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Electrical Standards based on quantum effects

Beat Jeckelmann

Outline

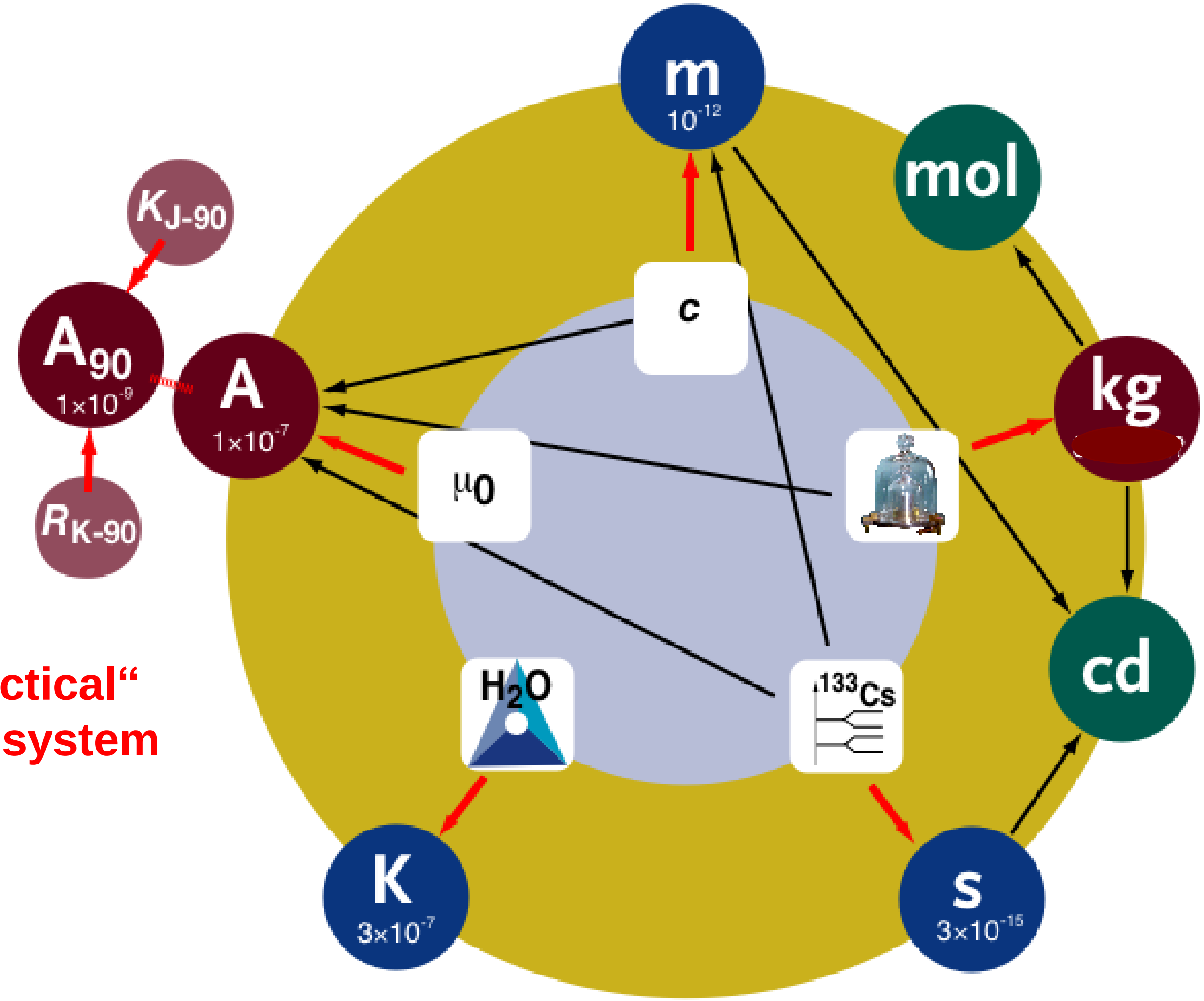
Introduction

- Electrical units in the SI today and in the future

Part I: Josephson voltage standards and applications

Part II: Quantum Hall resistance standards and applications

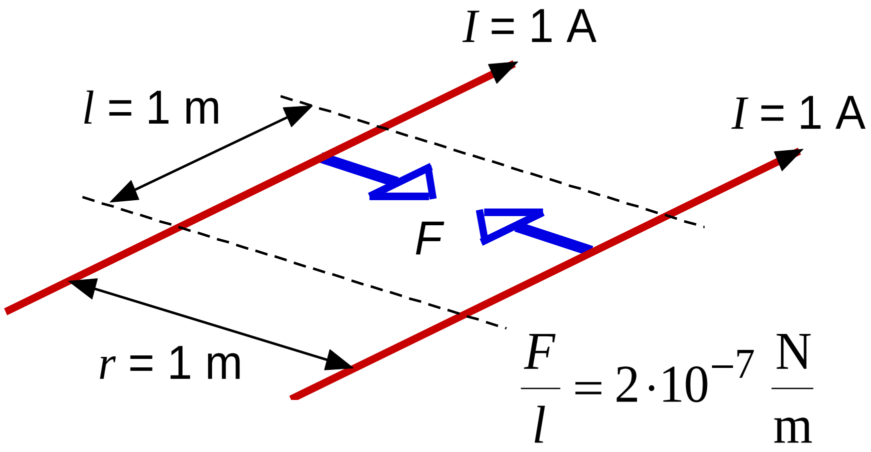
The SI today



Stability?

„practical“
sub-system

The ampere definition



$$\frac{F}{l} = 2 \cdot 10^{-7}\ \frac{\mathrm{N}}{\mathrm{m}}$$

"The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 ×10^{-7} Newton per metre of length."

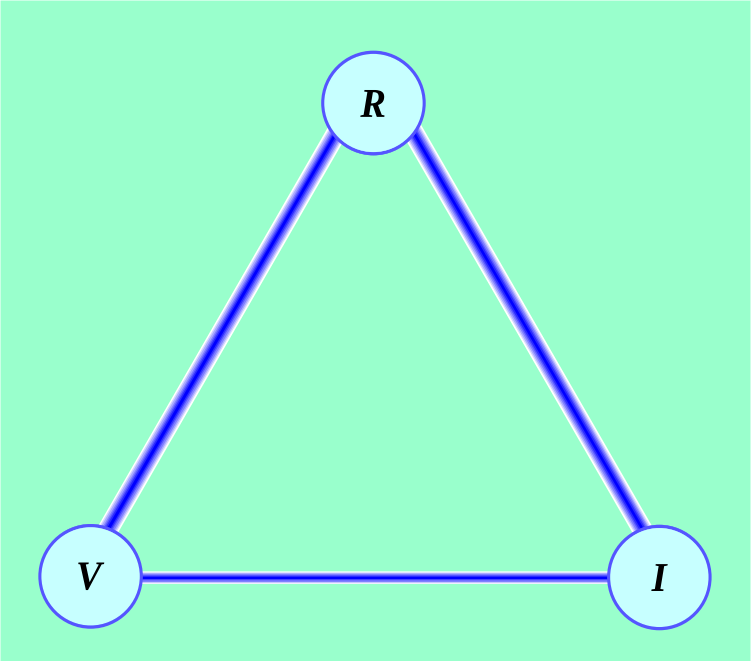
Ampère's law for the idealized case:

$$\frac{F}{l} = \mu_0 \frac{I^2}{2\pi r}$$

With the ampere definition and equating mechanical and electrical power, one obtains for the vacuum permeability:

$$\mu_0 = 2\pi \frac{F}{I^2}\frac{r}{l} = 4\pi \cdot 10^{-7}\ \frac{\mathrm{N}}{\mathrm{A}^2}$$

Electrical units in the SI



Ohm‘s law: $V = R \cdot I$

Link to mechanical units

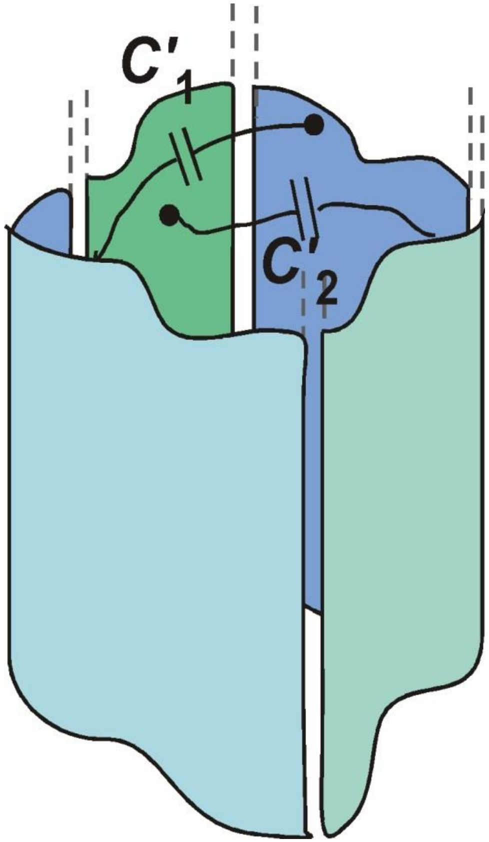
Ampere definition introduces dimension „A“ and fixes the value for μ_0:

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\mathrm{N}}{\mathrm{A}^2}$$

Electrical units:
Two realisations in terms of mechanical units necessary

Today:
Ohm: calculable capacitor (10^{-8})
Watt: watt balance (10^{-8})

The SI realisation of the ohm: the calculable capacitor



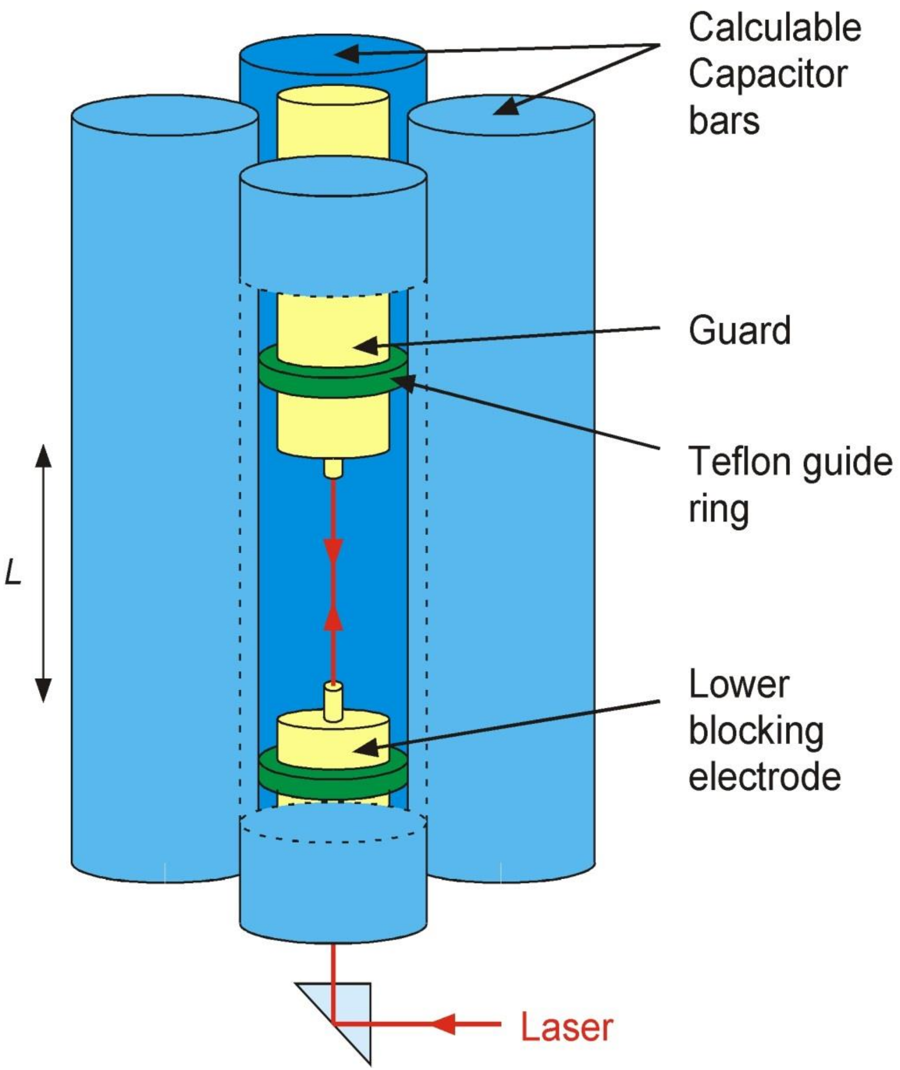
Thompson-Lampard Theorem (1956):

$$\exp\left(-\frac{\pi C_1'}{\varepsilon_0}\right) + \exp\left(-\frac{\pi C_2'}{\varepsilon_0}\right) = 1$$

Cross-capacitance identical:

$$C' = \frac{\varepsilon_0 \ln(2)}{\pi} \cong 1.95\,\mathrm{pFm}^{-1}$$

Calculable capacitor: Practical Realisations



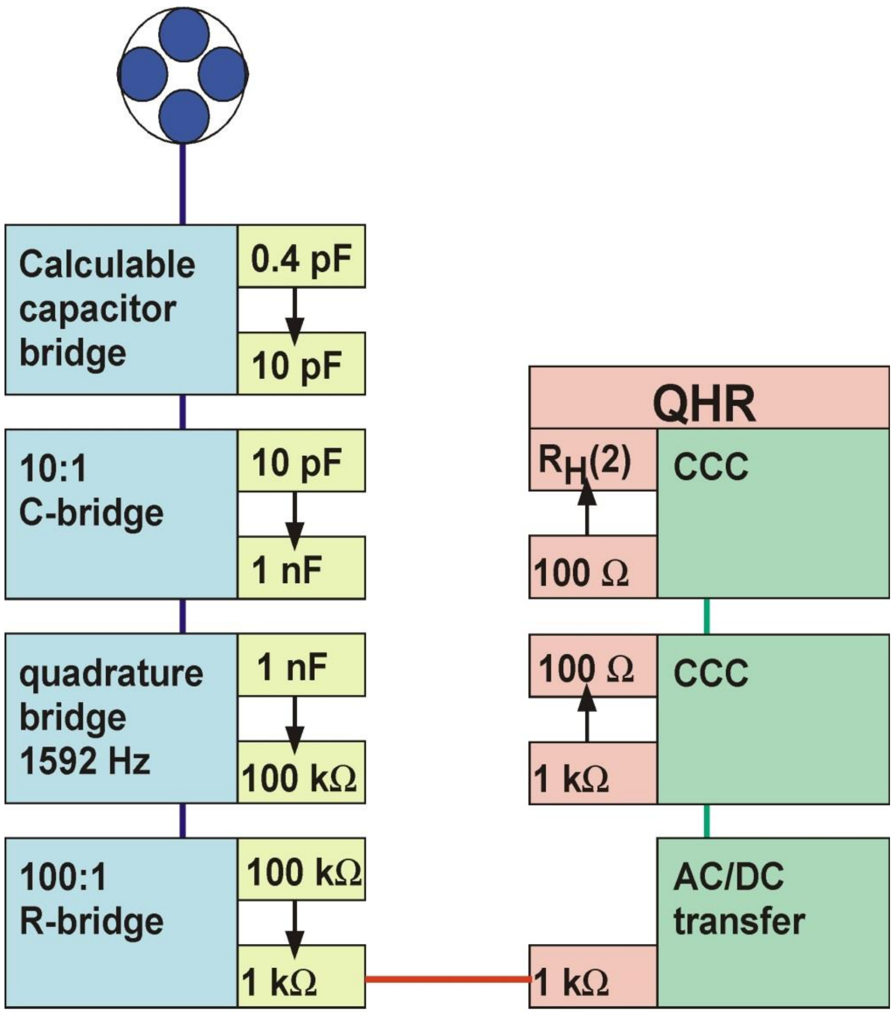
Measurements:
ΔL = 5 - 50 cm
ΔC = 0.1 - 1 pF

u: several parts in 10^8

Running projects
NMIA, BIPM, NRC, LNE

$u < 10^{-8}$

Link from the calculable capacitor to the ohm



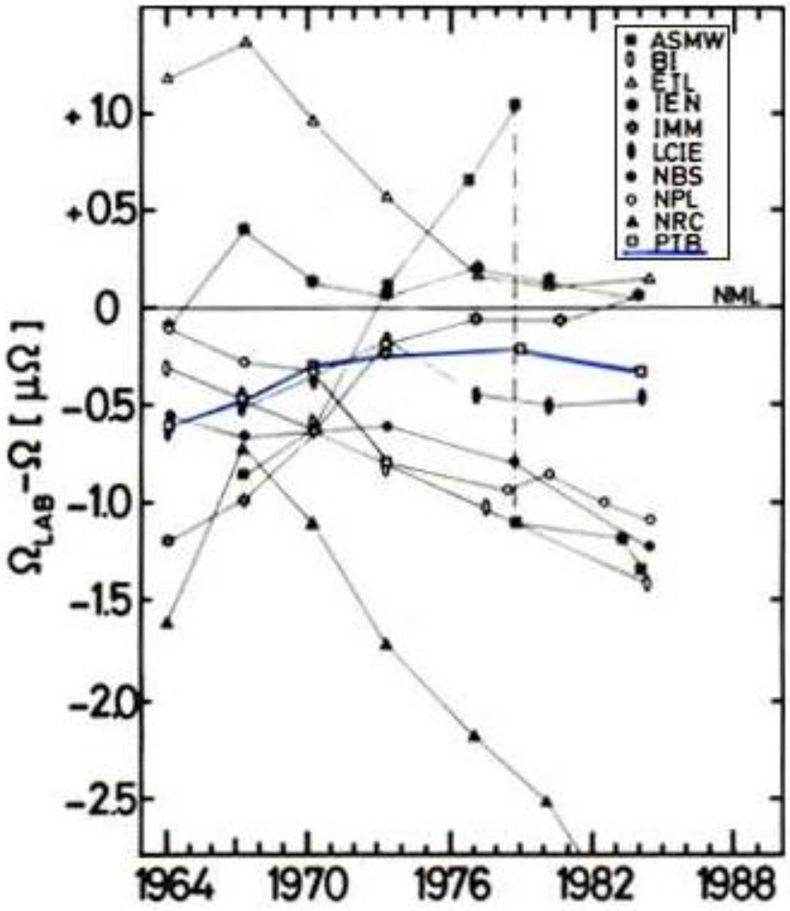
CODATA 2014:

- NPL-88: $u = 5.4 \times 10^{-8}$
- NIST-97: $u = 2.4 \times 10^{-8}$
- NMI-97: $u = 4.4 \times 10^{-8}$
- NIM-95: $u = 1.3 \times 10^{-7}$
- LNE-01: $u = 5.3 \times 10^{-8}$

New Projects:

- BIPM: $u \leq 10^{-8}$ expected 2016

Realization of the ohm before 1990



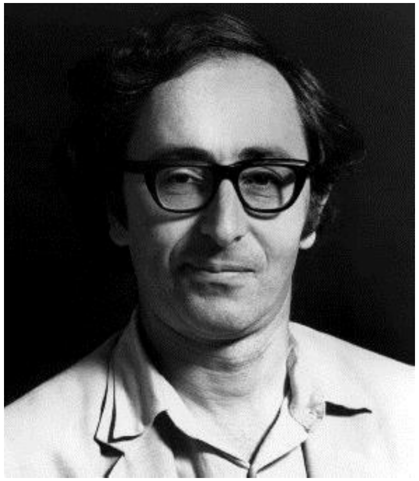
Complicated electro-mechanical experiments needed to realize the ohm

Artifacts were used to maintain the unit:

→ drift in time
→ differences of up to several ppm from country to country

Electrical quantum standards

Quantum mechanical effects allow the realization of highly reproducible electrical standards:

