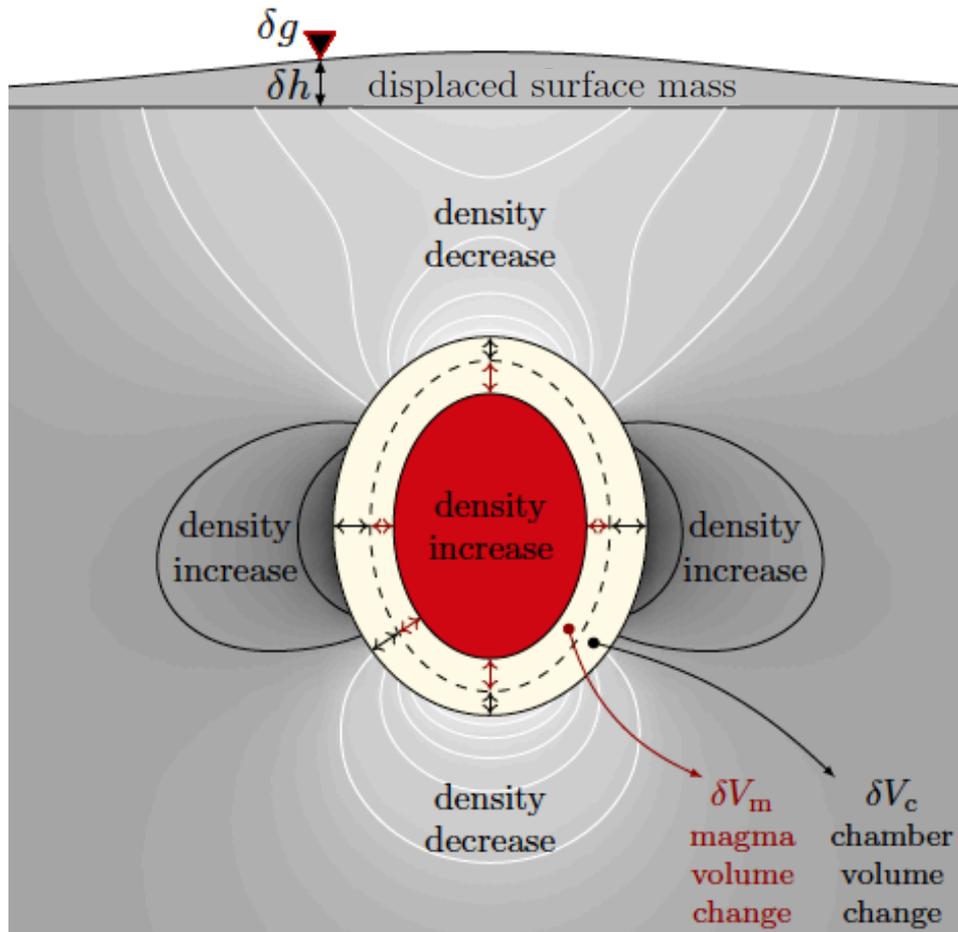


Joint inversions of deformation and gravity changes due to the inflation of a magmatic reservoir of arbitrary shape



Mehdi Nikkhoo¹,
Eleonora Rivalta^{1,2}

¹ GFZ, Potsdam, Germany

² University of Bologna, Italy

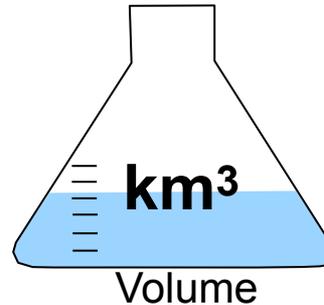
107° Congresso Nazionale
Società Italiana di Fisica
15 September 2021

Motivation

Motivation: deformation & gravity change modelling

• Deformation modelling:

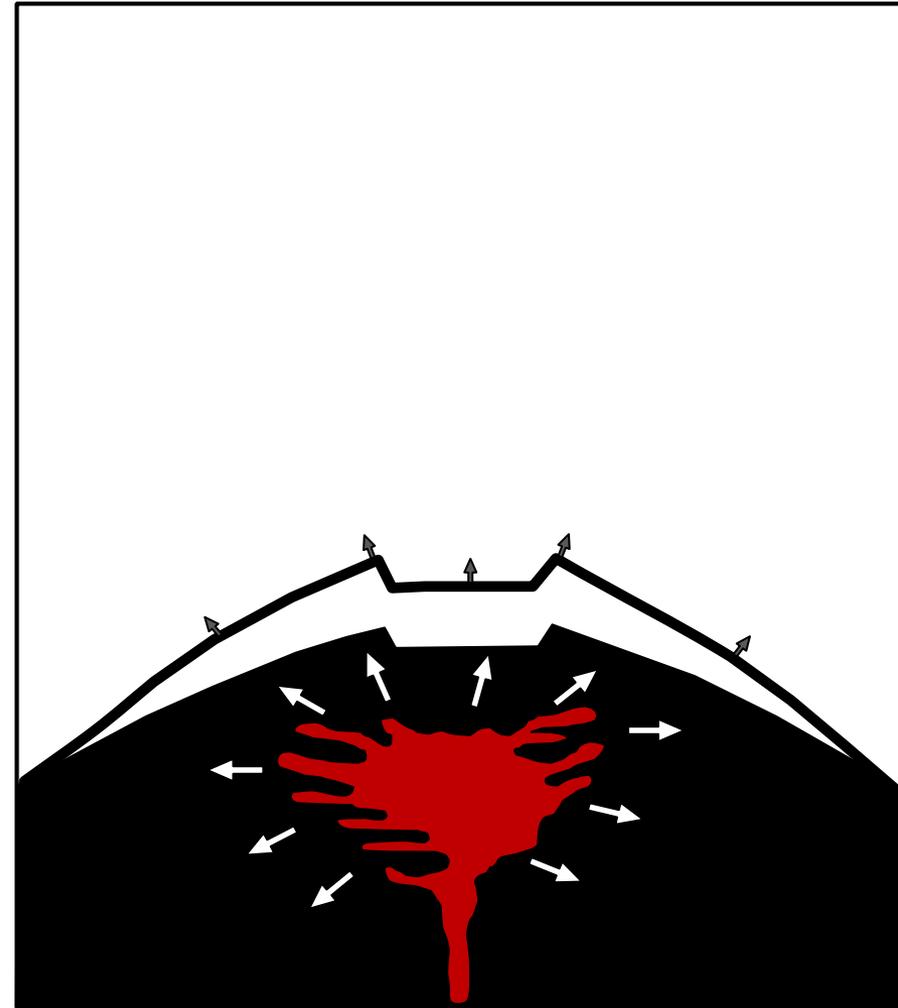
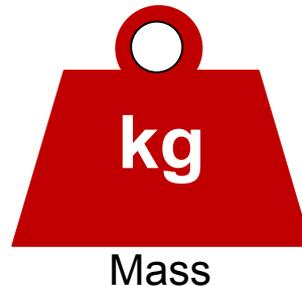
- 1) Location,
- 2) Shape,
- 3) Spatial orientation,
- 4) Volume change



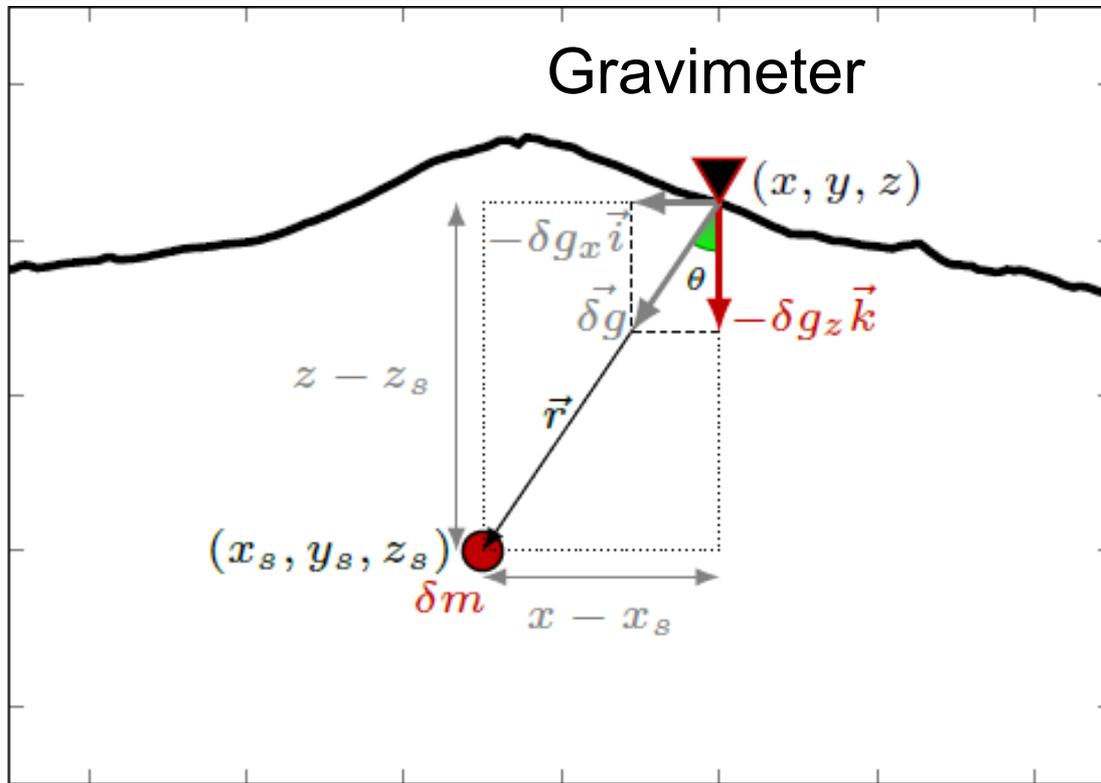
- No answer for the causes:
New magma? Degassing? Hydrothermal?

• Gravity change modelling:

- 5) Mass change



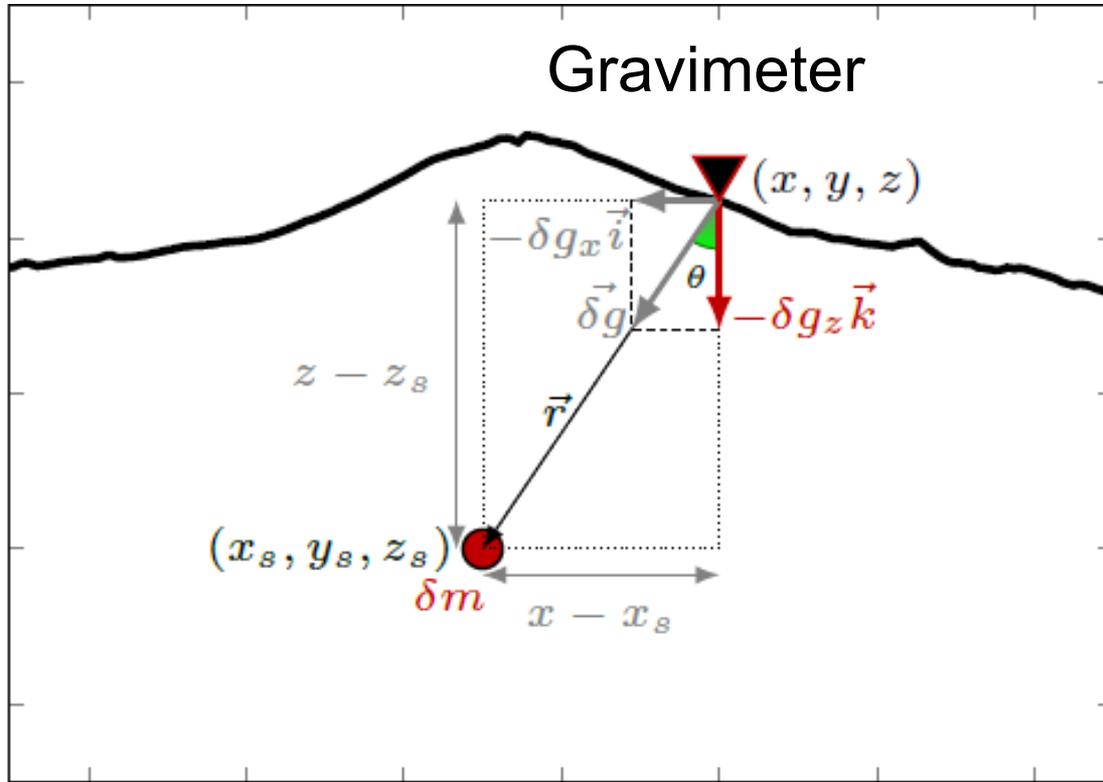
Motivation: deformation & gravity change modelling



$$\text{NEWTON-g} \cdot \vec{g} = G \frac{\delta m d}{r^3}$$

$$d = z - z_s$$

Motivation: deformation & gravity change modelling



AQG-B gravimeter



source: newton-g.eu

Resolution:

$$1 \mu\text{Gal} = 10^{-8} \text{ m/s}^2$$

$$\text{NEWTON-g} \cdot \vec{g} = G \frac{\delta m d}{r^3}$$

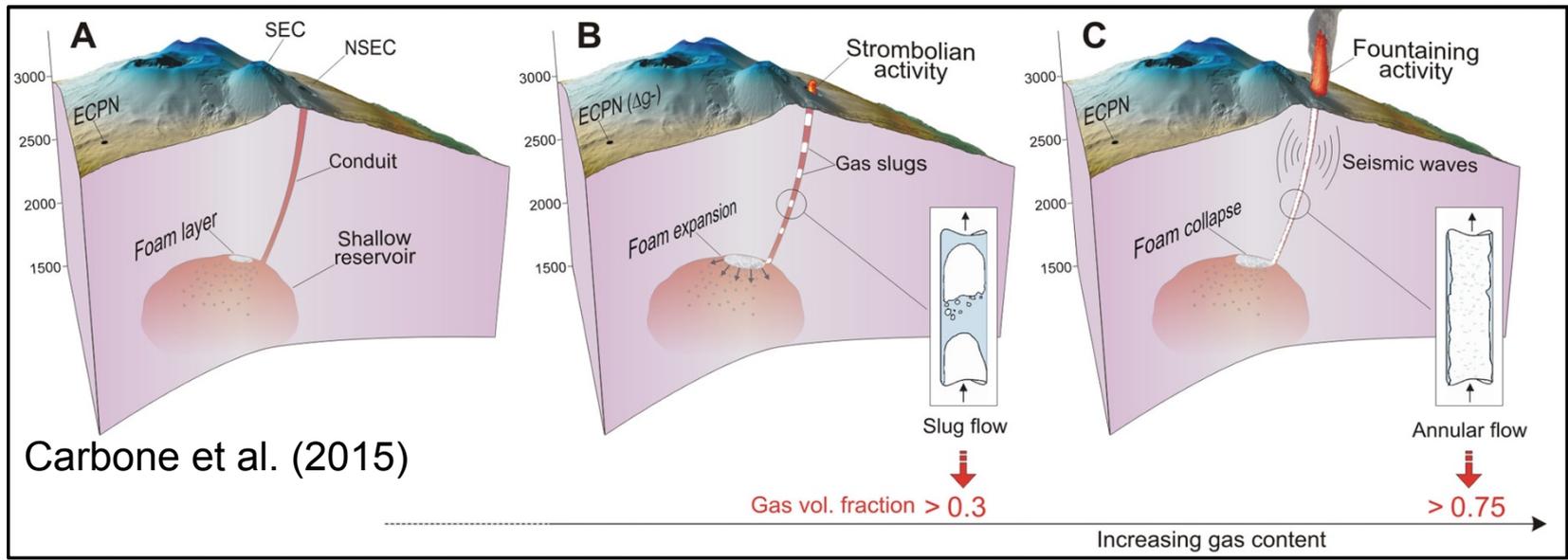
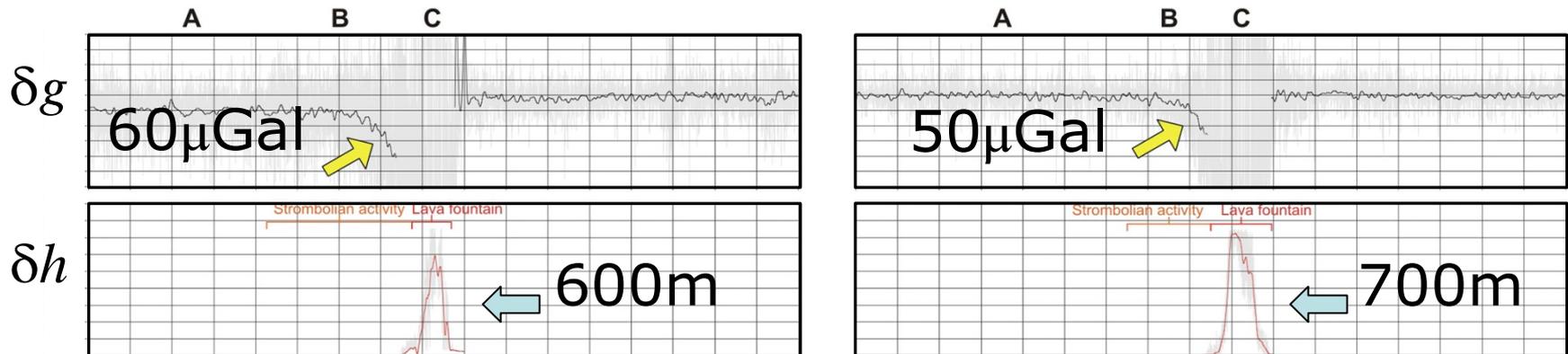
A gravity change of $6.7 \mu\text{Gal}$ for:

