How do you ***really*** know what the temperature is?

Michael de Podesta

Varenna
6th July 2016

Talk#1

Michael de Podesta

Age: 20,645 Earth rotations

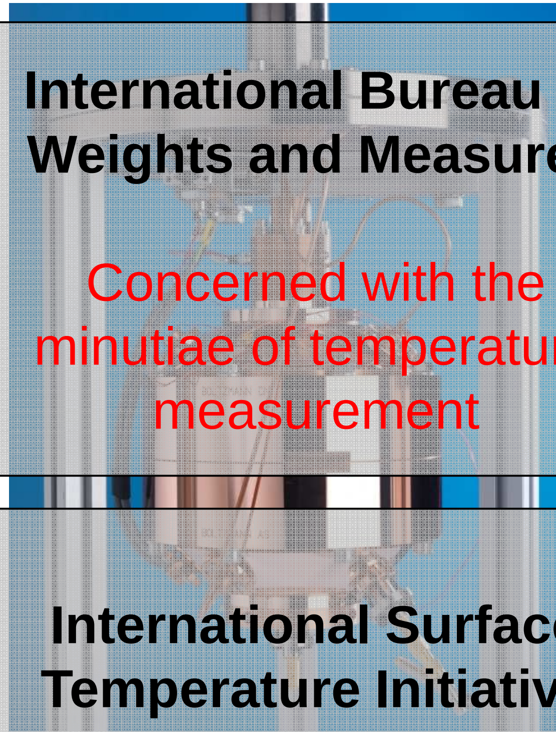
: More than 56 complete solar orbits

Work

- Lecturer in Physics for 13 years
- Understanding the Properties of Matter
- NPL for 16 years.
- BIPM CCT TG-CTh (Old WG4)
- ISTI
- Most accurate thermometer in the world

Not Work

- Married with two sons (17 & 19)
- *Protons for Breakfast*
  - *http://blog.protonsforbreakfast.org/*

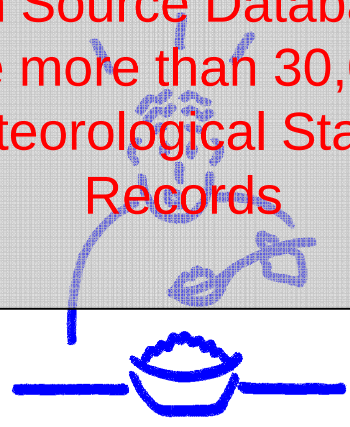


International Bureau of Weights and Measures

Concerned with the minutiae of temperature measurement

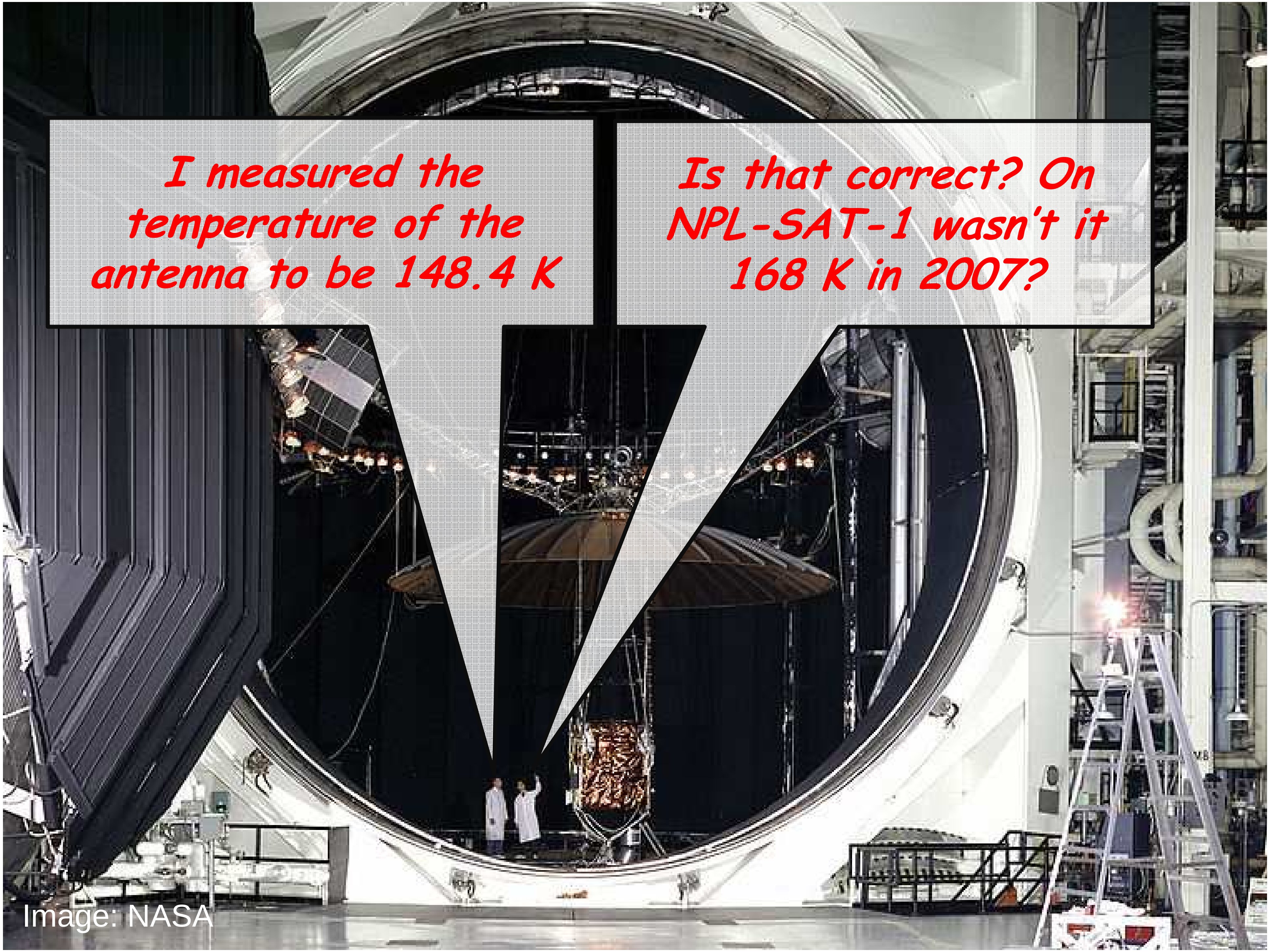
International Surface Temperature Initiative

Open Source Databank of the more than 30,000 Meteorological Station Records



How do you ***really*** know what the temperature is?

1. **What this talk is about**
2. **The International Temperature Scale of 1990**
3. **Primary Thermometry**
4. **Measuring the speed of sound *really* accurately**
5. **What next?**


I measured the temperature of the antenna to be 148.4 K
Is that correct? On NPL-SAT-1 wasn't it 168 K in 2007?
Image: NASA

Temperature Measurement Matters

We need the temperature measurement system to be:

- **Stable**
  - Enables comparability over time
- **Consistent**
  - Enables comparability over distance and across boundaries

Additionally the system should:

- **Not be too complicated or expensive**
  - So that people actually use it
- **Provide measurement uncertainty required**
  - Subject to the historical trend to always make things better!

But does it need to tell us the <u>right</u> temperature?

Why are they measuring the temperature?

'Cooking'

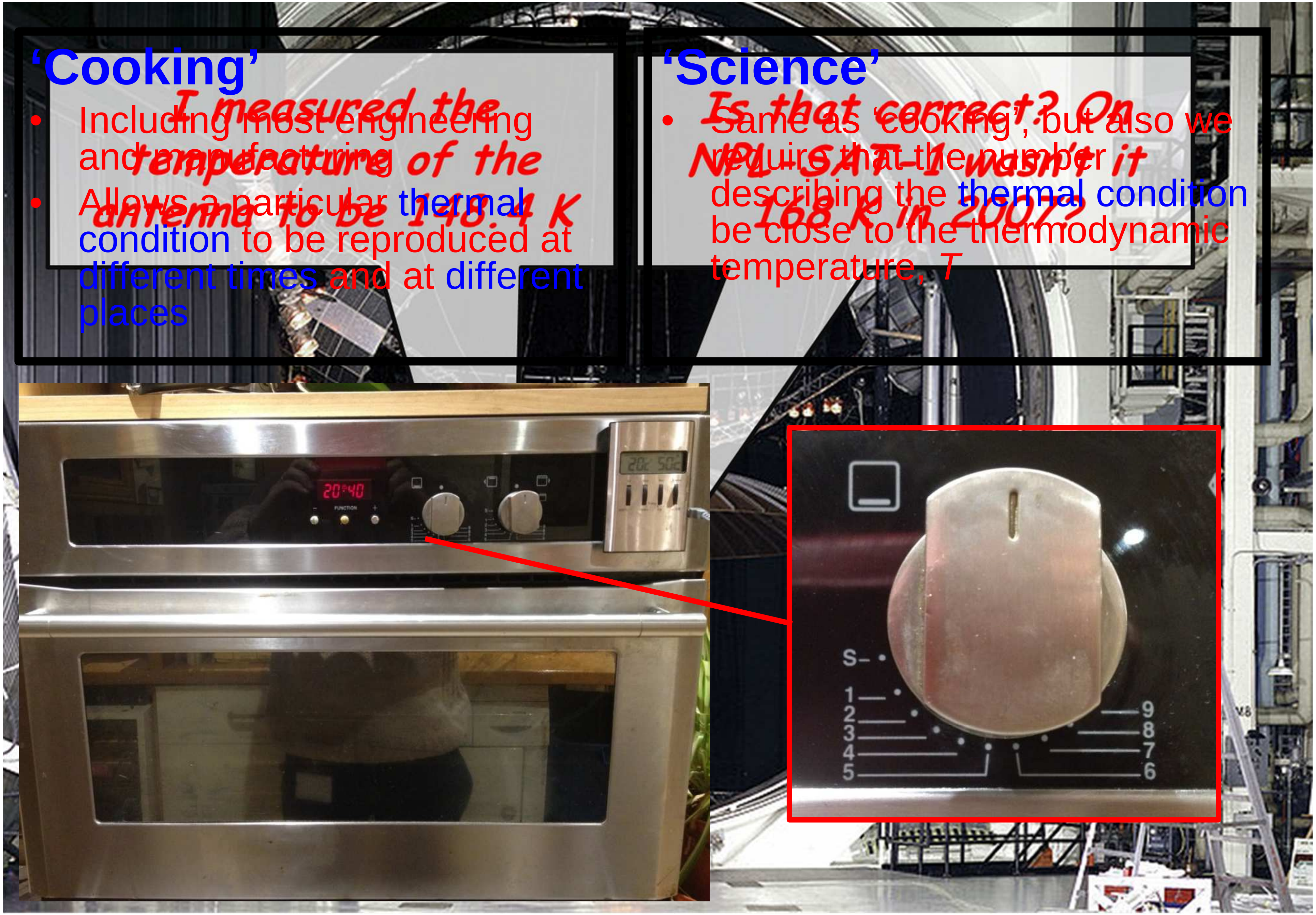
- Including most engineering and manufacturing
- Allows a particular thermal condition to be reproduced at different times and at different places

I measured the temperature of the antenna to be 148.4 K

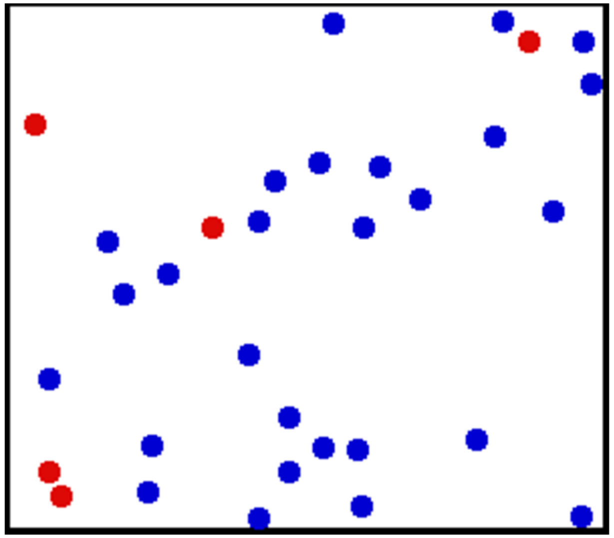
'Science'

- Same as 'cooking', but also we require that the number describing the thermal condition be close to the thermodynamic temperature, *T*

Is that correct? On NPL-SAT-1 wasn't it 168 K in 2007?



Science



Molecular Motion

Rock

Thermometer

How do we relate the number produced by a thermometer (e.g. 20 ° C) to the basic physics describing the jiggling of molecules?

How do you *really* know what the temperature is?

1. What this talk is about
2. The International Temperature Scale of 1990
3. Primary Thermometry
4. Measuring the speed of sound *really* accurately
5. What next?

How do you know what the temperature is?

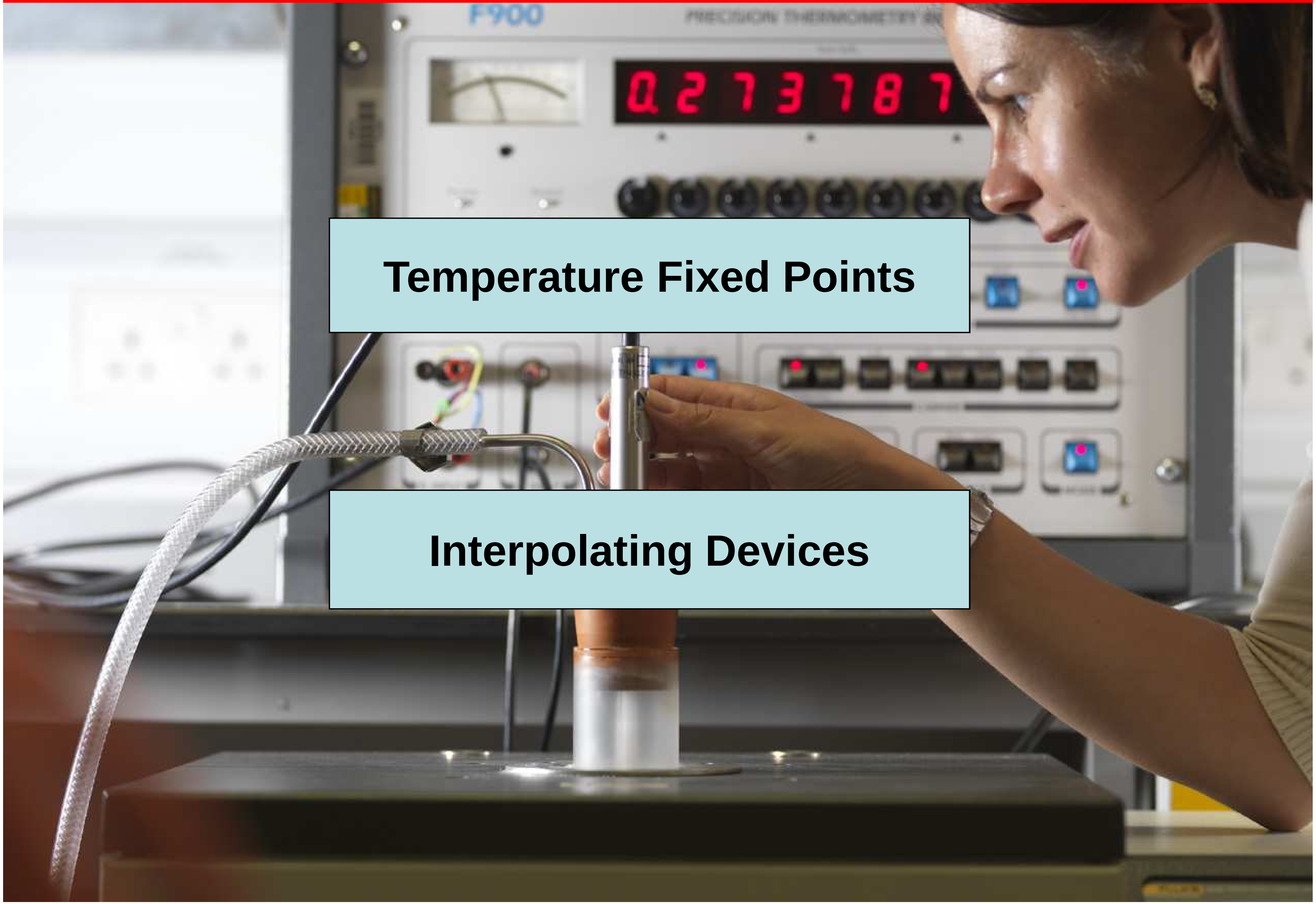
You use a thermometer…

Michael,
Don't
forget to
put the
heater on

…And you get it calibrated!

International Bureau of Weights and Measures, BIPM

The International Temperature Scale of 1990



Temperature Fixed Points

Interpolating Devices

Temperature Fixed Points of ITS90

	T(K)	T (ºC)
Triple point of hydrogen	13.8033	−259.3467
Triple point of neon	24.5561	−248.5939
Triple point of oxygen	54.3584	−218.7916
Triple point of argon	83.8058	−189.3442
Triple point of mercury	234.3156	−38.8344
Triple point of water	273.16	0.01
Melting point[1] of gallium	302.9146	29.7646
Freezing poi		156.5985
Freezing poi		231.928
Freezing poi		419.527
Freezing poi		660.323
Freezing poi		961.78
Freezing poi		1064.18
Freezing poi		1084.62

Now Supplemented by

Metal-Carbon Eutectic Fixed Points e.g.

Pd-C 1492 °C

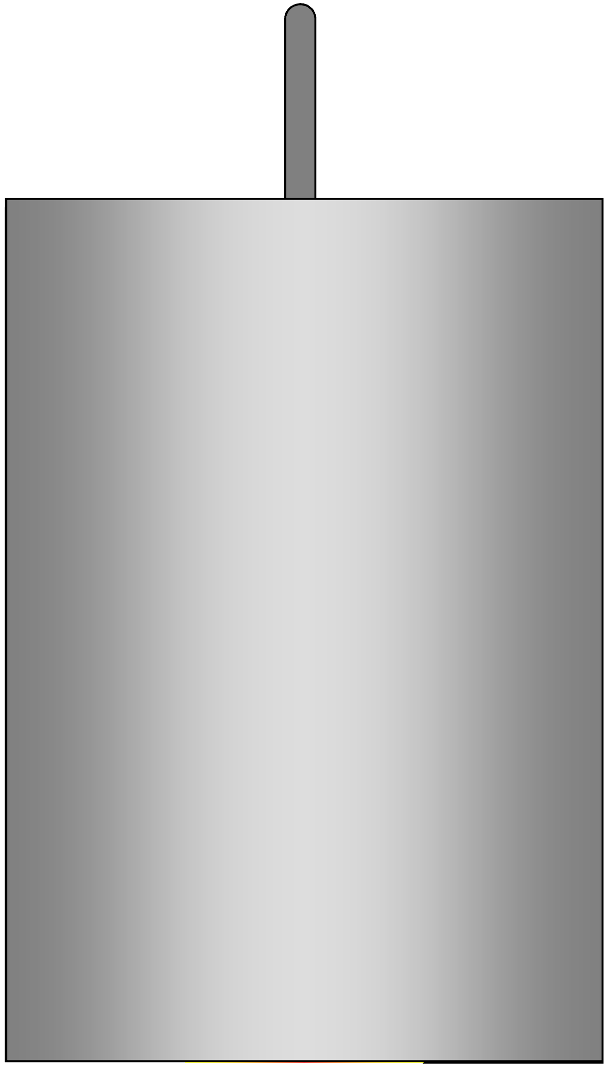
Pt-C 1737 °C

Ir-C 2290 °C

Re-C 2474 °C

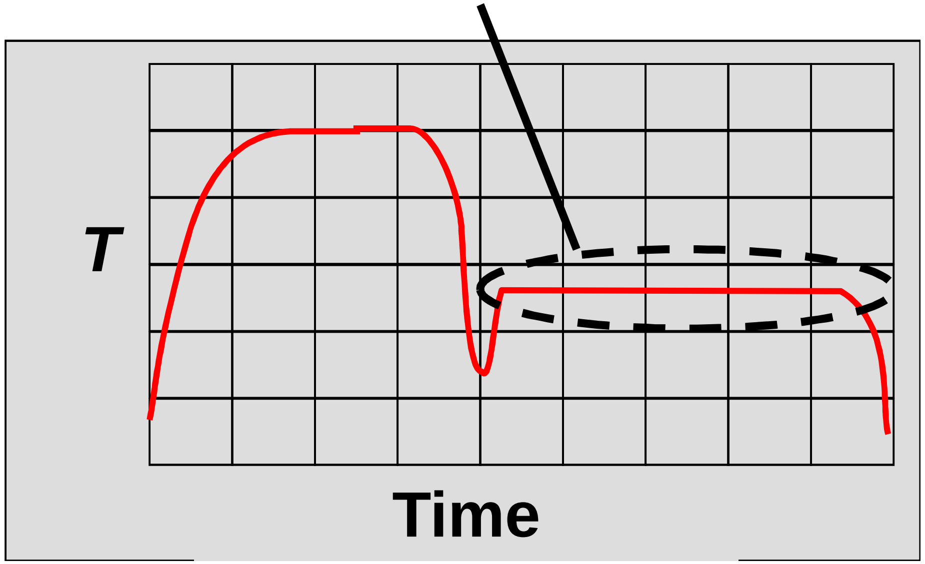
Tin Fixed-Point Cell

A Temperature Fixed-Point Cell

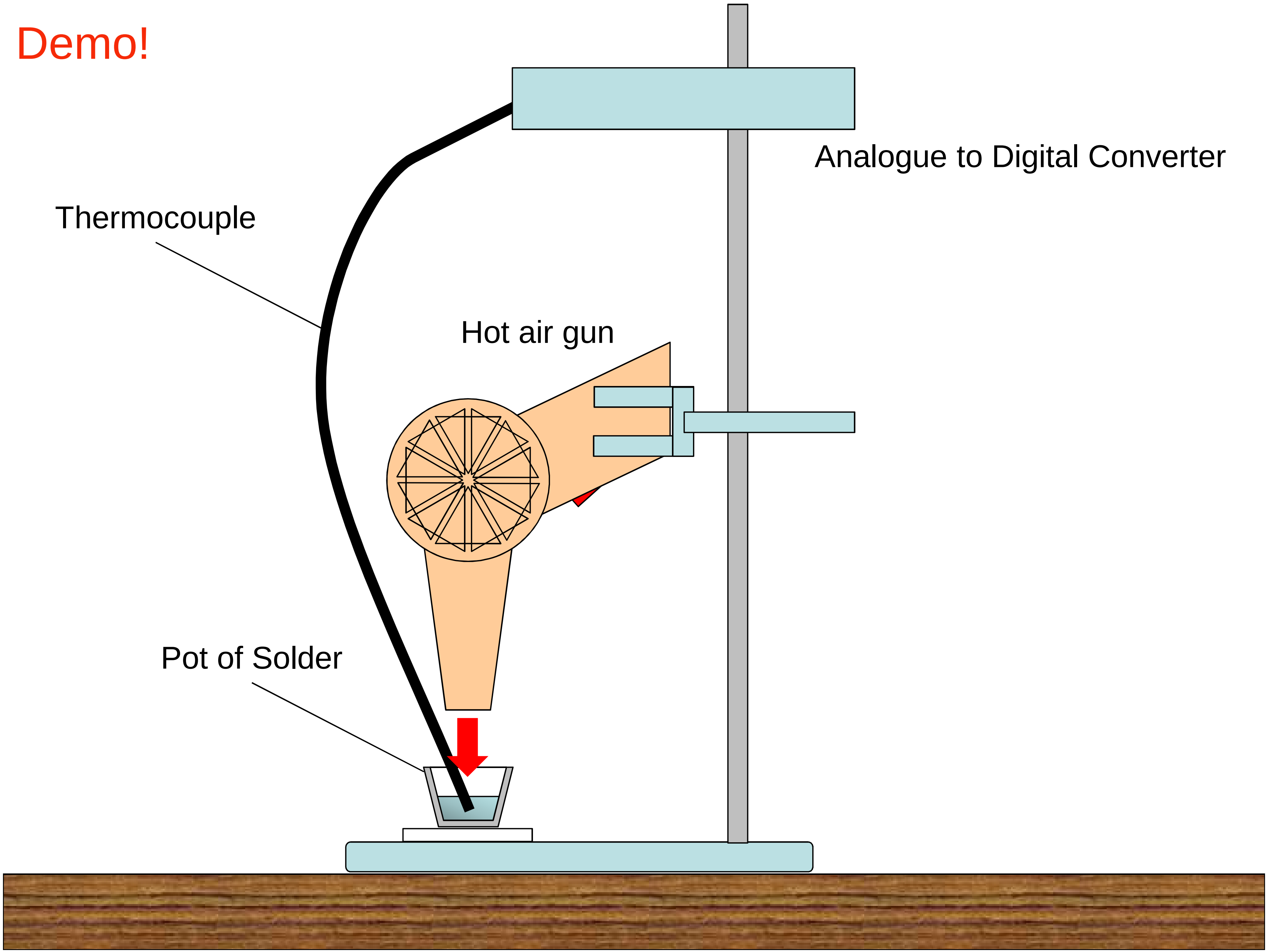


Furnace

- Heat Furnace
- Insert Thermometer
- Melt Crucible of Pure Metal
- Allow it to cool slowly
- While the crucible freezes, the temperature is 'stable'



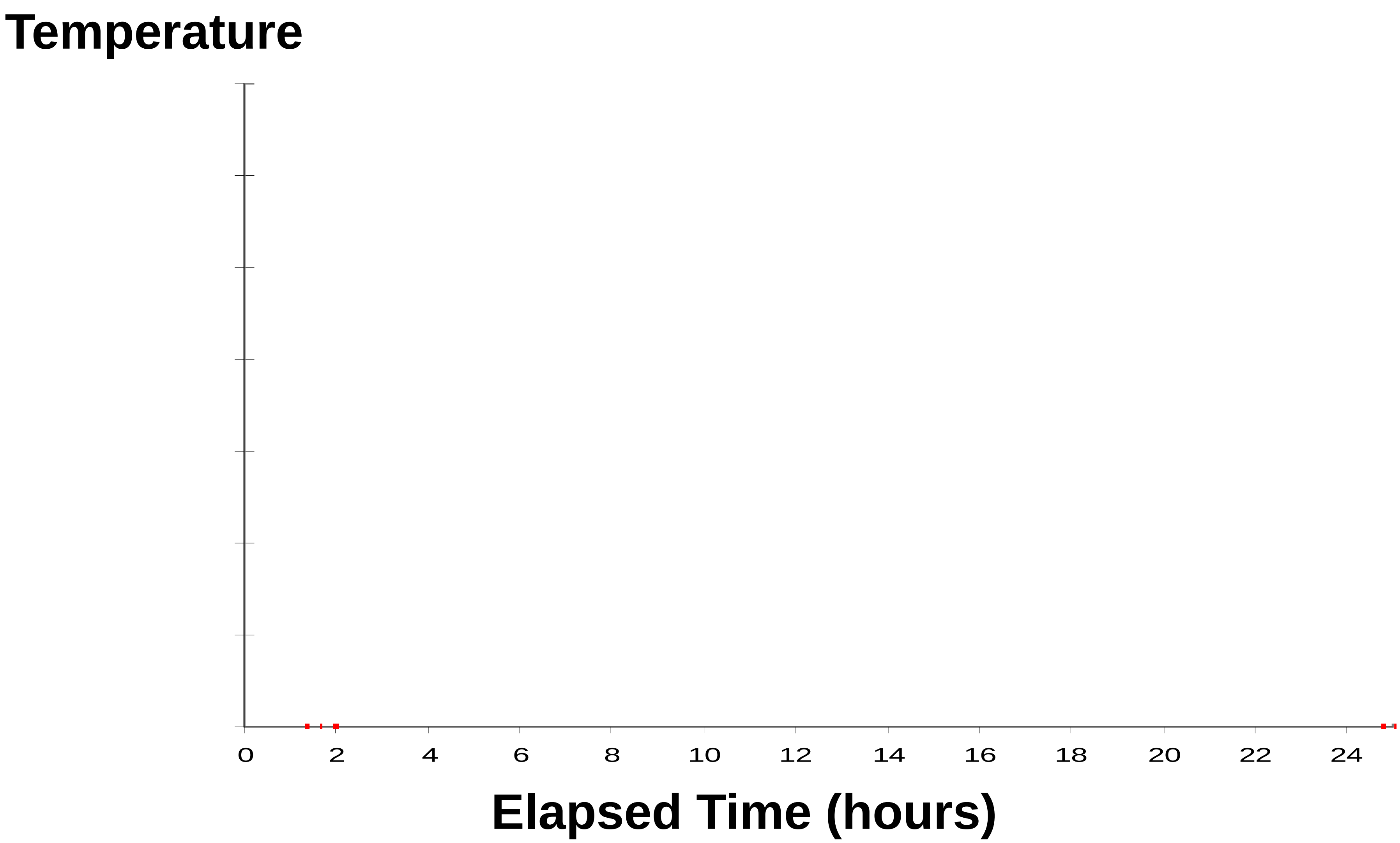
Demo!



