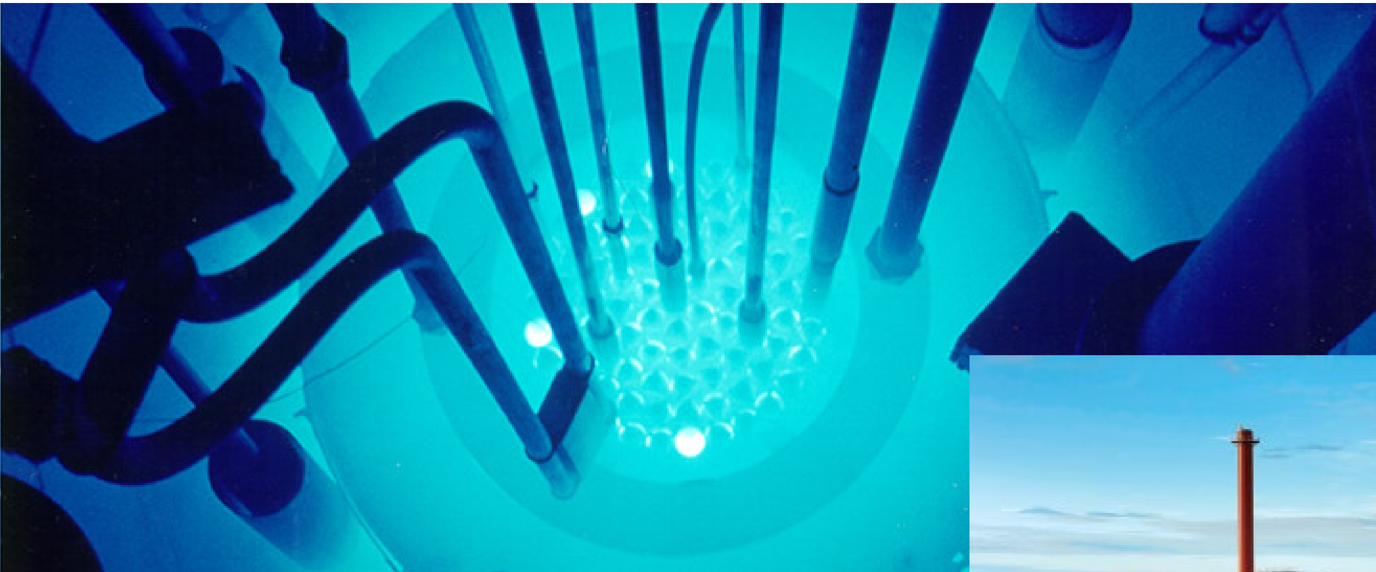
Energy from nuclear fission



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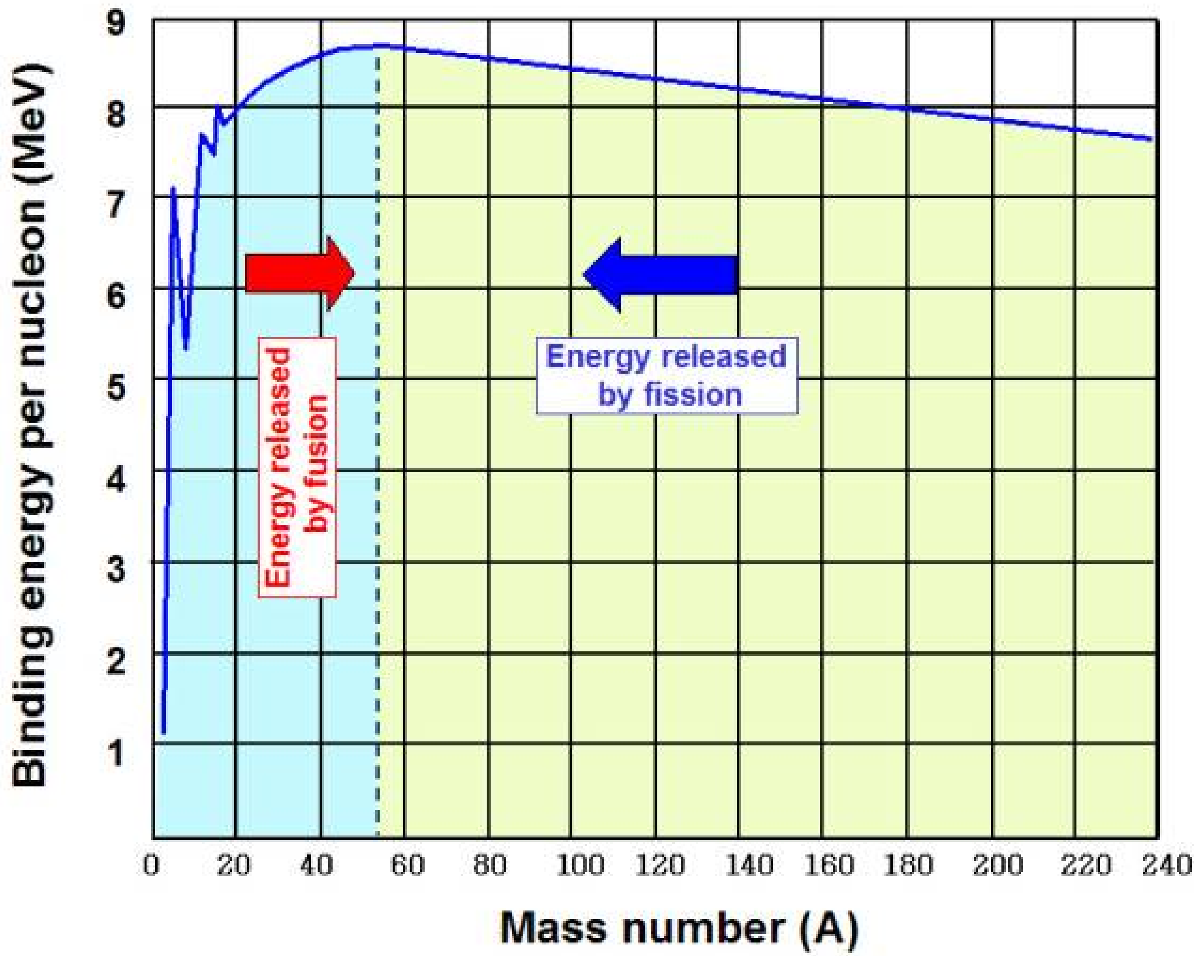
Why can fission produce energy ?

Nuclear mass and **nuclear binding energy** $M(Z,A) = ZM_p + (A - Z)M_n + \mathbf{B(Z,A)}$

B(Z,A) < 0 !!! i.e. a nucleus weighs <u>less</u> than the sum of proton and neutron masses

$$\varepsilon \equiv \frac{|B(Z,A)|}{A}$$



How can fission produce energy ?

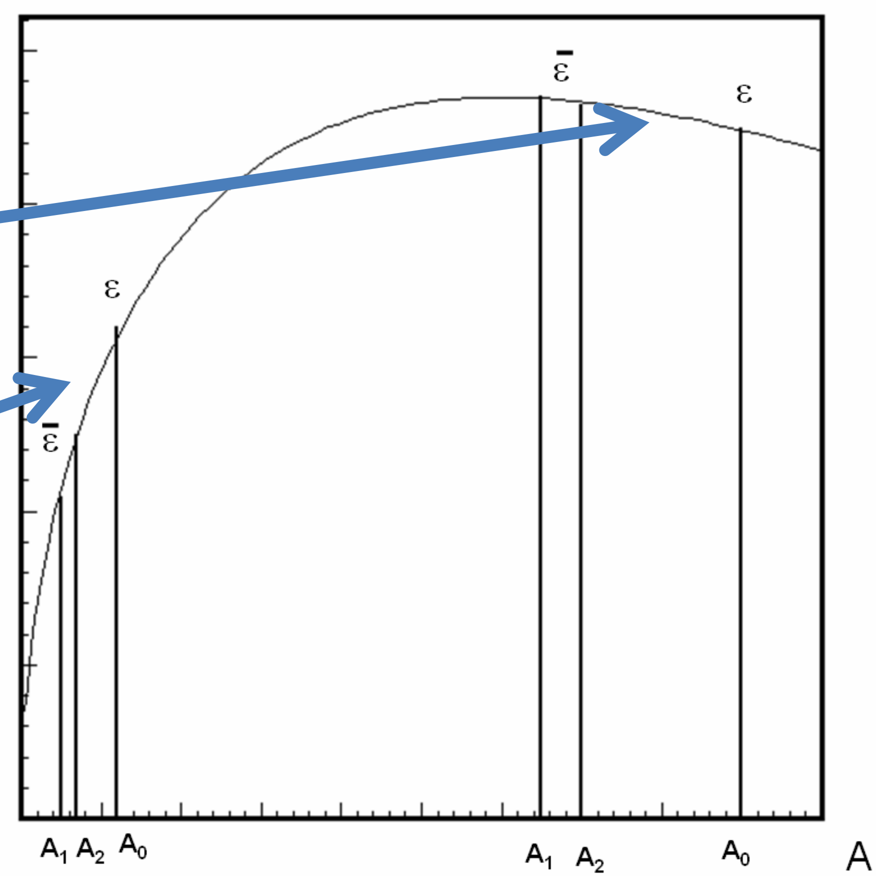
Nucleus splits into 1 and 2: $M(Z_0,A_0) \rightarrow M(Z_1, A_1) + M(Z_2, A_2)$; $Z_0 = Z_1 + Z_2$; $A_0 = A_1 + A_2$

Energy balance (Q-value) $Q_{fiss} = M(Z_0, A_0) - M(Z_1, A_1) - M(Z_2, A_2)$

$$= B(Z_0, A_0) - B(Z_1, A_1) - B(Z_2, A_2) = -\varepsilon A + \varepsilon_1 A_1 + \varepsilon_2 A_2 = -\varepsilon A + \bar{\varepsilon} A = (\bar{\varepsilon} - \varepsilon) A$$

