Probing and Controlling Quantum Matter in Optical Lattices

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erc synergy

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Course Outline

Introduction - Brief Review Lattice Basics

Hubbard models

LECTURE 1 - Single Atom Imaging/Control

Single Atom Imaging

Probing Thermal and Quantum Fluctuations

String Order - a Hidden Order Parameter

'Higgs' Amplitude Mode

Single Spin Manipulation

LECTURE 2 - Quantum Magnetism

Superexchange - from double wells to RVB/d-wave states on plaquettes

Probing Spin Correlations

Single Spin Impurity

Bound Magnons

Non-Equilibrium Dynamics in Heisenberg Quantum Magnets

Quantum Magnetism with Rydberg atoms

LECTURE 3 - Artificial Gauge Fields

SSH model - the simplest Topological Insulator

Probing the Zak Phase in the SSH model
- Bulk-Edge correspondence in 1d -

'Aharonov Bohm' Interferometry for Measuring Band Geometry

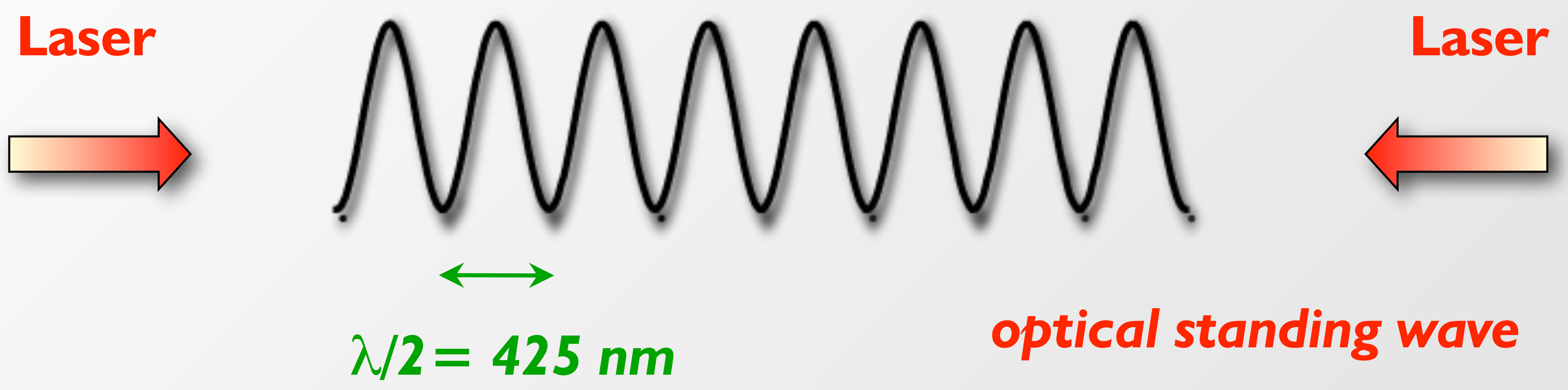
- Berry connection/Berry curvature
- pi-flux Singularity in Graphene
- Stückelberg Interferometry

(non-Abelian Berry connection, Wilson loops)

Realizing Staggered Flux, Hofstadter & QSH Hamiltonian

Hall Response and Chern Number in Hofstadter Bands

Optical Lattice Potential – Perfect Artificial Crystals

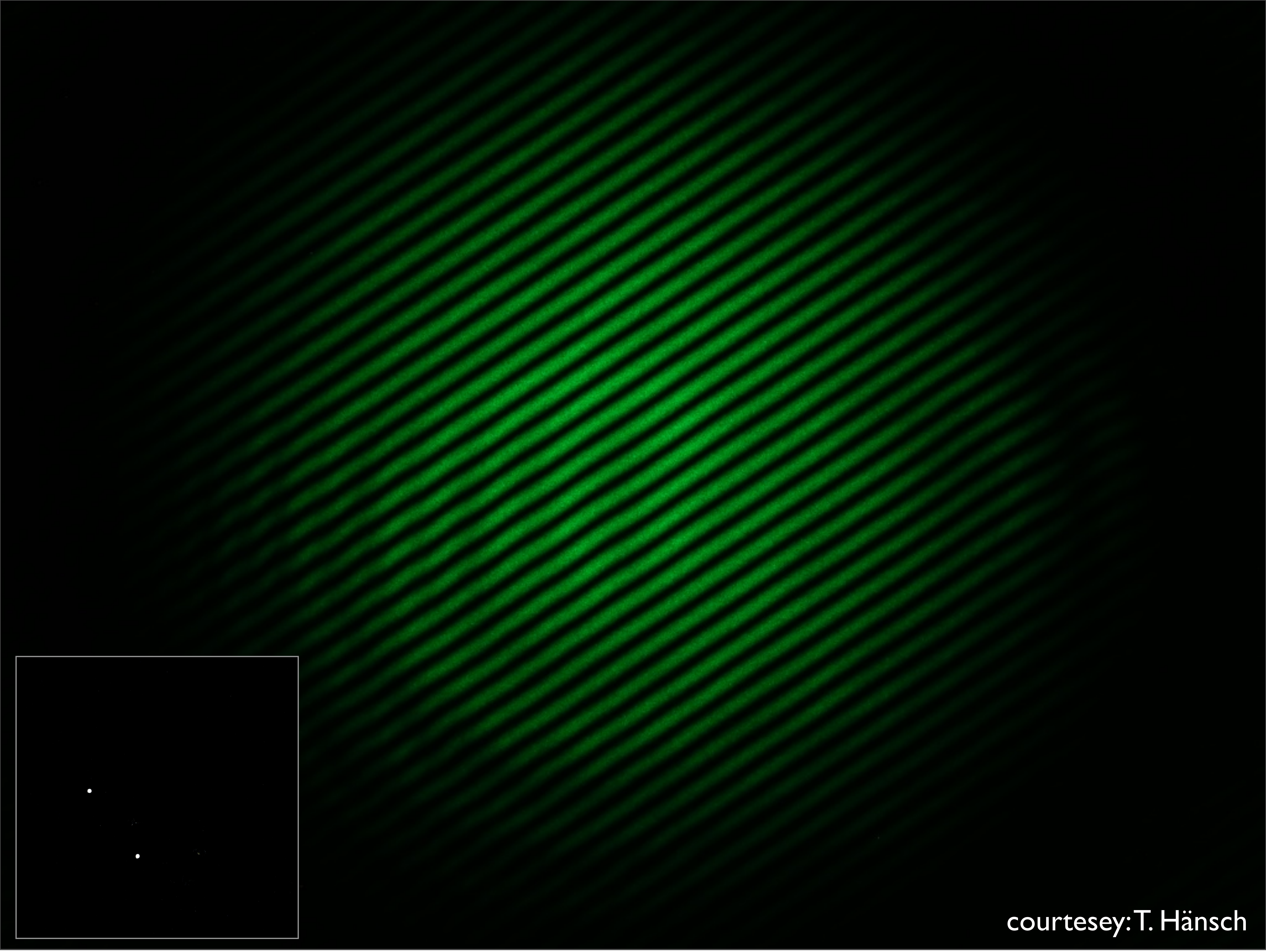


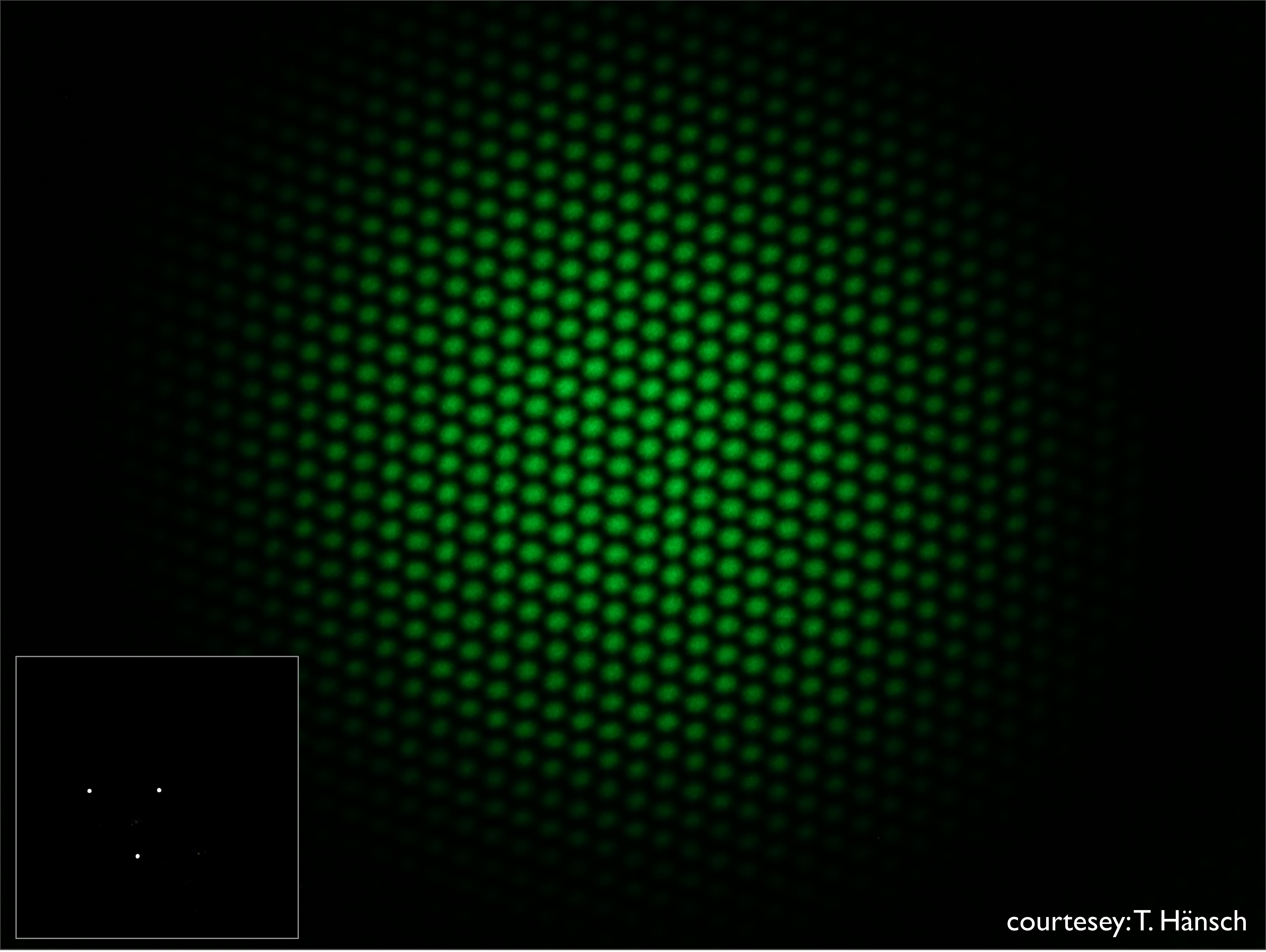
Fourier synthesize aribtrary lattices:

- Square
- Hexagonal/Triangular/Brick Wall
- Kagomé
- Superlattices
- *Spin dependent lattices*
- ...

Special case: flux lattices...

Full **dynamical** control over **lattice depth, geometry, dimensionality!**


courtesey: T. Hänsch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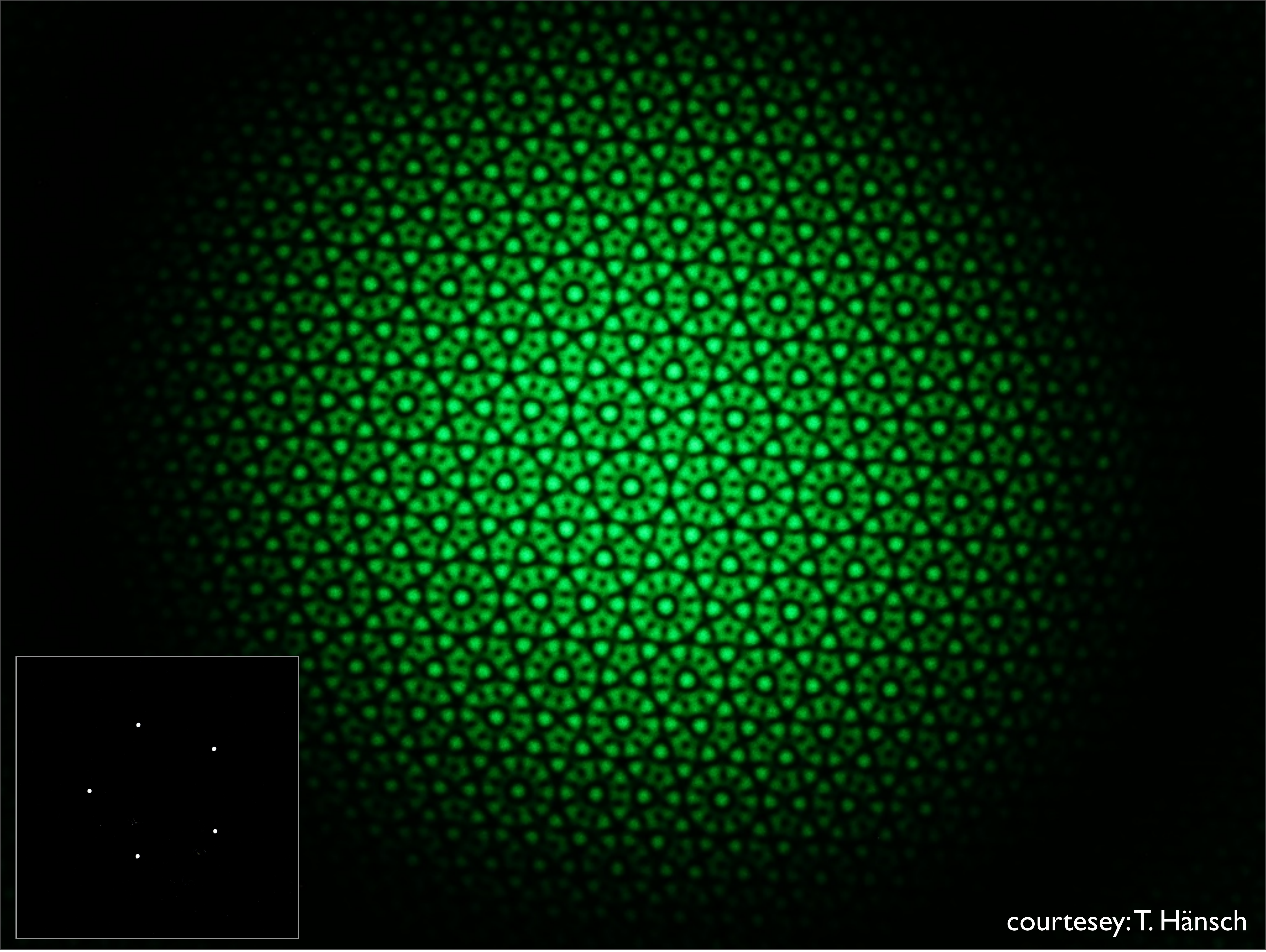
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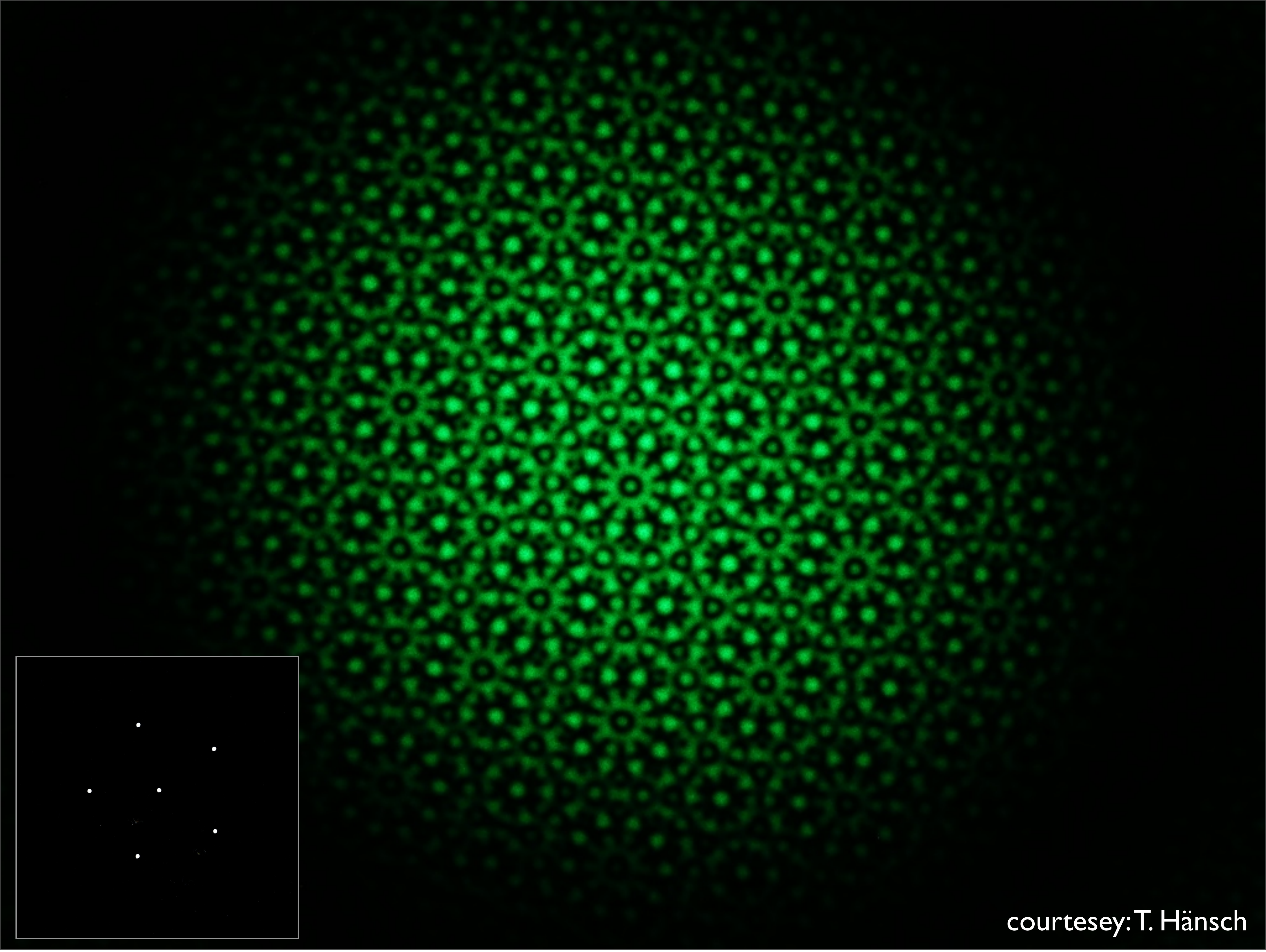

courtesey: T. Hänsch

Atoms in Periodic Potentials

Single Particle in a Periodic Potential - Band Structure (1)

$$H\phi_q^{(n)}(x) = E_q^{(n)}\phi_q^{(n)}(x) \qquad \text{with} \qquad H = \frac{1}{2m}\hat{p}^2 + V(x)$$

Solved by Bloch waves (periodic functions in lattice period)

$$\phi_q^{(n)}(x) = e^{iqx} \cdot u_q^{(n)}(x)$$

q = Crystal Momentum or Quasi-Momentum

n = Band index

Plugging this into Schrödinger Equation, gives:

$$H_B u_q^{(n)}(x) = E_q^{(n)} u_q^{(n)}(x) \qquad \text{with} \qquad H_B = \frac{1}{2m}(\hat{p}+q)^2 + V_{lat}(x)$$

Use Fourier expansion

$$V(x) = \sum_r V_r e^{i2rkx} \qquad \text{and} \qquad u_q^{(n)}(x) = \sum_l c_l^{(n,q)} e^{i2lkx}$$

yields for the potential energy term

$$V(x) u_q^{(n)}(x) = \sum_l \sum_r V_r e^{i2(r+l)kx} c_l^{(n,q)}$$

and the kinetic energy term

$$\frac{(\hat{p}+q)^2}{2m} u_q^{(n)}(x) = \sum_l \frac{(2\hbar kl+q)^2}{2m} c_l^{(n,q)} e^{i2lkx}.$$

In the experiment standing wave interference pattern gives

$$V(x) = V_{lat} \sin^2(kx) = -\frac{1}{4}\left(e^{2ikx} + e^{-2ikx}\right) + \text{c.c.}$$

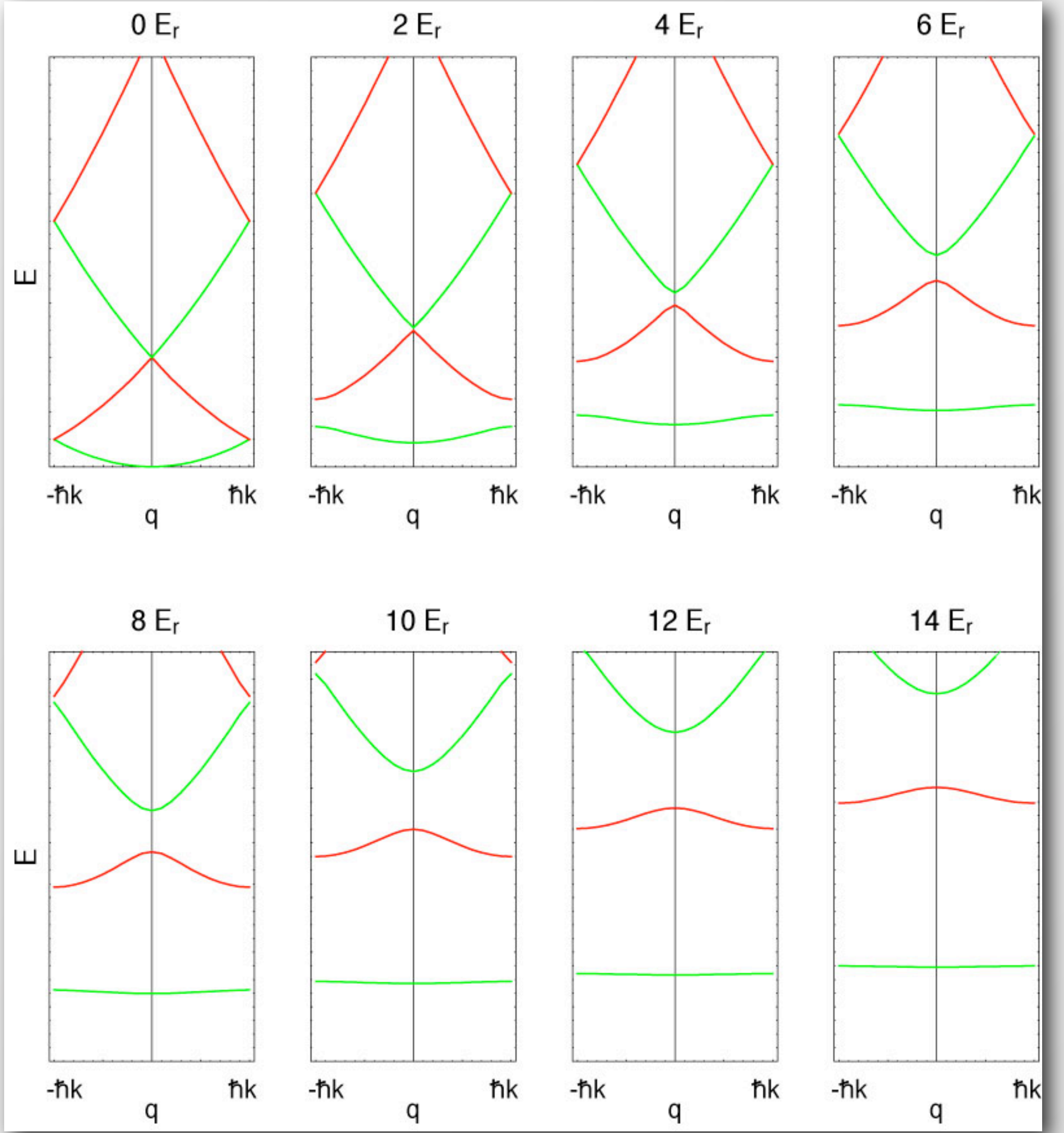
Use Fourier expansion

$$\sum_l H_{l,l'} \cdot c_l^{(n,q)} = E_q^{(n)} c_l^{(n,q)} \qquad \text{with} \qquad H_{l,l'} = \begin{cases} (2l + q/\hbar k)^2 E_r & \text{if } l = l' \\ -1/4 \cdot V_0 & \text{if } |l - l'| = 1 \\ 0 & \text{else} \end{cases}$$

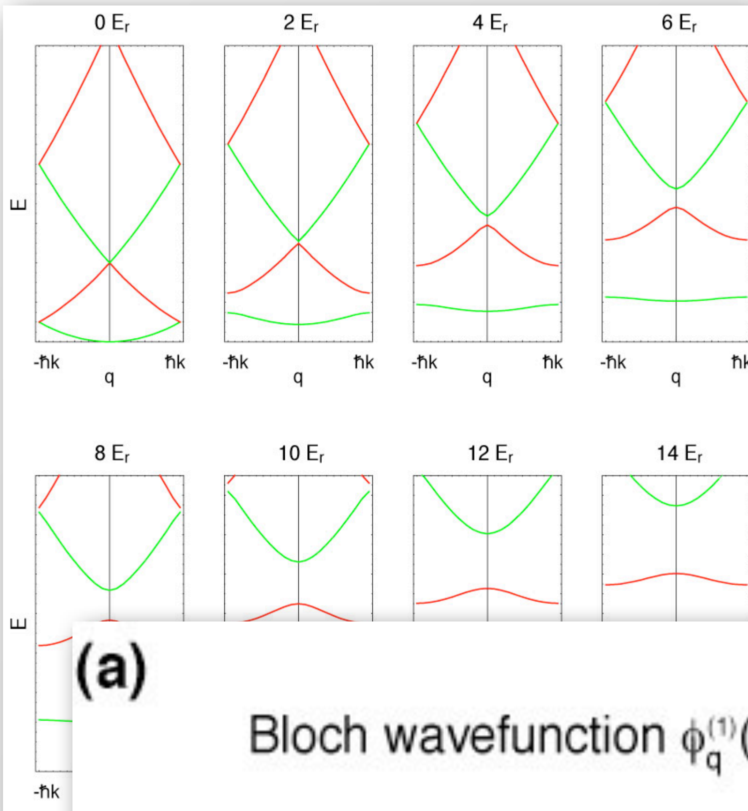
$$\begin{pmatrix} (q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & 0 & 0 & \cdots \\ -\frac{1}{4}V_0 & (2+q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & 0 & \\ 0 & -\frac{1}{4}V_0 & (4+q/\hbar k)^2 E_r & -\frac{1}{4}V_0 & \\ & & -\frac{1}{4}V_0 & \ddots & \\ & & & & \end{pmatrix} \begin{pmatrix} c_0^{(n,q)} \\ c_1^{(n,q)} \\ c_2^{(n,q)} \\ \\ \vdots \end{pmatrix} = E_q^{(n)} \begin{pmatrix} c_0^{(n,q)} \\ c_1^{(n,q)} \\ c_2^{(n,q)} \\ \\ \vdots \end{pmatrix}$$

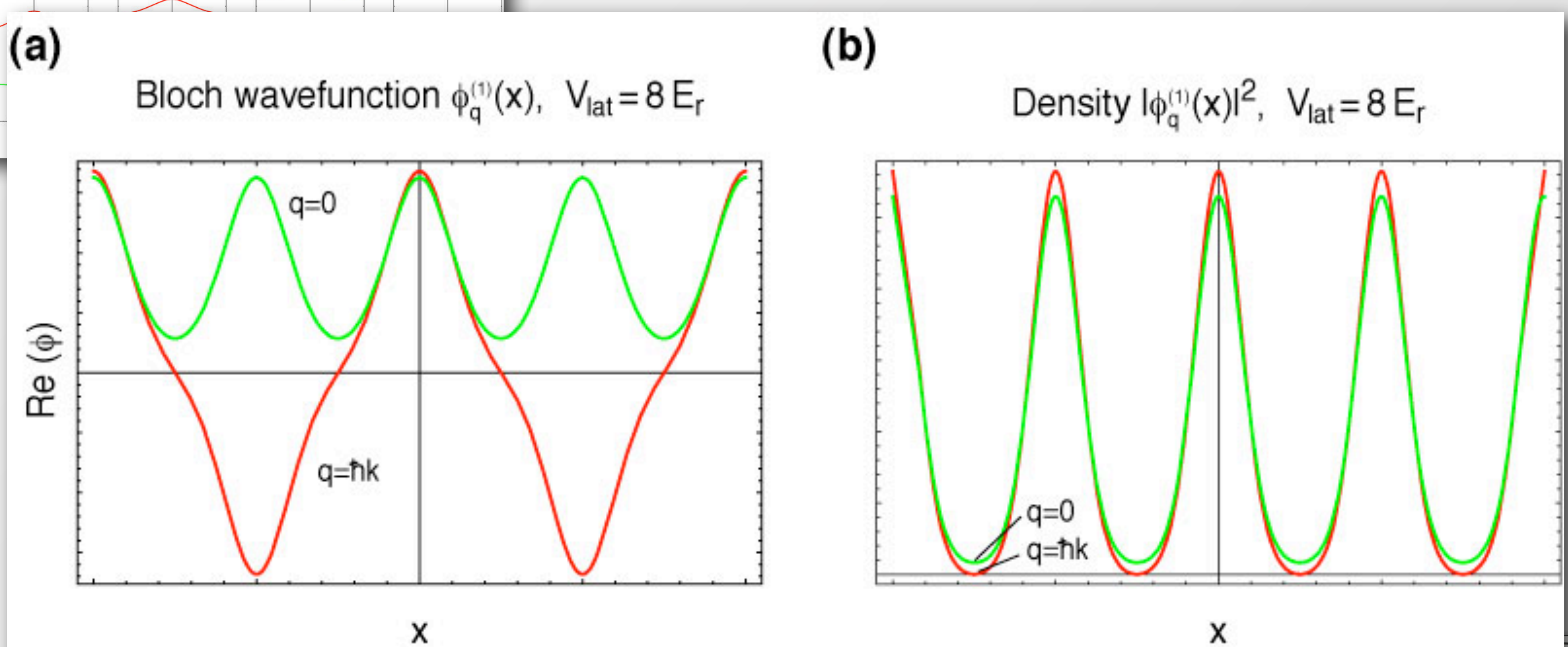
Diagonalization gives us Eigenvalues and Eigenvectors!

