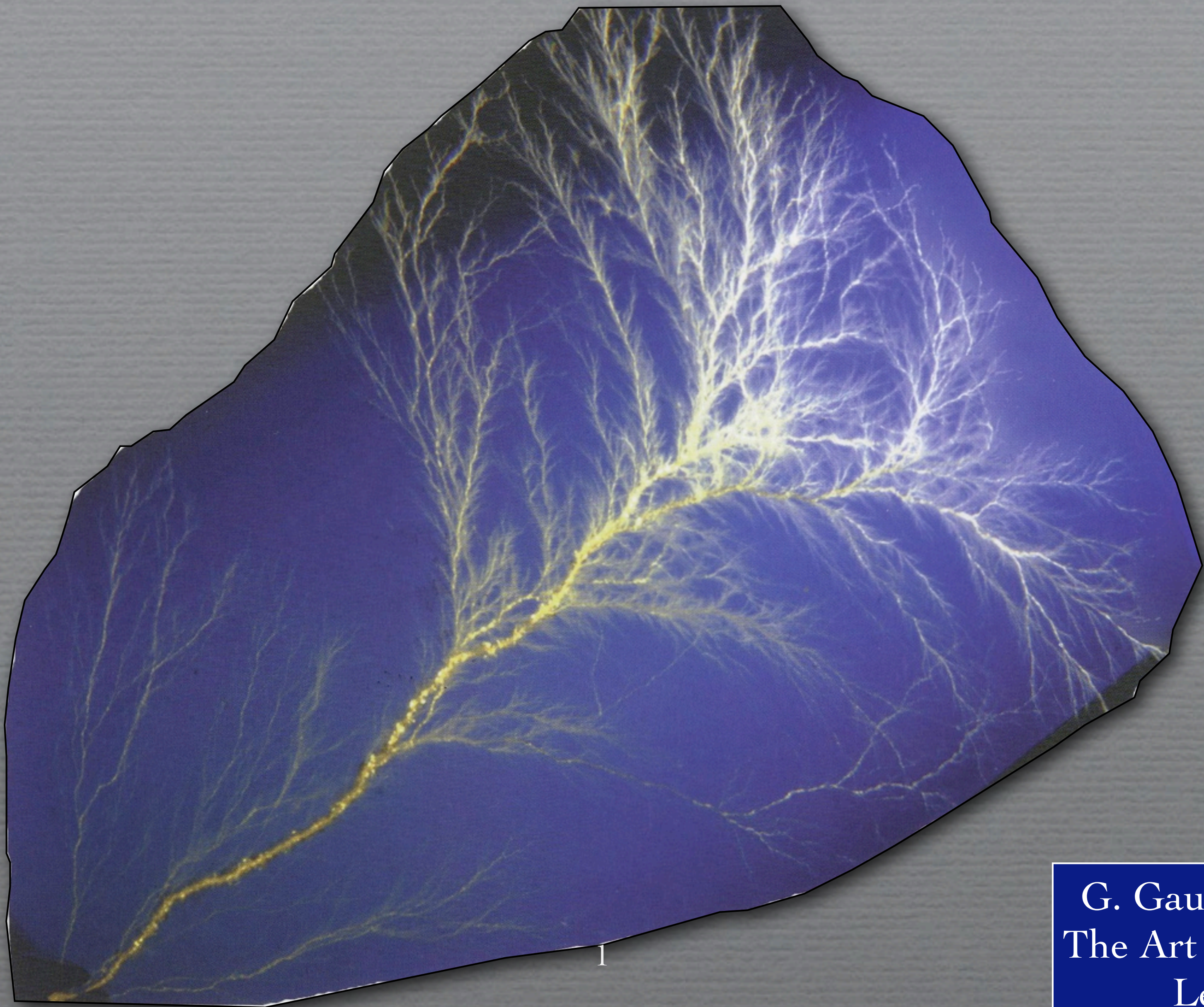
Electromagnetic and hadronic showers development



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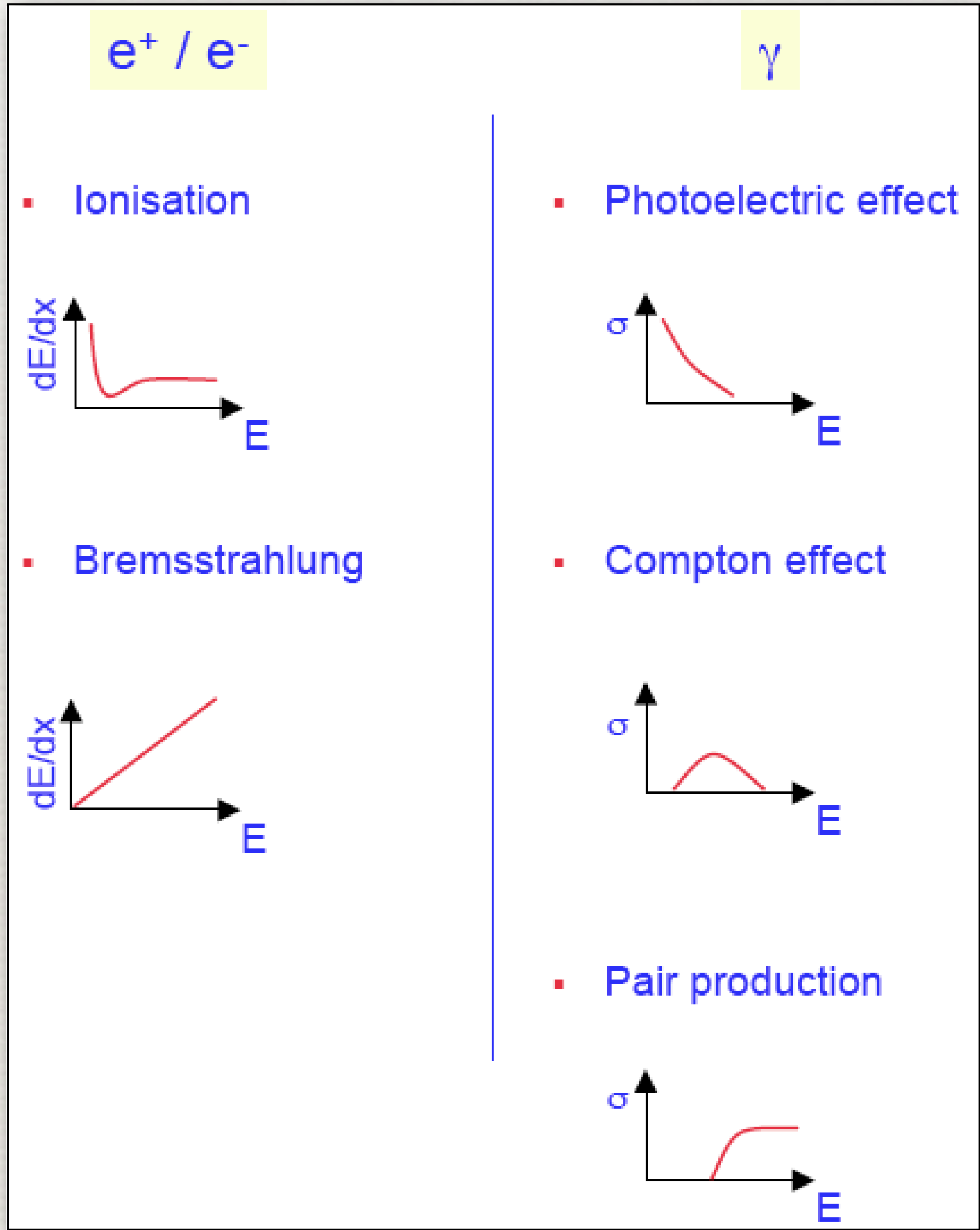
G. Gaudio, M. Livan
The Art of Calorimetry
Lecture II

Summary (Z dependence)

Z

$Z(Z+1)$



$Z^{4\div5}$

Z

$Z(Z+1)$

A simple shower

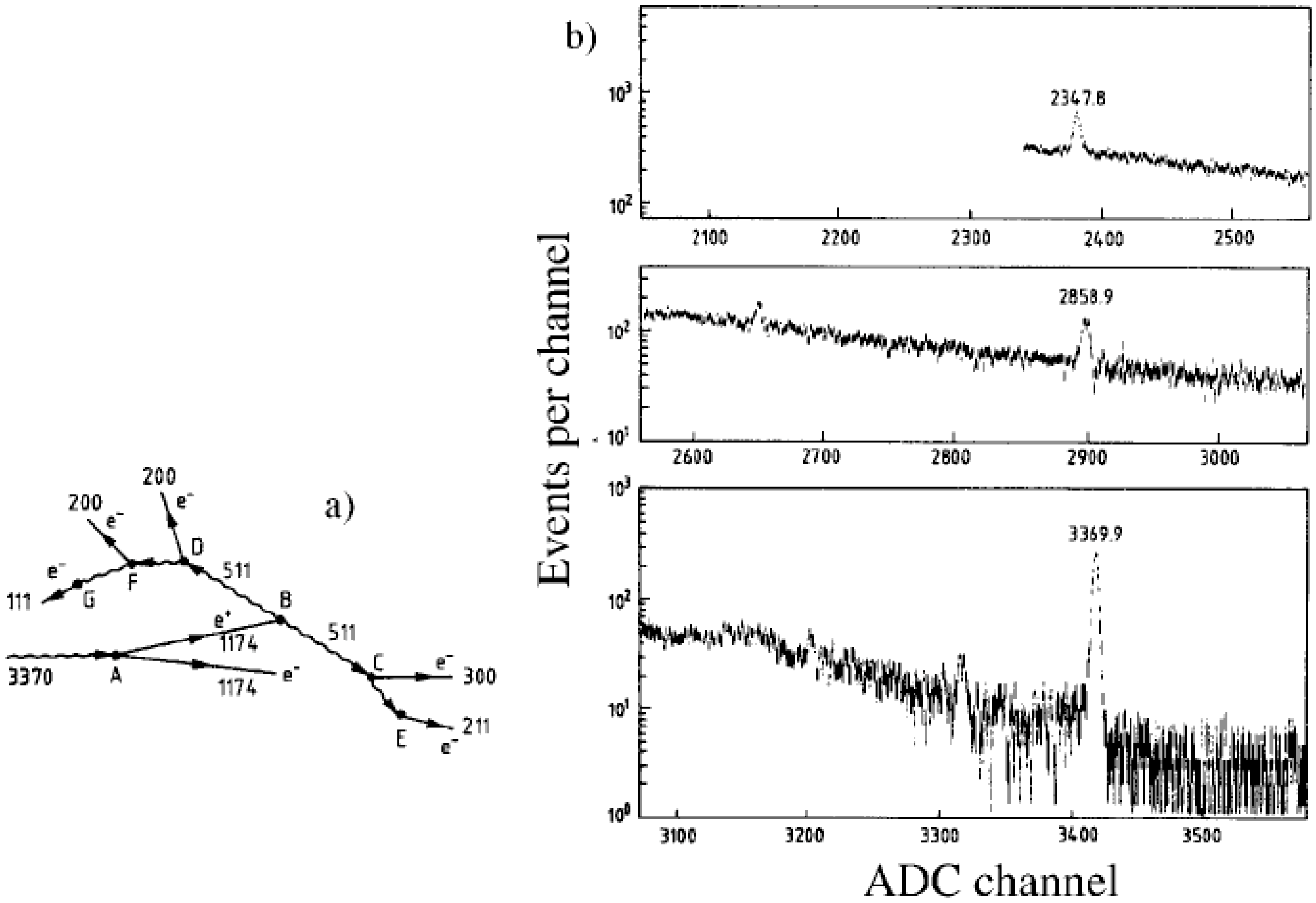


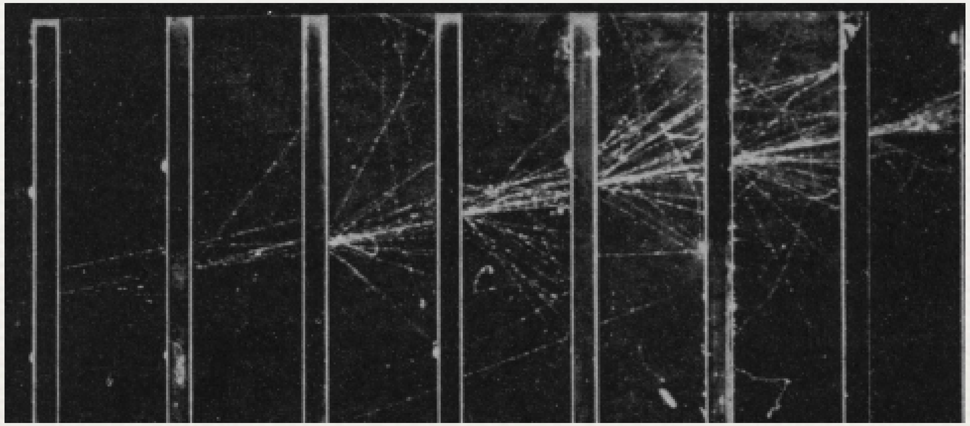
FIG. 2.8. Shower development induced by nuclear γs with an energy of 3370 keV, produced in the decay of ^{65}Ga. In (*a*), one possible sequence of absorption processes is depicted, with the energies of the positron, the electrons and the photons given in keV. The γ-ray spectrum, measured with a (small) Ge(Li) crystal in which these γs (and others of different energies) interacted is shown in (*b*). The total-containment peak (3369.9 keV), the single-(2558.9 keV) and double-escape peak (2347.8 keV) and the continuum background reflect the different degrees of absorption that occur in this crystal. See text for details.

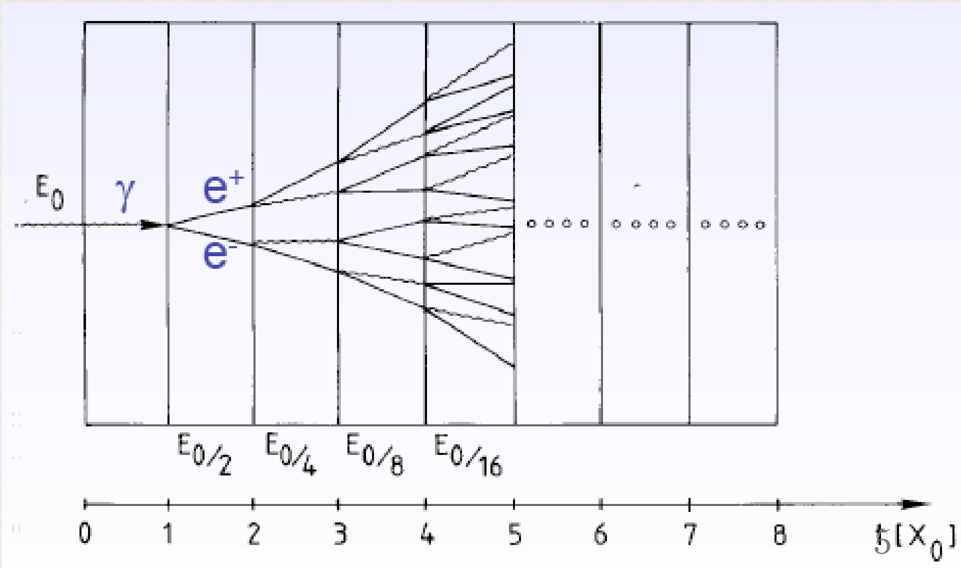
Electromagnetic Showers

- Differences between high-Z/low-Z materials
  - Energy at which radiation becomes dominant
  - Energy at which photoelectric effect becomes dominant
  - Energy at which pair production becomes dominant
- Showers ⇒ Particle multiplication ⇒ little material needed to contain shower
  - 100 GeV electrons: 90% of shower energy contained in 4 kg of lead
- Shower particle multiplicity reaches maximum at shower maximum
  - Depth of shower maximum shifts logarithmically with energy

Simple shower model

- Consider only Bremsstrahlung and (symmetric) pair production
- Assume $X_0 \sim \lambda_{pair}$
- After t X_0:
  - $N(t) = 2^t$
  - $E(t)/\text{particle} = E_0/2^t$
- Process continues until $E(t)<E_c$
- $E(t_{max}) = E_0/2^{t_{max}} = E_c$
- $t_{max} = \ln(E_0/E_c)/\ln 2$
- $N_{max} \approx E_0/E_c$





Electromagnetic shower profiles (longitudinal)