- 10^3 kg at depth = 1m
- 10^9 kg at depth = 1000m

$$d = z - z_s$$

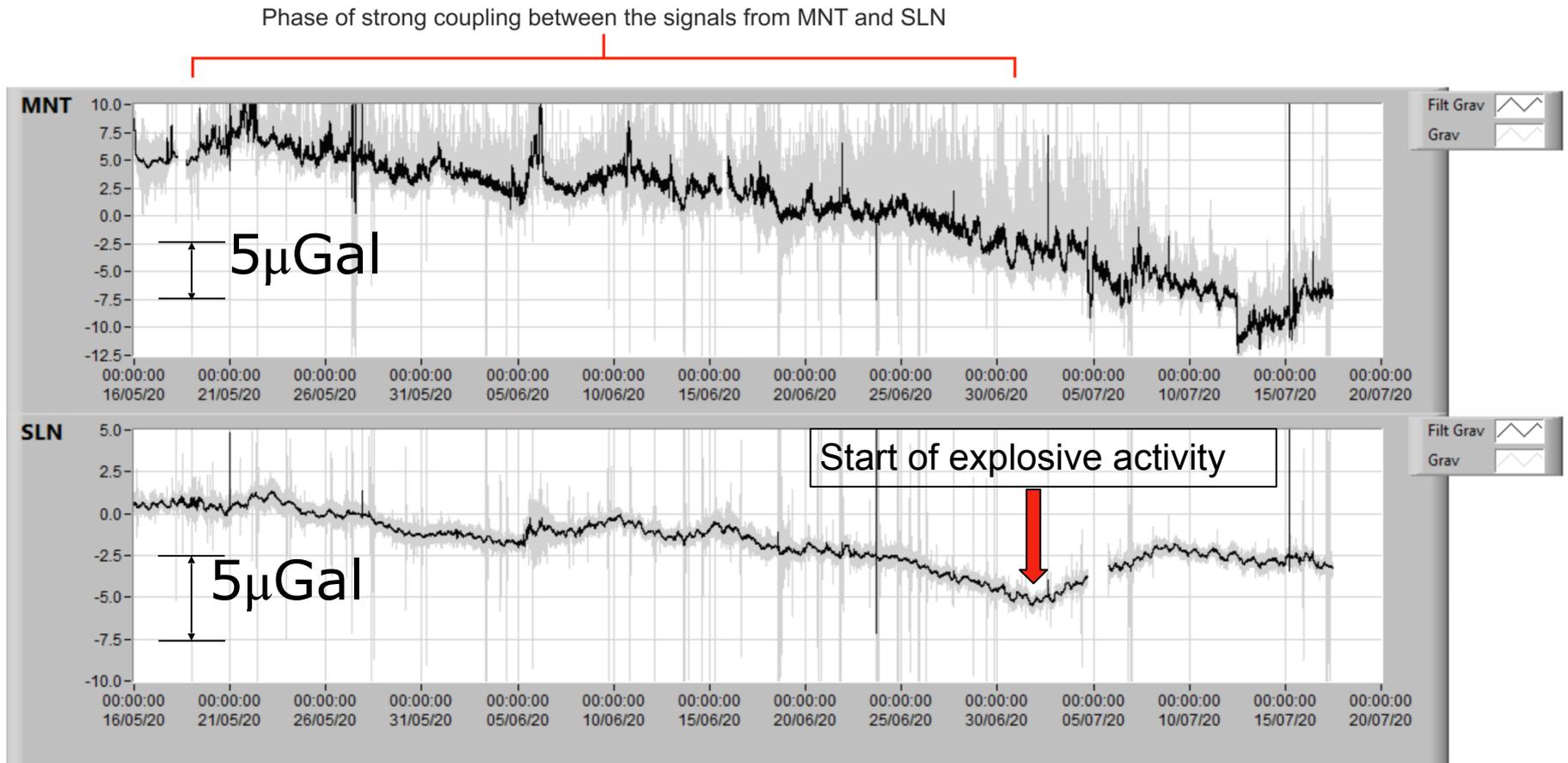
Motivation for volcano gravimetry: Mount Etna

- The 2011 lava fountaining episode:



Motivation for volcano gravimetry: Mount Etna

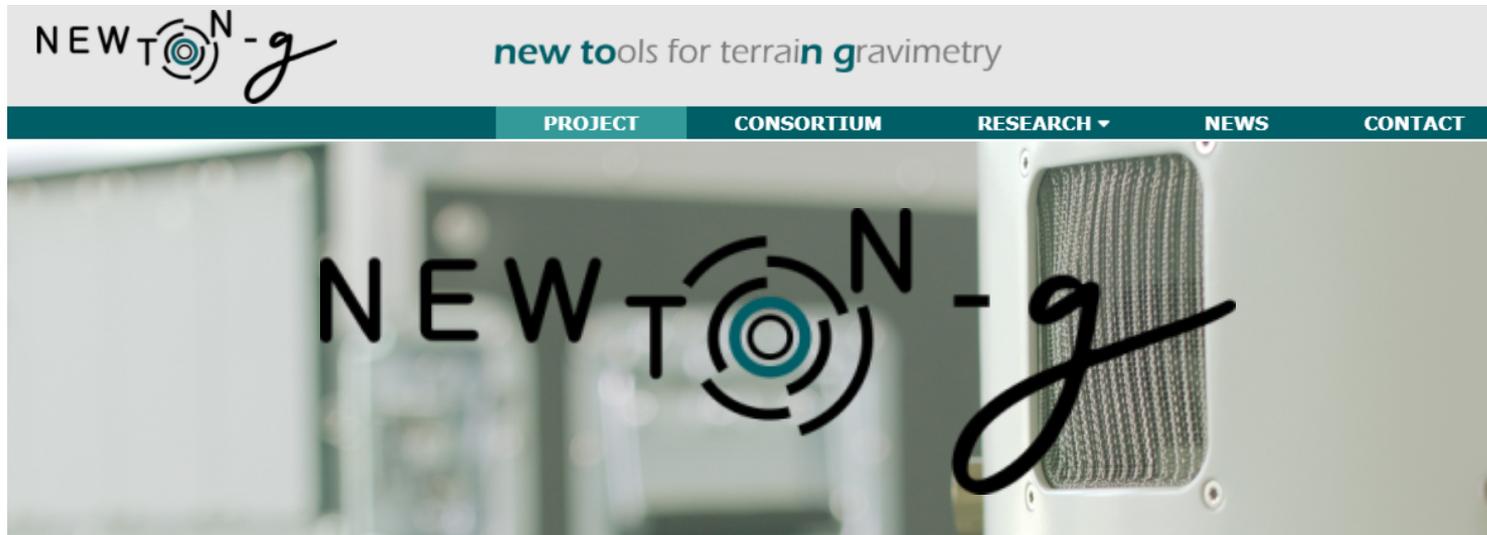
- The mid-2020 eruptive phase:



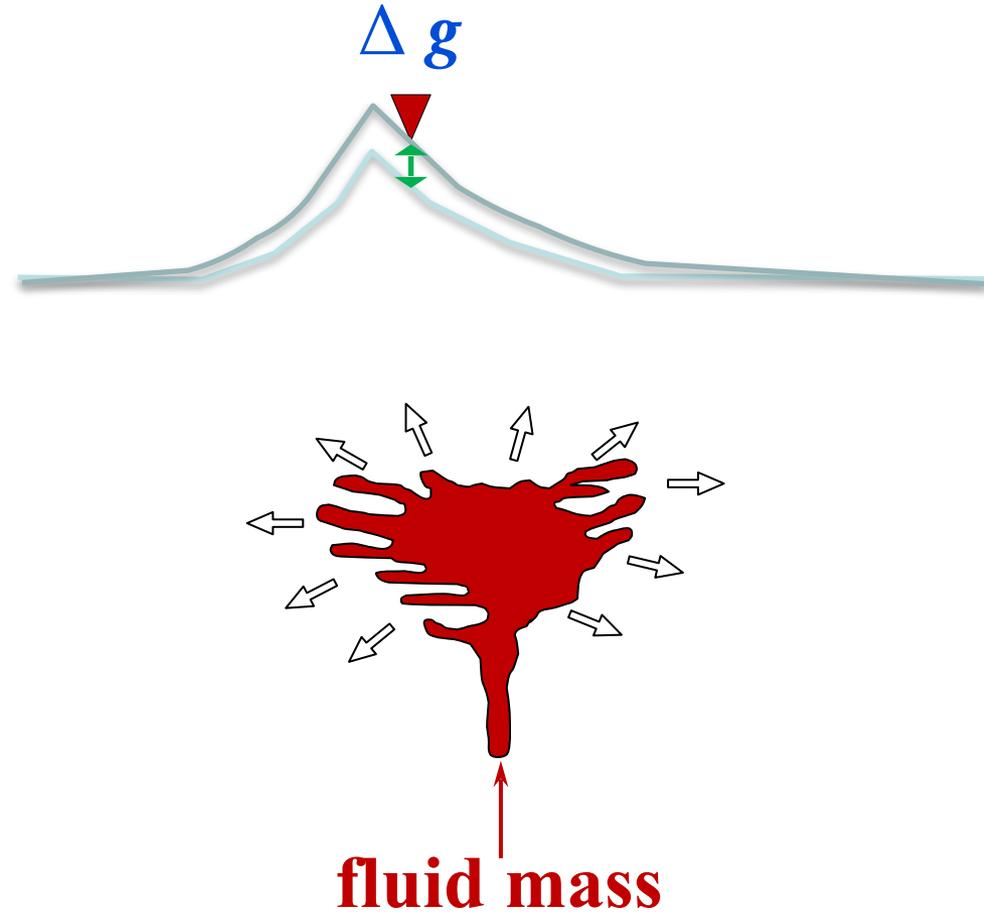
Source: Daniele Carbone, INGV

Volcano gravimetry: The main challenges

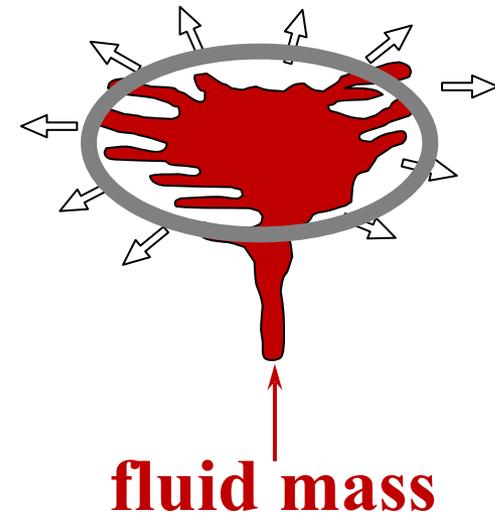
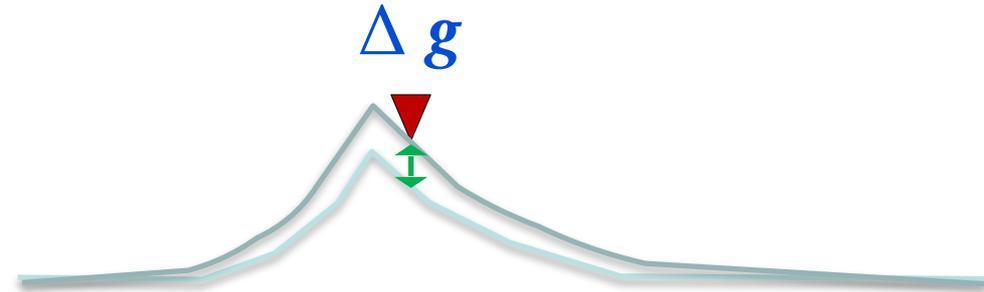
- 1) Instrument costs,
- 2) Volcano-specific gravimeters,
- 3) Develop a gravity imager,
- 4) Deformation effects and modelling? Our task at GFZ...



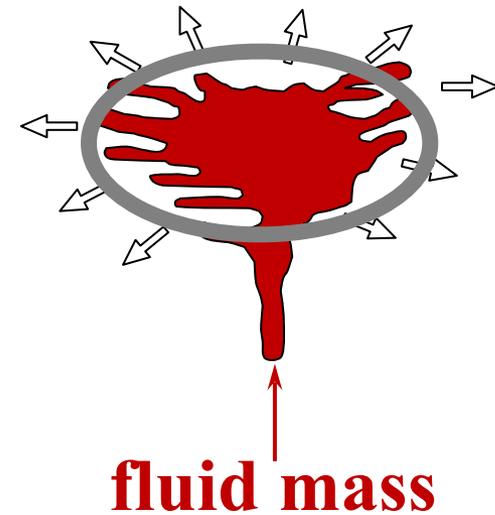
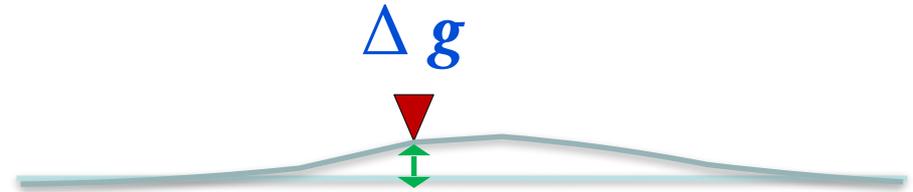
Main challenge: Deformation-induced gravity changes



Main challenge: Deformation-induced gravity changes



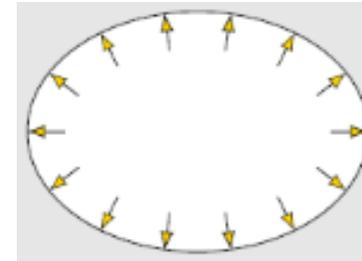
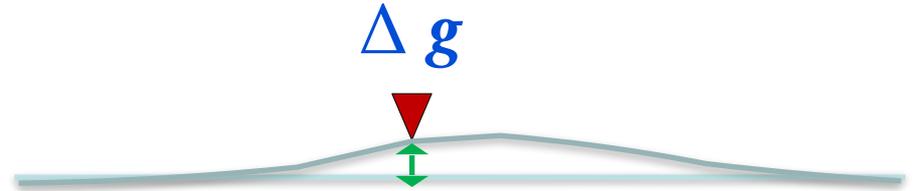
Main challenge: Deformation-induced gravity changes



Main challenge: Deformation-induced gravity changes

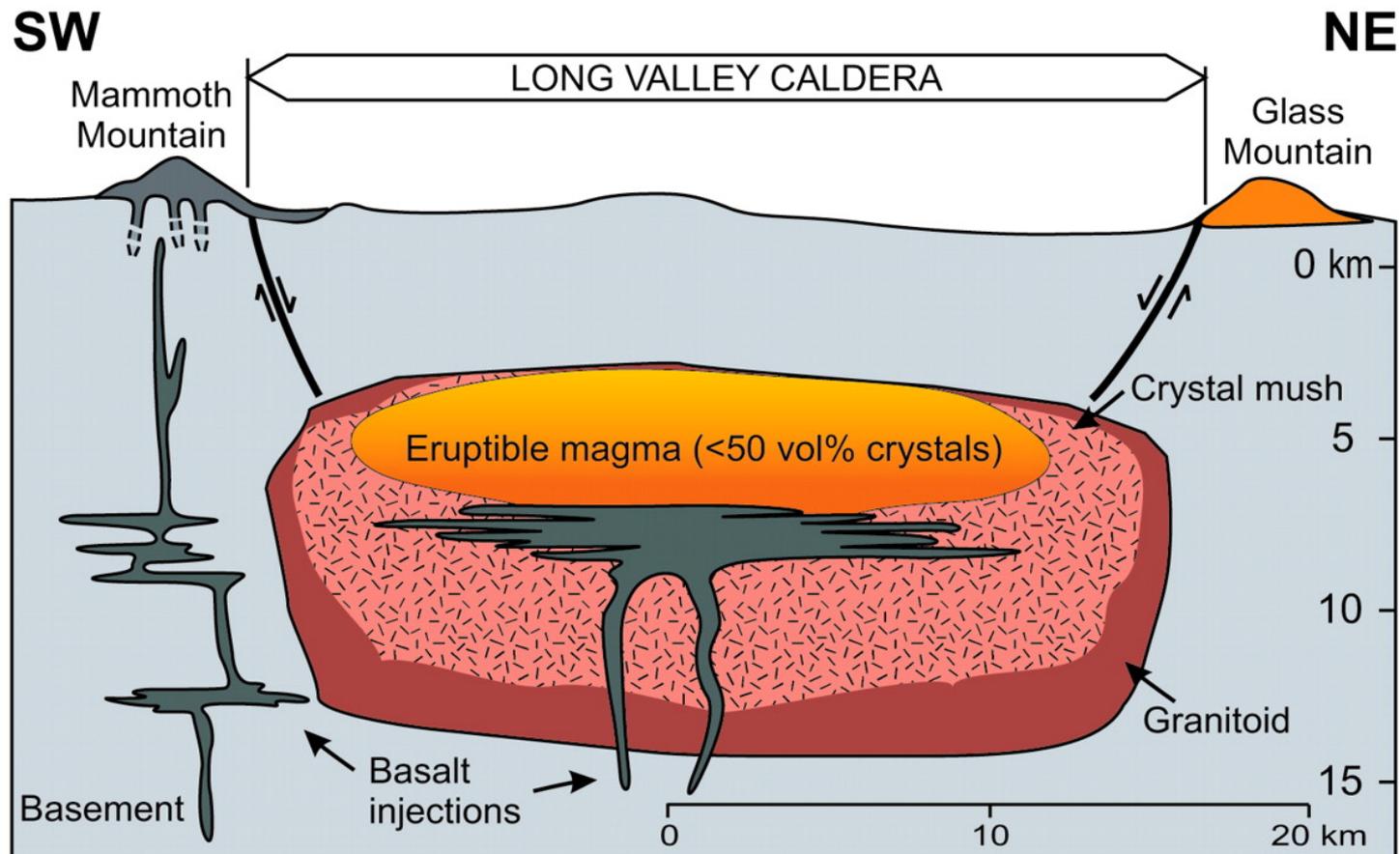
Available analytical solutions:

- Hagiwara (1977),
Walsh and Rice (1979)
for spherical sources
- Okubo (1991, 1992)
for dislocations
- **This study: analytical solutions
for gravity changes caused by
triaxial deformation sources**



Why triaxial sources?

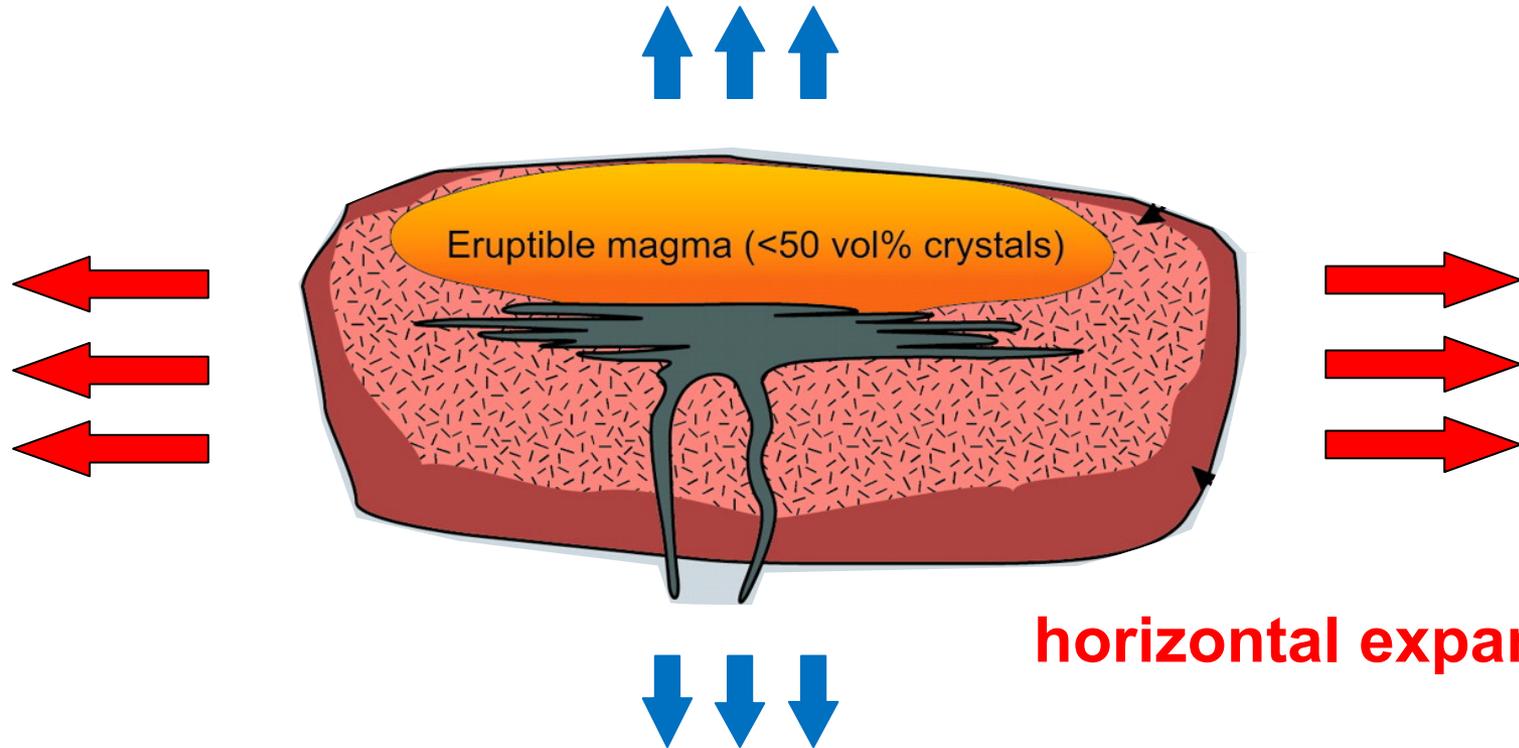
What are triaxial deformation sources?