Bandstructure - Blochwaves



Bandstructure - Blochwaves

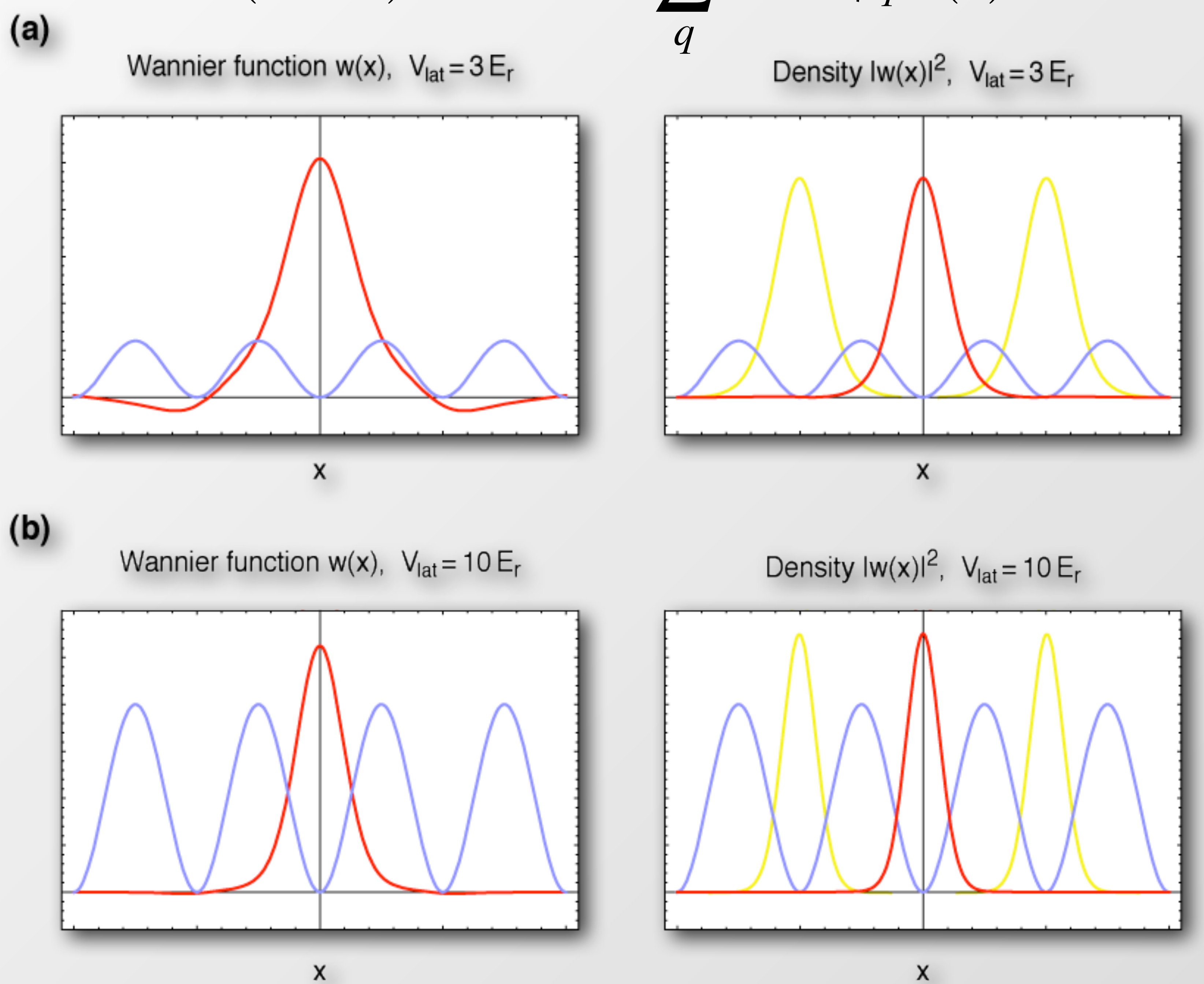




Wannier Functions

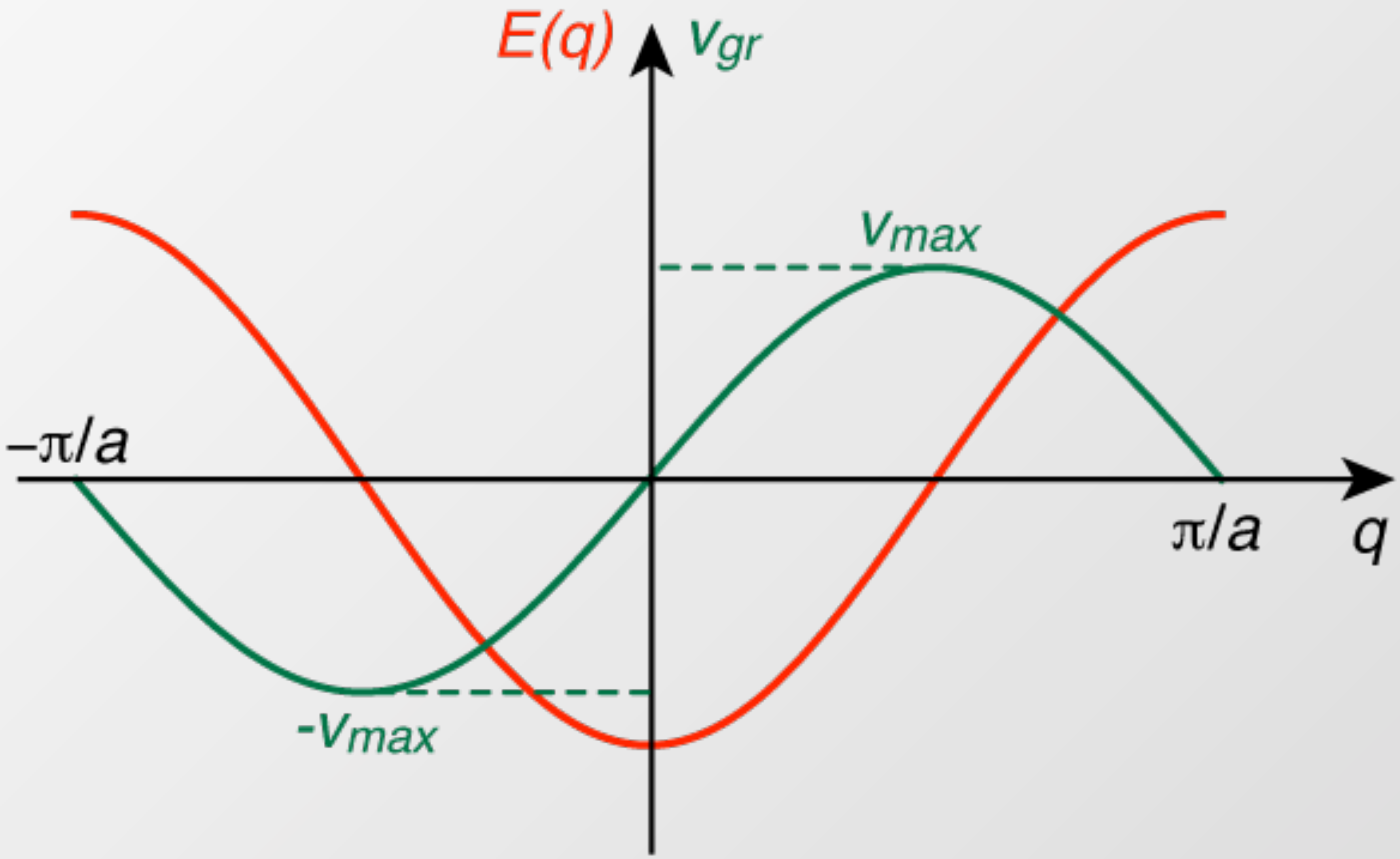
An alternative basis set to the Bloch waves can be constructed through localized wave-functions: Wannier Functions!

$$w_n(x - x_i) = \mathcal{N}^{-1/2} \sum_q e^{-iqx_i} \phi_q^{(n)}(x)$$

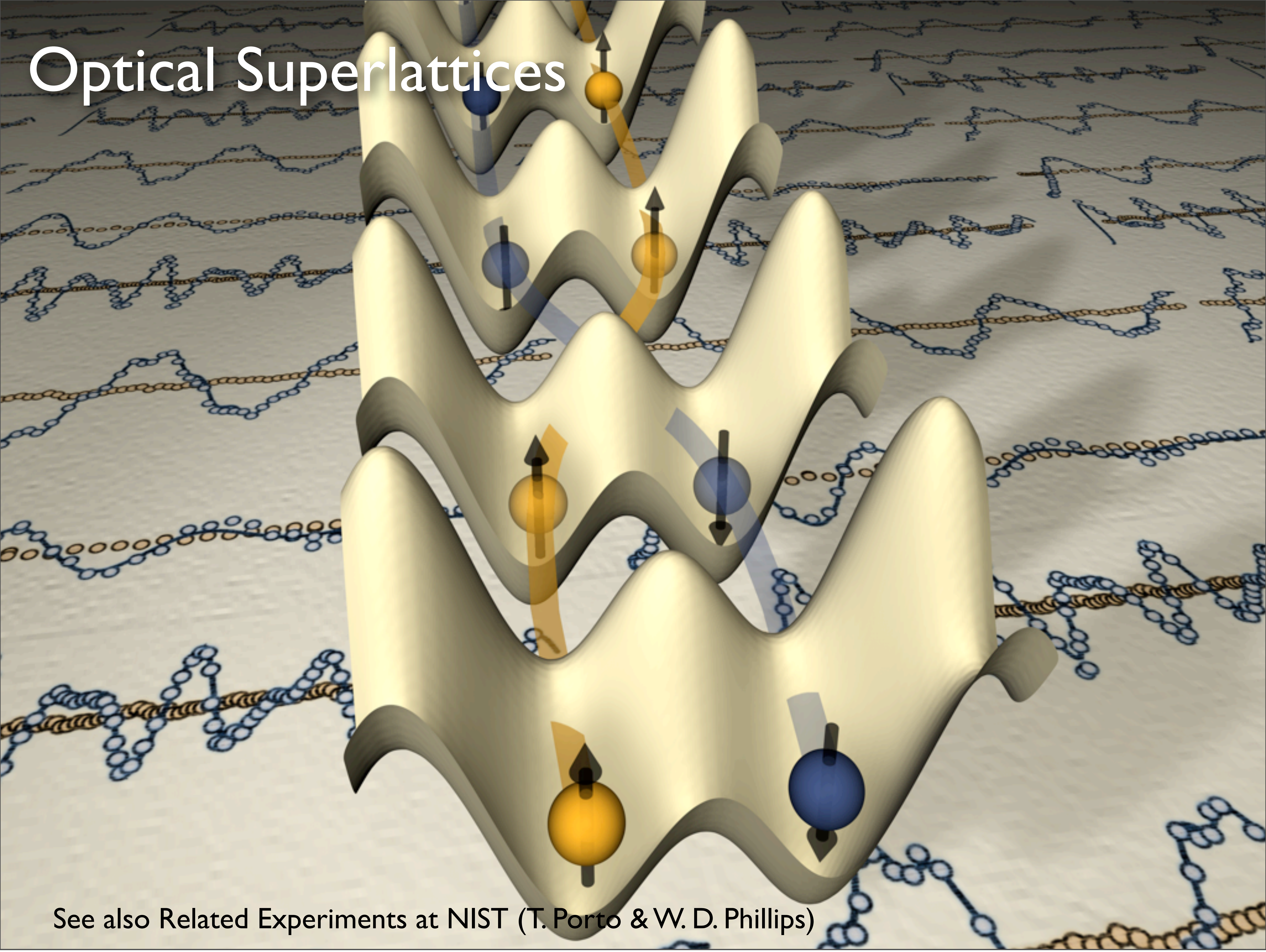


Dispersion Relation in a Square Lattice

$$E(q) = -2J\cos(qa)$$



Optical Superlattices



See also Related Experiments at NIST (T. Porto & W. D. Phillips)

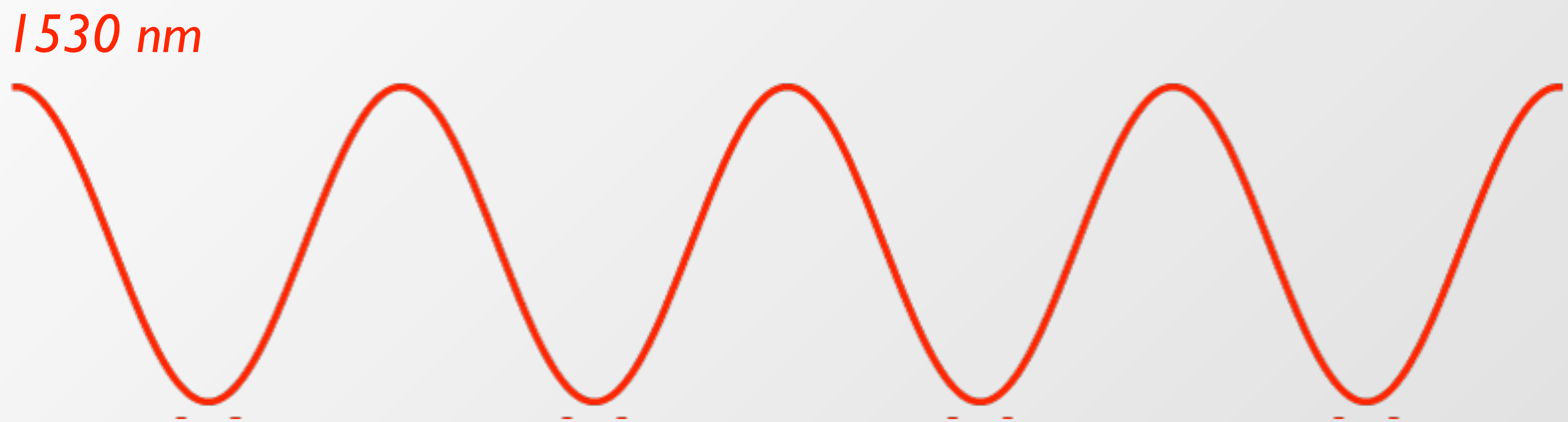
Optical Superlattices

Superimpose two standing waves **with controllable phase & amplitude.**

Note: two non-equivalent sites in unit cell!

Optical Superlattices

Superimpose two standing waves **with controllable phase & amplitude.**

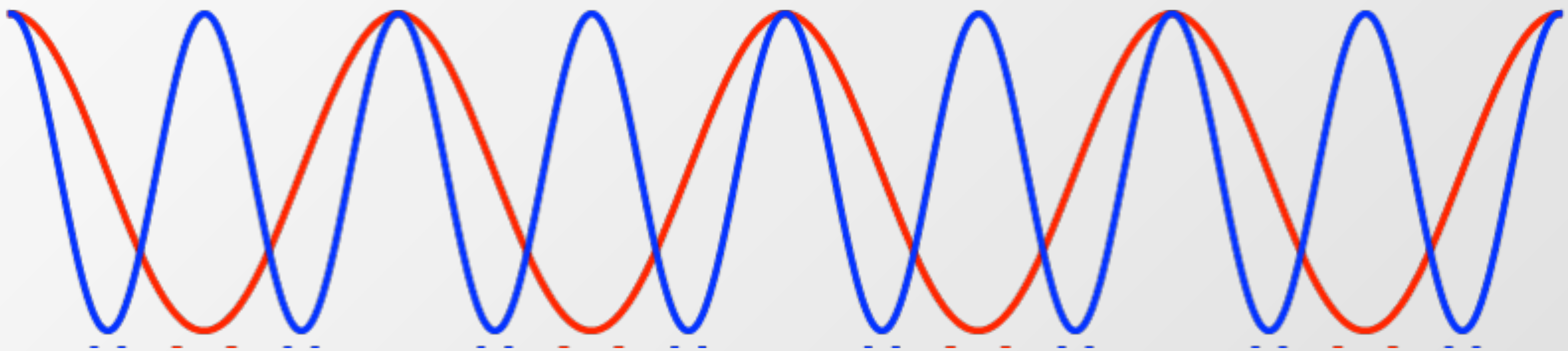


Note: two non-equivalent sites in unit cell!

Optical Superlattices

Superimpose two standing waves **with controllable phase & amplitude.**

1530 nm + *765 nm*

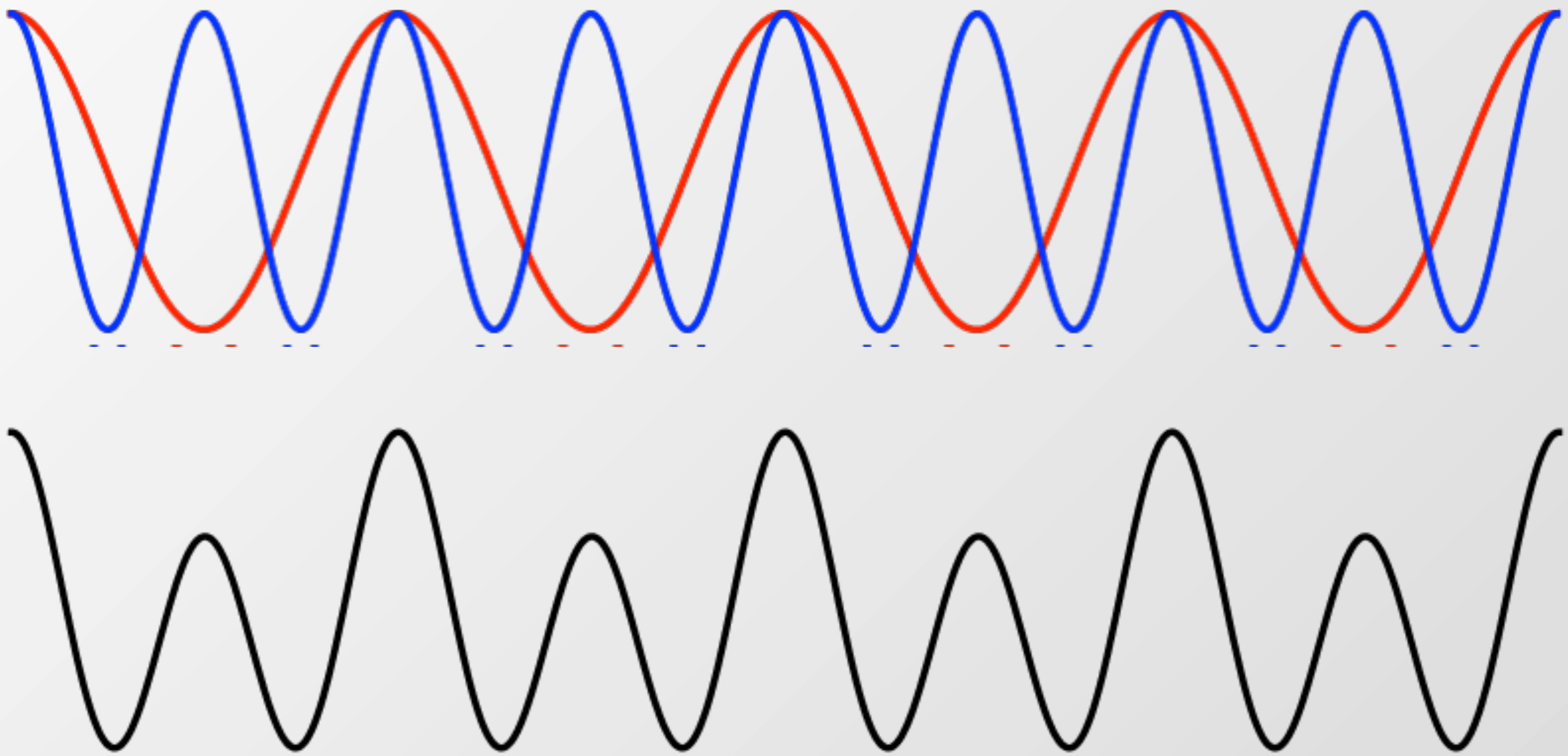


Note: two non-equivalent sites in unit cell!

Optical Superlattices

Superimpose two standing waves **with controllable phase & amplitude.**

1530 nm + 765 nm



Array of double wells

Note: two non-equivalent sites in unit cell!

Controllable Parameters

- **Original**

All parameters can be changed dynamically & in-situ!

Controllable Parameters

- **Original**

- **Intra & Interwell Barrier Depth**

All parameters can be changed dynamically & in-situ!

Controllable Parameters

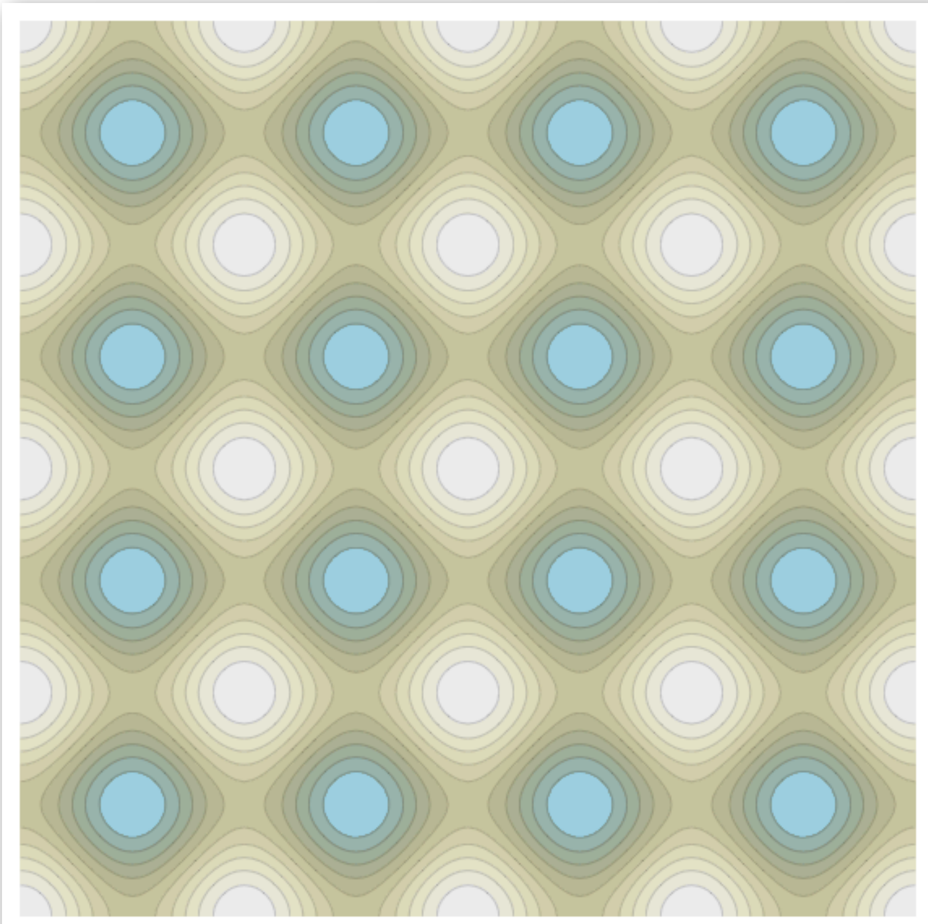
- **Original**

- **Intra & Interwell Barrier Depth**

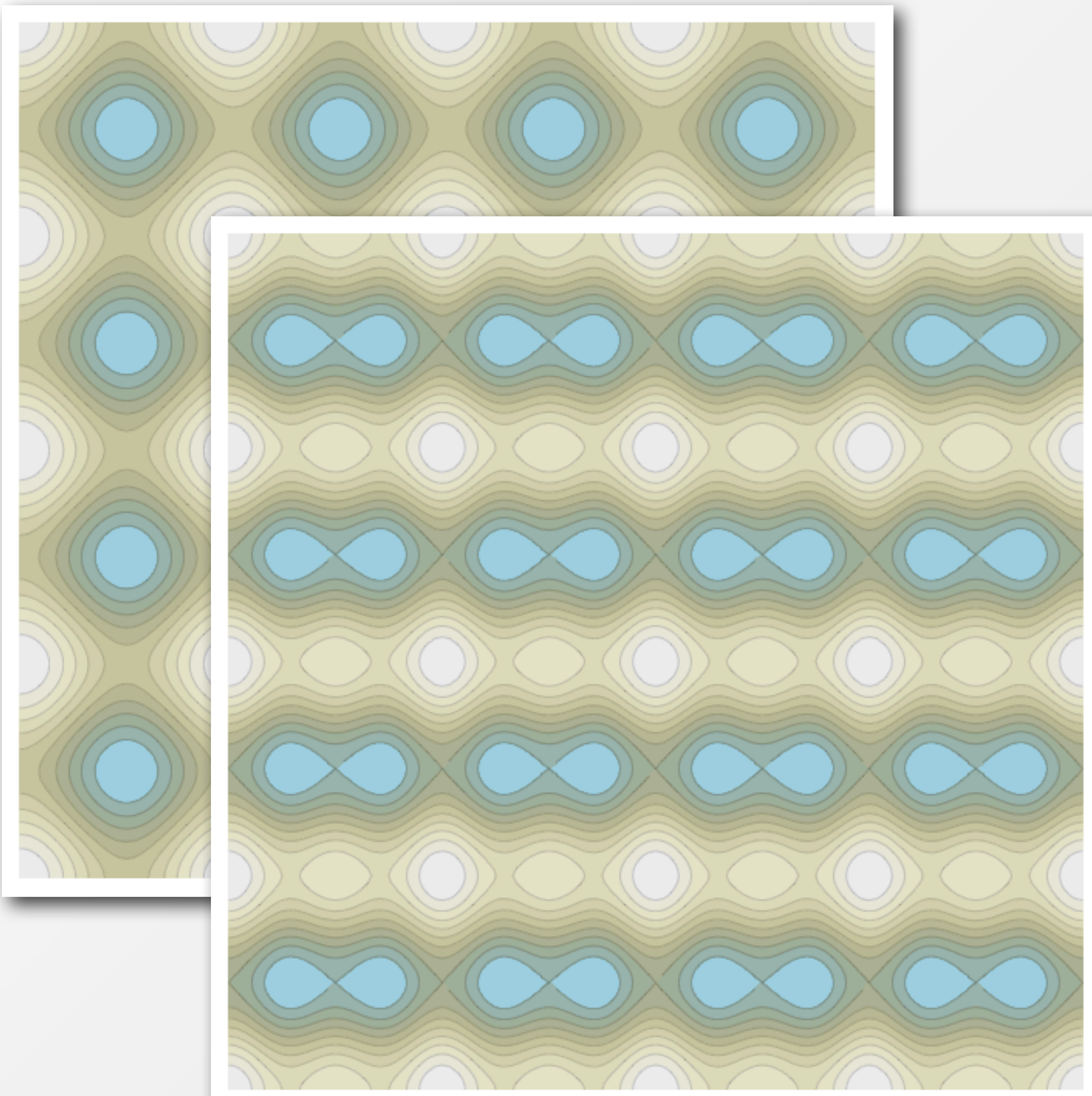
- **Potential Bias**

All parameters can be changed dynamically & in-situ!

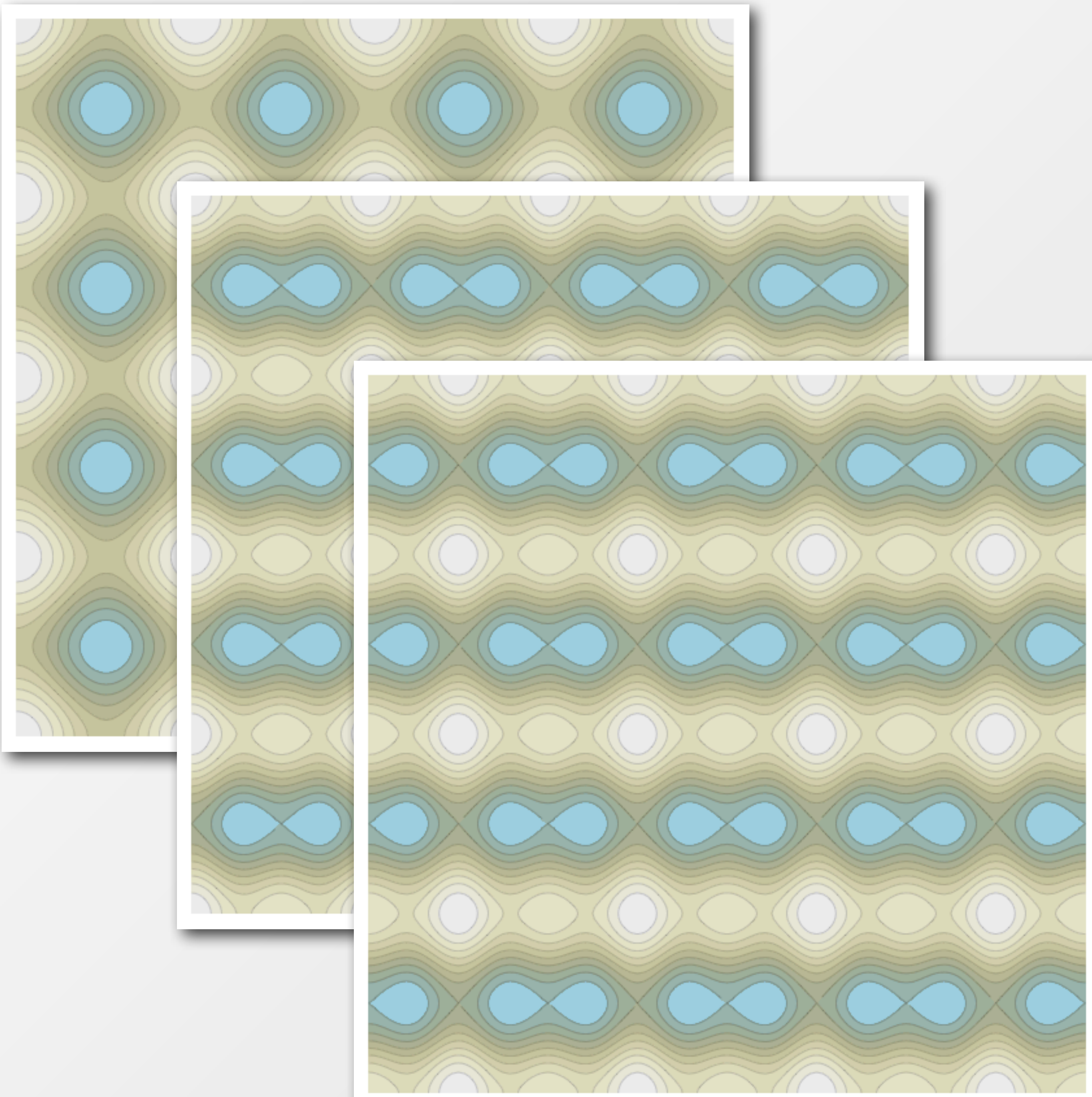
2D Superlattice Geometries (1 SL)



2D Superlattice Geometries (1 SL)



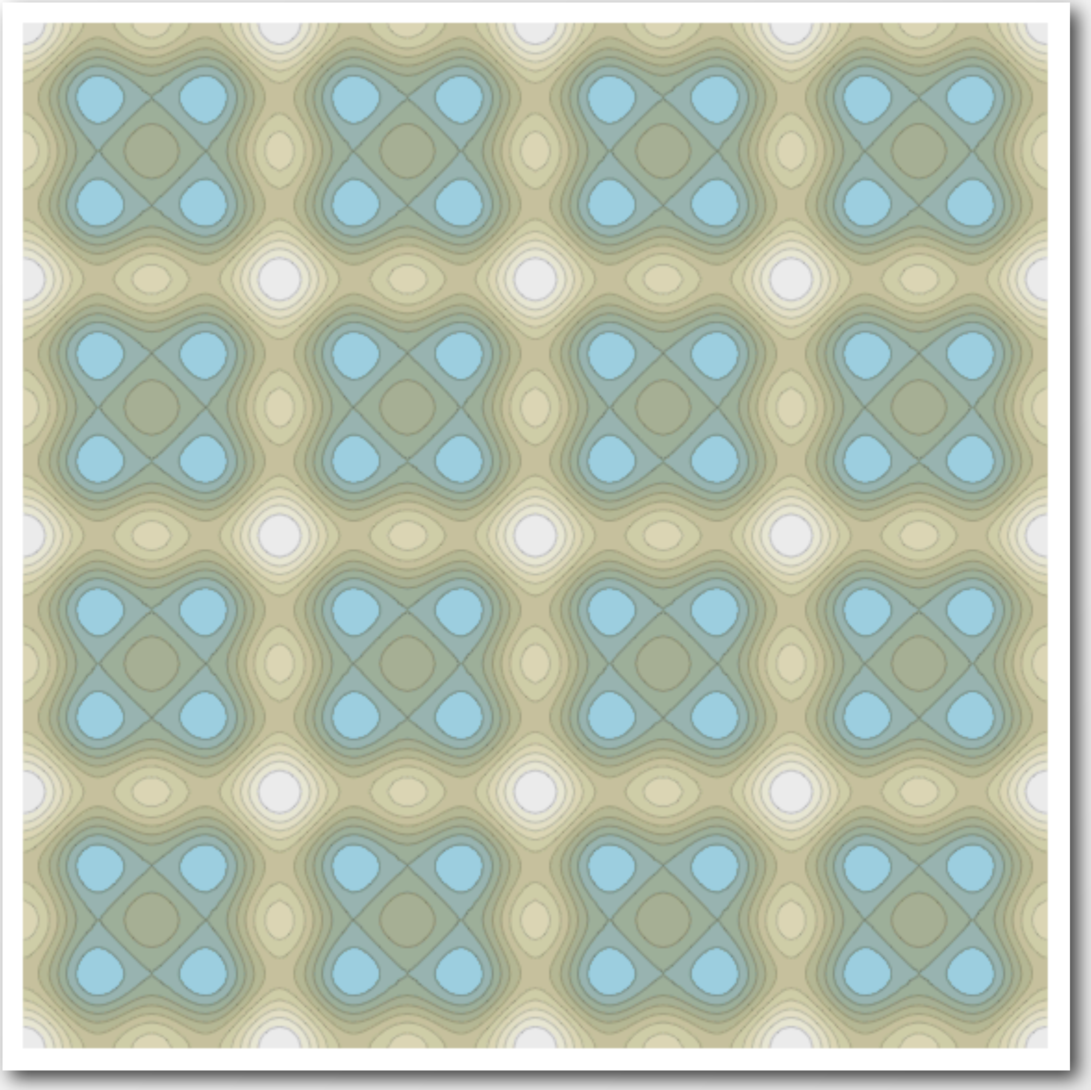
2D Superlattice Geometries (1 SL)



Controllable couplings and dynamics

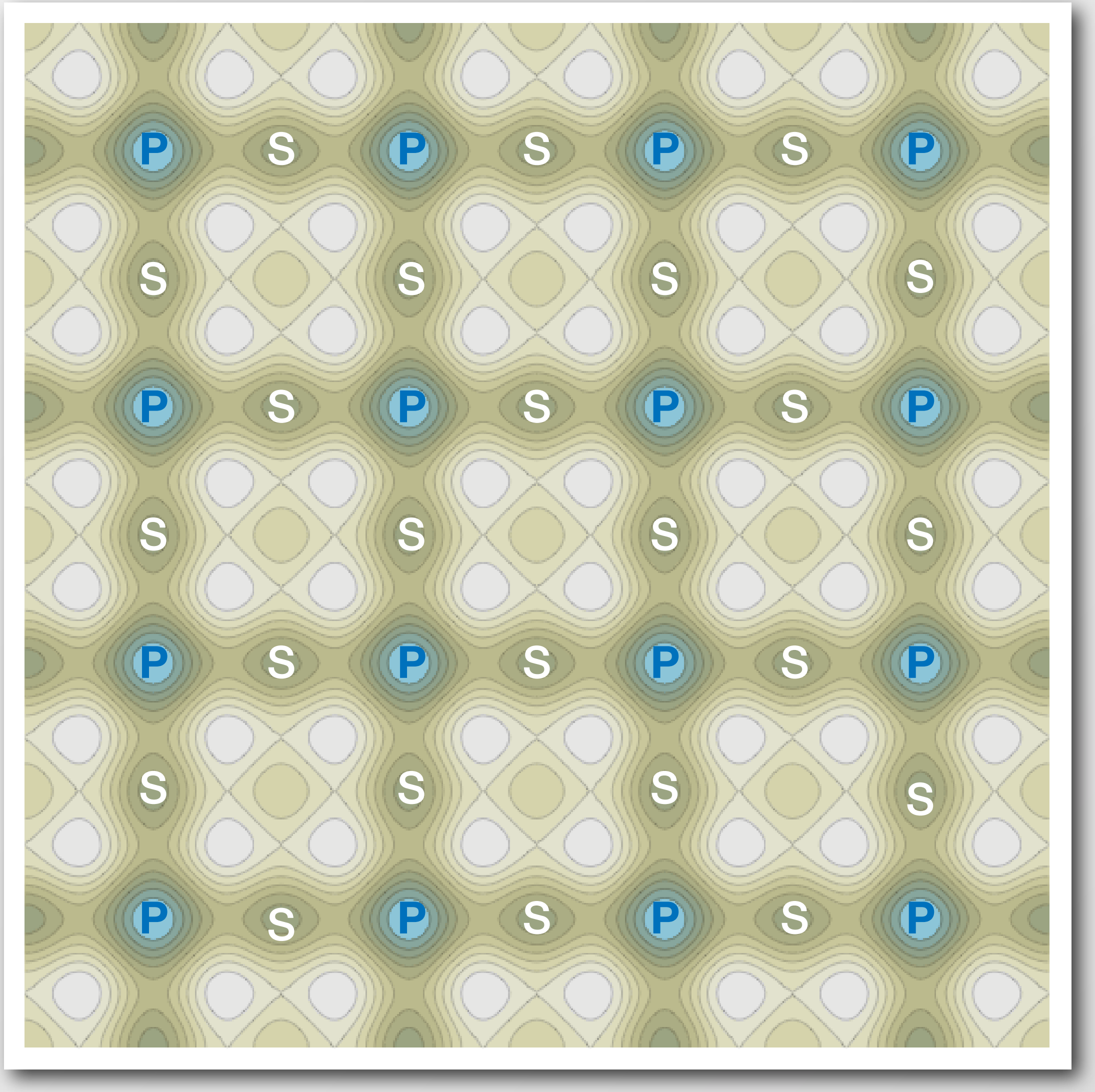
Superlattice useful for controlling and detecting spin correlations

2D Superlattice Geometries (2 SL)



Coupled Plaquette Systems

see B. Paredes & I. Bloch, PRA **77**, 23603 (2008)
S. Trebst et al., PRL **96**, 250402 (2006)



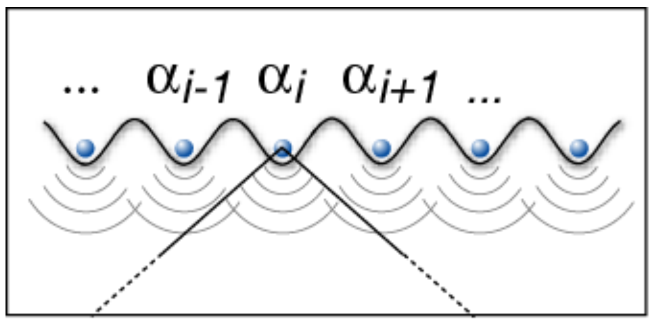
Higher Lattice Orbital Physics

see V. Liu, A. Ho, C. Wu and others work
exp: related to A. Hemmerich's exp.

