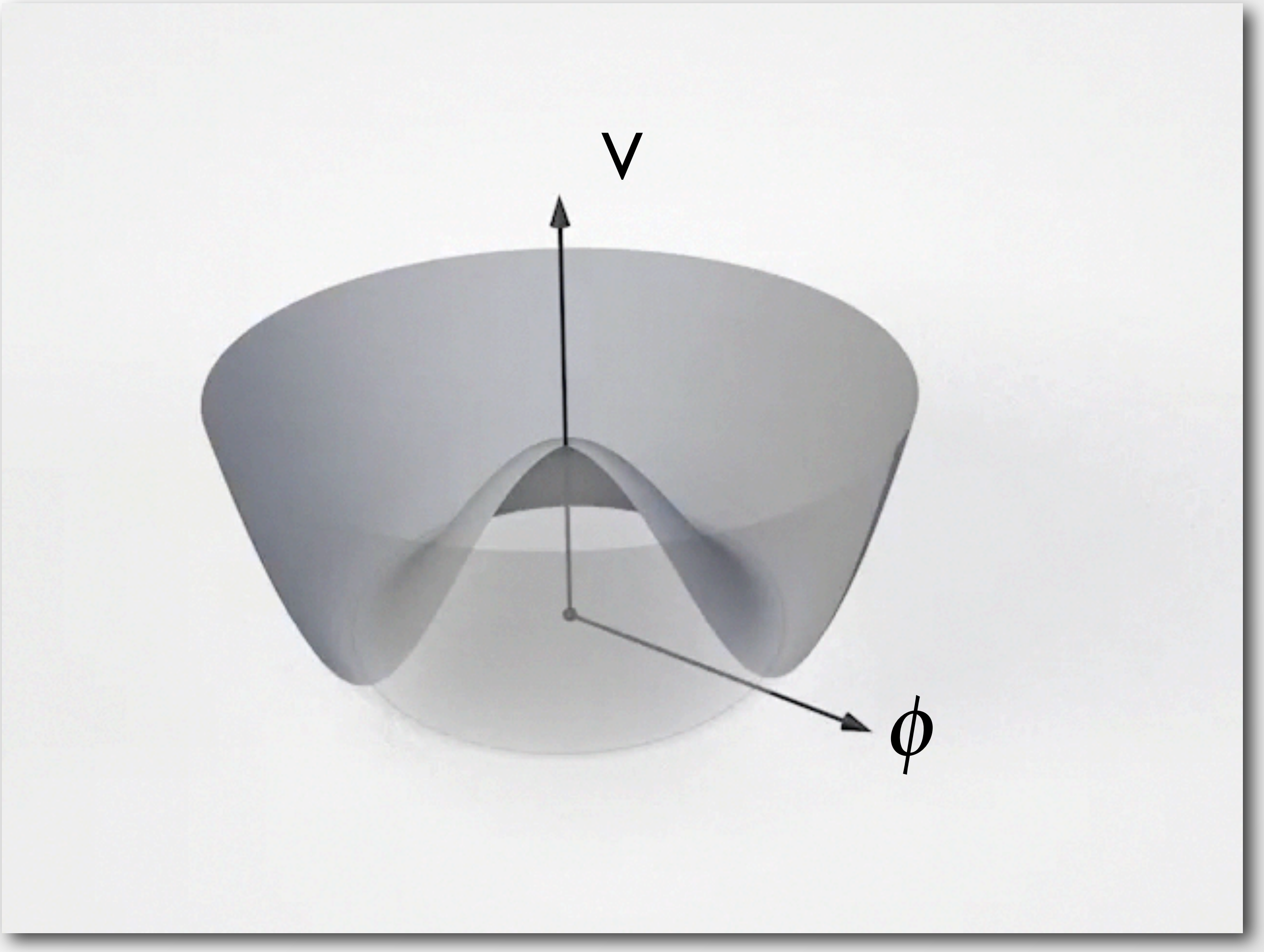
Absorbed energy

$$E = 2\pi(\delta J)^2 S(\omega)\omega T_{\mathrm{mod}}$$

Very low modulation amplitude!

Very sensitive temperature measurement!

Exciting the Amplitude Mode



Modulate coupling strength close to Quantum Phase Transition!

$$j = j + \delta j \sin(\omega t)$$

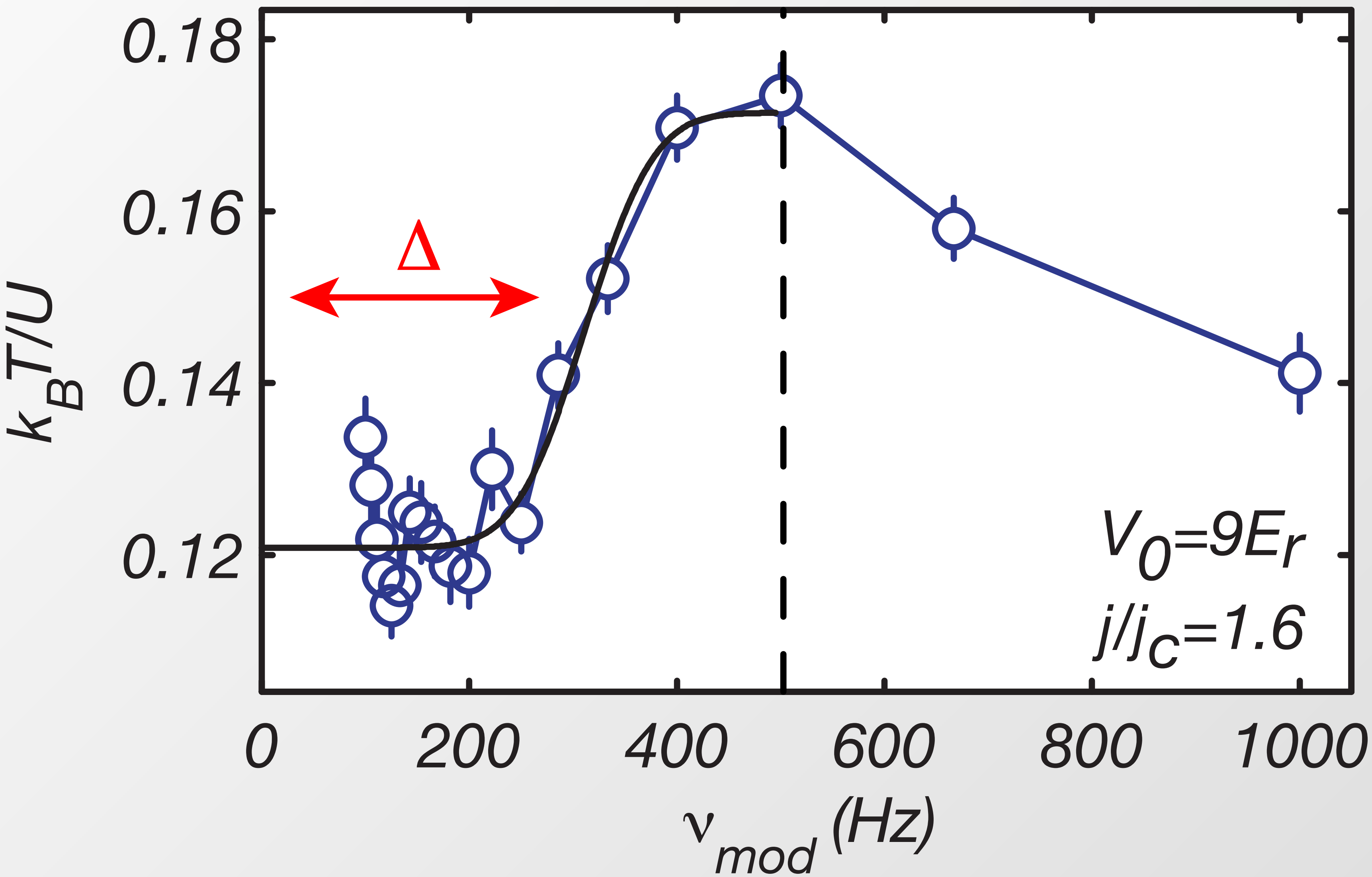
$$j = J/U$$

Amplitude Modulation of Lattice

Bragg spectroscopy: couples mainly to phonons

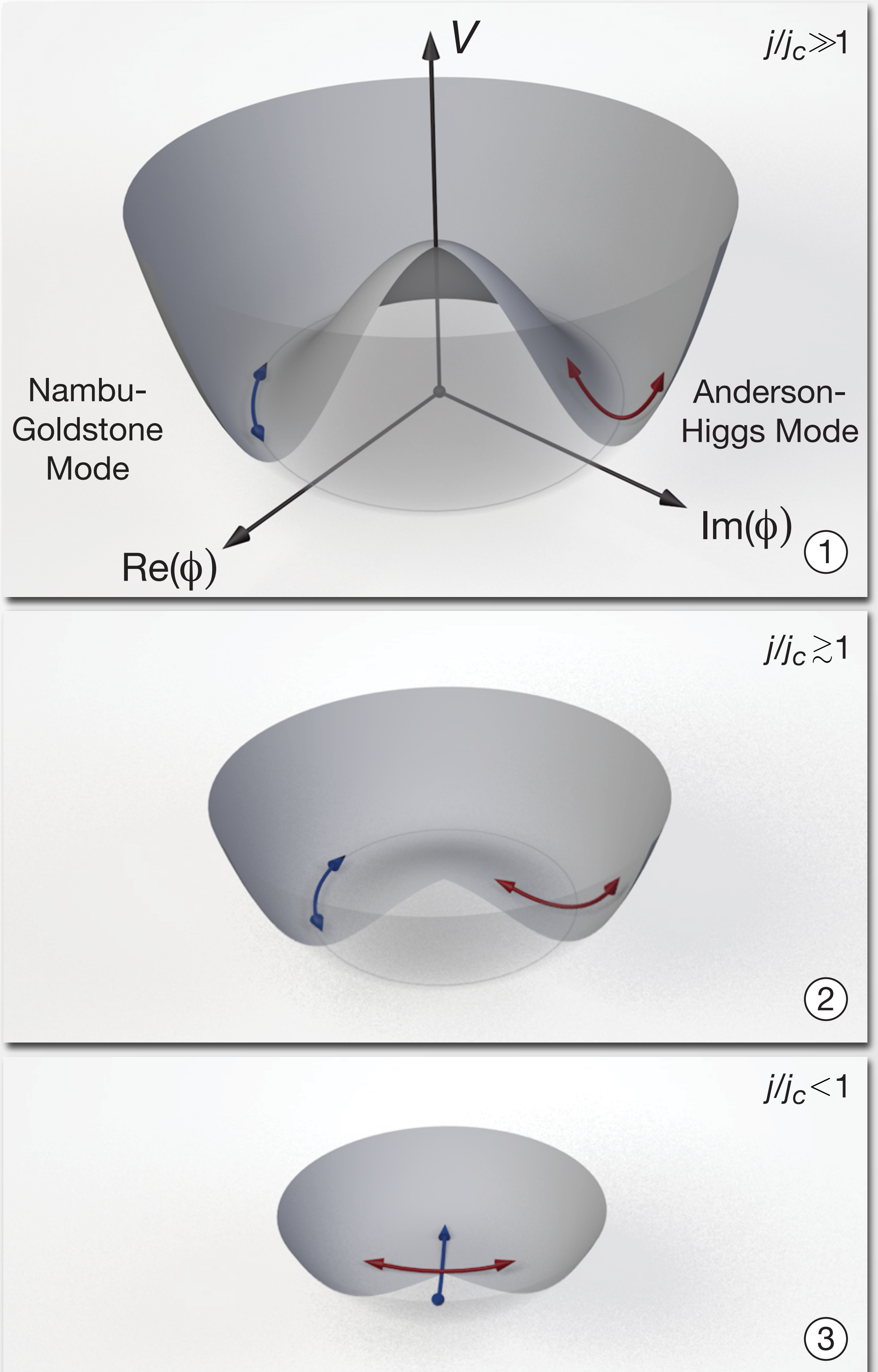
Exp.: Ch. Schori et al. Phys. Rev. Lett. (2004) (ETHZ), Theory: C. Kollath et al., Phys. Rev. Lett (2006)
U. Bissbort et al. Phys. Rev. Lett. (2011) (Frankfurt, Hamburg)

Single Spectrum



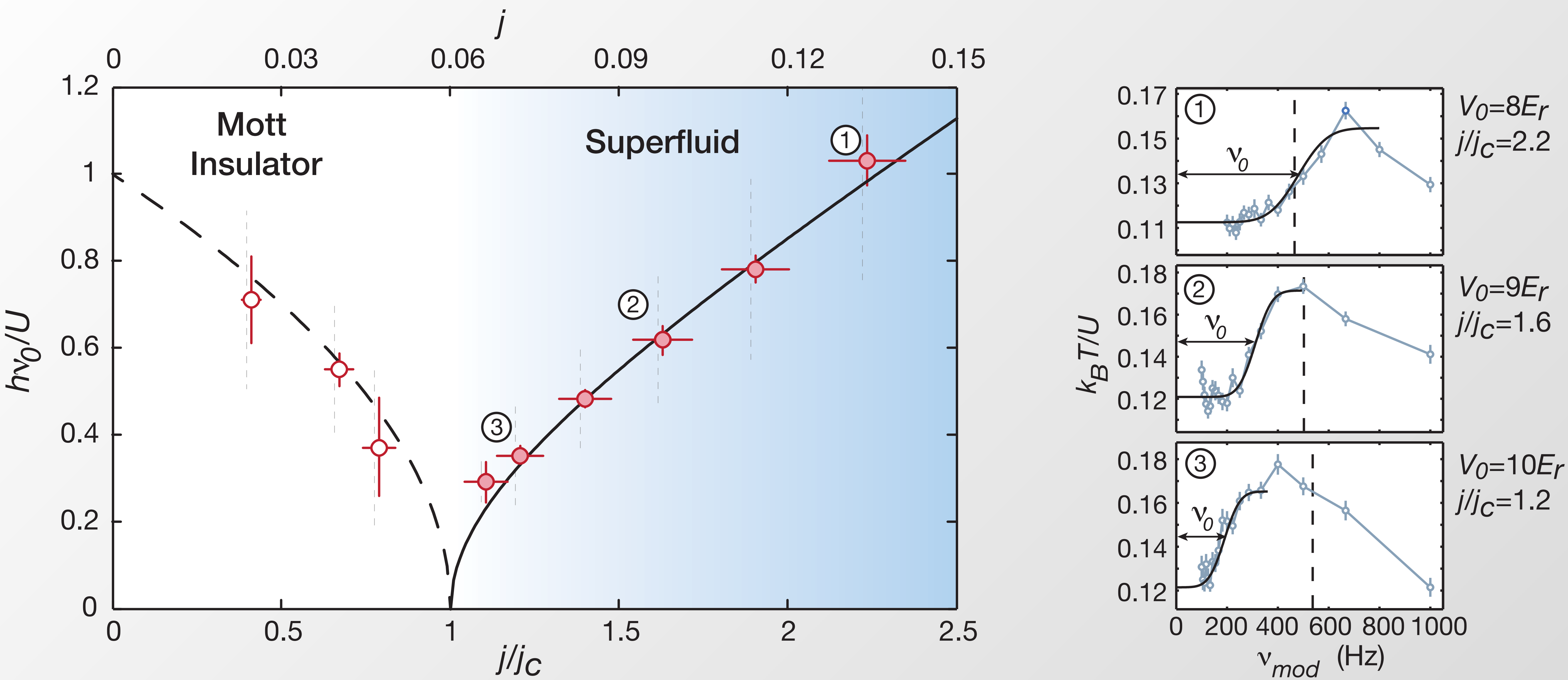
Use fit with error function to find minimum excitation frequency!
(also avoids inhomogeneous trap effects)

Evolution Across Critical Point



Higgs mode softens towards critical point!

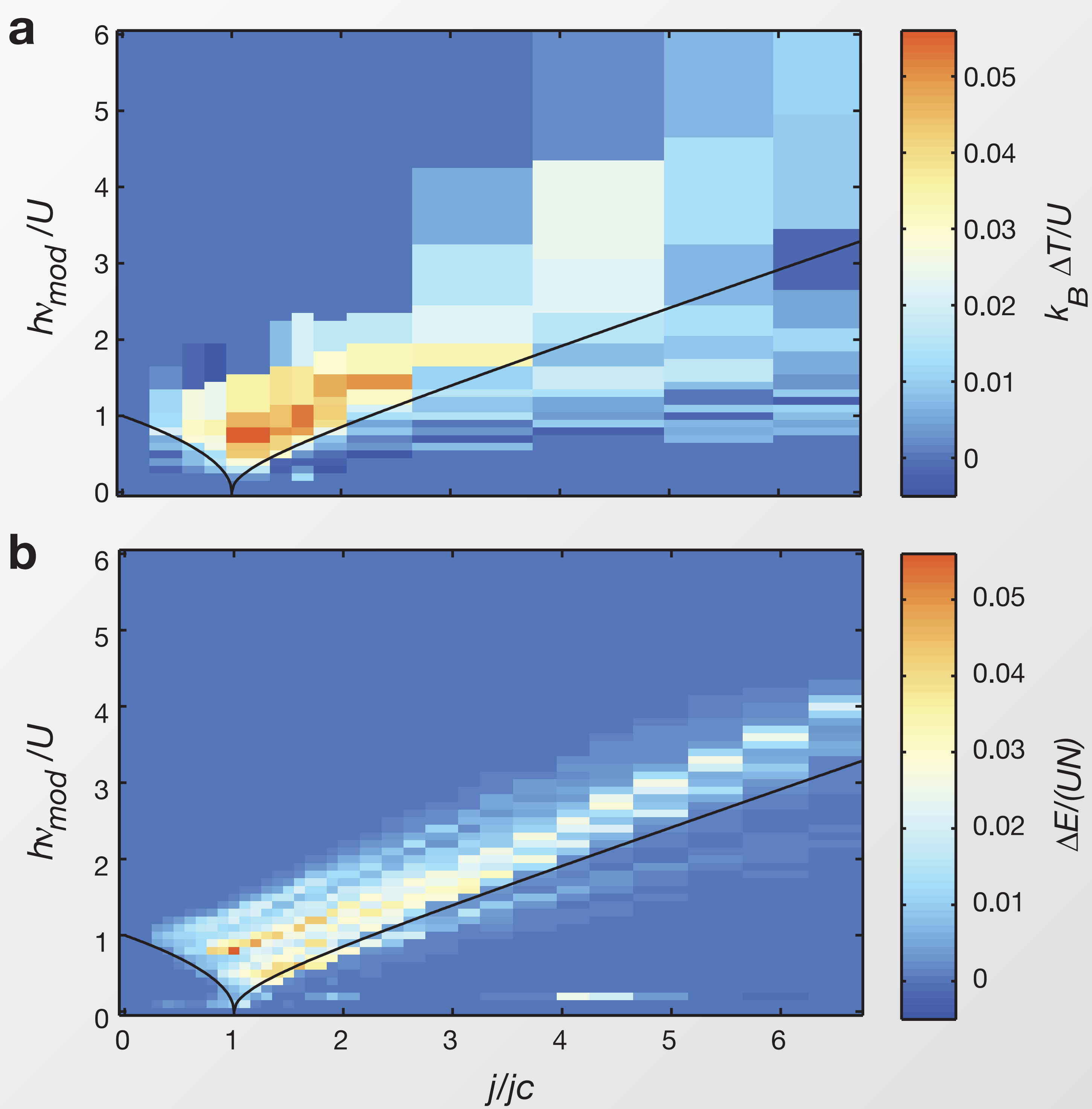
Measuring Across the QCP



'Higgs' mode softens towards critical point and turns into opening gap of Mott Insulator!

Theory in SF (S. Huber et al. PRB 2007) $\Delta_m = \sqrt{3\sqrt{2} - 4}\sqrt{(j/j_c)^2 - 1}$

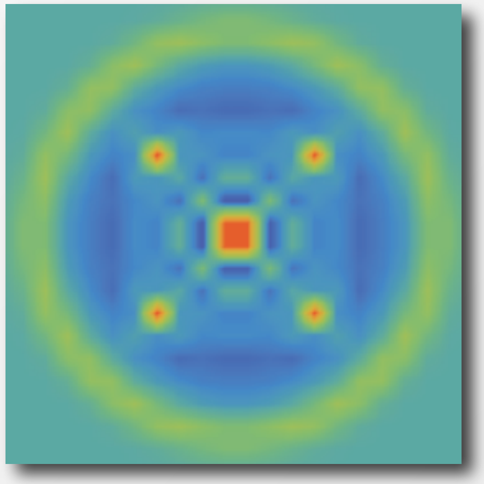
Full Spectral Response



Open theory question: what is the fate of Higgs mode towards weaker interactions?

Outlook

✓ Selectively excite Higgs eigenmodes (larger system, spatial modulation)
✓ Probe Quantum Critical behaviour via Dynamical critical scaling



Higgs drum, spatial eigenmodes!

✓ Fate of mode at weaker interactions (towards GPE)
✓ Ratio of 'Higgs' mass to Mott gap
✓ Well defined mode down to critical point?
✓ Anderson-Higgs Mechanism via Coupling to (Dynamical) Gauge Field

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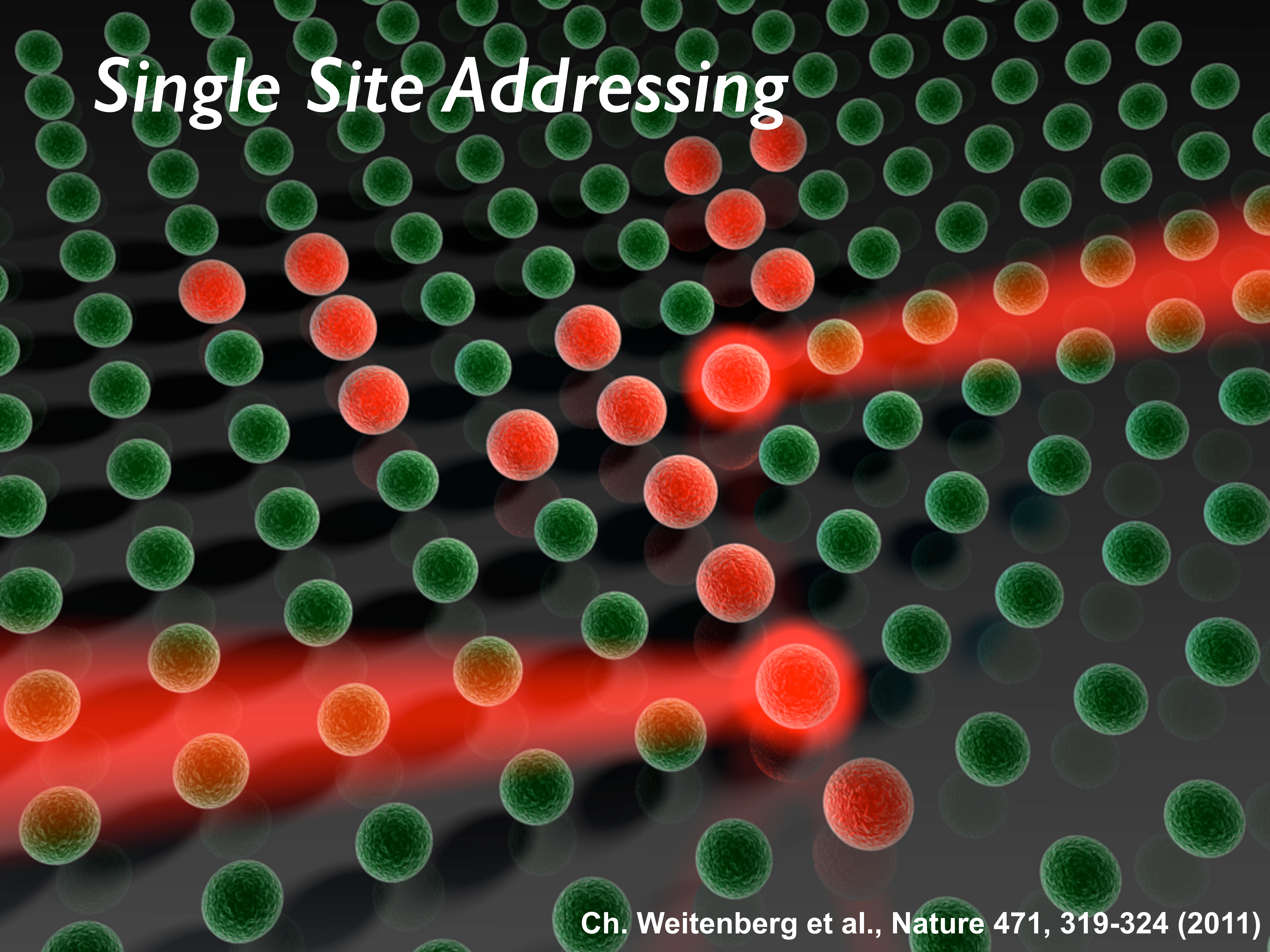
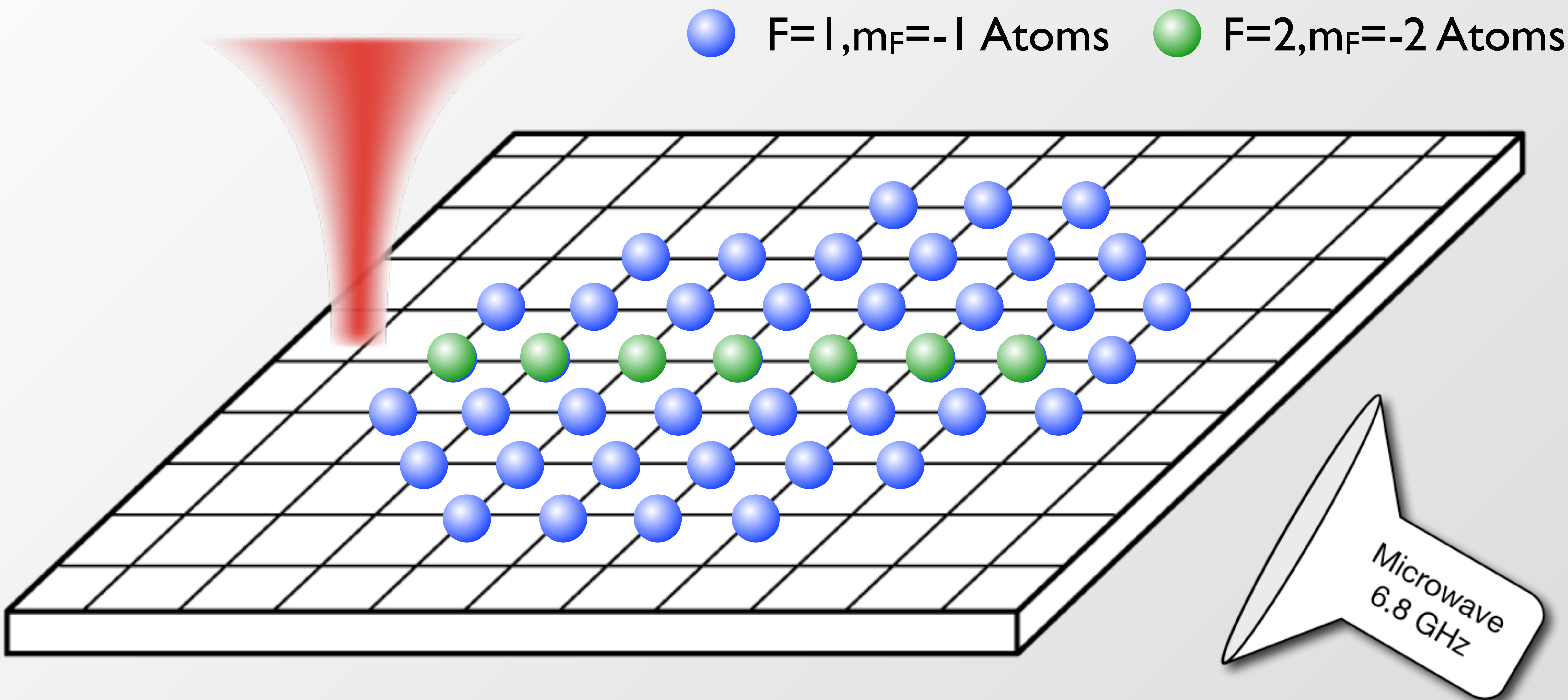
Single Site Addressing



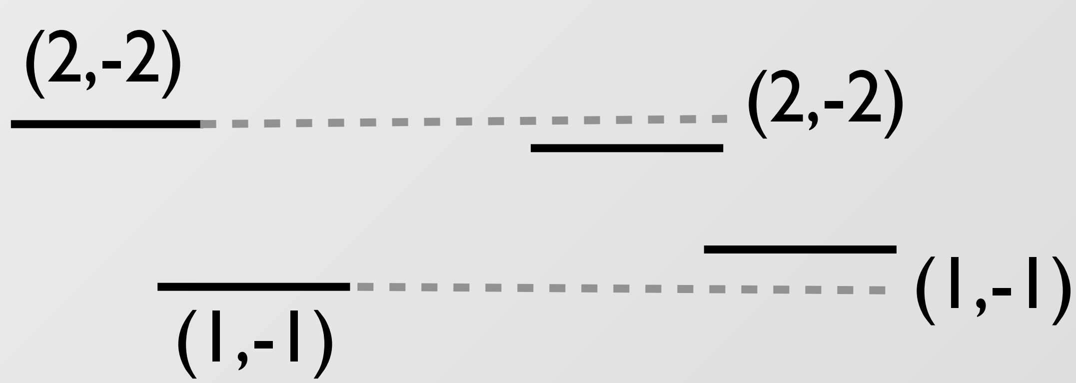
Ch. Weitenberg et al., Nature 471, 319-324 (2011)

Coherent Addressing of Atoms



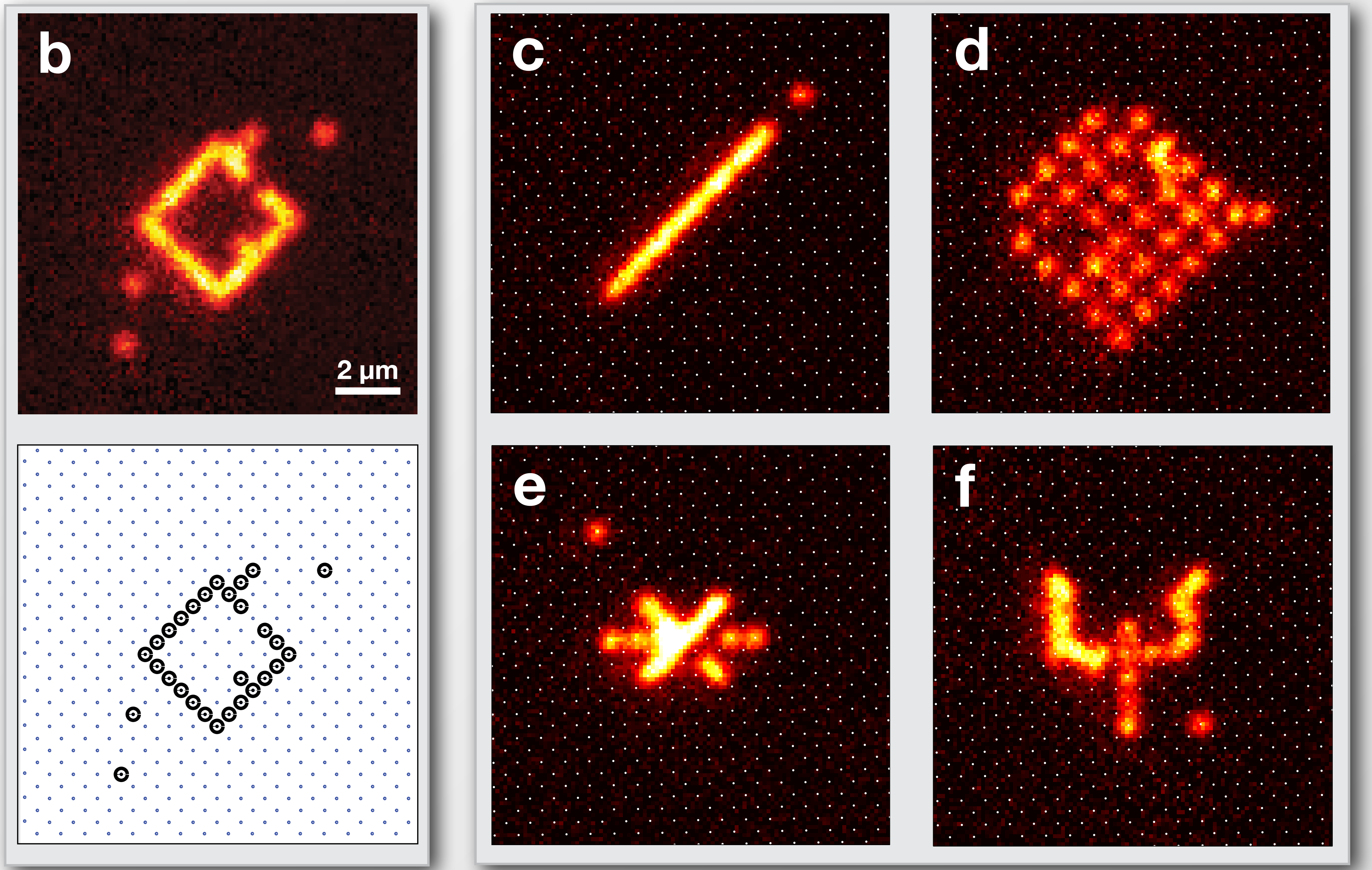
Differential light shift allows to coherently address single atoms!