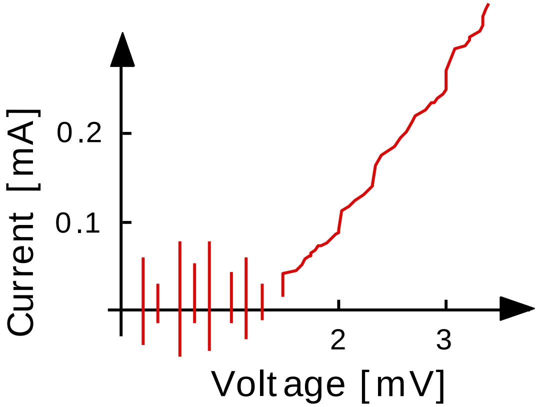


B. Josephson predicts quantized voltage steps in superconductors (1962)
→ voltage standard

K. Von Klitzing discovers the quantum Hall effect in 1980 → Resistance standard

Electrical standards based on fundamental constants

Josephson effect

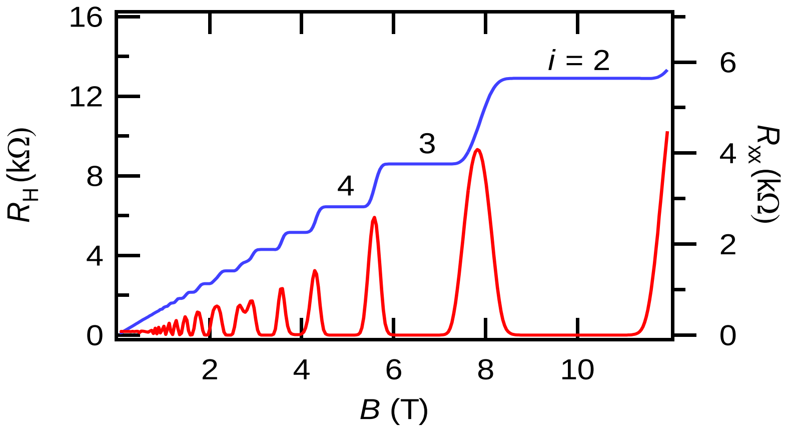


Weakly coupled superconductors

$$U = \frac{h}{2e} i \cdot f_{\mathrm{J}} = \frac{i}{K_J} f_J$$

K_{J}: Josephson constant

Quantum Hall effect



2D electron gas in high magnetic field

$$R_{\mathrm{H}} = \frac{h}{i \cdot e^2} = \frac{R_K}{i}$$

R_{K}: von Klitzing constant

Practical electrical units

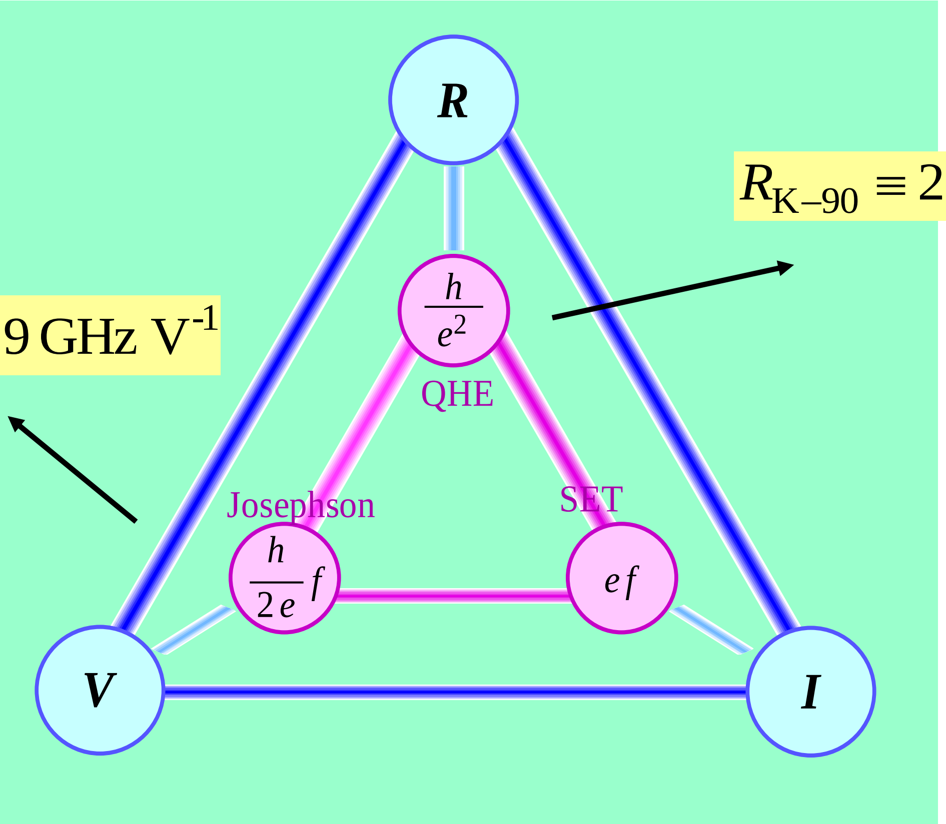
- To make best use of the good reproducibility and worldwide availability of the quantum standards, the CIPM introduced conventional values for the Josephson- and von Klitzing constants as of January 1, 1990.

- $K_{J\text{-}90}$ = 483 597.9 GHz/ V → rel. uncertainty in the SI : 0.4 ppm
  $R_{K\text{-}90}$ = 25 812.807 Ω 0.2 ppm
  (now: 0.1 ppm)

- Worldwide uniformity and improvement of electrical calibrations as a consequence of the conventional units.

- The uncertainty of the constants does only apply if electrical units are linked with mechanical units.

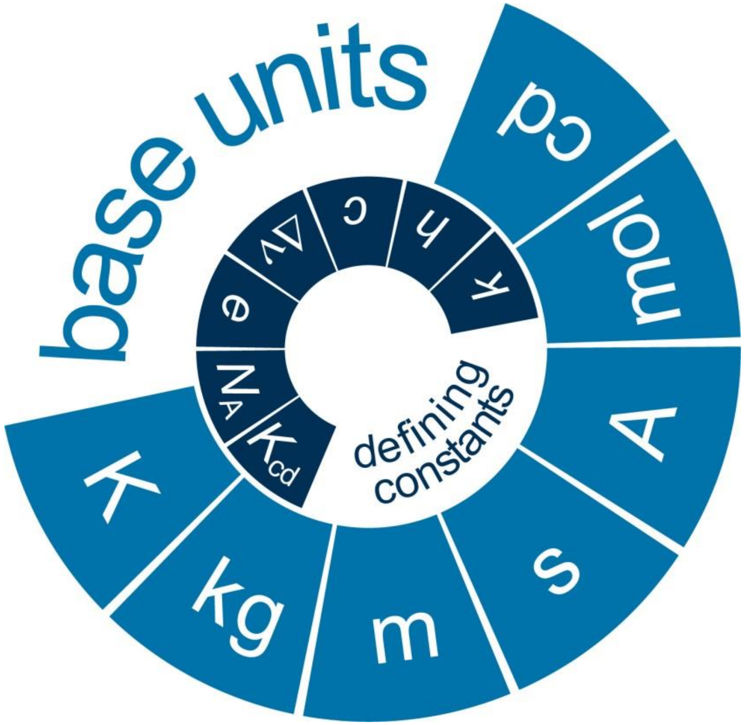
Quantum effects and „Practical“ electrical units



$$R_{K-90} \equiv 25\,812.807\ \Omega$$

$$K_{J-90} \equiv 483\,597.9\ \mathrm{GHz\ V^{-1}}$$

Electrical units in th «new» SI

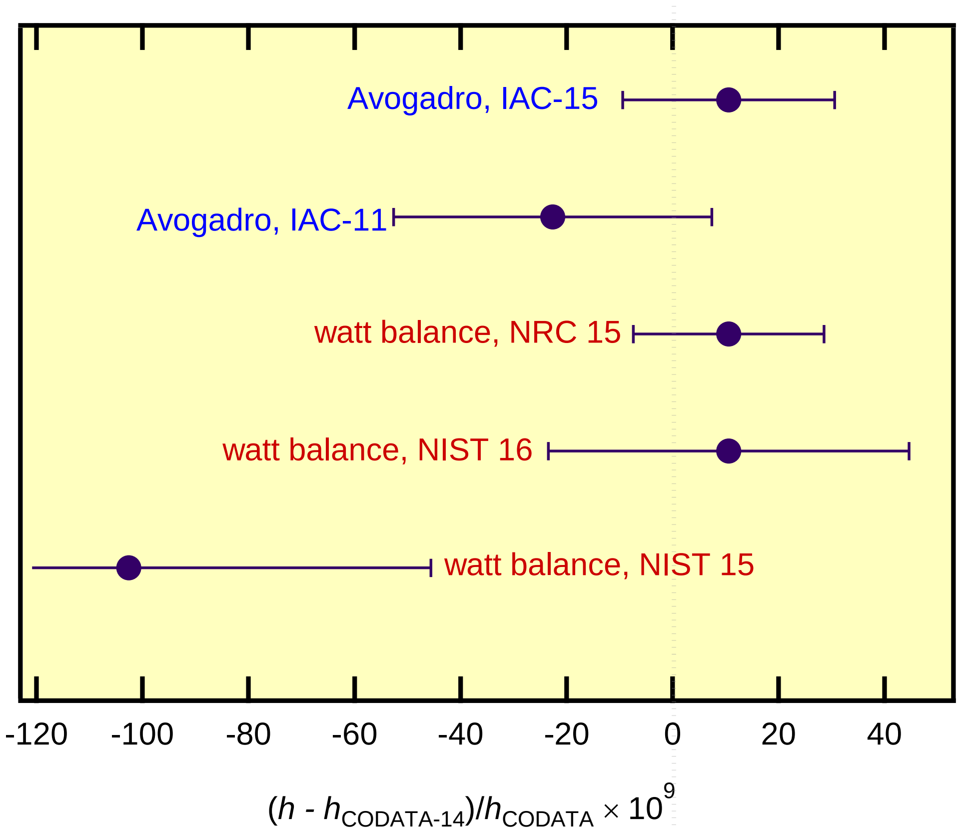


***h* and *e* have fixed values (defining constants)**

- R_K (h/e^2) and K_J ($2e/h$) are fixed
- Quantum Hall and Josephson standards realize the ohm and the volt in the new SI directly (assuming that the QHE and JV relations are correct!)
- μ_0 has to be measured

Validity of R_K and K_J relations

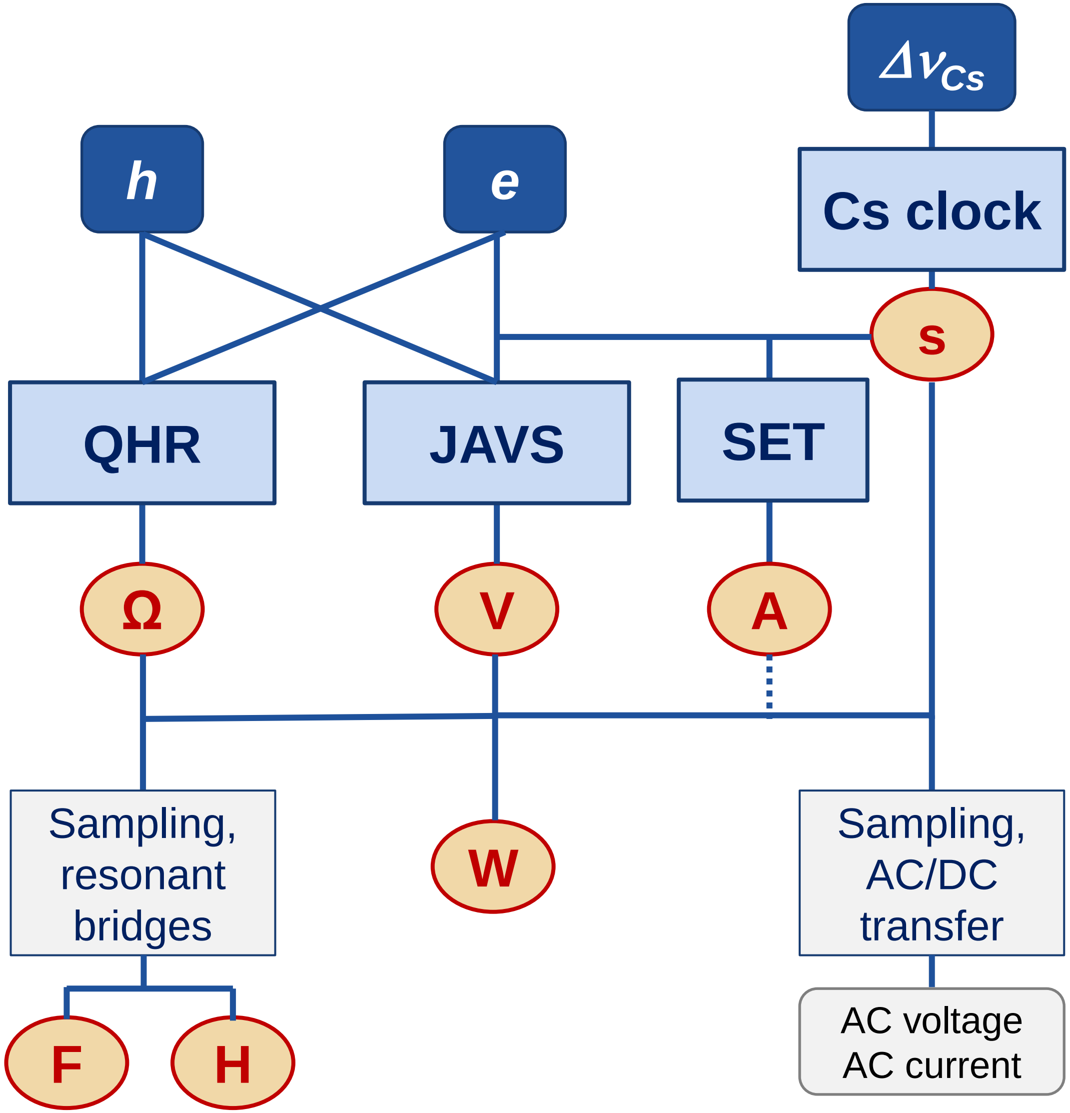
Determination of the Planck constant



Watt balance results rely on QHE and JV relations

Δ Watt balance - Avogadro $< \sim 10^{-8}$

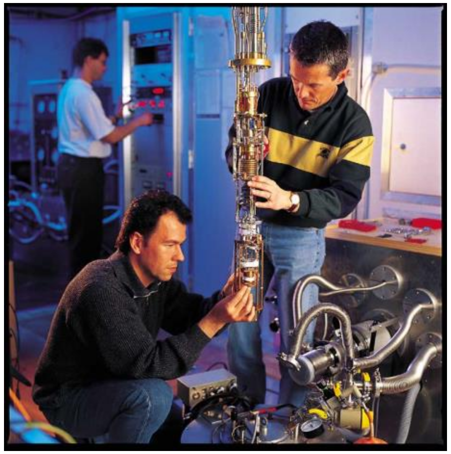
Realization of electrical units



Outline

Introduction

- Electrical units in the SI today

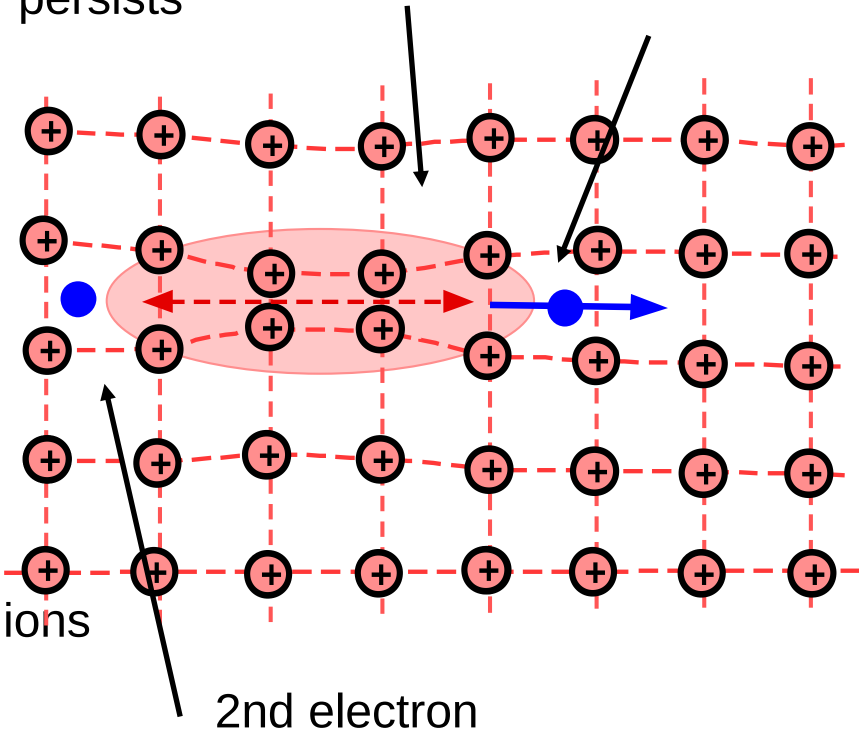


Part I: Josephson effect

- DC and AC Josephson effects
- Different types of Josephson junctions
- Hysteretic Josephson Arrays and their applications
- Programmable arrays
- Pulsed driven arrays

Electrons in a superconductor

Cooper pairing mechanism



Region of pos. charge persists

passage of electron

ions

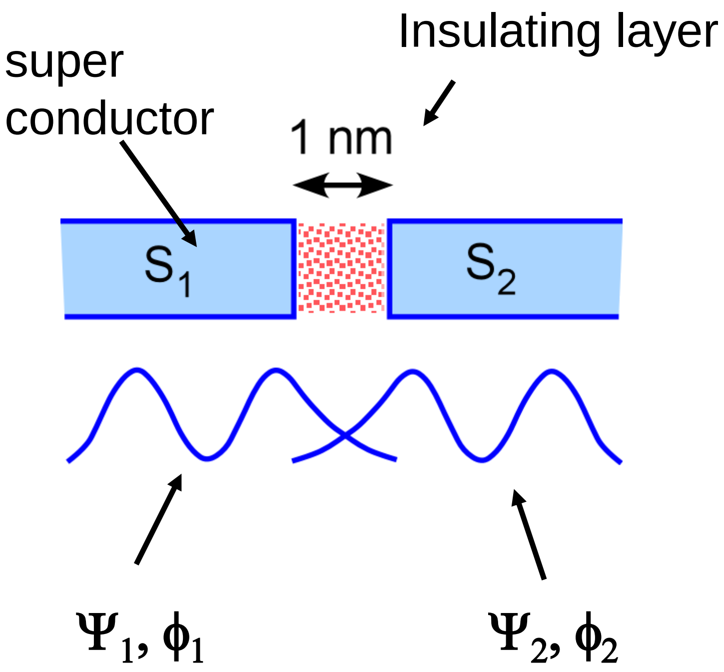
2nd electron attracted by charge density

- Conduction electrons pair up through exchange of “virtual” phonons
- Interaction is isotropic
- Macroscopic wave function describes entire electronic system

$$\psi = |\psi| e^{i\phi}$$

$$|\psi| = n = \text{cooper pair density}$$

DC Josephson effect



Quantum states in superconductor described by Schrödinger equation:

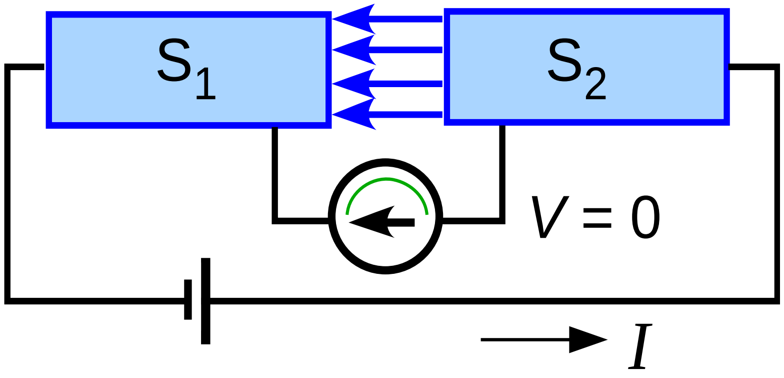
$$\frac{d\psi}{dt} = \frac{-i}{\hbar} E\psi$$

Two weakly coupled superconductors:
Phase coherent transfer of Cooper pairs

$$\frac{d\psi_1}{dt} = \frac{-i}{\hbar} E_1\psi_1 + K\psi_2$$

$$\frac{d\psi_2}{dt} = \frac{-i}{\hbar} E_2\psi_2 + K\psi_1$$

DC Josephson effect (2)



A small ***supercurrent*** flows through the weak link with a corresponding phase shift:

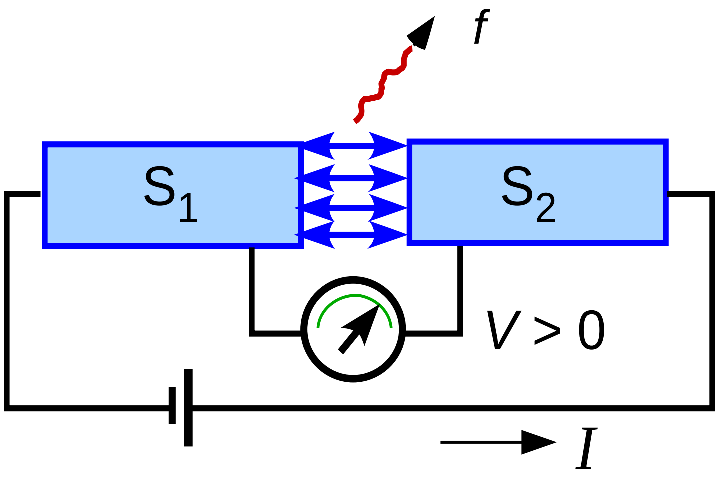
$$I = I_c \cdot \sin(\phi_2 - \phi_1) = I_c \cdot \sin(\phi)$$