Measuring
Momentum Distributions

Time of flight interference pattern

- Interference between all waves coherently emitted from each lattice site

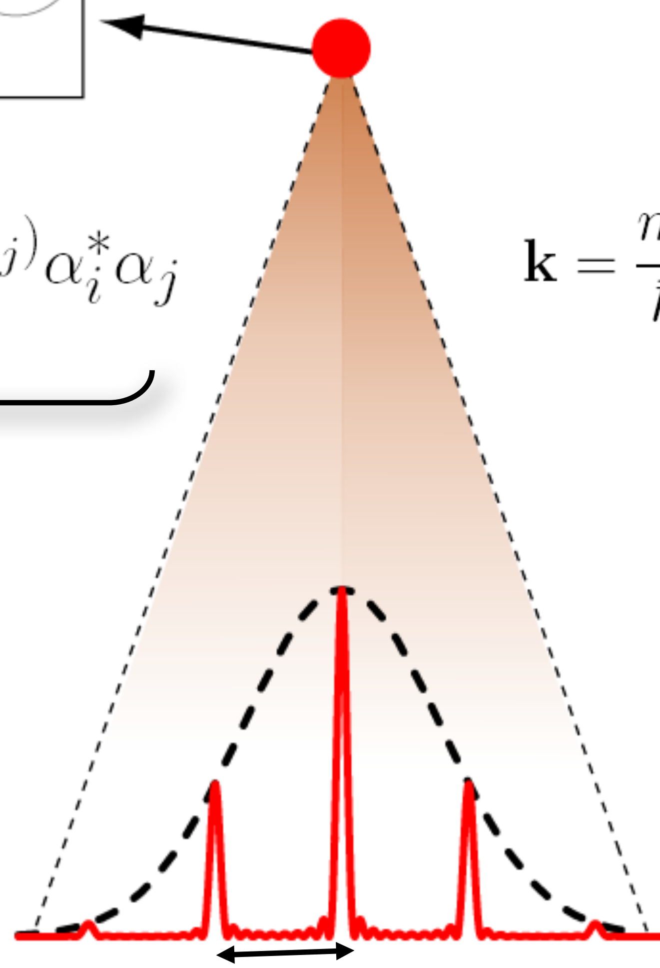


$$\tilde{n}(\mathbf{k}) = |\tilde{w}(\mathbf{k})|^2 \sum_{i,j} e^{i\mathbf{k}.(\mathbf{r}_i - \mathbf{r}_j)} \alpha_i^* \alpha_j$$

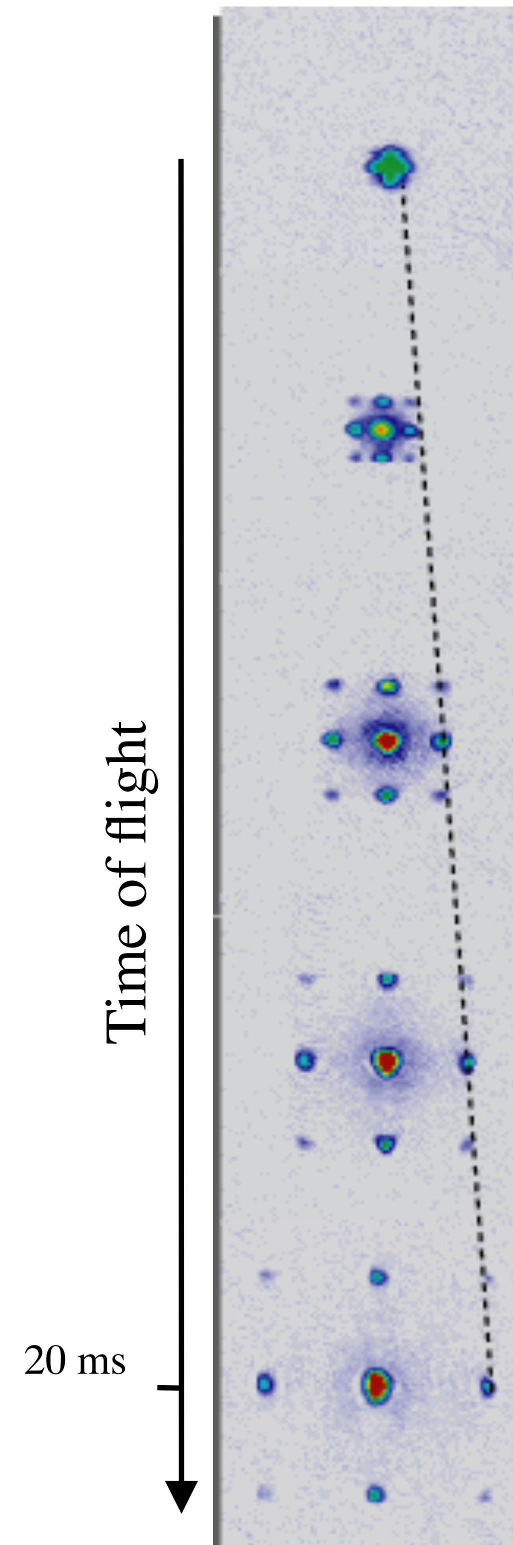
$$\mathbf{k} = \frac{m\mathbf{r}}{\hbar t}$$

Wannier envelope

Grating-like interference



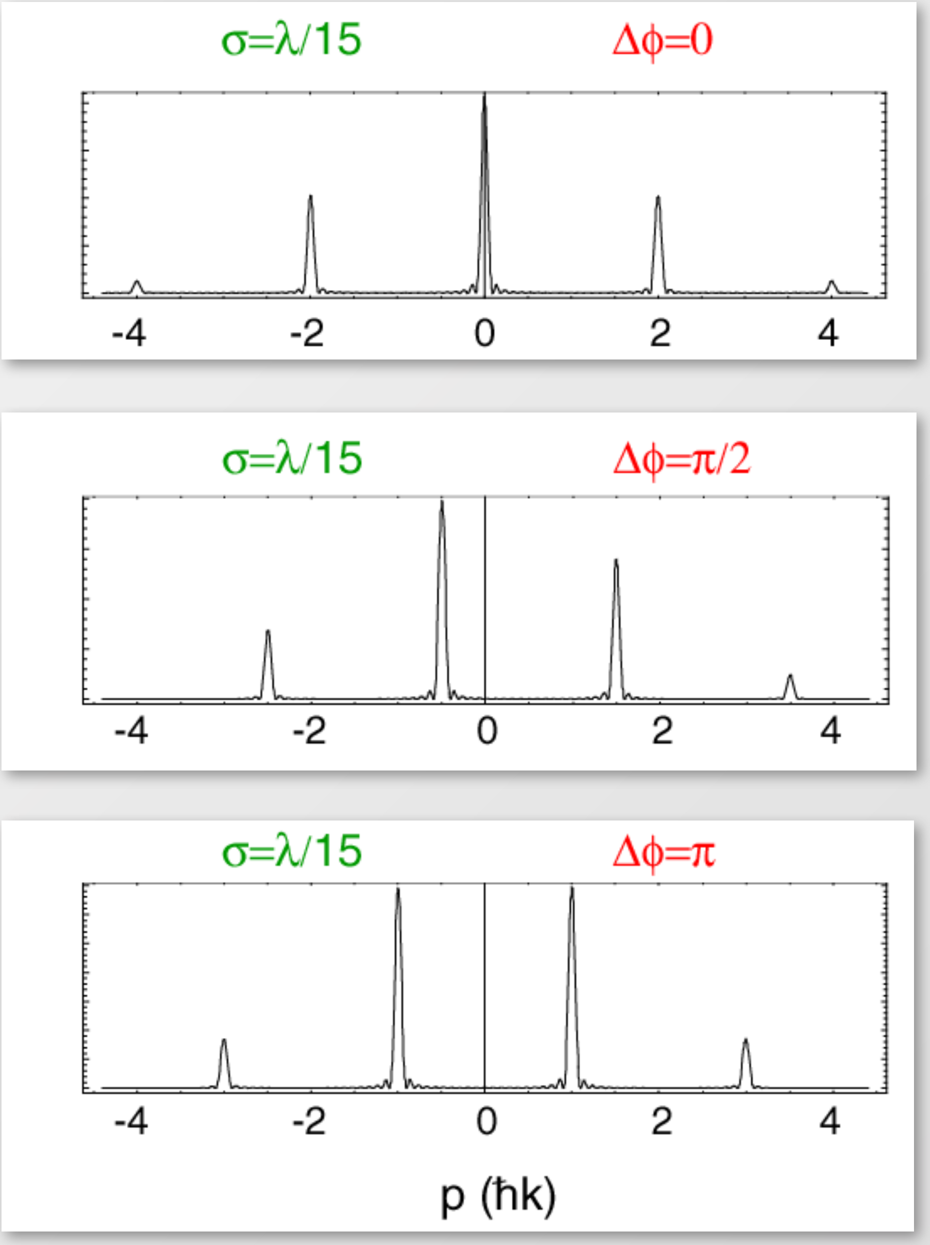
Periodicity of the reciprocal lattice $l = \frac{2\hbar k_L t}{m}$



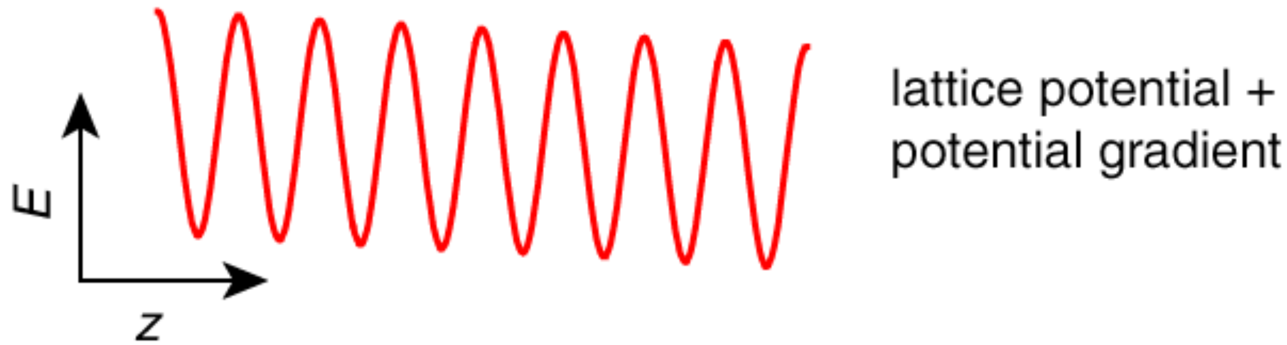
Momentum Distributions – 1D

Momentum distribution can be obtained by Fourier transformation of the macroscopic wave function.

$$\Psi(x) = \sum_i A(x_j) \cdot w(x - x_j) \cdot e^{i\phi(x_j)}$$



Preparing Arbitrary Phase Differences Between Neighbouring Lattice Sites



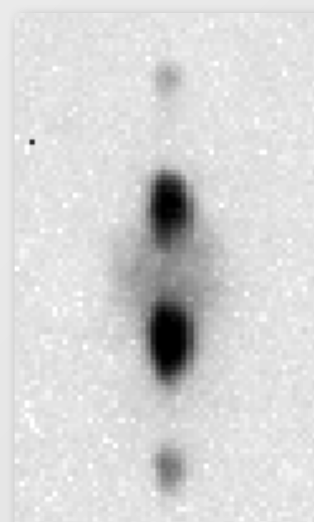
$$\phi_j = E_j \cdot t / \hbar$$

Phase difference between neighboring lattice sites

$$\Delta\phi_j = (V'\lambda/2)\,\Delta t$$

(cp. Bloch-Oscillations)

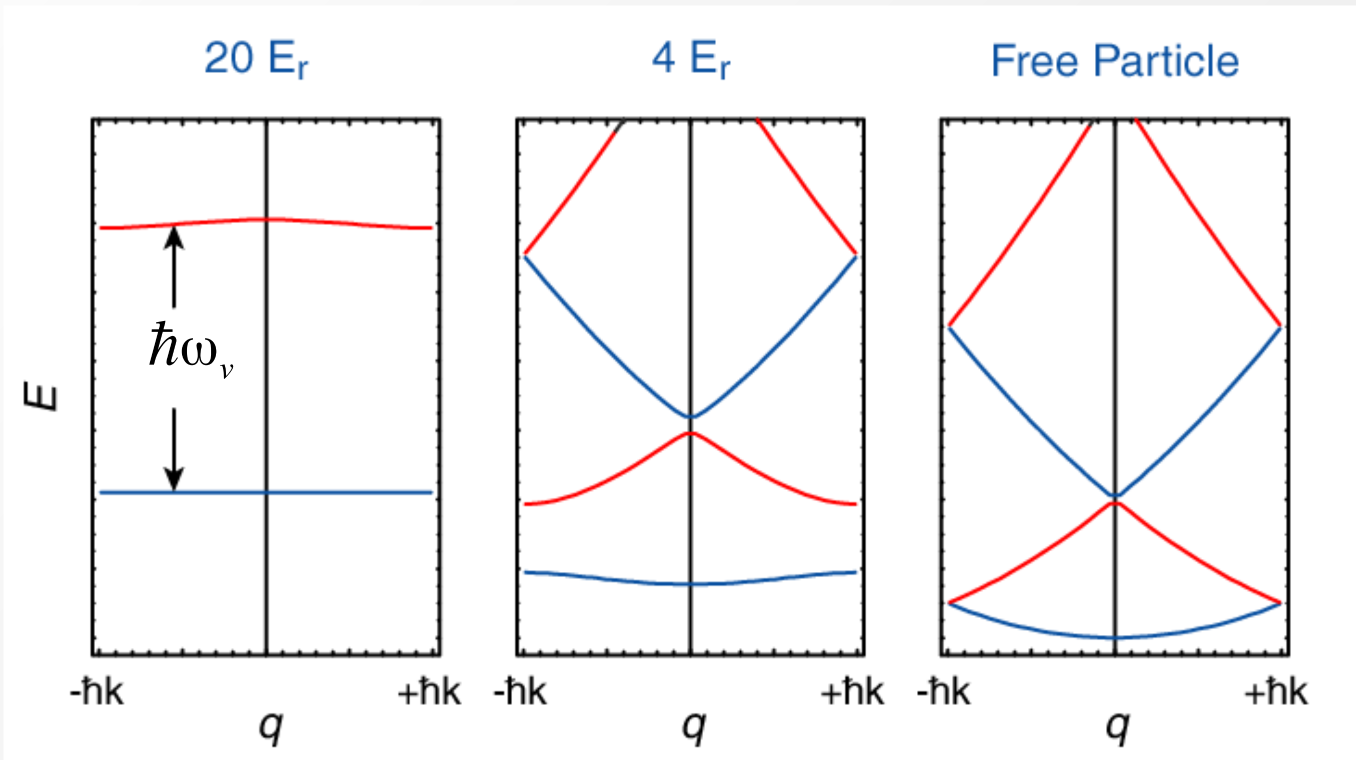
$\Delta\phi = 0$



$\Delta\phi = \pi$

But: dephasing if gradient is left on for long times !

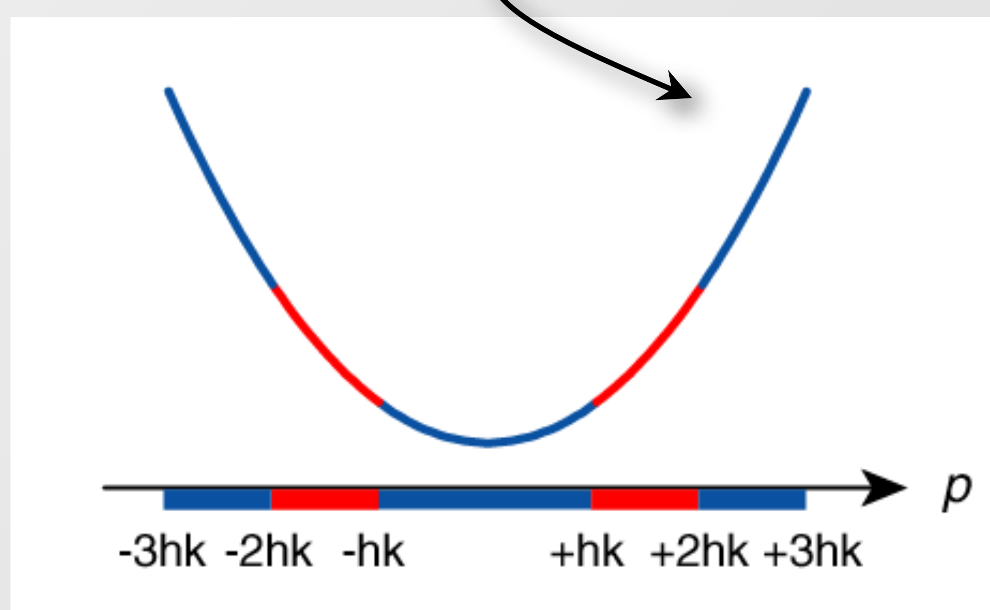
Mapping the Population of the Energy Bands onto the Brillouin Zones



Crystal momentum

Crystal momentum is conserved while lowering the lattice depth adiabatically !

Population of nth band is mapped onto nth Brillouin zone !

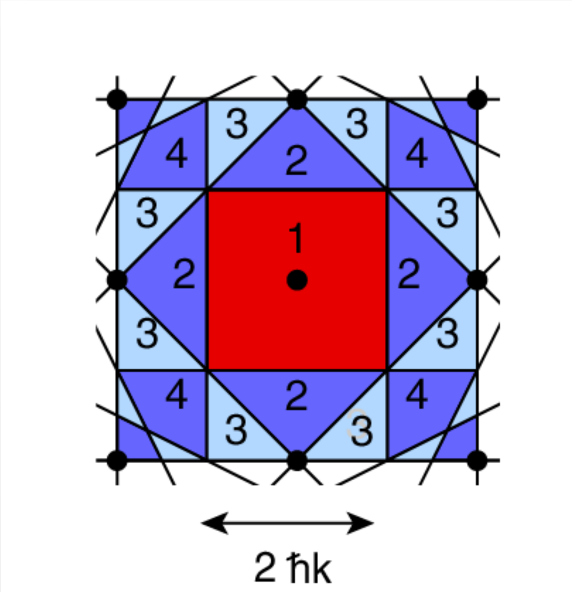


Free particle momentum

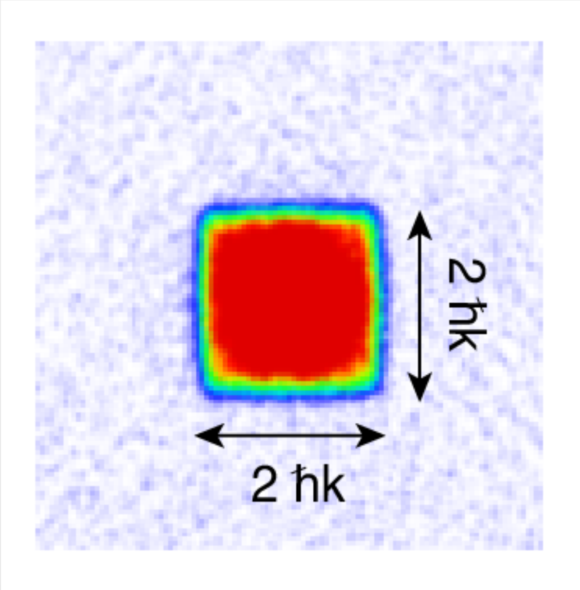
A. Kastberg et al. PRL 74, 1542 (1995)
M. Greiner et al. PRL 87, 160405 (2001)

Experimental Results

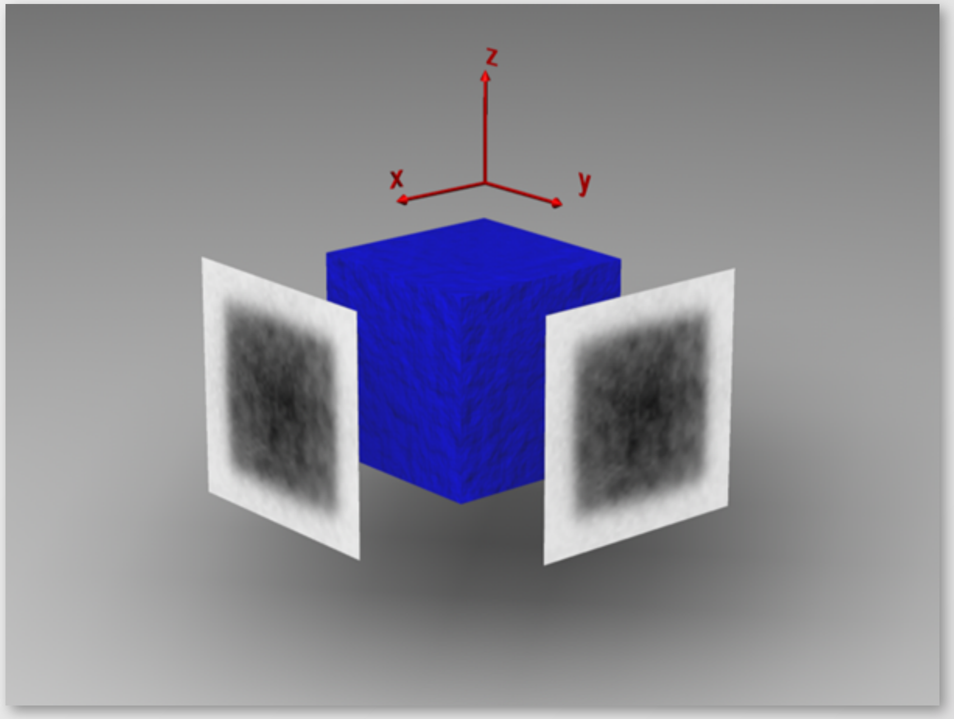
Brillouin Zones in 2D



Momentum distribution of a dephased condensate after turning off the lattice potential adiabtically



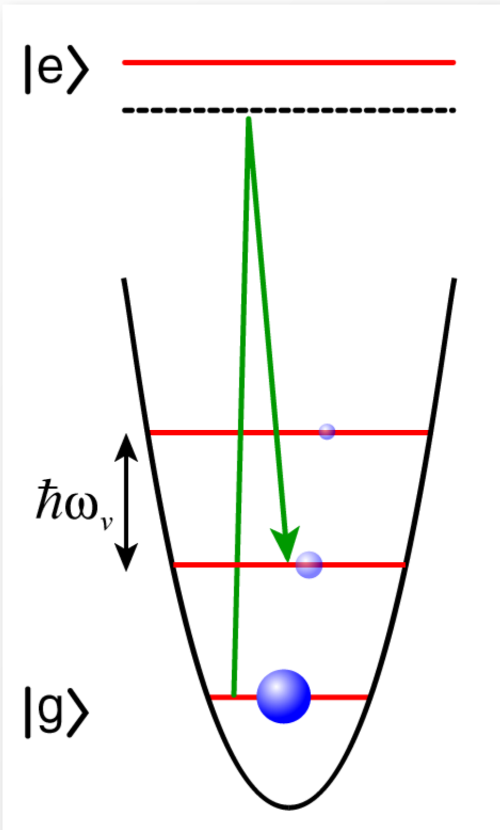
2D



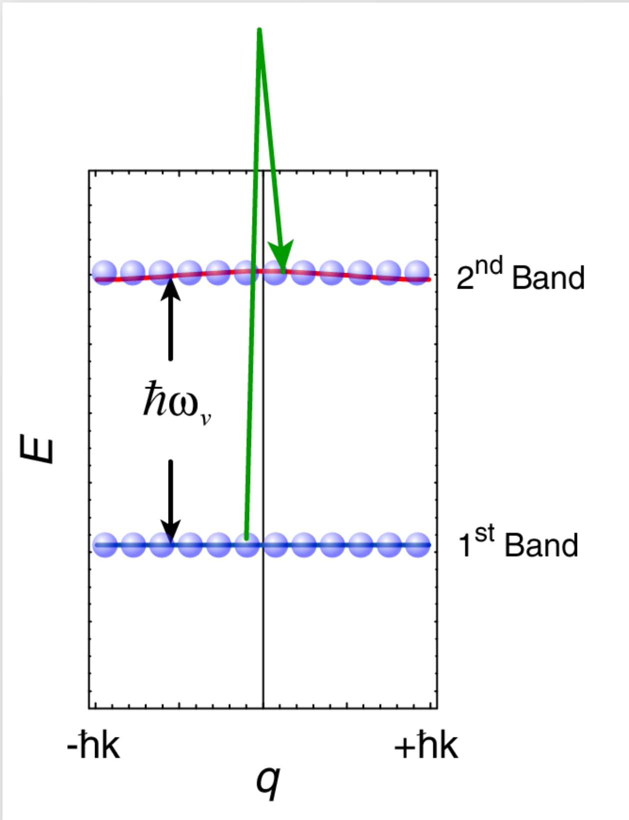
3D

Populating Higher Energy Bands

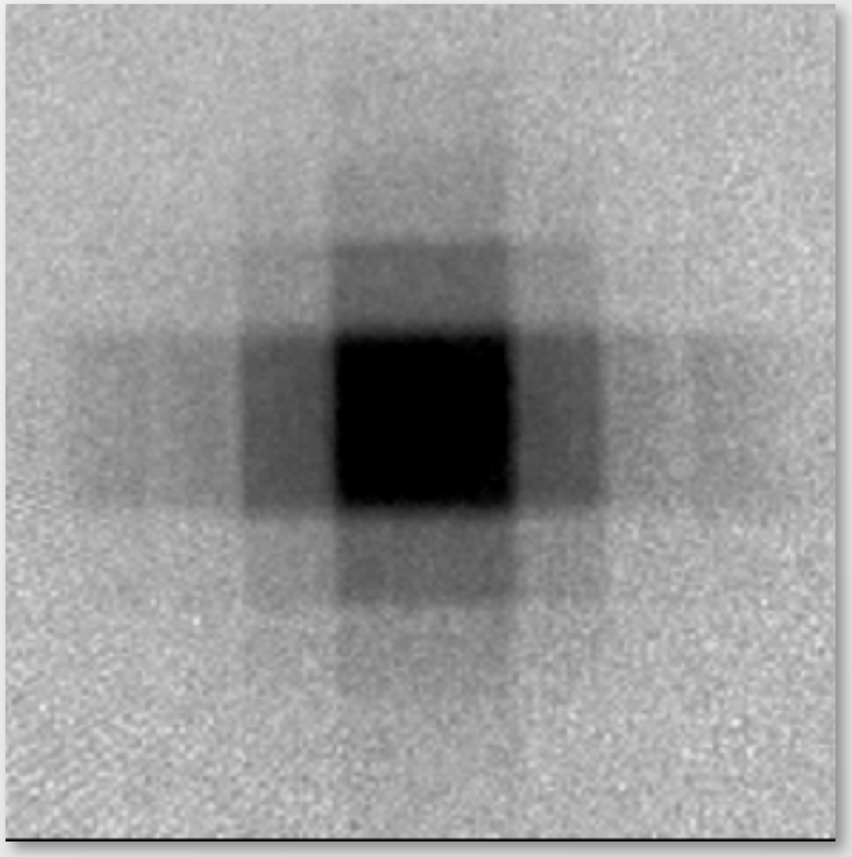
Single lattice site



Energy bands

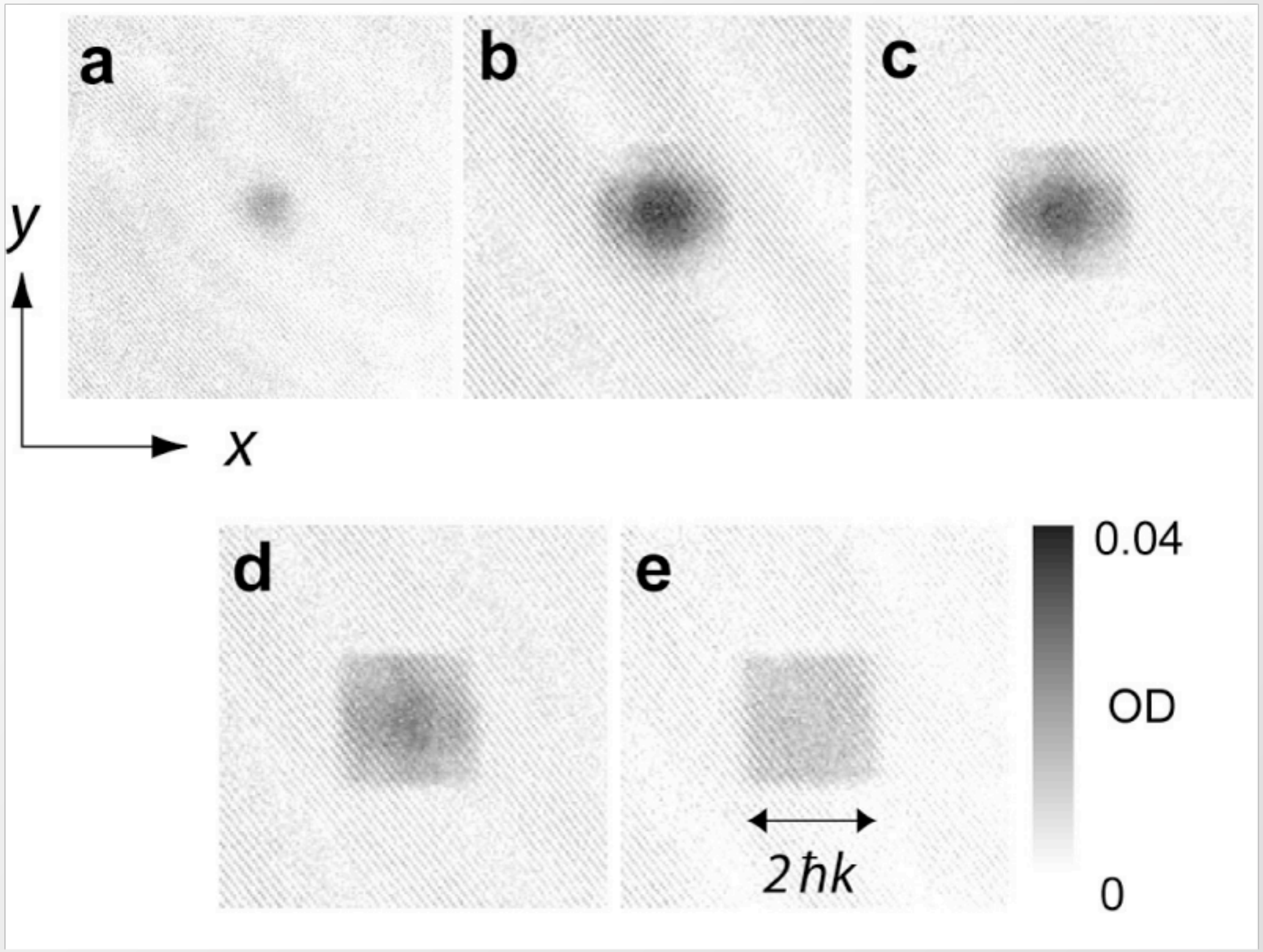


Stimulated Raman transitions between vibrational levels are used to populate higher energy bands.



Measured Momentum Distribution !

From a Conductor to a Band Insulator



Fermi Surfaces become directly visible!

M. Köhl et al. Physical Review Letters (2005)

Bose-Hubbard Hamiltonian

Expanding the field operator in the Wannier basis of localized wave functions on each lattice site, yields :

$$\hat{\psi}(\boldsymbol{x}) = \sum_i \hat{a}_i w(\boldsymbol{x} - \boldsymbol{x}_i)$$

Bose-Hubbard Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Tunnelmatrix element/Hopping element

$$J = -\int d^3x \, w(\boldsymbol{x} - \boldsymbol{x}_i) \left(-\frac{\hbar^2}{2m} \nabla^2 + V_{lat}(\boldsymbol{x}) \right) w(\boldsymbol{x} - \boldsymbol{x}_j)$$

Onsite interaction matrix element

$$U = \frac{4\pi \hbar^2 a}{m} \int d^3x \, |w(\boldsymbol{x})|^4$$

M.P.A. Fisher et al., PRB 40, 546 (1989); D. Jaksch et al., PRL 81, 3108 (1998)

Mott Insulators now at: Munich, Mainz, NIST, ETHZ, Texas, Innsbruck, MIT, Chicago, Florence,...
see also work on JJ arrays H. Mooij et al., E. Cornell,...

