Weak and strong Anderson Localization of ultra-cold atoms: an introduction

Varenna school on
Quantum matter at ultralow temperature
JULY 11, 2014

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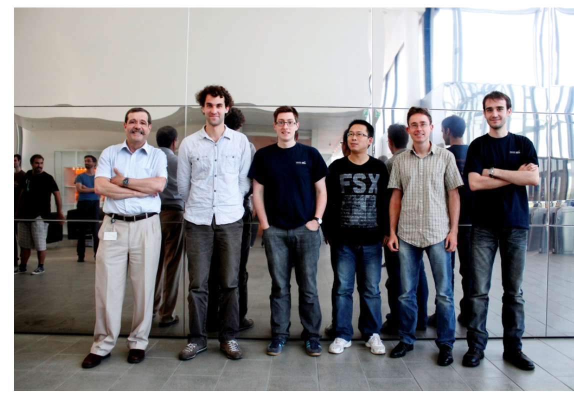
Anderson localisation in the Atom Optics group at Institut d'Optique

Experiments **(Philippe Bouyer →Bordeaux)**

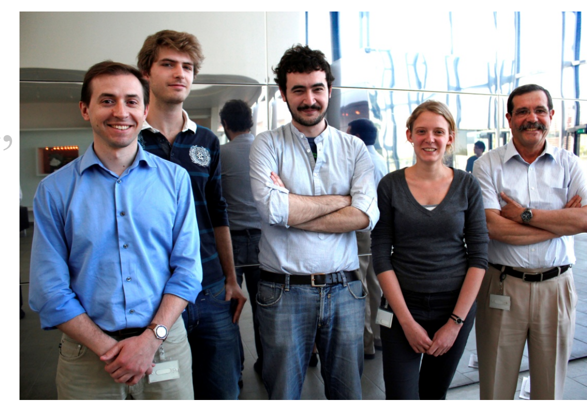
1. **Vincent Josse,** J. Billy, A. Bernard, P. Cheinet, F. Jendrzejewski, K. Müller, J. Richard, V. Volchkov
2. **Thomas Bourdel,** J. P. Brantut, M. Robert dSV, B. Allard, T. Plisson , G. Salomon, L. Fouché

and our electronic wizards: A.Villing, F. Moron



Theory **(Laurent Sanchez Palencia)**: M. Piraud, L.Pezze, G. Carleo, S Lellouch, G. Boeris

Collaborations: M Lewenstein, G Shlyapnikov, Thierry Giamarchi, M Holzmann



Ultra cold atoms in a laser speckle: a quantum simulator for Anderson localization

1. Anderson localization: why, what?
2. Cold atoms in laser speckle?
3. 1D Anderson Localization?
4. 3D Anderson Localization?
5. Coherent Back Scattering?
6. Outlook and open questions

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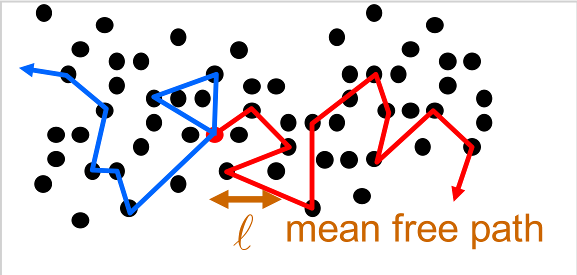
Anderson localization: a model for metal/ insulator transition induced by disorder

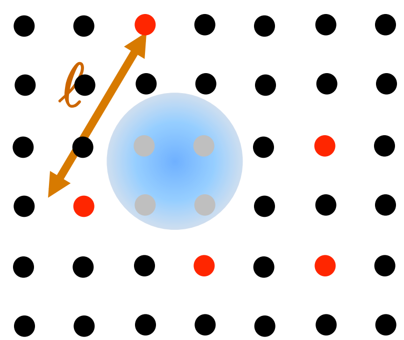
Classical model of metal: disorder hinders, does not cancel, conduction



- Classical particles bouncing on impurities
- Matter waves scattering on impurities + incoherent addition

⇒ delocalized (extended) states: conductor

Anderson L. (1958): disorder can totally cancel conduction



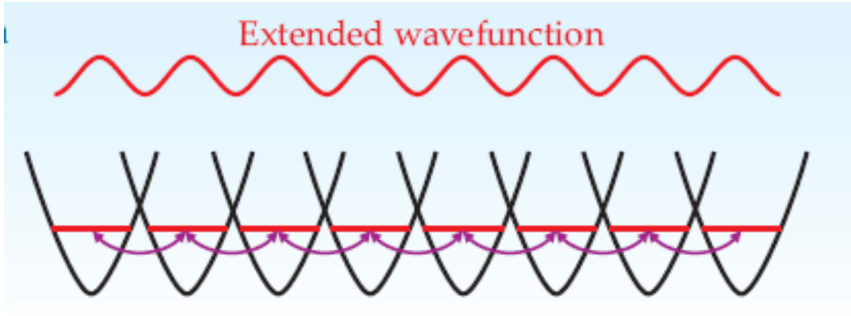
Tight binding model with disorder : ⇒

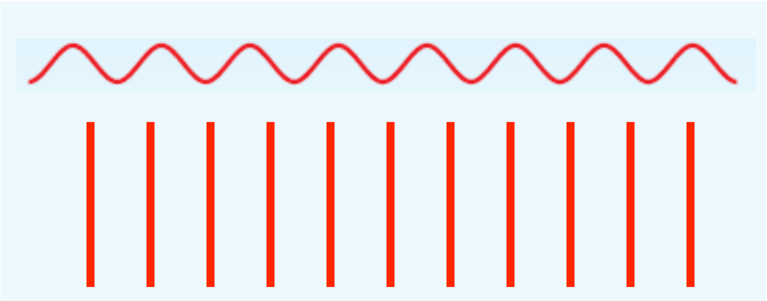
⇒ localized states: insulator

Quantum effect: addition of quantum amplitudes of hopping

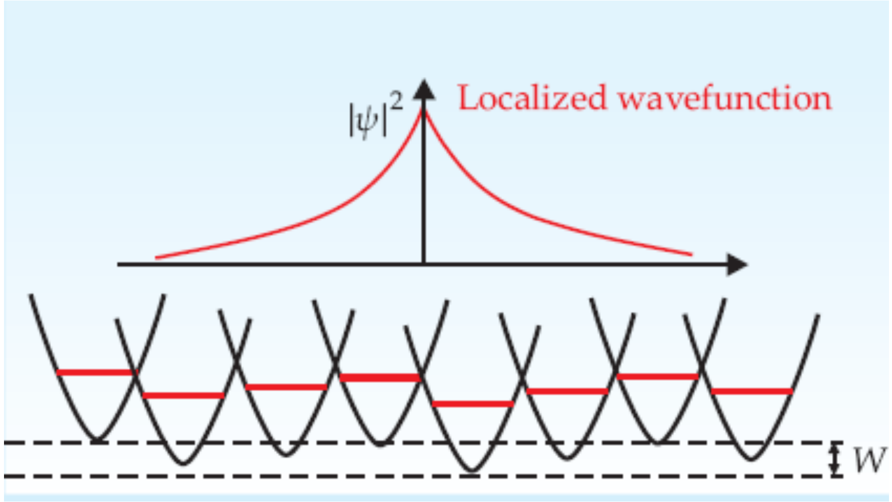
3D mobility edge (Ioffe Regel, Mott) $E \leq \dfrac{\hbar^2}{2m\ell^2}$

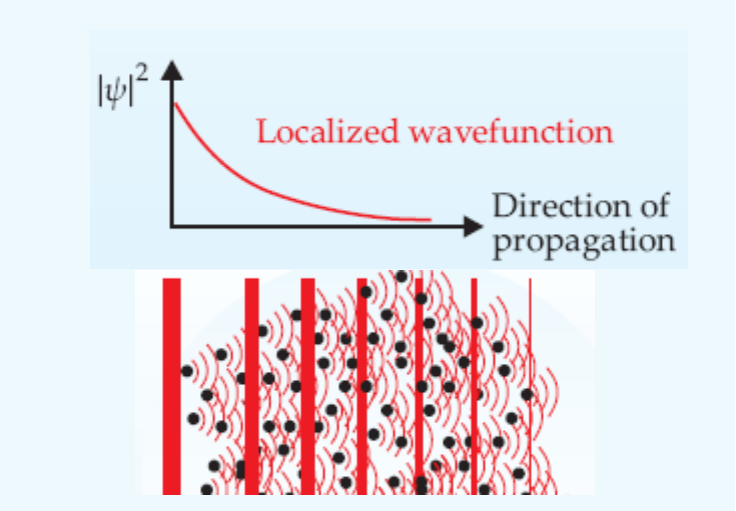
Tight binding model vs. wave model
Condensed matter vs. AMO physics