I_c: critical current of the weak link

AC Josephson effect

With $E_1 - E_2 = 2eV(t)$ Schrödinger eq. also gives:

$$V(t) = \frac{\hbar}{2e}\frac{d\phi}{dt}$$



DC external current $I > I_c$

- Direct voltage across junction
- Oscillating supercurrent flows with frequency *f*

$$\boxed{f_J = \frac{2e}{h}V}$$

Mean voltage: $V = \frac{h}{2e} f_J = \phi_0 \cdot f_J$

→Voltage driven oscillator

Josephson Voltage Standard

Microwave irradiation, frequency *f*, applied to junction:

- Cooper pair current synchronizes with *f* and its harmonics
- Direct voltage appears at the terminals

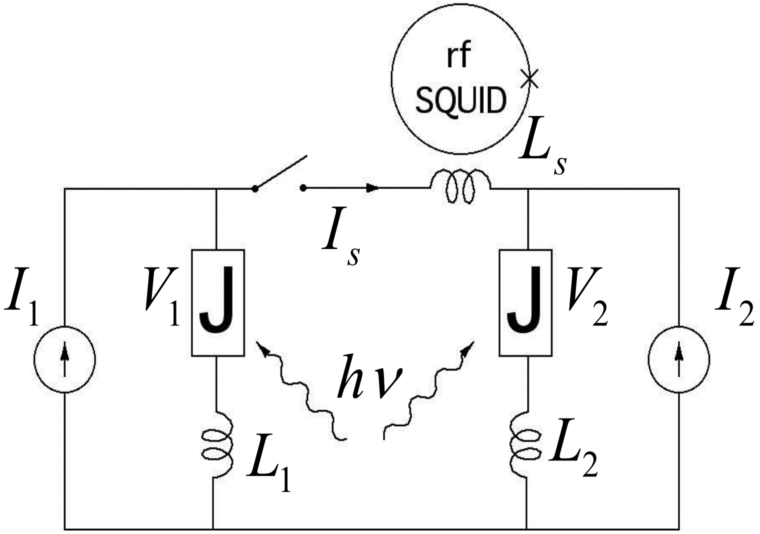
$$\boxed{V = n\frac{h}{2e}f}$$

$V_1 \sim 145\ \mu\mathrm{V}$ @ 70 GHz

$V < 2.5$ mV (gap energy in Nb)

- Relationship independent of
  - Temperature, material, polarization current…
- **Tested at a level of 3×10^{-19}**

Universality Tests



→ Extremely sensitive method

J-S. Tsai et al.,PRL, **51**, 316 (1983)

- Test of the material independence of the Josephson relationship
- Two different superconductors (Nb, In) and different weak links

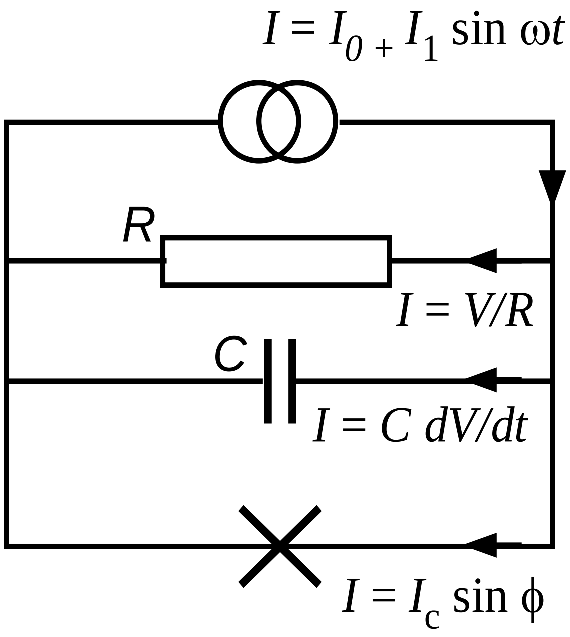
$$I_s = \frac{1}{L}\int (V_1 - V_2)dt$$

$$L = L_s + L_1 + L_2$$

No difference of the Josephson voltages (when biased with the same microwave frequency on the same step) **at the level of 1 × 10^{-16}**

Most precise test (A. K. Jain et al, PRL 58 (1987)): **3 × 10^{-19}**

Real Josephson junction



Cooper pair current

$$I_0 + I_1 \sin \omega t = I_c \sin \phi + \frac{V(t)}{R} + C \frac{dV(t)}{dt}$$

With Josephson relation

$$V(t) = (\hbar/2e)\, d\phi/dt$$

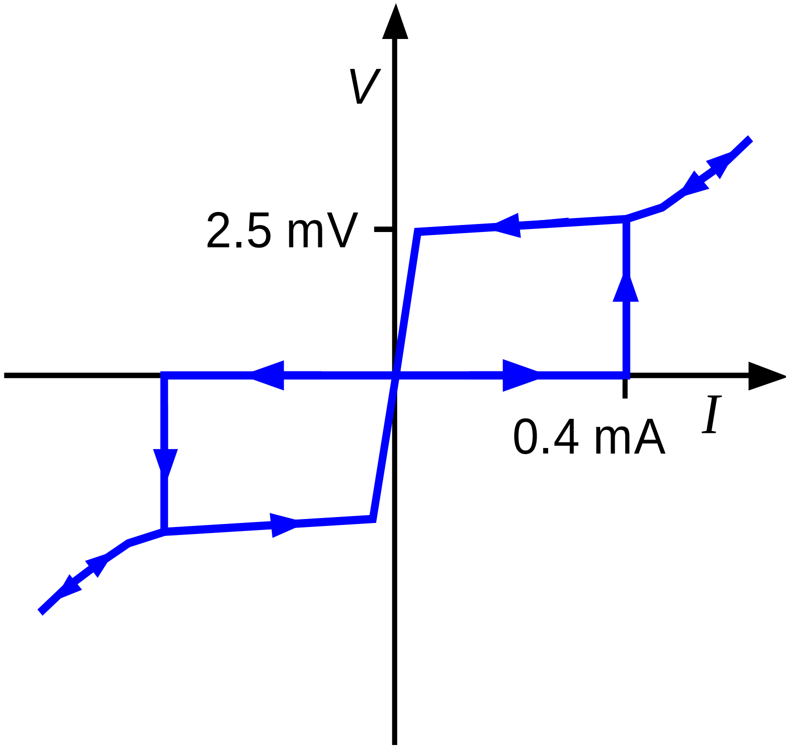
differential equation of a driven damped oscillator →

$$\beta_c \frac{d^2\phi}{dt^2} + \frac{d\phi}{dt} + \sin \phi = i_0 + i_1 \sin(\Omega t')$$

$$\beta_c = \frac{2e}{h} I_c R^2 C$$

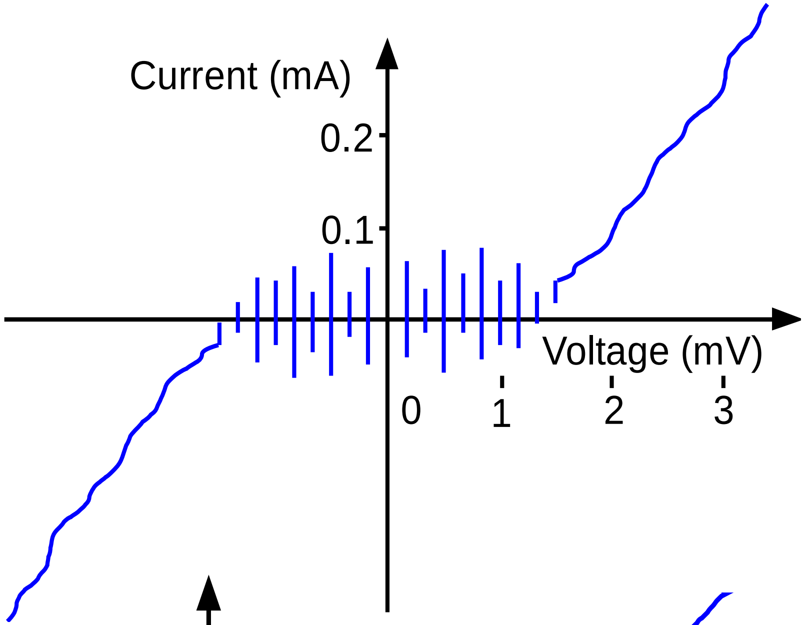
- **β_c (McCumber parameter) describes damping** of the Josephson oscillator
  $(\beta_c)^{1/2}$ quality factor LCR resonator (Josephson junction: role of *L*)
- Chaotic properties; stable operation only in limited parameter space

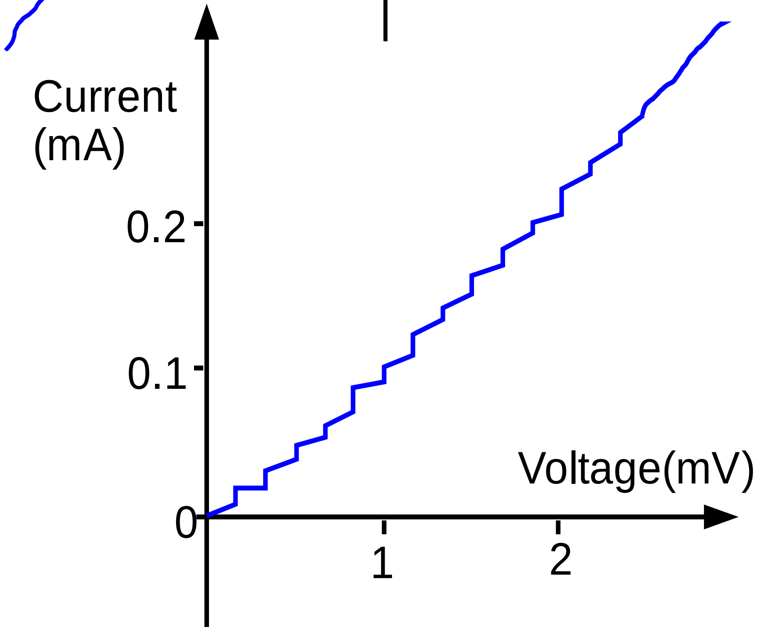
I-V Characteristics



Weakly damped Nb-Al_2O_3-Nb Josephson junction without microwave power

I-V Characteristics with microwave power (*f* = 70 GHz)





Weakly damped junction

β_c > 100

- Voltage steps at zero current (“zero crossing” steps)
- Hysteretic

Highly damped junction

β_c < 1

- Different current for every voltage step

Junction Arrays

Idea: **Increase the voltage by cascading an array of junctions in series**

- **Early days**: not possible to produce overdamped arrays with sufficient uniformity for polarization of the entire array on the same voltage step

Solution: Levinsen (1977) proposes zero-crossing steps (**β_c > 100)**
(SIS junctions: superconductor-insulator-superconductor)

- 1985, first 1 V array: Niemeyer, Hamilton, Kautz, NIST
- 1987, 10 V array, NIST, 14’484 junctions, ~ 150’000 voltage steps

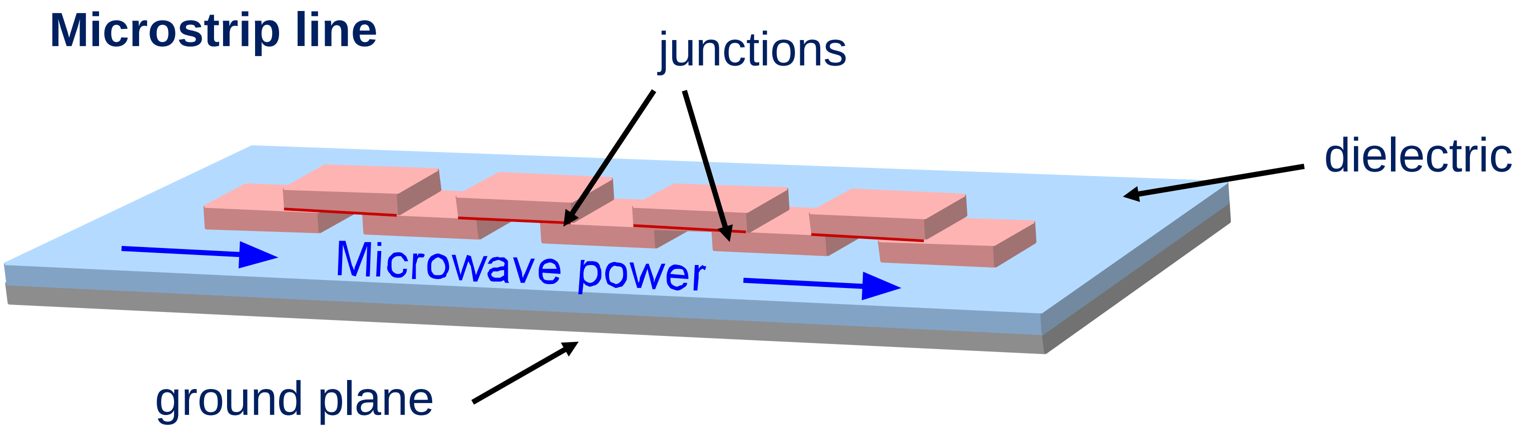
Limited parameter space available

- External frequency has to be well above resonant frequency of the junctions, to prevent chaotic behaviour (70 GHz)
- Current step width > induced current noise
- Dependence of non-chaotic regime on microwave power

Junction Arrays (2)

Problems to be solved:

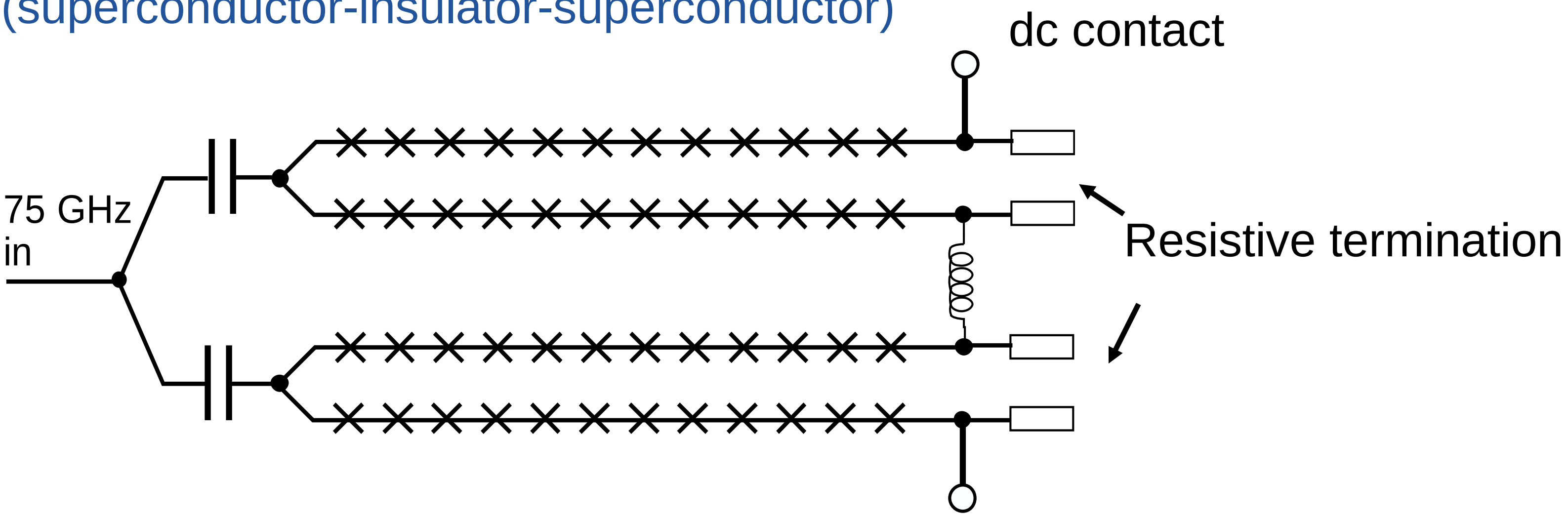
- Homogeneous distribution of microwave power to all junctions
- Fabrication of large junction arrays with little variation in parameters



Impedance 2 to 5 Ω
→ very low attenuation

SIS Arrays

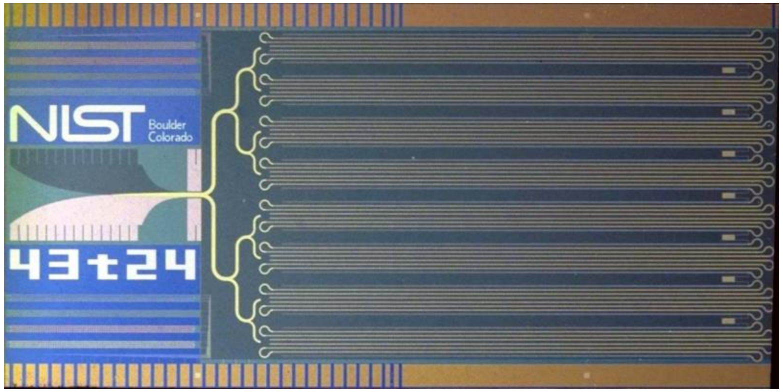
(superconductor-insulator-superconductor)



dc contact

75 GHz in

Resistive termination

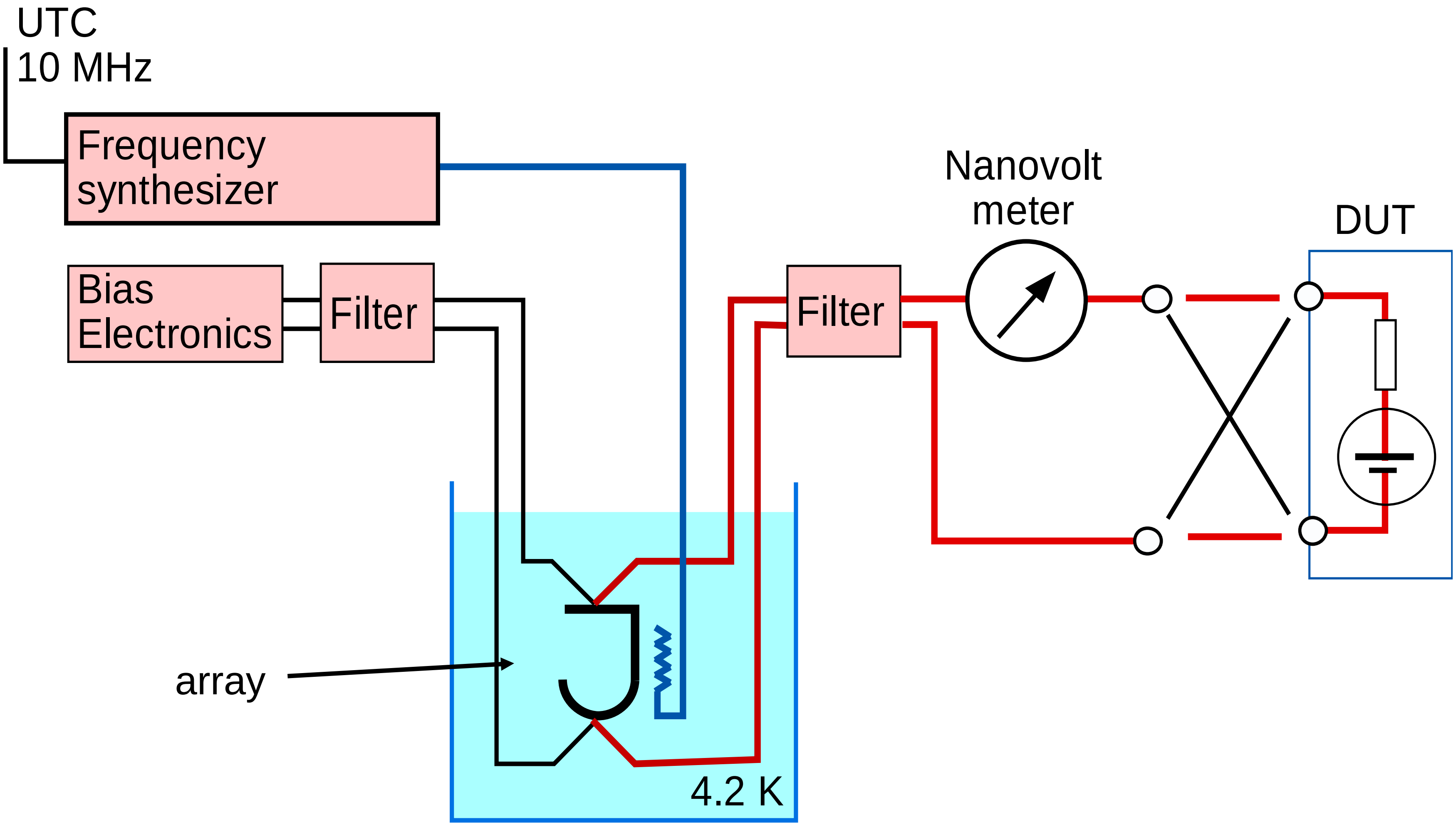


NIST design (similar to PTB design):