$$\bar{\varepsilon} = \frac{\varepsilon_1 A_1 + \varepsilon_2 A_2}{A_1 + A_2}$$

$\varepsilon = |B|/A$

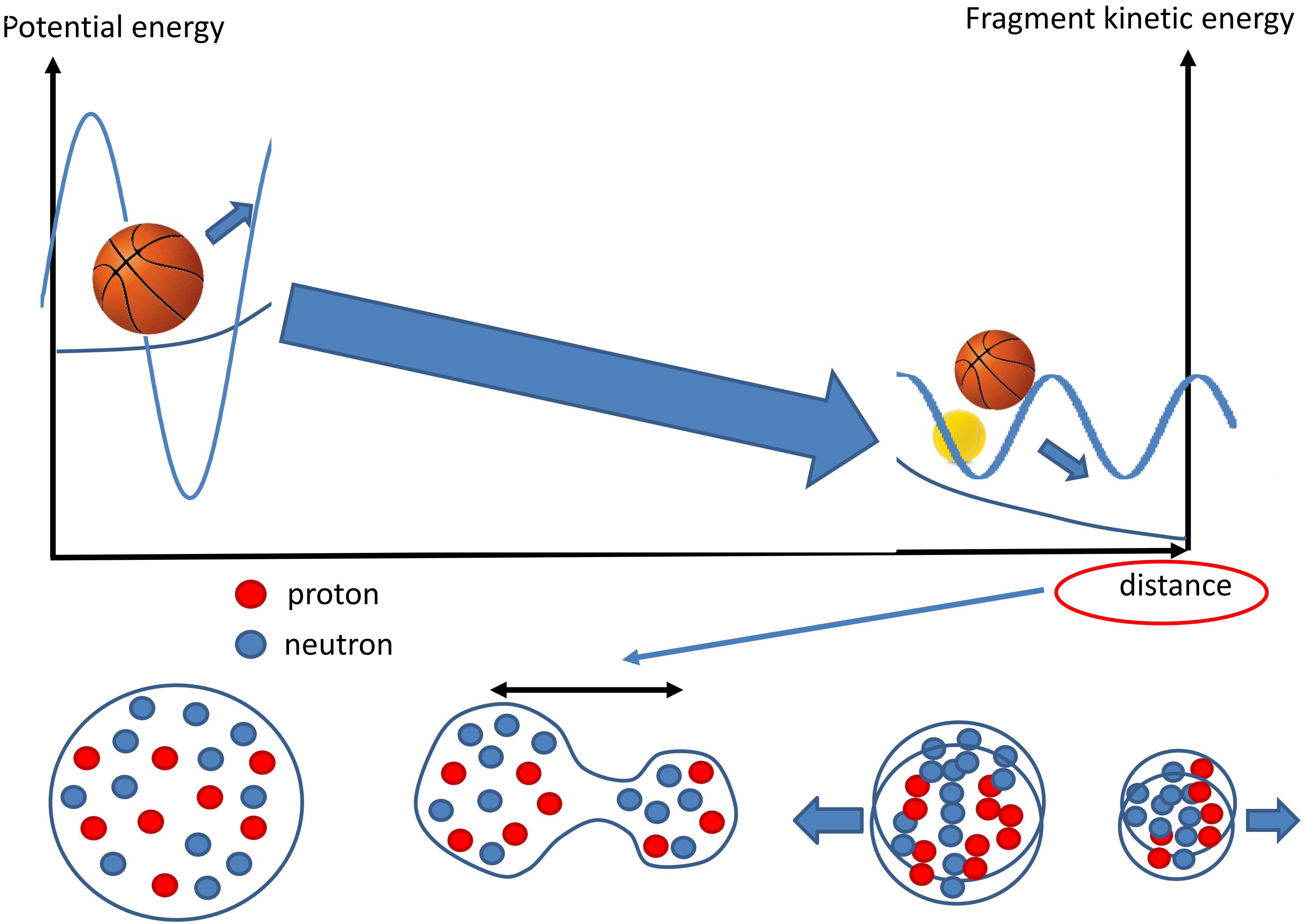


$Q_{fiss} > 0$

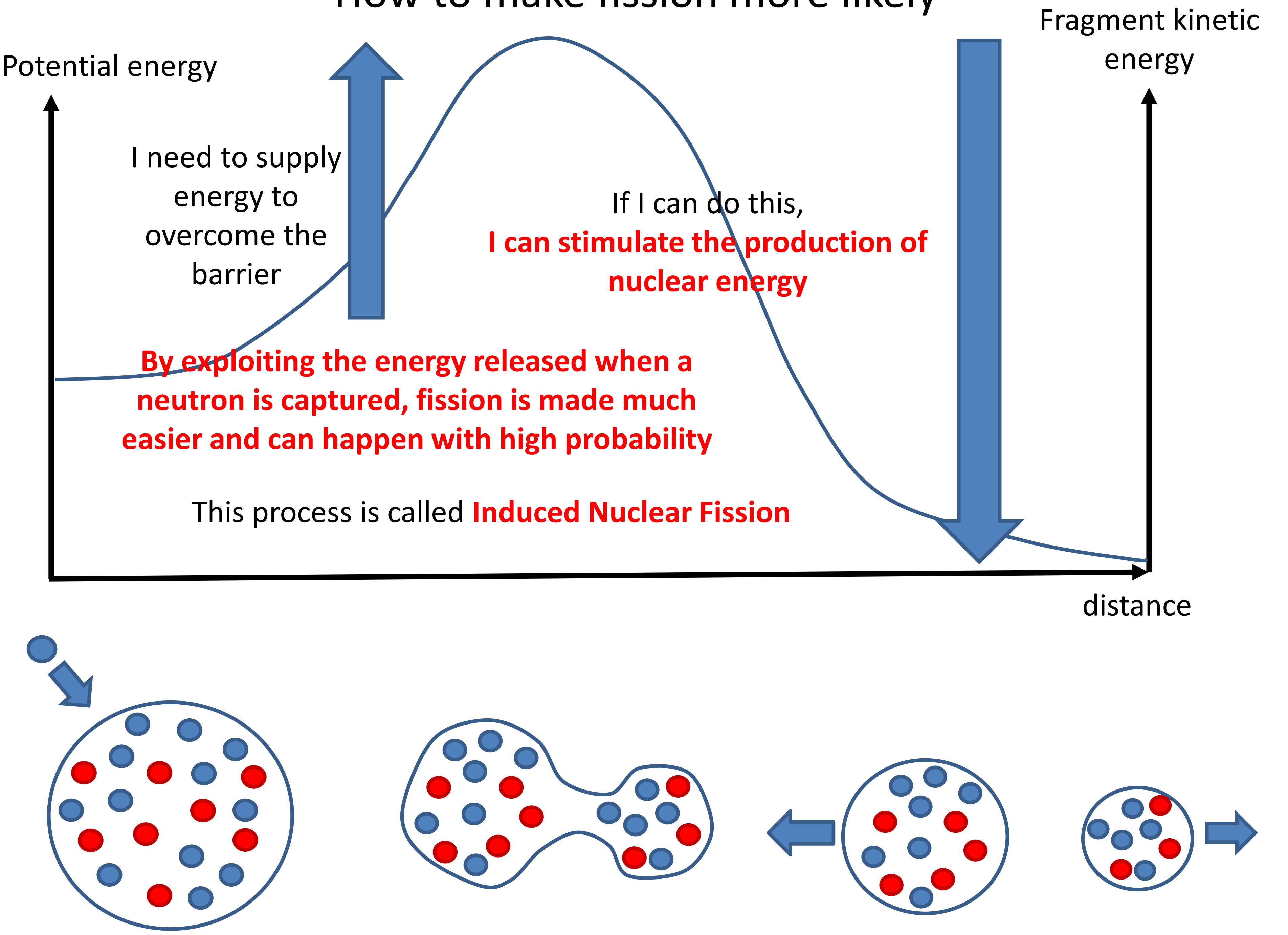
$Q_{fiss} < 0$

Q > 0 means that the reaction is spontaneously producing energy

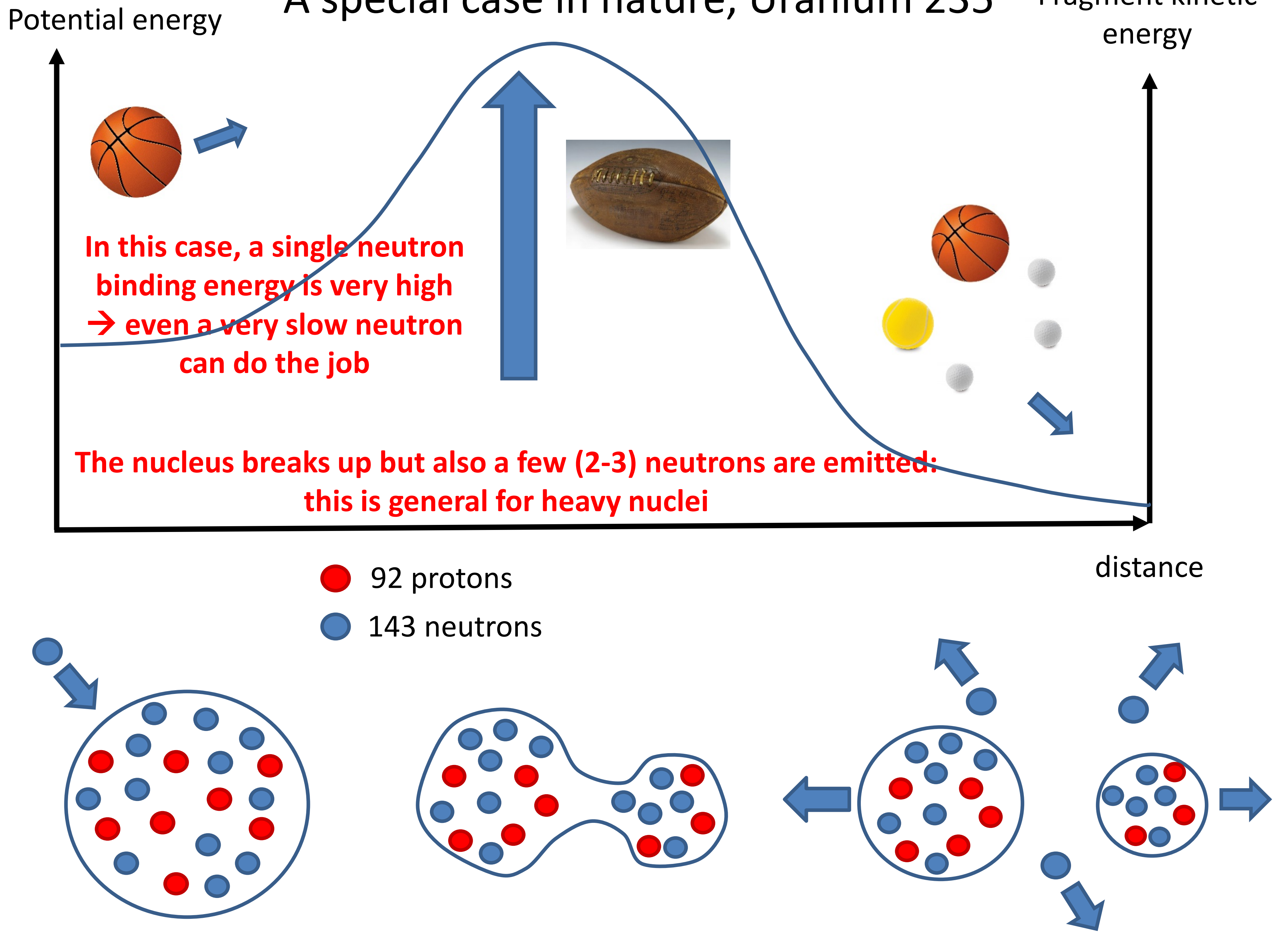
Energy balance is not the whole story, what is the reaction dynamics ?



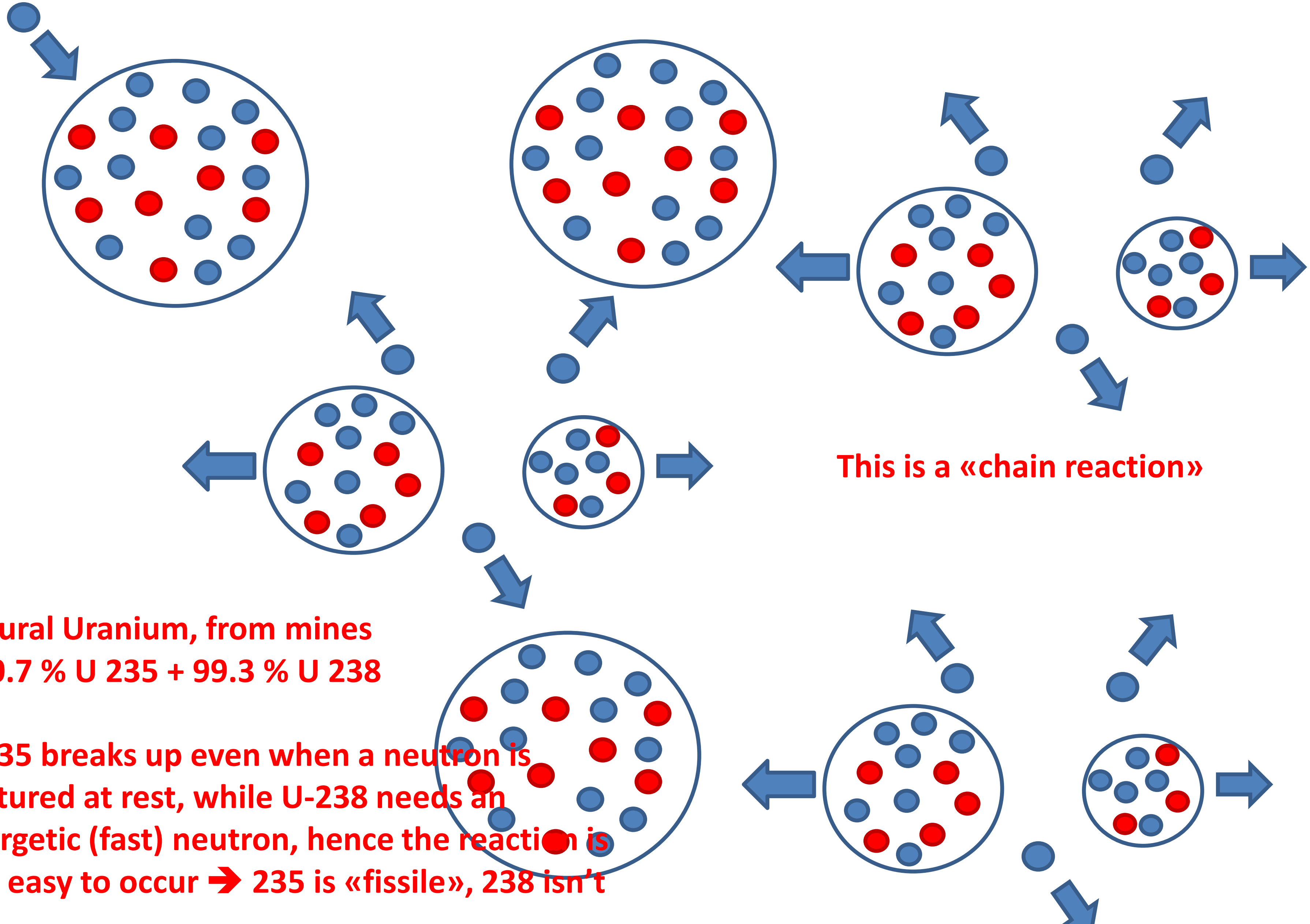
How to make fission more likely



A special case in nature, Uranium 235



The chain reaction



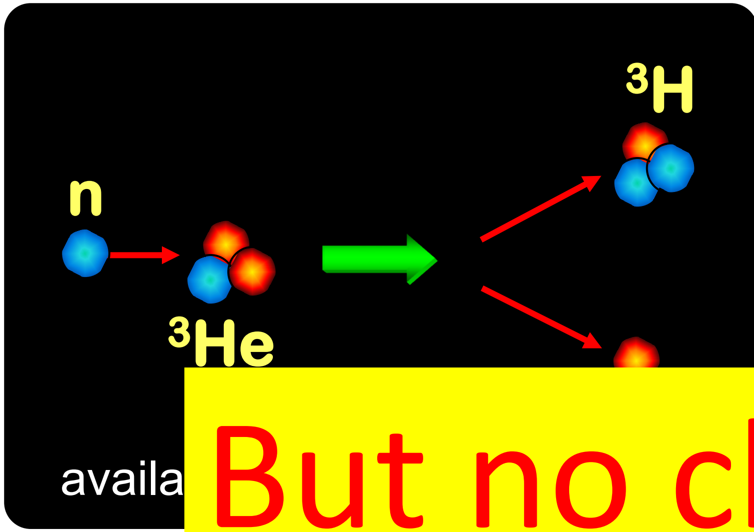
This is a «chain reaction»

Natural Uranium, from mines
→ 0.7 % U 235 + 99.3 % U 238

U-235 breaks up even when a neutron is captured at rest, while U-238 needs an energetic (fast) neutron, hence the reaction is less easy to occur ➔ 235 is «fissile», 238 isn't

Fragments are moving ➔ kinetic energy ➔ transferred to atoms ➔ heat

Other neutron absorption processes yielding energy



σ(thermal neutrons)
≈ 5330 b (barn, 1 b=10^{-24} cm^2, σ is proportional to the reaction probability, see later)

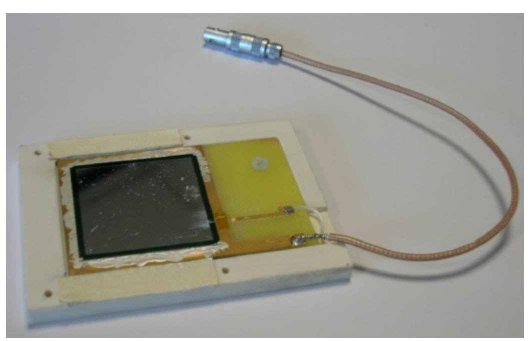
1 MeV= 1 MegaelectronVolt =
10^6 electronVolt = 10^6 x 1.6x10^{-19} Coulomb Volt =