Bloch wave in a perfect crystal ⇔ Freely propagating wave





Disordered crystal ⇔ Scattering from impurities

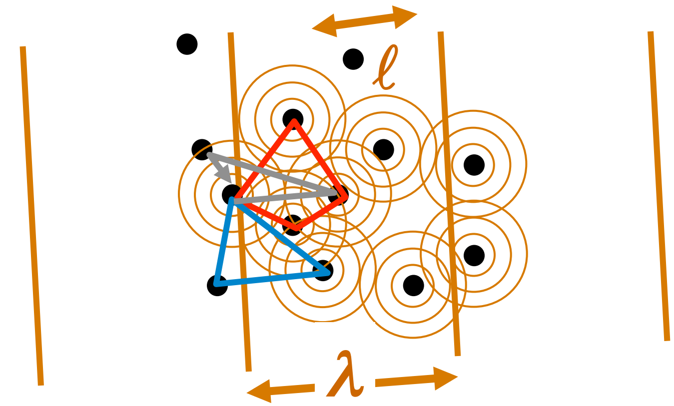
A.A, M Inguscio, Physics today August 2009

Anderson localization: the point of view of an AMO physicist

Coherent addition of waves scattered on impurities. If mean free path ℓ smaller than wavelength:

- coherent addition of trajectories returning to origin
- destructive interference in any direction

⇒ Localized states: insulator



3D mobility edge (Ioffe Regel) $\ell \leq \dfrac{\lambda}{2\pi}$

Akkermans&Montambaux, B. Van Tiggelen (Les Houches 1999)

Main features:

- Interference of many scattered wavelets ⇒ localization
- Single particle quantum effect (no interaction)
- Role of dimensionality (probability of return to origin)

The quest of AL with classical waves

Waves scattering on impurities:

AL not easy to discriminate from ordinary absorption

Microwaves (cm) on dielectric spheres:

- Chabonov et al., Nature 404, 850 (2000)

Light on dielectric microparticles (TiO_2):

- Wiersma et al., Nature 390, 671 (1997)
- Störzer et al., Phys Rev Lett 96, 063904 (2006); Sperling et al. Nature Photonics 7, 48 (2013)

Difficult to obtain $\ell < \lambda / 2\pi$ (Ioffe-Regel)

Light on 2D or 1D photonic lattices:

- T. Schwartz et al. (M. Segev), Nature 446, 52 (2007)
- Lahini et al. (Silberberg), PRL 100, 013906 (2008).

Acoustic (and seismic) waves:

- RL Weaver ; J.H. Page (coll. Van Tiggelen, Skipetrov); Fink

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Ultra cold atoms (matter waves) Good candidate to observe AL

Good features

- Controllable dimensionality (1D, 2D, 3D)
- Wavelength λ_{dB} "easily" controllable (1 nm to 10 μm)
- Pure potentials (no absorption), controllable disorder
- Many observation tools: absorption or fluorescence imaging, time of flight, Bragg spectroscopy, …

A new feature: interactions between atoms

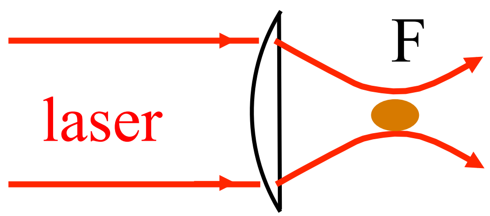
- A hindrance to observe AL (pure wave effect for single particle)
- New interesting problems, many-body physics (T. Giamarchi, B. Altshuler, S. Skipetrov, D. Shepelyansky…)

Optical dipole potential: a true potential for the atoms

Inhomogeneous light field: $\mathbf{E}(\mathbf{r},t) = \mathbf{E_0}(\mathbf{r})\cos\left[\omega t - \varphi(\mathbf{r})\right]$

Induced atomic dipole: $\left\langle \mathbf{D}_{\text{at}}(t) \right\rangle_{\mathbf{r}_{\text{at}}} = \alpha\, \mathbf{E}(\mathbf{r}_{\text{at}},t)$

Interaction energy: $W = -\overline{\mathbf{E}(\mathbf{r}_{\text{at}},t)\left\langle \mathbf{D}_{\text{at}}(t) \right\rangle_{\mathbf{r}_{\text{at}}}} = -\alpha \dfrac{\left[\mathbf{E}_0(\mathbf{r}_{\text{at}})\right]^2}{2}$



Far from atomic resonance, α real

- $\alpha < 0$ above resonance: repulsion from F
- $\alpha > 0$ below resonance: attraction to F

Atoms experience a (mechanical) potential proportional to light intensity

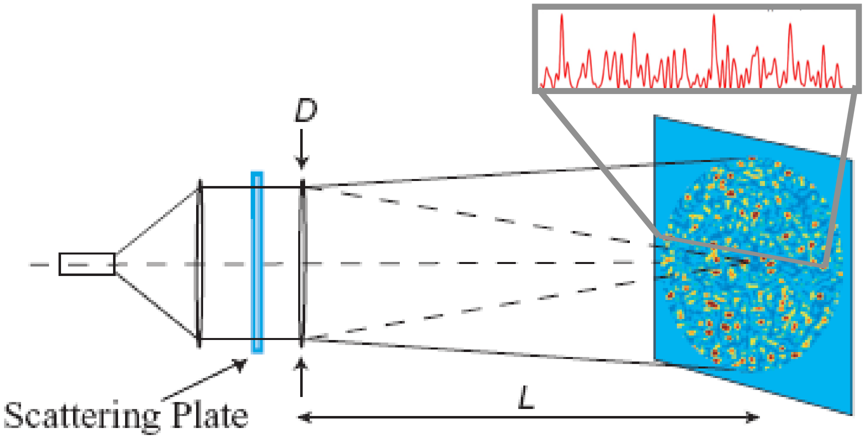
$$U_{\text{dip}}(\mathbf{r}) = -\alpha I(\mathbf{r})$$

- Attracted towards large intensity regions below resonance
- Repelled out of large intensity regions above resonance
- No dissipation for large enough detuning from resonance

Laser speckle: a well controlled disorder

Blue detuned ($\delta > 0$) light creates a repulsive potential for atoms, proportional to light intensity

$$V \propto \frac{I}{\delta} \propto \frac{|\mathrm{E}|^2}{\delta}$$



Laser speckle: well controlled random pattern

(Complex electric field = Gaussian random process, central limit theorem)

Intensity (*i.e.* disordered potential) is NOT Gaussian:

$$P(I) = \frac{1}{\overline{I}} \exp\{-\frac{I}{\overline{I}}\}$$

Intensity inherits some properties of underlying Gaussian process

Easy to calculate autocorrelation function

Speckle grain size $\sigma_R \simeq \frac{\lambda L}{\pi D}$

D. Clément *et al.*, New J. Phys. 8, 165 (2006)

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1D Anderson localization?

Theorist statement: all states localized in 1D. Why bother?

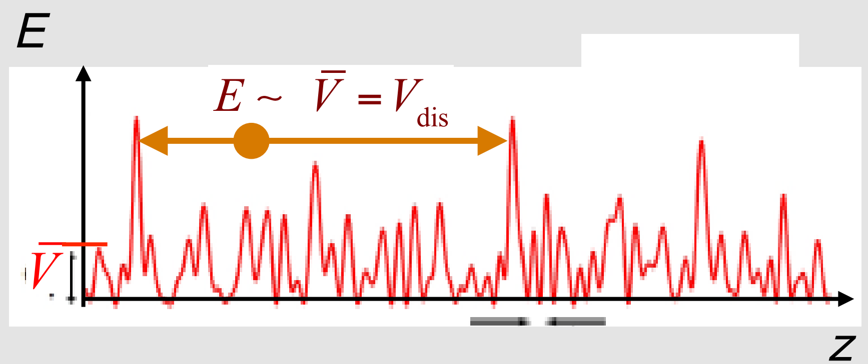
Abrahams, Anderson, Licciadello, Ramakkrishnan, PRL 1979

Some theorists care: Prigodin and Altshuler, 1989

Experimentalist: Can we observe Anderson-like localization?