Landau-Zener Microwave sweep to coherently convert atoms between spin-states.



D.S.Weiss et al., PRA (2004),
Zhang *et al.*, PRA (2006)

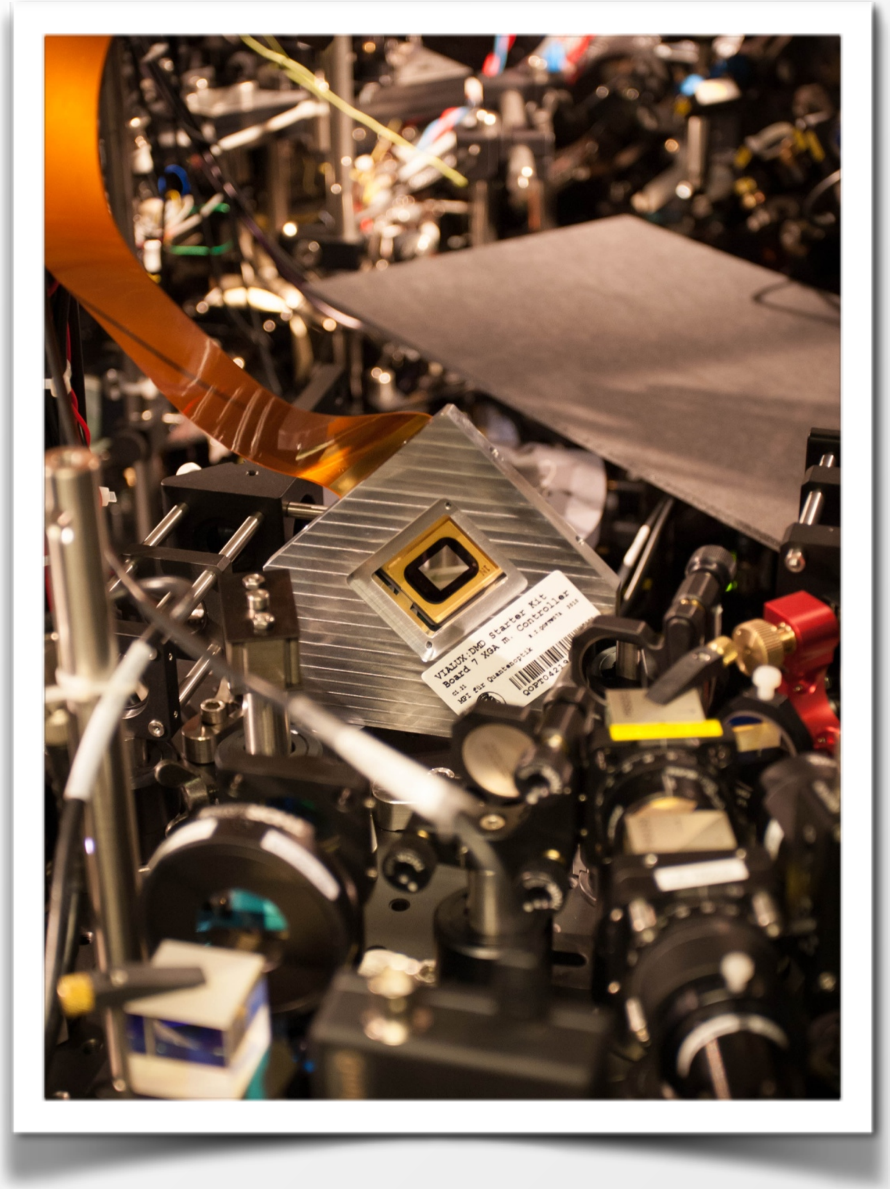
Coherent Spin Flips - Positive Imaging



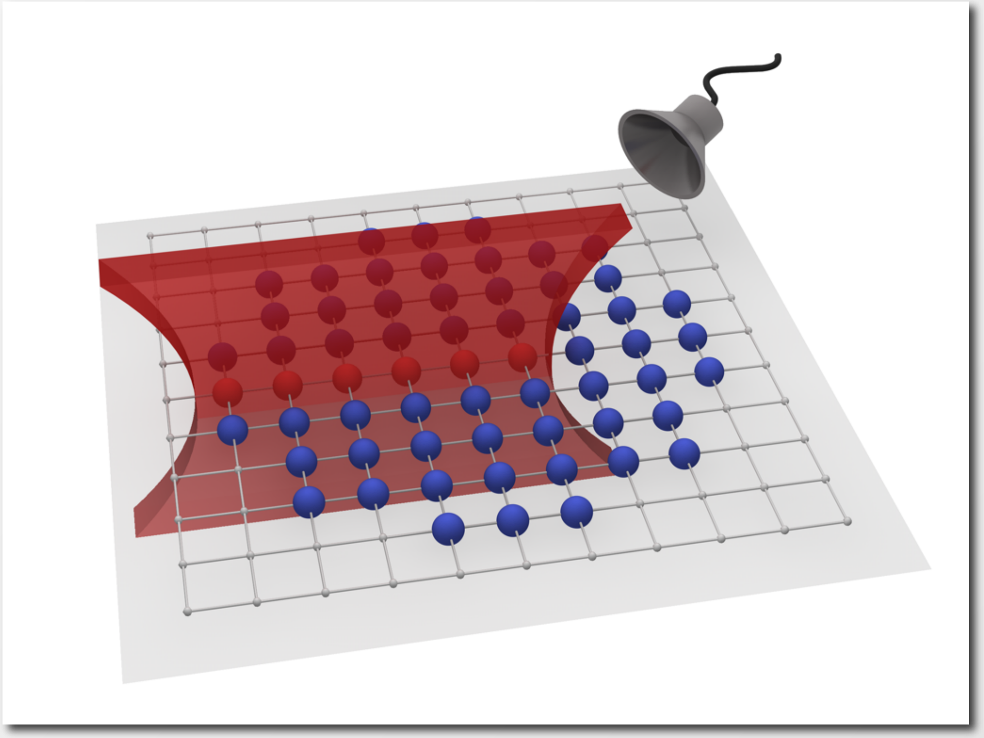
Subwavelength spatial resolution: 50 nm

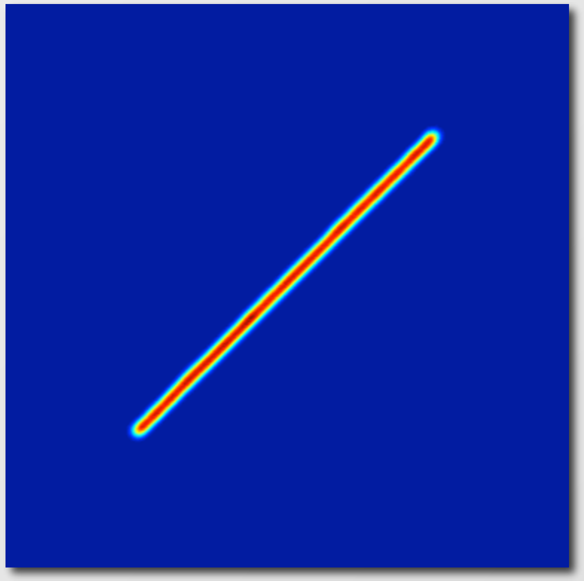
Ch. Weitenberg et al., Nature **471**, 319-324 (2011)

Arbitrary Light Patterns

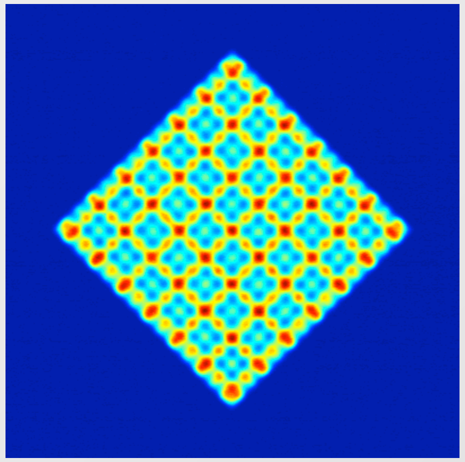


Digital Mirror Device (DMD)

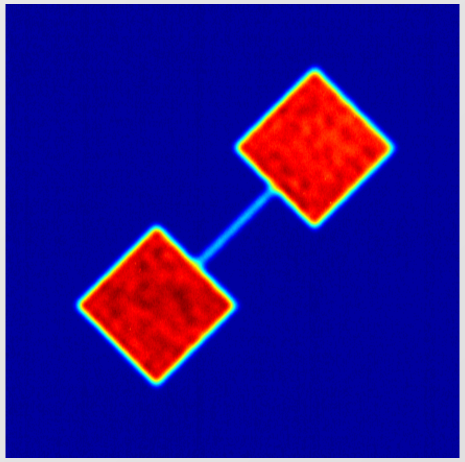




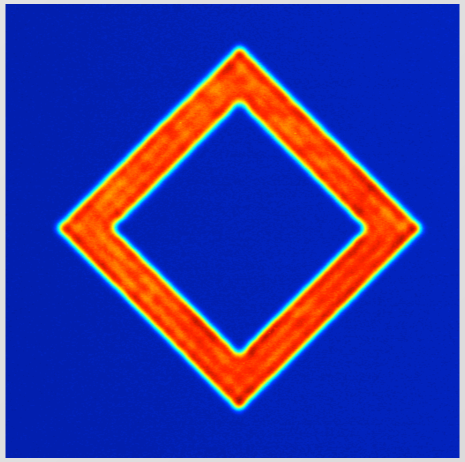
Measured Light Pattern



Exotic Lattices



Quantum Wires

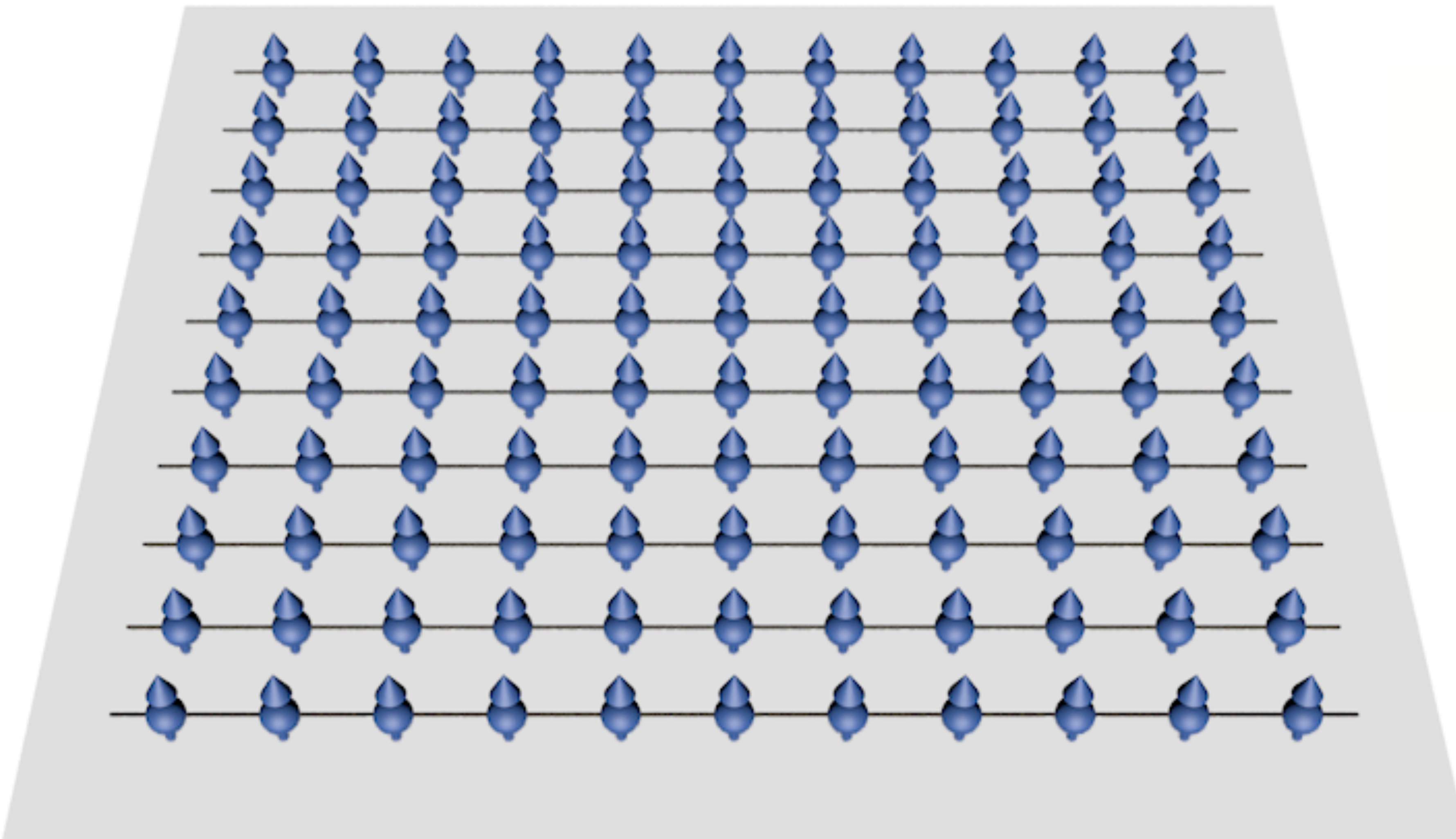


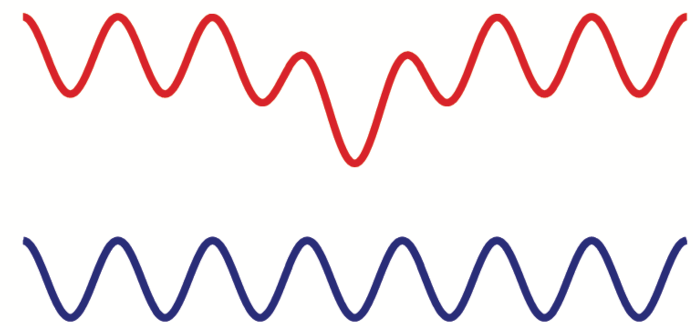
Box Potentials

Almost Arbitrary Light Patterns Possible!

Single Spin Impurity Dynamics, Domain Walls, Quantum Wires, Novel Exotic Lattice Geometries, ...

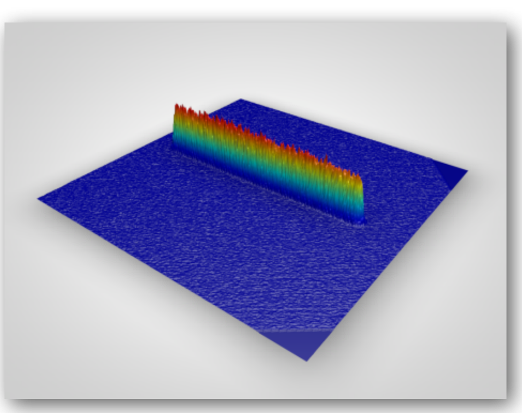
Spin impurity dynamics





$|2\rangle = |F=2, m_F=-2\rangle$

$|1\rangle = |F=1, m_F=-1\rangle$



Line-shaped light field created with DMD SLM

T. Fukuhara et al., Nature Physics **9**, 235 (2013)

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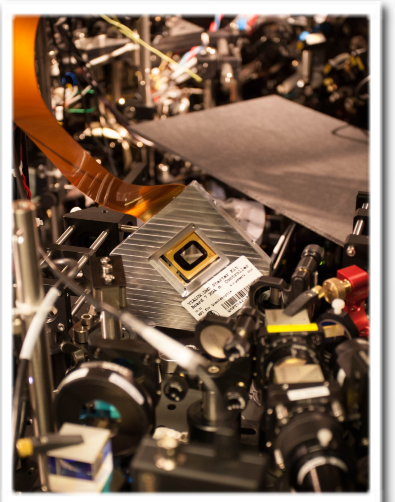
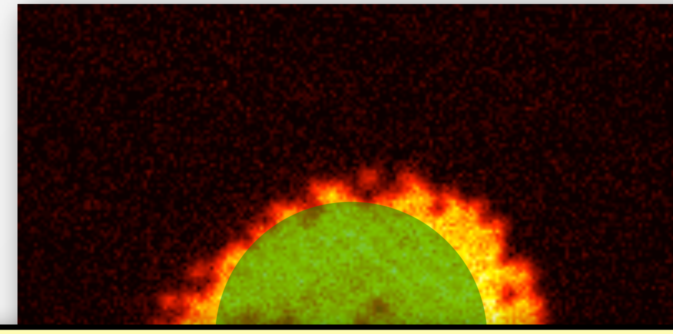
Ultimate Size Control in 2D

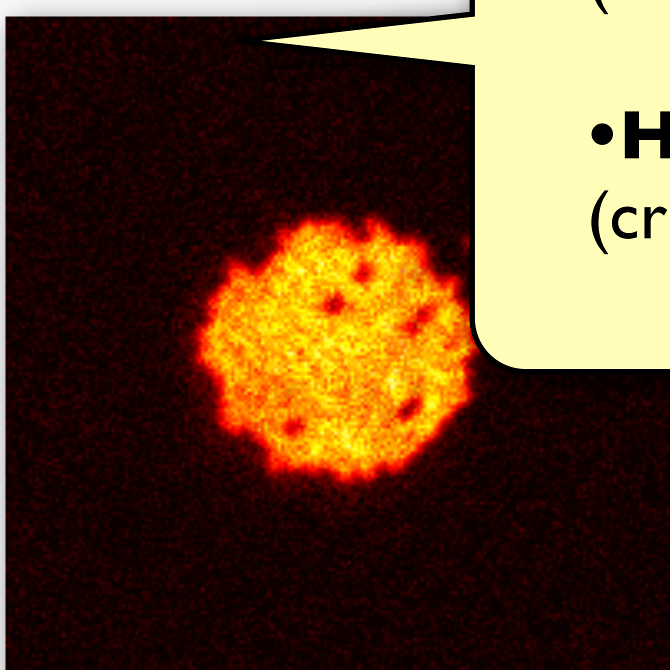


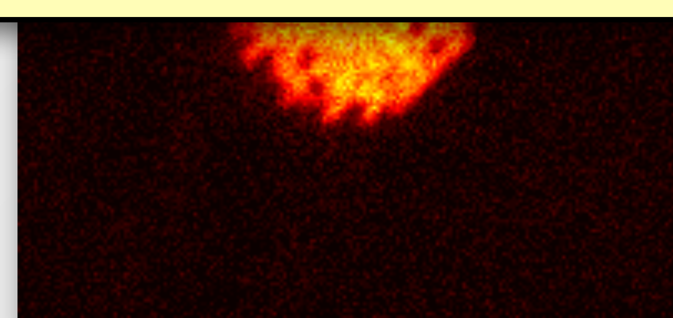
Digital Mirror Device (Size Control)

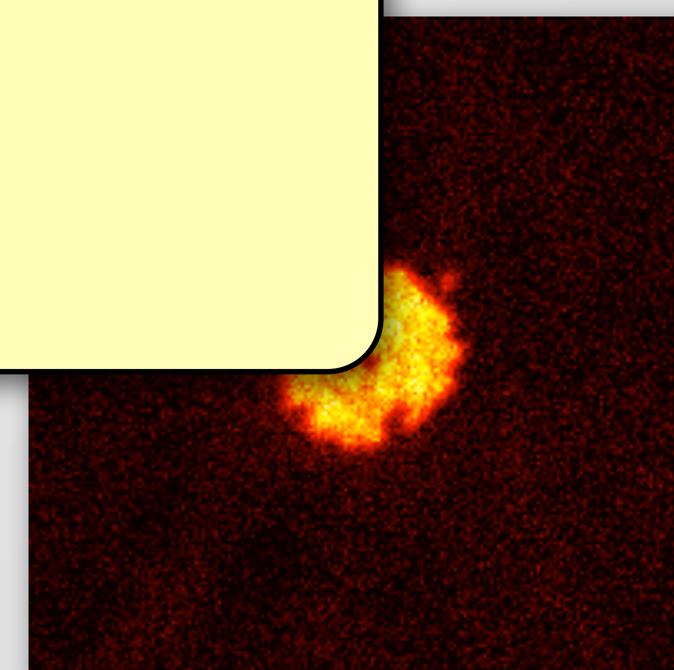


Fluctuating Size and

- **Sub Shot Noise Atom Number Preparation**
- **Geometric & atom number control**
(crucial e.g. for quantum criticality)
- **Hard wall potentials realized**
(crucial for edge states)

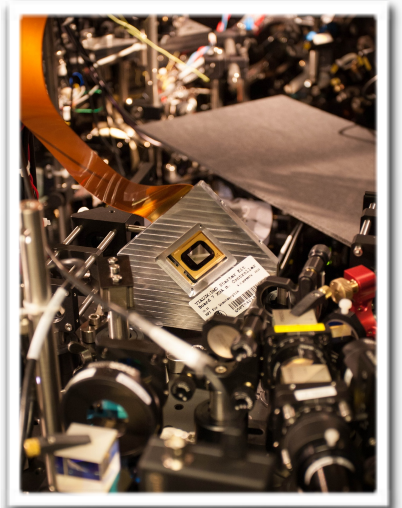




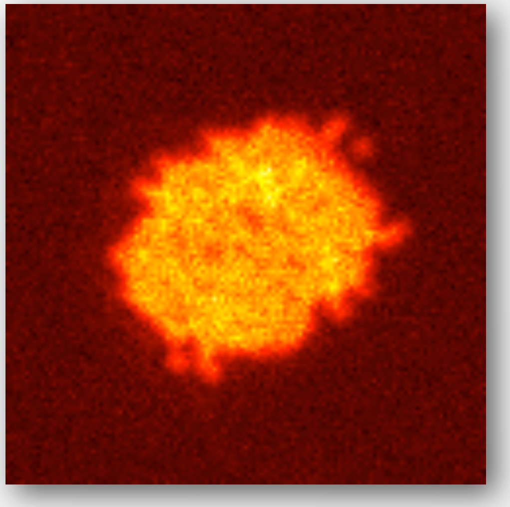


Size & atom number perfectly controlled

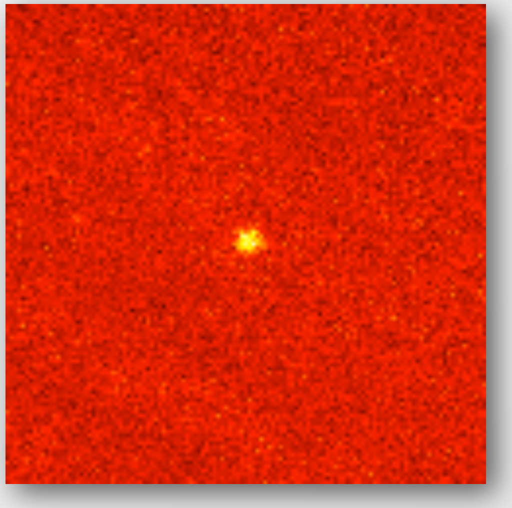
Ultimate Size Control in 2D



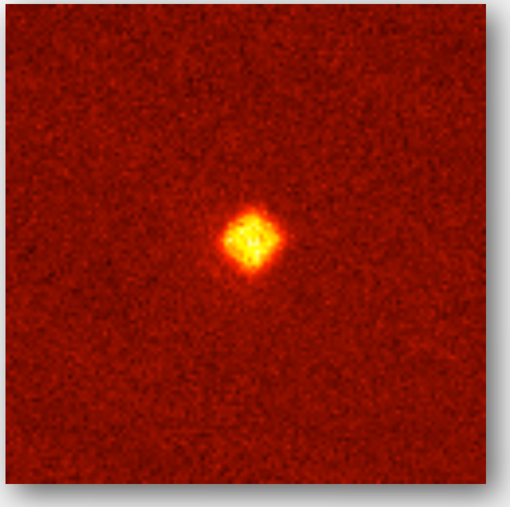
Digital Mirror
Device (Size Control)