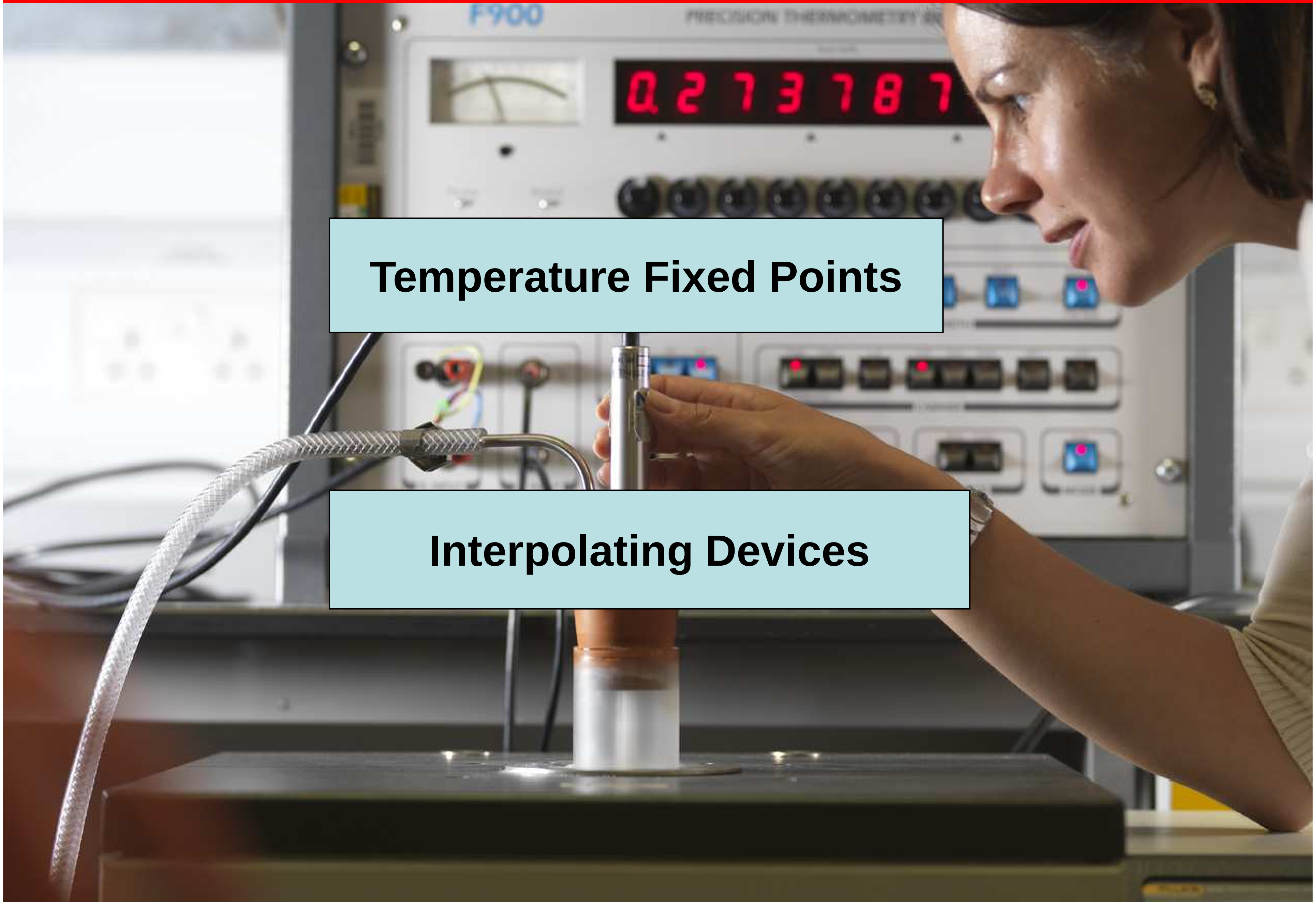
Analogue to Digital Converter
Thermocouple
Hot air gun
Pot of Solder

Tin Fixed-Point Behaviour

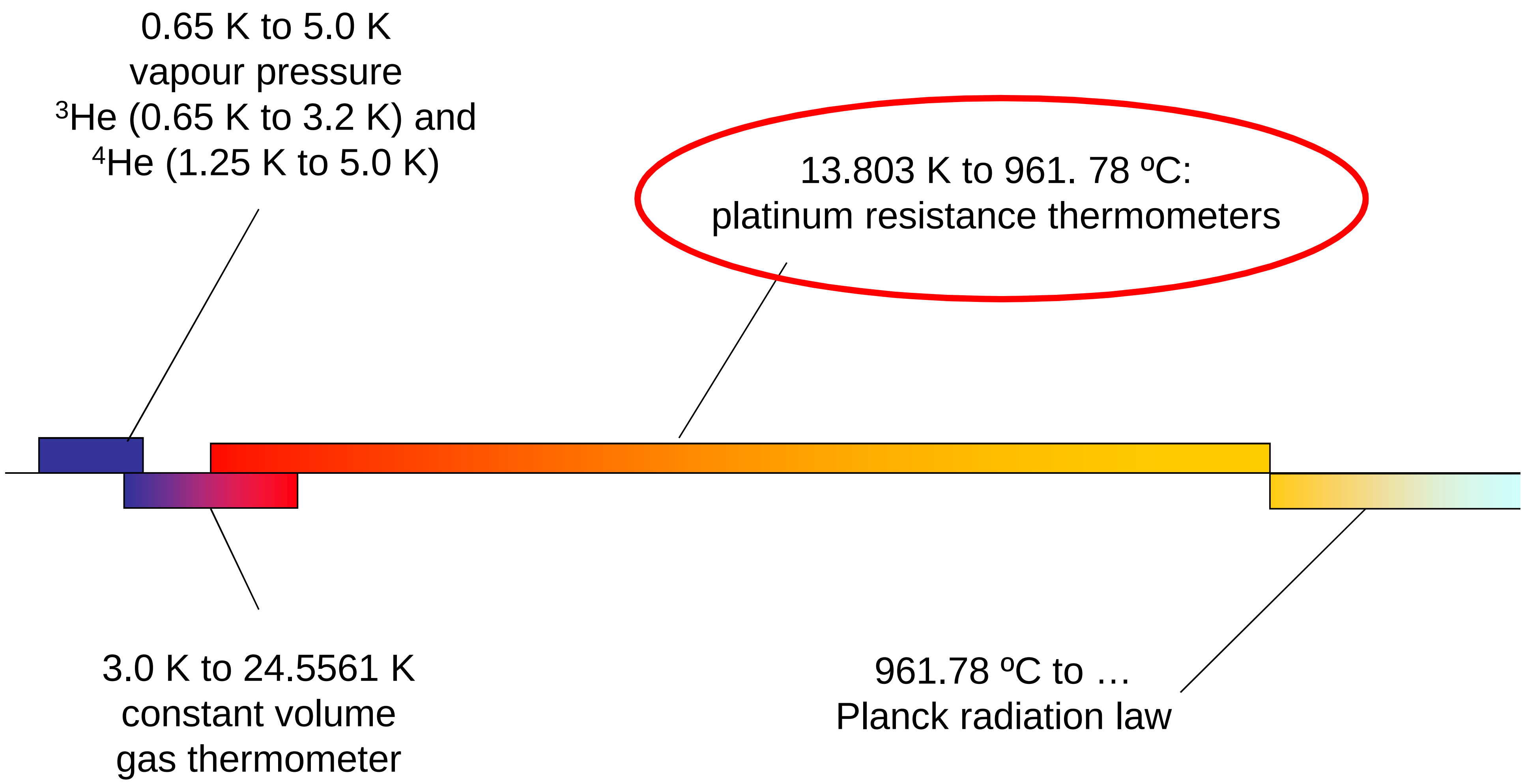
TIN Sn-A Freeze (24-25 August 2005)
furnace C5 Set pt 228.0 variac 22%



The International Temperature Scale of 1990



ITS 90 Interpolating Instruments



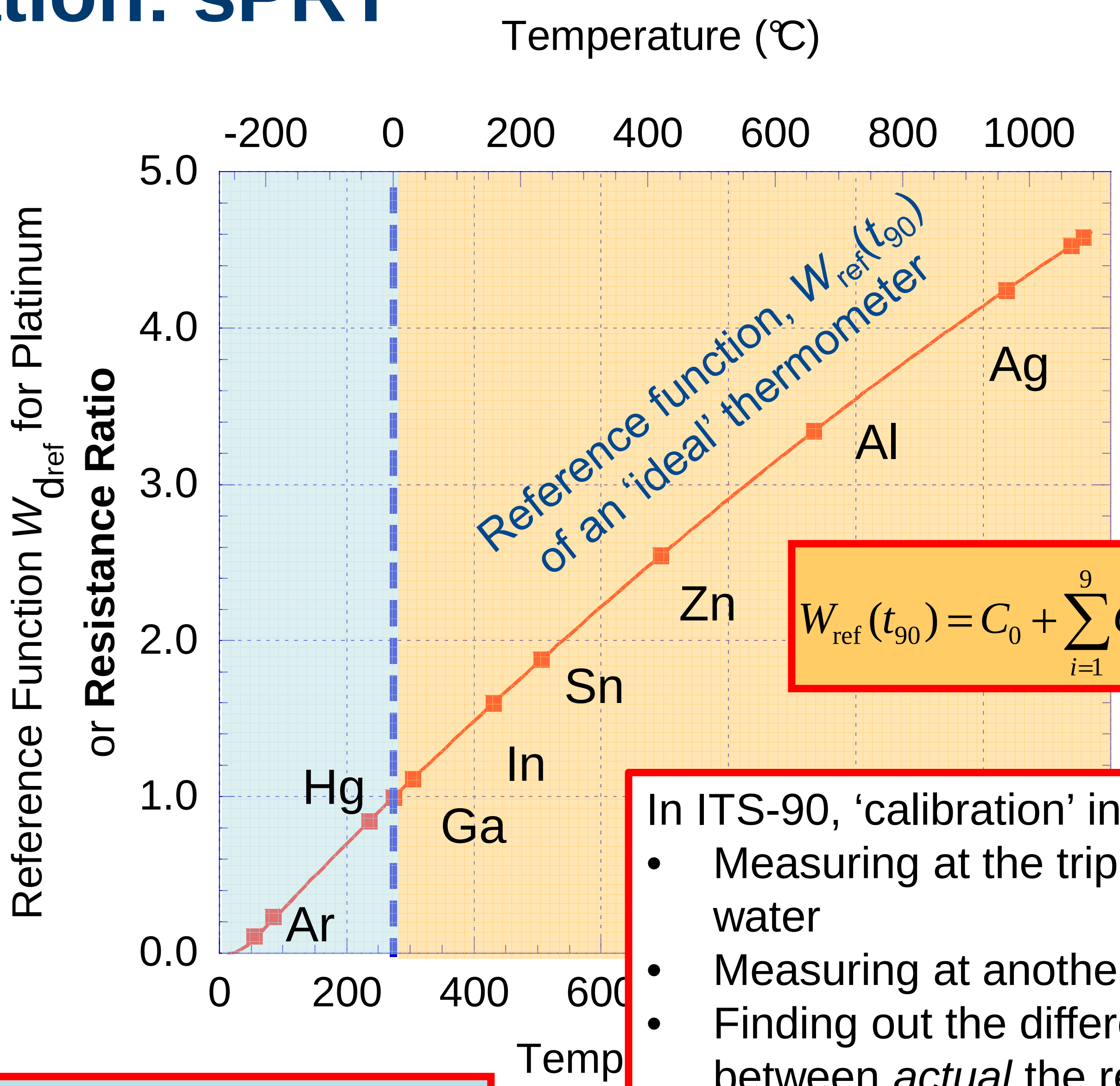
Interpolation: sPRTs Photographs

Resistance typically ~25 Ω

<u>Uncertainty of Measurement</u>

- Resistance: ~10 μΩ
- Temperature:~ few 10's of μK

Interpolation: sPRT



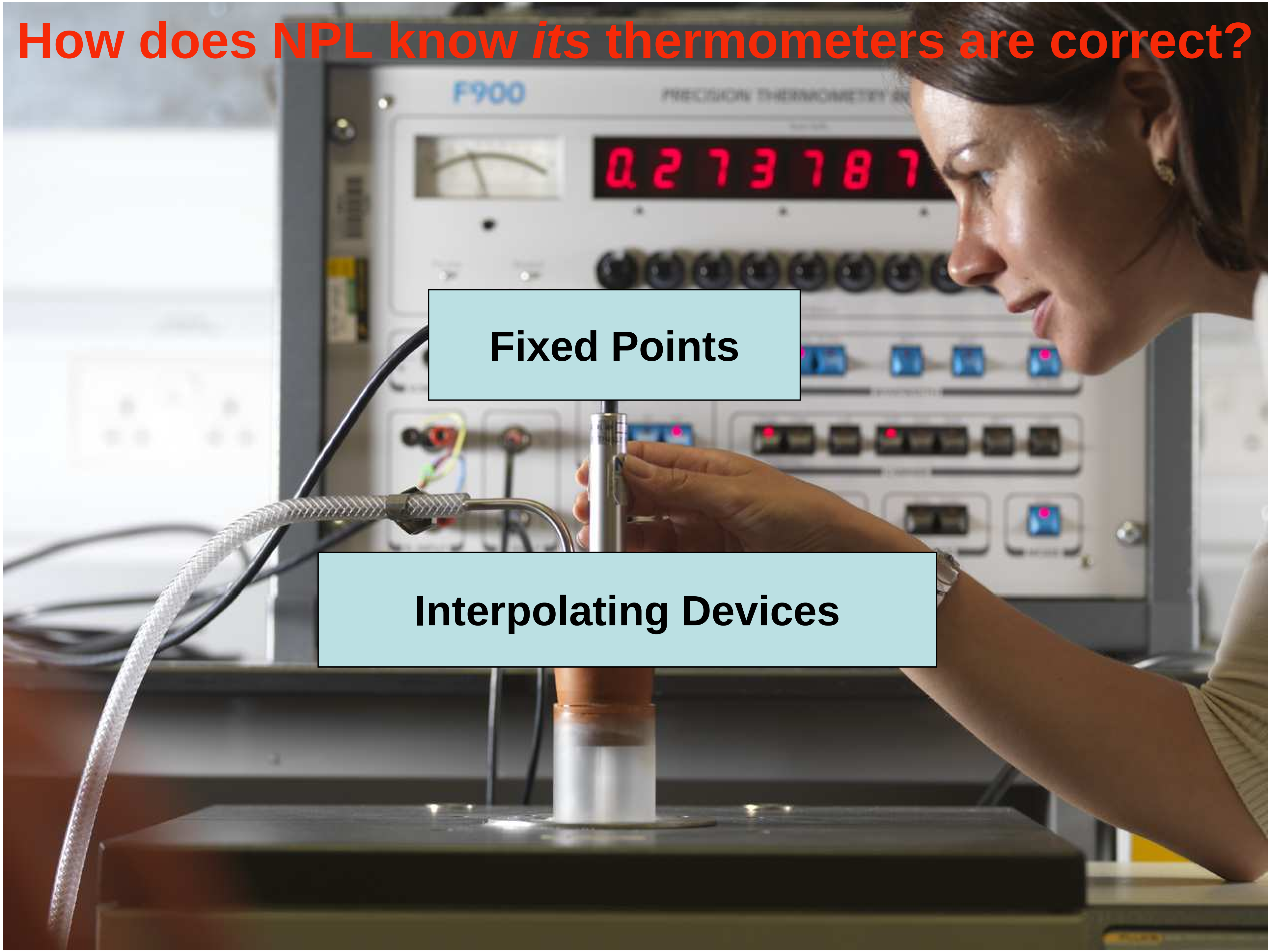
$$W_{ref}(t_{90}) = C_0 + \sum_{i=1}^{9} C_i \left[\frac{(t_{90}-481)}{481}\right]^i$$

$$\ln(W_{ref}) = A_0 + \sum_{i=1}^{12} A_i \left\{\frac{\ln(T_{90}/273.16)+1.5}{1.5}\right\}^i$$

In ITS-90, 'calibration' involves

- Measuring at the triple point of water
- Measuring at another fixed point
- Finding out the differences between _actual_ the resistance ratio and that of an '*ideal*' thermometer

How does NPL know *its* thermometers are correct?



International Temperature Scale of 1990

‘Cooking’

- Including most engineering and manufacturing
- Allows a particular thermal condition to be reproduced at different times and at different places

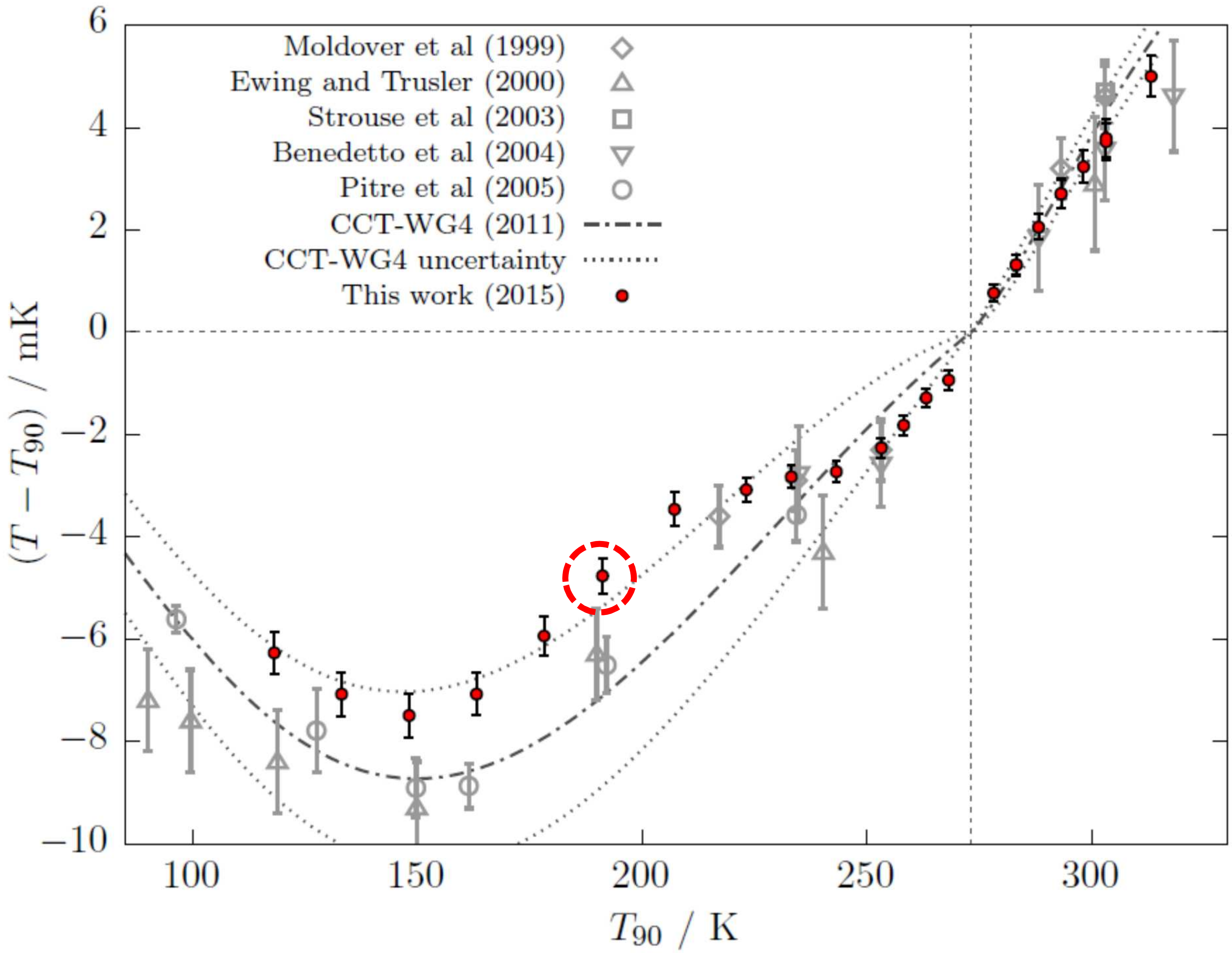
‘Science’

- Same as ‘cooking’, but also we require that the number describing the thermal condition be close to the thermodynamic temperature, T

- Describes ‘thermal condition’ with numbers called T_{90}
- Answers the ‘cooking’ requirements really well
  - Practical, Precise, Reproducible
- Answers the ‘science’ requirements modestly well
  - T_{90} is *‘close’* to thermodynamic temperature T
  - Based on best understanding in 1990

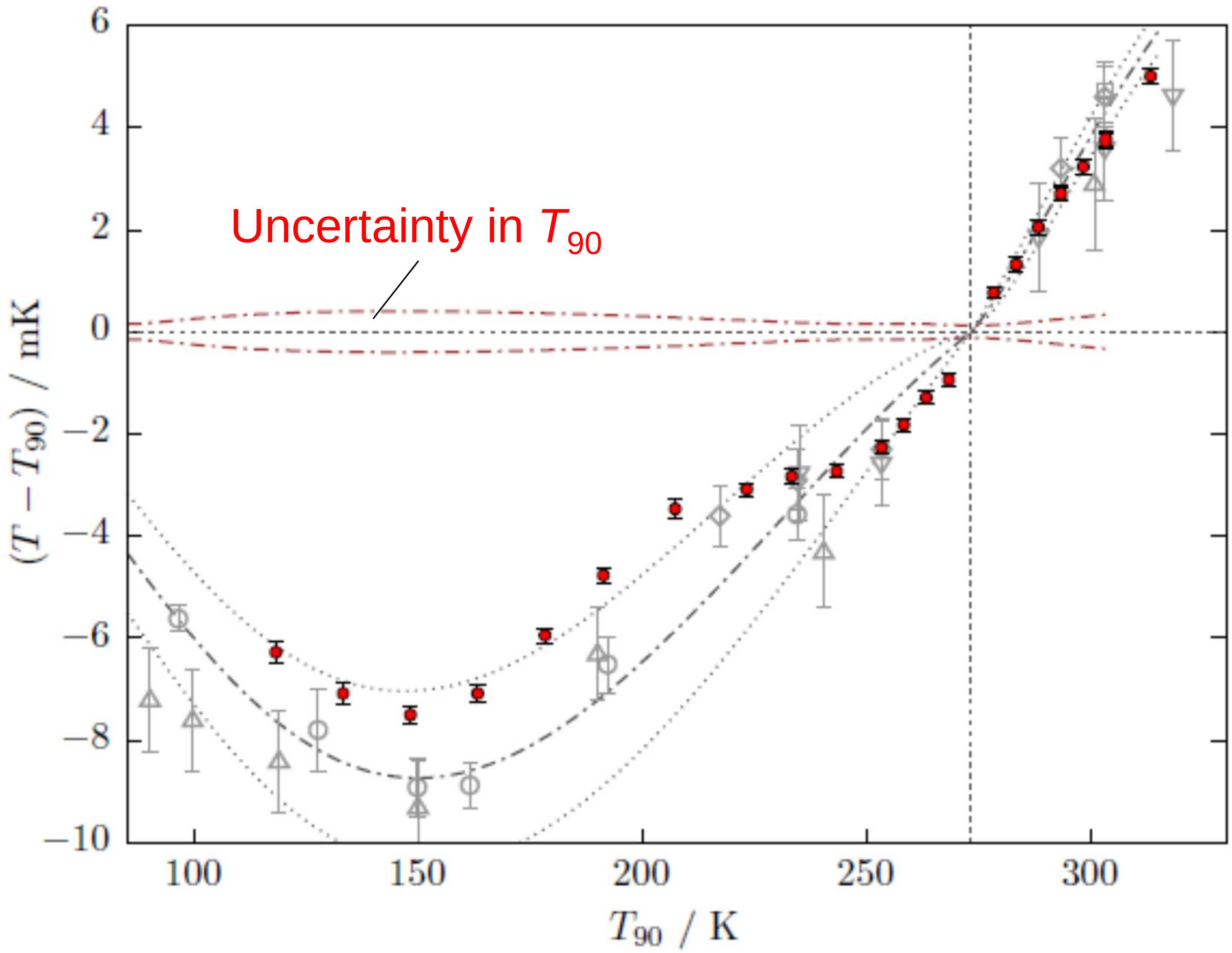
How big is the difference between T and T_{90}?

$T - T_{90}$

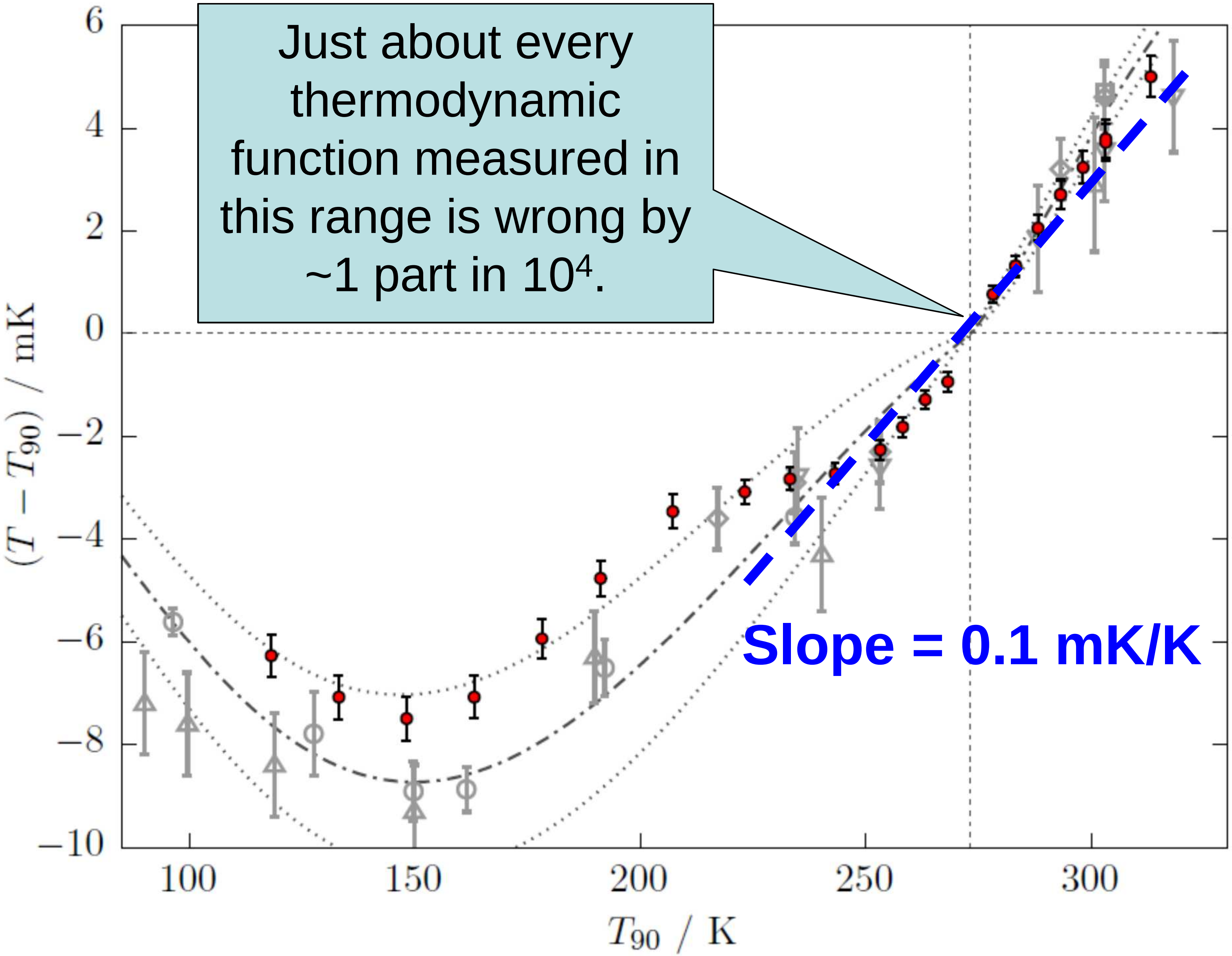


Moldover et al (1999)
Ewing and Trusler (2000)
Strouse et al (2003)
Benedetto et al (2004)
Pitre et al (2005)
CCT-WG4 (2011)
CCT-WG4 uncertainty
This work (2015)
(T − T90) / mK
T90 / K
6
4
2
0
−2
−4
−6
−8
−10
100
150
200
250
300

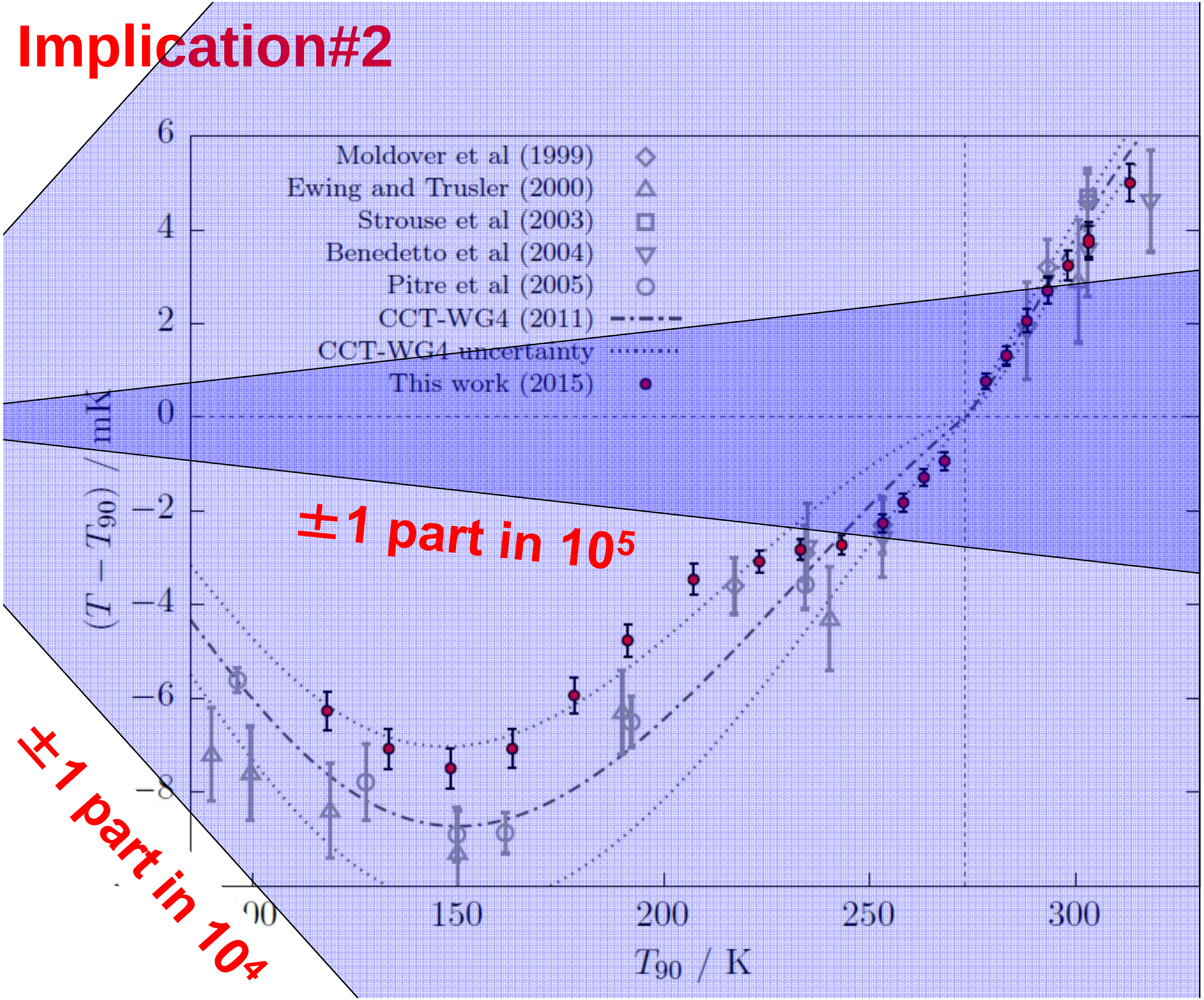
$T - T_{90}$ showing only uncertainty in T



Implication#1: The size of the kelvin



Implication#2



How do you ***really*** know what the temperature is?

