ITALIAN PHYSICAL SOCIETY
SOCIETÀ ITALIANA DI FISICA

107 National Congress
13-17 Settembre, 2021 (virtual)

SEZIONE 2
Fisica della materia

Metastability and signatures of topology in open quadratic bosonic dynamics

Lorenza Viola
Dept. Physics & Astronomy
Dartmouth

Vincent P. Flynn
Dartmouth

Emilio Cobanera
SUNY Poly & Dartmouth

NSF
SUNY POLYTECHNIC INSTITUTE
Quantum Information Science at Dartmouth
DARTMOUTH Guarini School of Graduate and Advanced Studies
DARTMOUTH

Prelude...

Challenge: To fully characterize the *roles of topology* in quantum phases of matter and light – naturally occurring and engineered, in equilibrium and beyond...

- Fundamental and [growing!] practical significance across quantum science – can topology
  - ➞ Provide an *organizing principle* for classification (beyond Landau's paradigm)?
  - ➞ Mandate the existence of *protected* edge/surface localized states with exotic properties?...
  - ➞ Enable new types of *device applications* (quantum sensors, amplifiers, lasers)?...
  - ➞ Point to novel approaches for *robust* quantum information storage and processing?...

Qi & Zhang, RMP **83** 2011... Chiu et al, ibid **88** 2016 ... Ozawa et al, ibid. **91** 2019

Prelude...

Challenge: To fully characterize the *roles of topology* in quantum phases of matter and light – naturally occurring and engineered, in equilibrium and beyond...

❖ Fundamental and [growing!] practical significance across quantum science – can topology

➞ Provide an *organizing principle* for classification (beyond Landau's paradigm)?

➞ Mandate the existence of *protected* edge/surface localized states with exotic properties?...

➞ Enable new types of *device applications* (quantum sensors, amplifiers, lasers)?...

➞ Point to novel approaches for *robust* quantum information storage and processing?...

Qi & Zhang, RMP **83** 2011... Chiu et al, ibid **88** 2016 ... Ozawa et al, ibid. **91** 2019

❖ Stepping stone: Symmetry-protected topological (SPT) phases

➞ Phases of particle-non-conserving *fully gapped*, *stable free-fermion* (mean-field) Hamiltonians

➞ Fully topologically classified ⇒ 'tenfold way'

- 1-1 correspondence between distinct ensembles of Bloch Hamiltonians and distinct SPT phases
- *Bulk-boundary correspondence:* Relation between bulk topological invariants and appearance of protected boundary zero modes...

TABLE VI. Symmetry classes that support topologically nontrivial line defects and their associated protected gapless modes.

Symmetry	Topological classes	1D gapless fermion modes
A	$\mathbb{Z}$	Chiral Dirac
D	$\mathbb{Z}$	Chiral Majorana
DIII	$\mathbb{Z}_2$	Helical Majorana
AII	$\mathbb{Z}_2$	Helical Dirac
C	$2\mathbb{Z}$	Chiral Dirac

Chiu et al, RMP **88** 2016

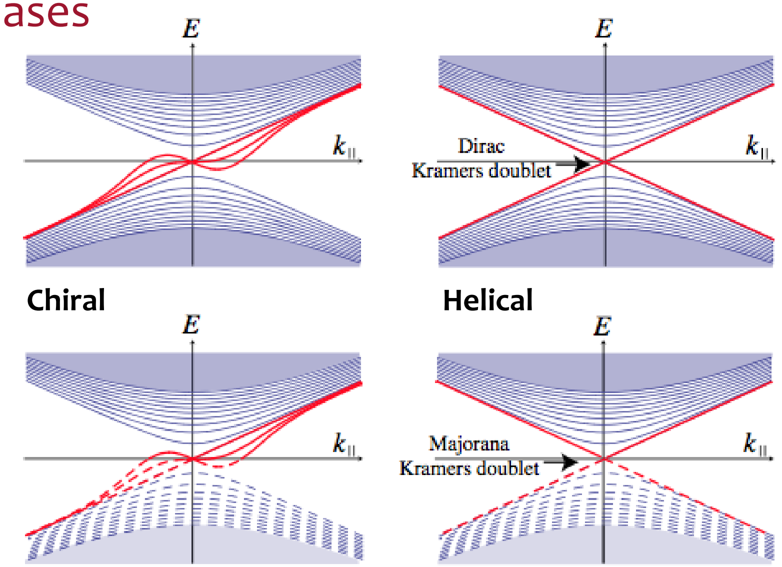
Prelude...