Bachmann and Bergantz (2008)

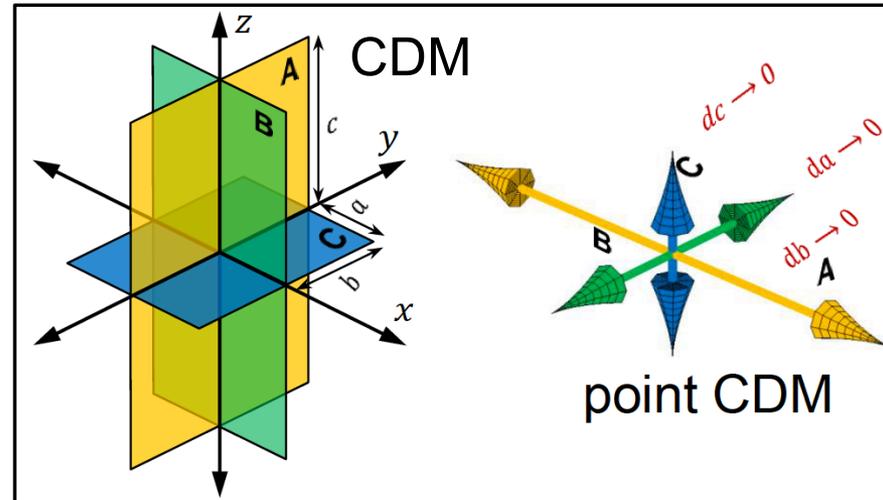
What are triaxial deformation sources?

vertical expansion

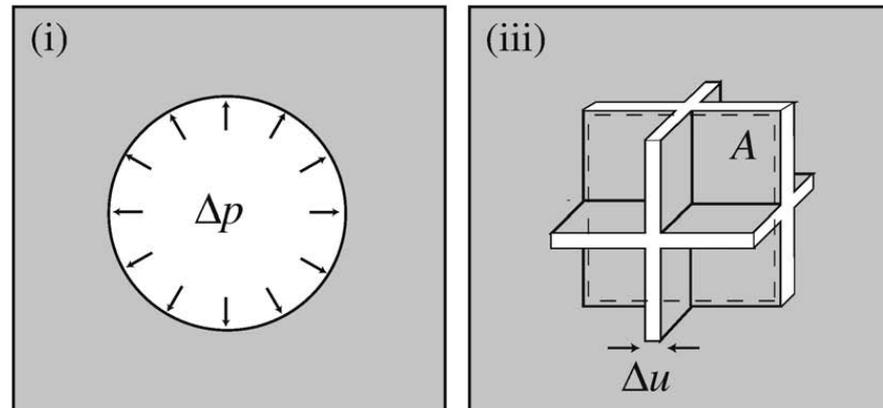


horizontal expansion

Analytical solution for triaxial deformation sources

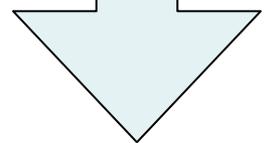
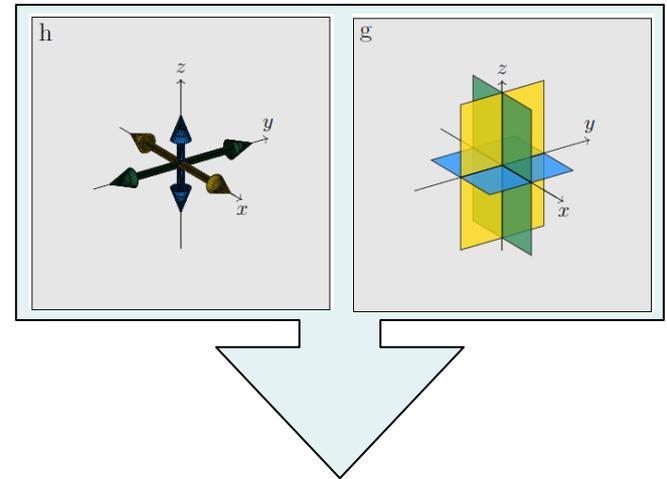


Nikkhoo et al. (2017)



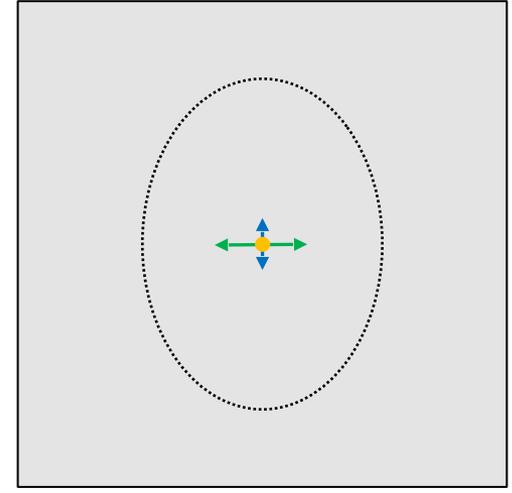
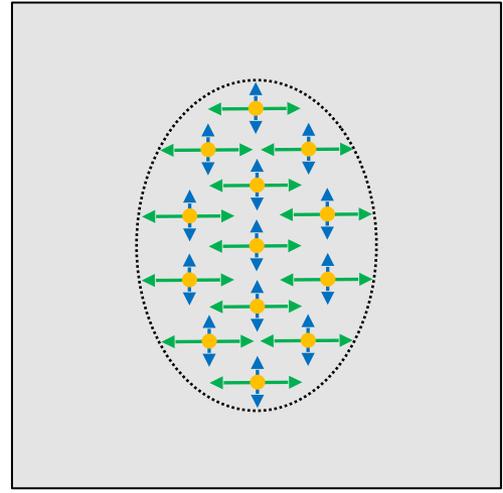
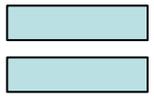
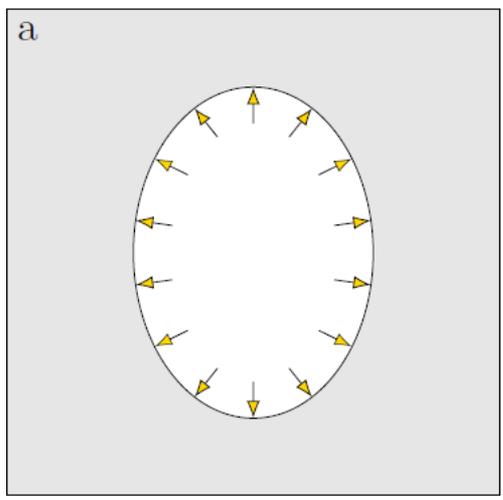
Bonafede and Ferrari (2009)

- The point source \equiv Far-field solution
- The finite source \equiv Near-field solution

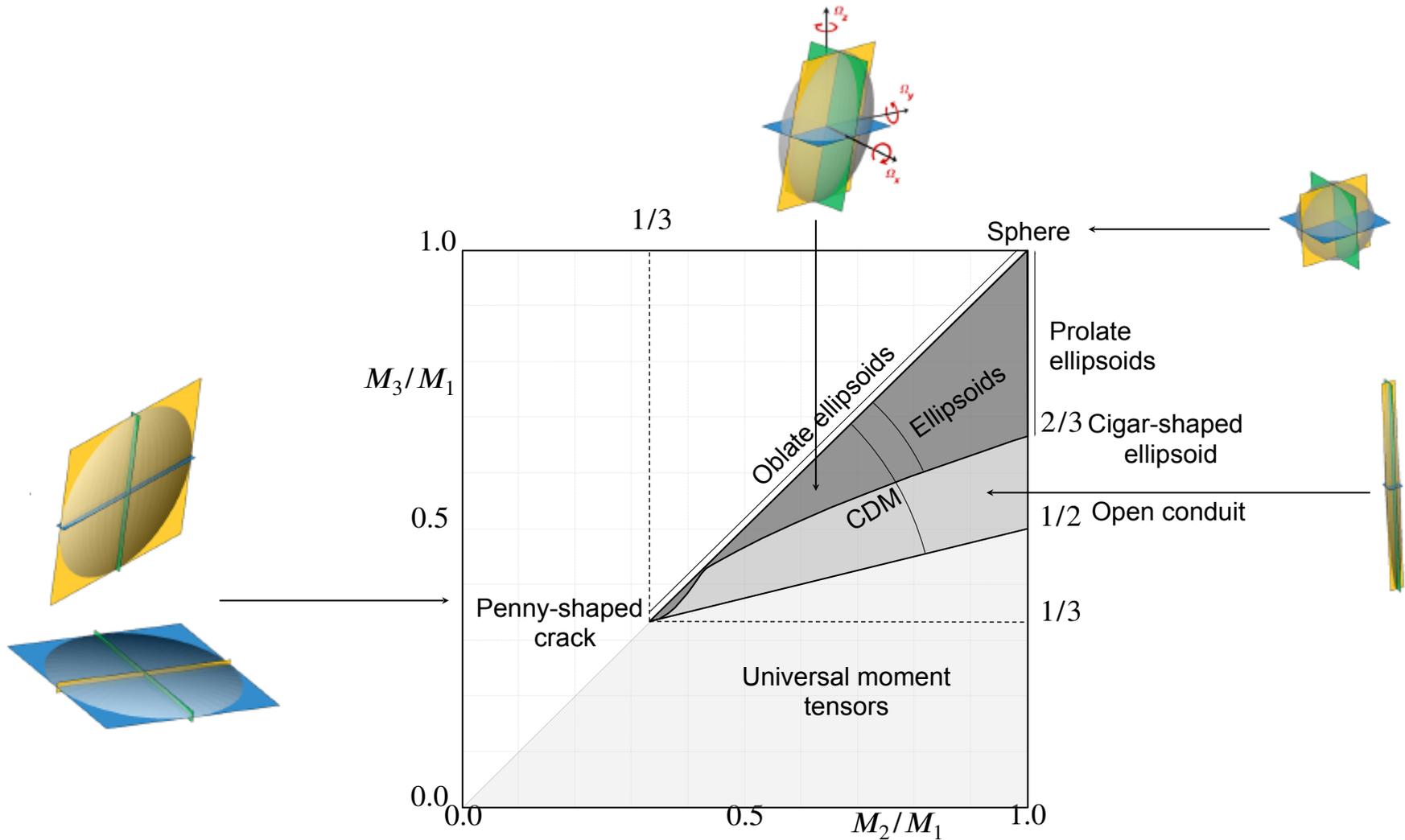


Near field

Far field



The point CDM and ellipsoidal cavity models

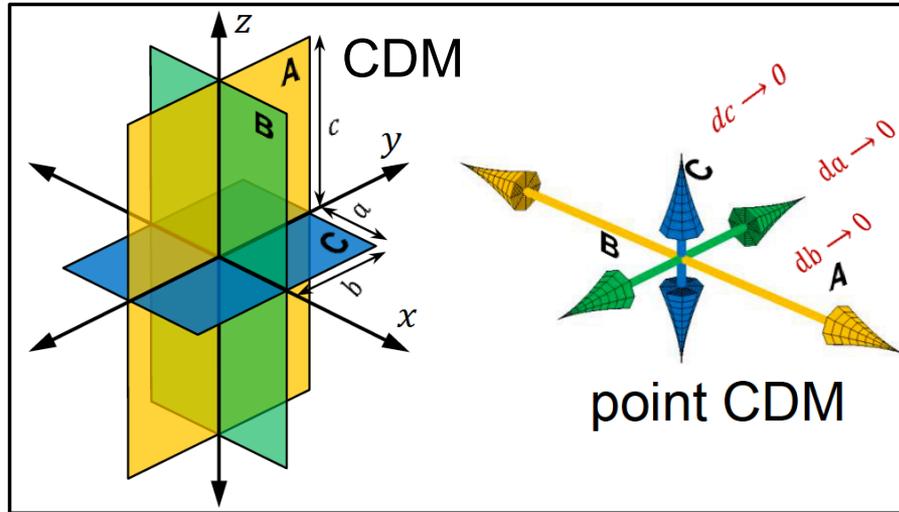


Analytical solution for gravity changes

Nikkhoo and Rivalta (2021), under review

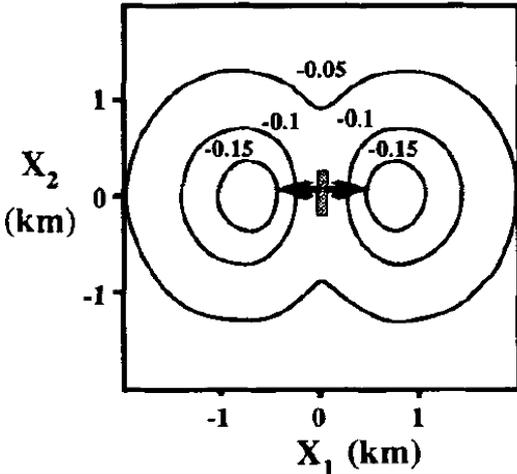
preprint on **ESSAr**
Earth and Space Science Open Archive

Analytical solution for triaxial deformation sources

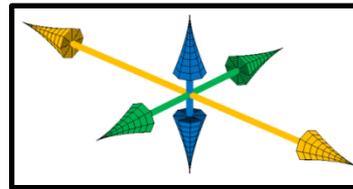


Nikkhoo et al. (2017)

Okubo (1991)