Localization in a strong disorder: particle trapped between two large peaks

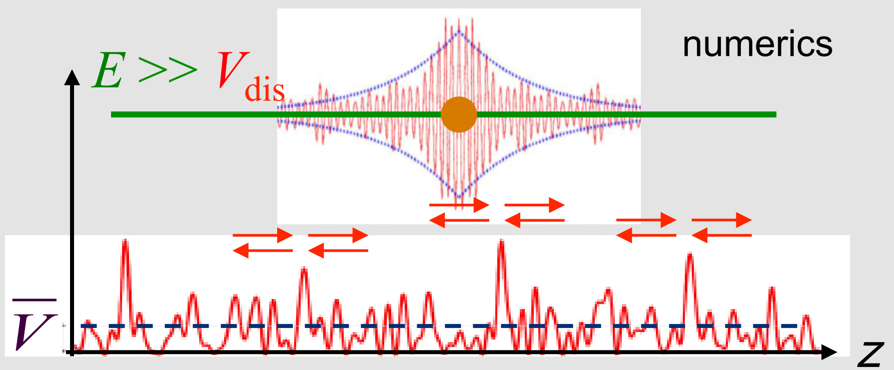
Classical localization, not Anderson



Localization in weak disorder (numerics/analytics): interference of many scattered wavelets

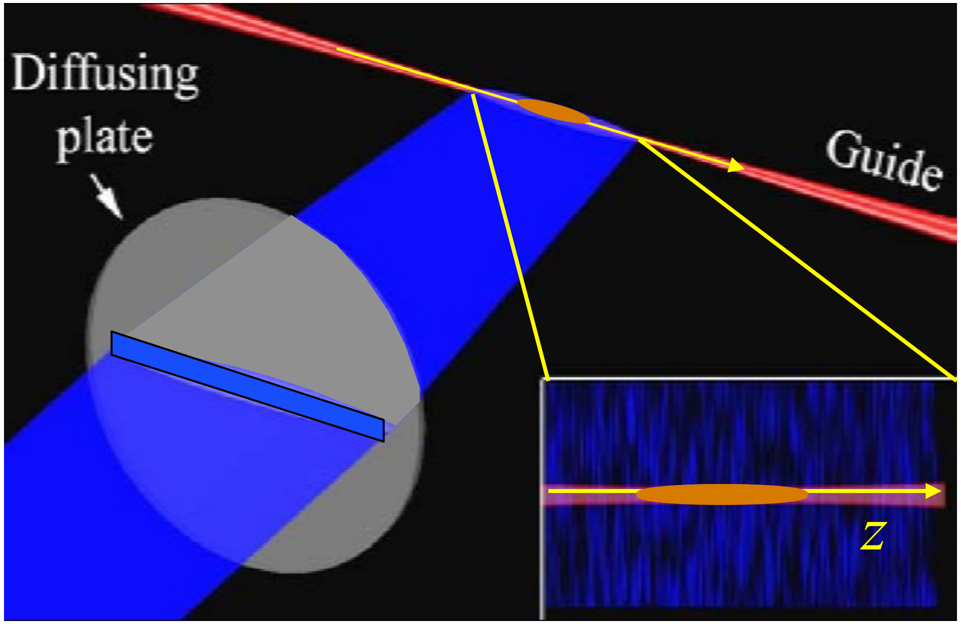
Looks like Anderson localization!

Worth testing it

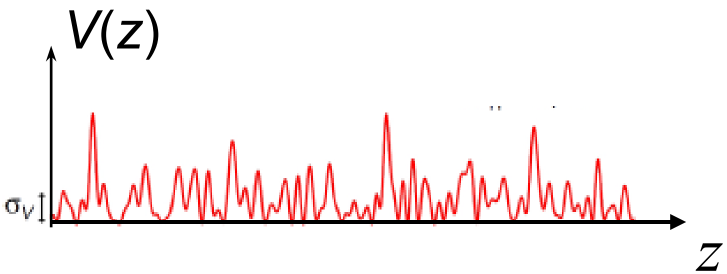


A 1D random potential for 1D guided atoms

Atoms tightly confined in x-y plane, free along z: 1D matterwaves



Anisotropic speckle
- elongated along *x-y*
- fine along *z*



$$\sigma_x, \sigma_y \approx 50\,\mu\text{m} \;;\; \sigma_z \approx 1\,\mu\text{m}$$

BEC elongated and guided along *z* (focused laser, $\delta < 0$)

$$2R_z^{\text{TF}} \approx 300\,\mu\text{m} \;;\; 2R_\perp^{\text{TF}} \approx 3\,\mu\text{m}$$

Imaged with resonant light: many times single atom observation

1 D situation for the elongated BEC.

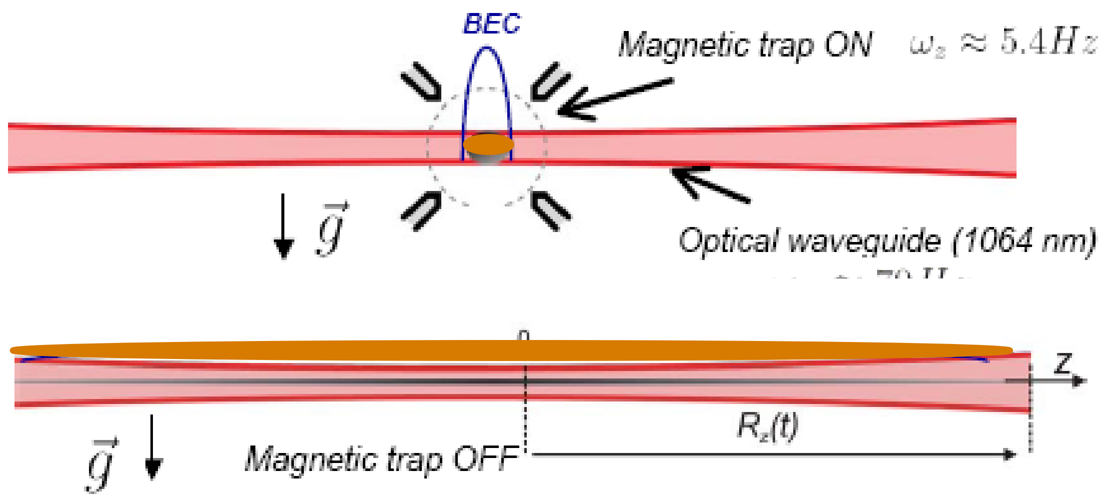
Many speckle grains covered (self averaging system = ergodic)

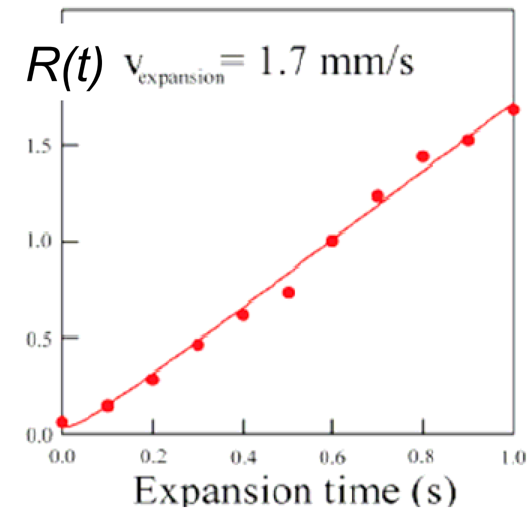
1D situation: invariant transversely to *z*

Ballistic expansion of a 1D BEC

Cloud of trapped ultracold atoms (dilute BEC) observable on a single shot: N atoms with the same confined wave function.

Release of trapping potential along z: expansion in the 1D atom guide





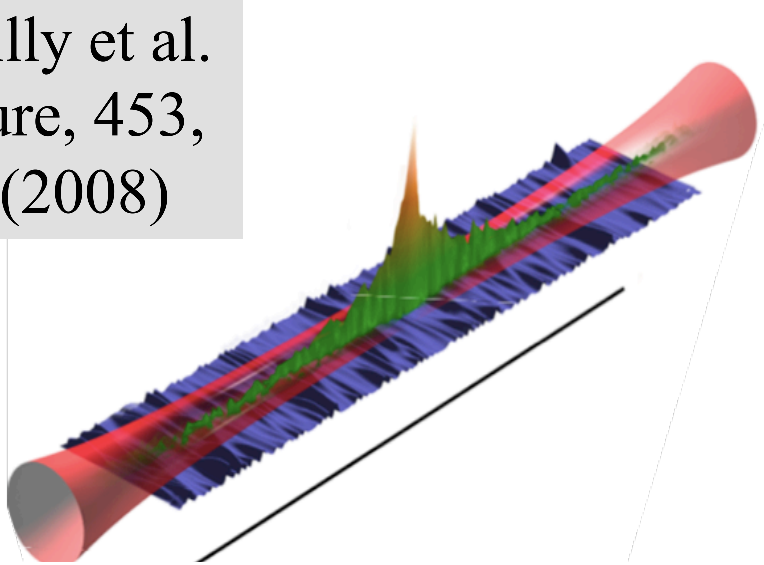
Initial interaction energy μ_{in} converted into kinetic energy

