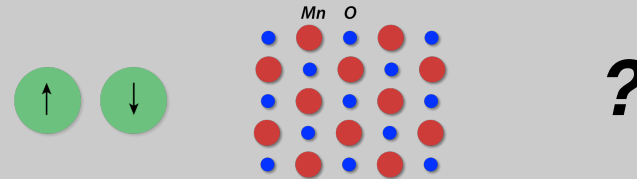


Superexchange Interactions

Origin of Spin-Spin Interactions – Exchange Interactions

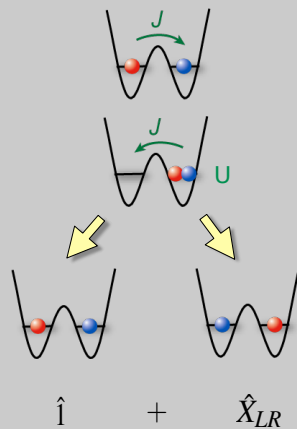


Important ionic solids with **no direct exchange** between magnetic ions show magnetic ordering (**MnO, CuO**)!

„Super“-exchange interactions must be at work!

Deriving the Effective Spin Hamiltonian (1)

How do we get from $-J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$ to $H = -J_{ex} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$?



Deriving the Effective Spin Hamiltonian (2)

Second order hopping can be written as

$$H = -2 \frac{J^2}{U} (1 + \hat{X}_{LR})$$

$$\hat{X}_{LR} \left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \right] = - \left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \right]$$

$$\hat{X}_{LR} \left[\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} \right] = + \left[\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} \right]$$

$$H = -J_{ex} \hat{P}_{\text{triplet}}$$

- 0 Singlet
- ≡ -J Triplet

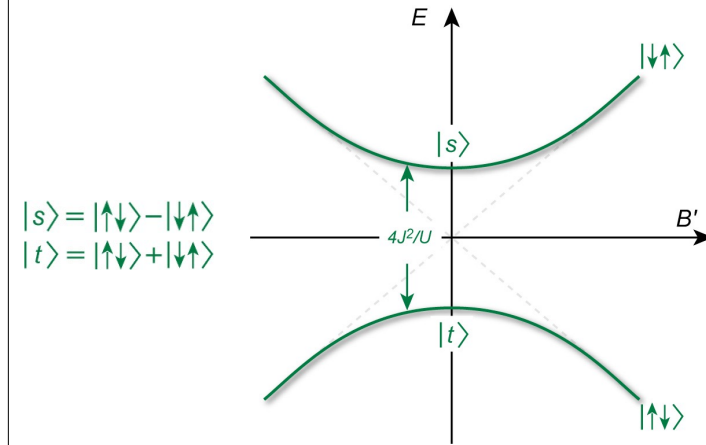
Deriving the Effective Spin Hamiltonian (3)

$$\hat{P}_{\text{triplet}} = \hat{P}_{S=1}$$

$$\begin{aligned} \mathbf{S}_L \cdot \mathbf{S}_R &= \frac{(\mathbf{S}_L + \mathbf{S}_R)^2}{2} - \frac{3}{4} \\ &= \frac{S(S+1)}{2} - \frac{3}{4} \\ &= \hat{P}_{S=1} - \frac{3}{4} \end{aligned}$$

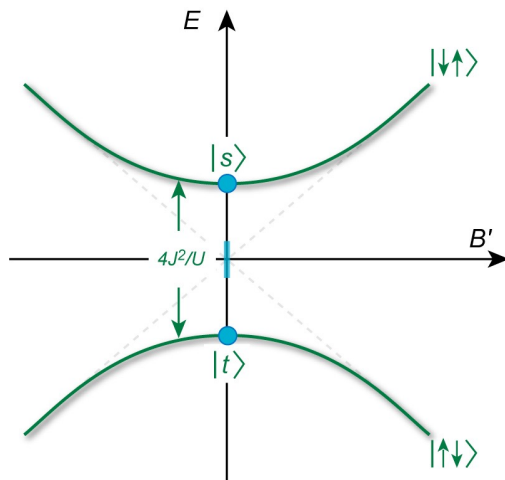
$$H = -J_{ex} \left(\mathbf{S}_L \cdot \mathbf{S}_R + \frac{3}{4} \right)$$

Direct Detection of Superexchange Interactions

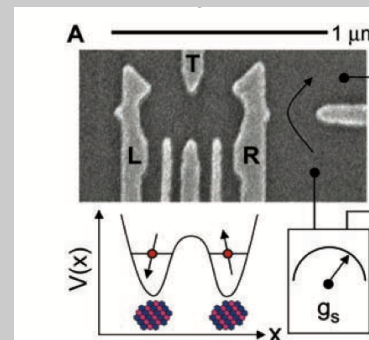


$$H_{\text{eff}} = -J_{ex} \vec{S}_L \cdot \vec{S}_R - \mu_B B' (S_{z,L} - S_{z,R})$$

Direct Detection of Superexchange Interactions (2)



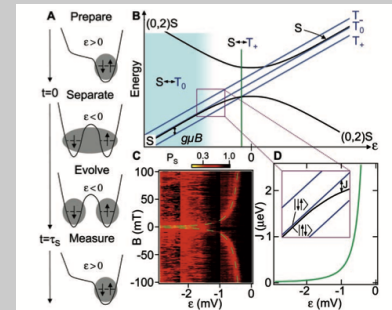
Superexchange Coupling in Quantum Dots



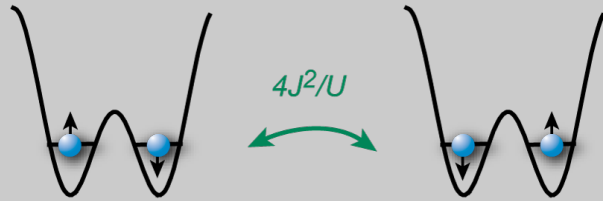
Local control of spin states & interactions between spin states.

J.R. Petta et al., Science **309**, 2180 (2005)