Challenge: To fully characterize the *roles of topology* in quantum phases of matter and light – naturally occurring and engineered, in equilibrium and beyond...

- Fundamental and [growing!] practical significance across quantum science – can topology
  - ➞ Provide an *organizing principle* for classification (beyond Landau's paradigm)?
  - ➞ Mandate the existence of *protected* edge/surface localized states with exotic properties?...
  - ➞ Enable new types of *device applications* (quantum sensors, amplifiers, lasers)?...
  - ➞ Point to novel approaches for *robust* quantum information storage and processing?...

Qi & Zhang, RMP **83** 2011... Chiu et al, ibid **88** 2016 ... Ozawa et al, ibid. **91** 2019

- Stepping stone: Symmetry-protected topological (SPT) phases
  - ➞ Phases of particle-non-conserving *fully gapped*, *stable free-fermion* (mean-field) Hamiltonians
  - ➞ Fully topologically classified ⇒ 'tenfold way'
    - 1-1 correspondence between distinct ensembles of Bloch Hamiltonians and distinct SPT phases
    - *Bulk-boundary correspondence:* Relation between bulk topological invariants and appearance of protected boundary zero modes...



Motivation: SPT phases of free bosons

Q1: What are equivalent results for *fully gapped, stable free-boson* Hamiltonians?
Q2: Are there topologically mandated bosonic zero modes?...

❖ No-go theorems can be proved for any thermodynamically stable (bounded from below) *quadratic bosonic Hamiltonian* (QBH)

$$H = H^\dagger \equiv \frac{1}{2}\Phi^\dagger \mathbf{H}\Phi, \quad \Phi \equiv [a_1, a_1^\dagger, \ldots, a_N, a_N^\dagger]^T, \quad \mathbf{H} = \mathbf{H}^\dagger, \mathbf{H} > 0$$

(1) *No parity switches:* The boson parity of the ground state is always *even*.

(2) *No non-trivial SPT phases:* All Hamiltonians are *adiabatically connected* regardless of the choice of protecting symmetries.

(3) *No localized zero modes:* The system subject to open boundary conditions *cannot* develop localized zero modes (zero is *not* in the spectrum).

Obstruction: *Stability constraint* $\mathbf{H} > 0$ implies many-body gap – no counterpart for fermions...

Xu, Flynn, Alase, Cobanera, LV & Ortiz, PRB **102** 2020.

Motivation: SPT phases of free bosons

Q1: What are equivalent results for *fully gapped, stable free-boson* Hamiltonians?
Q2: Are there topologically mandated bosonic zero modes?...

- No-go theorems can be proved for any thermodynamically stable (bounded from below) *quadratic bosonic Hamiltonian* (QBH)

$$H = H^\dagger \equiv \frac{1}{2}\Phi^\dagger \mathbf{H}\Phi, \quad \Phi \equiv [a_1, a_1^\dagger, \ldots, a_N, a_N^\dagger]^T, \quad \mathbf{H} = \mathbf{H}^\dagger, \mathbf{H} > 0$$

(1) *No parity switches:* The boson parity of the ground state is always *even*.

(2) *No non-trivial SPT phases:* All Hamiltonians are *adiabatically connected* regardless of the choice of protecting symmetries.

(3) *No localized zero modes:* The system subject to open boundary conditions *cannot* develop localized zero modes (zero is *not* in the spectrum).

Obstruction: *Stability constraint* $\mathbf{H} > 0$ implies many-body gap – no counterpart for fermions...

Xu, Flynn, Alase, Cobanera, LV & Ortiz, PRB **102** 2020.

- Non-trivial manifestations of topology *can* emerge in equilibrium if one allows for gapless phases, strong (beyond mean-field) interactions, or instabilities...