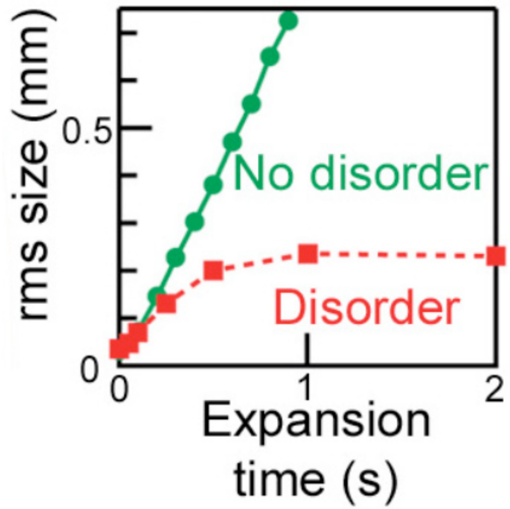
$\Rightarrow$ After a while, interaction free ballistic expansion

$\Rightarrow$ Superposition of plane waves with $p \leq p_{max} = \sqrt{2M\mu_{in}}$

1D Anderson localization in a weak speckle

J. Billy et al. Nature, 453, 891 (2008)

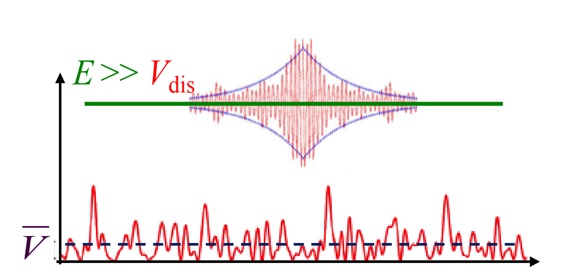




Expansion stops. Exponential localization?

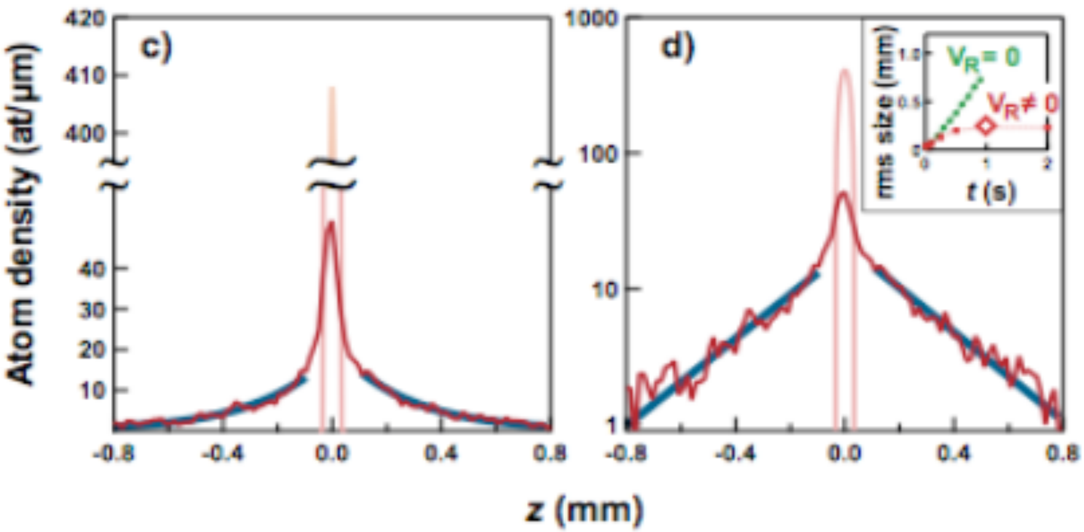
Weak disorder regime

$V_R = 0.1\ \mu_{ini}$

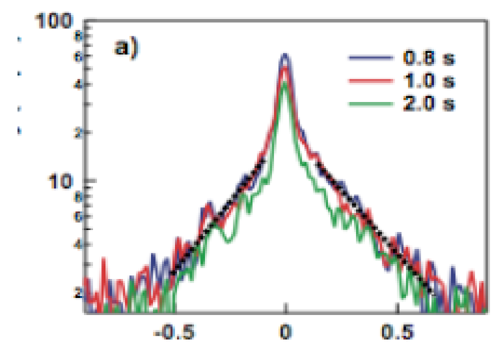


1D Anderson localization in a weak speckle

Direct observation of the wave function (squared modulus)



Profile stops evolving



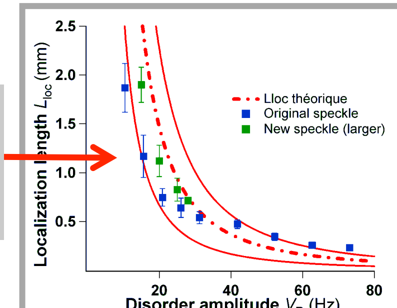
Exponential fit in the wings $\Rightarrow$ Localization length

Agreement with (Born 1^{rst} order) calculation

Speckle 1D disorder: effective mobility edge

L Sanchez Palencia et al., PRL 98, 210401 (2007)

Ask a question



Related results in Florence (Inguscio), Austin (Raizen), Lille (Garreau), Hannover (Ertmer), Rice (Hulet)

J. Billy et al. Nature 453, 891 (2008)

1D Anderson localisation in a weak speckle disorder: effective mobility edge

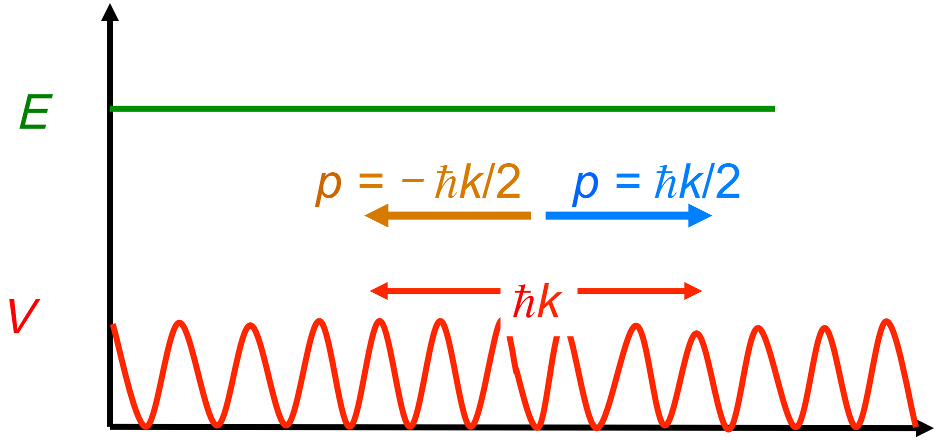
In order to observe 1D Anderson localisation in a weak disorder, we had to reduce the atoms momentum below an effective mobility edge

$$p < \frac{h}{\sigma_{\mathrm{R}}}$$

σ_{R} = correlation length of the disorder

How to understand it?

1D localization in a weak disorder as Bragg reflection



Periodic potential

$$V = V_0 \cos kz$$

No propagation of matter wave

$$\psi = A \exp\left(\pm i \frac{p}{\hbar} z\right) \text{ if } p \sim \hbar k/2$$

even in the case of $E = \dfrac{p^2}{2M} >> V_0$ (weak perturbation)

- Bragg reflection of $p \sim \hbar k/2$ on modulation $\cos kz$
- No propagation at band edge

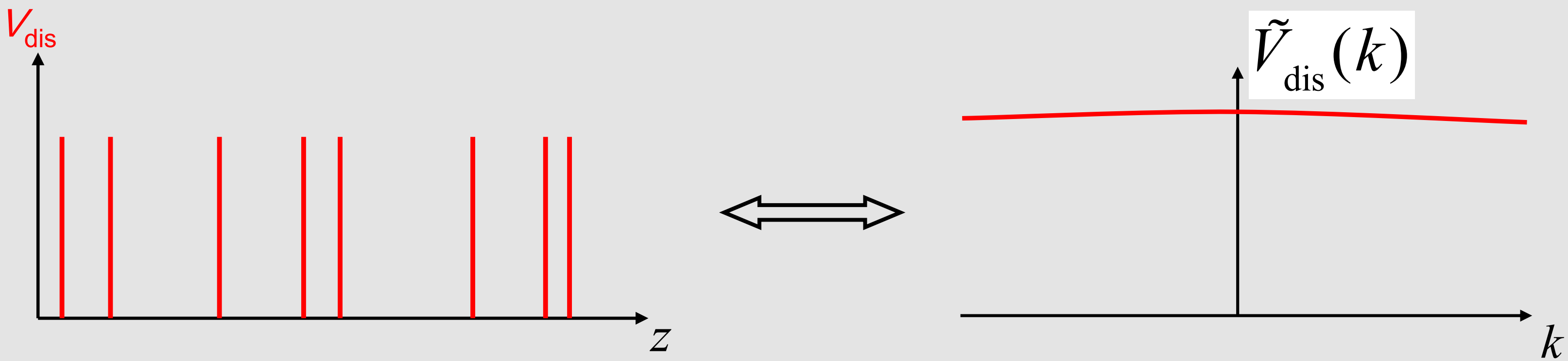
Disordered potential: many k components.