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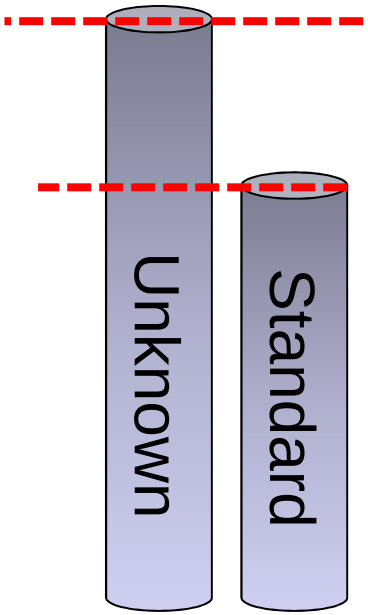
Measurement is…

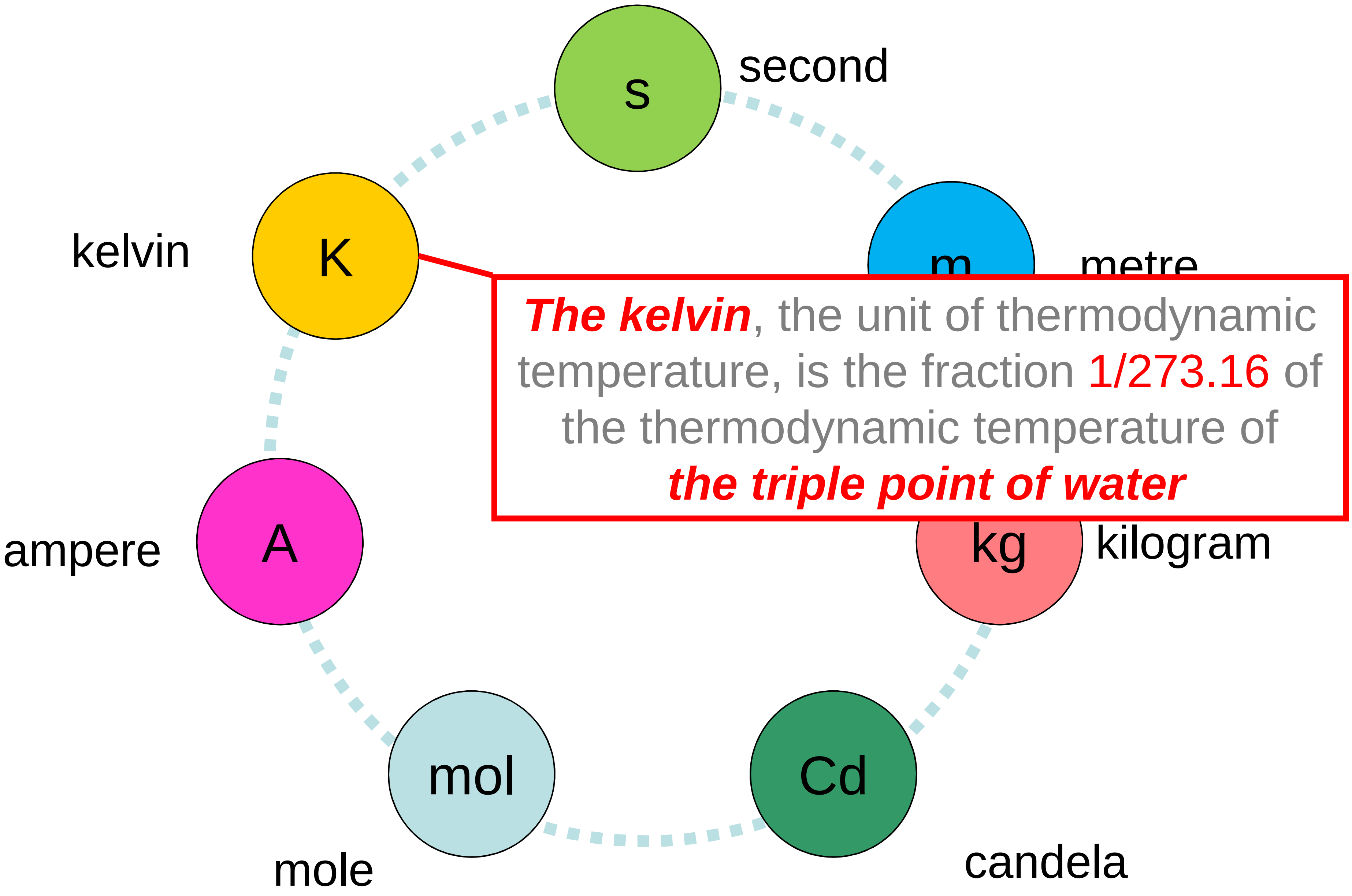
Quantitative *Comparison…*



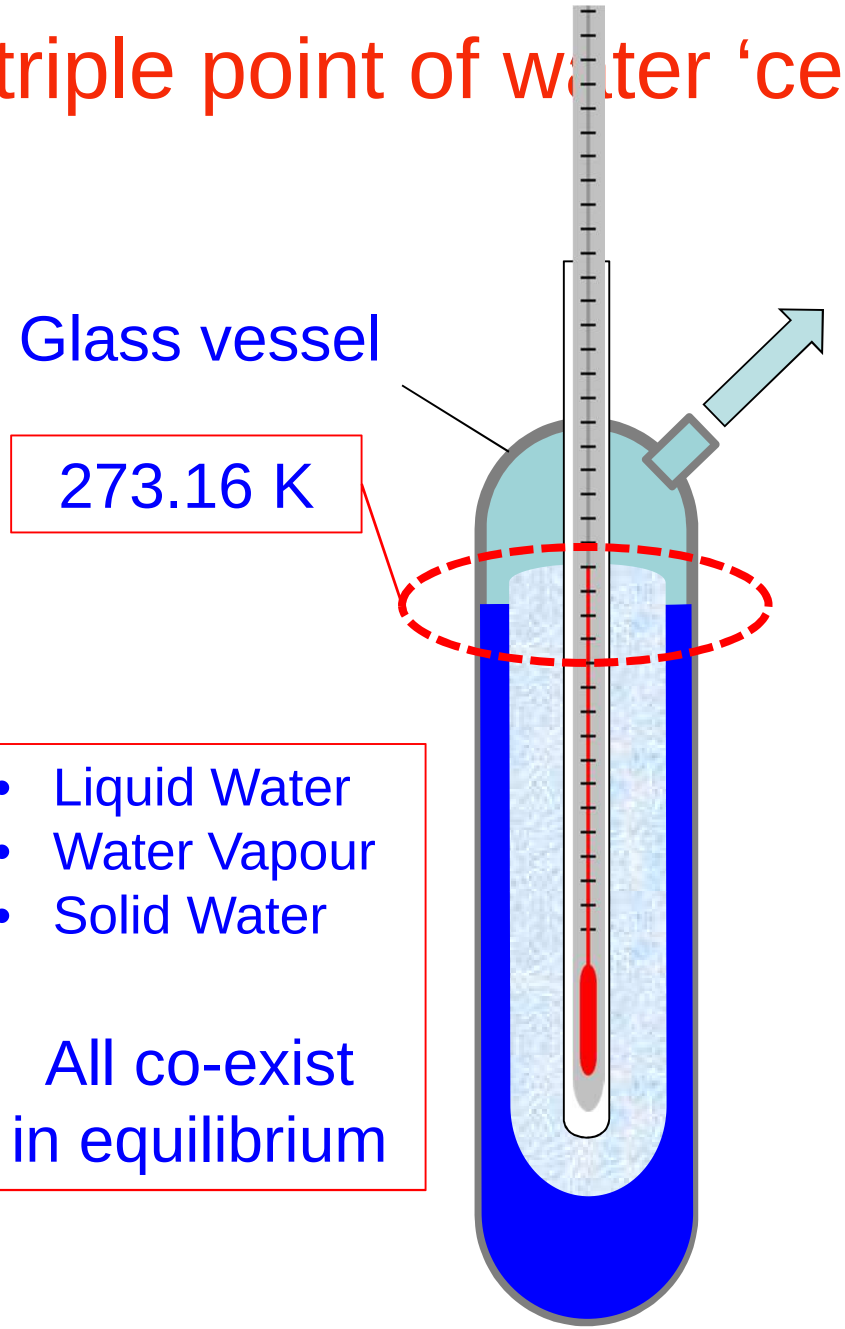
…of an unknown quantity with a standard quantity

The temperature of the ‘triple-point’ of water is humanity’s temperature standard

The International System of Units



A triple point of water 'cell'



- Liquid Water
- Water Vapour
- Solid Water

All co-exist
in equilibrium

1. Fill with 'water'
2. Remove the air
3. Seal
4. Water Vapour fills the space
5. Chill the middle
6. Leave to equilibrate

Triple point of water

Every **temperature measurement is a quantitative comparison of**

the ‘level of molecular jiggling’

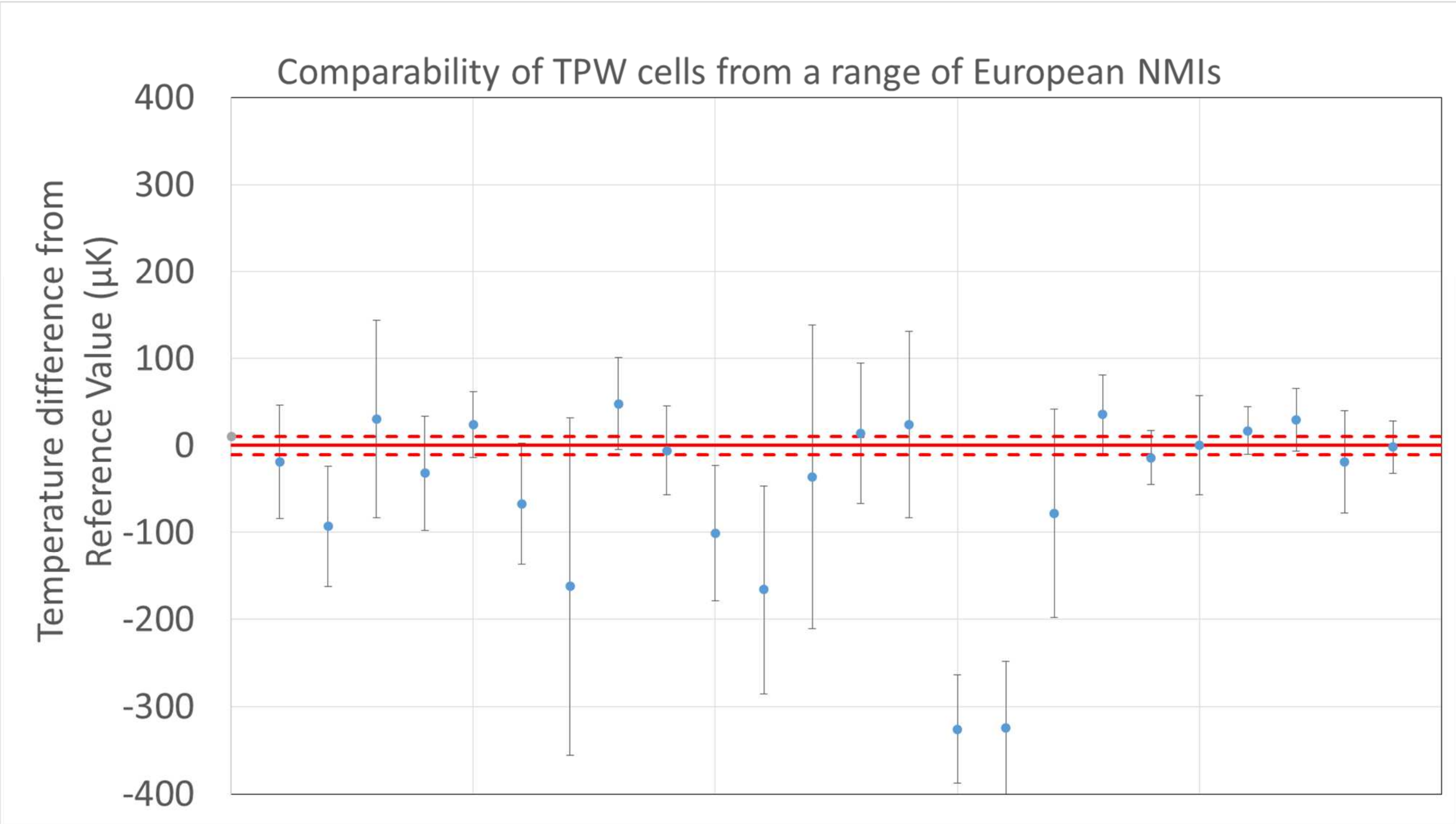
in the target with

the ‘level of molecular jiggling’

in a triple-point cell

The reproducibility of TPW

- No temperature measurement can ever be more accurate than the uncertainty with which the triple point of water can be realised.
  - $u_{TPW}(k = 1)$ is typically 0.05 mK
  - $u_R(k = 1) \sim 0.2$ parts in 10^6



Well that's great.
But how do you measure the temperature of something that is not at 273.16 K (0.01°C)

Good Question.
It turns out to be difficult!

Why?

You need a 'thermometer' whose physics you understand completely.

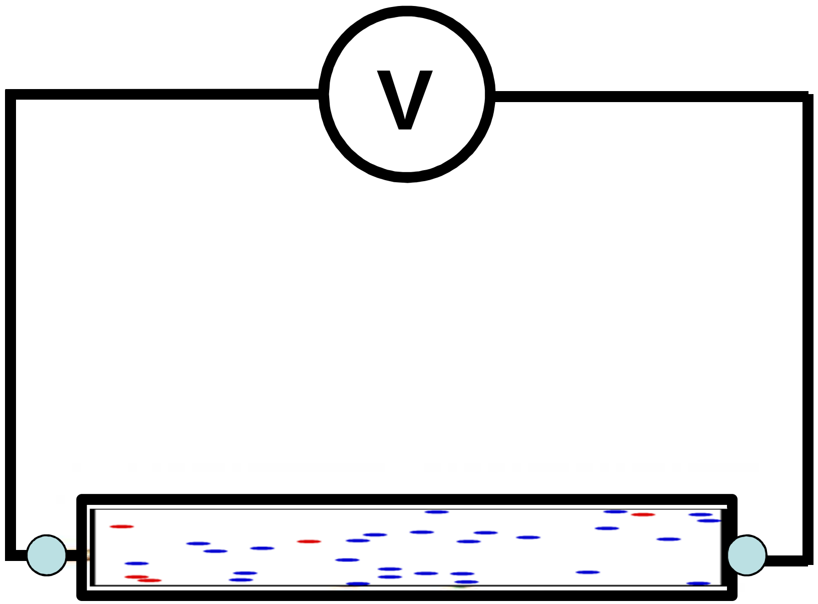
Such as…?

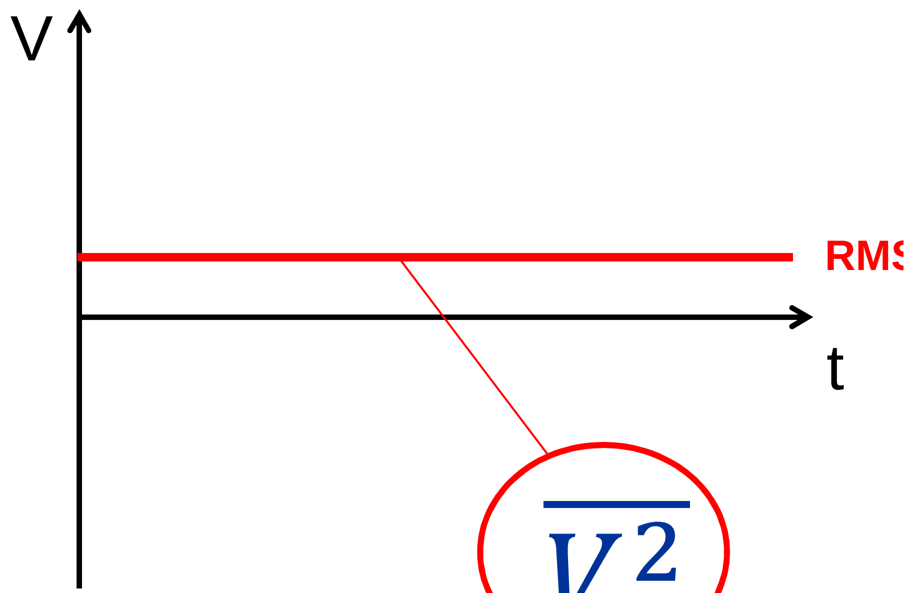
Primary Thermometers

1. *The electrical noise generated by a resistor.*
2. *The pressure of a known amount of gas in a known volume*
3. *The limiting low pressure speed of sound in a gas*
4. *The speed distribution of the molecules in a gas*
5. *The dielectric constant of a low pressure gas*
6. *The refractive index of a low pressure gas*
7. *The brightness of a hot surface*
8. *The Rayleigh Scattering of light from a gas*

Example#1: Johnson Noise Thermometry

- Random thermal motion of electrons creates minute voltage fluctuations
- The fluctuations are small (typically microvolts) and hard to measure.
- But their magnitude is predicted exactly by the Nyquist formula





$$T = \frac{\overline{V^2}}{4k_B R \Delta f}$$

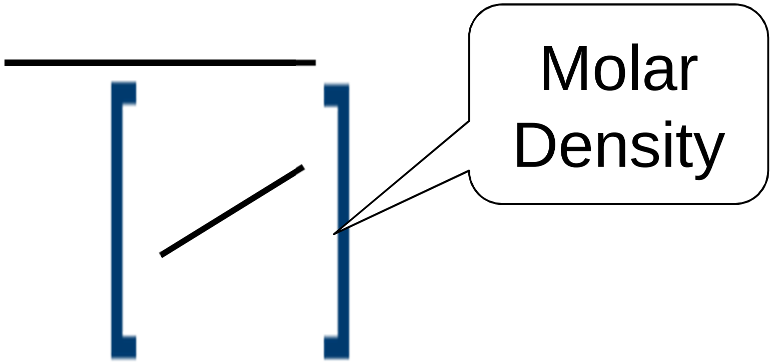
Additionally one needs to know:

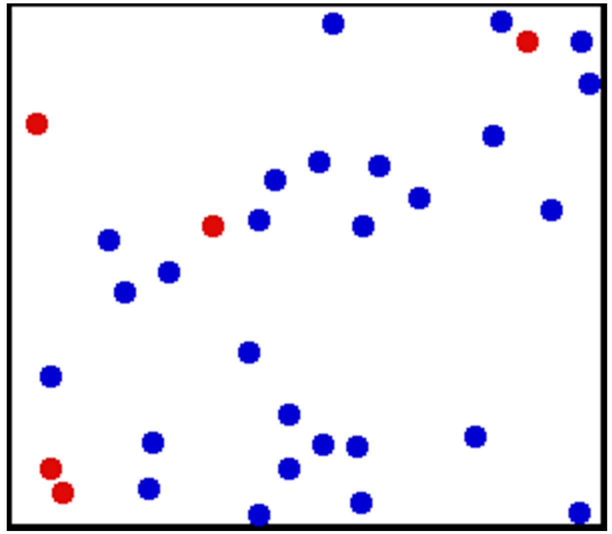
- The measurement bandwidth
- The resistance of the resistor
- The Boltzmann constant

Example#2: **Constant Volume Gas Thermometry**

- Low pressure gases are nearly ideal

$$P\ V = n\ R\ T$$





- Measure a ‘known’ amount of gas into a ‘bulb’ of ‘known’ volume.
- Measure the resulting pressure in the limit of low density
- Main technique used as basis for ITS-90, but uncertainties were probably underestimated.

Example#3: **Acoustic thermometry**

- Exact in the limit of low density

(Speed of Sound)²

The 'Molar' gas constant

Measure the (speed of sound)²...

$$c^2 = \frac{{}^{5}/_{3}\, R\, T}{M}$$

Molar mass of the gas

To work out the absolute temperature in kelvins…

… in a pure gas of known molar mass

Acoustic thermometry

- Exact in the limit of a low-density monatomic gas

(Speed of Sound)²

The Boltzmann constant

$$c^2 = \frac{{}^5\!/_3\, k_B T}{m}$$

Measure the (speed of sound)²…

Mass of a molecule

To work out the absolute temperature in kelvins…

… in a pure gas of known molar mass

Primary Thermometers

1. *The electrical noise generated by a resistor.*
2. *The pressure of a known amount of gas in a known volume*
3. *The limiting low pressure speed of sound in a gas*
4. *The speed distribution of the molecules in a gas*
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8. *The Rayleigh Scattering of light from a gas*

Acoustic thermometry

(Speed of Sound)²

The Boltzmann constant

$$c^2 = \frac{{}^5/_3\, k_B T}{m}$$

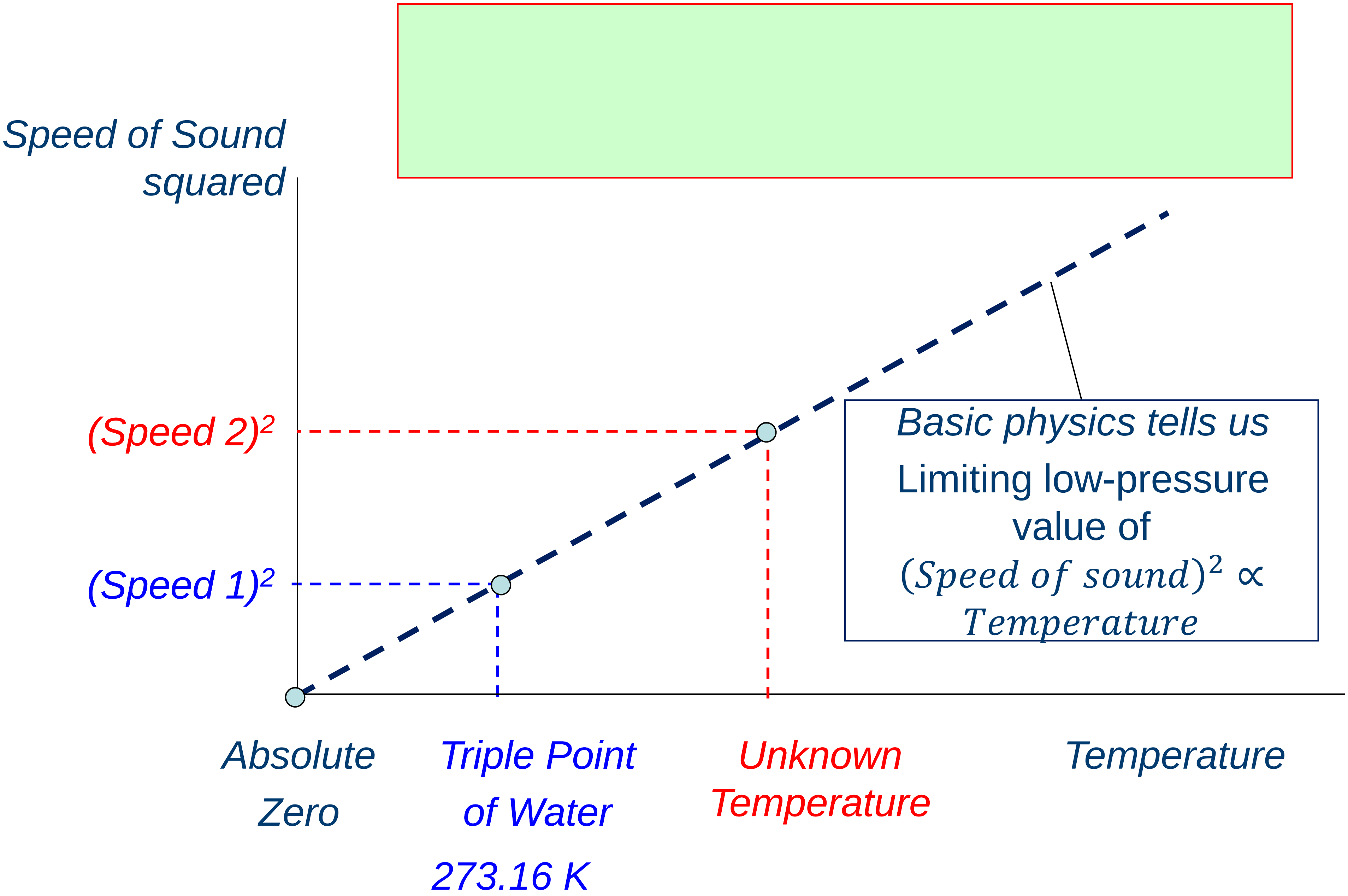
Mass of a molecule

- Exact in the limit of a low-density monatomic gas

- If the same gas is used at both temperatures

$$\frac{T_1}{T_{TPW}} = \frac{c_1^2}{c_{TPW}^2}$$

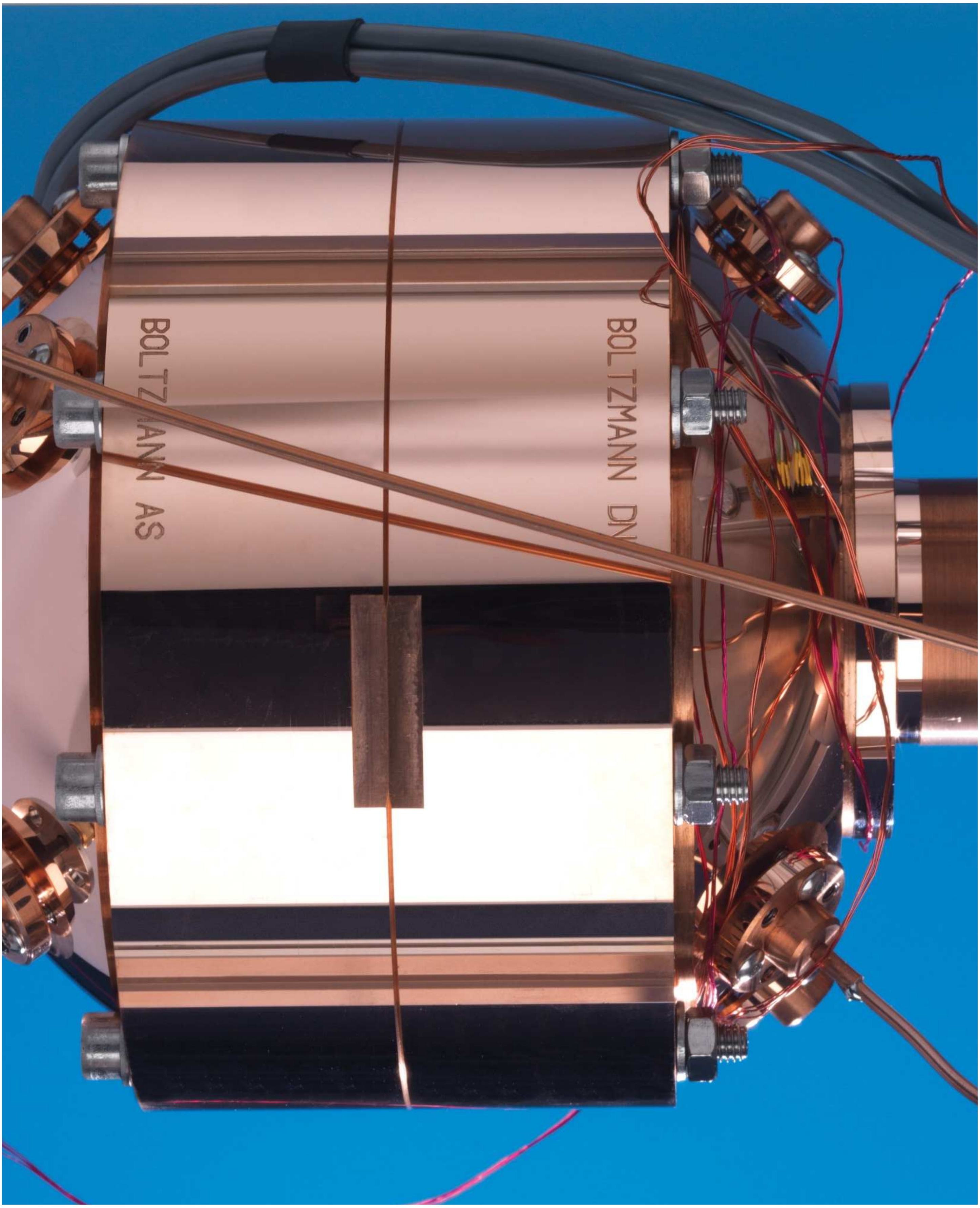
How to work out an unknown temperature



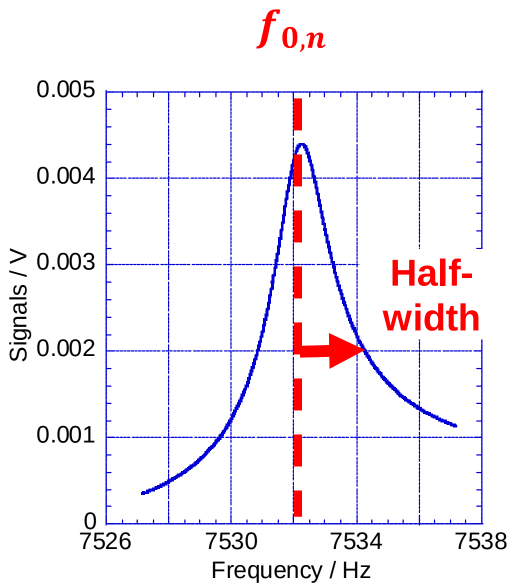
How do you ***really*** know what the temperature is?