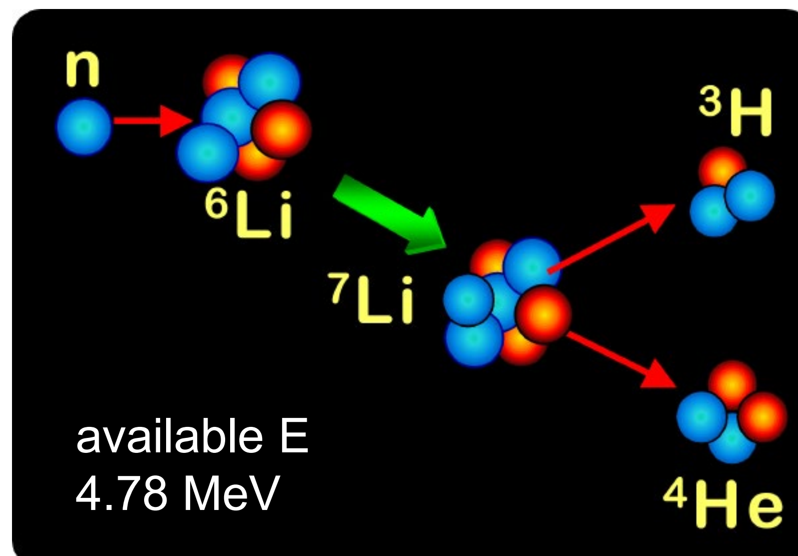
But no chain reaction

≈ 940 b

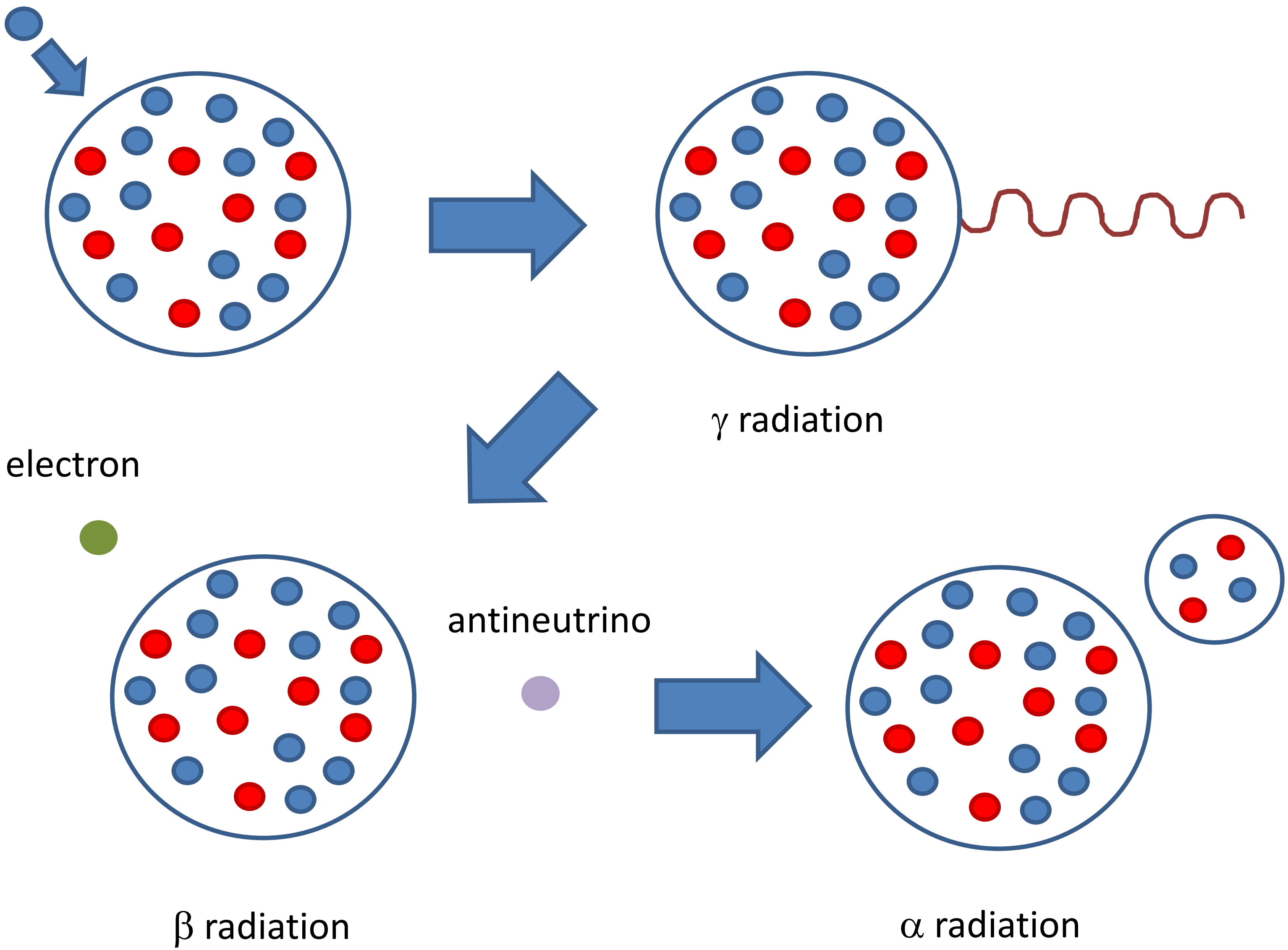
^{3}He neutron detector

neutron detector based on a LiF film

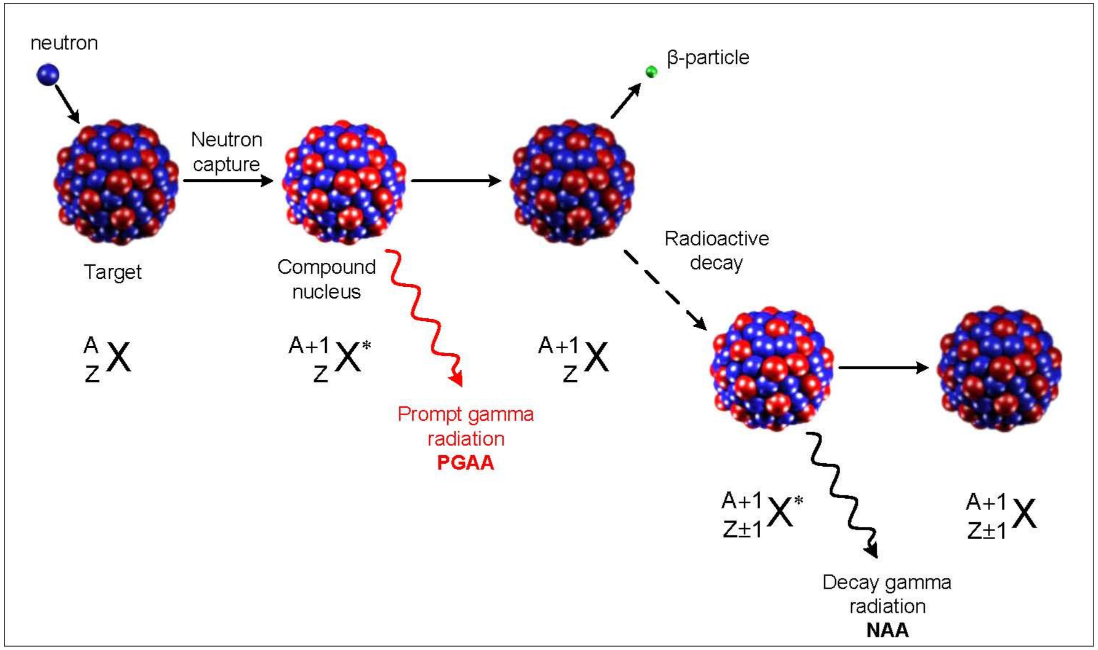




Radiative neutron capture



- **For nuclei that can undergo fission, Radiative Neutron Capture is a competitive mechanism**



- **For other materials (e.g. steel), RNC can lead to the formation of radioactive nuclei → this process is called activation**
- **γ radiation can also occur due to neutron scattering**
- **Activation can also occur due e.g. due to (n, 2n) reactions**

Example: Plutonium production from Uranium

n+ ^{238}U → ^{239}U + γ → ^{239}Np + β + anti-ν → **^{239}Pu** + β + anti-ν

Example: ^{60}Co production in steel

n + ^{59}Co → **^{60}Co** + γ → ^{60}Ni + β + anti-ν

Amount of energy and reaction products

When a uranium nucleus fissions into two daughter nuclei fragments, about **0.1 % of uranium mass appears as fission energy of ~200 MeV ($E=Mc^2$)**

➔ **much bigger than any other exoenergetic nuclear reaction** (in absolute terms)

- ~**170 MeV** appears as the **kinetic energy of the daughter nuclei**,
-which fly apart at about 3% of the speed of light

- an **average of 2.5 prompt neutrons are emitted**, with a **mean kinetic energy per neutron of ~2 MeV** (total of 4.8 MeV)

- the **average number of neutrons emitted is called ν (order of 2-3)**

- **~25 MeV** are released in form of **prompt gamma ray photons and fission product β decay**

Chemical reactions vs nuclear fission

$C+O_2 = CO_2 + Q (Q = 3.6\ eV)$,

$CH_4 + 2O_2 = CO_2 + 2H_2O + Q (Q = 9.22\ eV)$,

${}^{235}U + n = F_1 + F_2 + \nu n + Q (Q \sim 200\ MeV)$

→ *Fission gives between 20 and 50 million times more energy*

Comparison of energy sources

Material	Specific energy (MJ/kg)	Energy density (MJ/L)
Deuterium-Tritium (50 % mix)	3.4×10^8	7.65×10^4 (at room temperature and standard pressure)
Uranium	8.2×10^7	1.56×10^9
Hydrogen	142	8.9 (at room temperature and 700 bar pressure)
Methane or natural gas	55.5	0.0364
Diesel / Fuel oil	48	35.8
LPG (including Propane / Butane)	46.4	26
Gasoline (petrol)	46.4	34.2
Ethanol fuel (E100)	26.4	20.9
Coal, anthracite	26-33	34-43
Wood	~13-20	~10-16

Source: Wikipedia plus some changes and corrections

Yellow background: nuclear energy (fusion and fission)

Physics: nuclear cross sections

Cross section: quantity that characterizes a nuclear reaction (elastic, inelastic scattering, etc.) connected to the range of the involved forces; **effective area of a nuclear target**

Here we will consider the **total cross section**, defined as follows:

Given a **flux** $$\frac{dN_{in}}{dSdt}$$

number of incident particles per unit surface and unit time on a single nucleus (target)

and given an **interaction rate** $$\frac{dN_{reac}}{dt}$$

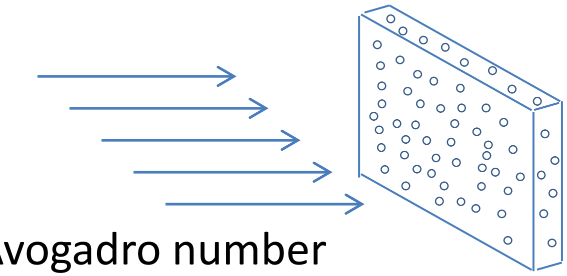
number of interacting particles (scattered or absorbed projectiles) per unit time, then

$$\sigma = \frac{\dfrac{dN_{reac}}{dt}}{\dfrac{dN_{in}}{dSdt}}$$

σ → physical dimensions of a surface

Nuclear cross sections

Macroscopic target comprising several nuclei with **density ρ (es. gr/cm3) and thickness x**, struck by a particle beam of intensity I (particles/sec) →



$$R = \frac{dN_{reac}}{dt} = I\,\frac{\rho\, x}{A} N_A \sigma$$

where A is the target atomic weight (es. in gr.) e N_A is the Avogadro number

$\frac{\rho}{A} N_A$ is the **number density of nuclei** in the target (i.e. number of nuclei per unit volume)

This is all valid for a <u>small thickness</u> x

For a target of <u>arbitrary thickness</u>, first divide it in thin slices of thickness dx →

$$dR = \frac{dN_{reac}}{dt} = I(x)\frac{\rho}{A} N_A \sigma\, dx$$

$$dI = -I(x)\frac{\rho}{A} N_A \sigma\, dx$$

$$I(x) = I(0)\exp(-\frac{\rho}{A} N_A \sigma\, x)$$

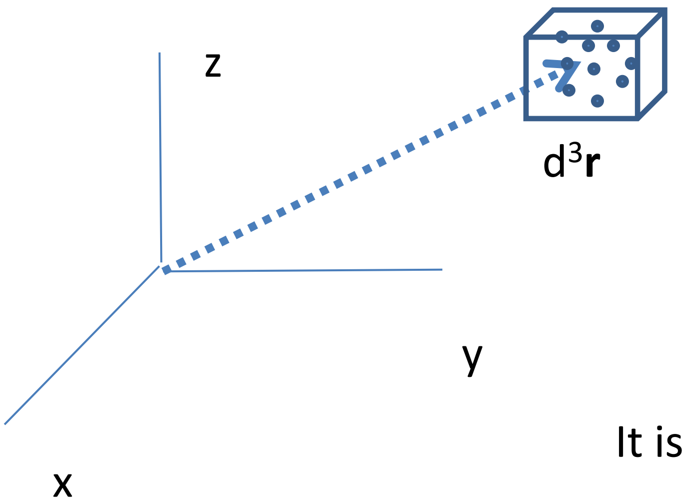
⇒ $\boxed{\Sigma \equiv \frac{\rho}{A} N_A \sigma}$ **Macroscopic cross section = prob.ty of interaction per unit length**

$1/\Sigma$ **= Mean free path**

$\Sigma\, \mathrm{v} =$ **Frequency with which reactions occur, v= projectile speed**

Neutron density and flux

Neutron density ≡ n(**r**,E,t) [cm^{-3}] ≡
expected number of neutrons with energy between E and E+dE, in the volume d^3**r** about **r**, at a time t



Reaction density ≡ R(**r**,E,t) ≡
Number of reactions in the volume d^3**r** about **r**, at a time t, initiated by neutrons with energy between E and E+dE = n(**r**,E,t) Σv

We give a <u>special name</u> to the quantity n(**r**,E,t)v
It is called the **neutron "flux" ϕ(r,E,t) ≡ n(r,E,t)v [cm^{-2} s^{-1}]**

Reaction density ≡ number of reactions per unit volume ≡ R(r,E,t) = Σ ϕ

Suppose you've got a reactor with 1 GW thermal power = 10^9 Joule/sec
Assume each fission releases order of 200 MeV energy = 3.2×10^{-11} Joule
→ In the reactor the fission rate is about 3×10^{19} fissions/sec
→Almost 10^{20} neutrons/sec emitted, about 2×10^{20} neutrinos/sec
→ $\phi \sim 10^{14}$ neutrons cm^{-2} s^{-1}

A lot of neutrons ! → structural material damage has to be taken into consideration
→ the same in fusion

Nuclear cross sections

Since the nuclear radius is roughly 10^{-12} cm, the geometrical cross sectional area of the nucleus is roughly **10^{-24} cm^2 = 1 barn**

Hence we might expect that nuclear cross sections are of the order of 10^{-24} cm$^2 \equiv$ 1 barn