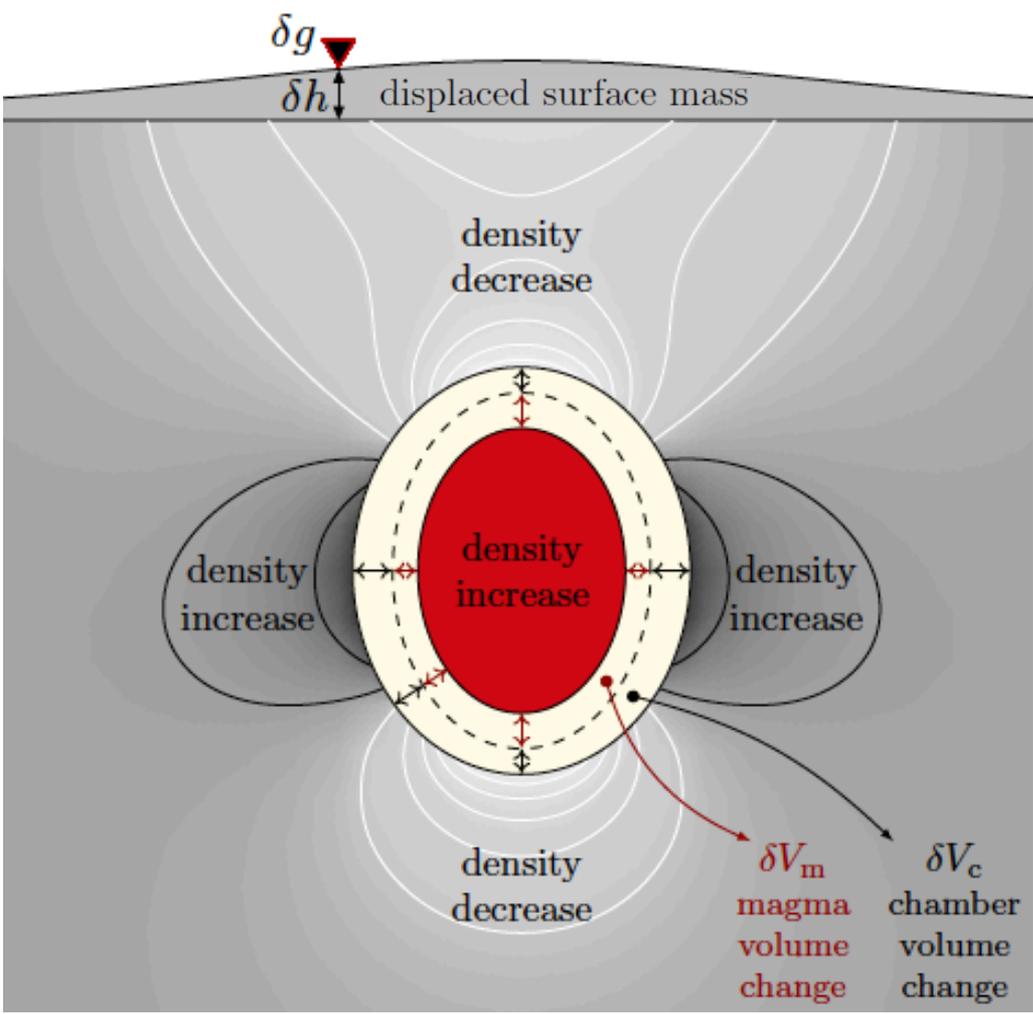


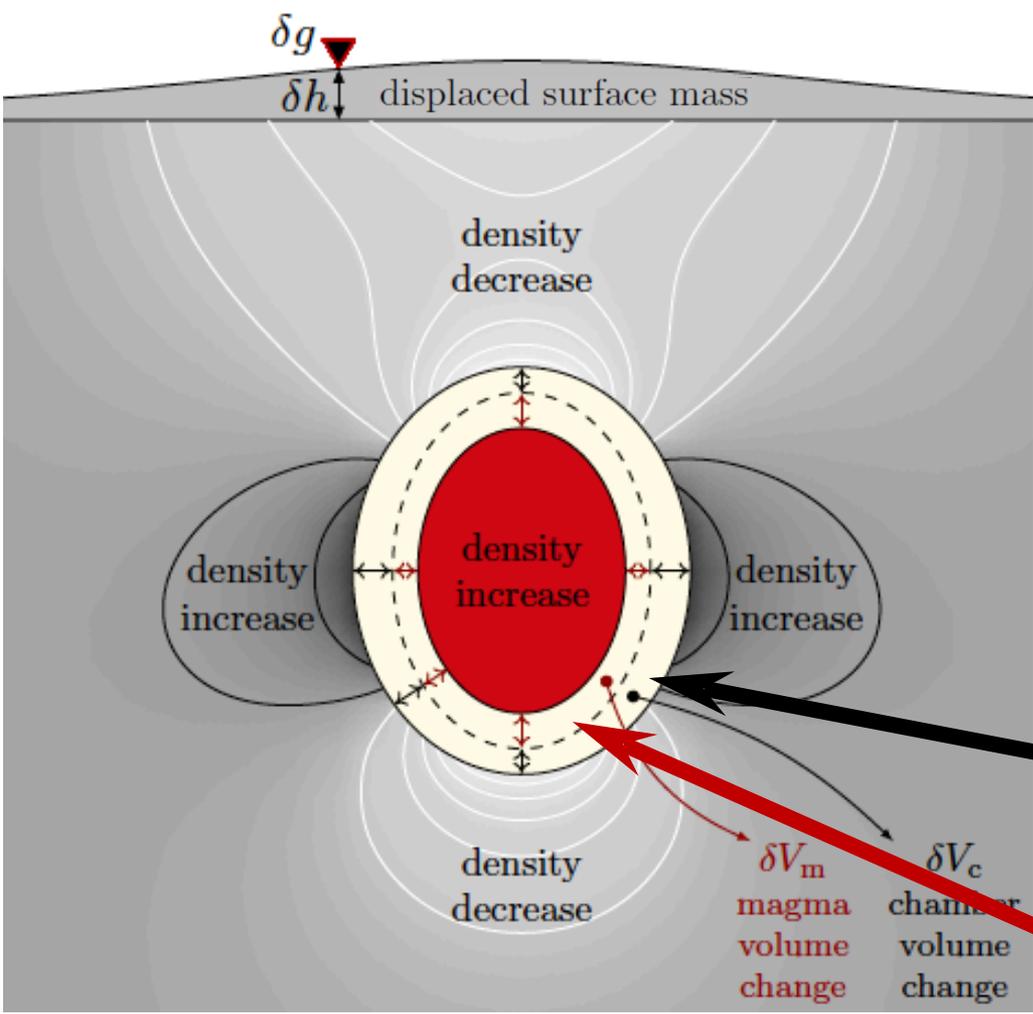
point CDM



The analytical solution
for gravity changes

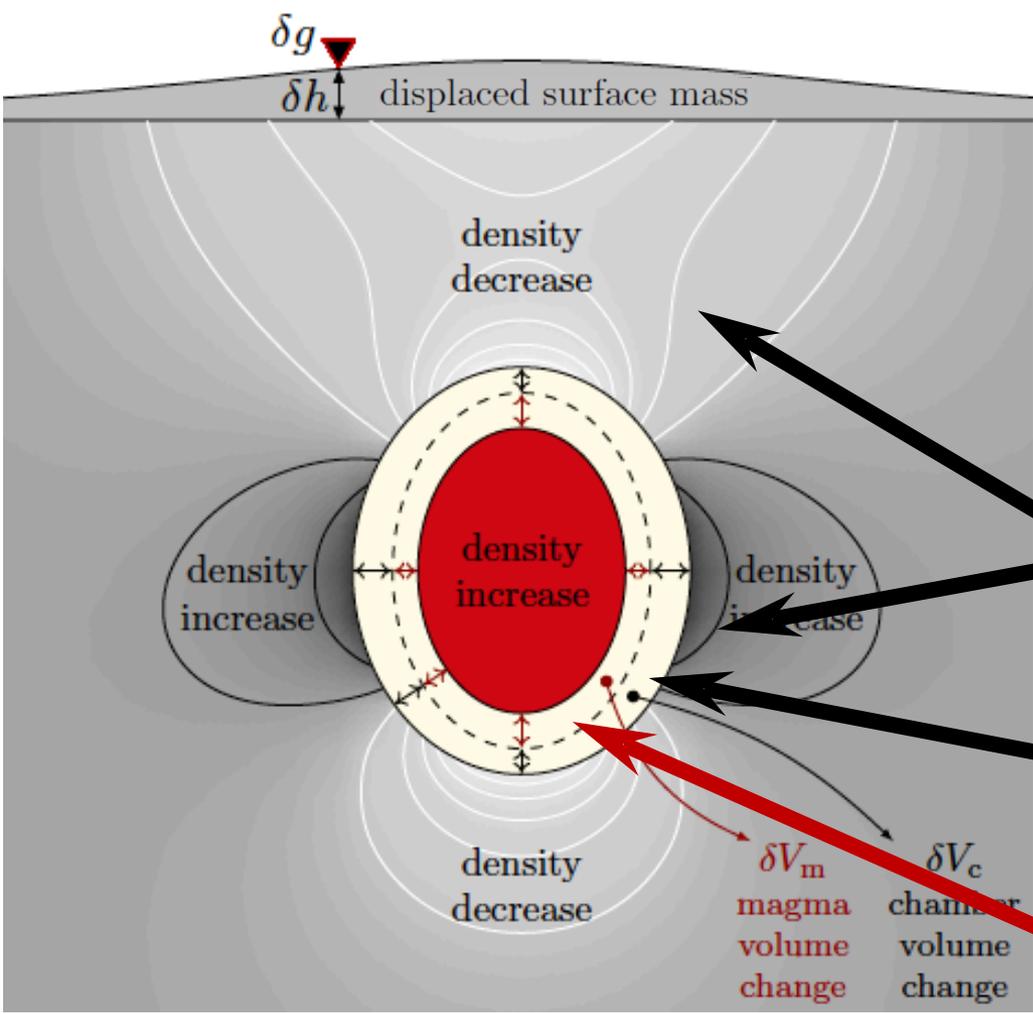
Nikkhoo and Rivalta (2021), under review



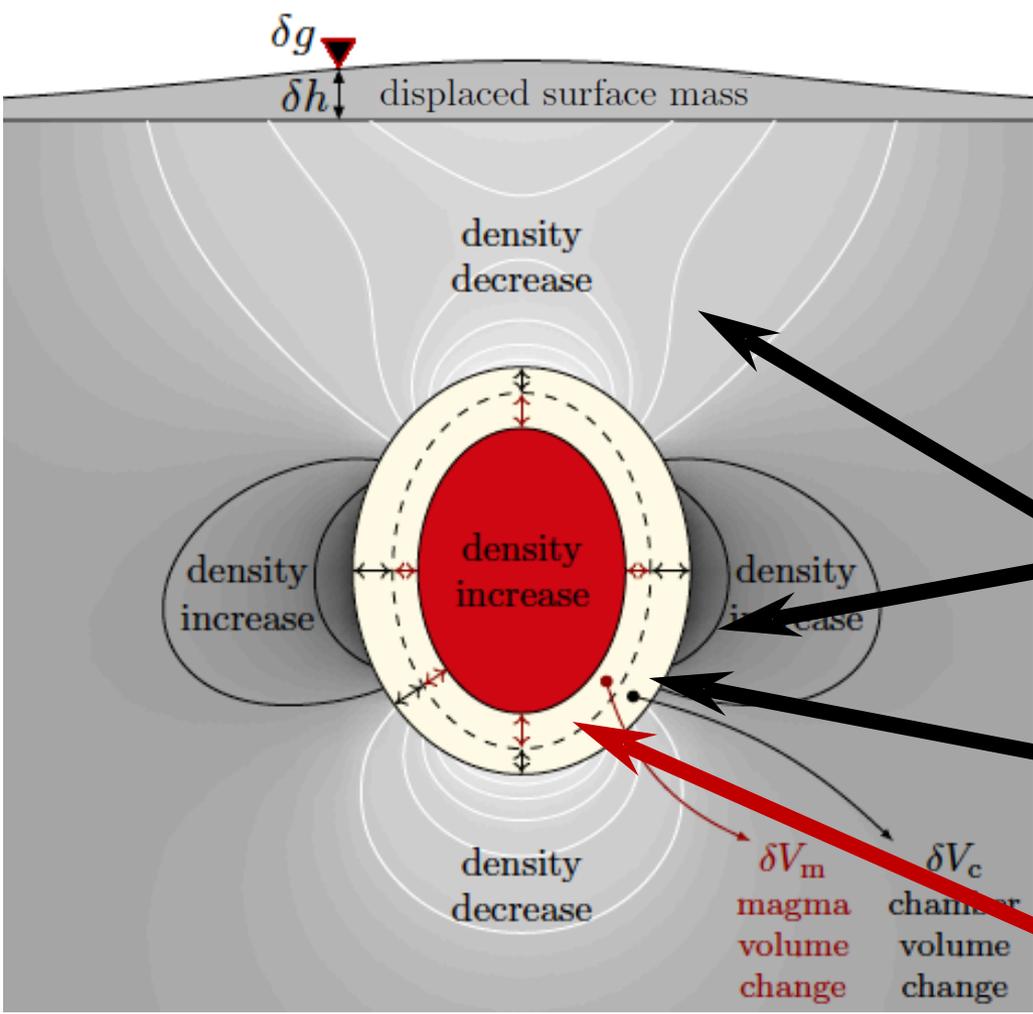


② $\Delta g_{\Delta V}$ **Interface volume change**

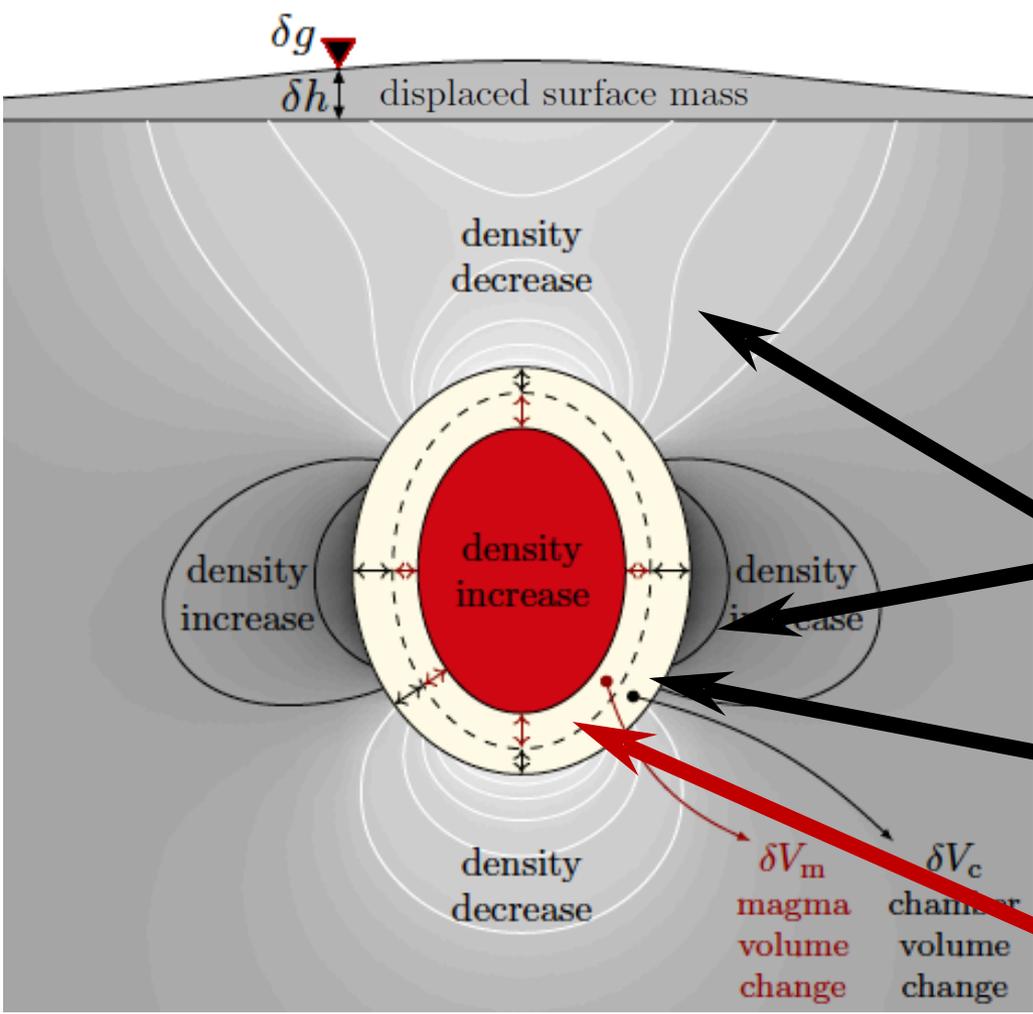
① $\Delta g_{\delta m}$ **Fluid mass**



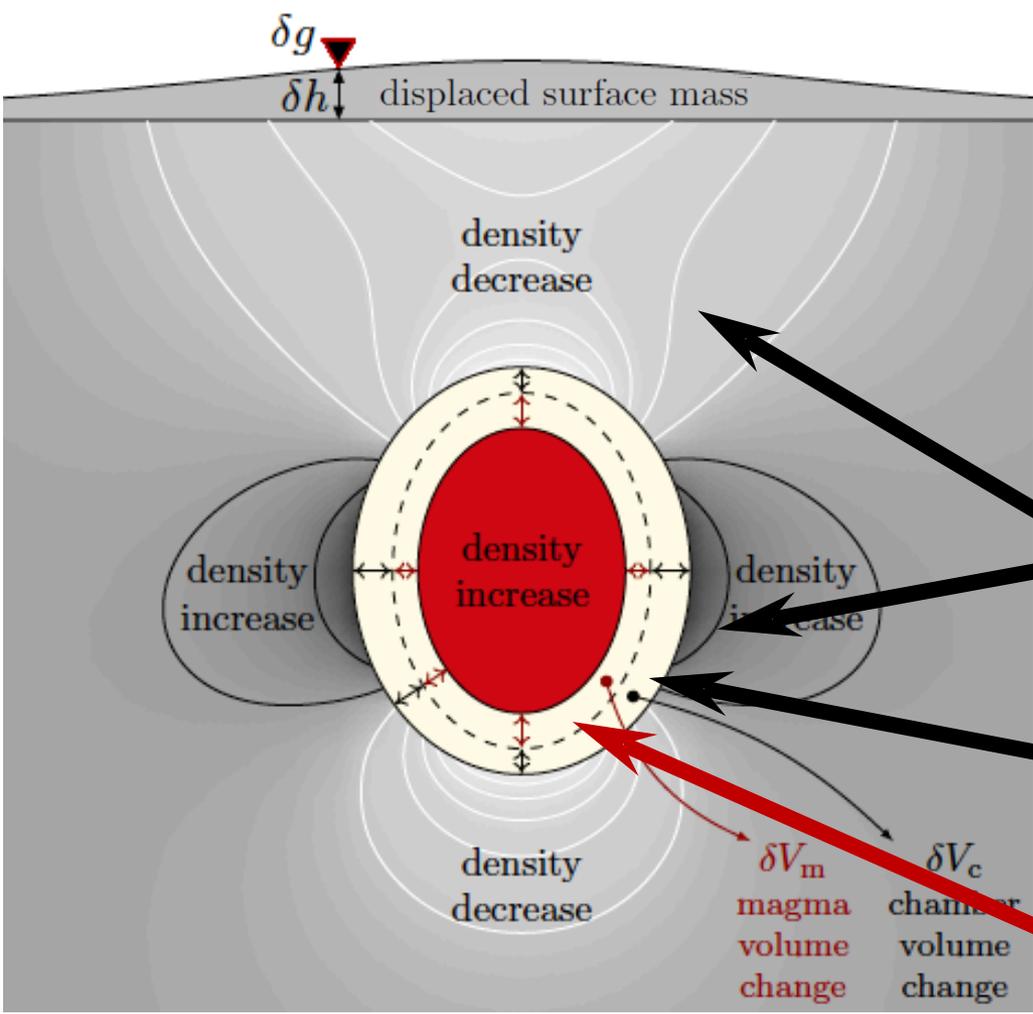
- ③ $\Delta g_{\epsilon_{kk}}$ Host-rock dilatation
- ② $\Delta g_{\Delta V}$ Interface volume change
- ① $\Delta g_{\delta m}$ Fluid mass



- ④ Δg_{SM} Surface mass
- ③ $\Delta g_{\epsilon_{kk}}$ Host-rock dilatation
- ② $\Delta g_{\Delta V}$ Interface volume change
- ① $\Delta g_{\delta m}$ Fluid mass

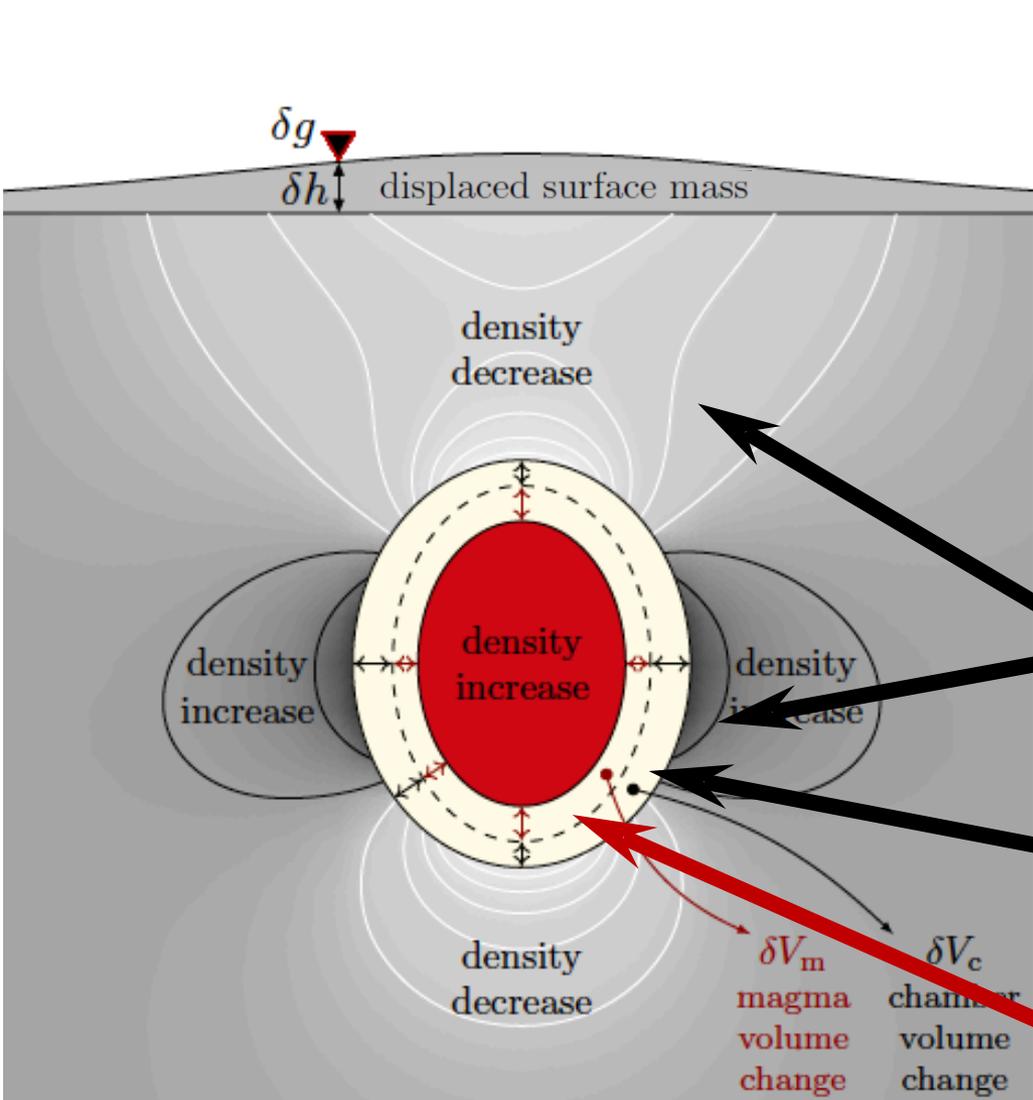


- ⑤ Δg_{FA} Free air
- ④ Δg_{SM} Surface mass
- ③ $\Delta g_{\epsilon_{kk}}$ Host-rock dilatation
- ② $\Delta g_{\Delta V}$ Interface volume change
- ① $\Delta g_{\delta m}$ Fluid mass