1. **The problem**
2. **The International Temperature Scale of 1990**
3. **Primary Thermometry**
4. **Measuring the speed of sound *really* accurately**
5. **What next?**

The NPL-Cranfield Acoustic Resonator



Measure the speed of sound in a spherical resonator

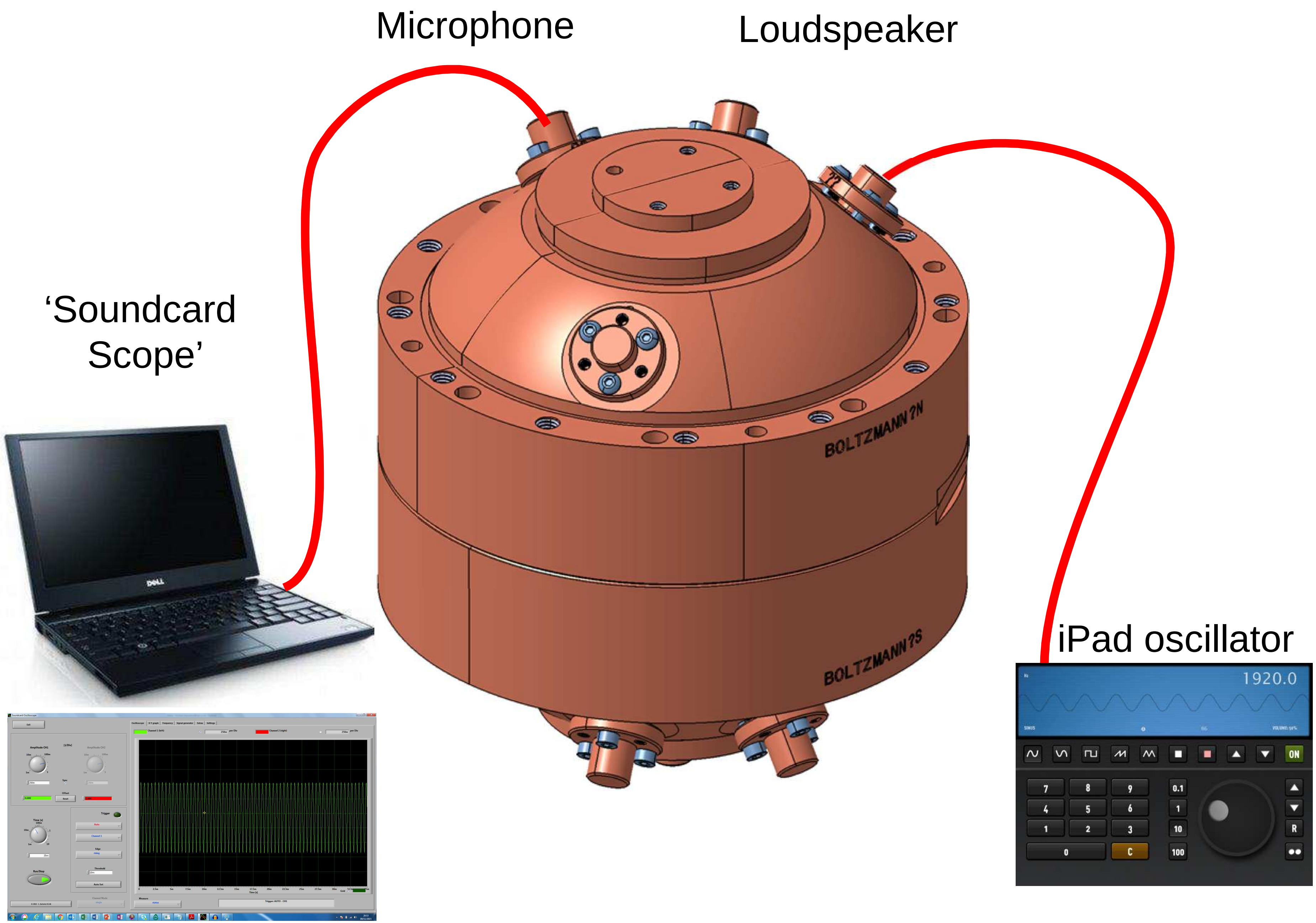


$$Speed\ of\ Sound = Resonant\ Frequency \times \frac{Radius}{constant\ for\ each\ resonance}$$

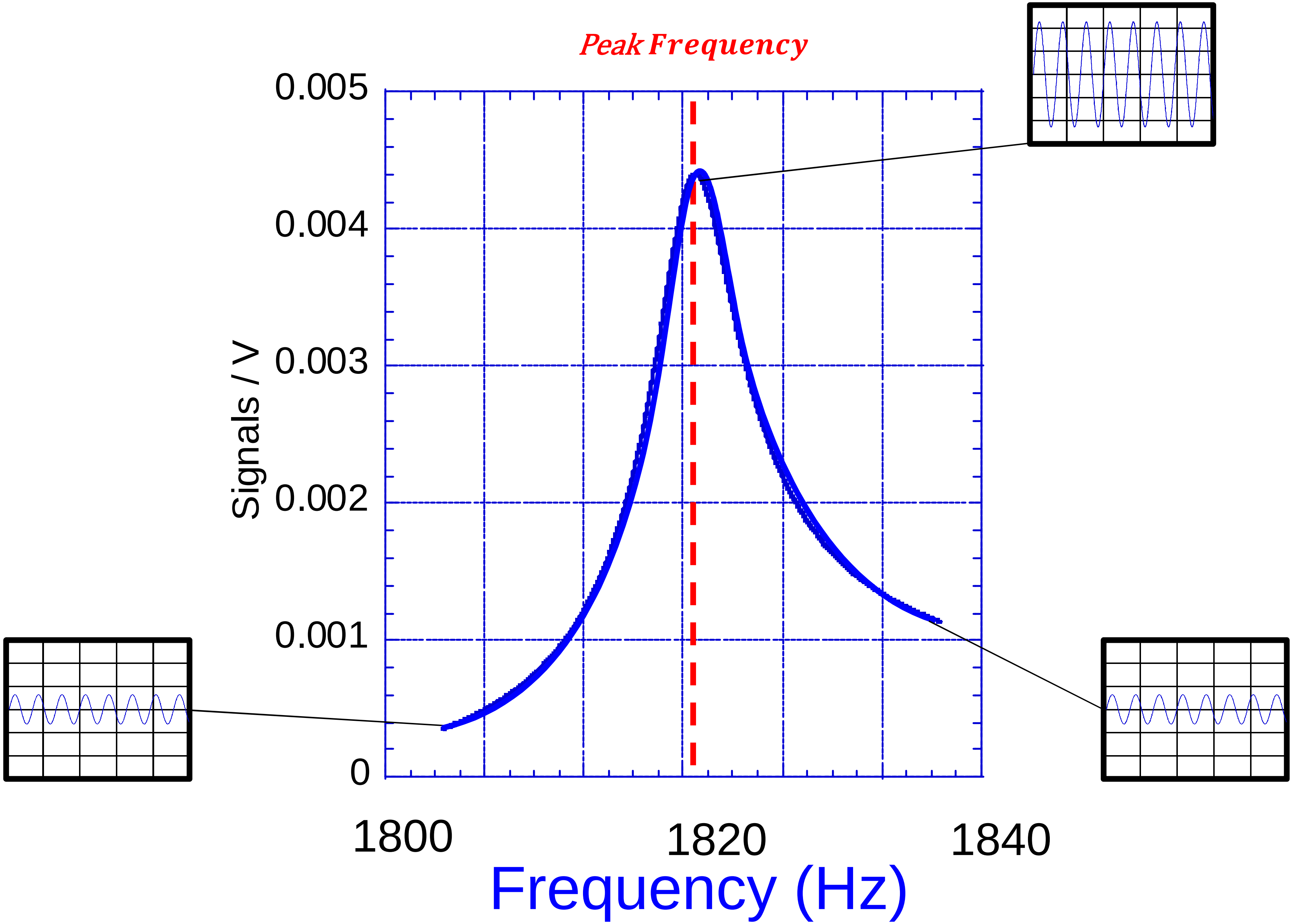
First three constants for spheres are 3.018, 1.880, 1.396

Demonstration!

Measure the speed of sound in a spherical resonator



Measure the speed of sound in a spherical resonator



End of Demonstration.

To get very low uncertainty…

- Measure the radius *in situ* at all pressures and temperatures
  - Simultaneously measure microwave resonances
  - Requires a tri-axially ellipsoidal resonator with a conducting inner surface

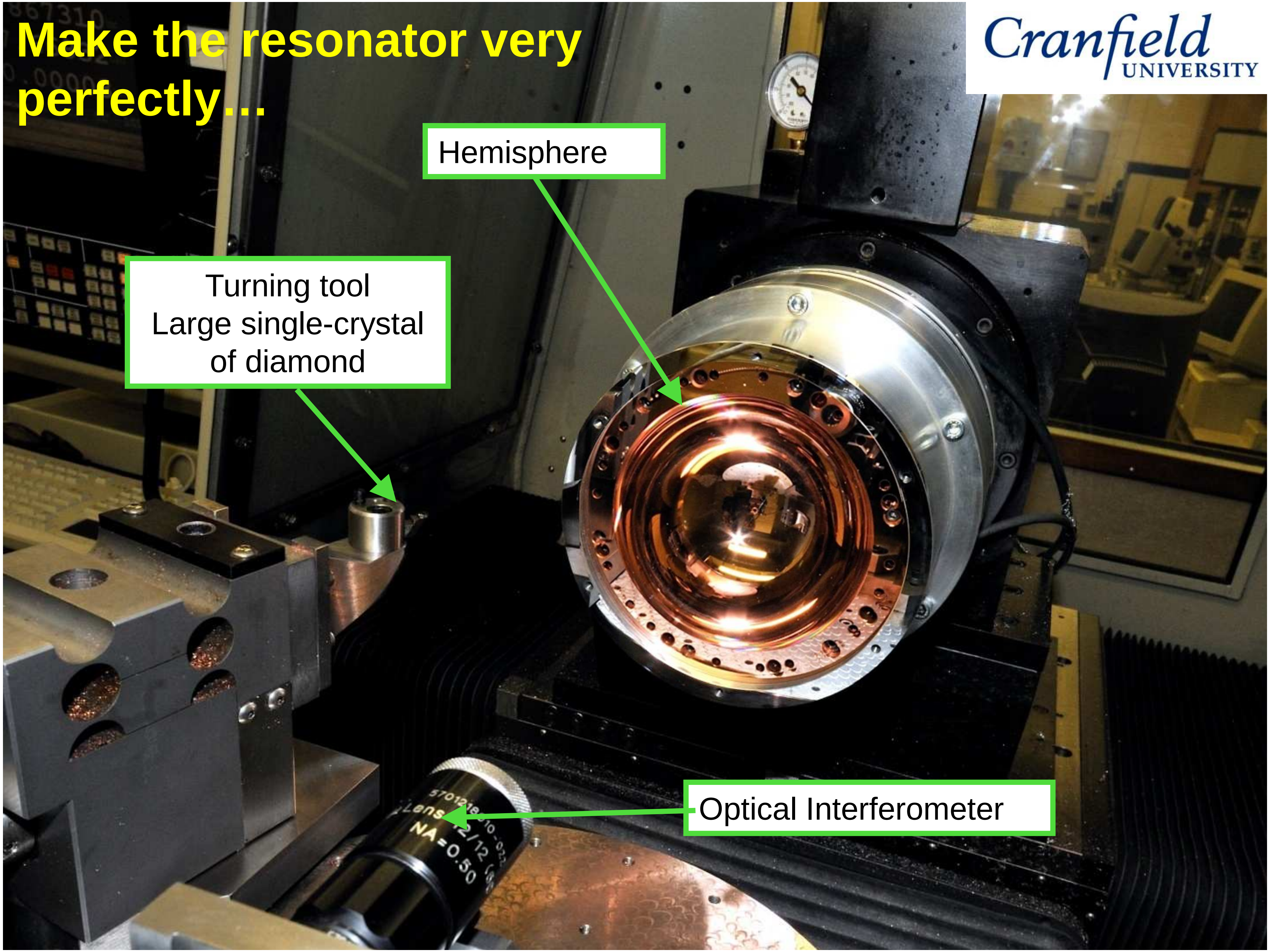
- Build resonator to be very perfectly the right shape
  - The microwave and acoustic resonances occur at the frequencies expected from their calculated eigenvalues
  - Self Check#1: Variability of radius inferred from different microwave resonances.
  - Self Check#2: Check the variability of c_0^2 inferred from different acoustic resonances.

- Build resonator out of copper
  - High thermal conductivity leads to low temperature gradients
  - High electrical conductivity leads to sharp microwave resonances.

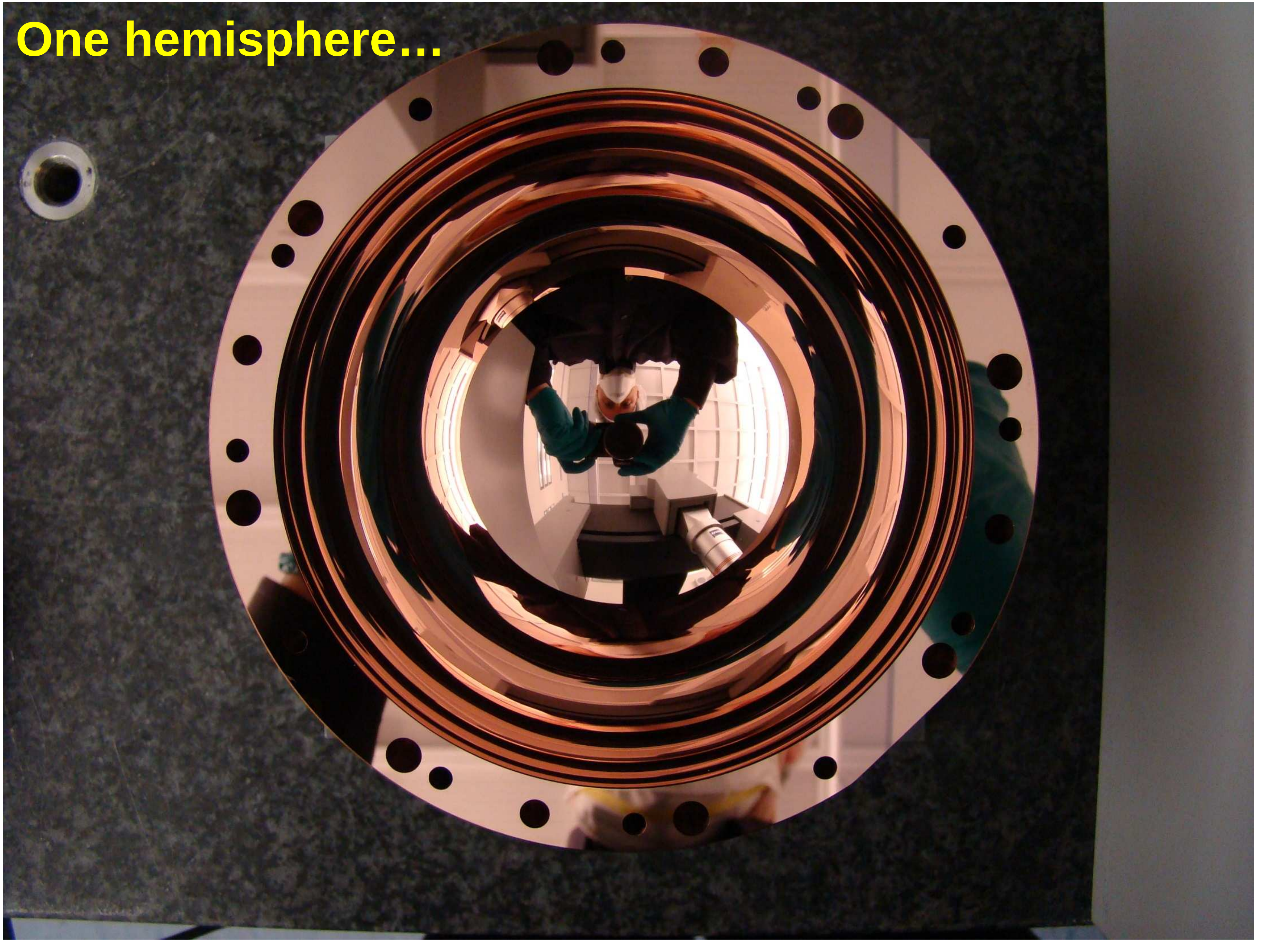
- Measure the half-widths of the resonances
  - Check that nothing unexpected is happening.