Ozawa et al, RMP **91** 2019; Chen et al, Science **338** 2012, Bardyn & Imamoglu, PRL **109** 2012; Peano et al, Nat. Comm. **7** 2016, McDonald et al, PRX **9** 2018; Flynn, Cobanera & LV, NJP **22** 2020...

Topology beyond closed systems…

❖ Topological phenomena have been – and increasingly are being – characterized away from equilibrium, in the context of non-unitary, *open quantum dynamics*:

➡ *Semiclassical* – Non-Hermitian effective Hamiltonians…

Minganti et al, PRA **100** 2019… McDonald & Clerk, Nat. Comm. **11** 2020;
Bergholtz et al, RMP **93** 2021; Liu et al, arXiv 2104.07335…

➡ *Fully quantum* – Markovian dynamics, via Lindblad or input-output formalism…

Diehl et al, Nat. Phys. **7** 2011; Bardyn & Imamoglu, PRL **109** 2012;
Porras & Fernandez-Lorenzo, PRL **122** 2019; Wanjura et al Nat. Comm. **11** 2020…

Topology beyond closed systems...

❖ Topological phenomena have been – and increasingly are being – characterized away from equilibrium, in the context of non-unitary, *open quantum dynamics*:

➞ *Semiclassical* – Non-Hermitian effective Hamiltonians...

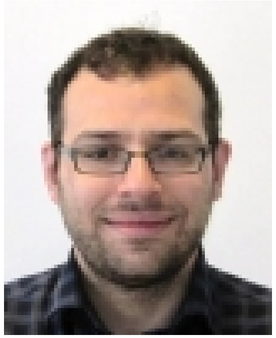
Minganti et al, PRA **100** 2019... McDonald & Clerk, Nat. Comm. **11** 2020; Bergholtz et al, RMP **93** 2021; Liu et al, arXiv 2104.07335...

➞ *Fully quantum* – Markovian dynamics, via Lindblad or input-output formalism...

Diehl et al, Nat. Phys. **7** 2011; Bardyn & Imamoglu, PRL **109** 2012; Porras & Fernandez-Lorenzo, PRL **122** 2019; Wanjura et al Nat. Comm. **11** 2020...

❖ Outline: Q1': Are there SPT phases for non-interacting dissipative bosonic systems?
Q2': Are there topologically mandated analogues of Majorana fermions?...

➞ Case study: Driven-dissipative Markovian dynamics, described by Lindblad master equation ⇒ *Quadratic Bosonic Lindbladian* (QBLs)



Key insight: Shift focus from steady-state to *transient* behavior ⇒ *Metastable QBLs*

Flynn, Cobanera & LV, ''Topology by dissipation: Majorana bosons in metastable quadratic Markovian dynamics,'' arXiv:2104.03985.

Quadratic bosonic Lindbladians

- For a QBL on N-mode bosonic Fock space, the Hamiltonian and dissipative components are *at most quadratic* in the canonical annihilation/creation operators:

$$\dot{\rho}(t) = \mathcal{L}(\rho(t)), \quad \dot{A}(t) = \mathcal{L}^\star(A(t)) \qquad \mathcal{L} \equiv \mathcal{L}(H, \{L_k\})$$

$$\dot{A}(t) = i[H, A(t)] + \sum_{i,j=1}^{2N} \mathbf{M}_{ij} \left(\Phi_i^\dagger A(t) \Phi_j - \frac{1}{2} \left\{ \Phi_i^\dagger \Phi_j, A(t) \right\} \right)$$

➡ Equations of motion of *linear* observables are determined by the *dynamical matrix*

$$\mathbf{G} \equiv \tau_3 \mathbf{H} - i\tau_3(\mathbf{M} - \tau_1 \mathbf{M}^T \tau_1)/2$$