- 

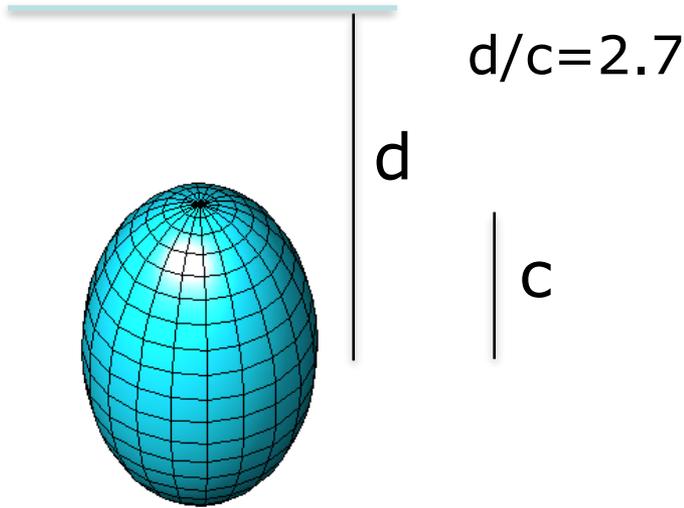
- ⑤ Δg_{FA} Free air
 - ④ Δg_{SM} Surface mass
- $$\int f(x) dx$$
- ③ $\Delta g_{\epsilon_{kk}}$ Host-rock dilatation
 - ② $\Delta g_{\Delta V}$ Interface volume change
 - ① $\Delta g_{\delta m}$ Fluid mass



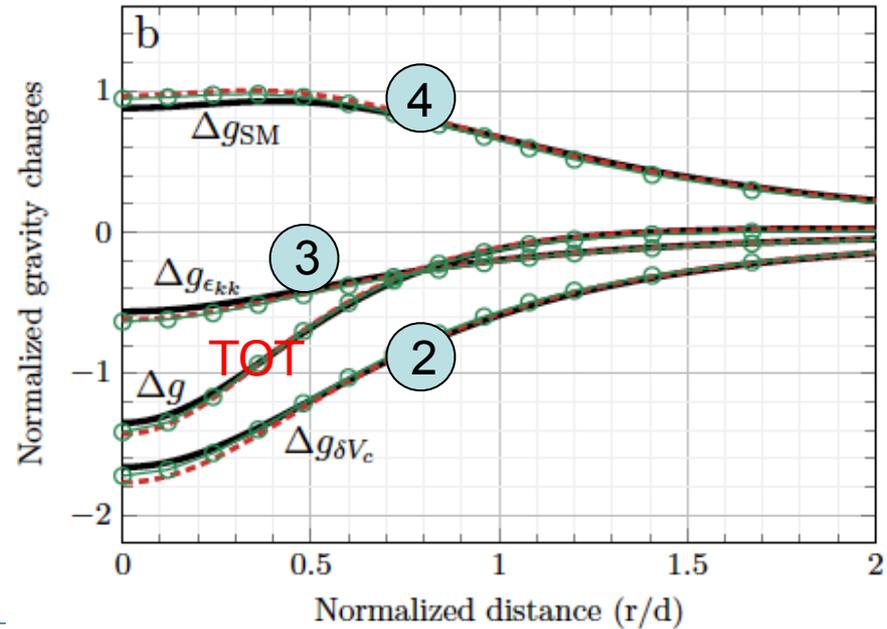
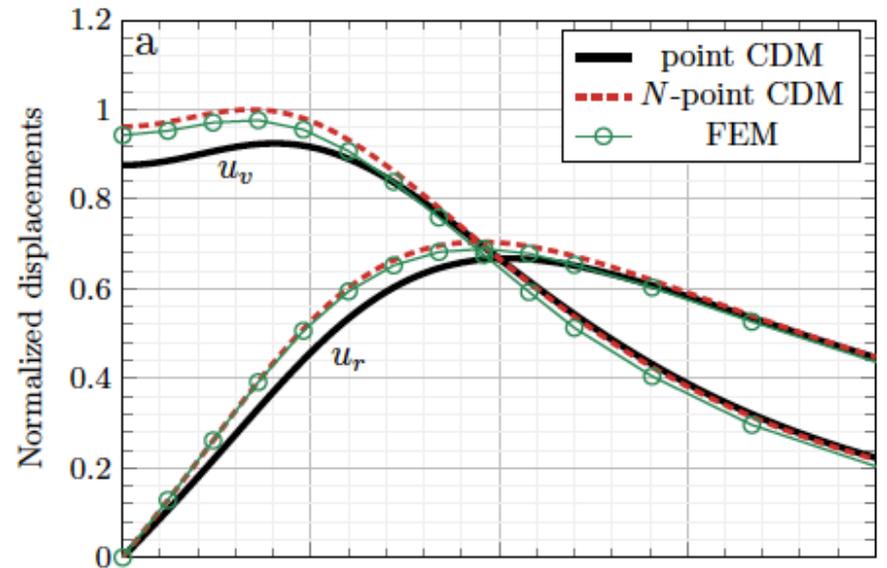
- 5 Δg_{FA} Free air
 - 4 Δg_{SM} Surface mass
 - 3 $\Delta g_{\epsilon_{kk}}$ Host-rock dilatation
 - 2 $\Delta g_{\Delta V}$ Interface volume change
 - 1 $\Delta g_{\delta m}$ Fluid mass
- $\int f(x)dx$

Comparison to published solutions and applications

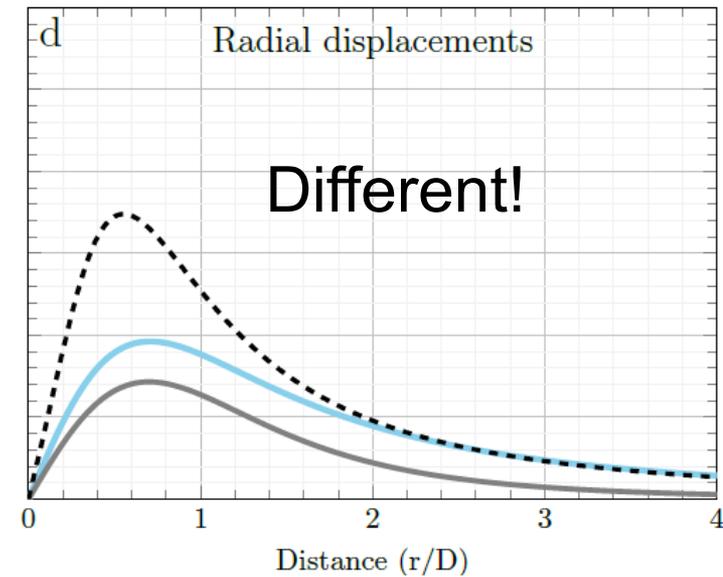
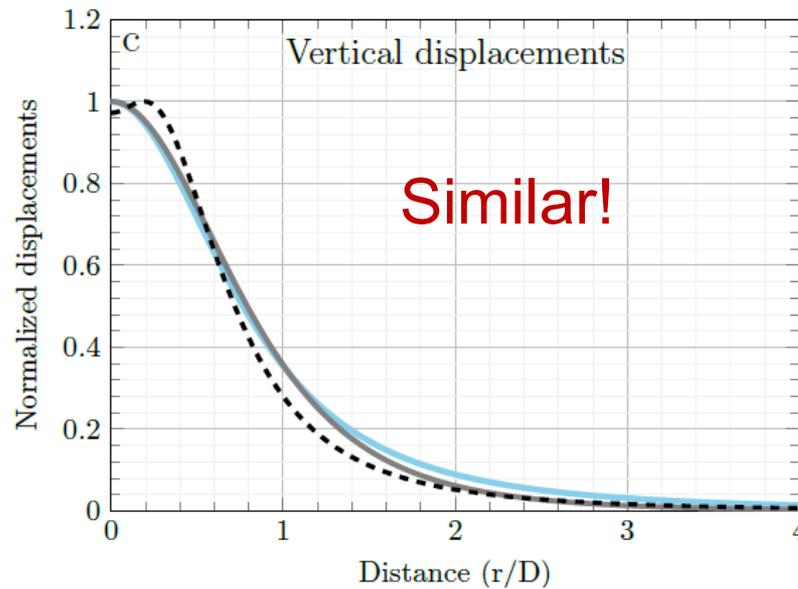
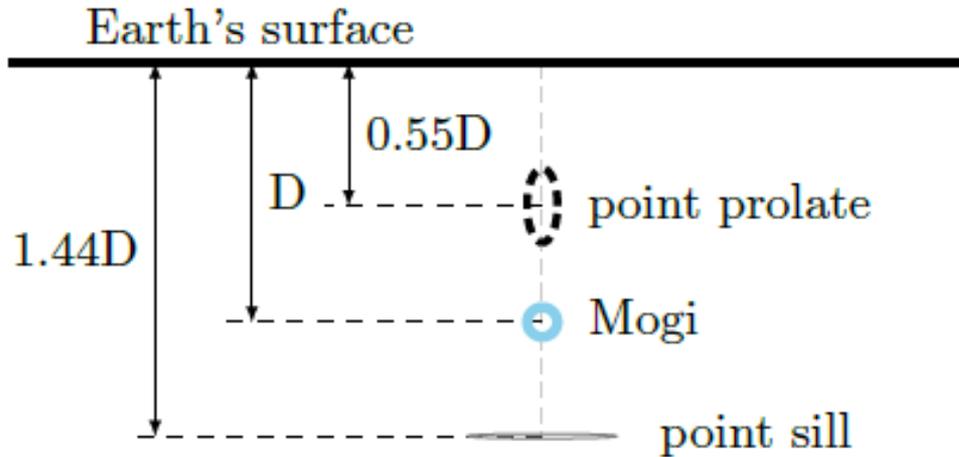
- FEM solution from:
Trasatti and Bonafede (2008)



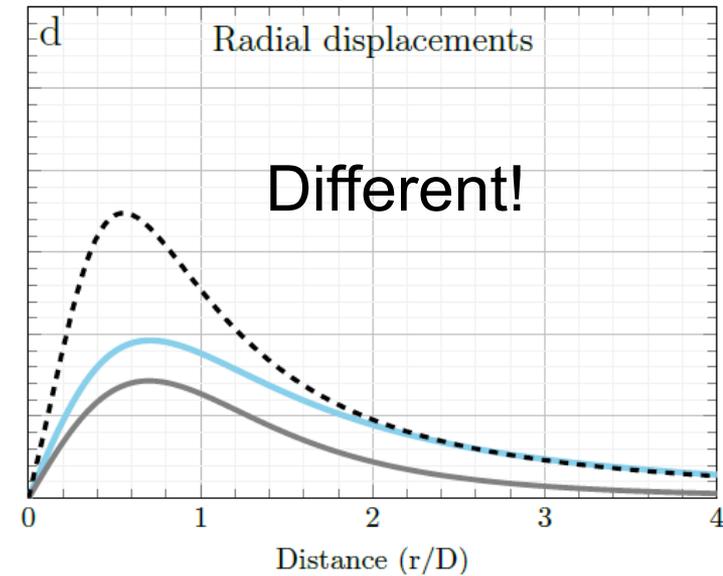
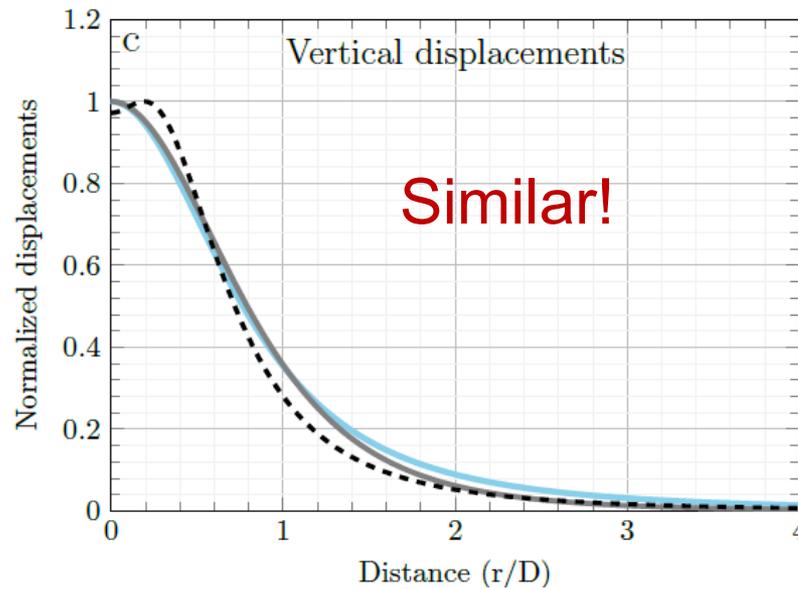
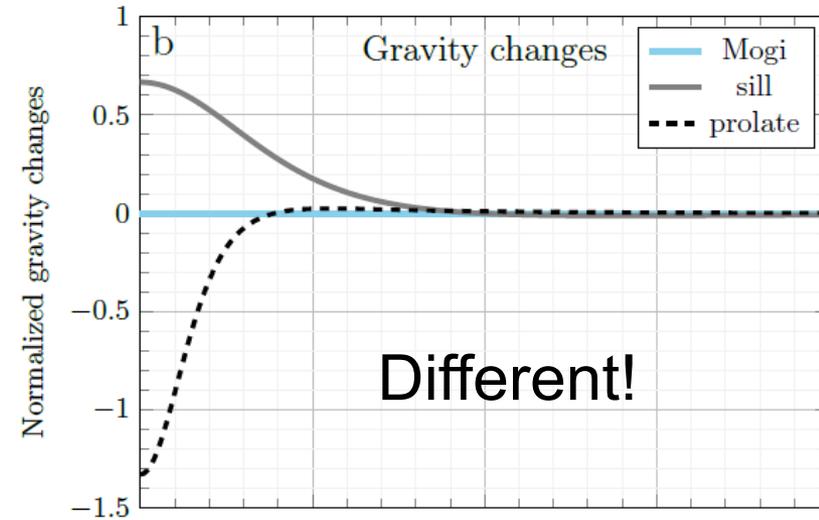
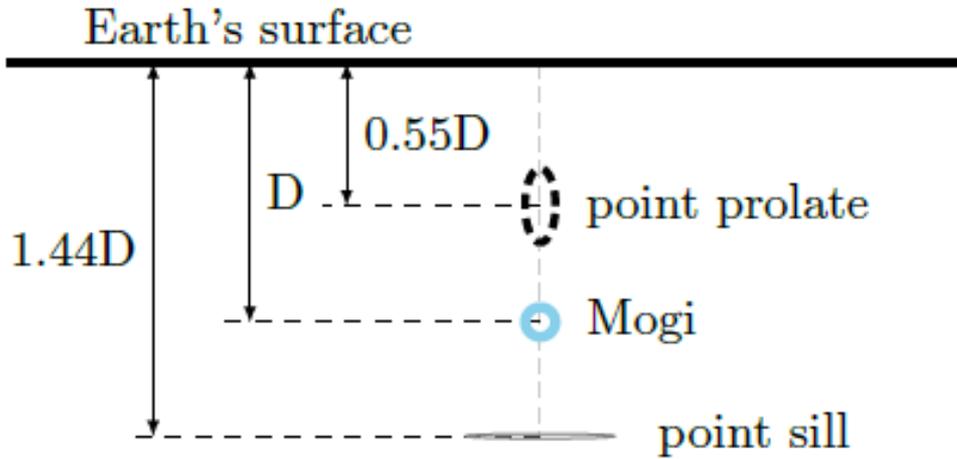
- ① ~~Fluid mass~~
- ② Volume change
- ③ Host-rock dilatation
- ④ Surface mass
- ⑤ ~~Free air~~



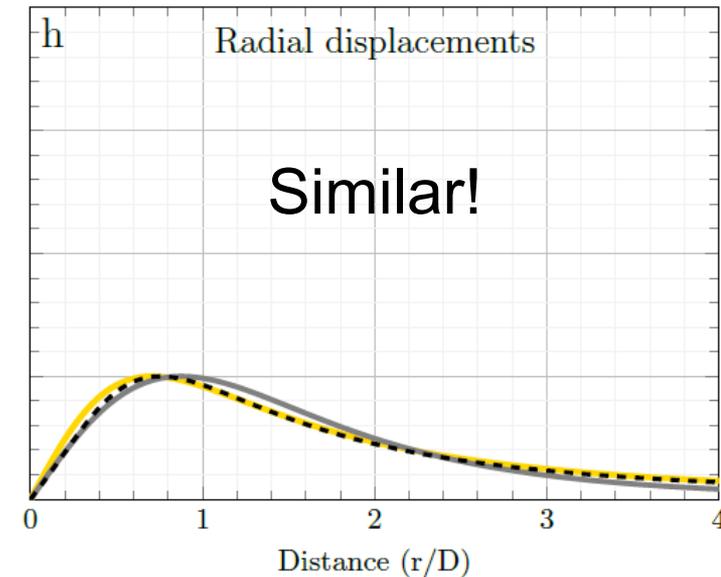
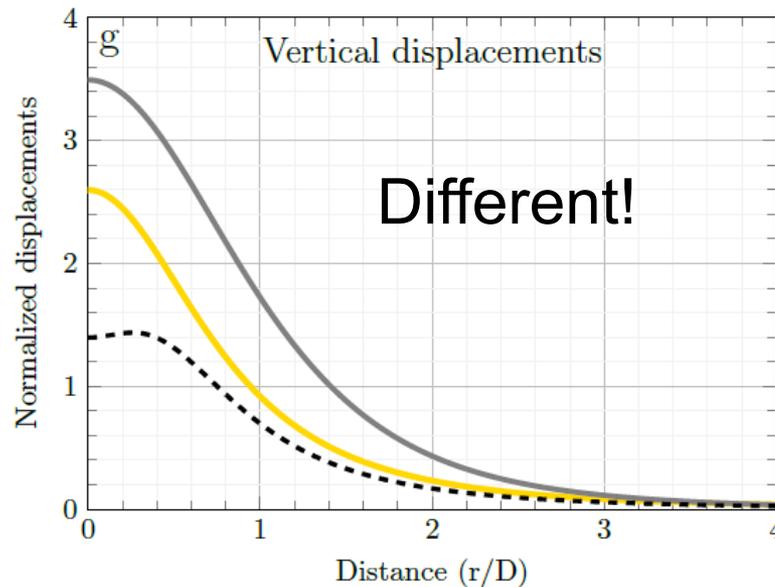
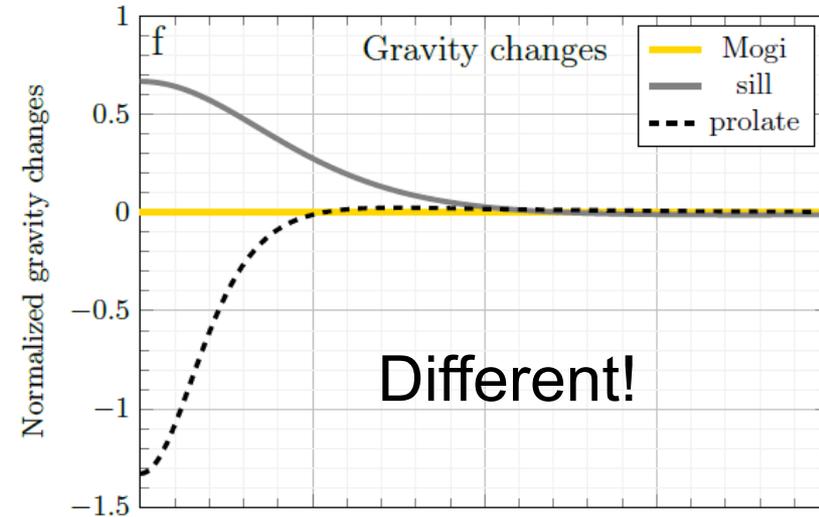
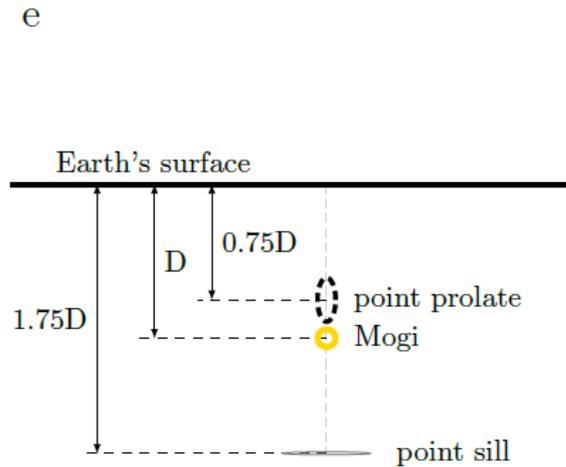
Coupled inversions of displacements and gravity changes



