107° CONGRESSO NAZIONALE
della SOCIETÀ ITALIANA DI FISICA

Quantum decay at short, intermediate and long times: Observation in integrated photonics

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in collaboration with
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Decay and quantum evolution

Exponential decay: classical statistical law on the behavior of the survival probability $p(t)$ of an unstable state/system

$$\frac{p(t+\Delta t)-p(t)}{p(t)}=-\gamma\Delta t \qquad (\Delta t \longrightarrow 0) \Longrightarrow \frac{p(t)}{p(t_0)}=e^{-\gamma(t-t_0)}$$

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Quantum decay: determined by Hamiltonian evolution

$$|\psi_0\rangle \Rightarrow |\psi(t)\rangle = \exp(-\mathrm{i}Ht)\,|\psi_0\rangle$$

SURVIVAL PROBABILITY

$$p(t)=|\langle\psi_0|\psi(t)\rangle|^2=\left|\langle\psi_0\middle|\mathrm{e}^{-\mathrm{i}Ht}\middle|\psi_0\rangle\right|^2\xrightarrow{t\to\infty}0$$

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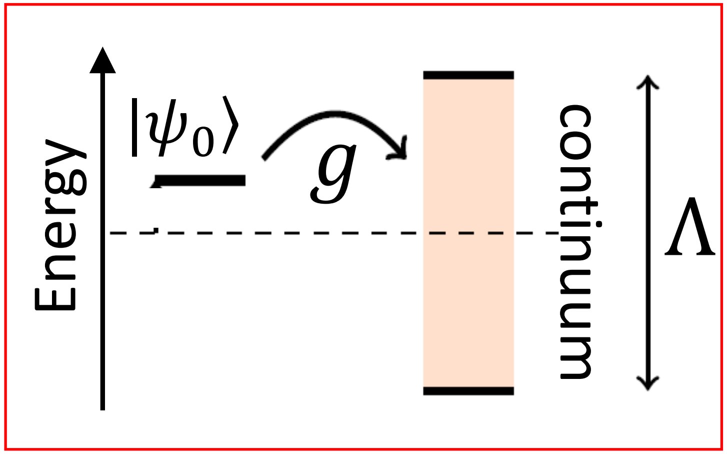
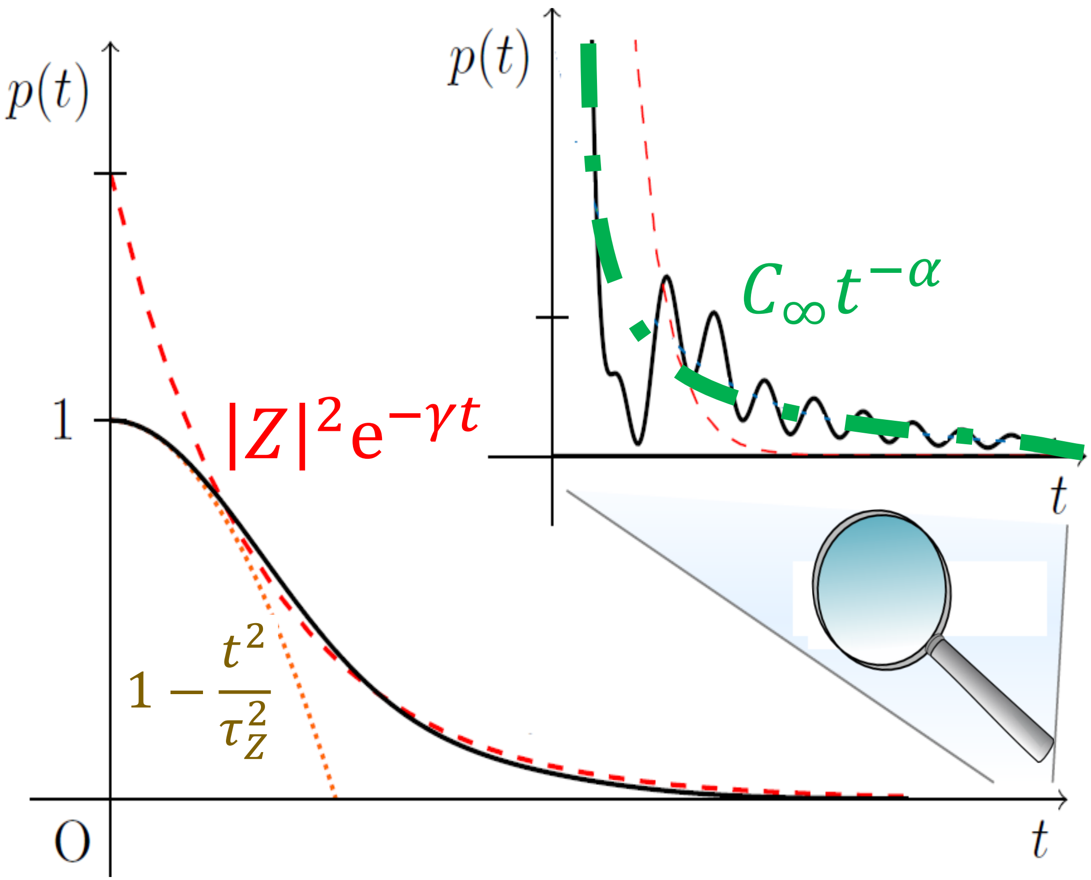


is quantum decay always well described by an exponential law?