Quadratic bosonic Lindbladians

- For a QBL on N-mode bosonic Fock space, the Hamiltonian and dissipative components are *at most quadratic* in the canonical annihilation/creation operators:

$$\dot{\rho}(t) = \mathcal{L}(\rho(t)), \quad \dot{A}(t) = \mathcal{L}^{\star}(A(t)) \qquad \mathcal{L} \equiv \mathcal{L}(H, \{L_k\})$$

$$\dot{A}(t) = i[H, A(t)] + \sum_{i,j=1}^{2N} \mathbf{M}_{ij} \left(\Phi_i^{\dagger} A(t) \Phi_j - \frac{1}{2} \left\{ \Phi_i^{\dagger} \Phi_j, A(t) \right\} \right)$$

➞ Equations of motion of *linear* observables are determined by the *dynamical matrix*

$$\mathbf{G} \equiv \tau_3 \mathbf{H} - i\tau_3(\mathbf{M} - \tau_1 \mathbf{M}^T \tau_1)/2$$

- A QBL is *dynamically stable* if expectation value of *any* observable in any state is bounded $\forall t$.

➞ If $\mathbf{G}$ is diagonalizable, $\mathcal{L}$ is as well, and its spectrum follows from the *rapidity spectrum*,

$$\sigma(-i\mathbf{G}) \equiv \{\mathrm{Eig}(-i\mathbf{G})\}$$

➞ Sufficient condition for stability: $-i\mathbf{G}$ is a *Hurwitz matrix*, i.e., $\mathrm{Re}(\sigma(-i\mathbf{G})) < 0$. The dynamics then converges to a *unique, Gaussian steady state*, $\mathcal{L}(\rho_{\rm ss}) = 0$.

Prosen, NJP **10** 2008; Prosen & Seligman, JPA **43** 2010.

➞ For a stable QBL, the convergence to the steady state is *exponential*, with worst-case distance upper-bounded by the *mixing time*:

$$d_{\max}(t) \equiv \sup_{\rho(0)} ||\rho(t) - \rho_{\rm ss}|| \leq K e^{-\Delta_{\mathcal{L}} t}, \quad d_{\max}(t_{\rm mix}(\delta)) < \delta$$

Quadratic bosonic Lindbladians: Model systems

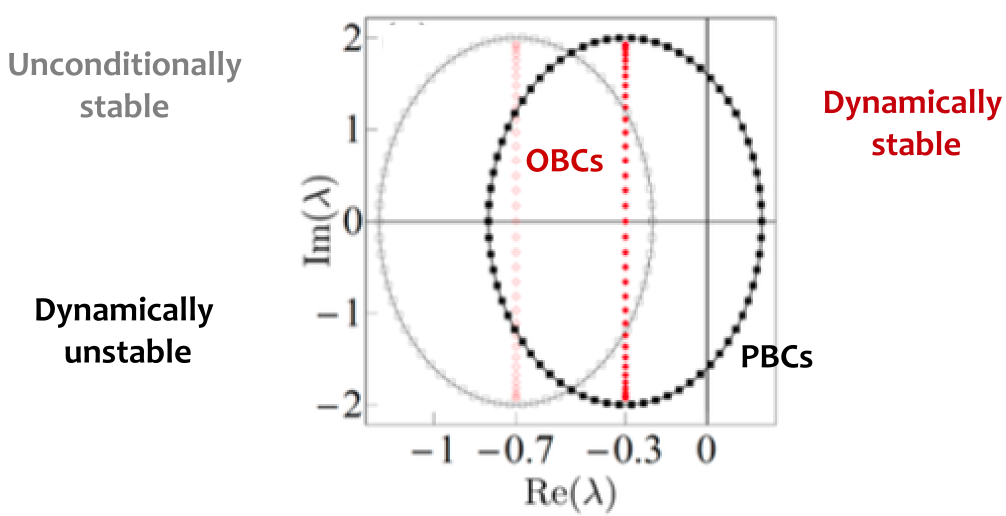
- Focus on one-dim QBLs, that are *translation-invariant* up to boundary conditions (BCs):
  - ➞ The dynamical matrix $\mathbf{G}$ is *block-Toeplitz* under OBCs, and *block-circulant* under PBCs.
  - ➞ The rapidities for PBC form closed curves, *rapidity bands*, as $N \to \infty$.