Depth of shower max increases logarithmically with energy

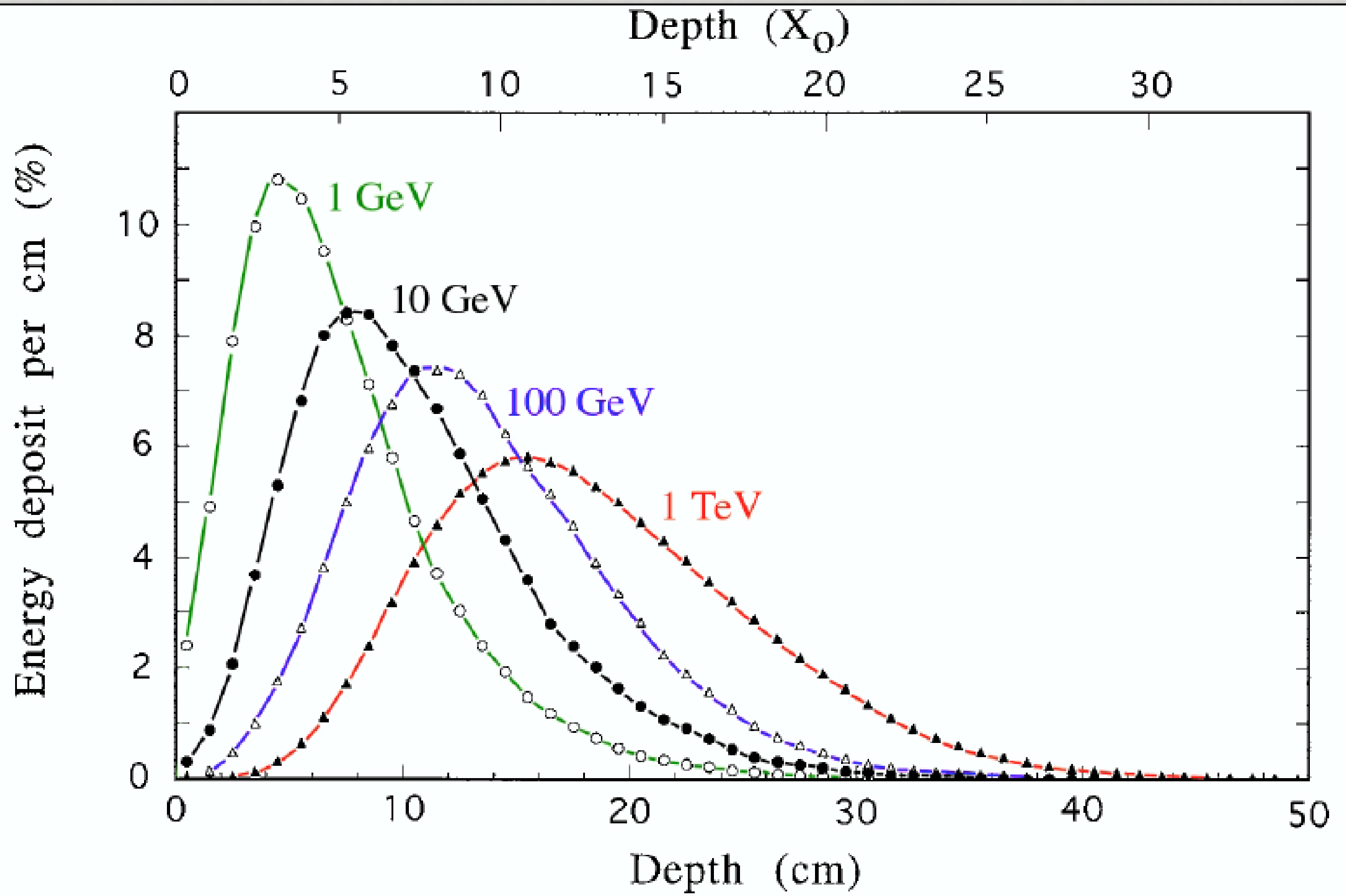
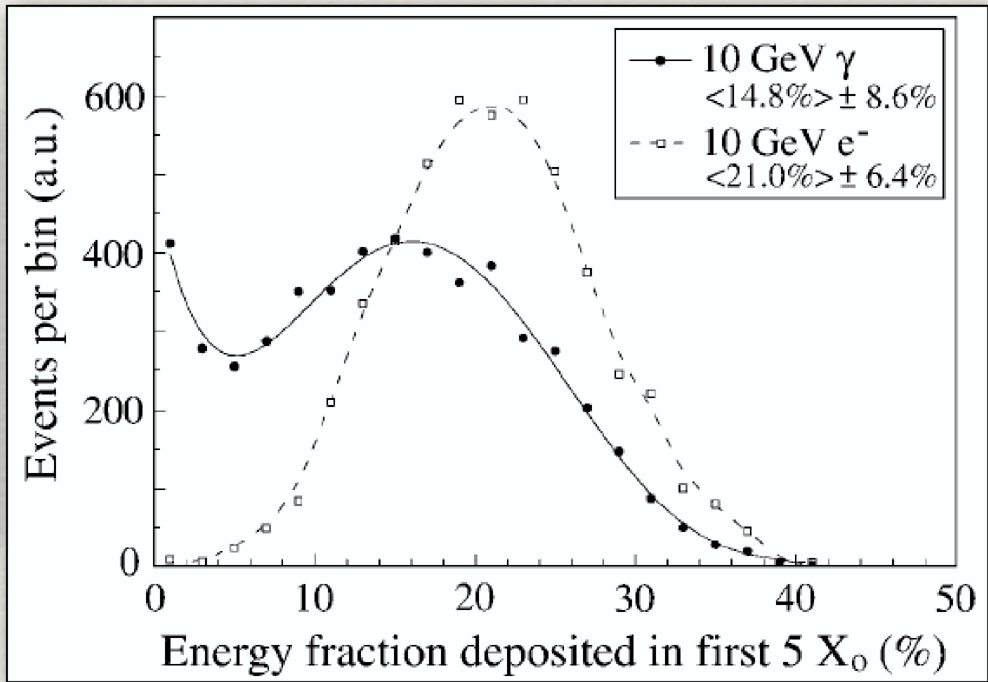


FIG. 2.9. The energy deposit as a function of depth, for 1, 10, 100 and 1000 GeV electron showers developing in a block of copper. In order to compare the energy deposit profiles, the integrals of these curves have been normalized to the same value. The vertical scale gives the energy deposit per cm of copper, as a percentage of the energy of the showering particle. Results of EGS4 calculations.

Electromagnetic Showers

- Longitudinal development governed by radiation length (X_0)
  - Defined only for GeV regime
- **there are important differences between showers induced by e, γ:**
  - e.g. Leakage fluctuations, effects of material upstream, ...
  - Mean free path of γs = 9/7 X_0

Distribution of energy fraction deposited in the first 5 X_0 by 10 GeV electrons and γs showering in Pb. Results of EGS4 simulations



Electromagnetic Showers

- Scaling with X_0 is not perfect
  - In high-Z materials, particle multiplication continues longer and decreases more slowly than in low-Z materials
    - $E_C \propto Z^{-1}$
  - The number of positrons strongly increases with the Z value of the absorber material
    - Example: number of e^+/GeV in Pb is 3 times larger than in Al
  - Need more X_0 of Pb to contain shower at 90% level

Scaling is NOT perfect

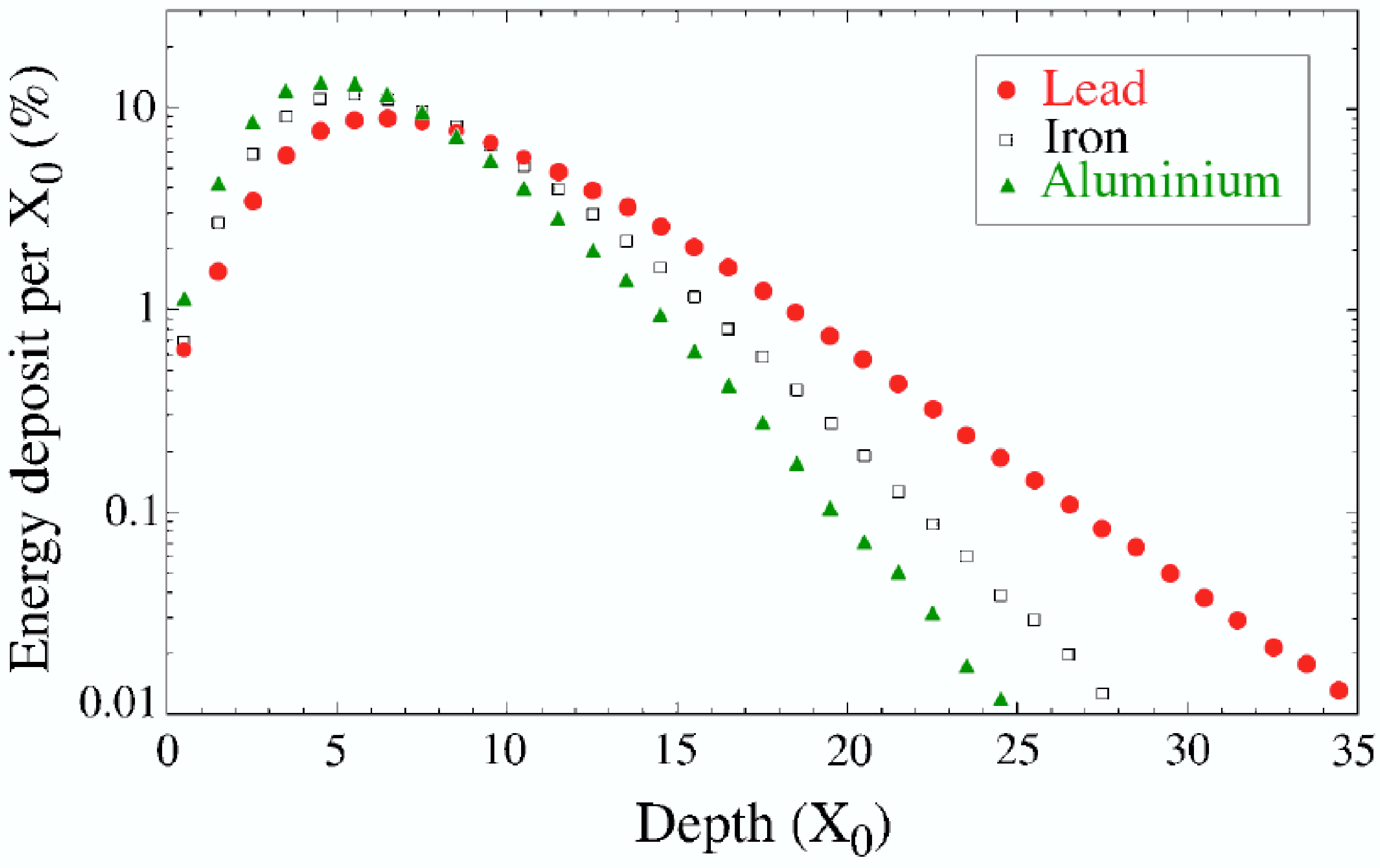
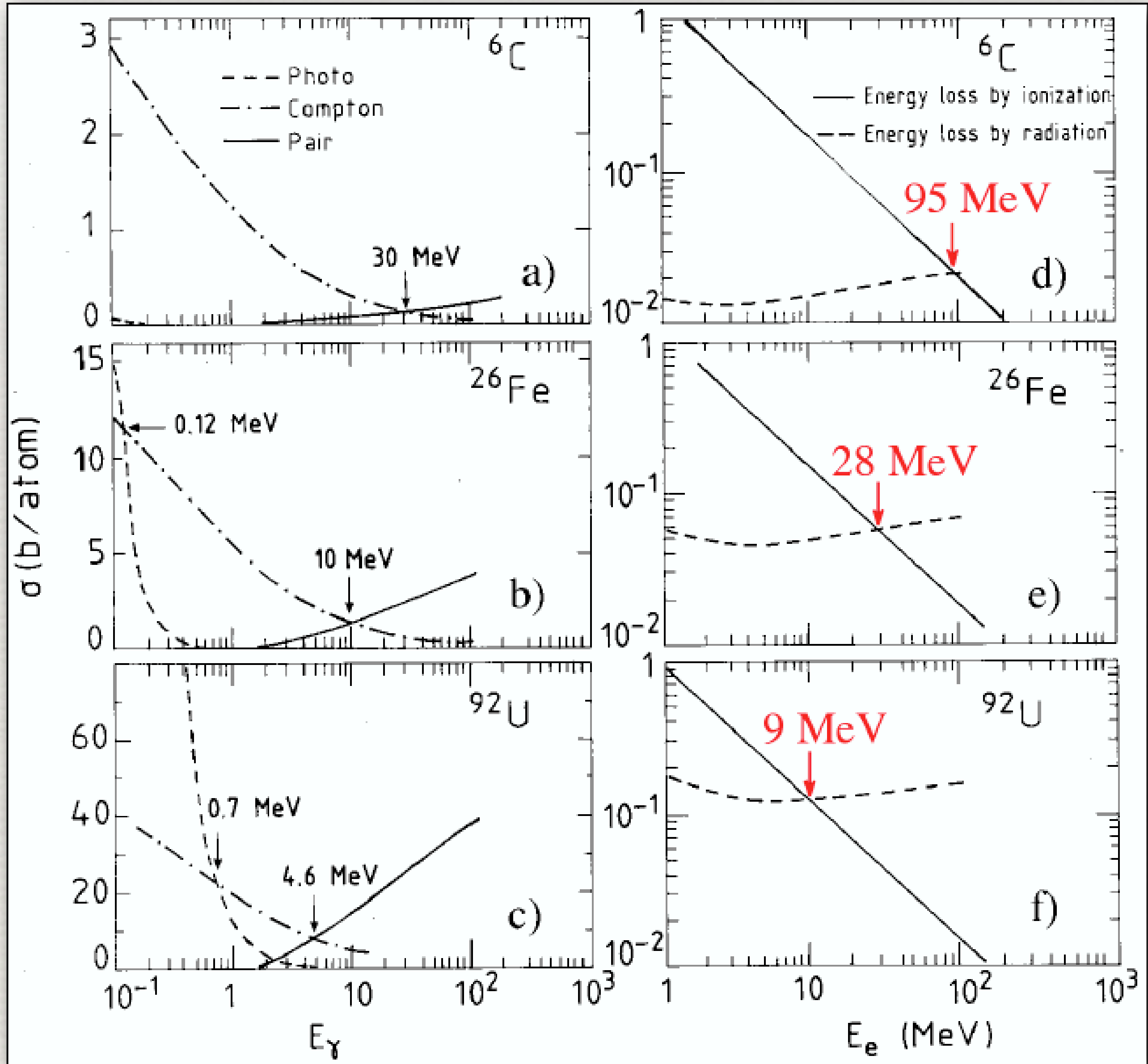


FIG. 2.12. Energy deposit as a function of depth, for 10 GeV electron showers developing in aluminium, iron and lead, showing approximate scaling of the longitudinal shower profile, when expressed in units of radiation length, X_0. Results of EGS4 calculations.

Pb Z = 82
Fe Z = 26
Al Z = 13

Z dependence

Gammas



Electrons

Electromagnetic shower leakage (longitudinal)

- The absorber thickness needed to contain a shower increases logarithmically with energy
- The number of X_0 needed to fully contain the shower energy can be as much as 10 X_0 going from high Z to low Z absorbers
- More X_0 needed to contain γ initiated showers