Coherent Manipulation of Coupled Electron Spins in Semiconductor Quantum Dots



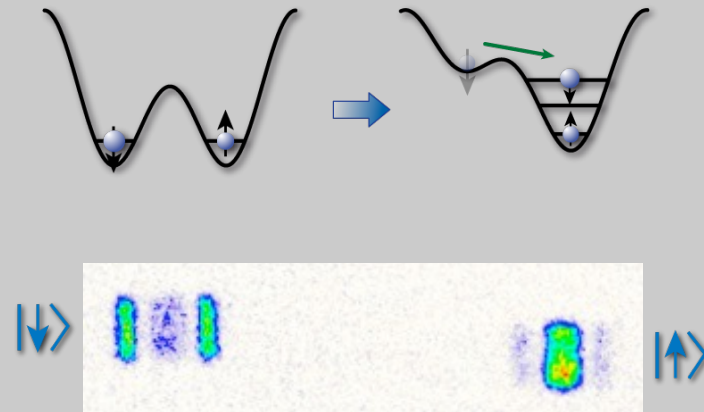
Superexchange induced flopping



$$H_{eff} = -J_{ex} \vec{S}_i \cdot \vec{S}_j$$

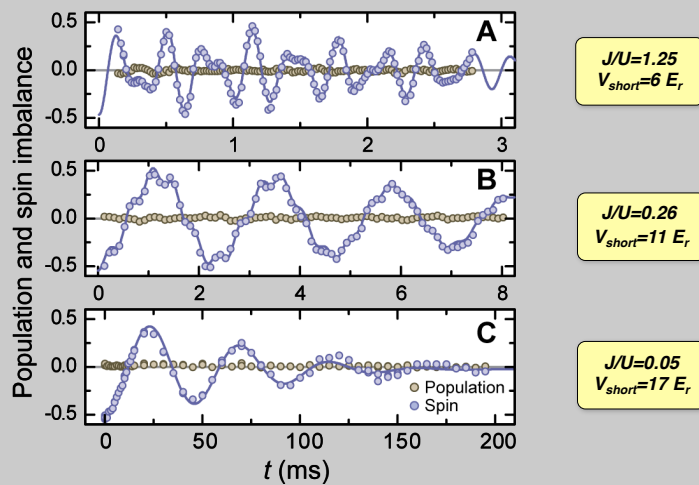
$$= -\frac{J_{ex}}{2} (\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+) - J_{ex} \hat{S}_i^z \hat{S}_j^z$$

Mapping the Spins



Initial AF order verified in the experiment!

Superexchange induced flopping



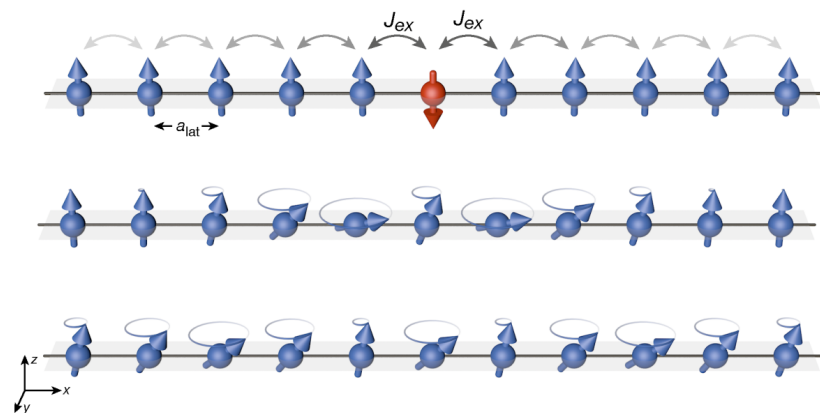
Quantum Dynamic of Mobile Single Spin Impurity

T. Fukuhara, M. Endres, M. Cheneau P. Schauss, Ch. Gross, I. Bloch, S. Kuhr,
U. Schollwöck, A. Kantian, Th. Giamarchi

Sherson et al. Nature 467, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

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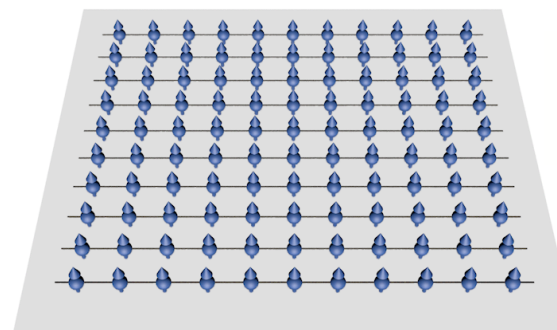
Spin impurity dynamics



$$-J_{ex} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \quad \text{Ferromagnetic Heisenberg Interaction}$$

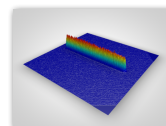


Spin impurity dynamics



$$|2\rangle = |F=2, m_F=-2\rangle$$

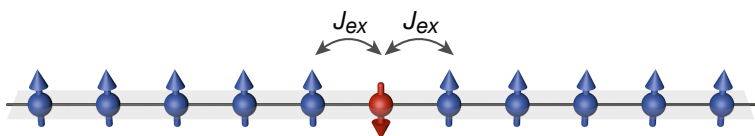
$$|1\rangle = |F=1, m_F=-1\rangle$$



Line-shaped light field created with DMD SLM



Spin impurity dynamics



Heisenberg Hamiltonian

$$H = -J_{ex} \sum \mathbf{S}_i \cdot \mathbf{S}_j = -J_{ex} \sum \left(S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z \right)$$

$$= -\frac{J_{ex}}{2} \sum \left(S_i^+ S_j^- + S_i^- S_j^+ \right) - J_{ex} \sum S_i^z S_j^z$$

$$J_{ex} = 4 \frac{J^2}{U}$$

$$H = -J \sum \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger \right) \quad \text{single particle tunneling}$$



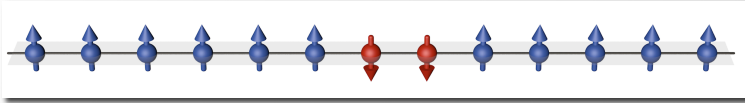
Direct Observation of Magnon Bound States

T. Fukuhara, P. Schauss, S. Hild, J. Zeiher, M. Cheneau, M. Endres, I. Bloch, Ch. Gross

T. Fukuhara et al., Nature **502**, 76 (2013)

for photons: O. Firstenberg et al., Nature **502**, 71 (2013)

Magnon Bound States

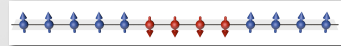


Hans Bethe (1906-2005)

There can be bound states in a Heisenberg spin chain!
Development of **Bethe Ansatz**.

$$H = -J_{ex} \sum_i (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

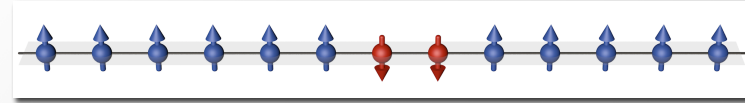
General **l-string bound states**



H. Bethe, Z. Phys. (1931)
M. Wortis, Phys. Rev. (1963)
M. Takahashi & M. Suzuki Prog. Th. Phys. (1972)
M. Karbach, G. Müller (1997)
see also: **repulsively bound pairs & interacting atoms**
K. Winkler et al. Nature (2006); S. Fölling et al. Nature (2007); Y Lahini et al. PRA (2012)



Magnon Bound States



Hans Bethe (1906-2005)

There can be bound states in a Heisenberg spin chain!
Development of **Bethe Ansatz**.

$$H = -J_{ex} \sum_i (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

$$H = -\frac{J_{ex}}{2} \sum_i (\hat{S}_i^+ \hat{S}_{i+1}^- + \hat{S}_i^- \hat{S}_{i+1}^+) - \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

H. Bethe, Z. Phys. (1931)
M. Wortis, Phys. Rev. (1963)
M. Takahashi & M. Suzuki Prog. Th. Phys. (1972)
M. Karbach, G. Müller (1997)
see also: **repulsively bound pairs & interacting atoms**
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Bound l-string

Eigenenergies:

$$E(k) = -J_{ex} \frac{\sin(\nu)}{\sin(l\nu)} \left\{ \cos(l\nu) - (-1)^l \cos k \right\}$$