- 20’000 junctions
- Nb/ Al_2O_3/ Nb technology
- V_{max} = 10 V

Disadvantage of SIS arrays: steps unstable and difficult to select

Measurement system


UTC
10 MHz

Frequency synthesizer

Bias Electronics

Filter

Filter

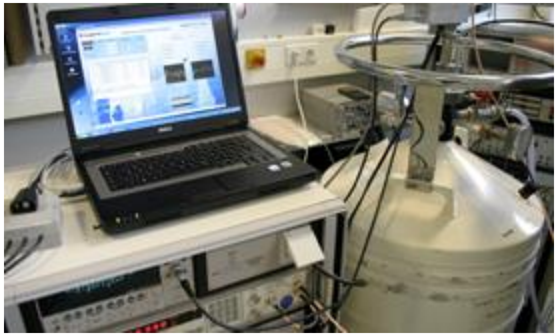
Nanovolt meter

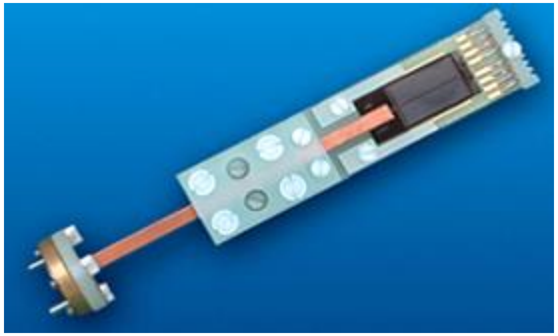
DUT

array

4.2 K

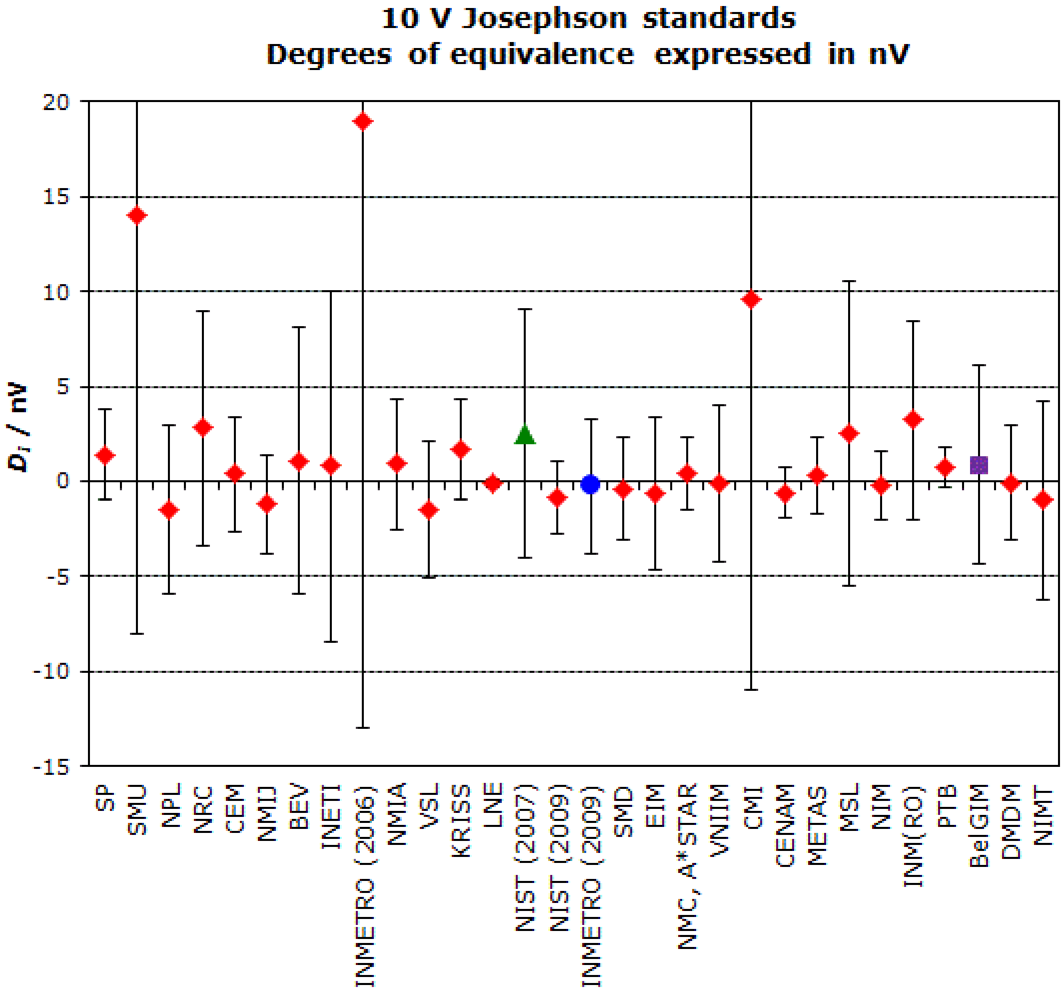
Josephson Array Standard





10 V systems commercially available
- Hypres (USA): NIST array technology
- SupraCon (Germany): PTB array technology

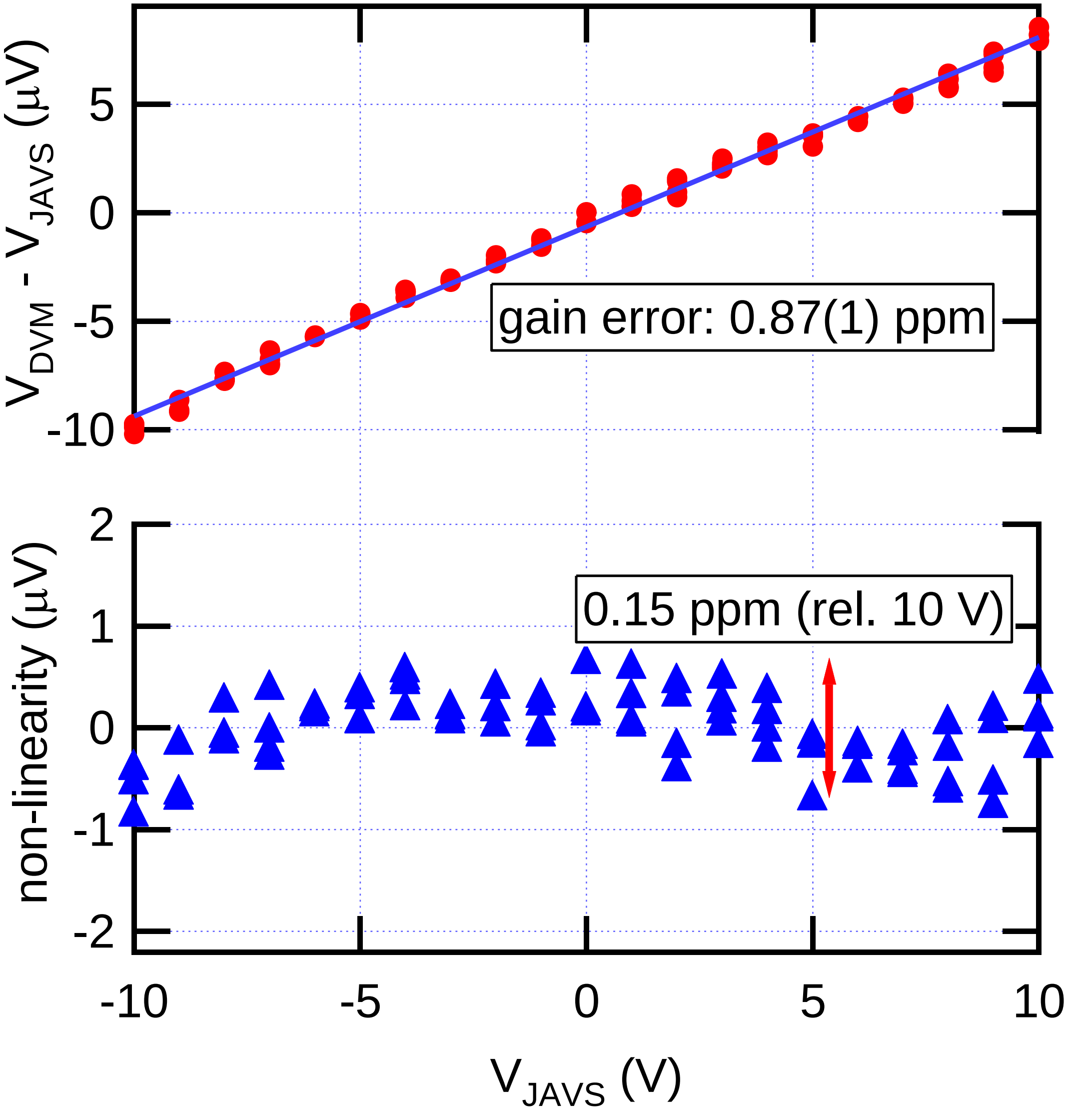
Comparison of JAVS



BIPM key comparison
Direct comparison of 10 V JAVS against BIPM transportable standard

Agreement to a few parts in 10^{10}

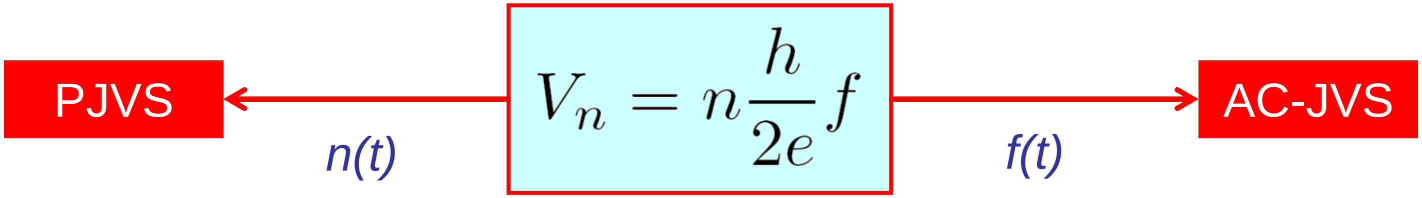
JAVS Application



Linearity check of a high-end DVM

Agilent 3458 A
10 V range

Josephson Standard for AC voltages



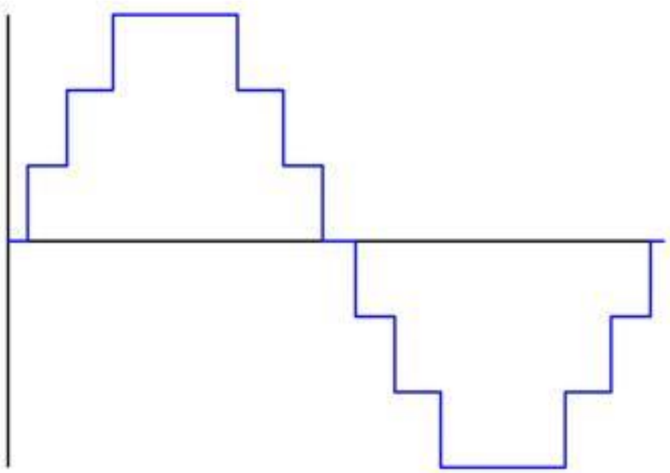
PJVS

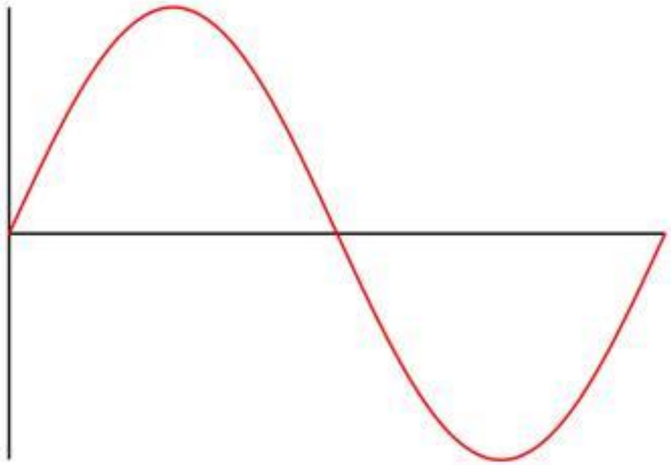
$$V_n = n\frac{h}{2e}f$$

AC-JVS

$n(t)$

$f(t)$





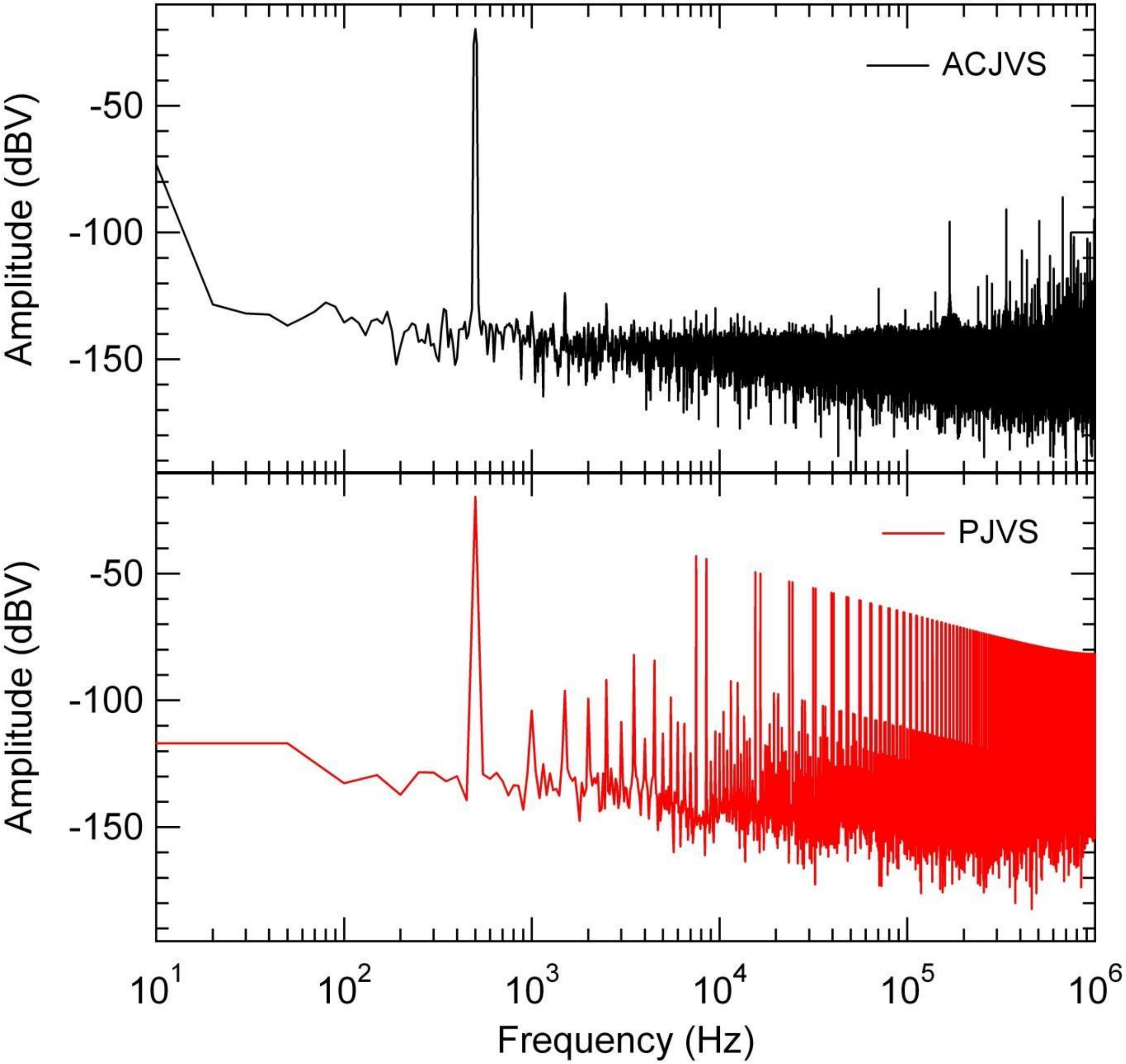
Programmable JVS:

- How to deal with transients?

Pulse driven JVS:

- Best AC source available
- Suitable for impedance measurements

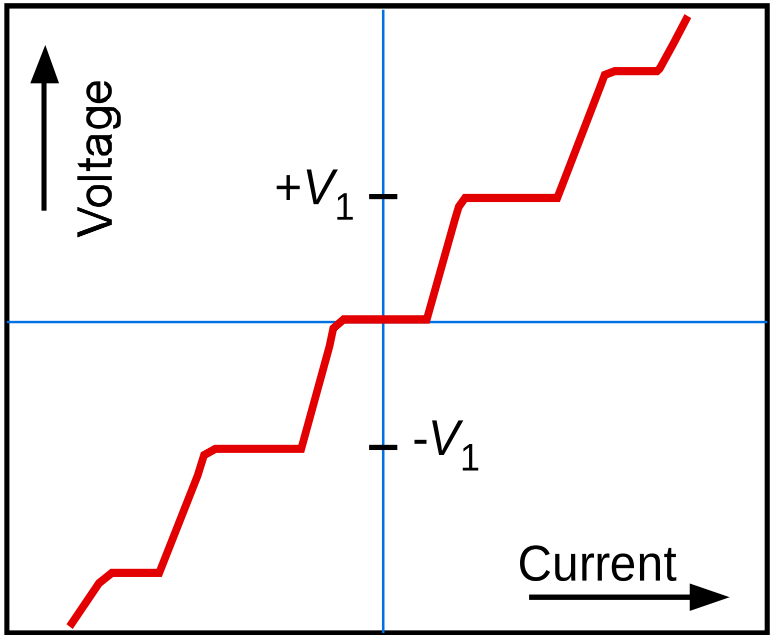
Josephson Standard for AC voltages



Programmable Josephson Arrays (PJVS)

Advances in nanotechnologies:

- Several thousand non-hysteretic junctions with same characteristics can be made



Overdamped junctions →

- **SNS junctions** (superconductor/ normal metal/ superconductor)
- **SINIS junctions** (supercond./insulator/normal/ insulator/supercond.)
- **Externally shunted SIS junctions**

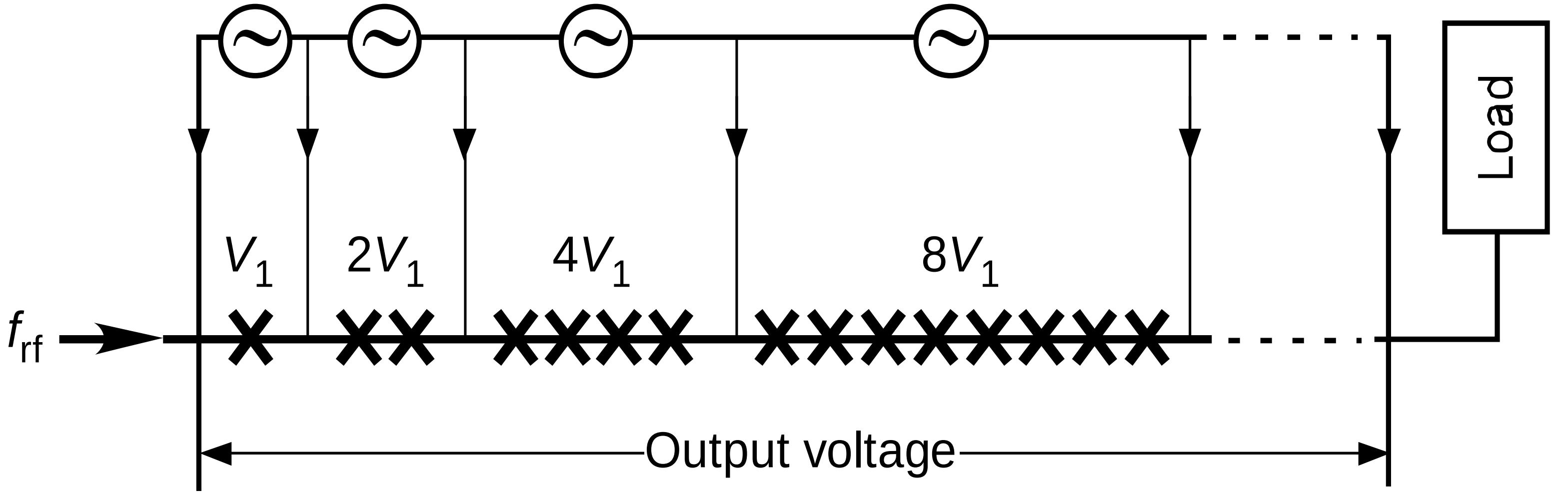
Advantage: voltage steps can be selected precisely (by choice of bias current) and very rapidly.

$$\Rightarrow \quad \boxed{V(t) = n(t)\frac{h}{2e}f}$$

Programmable Arrays (2)