Coupled inversions of displacements and gravity changes



Coupled inversions of displacements and gravity changes



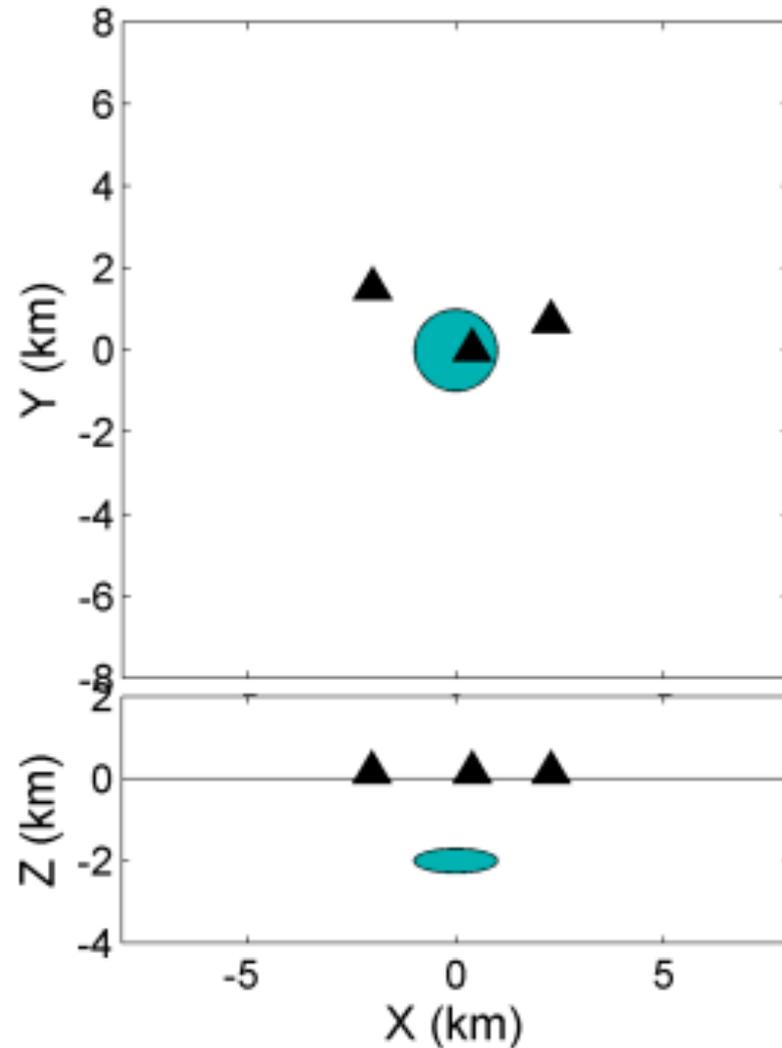
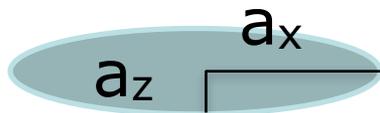
Coupled inversions of displacements and gravity changes

- Gravity changes contain information on both the mass changes and the deformation source parameters
- Gravity changes help better constrain all source parameters
- Coupled inversions of gravity changes and either **vertical** or **horizontal** displacements are possible

The issue of free air correction: A synthetic test

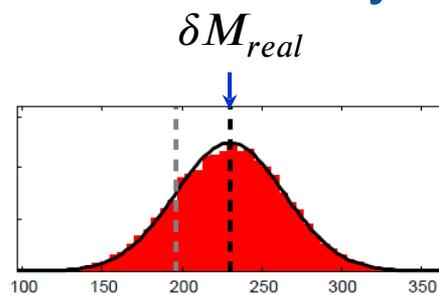
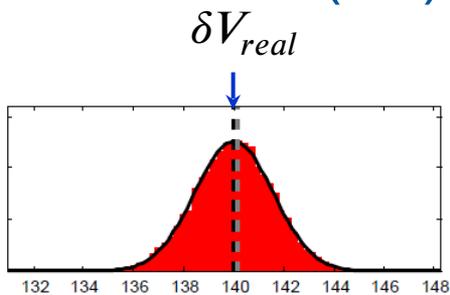
Errors

- Oblate source
- depth = 2000 m
- $a_x = 1000$ m
- $a_z = 300$ m
- $\sigma_x = \sigma_y = 15$ mm
- $\sigma_z = 45$ mm
- $\sigma_{\Delta g} = 2 \mu\text{Gal}$



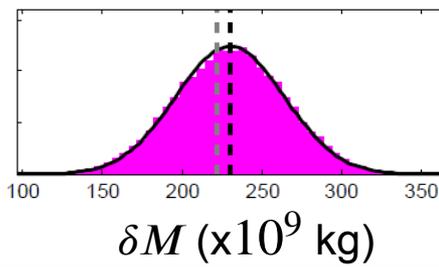
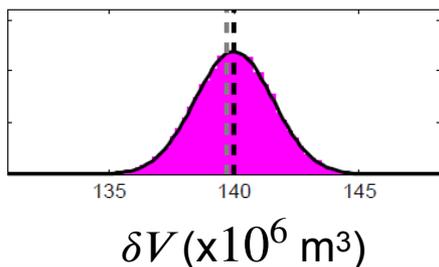
The issue of free-air (FA) correction: A synthetic test

FA + Inverted separately



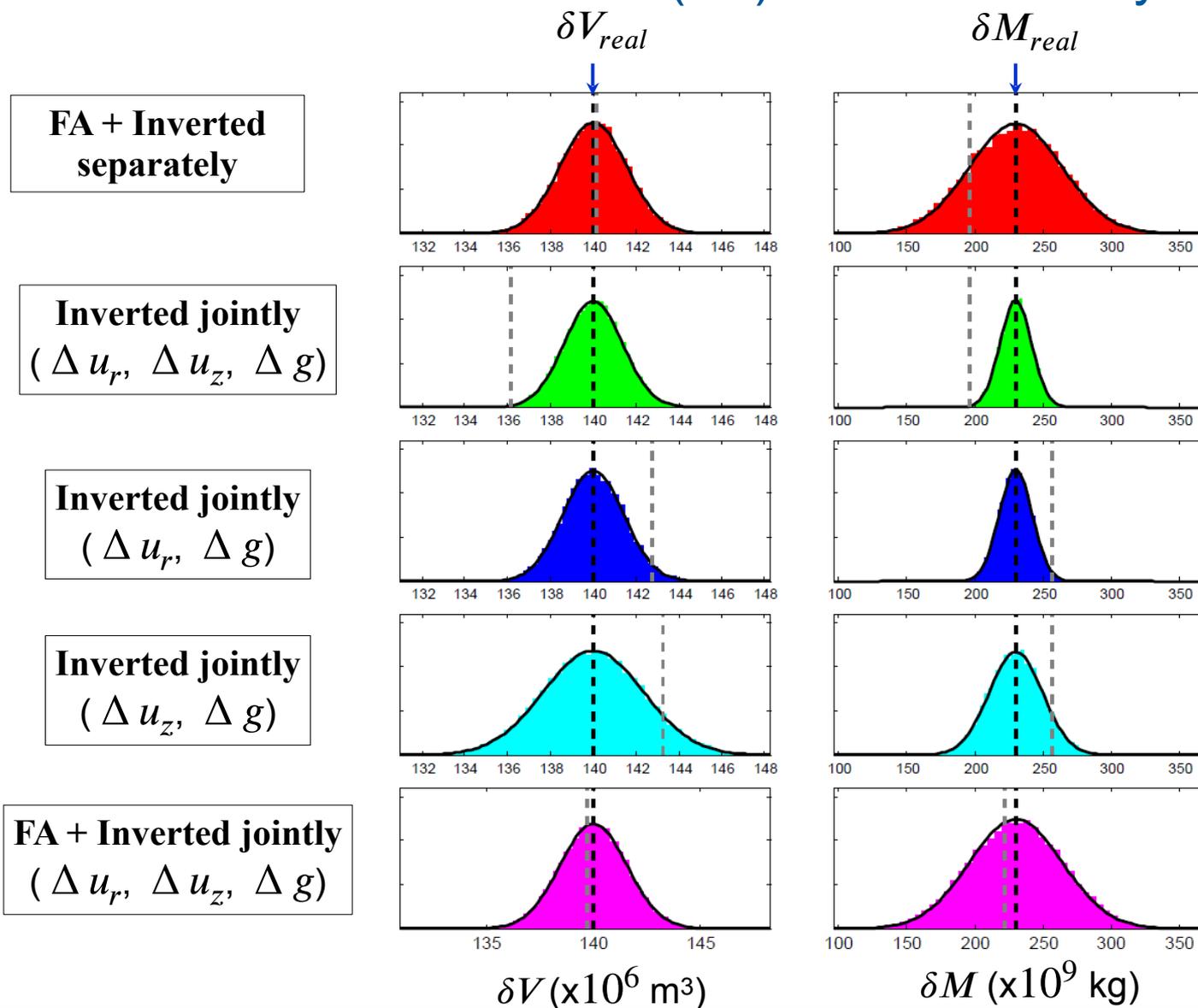
Standard procedure

FA + Inverted jointly
(Δu_r , Δu_z , Δg)



Amoruso & Crescentini (2008)

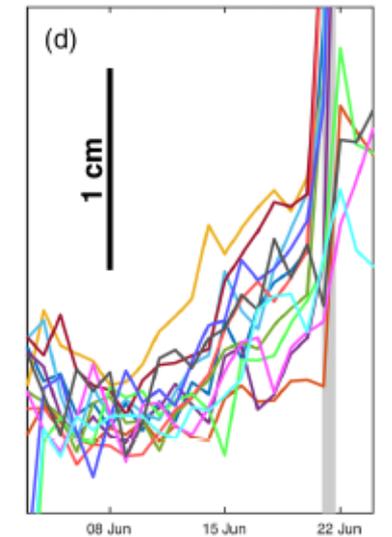
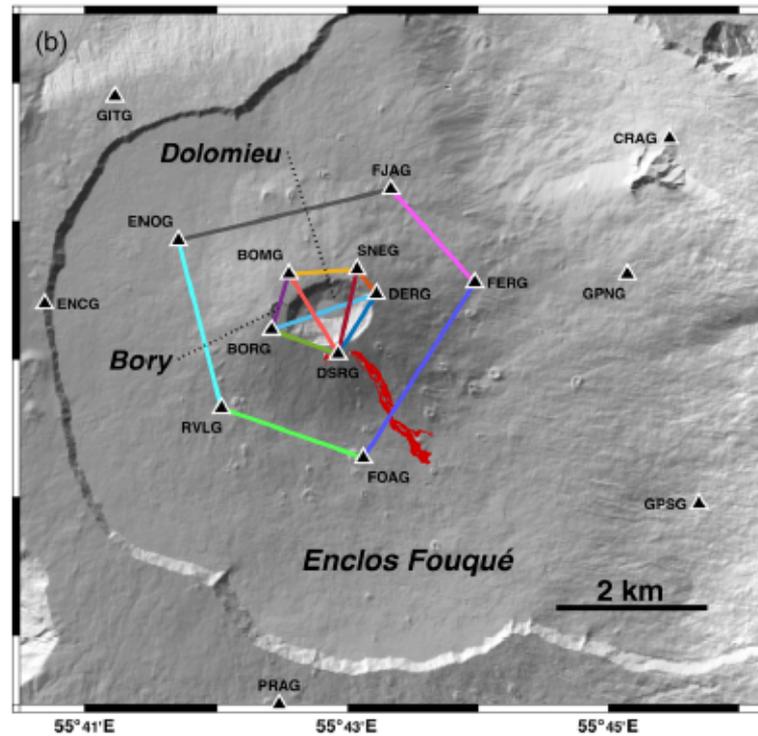
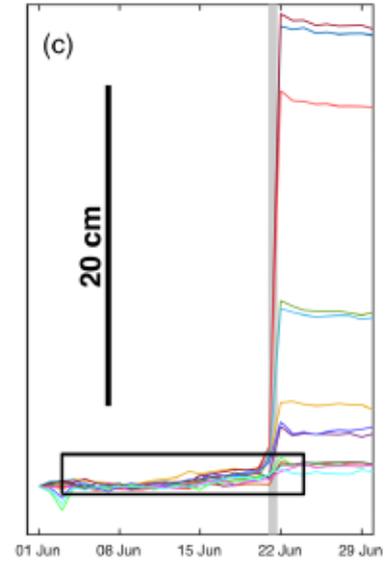
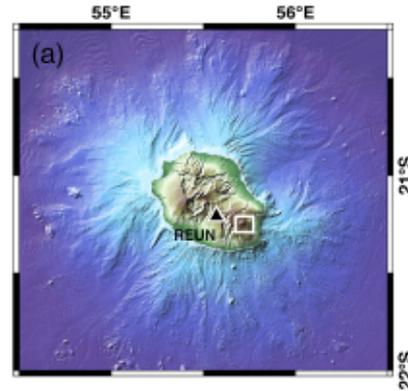
The issue of free-air (FA) correction: A synthetic test



Implications for hazard estimation

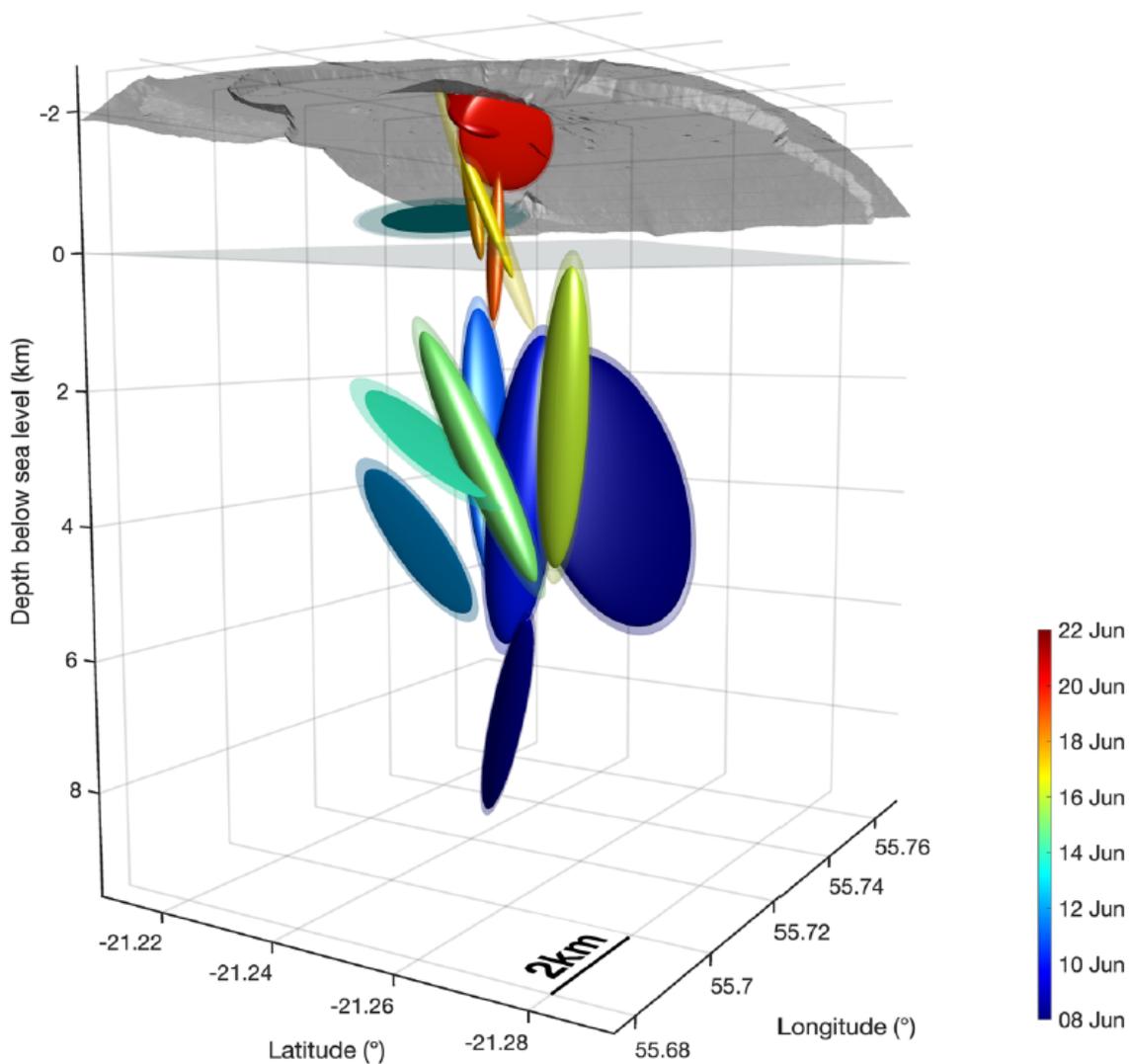
Mechanical imaging of a volcano plumbing system

Piton de la Fournaise
La Réunion Island



Beauducel et al. (2020)