Quantum decay: general results

$\boldsymbol{g}$ coupling constant, $\boldsymbol{\Lambda}$ continuum bandwidth



- **SHORT TIMES** $(\boldsymbol{t} \lesssim \boldsymbol{\Lambda}^{-1})$: quadratic survival probability (Zeno region), observed in different experiments
- **LONG TIMES** $(\boldsymbol{t} \sim \boldsymbol{g}^{-2} \log \boldsymbol{g})$: power-law tail, very elusive in isolated quantum systems (but see Rothe *et al*. PRL 2006)
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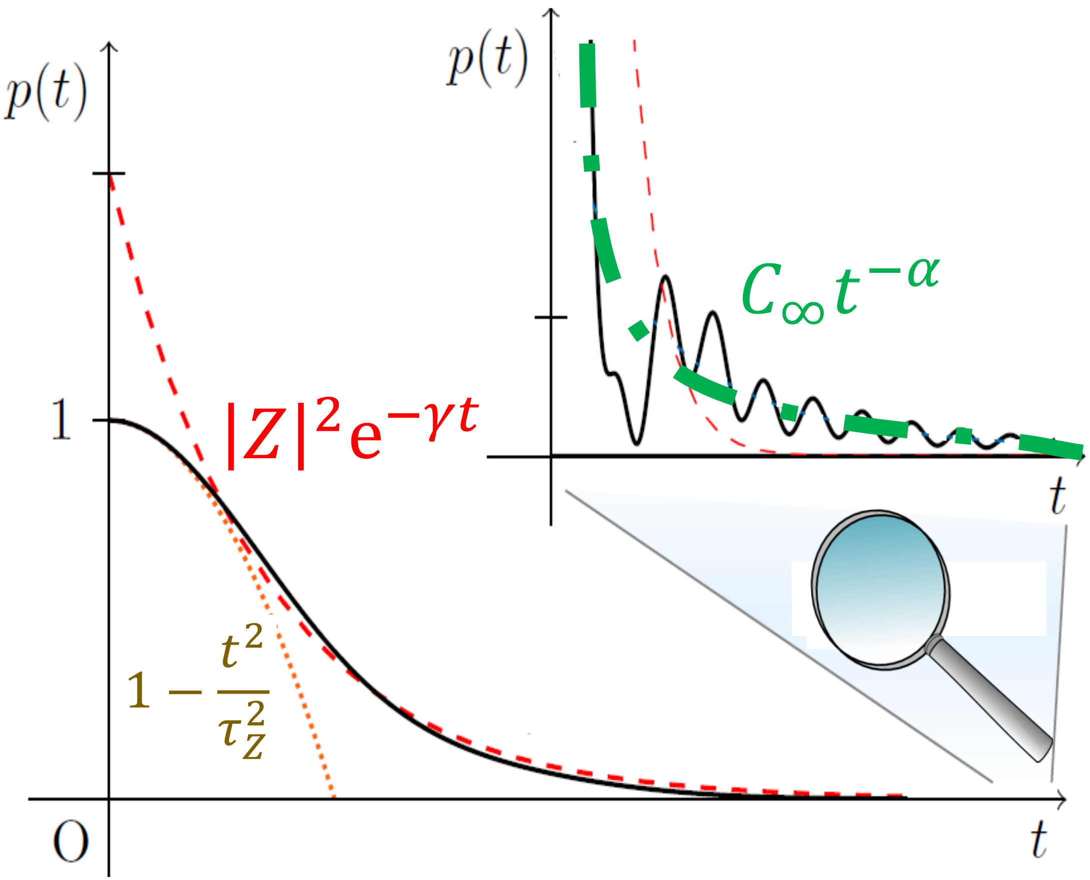

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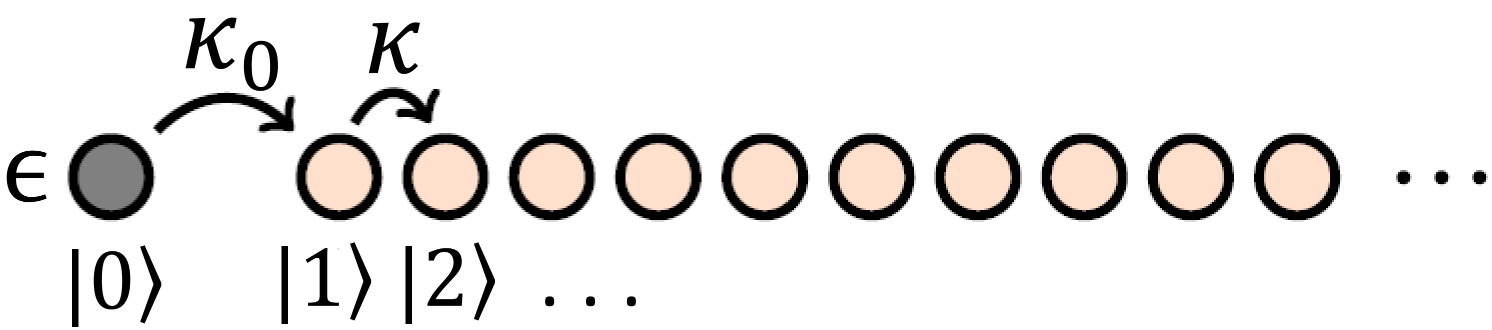
2P → 1S electric dipole transition in Hydrogen (**natural system, weak coupling**)

