Vacancy-like dressed states in Topological Waveguide QED

Angelo Carollo

Outline

Part I: Topological protected Photon-atom bound states

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

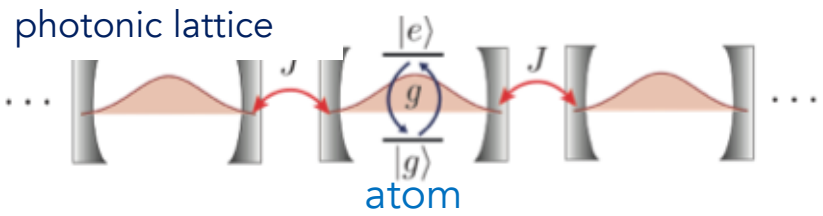
Part II: Exotic interaction mediated by a non-Hermitian photonic bath

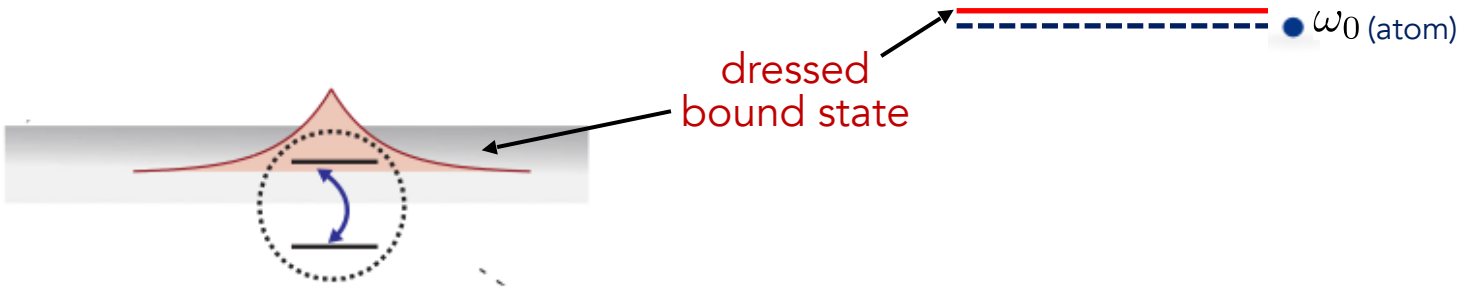
Part I: Topological protected Photon-atom bound state

introduction &
motivations

context

«waveguide QED» in structured photonic baths





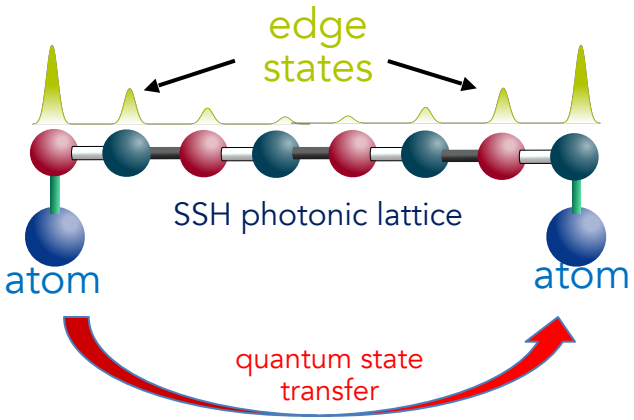
J. S. Douglas et al, Nat. Phot. 9, 326 (2015)
A. Gonzalez-Tudela et al, Nat. Photonics 9, 320 (2015).
G. Calajò, F. Ciccarello, D. Chang, and P. Rabl, PRA 93, 033833 (2016)
A. Gonzalez-Tudela and J. I. Cirac, PRL 119, 143602 (2017)

experiments:

J. D. Hood et al, PNAS 113, 10507 (2016)
Y. Liu and A. A. Houck, Nat. Phys. 13, 48 (2017)
L. Krinner et al, Nature 559, 589 (2018)
N. M. Sundaresan et al, PRX 9, 011021 (2019)

Topological photonic lattices

1D



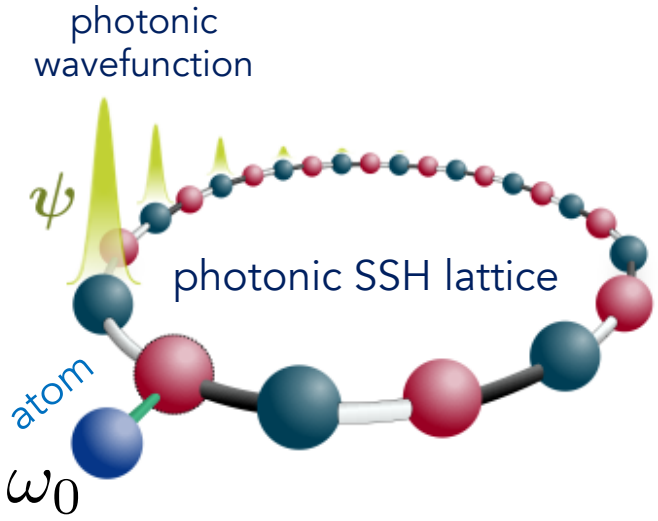
G. M. A. Almeida, F. Ciccarello, T. J. G. Apollaro, and A. M. C. Souza, PRA 93, 032310 (2016)

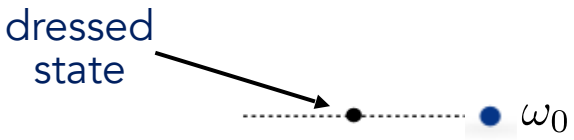
2D

state transfer, chiral emission,…

N. Y. Yao et al, Nat. Commun. 4, 1 (2013)
S. Barik et al, Science 359, 666 (2018)
M.-A. Lemonde, V. Peano, P. Rabl, and D. G. Angelakis, NJP 21, 113030 (2019)
M. J. Mehrabad et al, Optica 7, 1690 (2020)

Topologically-protected dressed states





experimentally observed:
E. Kim et al, PRX 11, 011015 (2021)

RESEARCH ARTICLE | PHYSICS

Unconventional quantum optics in topological waveguide QED

M. Bello[1], G. Platero[1], J. I. Cirac[2] and A. González-Tudela[2,3,*]
+ See all authors and affiliations

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DOI: 10.1126/sciadv.aaw0297

dressed bound state

$$|\Psi\rangle \propto \varepsilon|e\rangle|\mathrm{vac}\rangle + |g\rangle|\psi\rangle$$

topologically protected

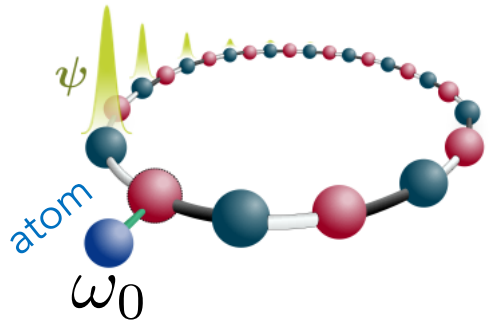
basic questions

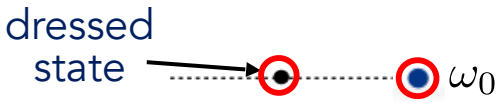
- general criteria for occurrence of topologically-protected atom-photon dressed states

- general properties of topologically-protected atom-photon dressed states

clues from SSH

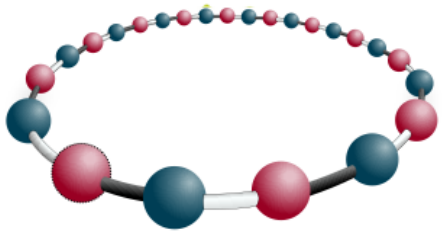
1

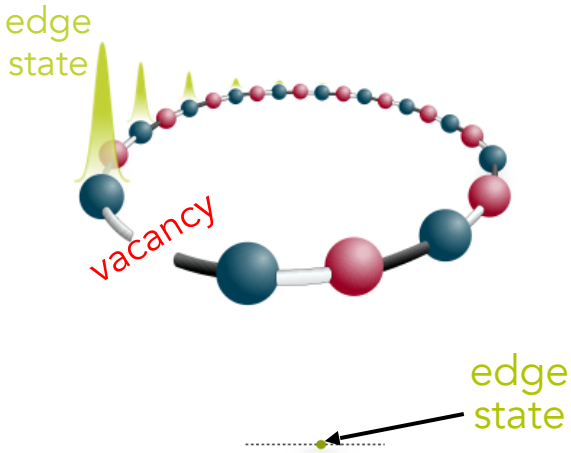




energy of Ψ pinned to the atomic frequency

2





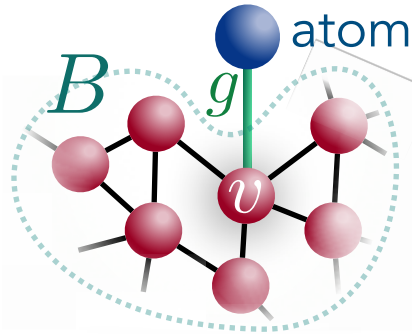
Vacancy-like Dressed States (VDS)

$$H = \omega_0 \sigma_+ \sigma_- + H_B + g\left(b_v^\dagger \sigma_- + b_v \sigma_+\right)$$

atom — RWA coupling

$$H_B = \sum_i \omega_i b_i^\dagger b_i + \sum_{i \neq j} J_{ij} b_i^\dagger b_j$$

photonic bath



generic dressed state:

$$|\Psi\rangle \propto \varepsilon |e\rangle|\mathrm{vac}\rangle + |g\rangle|\psi\rangle$$

single-photon state

def VDS: $H|\Psi\rangle = \omega_0 |\Psi\rangle$

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

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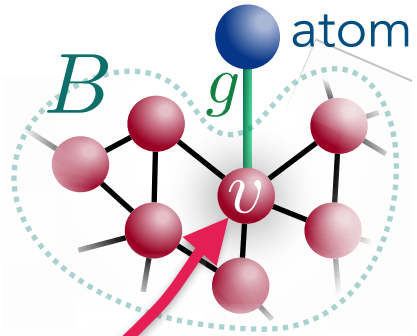

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def VDS: $H|\Psi\rangle = \omega_0|\Psi\rangle \Leftrightarrow \langle v|\psi\rangle = 0$

field's node

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

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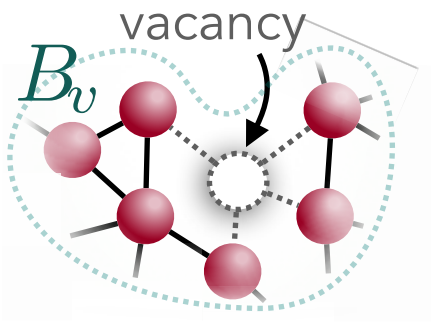

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L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

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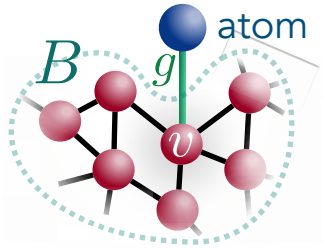