Make the resonator very perfectly…

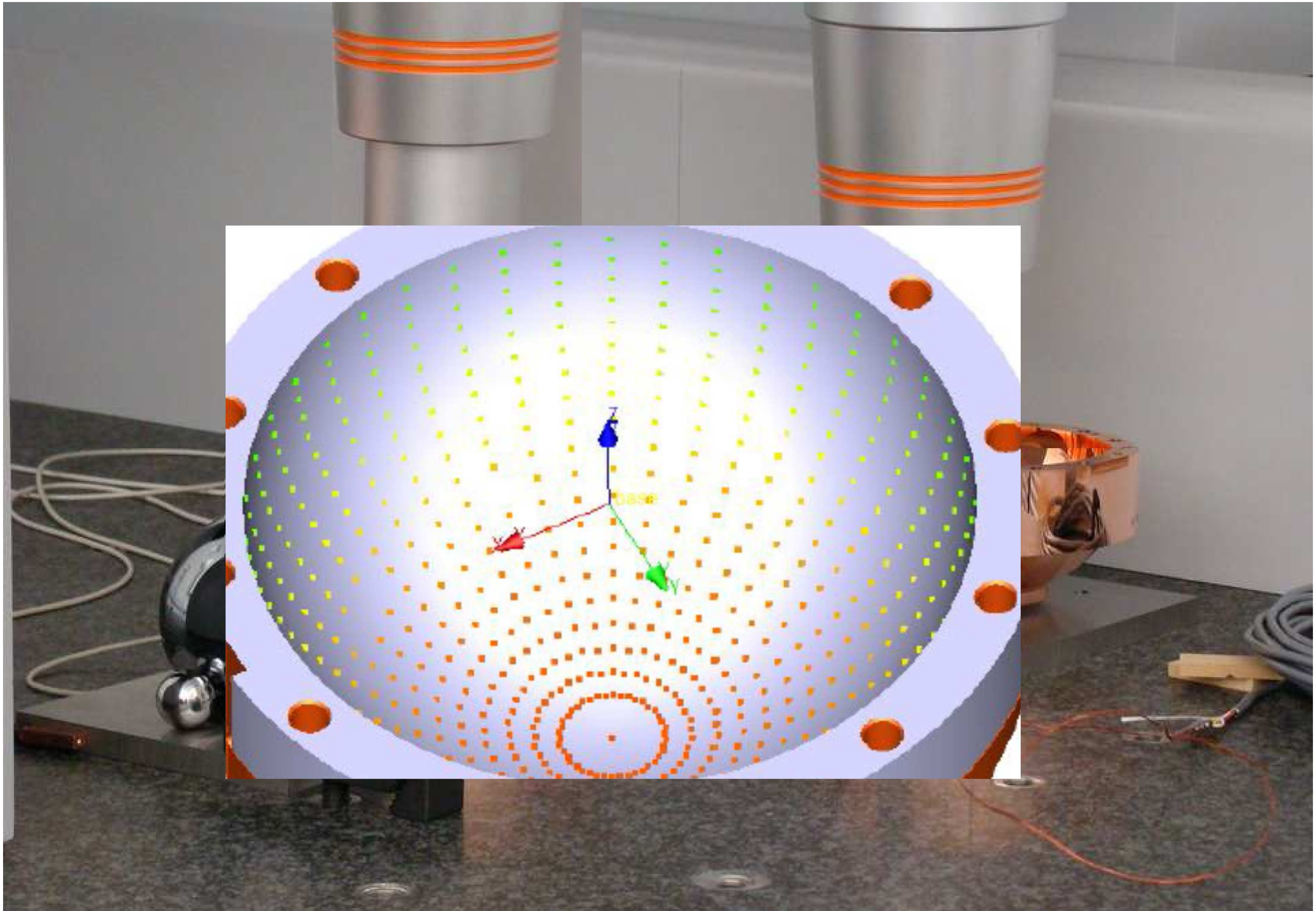
Cranfield UNIVERSITY



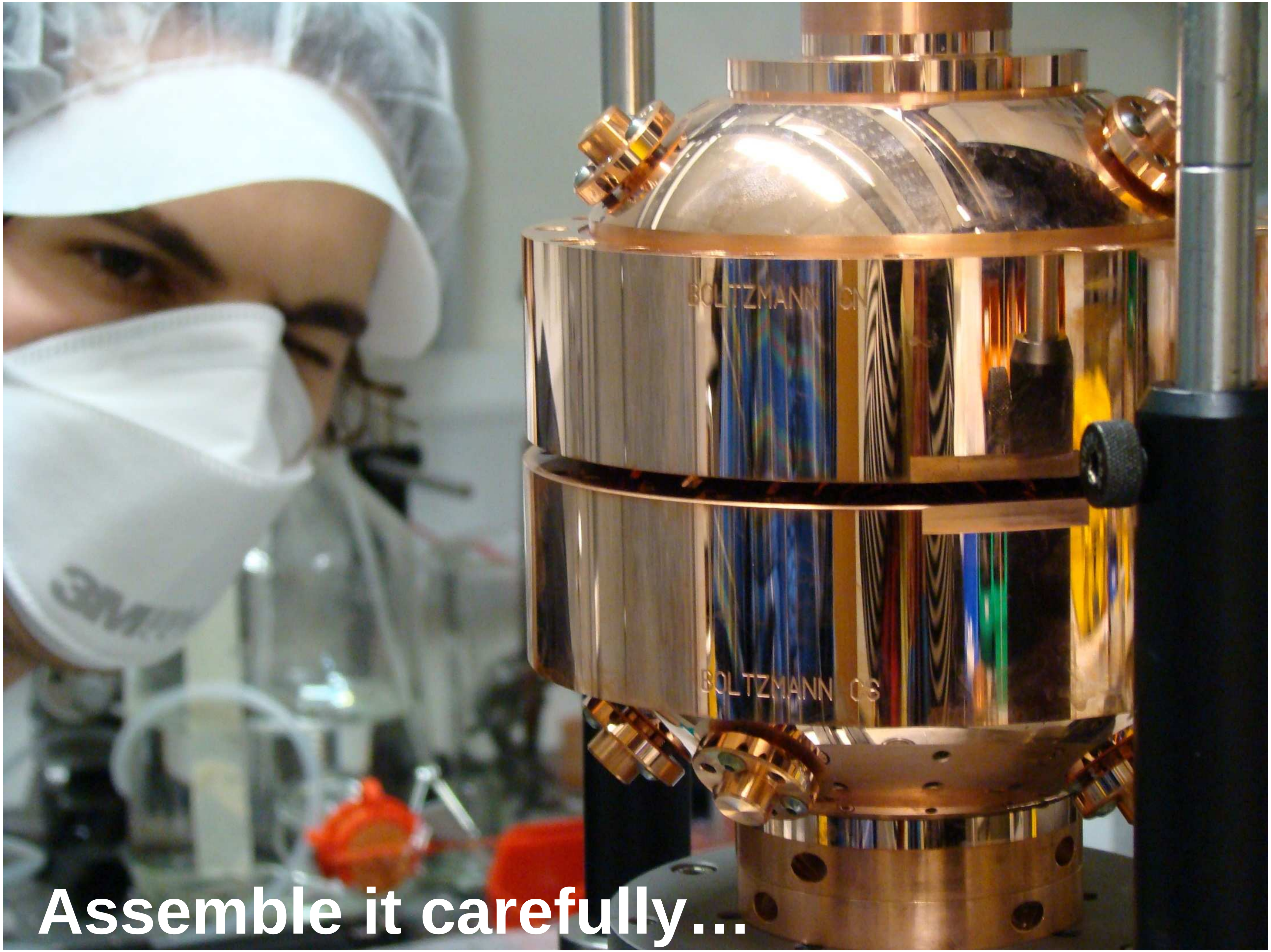
One hemisphere…



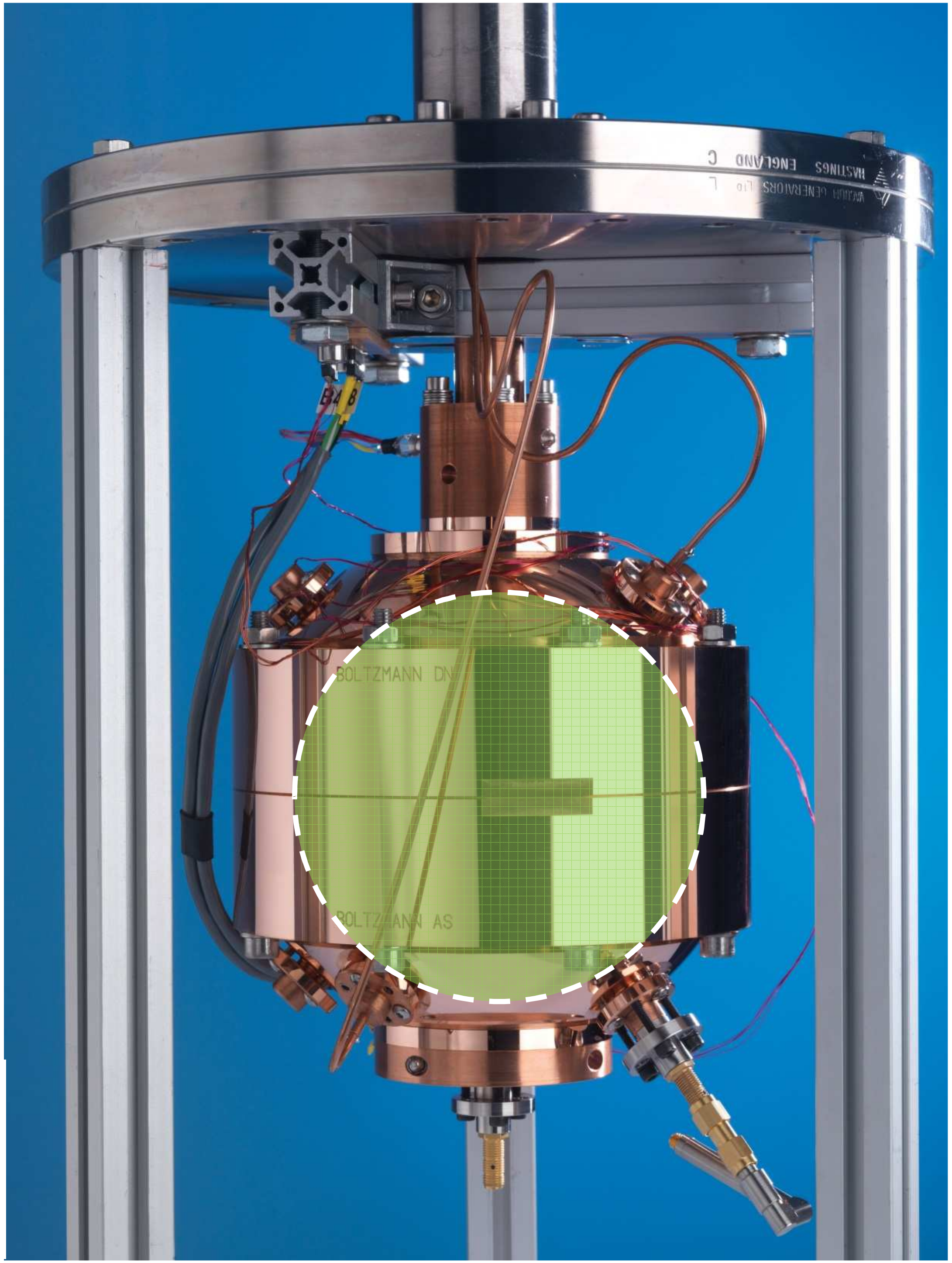
Measure it's shape…



…with a Coordinate Measuring Machine


BOLTZMANN CN
BOLTZMANN CS
3M
Assemble it carefully…

Wire it up…



Copper Resonator

- 124 mm inner diameter
- Argon Gas

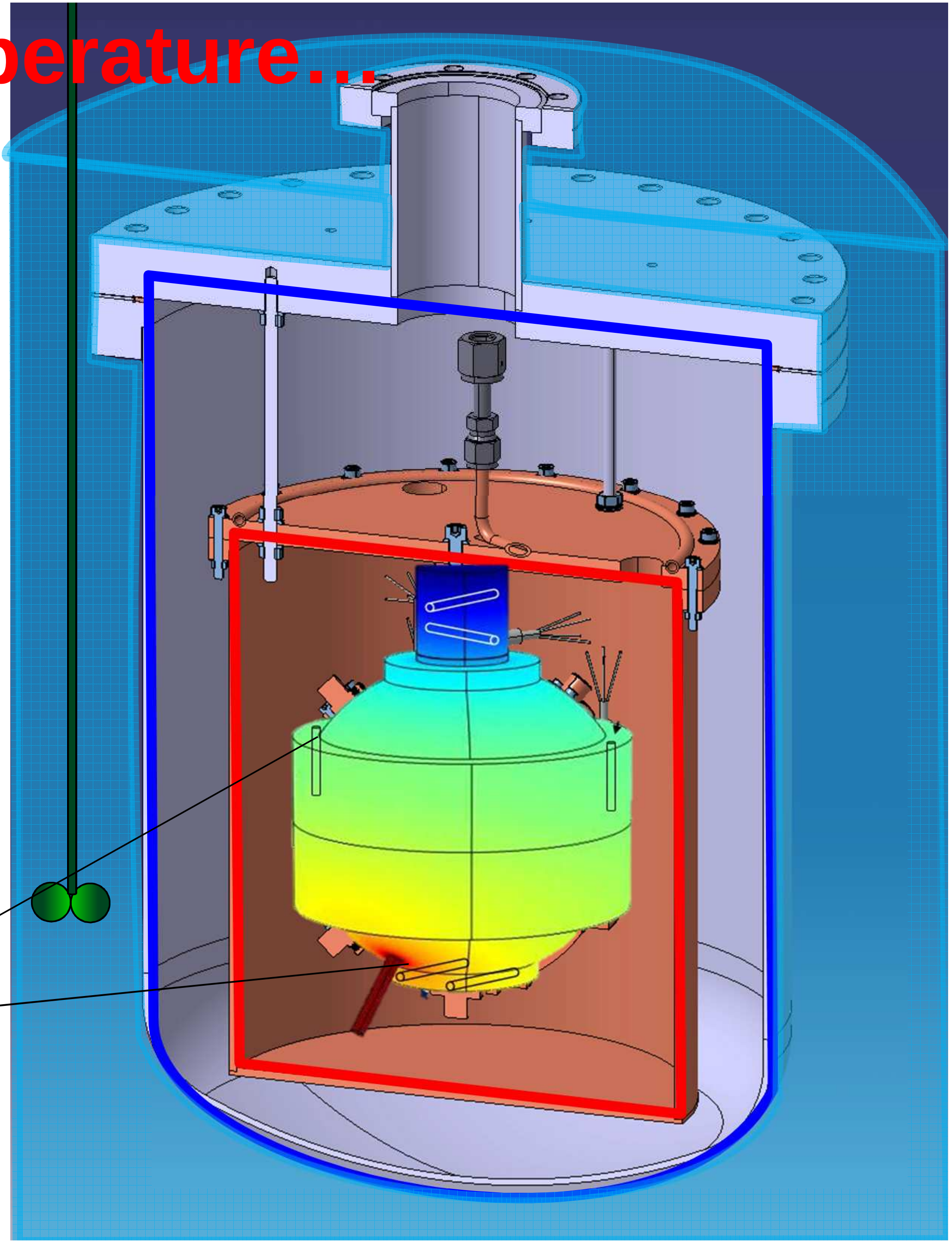
Cranfield UNIVERSITY

NPL National Physical Laboratory

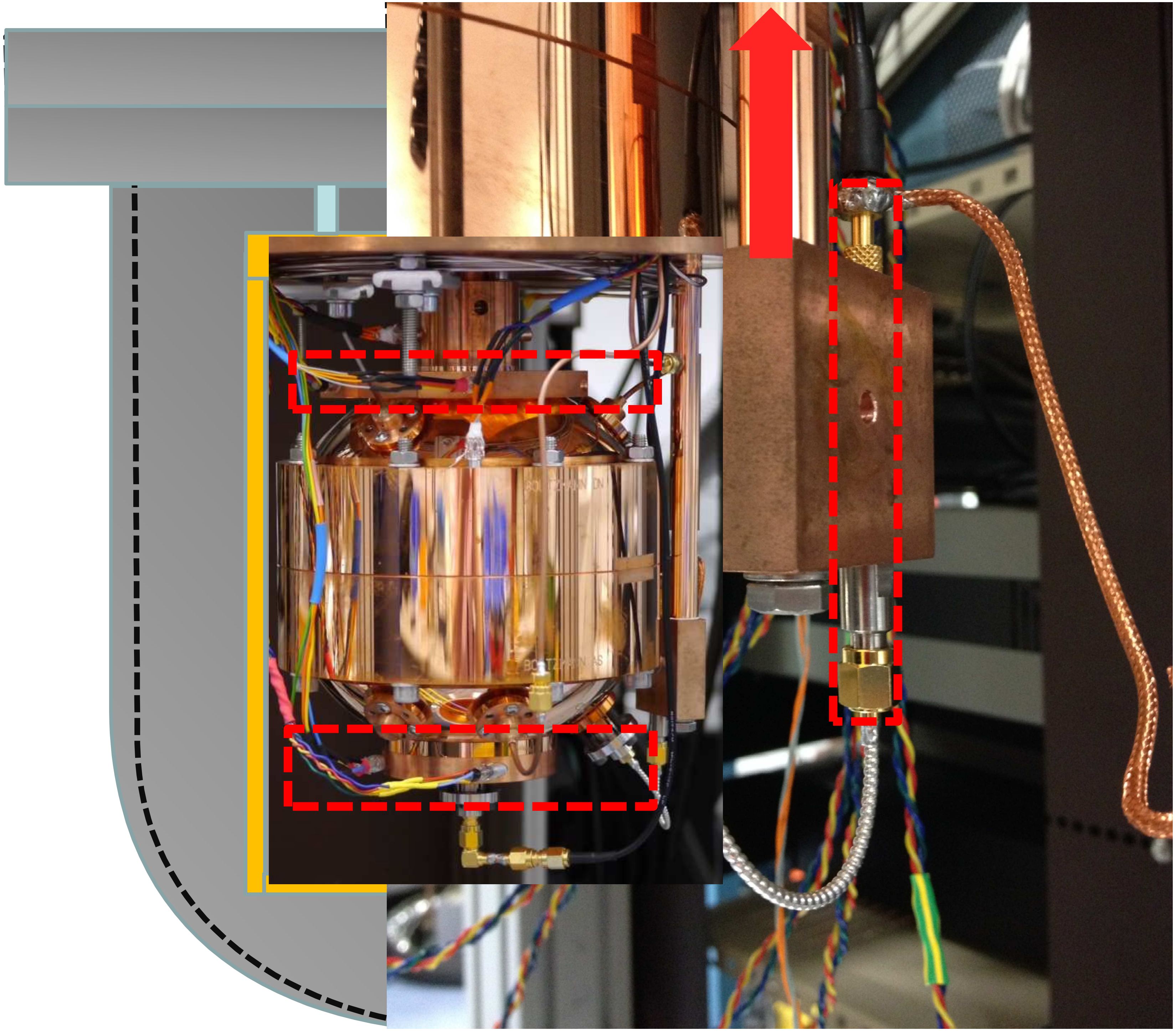
Stabilise the temperature…

Resonator

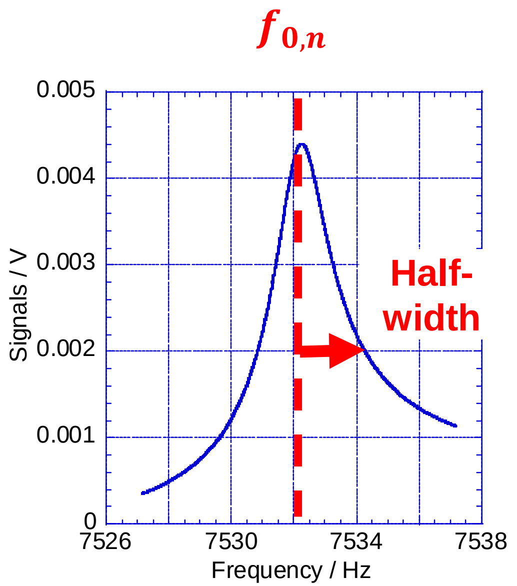
- **Placed inside a isothermal vessel**
- **Held inside a pressure vessel**
- **Dunked in bucket**
- **Liquid Stirred**



Observed
ΔT = 91 μK



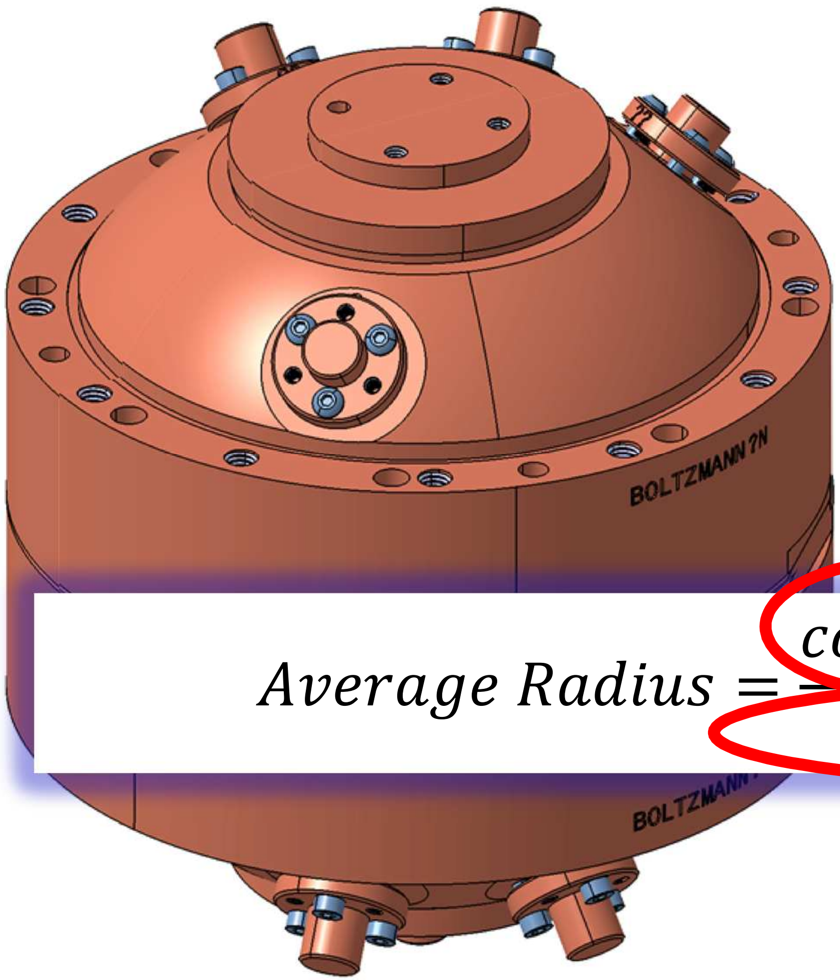
Measure the speed of sound in a spherical resonator



$$\text{Speed of Sound} = \text{Resonant Frequency} \times \frac{\text{Radius}}{\text{constant for each resonance}}$$

First three constants for spheres are 3.018, 1.880, 1.396

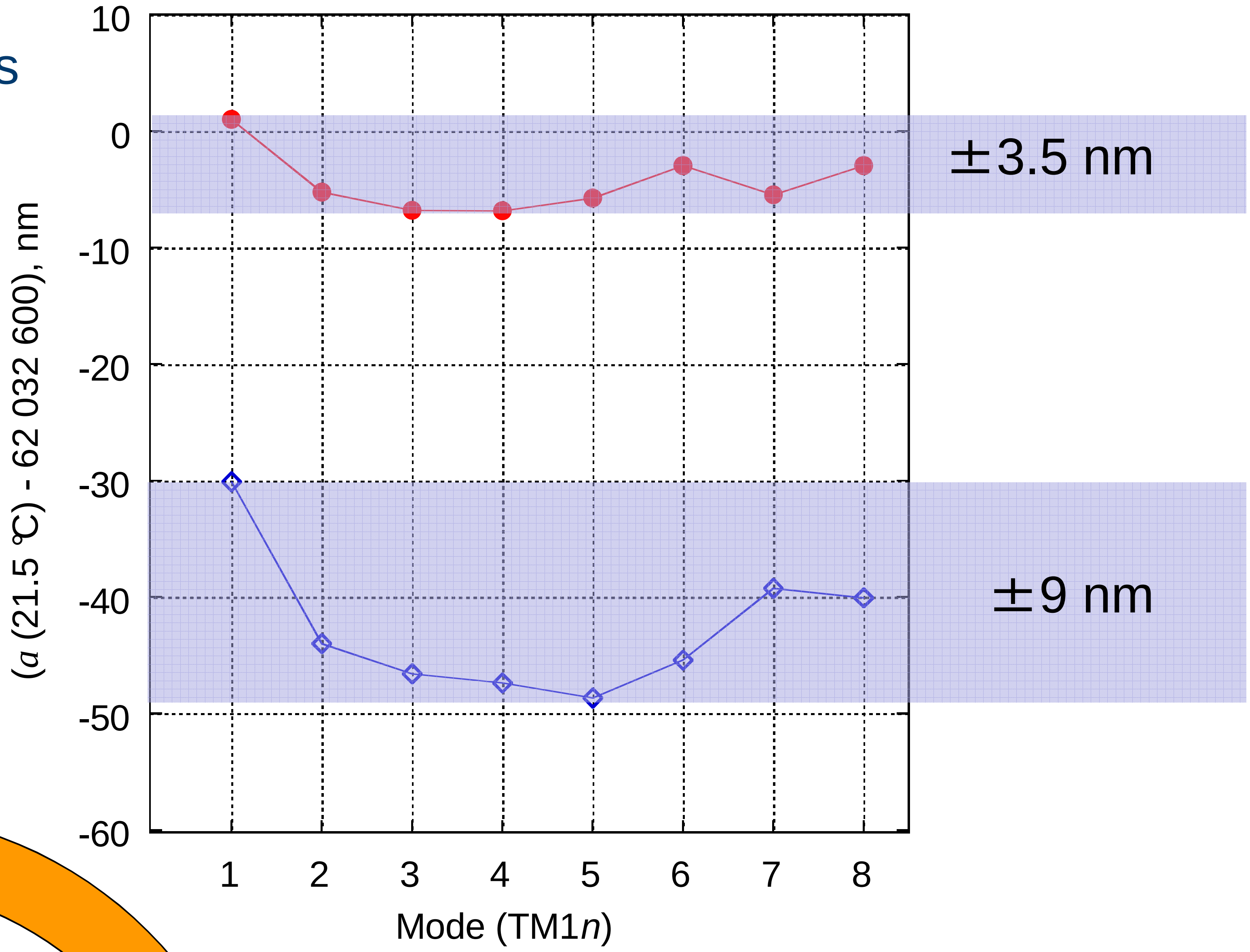
Measure the Average Radius using Microwaves

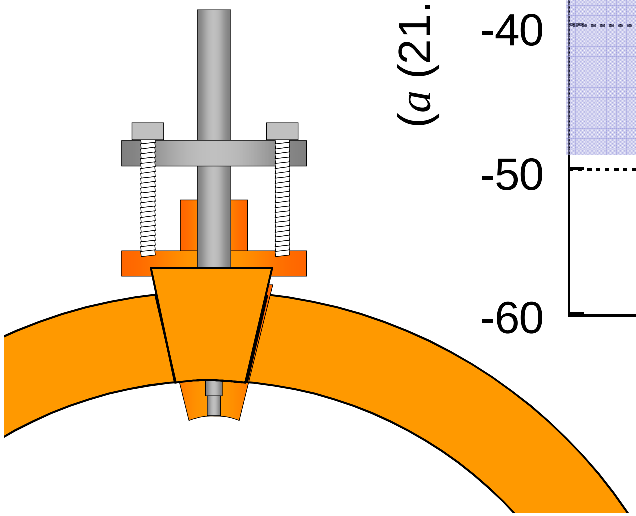


$$\text{Average Radius} = \frac{\text{constant} \times \text{speed of light}}{\text{Resonant Frequency}}$$

Microwave Radius Estimates ***in Vacuum***

nanometres





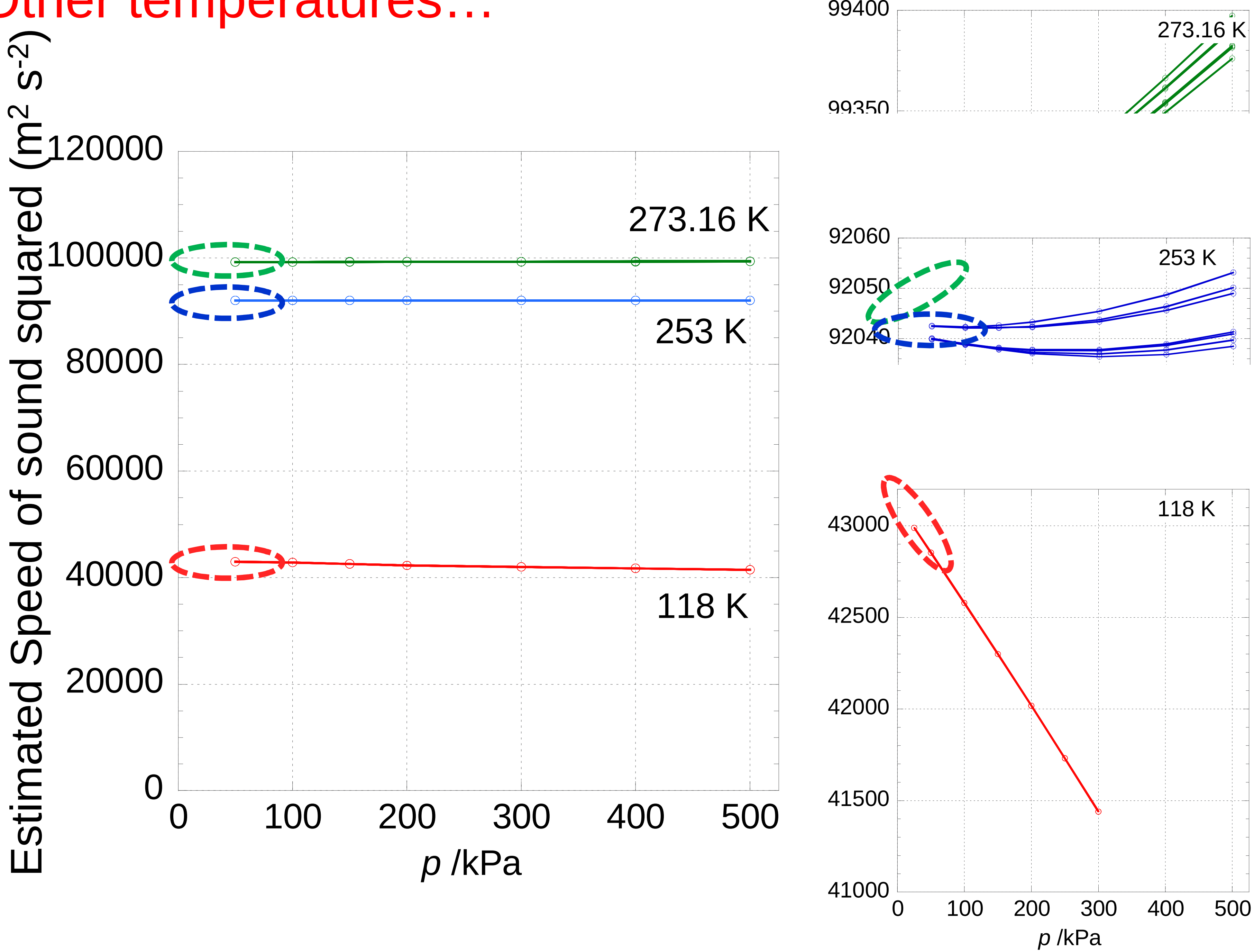
Measure

the speed of sound

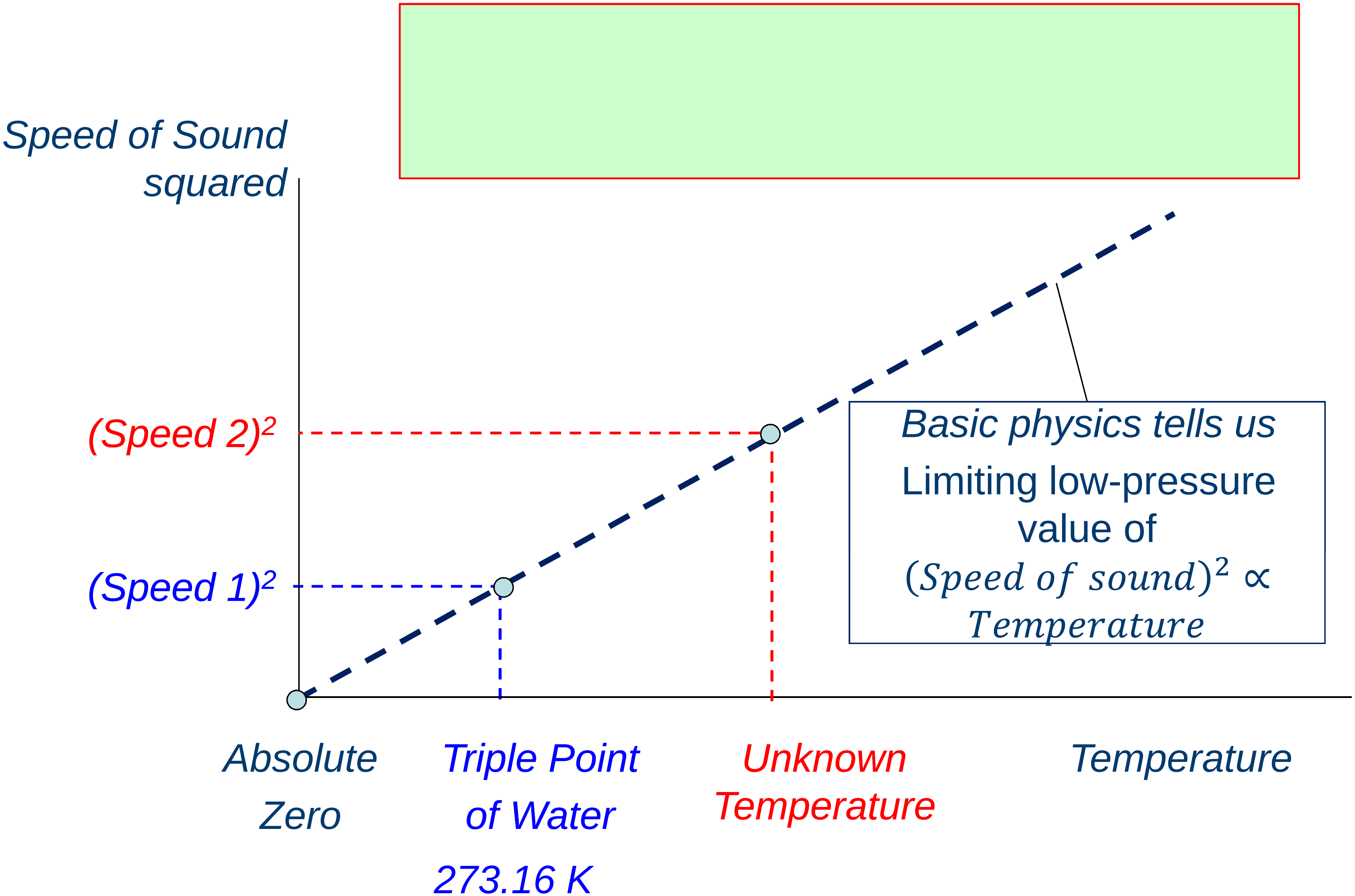
versus

pressure.

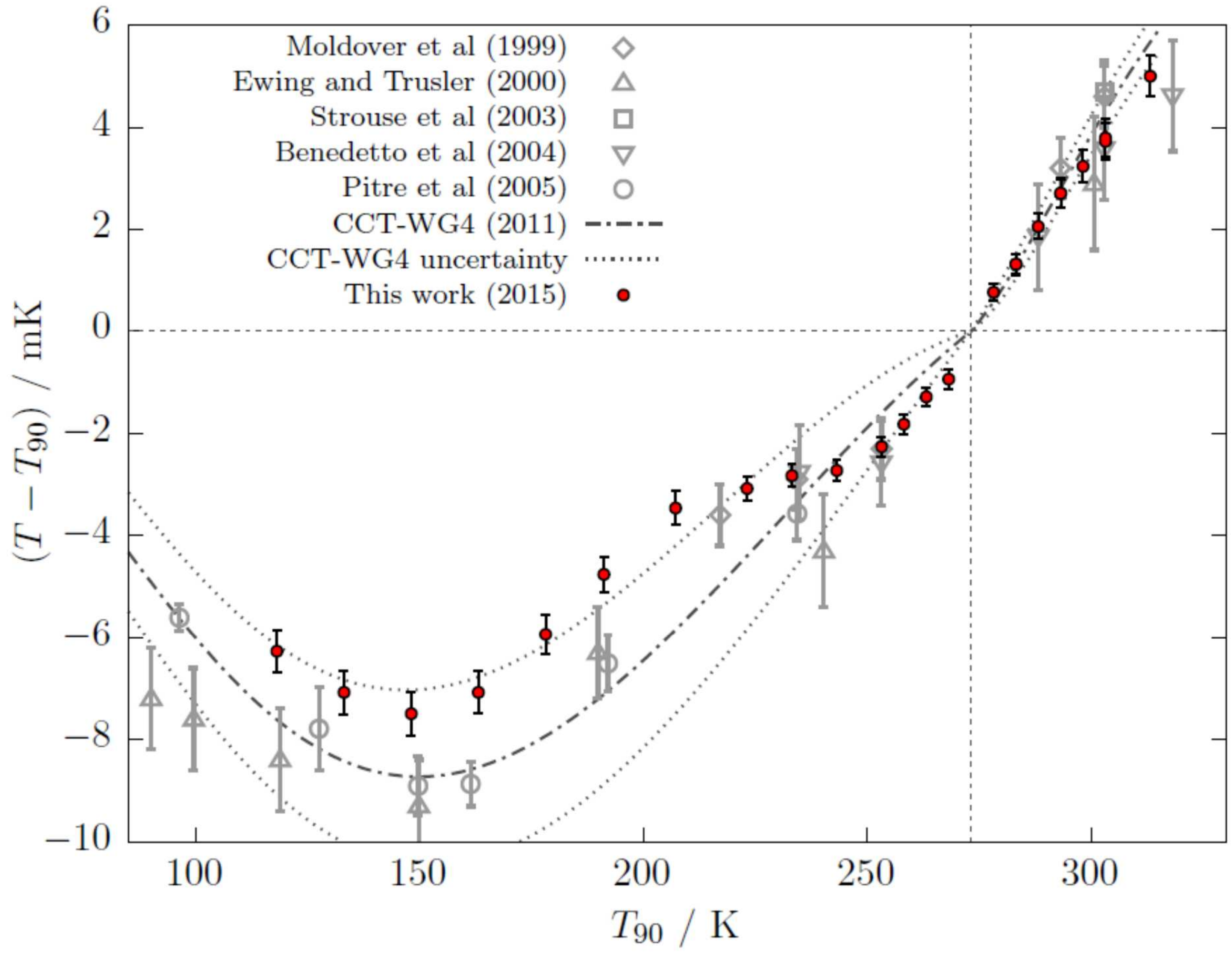
Other temperatures…



How to work out an unknown temperature



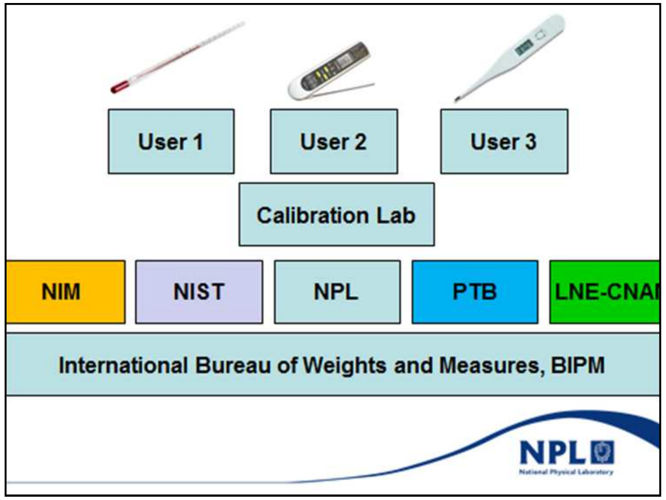
NPL results



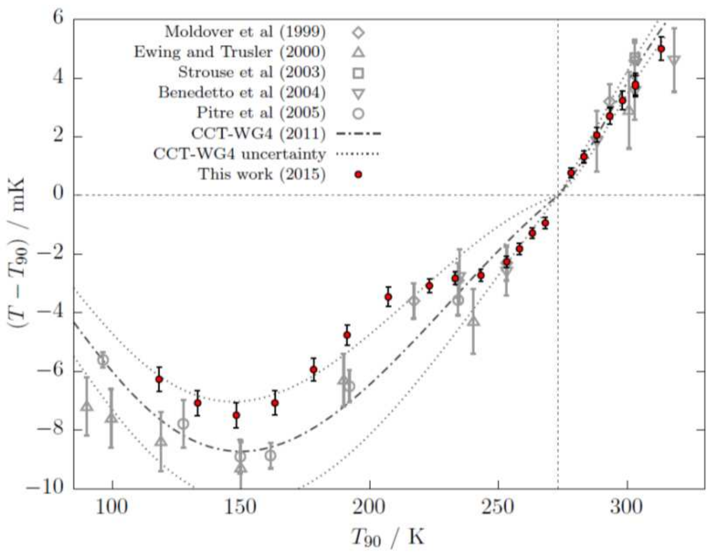
Moldover et al (1999)
Ewing and Trusler (2000)
Strouse et al (2003)
Benedetto et al (2004)
Pitre et al (2005)
CCT-WG4 (2011)
CCT-WG4 uncertainty
This work (2015)
$(T - T_{90})$ / mK
T_{90} / K
6
4
2
0
−2
−4
−6
−8
−10
100
150
200
250
300

How do you really know what the temperature is?

- **Inter-comparisons and calibrations according to ITS-90**
  - **Make sure everyone agrees**



- **Fundamental Measurements**
  - **Measure the errors in ITS-90**



How do you ***really*** know what the temperature is?

1. **What this talk is about**
2. **The International Temperature Scale of 1990**
3. **Primary Thermometry**
4. **Measuring the speed of sound *really* accurately**
5. **What next?**

The Second Talk!