$\tau = \gamma^{-1} = 1.6 \times 10^{-9}$ s $\qquad t_{\text{quadratic}} \sim \Lambda^{-1} \sim 10^{-19}$ s $\qquad t_{\text{power}} \sim 100\,\tau$

One-dimensional hopping model (nearest-neighbor)

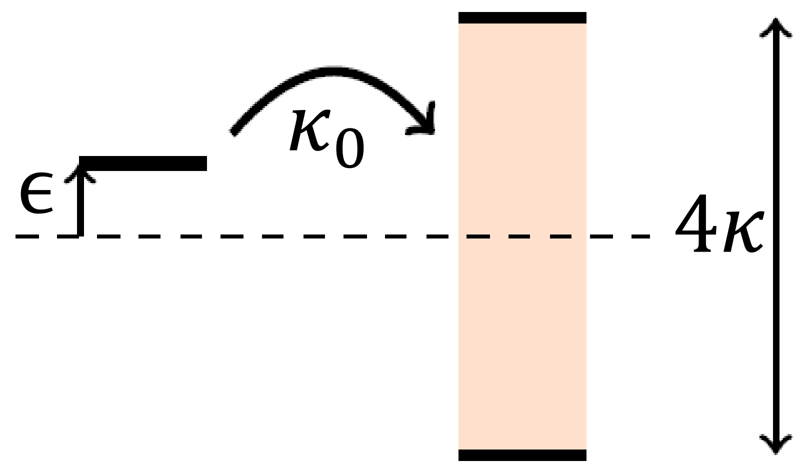


$$H = H_0 + H_1 + H_{\text{int}}$$

$$H_0 = \epsilon|0\rangle\langle 0|$$

$$H_1 = \kappa \sum_{n=1}^{\infty} |n\rangle\langle n+1| + \text{H.c.}$$

$$H_{\text{int}} = \kappa_0(|0\rangle\langle 1| + |1\rangle\langle 0|)$$



$$\chi = \frac{\kappa_0^2}{2\kappa^2}$$

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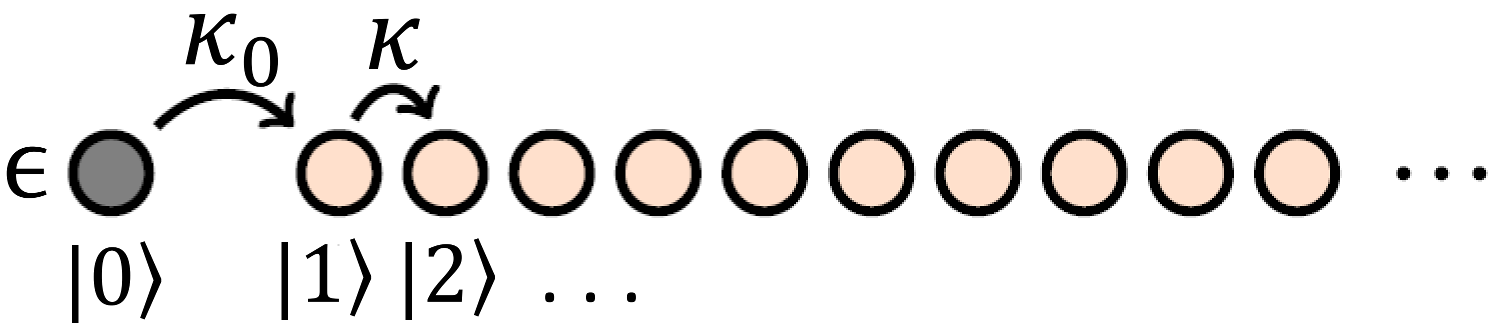
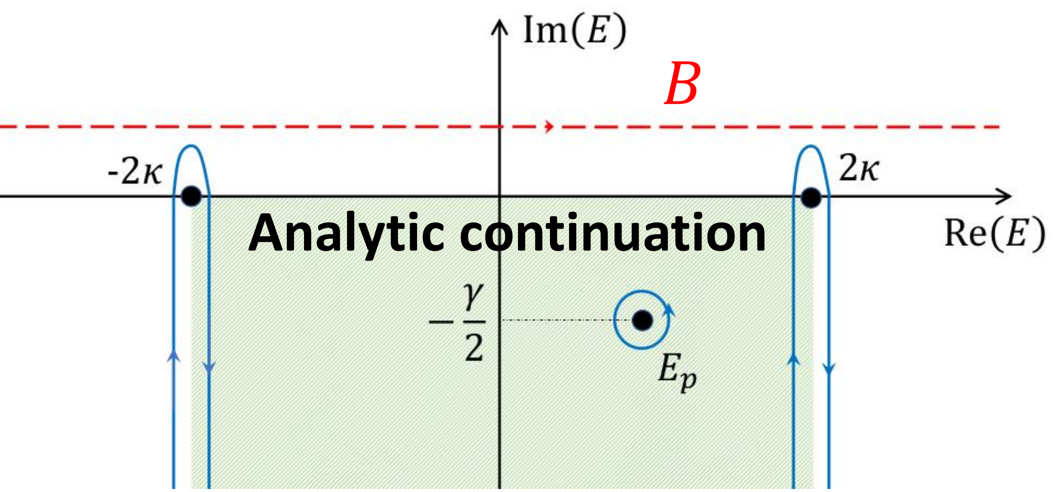