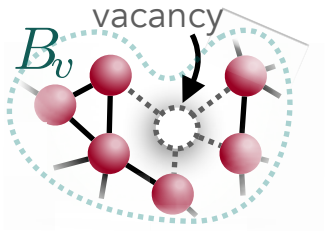

$|\psi\rangle \in B_v$

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

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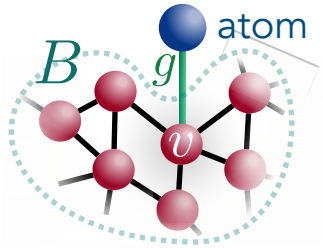
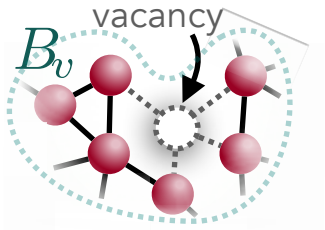
VDS: $|\Psi\rangle \propto \varepsilon|e\rangle|\mathrm{vac}\rangle + |g\rangle|\psi\rangle$ $\qquad H|\Psi\rangle = \omega_0|\Psi\rangle$





$$H_{B_v}|\psi\rangle = \omega_0|\psi\rangle$$
$$g\varepsilon + \langle v|H_B|\psi\rangle = 0$$

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

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bound VDS: general form

$$|\Psi\rangle = \cos\theta\,|e\rangle|\mathrm{vac}\rangle + e^{i\varphi}\sin\theta\,|g\rangle|\psi\rangle$$

$$H_{B_v}|\psi\rangle = \omega_0|\psi\rangle$$

$$\theta = \arctan|\eta|\,,\ \varphi = \arg\eta \quad \text{with } \eta = -\frac{g}{\langle v|H_B|\psi\rangle}$$

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

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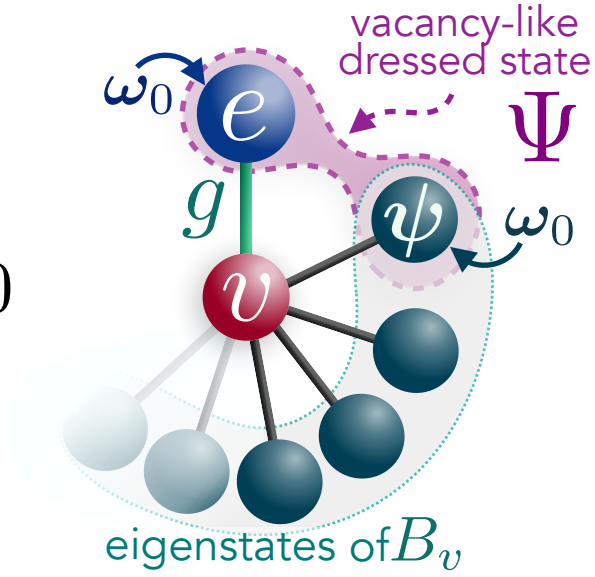

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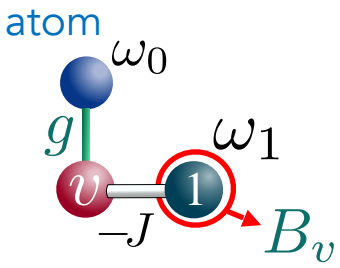
(non-topological) instances of VDS

2 cavities:



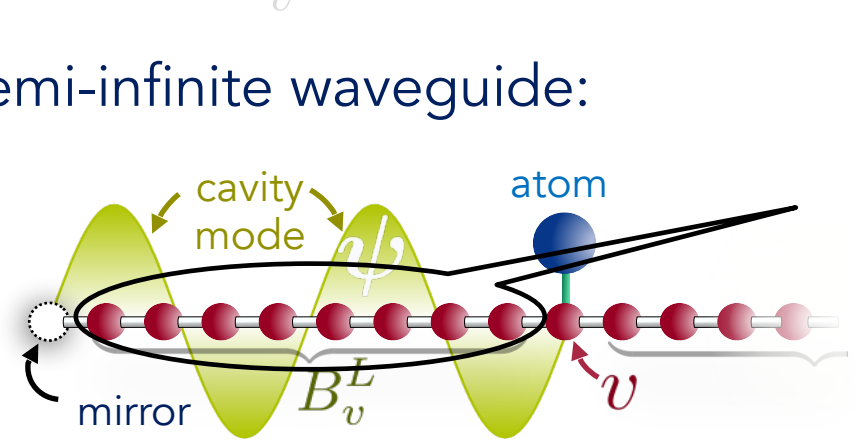
$$\omega_1 = \omega_0 \Rightarrow |\Psi\rangle \propto -J|e\rangle + g|1\rangle$$

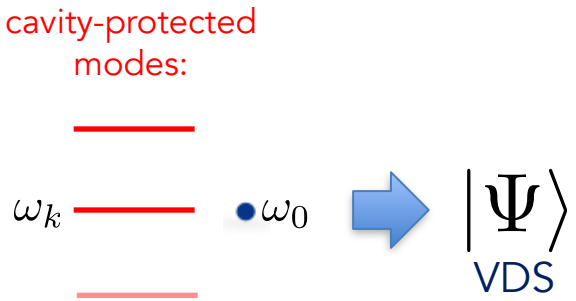
(non-topological) instances of VDS

2 cavities:

$\omega_1 = \omega_0 \Rightarrow |\Psi\rangle \propto -J|e\rangle + g|1\rangle$

semi-infinite waveguide:





dressed BICs:

T. Tufarelli, F. Ciccarello, and M. S. Kim, PRA 87, 013820 (2013)
C. Gonzalez-Ballestero, F. J. Garcia-Vidal, and E. Moreno, NJP 15, 073015 (2013)
G. Calajò, Y.-L. L. Fang, H. U. Baranger, and F. Ciccarello, PRL 122, 073601 (2019)
A. Feiguin, J. Jose Garcia-Ripoll, and A. Gonzalez-Tudela, PRR 2, 023082 (2020)

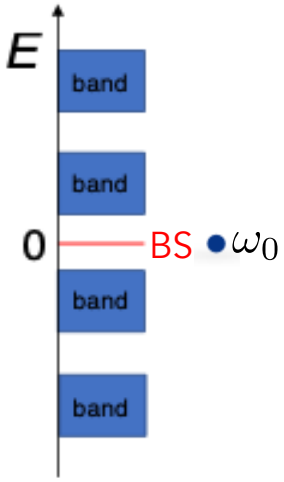
topological VDS

photonic lattice + 0-dim defect

class	$\mathcal{T}$	$\mathcal{C}$	$\mathcal{S}$	topol invariant $\mathcal{I}$
A	0	0	0	0
AIII	0	0	1	$\mathbb{Z}$
AI	+1	0	0	0
BDI	+1	+1	1	$\mathbb{Z}$
D	0	+1	0	$\mathbb{Z}_2$
DIII	−1	+1	1	$\mathbb{Z}_2$
AII	−1	0	0	0
CII	−1	−1	1	$2\mathbb{Z}$
C	0	−1	0	0
CI	+1	−1	1	0

C.-K. Chiu, J. C. Y. Teo, A. P. Schnyder, and S. Ryu, RMP 88, 035005 (2016)

$\mathcal{I} \neq 0$



topologically protected VDS

many atoms & weak coupling:

$$H_{\text{eff}} = \sum_{\nu\nu'} K_{\nu'\nu}\sigma_{\nu+}\sigma_{\nu'-} + \text{H.c.}$$

atom-atom couplings:

$$K_{\nu'\nu} = -\frac{g^2}{2\langle\nu'|H_B|\psi^\nu\rangle}\,\psi^\nu_{\nu'}$$

theorem

topologically-protected dressed state ⇒ VDS

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

search of vacancy-induced BS

theorem for 1D lattices

R·d edge states exist under open BCs ⇒ 1 vacancy-induced BS always exists

R: interaction range, d: number of bands

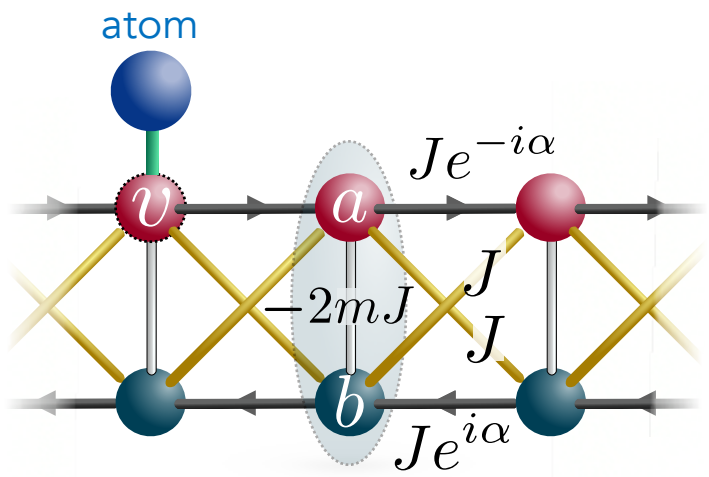
using methods in:

A. Alase, E. Cobanera, G. Ortiz, and L. Viola, PRB 96, 195133 (2017)

E. Cobanera, A. Alase, G. Ortiz, and L. Viola, PRB 98, 245423 (2018)

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

Creutz-ladder photonic lattice



vacancy-induced BS: $\psi_{a_n} = \frac{1}{2}\sqrt{1-m^2}\left(e^{i\alpha}m^{n-2}+e^{-i\alpha}m^{N-n}\right)$

atom-atom couplings: $K^{(aa)}_{n,n'} = \frac{g^2}{2m}\, e^{i\alpha}m^{n-n'}$

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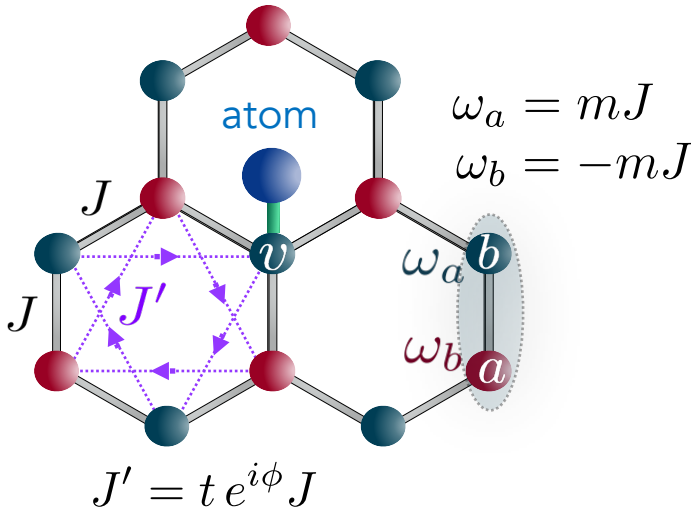
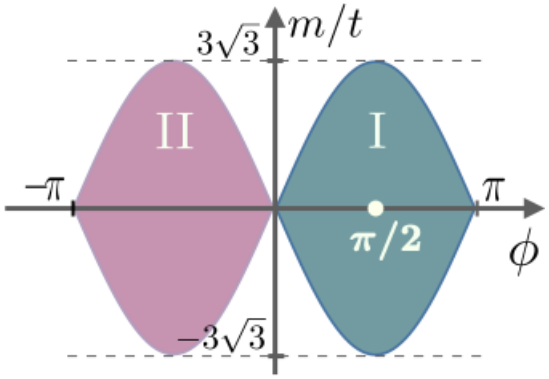
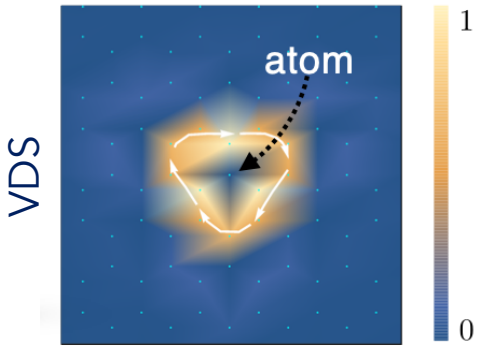
Haldane photonic lattice





parameters space



atom dressed by
a photon orbiting around it

L. Leonforte, A. Carollo and F. Ciccarello, PRL 126, 063601 (2021)

Conclusions

- identified a class of dressed states (VDS)
- mirror-like behavior of atoms in waveguide QED
- VDS : key role in topological quantum optics

outlook

- many-photon VDS ?
  (see arXiv:2105.08833 by Ashida, Imamoglu & Demler)

teammates

Luca Leonforte

Università degli Studi di Palermo

Francesco Ciccarello

Università degli Studi di Palermo

CNR NANO PISA NEST

Part II: Exotic interaction mediated by a non-Hermitian photonic bath

introduction &
motivations

non-Hermitian Hamiltonian

$$H^{\dagger} \neq H$$

Physical Context

- Approximate **Dissipative** Quantum systems
- Optical systems
- Mechanical oscillators