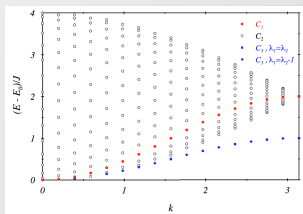
l-string

$$\Delta = \cos(\nu)$$

Maximum propagation velocity:

$$v_{max,1} = \frac{\sin(\nu)}{\sin(l\nu)}$$

$$v_{max,2} = \frac{J}{2\Delta}$$



M. Karbach & G. Müller (1997)

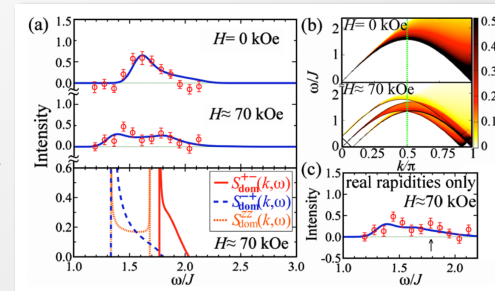
Bound magnon



A Challenge for CM Physics

Very difficult to observe in spectroscopic data in real materials!

theory with bound states



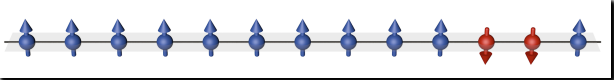
theory without bound states

M. Kohno, Phys. Rev. Lett. 102, 037203 (2009)

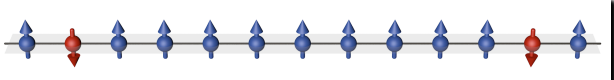


Magnon Bound State Dynamical Evolution


Bound Magnon Motion




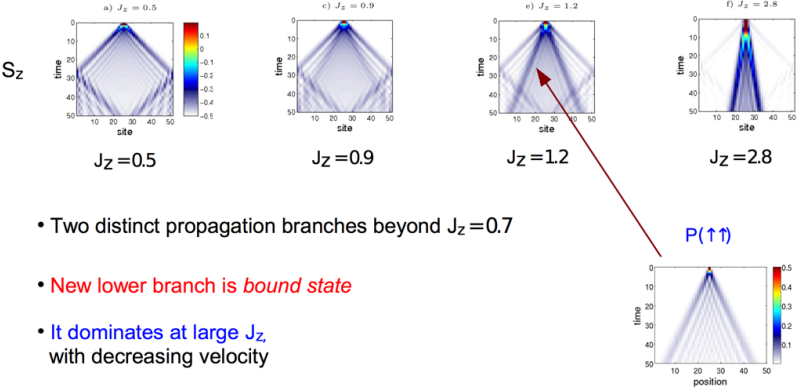
Breakup and Single Spin Motion



M. Ganahl et al., Phys. Rev. Lett. (2012)



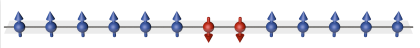
Two-spin excitation in FM 



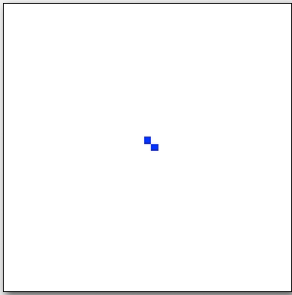
- Two distinct propagation branches beyond $J_z = 0.7$
- **New lower branch is bound state**
- **It dominates at large J_z with decreasing velocity**
- Low entanglement entropy (see below)

From: H.G. Evertz
M. Ganahl et al., Phys. Rev. Lett. (2012)

Magnon Bound State Dynamical Evolution

Initial State: 


Pair distribution evolution $P(x_1, x_2)$



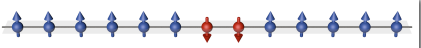
$\Delta = 0$
Non-Interacting

x_1 x_2

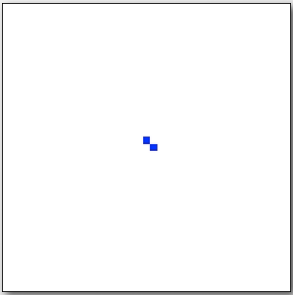
see also: two interacting atoms
Y. Lahini et al., PRA 86, 011603 (2012)



Magnon Bound State Dynamical Evolution


Initial State: 

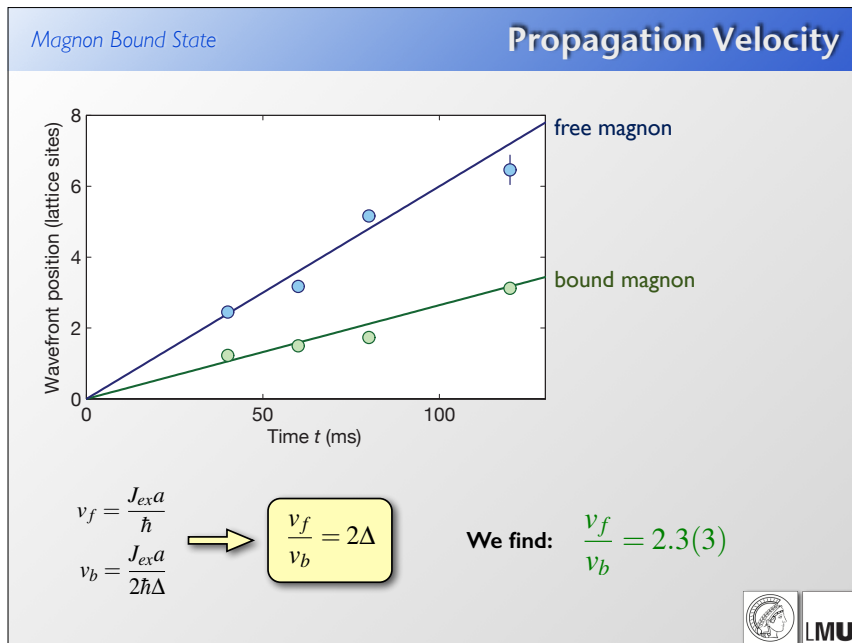
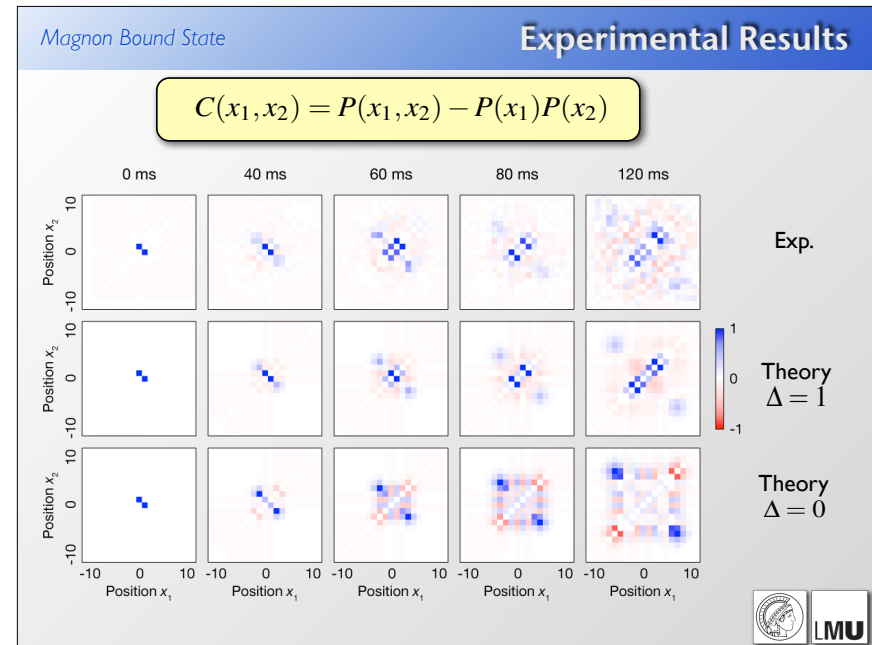
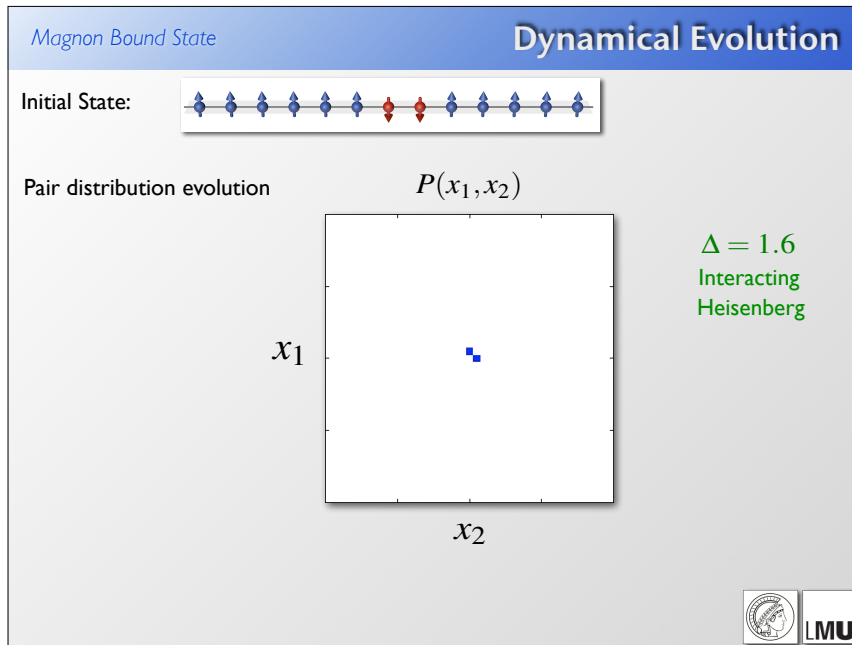
Pair distribution evolution $P(x_1, x_2)$