Initial state: $|\psi_0\rangle = |0\rangle$ **SURVIVAL PROBABILITY**

$$p(t) = \left|\frac{1}{2\pi}\int_B dE\, e^{-iEt}\langle 0|(E-H)^{-1}|0\rangle\right|^2 = \left|Ze^{-i\epsilon_p t-\gamma t/2} + \tilde{A}(t)\right|^2$$



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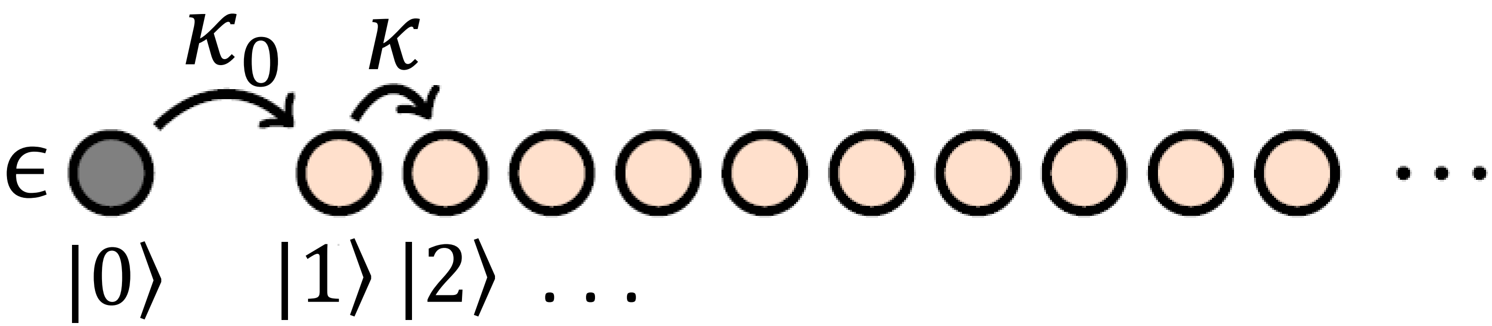

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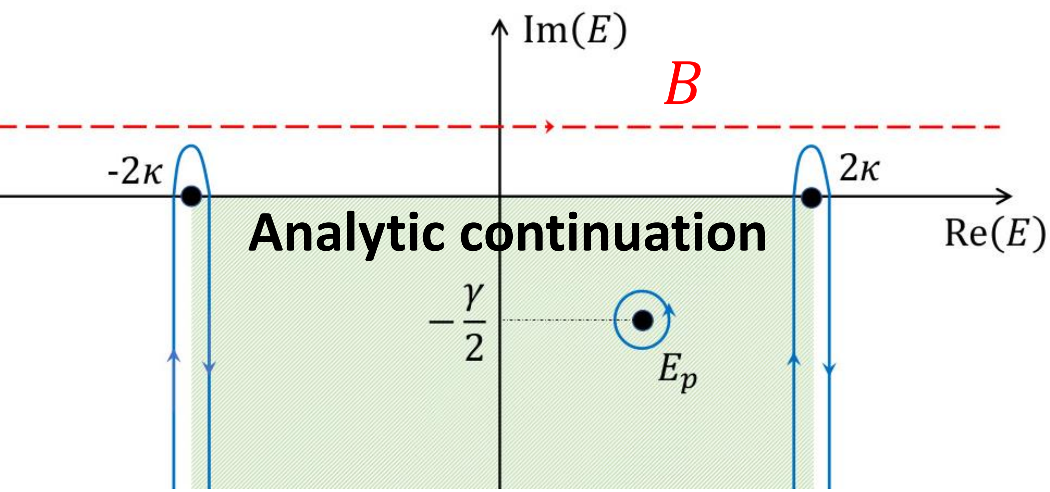
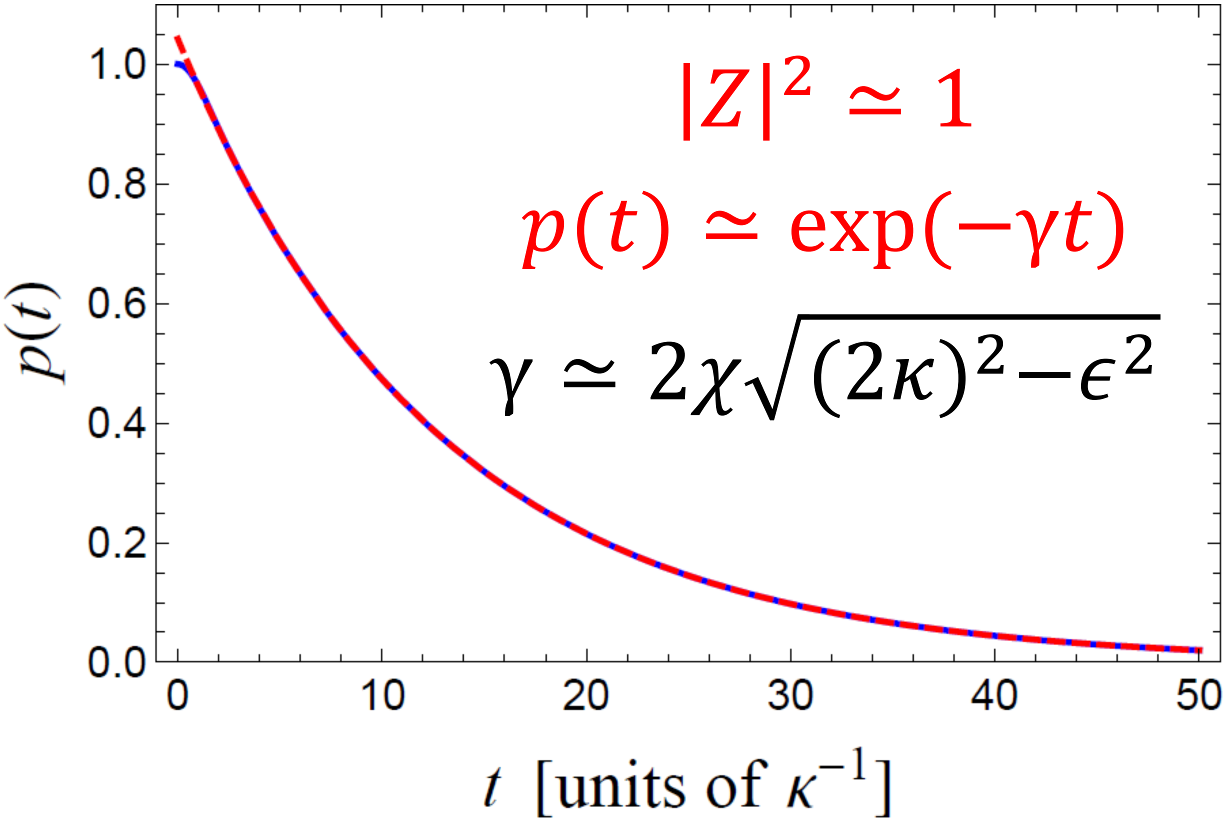