- Illustrative setting: Bosonic Kitaev chain with Markovian dissipation

$$H_{\text{BKC}} = \frac{i}{2}\sum_{j=1}^{N-1}\left(J a_{j+1}^\dagger a_j + \Delta a_{j+1}^\dagger a_j^\dagger\right) + \frac{i\mu}{2}\sum_{j=1}^{N}(a_j^\dagger)^2 + \text{H.c.}$$

J: NN hopping
Δ: Non-degenerate PA
μ: Degenerate PA

- ➞ This QBH is *never thermodynamically stable*. Construct two QBLs that differ in the dissipator:
  - ➢ **Model 1**: Add uniform onsite dissipation to the BKC, with strength $\kappa > 0$;



Quadratic bosonic Lindbladians: Model systems

❖ Focus on one-dim QBLs, that are translation-invariant up to boundary conditions (BCs):

- ➞ The dynamical matrix $\mathbf{G}$ is *block-Toeplitz* under OBCs, and *block-circulant* under PBCs.
- ➞ The rapidities for PBC form closed curves, *rapidity bands*, as $N \to \infty$.

❖ Illustrative setting: Bosonic Kitaev chain with Markovian dissipation

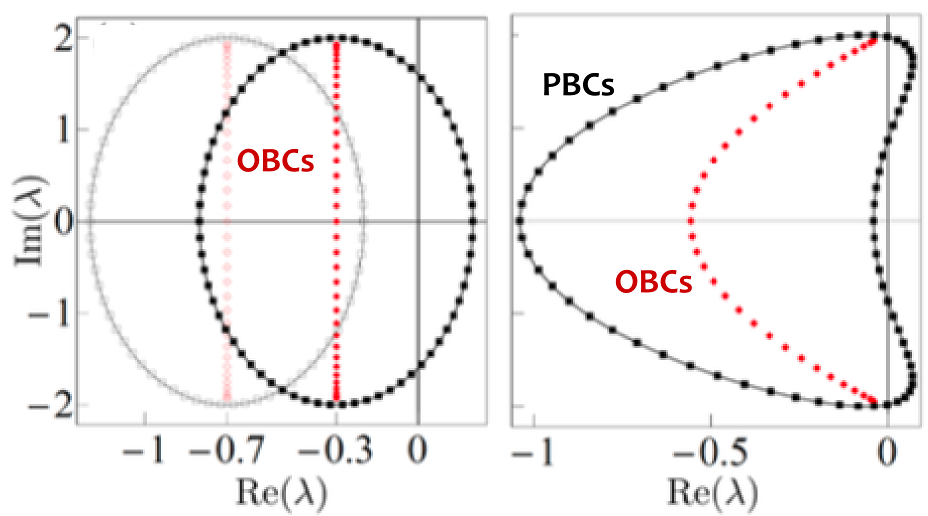
$$H_{\text{BKC}} = \frac{i}{2}\sum_{j=1}^{N-1}\left(Ja^\dagger_{j+1}a_j + \Delta a^\dagger_{j+1}a^\dagger_j\right) + \frac{i\mu}{2}\sum_{j=1}^{N}(a^\dagger_j)^2 + \text{H.c.}$$

J: NN hopping
Δ: Non-degenerate PA
μ: Degenerate PA

- ➞ This QBH is *never thermodynamically stable*. Construct two QBLs that differ in the dissipator:
  - ➢ **Model 1**: Add uniform onsite dissipation to the BKC, with strength $\kappa > 0$;
  - ➢ **Model 2**: Add next-nearest-neighbor dissipation to Model 1, with strength $\Gamma > \kappa/2$.

Unconditionally stable

Dynamically unstable



Dynamically stable

Dynamically unstable
<u>No winding around 0</u>

Metastable QBLs

- If a QBL is dynamically stable for OBCs and unstable for PBCs ⇒ OBC system is *metastable.*
  - ➞ Dynamical metastability need not be captured by spectral properties since $\mathbf{G}$ *is not normal* ⇒ Appropriate mathematical tool is provided by *pseudospectra*:

$$\sigma_\epsilon(\mathbf{X}) \equiv \{\lambda \in \mathbb{C} : \exists \vec{v},\ ||v|| = 1,\ ||(\mathbf{X} - \lambda \mathbf{1})\vec{v}|| < \epsilon\}$$

Trefethen & Embree, *Spectra and Pseudospectra* (2005).