From Weak to Strong Interactions

$$\gamma = \frac{\text{Interaction Energy}}{\text{Kinetic Energy}} >> 1$$

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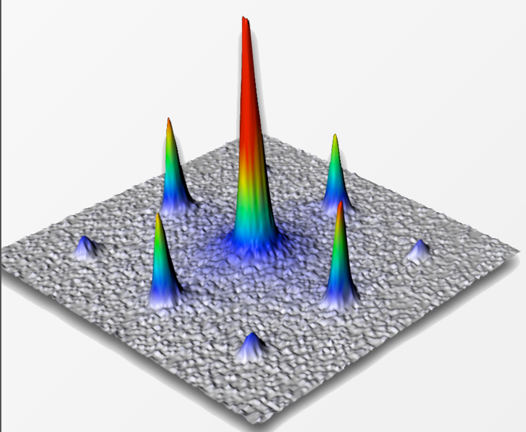
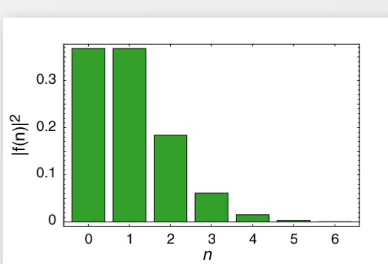
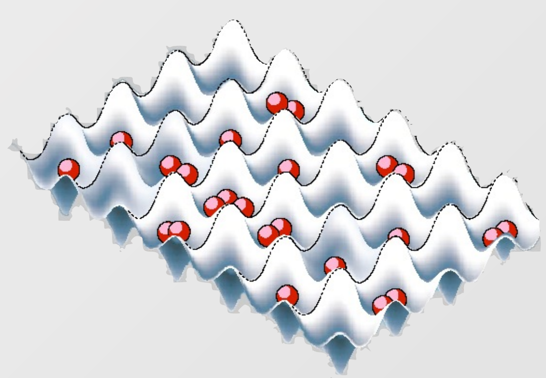


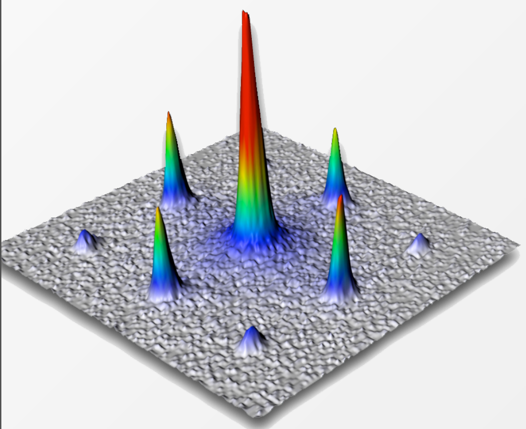
Weak Interactions

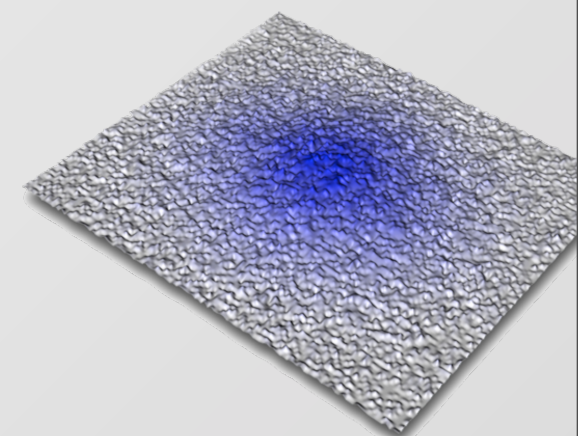




From Weak to Strong Interactions

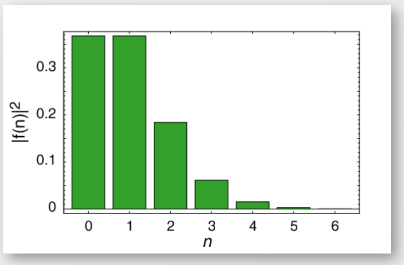
$$\gamma = \frac{\text{Interaction Energy}}{\text{Kinetic Energy}} >> 1$$

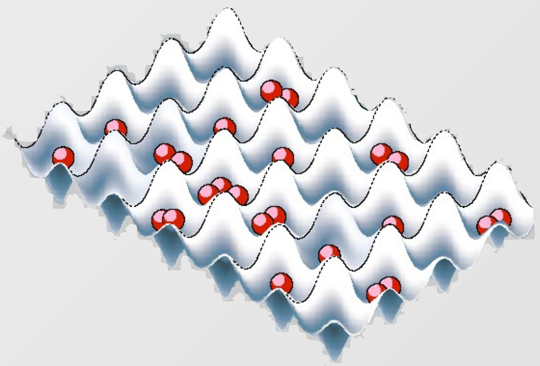


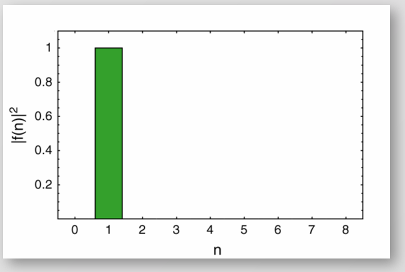


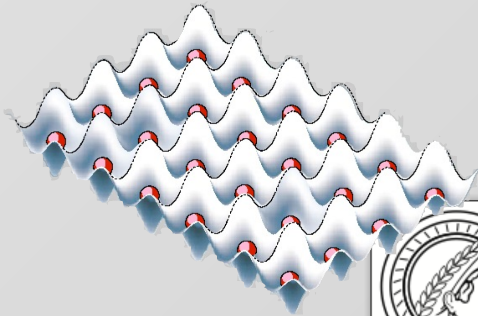
Weak Interactions

Strong Interactions



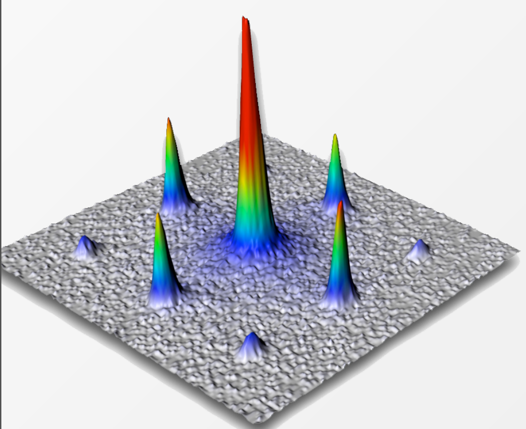


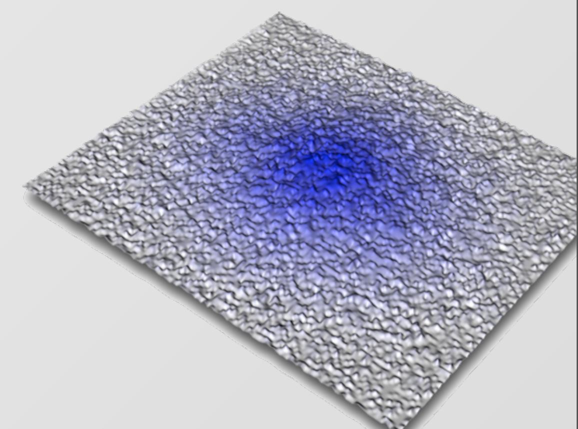




From Weak to Strong Interactions

$$\gamma = \frac{\text{Interaction Energy}}{\text{Kinetic Energy}} >> 1$$



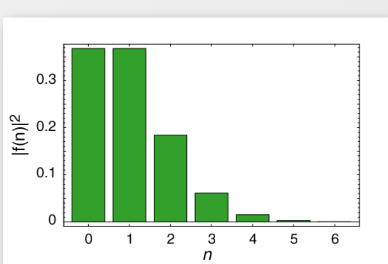


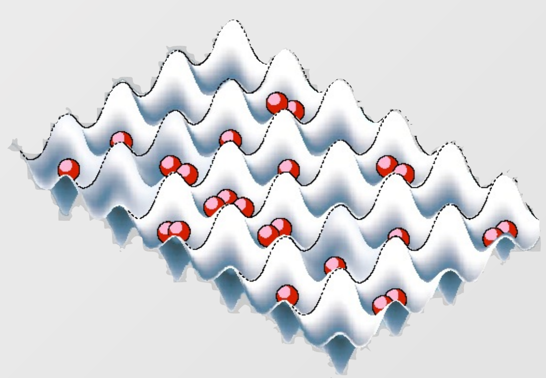
Weak Interactions

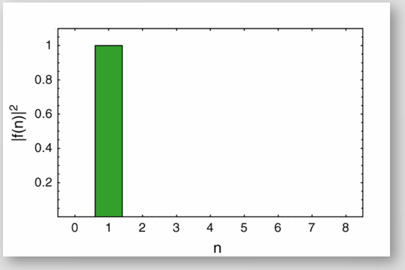
Quantum Phase Transition

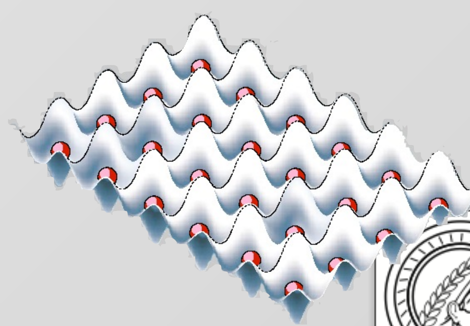
See S. Sachdev & B. Keimer Phys. Today 2011

Strong Interactions





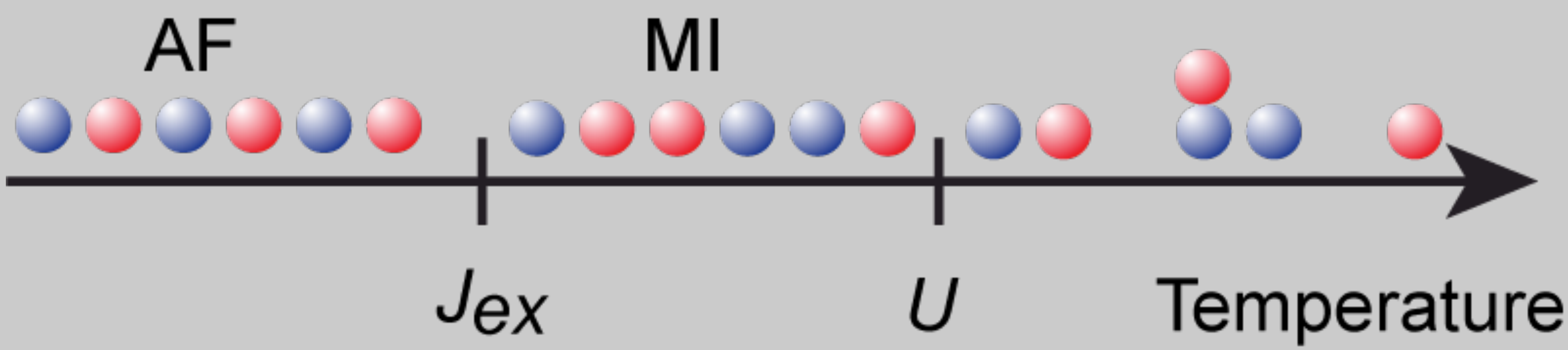




Strongly Interacting Fermions in Optical Lattices

$$\hat{H} = -J \sum_{\langle i,j \rangle,\sigma} \hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\downarrow} \hat{n}_{i,\uparrow} + V_t \sum_{i,\sigma} i^2 \hat{n}_{i,\sigma}$$

Predicted phases at half filling for strong interactions $U/12J > 1$



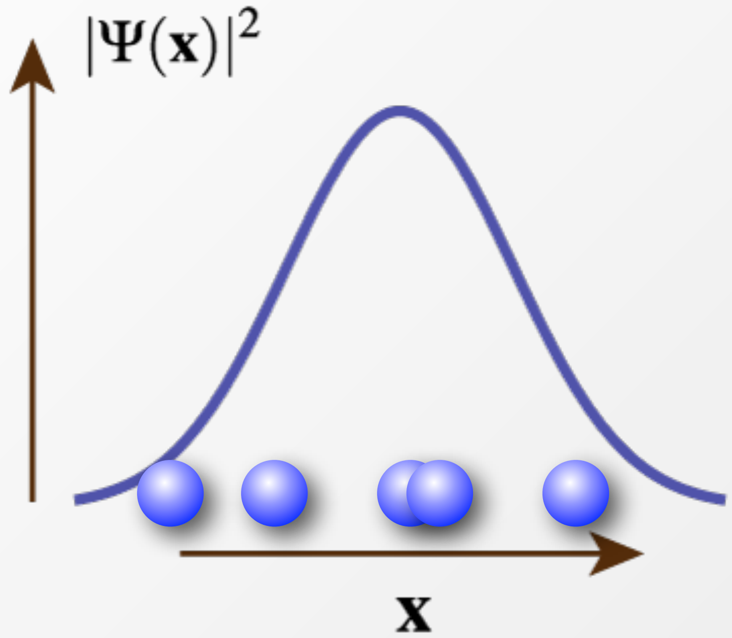
max. Entropy
$S/N = k_B\, 2 \ln 2$

R. Jördens et al., Nature **455**, 204 (2008), U. Schneider et al., Science **322,** 1520 (2008), D. Greif et al., Science **340**, 1307 (2013)

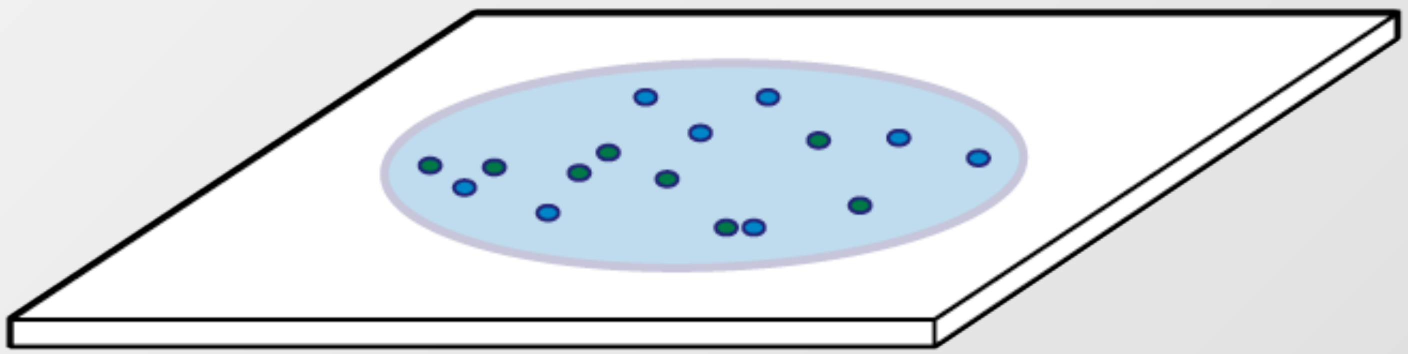
Single Atom Detection in a Lattice

Sherson et al. Nature 467, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

Measuring a Quantum System



Single Particle



Correlated 2D Quantum Liquid

$\Psi(\mathbf{x})$ wave function

$|\Psi(\mathbf{x})|^2$ probability distribution

$\Psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_N)$

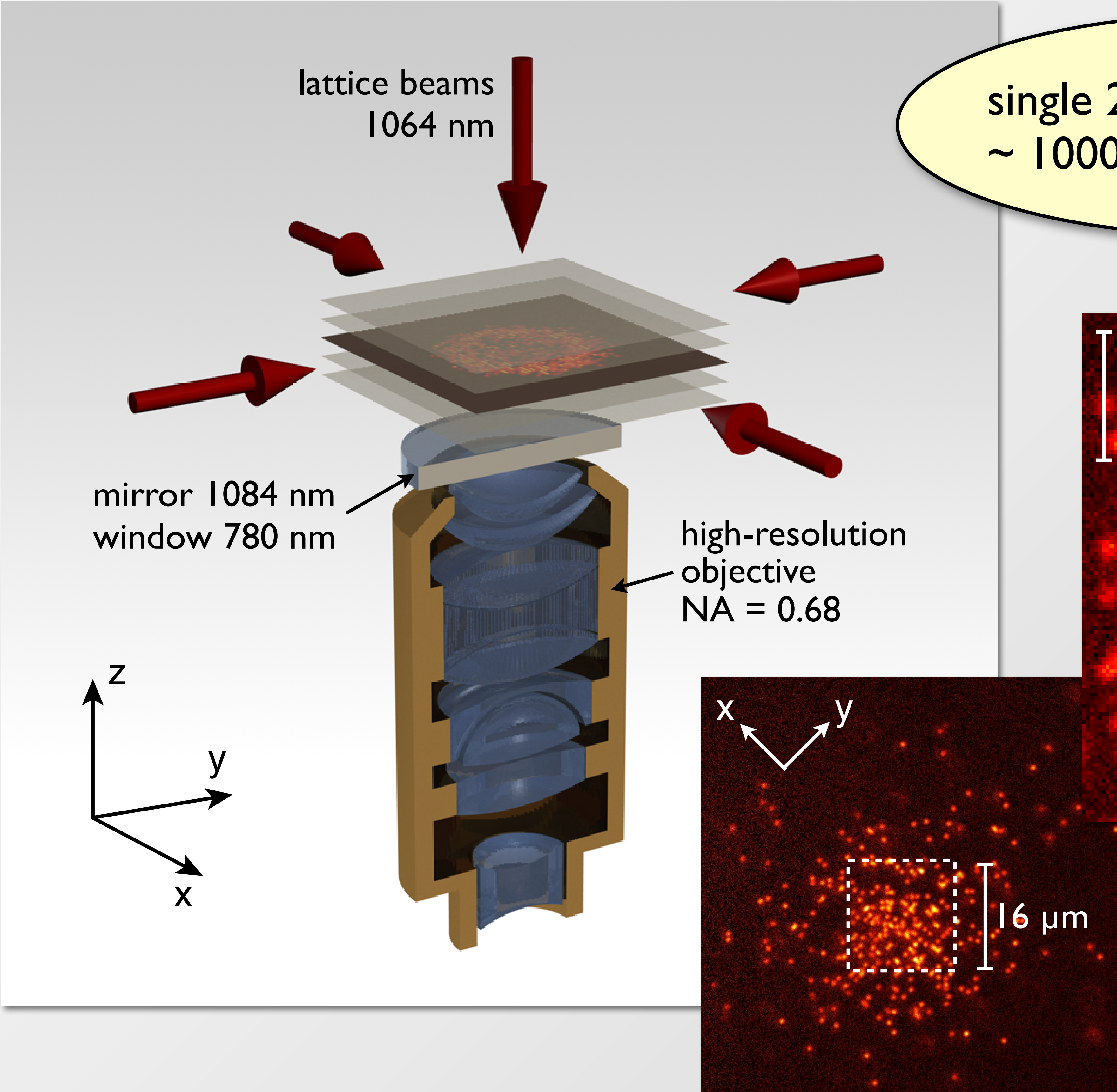
$|\Psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_N)|^2$

averaging over single-particle measurements, we obtain $|\Psi(\mathbf{x})|^2$

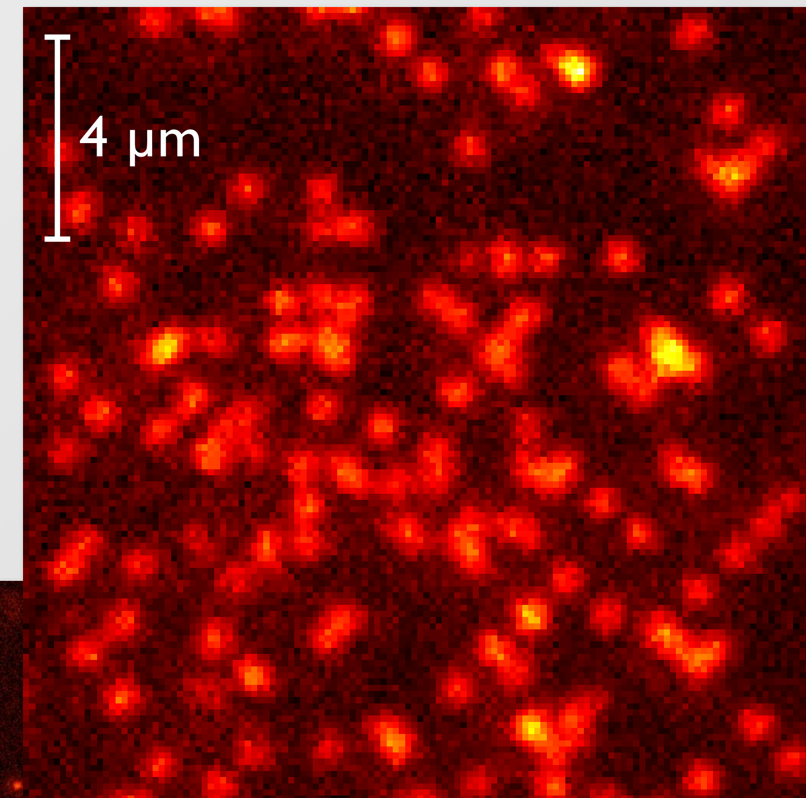
For many-body system: need access to single snapshots of the many-particle system!

Enables Measurement of Non-local Correlations

Experimental Setup

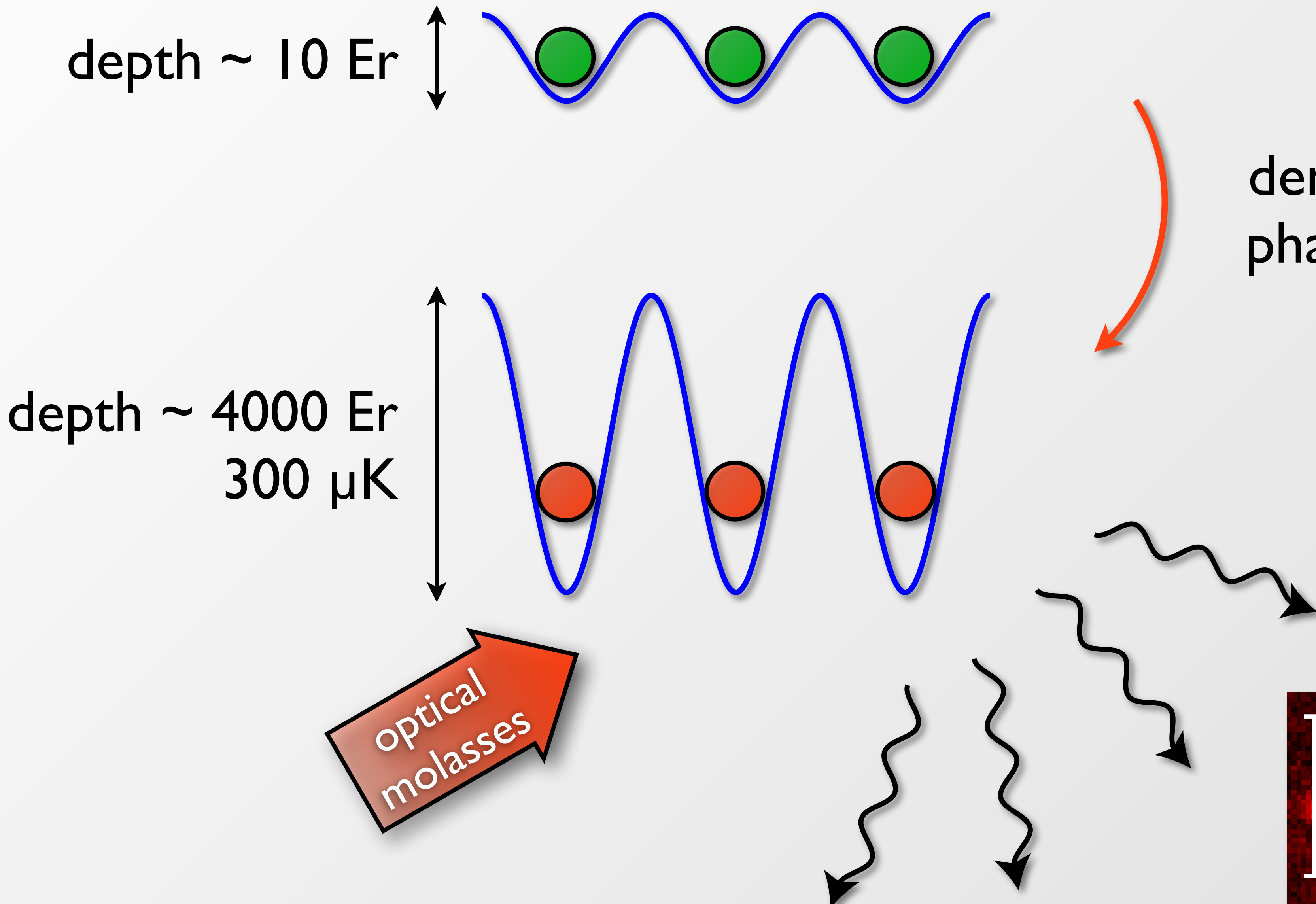


single 2D degenerate gas
~ 1000 ^{87}Rb atoms (bosons)

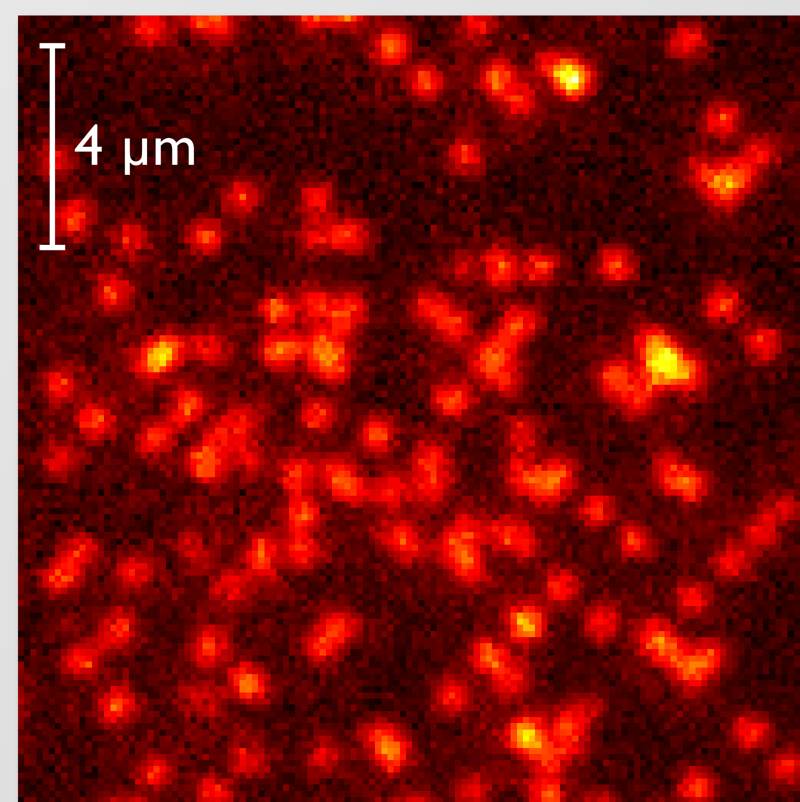


resolution of the imaging system: ~700 nm

Fluorescence imaging

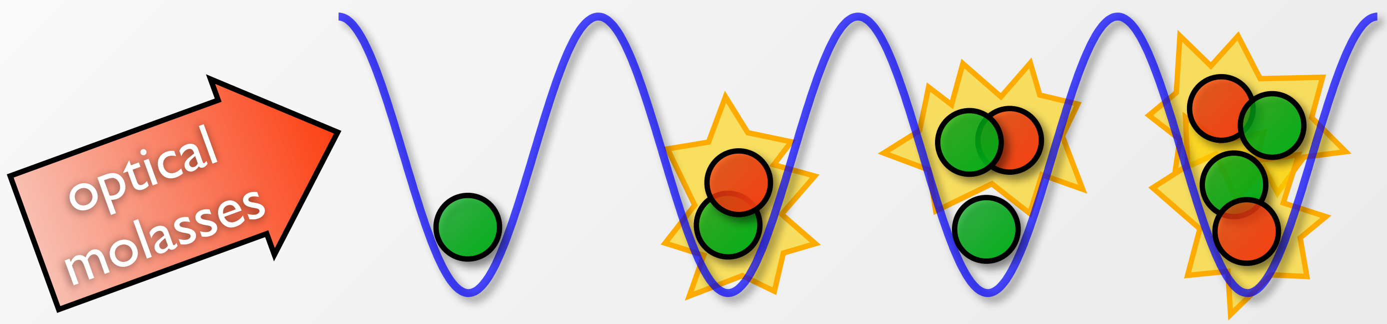


density distribution frozen
phase coherence lost



fluorescence rate / atom: 60 kHz
~ 5000 photons / atom collected in 900ms

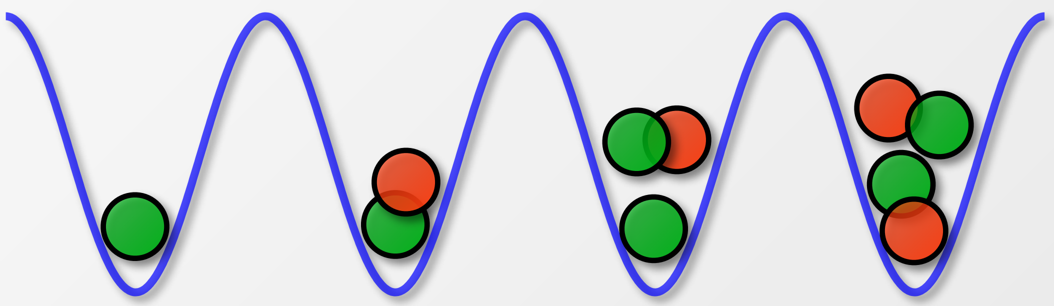
Parity projection



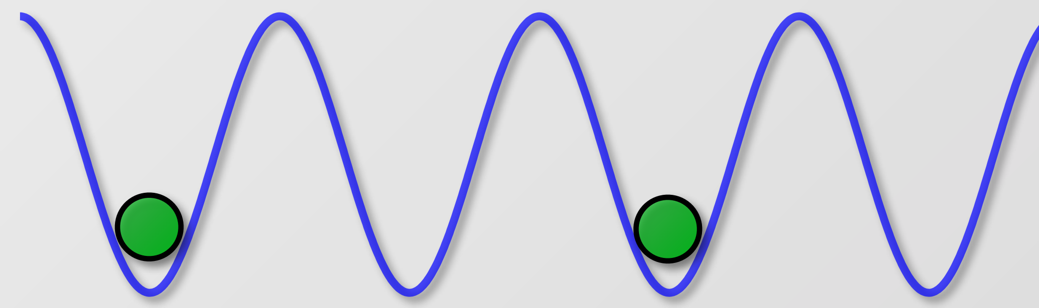
dePue *et al.*, PRL **82**, 2262 (1999)

Light-induced collisions

initial density distribution



measured density distribution



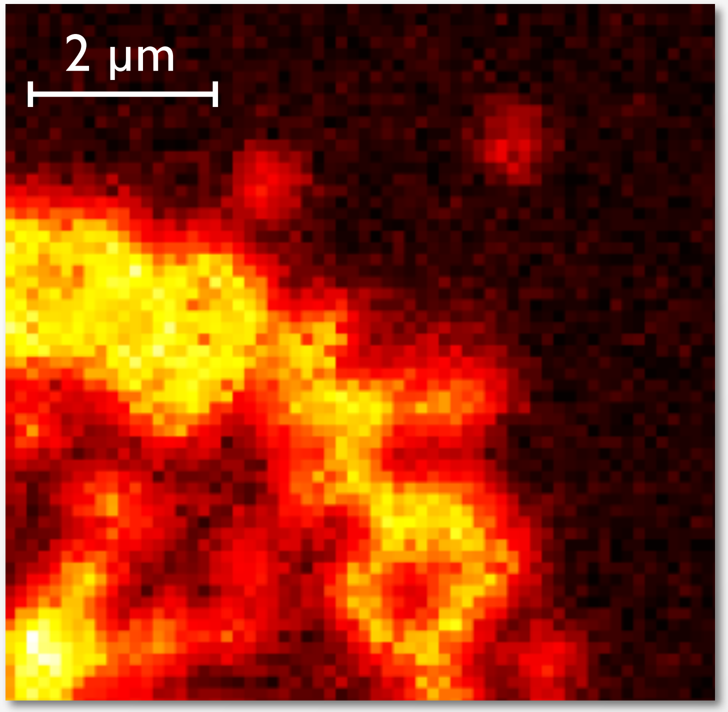
measured occupation: $n_{\mathrm{det}} = \mathrm{mod}_2 n$

measured variance: $\sigma^2_{\mathrm{det}} = \langle n^2_{\mathrm{det}} \rangle - \langle n_{\mathrm{det}} \rangle^2$

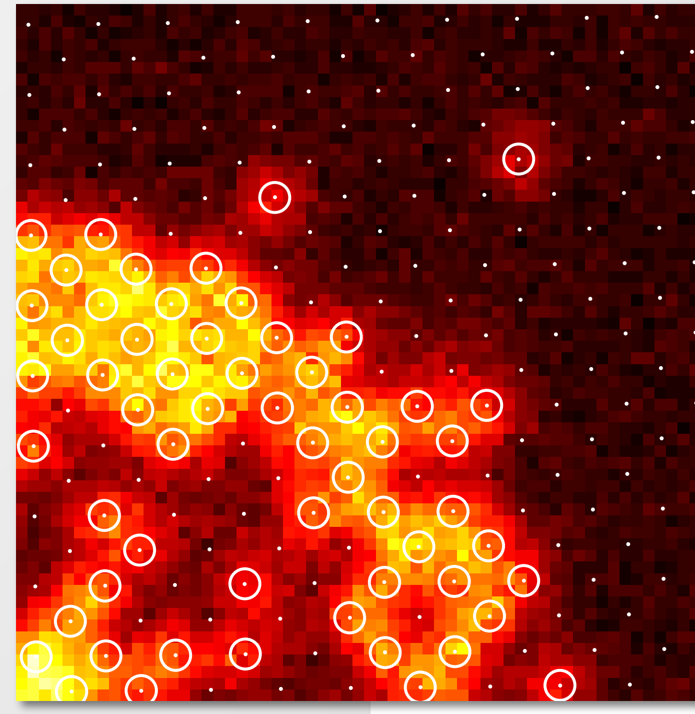
parity projection $\Rightarrow$ $\langle n^2_{\mathrm{det}} \rangle = \langle n_{\mathrm{det}} \rangle$