Exponential
(pole contribution)

Deviations
(cut contribution)

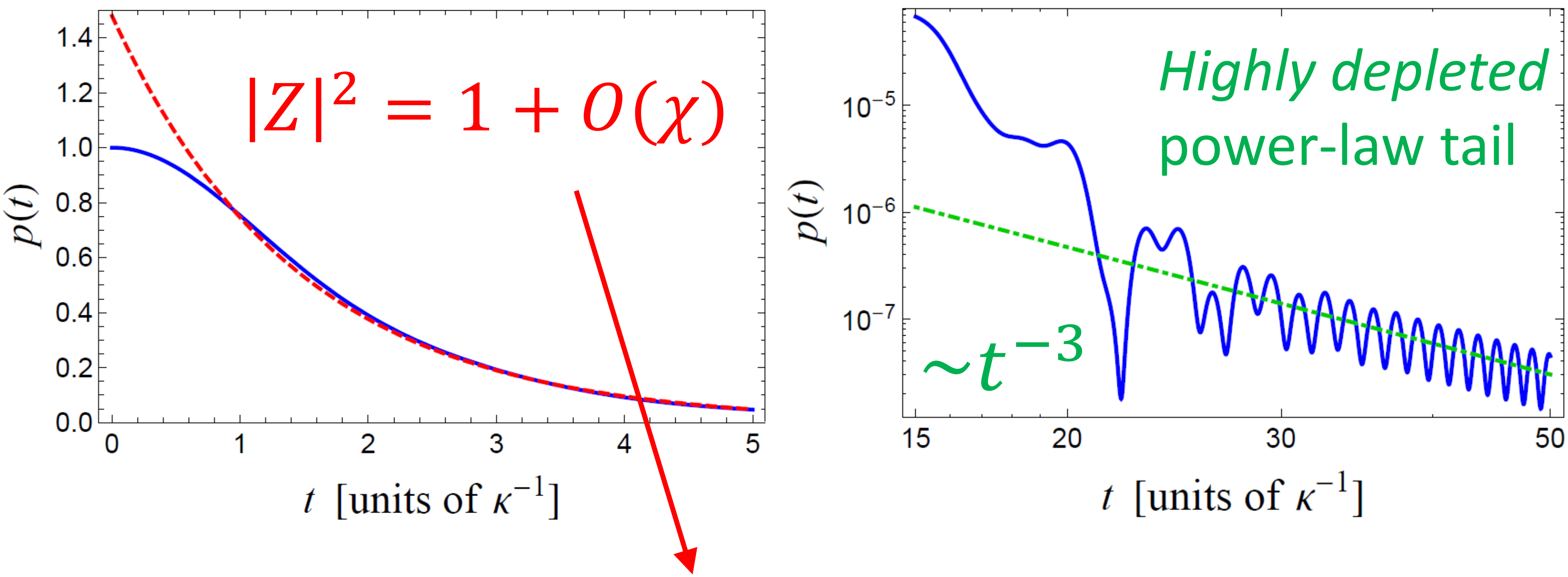
Decay regimes

$\chi = \frac{\kappa_0^2}{2\kappa^2} \ll 1$: **exponential decay** with small corrections