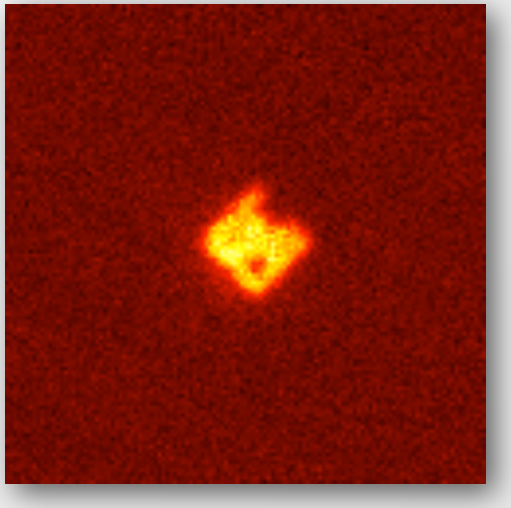
Initial MI



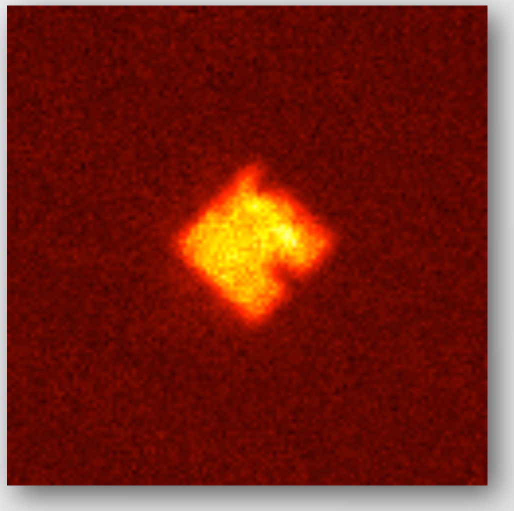
Single Atom



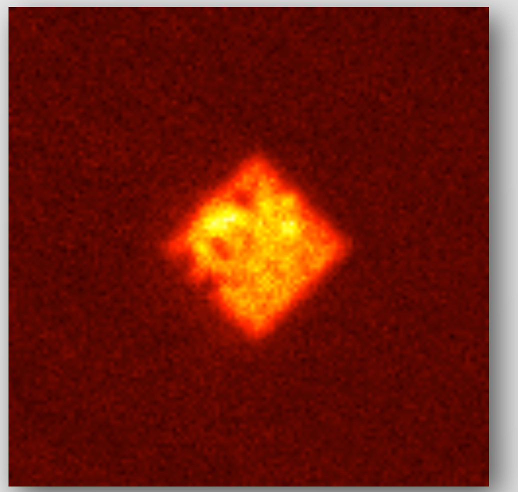
3x3



5x5



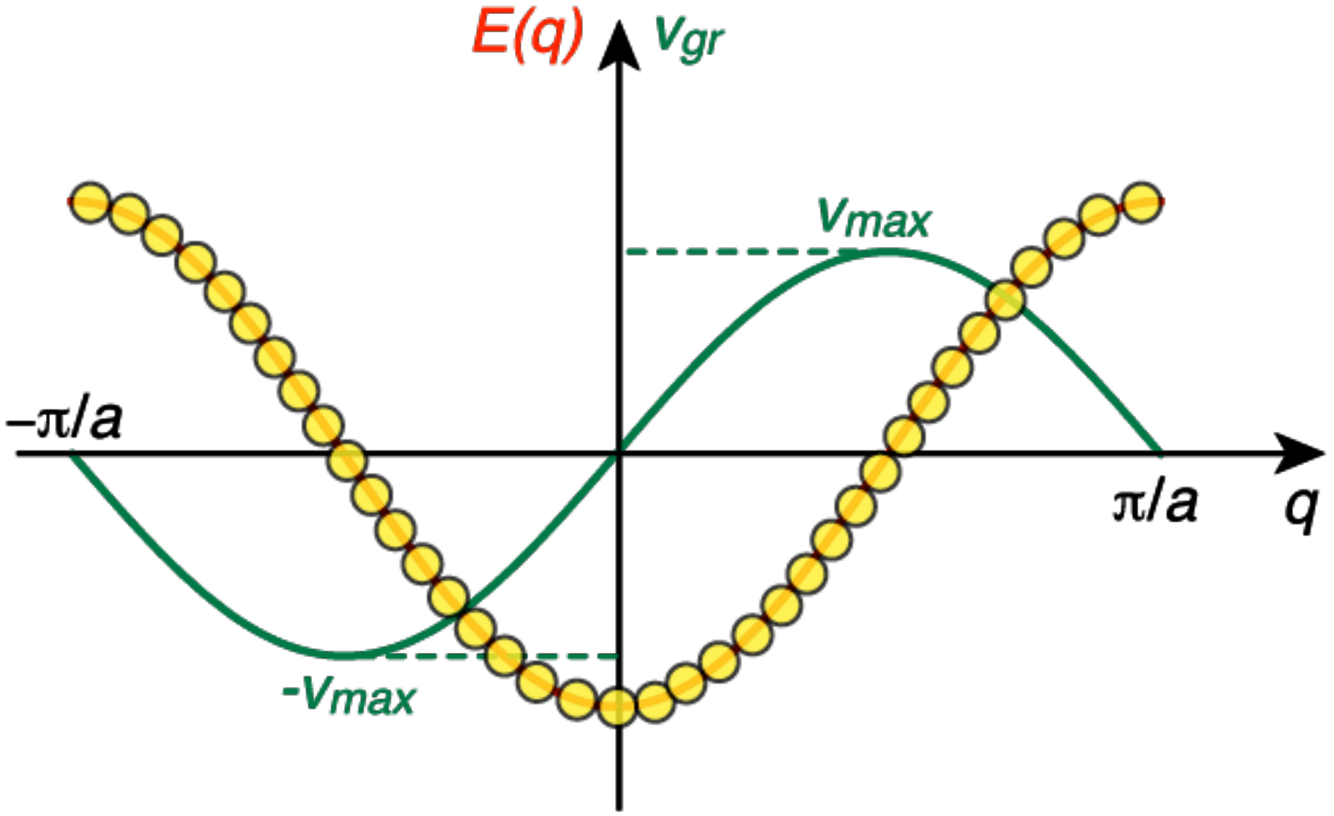
7x7



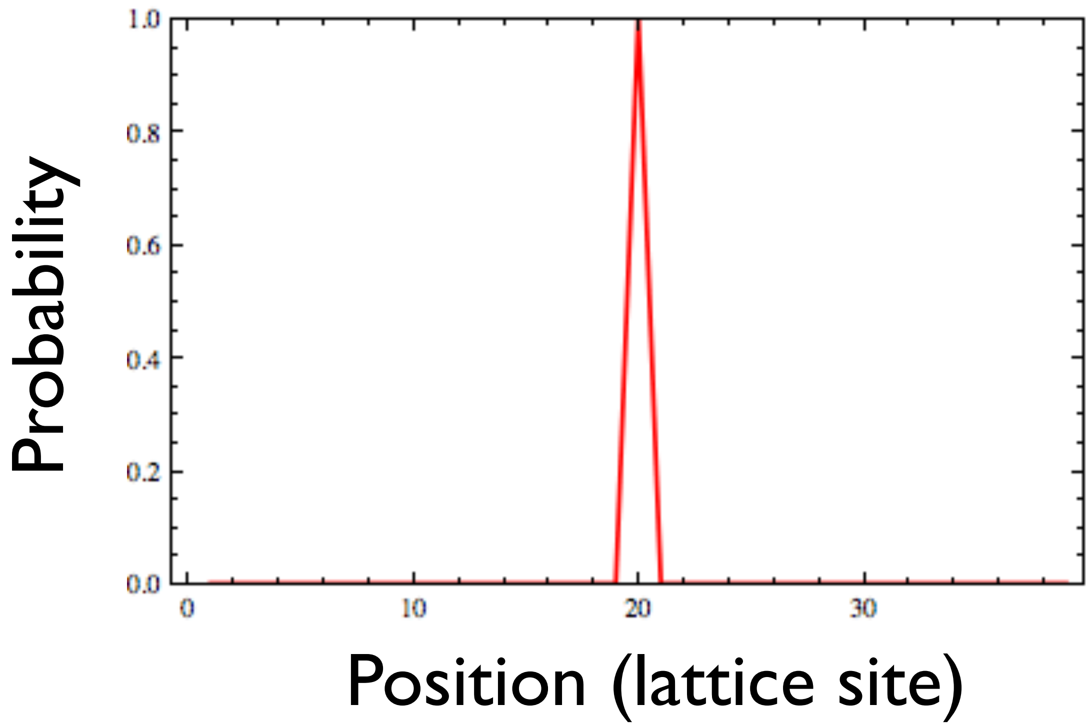
8x8

atoms

Tunneling of a Single Atom

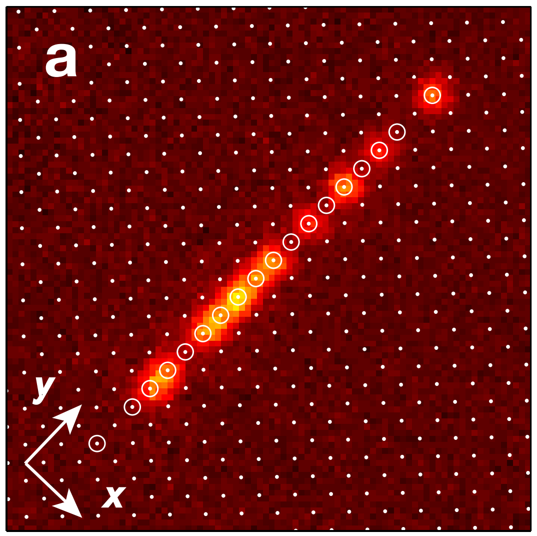


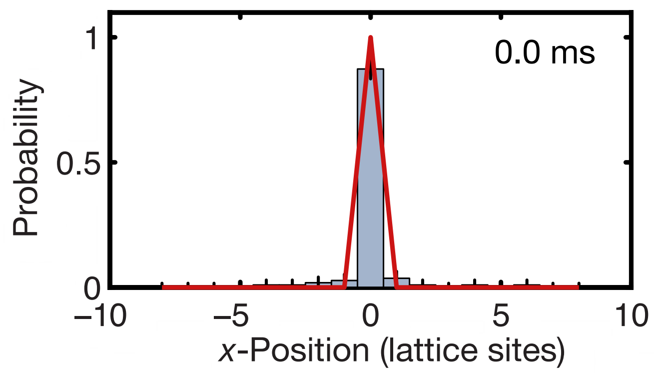
$$v_{max} = \frac{2Ja}{\hbar}$$



$$H = -J^{(0)} \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \frac{1}{2} m\omega^2 a_{\mathrm{lat}}^2 i^2 \hat{n}_i$$

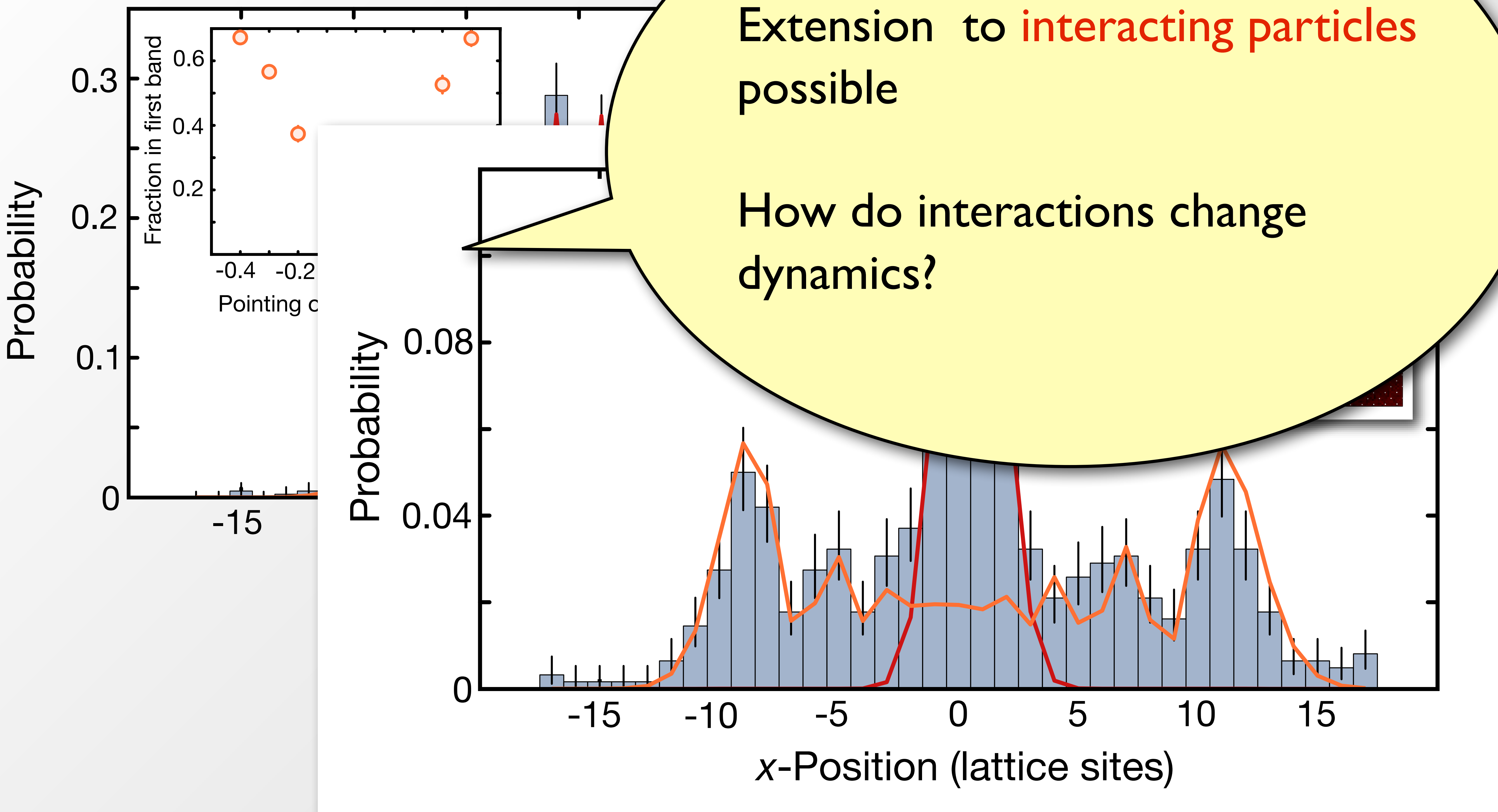
Motional State Affected?





see exp: Y. Silberberg (photonic waveguides), D. Meschede & R. Blatt (quantum walks)...

nneling



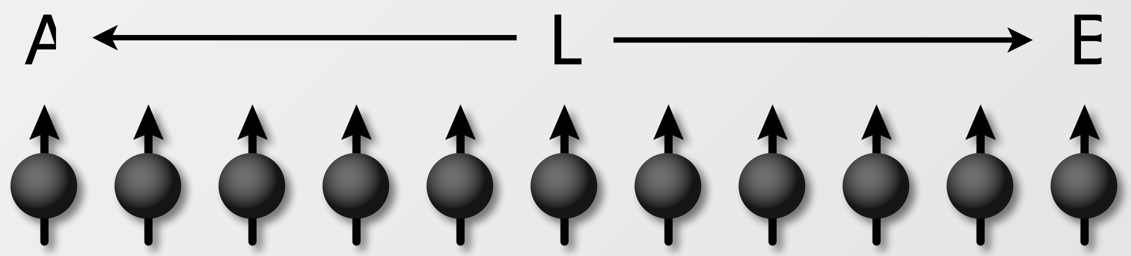
Extension to interacting particles possible

How do interactions change dynamics?

Excellent agreement with simulation.

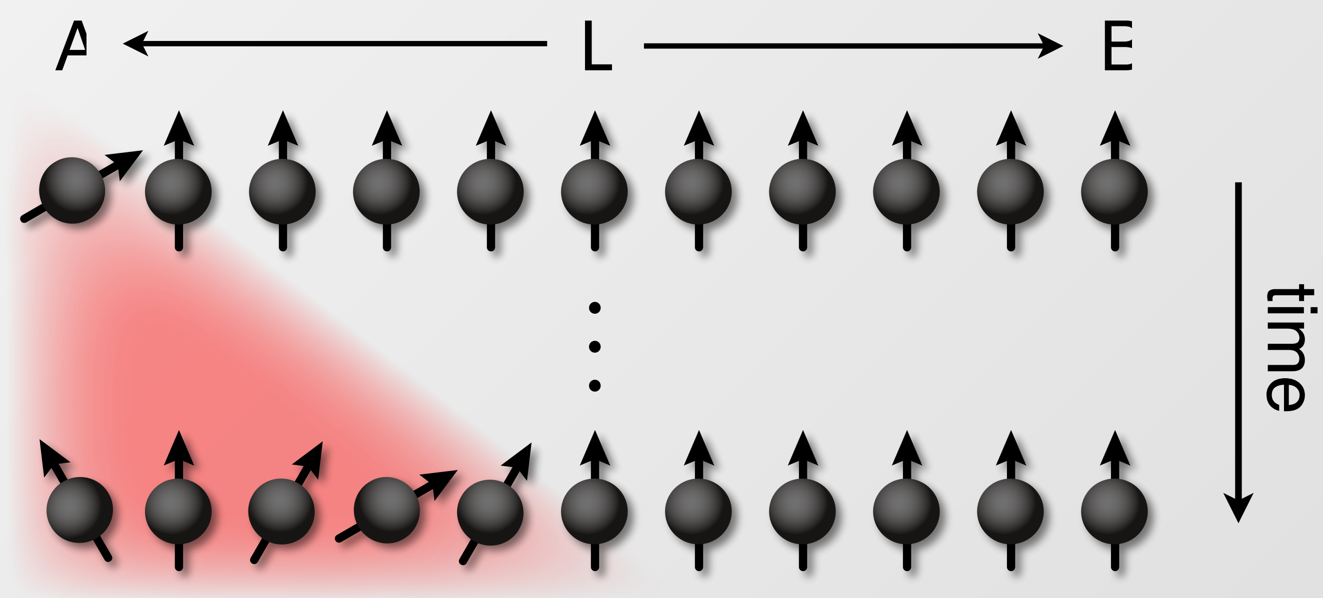
Lieb-Robinson bounds

Spin chain
short-range
interactions



Lieb-Robinson bounds

Spin chain
short-range
interactions

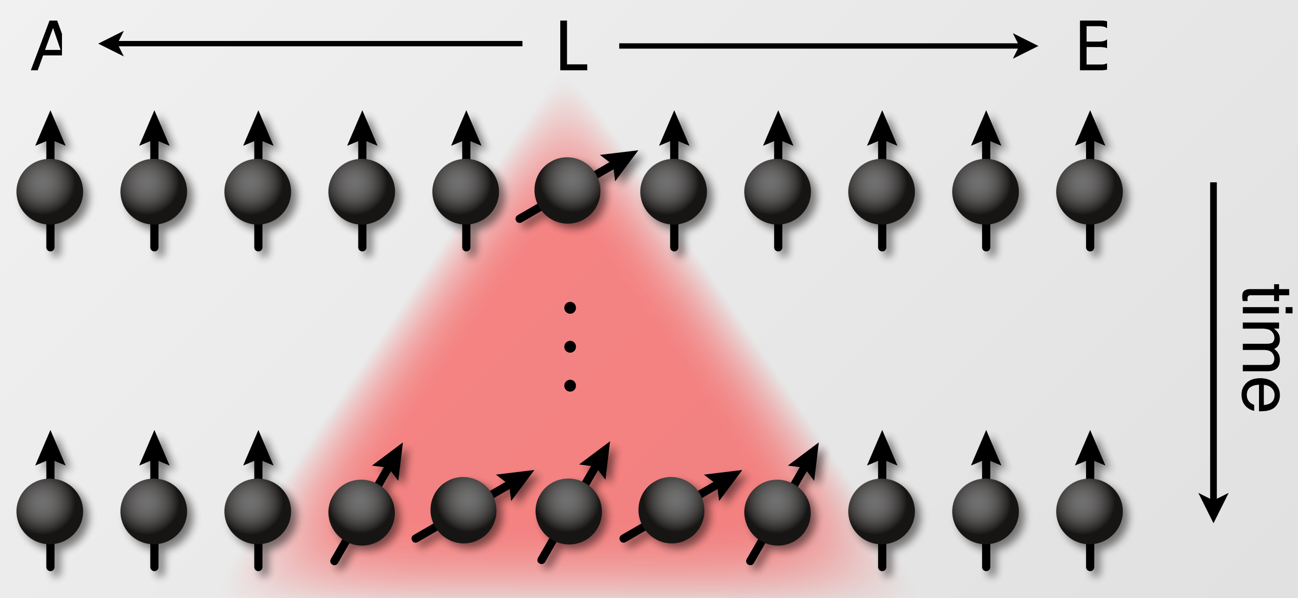


Lieb and Robinson (1972)

$$\left|[A, B(t)]\right| \leq \lambda \exp\left(\frac{vt - L}{\zeta}\right)$$

Lieb-Robinson bounds

Spin chain
short-range
interactions



Bravyi, Hastings and Verstraete (2006)
Calabrese and Cardy (2006)
Eisert and Osborne (2006)
Nachtergaele, Ogata and Sims (2006)
... *and many others since then*

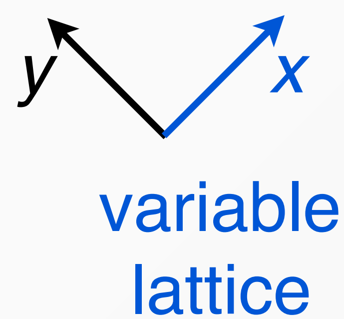
$$\left|\langle A(t)B(t)\rangle - \langle A(t)\rangle\langle B(t)\rangle\right| \leq \lambda' \exp\left(\frac{vt - L/2}{\zeta'}\right)$$

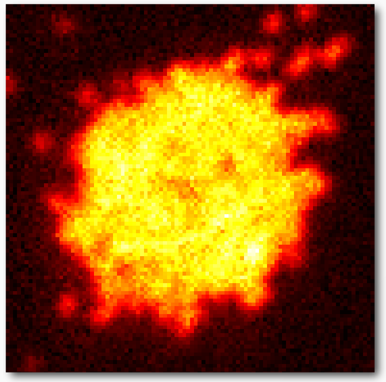
the propagation of correlations is bounded by an effective light cone

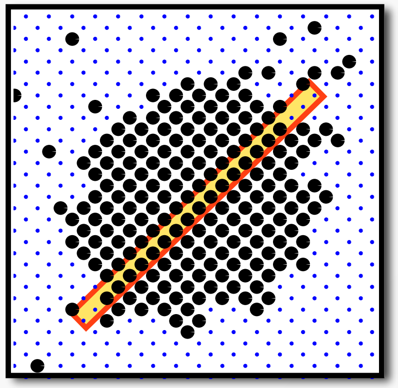
1D Mott insulator out of equilibrium

1. Prepare 1D Mott insulator with U/J ≫ 1

deep lattice (20 E_r)



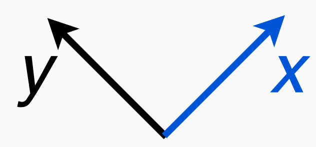




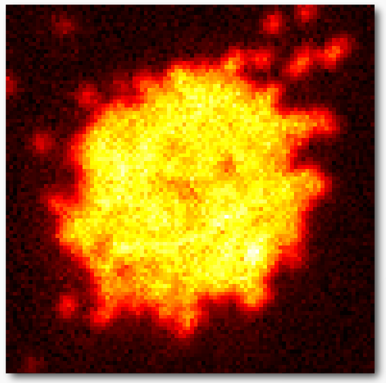
1D Mott insulator out of equilibrium

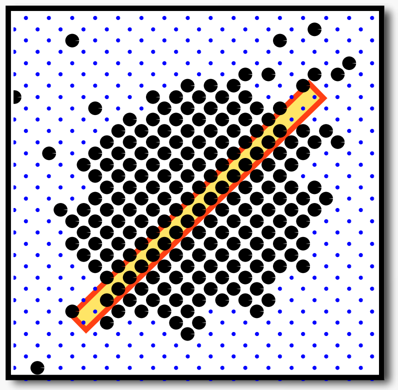
1. Prepare 1D Mott insulator with U/J ≫ 1

deep lattice (20 E_r)

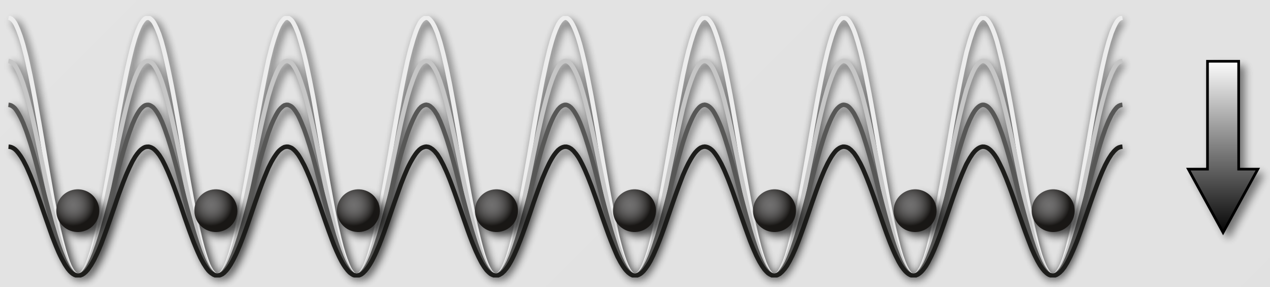


variable lattice





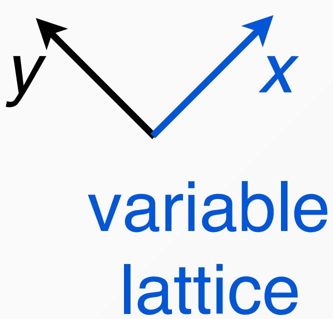
2. Lower U/J abruptly

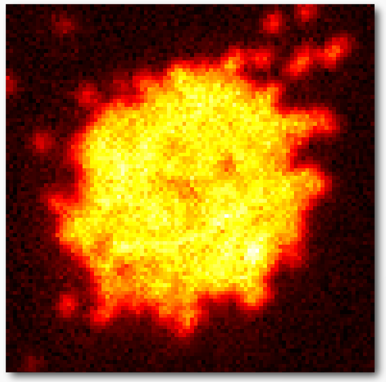


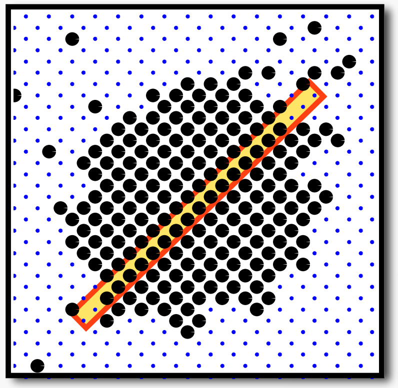
1D Mott insulator out of equilibrium

1. Prepare 1D Mott insulator with U/J ≫ 1

deep lattice (20 E_r)







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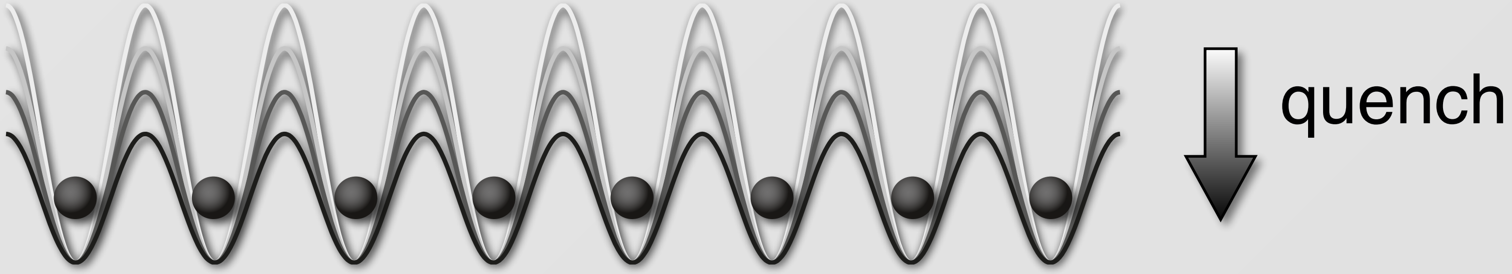


3. Record the dynamics

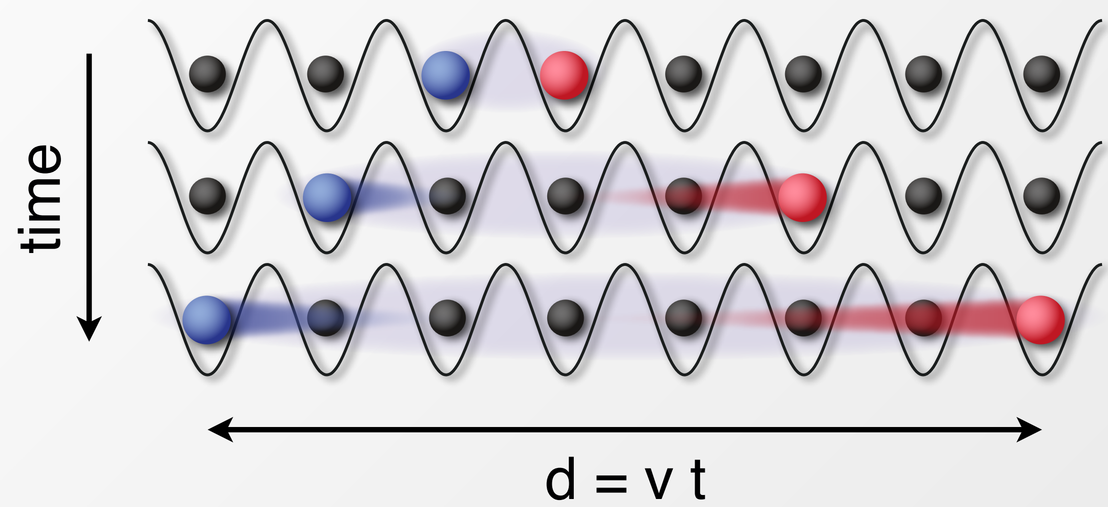
The initial state is highly excited.

Calabrese and Cardy (2006)

Quasiparticles are emitted and propagate ballistically, carrying correlations across the system.