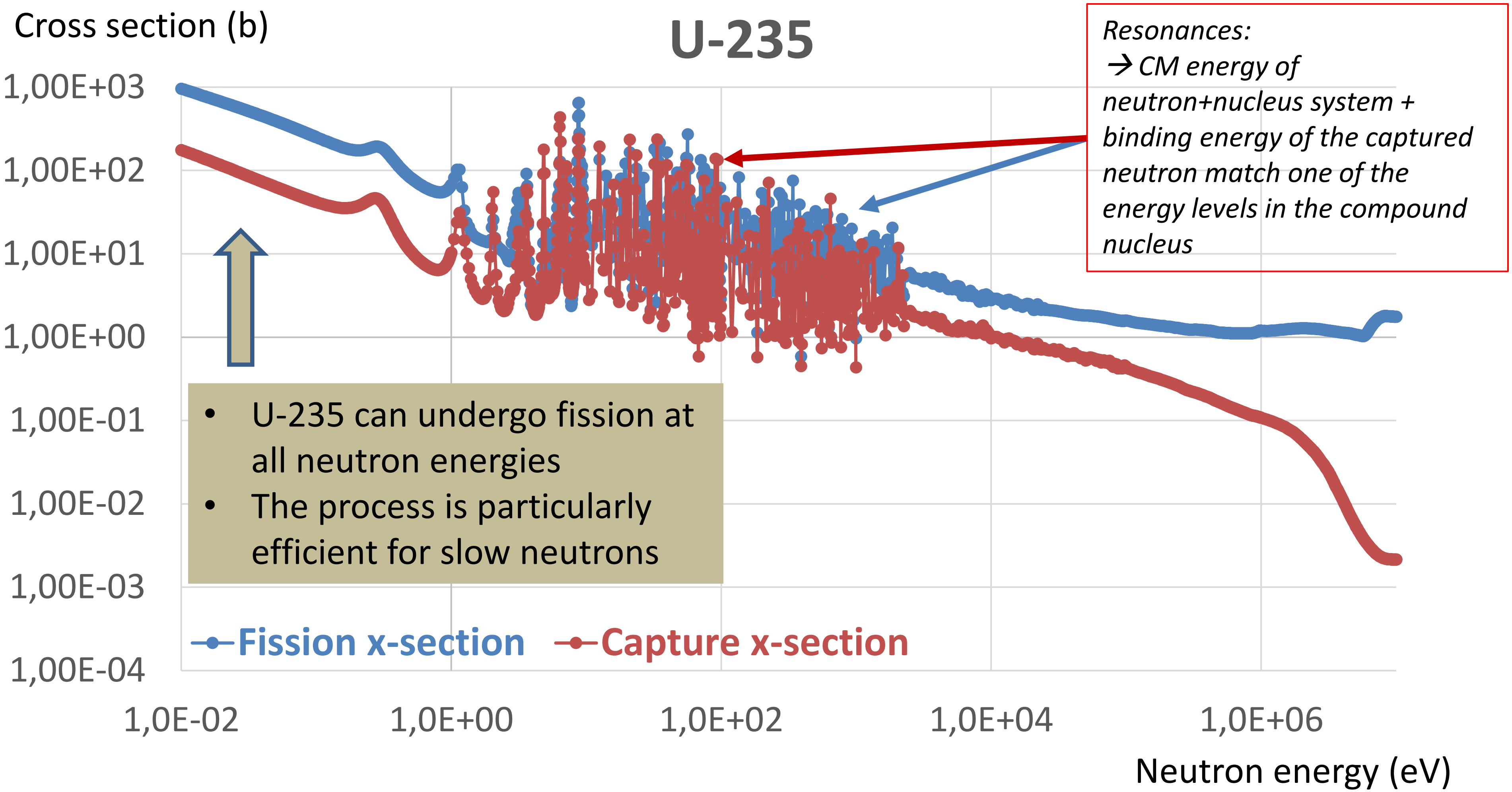
However, for both fission and radiative neutron capture, due to quantum mechanical effects

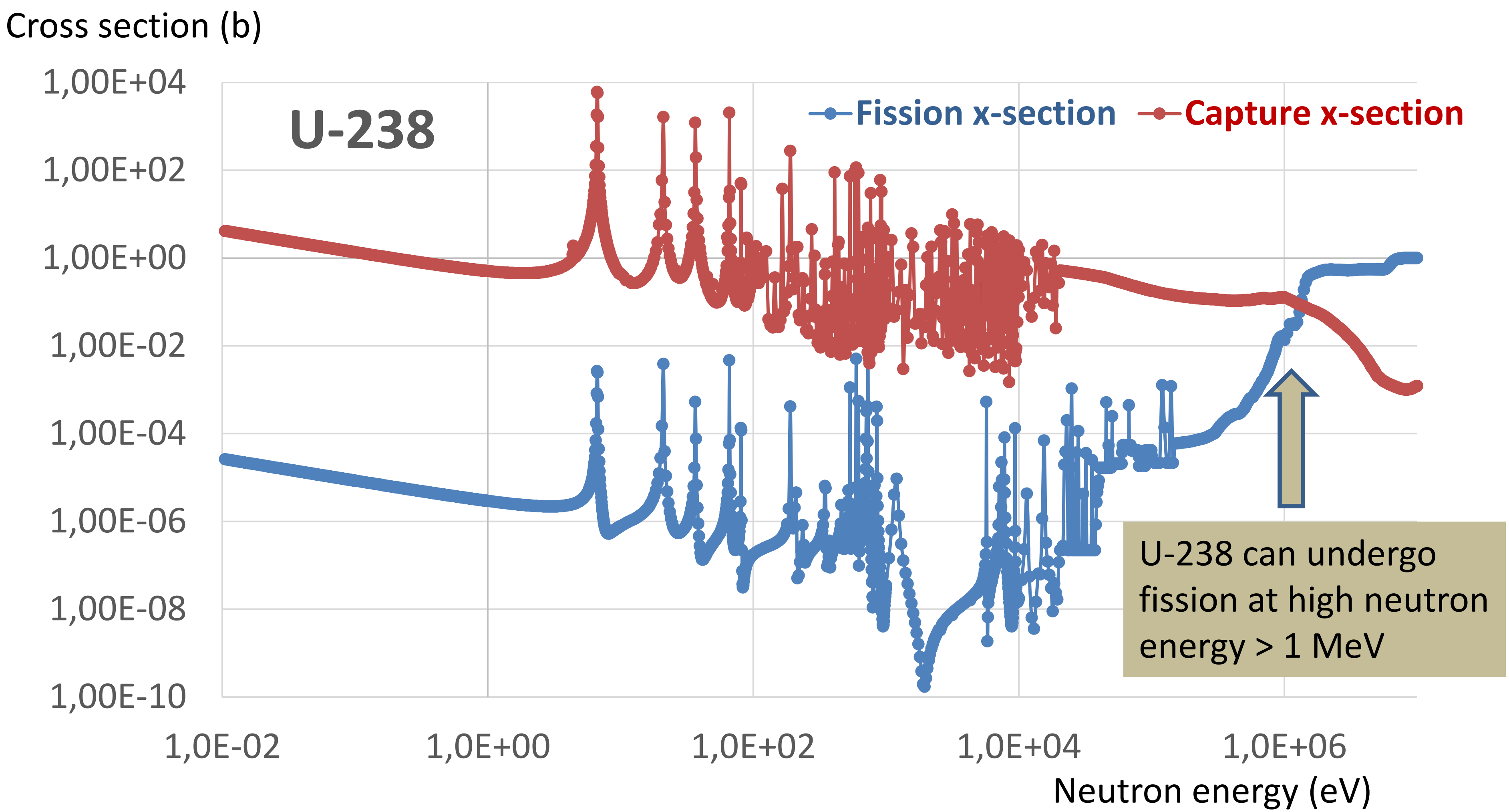
→ cross section follows a 1/v law, with v being the relative speed (essentially the n speed)

→ "slow" neutrons are much more effective at producing fission or radiative capture

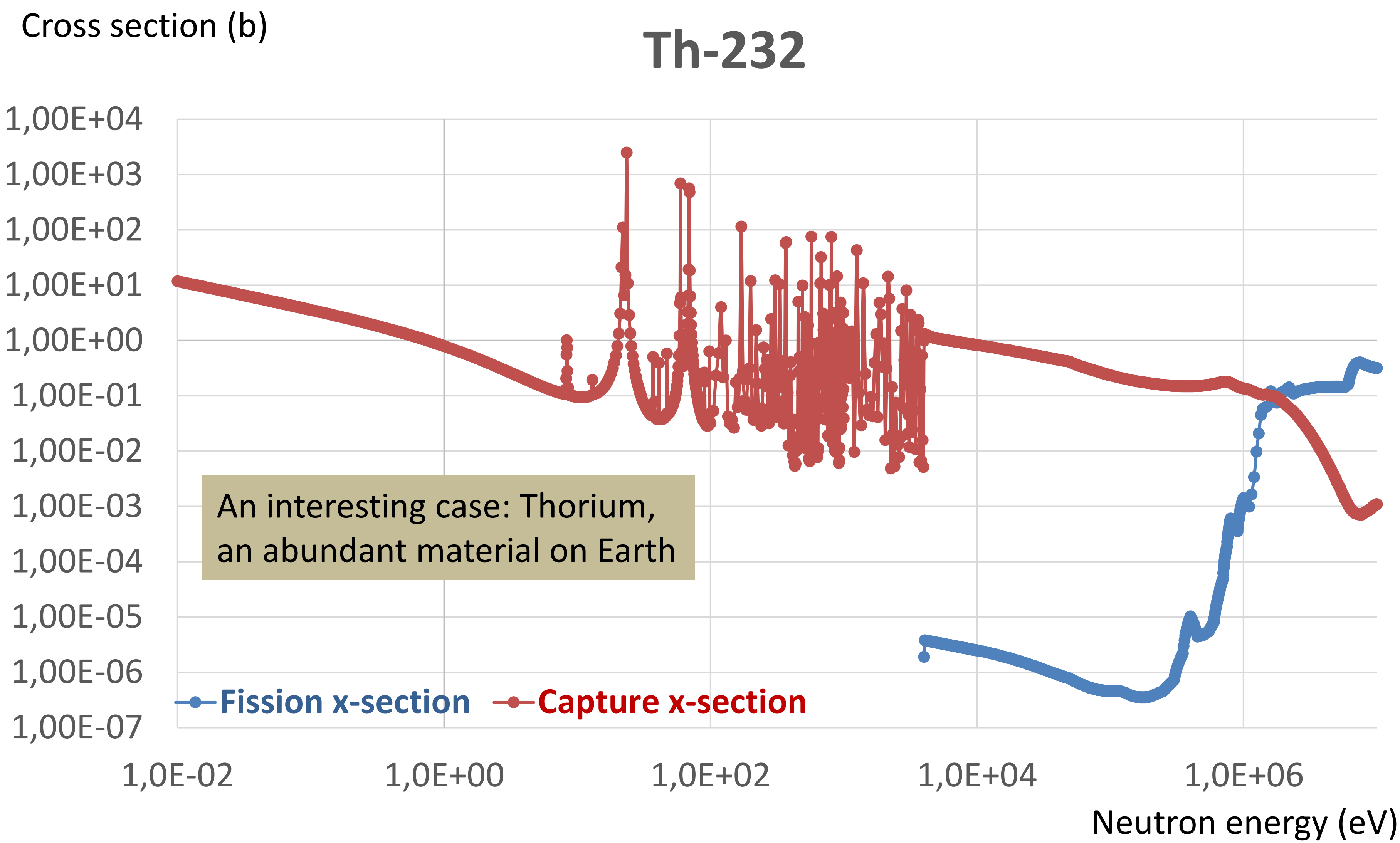
→ however, for certain nuclei fission can have a threshold, i.e. it happens only if neutron energy is above some minimum value (e.g. ^{238}U)

Nuclear cross sections





Cross section (b)
U-238
Fission x-section
Capture x-section
1,00E+04
1,00E+02
1,00E+00
1,00E-02
1,00E-04
1,00E-06
1,00E-08
1,00E-10
1,0E-02
1,0E+00
1,0E+02
1,0E+04
1,0E+06
Neutron energy (eV)
U-238 can undergo fission at high neutron energy > 1 MeV


Cross section (b)
Th-232
1,00E+04
1,00E+03
1,00E+02
1,00E+01
1,00E+00
1,00E-01
1,00E-02
1,00E-03
1,00E-04
1,00E-05
1,00E-06
1,00E-07
An interesting case: Thorium, an abundant material on Earth
Fission x-section
Capture x-section
1,0E-02
1,0E+00
1,0E+02
1,0E+04
1,0E+06
Neutron energy (eV)

Fissile, fissionable, fertile isotopes

- Heavy nuclei with a high fission cross section at low (thermal) neutron energies are called **fissile** (e.g. ^{233}U, ^{235}U, ^{239}Pu,…)

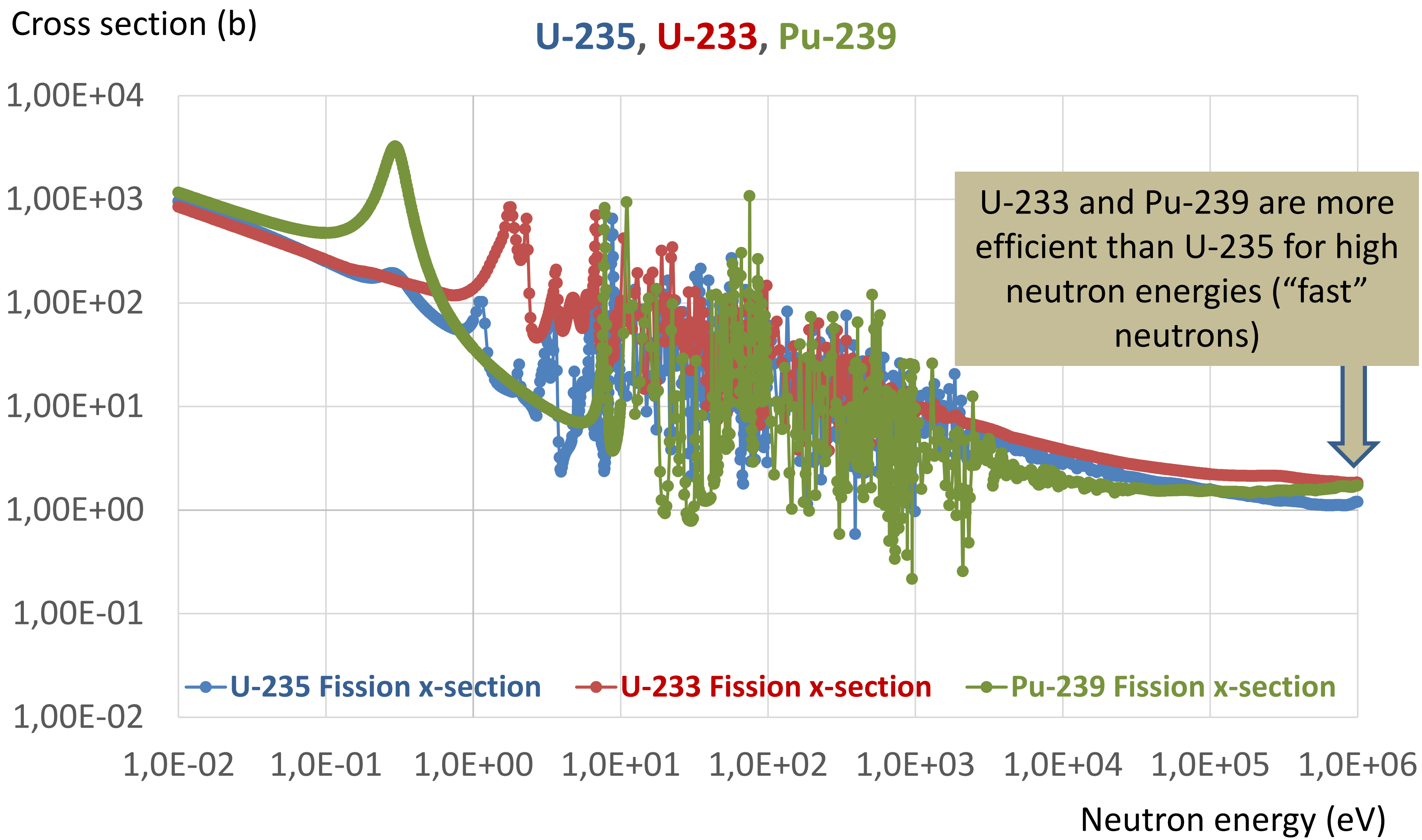
- Those with a non-zero fission cross section only at higher neutron energies are called **fissionable** (e.g. ^{238}U,…)