Decay regimes

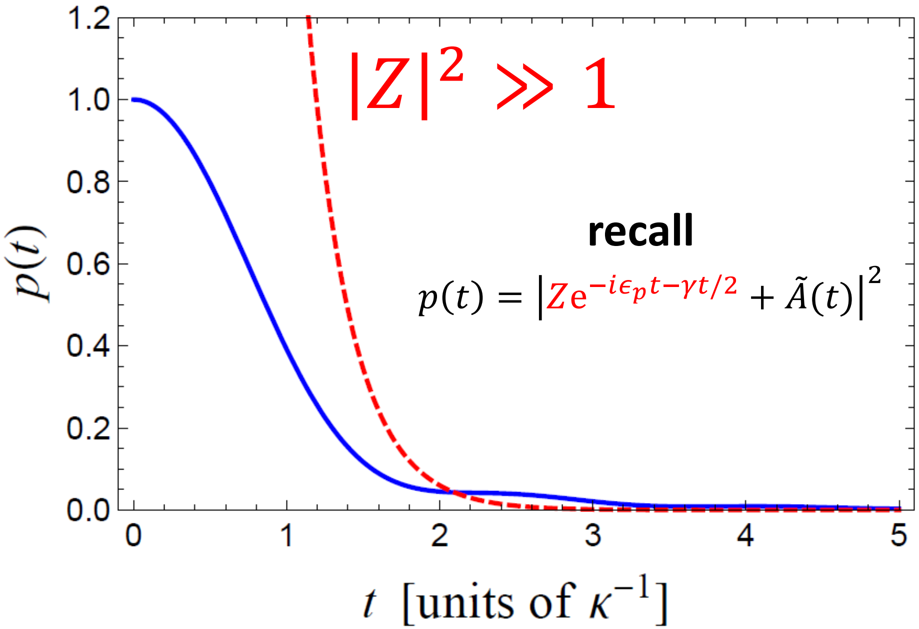
$\chi \lesssim \frac{1-|\epsilon|/2\kappa}{2-|\epsilon|/2\kappa}$: exponential decay with **relevant deviations**

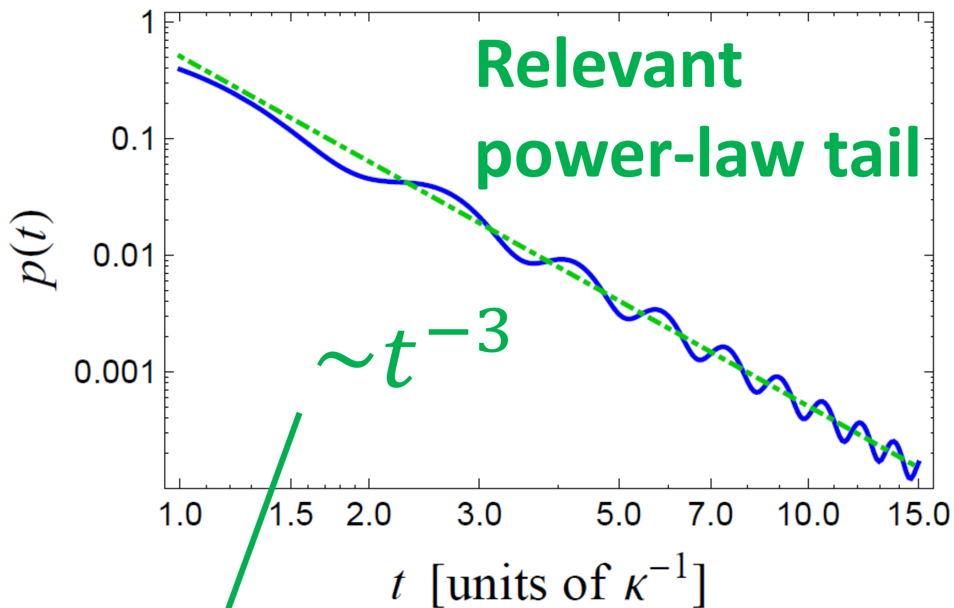


$$|Z|^2 = 1 + \frac{2\chi}{1-2\chi}\frac{1-3\chi/2-(\epsilon/2\kappa)^2}{1-2\chi-(\epsilon/2\kappa)^2}$$

Decay regimes

$\frac{1-|\epsilon|/2\kappa}{2-|\epsilon|/2\kappa} \lesssim \chi \leq 1-|\epsilon|/2\kappa$: **non-exponential decay**