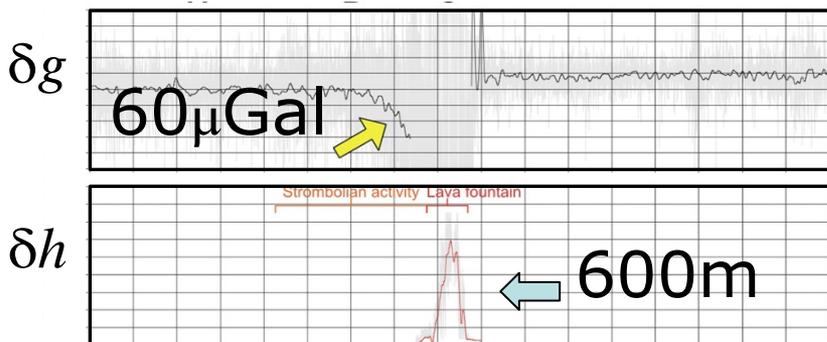
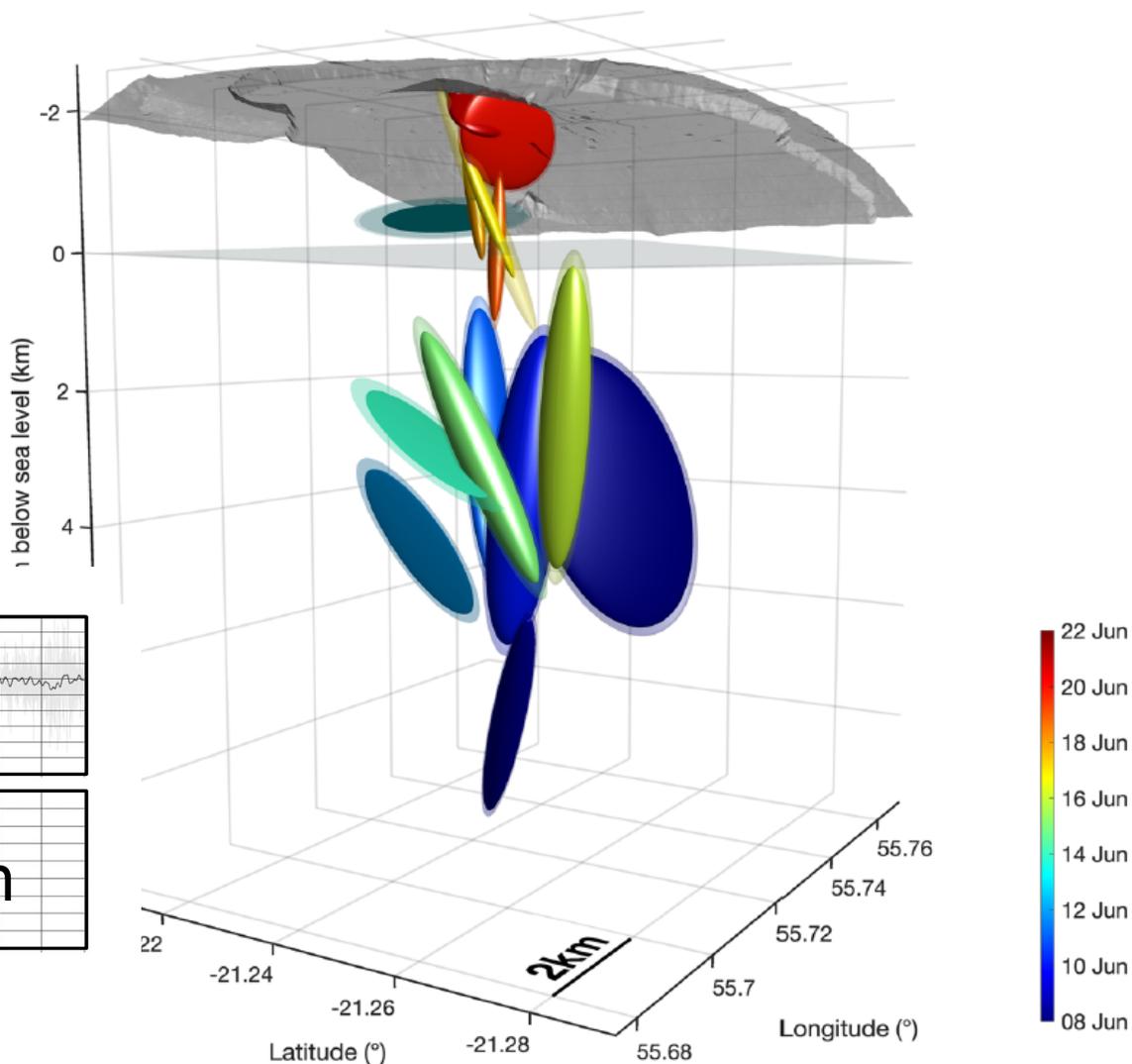
Mechanical imaging of a volcano plumbing system



Beauducel et al. (2020)

Mechanical imaging of a volcano plumbing system

By combining GNSS data and gravity changes, we can retrieve the evolving mass and density of the ascending magma batch, and thus forecast the eruption's size and style.



Beauducel et al. (2020)

Summary

- The point CDM allows for inferring the parameters of arbitrary triaxial deformation sources in real time
- Coupled inversions of surface displacements and gravity changes better exploit the gravity changes
- The new analytical solutions allow modelling of deformation-induced gravity changes and can help with process understanding and crisis management

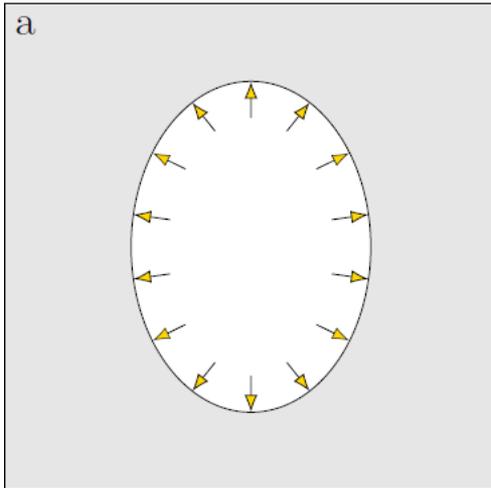
Thank you!

?

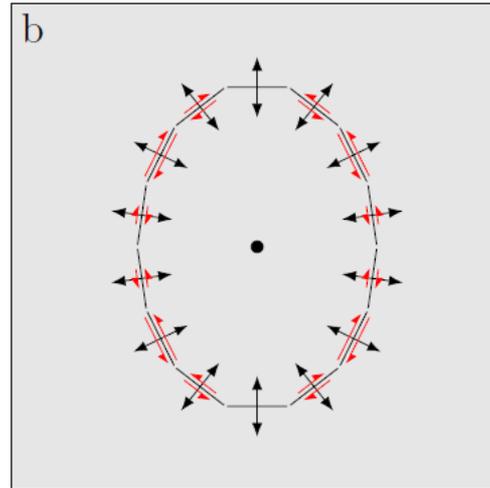
?

Volumetric sources: Why does the point CDM work well?

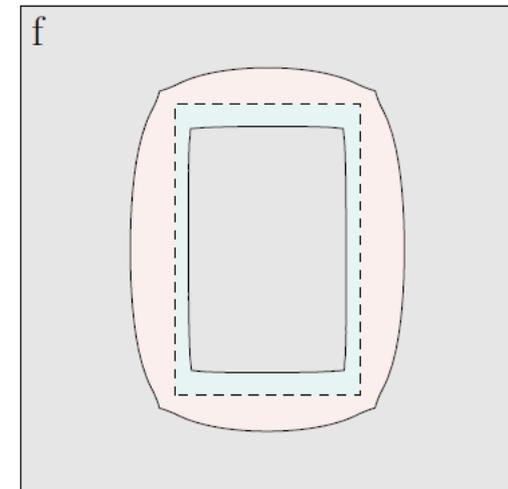
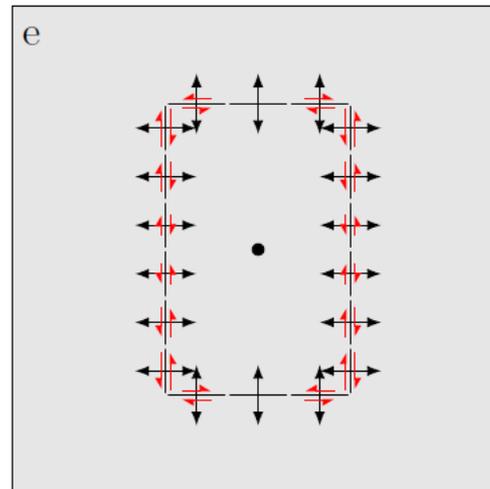
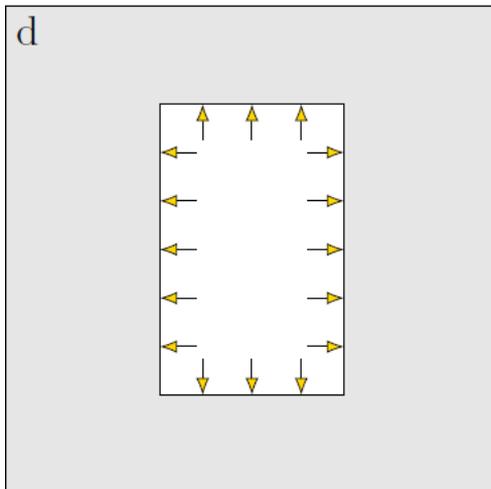
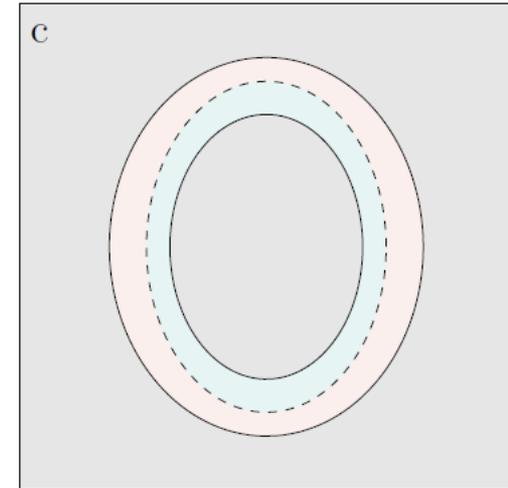
Initial pressurized cavity



Initial dislocation model

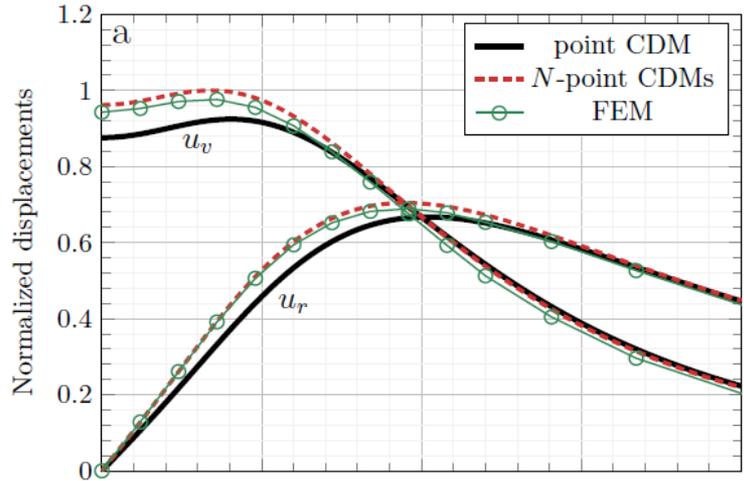


Final dislocation model

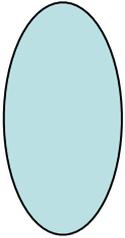
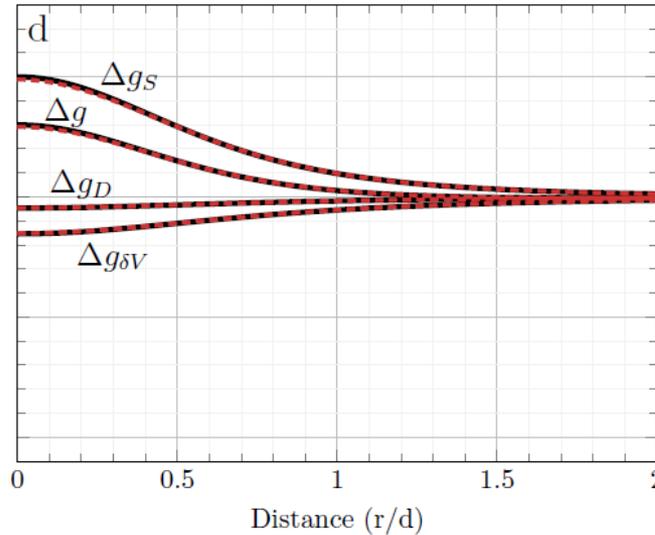
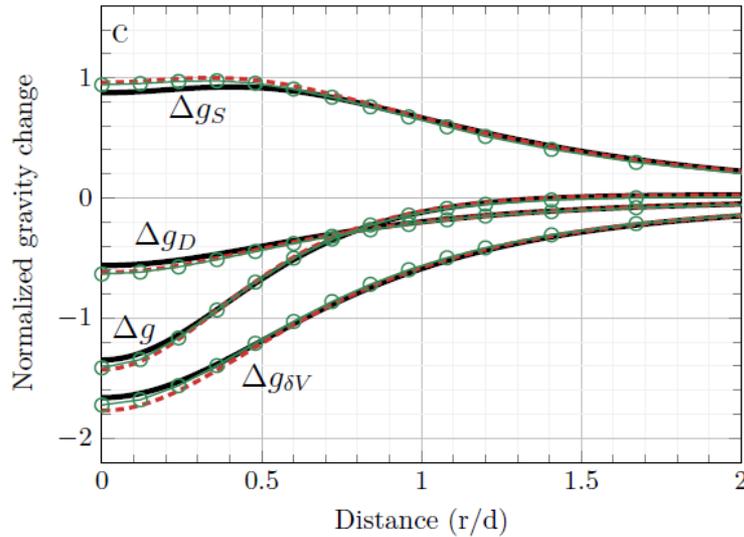
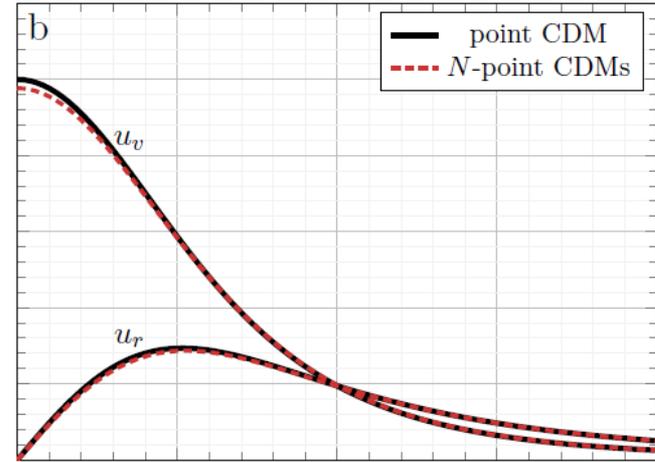


Deformation-induced gravity changes: benchmarking

Prolate ellipsoid

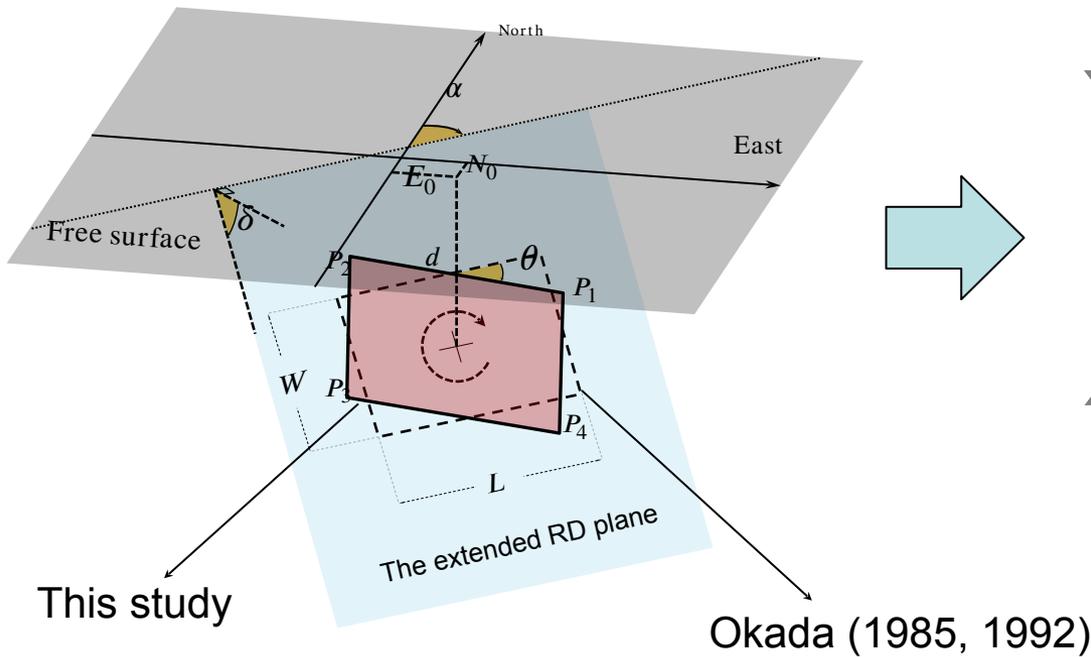


Oblate ellipsoid

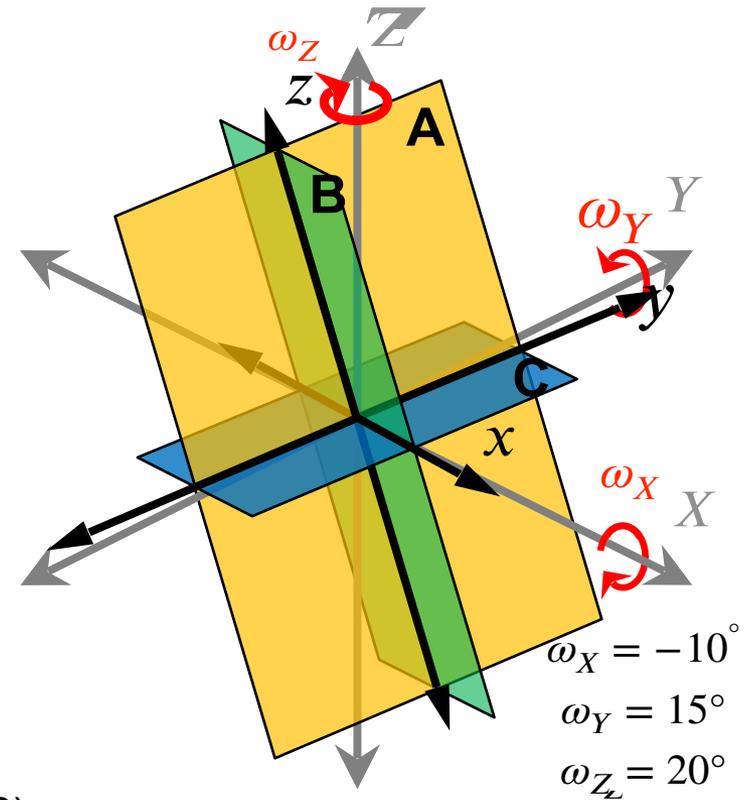


The compound dislocation model (CDM)