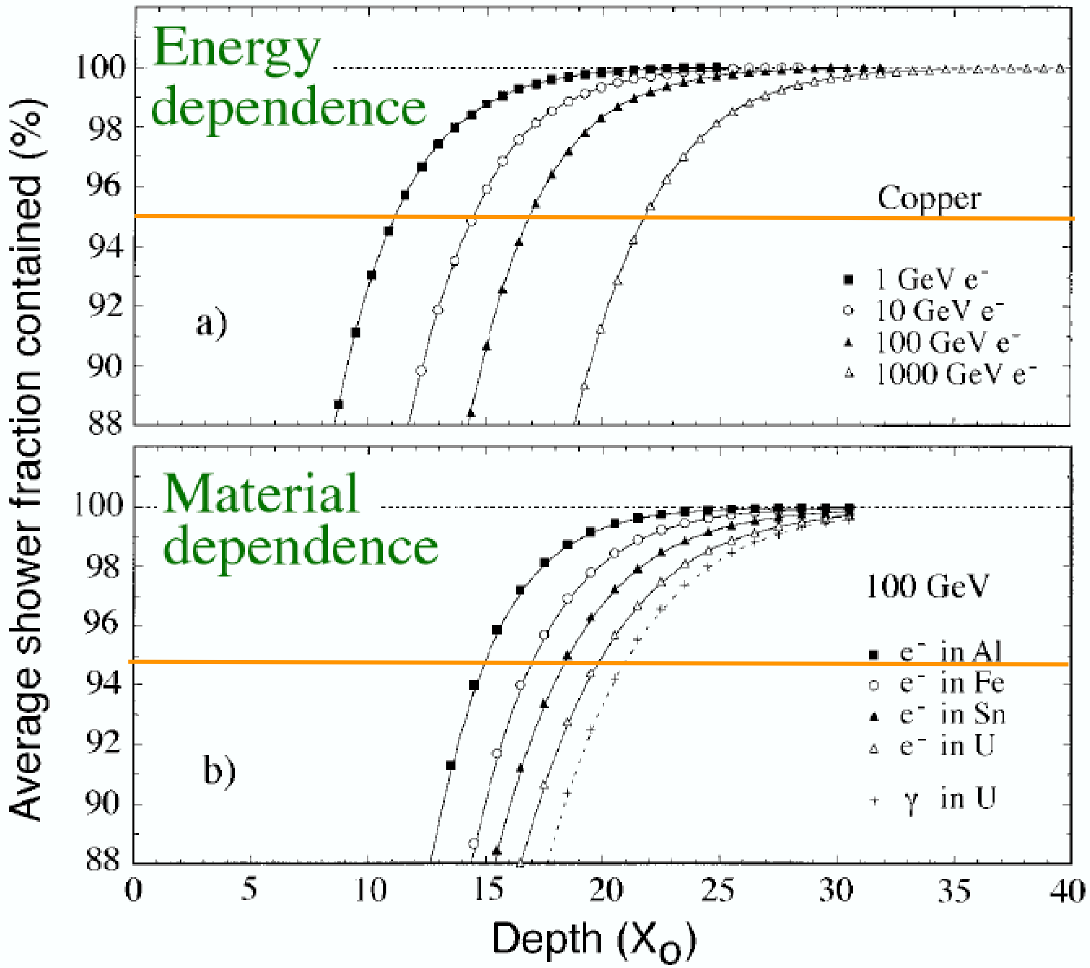
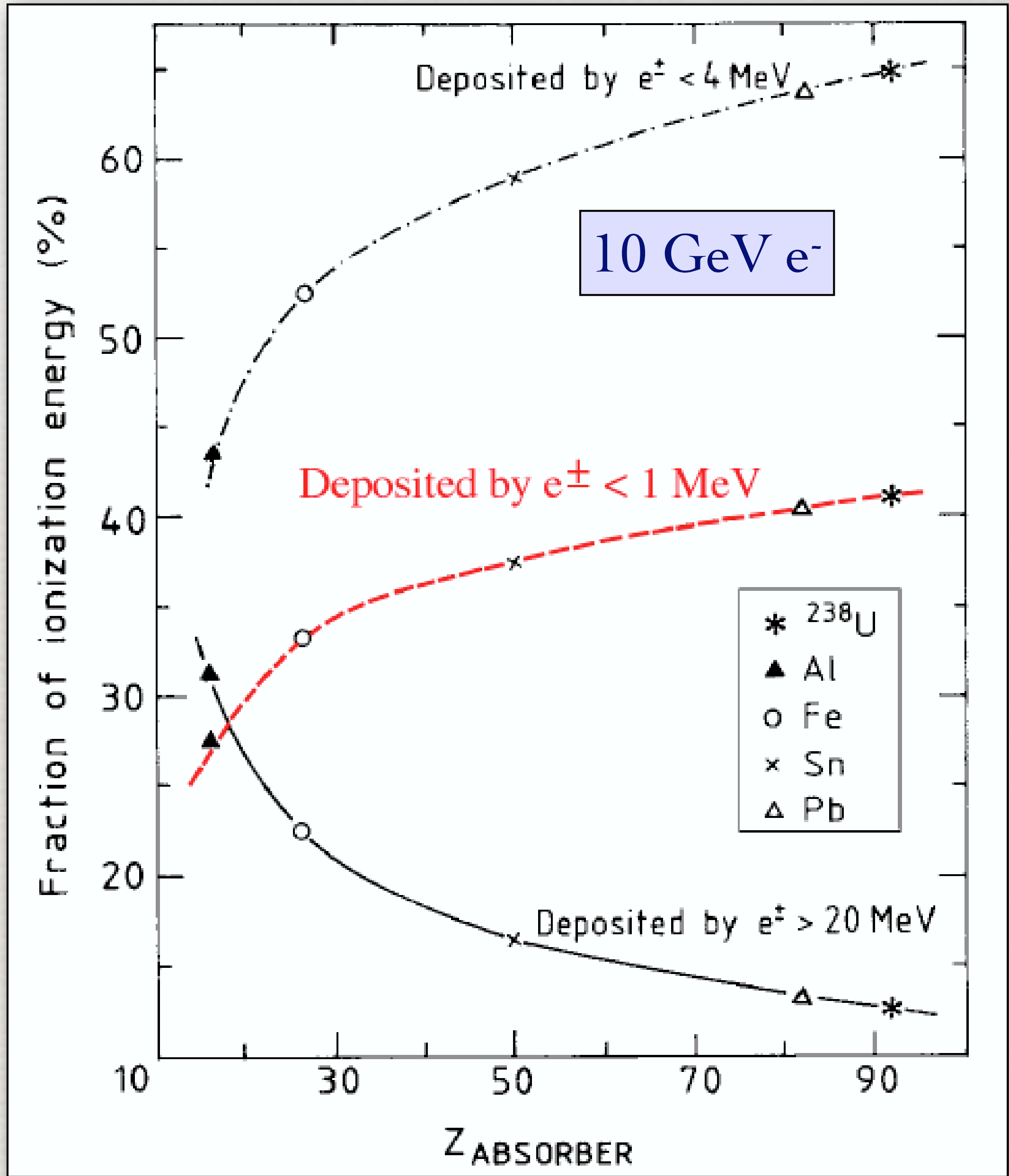


FIG. 2.17. Average energy fraction contained in a block of matter with infinite transverse dimensions, as a function of the thickness of this absorber. Shown are results for showers induced by electrons of various energies in a copper absorber (a) and results for 100 GeV electron showers in different absorber materials (b). The lower figure also shows the results for 100 GeV γ showers in ^{238}U. Results of EGS4 calculations.

Importance of SOFT particles

- The composition of em showers. Shown are the percentages of the energy of 10 GeV electromagnetic showers deposited through shower particles with energies below 1 MeV, below 4 MeV or above 20 MeV as function of the Z of the absorber material.
- Results of EGS4 simulations



Electromagnetic Showers

- Phenomena at $E < E_c$ determine important calorimeter properties
  - In lead > 40% of energy deposited by $e^{\pm}$ with E < 1 MeV
  - Only 1/4 deposited by e^+, 3/4 by e^- (Compton, photoelectrons!)
  - The e^+ are closer to the shower axis, Compton and photoelectrons in halo

Electromagnetic Showers

- Lateral shower width scales with Molière radius ϱ_M

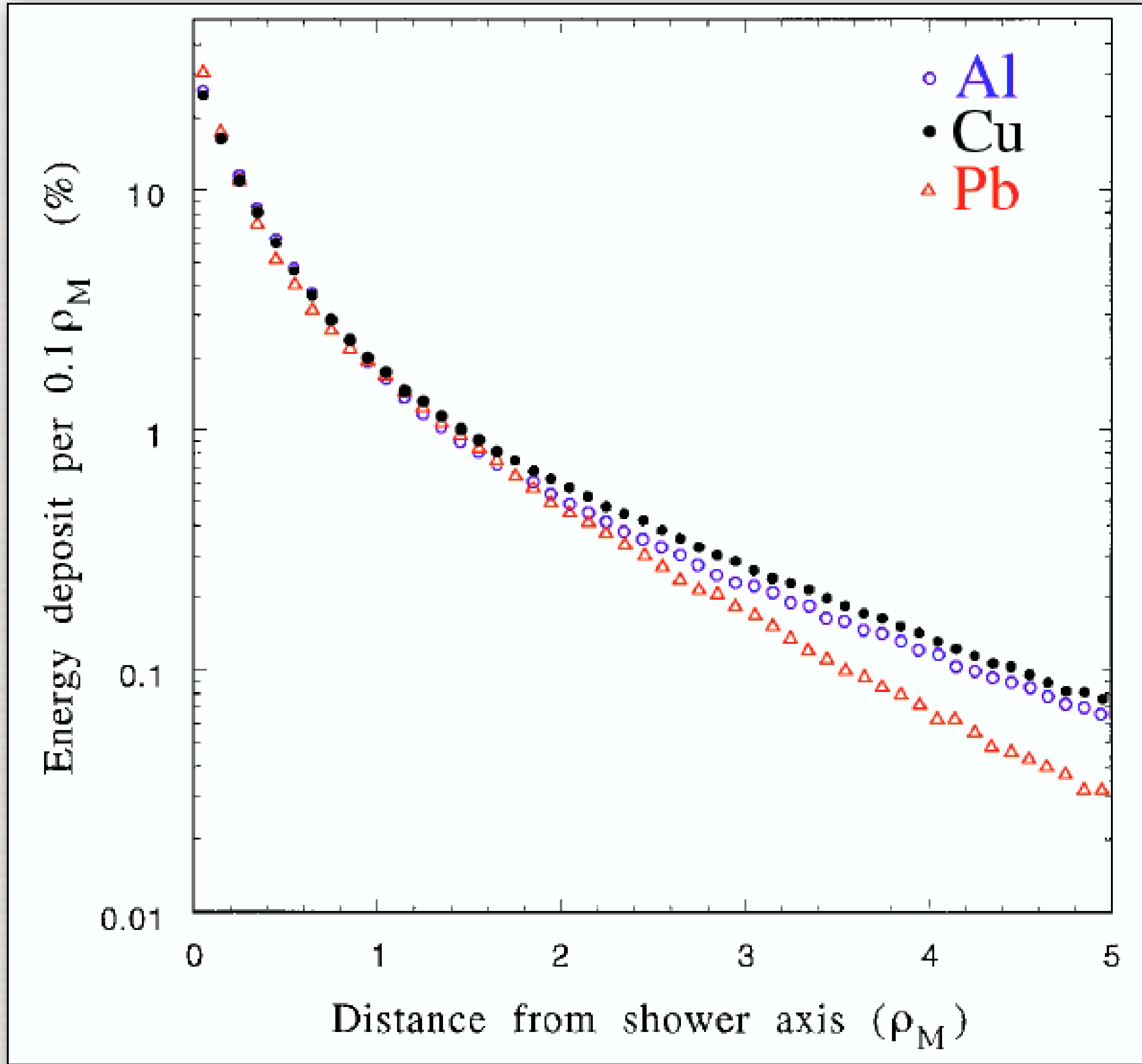
$$\rho_M = E_s \frac{X_0}{E_c} \qquad E_s = m_e c^2 \sqrt{4\pi/\alpha}$$

$$X_0 \propto A/Z^2, \quad E_c \propto 1/Z \quad \Rightarrow \quad \rho_M \propto A/Z$$

- ϱ_M much less material dependent than X_0
- Lateral shower width determined by:
  - Multiple scattering of $e^{\pm}$ (early, 0.2 ϱ_M)
  - Compton γs travelling away from axis (1 - 1.5 ϱ_M)

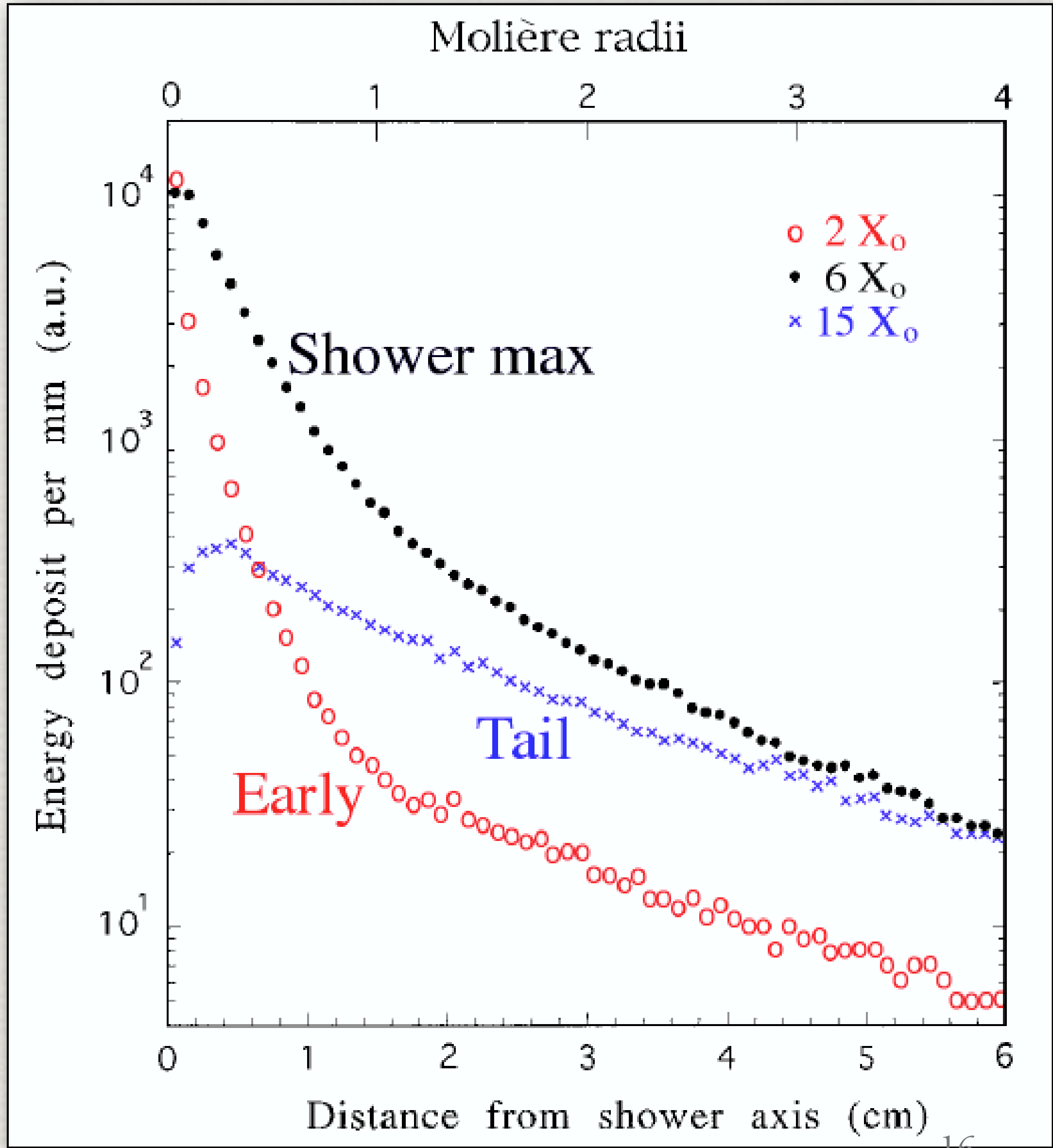
Lateral profile

- Material dependence
- Radial energy deposit profiles for 10 GeV electrons showering in Al, Cu and Pb
- Results of EGS4 calculations



Lateral profile

Radial distributions of the energy deposited by 10 GeV electron showers in Cu.
Results of EGS4 simulations



Electrons and positrons

Table 2.1 *The numbers of positrons that are generated in em shower development and the fraction of the total energy deposited by these particles. Results of EGS4 simulations.*

Shower energy →	10 GeV		100 GeV	
Absorber ↓	$\#e^+$	E^+/E_{tot}	$\#e^+$	E^+/E_{tot}
Aluminium ($Z = 13$)	191	26%	1750	27%
Iron ($Z = 26$)	285	27%	2920	26%
Tin ($Z = 50$)	427	24%	4330	25%
Lead ($Z = 82$)	554	22%	5730	23%
Uranium ($Z = 92$)	612	23%	5970	23%

- The number of positrons increases by more than a factor 3 going from Al to U
  - Aluminum (~18 e^+/GeV)
  - Uranium (~60 e^+/GeV)
- Increase due to the fact that particle multiplication in showers developing in high-Z absorber materials continues down to much lower energies than in low-Z materials

Contributions to signal

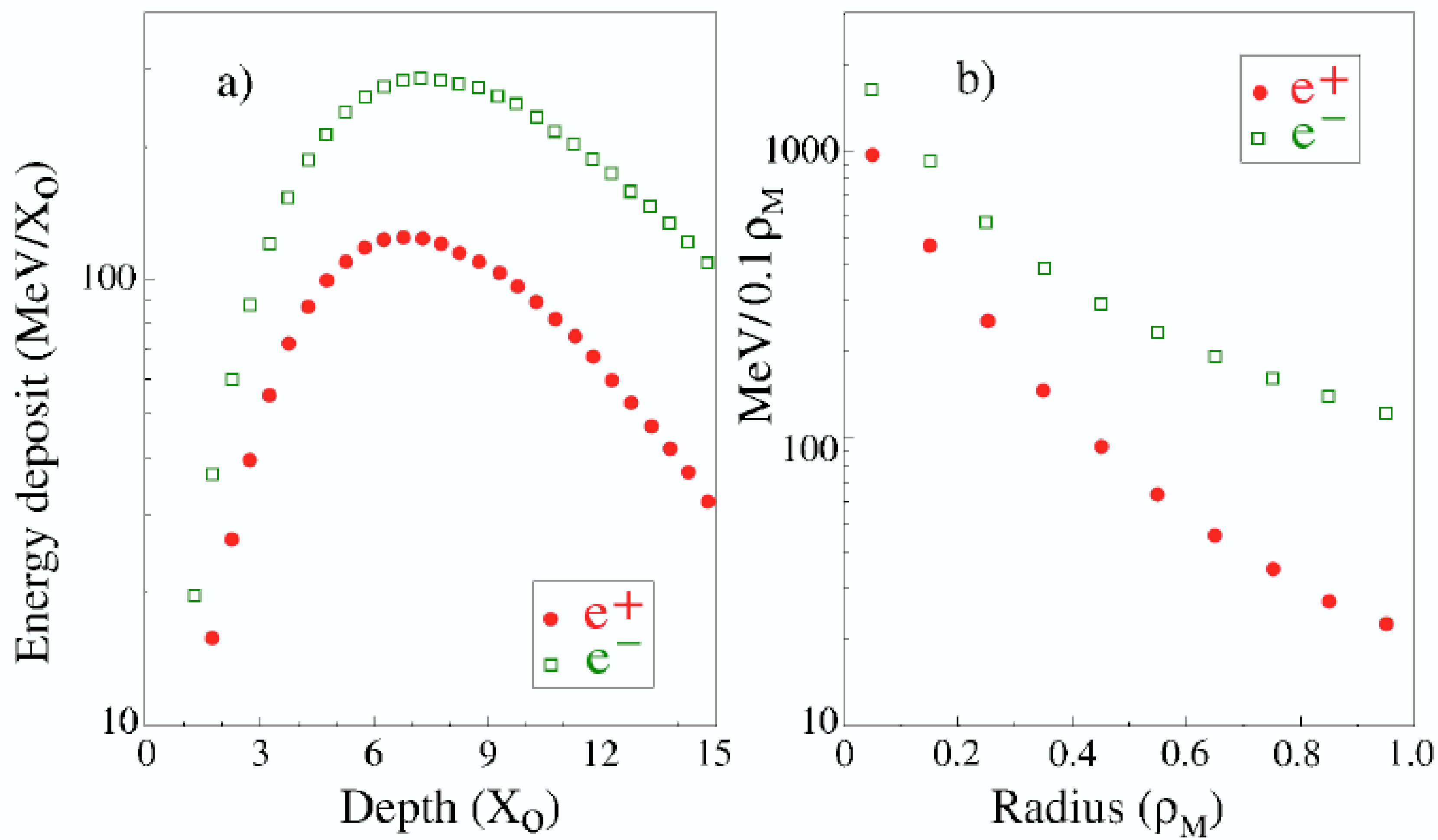


FIG. 2.11. Comparison of the longitudinal (*a*) and lateral (*b*) profiles of the energy deposited by electrons and positrons in 10 GeV em showers developing in lead. Note the logarithmic vertical scale. Results from EGS4 simulations.