Or more details…

1. **Triaxial Ellipsoids**

2. **Boltzmann Maths**

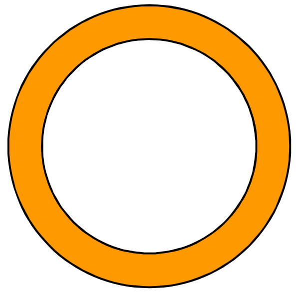
3. **More on Half-widths** **(How we know we are right!)**

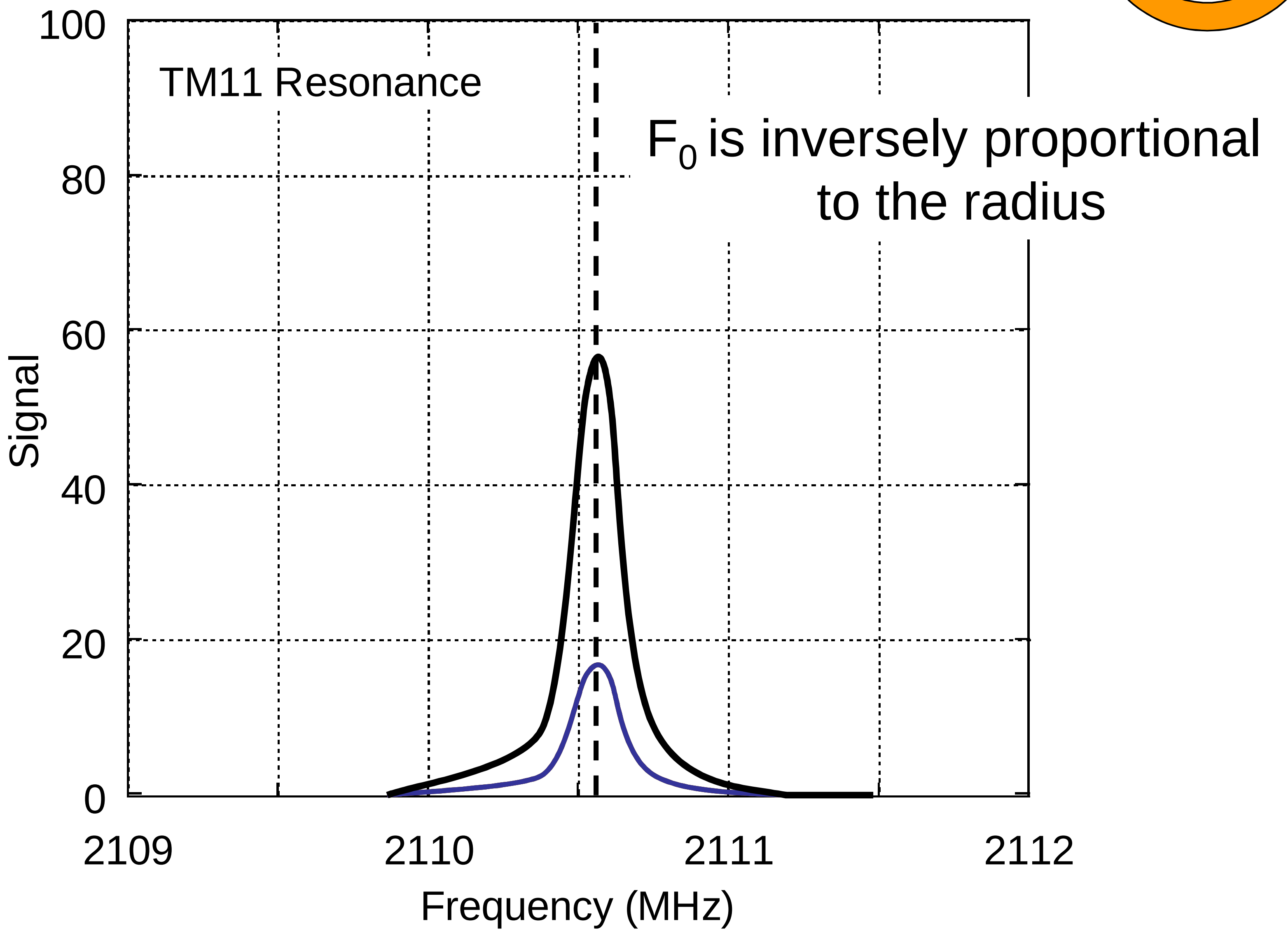
4. **Why the gas doesn't matter**

5. ***$T - T_{90}$***

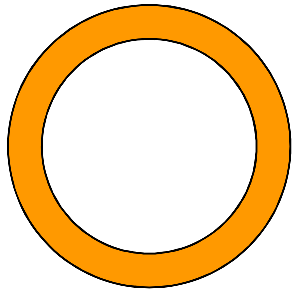
Slides about triaxial ellipsoids

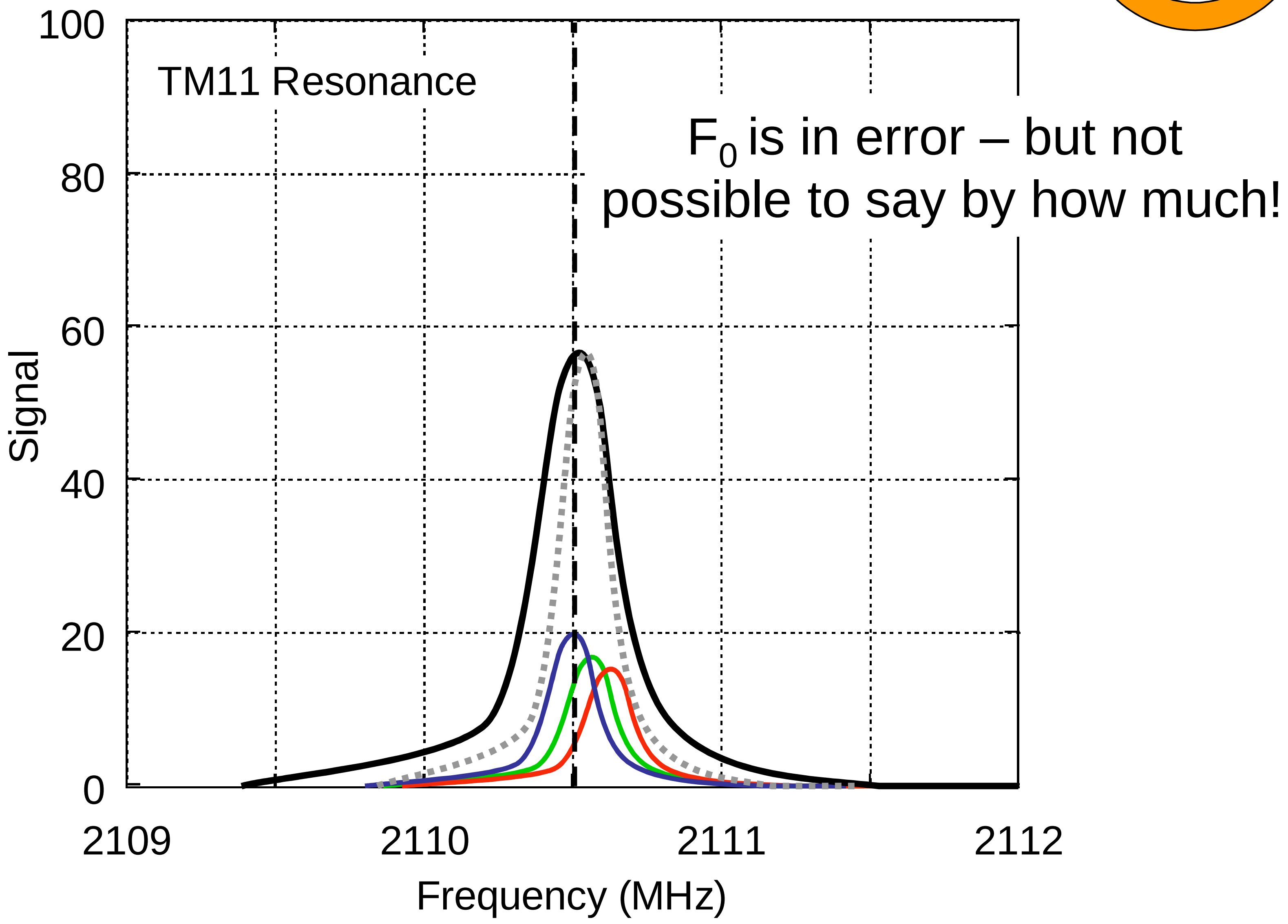
Microwave resonance
in a perfect sphere



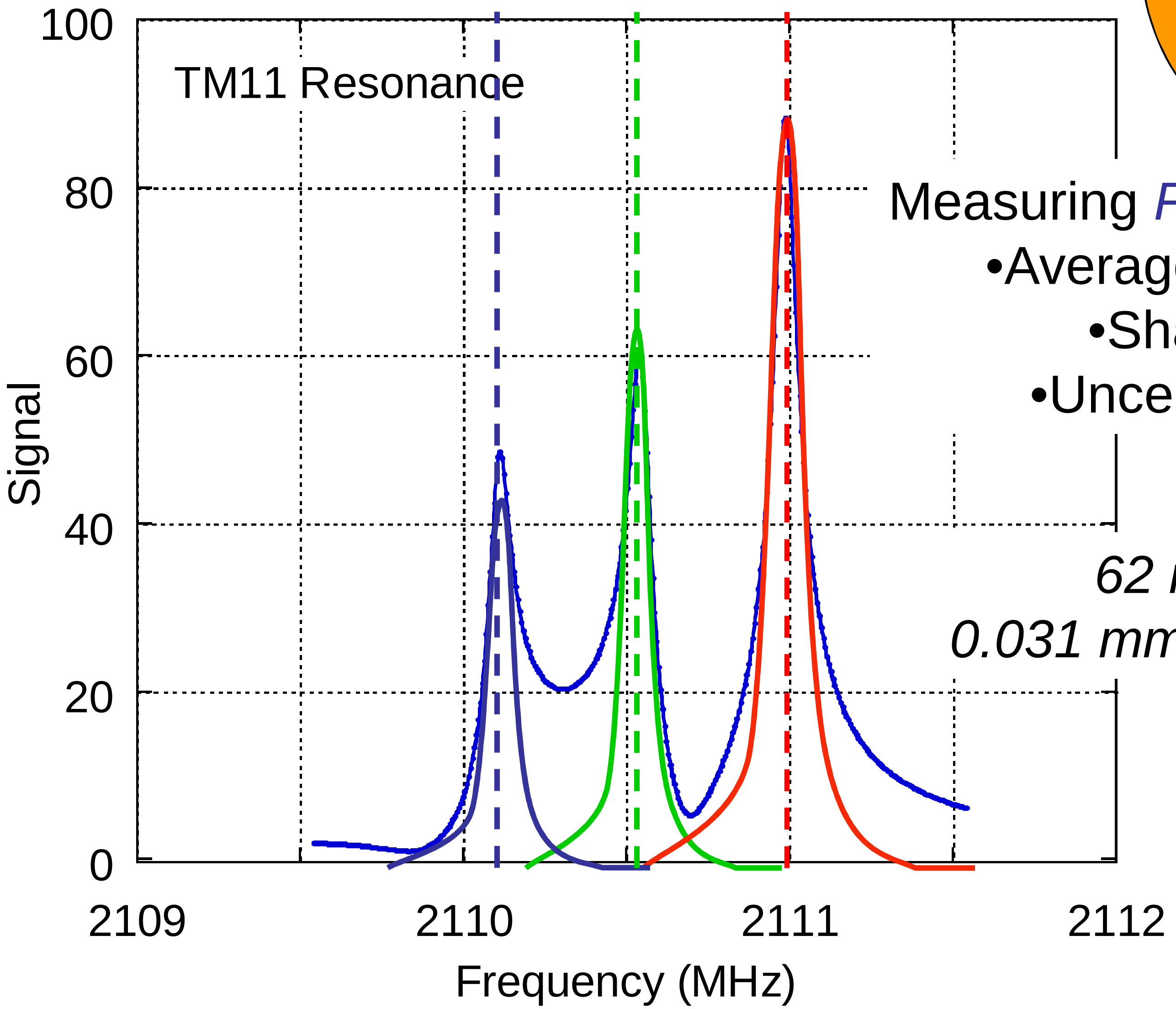


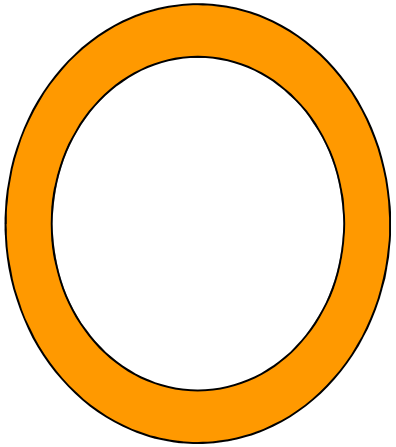
Microwave resonance
in a ***nearly*** perfect sphere





Microwave resonance
in a triaxial ellipsoid





Measuring F_0, F_1 and F_2

- Average radius
- Shape
- Uncertainty

62 mm

0.031 mm deviation

Back

Heat Contact
(Why the gas used in a primary thermometer doesn't matter)

Heat

Hot Object

Cold Object

When materials 'touch',

fast atoms slow down,

slow atoms speed up

until…

Average energy per molecule is equal.

[Actually what equalises is the average energy per accessible degree of freedom]

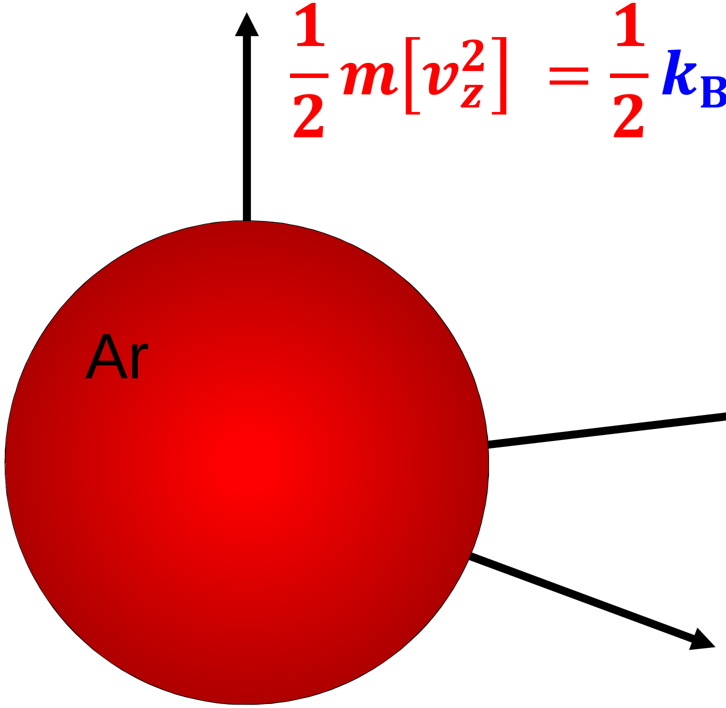
Back

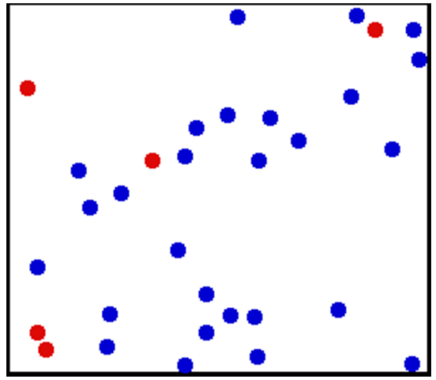
Boltzmann Maths

We measure in argon gas

- Molecular motions are simple in a gas
- We can approach ‘ideal gas’ conditions at low pressure
- In an ideal gas the internal energy is just the kinetic energy of the molecules

$$\frac{1}{2}m[v_z^2] = \frac{1}{2}k_B T$$

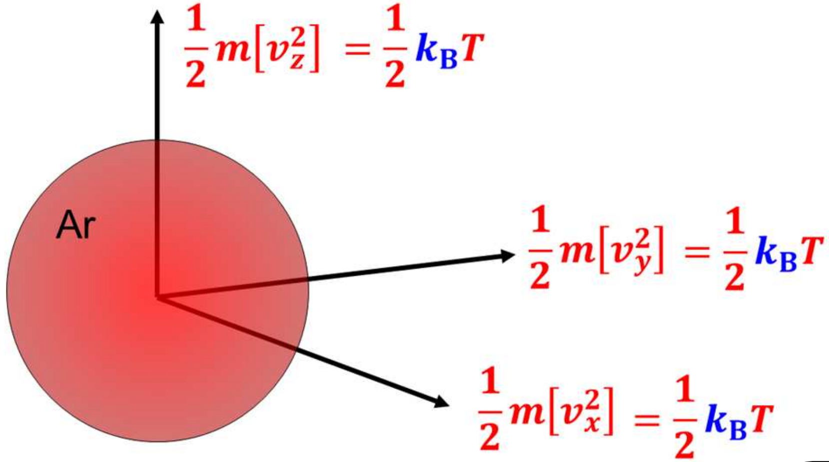


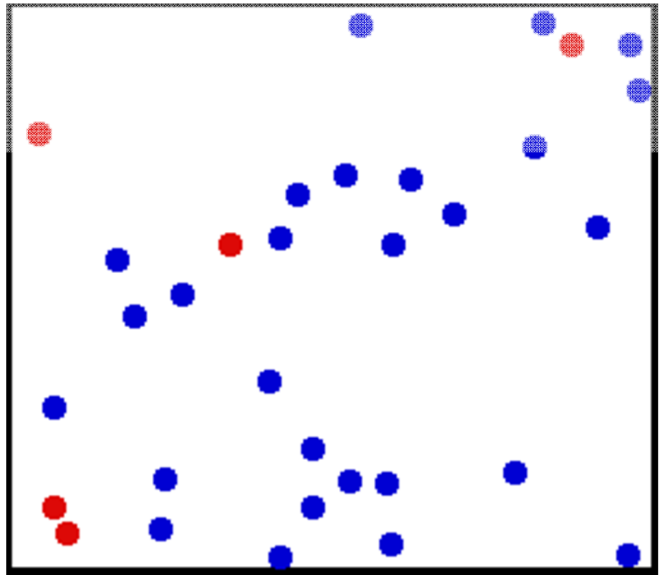


$$\frac{1}{2}m[v_y^2] = \frac{1}{2}k_B T$$

$$\frac{1}{2}m[v_x^2] = \frac{1}{2}k_B T$$

The big idea…





Look up mass of an argon molecule

$$k_B = \frac{3m}{5T}\,[\,speed\ of\ sound]^2$$

$$\underbrace{\qquad}_{\frac{9}{5}(speed\ of\ sound)^2}$$

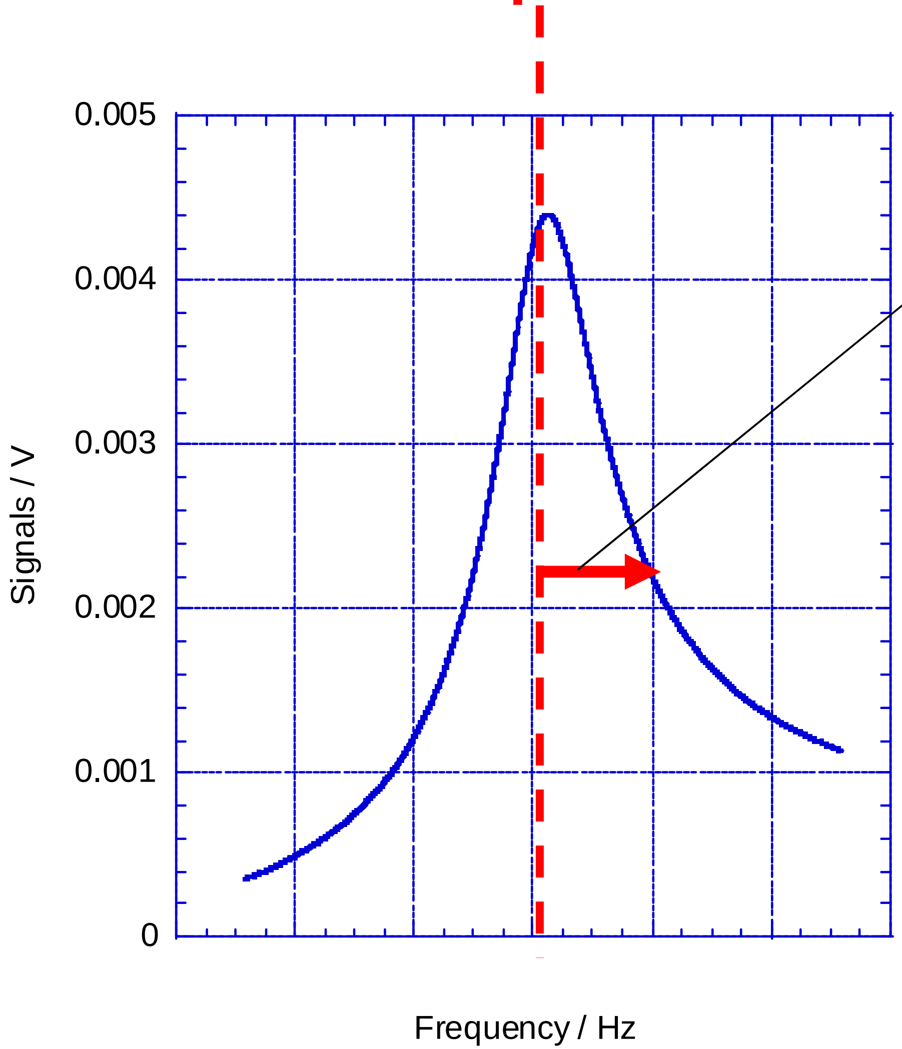
Back

Carry out experiment at T_{TPW}

Measure the speed of sound

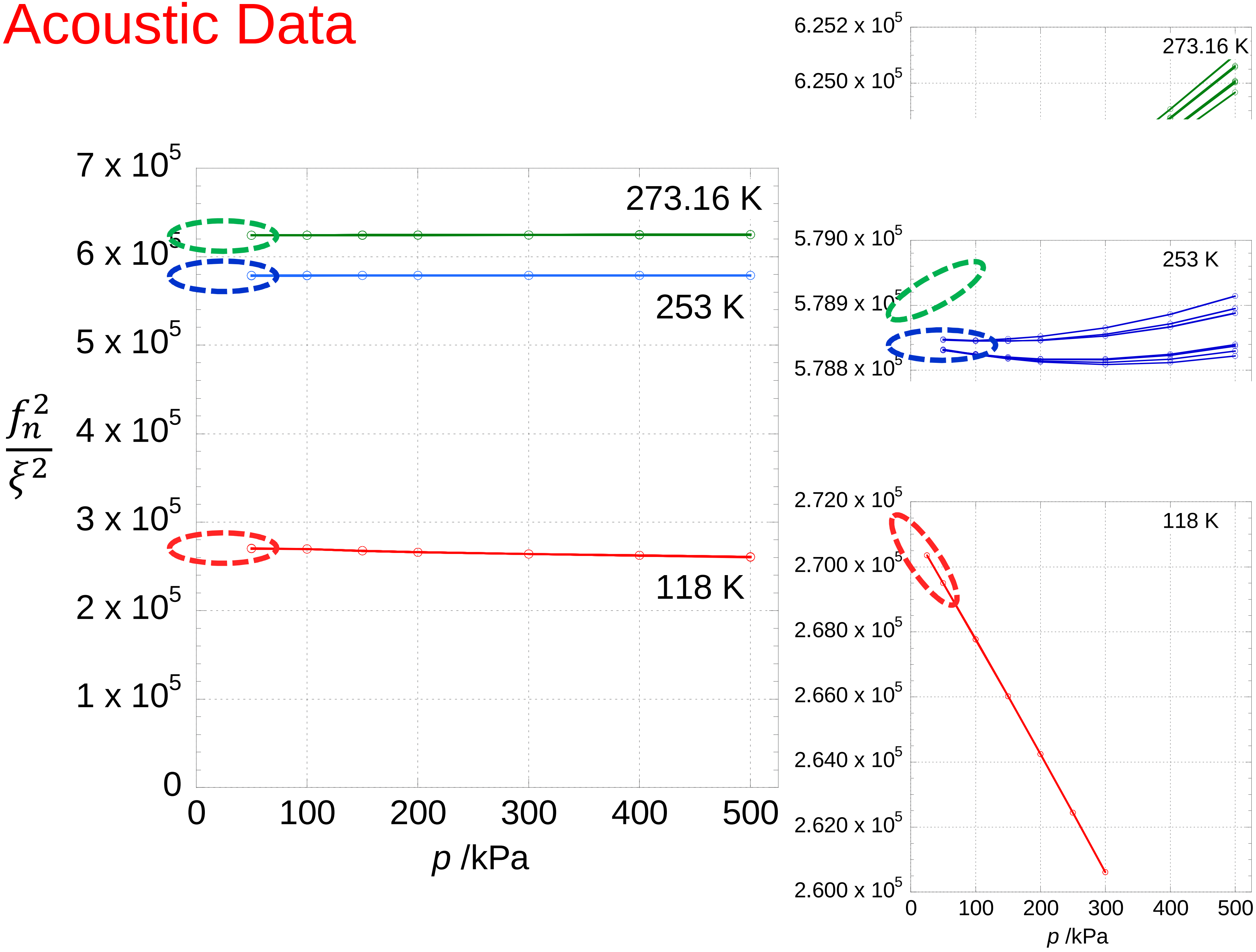
Half-widths

Central frequency changes with temperature



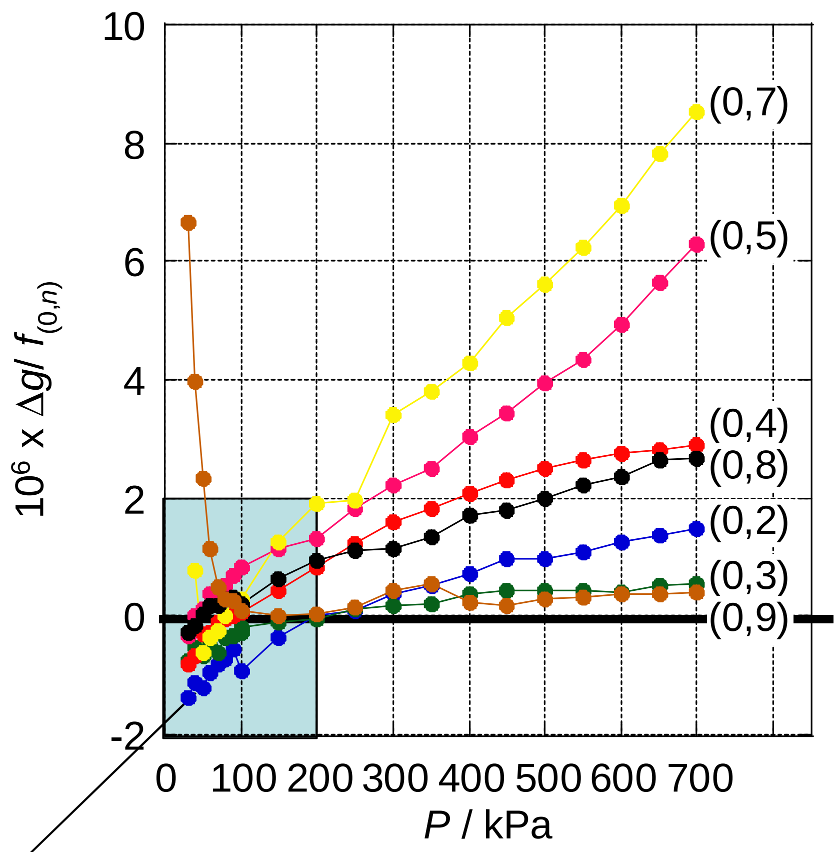
Half-Width should be *exactly* what we expect

Acoustic Data



Half-Width (*Experiment – Theory*)

Parts per million of resonance frequency

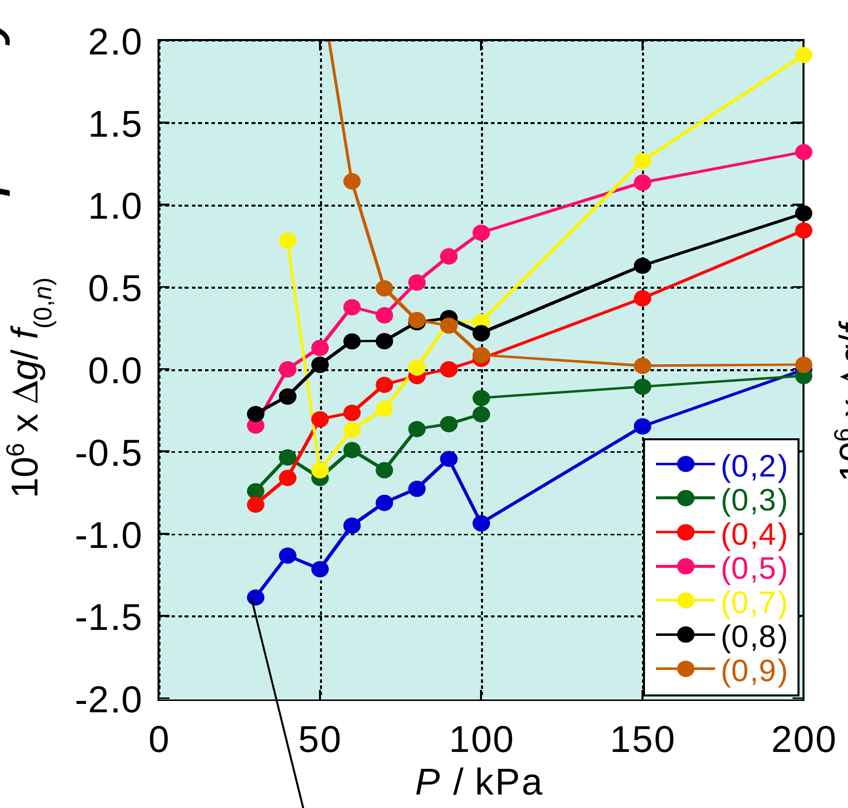


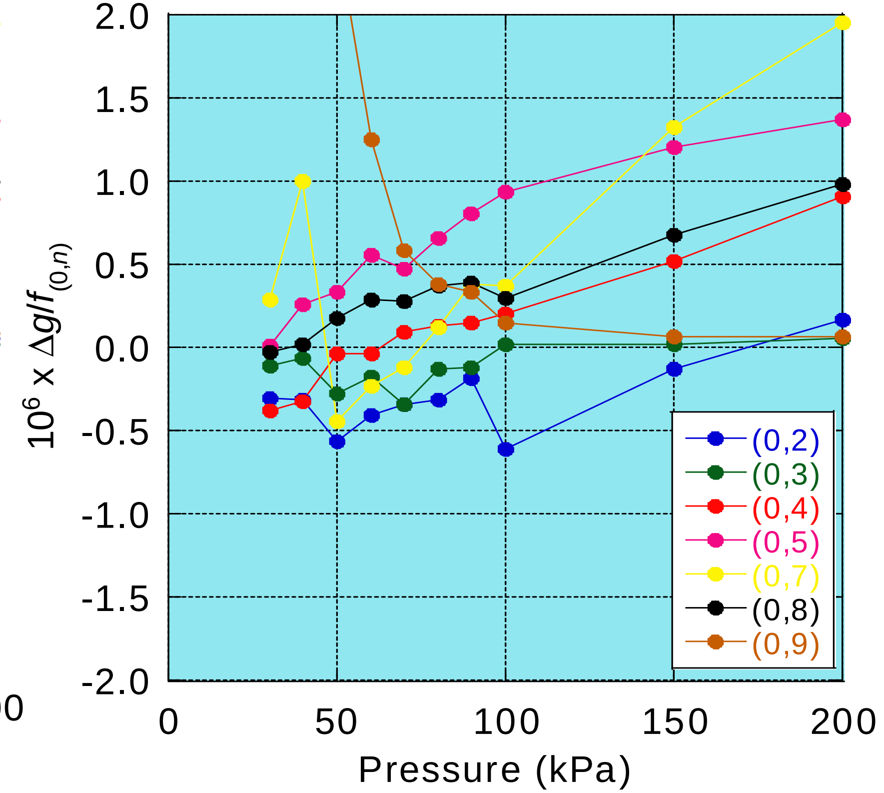
f_0 =3548.8095 Hz
expected width = 2.864 Hz
measured width = 2.858 Hz