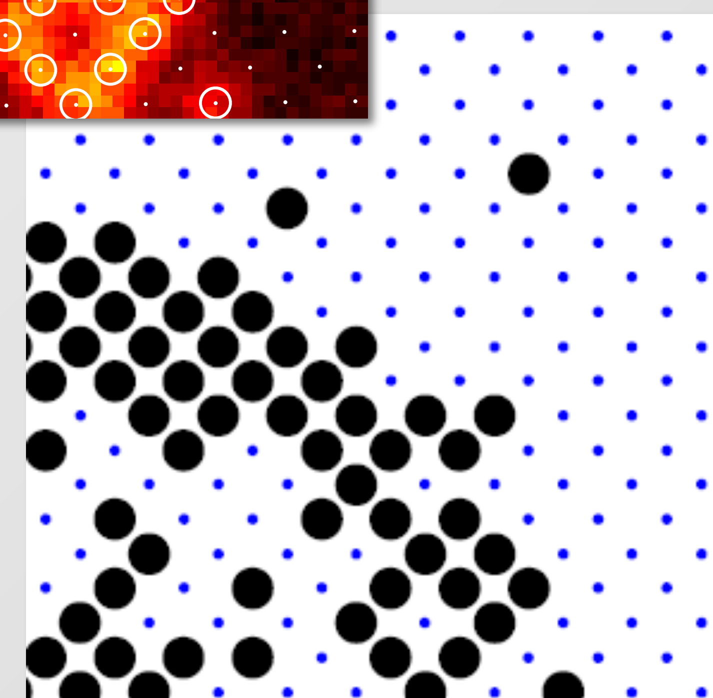
see also E. Kapit & E. Mueller, Phys. Rev. A **82**, 013644 (2010)

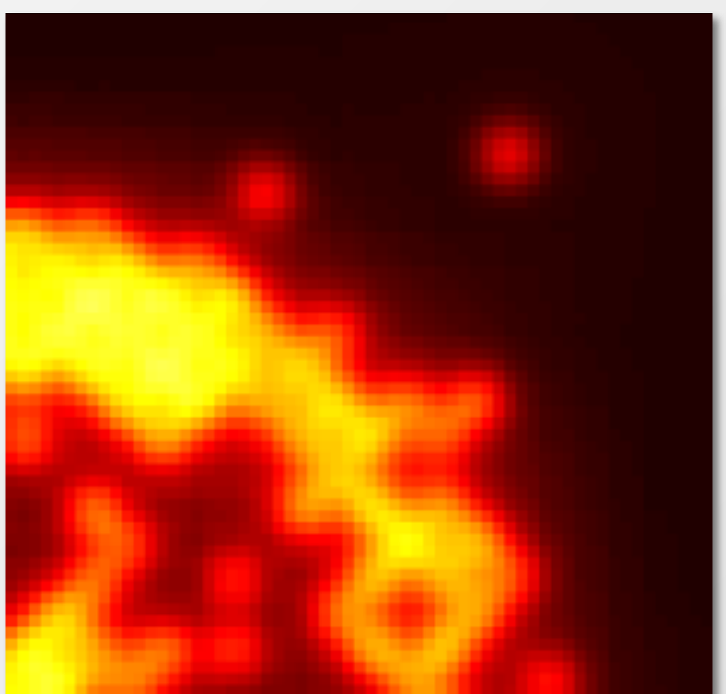
Reconstruction of site occupation



Reconstruction algorithm







Digitized image convoluted with point-spread function

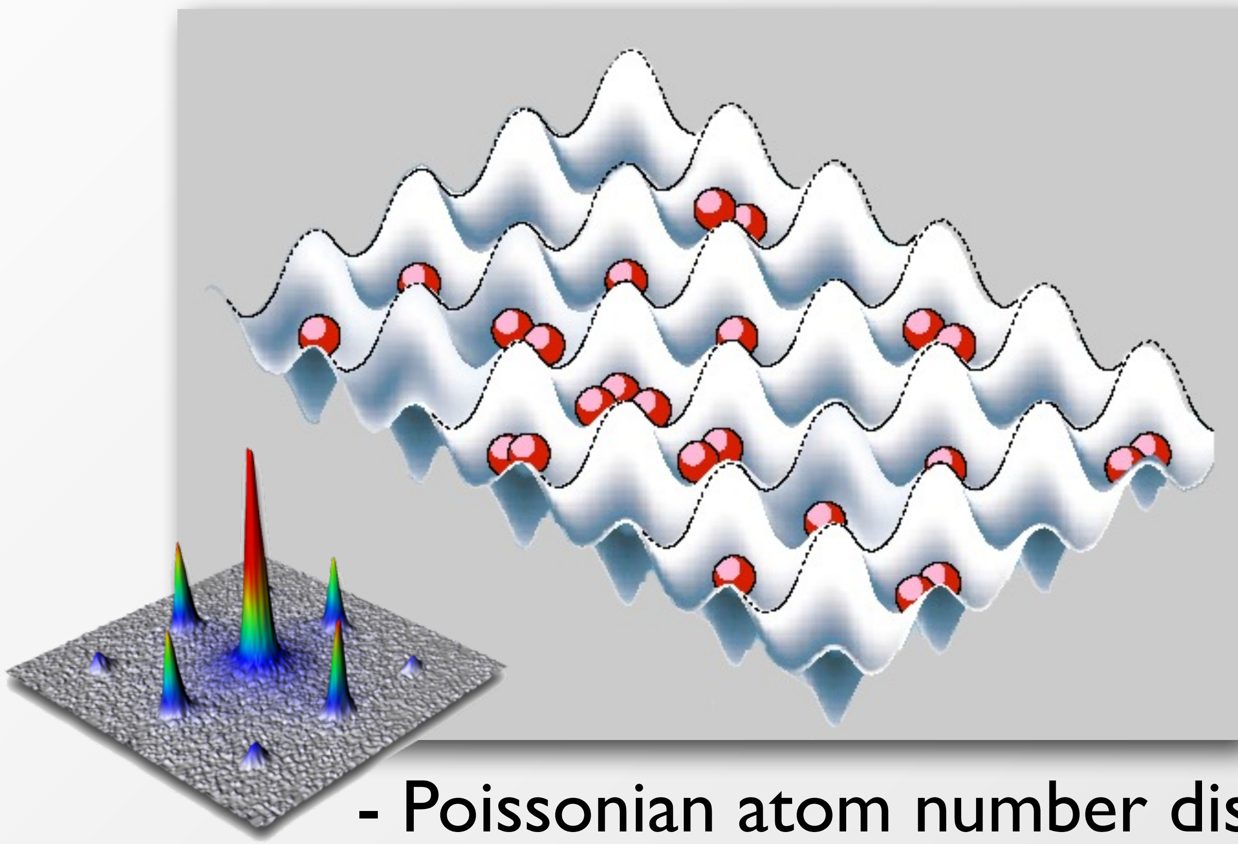
digitized image
no experimental noise

In-Situ Imaging of a Mott Insulator

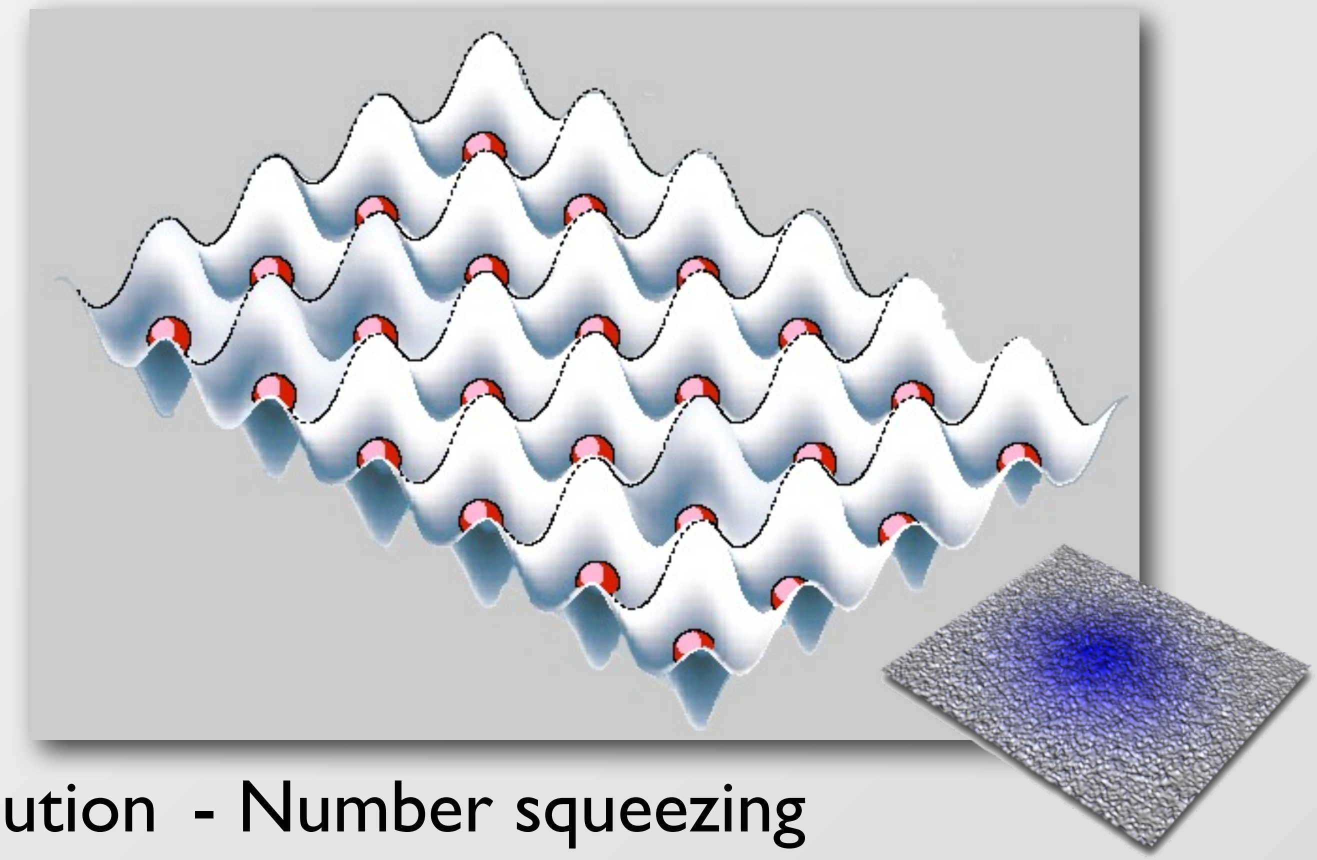
J. Sherson et al. Nature **467**, 68 (2010),
see also S. Fölling et al. Phys. Rev. Lett (2006), G.K. Campbell et al. Science (2006)
N. Gemelke et al. Nature (2009), W. Bakr et al. Science (2010)

Mott Insulators

Superfluid

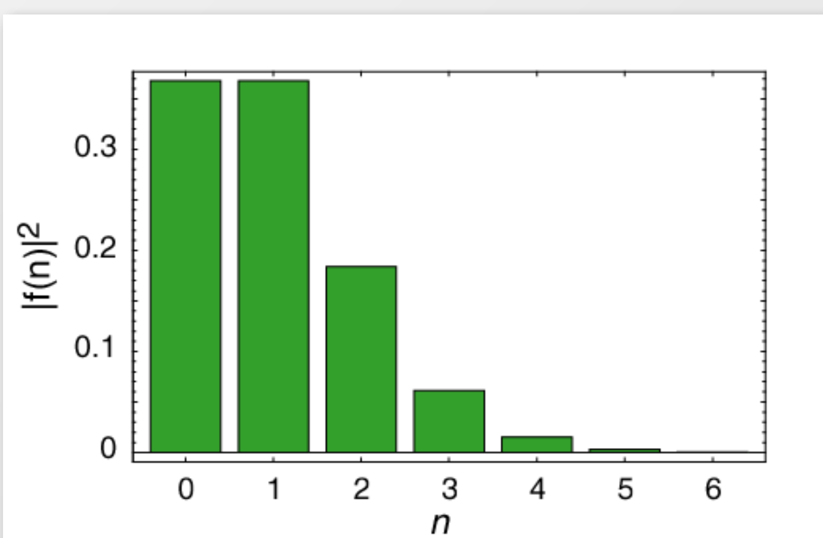


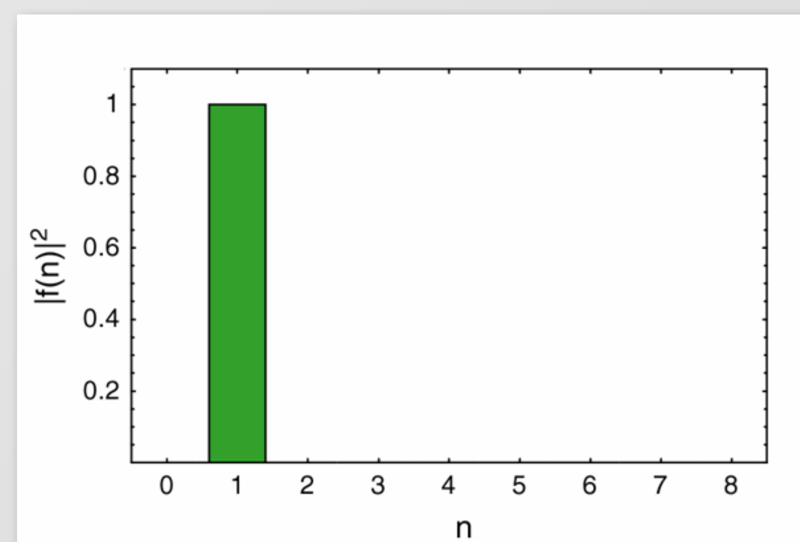
Mott-Insulator



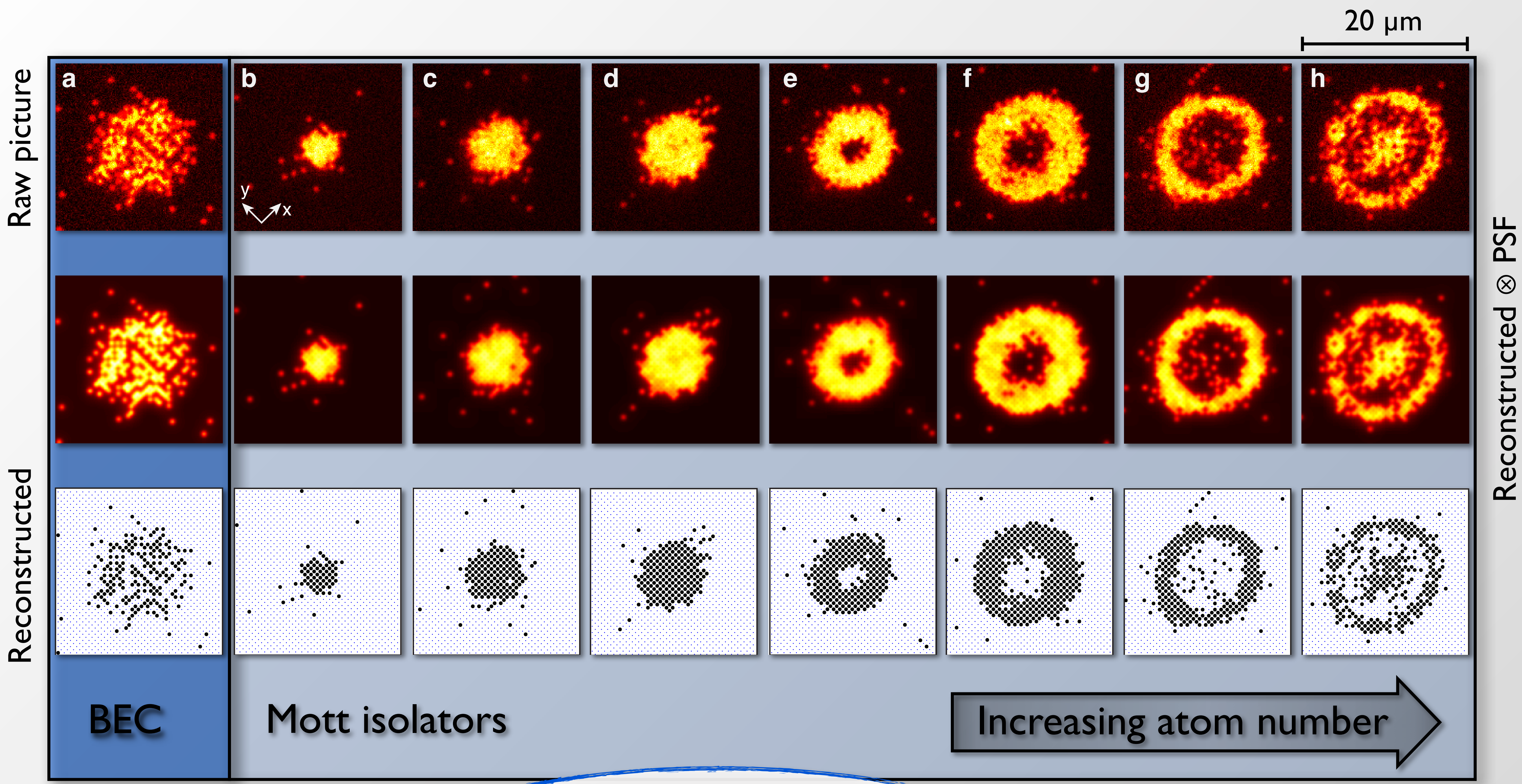
- Poissonian atom number distribution
- Long range phase coherence

- Number squeezing
- No phase coherence





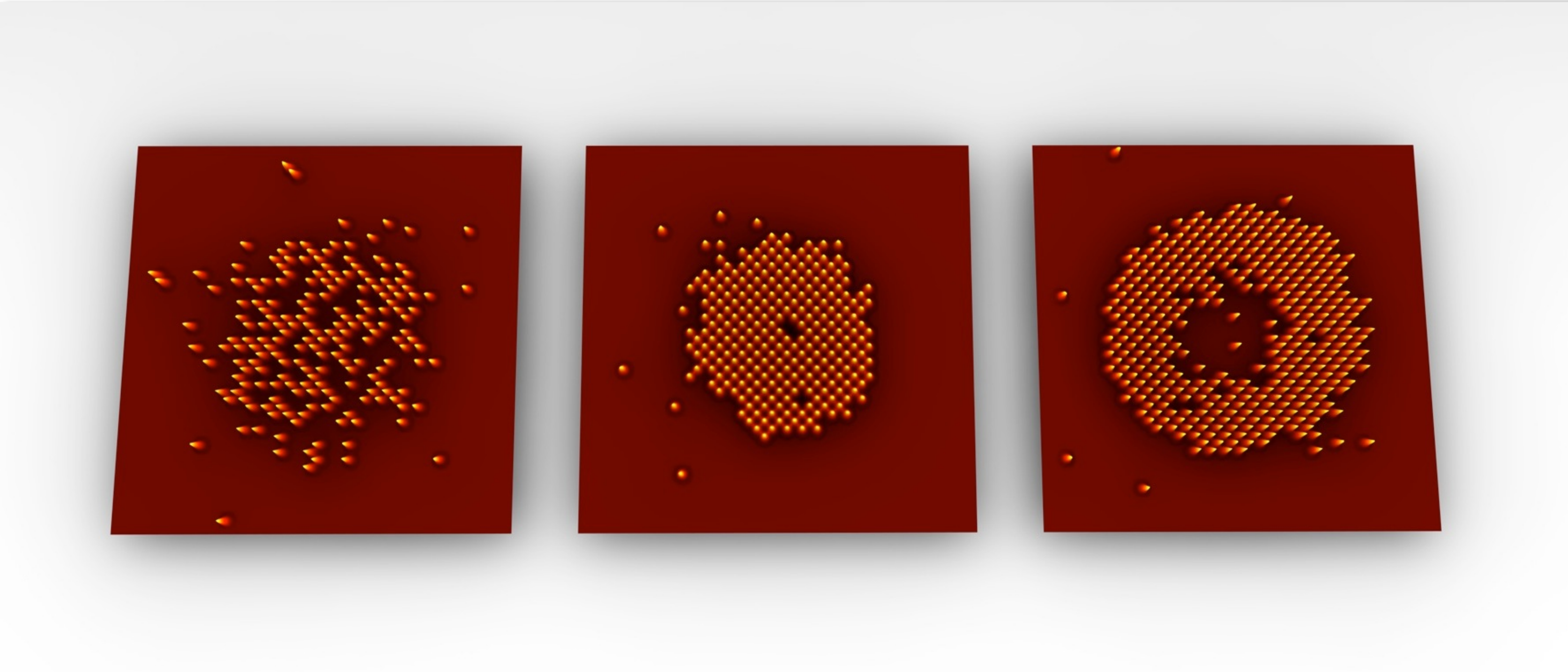
In-situ observation of a Mott insulator



for the Mott insulators: U/J ~ 300 (critical U/J ~ 16)

⇒ only thermal fluctuations

Snapshot of an Atomic Density Distribution



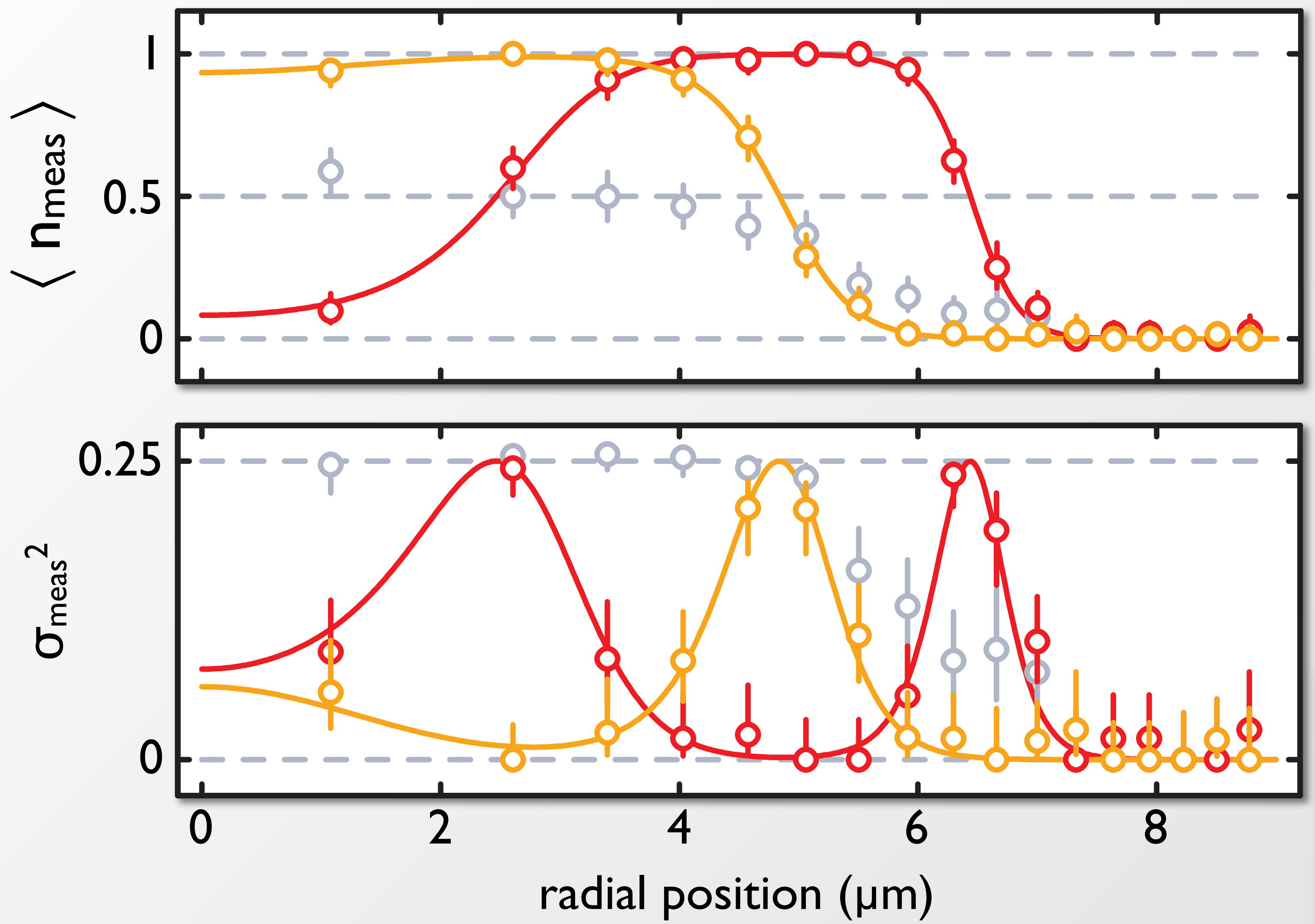
BEC

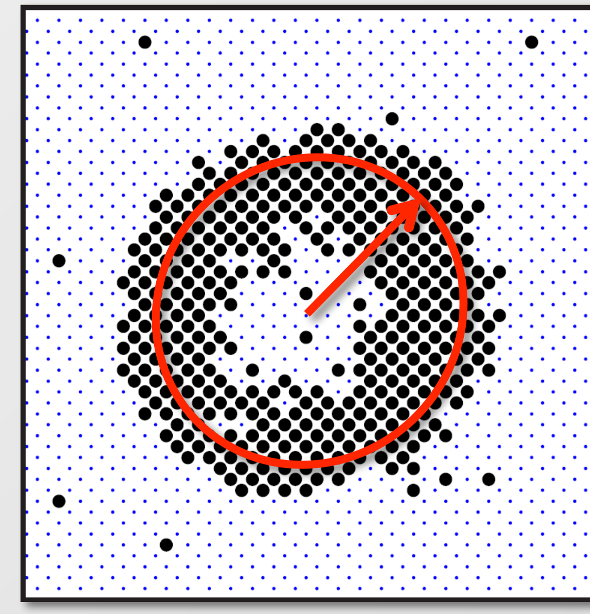
n=1
Mott Insulator

n=1 & n=2
Mott Insulator

J. Sherson *et al.* Nature **467**, 68 (2010)

Single Shot Thermodynamics



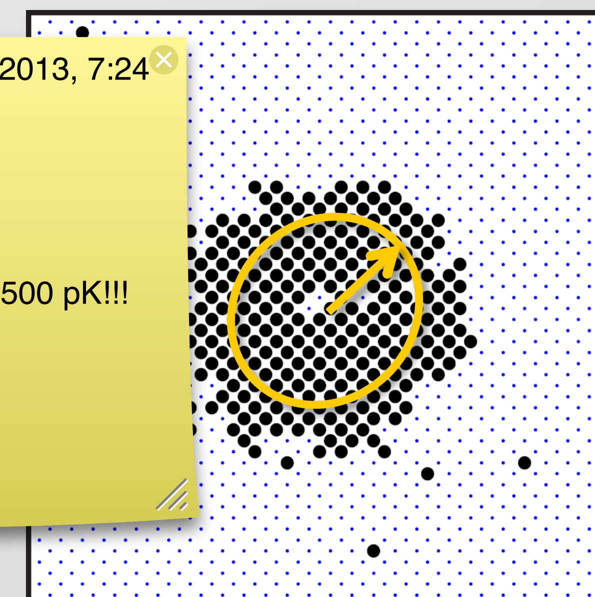


T = 0.074(5) *U*/k_B, *μ* = 1.17(1) *U*
N = 610(20)

Imported Author 23 Oct 2013, 7:24
2 kHz=100nK
1 kHz=50 nK

0.1 U approx 5 nK

measurement precision 500 pK!!!



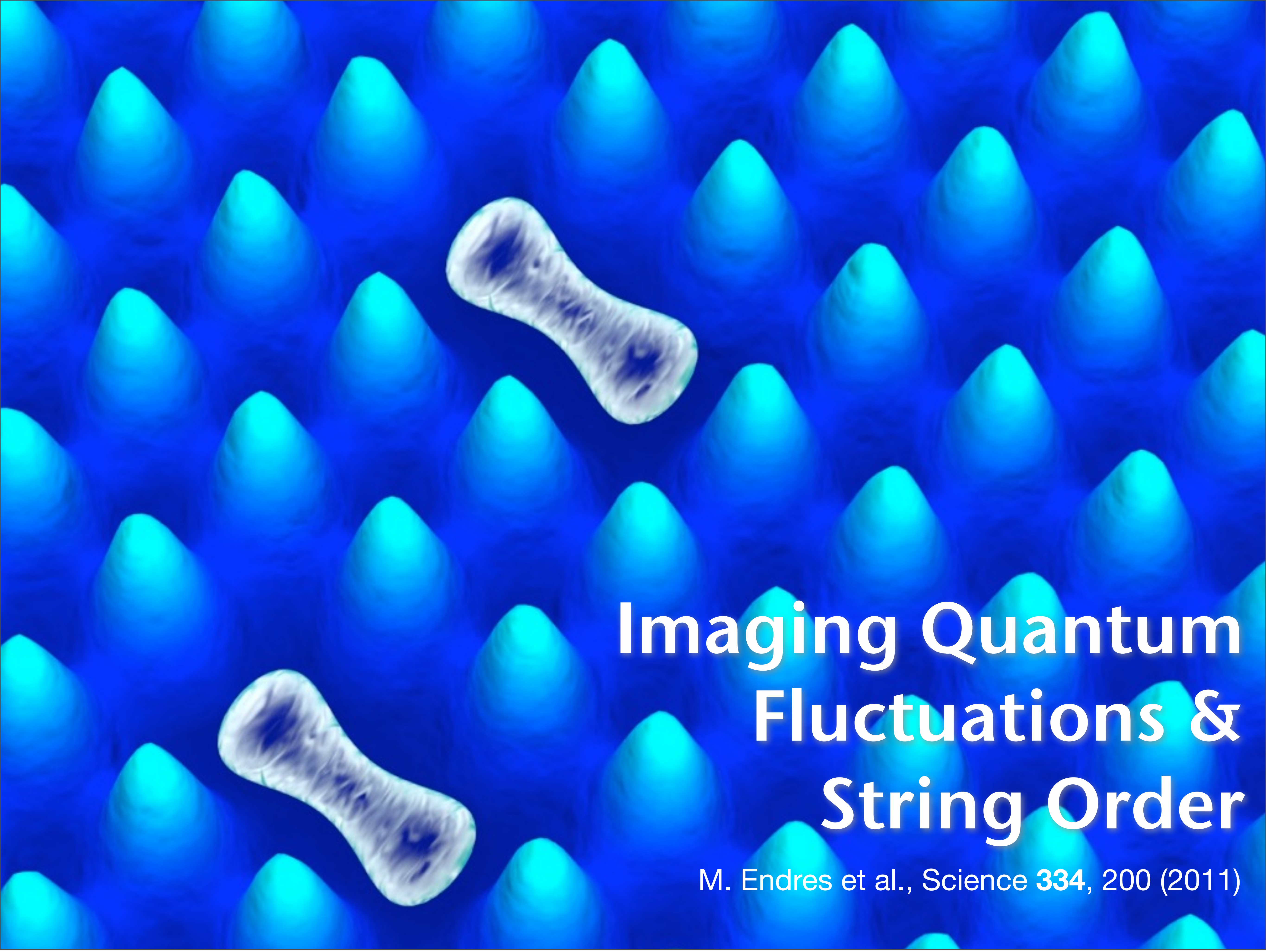
T = 0.090(5) *U*/k_B, μ = 0.73(3) U
N = 300(20)

Simple Theory - Atomic Limit Mott Insulator

occupation probability: $p_n(r) = \dfrac{e^{-\beta(E_n-\mu(r)n)}}{Z(r)}$

interaction energy: $E_n = \dfrac{1}{2}Un(n-1)$

fit parameters: $T/U, \mu/U, U/\omega^2$



Imaging Quantum Fluctuations & String Order

M. Endres et al., Science **334**, 200 (2011)

Probing Hidden Non-Local String Order

M. Endres, M. Cheneau, T. Fukuhara, Ch. Weitenberg, P. Schauss,
L. Mazza, M.C. Bañuls, L. Pollet, I. Bloch, S. Kuhr

discussions: Emanuele Dala Torre, Ehud Altman

E. G. Dalla Torre et al. Phys. Rev. Lett. **97**, 260401 (2006),
E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008).

Order Parameters

Typical Order Parameter in Landau Paradigm of Phase Transition

$$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \langle \hat{A}(\mathbf{x})\hat{A}(\mathbf{y})\rangle = c$$

Order Parameter: $\langle \hat{A} \rangle = \sqrt{c}$

Examples:

$m(\mathbf{r})$ Magnetization (Ferromagnetism, AFM,...)

$\psi(\mathbf{r})$ BEC (Condensate Wave Function)

$\Delta(\mathbf{r})$ BCS Superconductor

Order Parameter Characterizes Ground State Correlations

Local ordering!

Order Parameters

Typical Order Parameter in Landau Paradigm of Phase Transition

$$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \langle \hat{A}(\mathbf{x})\hat{A}(\mathbf{y})\rangle = c$$

Order Parameter:

Examples: gnetism, AFM,...)

Function)

or

General classification scheme for all phases of matter ???

Order Parameter Chara ound State Correlations

Local ordering!

String Order in 1D Systems

E.g. in 1D gapped systems where $\langle \hat{A}(\mathbf{x})\hat{A}(\mathbf{y})\rangle$ decays exponentially with distance

However, they can show hidden non-local order:

$$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \langle \hat{A}(\mathbf{x}) \left(\prod_{\mathbf{z}\in S(\mathbf{x},\mathbf{y})} \hat{B}(\mathbf{z}) \right) \hat{A}(\mathbf{y})\rangle = c$$

We say the order is hidden, because a "global view" of the underlying state is required. (**Topological Order:** X.-G. Wen)

Allows us to characterize state only via its ground state correlations!