Anderson localization: all p components are Bragg reflected.

Demands broad spectrum of disordered potential

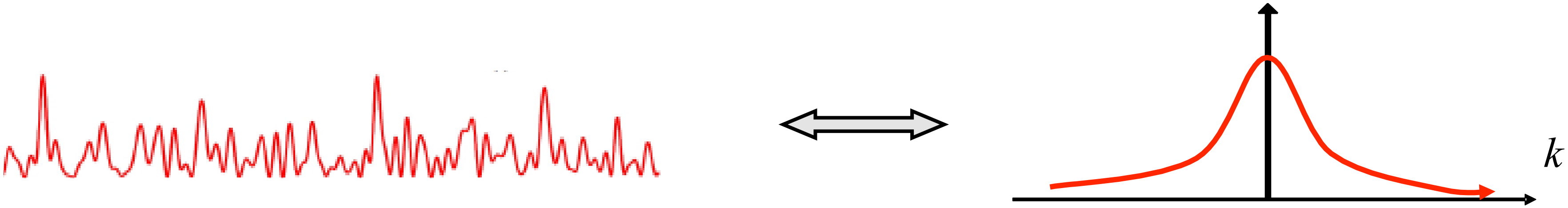
1D Anderson localization in a weak uncorrelated disorder

Disordered potential with a white spectrum of k vectors

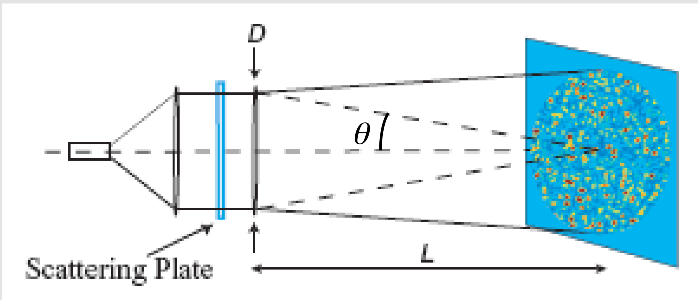


Anderson Loc: all p components Bragg reflected

What happens for a correlated potential (finite spectrum)?

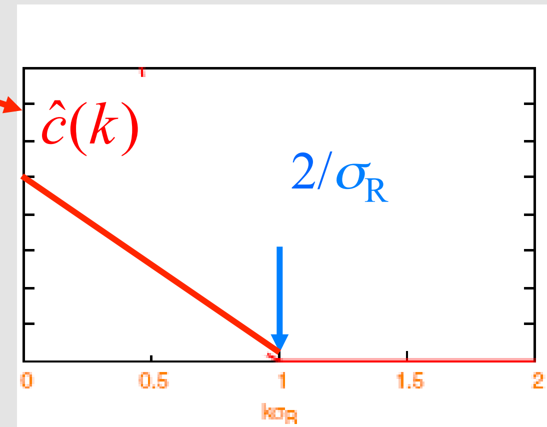


Case of a speckle disorder: cut off in the spatial frequency spectrum



Spatial frequencies spectrum of speckle potential

FT of auto correlation



Speckle potential, created by diffraction from a scattering plate: no k component beyond a cut off $\frac{2}{\sigma_R} = \frac{\sin\theta}{k_{\text{light}}}$

Only matter waves with $2p < 2\hbar/\sigma_R$ localize

Effective mobility edge

First order calculation (phase formalism)

Lyapunov coefficient $\gamma(p) = \frac{1}{L_{\text{loc}}(p)} \propto \hat{c}(2\frac{p}{\text{h}}) \Rightarrow \gamma(p) = 0$ for $p > \frac{\text{h}}{\sigma_R}$

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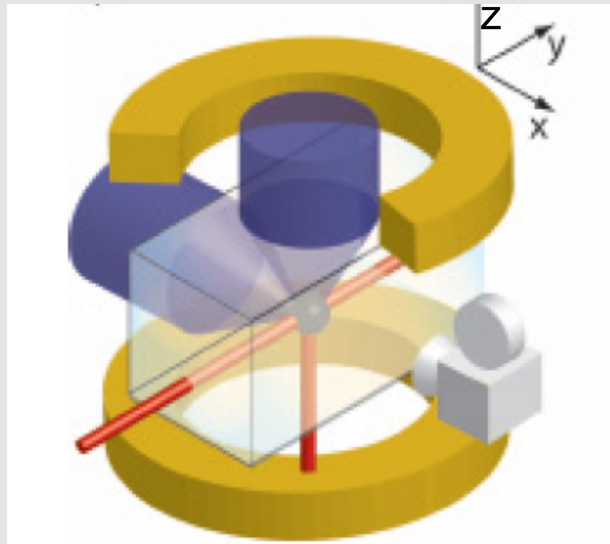
3D localization of ultra-cold atoms in a laser speckle disordered potential: apparatus

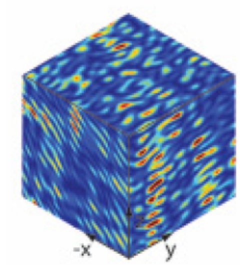
Ultra-cold atoms (BEC), suspended against gravity (mag field gradient) ; released in repulsive disordered potential



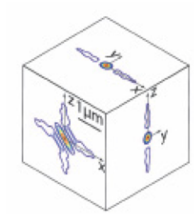
F. Jendrzejewski et al., Nature Physics 8, 398 (2012)

Fluorescence integrated along x: column density $n(y,z)$

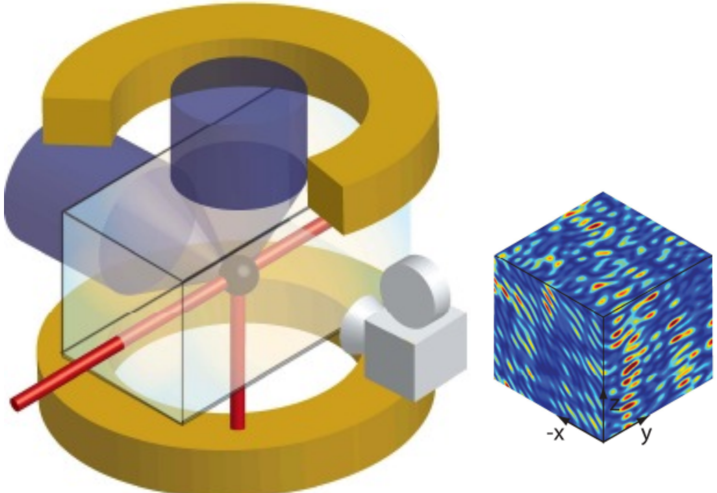
Disordered optical potential: Two crossed coherent speckle fields, with same polarization:



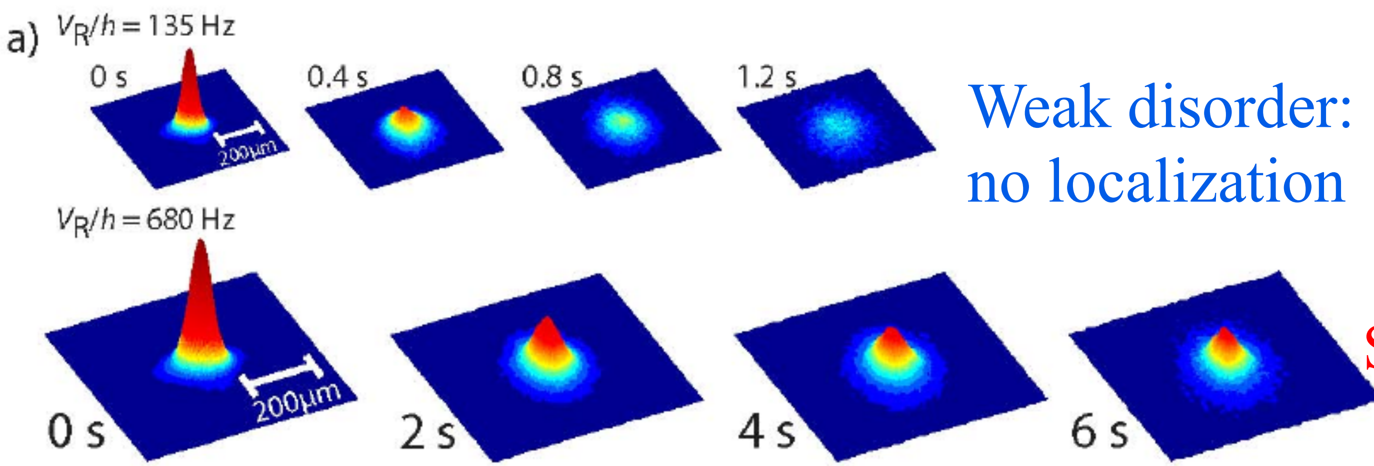
- Tunable amplitude of disorder $(0 < \overline{V_{\text{dis}}} / h < 1.1\,\text{kHz})$
- Small correlation lengths in all directions ($\sigma_u = 0.11, 0.27, 0.08\ \mu\text{m}$), but not isotropic
- Low classical percolation threshold $(< 10^{-2}\,\overline{V_{\text{dis}}})$