Half-Width

Experiment – Theory *Experiment – New Theory*

Parts per million of resonance frequency





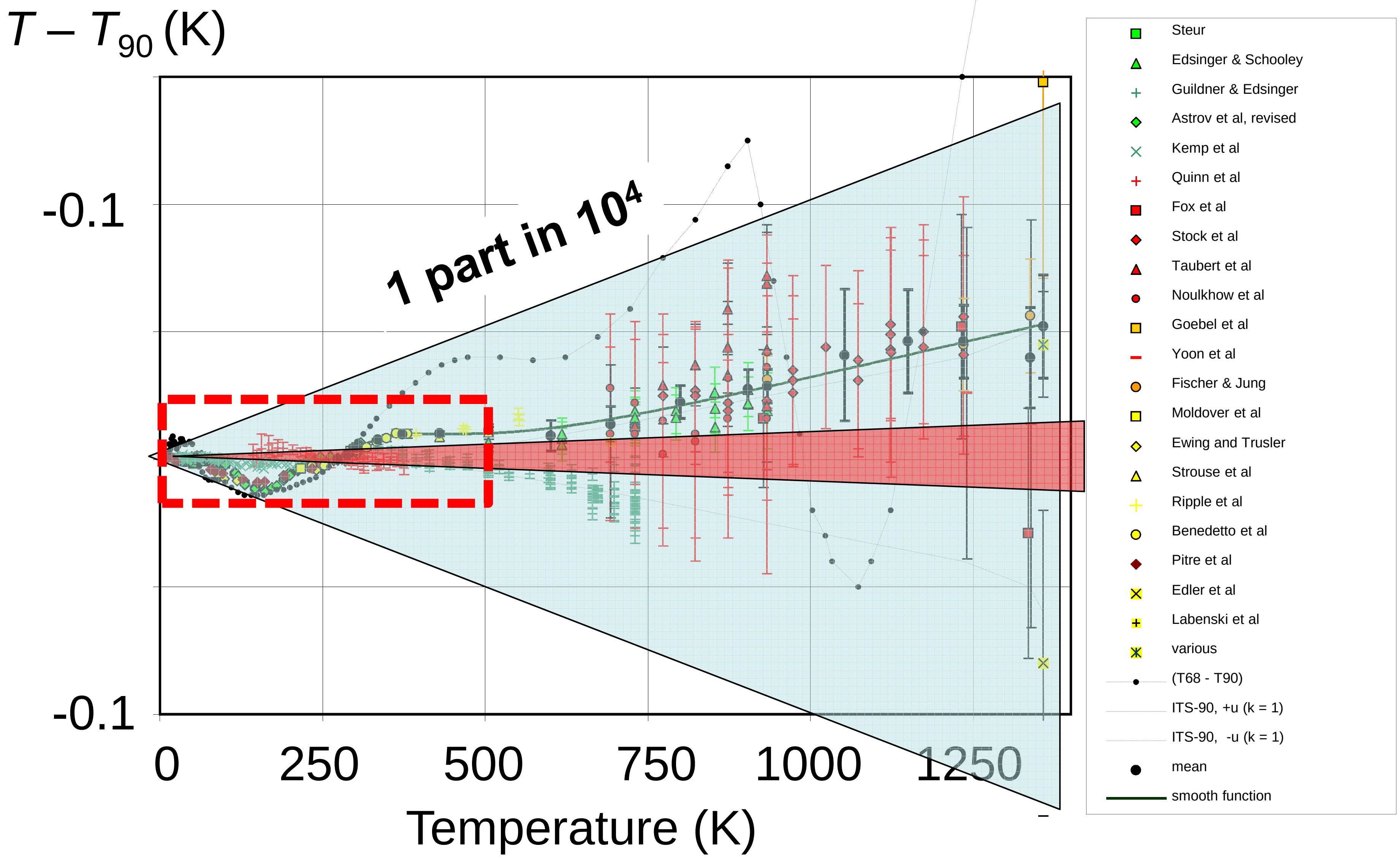
f_0 =3548.8095 Hz
expected width = 2.864 Hz
measured width = 2.858 Hz

Back

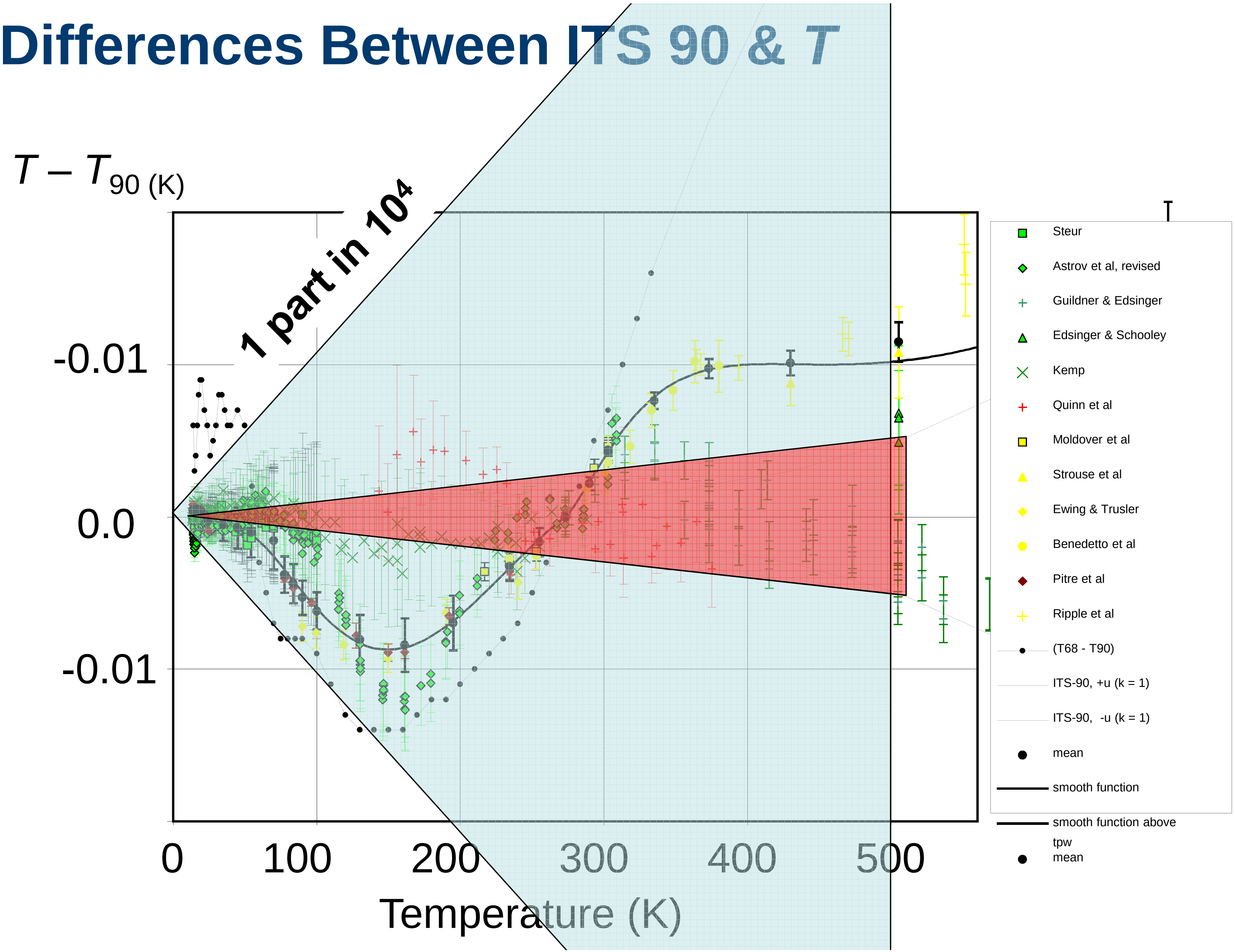
$T - T_{90}$ in 2010

Differences Between ITS90 & *T*

2010



Differences Between ITS 90 & *T*



Acoustic data since 1990

Moldover 1999

Maraging Steel Resonator

- 180 mm inner diameter
- Argon Gas
- Estimated radius with microwaves at $p = 0$

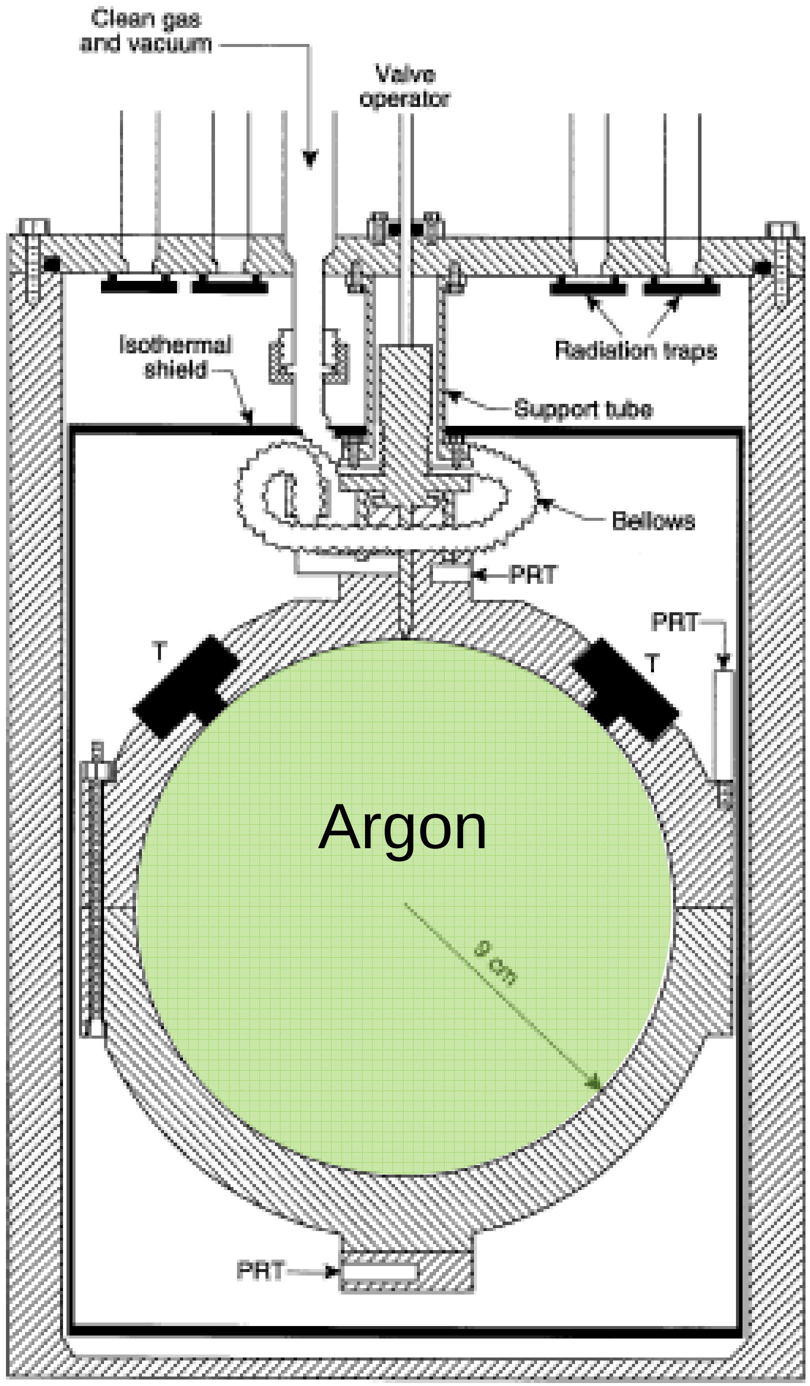
Thermodynamic Temperatures of the Triple Points of Mercury and Gallium and in the Interval 217 K *to* 303 K

Volume 104 Number 1 January–February 1999

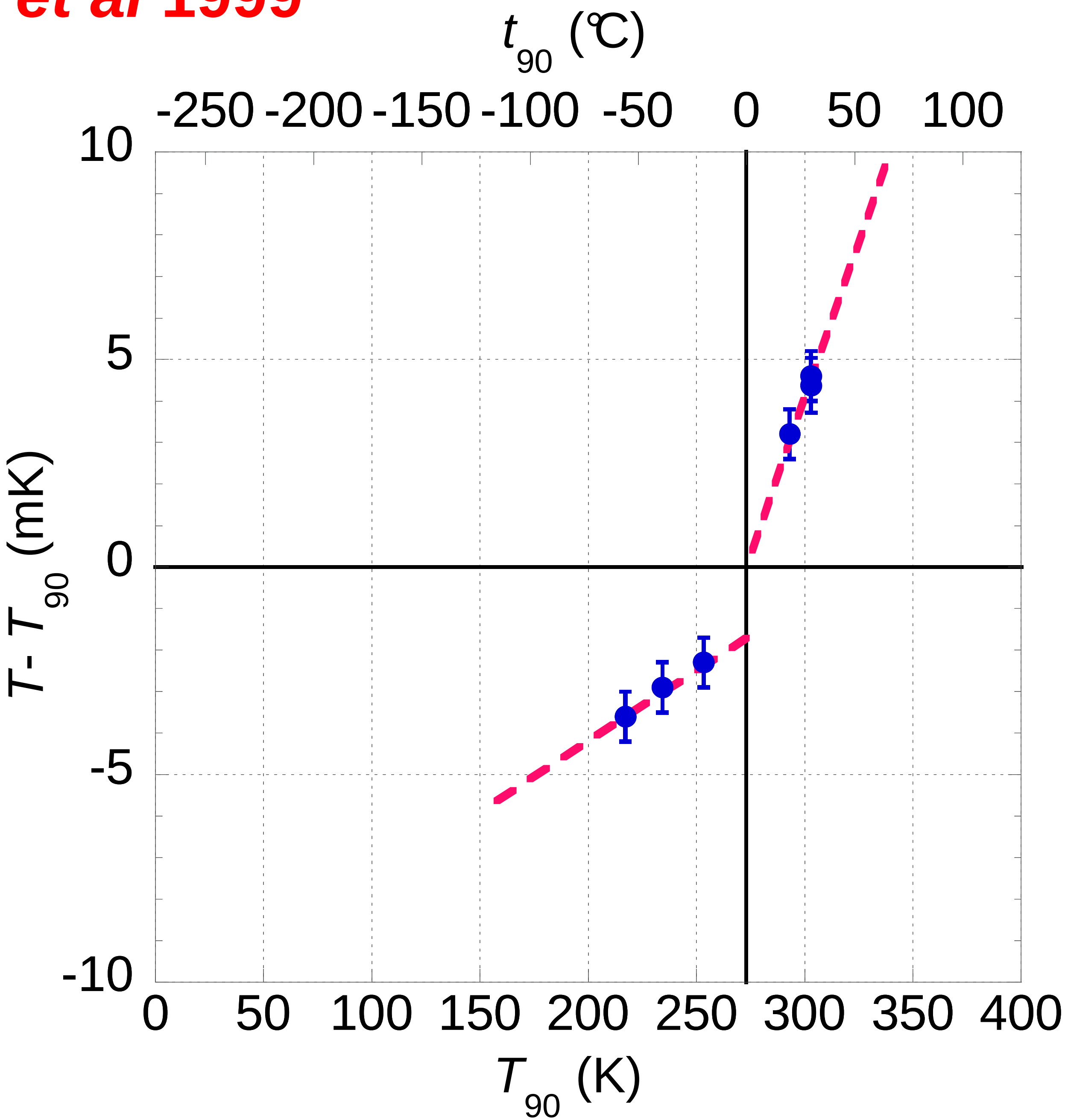
M. R. Moldover, S. J. Boyes[1], C. W. Meyer, and A. R. H. Goodwin[2]

National Institute of Standards and Technology, Gaithersburg, MD 20899-0001

We measured the acoustic resonance frequencies of an argon-filled spherical cavity and the microwave resonance frequencies of the same cavity when evacuated. The microwave data were used to deduce the thermal expansion of the cavity and the acoustic data were fitted to a temperature-pressure surface to deduce zero-pressure speed-of-sound ratios. The ratios determine $(T - T_{\mathrm{g}})$, the difference be- been used to redetermine both the universal gas constant R, and T_{g}. However, the present value of T_{g} is (4.3 ± 0.8) mK larger than that reported earlier. We suggest that the earlier redetermination of T_{g} was erroneous because a virtual leak within the resonator contaminated the argon used at T_{g} in that work. This suggestion is supported by new acoustic data taken when the resonator was filled with xenon. For-



Moldover *et al* 1999



Ewing and Trusler 2000

Aluminium Resonator

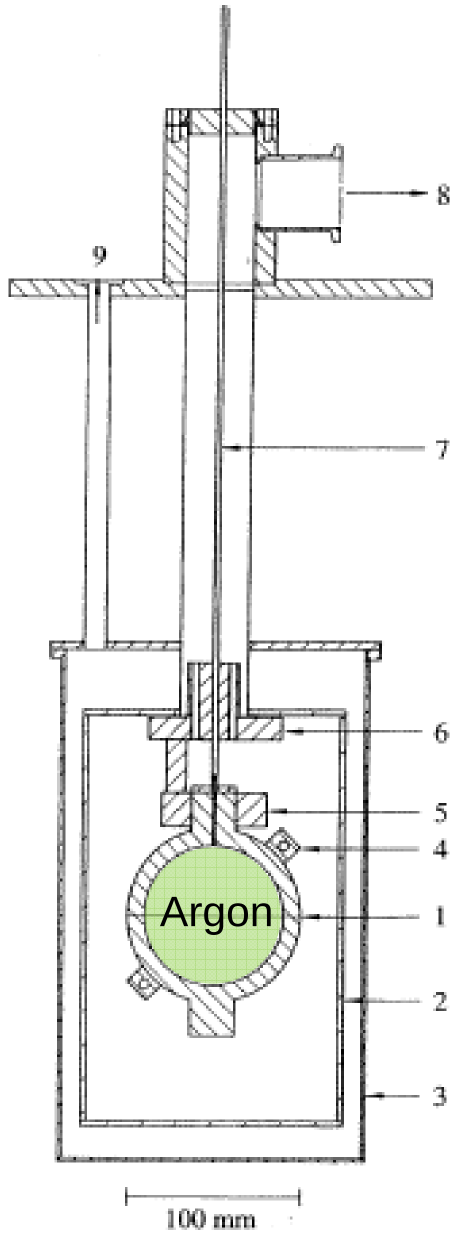
- 80 mm inner diameter
- Argon Gas
- Estimated radius with microwaves at $p = 0$

J. Chem. Thermodynamics **2000**, 32, 1229–1255
doi:10.1006/jcht.1999.0606
Available online at http://www.idealibrary.com on IDEAL®

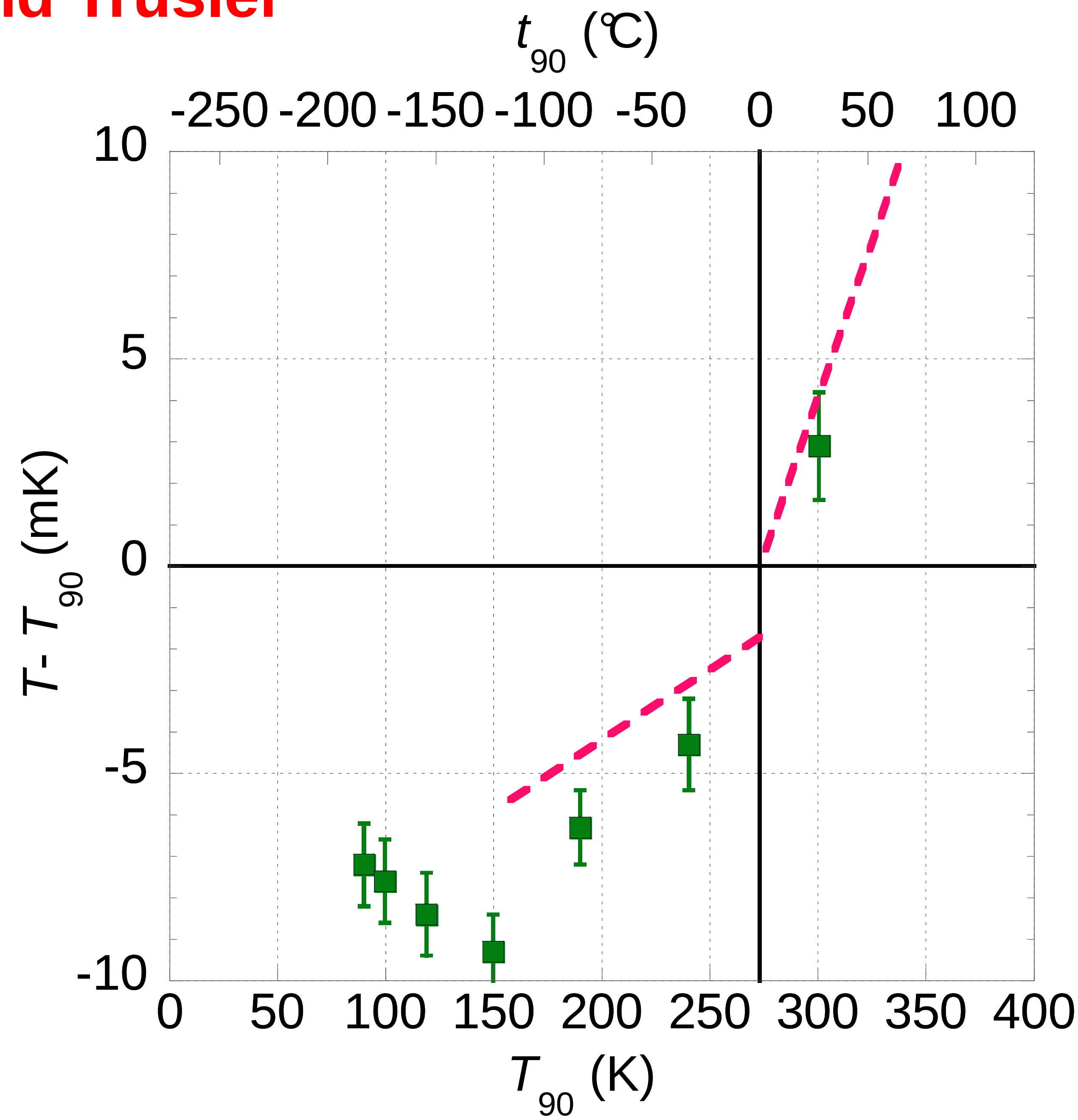
Primary acoustic thermometry between $T = 90$ K and $T = 300$ K

M. B. Ewing
Department of Chemistry, University College London, 20 Gordon Street, London WC1H 0AJ, U.K.

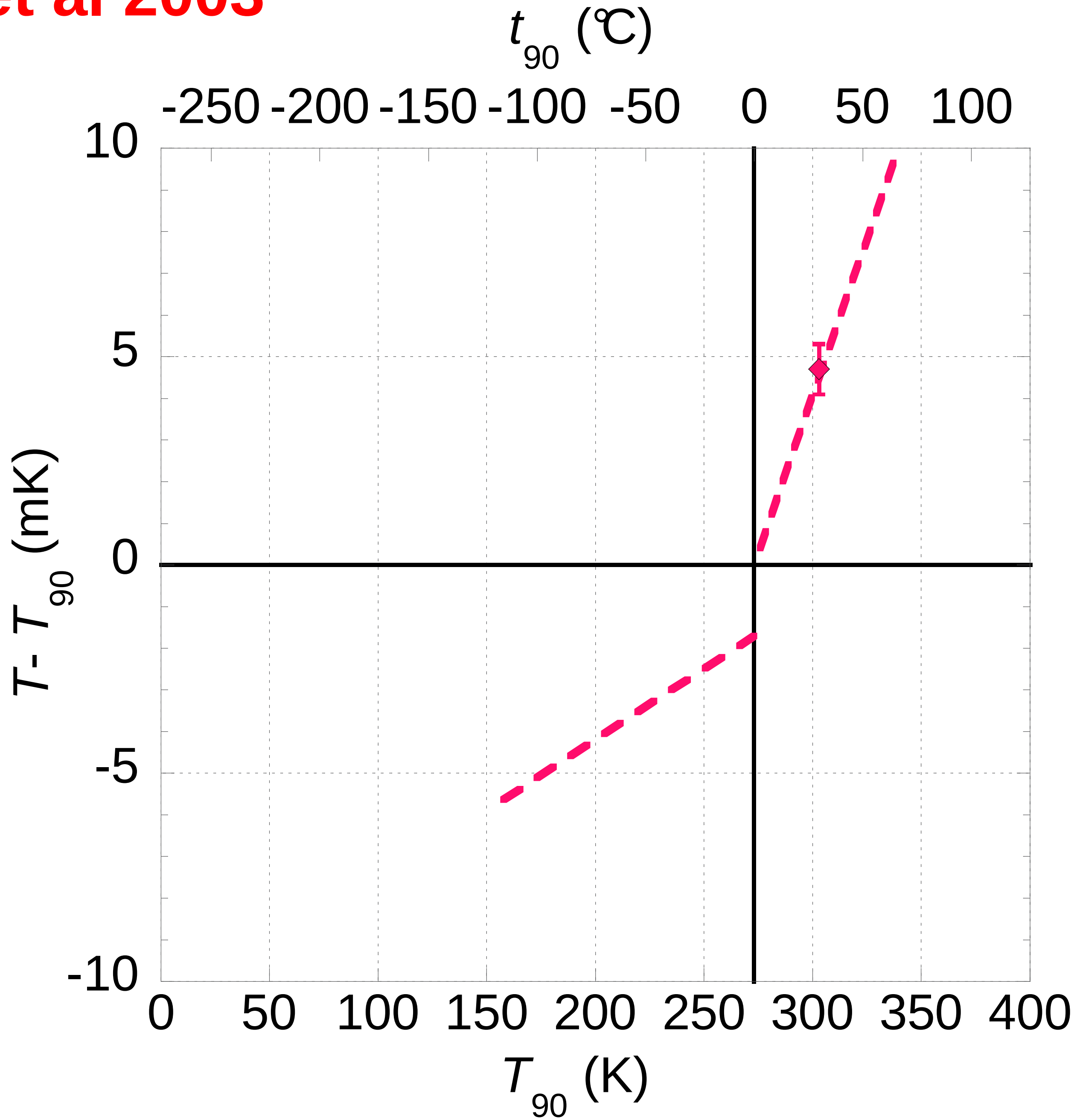
and J. P. M. Trusler
Department of Chemical Engineering and Chemical Technology, Imperial College, London SW7 2BY, U.K.



Ewing and Trusler



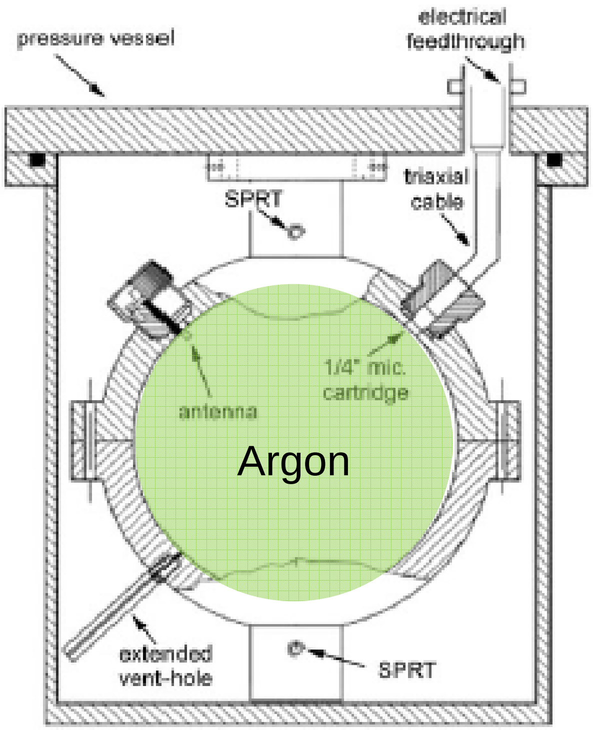
Strouse et al 2003



Benedetto et al 2004

Stainless Steel Resonator

- 120 mm inner diameter
- Argon Gas
- Estimated radius with microwaves at $p = 0$

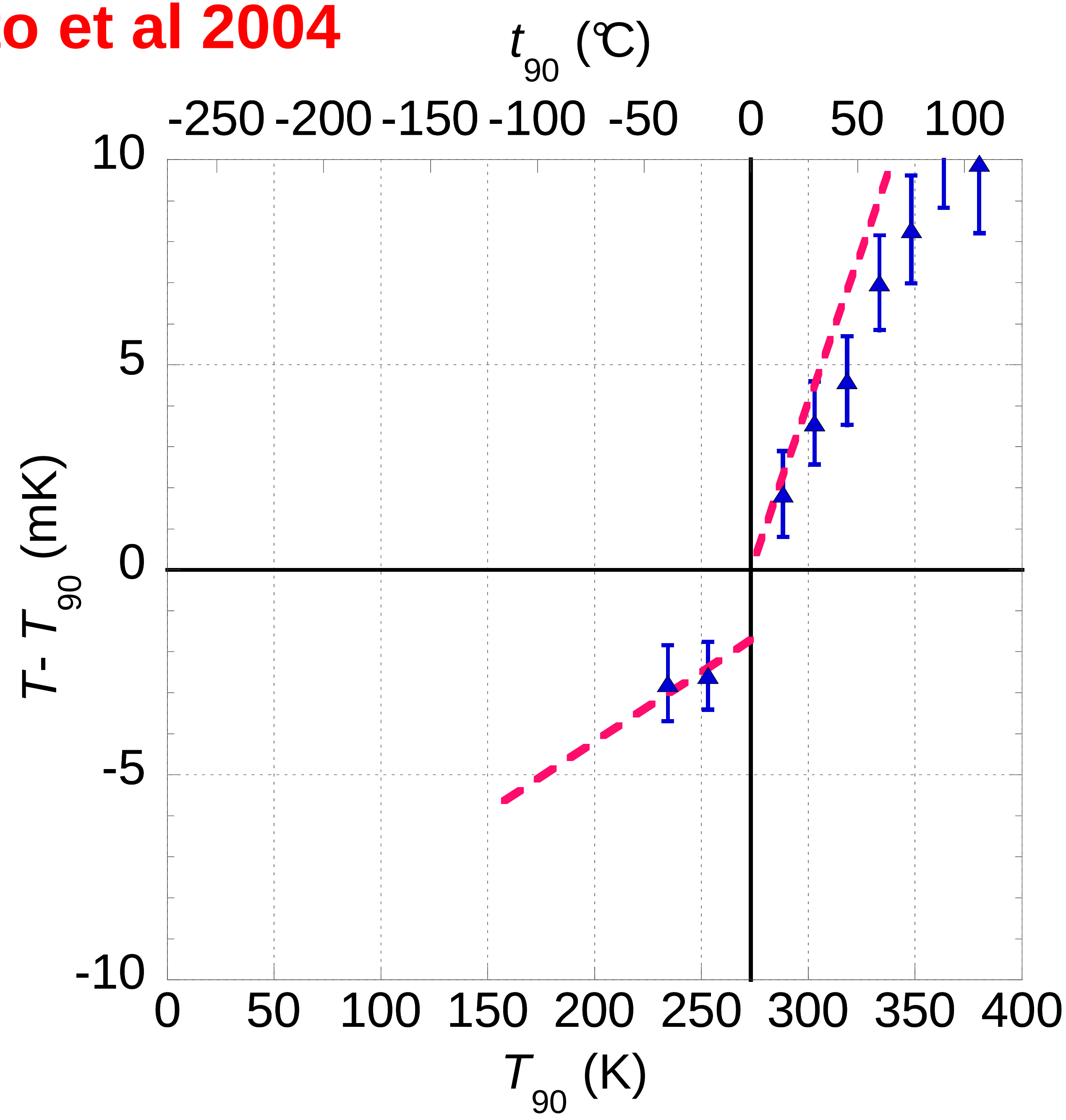


INSTITUTE OF PHYSICS PUBLISHING METROLOGIA
Metrologia 41 (2004) 74–98 PII: S0026-1394(04)72948-5

Acoustic measurements of the thermodynamic temperature between the triple point of mercury and 380 K

G Benedetto[1], R M Gavioso[1], R Spagnolo[1], P Marcarino[2] and A Merlone[2]

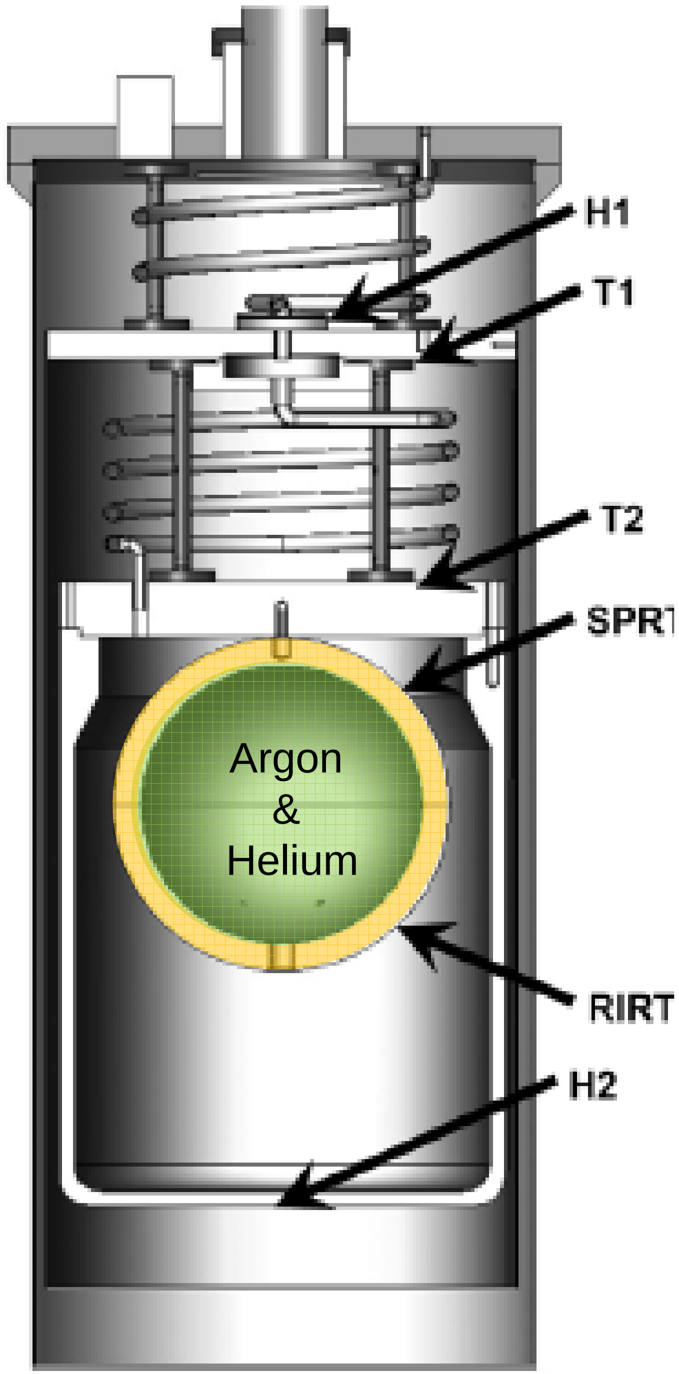
Benedetto et al 2004



Pitre et al 2006

Copper Resonator

- Quasi-spherical
- 120 mm inner diameter
- Argon AND Helium Gas
- Estimated radius with microwaves at all pressures.