- Array is divided into segments (binary sequence)
- Each segment controlled by its own bias source
- Steps $-V_1$, 0 and $+V_1$ in each segment selected

Computer controlled bias sources



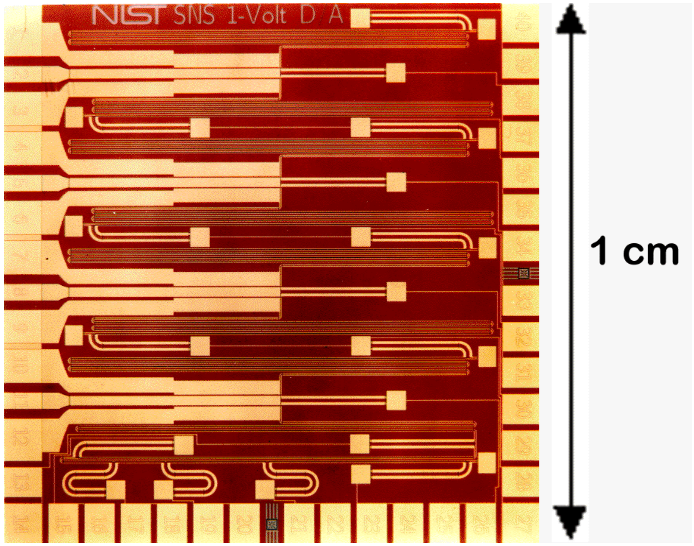
➔ D/A converter with fundamental accuracy (Hamilton 1995)

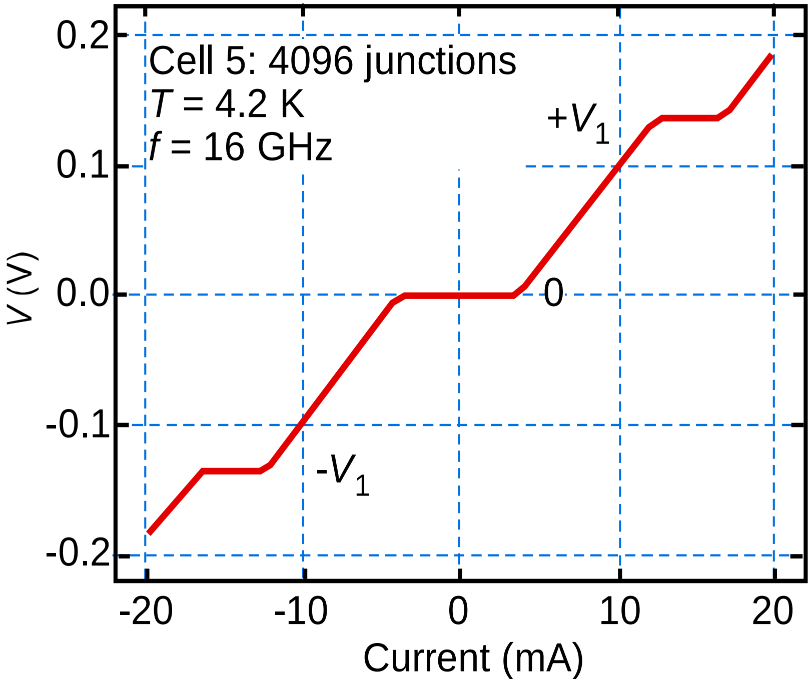
SNS array (NIST, 1997)

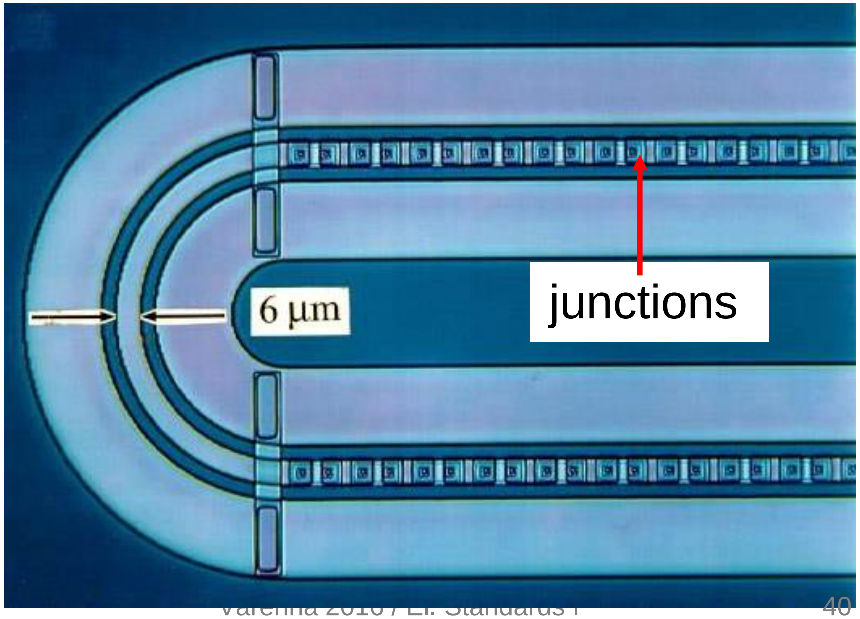
Nb /PdAu / Nb technology

1 V array; *f* = 16 GHz

- 32’768 junctions
- 33 µV/junction
- LSB (128 junctions) : 4.23 mV

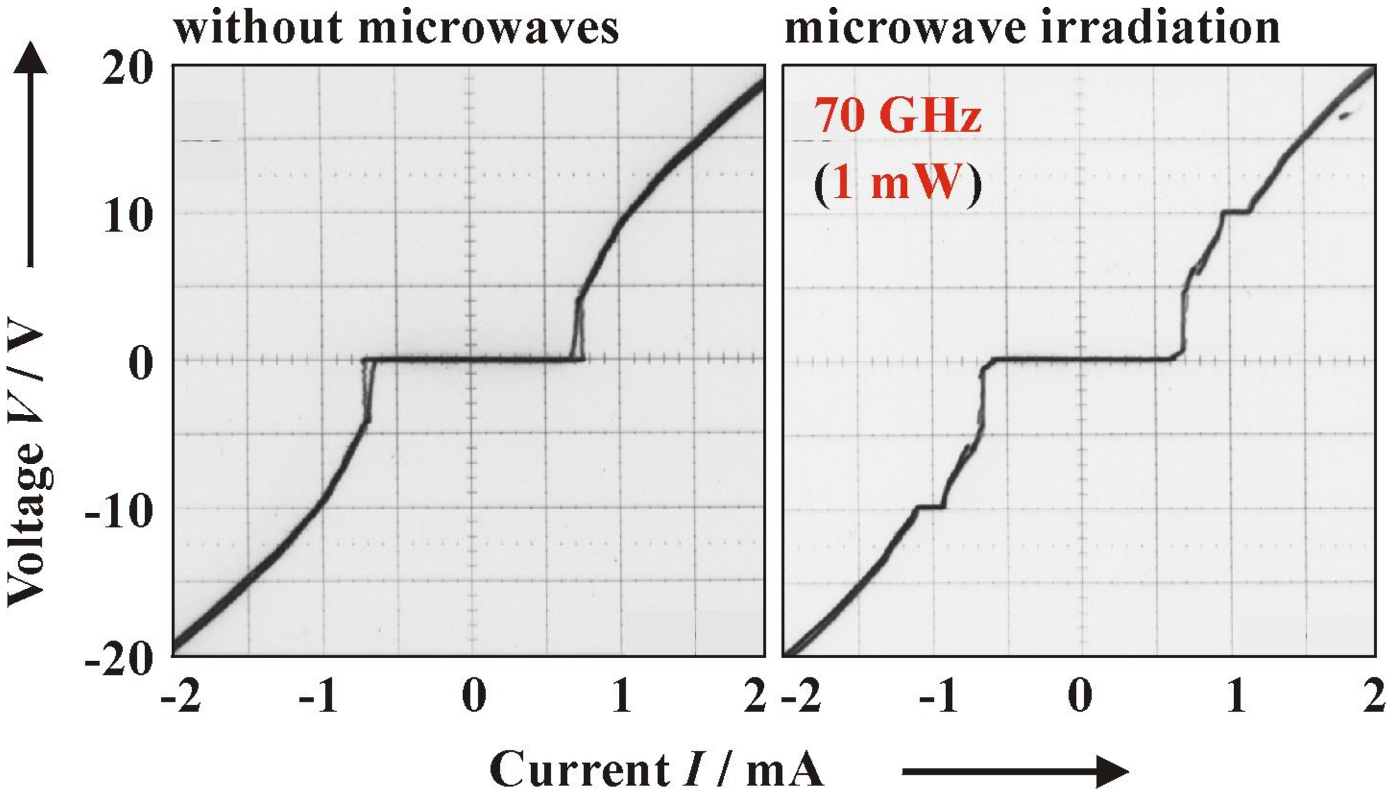






10 V SINIS Array PTB

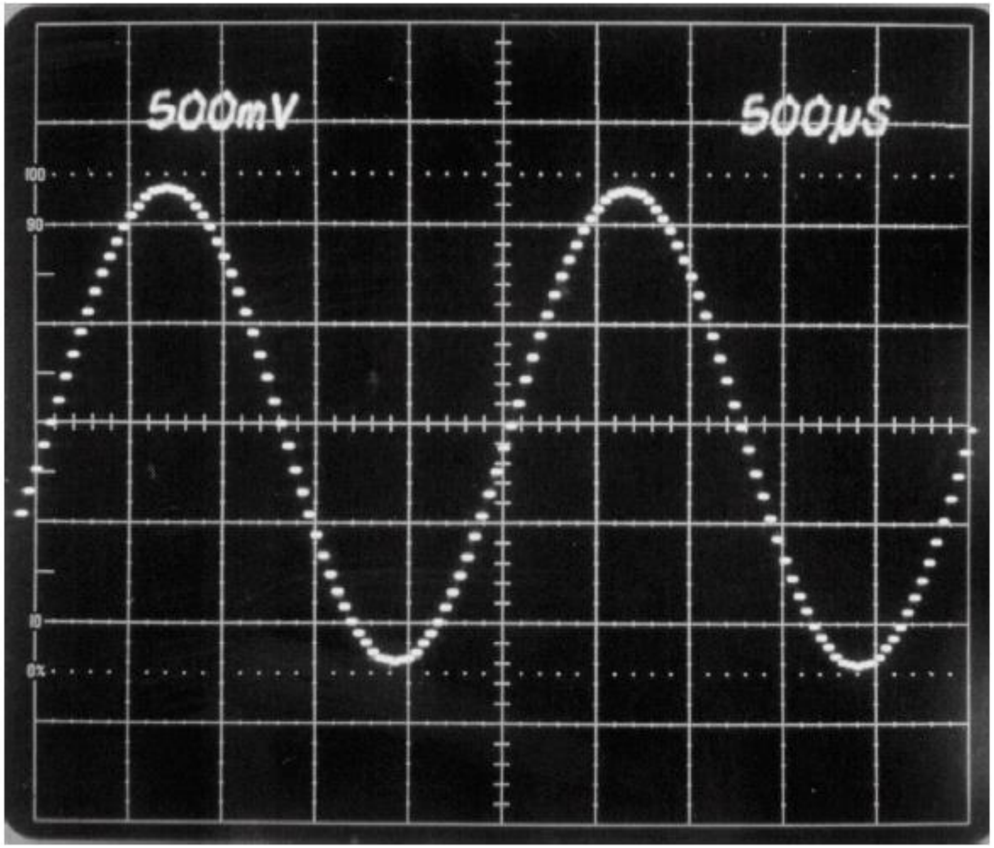
- Series array consisting of 69 120 SINIS Josephson junctions
- Step at 10 V (step width: 200 µA)



J. Kohlmann *et al.*, *IEEE Trans. Instrum. Meas.* **50** (2001) 192-194.

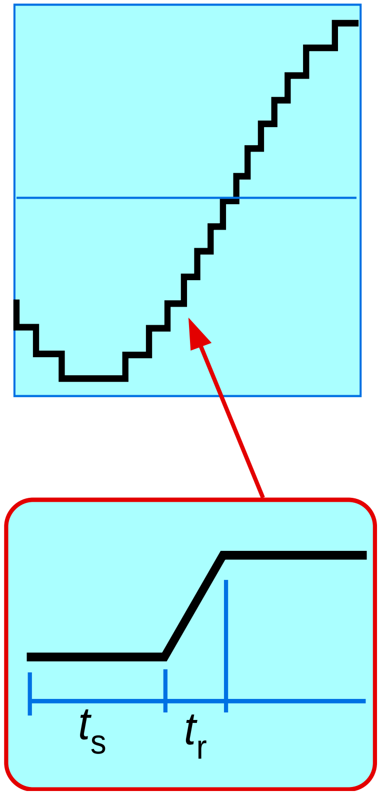
Waveform Synthesis

Synthesized sine wave with a 13 bit PTB Josephson array: V = 1.2 Vpp, f = 400 Hz



R. Behr et al, *IEEE IM* 54, 2005

Uncertainty



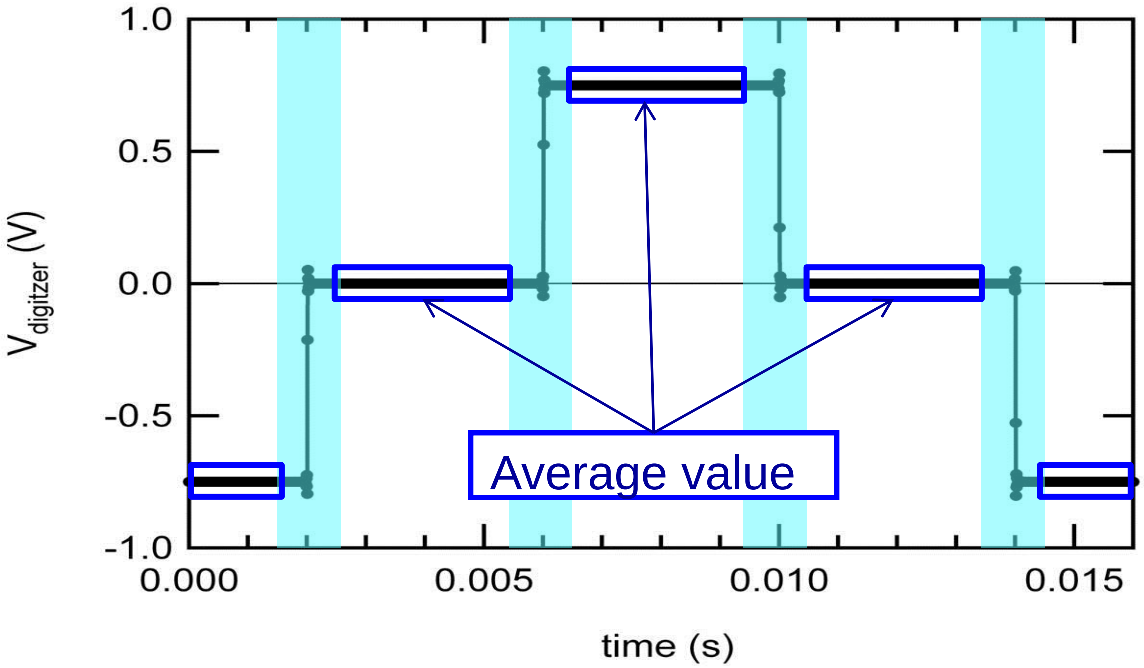
N = 64

t_s = 39 µs

t_r = 250 ns

$$\Delta_{rms} \propto \frac{16 t_r}{6 N t_s}$$

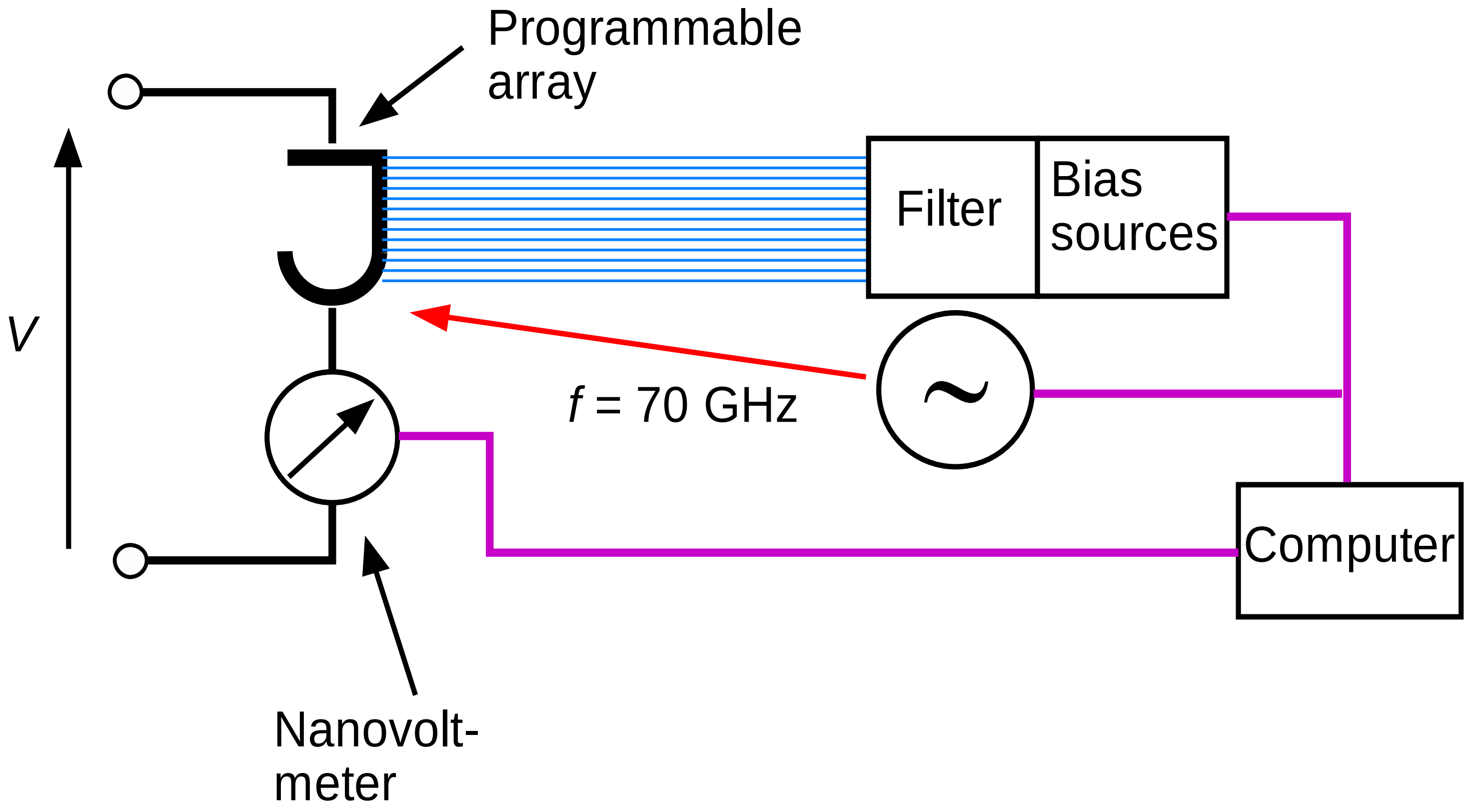
Suppression of transients: Sampling and signal reconstruction



- Accurate synchronization → **possible to remove data points during the transients**
- Digitizer digital filter → remove 50 points for each transition
→ limits the frequency