$\Delta = 1$
Interacting
Isotropic Heisenberg

x_1 x_2





Quantum Magnetism Outlook

Quantum Dynamics of Interacting Atoms/Spins

- Effect of Temperature/Holes on Dynamics
- Dynamics of **l-string bound states**
- Domain Walls
- Higher Dimensions (1D, 2D, 3D)
- Entropy Transport
- Probe for **Quantum Critical Transport**
- Direct measurement of **Green's function**

$$G(x_i, x_j, t) \propto \langle \uparrow | \hat{S}^\dagger(x_j, t) \hat{S}^-(x_i, 0) | \uparrow \rangle$$

M. Knap et al. PRL 111, 147205 (2013)

Controlling and Detecting Spin Correlations

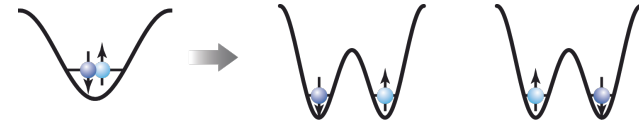
S. Trotzky et al., Phys. Rev. Lett 105, 265303 (2010)

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Splitting a spin pair

- **Spin pairs** in $|F=1, m_F=\pm 1\rangle = |\uparrow\rangle, |\downarrow\rangle$ (repulsive)
- Barrier raised *slowly* to split
→ Crossing a miniature Mott-transition: $n_{\text{Left}} = n_{\text{Right}} = 1$

J. Sebby-Strabley et al., PRL 98 (2007)



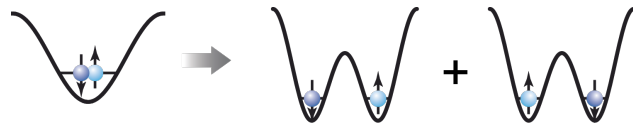
Details on the loading of the Spin-pairs:
S.T., P. Cheinet et al., Science 319 (2008)



Splitting a spin pair

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J. Sebby-Strabley et al., PRL 98 (2007)



- **Bosons:** Symmetric wavefunction → Triplet $|t_0\rangle$
- (Fermions: Antisymmetric wavefunction → Singlet $|s\rangle$)

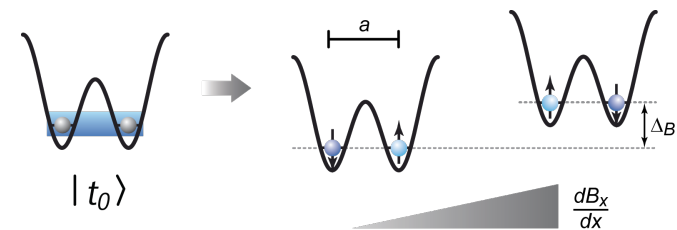
Details on the loading of the Spin-pairs:
S.T., P. Cheinet et al., Science 319 (2008)



Driving Triplet-Singlet oscillations

- **Magnetic field gradient** lifts degeneracy:

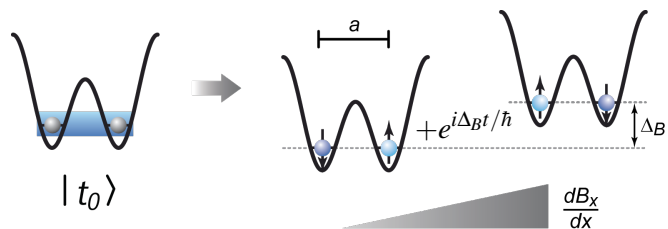
$$\Delta_B \propto a \cdot \partial_x B_x$$



Driving Triplet-Singlet oscillations

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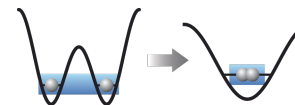


- **Triplet-Singlet oscillations** with frequency Δ_B/\hbar



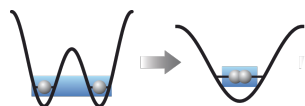
How to detect triplets and singlets

- Barrier lowered slowly to **merge** double-wells
 → **Triplet**: both atoms reach the **ground state**

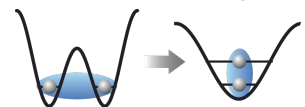


How to detect triplets and singlets

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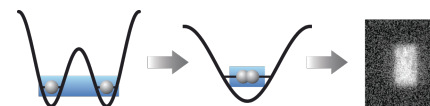


- **Singlet**: needs anti-symm. spatial wavefunction (Bosons)
 One atom transferred to **higher vibrational band**

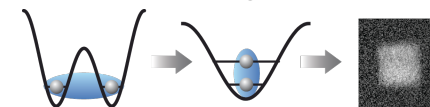


How to detect triplets and singlets

- Barrier lowered slowly to **merge** double-wells
 → **Triplet**: both atoms reach the **ground state**



- **Singlet**: needs anti-symm. spatial wavefunction (Bosons)
 One atom transferred to **higher vibrational band**



Band-mapping reveals **singlet-contribution**
 in higher Brillouin-Zone



A sensitive probe of next-neighbor spin-correlations in Mott-insulator type many-body systems