Lateral shower leakage

- No energy dependence
- A (sufficiently long) cylinder will contain the same fraction of energy of a 1 GeV or 1 TeV em shower
- Results of EGS4 simulations

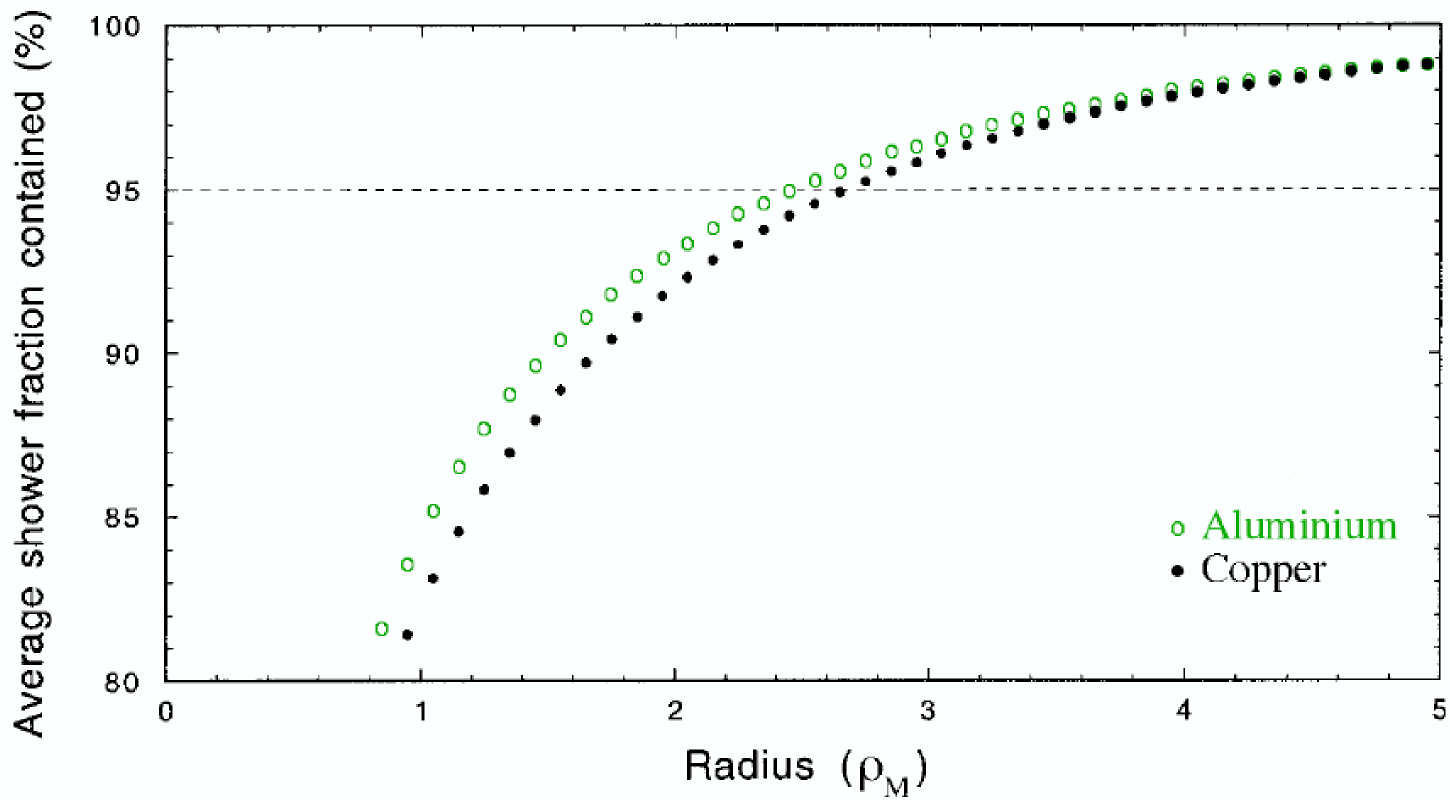


FIG. 2.18. Average energy fraction contained in an infinitely long cylinder of absorber material, as a function of the radius of this cylinder. Results of EGS4 calculations for various absorber materials and different energies.

Muons in calorimeters

- Muons are not minimum ionizing particles

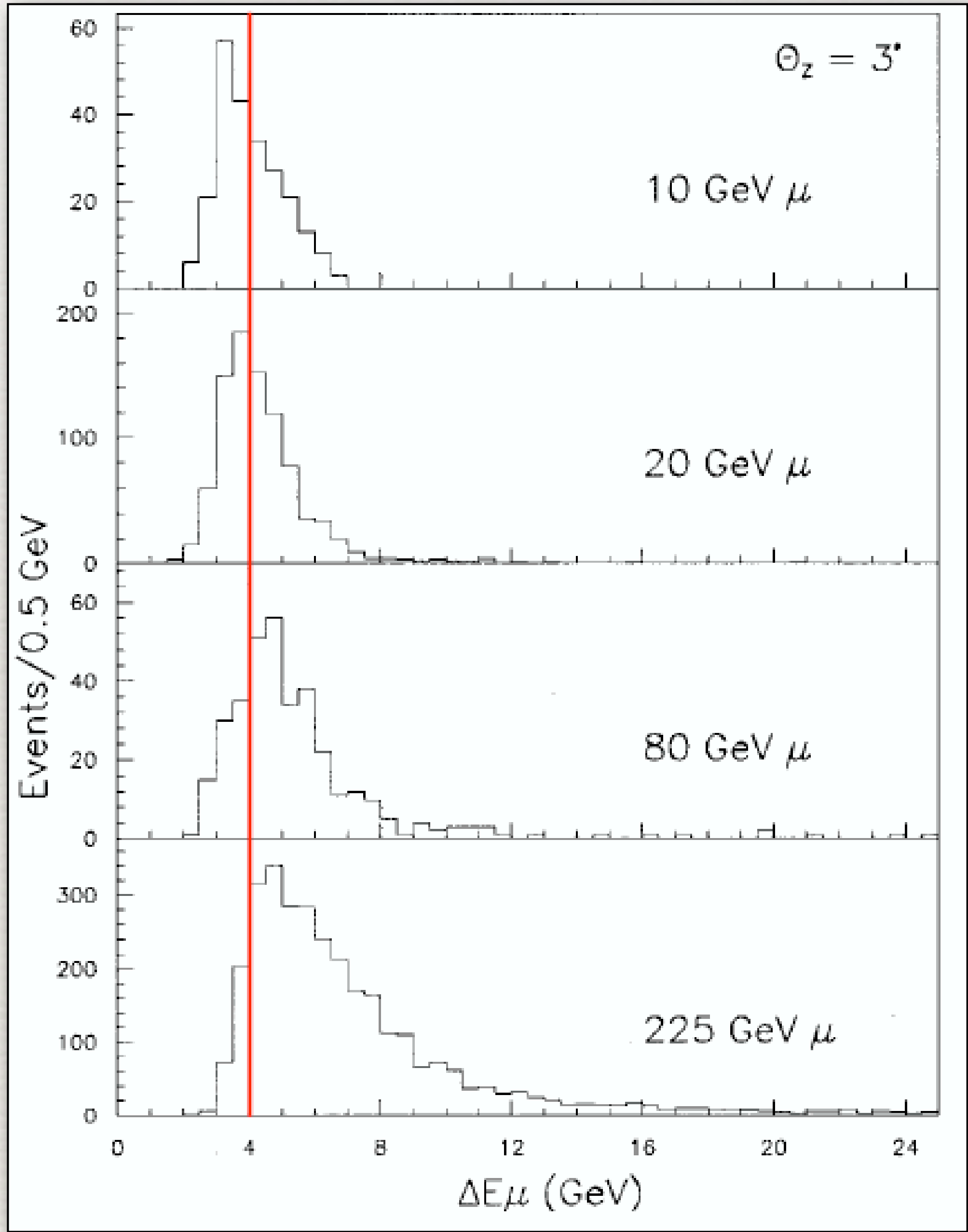
$$E_c(\mu) = \left[\frac{m_\mu}{m_e}\right]^2 \times E_c(e)$$

$$\Longrightarrow E_c(\mu) \approx 200\ GeV\ in\ Pb$$

- The effects of radiation are clearly visible in calorimeters, especially for high-energy muons in high-Z absorber material

Muon signals in a calorimeter

- Signal distributions for muons of 10, 20, 80 and 225 GeV traversing the SPACAL detector at 3 degrees



Hadron showers

- Extra complication: the strong interaction
- Much larger variety may occur both at the particle level and at the level of the stuck nucleus
  - Production of other particles, mainly pions
  - Some of these particles (π^0, η) develop electromagnetic showers
  - Nuclear reactions: protons, neutrons released from nuclei
  - Invisible energy (nuclear binding energy, target recoil)

Hadron vs em showers

- Hadron showers ⇒ much more complex than em showers
  - Invisible energy
    - em showers: all energy carried by incoming e or γ goes to ionization
    - had showers: certain fraction of energy is fundamentally undetectable
  - em showers
    - $e^{\pm}$ ⇒ continuous stream of events (ionization + bremsstrahlung)
    - γ ⇒ can penetrate sizable amounts of material before losing energy
  - had showers
    - ionization (as a μ) then interaction with nuclei
    - development similar to em shower but different scale (λ vs. X_0)
    - Particle sector
    - Nuclear sector

The electromagnetic fraction, f_{em}

- em decaying particles : $\pi^0, \eta^0 \Rightarrow \gamma\,\gamma$
  - % of hadronic energy going to em fluctuates heavily
  - On average 1/3 of particles in first generation are π^0s
  - π^0s production by strongly interacting particles is an irreversible process (a "one-way street")
  - Simple model
    - after first generation f_{em} = 1/3
    - after second generation f_{em} = 1/3 + 1/3 of 2/3 = 5/9
    - after third generation f_{em} = 1/3 + 1/3 of 2/3 + 1/3 of 4/9 = 19/27
    - after n generations $f_{em} = 1 - (1- 1/3)^n$
    - the process stops when the available energy drops below the pion production threshold and n depends on the average multiplicity of mesons produced per interaction <m> $\Rightarrow$ n increases by one unit every time E increases by a factor <m>
    - f_{em} increases with increasing incoming hadron energy

The electromagnetic fraction, f_{em}

- But
  - other particles than pions are produced (factor 1/3 wrong)
  - <m> is energy dependent
  - barion number conservation neglected → lower f_{em} in proton induced showers than in pion induced ones
- Using a more realistic model
  - $< f_{em} > = 1 - (E/E_0)^{(k-1)}$
  - E_0= average energy needed to produce a π^0
  - (k-1) related to the average multiplicity
  - $< f_{em} >$ slightly Z dependent
- Consequences:
  - Signal of pion < signal of electron (non-compensation)
  - e/π signal ratio energy dependent (non-linearity)

Energy dependence em component

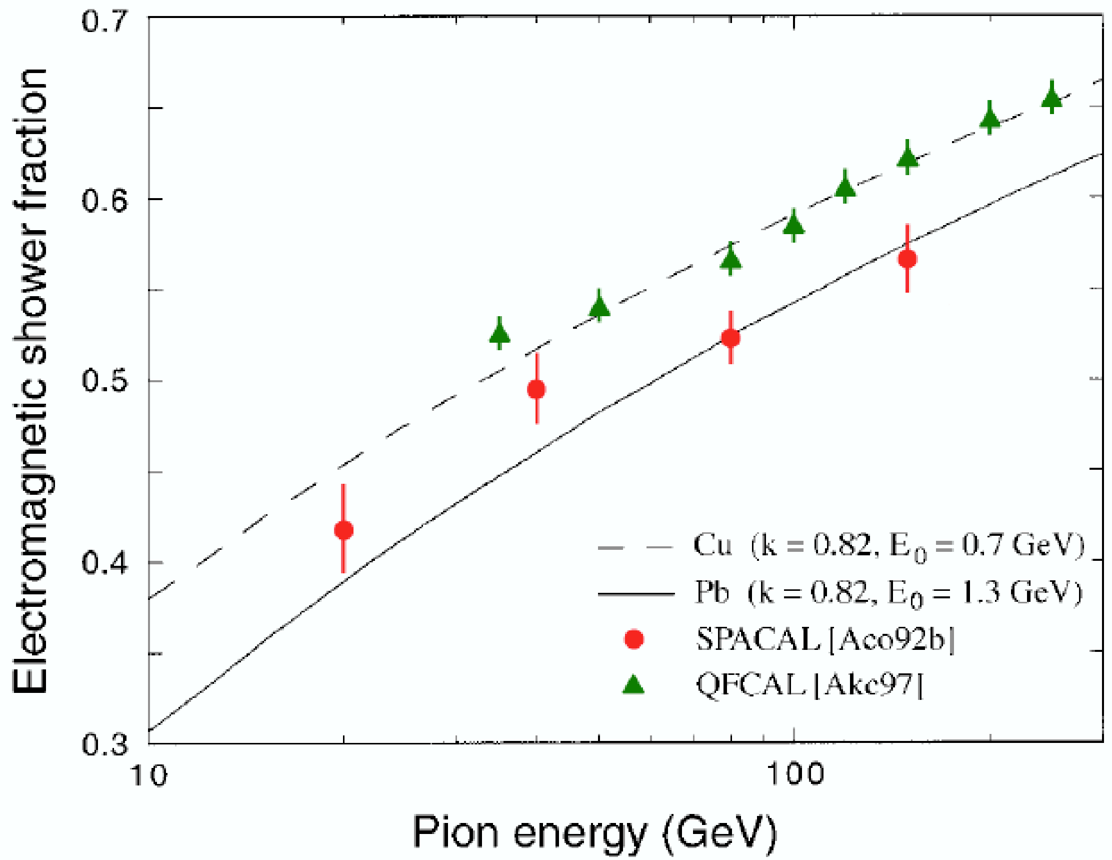


FIG. 2.22. Comparison between the experimental results on the em fraction of pion-induced showers in the (copper-based) QFCAL and (lead-based) SPACAL detectors. Data from [Akc 97] and [Aco 92b].