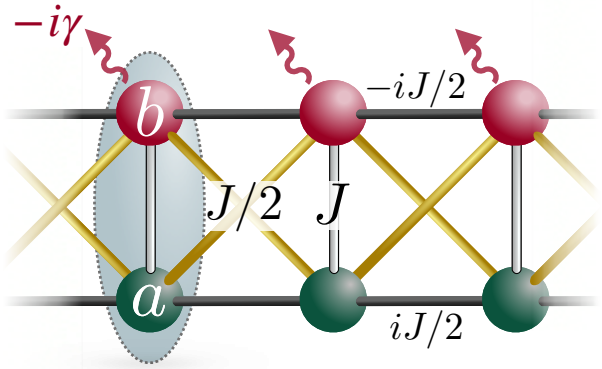
Exotic NH Properties

- Complex Energies
- Non-Orthogonal Eigenstates
- Coalescence of eigenstates (Exceptional points)
- Extreme sensitivity to BC (NH Skin effect)
- Exotic topological properties

Bender, C. M., "Making sense of non-Hermitian Hamilto- nians," Rep. Prog. Phys. 70, 947–1018 (2007)

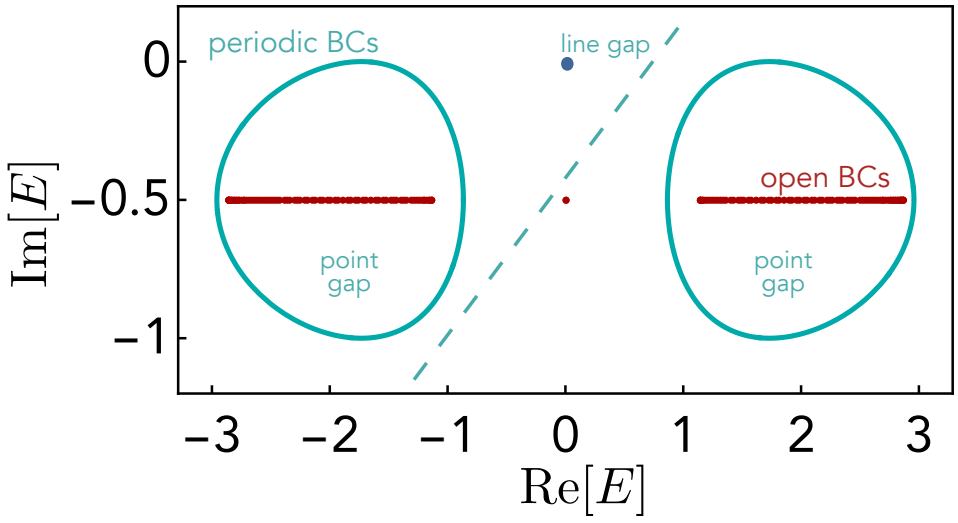
Bergholtz, E. J., et al. Exceptional topology of non-Hermitian systems. Rev. Mod. Phys. 93, 015005 (2021).

Setup: Lee Photonics Lattice



$$H_B^\dagger \neq H_B$$ (due to Dissipation)

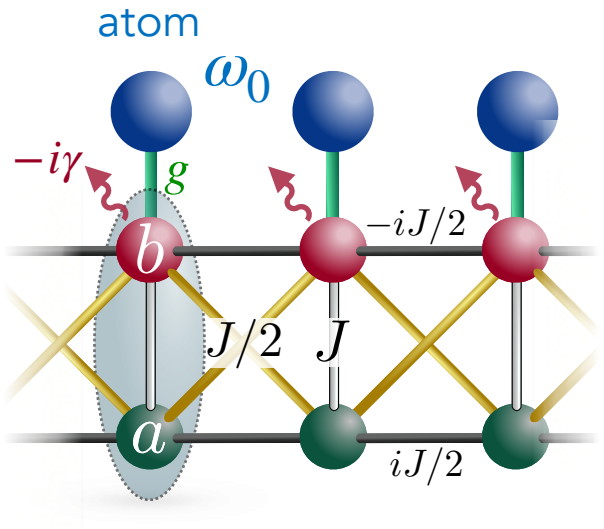
Photonic Lattice Band



$(\gamma = J)$

Lee, T. E. Anomalous Edge State in a Non-Hermitian Lattice. Phys. Rev. Lett. 116, 133903 (2016).

Setup: Lee Photonics Lattice

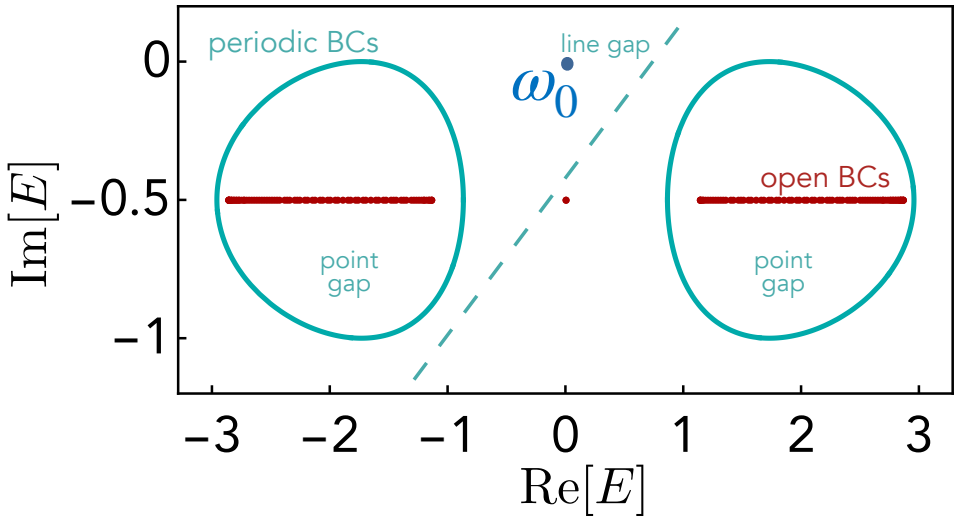


$H_B^\dagger \neq H_B$ (due to Dissipation)

$$H = H_B + \omega_0 \sum_{i\ \text{atoms}} \sigma_+^i \sigma_-^i + g(\sigma_+ b_{n_i} + \sigma_- b_{n_i}^\dagger)$$

RWA coupling

Photonic Lattice Band



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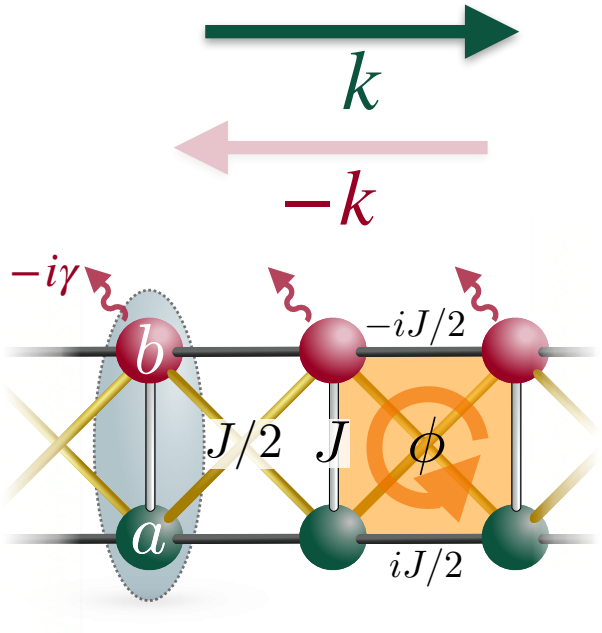
Lattice non-reciprocity

Photon propagate preferably leftwards



Necessary ingredients:

- (Structured) Dissipation
- Effective Magnetic field
  (Complex coupling)

Lattice non-reciprocity

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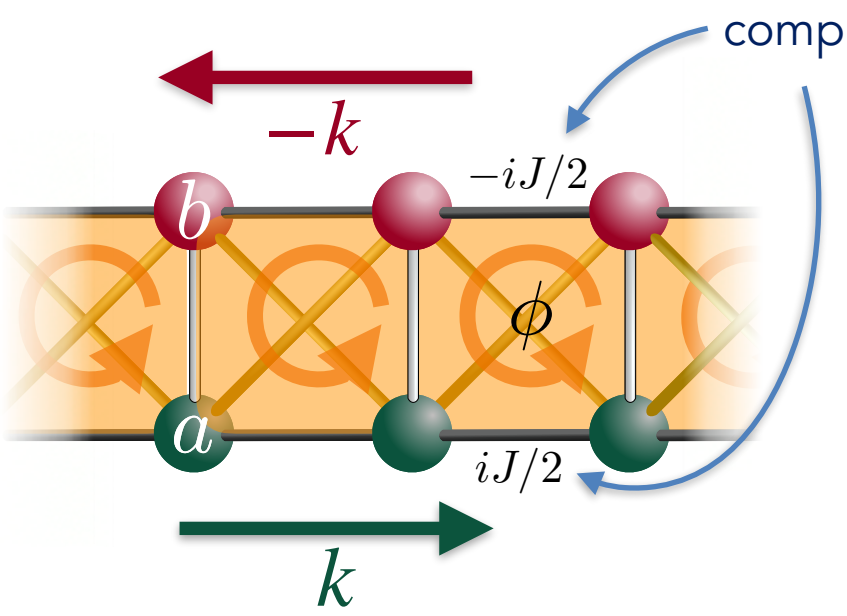
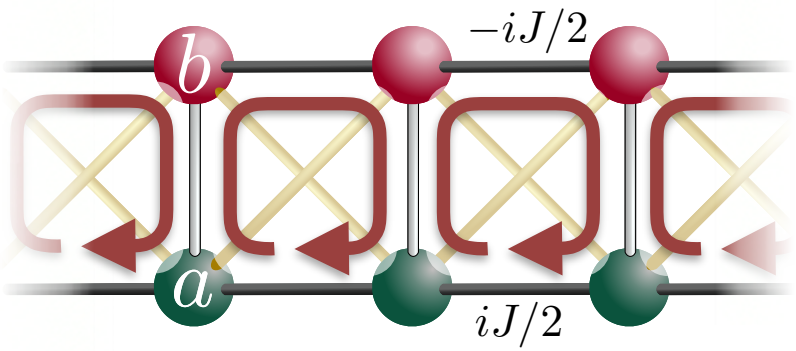