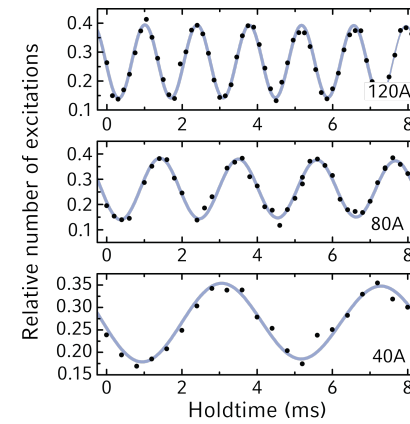
	band excitations		STO amplitude	
	bosons	fermions	bosons	fermions
$ t\rangle$	0%	50%	50%	50%
$ s\rangle$	50%	0%	50%	50%
$ \downarrow, \uparrow\rangle$	25%	25%	0%	0%
$ \uparrow, \downarrow\rangle$	25%	25%	0%	0%
$ \uparrow, \uparrow\rangle$	0%	50%	0%	0%
$ \downarrow, \downarrow\rangle$	0%	50%	0%	0%

→ Capable of probing spin-order in strongly correlated phases at low temperatures

Band-mapping reveals singlet-contribution in higher Brillouin-Zone



Singlet-Triplet oscillations



- Load system and create **spin pairs**
- Split pairs into **triplets**
- Induce **STO** via gradient
- **Merging** and **band-mapping** for detection

→ **Traces of STO** versus holdtime with gradient

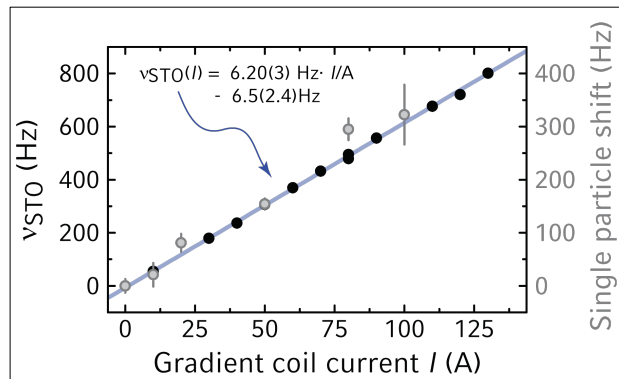
- Vary gradient coil current

S. Trotzky et al., Phys. Rev. Lett. 105, 265303 (2010) & D. Greif et al., Science 340, 1307-1310 (2013)



Singlet-Triplet oscillations

- **Linear increase** in Frequency with gradient strength
- Frequency = **2x single particle shift** (independently meas.)
- confirms **2-particle nature** of oscillations

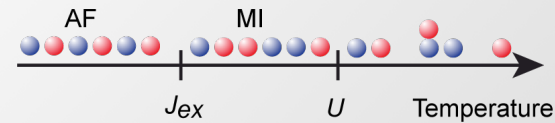


AFM

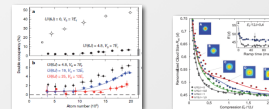
The Quest for AFM Spin Order

Predicted phases at half filling for strong interactions $U/12J > 1$

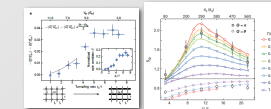
$$\hat{H} = \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + \text{h.c.}) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$



Fermonic Mott Insulator

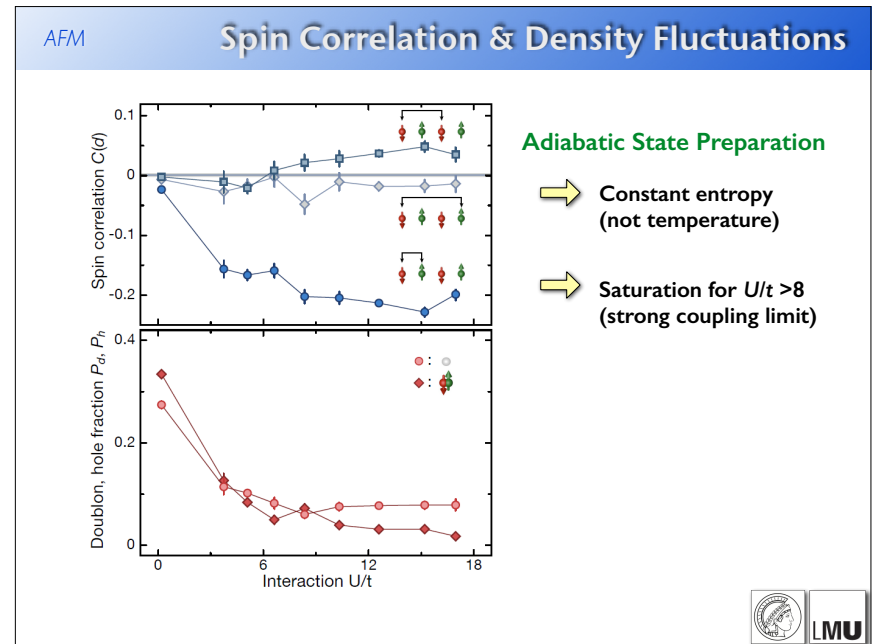
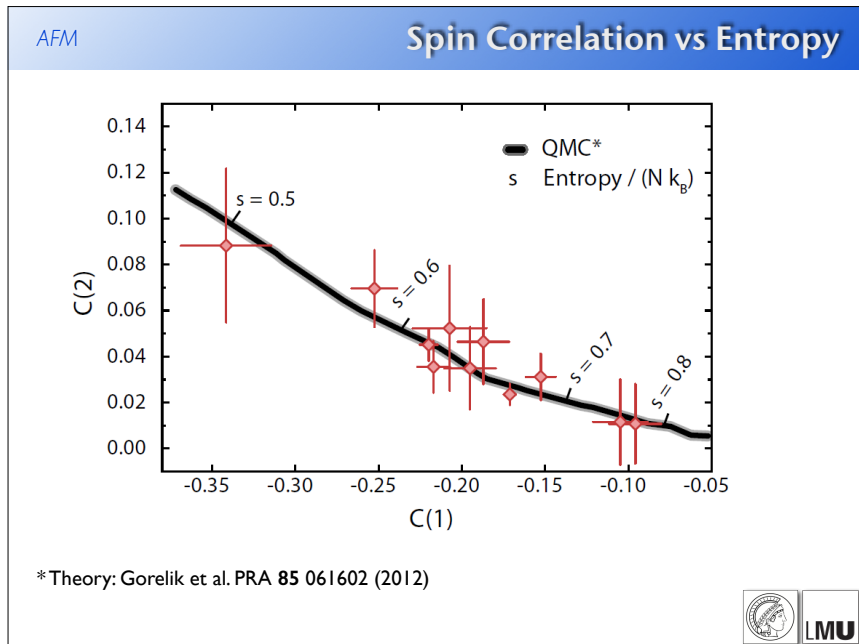
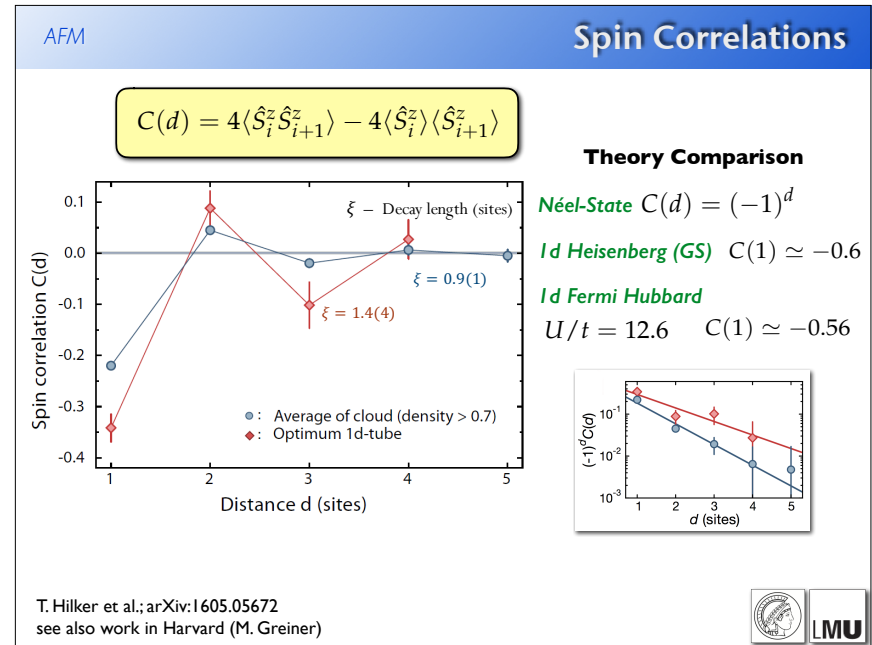
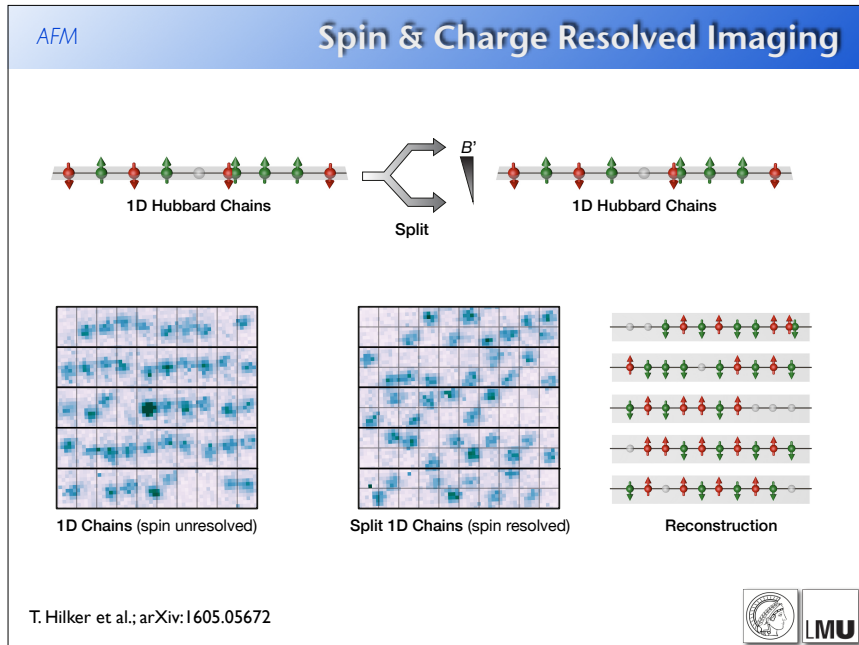