Light-cone like spreading of correlations

- Quasiparticle dynamics



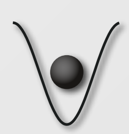
- Two-point parity correlation function

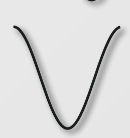
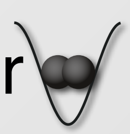
$$C_d(t) = \langle s_j(t) s_{j+d}(t) \rangle - \langle s_j(t) \rangle \langle s_{j+d}(t) \rangle \longrightarrow$$

$\simeq 0$ in the initial state

>0 when $t \simeq d/v$

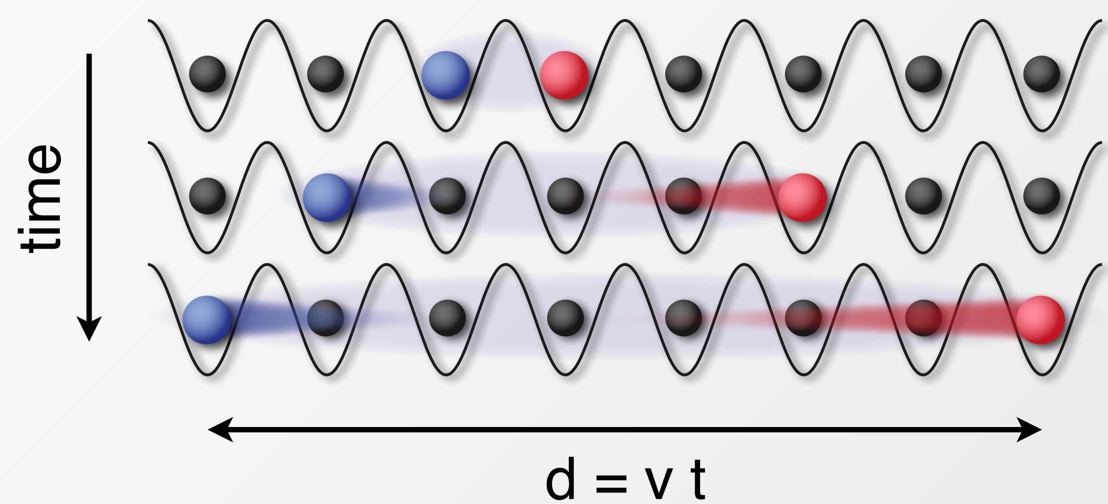
$$s_j(t) = e^{i\pi[n_j(t) - \bar{n}]}$$

{ +1 if 

{ -1 if  or 

Light-cone like spreading of correlations

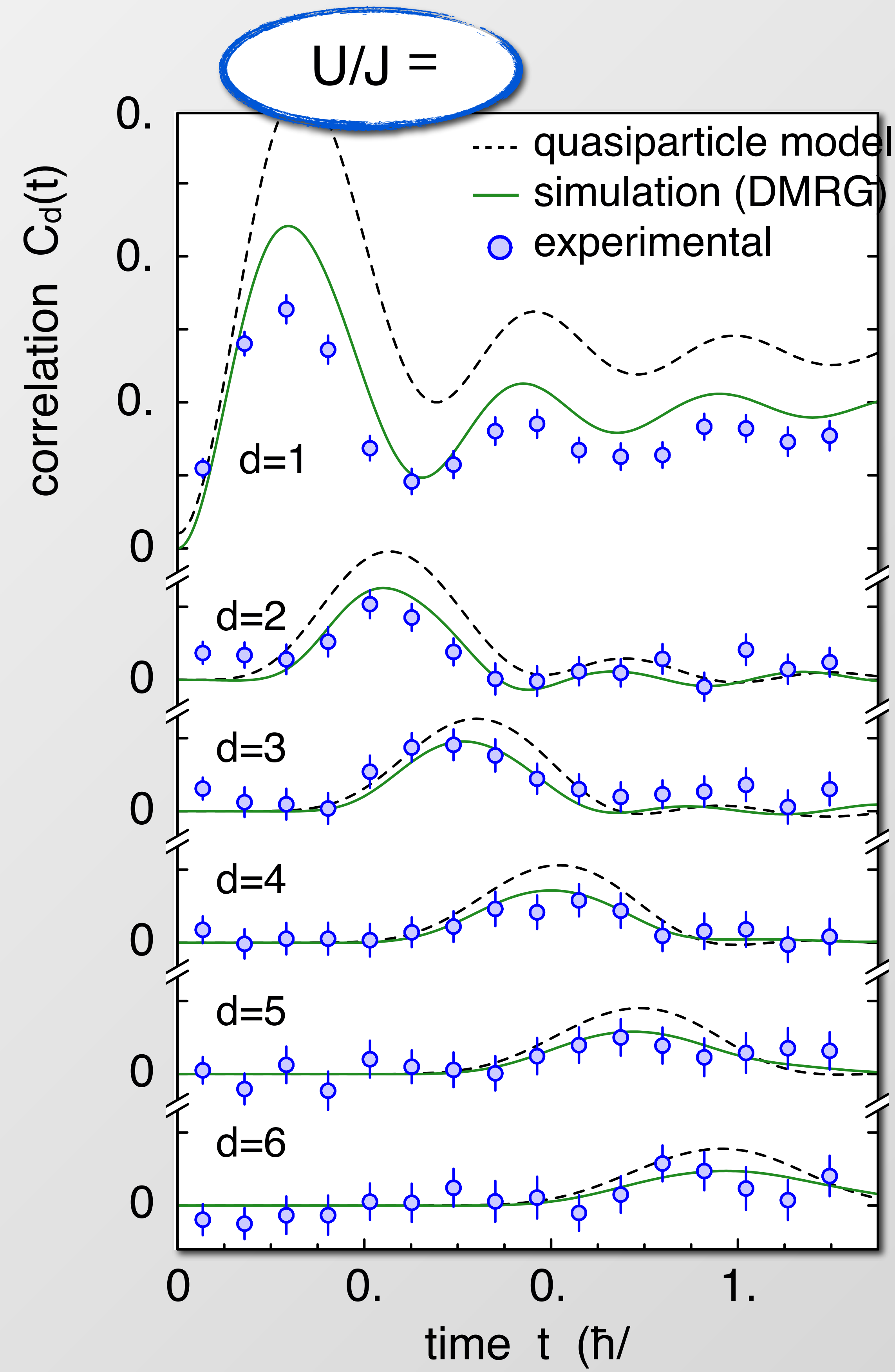
- Quasiparticle dynamics



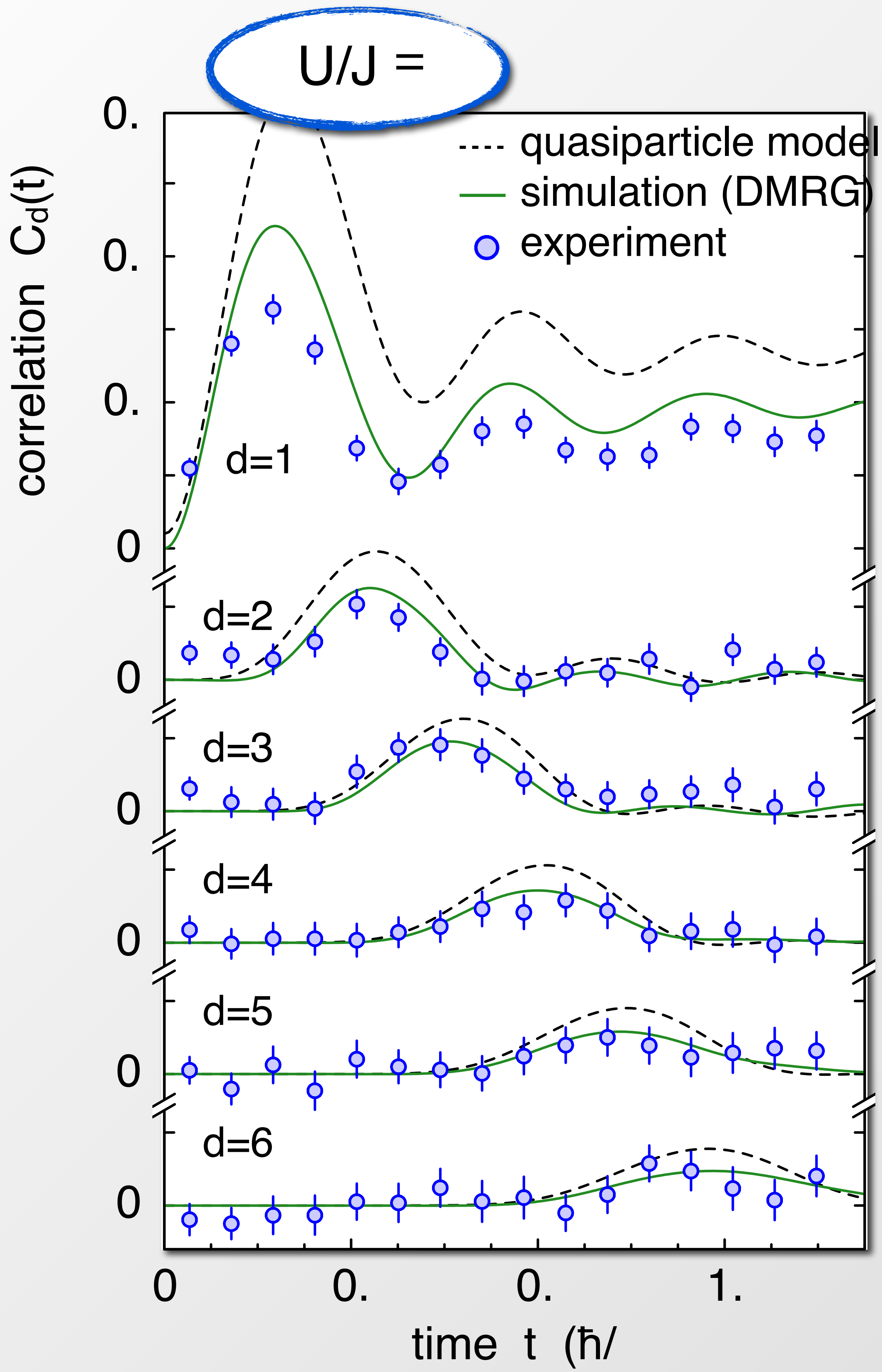
- Two-point parity correlation function

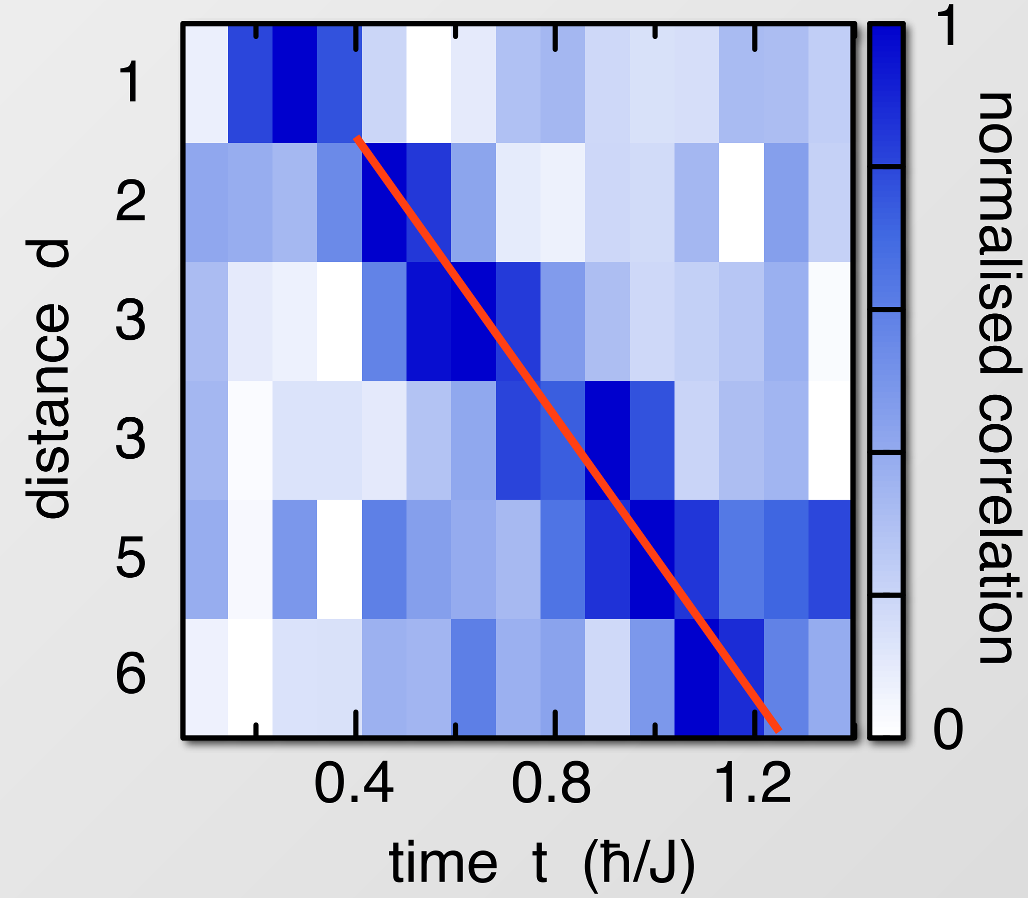
$$C_d(t) = \langle s_j(t) s_{j+d}(t) \rangle - \langle s_j(t) \rangle \langle s_{j+d}(t) \rangle$$

$$s_j(t) = e^{i\pi[n_j(t) - \bar{n}]} \quad \begin{cases} +1 & \text{if} \\ -1 & \text{if} \quad \text{or} \end{cases}$$



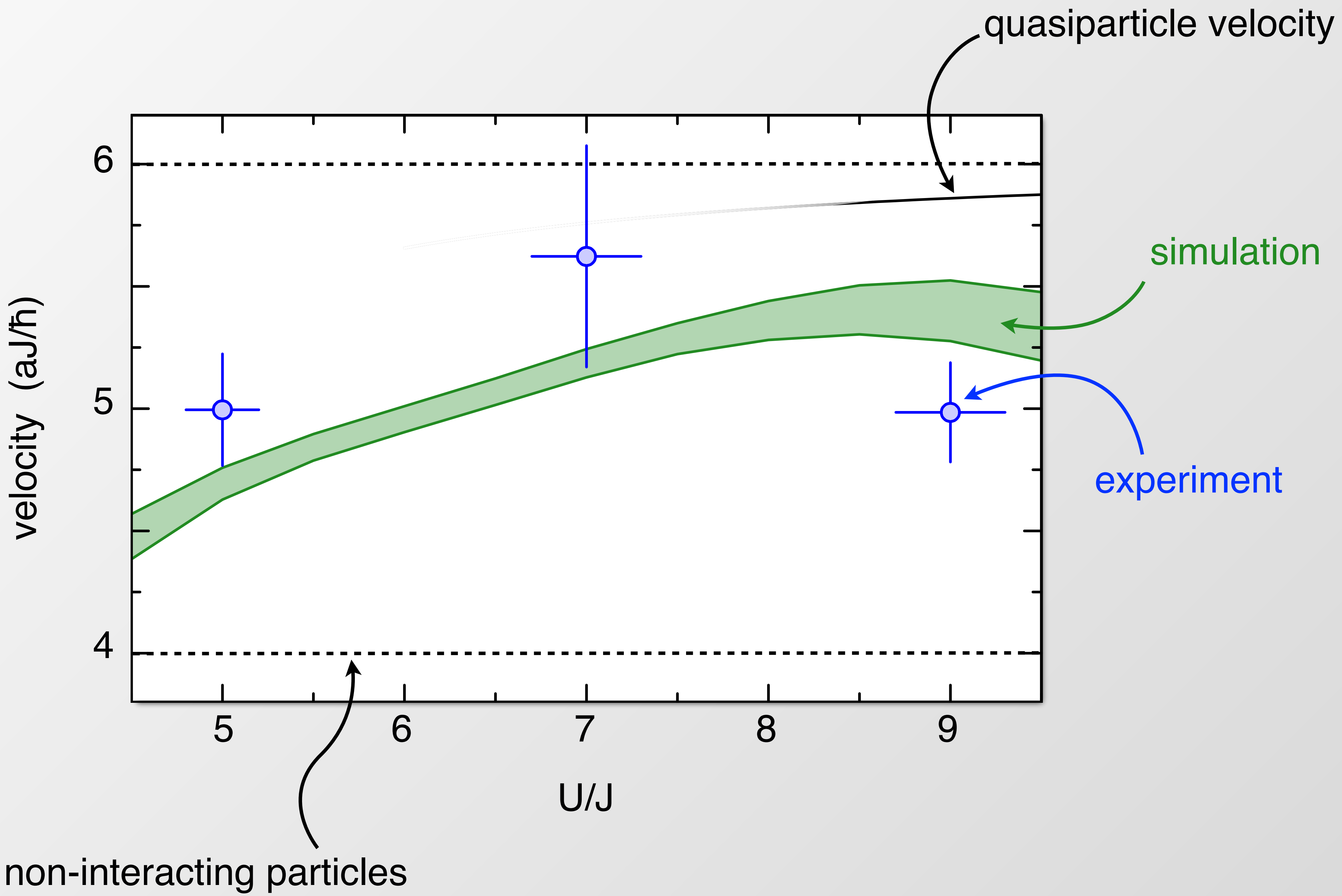
Light-cone like spreading of correlations





effective light-cone!

Spreading velocity



Noise Correlations

Information ?

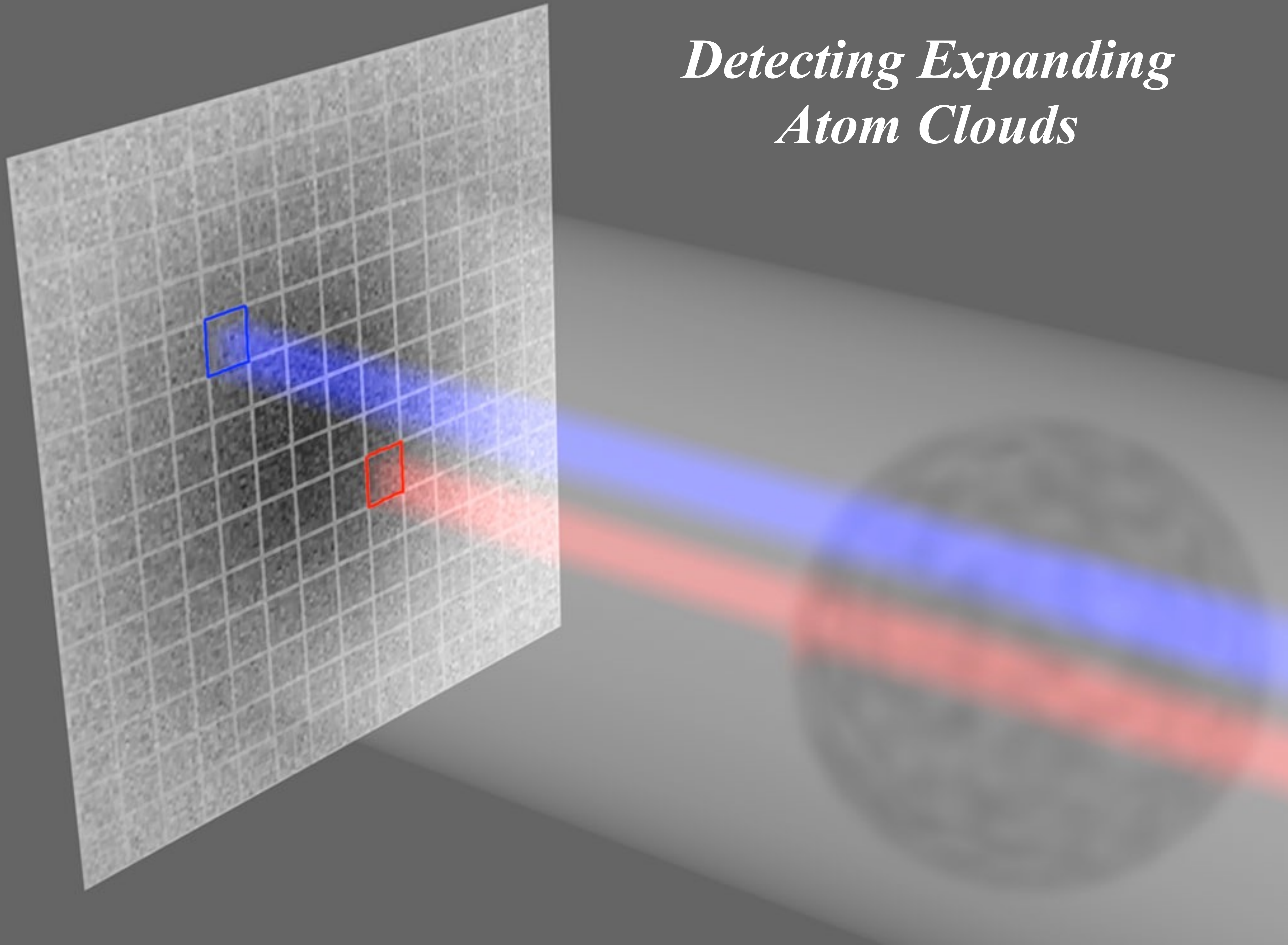
Proposal:
E. Altman, E. Demler & M. Lukin PRA (2004)
A. Polkovnikov et al., PNAS (2006)

Experiment:
Fölling et al., Nature (2005),
Greiner et al., PRL (2005)
Rom et al., Nature (2006)
Guarrera et al., PRL (2008)

related work:
Bach & Rzazewski, PRA (2004)
Z. Hadzibabic et al. PRL (2004),

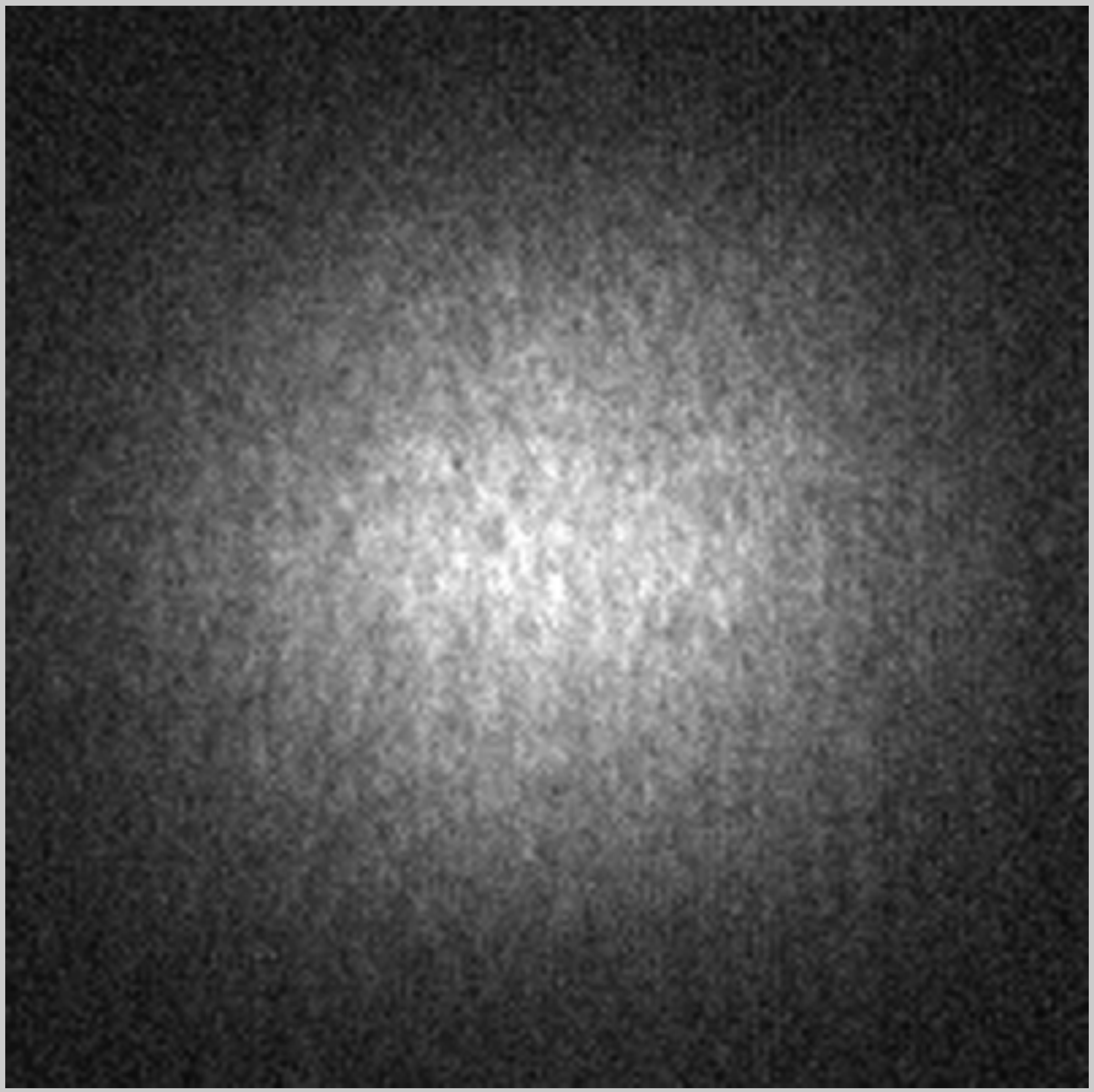
Yasuda & Shimizu, PRL (1996),
Schellekens et al., Science (2005),
Jeltes et al., Nature (2007)
Öttl et al., PRL (2005),
Estève et al., PRL (2006),
K. Eckert et al., Nat. Phys. (2008)

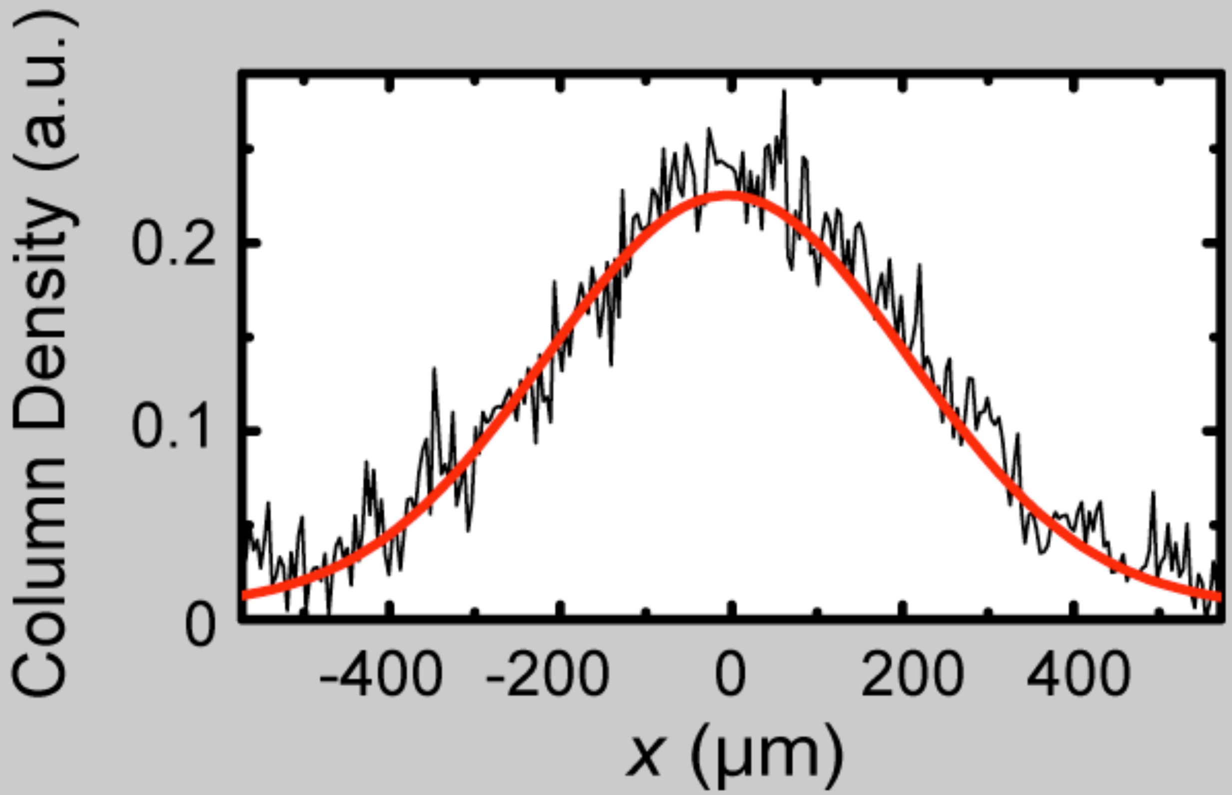
Detecting Expanding Atom Clouds



Typically Noise in Images of a Mott Insulator

Single Image

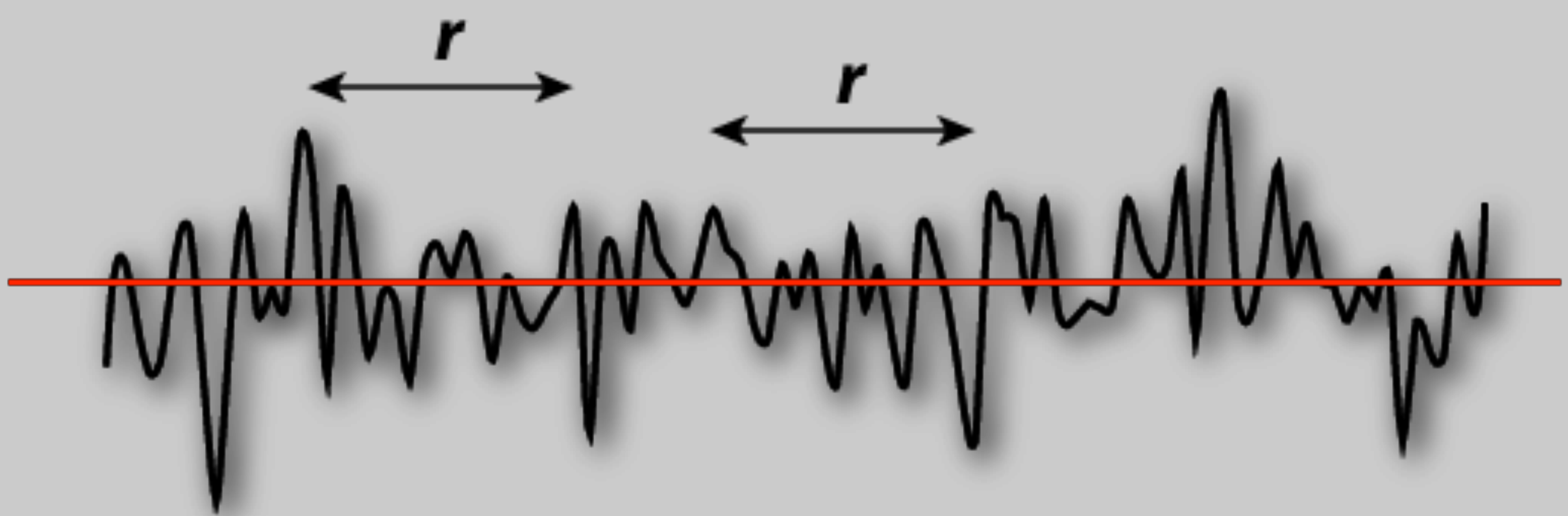




Fluctuations due to Atomic Shot Noise

$$\sigma \sim \sqrt{N_{bin}}$$

Correlations in Noise?

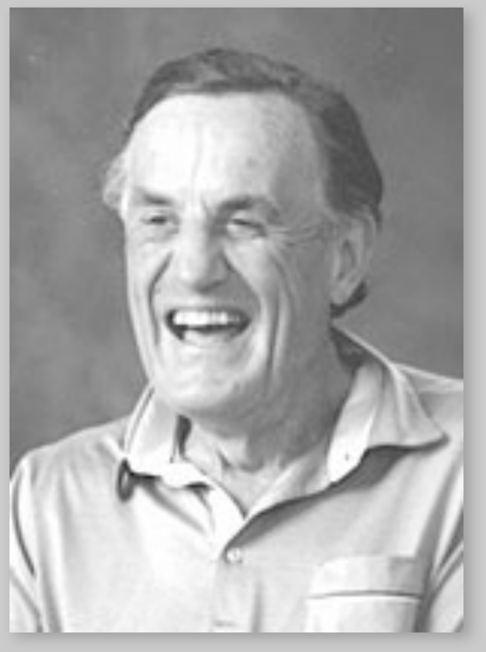


Hanbury-Brown Twiss effect correlates fluctuations at special distances r!