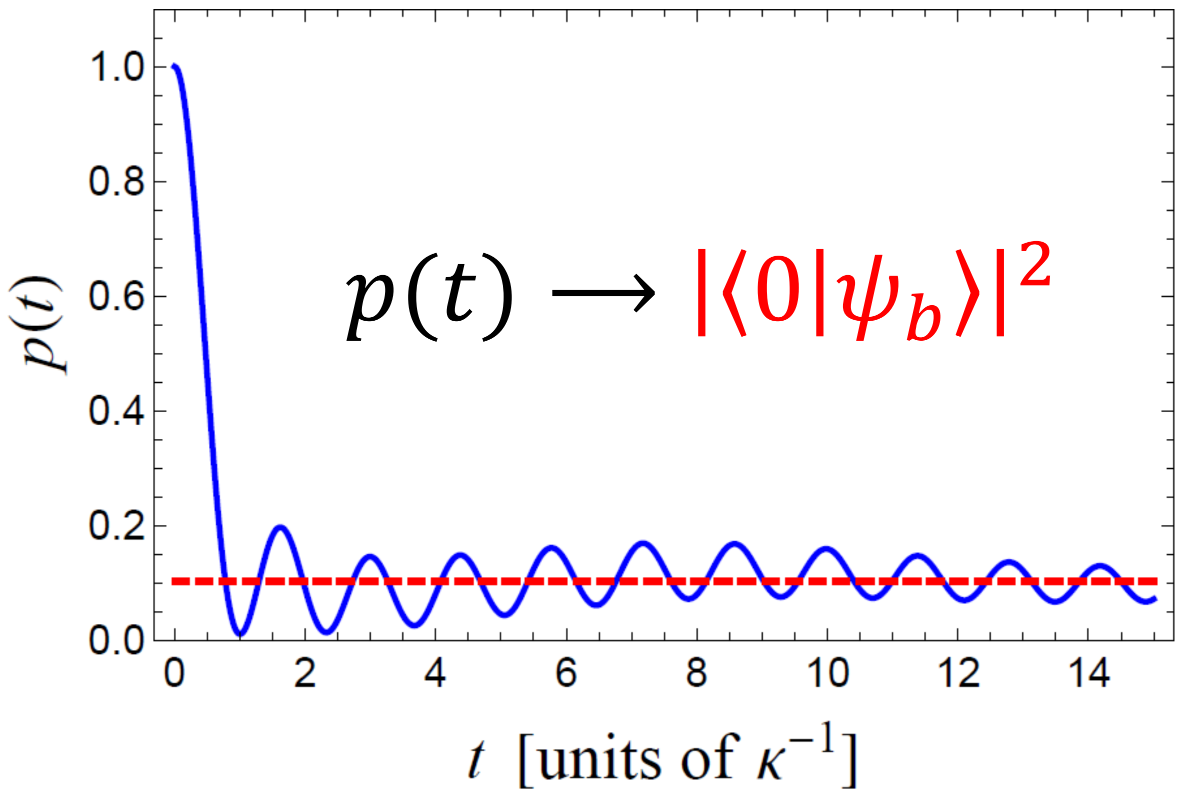


$$p(t) \simeq \frac{\chi^2\kappa}{\pi t^3}\left|\frac{e^{i(2\kappa t+\pi/4)}}{(2\kappa(1-\chi)+\epsilon)^2} + \frac{e^{-i(2\kappa t+\pi/4)}}{(2\kappa(1-\chi)-\epsilon)^2}\right|^2$$

Decay regimes

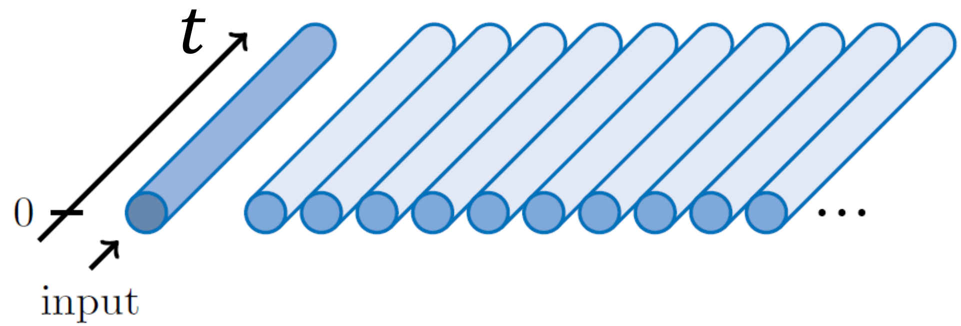
$\chi > 1 - |\epsilon|/2\kappa$: **no instability**

Emergence of a **bound state** $|\psi_b\rangle$ of the total Hamiltonian



Experimental implementation of the hopping model

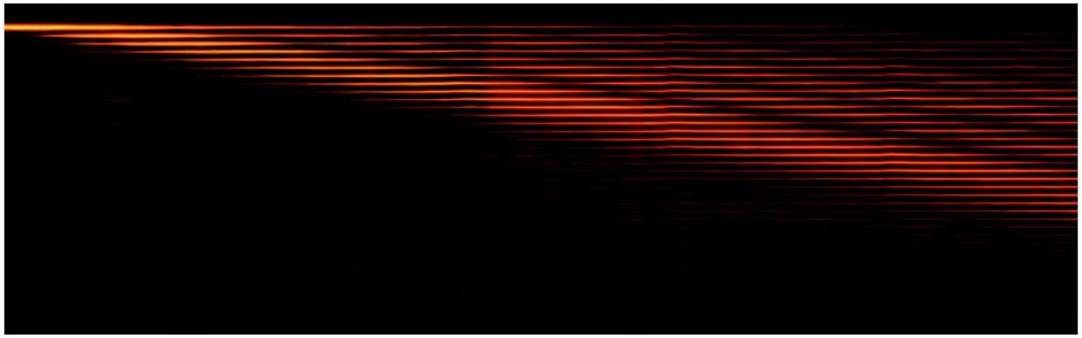
Array of **femtosecond laser-written integrated waveguides** (IFN-CNR Milano)



Quantum-optical analogy (S. Longhi, 2009): stationary paraxial optics in photonic structures can simulate Schrödinger dynamics.