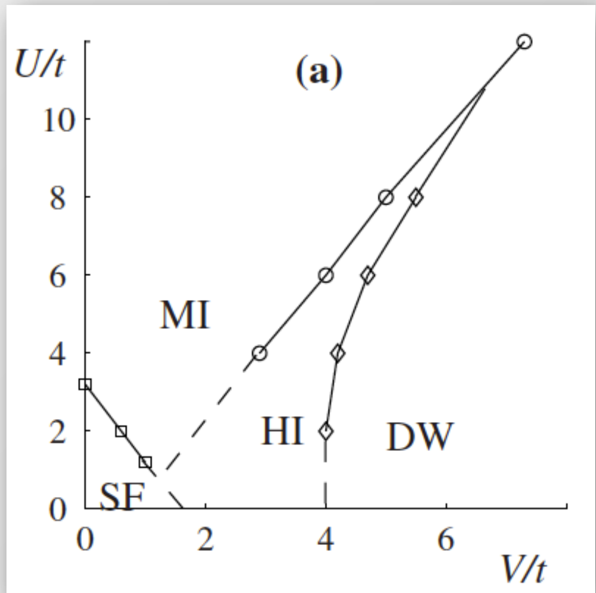
M. den Nijs, K. Rommelse, Phys. Rev. B 40, 4709 (1989).
E. Kim, G. Fa´th, J. So´lyom, D. Scalapino, Phys. Rev. B 62, 14965 (2000)
E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
F. Anfuso, A. Rosch, Phys. Rev. B 75, 144420 (2007)
E. Berg, 1 E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

An Example: Haldane Insulator in 1D

E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
E. Berg, 1 E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

$$H = -J \sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}) + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) + V \sum \hat{n}_i \hat{n}_{i+1}$$

Bose-Hubbard with next-neighour interaction



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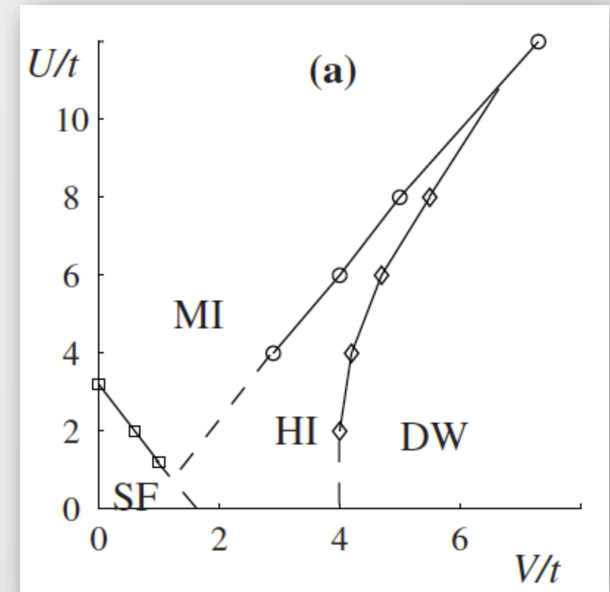
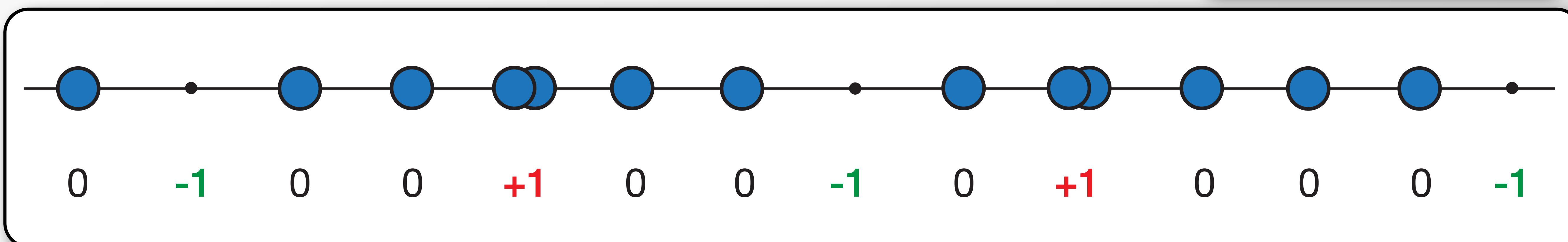
E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
E. Berg, 1 E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)



$$H = -J\sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1) + V\sum \hat{n}_i\hat{n}_{i+1}$$

Bose-Hubbard with next-neighour interaction



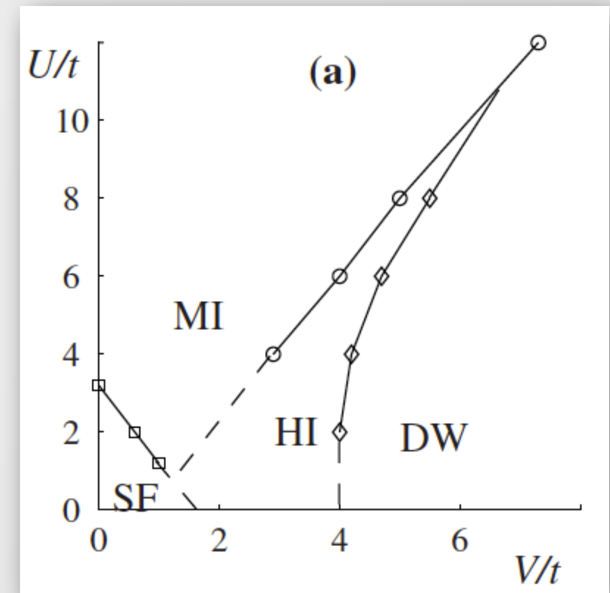
A Hidden Antiferromagnet!

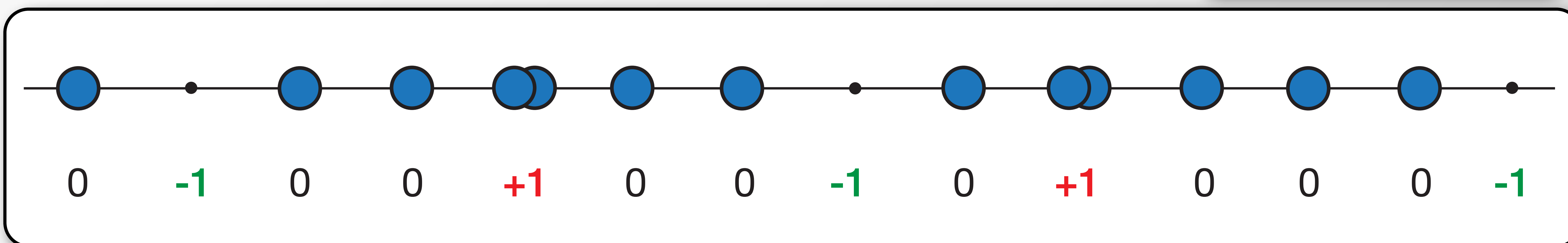
An Example: Haldane Insulator in 1D

E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
E. Berg, 1 E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

$$H = -J\sum_i(\hat{a}_i^\dagger\hat{a}_{i+1} + \text{h.c.}) + \frac{1}{2}U\sum_i\hat{n}_i(\hat{n}_i - 1) + V\sum\hat{n}_i\hat{n}_{i+1}$$

Bose-Hubbard with next-neighbour interaction





A Hidden Antiferromagnet!

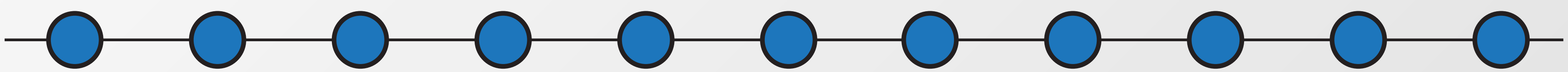
Hidden Non-local Order Captured by String Correlator

$$\mathcal{O}_S^2 = -\lim_{|i-j|\to\infty}\left\langle \delta\hat{n}_i \left(\prod_{i<k<j} e^{i\pi\delta\hat{n}_k}\right)\delta\hat{n}_j\right\rangle$$

Ground State of a MI

$$H = -J\sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

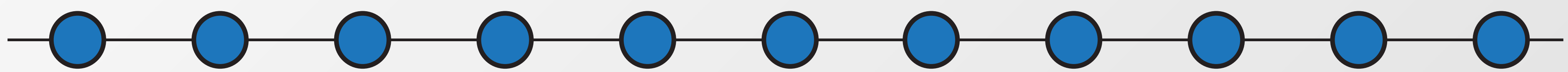
Starting Point: MI in Atomic Limit (*J*=0) ***No fluctuations!***



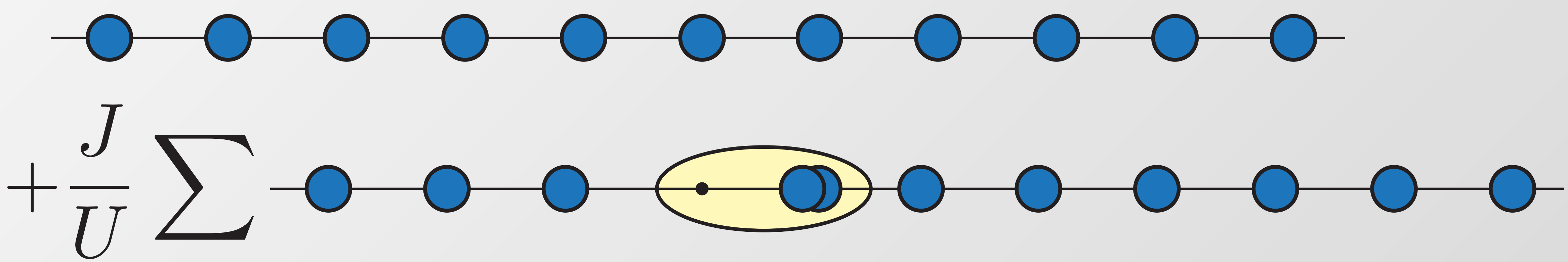
Ground State of a MI

$$H = -J\sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Starting Point: MI in Atomic Limit (J=0) ***No fluctuations!***



Small Tunneling (First order perturbation)



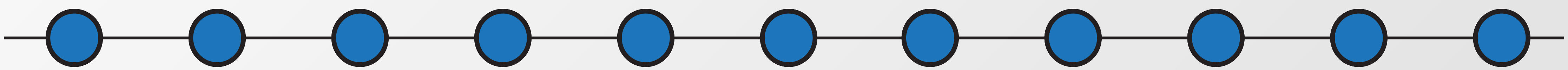
Quantum Fluctuations appear in form of

Quantum Correlated Particle Hole Pairs

In constrast: *thermal fluctuations appear as uncorrelated fluctuations!*

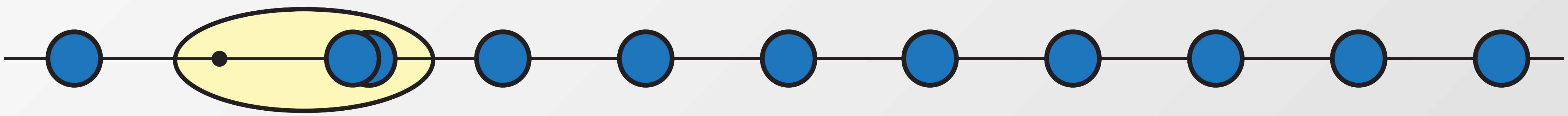
Towards Larger J/U

Increasing *J/U*



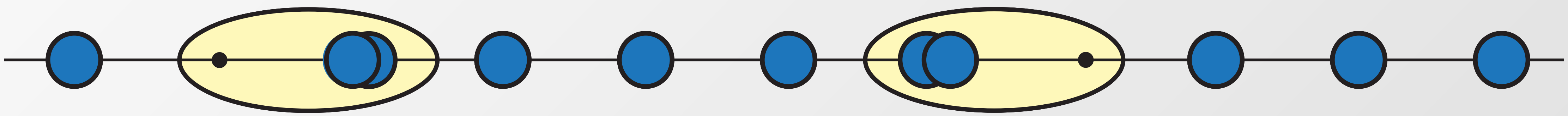
Towards Larger J/U

Increasing *J/U*



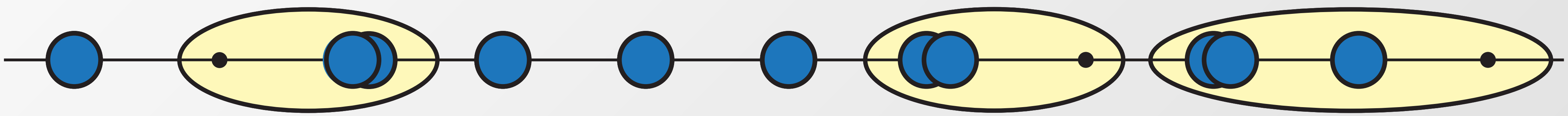
Towards Larger J/U

Increasing *J/U*



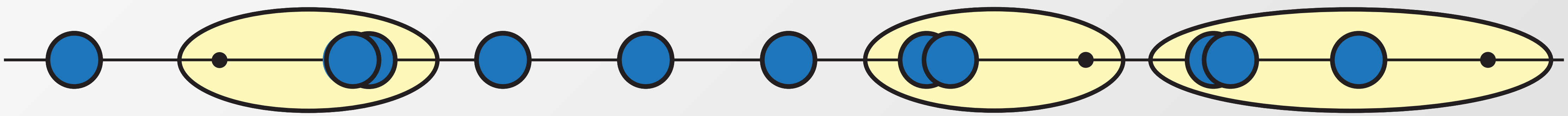
Towards Larger J/U

Increasing *J/U*



Towards Larger J/U

Increasing *J/U*



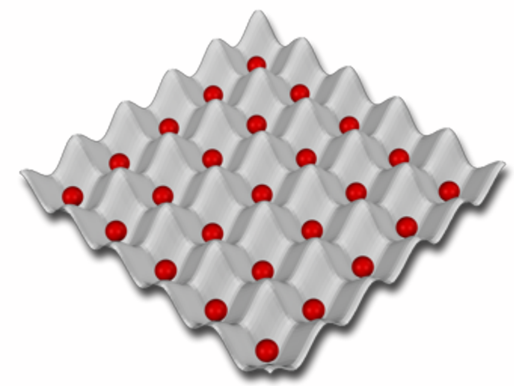
- Particle-Hole Pairs **Proliferate**
- Particle-Hole Pairs **Extend in Size**
  (leading to Deconfinement at Transition Point)

Deviations from the Atomic Mott Insulator

Ground state for J=0 :

``atomic´´ Mott insulator

$$|\Psi_0\rangle = \prod_i |n_0\rangle_i$$

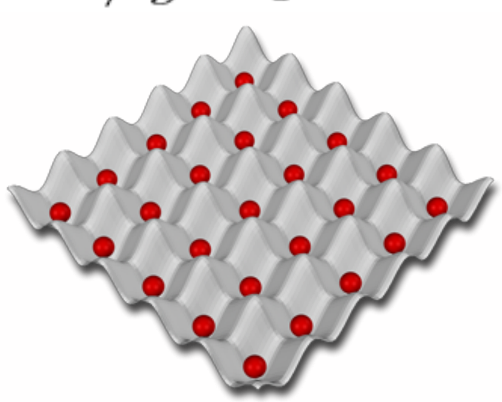


Ground state for finite J<<U :

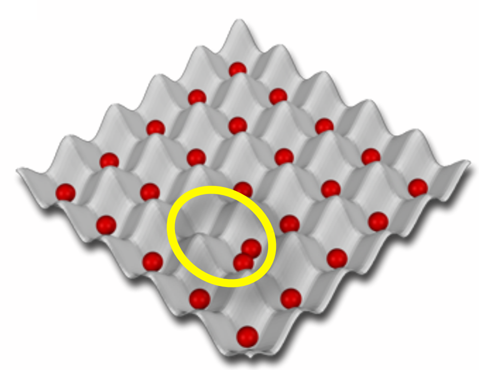
treat the hopping term H_{hop} in 1st order perturbation

$$|\Psi_1\rangle = -\sum_{n\neq g} \frac{H_{\rm hop}}{E_g^{(0)} - E_n^{(0)}} |\Psi_0\rangle$$

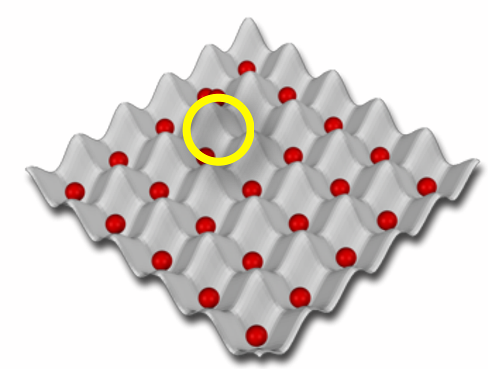
=



$+ \frac{J}{U}$



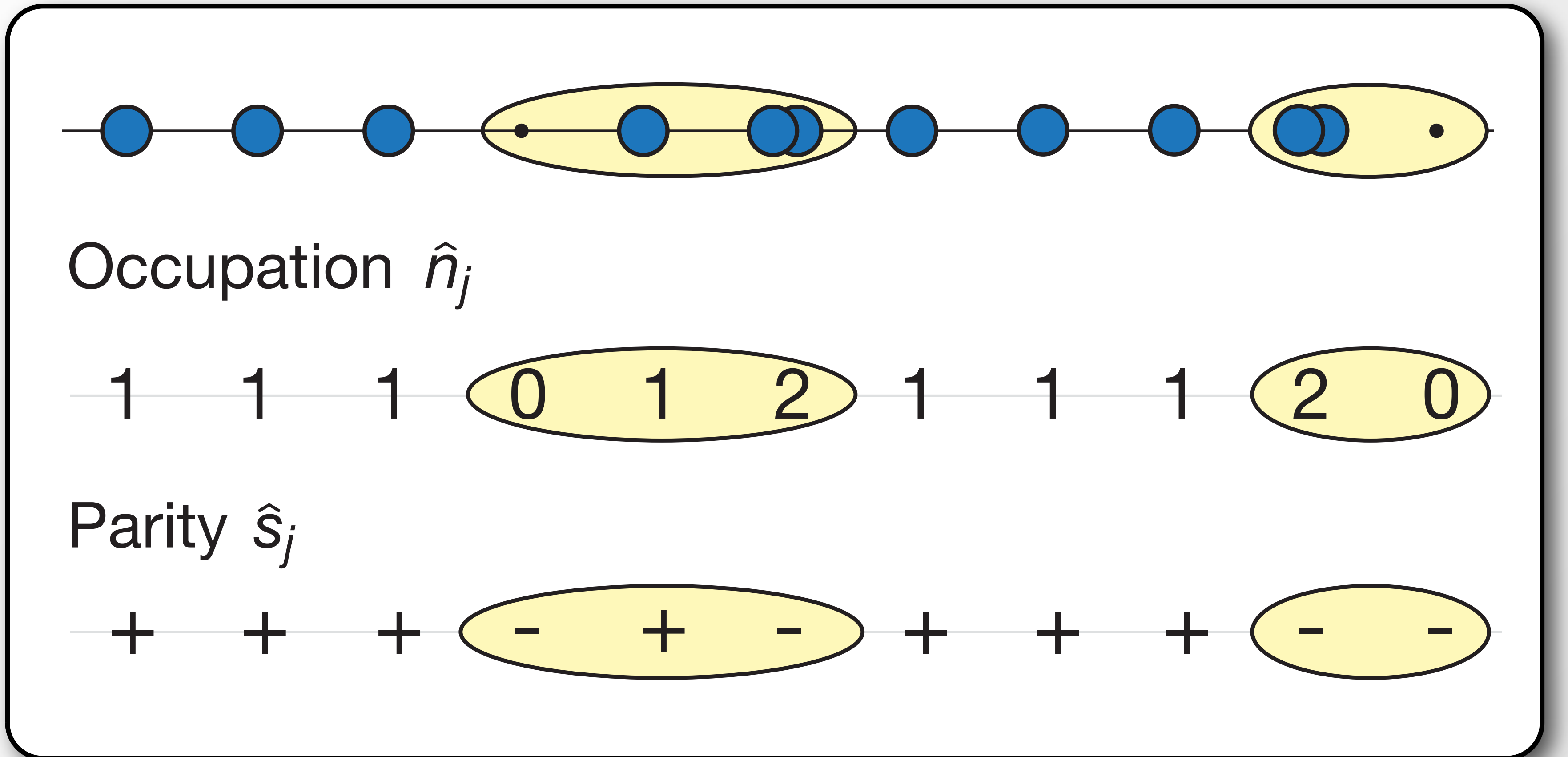
$+ \frac{J}{U}$



$+ \ldots$

Coherent admixture of particle/holes at finite J/U

String Order in a 1D Mott Insulator

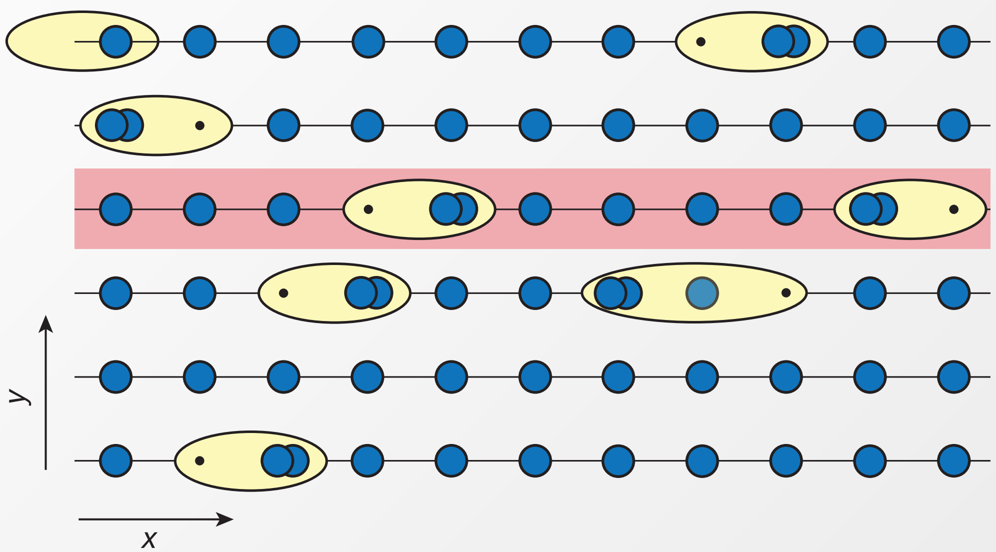


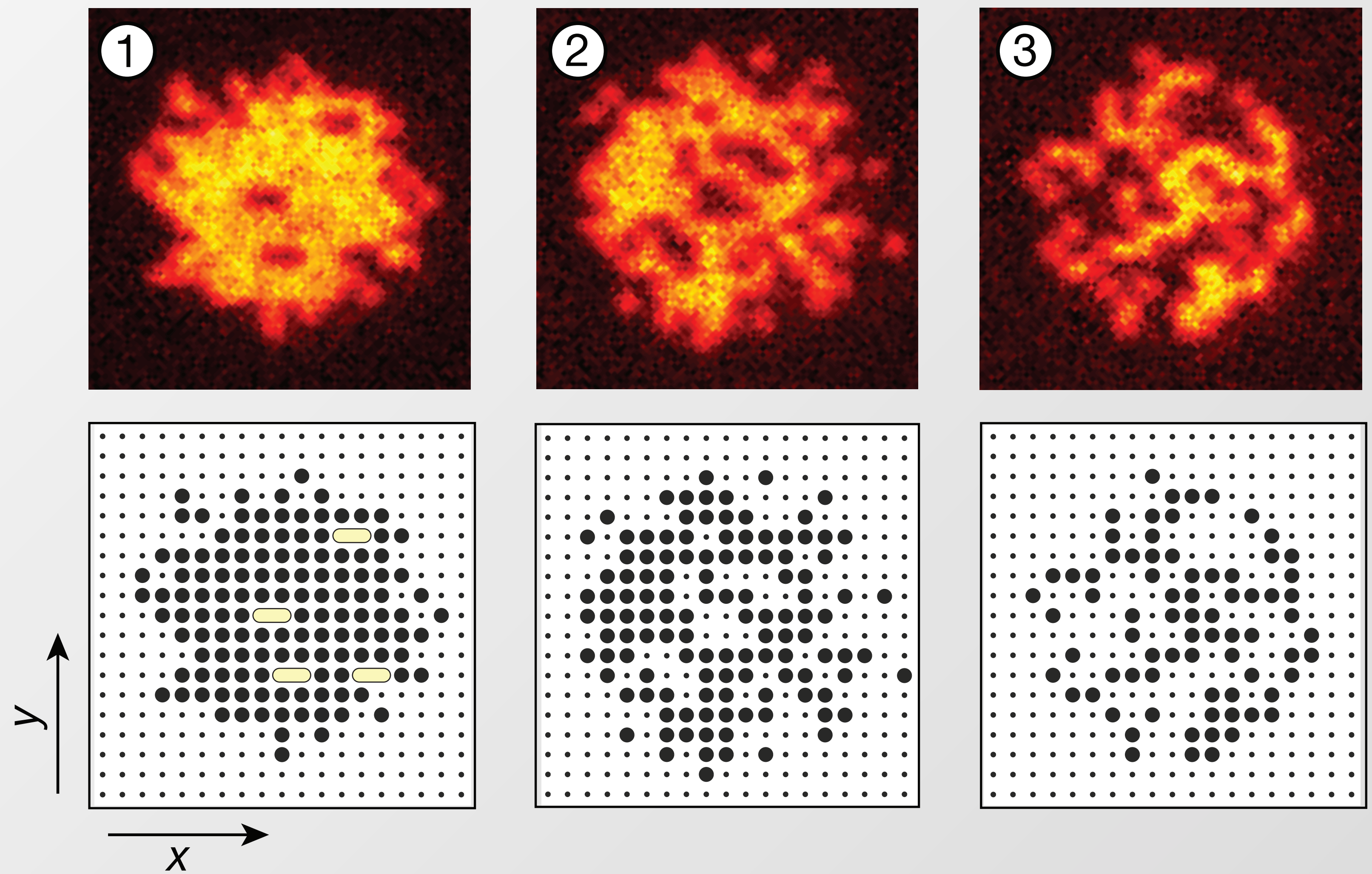
Hidden Non-Local Order Parameter of MI

$$\mathcal{O}_P^2 = \lim_{|i-j|\to\infty} \left\langle \prod_{i\leq k\leq j} e^{i\pi\delta\hat{n}_j} \right\rangle$$

E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
E. Berg, 1 E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

Quantum Correlated Particle Hole Correlations



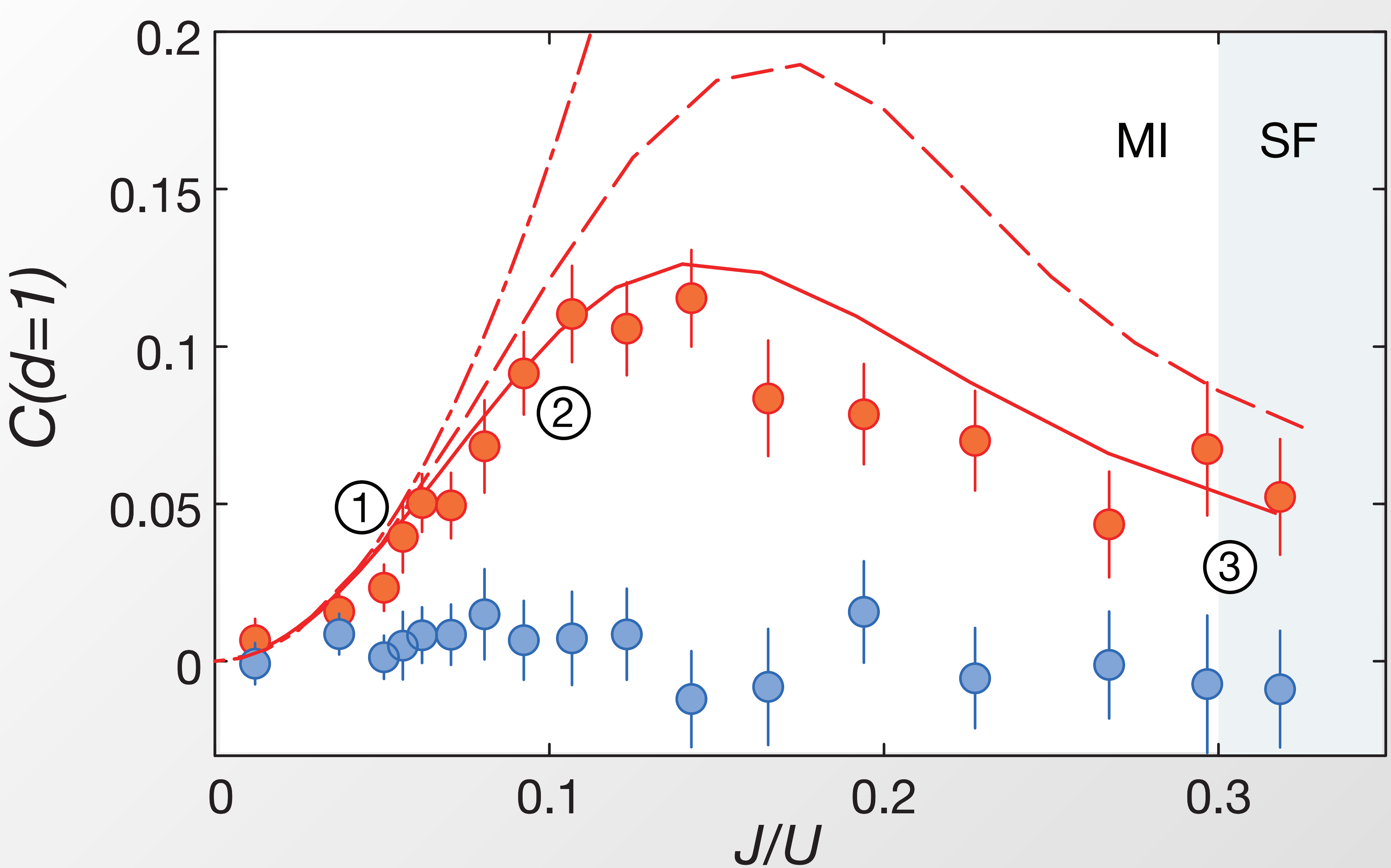


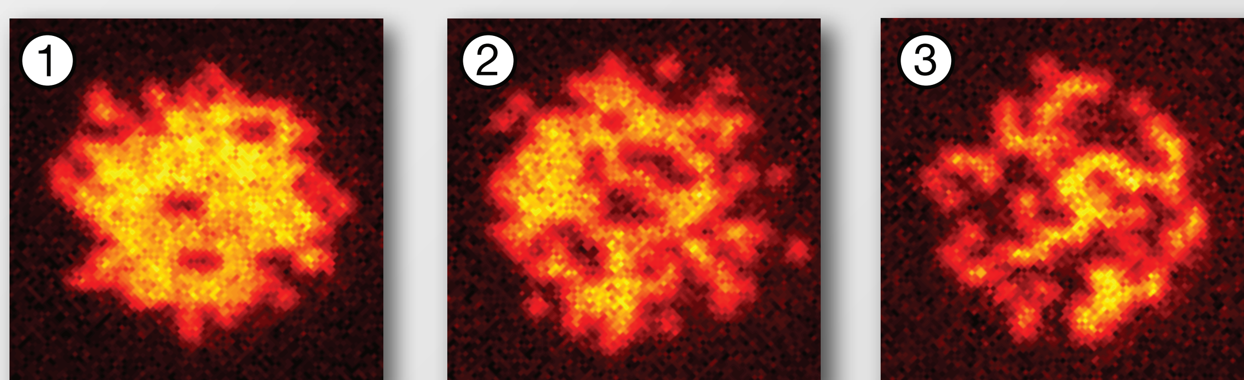
J/*U*=0.06 *J*/*U*=0.11 *J*/*U*=0.3

$$C(d) = \langle \hat{s}_k \hat{s}_{k+d} \rangle - \langle \hat{s}_k \rangle \langle \hat{s}_{k+d} \rangle$$