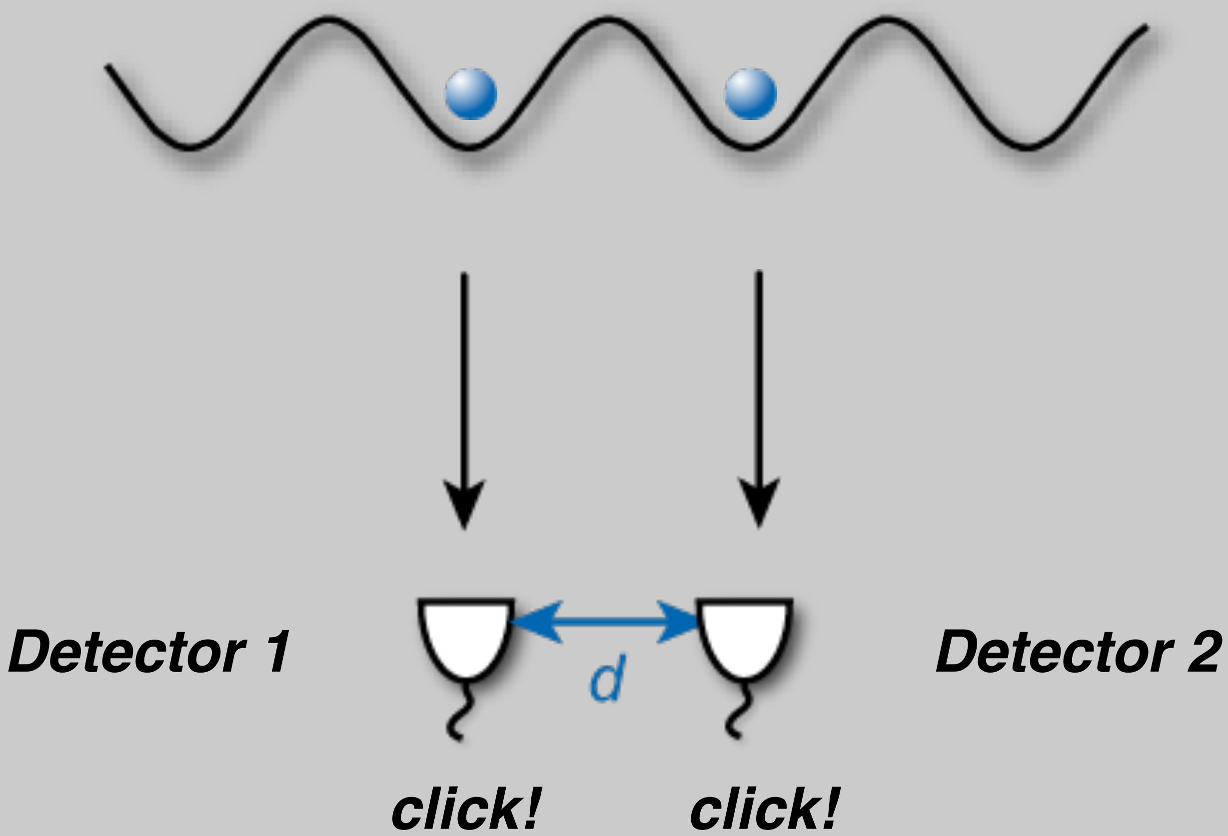
Quantitatively

$g^{(2)}(r)-1>0$	*Noise correlated (Bosons)*
$g^{(2)}(r)-1=0$	*Noise uncorrelated*
$g^{(2)}(r)-1<0$	*Noise anti-correlated (Fermions)*

- Hanbury Brown-Twiss Effect for Atoms (1) -

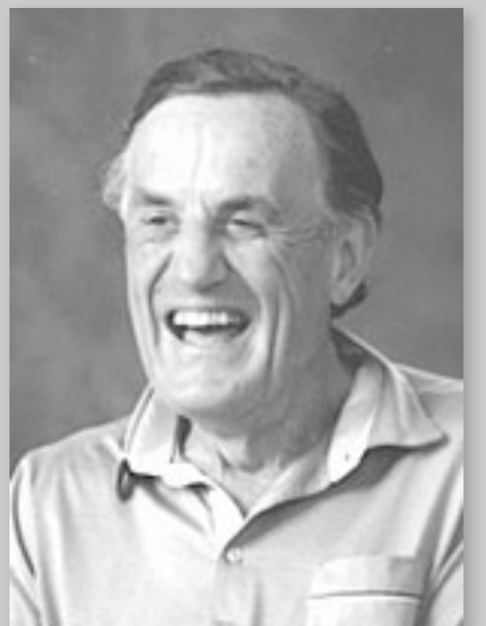


Hanbury Brown
1916-2002

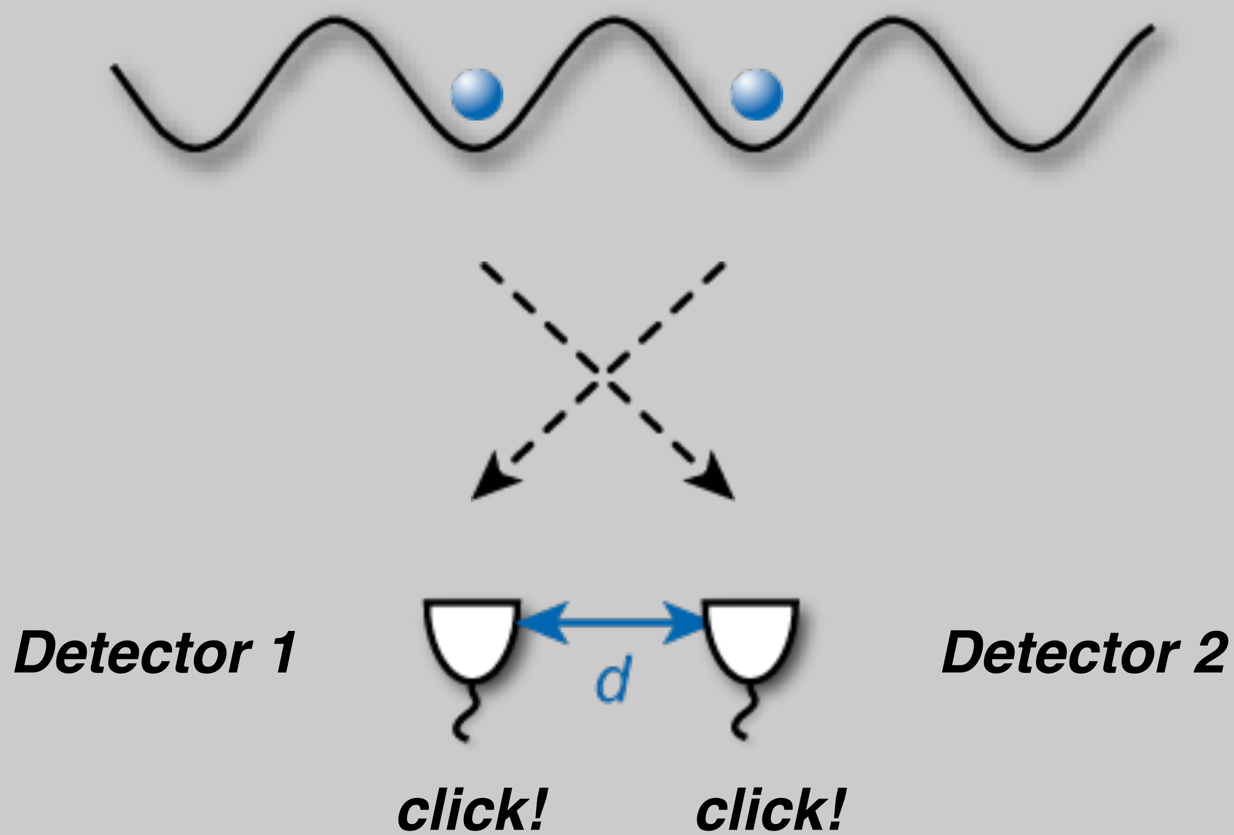


- Hanbury Brown-Twiss Effect for Atoms (2) -

There's another ways....

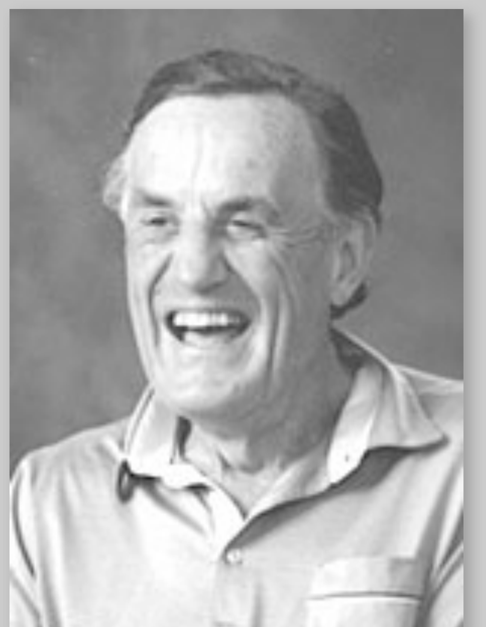


Hanbury Brown
1916-2002

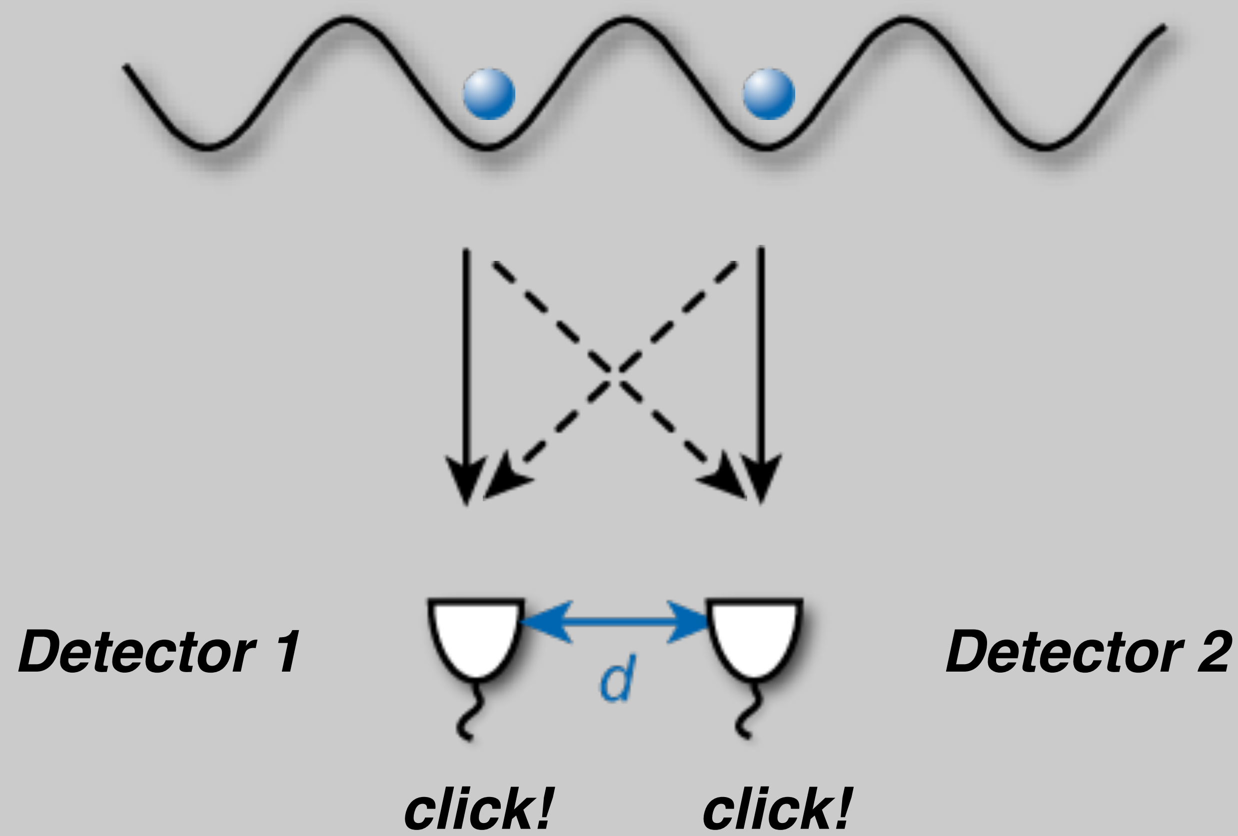


- Hanbury Brown-Twiss Effect for Atoms (3) -

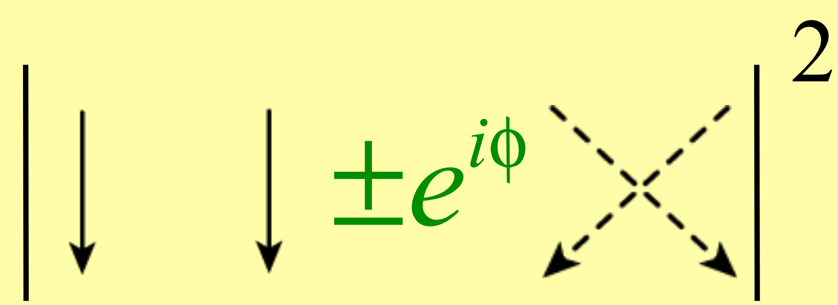
Cannot fundamentally distinguish between both paths...



Hanbury Brown
1916-2002

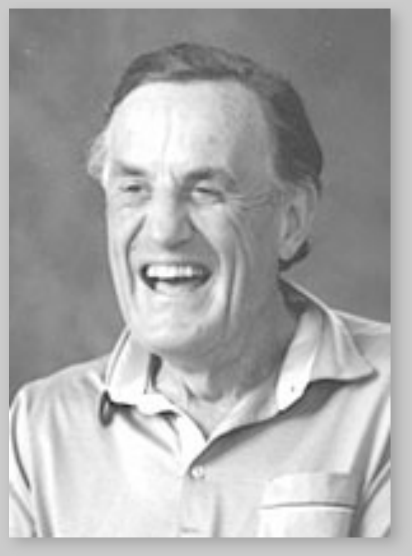


Two Particle Detection probability

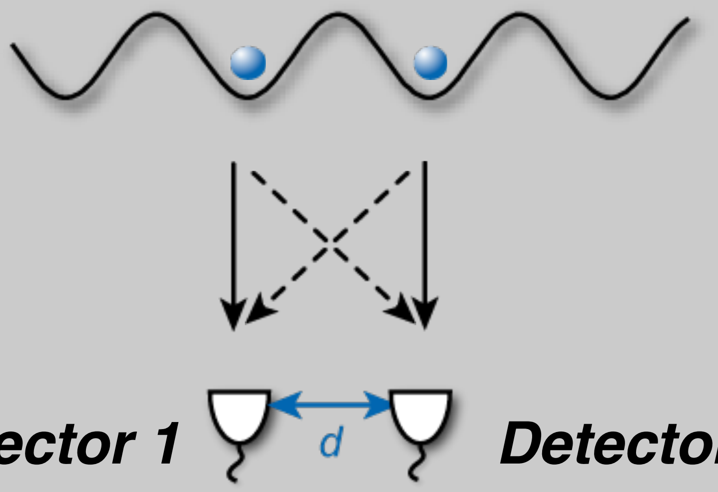


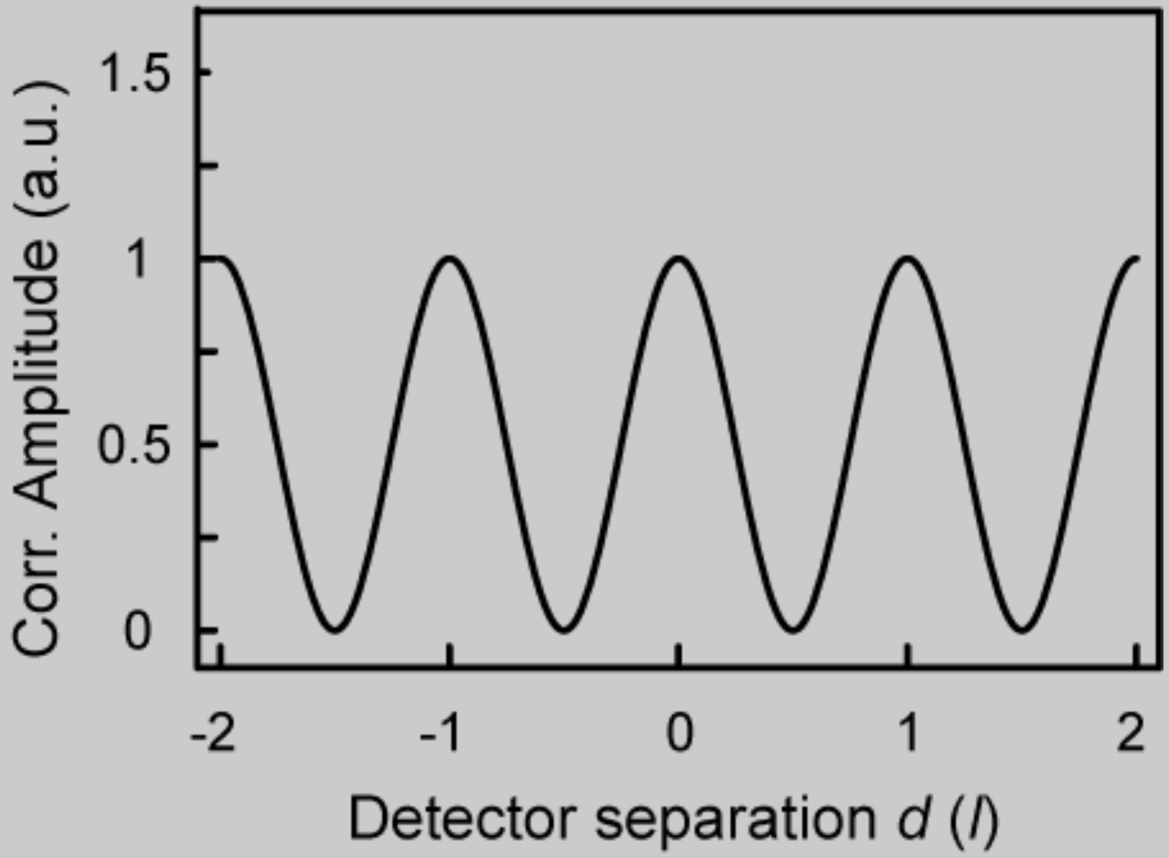
- Hanbury Brown-Twiss Effect for Atoms (4) -

Interference in Two-Particle Detection Probability!



Hanbury Brown
1916-2002

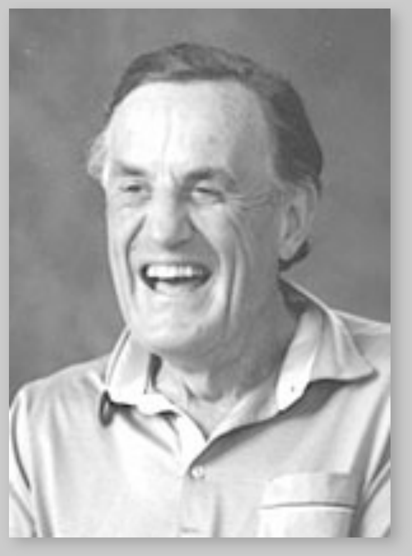




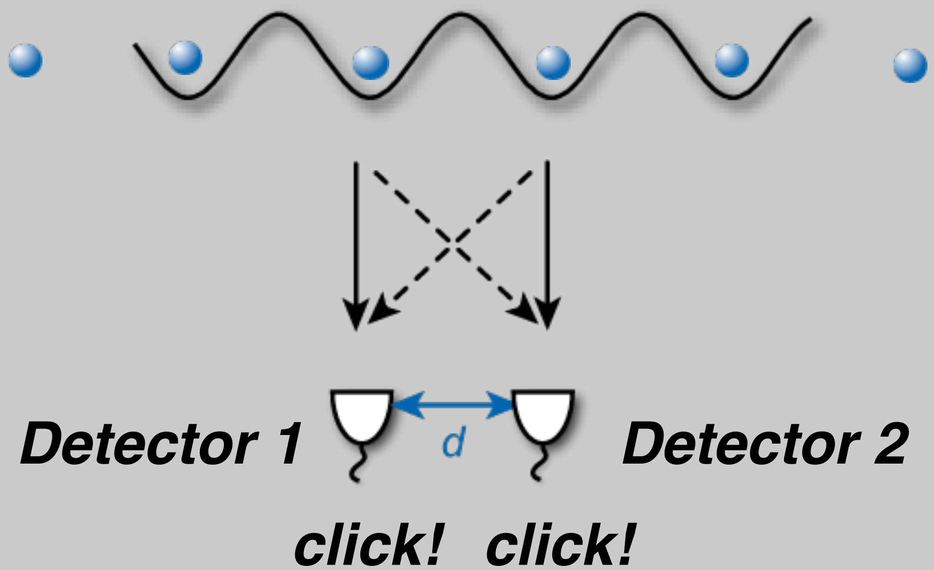
$$\ell = \frac{h}{m a_{lat}} t$$

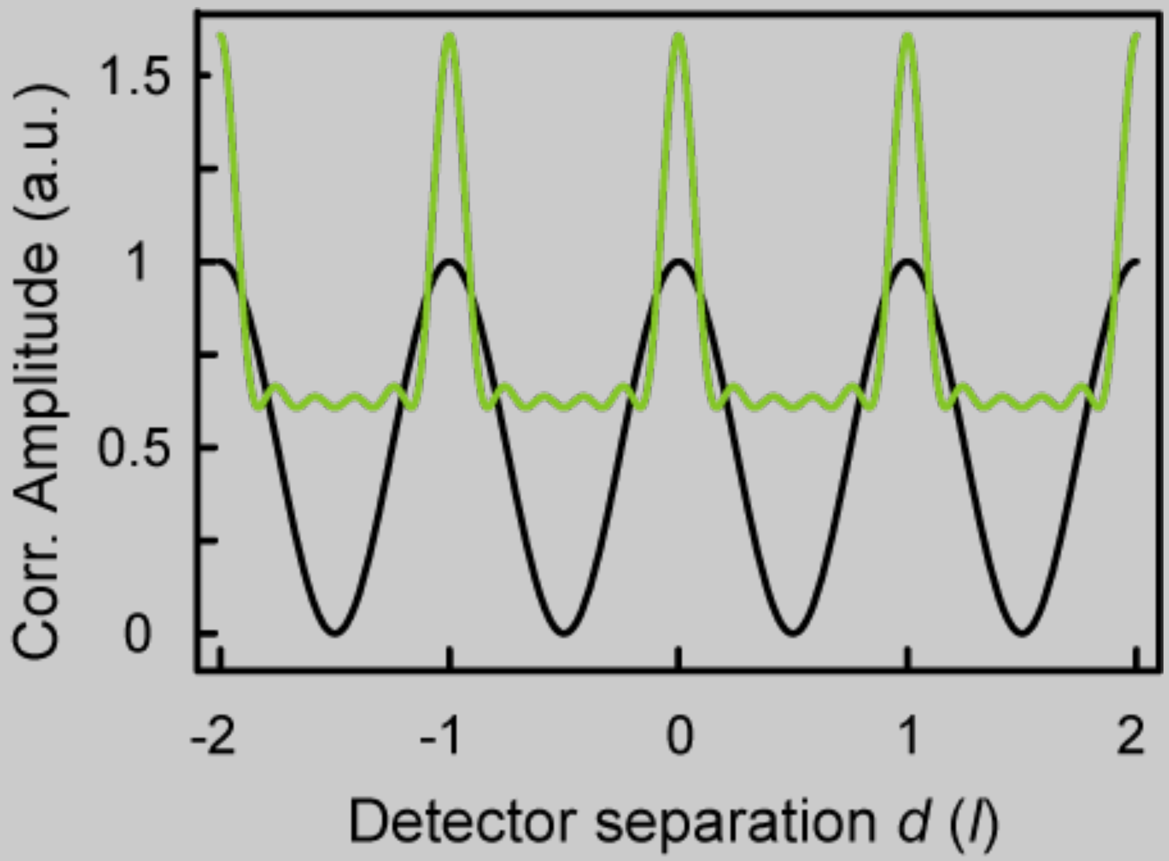
- Multiple Wave Hanbury Brown-Twiss Effect (4) -

Interference in Two-Particle Detection Probability!



Hanbury Brown
1916-2002





$$l = \frac{h}{m a_{lat} t}$$

Deriving the Noise Correlation Signal (1)

In Time of Flight we measure:

$$\begin{aligned}\langle \hat{n}_{3D}(\mathbf{x})\rangle_{\text{tof}} &= \langle \hat{a}^{\dagger}_{tof}(\mathbf{x})\hat{a}_{\text{tof}}(\mathbf{x})\rangle_{\text{tof}} \\ &\approx \langle \hat{a}^{\dagger}(\mathbf{k})\hat{a}(\mathbf{k})\rangle_{\text{trap}} = \langle \hat{n}_{3D}(\mathbf{k})\rangle_{\text{trap}}\end{aligned}$$

where $\mathbf{k} = M\mathbf{x}/\hbar t$

In Noise Correlations we measure:

$$\langle \hat{n}_{3D}(\mathbf{x})\hat{n}_{3D}(\mathbf{x}')\rangle_{\text{tof}} \approx \langle \hat{a}^{\dagger}(\mathbf{k})\hat{a}(\mathbf{k})\hat{a}^{\dagger}(\mathbf{k}')\hat{a}(\mathbf{k}')\rangle_{\text{trap}} = \langle \hat{a}^{\dagger}(\mathbf{k})\hat{a}^{\dagger}(\mathbf{k}')\hat{a}(\mathbf{k}')\hat{a}(\mathbf{k})\rangle_{\text{trap}} + \delta_{\mathbf{k}\mathbf{k}'}\langle \hat{a}^{\dagger}(\mathbf{k})\hat{a}(\mathbf{k})\rangle_{\text{trap}} \quad .$$

Deriving the Noise Correlation Signal (2)

$$\hat{a}(\mathbf{k}) = \int e^{-i\mathbf{k}\mathbf{r}} \hat{\psi}(\mathbf{r}) d^3 r$$ with $$\hat{\psi}(\mathbf{r}) = \sum_{\mathbf{R}} \hat{a}_{\mathbf{R}} w(\mathbf{r} - \mathbf{R})$$

$$\Rightarrow \quad \hat{a}(\mathbf{k}) = \tilde{w}(\mathbf{k}) \sum_{\mathbf{R}} e^{-i\mathbf{k}\mathbf{R}} \hat{a}_{\mathbf{R}}$$

Plug this into four operator correlator

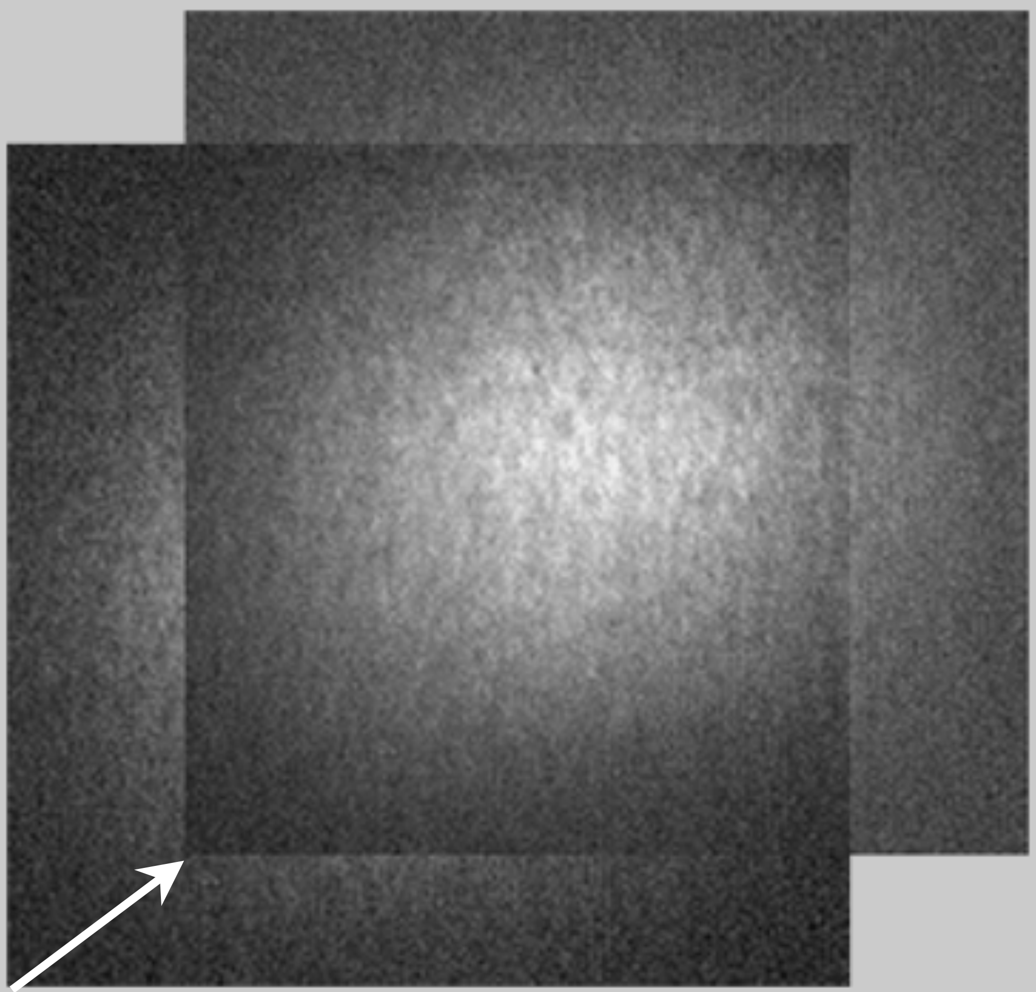
For Mott state or Fermi gas, one has $\langle \hat{a}^\dagger_{\mathbf{R}} \hat{a}_{\mathbf{R}'} \rangle = n_{\mathbf{R}} \, \delta_{\mathbf{R},\mathbf{R}'}$

which yields:

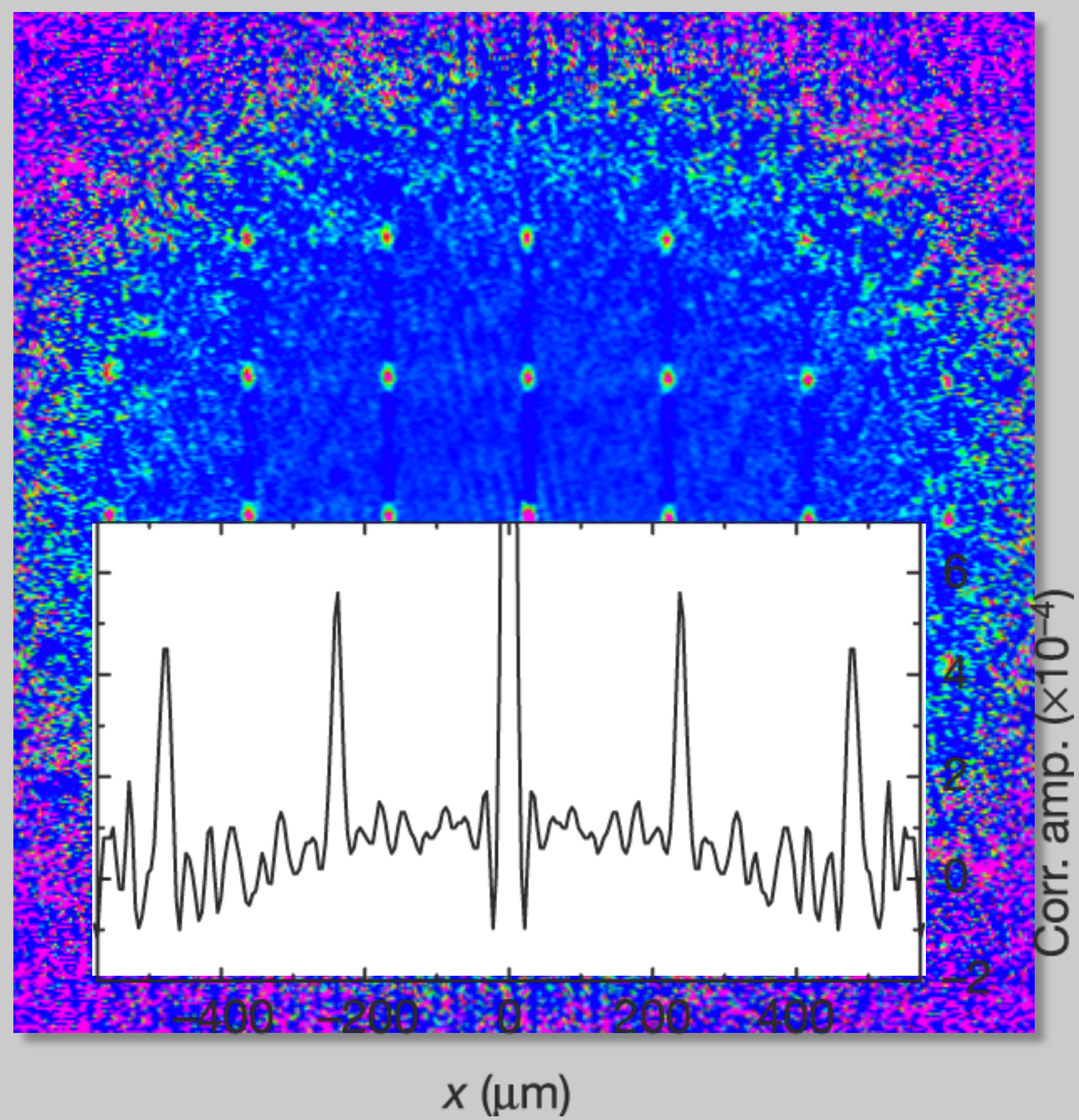
$$\langle \hat{n}_{3D}(\mathbf{x}) \hat{n}_{3D}(\mathbf{x}') \rangle = |\tilde{w}(M\mathbf{x}/\hbar t)|^2 |\tilde{w}(M\mathbf{x}'/\hbar t)|^2 N^2 \times \left[1 \pm \frac{1}{N^2} \left| \sum_{\mathbf{R}} e^{i(\mathbf{x}-\mathbf{x}')\cdot\mathbf{R}(M/\hbar t)} n_{\mathbf{R}} \right|^2 \right]$$

Information in the Noise – Correlations become visible!

$$g^{(2)}_{\text{exp}}(\mathbf{b}) = \frac{\int \langle n(\mathbf{x}+\mathbf{b}/2)\cdot n(\mathbf{x}-\mathbf{b}/2)\rangle\, d^2\mathbf{x}}{\int \langle n(\mathbf{x}+\mathbf{b}/2)\rangle \langle n(\mathbf{x}-\mathbf{b}/2)\rangle\, d^2\mathbf{x}}$$

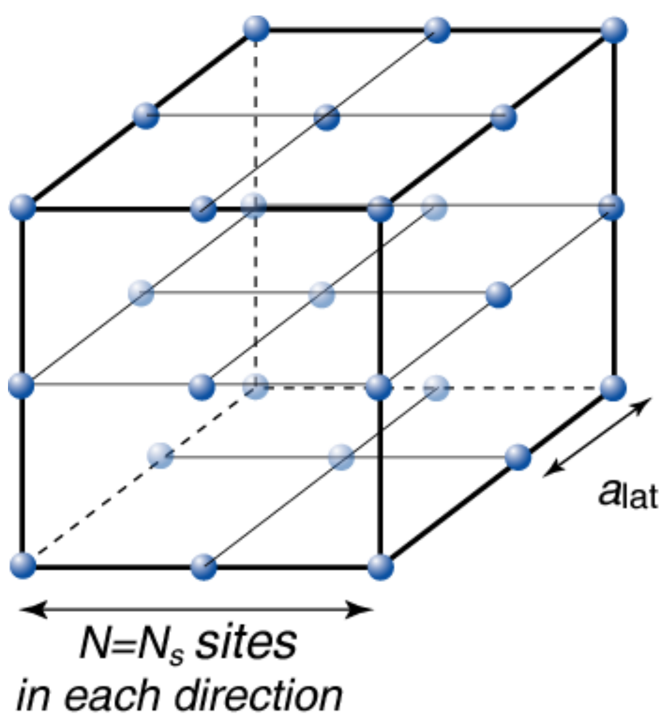


Fölling et al. Nature **434**, p. 481(2005)



Correlation function!