3D localization of ultra-cold atoms in a laser speckle disordered potential



- Ultra-cold atoms (BEC, μ_{ini} / h = 40 Hz; T = 1nK) released
- Disordered ramped up rapidly
- Cloud of atoms observed after variable delay



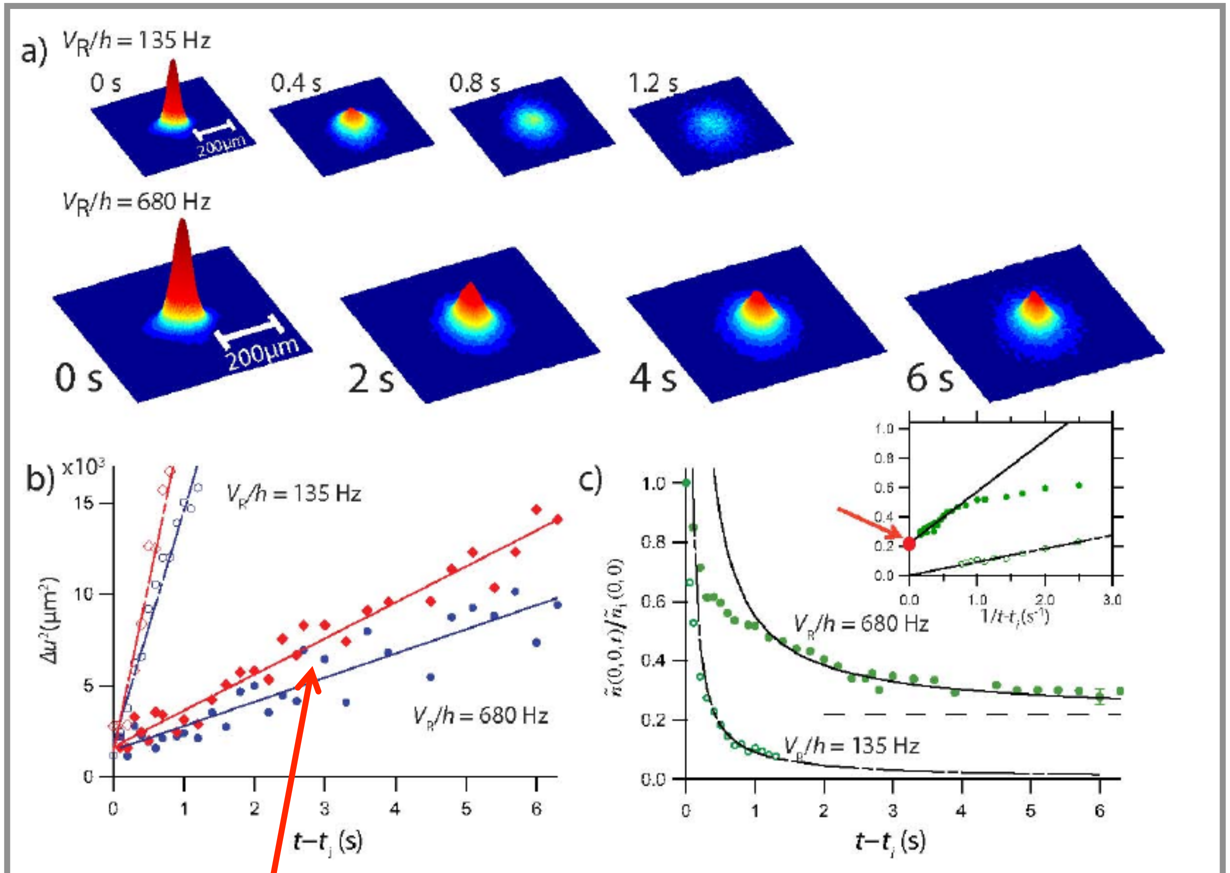
Weak disorder: no localization

F. Jendrzejewski et al., Nature Physics 8, 398 (2012)

Strong disorder: localization

Related experiment in DeMarco's group: Kondov et al., Science 334, 66 (2011); in Inguscio's group (G. Modugno, 2014)

Diffusive and localized components: phenomenological analysis



Column density taken as sum of:

⇒ localized peak, replica of initial profile

⇒ diffusing component

$\tilde{n}_D(0,0,t)$ decays as $1/t$ ⇒ localized fraction

Mean squared radii grow linearly with time: ⇒ determination of the diffusion coefficients

$$\tilde{n}(y,z,t) = f_{\mathrm{loc}} \times \tilde{n}_{\mathrm{i}}(y,z) + \tilde{n}_{\mathrm{D}}(y,z,t)$$

stationary diffusing

Theoretical analysis

Sudden application of the (strong) disorder

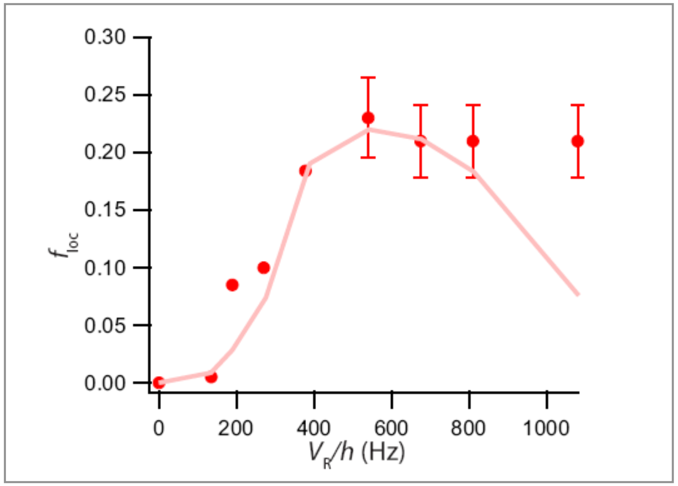
$\Rightarrow$ wide energy distribution of the atoms.

Results of calculation (self-consistent theory of localization)

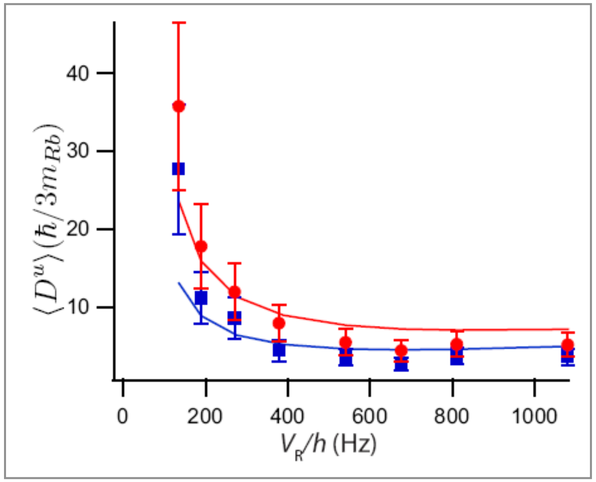
$\Rightarrow$ Always a fraction of the atoms above mobility edge

$\Rightarrow$ Localization lengths too small to be oberved ($\Rightarrow$ localized part a replica of the initial distribution)

Theory (one adjustable parameter, partly understood): solid curves



Localized fraction



Diffusion coefficients

M. Piraud, L. Pezzé, L. Sanchez-Palencia, EuroPhys. Lett. (2012)

Matterwave transport and AL in anisotropic 3D disorder

Localization of ultra cold atoms in 3D disordered potential: summary

- Direct observation of localized and diffusing components
- Phenomenological measurements (not theory dependent) of localized fraction and diffusion coefficients.