  - ➞ The semi-open (infinite-size) limit of a metastable QBL is *dynamically unstable.*

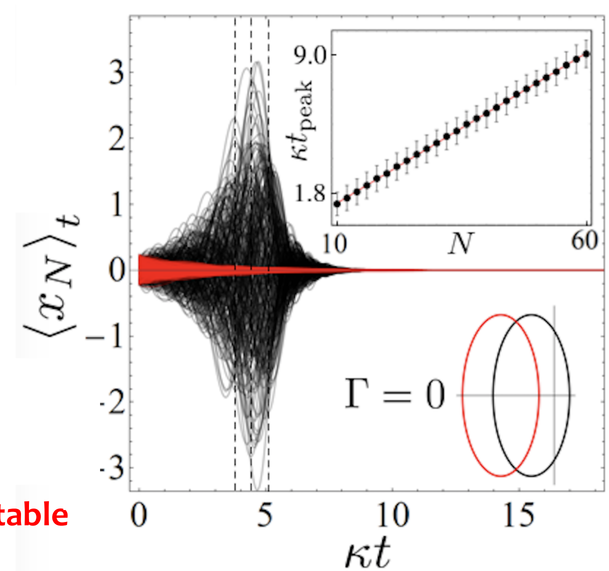
- Dynamical metastability elicits distinctive features in the relaxation dynamics:

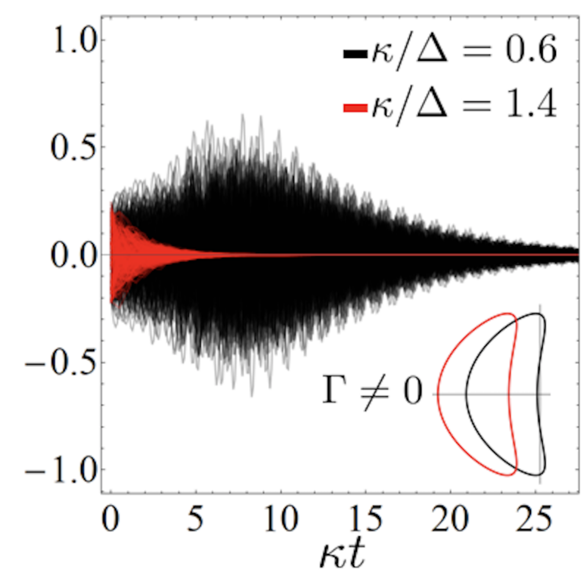
(1) *Divergent mixing times:* For a metastable QBL, the mixing time diverges with system size...

$$t_{\mathrm{mix}}(\delta, N) \gtrsim t_{\mathrm{lin}}(\delta, N) \sim O(N)$$

(2) *Transient directional amplification:* Observable expectations may be exponentially amplified...

Metastable
Unconditionally stable





Q1': Are there equivalent of SPT phases for non-interacting dissipative bosonic systems?
Q2': Are there topologically mandated analogues of Majorana fermions?...

- The steady state is unique and Gaussian, but is hard to reach. Look in the *transient* regime!
  - ➞ Hint: Approximate zero modes (ZMs) exist in the BKC Hamiltonian – but *not* in a stable regime...

$$J = \Delta : \quad \gamma_L = \sum_{j=1}^{N} \delta_0^{j-1} x_j, \quad \gamma_R = \sum_{j=1}^{N} \delta_0^{N-j} p_j, \quad \delta_0 \equiv -\frac{\mu}{J}$$

  - ➞ Adding uniform dissipation (Model 1) gives localized approximately conserved ZMs, *precisely* when rapidity bands winds around zero – but they *need not* be canonically conjugate...

$$J = \Delta : \quad \gamma_L^{\rm c} \equiv \sum_{j=1}^{N} \delta_-^{j-1} x_j, \quad \gamma_R^{\rm c} \equiv \sum_{j=1}^{N} \delta_+^{N-j} p_j, \quad \delta_\pm \equiv -\frac{\mu \pm \kappa}{J} \qquad \begin{aligned} \mathcal{L}^\star(\gamma_L^{\rm c}) &= -J\delta_-^N x_N \\ \mathcal{L}^\star(\gamma_R^{\rm c}) &= J\delta_+^N p_1 \end{aligned}$$

Hunting for Majoranas...