How large are the correlations ?



Coherence length :
also ideal peak width

$$L_{coh} \sim \frac{\hbar t}{m N_s a_{lat}} = \frac{l}{N_s}$$

Great spatial resolution :
fringe spacing $l \gg L_{coh} \gg$ **res.**

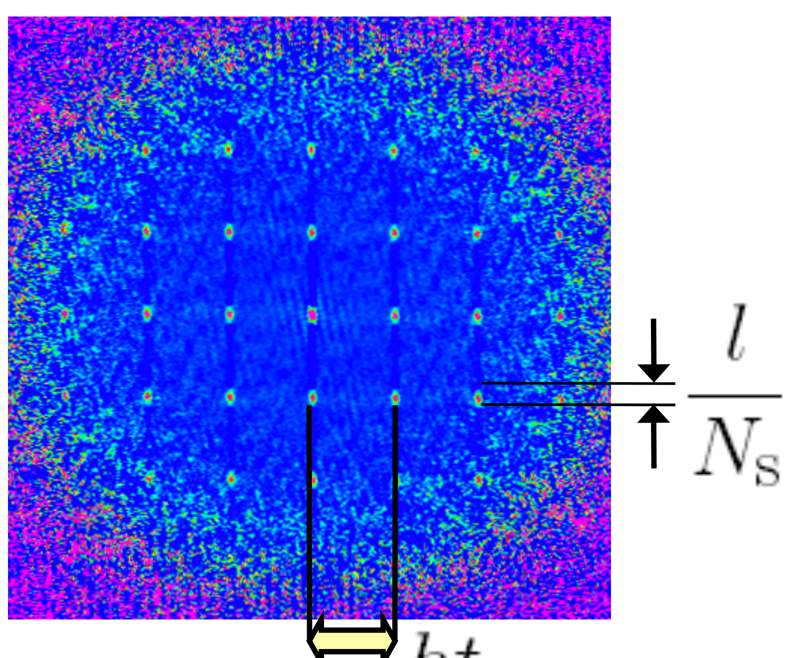
$$\mathcal{C}_{max} \approx 1 + 1$$

Poor spatial resolution :
res. >> fringe spacing $l \gg L_{coh}$

$$\mathcal{C}_{max} \approx 1 + \frac{1}{N_s^3}$$

Intermediate spatial resolution :
fringe spacing $l \gg$ **res.** $\gg L_{coh}$

$$\mathcal{C}_{max} \approx 1 + \left(\frac{L_{coh}}{\text{res}}\right)^3$$

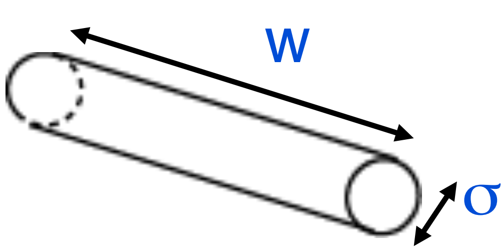


$$l = \frac{h t_{of}}{m a_{lat}}$$

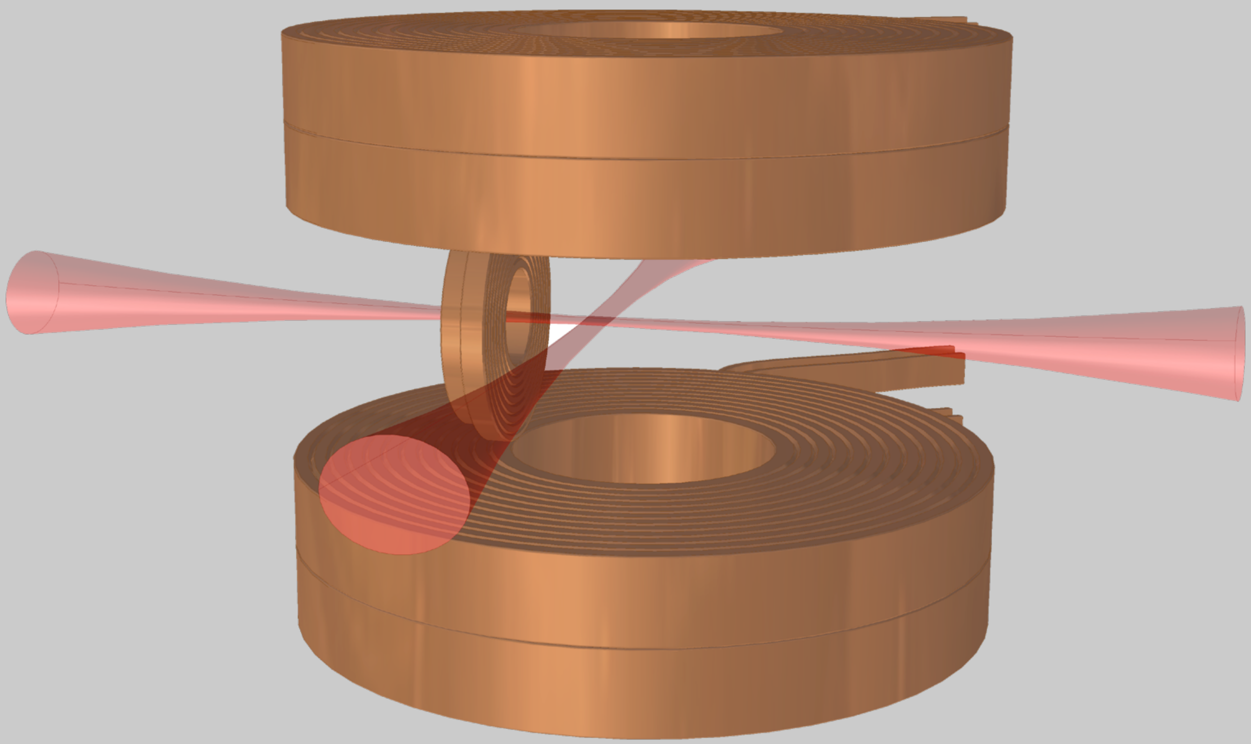
$$\mathcal{C}_{max} \approx 1 + \frac{1}{N_s} \cdot \frac{1}{N_s^2}\left(\frac{l}{\sigma}\right)^2$$

Imaging plane:
$l \gg \sigma > L_{coh}$

Probe direction : $w \gg l$



Sympathetic Cooling of ^{40}K-^{87}Rb in Crossed Dipole Trap:



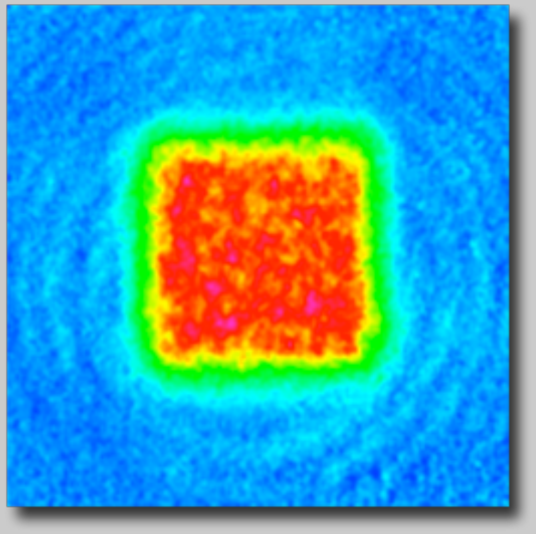
After final cooling in optical dipole trap

2×10^5 ^{87}Rb (almost pure condensate)

2.5×10^5 ^{40}K

After removal of ^{87}Rb

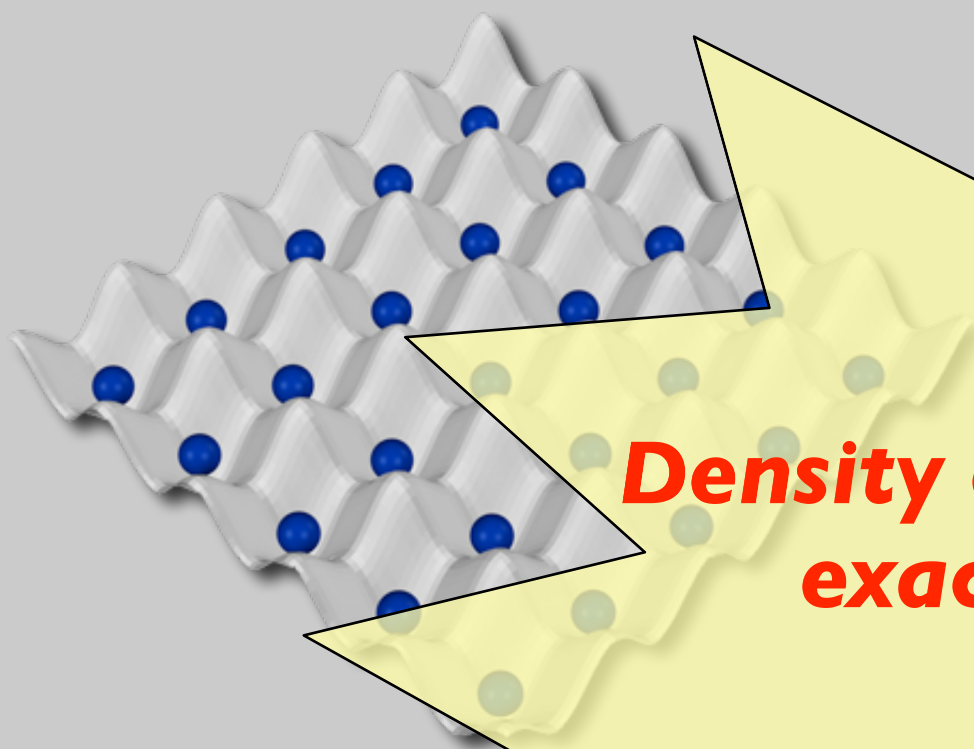
2×10^5 ^{40}K @ $T/T_F=0.2$

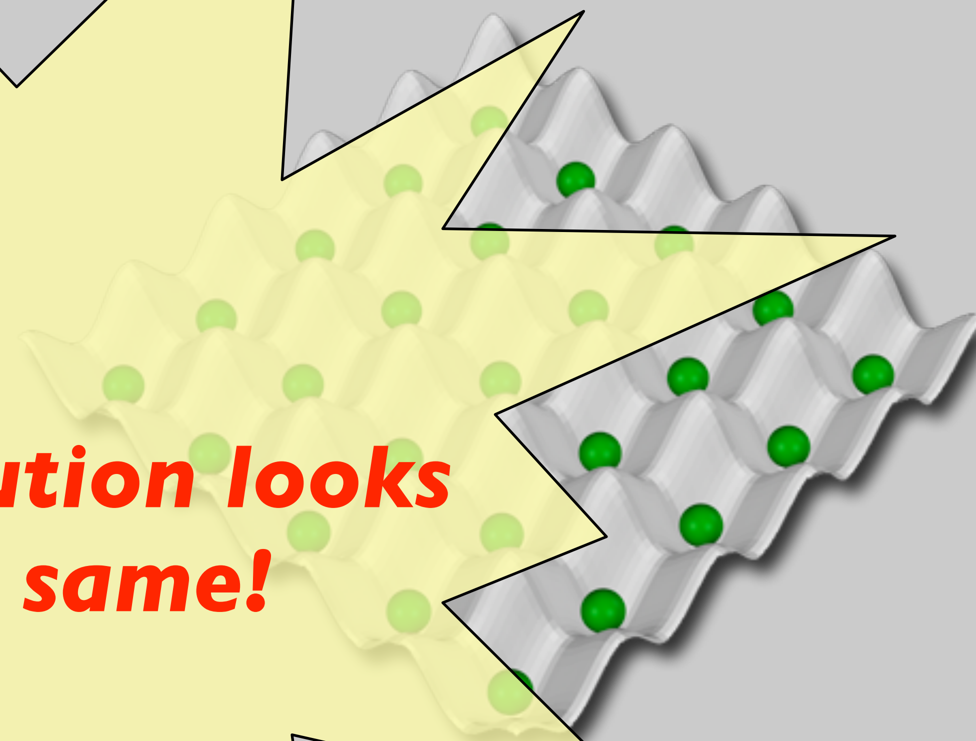


Then load into 3D optical lattice and create a fermionic band insulator!

Adiabatic mapping:
theory: A. Kastberg et al. PRL (1995)
exp: M. Greiner et al., PRL (2001), M. Köhl et al. PRL (2005)

Mott insulator – Fermionic Band Insulator





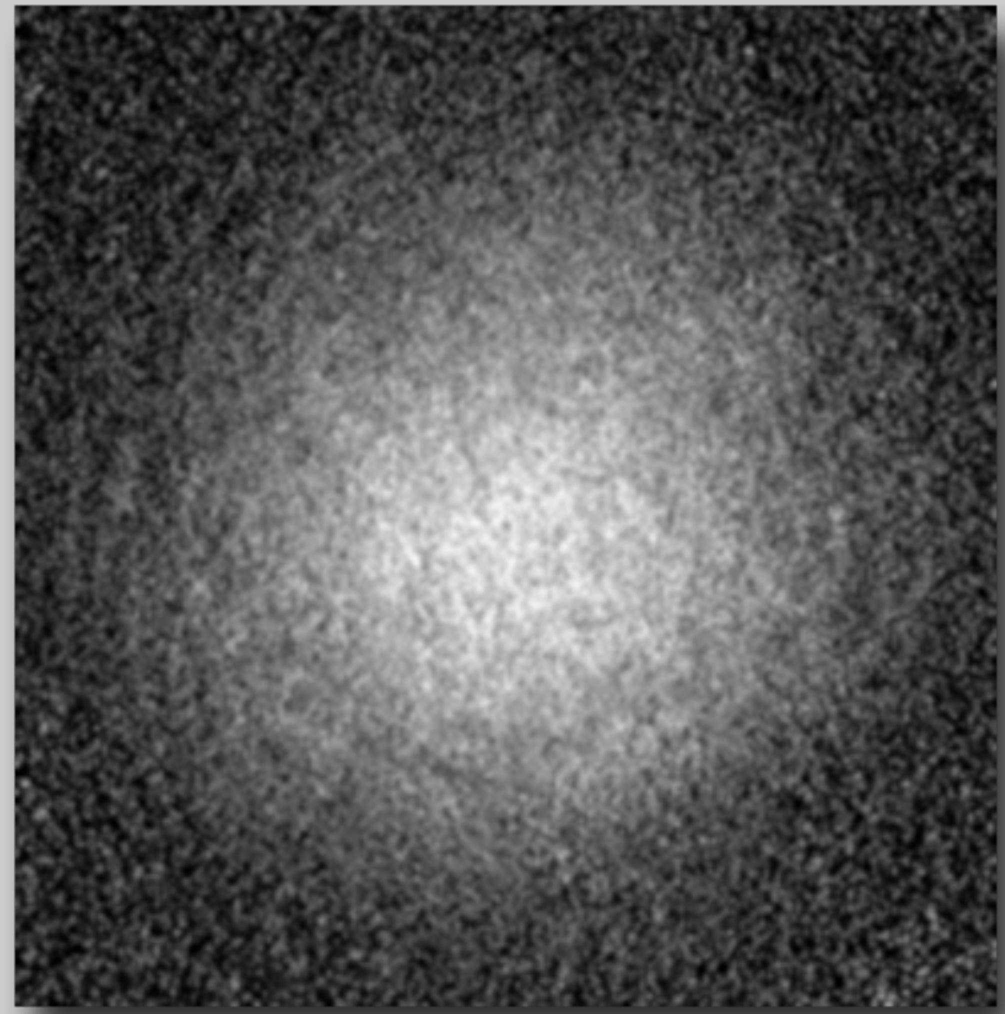
Density distribution looks exactly the same!

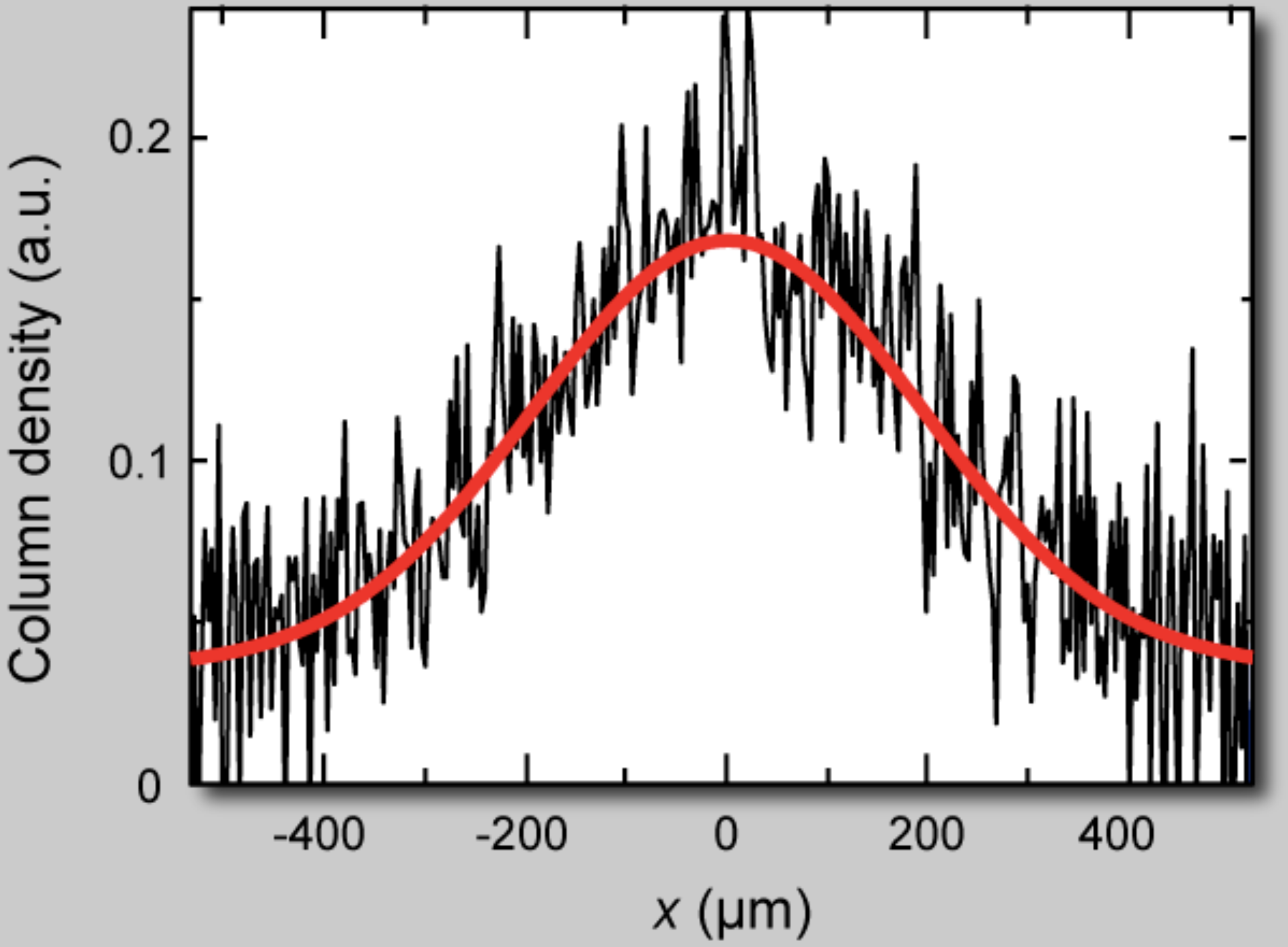
Mott insulator

Fermionic band insulator

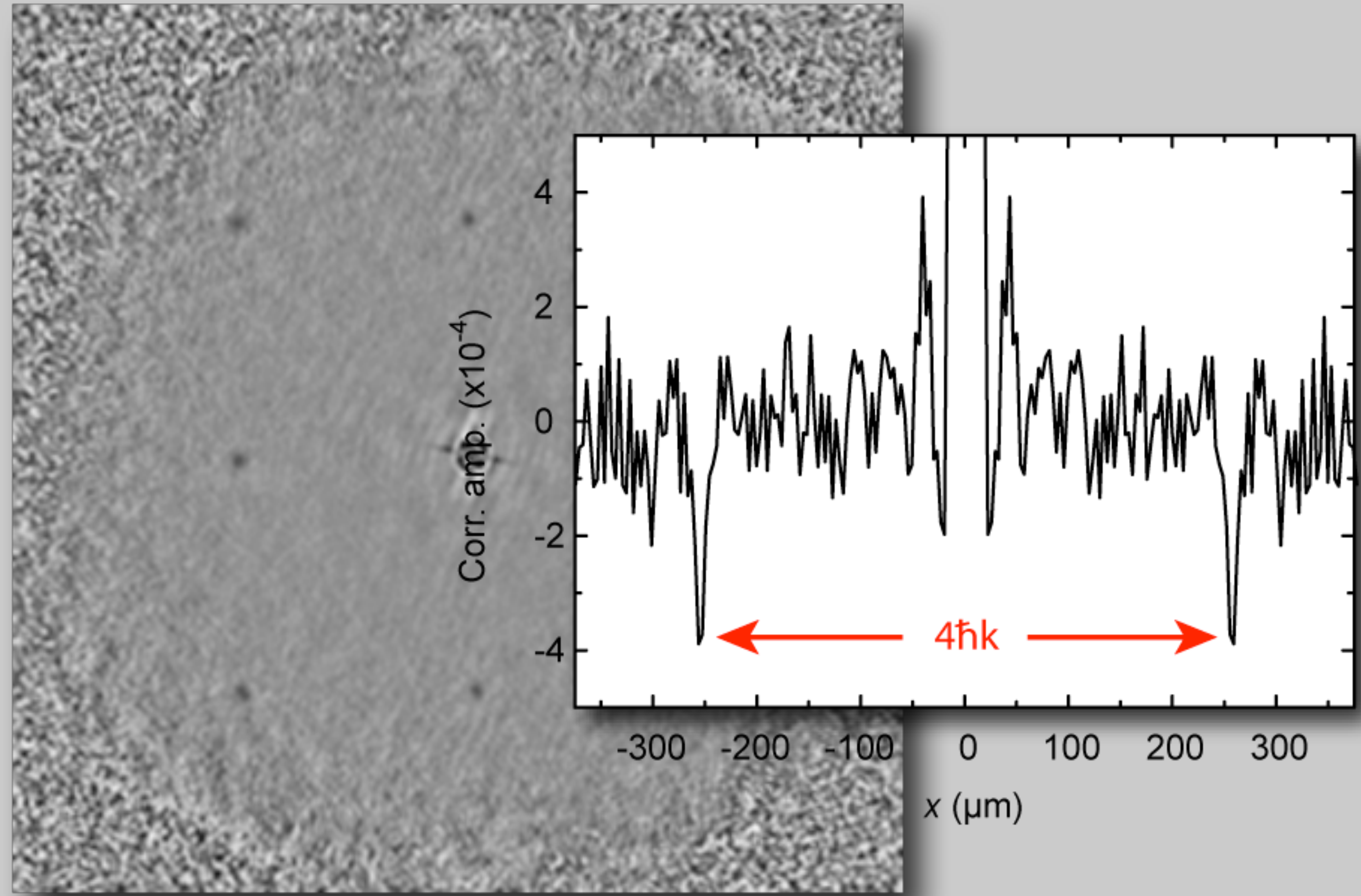
Dominating repulsive interactions mimic Pauli principle!