Localization cannot be explained

- by classical trapping below the classical percolation threshold
- by quantum trapping in local minima of the disorder

Fairly convincing theoretical description of the observations

Hard to conclude that it is not Anderson Localization

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Convincing theoretical description of the observations

Hard to conclude that it is not Anderson Localization

Still missing: a smoking gun of the role of coherence in the observed localisation

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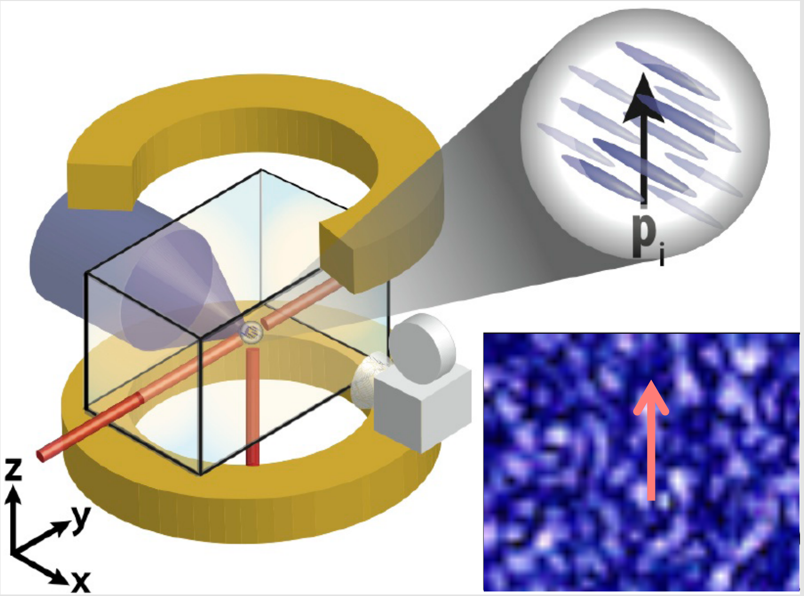
Ultra cold atoms with well defined momentum launched in 2D disorder

Atoms launched with a momentum $\mathbf{p}_i$ in a quasi 2D speckle* (elongated)

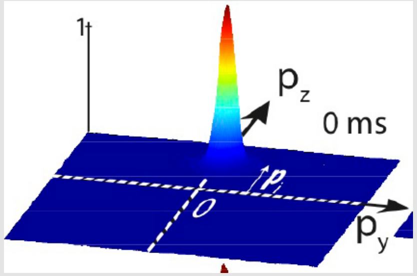
Narrow momentum distribution :

- Harmonic kick on expanding BEC→ stopped atoms
- Magnetic kick



Time of flight ⇒ initial velocities distribution:

$V_i = 3.3 \pm 0.2$ mm / s

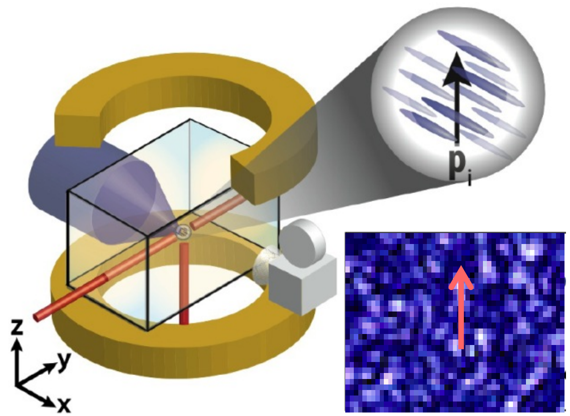


F. Jendrzejewski et al., PRL 109, 195302 (2012)

* Theoretical proposal: Cherroret et al., PRA85, 01604 (2012)

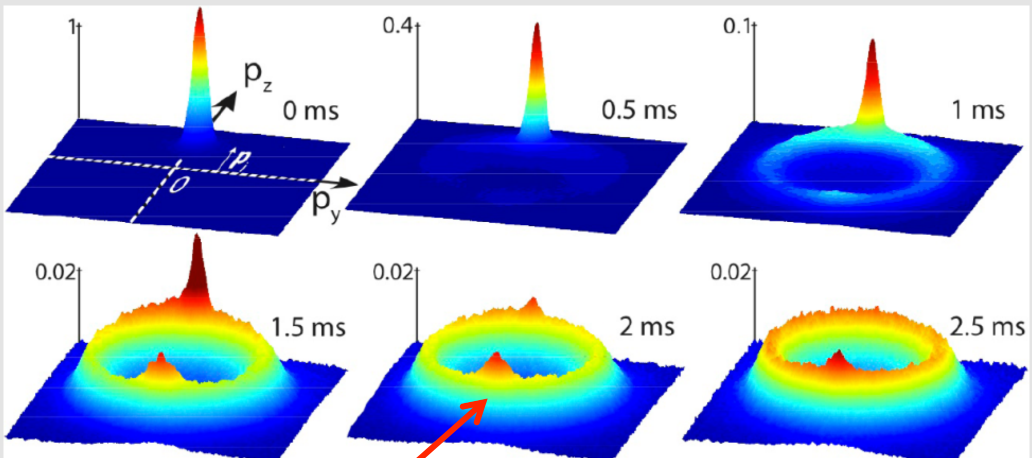
Evolution of momentum distribution in the disordered potential

Atoms launched in 2D disorder with well defined momentum $\mathbf{p_i}$



F. Jendrzejewski et al., PRL 109, 195302 (2012)

Momentum distribution after diffusion time *t*



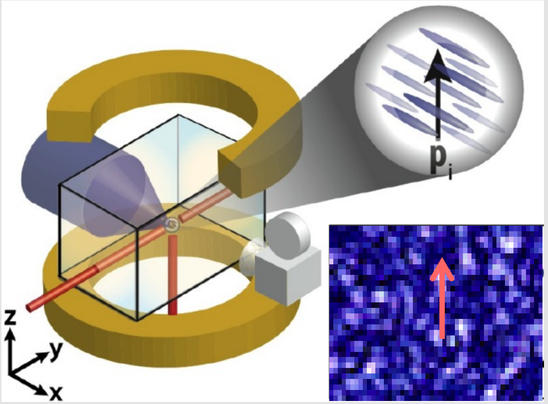
- Elastic scattering ring: determination of scattering and transport times
- Coherent Back Scattering peak

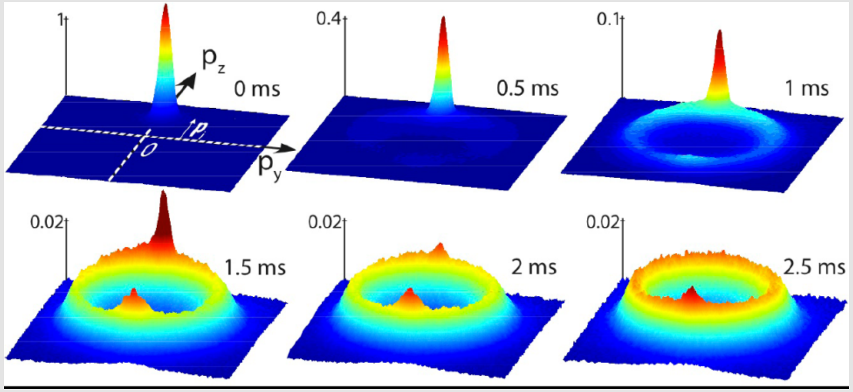
Related results G. Labeyrie et al., EPL 100, 66001 (2012)

Atomic Coherent Back Scattering as an evidence of coherence

Atoms launched with $\mathbf{p}_i$

Momentum distribution after diffusion time t





- Elastic scattering ring: determination of scattering and transport times
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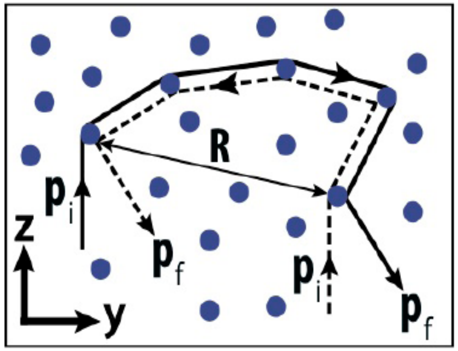
CBS peak: interference between counter propating multiple scattering paths for $\mathbf{p}_f = -\mathbf{p}_i$