Q1': Are there equivalent of SPT phases for non-interacting dissipative bosonic systems?
Q2': Are there topologically mandated analogues of Majorana fermions?...

- The steady state is unique and Gaussian, but is hard to reach. Look in the *transient* regime!
  - ➞ Hint: Approximate zero modes (ZMs) exist in the BKC Hamiltonian – but *not* in a stable regime...

$$J=\Delta: \quad \gamma_L = \sum_{j=1}^{N} \delta_0^{j-1} x_j, \quad \gamma_R = \sum_{j=1}^{N} \delta_0^{N-j} p_j, \quad \delta_0 \equiv -\frac{\mu}{J}$$

  - ➞ Adding uniform dissipation (Model 1) gives localized approximately conserved ZMs, *precisely* when rapidity bands winds around zero – but they *need not* be canonically conjugate...

$$J=\Delta: \quad \gamma_L^{\mathrm{c}} \equiv \sum_{j=1}^{N} \delta_-^{j-1} x_j, \quad \gamma_R^{\mathrm{c}} \equiv \sum_{j=1}^{N} \delta_+^{N-j} p_j, \quad \delta_\pm \equiv -\frac{\mu \pm \kappa}{J} \qquad \begin{aligned} \mathcal{L}^\star(\gamma_L^{\mathrm{c}}) &= -J\delta_-^N x_N \\ \mathcal{L}^\star(\gamma_R^{\mathrm{c}}) &= J\delta_+^N p_1 \end{aligned}$$

- Proposal: Bosonic analogues of Majorana fermions are the pairs $(\gamma_L^{c}, \gamma_R^{s})$, $(\gamma_L^{s}, \gamma_R^{c})$, where the *conjugate modes* needed to form delocalized boson DOFs are the *symmetry generators*

$$\gamma_L^{\mathrm{s}} \equiv \sum_{j=1}^{N} \delta_+^{j-1} x_j, \quad \gamma_R^{\mathrm{s}} \equiv \sum_{j=1}^{N} \delta_-^{N-j} p_j, \qquad \begin{aligned} \mathcal{L}^\star\left([\gamma_L^{\mathrm{s}}, A]\right) - [\gamma_L^{\mathrm{s}}, \mathcal{L}^\star(A)] &= -J\delta_+^N [x_N, A] \\ \mathcal{L}^\star\left([\gamma_R^{\mathrm{s}}, A]\right) - [\gamma_R^{\mathrm{s}}, \mathcal{L}^\star(A)] &= J\delta_-^N [p_1, A] \end{aligned}$$

$$\boxed{[\gamma_L^{\mathrm{c}}, \gamma_R^{\mathrm{s}}] = iN\delta_-^{N-1}, \; [\gamma_L^{\mathrm{s}}, \gamma_R^{\mathrm{c}}] = iN\delta_+^{N-1}}$$

Majorana bosons (MBs)

Topologically metastable QBLs

Metastability is necessary but not sufficient for MBs: *Non-vanishing winding number* of bulk rapidity spectrum *at zero* is additionally needed for *topological* metastability…

- Each conserved MB localized on one edge is paired, *in general*, with a *distinct* symmetry generator localized on the opposite edge – reflecting breakdown of Noether theorem…
  - ➞ MBs are *robust against disorder* thanks to the robustness of pseudospectra.
  - ➞ The approximate MB symmetries γ^S imply the existence of a *topologically sourced manifold of almost*-degenerate quasi-steady states…

Topologically metastable QBLs

Metastability is necessary but not sufficient for MBs: *Non-vanishing winding number* of bulk rapidity spectrum *at zero* is additionally needed for *topological* metastability...