Peierls substitution

Effective Magnetic field per plaquette

Lattice non-reciprocity

Photon propagate preferably leftwards

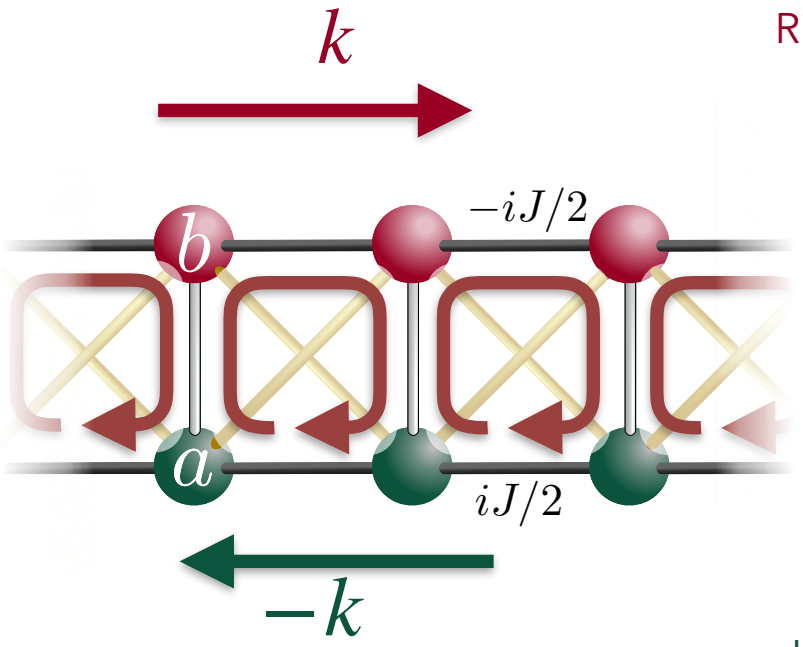


Effective Magnetic field per plaquette

Photons propagate counterclockwise

Lattice non-reciprocity

Photon propagate preferably leftwards

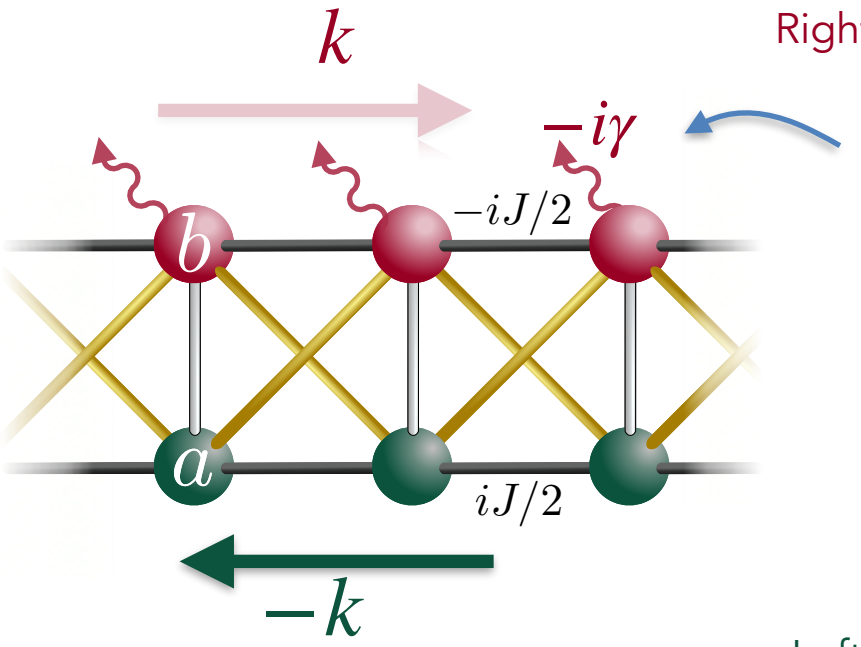


Right propagating photons in **b**

Left propagating photons in **a**

Lattice non-reciprocity

Photon propagate preferably leftwards

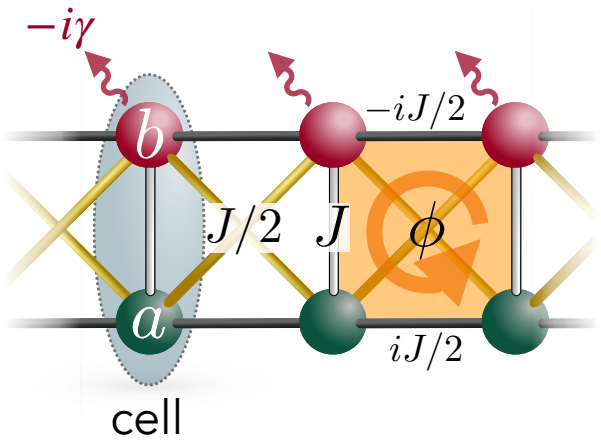


Right propagating photons in **b**

Adding Dissipation in **b** dissipate rightward photons

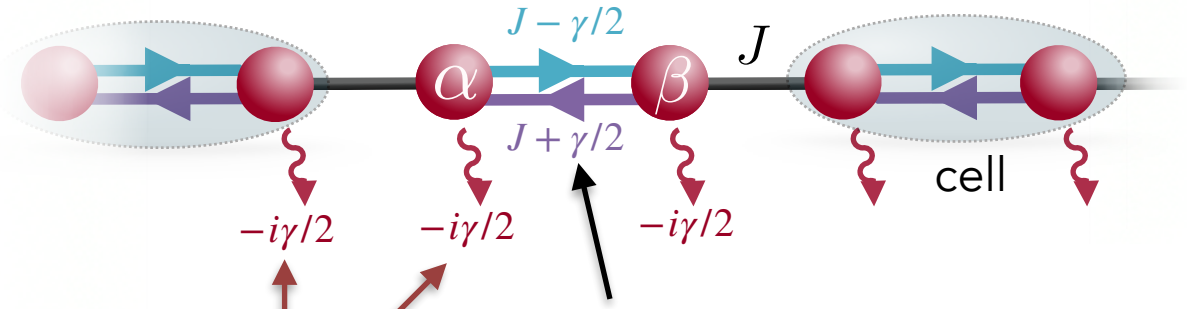
Left propagating photons in **a**

Unitary Cell Mapping



$$U : \begin{pmatrix} a_n \\ b_n \end{pmatrix} \rightarrow \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix}$$

Non-Hermitian SSH



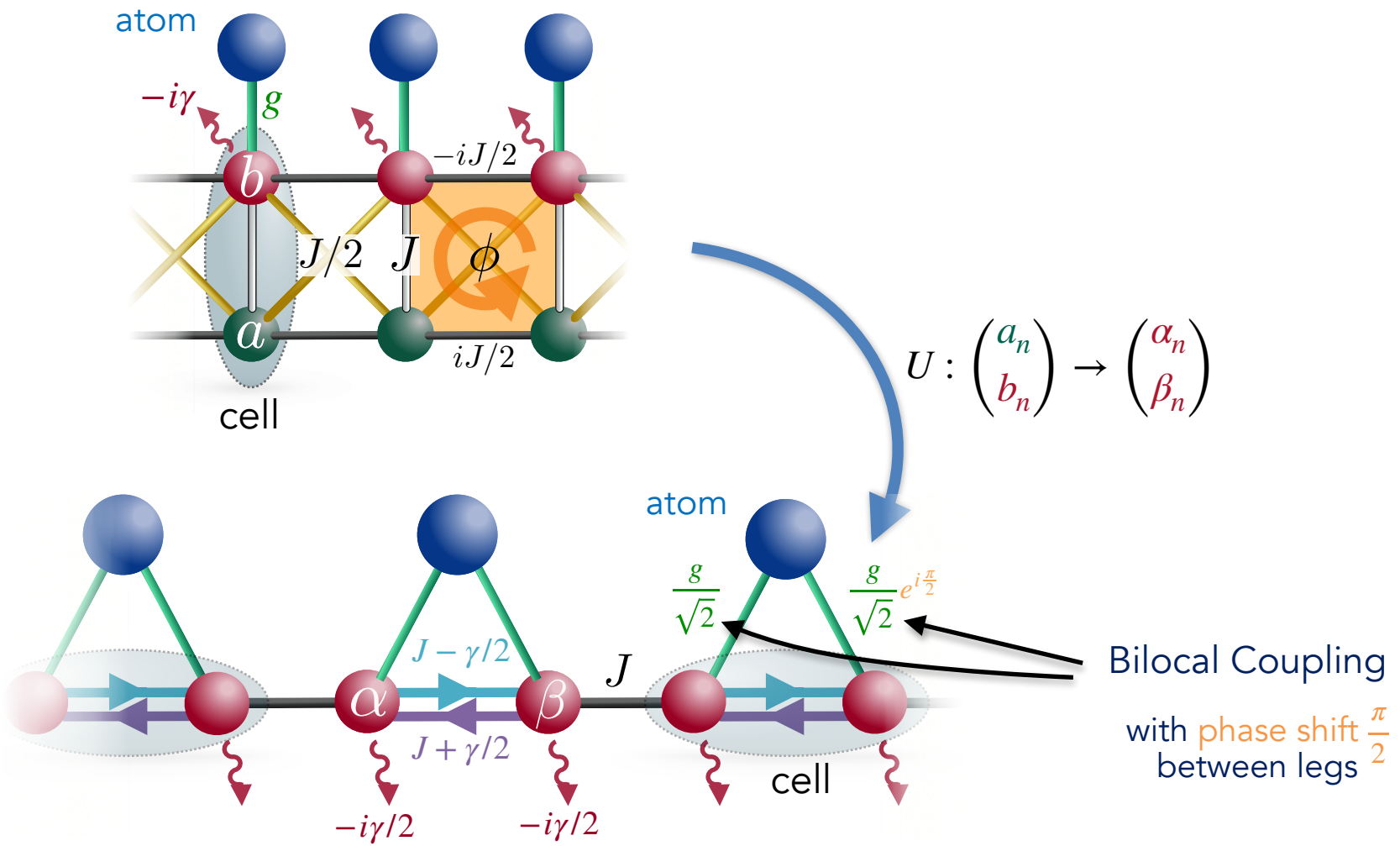
$$b_n = -\frac{i}{\sqrt{2}}(\alpha_n + i\beta_n)$$

$$a_n = \frac{1}{\sqrt{2}}(\alpha_n - i\beta_n)$$

Uniform losses

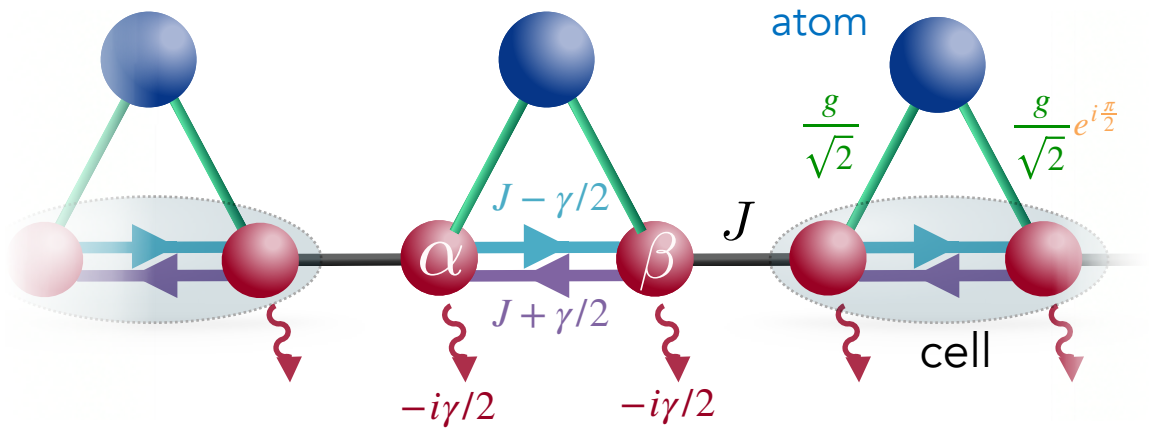
Non Reciprocity (asymmetric hopping)