- Those that can produce a fissile isotope via neutron radiative capture and β decay are called **fertile**, i.e. they can be used to **produce fuel** (e.g. ^{238}U,…)

- $n + {}^{238}U \rightarrow {}^{239}U + \gamma \rightarrow {}^{239}Np + \beta + \text{anti-}\nu \rightarrow \boxed{{}^{239}Pu} + \beta + \text{anti-}\nu$ fissile

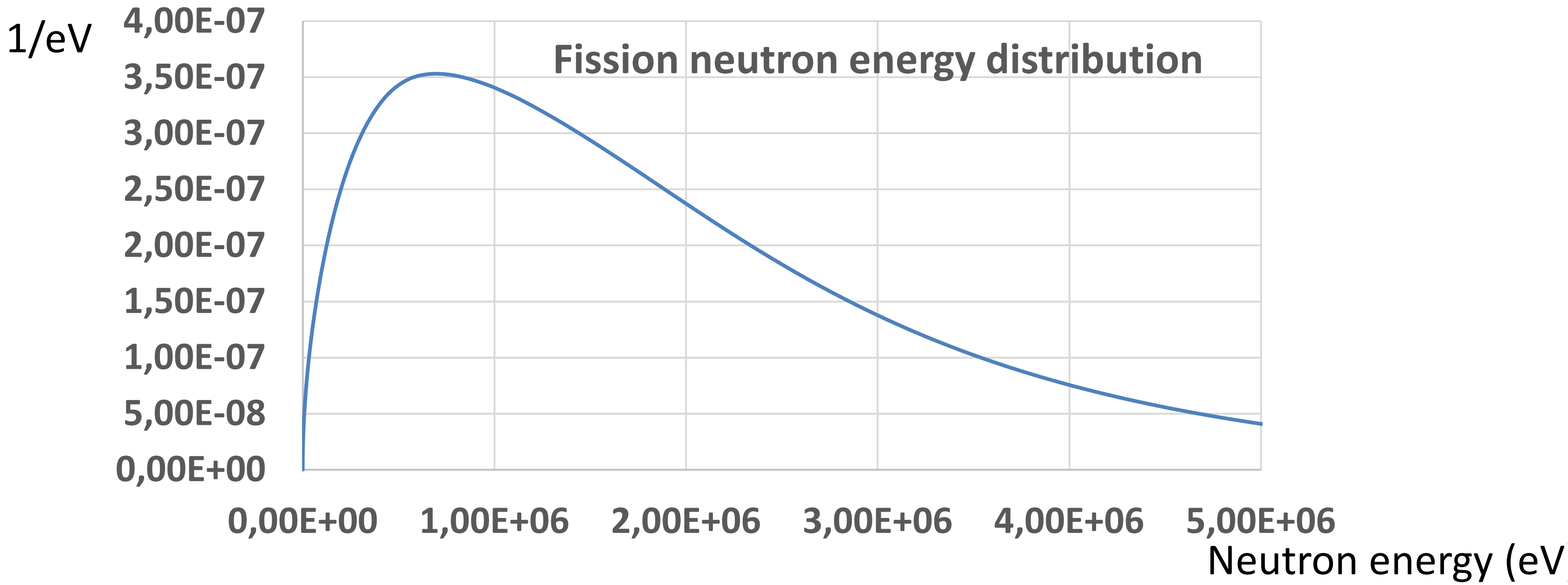
- $n + {}^{232}Th \rightarrow {}^{233}Th + \gamma \rightarrow {}^{233}Pa + \beta + \text{anti-}\nu \rightarrow \boxed{{}^{233}U} + \beta + \text{anti-}\nu$

- ✓ Natural Uranium → 0.7 % ^{235}U + 99.3 % ^{238}U → **most reactors need 3-5 % ^{235}U → "enrichment" process**
- ✓ Plutonium production is also called "breeding"
- ✓ Under certain conditions, a reactor can produce more Pu than it consumes → it is called "breeder"



Cross section (b)
U-235, U-233, Pu-239
1,00E+04
1,00E+03
1,00E+02
1,00E+01
1,00E+00
1,00E-01
1,00E-02
U-233 and Pu-239 are more efficient than U-235 for high neutron energies (“fast” neutrons)
U-235 Fission x-section
U-233 Fission x-section
Pu-239 Fission x-section
1,0E-02
1,0E-01
1,0E+00
1,0E+01
1,0E+02
1,0E+03
1,0E+04
1,0E+05
1,0E+06
Neutron energy (eV)

Fission spectrum, fast and slow neutrons



It is customary to adopt the following classification:

- **slow neutrons**: those with kinetic energy T_n < 1 eV
- in particular **thermal neutrons** have T_n around 0.025 eV or 25 meV (the value of kT, where k is the Boltzmann constant and T is the temperature
- **epithermal neutrons**: 1 eV < T_n < 100 keV (0.1 MeV)
- **fast neutrons**: 0.1 MeV < T_n < 20 MeV

Obviously neutrons in general can have energies above 20 MeV but this is an extreme limit in reactor physics (e.g. neutrons from D+T fusion have 14 MeV fixed energy)

Slowing down neutrons (moderation)

It is easy to show in non-relativistic kinematics that **after a scattering off a nucleus with mass number A**, the kinetic energy of the neutron changes according to the ratio

$$\frac{T'_n}{T_n} = \frac{m_n^2 + m_A^2 + 2m_n m_A cos\theta_{CM}}{(m_n + m_A)^2}$$

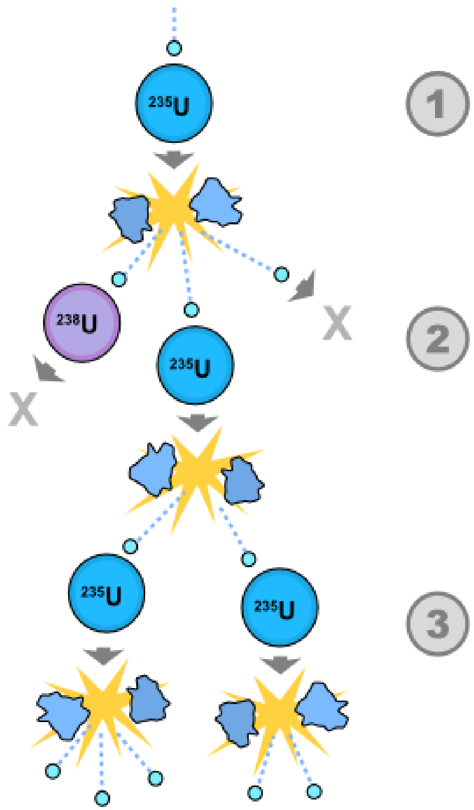
Assuming an isotropic CM cross section that does not depend on $\cos\theta_{CM}$, the corresponding term averages out to zero, so that we can write <u>on average</u>

$$\frac{T'_n}{T_n} = \frac{m_n^2 + m_A^2}{(m_n + m_A)^2}$$ ➔ Assuming $M_A \cong Am_n$ ➔ $$\frac{T'_n}{T_n} = \frac{1 + A^2}{(1 + A)^2}$$

For a **heavy nucleus A>>1 → $T_n' \cong T_n$** or in other words, the neutron has to undergo many collisions in order to significantly lose energy.
Consider instead the case **A=1** → (target containing hydrogen, i.e. protons as nuclei) **$T_n' = T_n/2$** i.e. on average a neutron will lose half of its energy at each collision and therefore few collisions are sufficient to rapidly decrease its energy

➔ **Moderators = light materials containing hydrogen = water, paraffin or graphite**

The chain reaction and the critical reactor



The chain reaction:
- must not diverge (more and more fissions at each "generation")
- must not die away (less and less fissions at each generation)

→ precisely one neutron from each fission has to induce another fission event

The remaining fission neutrons will then either be
- absorbed (e.g. by radiative capture) or
- will leak out from the system

Suppose we can count the number of neutrons in one generation and in the next one
Then

$$k \equiv \frac{\text{number of neutrons in one generation}}{\text{number of neutrons in the preceding generation}}$$

- The condition **k=1** corresponds to a **critical reactor** (steady state)
- **k>1** is a **supercritical reactor** (chain reaction diverges)
- **k<1** is a **subcritical reactor** (chain reaction dies away)

“Simple-minded” reactor kinetics

$$\frac{dn(t)}{dt} = P(t) - L(t)$$

n(t)=neutron population at time t
P(t)= neutron production at time t (mainly as fission products)
L(t)=neutron loss (fission+capture+leakage) at time t

All are functions of time as reactor evolves over time

⇨ Alternative definition $k \equiv \frac{P(t)}{L(t)}$ $\quad \text{Neutron lifetime} \equiv \tau \equiv \frac{n(t)}{L(t)}$

⇨ $\frac{dn(t)}{dt} = \frac{k-1}{\tau} n(t)$ Let’s assume k and τ are time independent (not true…)

$$\boxed{n(t) = n_0(t)\exp\left(\frac{k-1}{\tau} t\right)}$$

- **k=1 → steady state →** critical reactor
- **k>1 → increase →** supercritical
- **k<1 → decrease →** subcritical

$$\text{Time constant} \equiv \mathrm{T} \equiv \text{Reactor period} \equiv \frac{\tau}{k-1}$$

Delayed neutrons: crucial for reactor control

Typical neutron lifetime in a thermal power reactor $\sim 10^{-4}$ sec

If k=1.001 → **T=0.1 sec** → **power will increase by 2.7 in 0.1 sec !!**

Actually, **we neglected** the very small amount (< 1 %) of **delayed neutrons**

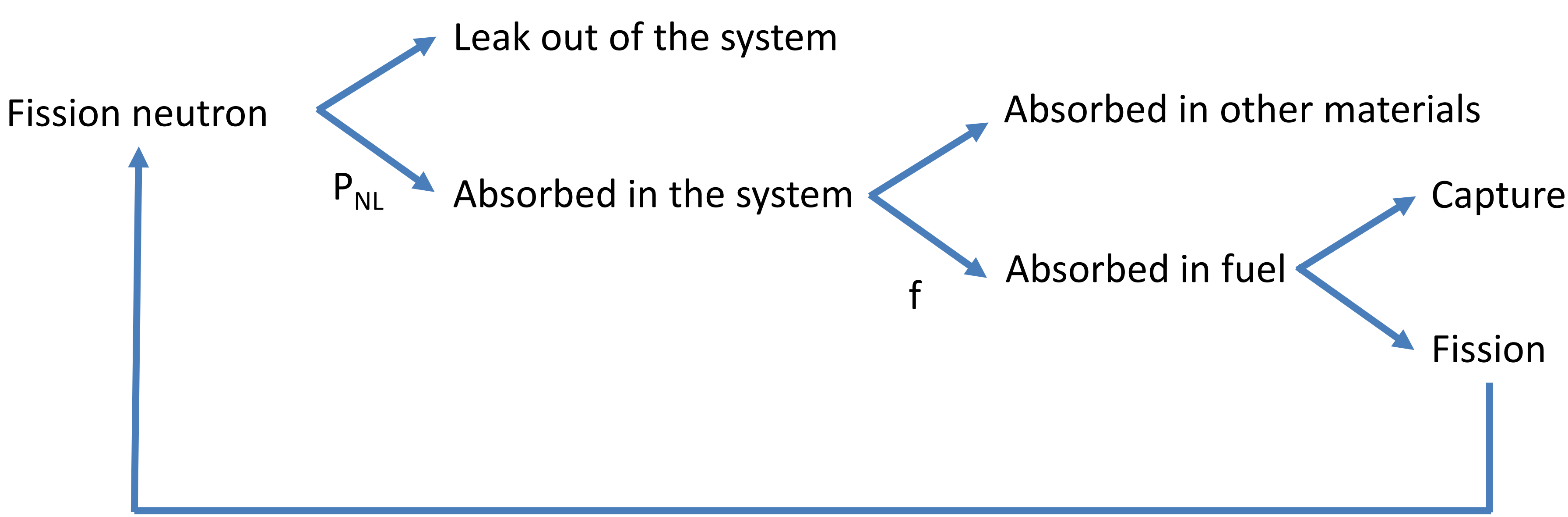
They are mitted by fragments after fission on **time scale from ms to sec**

The trick is to **make the reactor critical thanks to that small fraction of neutrons**

→ Delayed neutrons **dominate the reactor response time** making it much longer

→ Reactor control manageable by absorbers: "control rods"

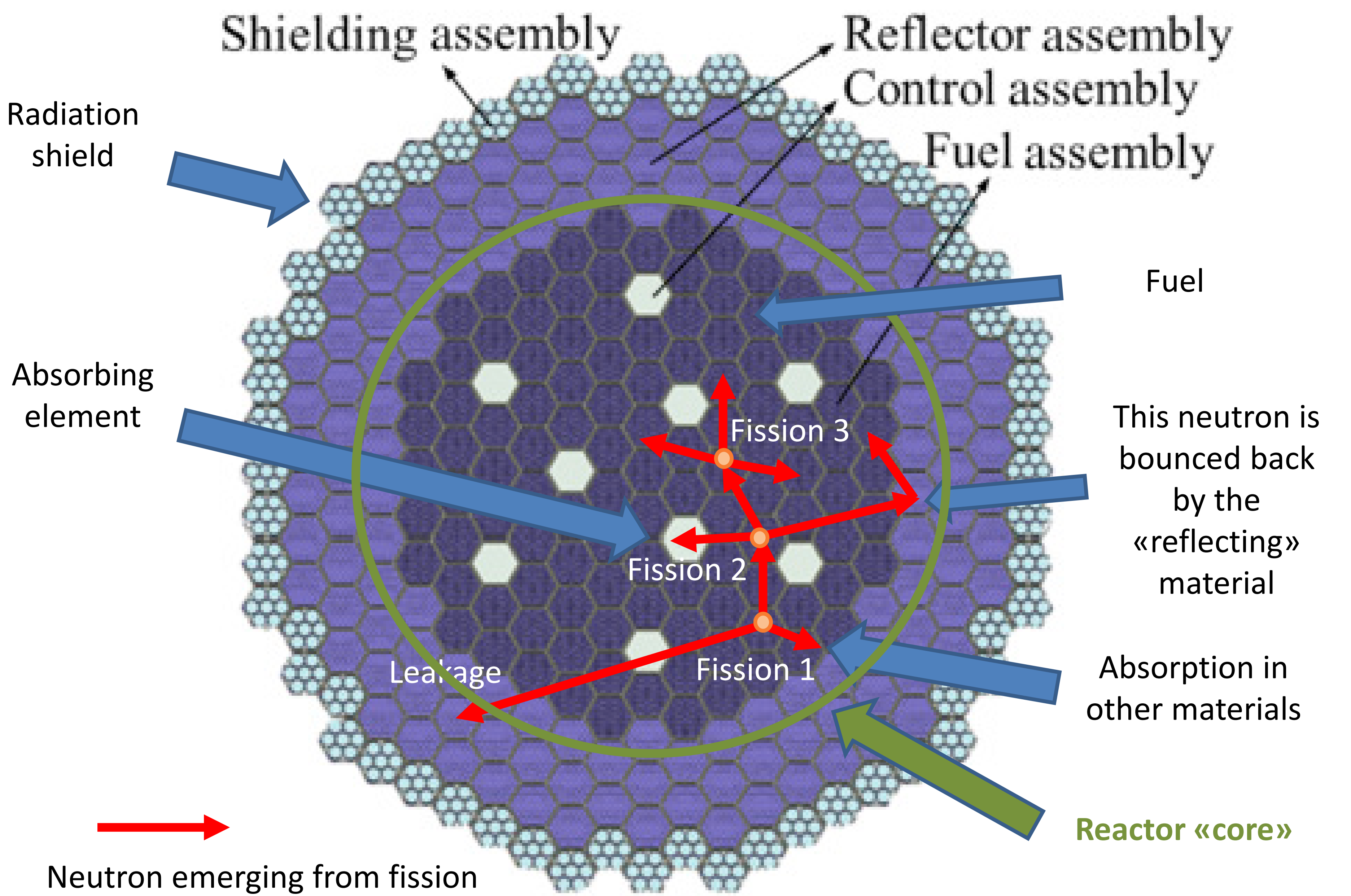
Physics of multiplication: path representation