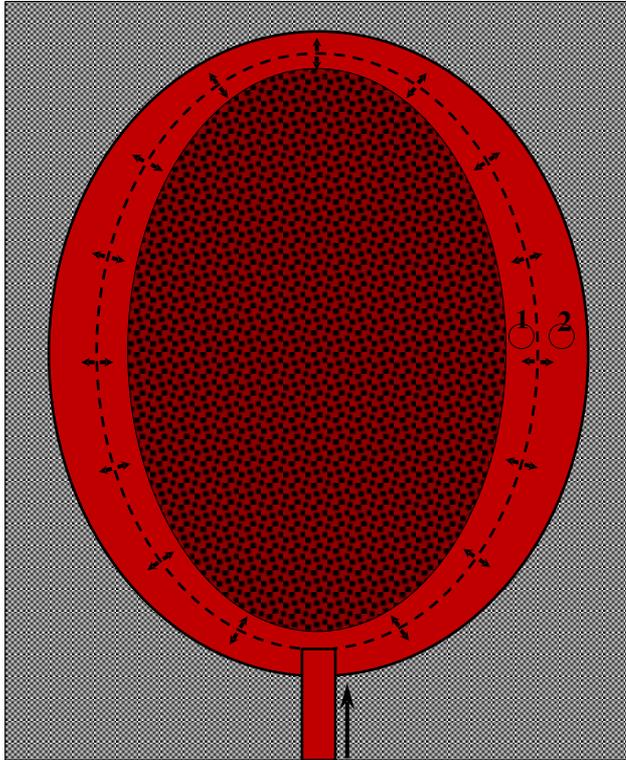
- Artefact-free solution with full rotational degrees of freedom



The CDM



The potency and volume change?



$$\Delta V^E = \delta V^E + \frac{pV}{K}$$

ΔV^E : Potency ① + ②

δV^E : Volume change ②

p : Pressure

V : Volume

K : Bulk modulus

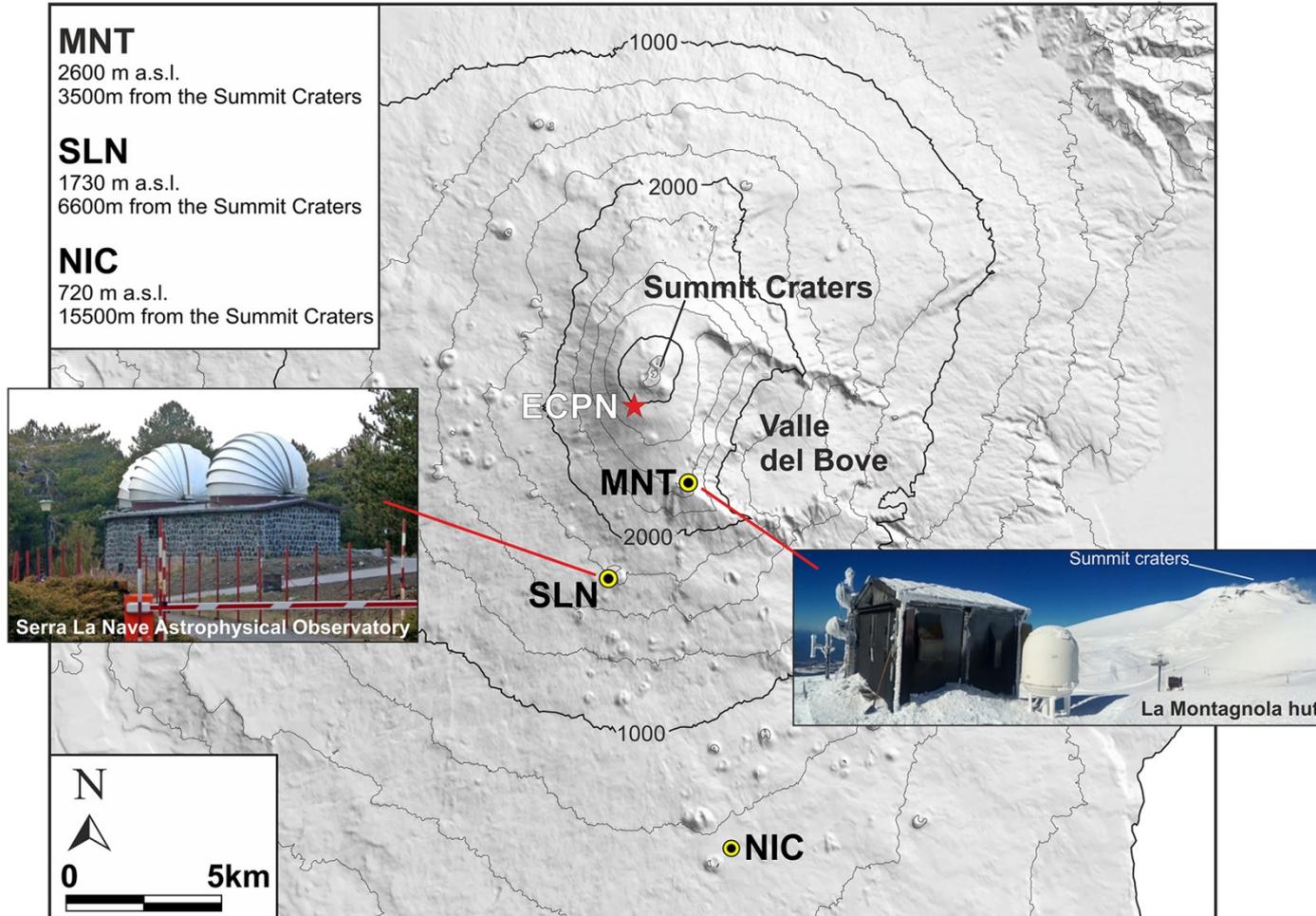
Two end members:

- For a Mogi source, if $\mu = \lambda$:
 $\Delta V = 1.8 \delta V$

- For a pressurized crack:
 $\Delta V = \delta V$

Motivation for volcano gravimetry: Mount Etna

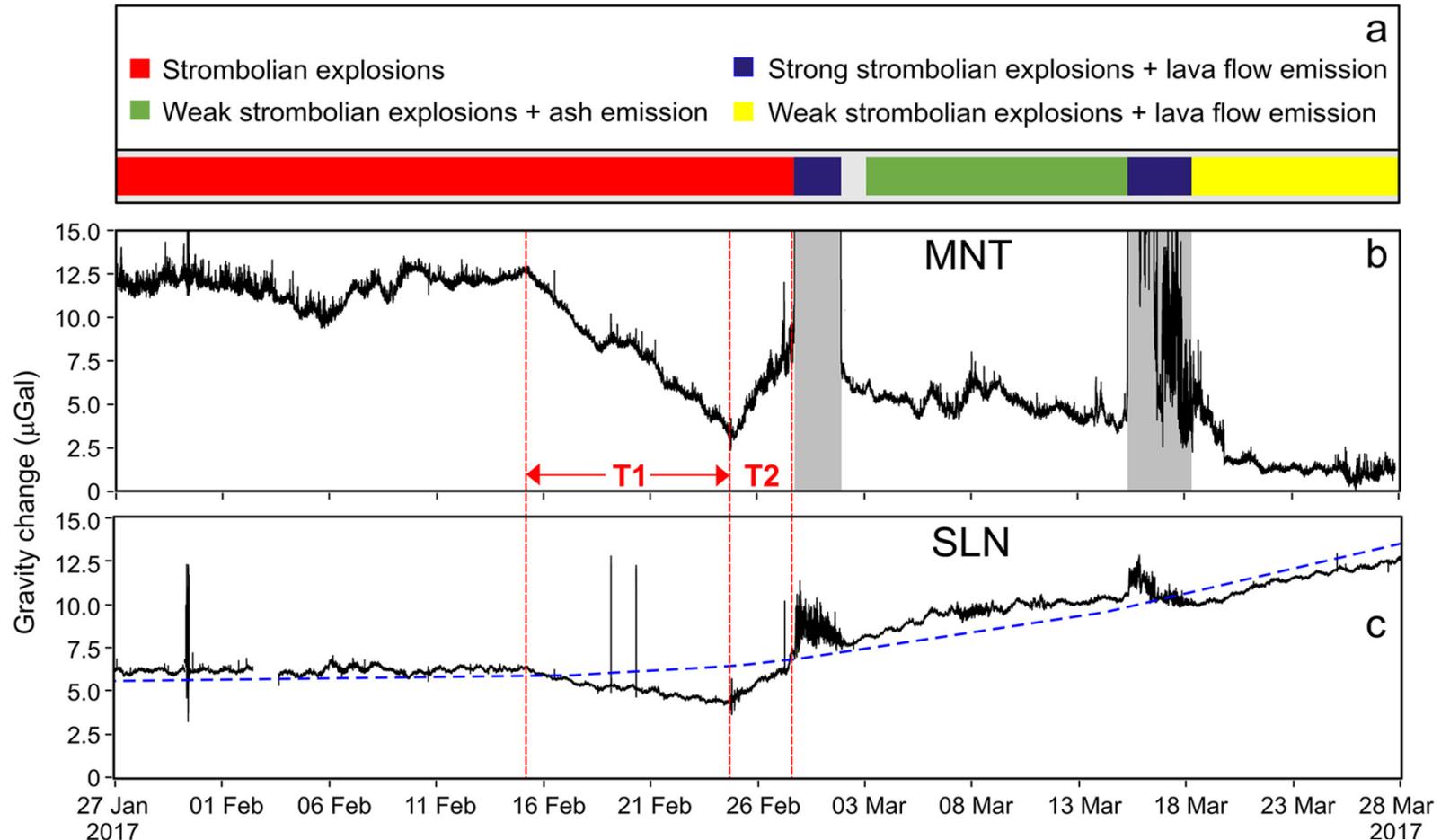
The Benefits of Using a Network of Superconducting Gravimeters to Monitor and Study Active Volcanoes



Carbone et al. (2017)

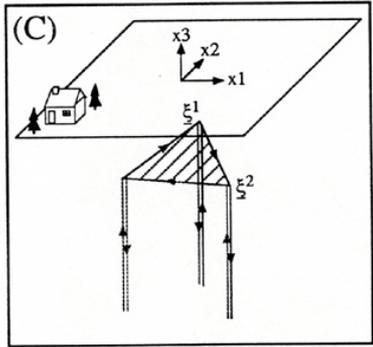
Motivation for volcano gravimetry: Mount Etna

- The early 2017 eruptive phase:

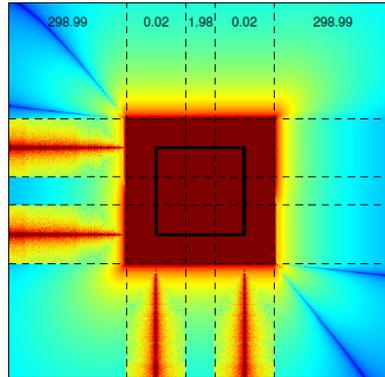


Carbone et al. (2017)

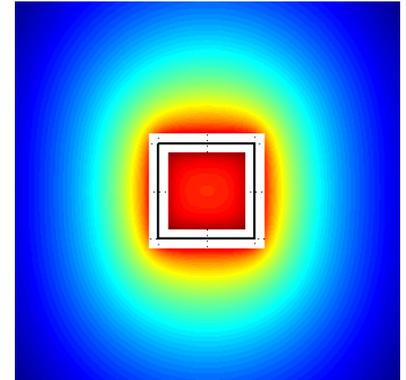
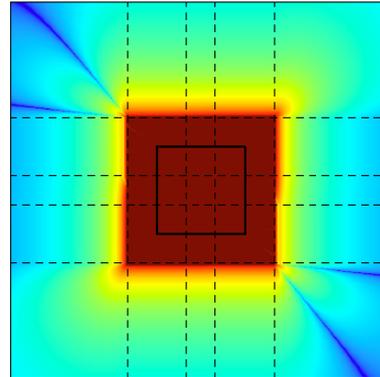
Artefact-singularities



Thomas (1993)

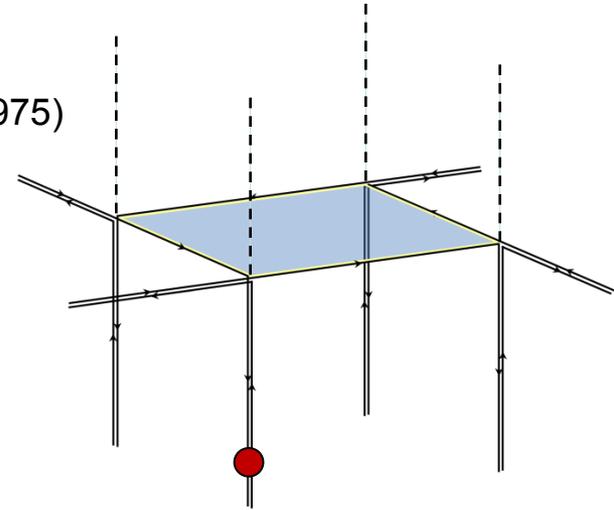


Bradley and Segall (2012)

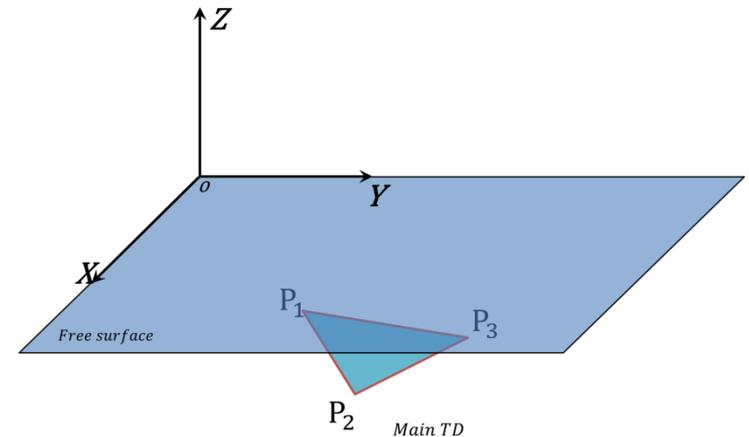


New solution for rectangular dislocations (RDs)

- Angular dislocations (Comninou and Dundurs 1975)
- Artefact singularities:
 - Along the sides
 - Underneath and above the vertices

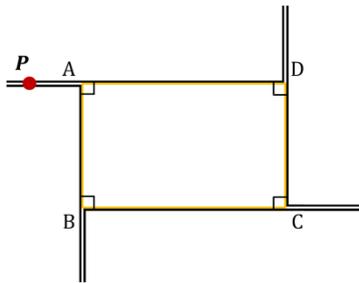


- Nikkhoo and Walter (2015) approach:
 - 1- Correct the full-space solution
 - 2- Correct the free surface term
 - 3- Superpose the three terms

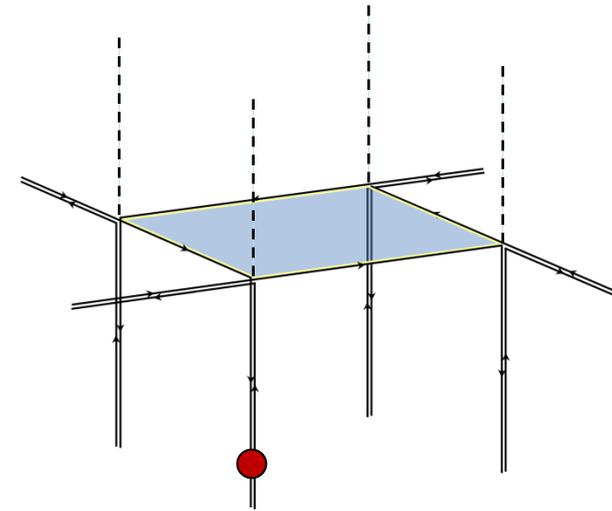
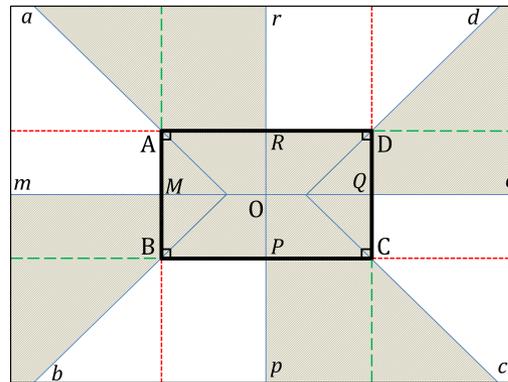
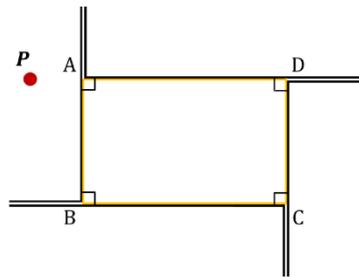


New solution for rectangular dislocations (RDs)

(a)

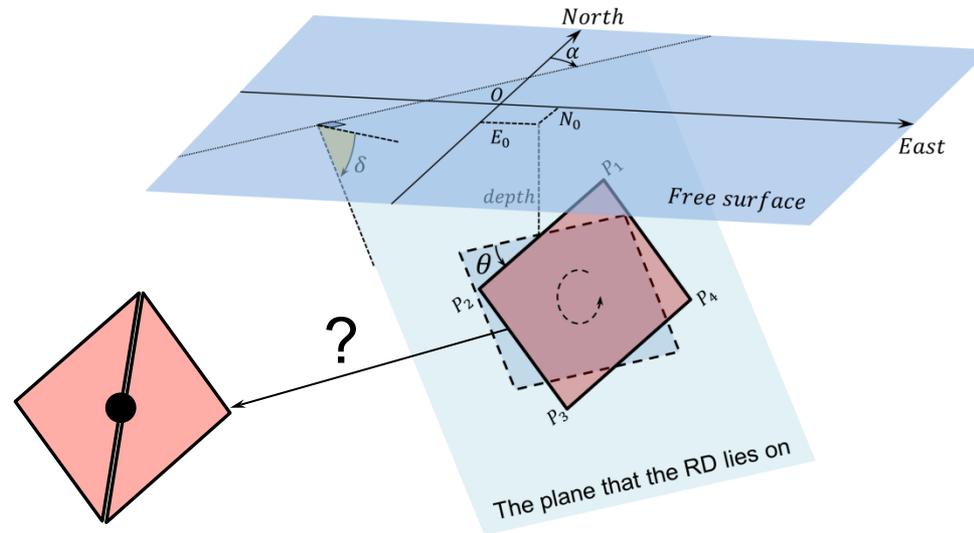


(b)



New solution for rectangular dislocations (RDs)

- Artefact-free solution with full rotational degrees of freedom



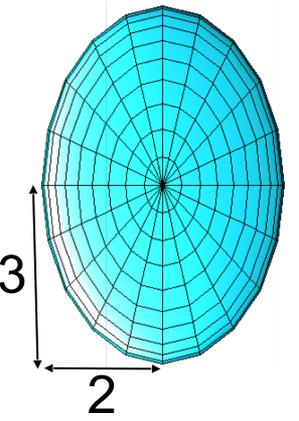
The CDM and point CDM comparison

Side view (XZ)

0.25



Top view (XY)



Depth to the centre = 3