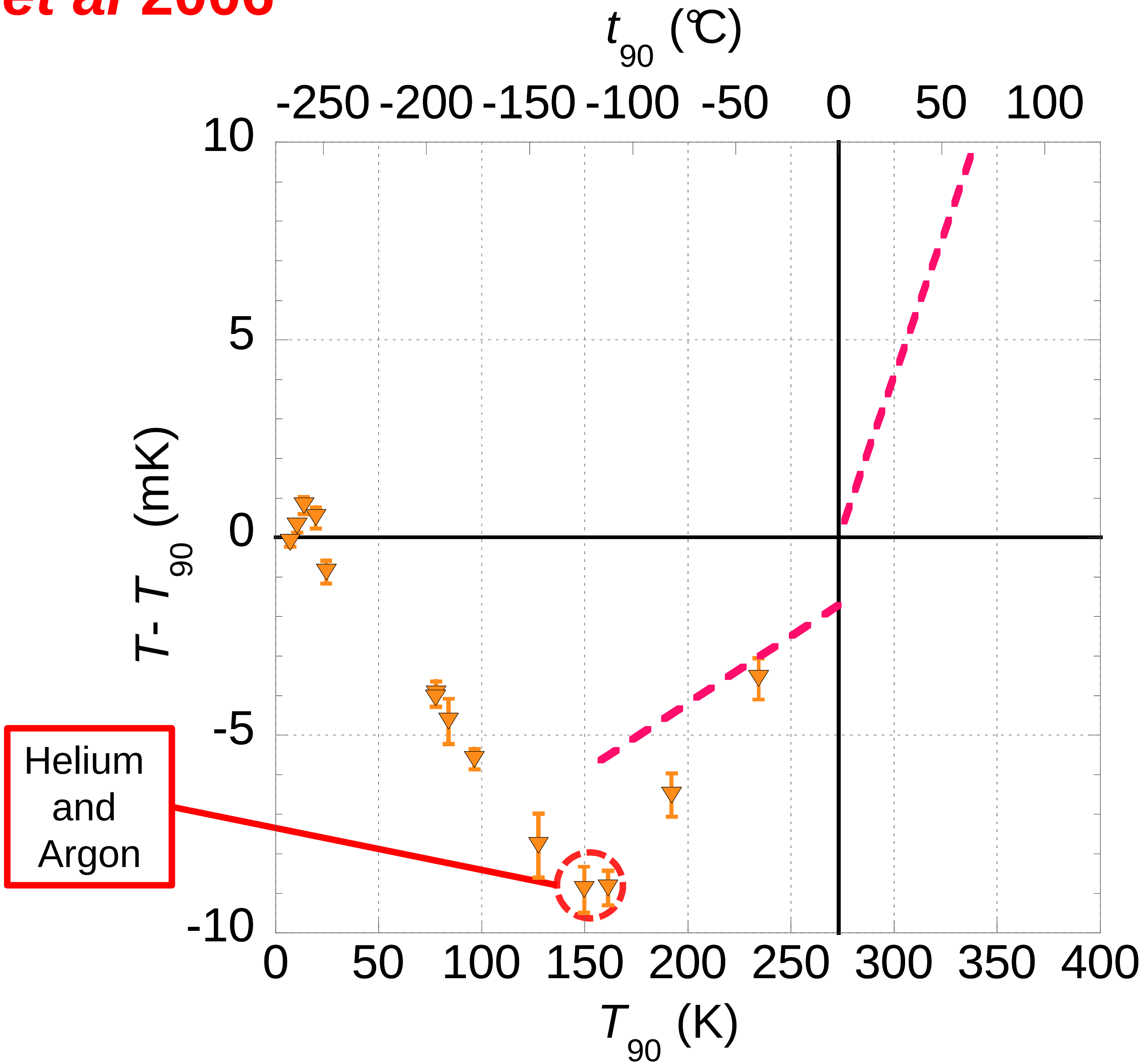
INSTITUTE OF PHYSICS PUBLISHING — METROLOGIA

Metrologia 43 (2006) 142–162 — doi:10.1088/0026-1394/43/1/020

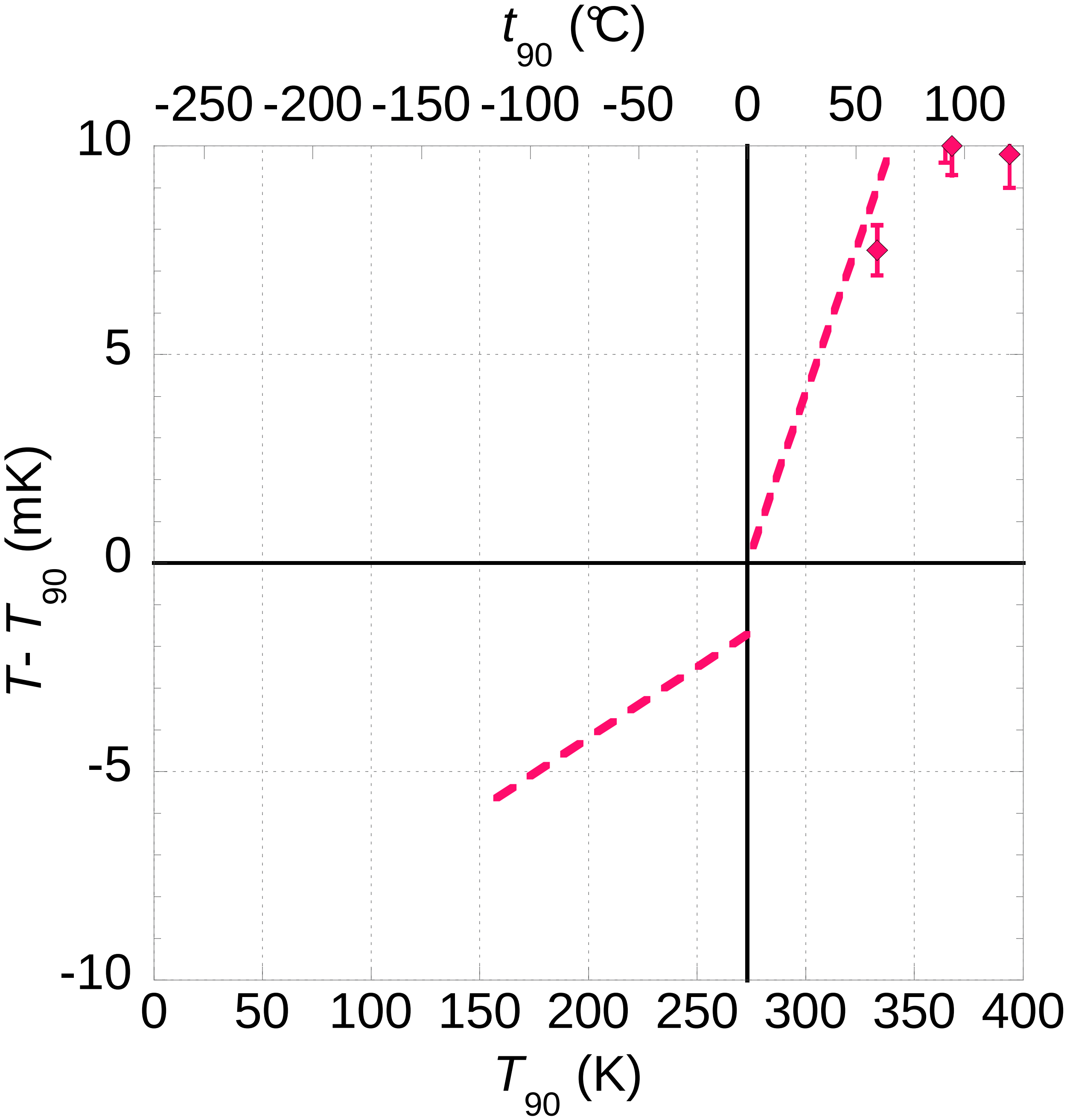
Acoustic thermometry: new results from 273 K to 77 K and progress towards 4 K

Laurent Pitre[1,2], Michael R Moldover[1] and Weston L Tew[1]

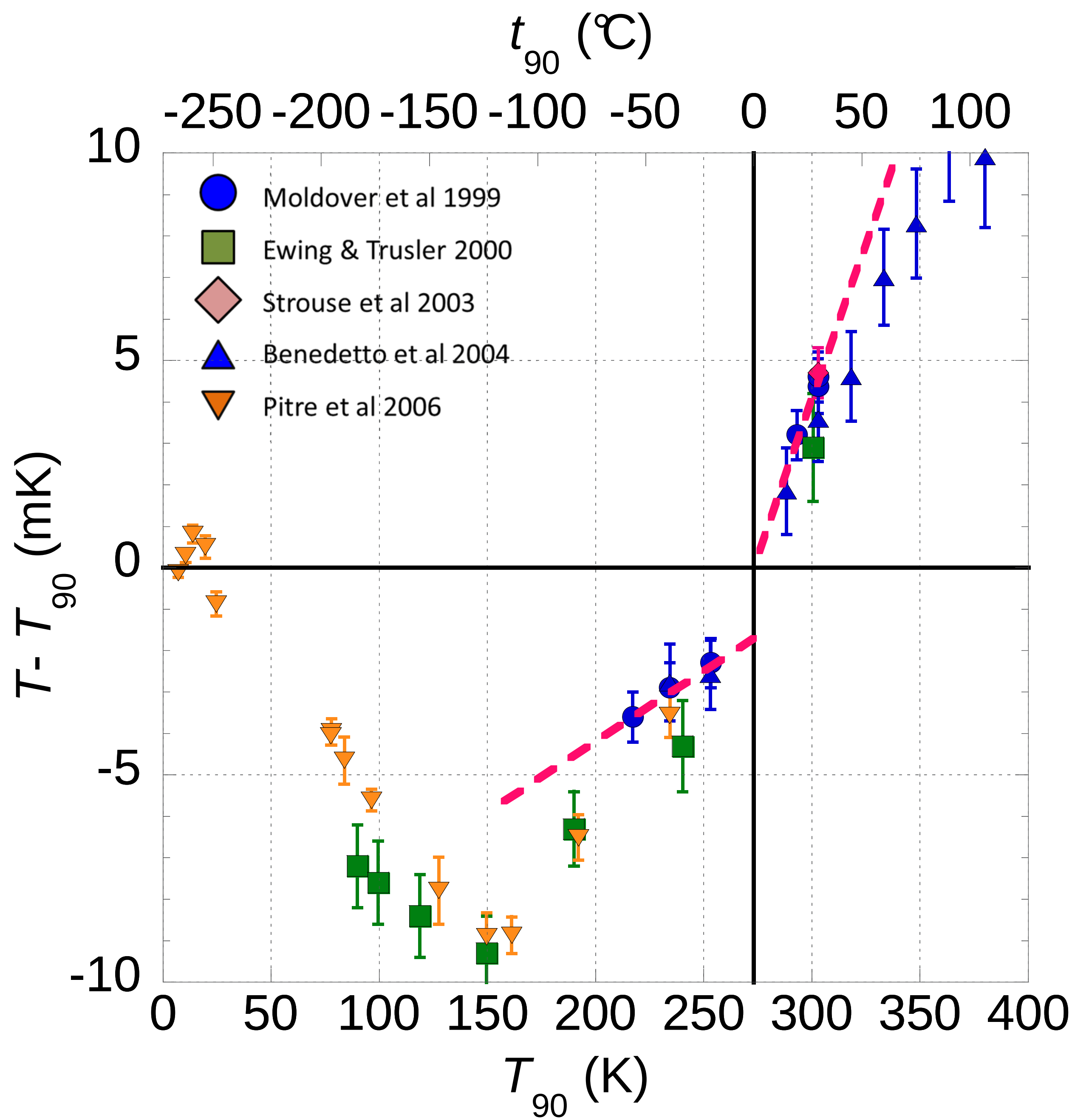
Pitre *et al* 2006



Ripple *et al* 2007

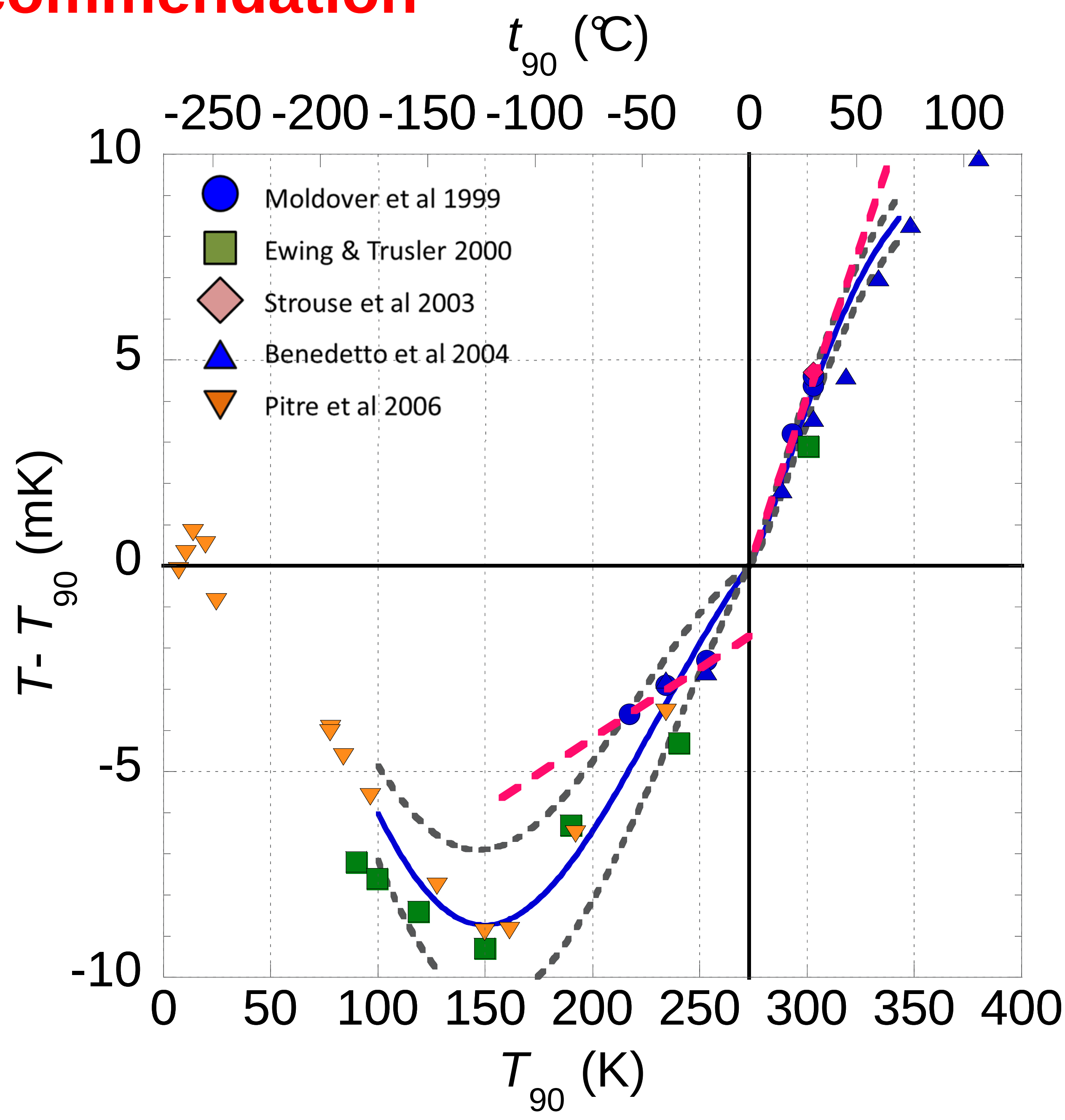


All Data



t90 (℃)
-250 -200 -150 -100 -50 0 50 100
10
5
0
-5
-10
T- T90 (mK)
0 50 100 150 200 250 300 350 400
T90 (K)
Moldover et al 1999
Ewing & Trusler 2000
Strouse et al 2003
Benedetto et al 2004
Pitre et al 2006

WG4 Recommendation



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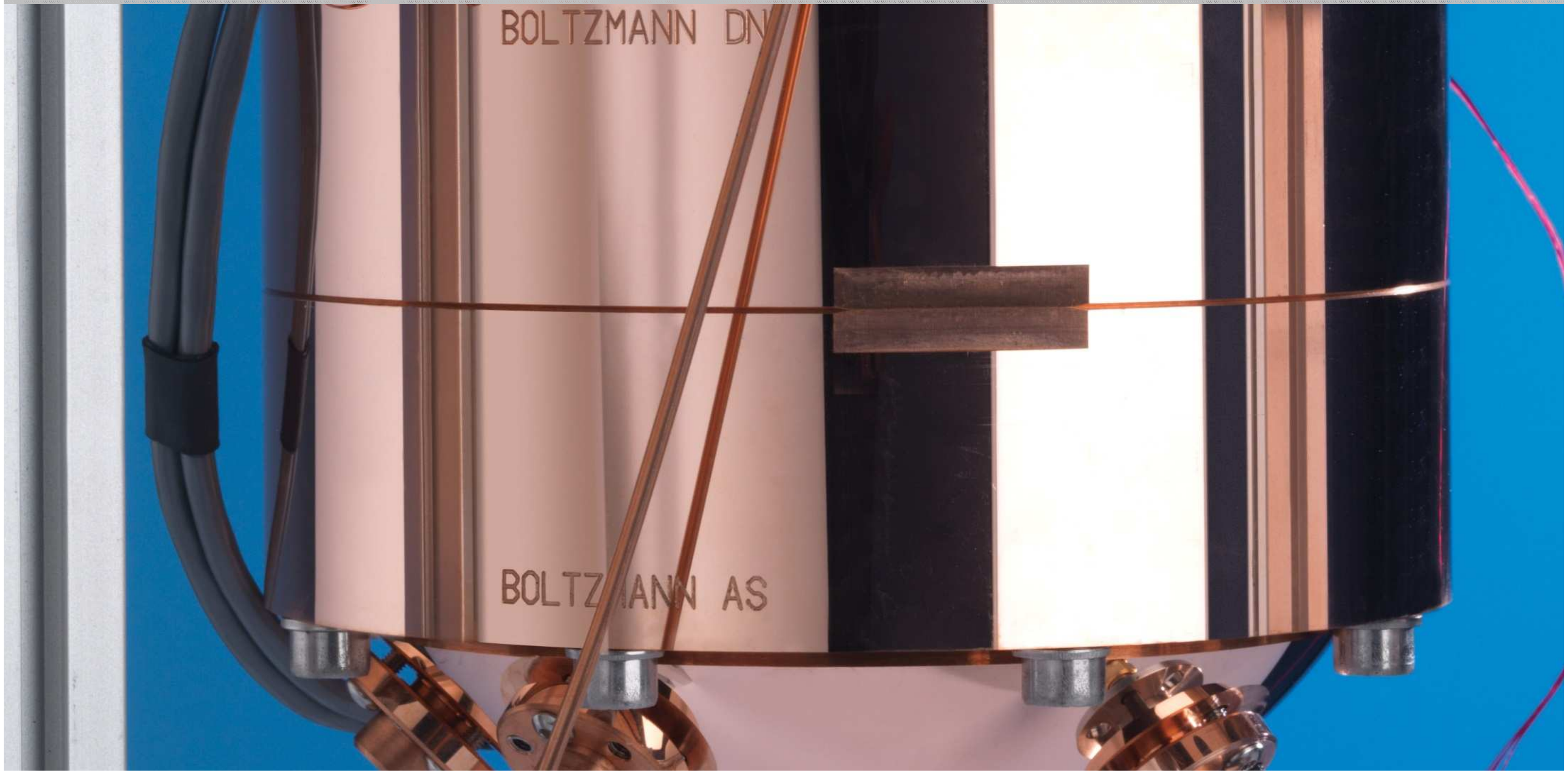
Questions

michael.depodesta@npl.co.uk

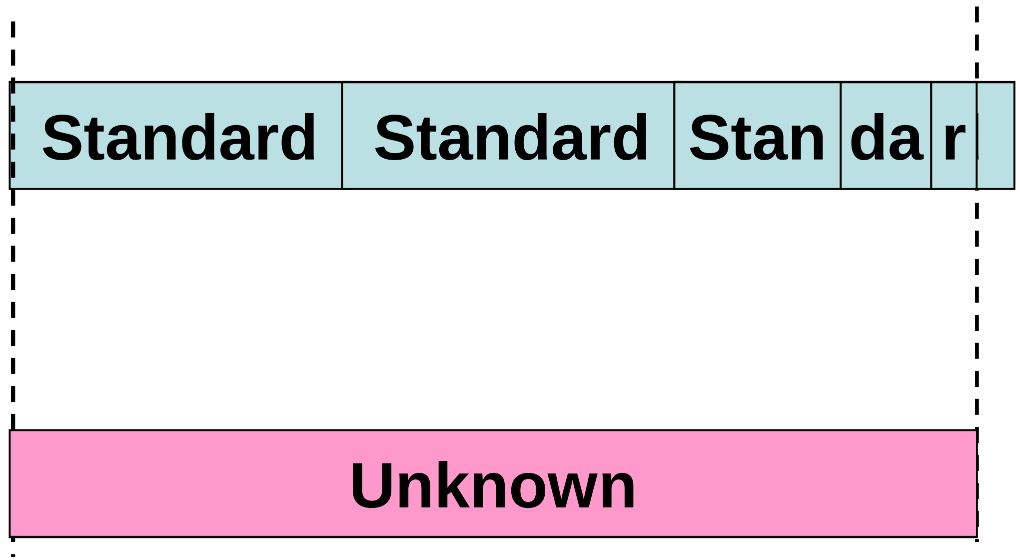
http://blog.protonsforbreakfast.org

#Protons4B



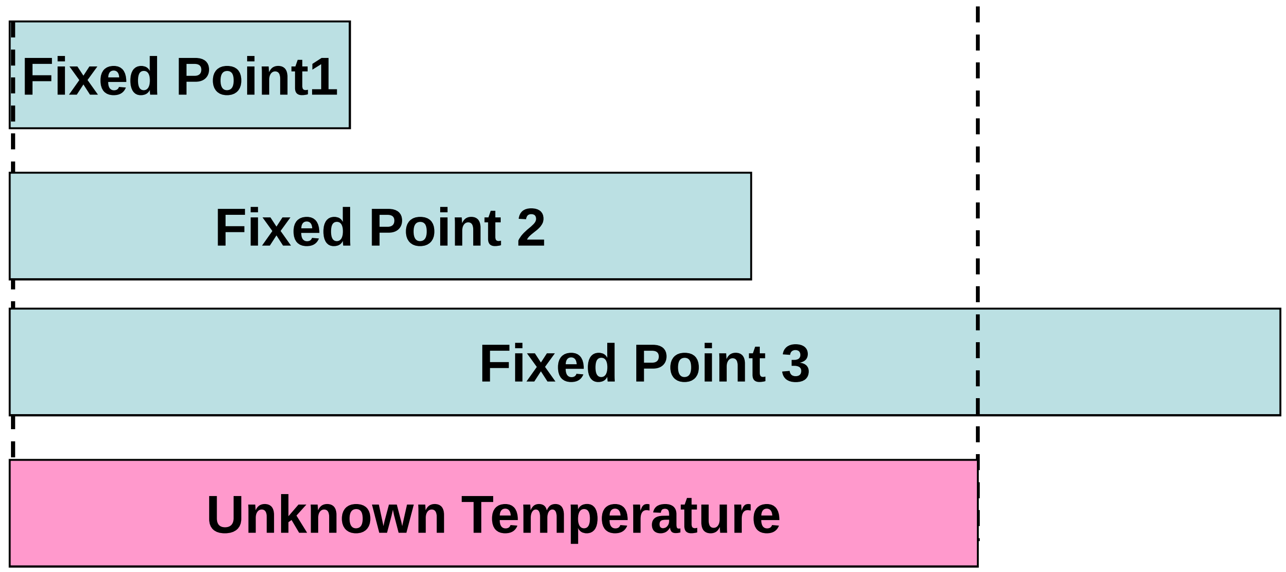
‘Fixed Points’ For length or mass standards

- We can make ‘standards’ with a convenient length or mass.
- We can create multiples and subdivisions of these ‘units’.



‘**Fixed Points**’ for temperature

- No possibility to create multiples or sub-divisions of units.
- Nature has given us only a few stable ‘fixed’ temperatures.
- There is no simple relationship between fixed points.



The Kelvin: SI Unit of Temperature

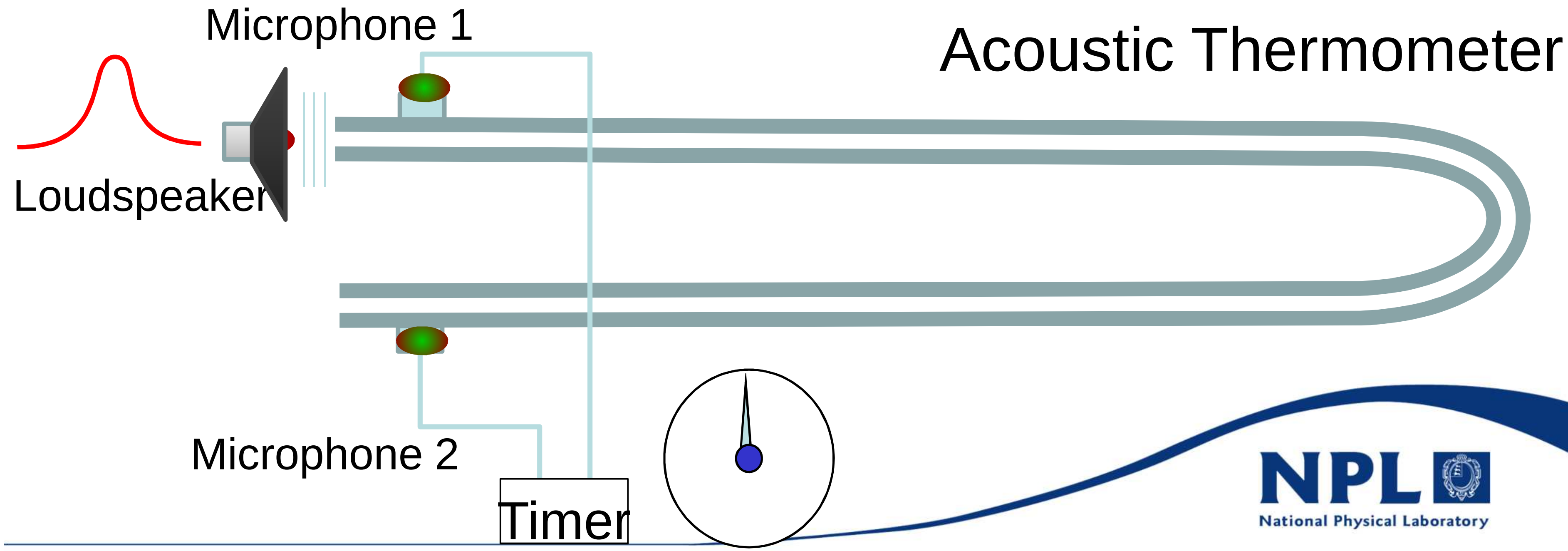
1/273.16

Chosen so that the magnitude of the kelvin would be close to the magnitude of the historical unit, the degree centigrade

Length
Mass
Time
Electrical Current
Temperature
Luminous Intensity
Mole

The kelvin, the unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of ***the triple point of water***

$$c^2 = \frac{\gamma k_B N_A T}{M}$$



How do we know we are right?

- Level of agreement between different acoustic modes
  - Different acoustic modes agree on the c_0^2 within 1 part in 10^6
- Simultaneously measure microwave resonances
  - Different microwave modes agree on the a_0 within 1 part in 10^7
- Excess half-width
  - Indistinguishable from zero.
- Results are reproducible over many years
  - Largest uncertainty $T - T_{90}$ is now measurement of T_{90}