Nearest Neighbour Spin Correlations



R. Jördens et al., Nature 455, 204 (2008).
U. Schneider et al., Science 322, 1520 (2008)

D. Greif et al., Science 340, 1307 (2013)
R. A. Hart et al., Nature 519, 211 (2015)



Summary & Outlook

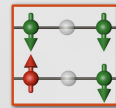
Summary

- ▶ Demonstrated spin & charge resolved imaging of 1d Hubbard chains
- ▶ Measure AFM spin correlations up to d=3-4
- ▶ Minimum Entropy per particle reached $S/N = 0.5 k_B$



Outlook

- ▶ Imaging directly extendable to 2d !
- ▶ Doped Mott insulators (away from half filling)
- ▶ Evaluation of almost arbitrary correlations
- ▶ Example: Spin correlations around holes
- 1d: domain walls, 2d: local ferromagnetism (?)
- Hole attraction ?
- ▶ Get colder



Spin correlations around holes (doped MI)

$$\langle \hat{S}_i^z \hat{n}_{i+1}^h \hat{S}_{i+2}^z \rangle$$



Dynamical Crystallization of Rydberg Atoms

P. Schauss, M. Cheneau, M. Endres, T. Fukuhara, T. Macri, Th. Pohl, I.B. & C. Gross

P. Schauss et al. Nature **491**, 87 (2012)
 P. Schauss et al. [arXiv:1404.9480]

www.quantum-munich.de

Rydberg atoms

- hydrogen-like wave function
- quantum defect

$$E_{nlj} = -\frac{Ry}{[n - \delta_{lj}(n)]^2}$$

⁸⁷Rb 43S_{1/2}

⁸⁷Rb 5S_{1/2}

Ø 0.5nm

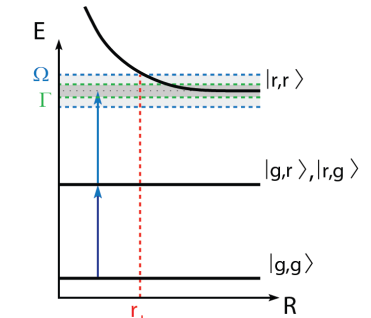
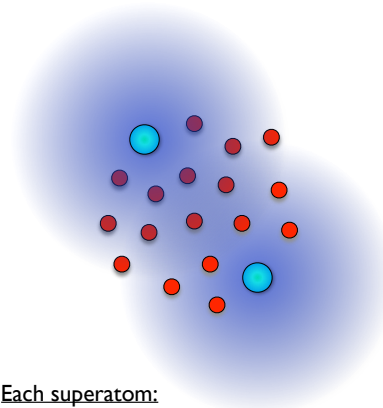
Ø 250 nm

- Strong switchable interactions

Property	Scaling	⁸⁷ Rb 43S
Radius	(n*) ²	2400 a ₀ = 127nm
Lifetime (dominated by black body radiation for large n)	(n*) ²	45 μs @ 20°C
van der Waals coefficient	(n*) ¹¹	C ₆ = -1.7 × 10 ¹⁹ a.u.
Blockade radius (Ω=2π 200 kHz)	(n*) ²	~ 5 μm

Rydberg Crystals

Rydberg blockade



Blockade condition

$$\mathcal{V}_{vdW} = \frac{C_6}{r^6} > \hbar \max(\Gamma, \Omega)$$

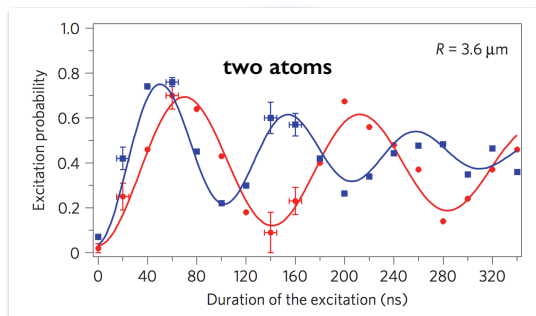
Each superatom:

$$\frac{1}{\sqrt{N}} (|r,0,0,0,\dots\rangle + |0,r,0,0,\dots\rangle + |0,0,0,\dots,r\rangle)$$

$$r_b \equiv \sqrt[6]{\frac{C_6}{\hbar\Omega}}$$



Rydberg blockade



blockade radius
larger than cloud size!