Two point correlator

Particle-Hole Correlations 1D

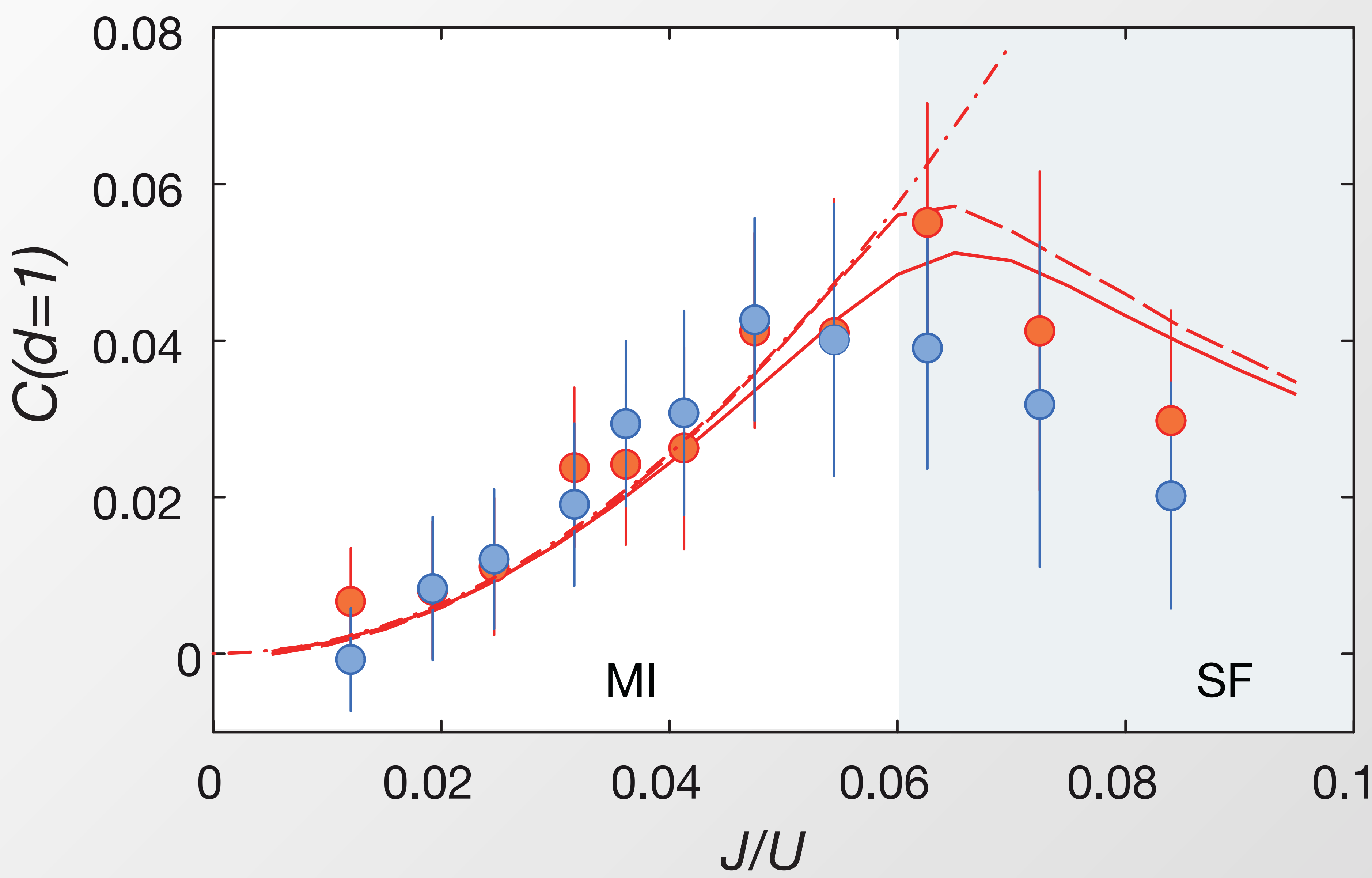




Single Shot Images

Theory: (dashed and solid line) DMRG & MPS by L. Mazza & M.C. Bañuls

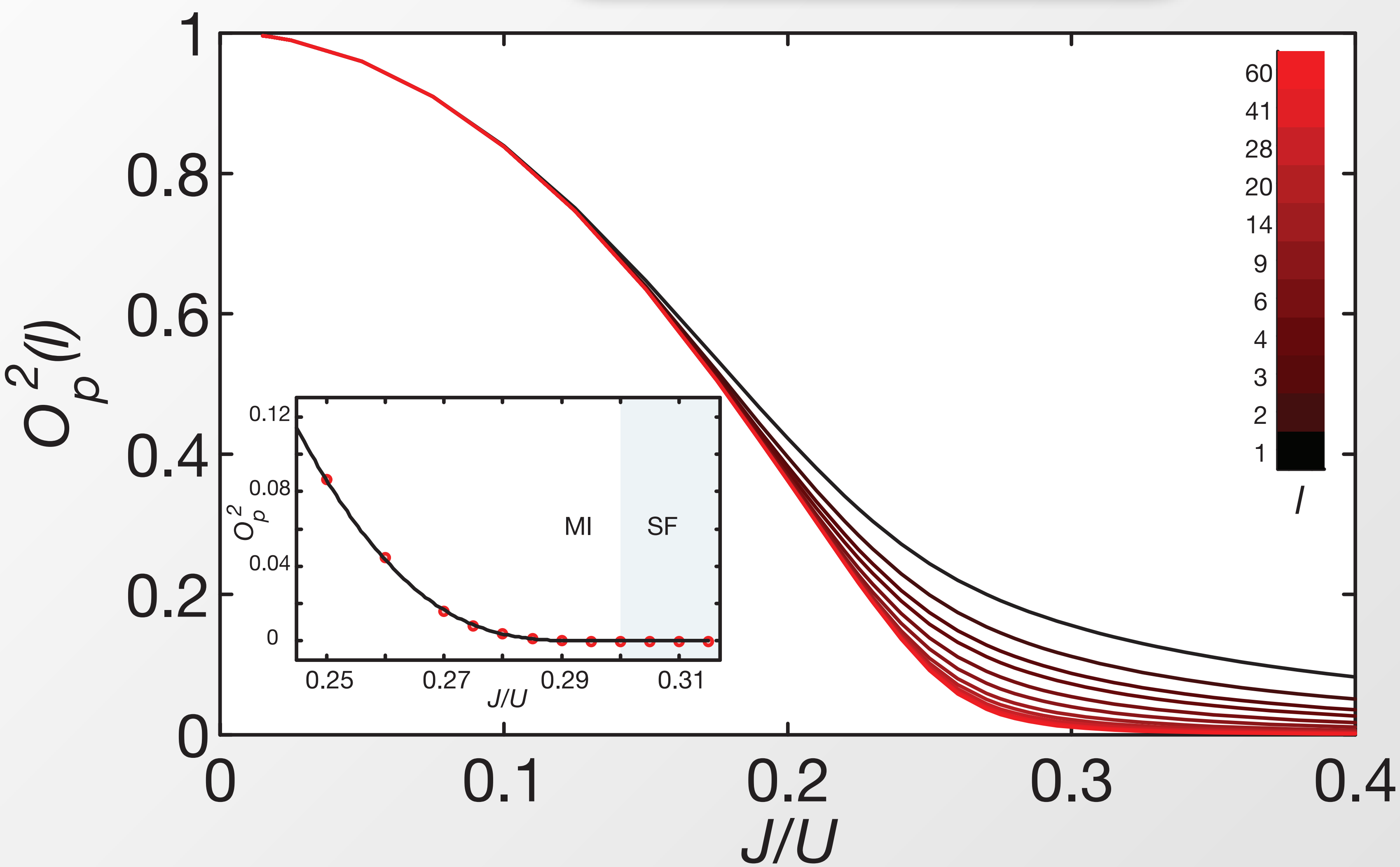
Particle-Hole Correlations 2D



Theory: (dashed and solid line) 2D QMC by Lode Pollet

String Order - an Order Parameter for the Mott Insulator?

$$\mathcal{O}_P^2(l) = \left\langle \prod_{j=k}^{k+l} \hat{s}_j \right\rangle$$



DMRG T=0
Simulations,
chain length 216,
nbar=1

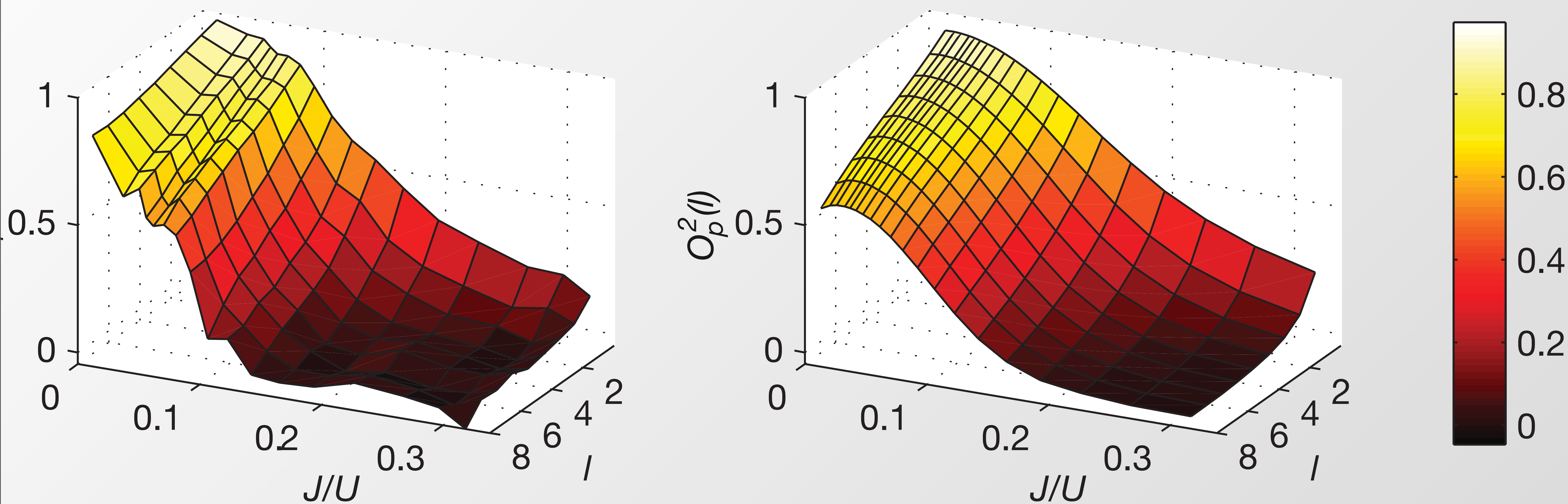
Inset: Finite size scaling of string correlator

Fit to $e^{-\frac{a}{\sqrt{(J/U)_c-(J/U)}}}$ Berezinskii-Kosterlitz-Thouless

$(J/U)_c = 0.295 - 0.32$

String Order

$$\mathcal{O}_P^2(l) = \left\langle \prod_{j=k}^{k+l} \hat{s}_j \right\rangle$$



Experiment

Theory (MPS *T*=0.09*U*)

Note:

- decay for larger string lengths due to thermal excitations
- shift of transition point due to inhomogenous trapping

(pointy Mott lobes in 1D)

Conclusion

- Mott Insulator contains many *quantum correlated particle-hole pairs*, induced by quantum fluctuations.
- Particle-hole pairs *deconfine* at Mott-Superfluid transition
- Deconfinement is captured by *hidden non-local order parameter*
- String Order useful concept for finite lengths
- Another Deconfinement Transition from *Mott Insulator to Haldane Phase* for next neighbour interactions
- **First Measurement of a Non-Local Order Parameter**

Extension to 2D:

S. P. Rath, W. Simeth, M. Endres, W. Zwerger, Annals of Physics, 334, p. 256-271

Light-Cone Like Spreading of Correlations in a Many-Body System

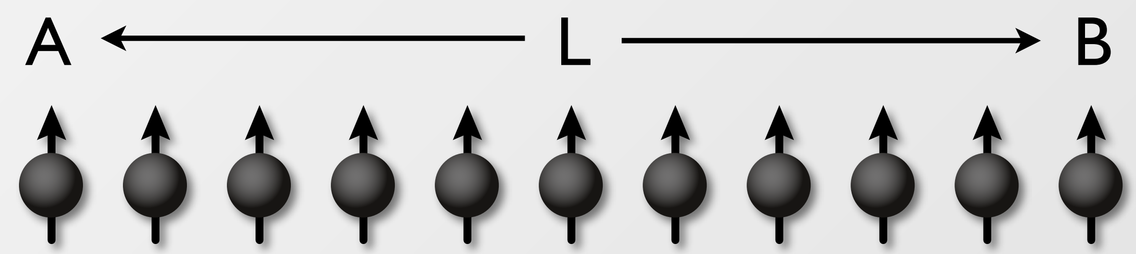
M. Cheneau, P. Barmettler, D. Poletti, M. Endres, P. Schauß, T. Fukuhara, Ch. Gross, I. Bloch, C. Kollath, S. Kuhr

M. Cheneau et al., Nature **481**, 484 (2012)
T. Langen et al. Nat. Physics **9**, 640 (2013)
P. Jurcevic et al. Nature (2014), Ph. Richerme et al. Science (2014)

www.quantum-munich.de

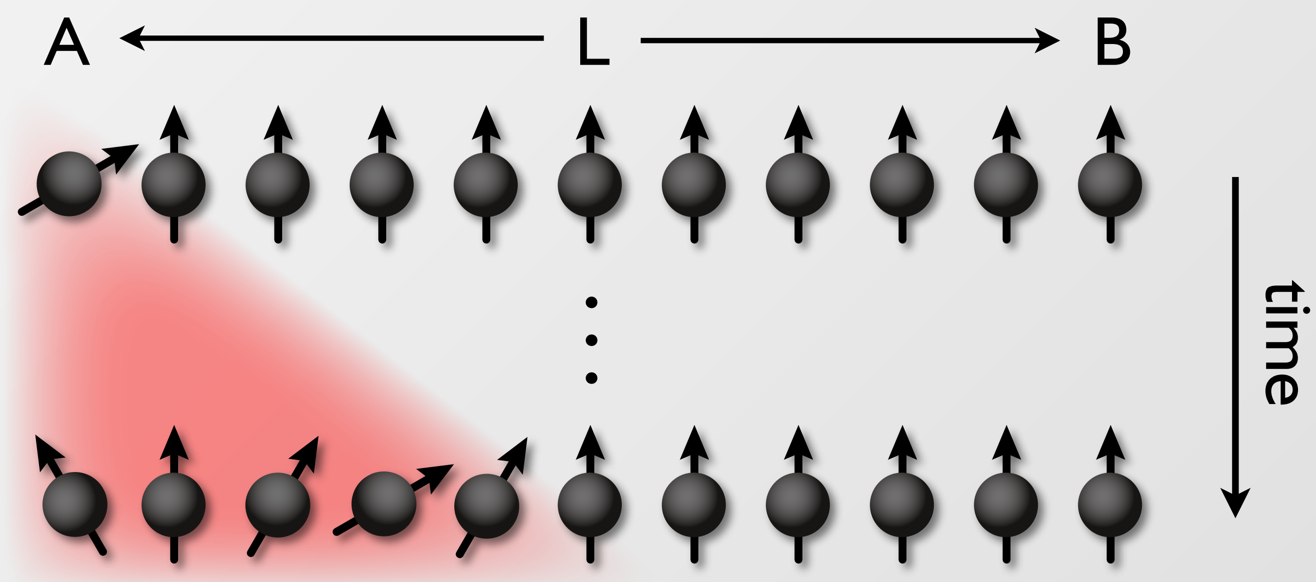
Lieb-Robinson bounds

Spin chain
short-range interactions



Lieb-Robinson bounds

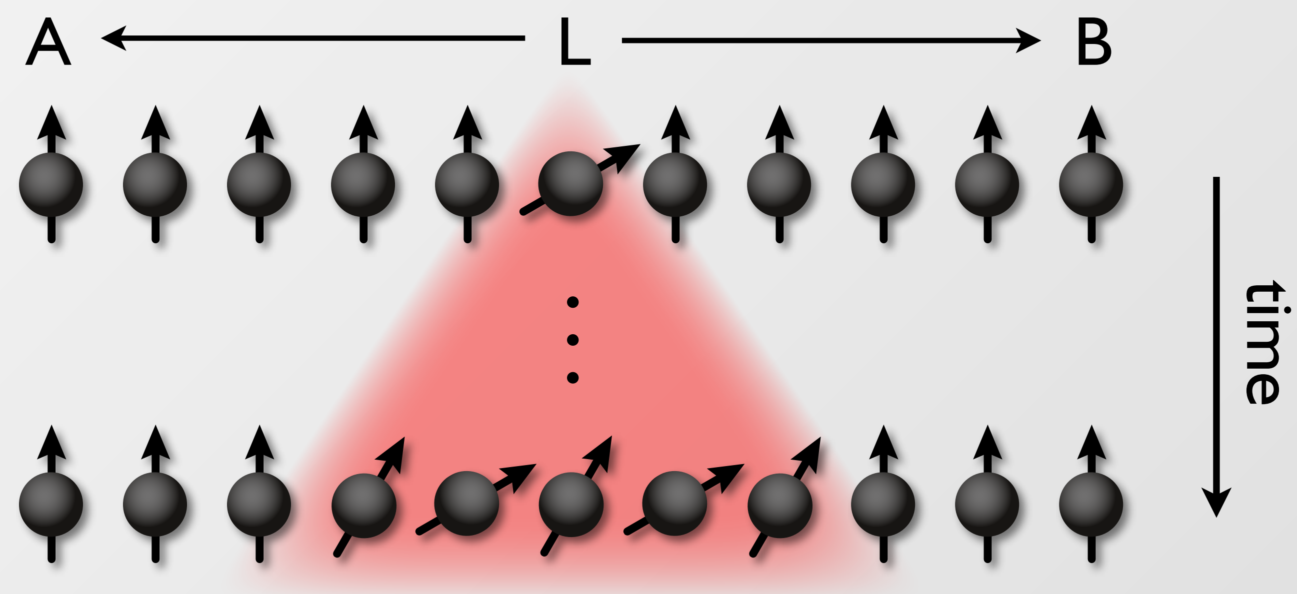
Spin chain
short-range interactions



Lieb and Robinson (1972)

$$\left|[A, B(t)]\right| \leq \lambda \exp\left(\frac{vt - L}{\zeta}\right)$$

Lieb-Robinson bounds



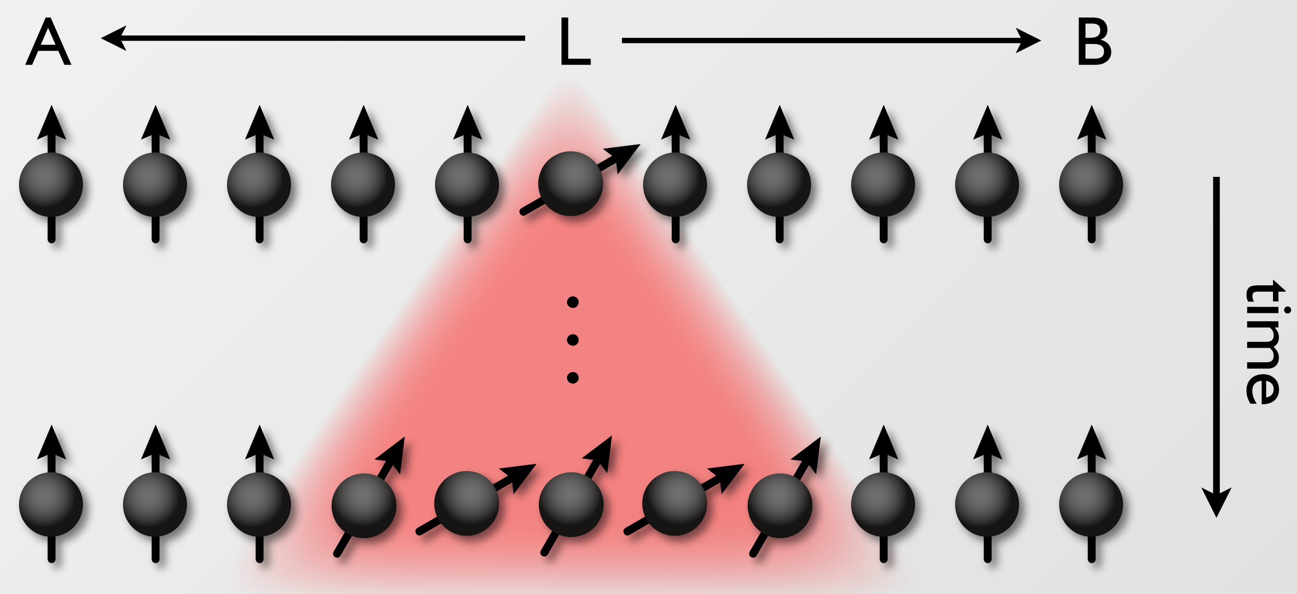
Spin chain
short-range interactions

Bravyi, Hastings and Verstraete (2006)
Calabrese and Cardy (2006)
Eisert and Osborne (2006)
Nachtergaele, Ogata and Sims (2006)
... ***and many others since then***

$$\left|\langle A(t)B(t)\rangle - \langle A(t)\rangle\langle B(t)\rangle\right| \leq \lambda' \exp\left(\frac{vt - L/2}{\zeta'}\right)$$

Lieb-Robinson bounds

Spin chain
short-range interactions



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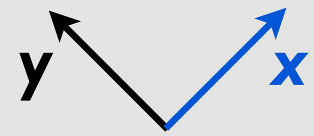
$$\left|\langle A(t)B(t)\rangle - \langle A(t)\rangle\langle B(t)\rangle\right| \leq \lambda' \exp\left(\frac{v\,t - L/2}{\zeta'}\right)$$

the propagation of correlations is bounded by an effective light cone

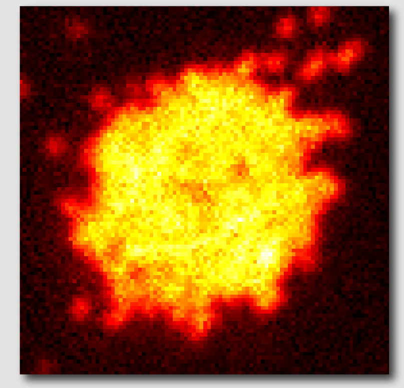
1D Mott insulator out of equilibrium

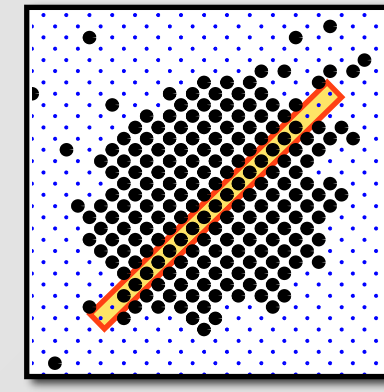
1. Prepare 1D Mott insulator with $U/J \gg 1$

deep lattice (20 E_r)
no tunnelling



variable
lattice depth





1D Mott insulator out of equilibrium

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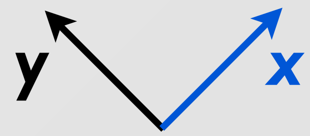
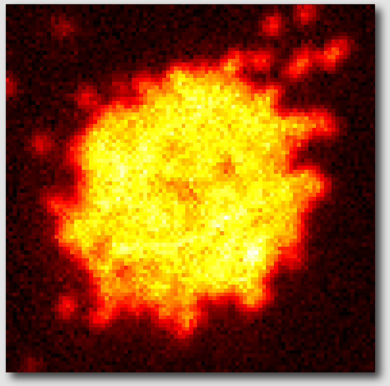
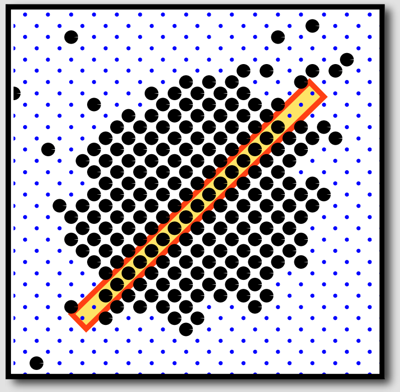
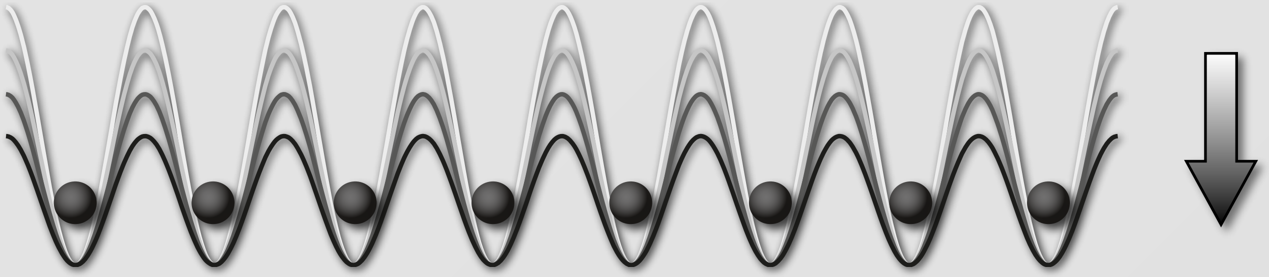


variable
lattice depth





2. Lower U/J abruptly

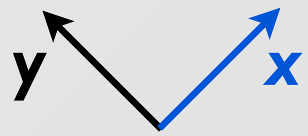


quench

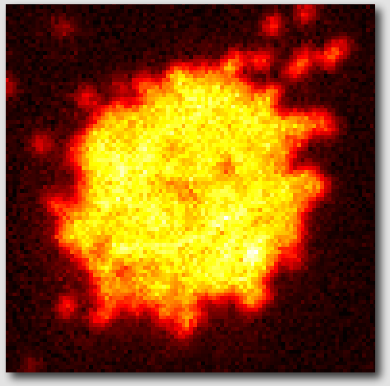
1D Mott insulator out of equilibrium

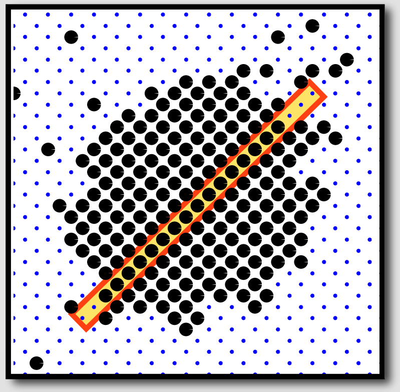
1. Prepare 1D Mott insulator with $U/J \gg 1$

deep lattice (20 E_r)
no tunnelling



variable lattice depth





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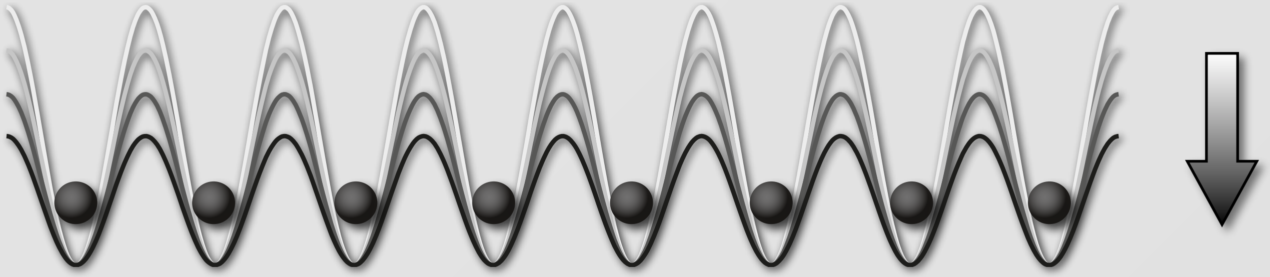


quench

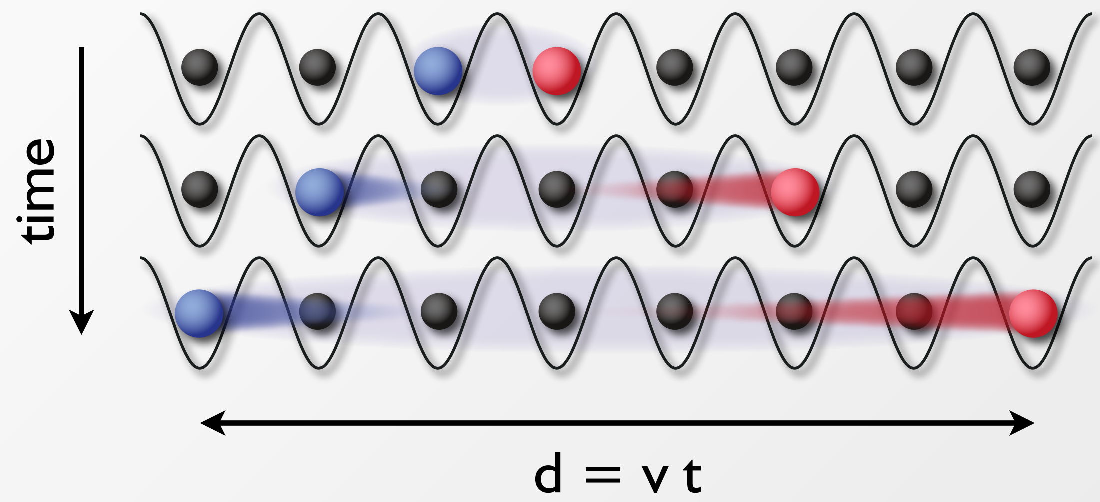
3. Record the dynamics

The initial state is highly excited. Calabrese and Cardy (2006)

Quasiparticles are emitted and propagate ballistically, carrying correlations across the system.

Light-cone like spreading of correlations

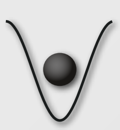
- Quasiparticle dynamics

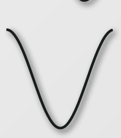
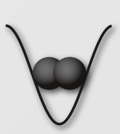


- Two-point parity correlation function

$$C_d(t) = \langle s_j(t) s_{j+d}(t) \rangle - \langle s_j(t) \rangle \langle s_{j+d}(t) \rangle \longrightarrow \begin{array}{l} \simeq 0 \text{ in the initial state} \\ >0 \text{ when } \boldsymbol{t \simeq d/v} \end{array}$$

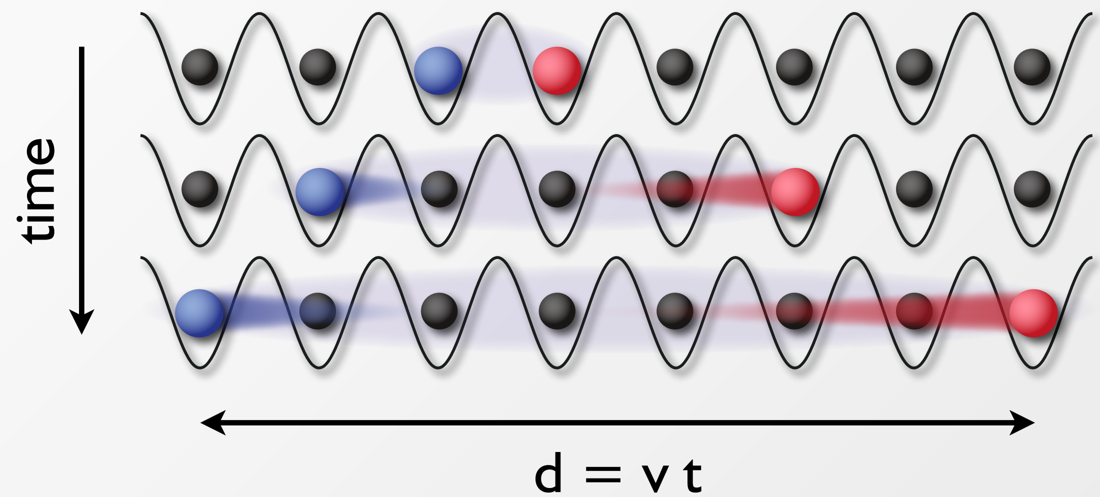
$$s_j(t) = e^{i\pi[n_j(t) - \bar{n}]} \quad \begin{cases} +1 \text{ if} \\ -1 \text{ if} \quad \text{or} \end{cases}$$