- SPACAL: Pb - scintillating fibers
- QFCAL: Cu - quartz fibers

Signal non-linearity

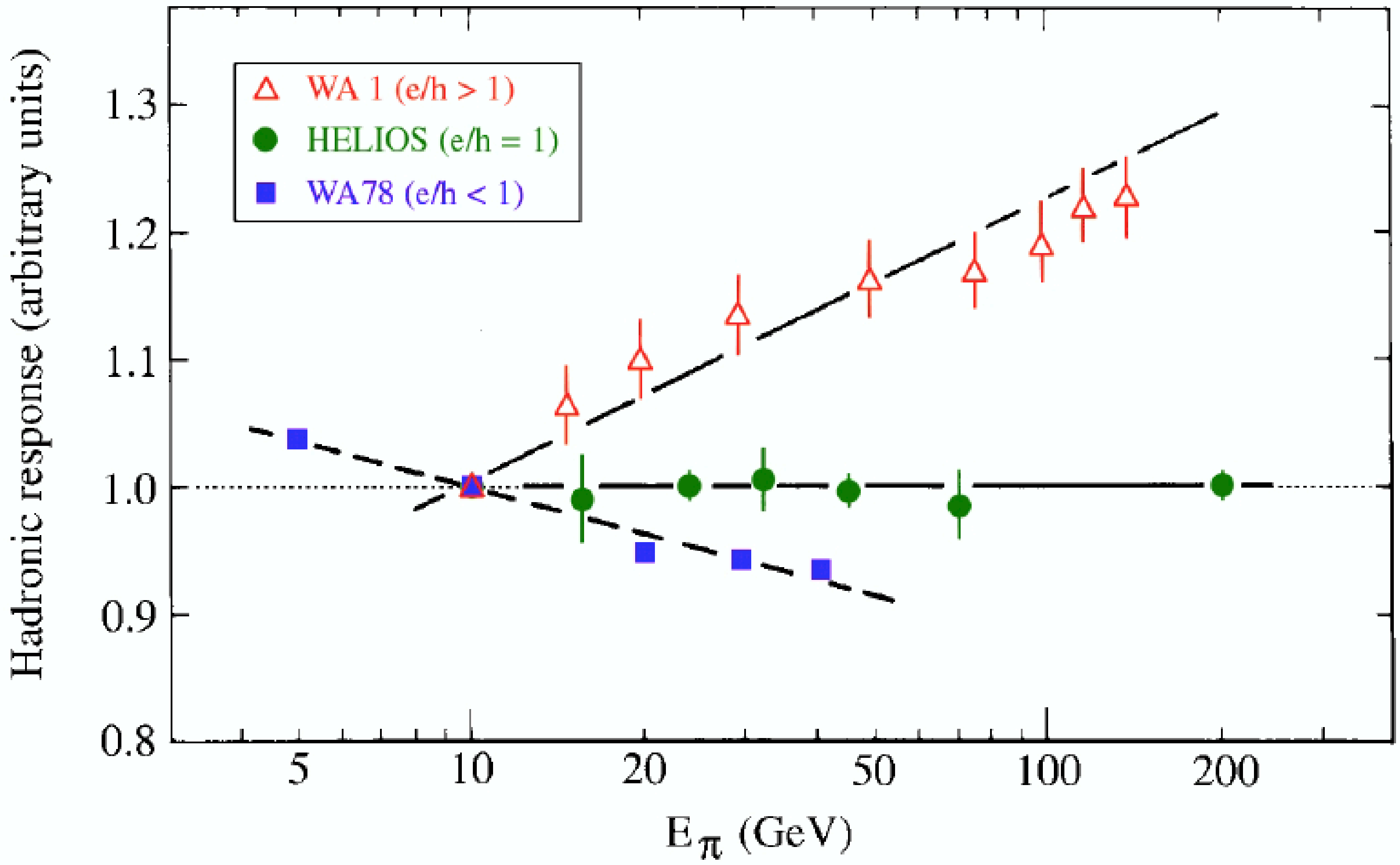


FIG. 3.14. The response to pions as a function of energy for three calorimeters with different e/h values: the WA1 calorimeter ($e/h > 1$, [Abr 81]), the HELIOS calorimeter ($e/h \approx 1$, [Ake 87]) and the WA78 calorimeter ($e/h < 1$, [Dev 86, Cat 87]). All data are normalized to the results for 10 GeV.

Non em component

- Breakdown of the non-em component in Lead

  - Ionizing particles 56% (2/3 from spallation protons)

  - Neutrons 10% (37 neutrons/GeV)

  - Invisible 34%

  - Spallation protons carry typically 100 MeV

  - Evaporation neutrons 3 MeV

Where does the energy go ?

- Energy deposit and composition of the non-em component of hadronic showers in lead and iron.
- The listed numbers of particles are per GeV of non-em energy

	Lead	*Iron*
Ionization by pions	19%	21%
Ionization by protons	37%	53%
Total ionization	56%	74%
Nuclear binding energy loss	32%	16%
Target recoil	2%	5%
Total invisible energy	34%	21%
Kinetic energy evaporation neutrons	10%	5%
Number of charged pions	0.77	1.4
Number of protons	3.5	8
Number of cascade neutrons	5.4	5
Number of evaporation neutrons	31.5	5
Total number of neutrons	36.9	10
Neutrons/protons	10.5/1	1.3/1

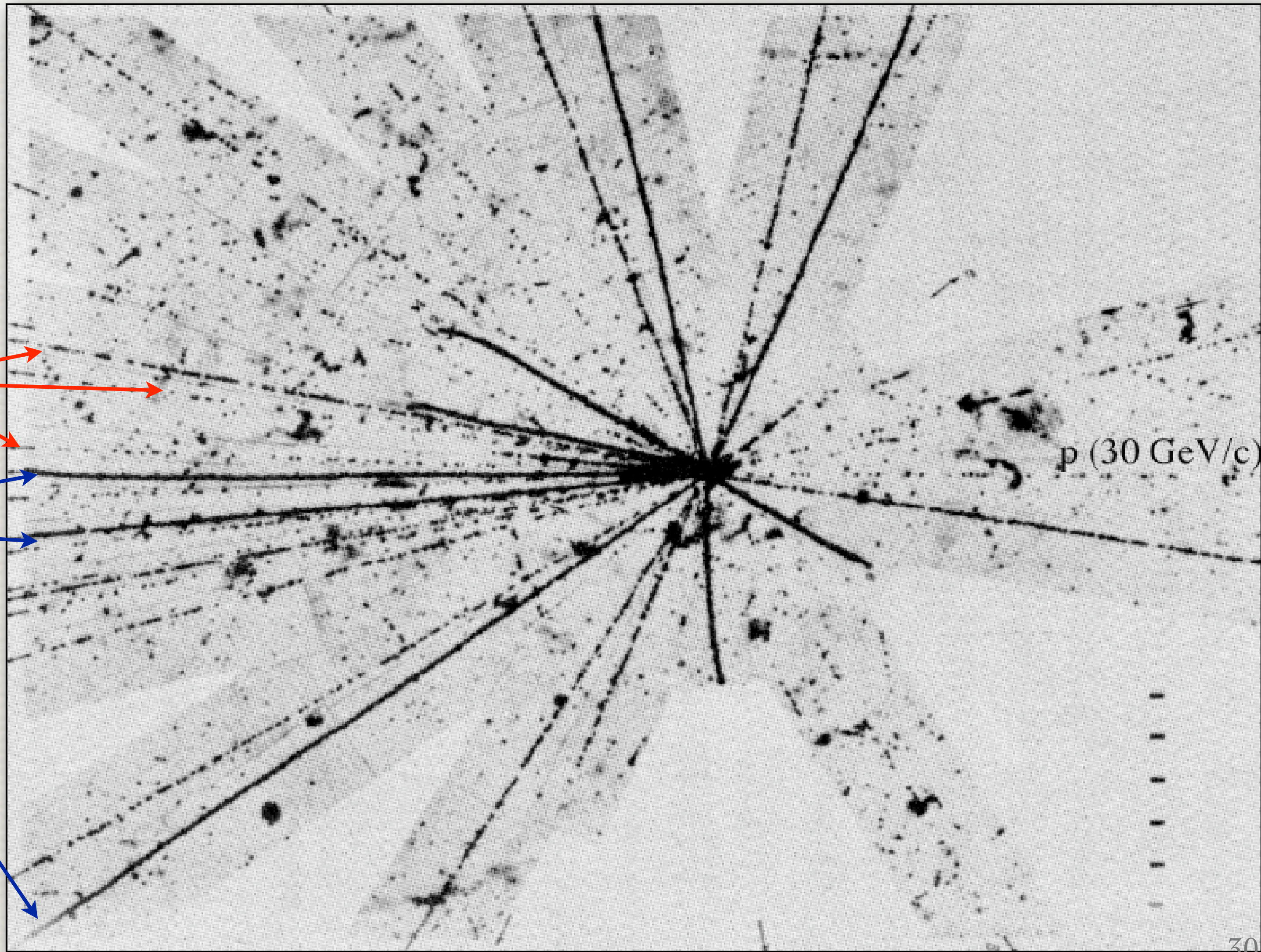
A typical process

- Nuclear interaction (nuclear star) induced by a proton of 30 GeV in a photographic emulsion

Spallation neutrons (non-visible)

Fast pions and fast spallation protons (non-isotropic)

Protons (isotropic)



Spallation

- Energy needed to release nucleons in nuclear reactions doesn't result in a measurable signal (binding energy-> invisible)
- Spallation is the most probable process in hadronic shower. It is a 2-stage process

Fast intranuclear cascade

Quasi-free collision of incoming hadron with nucleon
Nucleus excitation by distribution of nucleon energy
Cascade of fast nucleons, pions produced

Slower evaporation

Due to de-excitation of intermediate nucleus
Evaporation of nucleons
Remaining energy (few MeV) released through -rays

Neutron production spectra

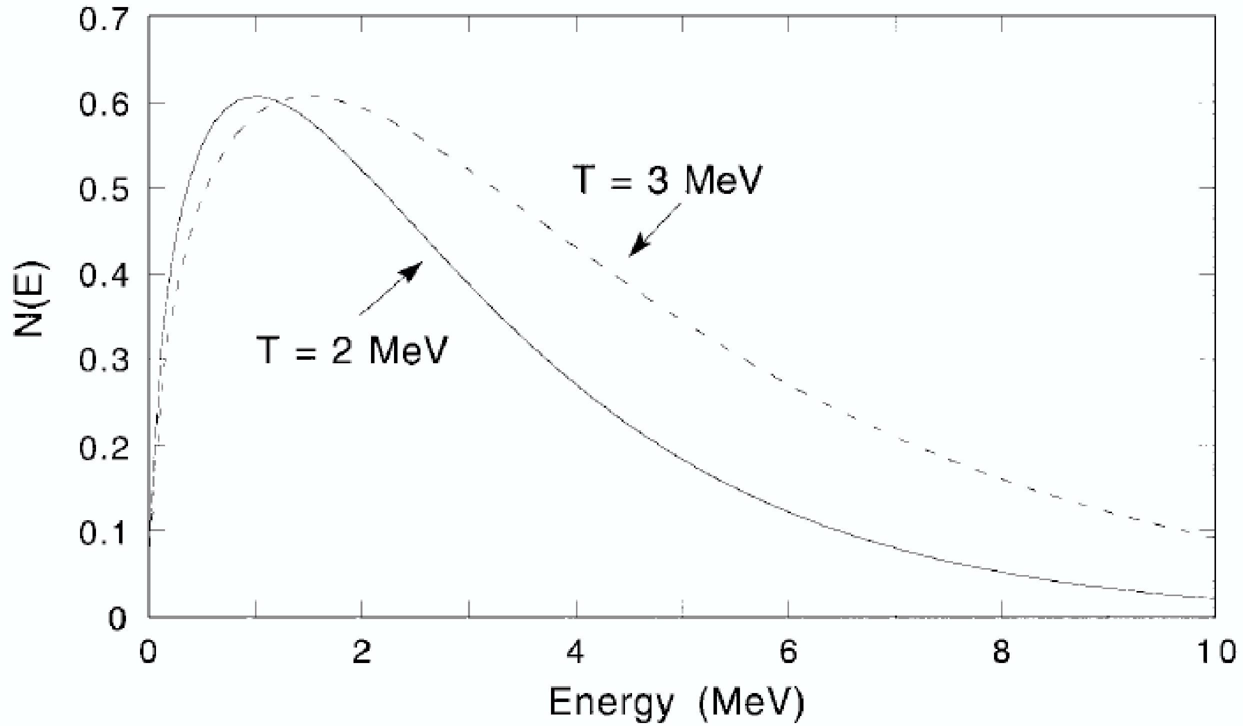


FIG. 2.29. Kinetic energy spectrum of evaporation neutrons, produced according to a Maxwell distribution with a temperature of 2 MeV. For comparison, the spectrum for a temperature of 3 MeV is given as well.

Kinetic energy spectrum of evaporation neutrons (Boltzmann-Maxwell distribution)

$$\frac{dN}{dE} = \sqrt{E}\exp(-E/T)$$

Hadronic shower profiles

- Shower profiles are governed by the
- Nuclear interaction lenght, λ_{int}
  - average distance a high-energy hadron has to travel inside a medium before a nuclear interaction occurs
  - λ_{int} (g cm^{-2}) $\propto A^{1/3}$
  - Fe 16.8 cm, Cu 15.1 cm, Pb 17.0 cm, U 10.0 cm

For comparison X_0 :

Fe 1.76 cm, Cu 1.43 cm, Pb 0.56 cm, U 0.32 cm

Longitudinal profile

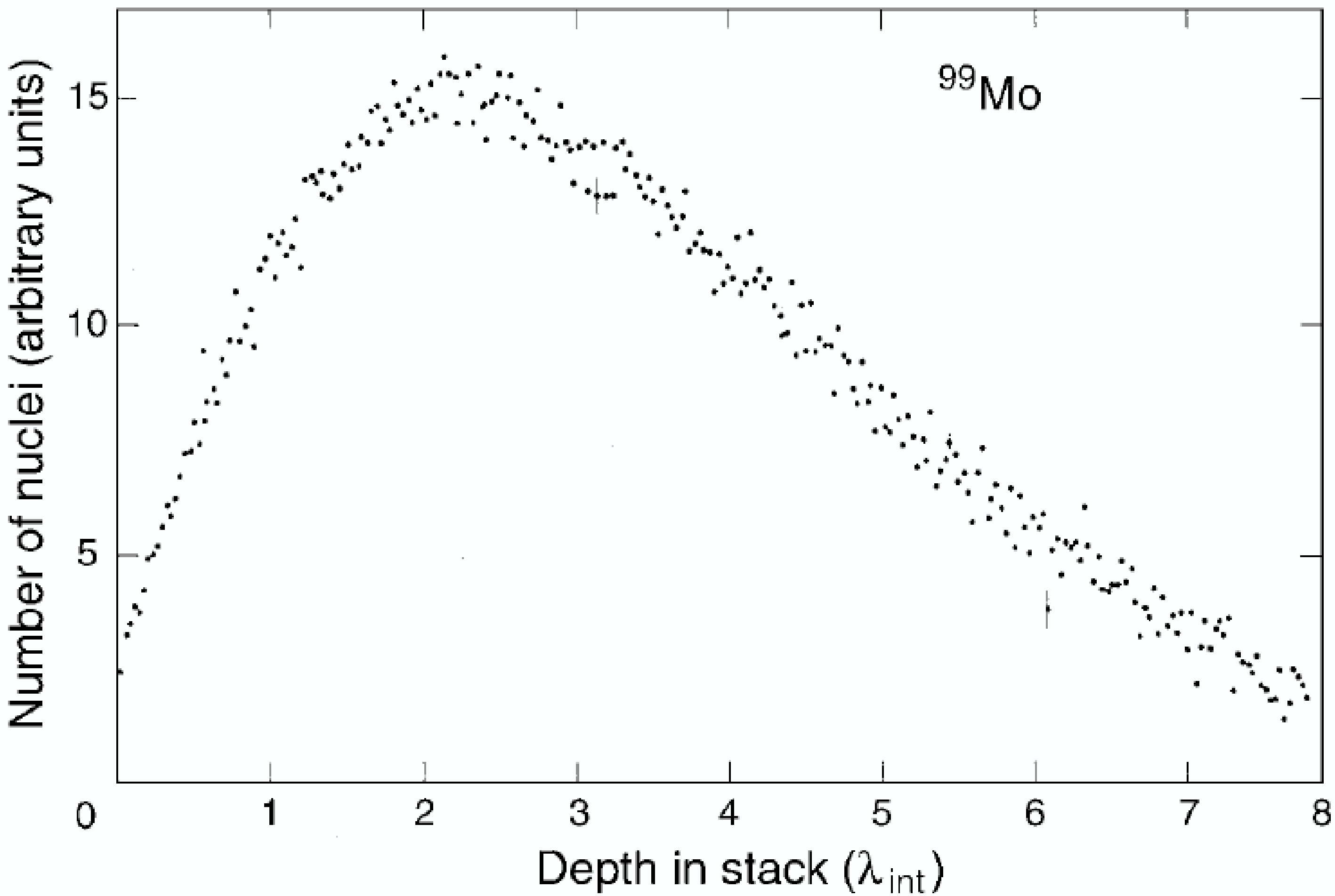


FIG. 2.31. Longitudinal shower profile for 300 GeV π^- interactions in a block of uranium, measured from the induced radioactivity. The ordinate indicates the number of radioactive decays of a particular nuclide, ^{99}Mo, produced in the absorption of the high-energy pions. Data from [Ler 86].