η = average number of neutrons produced per neutron absorbed in the fuel

P_{NL}= probability of non-leakage (for a **finite system**)
f = conditional probability that, if neutron will be absorbed, it will be absorbed in fuel

Physics of multiplication: visual representation



Physics of multiplication

Multiplication can be written as

$$k = \frac{N_2}{N_1} = \eta\, f\, P_{NL}$$

N_1, N_2 = number of neutrons in two subsequent generations

η = average number of neutrons produced per neutron absorbed in the fuel

where

$$\eta = \nu \frac{\sigma_f^F}{\sigma_a^F}$$

$\sigma_f^F =$ Fission cross section in the fuel

$\sigma_a^F =$ Absorption cross section in the fuel

$\nu =$ Average number of emitted neutrons

f = conditional probability that, if neutron will be absorbed, it will be absorbed in fuel

P_{NL}= probability of non-leakage

Infinite reactor → P_{NL}= 1 ⟹ $k_\infty = \eta\, f$

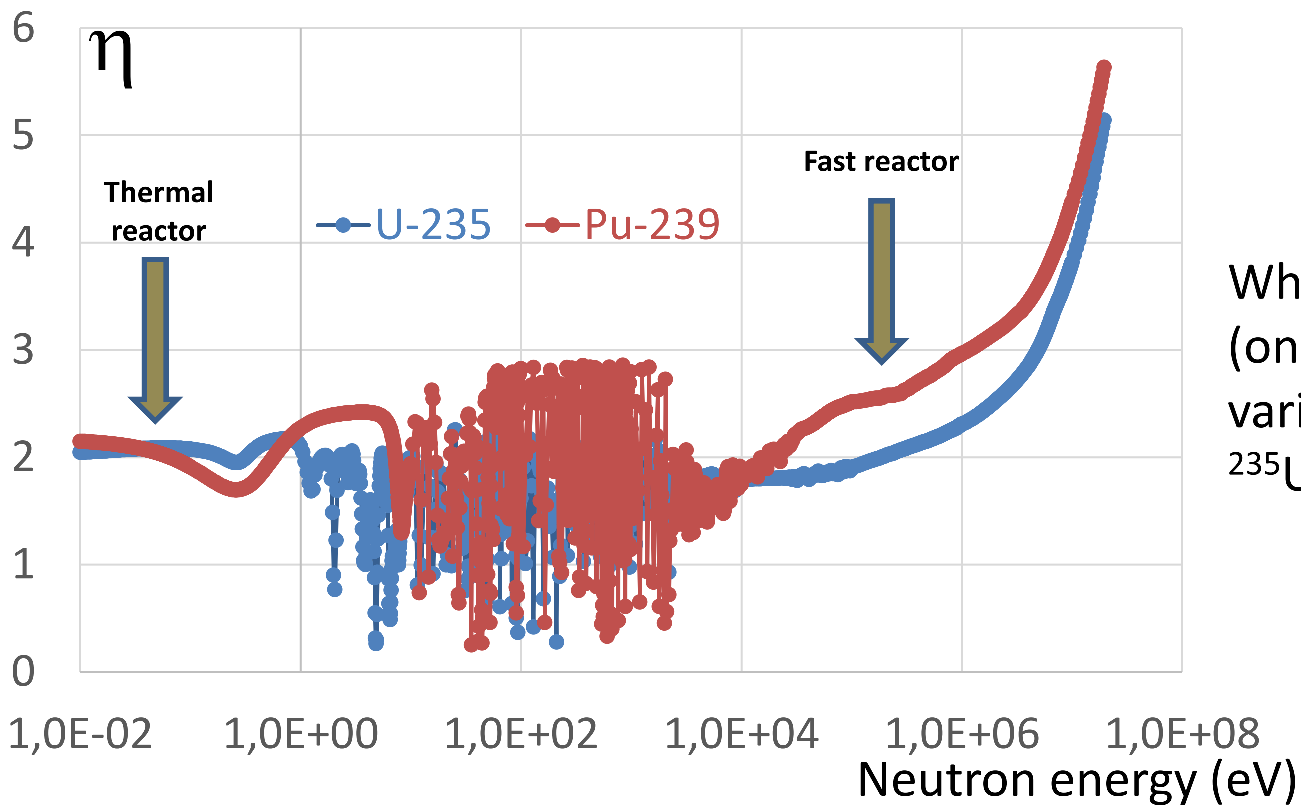
- ✓ This is a <u>property of the material</u>, not of the geometry
- ✓ For a finite, non-homogenous reactor → **effective k, or k_{eff}**

Simple considerations

$$k = \frac{N_2}{N_1} = \eta\, f\, P_{NL}$$

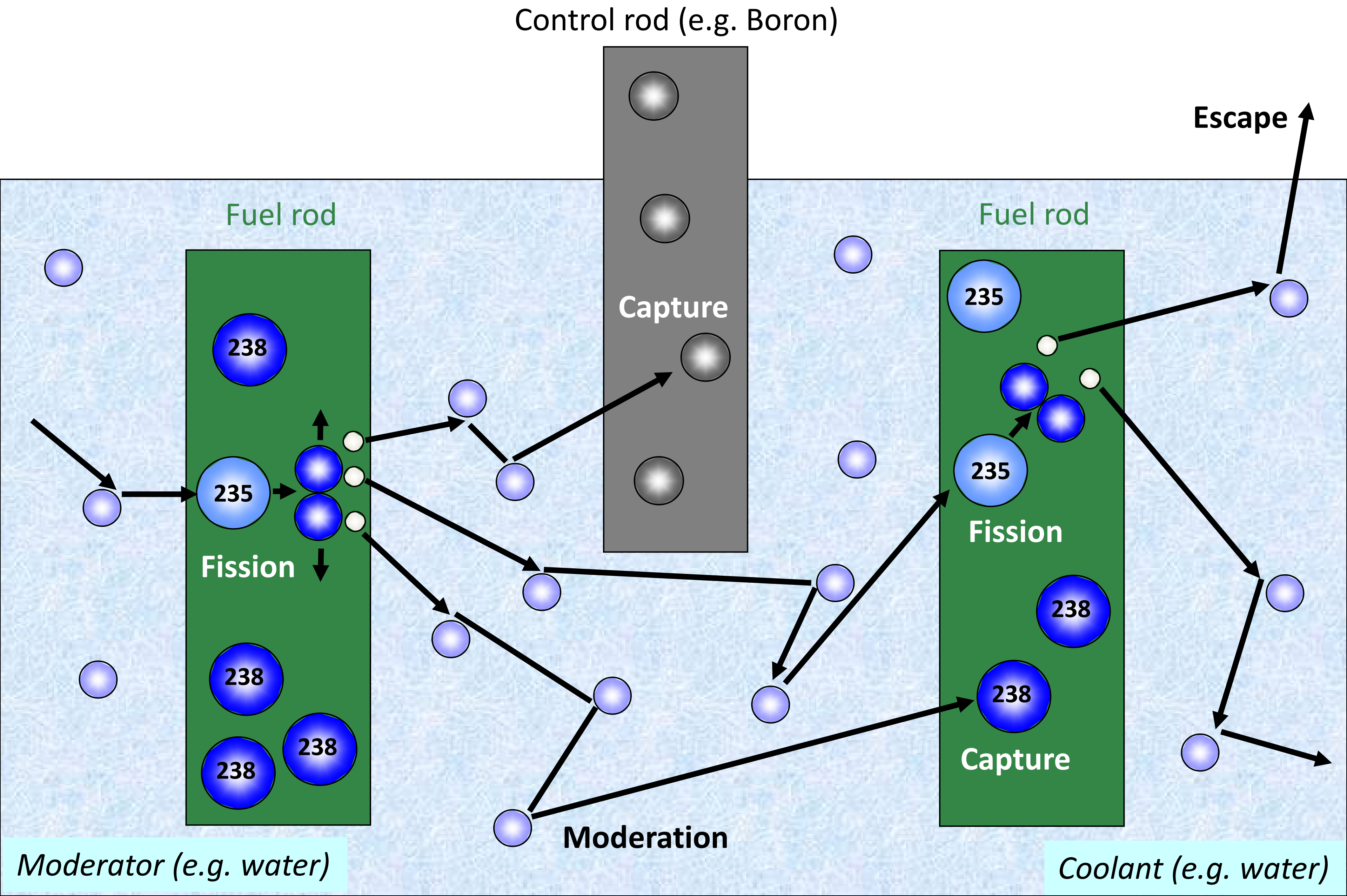
If n is absorbed, it is absorbed in the fuel

$f < 1$, $P_{NL} < 1$ ⇒ To have k~ 1 ➔ η significantly > 1

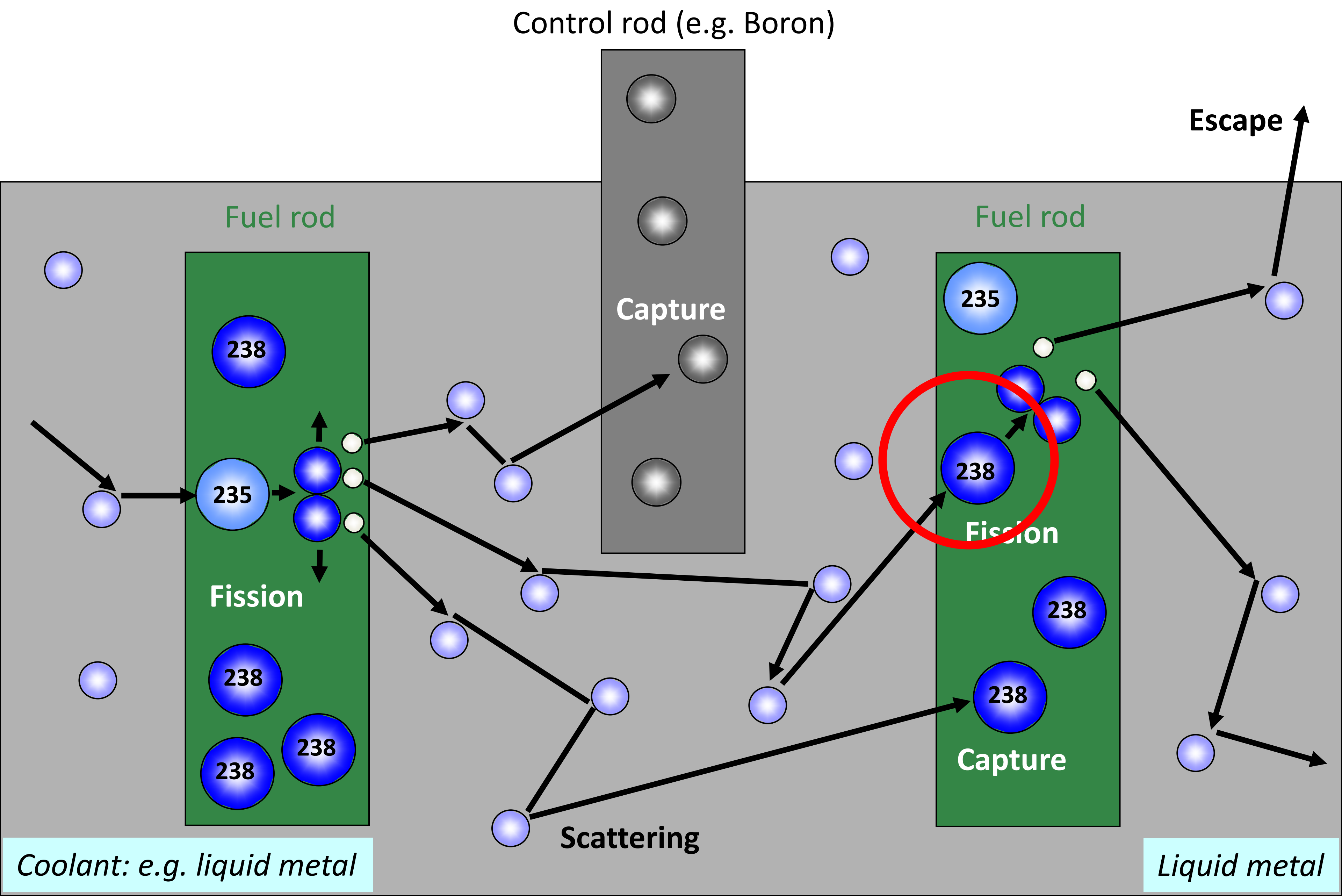


Which is indeed the case (on average): variation of η with energy for ^{235}U, ^{239}Pu