Longitudinal coordinate ≡ Time

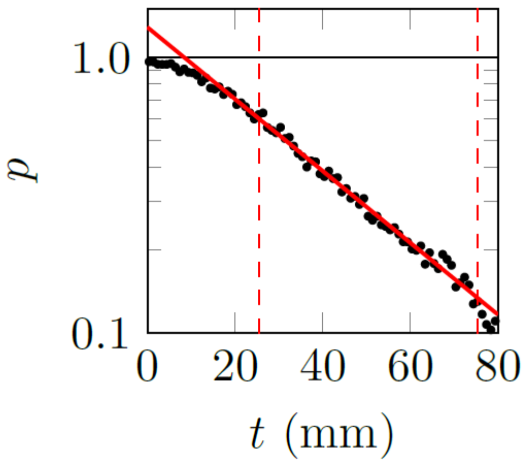
Scattered laser light
High dynamic range

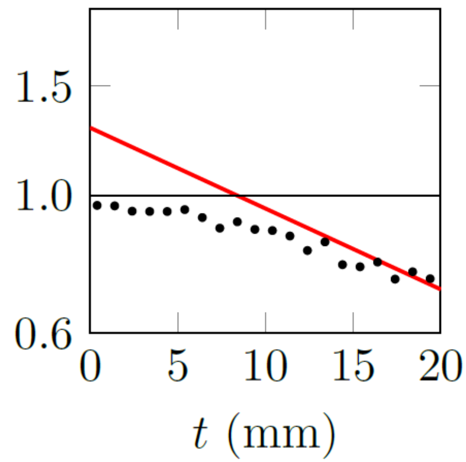


Next-to-nearest-neighbor hopping improves prediction accuracy

$$H_2 = q_0(|0\rangle\langle 2| + |2\rangle\langle 0|) + q\sum_{n=1}^{\infty}(|n\rangle\langle n+2| + |n+2\rangle\langle n|)$$

Observation of different decay regimes

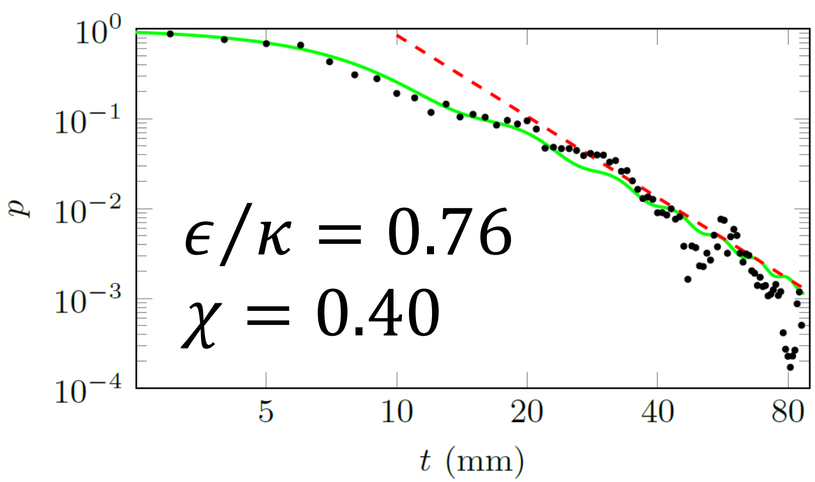




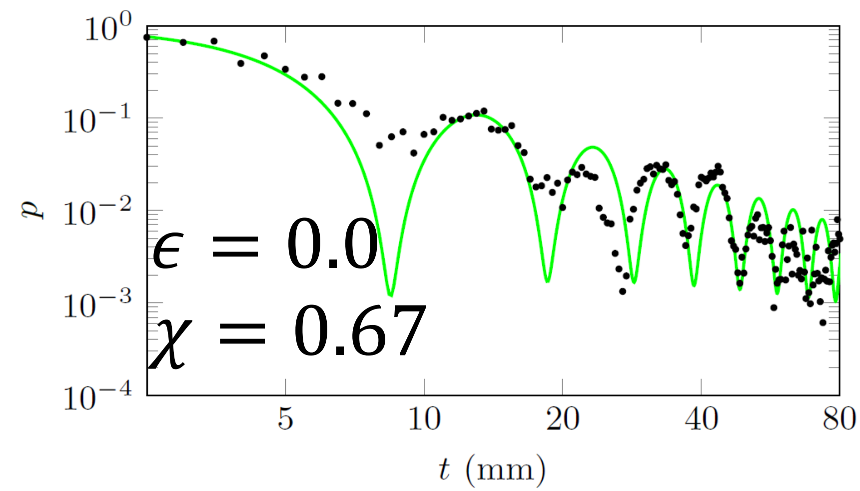
$\epsilon/\kappa = -0.67$

$\chi = 0.071$

Exponential decay with relevant *short-time* deviations



Non-exponential decay with **detectable power-law** (t^{-3}) tail



Non-exponential decay with no simple power-law tail

Conclusions and outlook

- ✓ Models defined on **one-dimensional arrays** represent a controllable testbed to study non-exponential decays and strong-coupling effects in general
- ✓ **Laser-written waveguides** provided a feasible, effective and *fully controllable* experimental platform to obtain evidence of power-law decay

A. Crespi *et al.*, Phys. Rev. Lett. **122**, 130401 (2019)

- ❑ Detailed characterization of next-to-nearest neighbor model (article in preparation)
- ❑ Effects of disorder on the semi-infinite array
- ❑ Observation of the transition *in time* from exponential to power-law decay

Thank you
for your attention