Hadronic showers fluctuations

- Very interesting measurements of the longitudinal energy deposition in em and hadronic showers were made with the "Hanging file calorimeter"

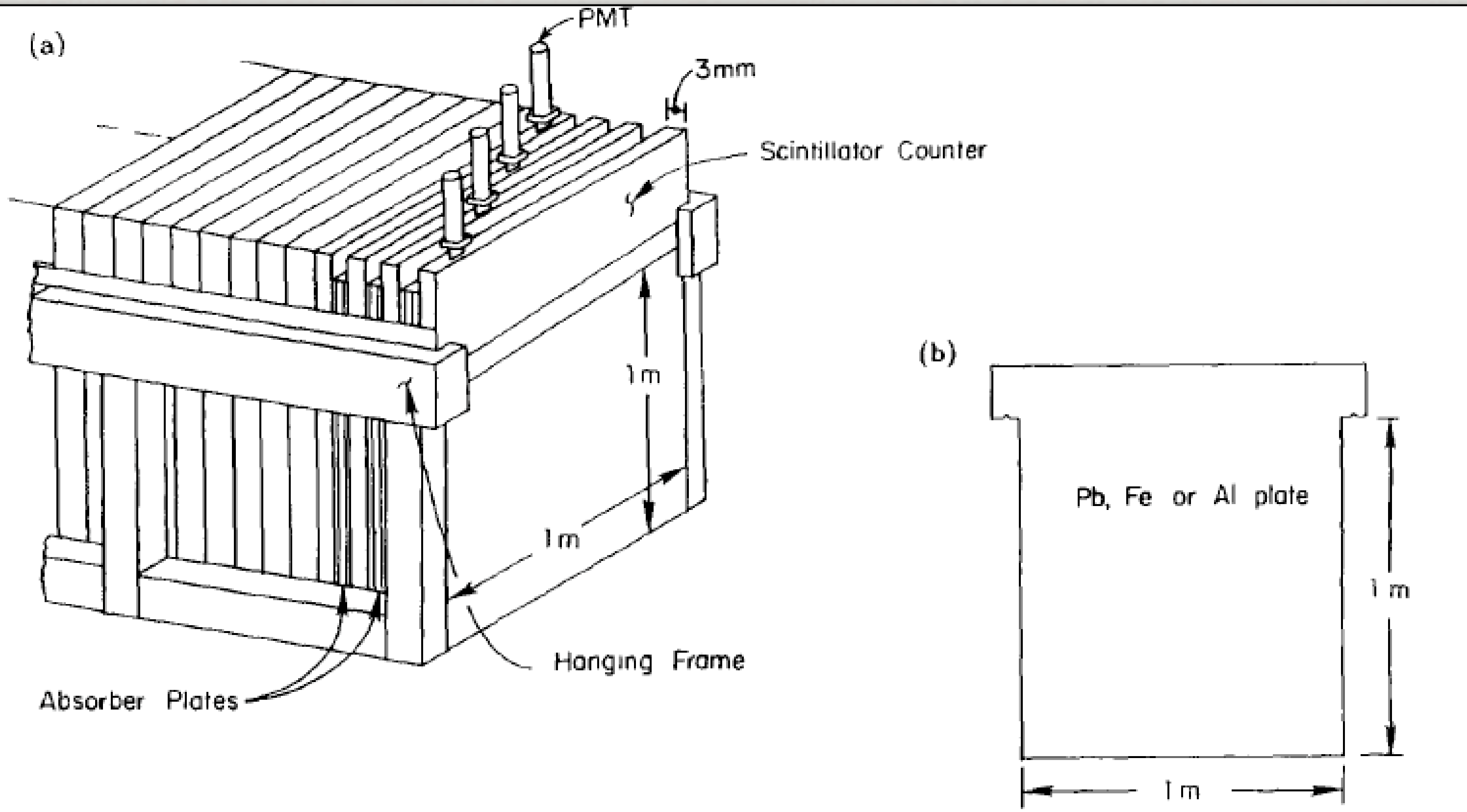
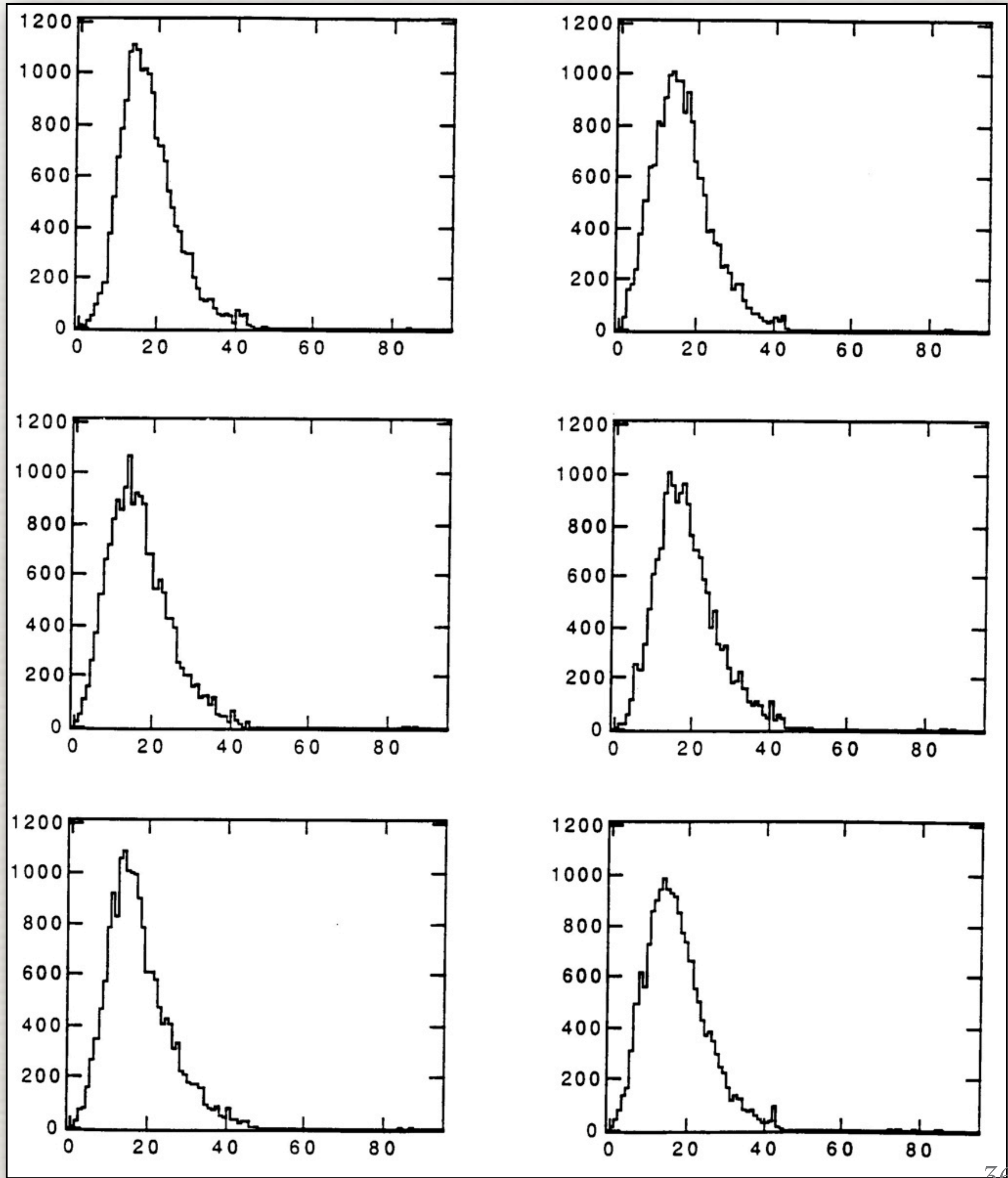


Fig. 1. (a) Schematic overview of the hanging file calorimeter (HFC). There was no transverse segmentation. The maximum depth of the calorimeter can be configured up to 2.2 m with a maximum number of 105 read-out planes. Each scintillator counter was read out separately. (b) Schematic drawing of the absorber plate.

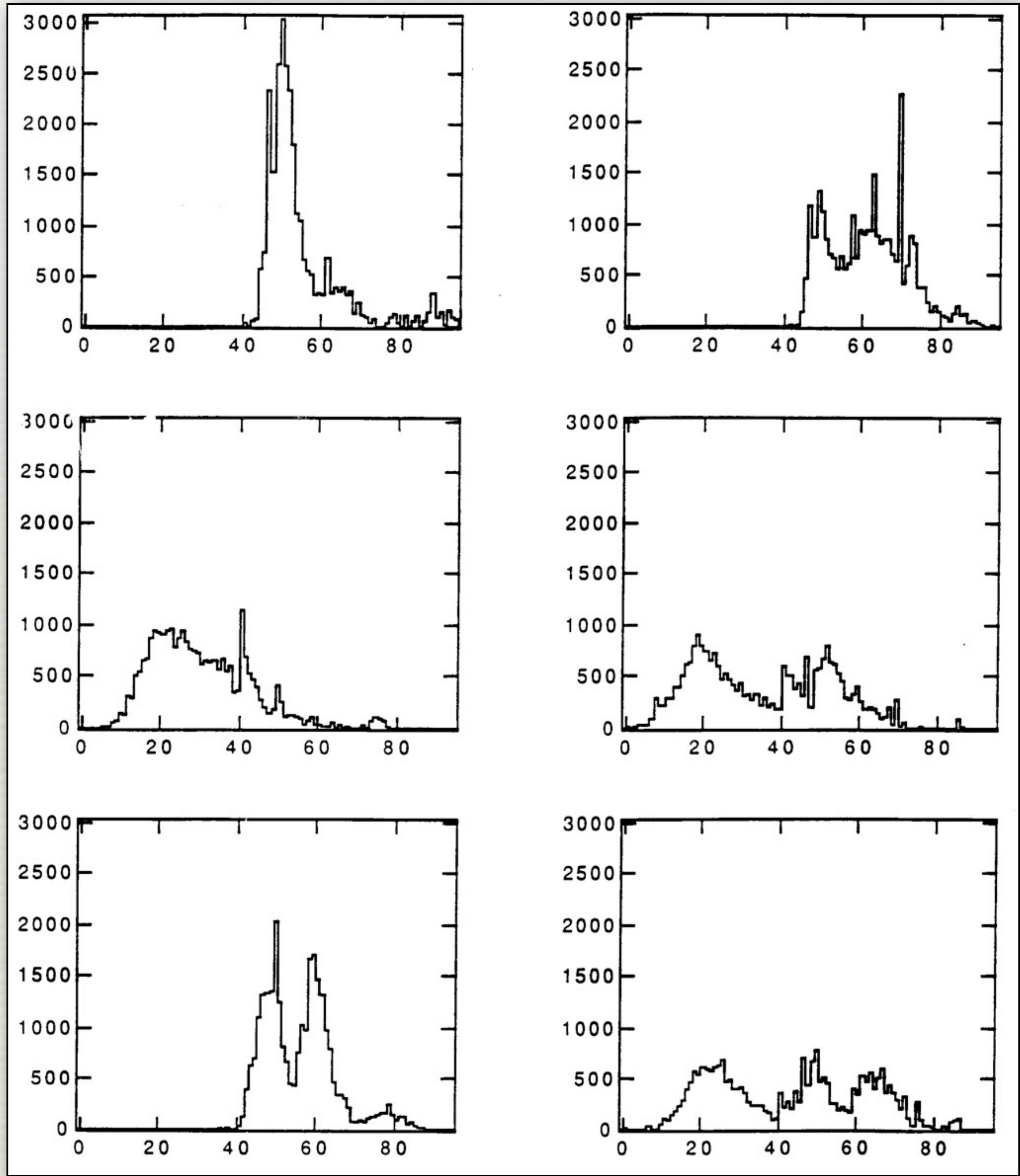
Fluctuations (em showers)

- Hanging file calorimeter
- 170 GeV electrons



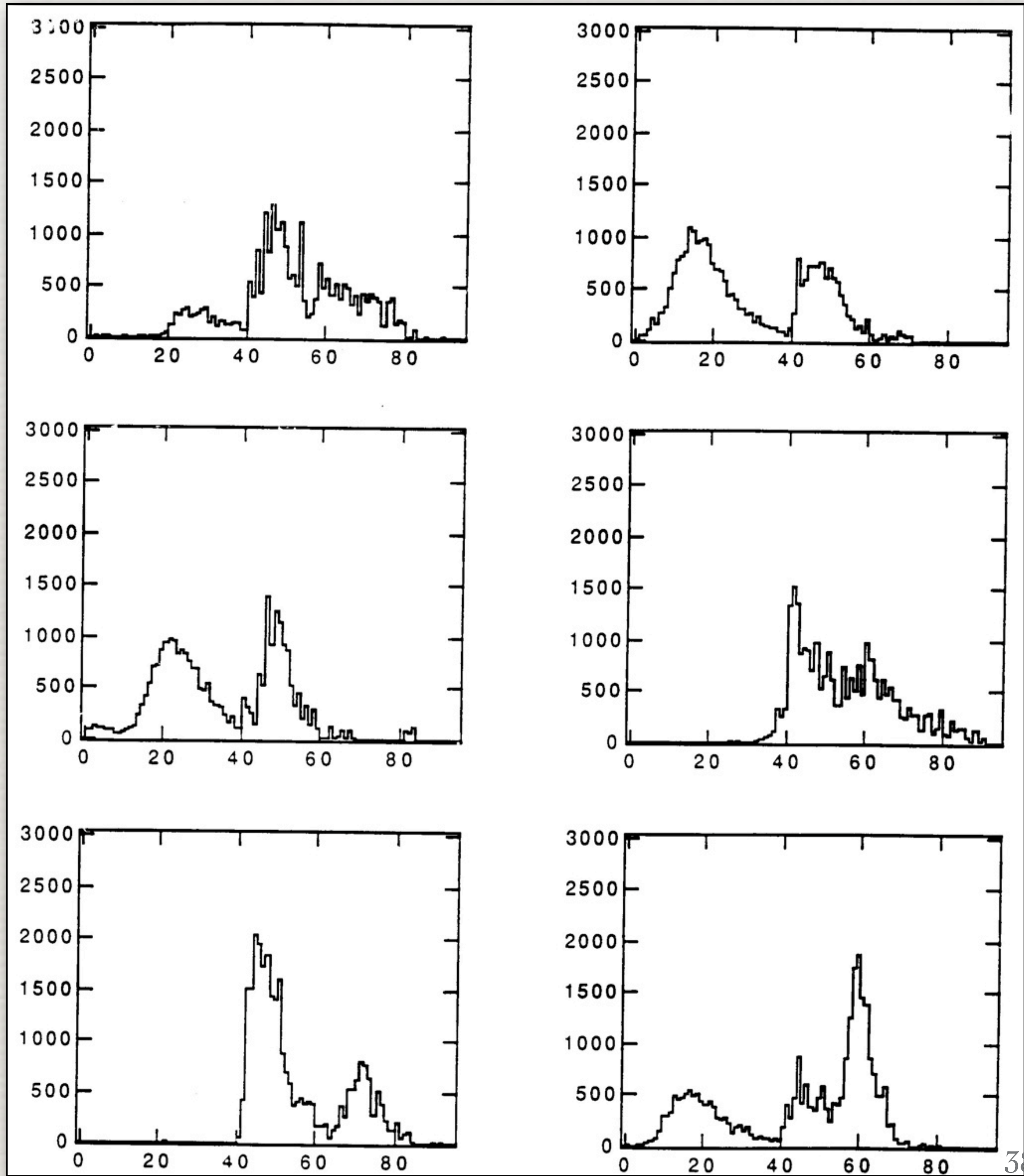
Fluctuations (hadronic showers)

- Hanging file calorimeter
- 270 GeV pions



Fluctuations (hadronic showers)

- Hanging file calorimeter
- 270 GeV pions



Hadronic lateral shower profiles

- Lateral shower profile has two components:
  - Electromagnetic core (π^0)
  - Non-em halo (mainly non-relativistic shower particles)
- Spectacular consequences for Čerenkov calorimetry
- Čerenkov light is emitted by particles with $\beta > 1/n$
  - e.g. quartz (n= 1.45) : Threshold 0.2 MeV for e, 400 MeV for p