Width decreases as $R^{-1} = (2Dt)^{-1/2}$



F. Jendrzejewski et al., PRL 109, 195302 (2012)

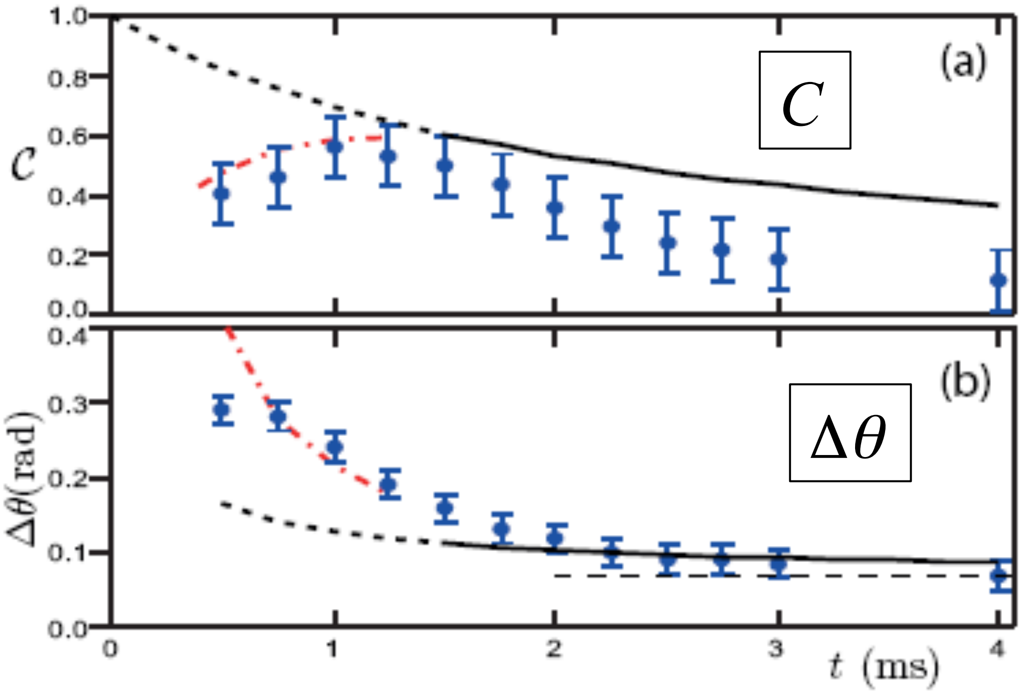
Peak width evolution agrees with model (no adjust. parameter)

Time resolved observation of atomic CBS

Evolution of CBS peak contrast C and width $\Delta\theta$, from short times to long times regime

Cross over

- from short times regime, where single scattering dominates (no CBS)
- to long times regime where multiple scattering dominates ($\Rightarrow$ pure CBS)



Excellent agreement, without adjustable parameter: direct evidence of the role of coherence in quantum transport

Ultra cold atoms in a laser speckle: a quantum simulator for Anderson localization

1. Anderson localization: why, what? The naïve view of an AMO experimentalist
2. Cold atoms in laser speckle? A well controlled system
3. 1D Anderson Localization? More interesting than expected
4. 3D Anderson Localization? A first evidence
5. Coherent Back Scattering? A smoking gun of coherence
6. Outllook and open questions A lot more to come!

Outlook and questions

3D AL
- measure the exact value of the mobility edge
- measure the localization lengths vs energy
- measure critical exponents

Demands a better conrol of the atom energy in disorder.
Adiabatic ramping of the disorder? Energy selective loading?

2D: AL will not be easy to observe, interesting results already obtained (PRL **104** 220602, NJP **13** 095015; PRA **84** 061606, **85** 033602)

Add controlled interactions: a big challenge for theorists

More evidence of role of coherence in quantum transport:
Scrambling disorder? Artificial gauge fields?

A quantum simulator?

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Many challenges for theorists:
genuine **quantum theorists stimulator**

Ultra cold atoms in a laser speckle: a quantum simulator for Anderson localization

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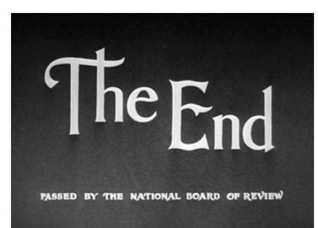
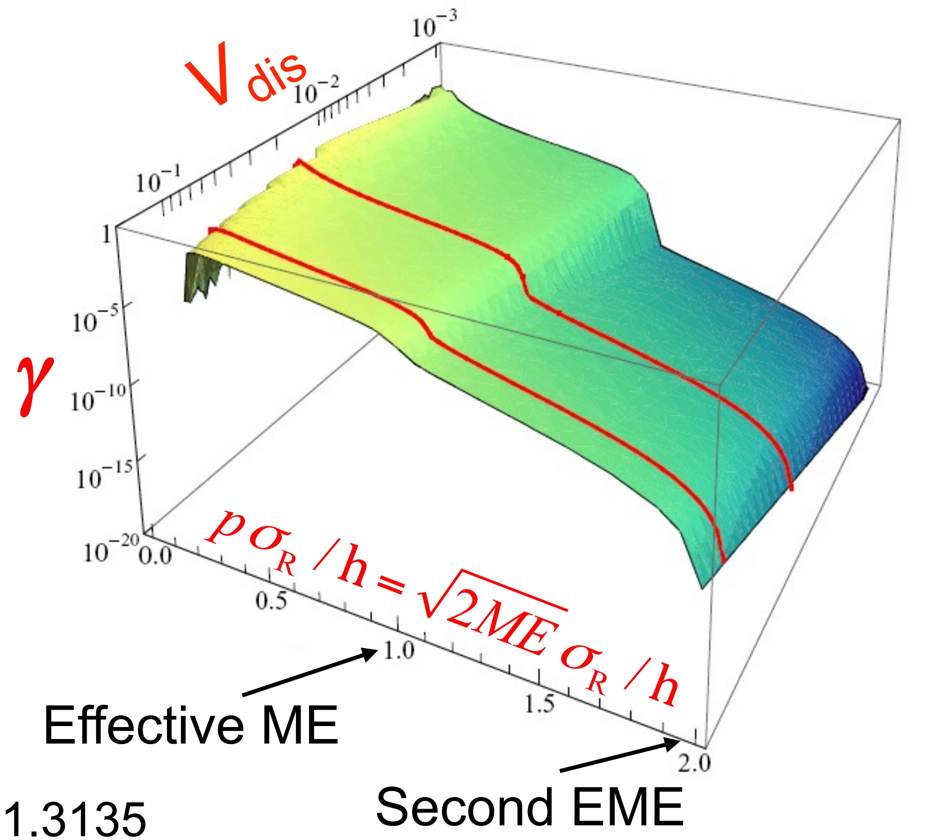


What happens beyond the effective 1D mobility edge?

Calculations (P. Lugan, L. Sanchez-Palencia) beyond the Born approximation (4th order) (agreement with numerics, D. Delande, and diagrams, C. Müller)

Pierre Lugan et al. PRA 80, 023605 (2009)

Lyapunov coefficient γ not exactly zero
but crossover to a much smaller value at effective mobility edge
Sharper crossover for weaker disorder



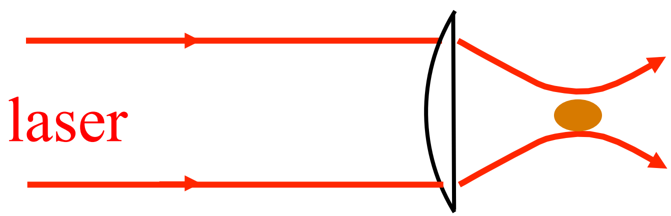
Effective transition in a finite size system

* analogous results by E. Gurevich, 0901.3135

Optical dipole potential

Inhomogeneous light field: $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r})\cos\left[\omega t - \varphi(\mathbf{r})\right]$

Induced atomic dipole: $\left\langle \mathbf{D}_{\mathrm{at}}(t)\right\rangle_{\mathbf{r}_{\mathrm{at}}} = \alpha\,\mathbf{E}(\mathbf{r}_{\mathrm{at}},t)$



Far from atomic resonance, α real

- $\alpha < 0$ above resonance
- $\alpha > 0$ below resonance

Interaction energy: $W = -\overline{\mathbf{E}(\mathbf{r}_{\mathrm{at}},t)\left\langle \mathbf{D}_{\mathrm{at}}(t)\right\rangle_{\mathbf{r}_{\mathrm{at}}}} = -\alpha\dfrac{\left[\mathbf{E}_0(\mathbf{r}_{\mathrm{at}})\right]^2}{2}$

Atoms experience a (mechanical) potential proportional to light intensity

$$U_{\mathrm{dip}}(\mathbf{r}) = -\alpha I(\mathbf{r})$$

- Attracted towards large intensity regions below resonance
- Repelled out of large intensity regions above resonance