...adding interacting atoms



Features

1) **Uniform losses**
2) **Non-Reciprocity** (asymmetric hopping)
3) **Bilocal Coupling** with phase shift $\frac{\pi}{2}$ between legs



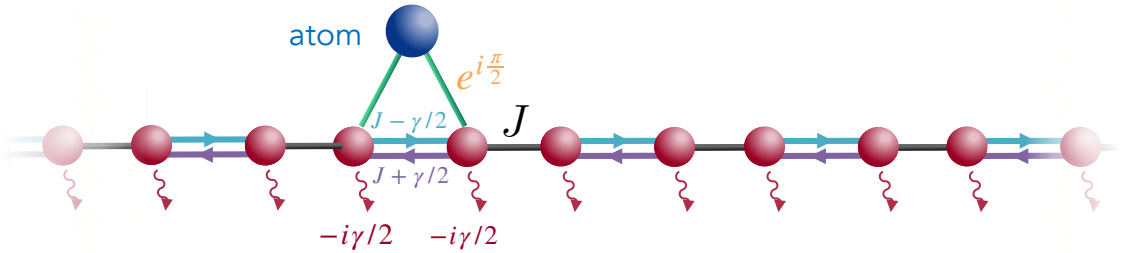
Spontaneous emission

Initial state

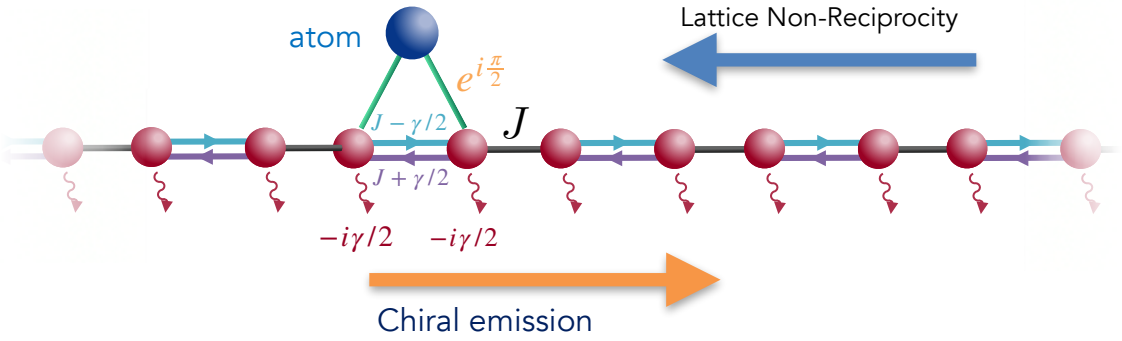
$$|\Psi_0\rangle = |e\rangle|\text{vac}\rangle$$

$$|\Psi_t\rangle = e^{-iHt}|\Psi_0\rangle$$

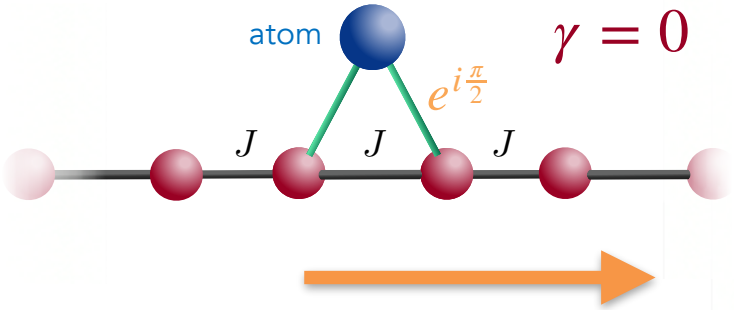
$$p_e(t) = |\langle e|\Psi_t\rangle|^2 = e^{-\Gamma t}$$



Spontaneous emission

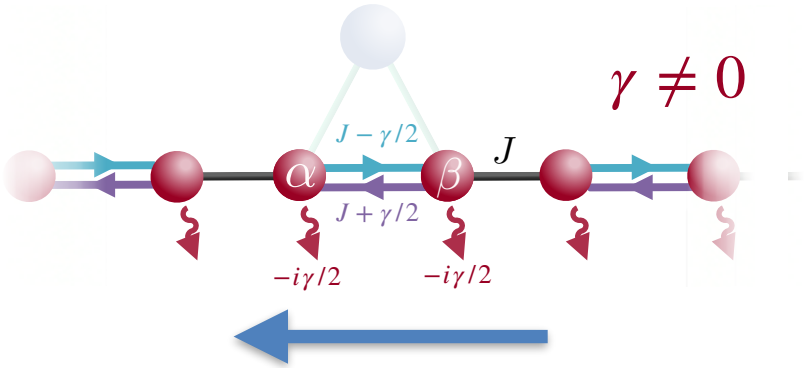


Two competing chiralities



Bilocal Coupling with phase shift $\frac{\pi}{2}$

Rightward chiral emission

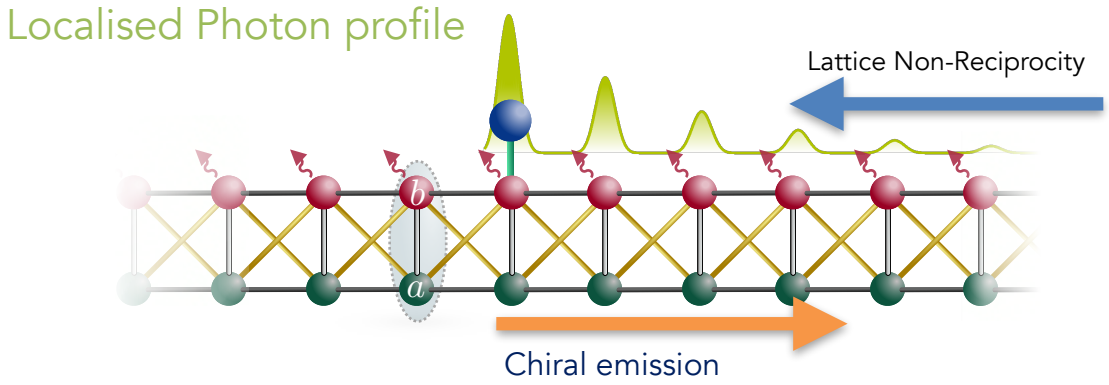


Lattice Non-Reciprocity

Leftwards propagation of photons

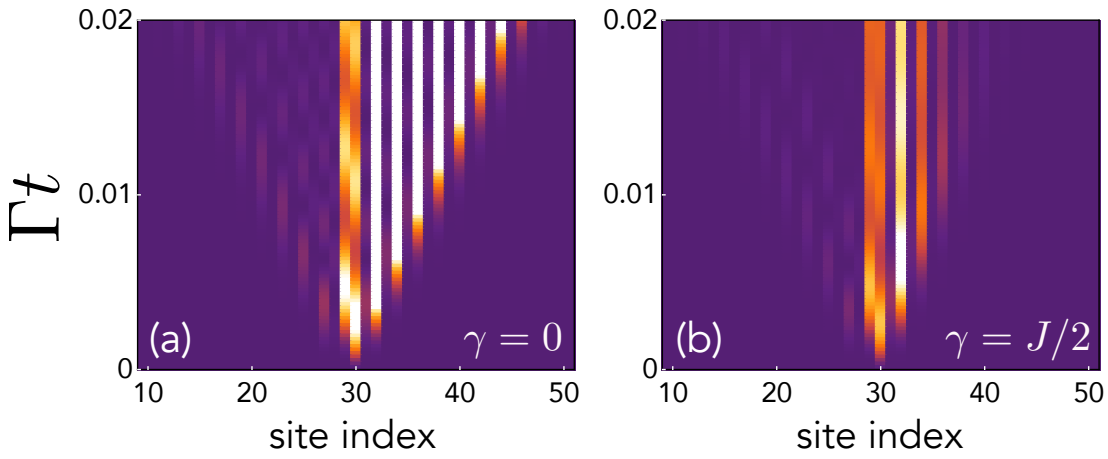
T.Ramos, et al. Phys. Rev. A 93, 062104 (2016).