Hadronic lateral shower profiles

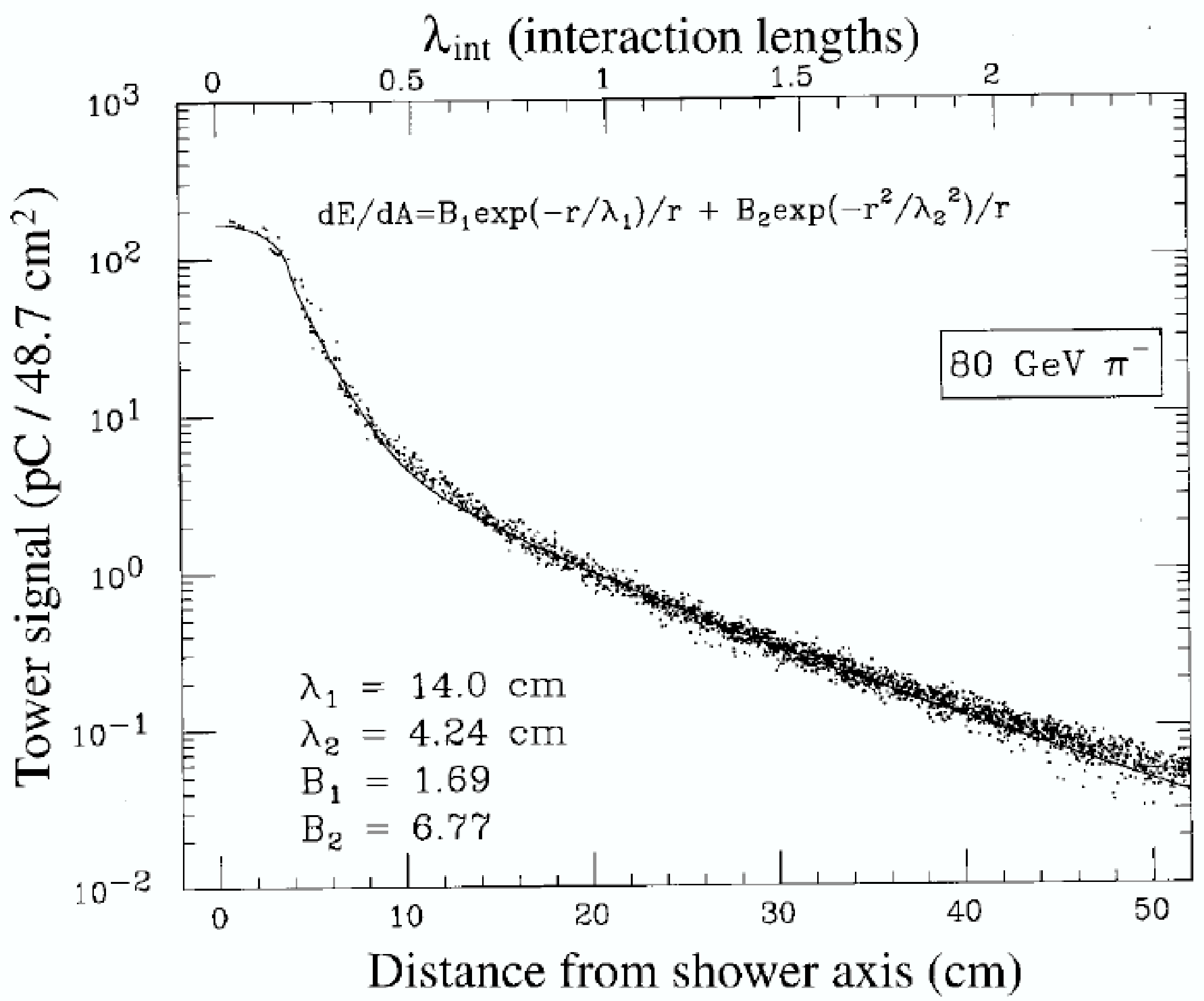
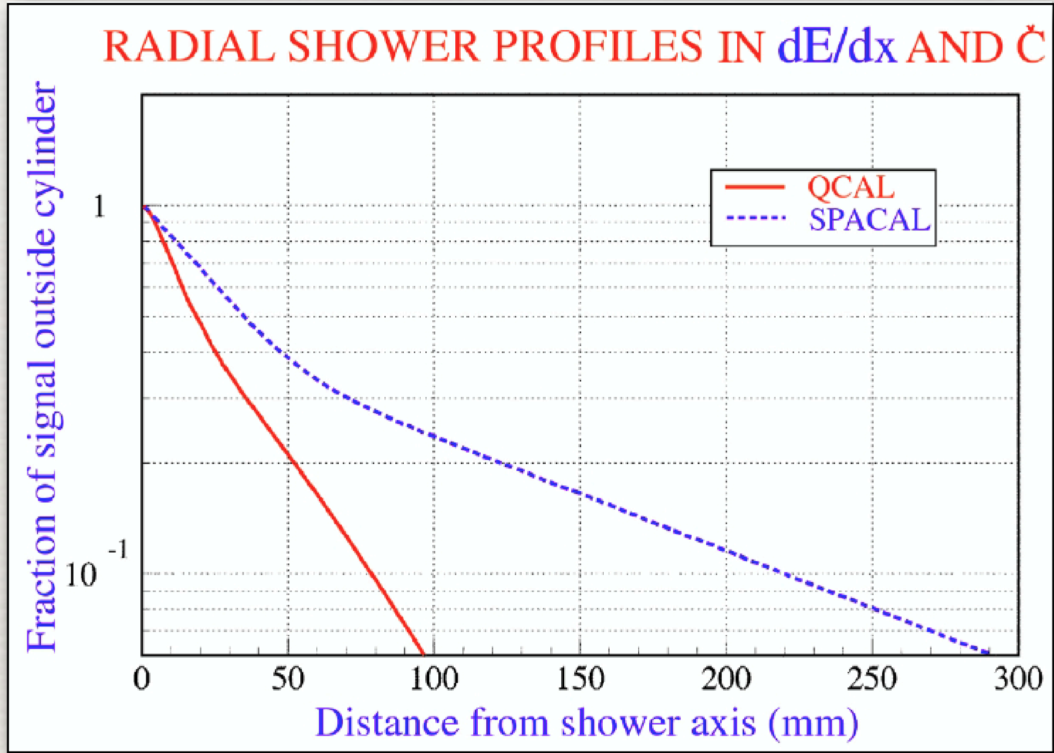


FIG. 2.32. Average lateral profile of the energy deposited by 80 GeV π^- showering in the SPACAL detector. The collected light per unit volume is plotted as a function of the radial distance to the impact point. Data from [Aco 92b].

Hadronic lateral shower profiles

- Nonrelativistic particles dominate tails in hadron showers



In material with n~1.4 Cherenkov light is emitted for:

E_e > 700 KeV

$E_{\pi^\pm}$ > 190 MeV

E_p >1.3 GeV

Lateral distribution shower particles

- Np produced by thermal neutron capture
- Mo fission product of U produced by non-thermal neutrons (MeV)
- U produced by γ (10 GeV) induced reactions present in the em core

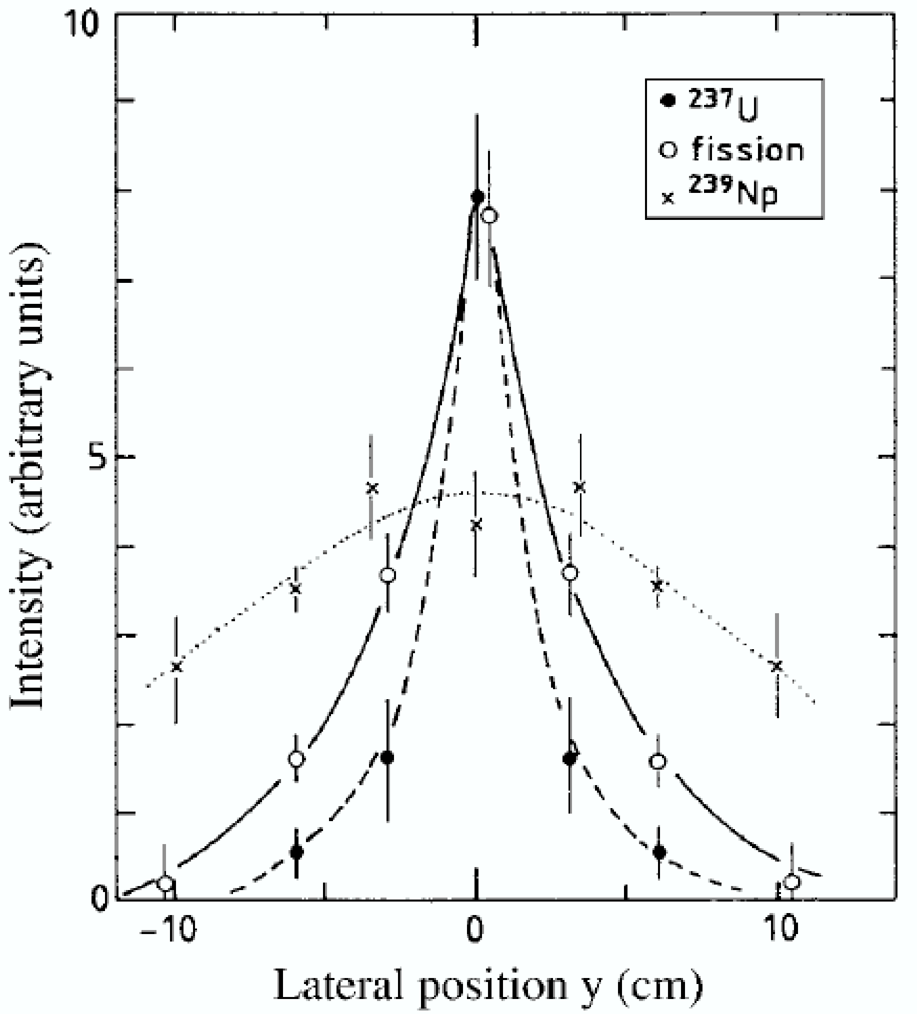


FIG. 2.34. Lateral profiles for 300 GeV π^- interactions in a block of uranium, measured from the induced radioactivity at a depth of $4\lambda_{int}$ inside the block. The ordinate indicates the decay rate of different radioactive nuclides, produced in nuclear reactions by different types of shower particles. Data from [Ler 86].

Shower containment

- Shower containment:

  - Depth to contain showers increases with log E

  - Lateral leakage decreases as the energy goes up !
    - $\langle f_{em} \rangle$ increases with energy
    - Electromagnetic component concentrated in a narrow cone around shower axis
    - ⇒ Energy fraction contained in a cylinder with a given radius increases with energy

Hadronic shower leakage (longitudinal)

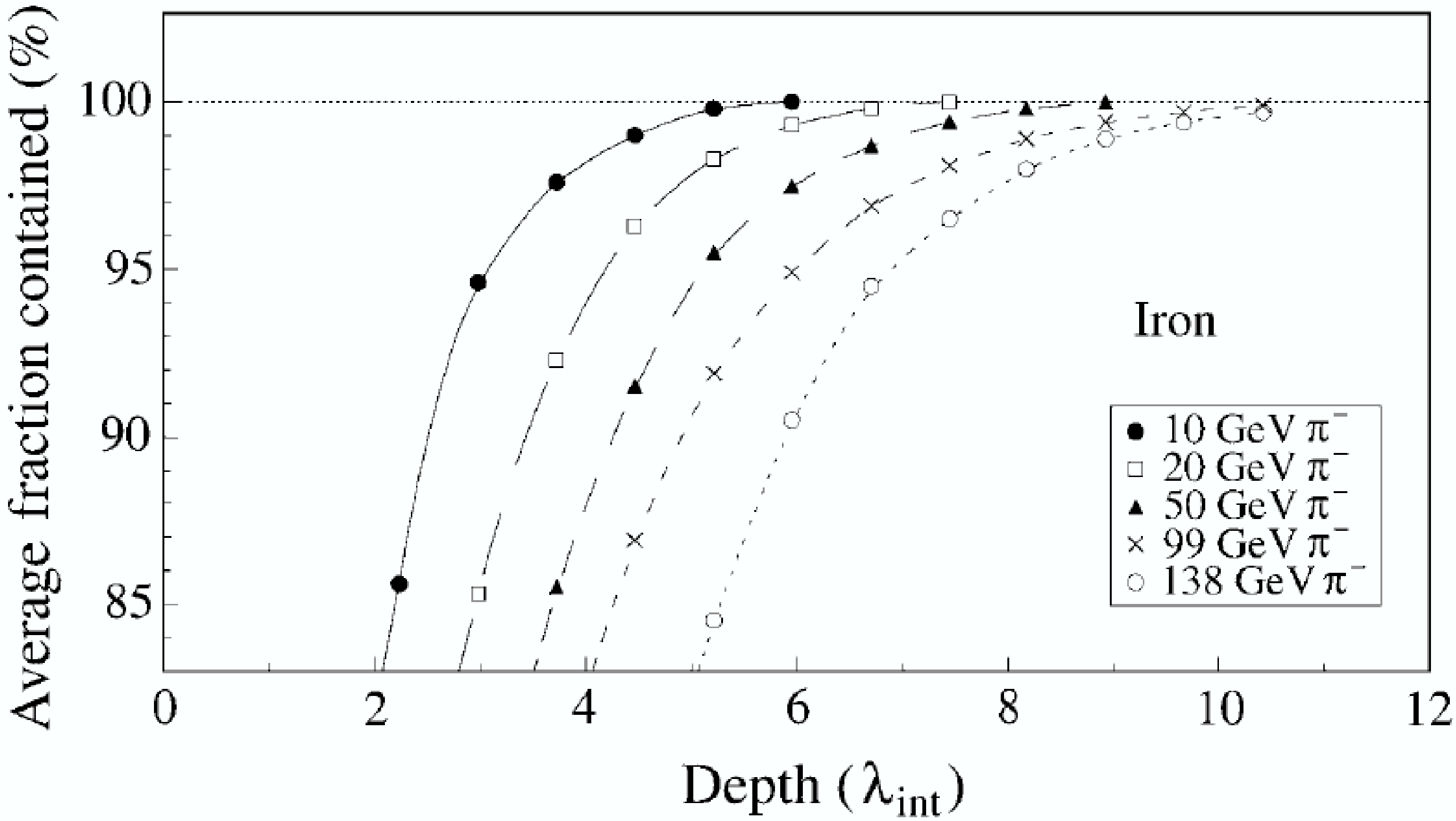


FIG. 2.37. Average energy fraction contained in a block of matter with infinite transverse dimensions, as a function of the thickness of this absorber, expressed in nuclear interaction lengths. Shown are results for showers induced by pions of various energies in iron absorber [Abr 81].

Hadronic shower leakage (lateral)

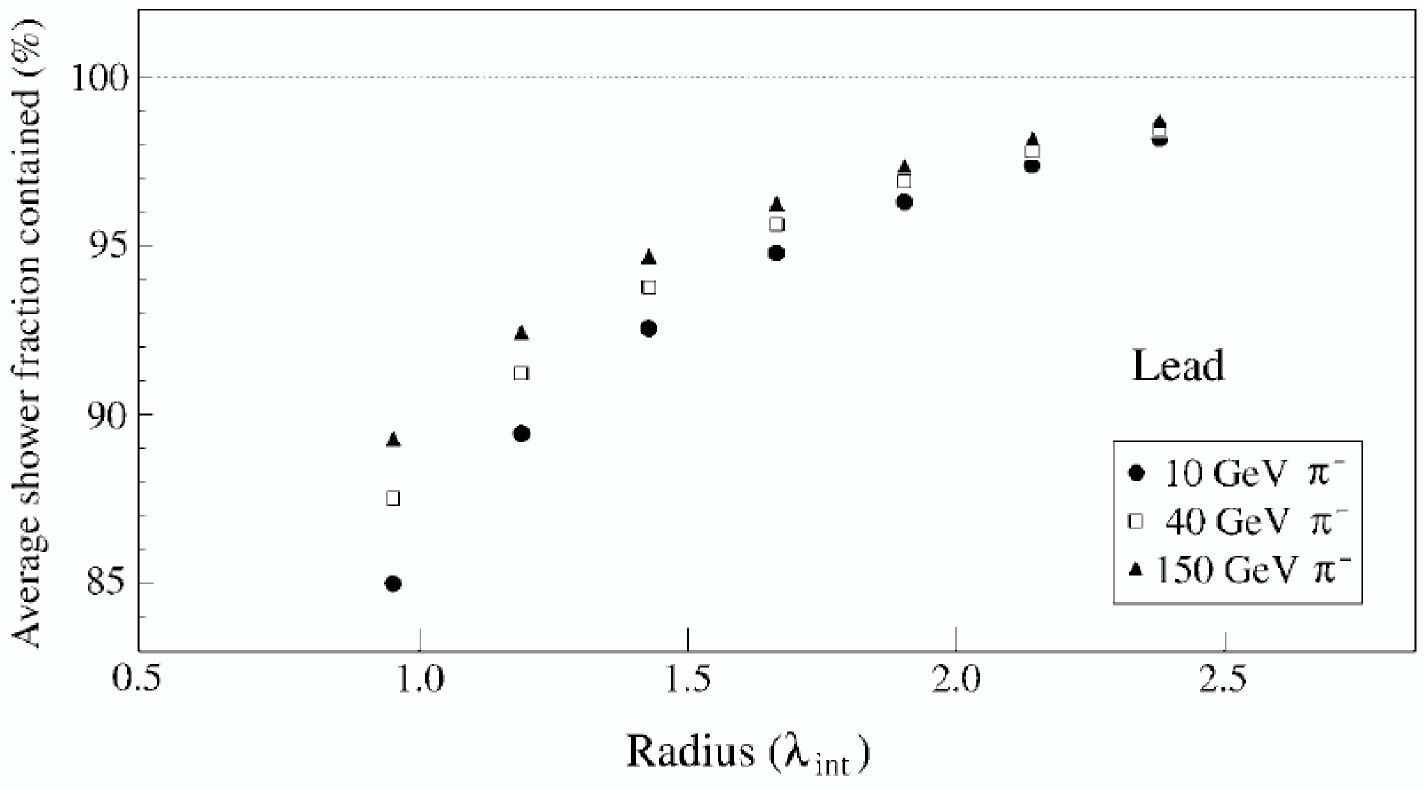


FIG. 2.38. Average energy fraction contained in an infinitely long cylinder of absorber material, as a function of the radius of this cylinder (expressed in nuclear interaction lengths), for pions of different energy showering in lead absorber [Aco 92b].