Releasing the Fermi Gas





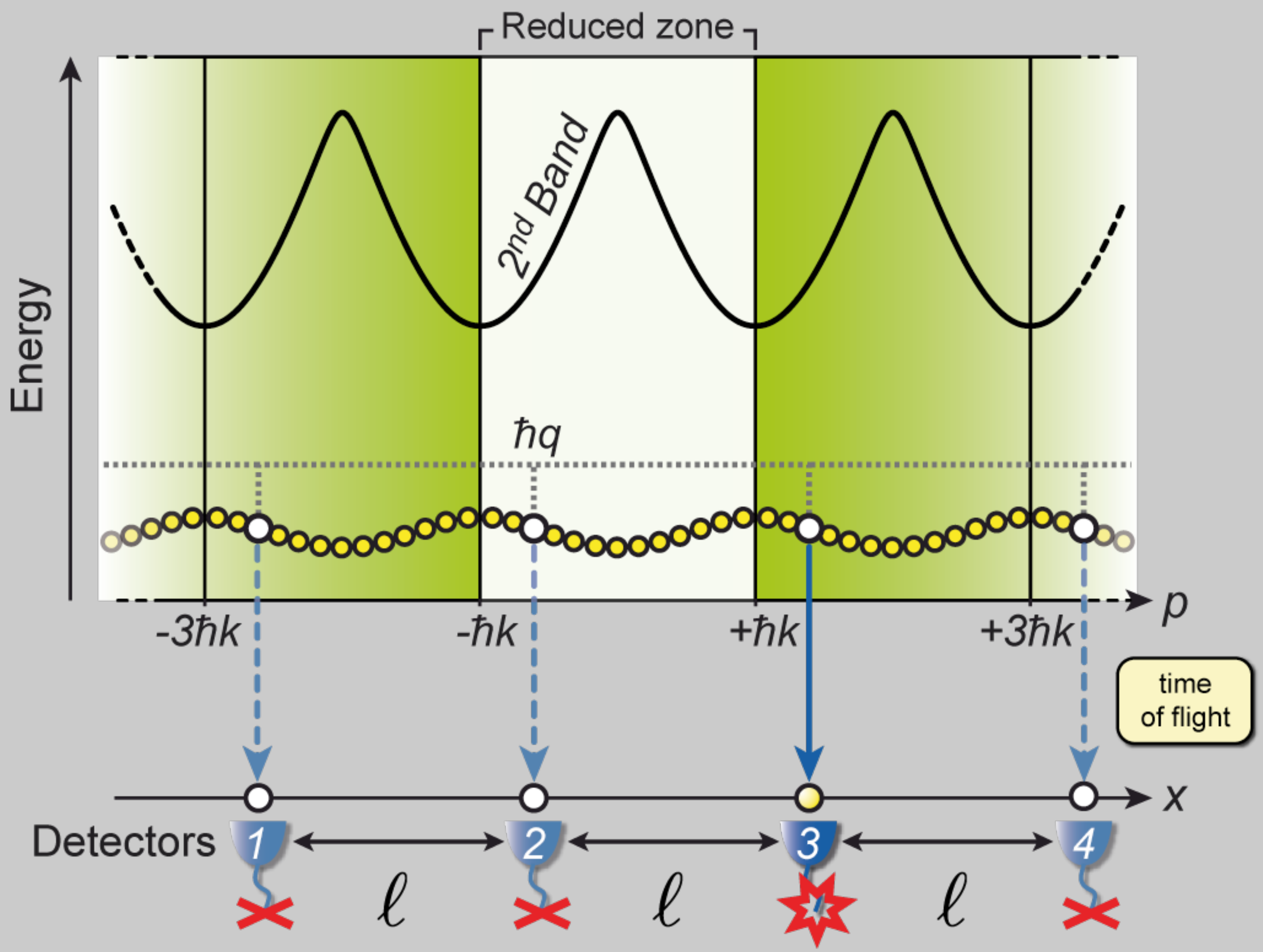
Noise Correlations of a Degenerate Fermi Gas



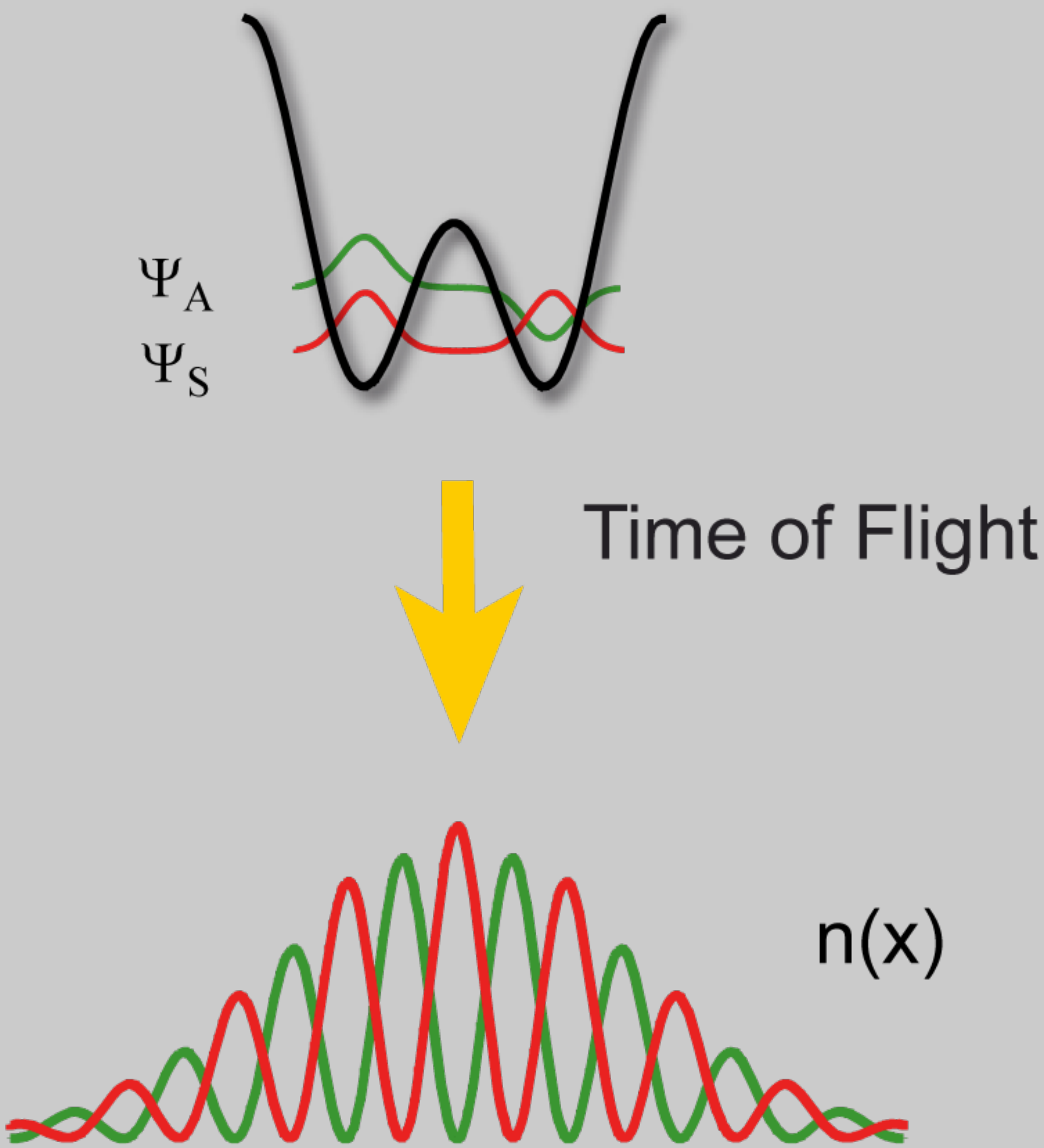
Rom et al.
Nature **444**, 733 (2006)

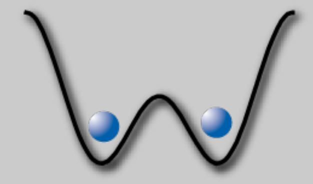
First observation of fermionic antibunching for neutral atoms (maybe neutral particles)! (see also Jeltes et al., Nature **445**, 402 (2007))

An Alternative Description



Why Bosons and Fermions are Different in their Correlations





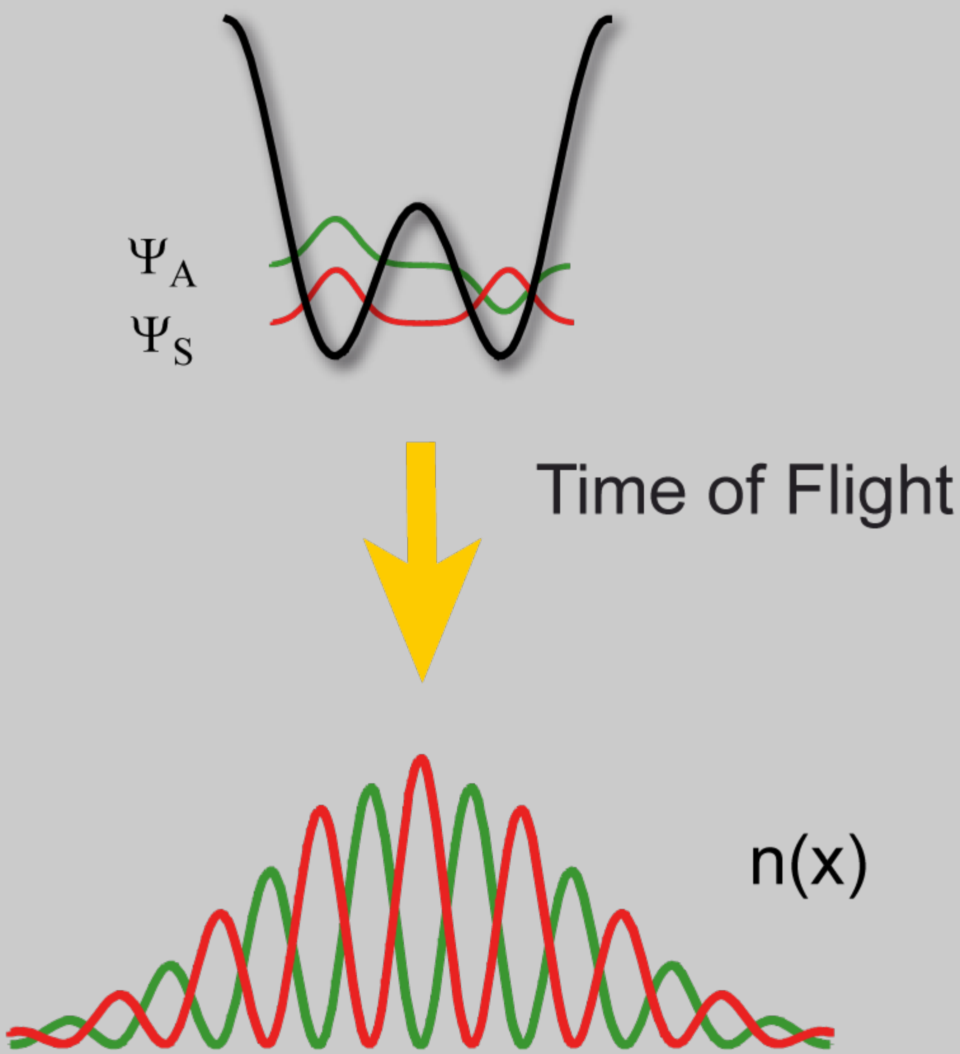
Bosons

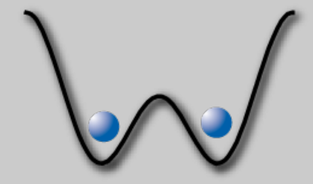
$$\Psi_L \otimes \Psi_R + \Psi_R \otimes \Psi_L$$

$$\Updownarrow$$

$$\Psi_S \otimes \Psi_S + \Psi_A \otimes \Psi_A$$

Why Bosons and Fermions are Different in their Correlations





Bosons

$$\Psi_L \otimes \Psi_R + \Psi_R \otimes \Psi_L$$

$$\Updownarrow$$

$$\Psi_S \otimes \Psi_S + \Psi_A \otimes \Psi_A$$

Fermions

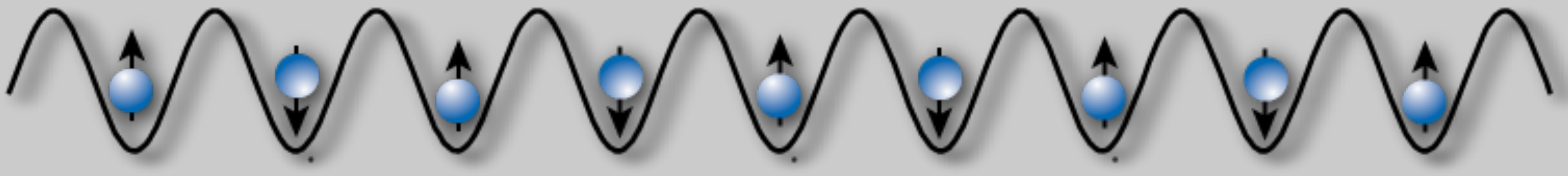
$$\Psi_L \otimes \Psi_R - \Psi_R \otimes \Psi_L$$

$$\Updownarrow$$

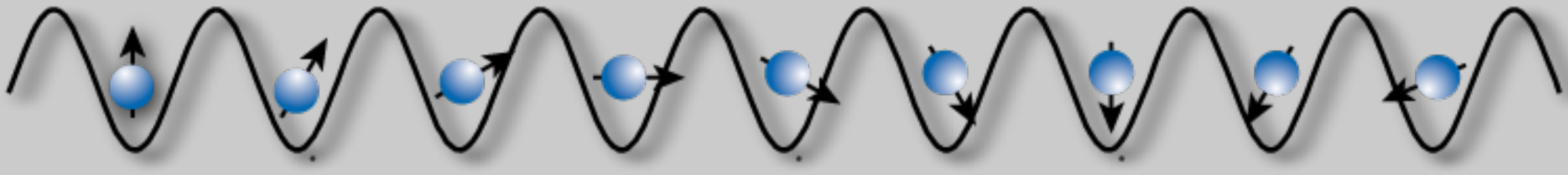
$$\Psi_S \otimes \Psi_A - \Psi_A \otimes \Psi_S$$

Now detection of many strongly correlated quantum states becomes possible!

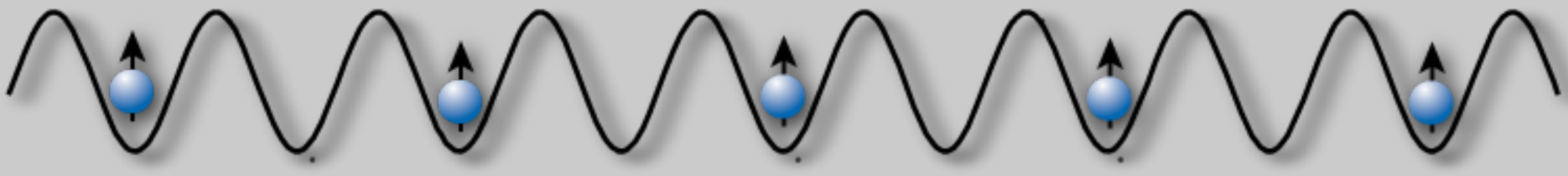
Antiferromagnet



Spin wave



Charge density wave



Atoms in Periodic Potentials

Single Particle in a Periodic Potential - Band Structure (1)

$$H\phi_q^{(n)}(x) = E_q^{(n)}\phi_q^{(n)}(x) \qquad \text{with} \qquad H = \frac{1}{2m}\hat{p}^2 + V(x)$$

Solved by Bloch waves (periodic functions in lattice period)

$$\phi_q^{(n)}(x) = e^{iqx} \cdot u_q^{(n)}(x)$$

q = Crystal Momentum or Quasi-Momentum

n = Band index

Plugging this into Schrödinger Equation, gives:

$$H_B u_q^{(n)}(x) = E_q^{(n)} u_q^{(n)}(x) \qquad \text{with} \qquad H_B = \frac{1}{2m}(\hat{p}+q)^2 + V_{lat}(x)$$

Single Particle in a Periodic Potential - Band Structure (2)

Use Fourier expansion

$$V(x) = \sum_r V_r e^{i2rkx} \qquad \text{and} \qquad u_q^{(n)}(x) = \sum_l c_l^{(n,q)} e^{i2lkx}$$

yields for the potential energy term

$$V(x) u_q^{(n)}(x) = \sum_l \sum_r V_r e^{i2(r+l)kx} c_l^{(n,q)}$$

and the kinetic energy term

$$\frac{(\hat{p}+q)^2}{2m} u_q^{(n)}(x) = \sum_l \frac{(2\hbar kl + q)^2}{2m} c_l^{(n,q)} e^{i2lkx}.$$

In the experiment standing wave interference pattern gives

$$V(x) = V_{lat} \sin^2(kx) = -\frac{1}{4}\left(e^{2ikx} + e^{-2ikx}\right) + \text{c.c.}$$

Single Particle in a Periodic Potential - Band Structure (3)

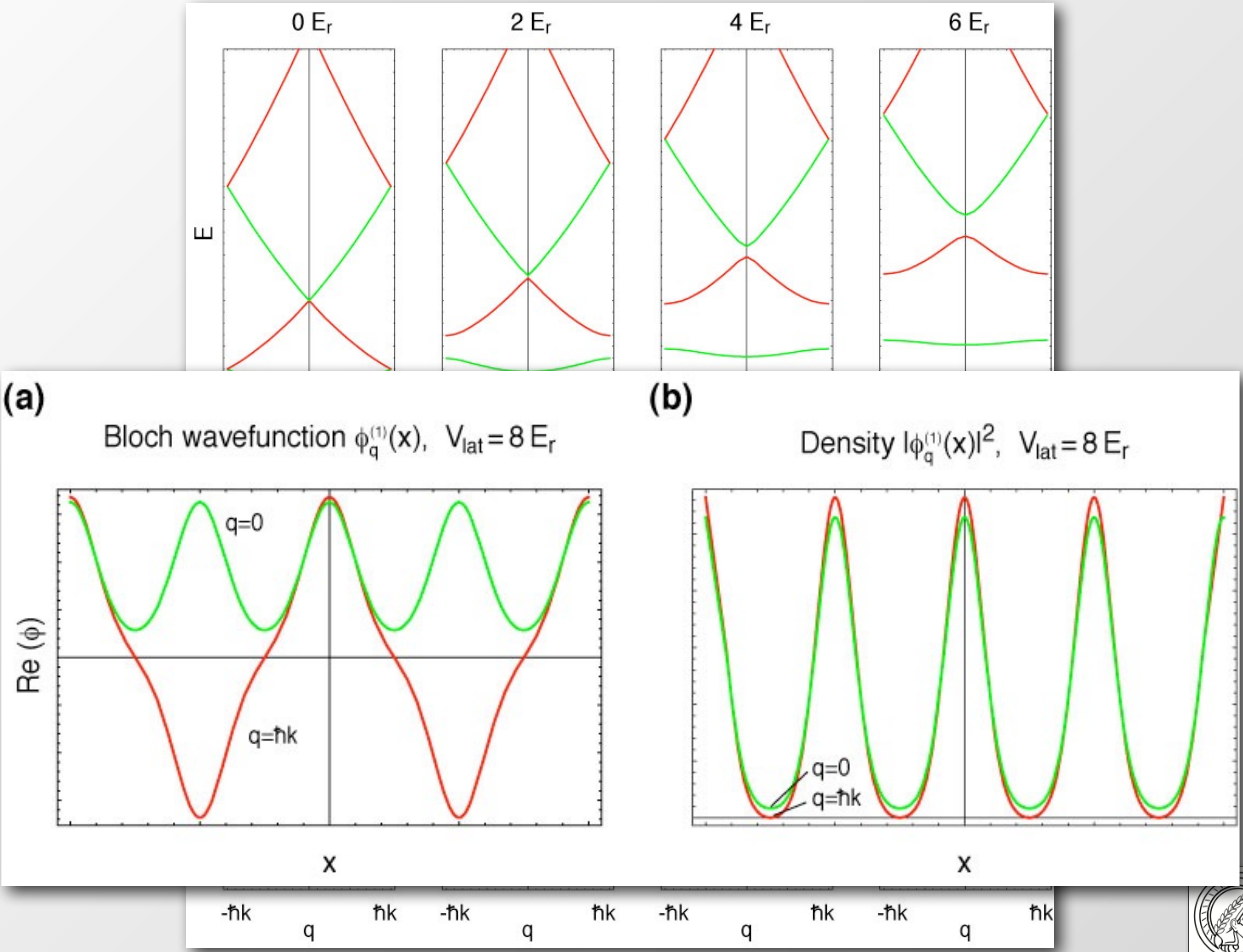
Use Fourier expansion

$$\sum_l H_{l,l'} \cdot c_l^{(n,q)} = E_q^{(n)} c_l^{(n,q)} \qquad \text{with} \qquad H_{l,l'} = \begin{cases} (2l + q/\hbar k)^2 E_r & \text{if } l = l' \\ -1/4 \cdot V_0 & \text{if } |l - l'| = 1 \\ 0 & \text{else} \end{cases}$$

$$\begin{pmatrix} (q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & 0 & 0 & \cdots \\ -\frac{1}{4}V_0 & (2+q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & 0 & \\ 0 & -\frac{1}{4}V_0 & (4+q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & \\ & & -\frac{1}{4}V_0 & \ddots & \\ & & & & \end{pmatrix} \begin{pmatrix} c_0^{(n,q)} \\ c_1^{(n,q)} \\ c_2^{(n,q)} \\ \\ \vdots \end{pmatrix} = E_q^{(n)} \begin{pmatrix} c_0^{(n,q)} \\ c_1^{(n,q)} \\ c_2^{(n,q)} \\ \\ \vdots \end{pmatrix}$$

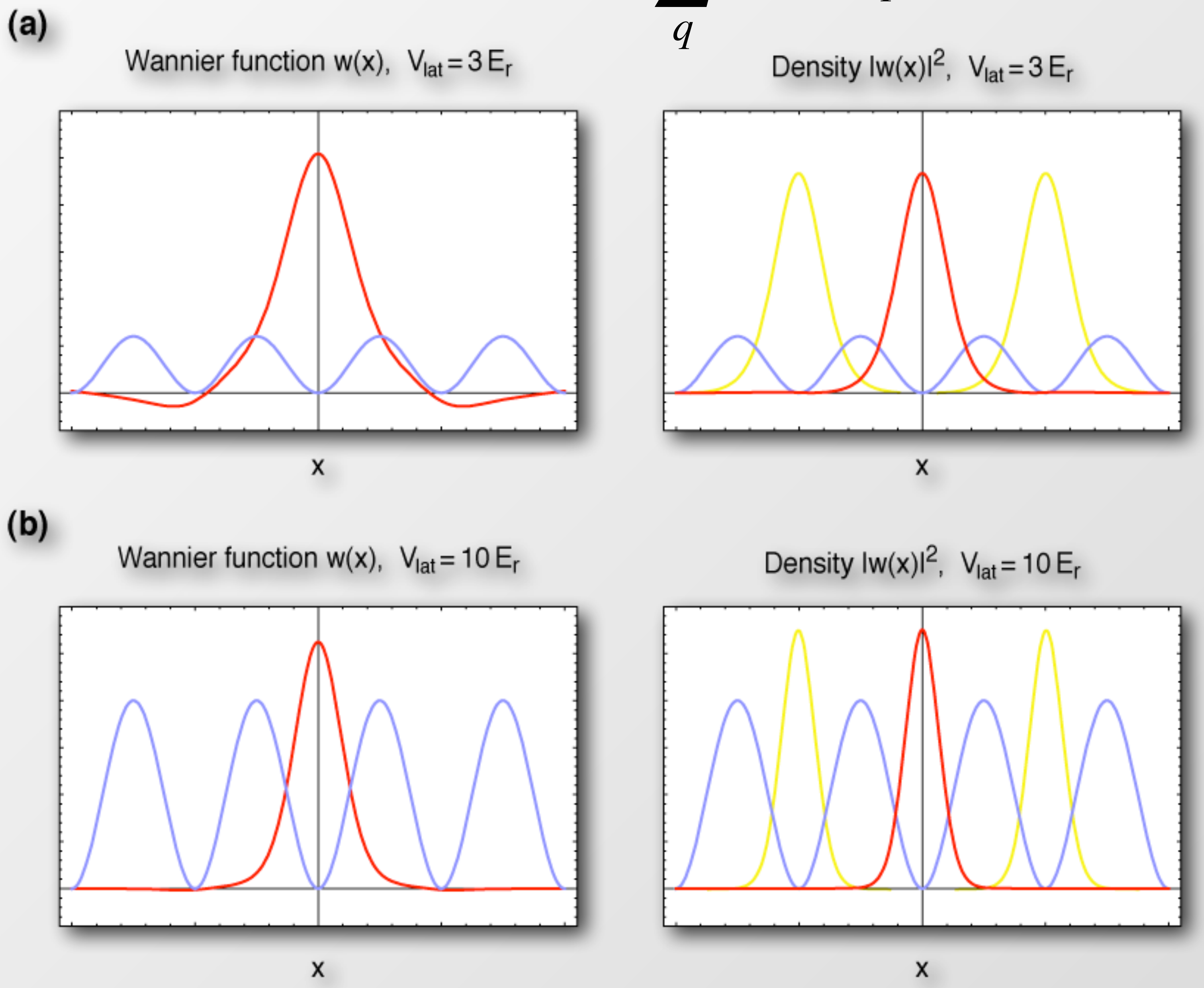
Diagonalization gives us Eigenvalues and Eigenvectors!

Bandstructure - Blochwaves



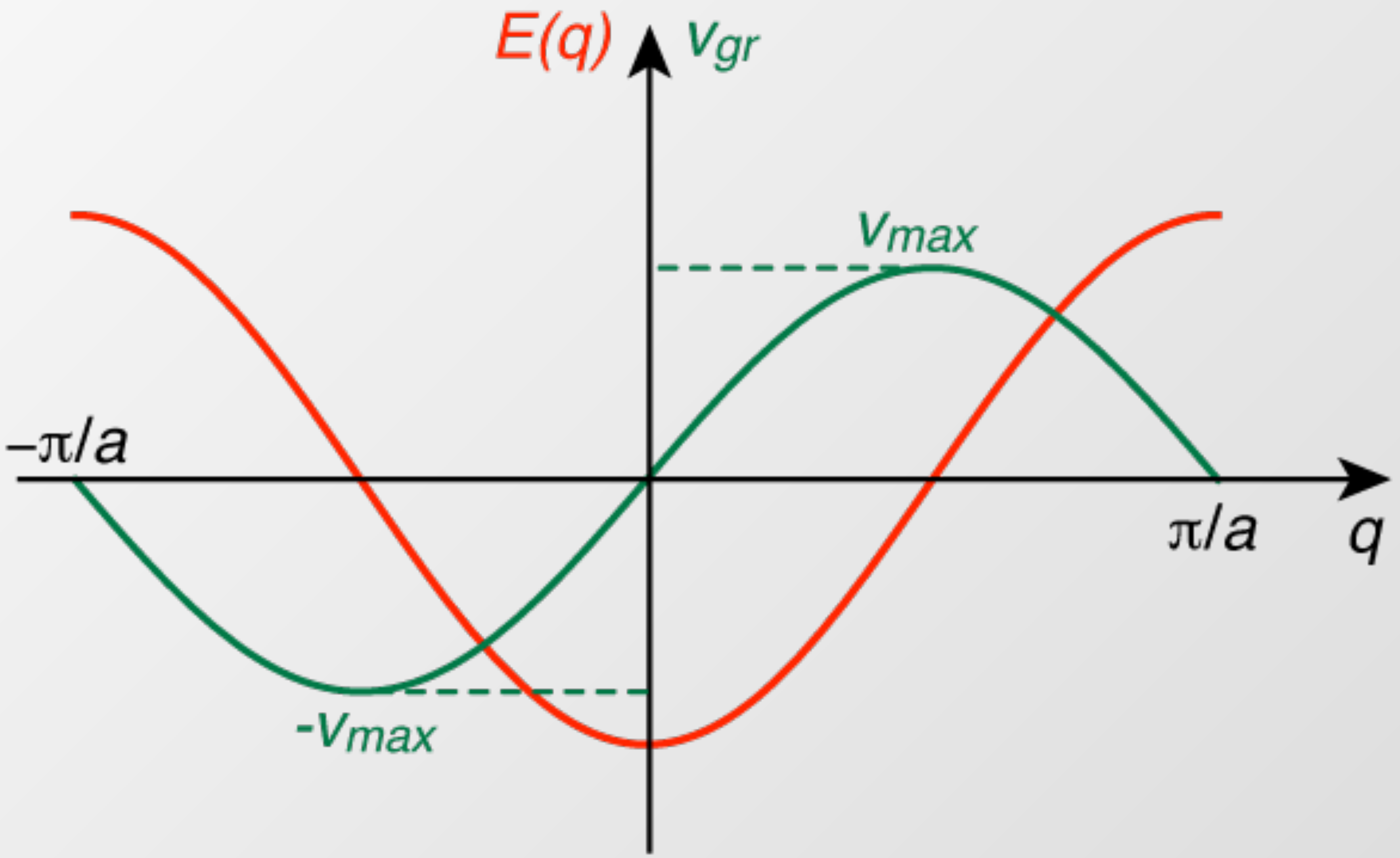
An alternative basis set to the Bloch waves can be constructed through localized wave-functions: Wannier Functions!

$$w_n(x - x_i) = \mathcal{N}^{-1/2} \sum_q e^{-iqx_i} \phi_q^{(n)}(x)$$



Dispersion Relation in a Square Lattice

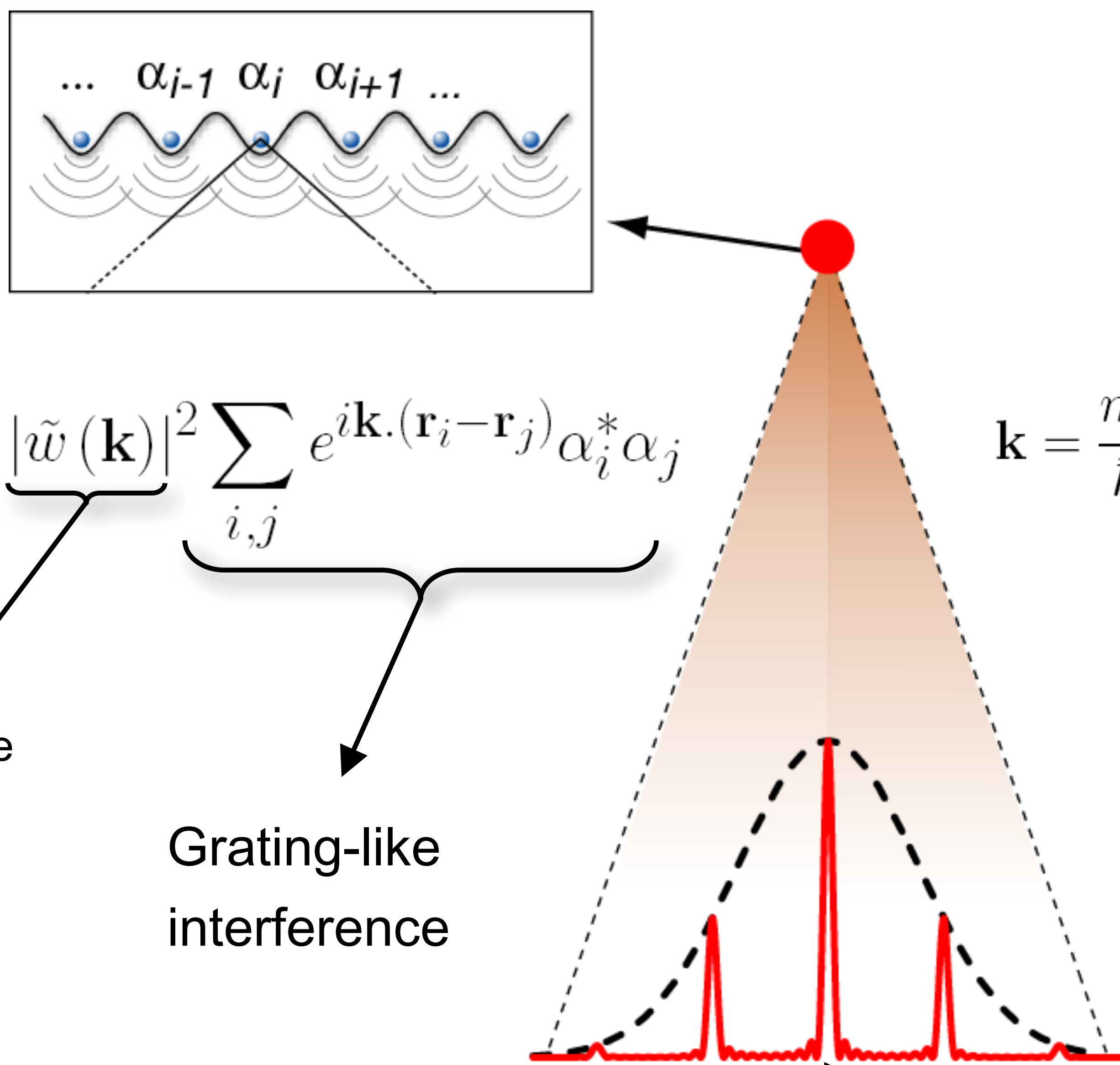
$$E(q) = -2J\cos(qa)$$



Measuring
Momentum Distributions

Time of flight interference pattern

- Interference between all waves coherently emitted from each lattice site



$$\tilde{n}(\mathbf{k}) = \underbrace{|\tilde{w}(\mathbf{k})|^2}_{\text{Wannier envelope}} \underbrace{\sum_{i,j} e^{i\mathbf{k}.(\mathbf{r}_i - \mathbf{r}_j)} \alpha_i^* \alpha_j}_{\text{Grating-like interference}}$$