Photon Localisation

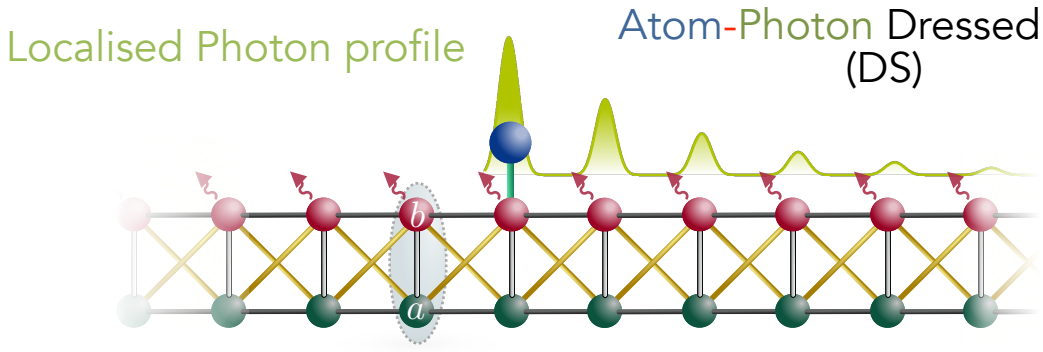


Time evolution of photon profile

$$p_{n,i}(t) = |\langle n, i | \Psi_t \rangle|^2 \qquad i = a, b$$



Photon Localisation



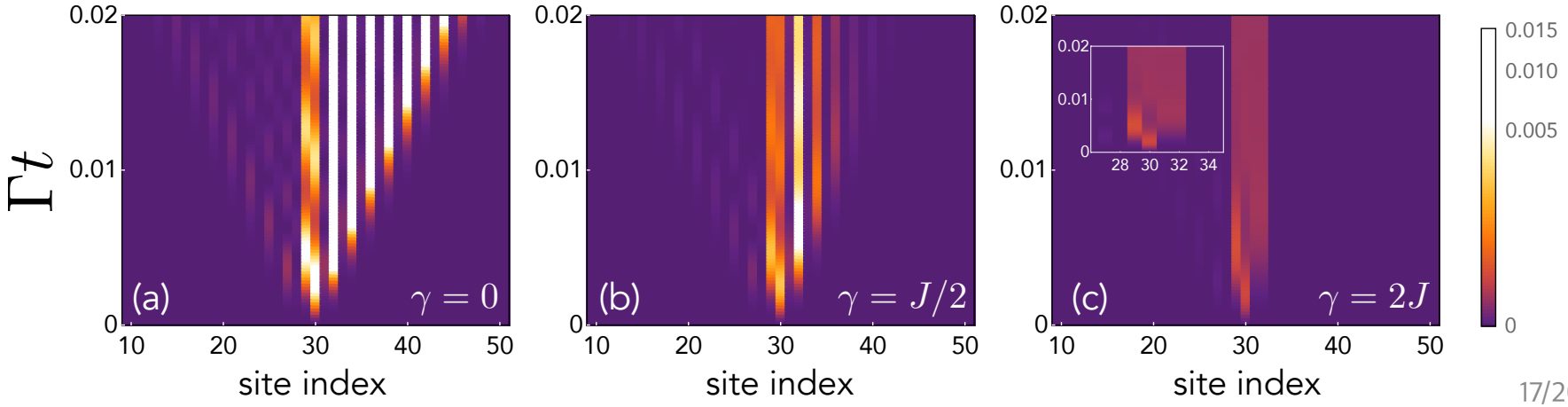
Long lifetime

$\Gamma = g^2/(4J) \gg \gamma$

Time evolution of photon profile

$p_{n,i}(t) = |\langle n, i | \Psi_t \rangle|^2$ $\quad i = a, b$

Maximal Localisation $\gamma = 2J$



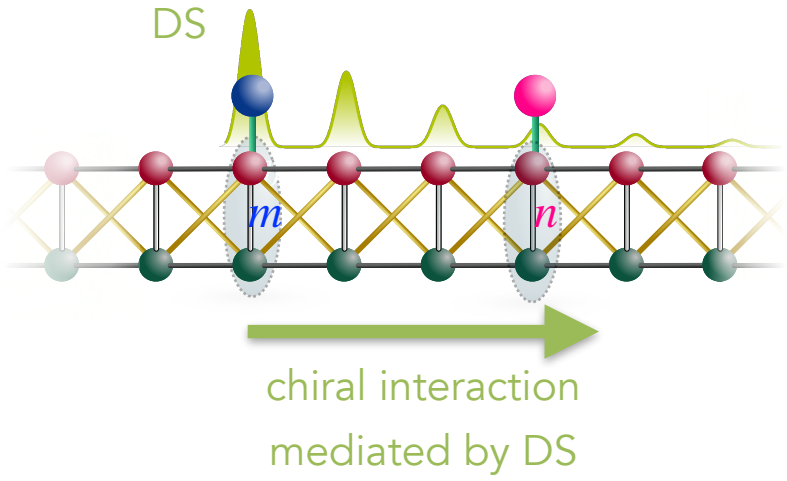
Many atoms Effective Hamiltonian

many atoms & weak coupling:

$$H_{\text{eff}} = \sum_{ij} K_{n_i n_j} \sigma_i^- \sigma_j^+$$

chiral atom-atom couplings:

$$K_{n>m} = 4g^2 J\, e^{-(n-m)/\lambda}$$



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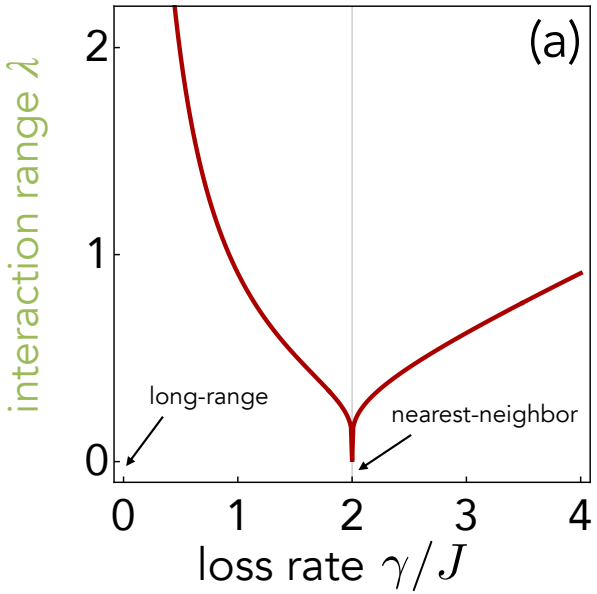
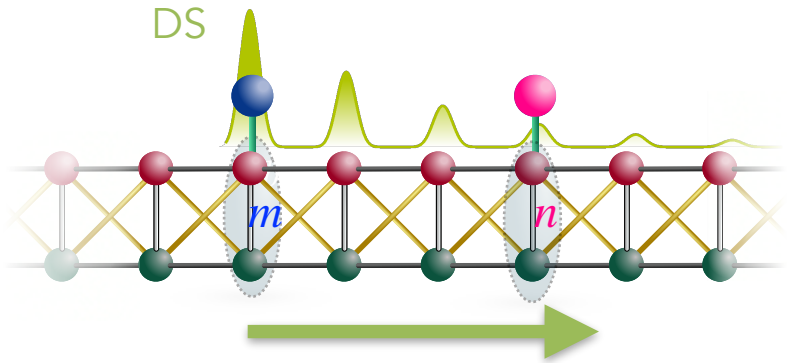




chiral interaction
mediated by DS

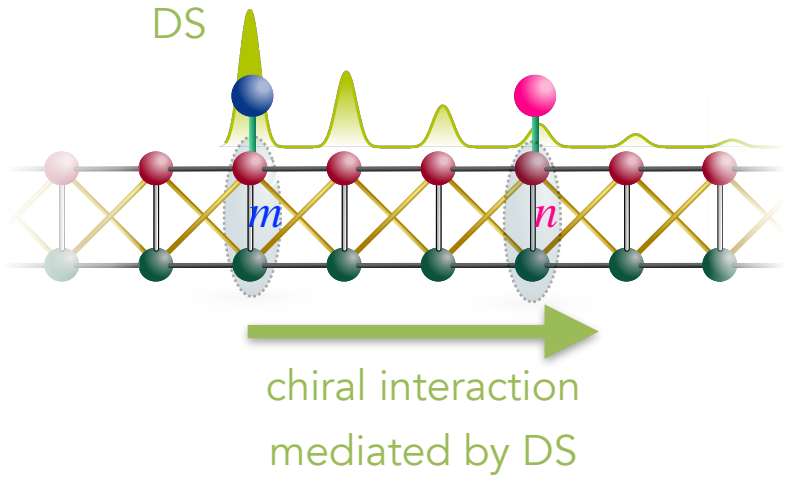
Many atoms Effective Hamiltonian

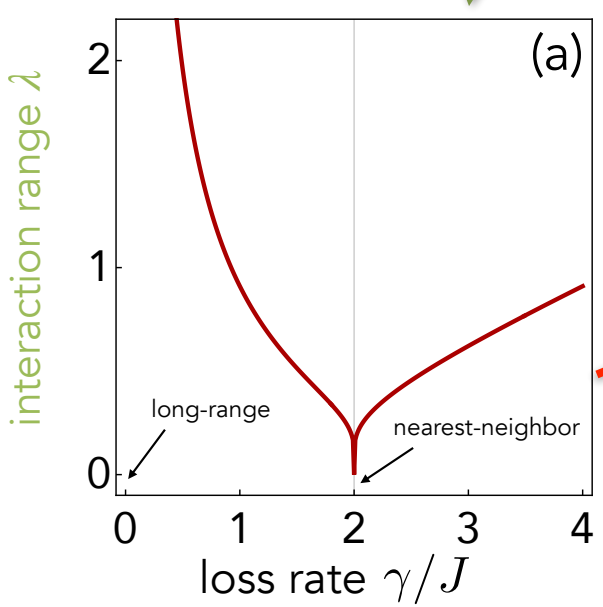
many atoms & weak coupling:

$$H_{\text{eff}} = \sum_{ij} K_{n_i n_j} \sigma_i^- \sigma_j^+$$

chiral atom-atom couplings:

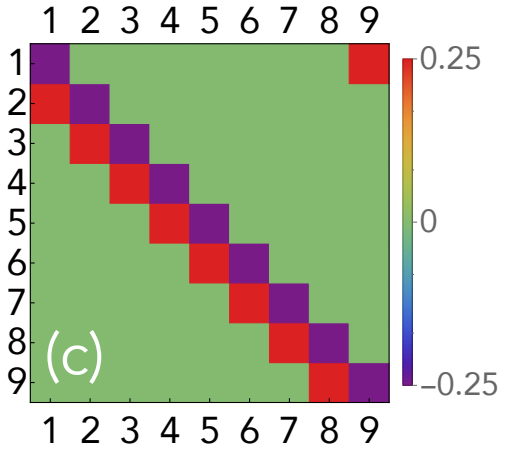
$$K_{n>m} = 4g^2 J\, e^{-(n-m)/\lambda}$$