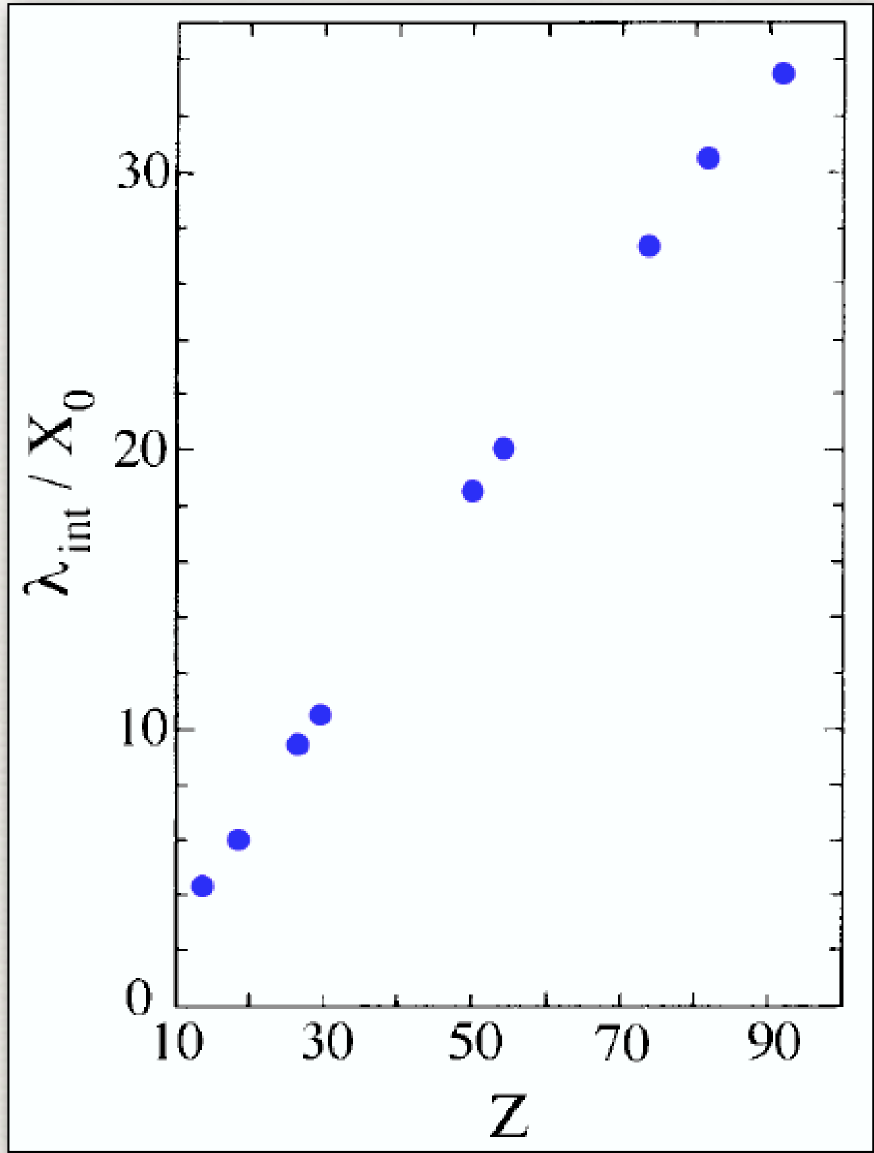
- This energy dependence is a direct consequence of the energy of <f_{em}>. The average energy fraction carried by the em shower component increases with energy and since this component is concentrated in a narrow cone around the shower axis, the energy fraction contained in a cylinder with a given radius increases with energy as well.

Hadronic shower profiles

- The λ_{int}/X_0 ratio is important for particle ID
  - In high-Z materials: $\lambda_{int}/X_0 \sim 30 \Rightarrow$ excellent e/π separator
  - 1 cm PB + scintillator plate makes spectacular preshower detector

Comparison em/hadronic calorimeter properties

- Ratio of the nuclear interaction lenght and the radiation lenght as a function of Z



Particle ID with a simple preshower detector

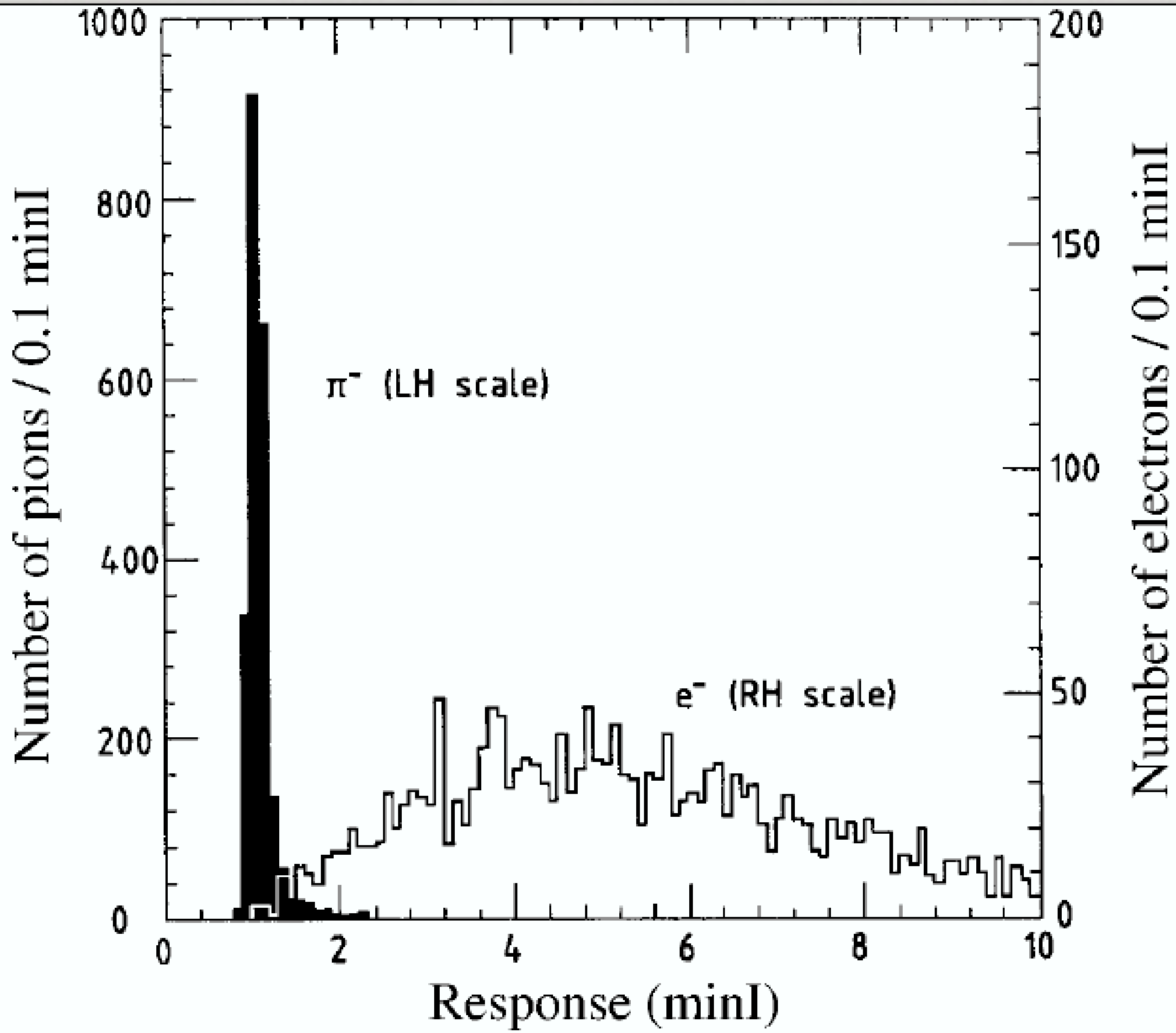


FIG. 7.35. Signal distributions for 75 GeV pions and electrons in a preshower detector used in beam tests of CDF calorimeters.

Lessons for calorimetry

- In absorption process, most of the energy is deposited by very soft particles
- Electromagnetic showers:
  - 3/4 of the energy deposited by e-, 2/4 of it by Compton, photoelectrons
  - These are isotropic, have forgotten direction of incoming particle ⇒ No need for sandwich geometry
  - The typical shower particle is a 1 MeV electron, range < 1mm ⇒ important consequences for sampling calorimetry
- Hadron showers:
  - Typical shower particles are a 50-100 MeV proton and a 3 MeV evaporation neutron
  - Range of 100 MeV proton is 1 - 2 cm
  - Neutrons travel typically several cm
  - What they do depends critically on detail of the absorber

Angular distributiom

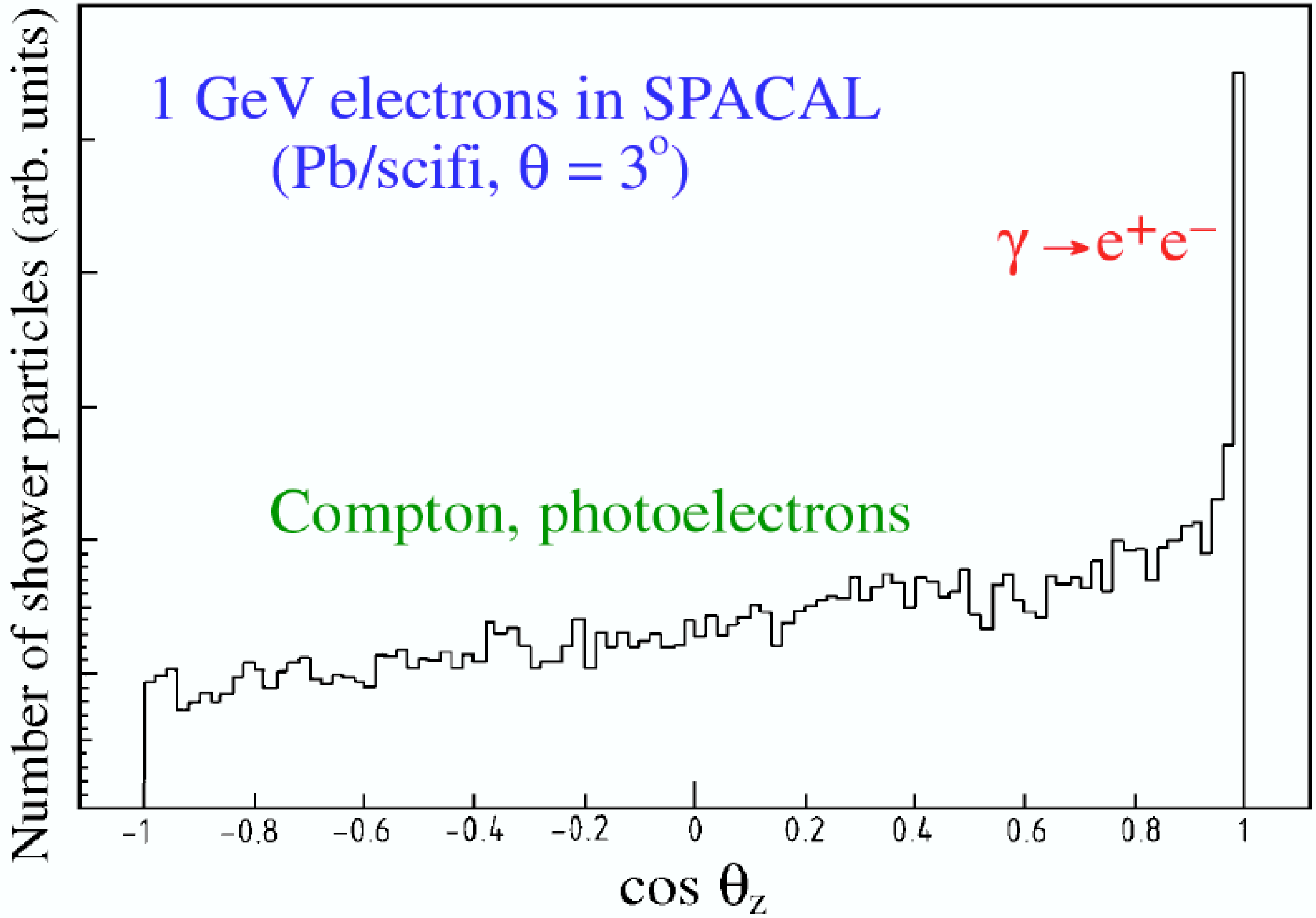
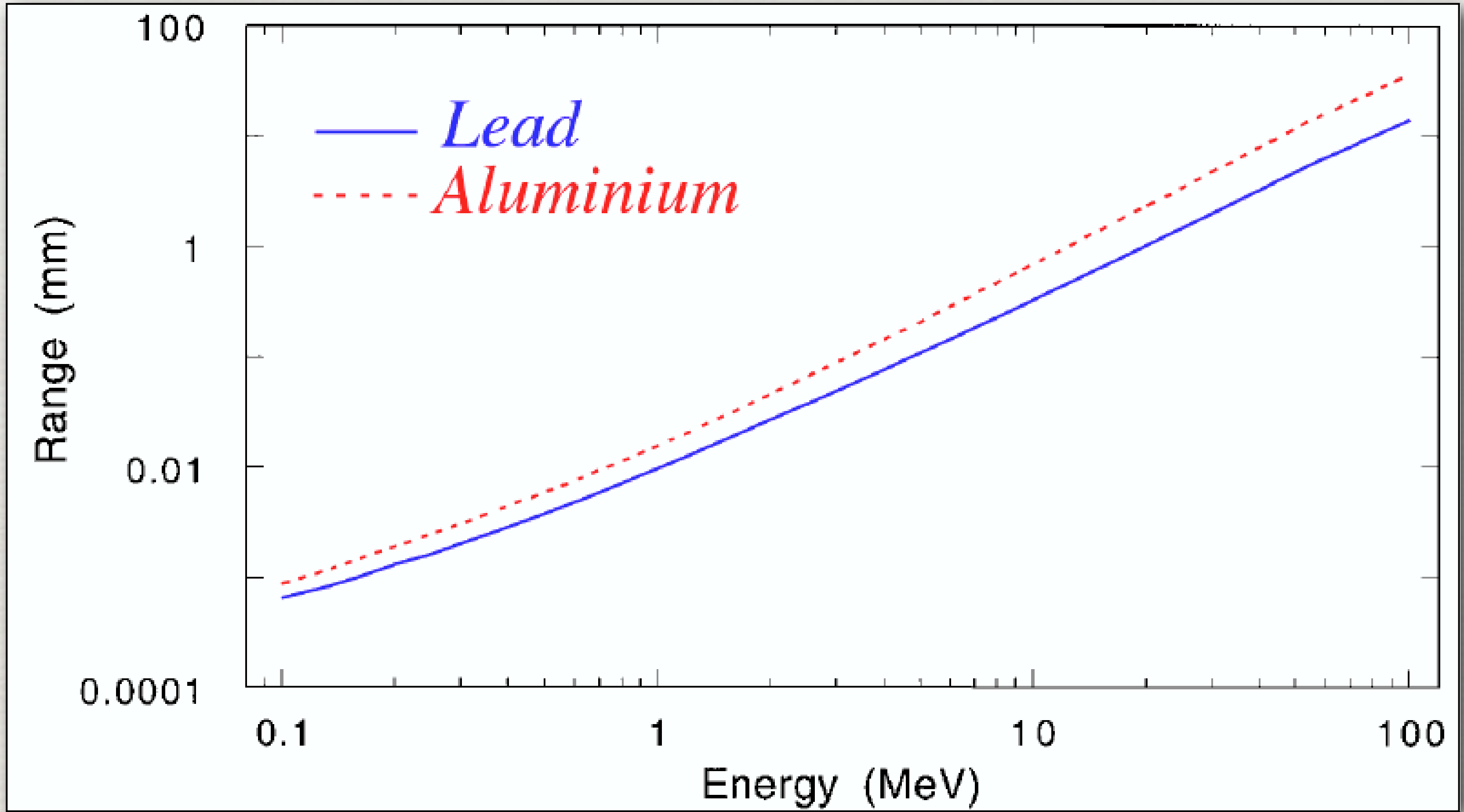


FIG. 2.39. Angular distribution of the shower particles (electrons and positrons) through which the energy of a 1 GeV electron is absorbed in a lead-based calorimeter. Results of EGS4 Monte Carlo simulations. From [Aco 90].

Range of protons generated in hadron showers



- Average range of protons in various absorber materials, as a function of energy