$\sqrt{N}\Omega_1$ Rabi Oscillations speed up!

Each superatom:

$$\frac{1}{\sqrt{N}} (|r, 0, 0, 0, \dots\rangle + |0, r, 0, 0, \dots\rangle + |0, 0, 0, \dots, r\rangle)$$

M. Lukin et al. PRL **87**, 037901 (2001)

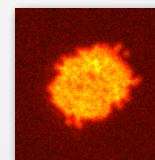
see work by A. Browaeys & Ph. Grangier, M. Saffman, A. Kuzmich, T. Pfau...



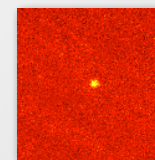
Ultimate Size Control in 2D



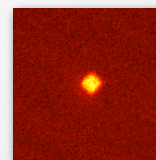
Digital Mirror Device (Size Control)



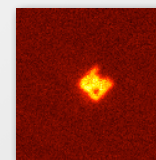
Initial MI



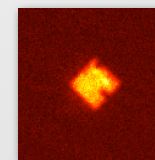
Single Atom



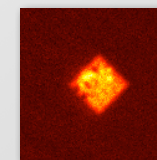
3x3



5x5



7x7

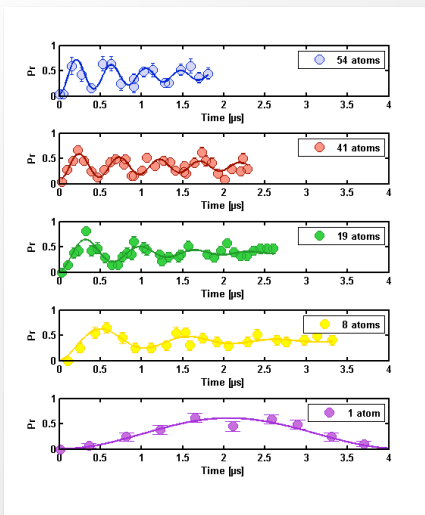


8x8

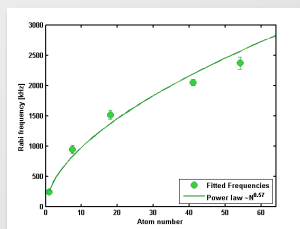
atoms



Collective Many-Body Rabi Oscillations



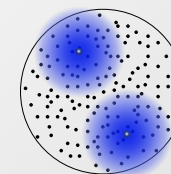
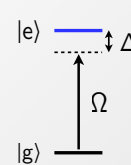
Preliminary Raw Data



Single atom non-linearity controls dynamics of >50 atoms!



The frozen Rydberg gas - long range QM



no mechanical motion on the timescale of the internal dynamics

$$H = \frac{\hbar\Omega}{2} \sum_i (\sigma_{eg}^{(i)} + \sigma_{ge}^{(i)}) + \sum_{i \neq j} \frac{V_{ij}}{2} \sigma_{ee}^{(i)} \sigma_{ee}^{(j)} - \Delta \sum_i \sigma_{ee}^{(i)}$$

coherent coupling

interaction between Rydberg atoms

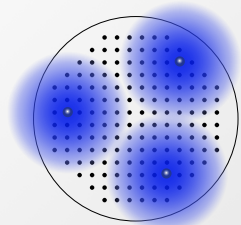
"chemical potential"

$$V_{ij} = C_\alpha |r_i - r_j|^{-\alpha}$$

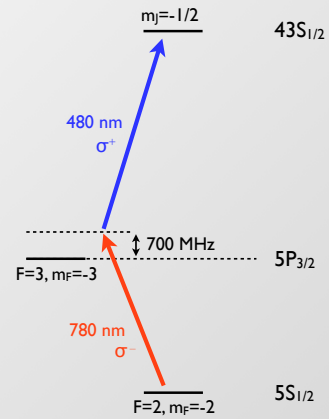
This work: $\alpha=6$, repulsive



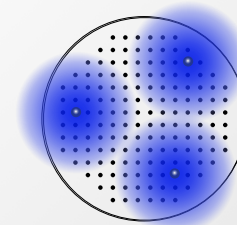
Excitation and detection of the Rydberg atoms



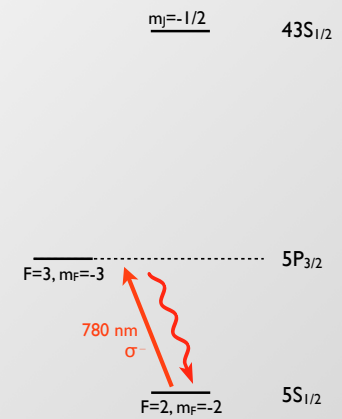
- two-photon Rabi frequency:
 $\Omega/2\pi = 170(20)$ kHz
- resonant excitation:
 $\Delta = 0$
- blockade radius:
 $R_b = 4.9(1)$ μm



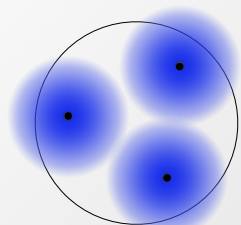
Excitation and detection of the Rydberg atoms



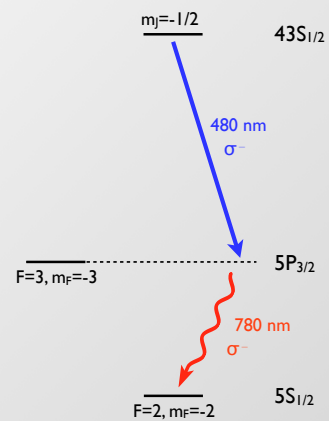
- removal pulse duration: 10 μs
- survival probability: 0.1 %



Excitation and detection of the Rydberg atoms

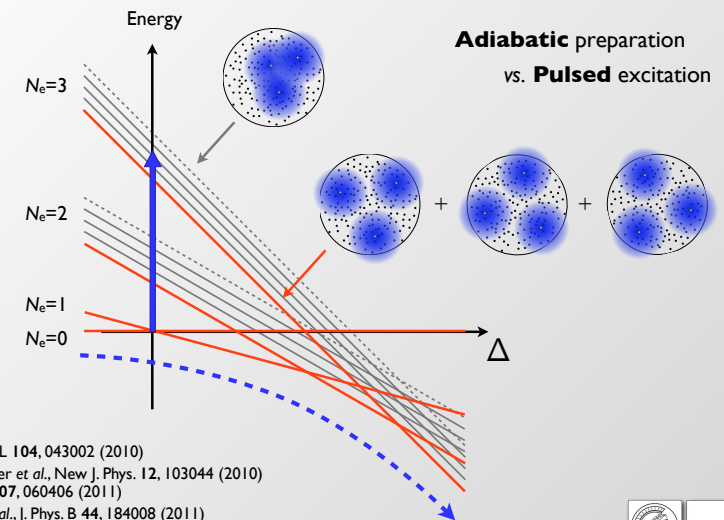


- deexcitation pulse duration: 2 μs
- detection efficiency: 75(10) %
- overall resolution: ~ 500 nm