For $\gamma = 2J$ nearest-neighbour

K_{nm}



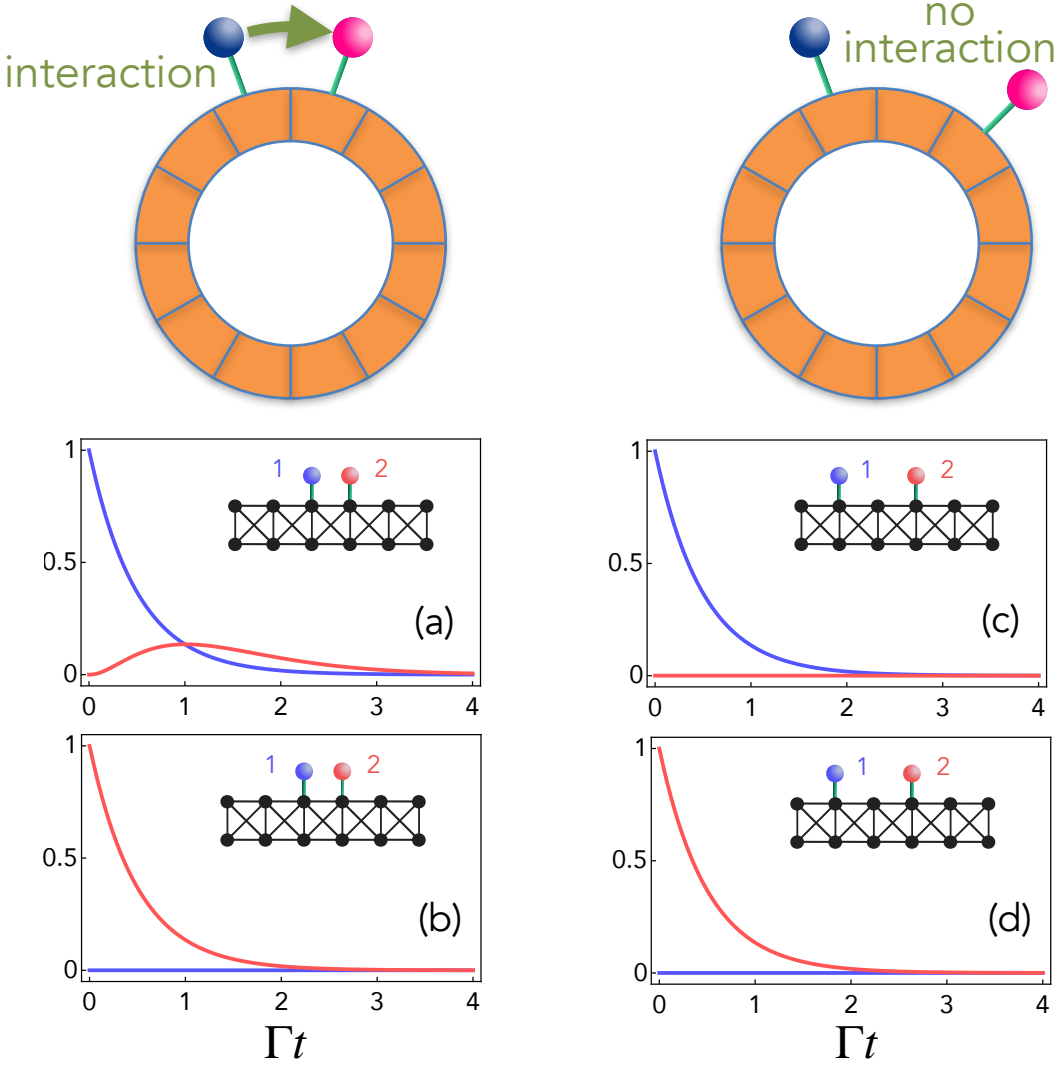
Insensitivity to Boundary Conditions

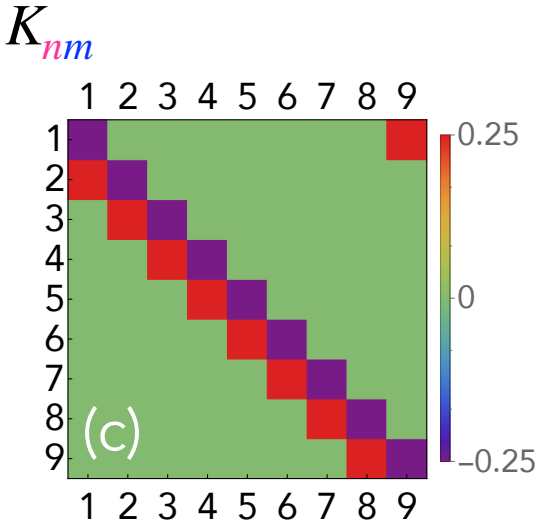
nearest-neighbour

non nearest-neighbour

For $\gamma = 2J$

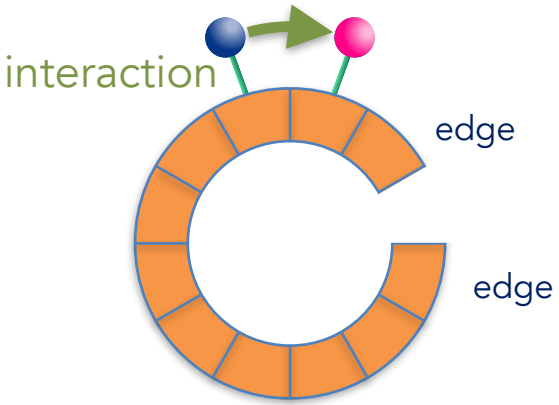
Periodic Boundaries



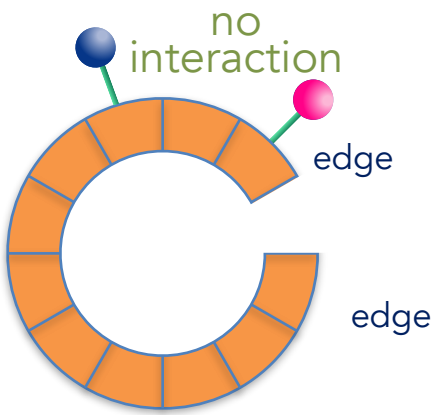


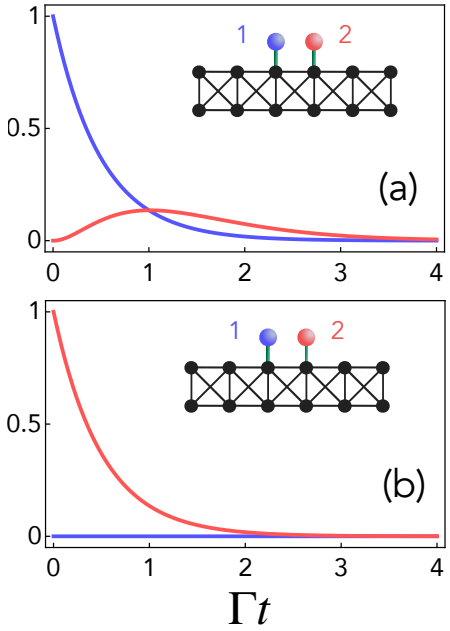
Insensitivity to Boundary Conditions

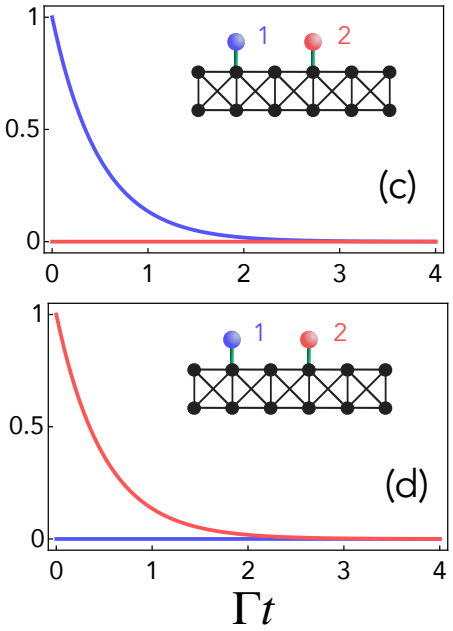
nearest-neighbour



non nearest-neighbour





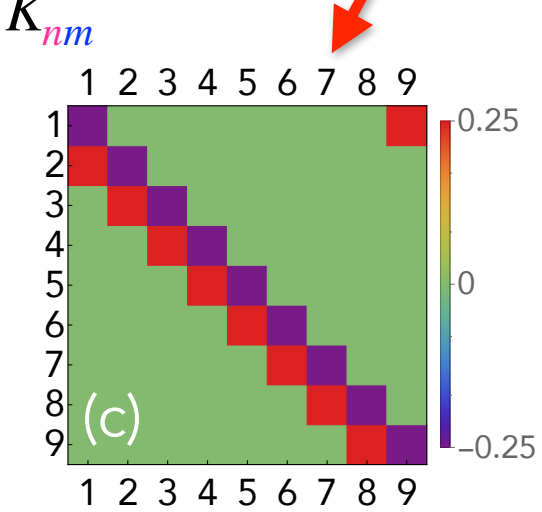


For $\gamma = 2J$

Periodic Boundaries

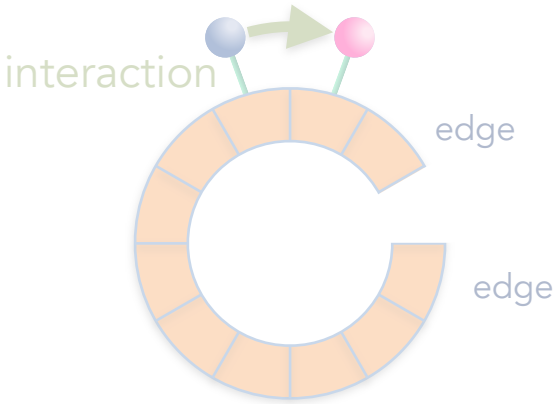
Same H_{eff}

Open Boundaries

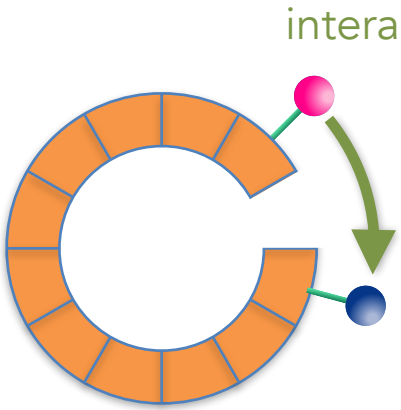


Insensitivity to Boundary Conditions

nearest-neighbour



across edge

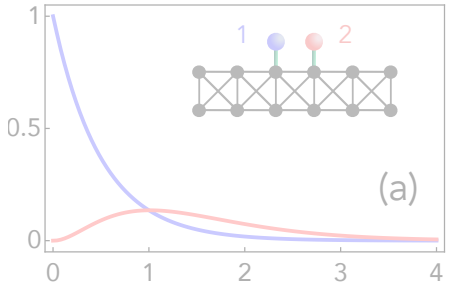


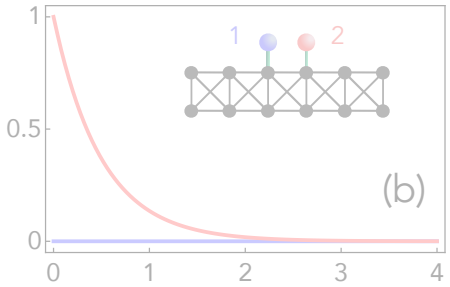
For $\gamma = 2J$

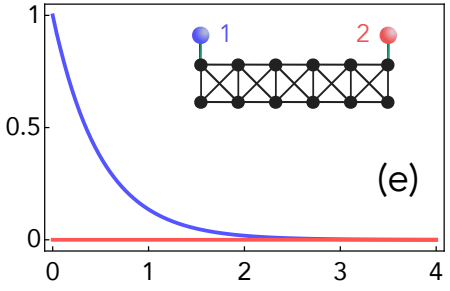
Periodic Boundaries

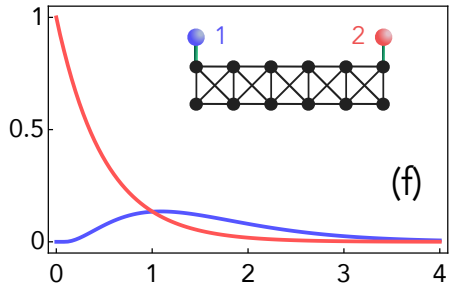
Same H_{eff}

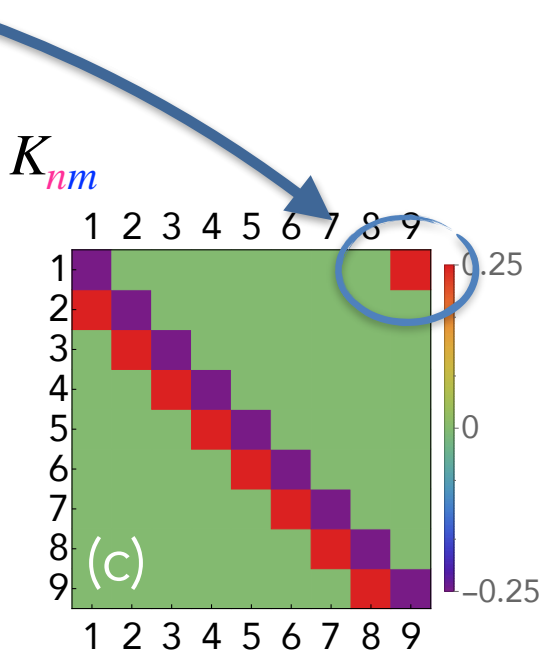
Open Boundaries







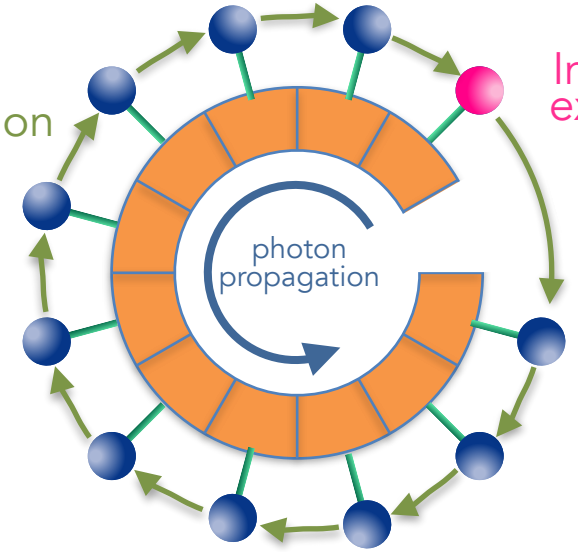


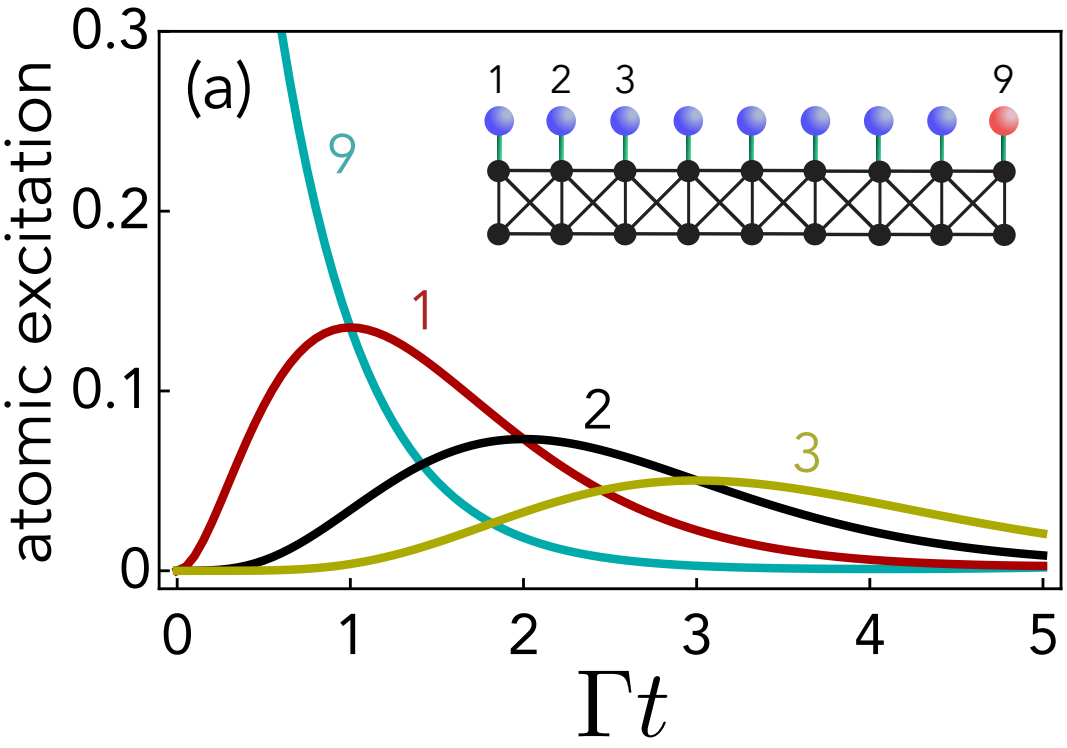


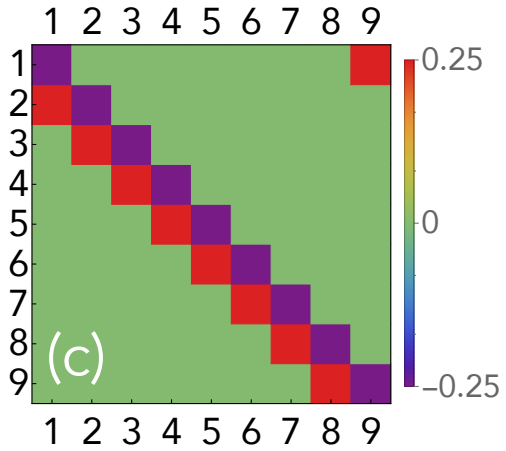
Insensitivity to Boundary Conditions

For $\gamma = 2J$

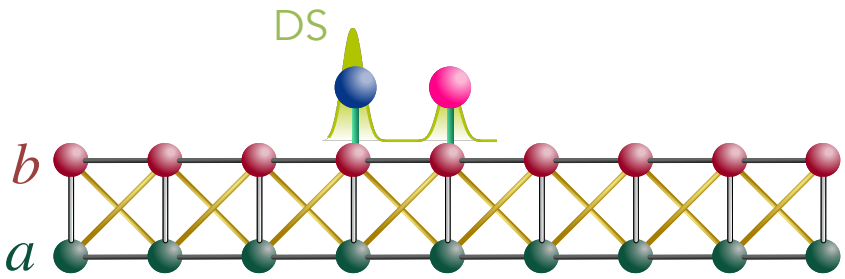