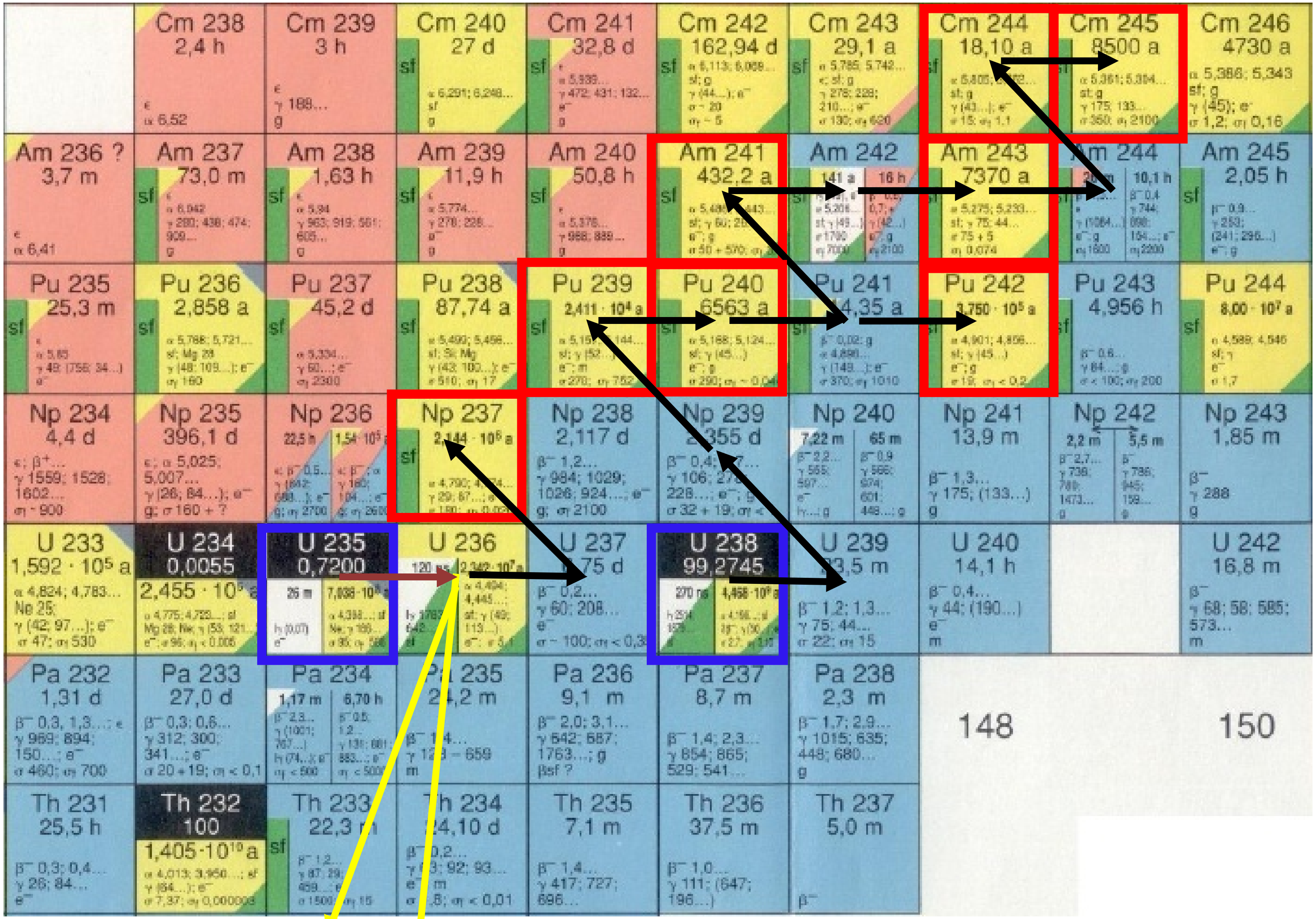
The thermal reactor



The fast reactor



Burning-breeding-burning: Uranium-Plutonium cycle and radioactive waste production (1 GW_e LWR)



$^{244, 245}Cm$ 1.5 Kg/yr

^{241}Am: 11.6 Kg/yr
^{243}Am: 4.8 Kg/yr

^{239}Pu: 125 Kg/yr

^{237}Np: 16 Kg/yr

LLFP 76.2 Kg/yr

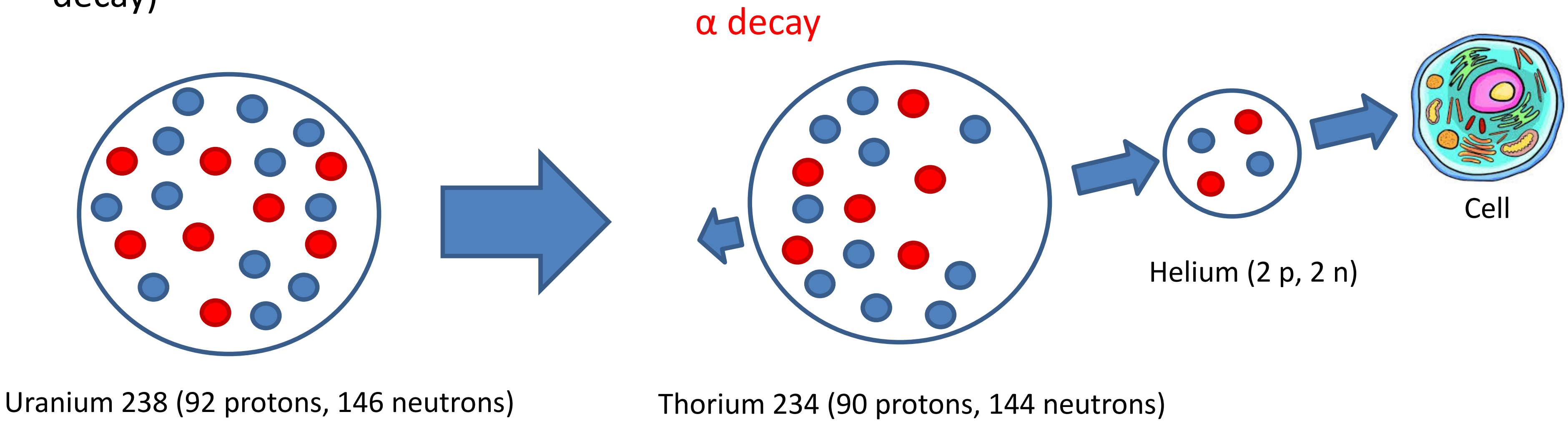
Fissile fuel

Fission Products (e.g. ^{90}Sr, ^{137}Cs → β emitters)
LLFP=Long Life Fission Products

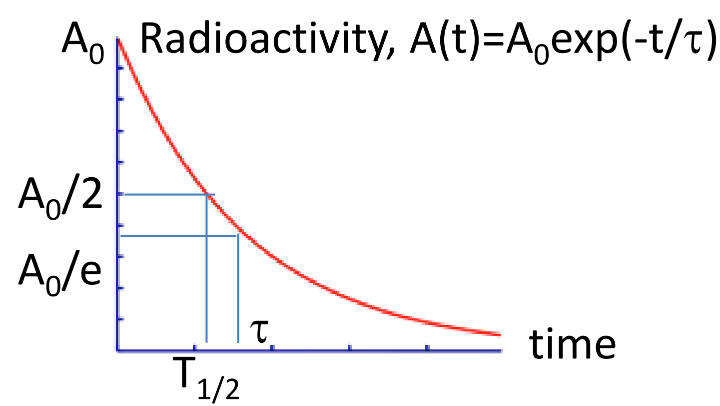
Transuranics = Minor Actinides + **Pu** (mostly α)

Transuranics and waste

- ✓ Operating a fission reactor means that radioactive nuclides (α, β, γ) are produced
- ✓ Many transuranics and some FP are long-lived
- ✓ Such radioactive nuclei can be dangerous for the environment and the human health (due to direct exposure in the case of γ's and due to ingestion or inhalation in the case of α and β decay)



Nuclide	Half-life $T_{1/2}$ (years)
Pu-239	24,000
Pu-242	3.7×10^5
Am-241	433



Breeding and burning: the Thorium-Uranium cycle

Fission products

Fission products

- Use an abundant element to «breed» new fuel
- Much lower production of Transuranics

Remember that

- ✓ *1 GW(th) = 1 GW thermal power*
- ✓ *1 GW(e) = 1 GW electrical power*
- ✓ *typically, for a fossil-fueled or nuclear power plant, a conversion factor between ~ 30 to 60 % has to be applied to go from thermal to electrical power*

How much fuel ?

Suppose you've got a **reactor with 1 GW thermal power (1 GW_{th} → ~ 300 MW_e) = 10^9 Joule/sec**
Assume each fission releases order of 200 MeV energy = 3.2×10^{-11} Joule
→In the reactor the fission rate is about 3×10^{19} fissions/sec
→ which means that e.g. 3×10^{19} (nuclei of ^{235}U)/sec disappear (actually a bit more because of radiative capture)

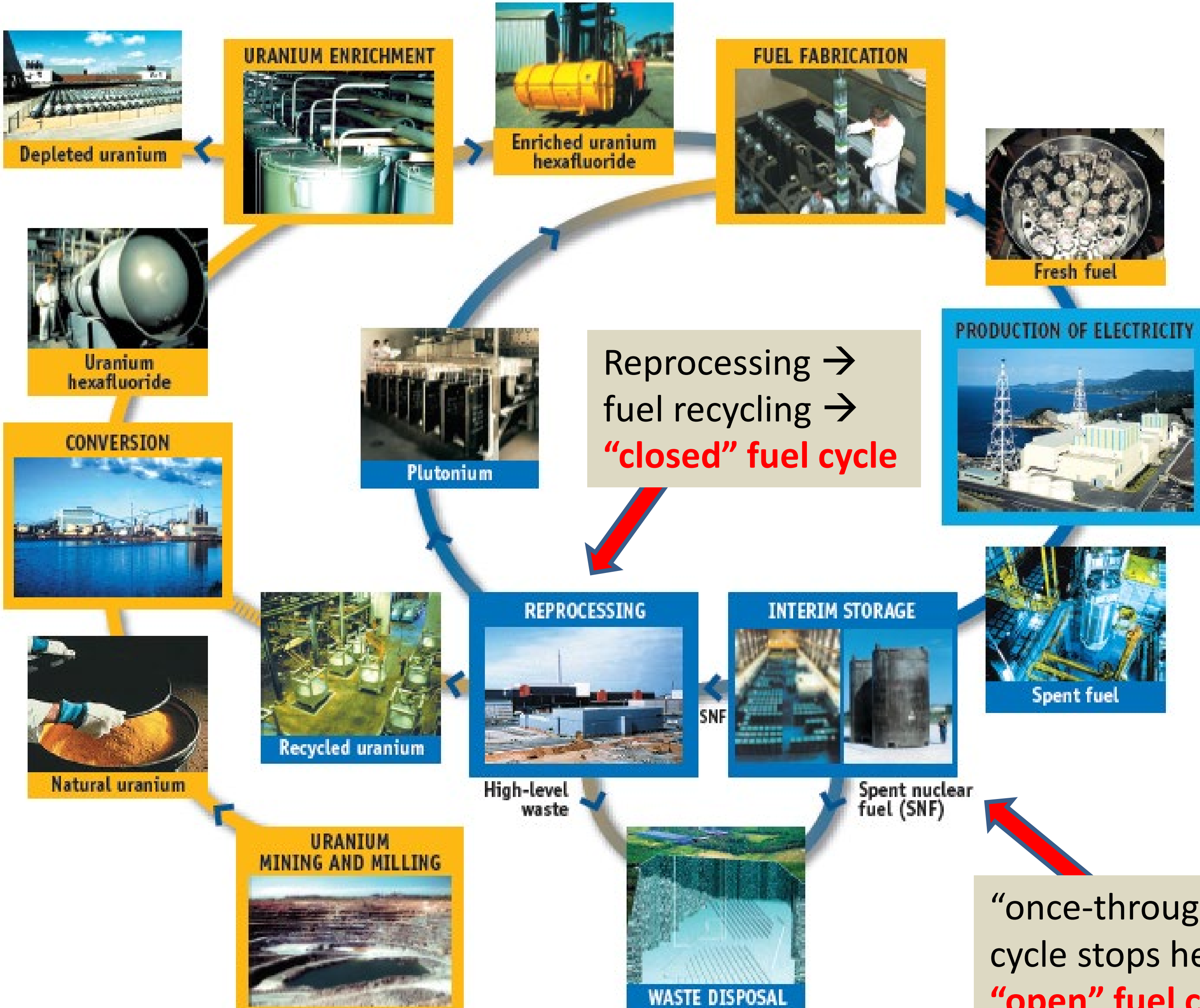
Fuel	Istantaneous consumption (per second)	Yearly consumption (@90 % load factor*)
Uranium	**0.012 g**	**340 Kg**
Natural Gas	27 m^3	766 million m^3
Crude oil (average)	22.5 Kg	0.6 million tons
Lignite (average)	67 Kg	1.9 million tons
Coal (average)	34 Kg	1 million tons

For a thermal reactor (see later) loaded with mixed UO_2 fuel (density about 11 gr/cm^3) comprising 4 % ^{235}U and 96 % ^{238}U, this corresponds to 8500 Kg of fuel → 0.8 m^3

In practice, there has to be much more as the chain reaction needs the presence of fissile nuclei at all times → the reactor has to be critical at all times
However, ^{235}U consumption is partly compensated by Plutonium (^{239}Pu) burn up

(*) load factor=percentage of time when the reactor is actually producing electricity

How does it actually work ? The nuclear fuel cycle

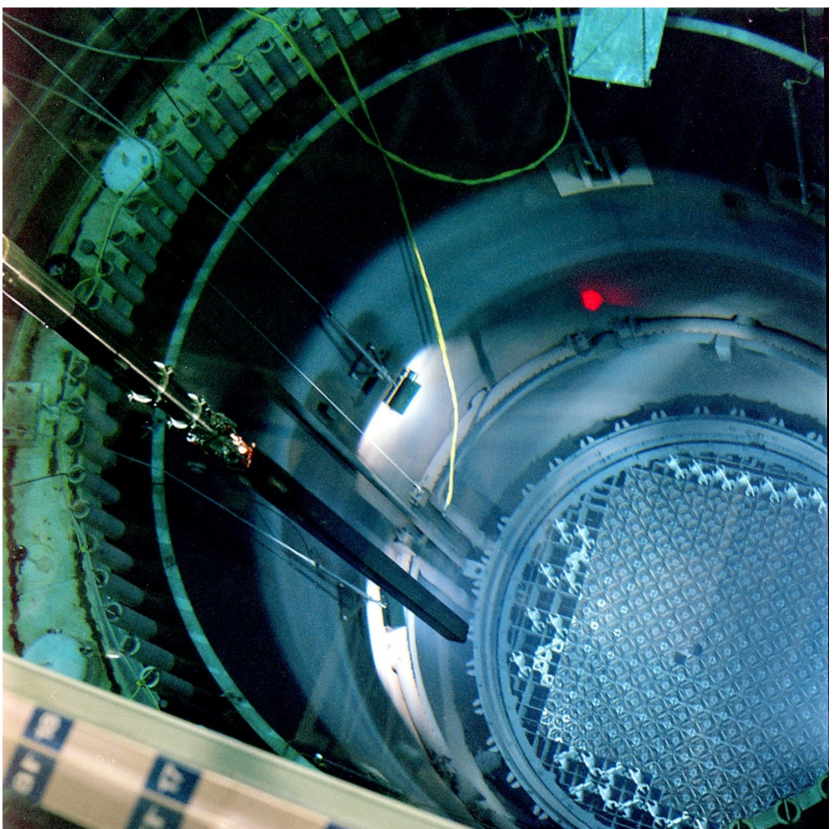


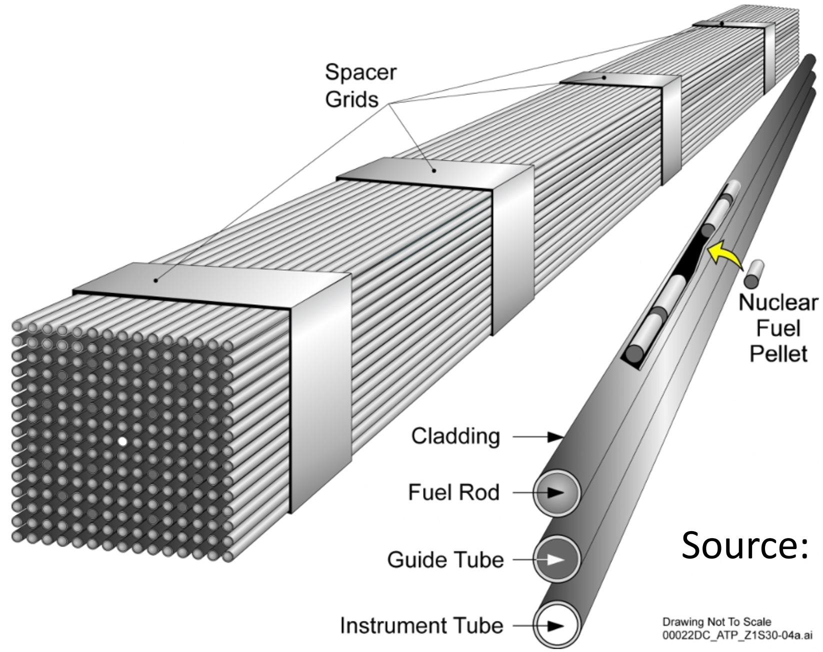
Reprocessing →
fuel recycling →
“closed” fuel cycle

“once-through”
cycle stops here →
“open” fuel cycle

How does it actually look like ?