Rydberg Crystal

Energy spectrum of the Rydberg gas



Pohl et al., PRL 104, 043002 (2010)
 Schachenmayer et al., New J. Phys. 12, 103044 (2010)
 Ji et al., PRL 107, 060406 (2011)
 van Bijnen et al., J. Phys. B 44, 184008 (2011)
 Gärtner et al., arXiv:1203.2884v2 (2012)

Dynamical Crystallization in the Dipole Blockade of Ultracold Atoms

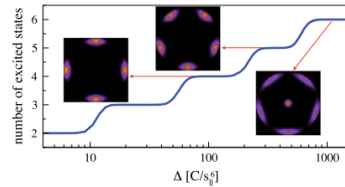
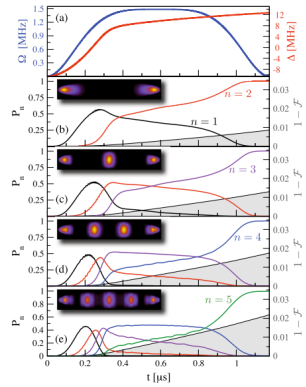
T. Pohl,^{1,2} E. Demler,^{2,3} and M. D. Lukin^{2,3}

¹Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany

²ITAMP-Harvard-Smithsonian Center for Astrophysics, Cambridge Massachusetts 02138, USA

³Physics Department, Harvard University, Cambridge Massachusetts 02138, USA

(Received 26 July 2009; revised manuscript received 23 October 2009; published 27 January 2010)



Coherent Control of Many-Body System through Adiabatic Sweeps

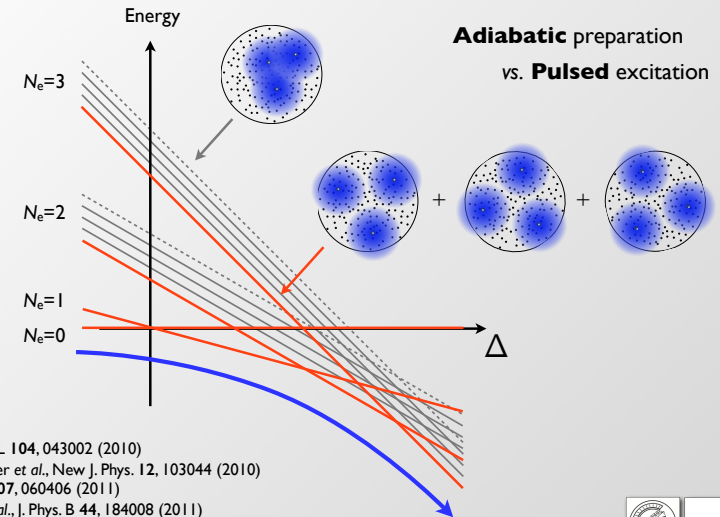
Theory see:

T. Pohl et al. PRL 2010; G. Pupillo et al. PRL 2010,

R.M.W. van Bijnen et al. J. Phys. B: At. Mol. Opt. Phys. (2011)

see also: H. Weimer et al., PRL 2008

Energy spectrum of the Rydberg gas



Pohl et al., PRL 104, 043002 (2010)

Schachenmayer et al., New J. Phys. 12, 103044 (2010)

Ji et al., PRL 107, 060406 (2011)

van Bijnen et al., J. Phys. B 44, 184008 (2011)

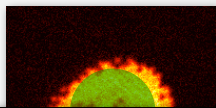
Gärtner et al., arXiv:1203.2884v2 (2012)



Ultimate Size Control in 2D



Digital Mirror Device (Size Control)

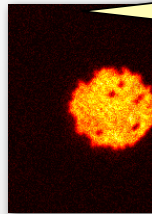


Fluctuating Size and

•Sub Shot Noise Atom Number Preparation

•Geometric & atom number control
(crucial e.g. for quantum criticality)

•Hard wall potentials realized
(crucial for edge states)

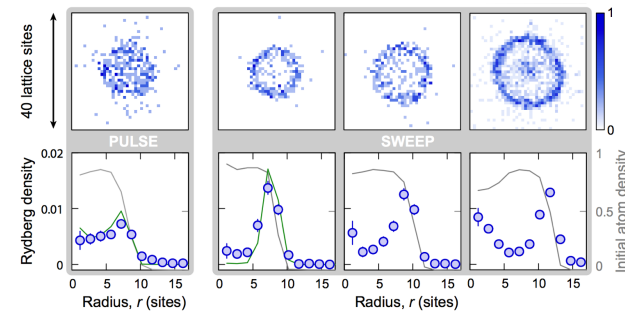


Size & atom number perfectly controlled

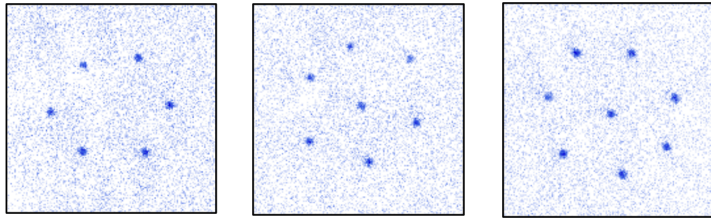


Adiabatic Sweeps in 2D

Pulsed vs swept excitation - localization of excitations to border of system!



Single-Shot Rydberg Crystal Configurations



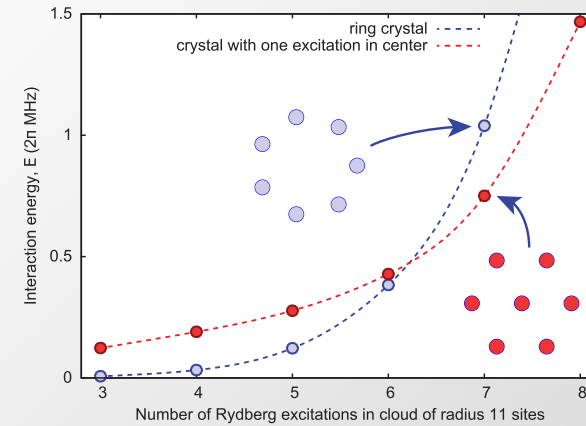
6

7

8

Rydberg Crystal configurations

Configurational Change



Outlook

Smaller Blockade/Larger Cloud

- ✓ Larger Rydberg Crystals
- ✓ Larger Rydberg Atoms cp. to Lattice Spacing
- ✓ Adiabatic Sweeps to Deterministically Prepare Crystal Structures
- ✓ Show Coherence of Crystalline Superpositions! a Quantum Crystal?

T. Pohl et al. (2010), van Bijnen et al. (2011), Gärtner et al. (2012),...

Larger Blockade/Smaller Cloud

- ✓ Collectively enhanced Rabi oscillations
- ✓ Large Entangled states (e.g. EIT schemes)

M. Lukin et al. (2001), D. Moller et al. (2008), M. Müller et al. (2009), H. Weimer et al. (2009),...

Dressed Rydberg Atom Regime

- ✓ Admix controlled long range interactions

G. Pupillo et al. (2010), Henkel et al. (2010), Schachenmeyer et al. (2010), Honer et al. (2010), Cinti et al. (2010), Johnson & Rolston (2010),...