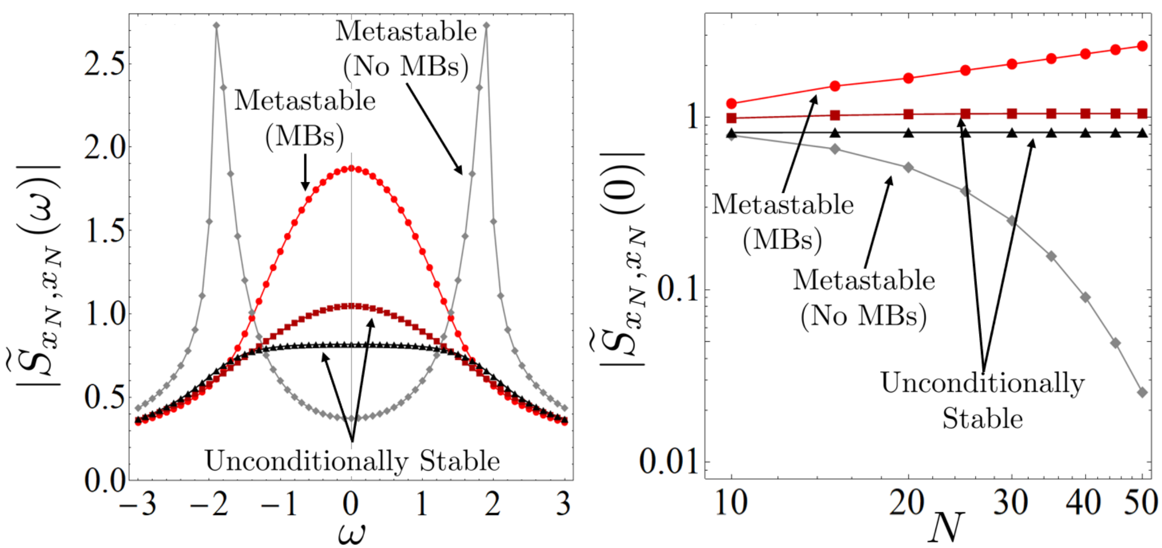
- Each conserved MB localized on one edge is paired, *in general*, with a *distinct* symmetry generator localized on the opposite edge – reflecting breakdown of Noether theorem...
  - ➞ MBs are *robust against disorder* thanks to the robustness of pseudospectra.
  - ➞ The approximate MB symmetries γ^S imply the existence of a *topologically sourced manifold of almost*-degenerate quasi-steady states...

- Signatures of MBs emerge in the *steady-state correlation functions* of linear observables and their [Fourier] power spectra:

$$C_{\alpha,\beta}(\infty,\tau) = \mathrm{Tr}[\alpha(\tau)\beta(0)\rho_{\rm ss}]$$

$$\widetilde{S}_{\alpha,\beta}(\omega) \equiv \frac{S_{\alpha,\beta}(\omega)}{C_{\alpha,\beta}(\infty,0)}$$

➞ Prediction: MBs lead to *zero-frequency peak*, whose height diverges with system size...



Conclusions & Outlook

- Tight bosonic analogues to Majorana fermions – 'halves-of-a-photon' (!) do exist and are linked to *non-trivial bulk topology* in *metastable, open* bosonic matter.

- In addition to *anomalously slow relaxation* and transient amplification, Majorana bosons manifest through distinctive signatures in steady-state *zero-frequency* spectra.

- Topological metastability is also tied to the existence of a *continuum of quasi-steady states* which can have *non-zero* first moments and can support long-lived *non-Gaussianity*...

- Our results point to *transient metastable regimes* as a new route to search for and realize SPT phases of non-interacting bosonic systems.

- (Some) Outstanding questions:
  - ➞ Nature and extent of symmetry protection mechanism?...
  - ➞ Extensions to higher-dimensional (2D) systems?...
  - ➞ Significance of the [generic] split between ZM and symmetry generator?...
  - ➞ Topological classification and dissipative bulk-boundary correspondence?...
  - ➞ Relevance to CV quantum information processing?...
  - ➞ Experimental implementations in AMO/photonic/SC platforms?...

  ⋮

Grazie per l'attenzione!