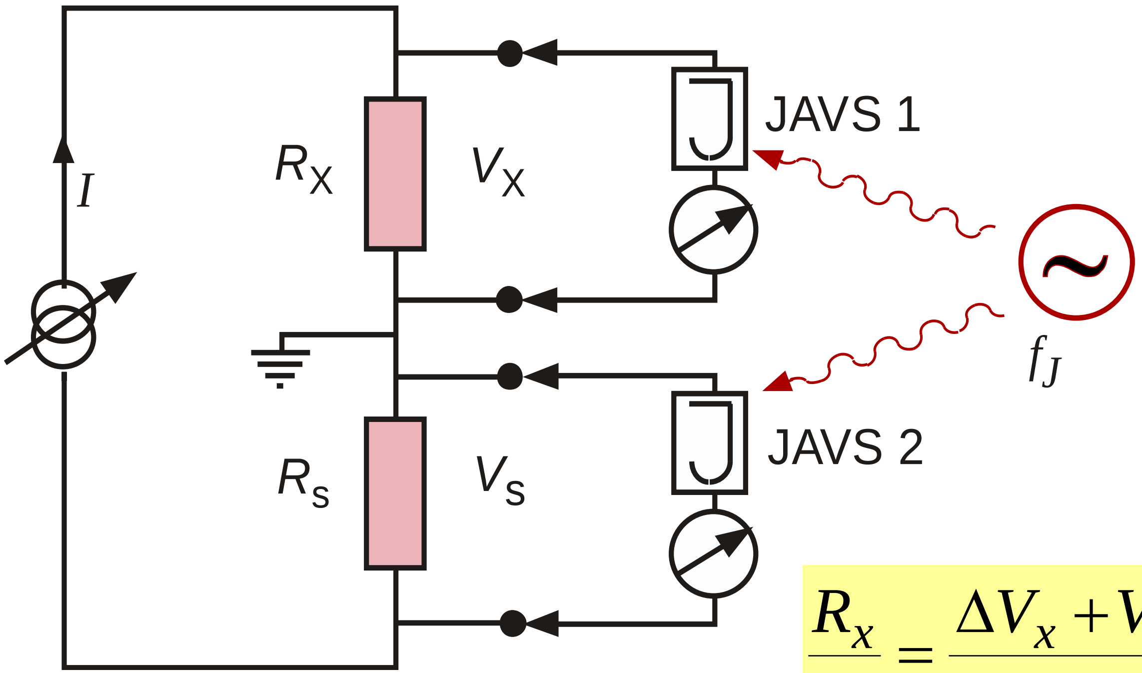
Applications

- The Quantum Voltmeter



Josephson Potentiometer

Comparison of resistance standards

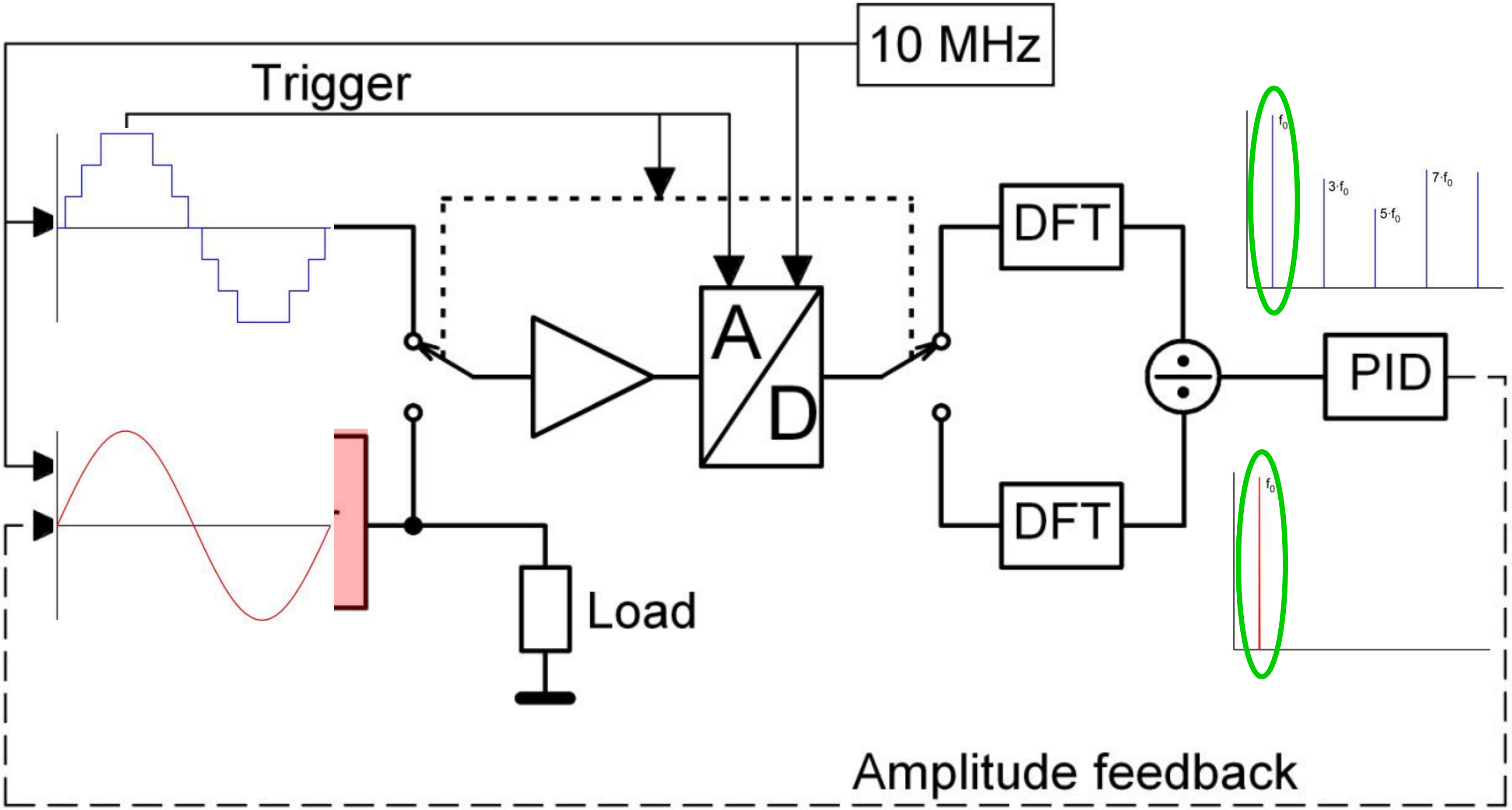


$$\frac{R_x}{R_s} = \frac{\Delta V_x + V_{J1}}{\Delta V_s + V_{J2}} = \frac{n_1}{n_2}\left(1 + \frac{\Delta V_x}{V_x} - \frac{\Delta V_s}{V_s}\right)$$

PTB, R. Behr et al., IEEE IM 52, 521 (2003)

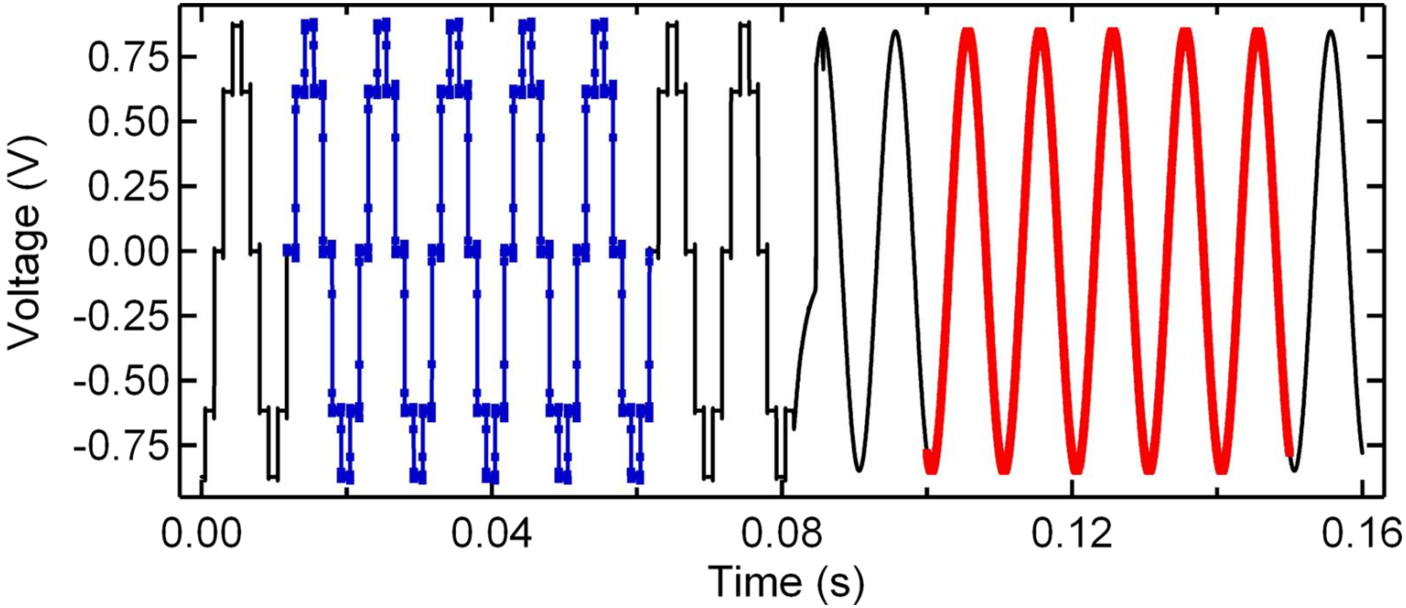
10 kΩ in terms of the QHR (12.9 kΩ) to 3 parts in 10^9

Josephson Locked Synthesizer (JoLoS)



$$V_{out} = V_{f_0}^{PJVS}$$

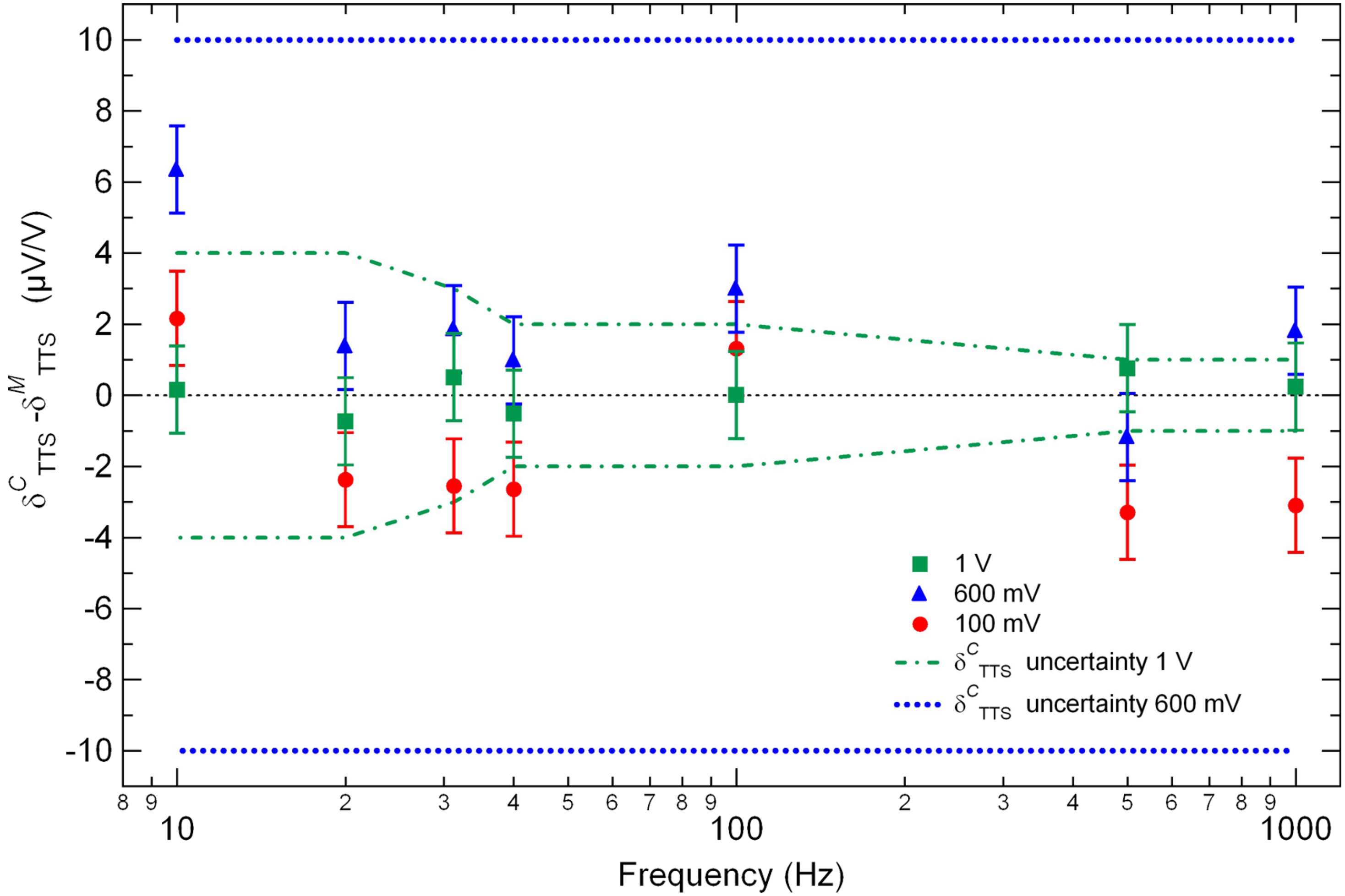
JoLoS: Data acquisiton and signal reconstruction



$$V_{\mathrm{SYN}} = \underbrace{\left| \frac{A_{\mathrm{SYN}}}{A_{\mathrm{PJVS}}} \right|}_{\equiv r} \cdot V_{\mathrm{PJVS}},$$

V_{PJVS}: Fundamental of the DFT of the theoretical ($K_{\text{J-90}}$) waveform (calculated)
A_{SYN}: Fundamental of the DFT of the synthesizer waveform (measured)
A_{PJVS}: Fundamental of the DFT of the reconstructed waveform (measured)

JoLoS Application: Thermal transfer measurements



→ A. Rüfenacht et al., IEEE Trans. Instrum. Meas. **60-8**, 2372-2377 (2011).

Pulse driven Josephson arrays

Due to undefined transitions between steps:

→ applications of binary programmable arrays are limited to < 1 kHz

Different approach:

→ Change frequency in time instead of number of junctions

$$\boxed{V(t) = N\frac{h}{2e}f(t)}$$

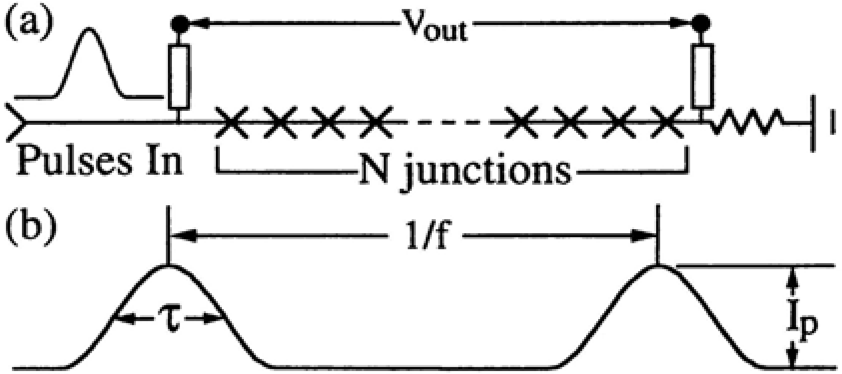
→ **Problem**: Sine-wave excitation: step amplitude decreases with frequency

→ Solution (Benz and Hamilton, 1995):
Replace sine wave with pulse excitation; in this case, step amplitude is independent of pulse repetition frequencies (simulations) for $f < f_c$

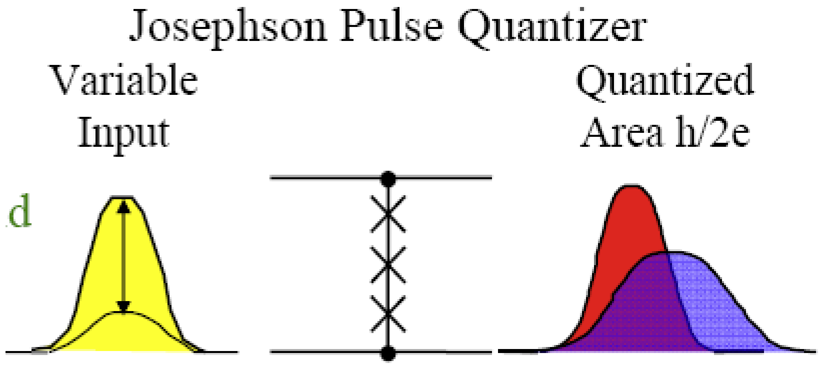
Pulse driven Josephson arrays (2)

→ Single large array with N junctions distributed along a wide bandwidth transmission line

→ A pulse train at frequency f generates an average voltage:



$$\boxed{\overline{V} = N\frac{h}{2e}f}$$



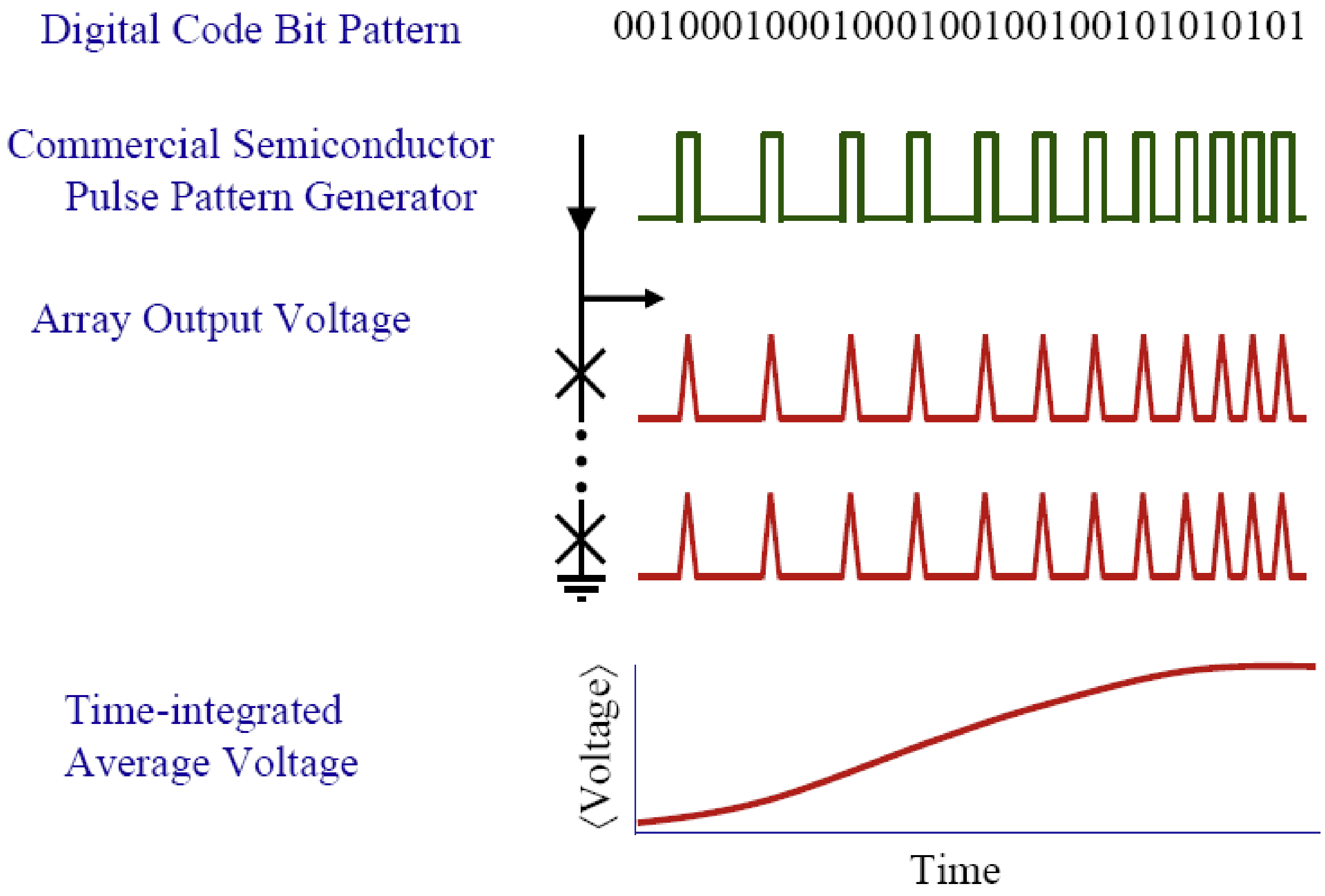
$$\int_{pulse} V(t)dt = \pm n\phi_0 = \frac{\pm n}{K_J}$$

$$\text{Flux quantum}: \frac{\mathrm{h}}{2\mathrm{e}} = \phi_0 = \frac{1}{\mathrm{K_J}}$$

H. Worsham, J.X. Przybysz, S. Benz, and C. Hamilton, NIST & Westinghouse, 1995

Pulse driven Josephson arrays (3)