Many atoms



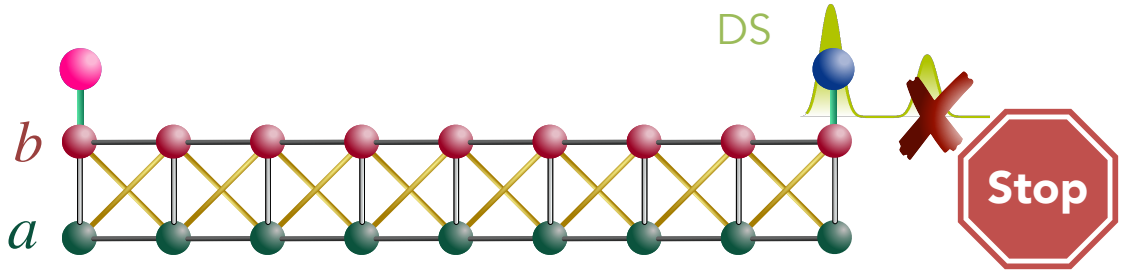




Explanation in terms of DS



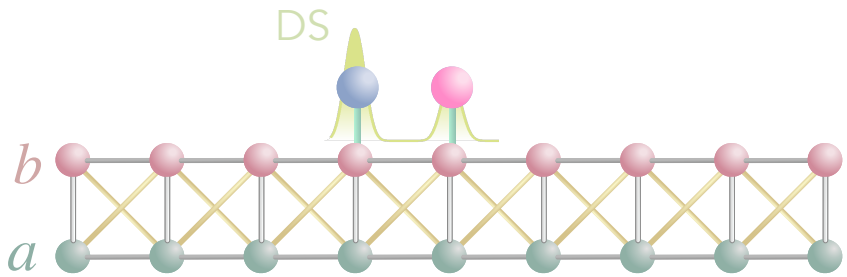
Interaction
in the Bulk



interaction
across the Edges

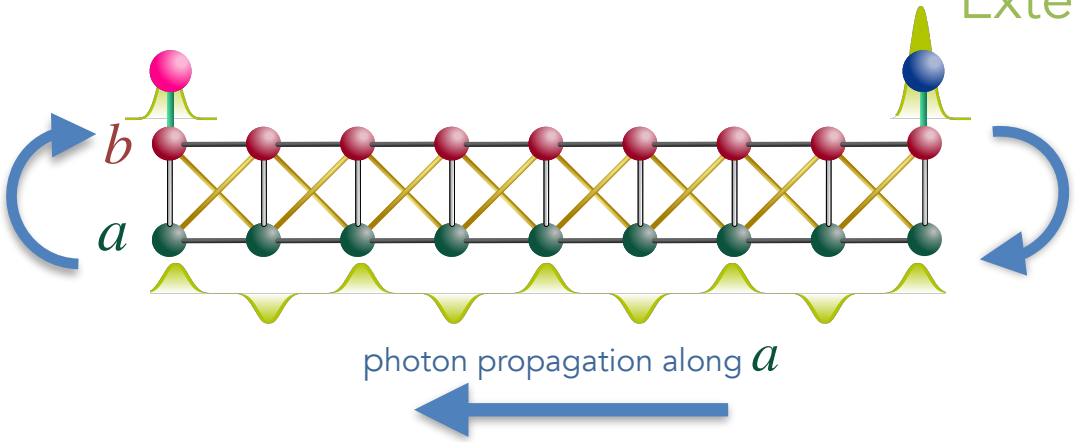
Breaking of Translational
Invariance

Explanation in terms of DS



Interaction
in the Bulk

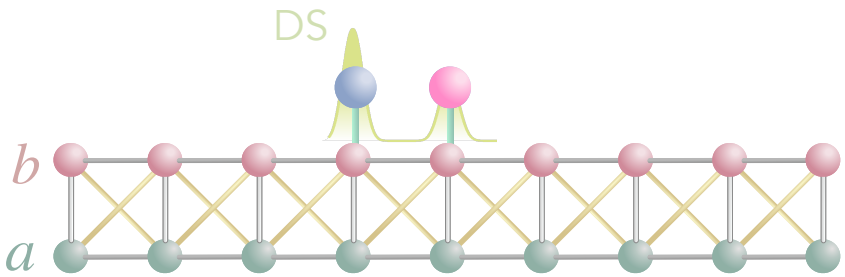
Extended DS



interaction
across the Edges

Neglect non Markovian effects (time delays…)

Explanation in terms of DS

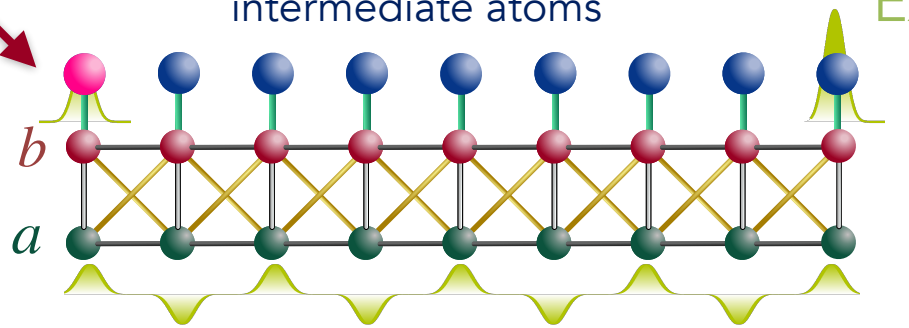


Interaction
in the Bulk

in b peaked
on the edge

uncoupled to the
intermediate atoms

Extended DS



interaction
across the Edges

Delocalised on sublattice a

Conclusions

Novel Non-Hermitian features due two competing chiralities:

1. Non-reciprocity due to patterned leakage
2. Chiral emission due to bilocal coupling

- Formation of Atom-Photon Dressed states
- Effective dipole-dipole Hamiltonian:
  - Chiral
  - Neighrest-neibouring (leakage-dependent range)
  - Insensitive to Boundary Conditions

Outlook

1. Application to non-Hermitian 2D lattices
2. Atom with multiple-local coupling/"Giant atoms"

teammates

Federico Roccati

Francesco Ciccarello

Salvatore Lorenzo

Giuseppe Calajo

Massimo G. Palma

Thanks for your attention!