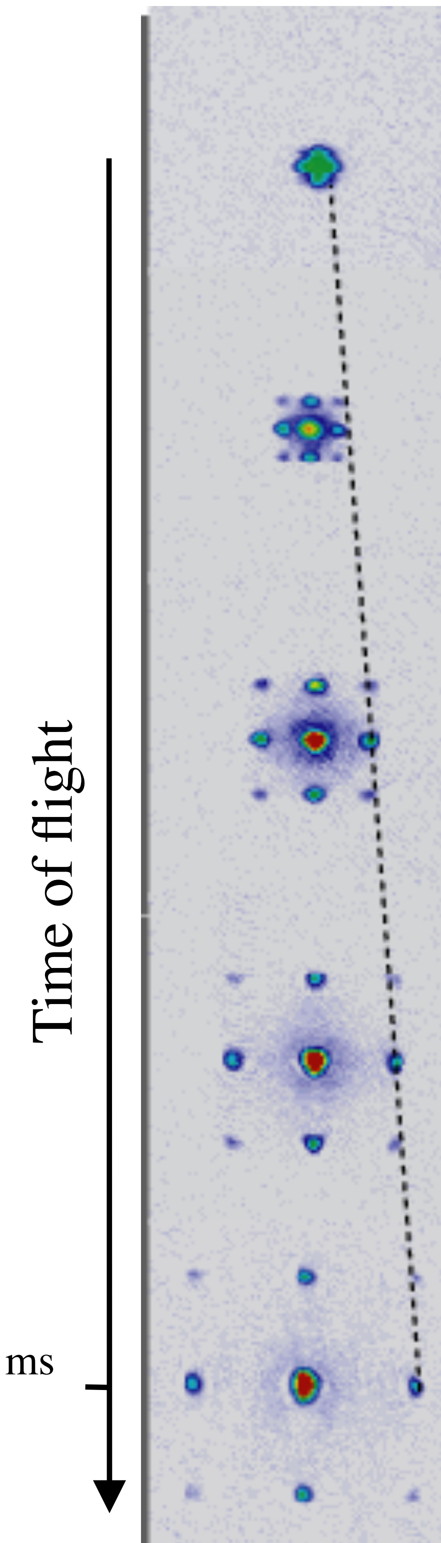
$$\mathbf{k} = \frac{m\mathbf{r}}{\hbar t}$$

Wannier envelope

Grating-like interference

Periodicity of the reciprocal lattice

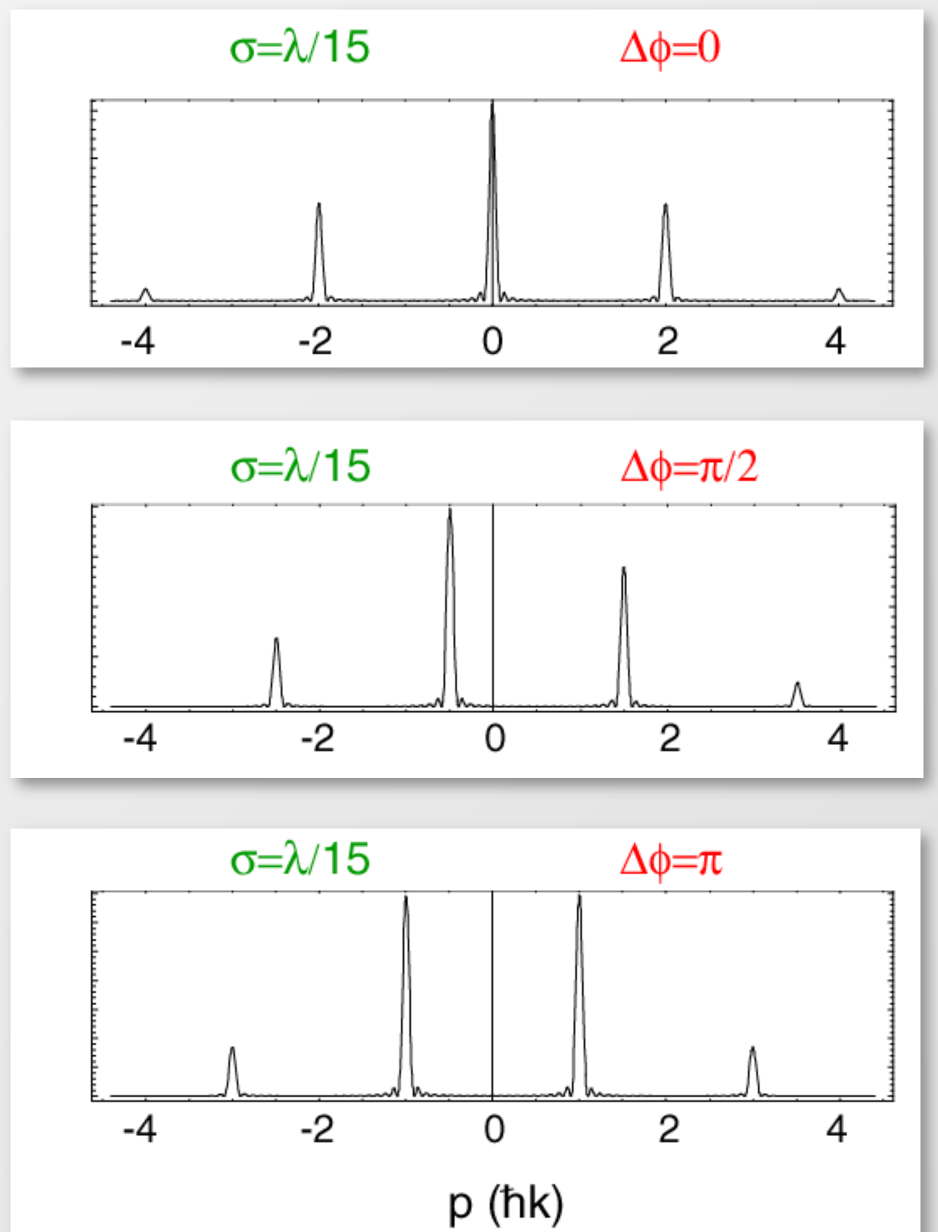
$$l = \frac{2\hbar k_{\mathrm{L}} t}{m}$$



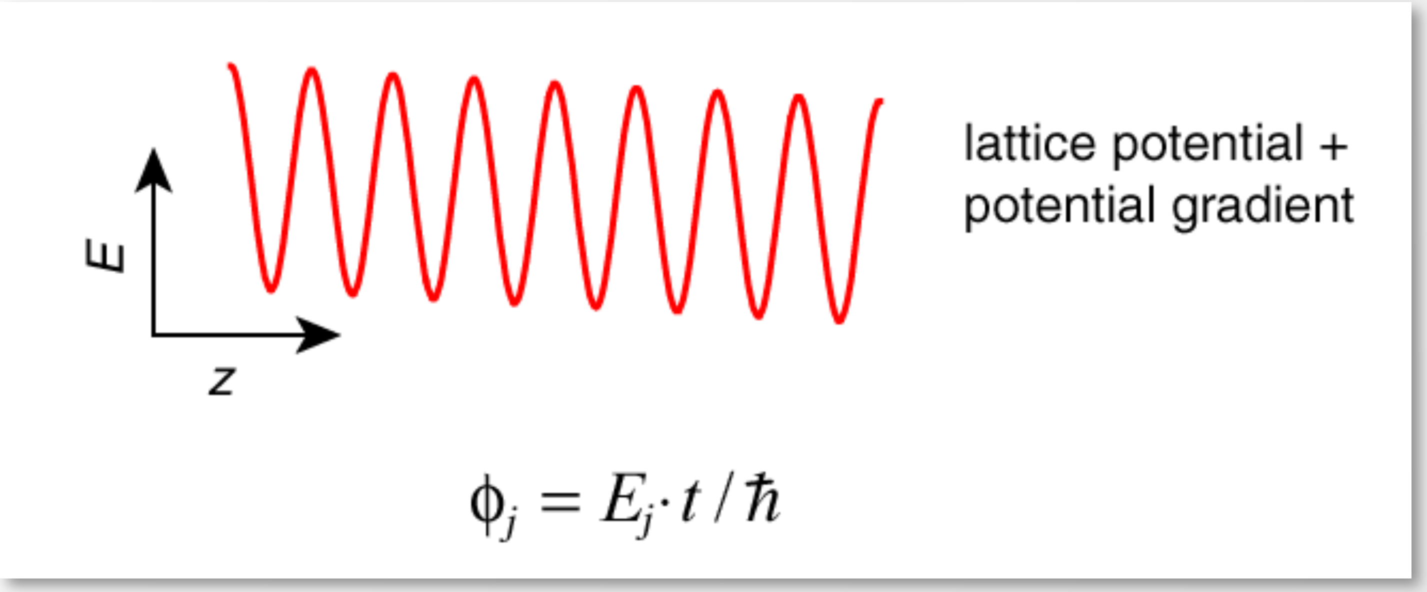
Momentum Distributions – 1D

Momentum distribution can be obtained by Fourier transformation of the macroscopic wave function.

$$\Psi(x) = \sum_i A(x_j) \cdot w(x - x_j) \cdot e^{i\phi(x_j)}$$



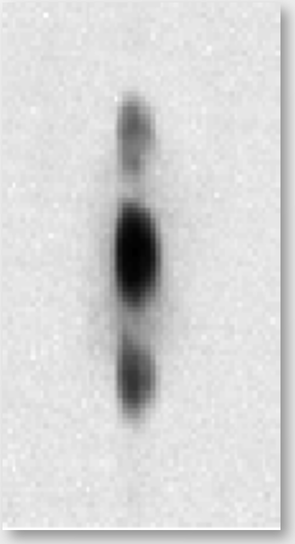
Preparing Arbitrary Phase Differences Between Neighbouring Lattice Sites



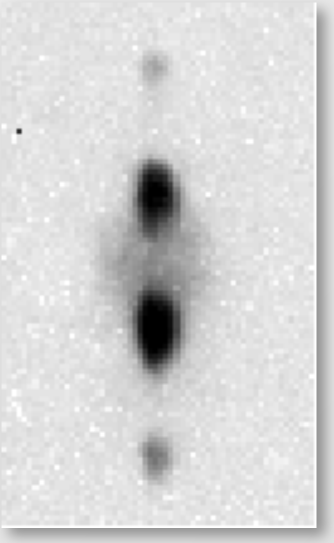
Phase difference between neighboring lattice sites

$$\Delta\phi_j = (V'\lambda/2)\,\Delta t$$

(cp. Bloch-Oscillations)



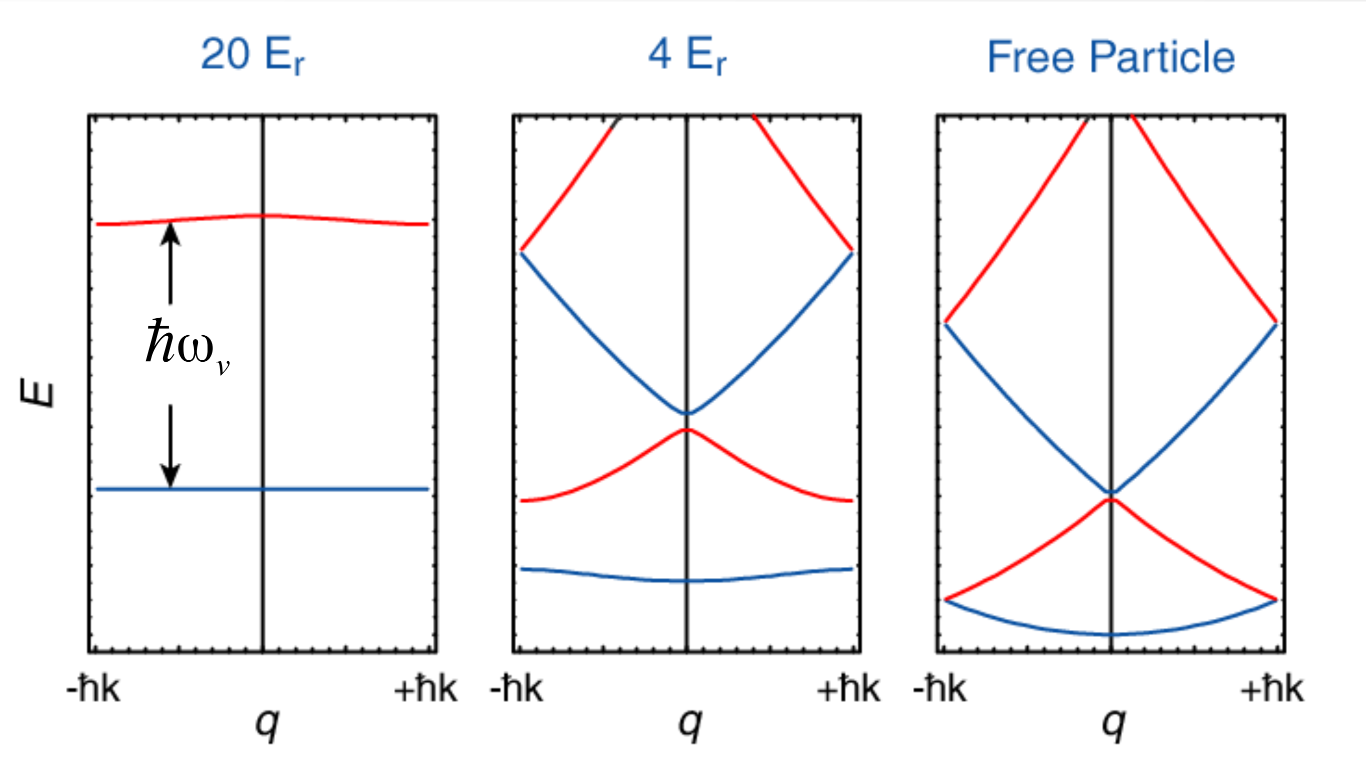
$\Delta\phi = 0$



$\Delta\phi = \pi$

But: dephasing if gradient is left on for long times !

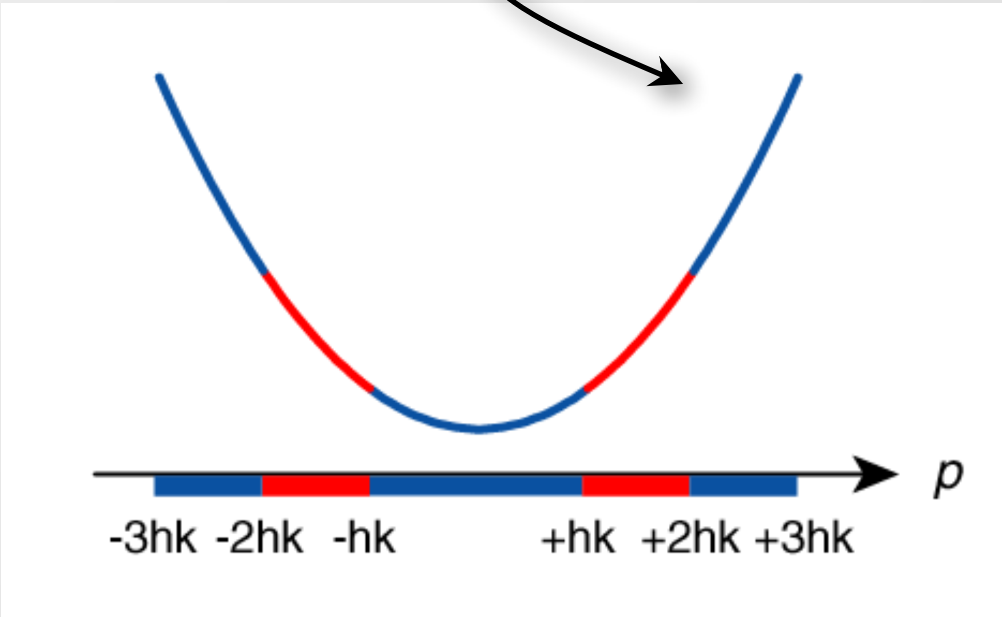
Mapping the Population of the Energy Bands onto the Brillouin Zones



Crystal momentum

Crystal momentum is conserved while lowering the lattice depth adiabatically !

Population of n^{th} band is mapped onto n^{th} Brillouin zone !

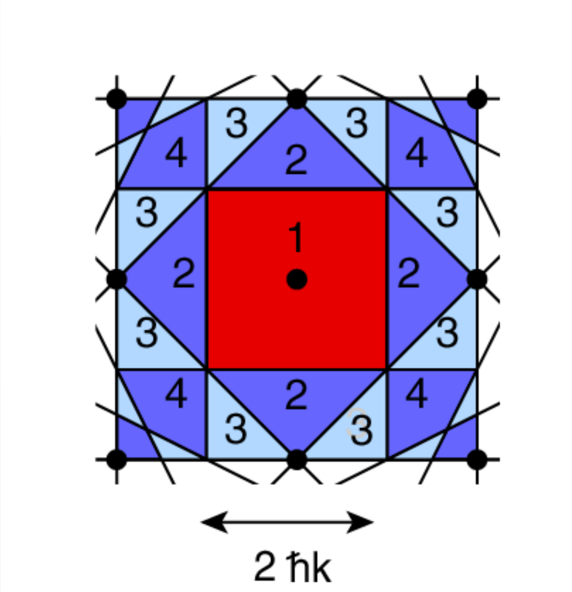


Free particle momentum

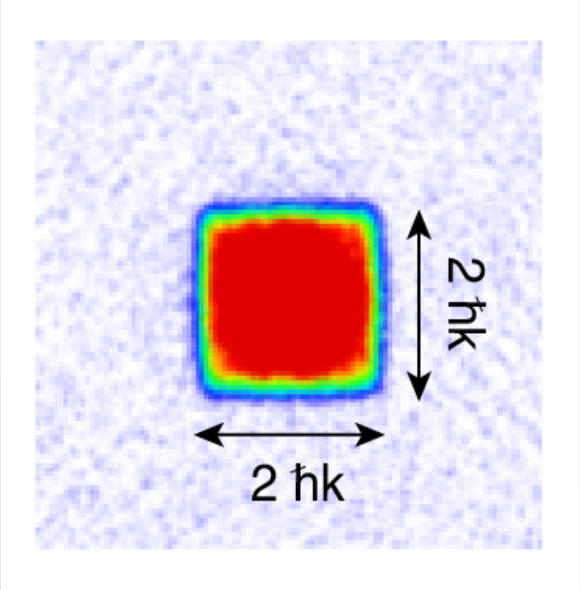
A. Kastberg et al. PRL 74, 1542 (1995)
M. Greiner et al. PRL 87, 160405 (2001)

Experimental Results

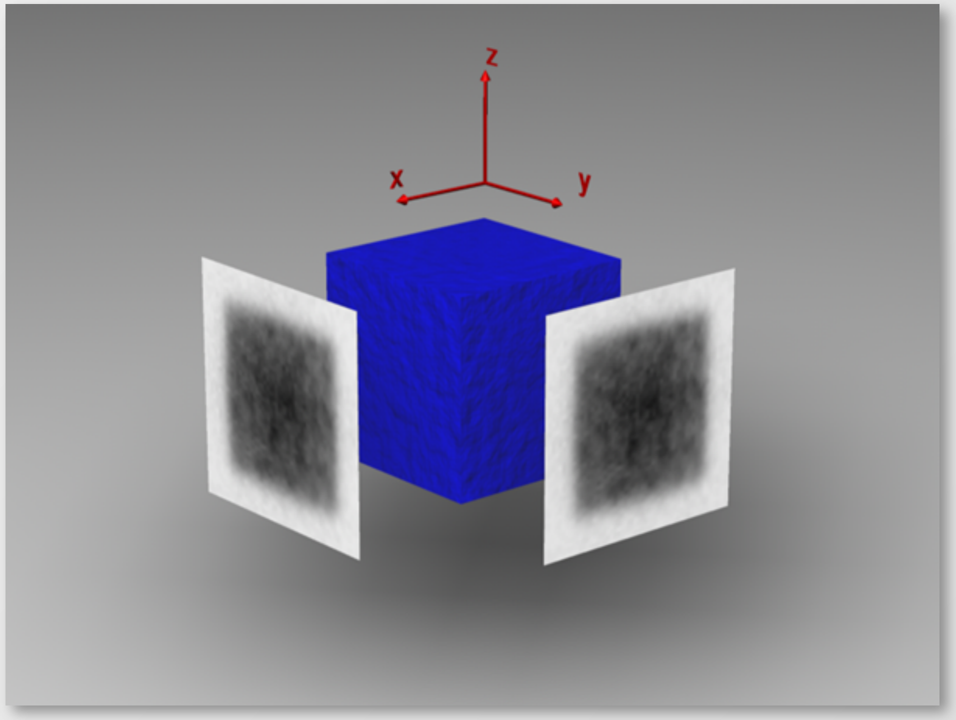
Brillouin Zones in 2D



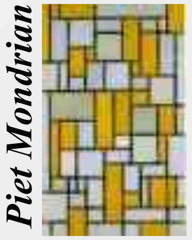
Momentum distribution of a dephased condensate after turning off the lattice potential adiabtically



2D

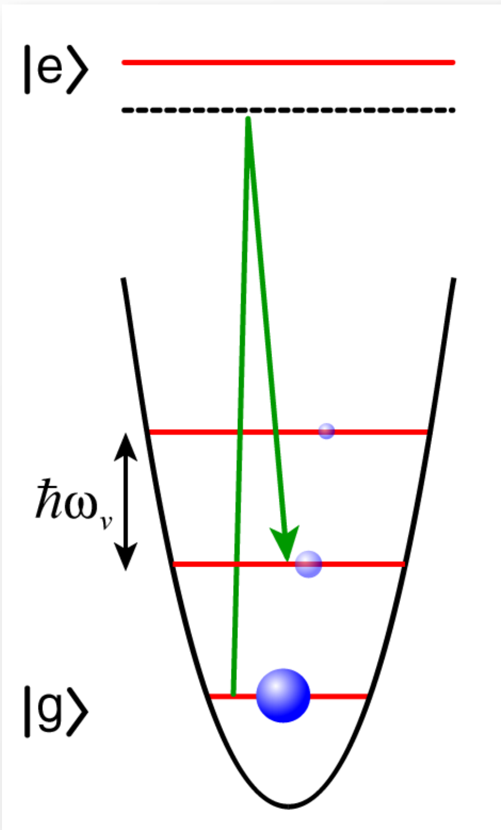


3D



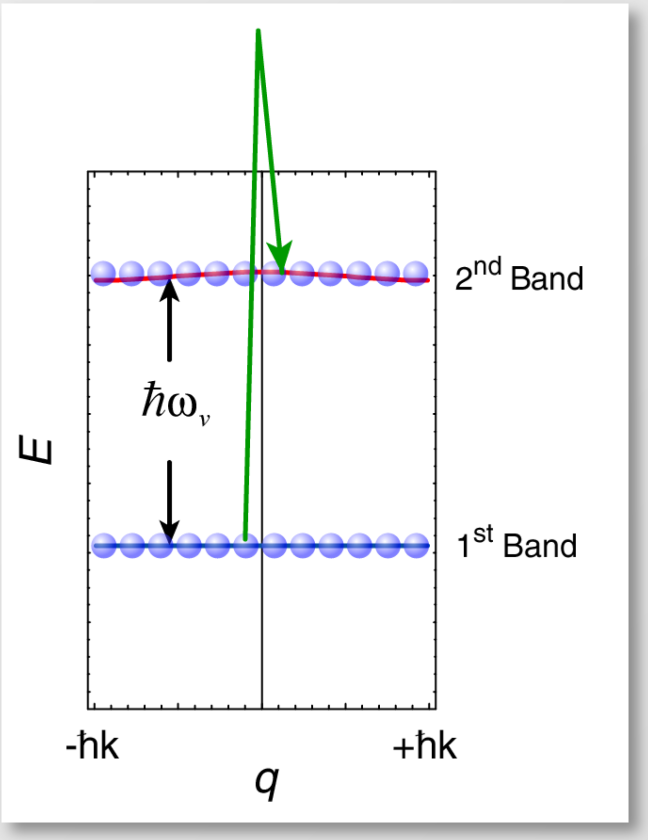
Populating Higher Energy Bands

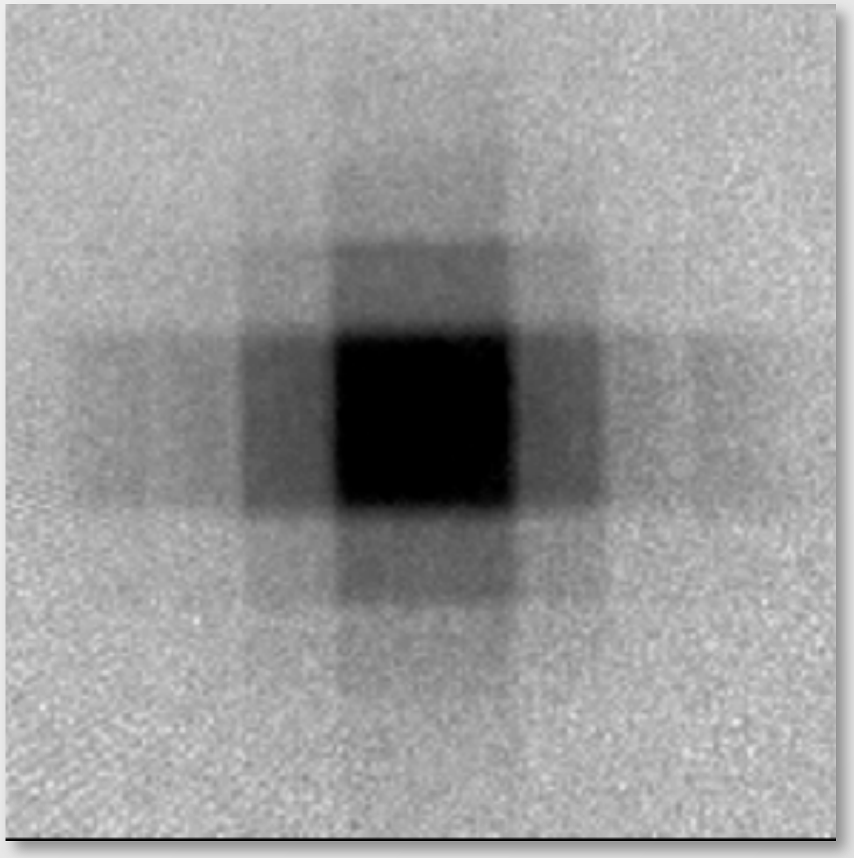
Single lattice site



Stimulated Raman transitions between vibrational levels are used to populate higher energy bands.

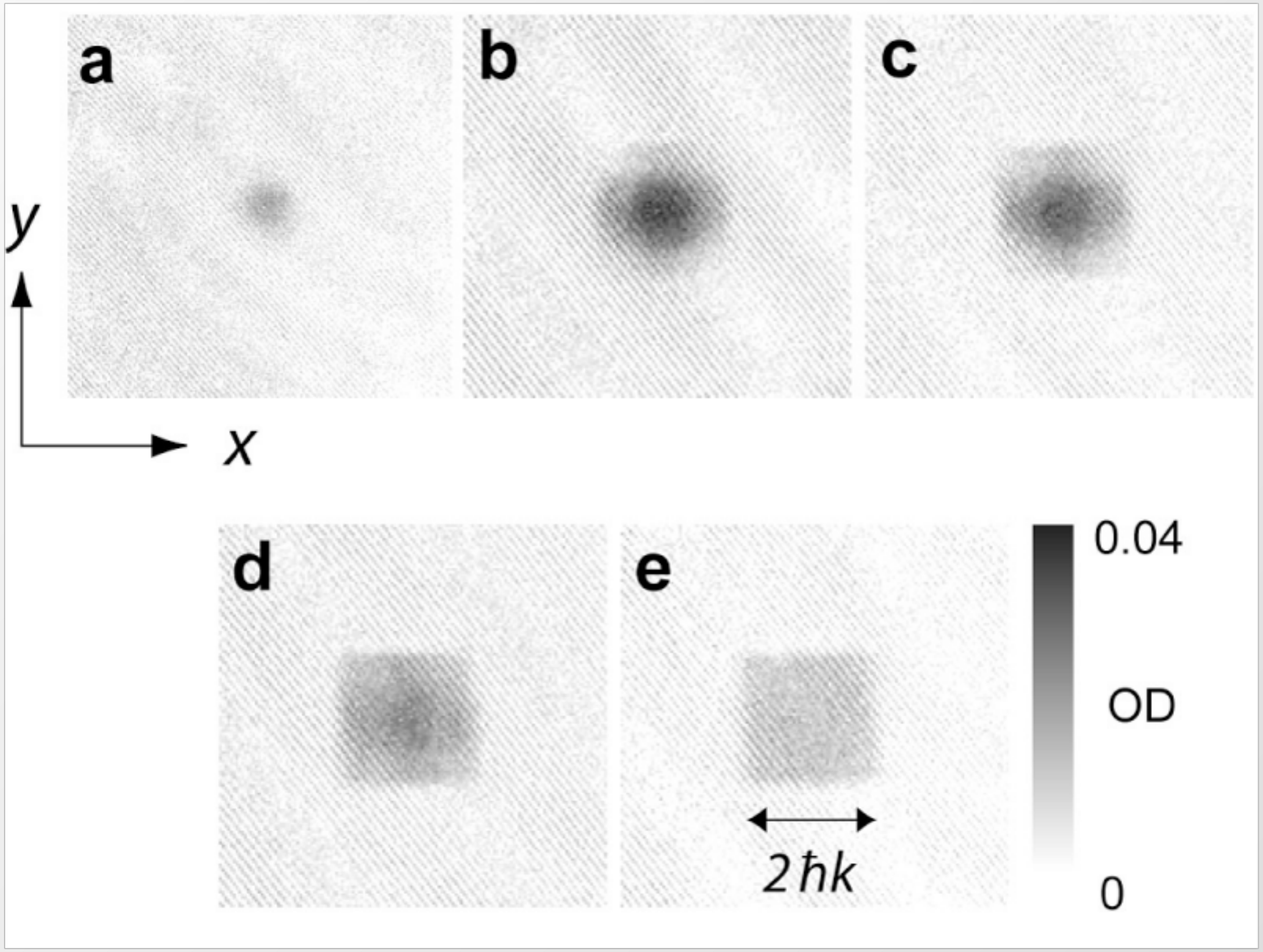
Energy bands





Measured Momentum Distribution !

From a Conductor to a Band Insulator



Fermi Surfaces become directly visible!

M. Köhl et al. Physical Review Letters (2005)