→ Generation of complex wave forms by modulating the pulse train



Digital Code Bit Pattern 00100010001000100100100101010101

Commercial Semiconductor
Pulse Pattern Generator

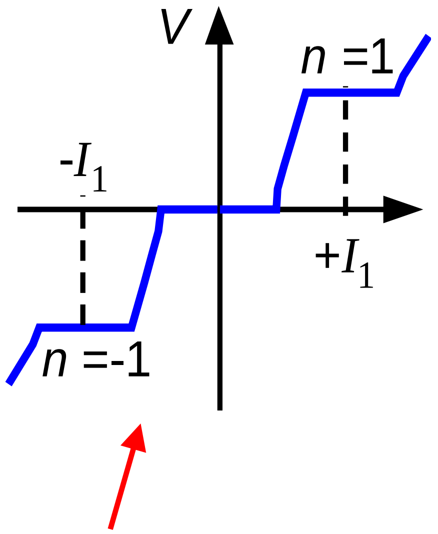
Array Output Voltage

Time-integrated
Average Voltage

Courtesy S. Benz, NIST

Bipolar operation

(Benz et al., 1998)
Combination of pulse train and sine wave bias

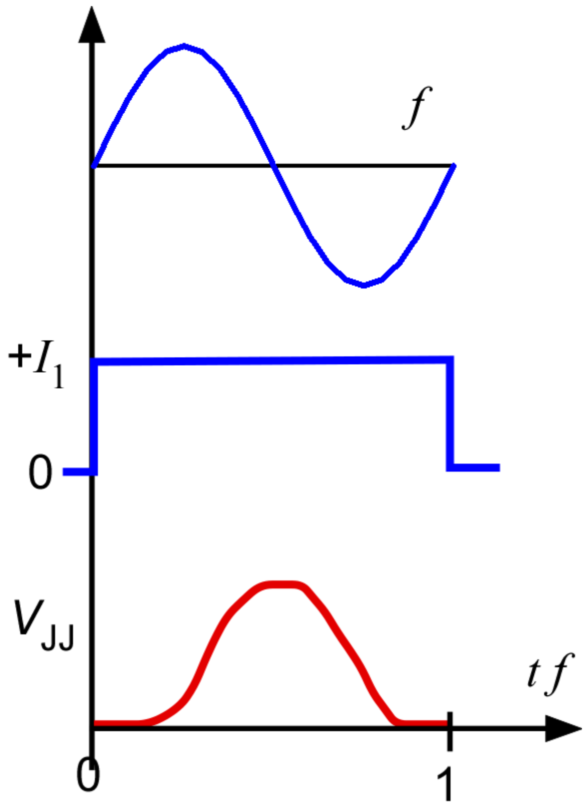


Resistively shunted JJ, driven by microwave, frequency *f*

Microwave bias

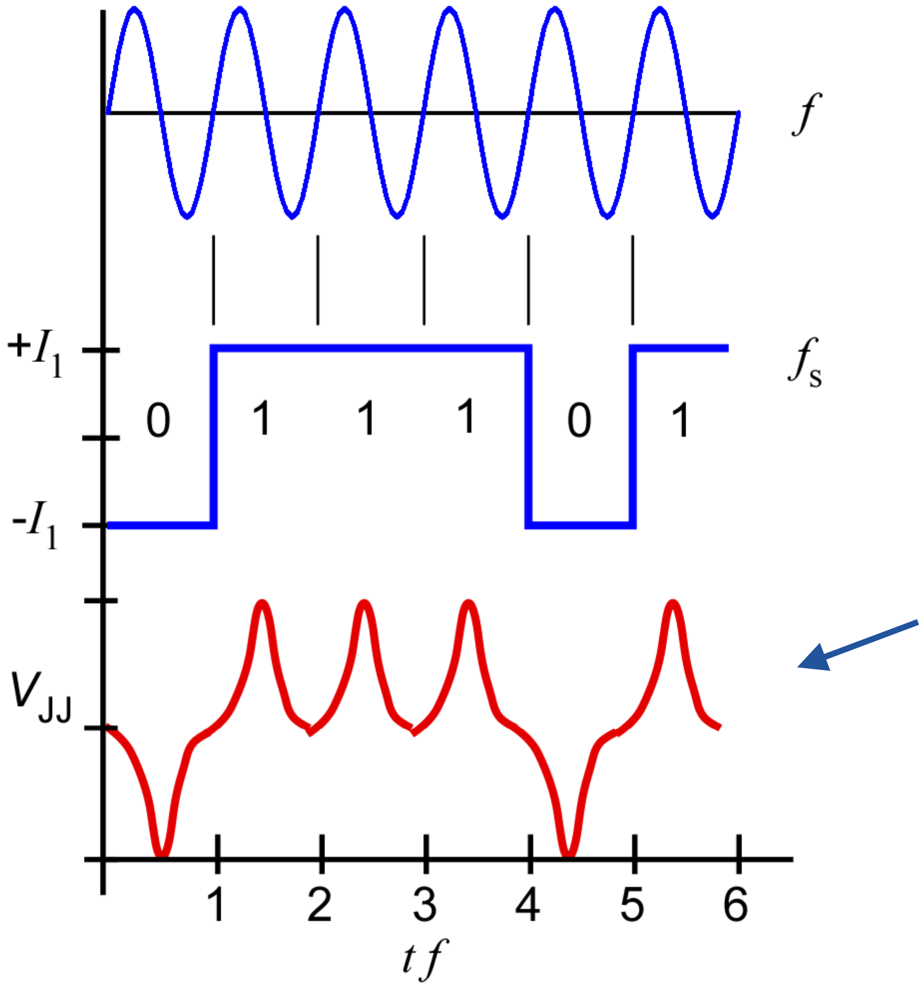
Synchronized current bias

Quantized JJ pulse



$$\int_0^T V_{JJ}(t)\cdot dt = \phi_0 = \frac{1}{K_J}$$

Bipolar pulse control



Fast switching

- Sampling frequency f_s
- Code levels $\pm I_1$

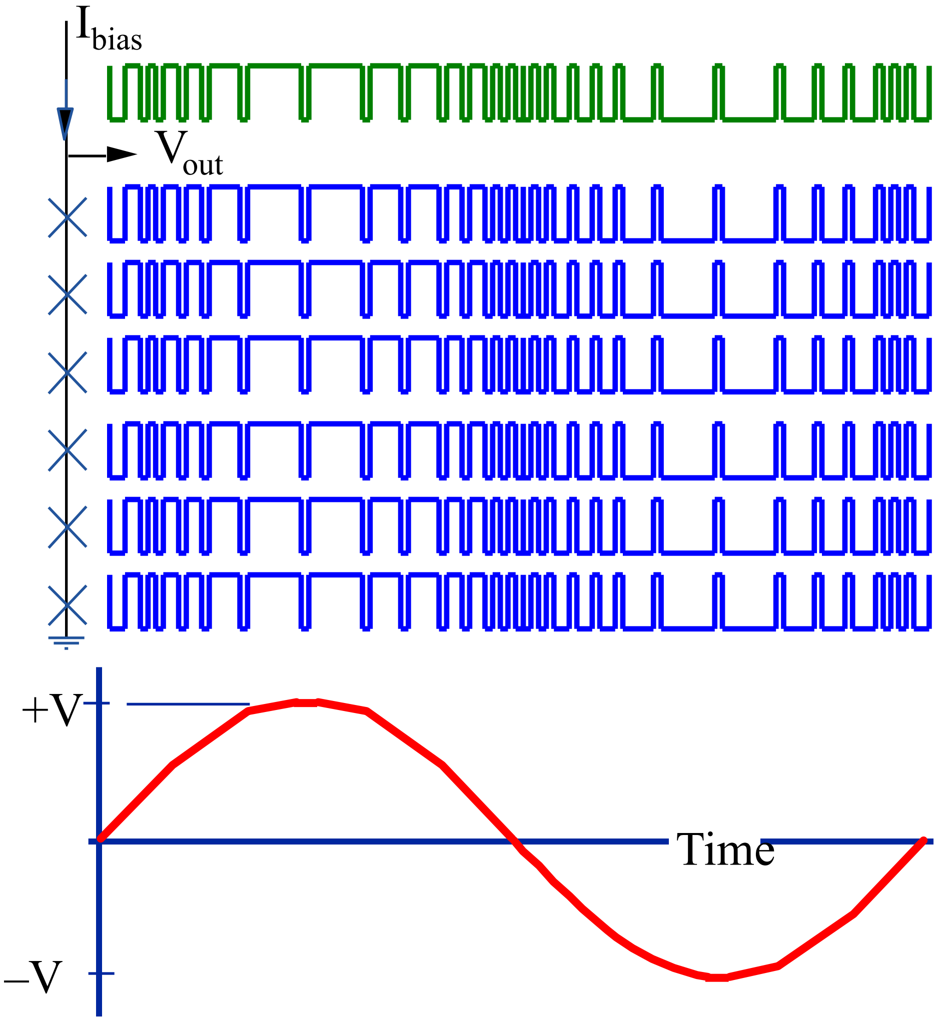
$$\langle V\rangle = \frac{p-q}{p+q}\frac{f}{K_J}$$

p: number of “1”
q: number of “0”

Specific frequency and phase relationships between sampling and drive frequencies required

$$f = mf_s/2; \quad m \geq 2$$

Digital waveform synthesis



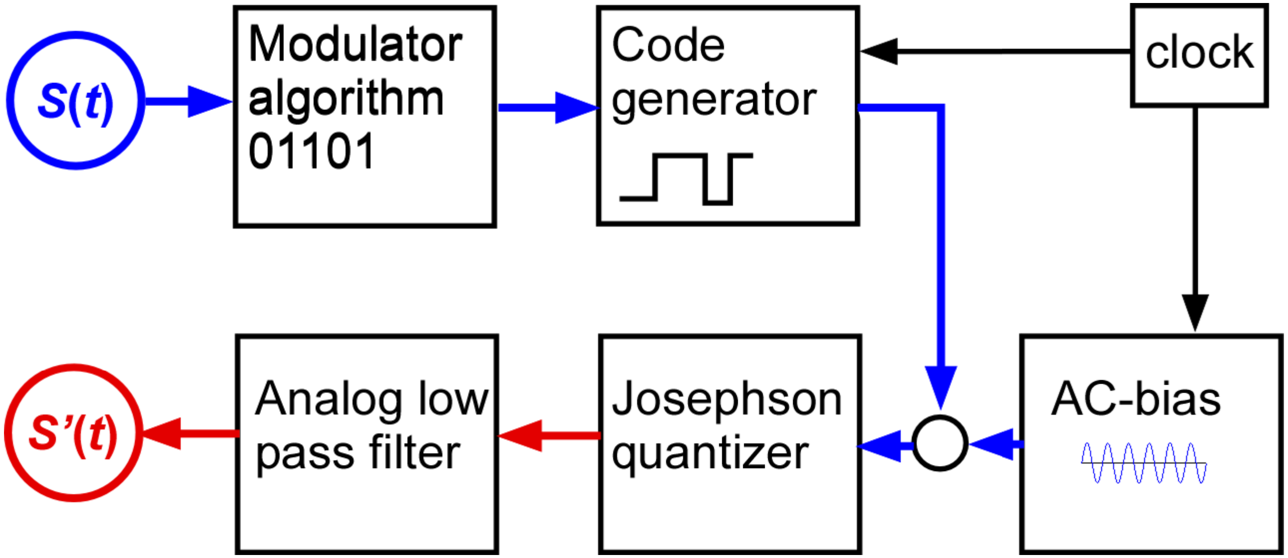
- Timing and polarity of the modulation signal precisely determine the voltage waveform

- Peak to peak voltage:

$$V_{pp-\max} = m \cdot N \frac{f_s}{K_J}$$

Number of junctions

Josephson Array Pulse Quantizer



RMS value of output determined by:

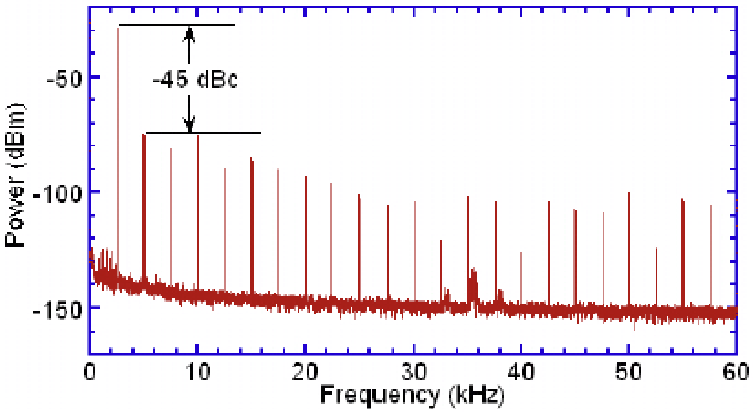
- **Digital code, sampling frequency and number of junctions**

Exact quantization if:

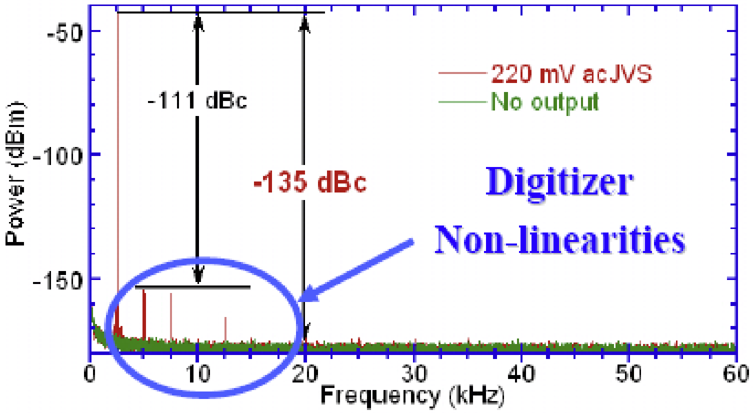
- Correct synchronisation of code and hf-drive, switching time << $1/f$
- Transmission path to every junction independent of frequency from dc to about 18 GHz

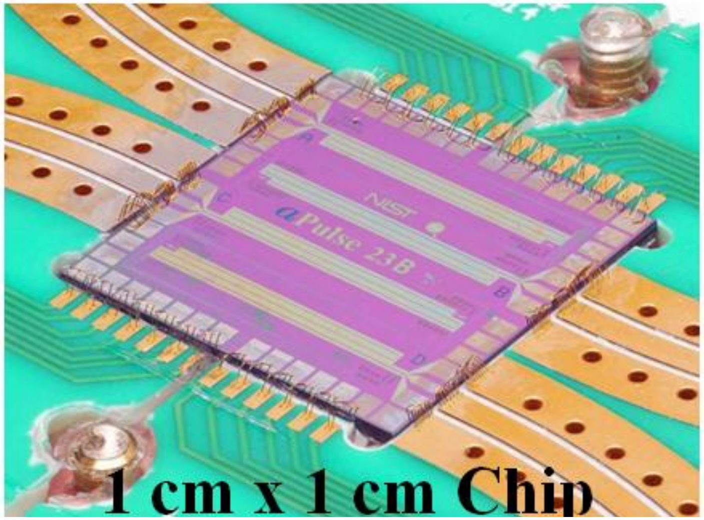
AC-JVS: Results

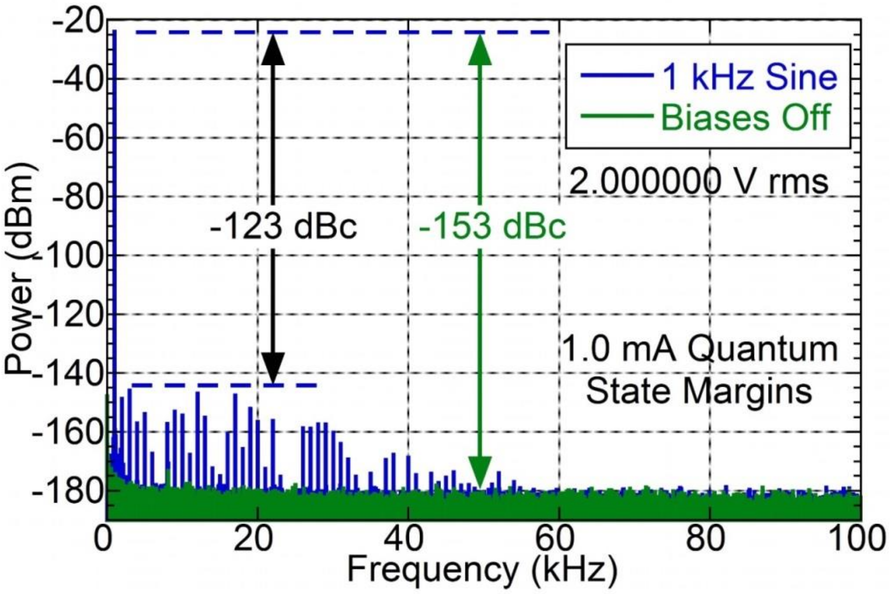
Semiconductor Code Generator Output



Josephson Junction Array Output





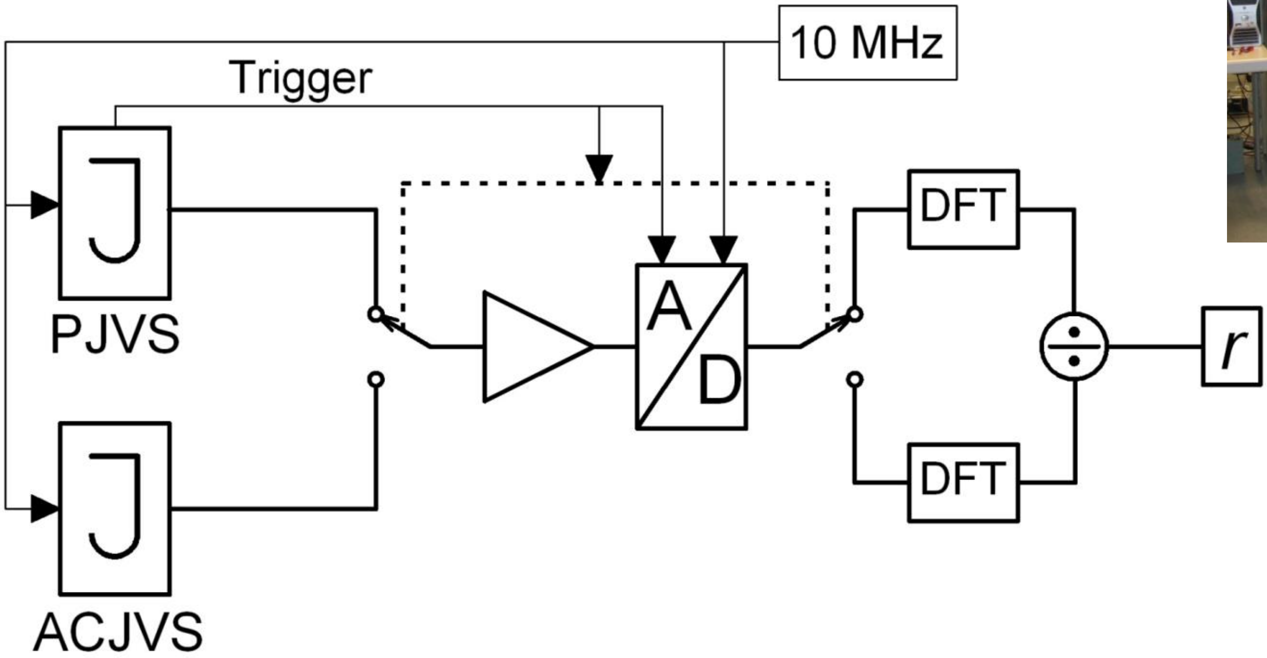


N. Flowers-Jacobs et al. IEEE Trans. Appl. Supercond., 2016.

Courtesy S. Benz, NIST

Comparison PJVS – AC-JVS

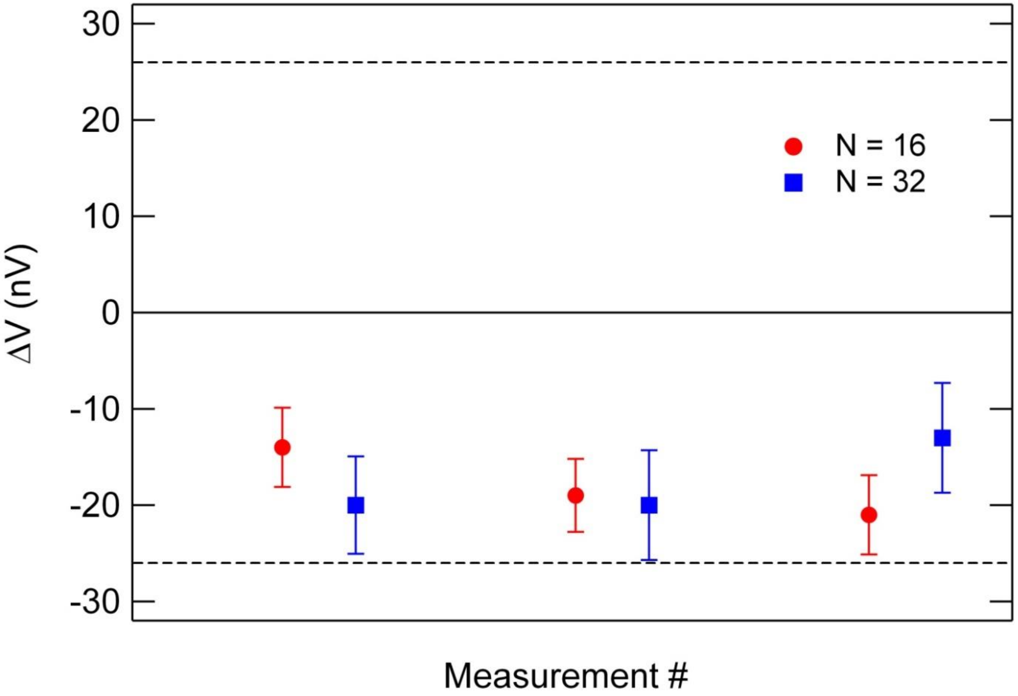
VSL, Delft



$$V_{\mathrm{ACJVS}}(\mathrm{METAS}) = \left|\frac{A_{\mathrm{ACJVS}}}{A_{\mathrm{PJVS}}}\right| \cdot V_{\mathrm{PJVS}} \equiv r \cdot V_{\mathrm{PJVS}}$$

$$\Delta V \equiv V_{\mathrm{ACJVS}}(\mathrm{VSL}) - V_{\mathrm{ACJVS}}(\mathrm{METAS})$$