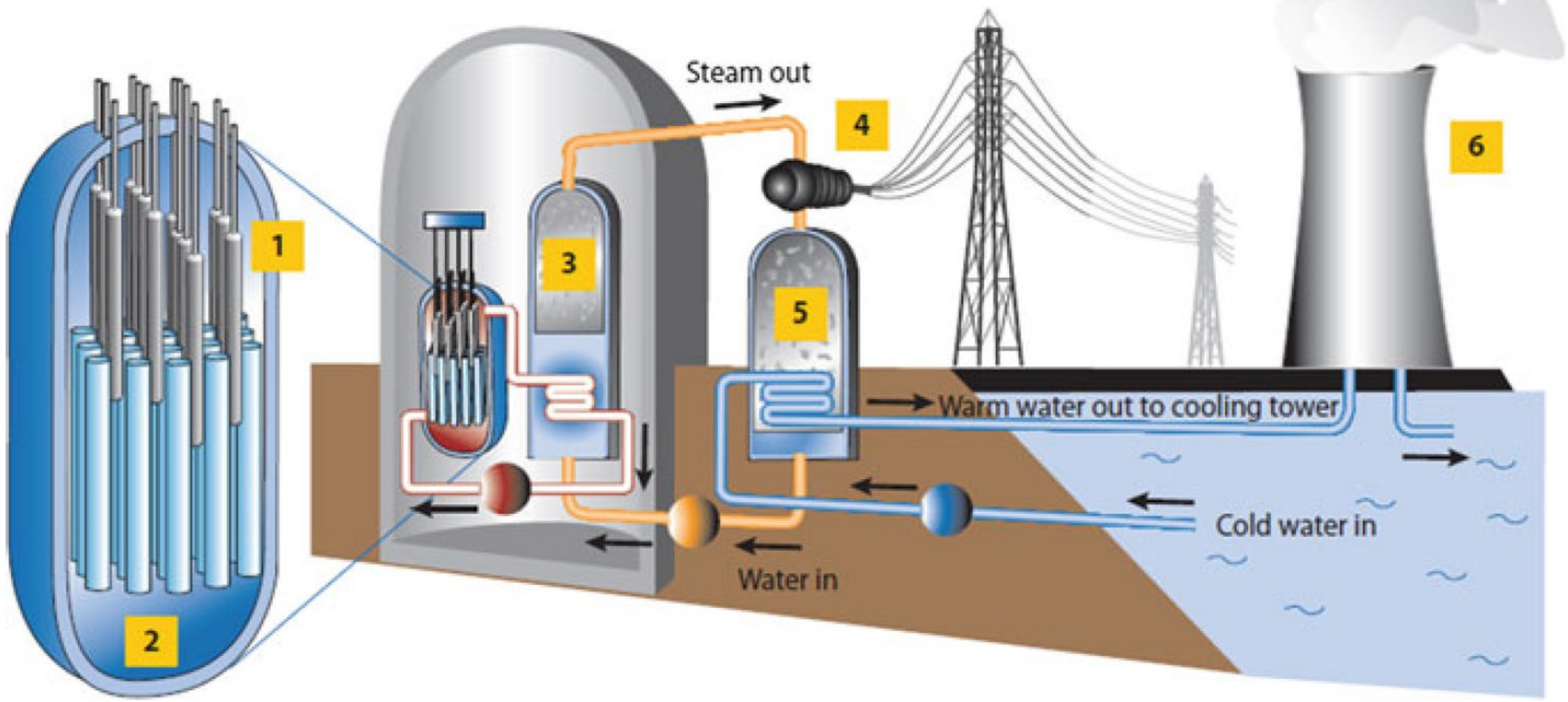
Fuel pellets. Photo: Areva/US NRC





Source: US DOE

The power plant



Basic components of a thermal nuclear power reactor (pressurised water reactor):

1-Reactor: fuel rods (light blue) heats up pressurised water. Control rods (grey) absorb neutrons to control or halt the fission process

2-Coolant and moderator: fuel and control rods are surrounded by water (primary circuit) that serves as coolant and moderator

3-Steam generator: water heated by the nuclear reactor transfers thermal energy through thousands of pipes to a secondary circuit of water to create high-pressure steam

4-Turbo-generator set: steam drives the turbine, which spins the generator to produce electricity just like in a fossil-fuel plant

5-Condenser: removes heat to convert steam back to water, which is pumped back to the steam generator

6-Cooling tower: removes heat from the cooling water that circulates through the condenser, before returning it to the source at near-ambient temperature

Source: OECD-NEA Nuclear Energy Today, 2nd edn. (2012), ISBN 978-92-64-99204-7. NEA Report No. 6885

Decay heat

Decay heat is the heat released as a result of radioactive decay: the energy of the alpha, beta or gamma radiation is converted into atomic motion

In nuclear reactors **decay of the short-lived radioisotopes created in fission continues at high power**, for a time after shut down

Heat production comes **mostly from β decay** of fission products

A practical approximation is given by the formula

$$\frac{P}{P_0} = 6.6 \cdot 10^{-2} \left[\frac{1}{(\tau - \tau_s)^{0.2}} - \frac{1}{\tau^{0.2}} \right]$$

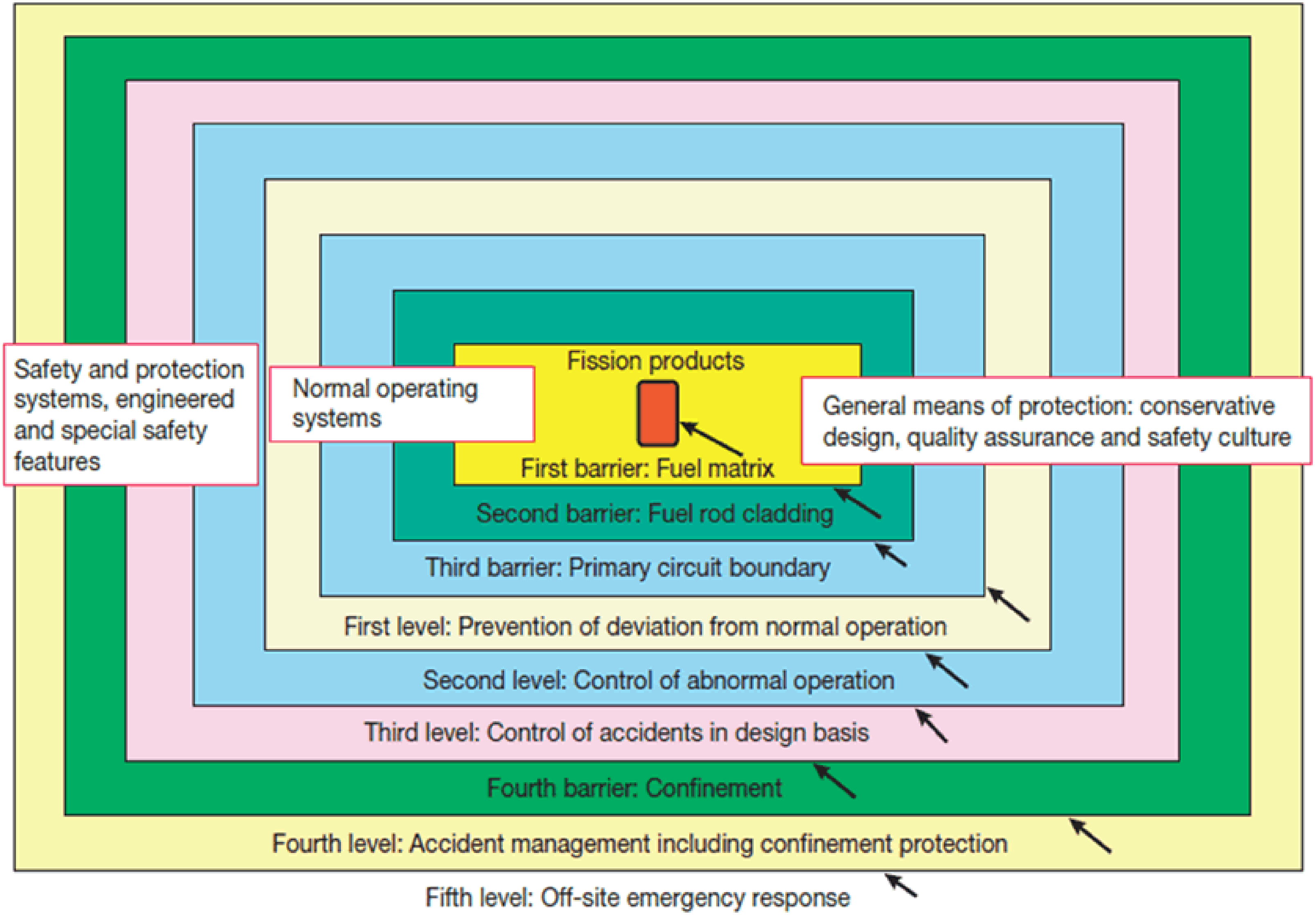
Where P is the decay power, P_0 is the reactor power before shutdown, τ is the time since reactor startup and τ_s is the time of reactor shutdown measured from the time of startup (in seconds)

➡ **At shutdown, the heat power is about 6.5 % (~200 MWth for a 1 GWe reactor) Sufficient to melt the core….**

About 1 hour after shutdown, the decay heat will be about 1.5% of the previous core power. After a day, the decay heat falls to 0.4%, and after a week it will be only 0.2%
Spent fuel rods are kept for long time in water pool, before storage or reprocessing.

Removal of decay heat very important → Fukushima…
"Heat sink" must not be compromised

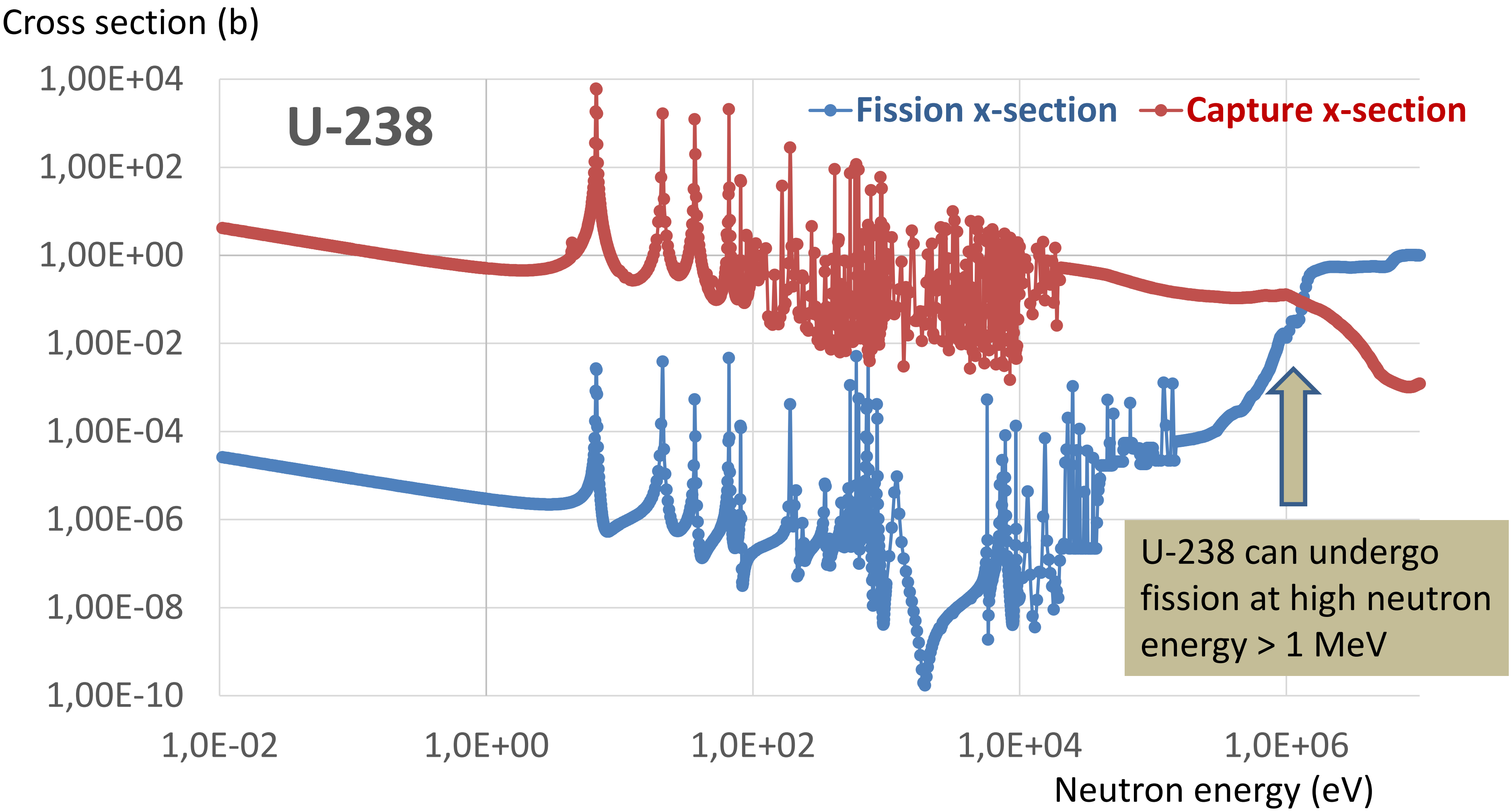
Safety: defence in depth



Control of abnormal operation should include some (negative) feedback mechanisms:
e.g. if temperature (power) goes up, reaction cross section goes down

Courtesy of IAEA

Fast spectrum systems and waste incineration (transmutation)



But also several **Minor Actinides** are characterized by a **fission threshold** around the **MeV**

➔ Such isotopes can be burnt in **fast reactors or in fast Accelerator Driven Systems (ADS)** (neutron spectrum from 10 keV to 10 MeV)

Past and future