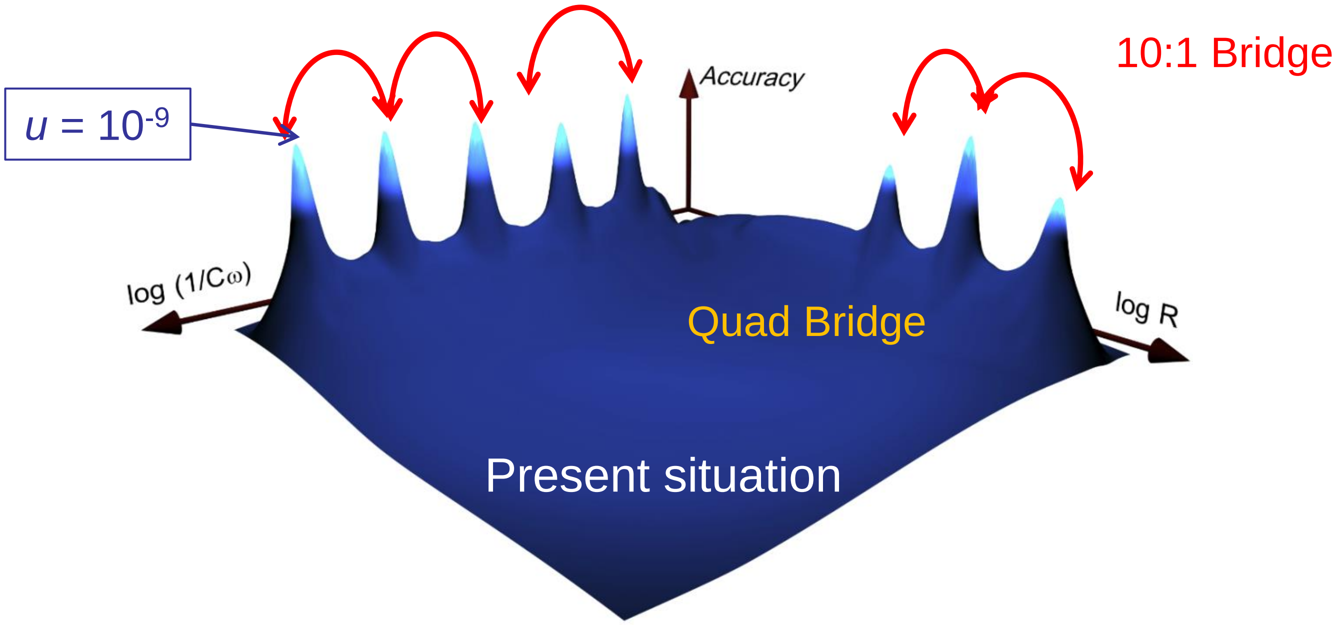
Comparison PJVS – AC-JVS (2)

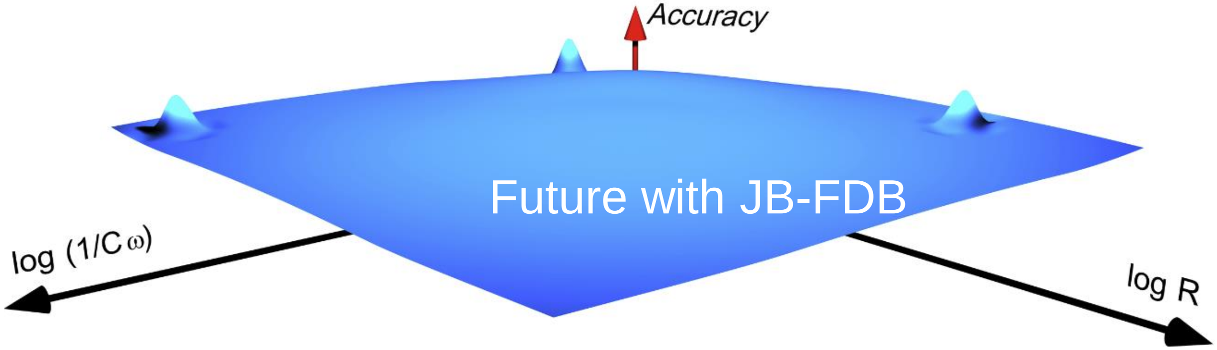


$$V_{\mathrm{ACJVS}}(\mathrm{VSL}) - V_{\mathrm{ACJVS}}(\mathrm{METAS}) = (-18 \pm 26)\ nV$$

→ B.Jeanneret et al., Metrologia 48, pp.311-316 (2011).

Application AC-JVS:
Josephson Impedance Bridge

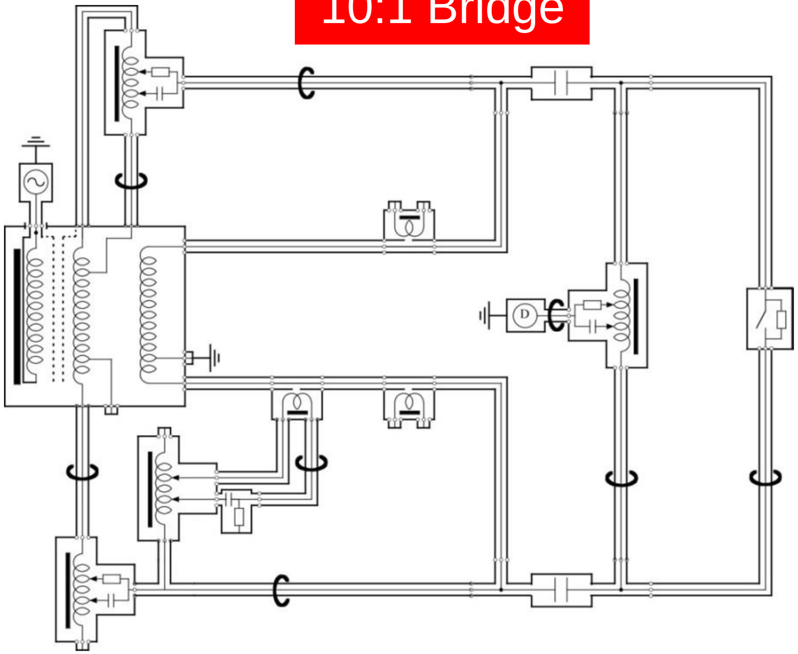




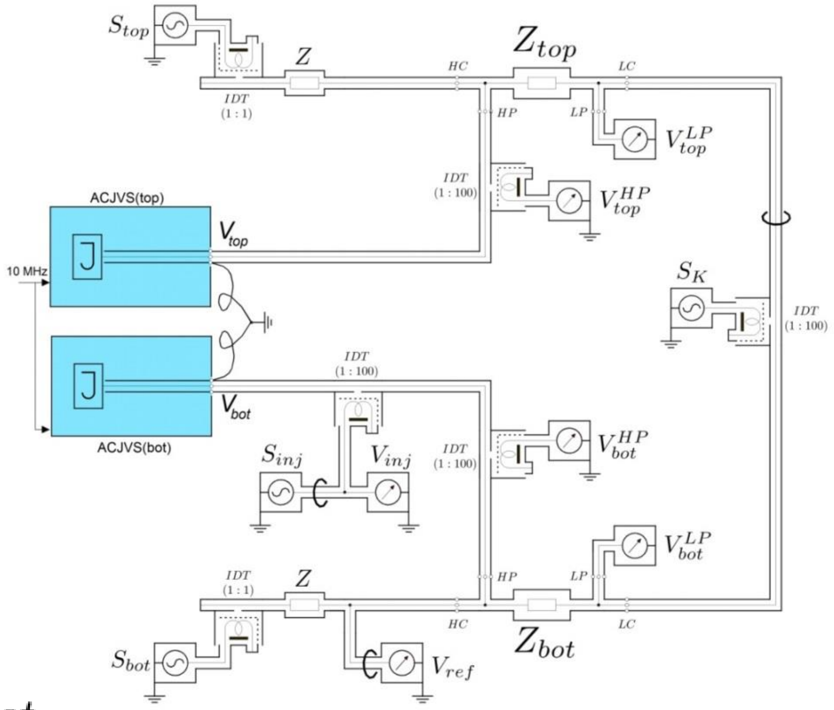
Courtesy L. Palafox, PTB

Josephson Impedance Bridge

10:1 Bridge



JB-FDB

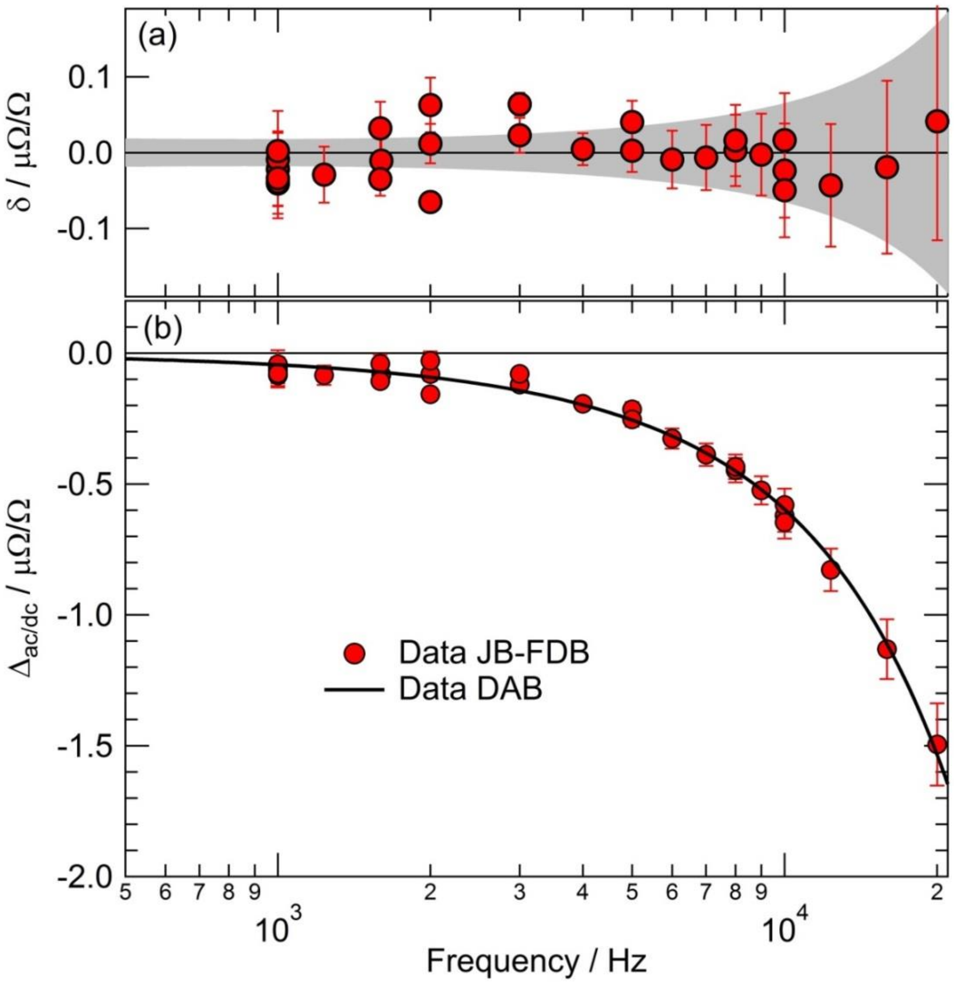


$$\Gamma = \frac{Z_{bot}}{Z_{top}} = -\frac{V_{bot}}{V_{top}}$$

- Fully manual
- Bandwidth 50 Hz – 5 kHz
- Accuracy: 10^{-8} @ 1 kHz

- Fully automated
- Bandwidth 50 Hz – 50 kHz **(10X)**
- Accuracy: 10^{-8} @ 1 kHz

The Josephson Bridge: Comparison R-R



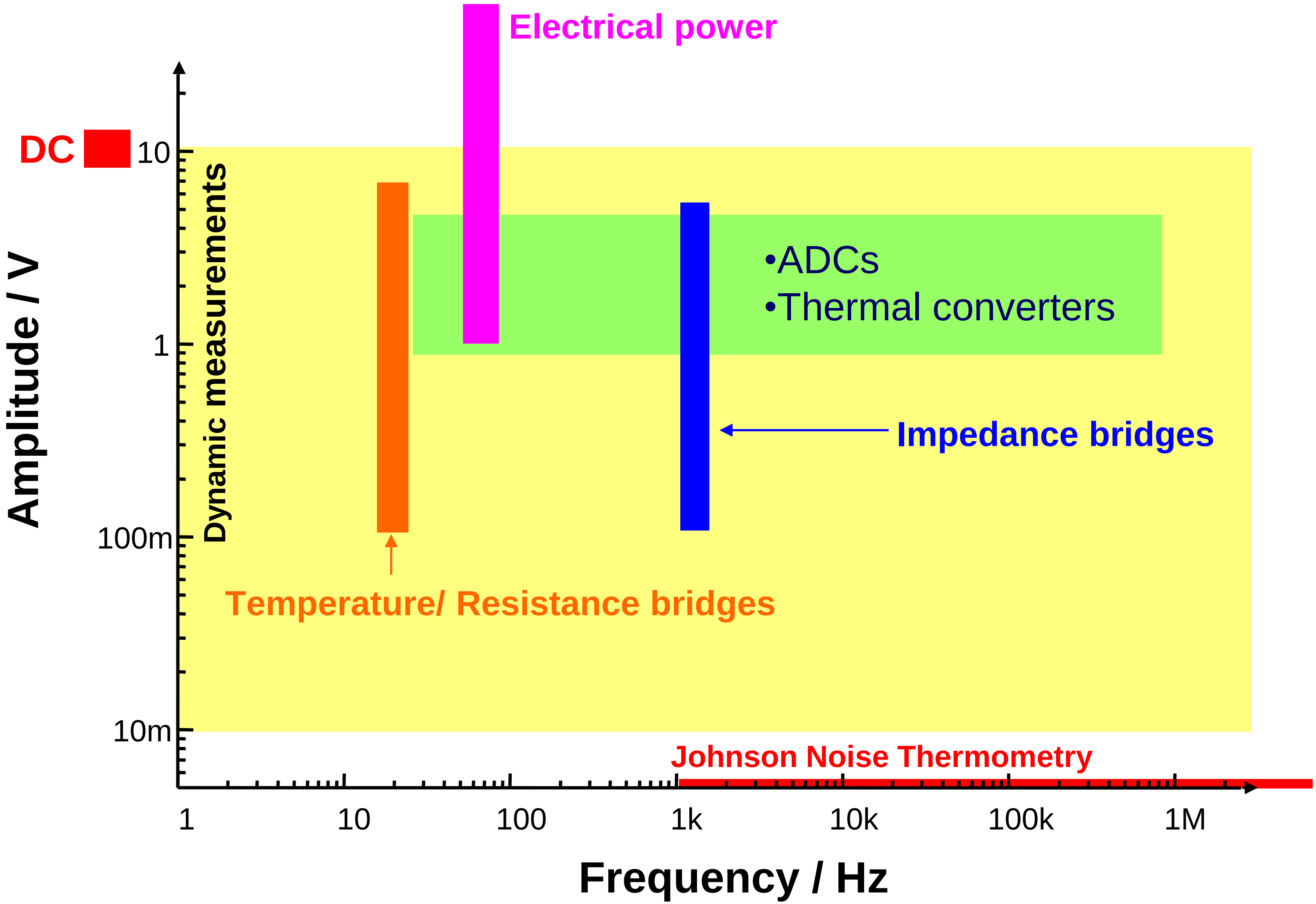
$$\Gamma = \frac{Z_{12k9}^{A}}{Z_{12k9}^{B}}$$

$$\Delta_{ac/dc} = \Gamma(f) - \Gamma(0)$$

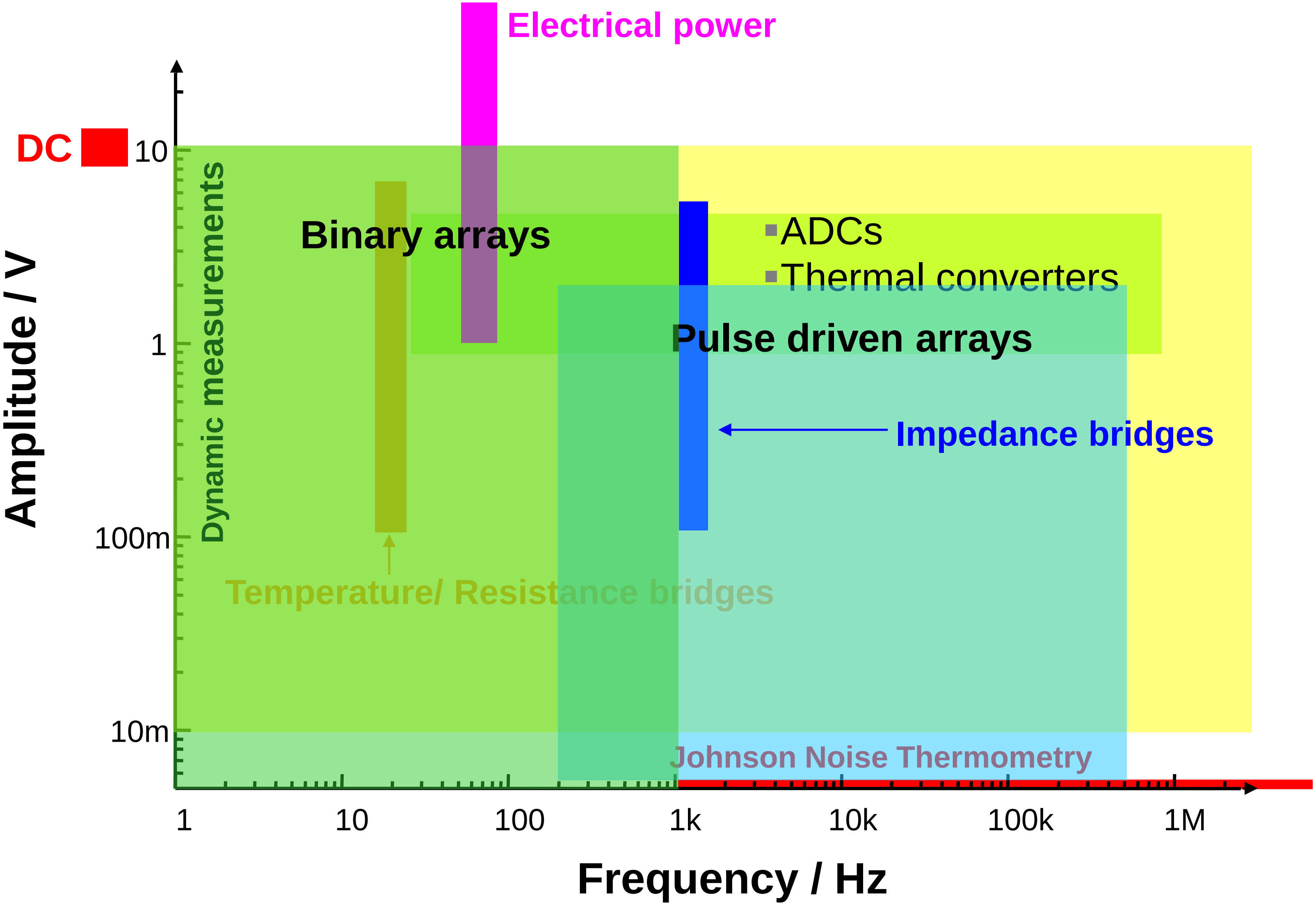
- Use Z_{12k9}^{A} as reference
- 12.9 kΩ thermostated resistors
- Range: 1 kHz to 20 kHz
- Agreement < 0.1 ppm

→ F. Overney et al., Metrologia (2016).

Range JVS applications



Range JVS applications



Summary Part I

- **Josephson Array voltage standards well established as primary standards for DC voltage in the range -10 V to 10 V**
  1. Reproducibility: parts in 10^9
  2. Two orders of magnitude better than realisation of the volt in the SI

- **Programmable standards well established**
  1. Low frequency arbitrary waveforms up to 10 V → better power standards
  2. Arbitrary waveforms DC to 1 MHz, with pulsed driven arrays; voltage up to 2 V
     → improved low voltage AC/DC transfer
     → impedance comparisons in the whole complex plane

Thank you very much for your attention