+1 if 

-1 if  or 

Light-cone like spreading of correlations

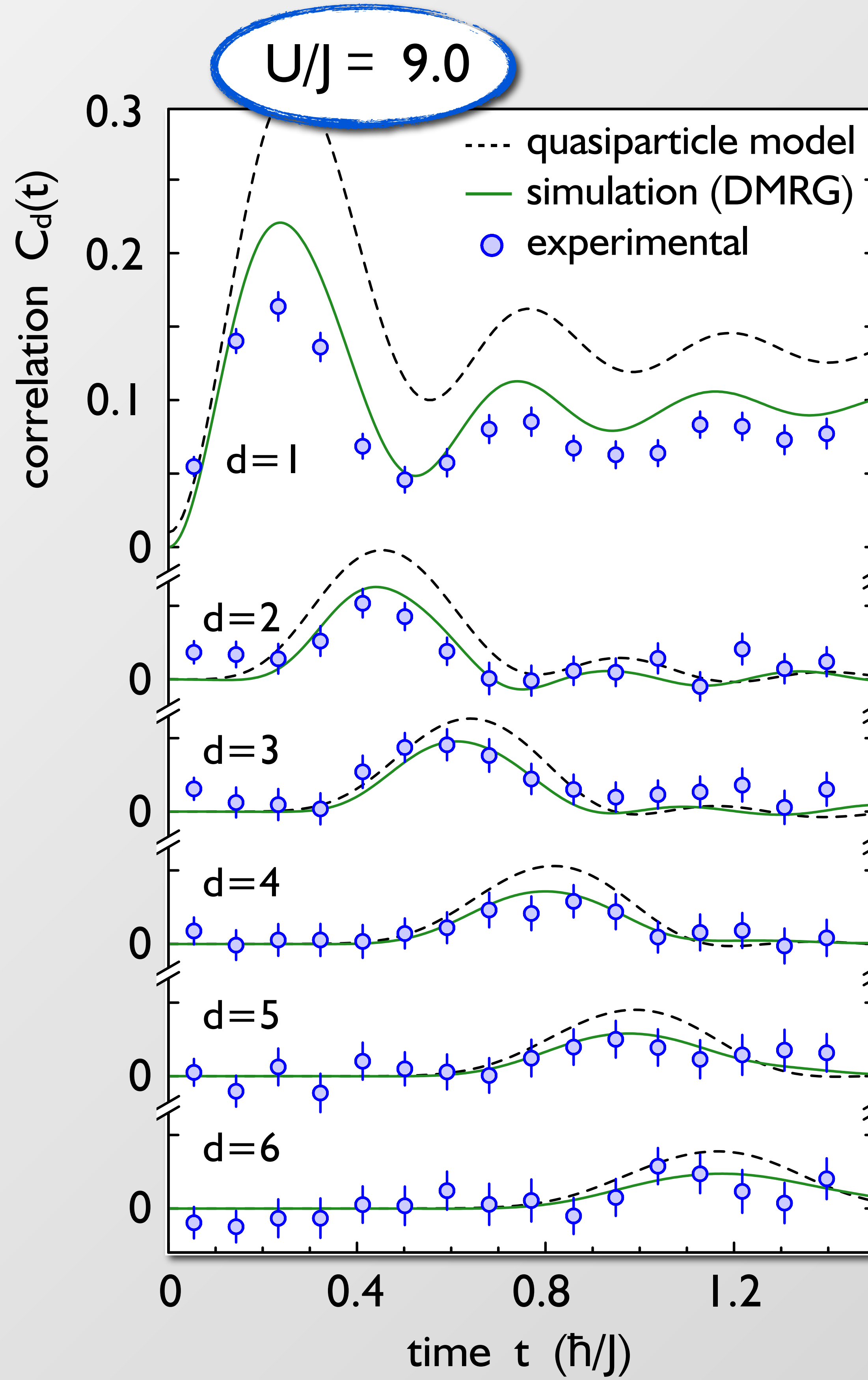
- Quasiparticle dynamics



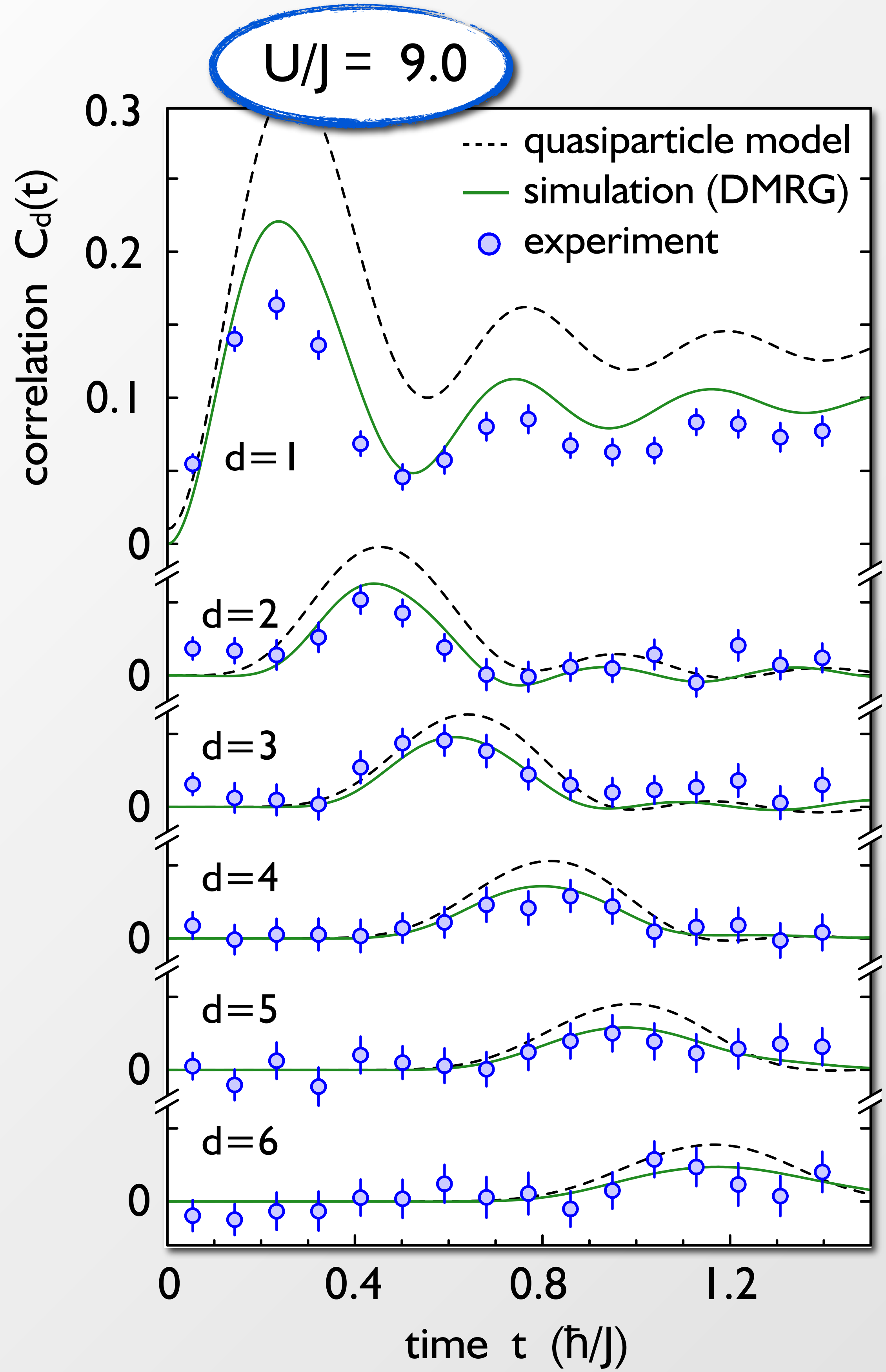
- Two-point parity correlation function

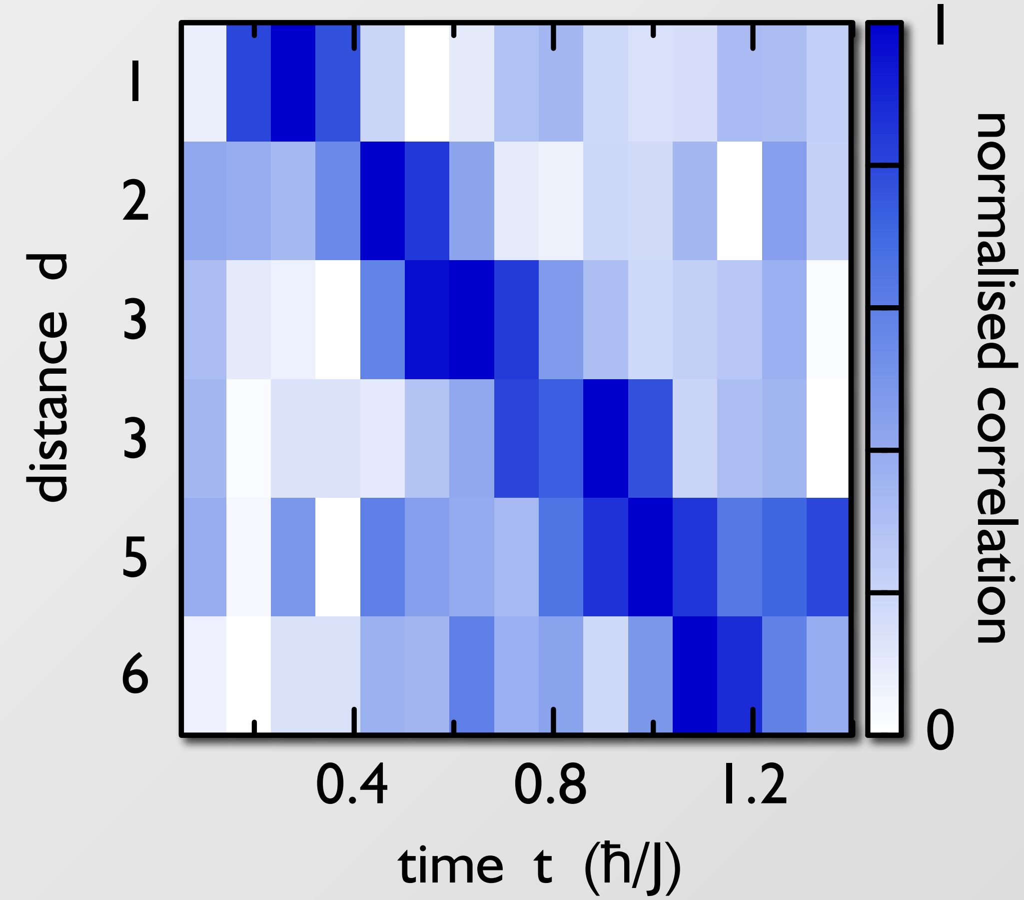
$$C_d(t) = \langle s_j(t) s_{j+d}(t) \rangle - \langle s_j(t) \rangle \langle s_{j+d}(t) \rangle$$

$$s_j(t) = e^{i\pi[n_j(t)-\bar{n}]} \begin{cases} +1 & \text{if (one atom in well)} \\ -1 & \text{if (empty well) or (two atoms in well)} \end{cases}$$

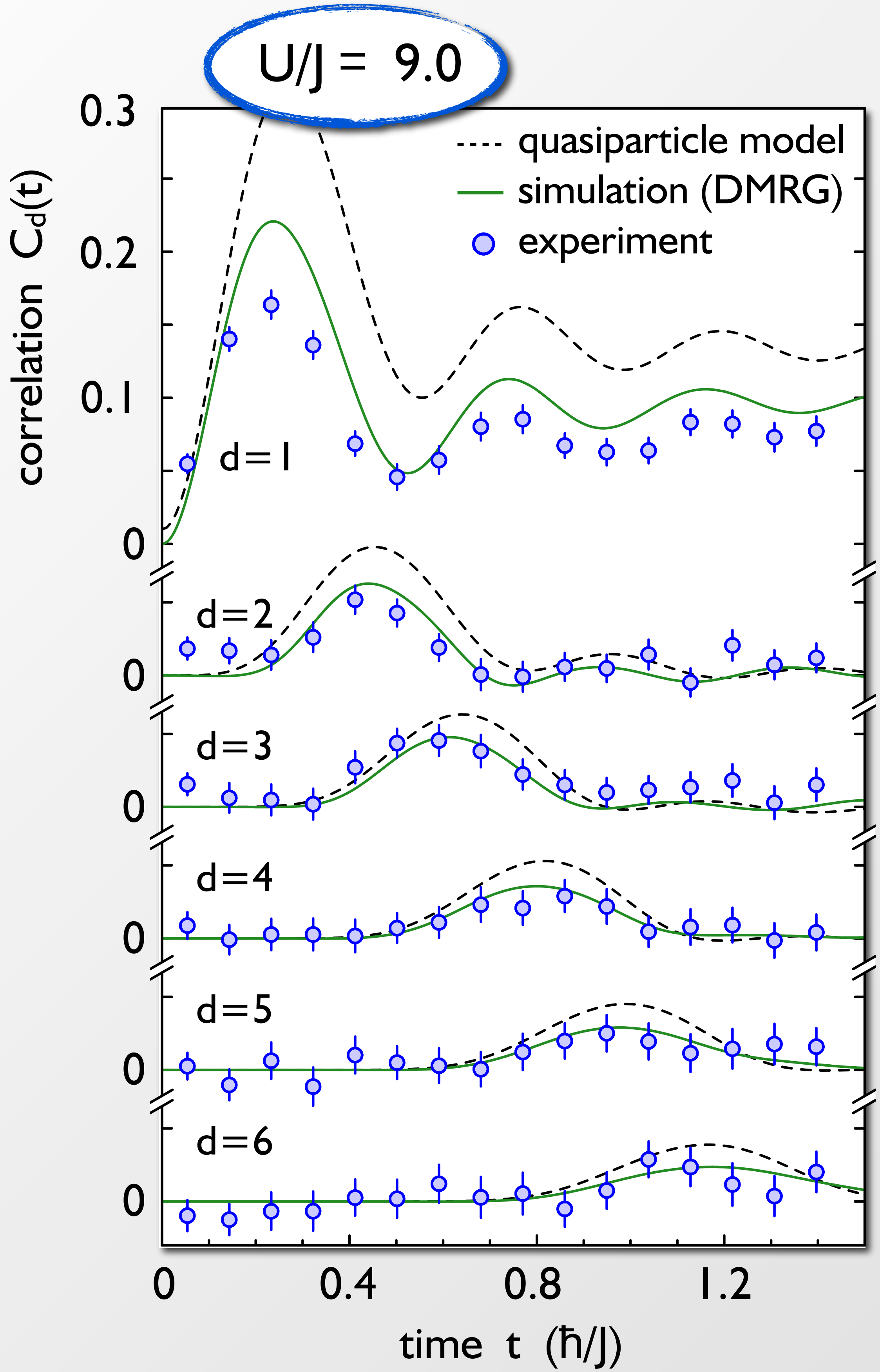


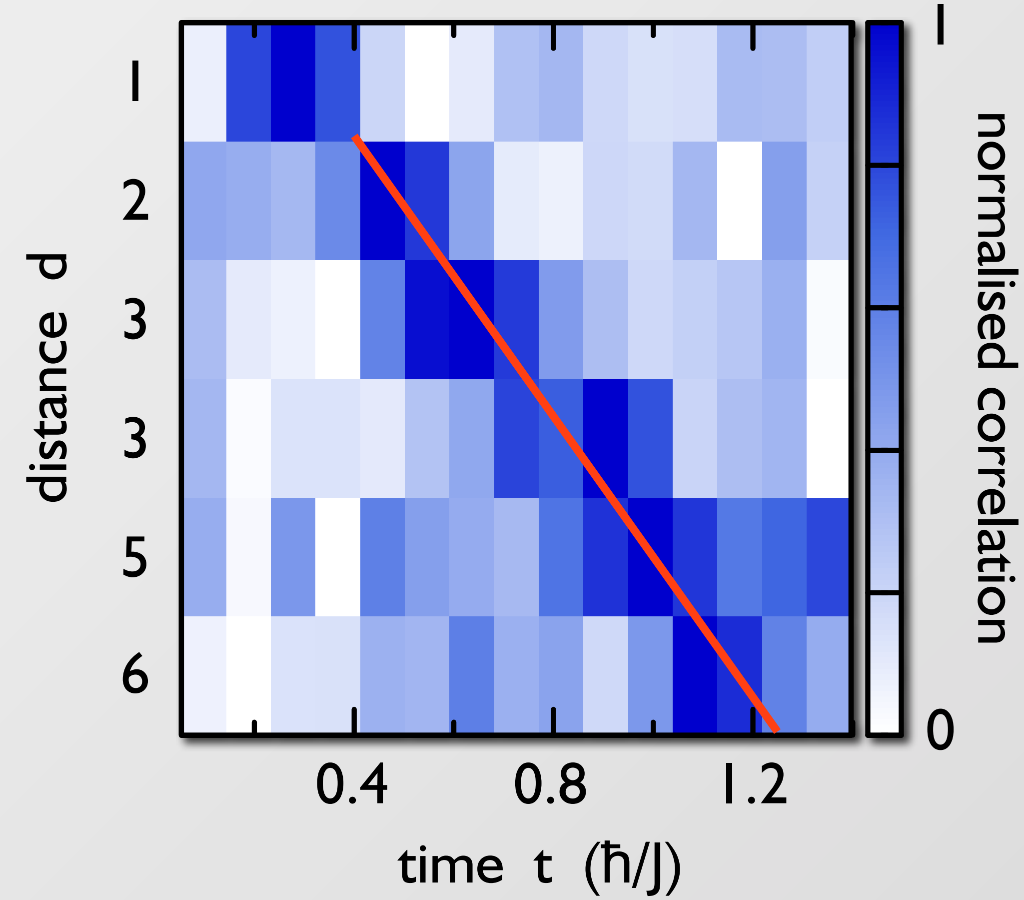
Light-cone like spreading of correlations



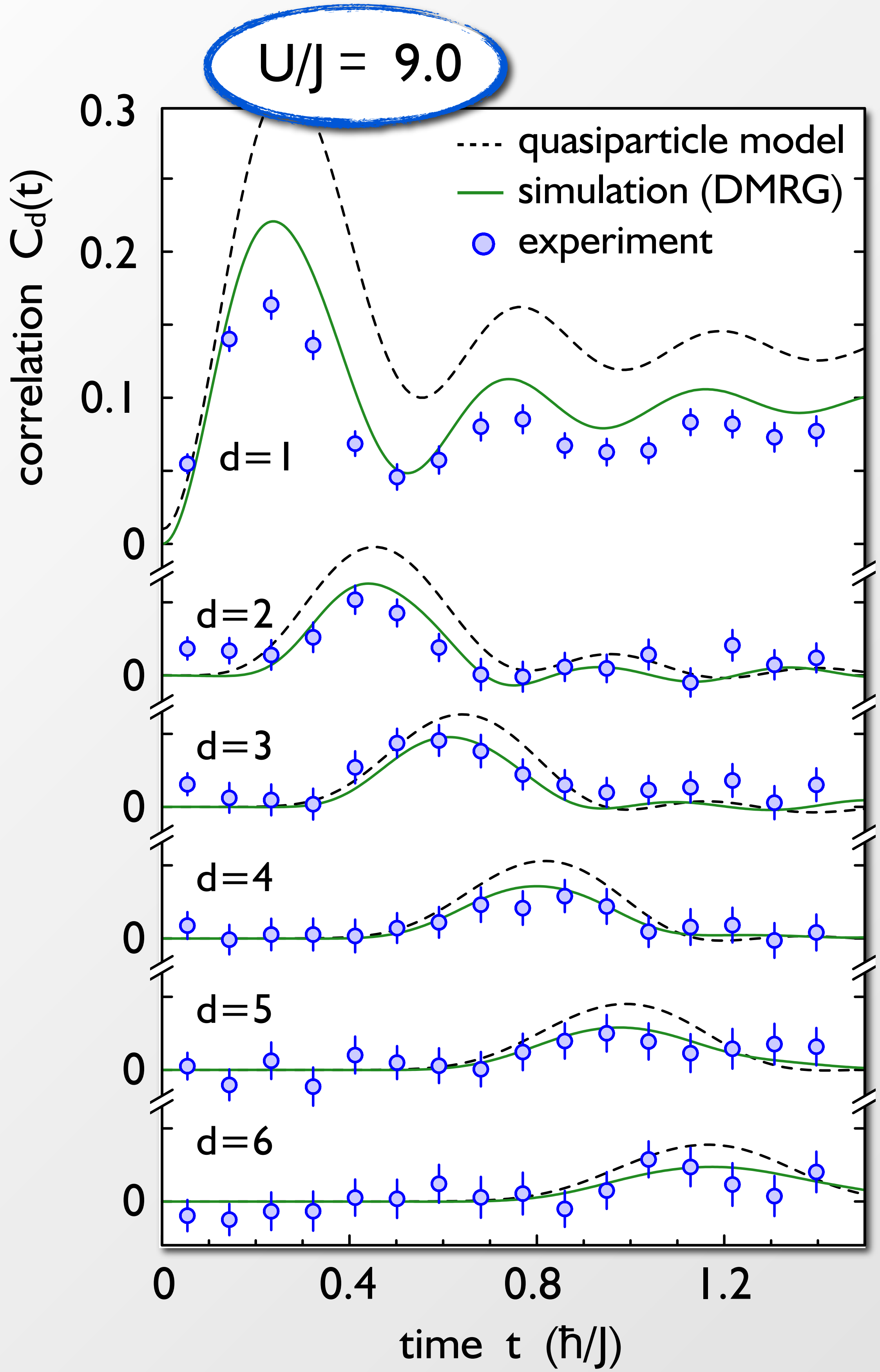


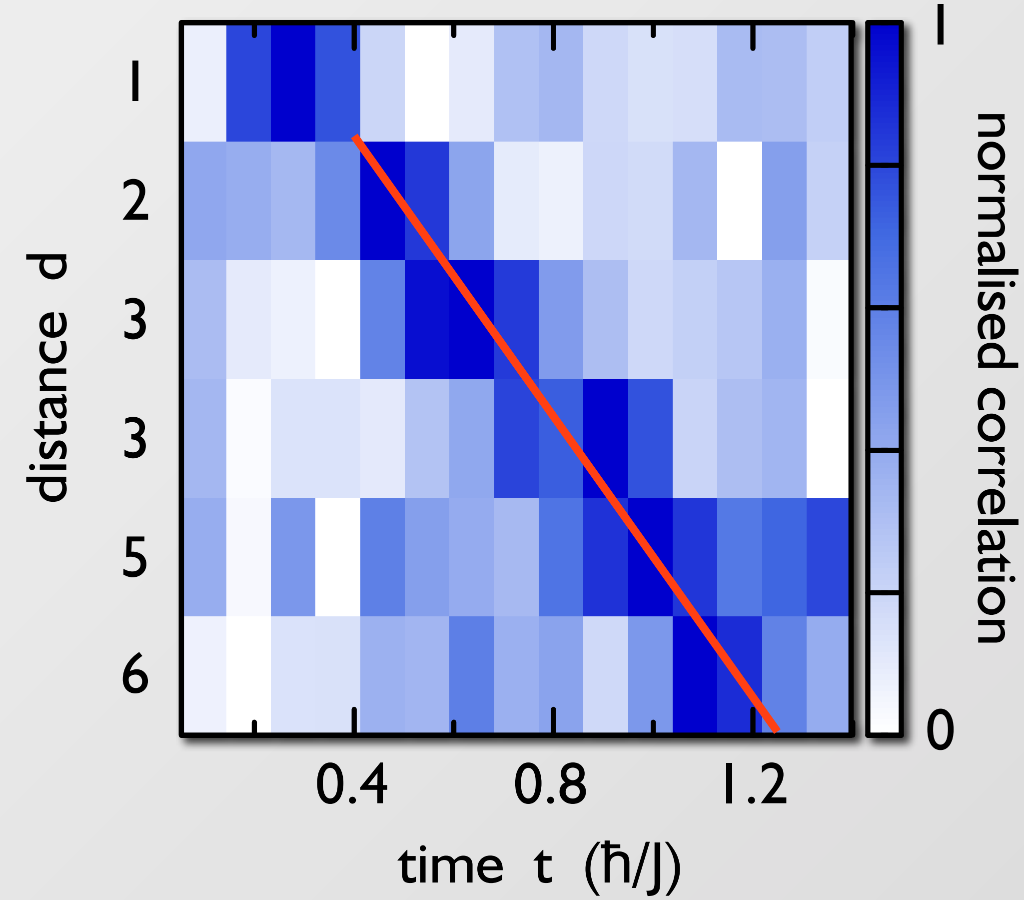
Light-cone like spreading of correlations





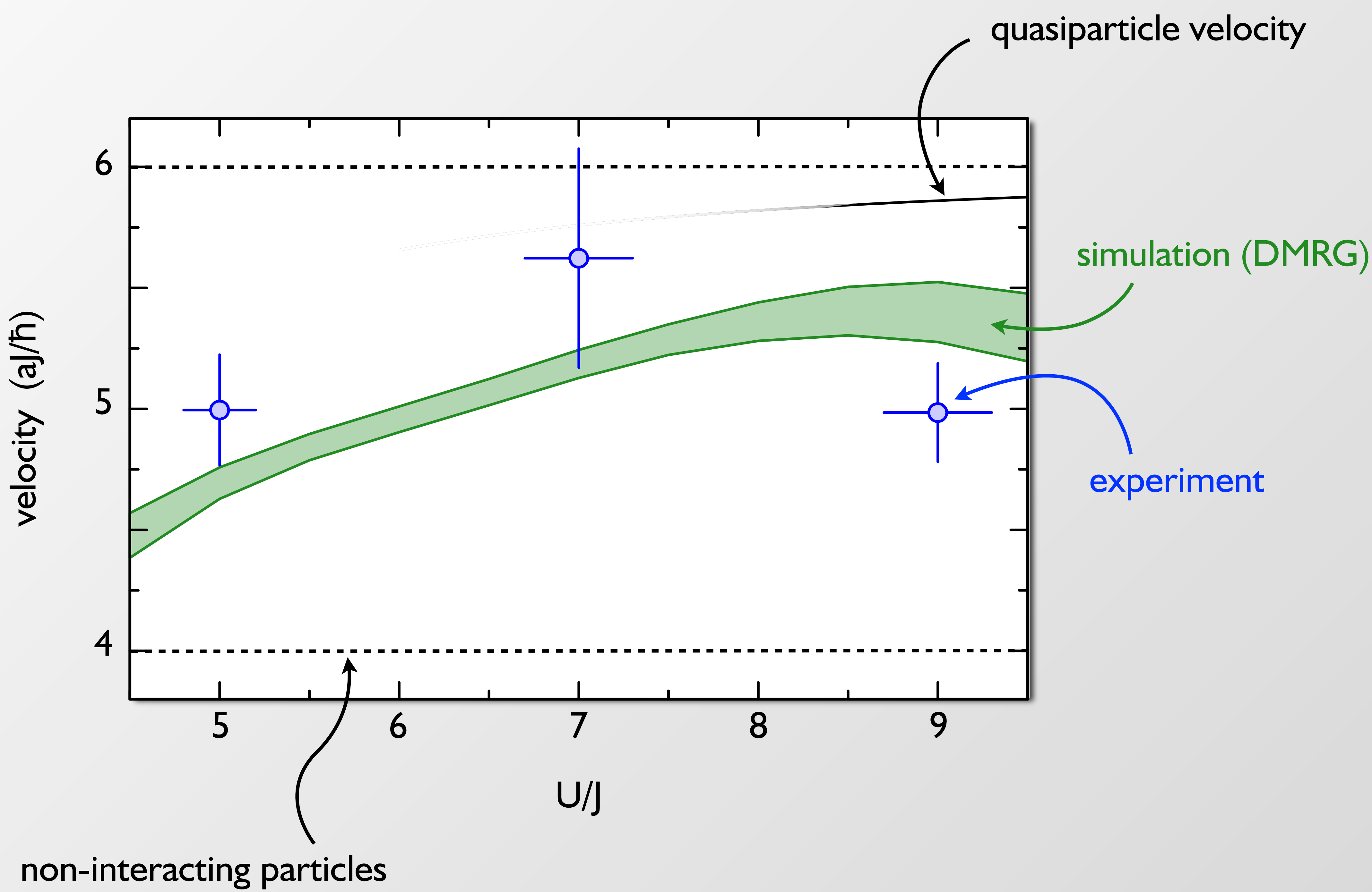
Light-cone like spreading of correlations

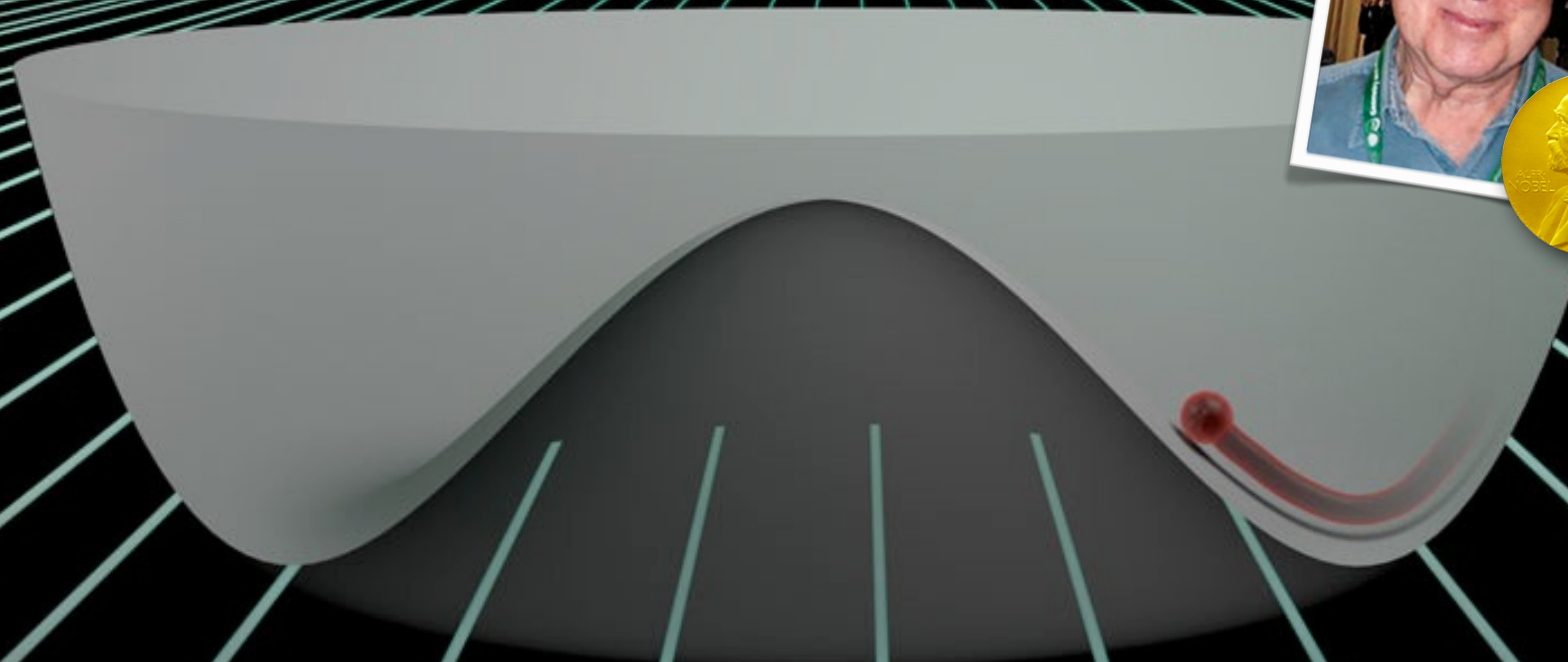




effective light-cone!

Spreading velocity





‘Higgs’ Amplitude Mode in Flatland

M. Endres, T. Fukuhara, M. Cheneau, P. Schauss, D. Pekker, E. Demler, S. Kuhr & I.B.

M. Endres et al. Nature (2012)
Chubukov & Sachdev, PRB 1993; Sachdev, PRB 1999; Zwerger, PRL 2004; Altman, Blatter, Huber, PRB 2007, PRL 2008; U. Bissbort et al. Phys. Rev. Lett. (2011); D. Podolsky, A. Auerbach, D. Arovas, PRB 2011

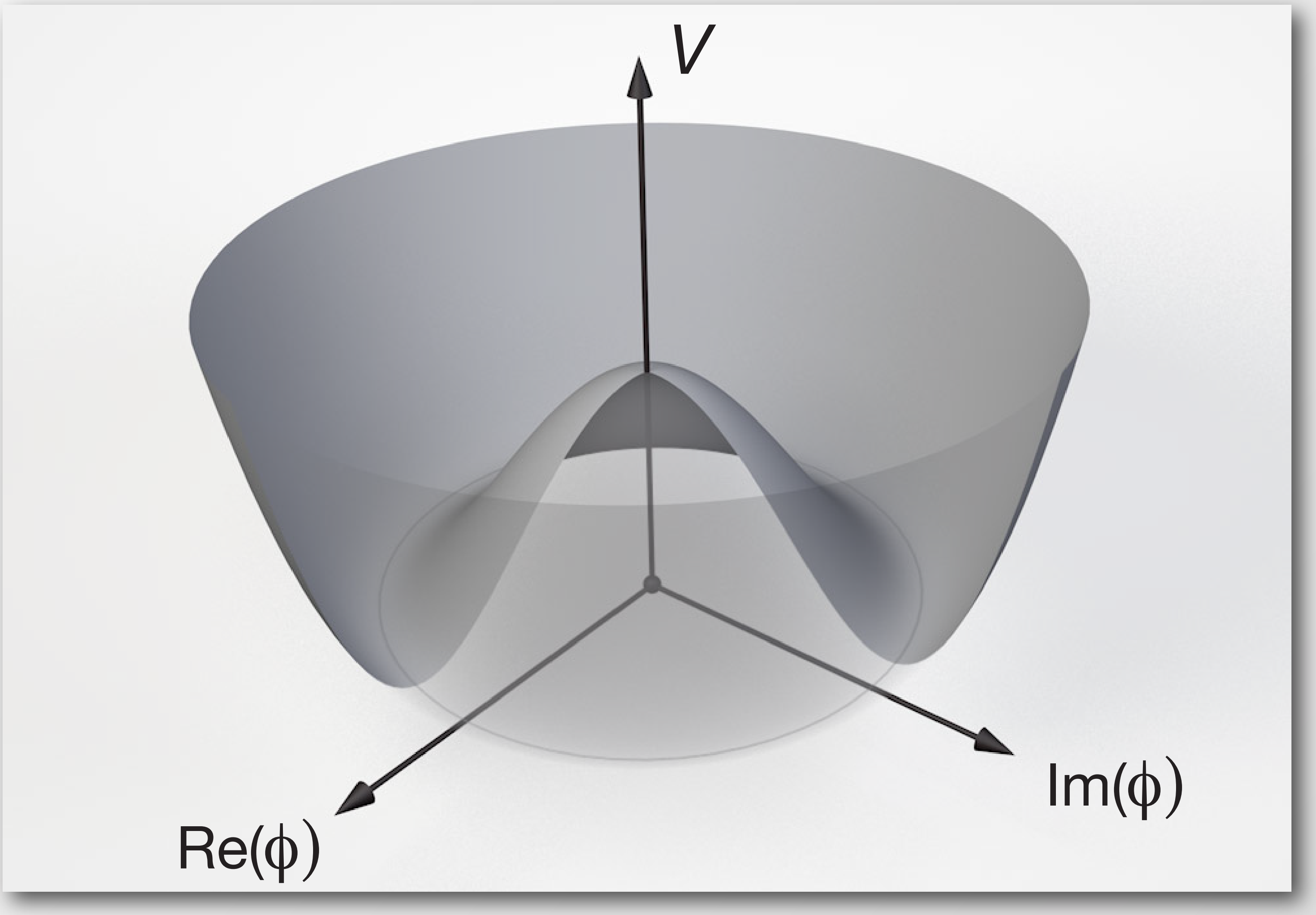
Spotaneous Symmetry Breaking

$$L = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{1}{2}\lambda (\phi^* \phi)^2$$

Relativistic Quantum Field-Theory of complex field ϕ with mass m.

$$L = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi - \frac{1}{2}\lambda (\phi^* \phi)^2$$

Imagine negative mass term.



$$L = \partial_\mu \phi^* \partial^\mu \phi - V(\phi)$$

$$\phi(x) \rightarrow \phi(x) e^{i\theta}$$

Lagrangian is U(1) symmetric

Spontaneous Symmetry Breaking - Modes

$$V(\phi) = -\frac{1}{2}\lambda v^2 \phi^* \phi + \frac{1}{2}\lambda(\phi^*\phi)^2 \qquad v^2 = -2m^2/\lambda$$

Minimum of Mexican Hat at: $|\phi|^2 = \frac{v^2}{2}$

Pick one vacuum state! Expand field around: $\phi = \frac{1}{\sqrt{2}}(v + \varphi_1 + i\varphi_2)$

$$L = \frac{1}{2}\left[(\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2)^2\right] - \frac{1}{2}\lambda v^2 \varphi_1^2 + \dots$$

φ_1, φ_2 real scalar fields

Spontaneous Symmetry Breaking - Modes

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Massless Nambu-Goldstone mode

φ_1, φ_2 real scalar fields

Spontaneous Symmetry Breaking - Modes

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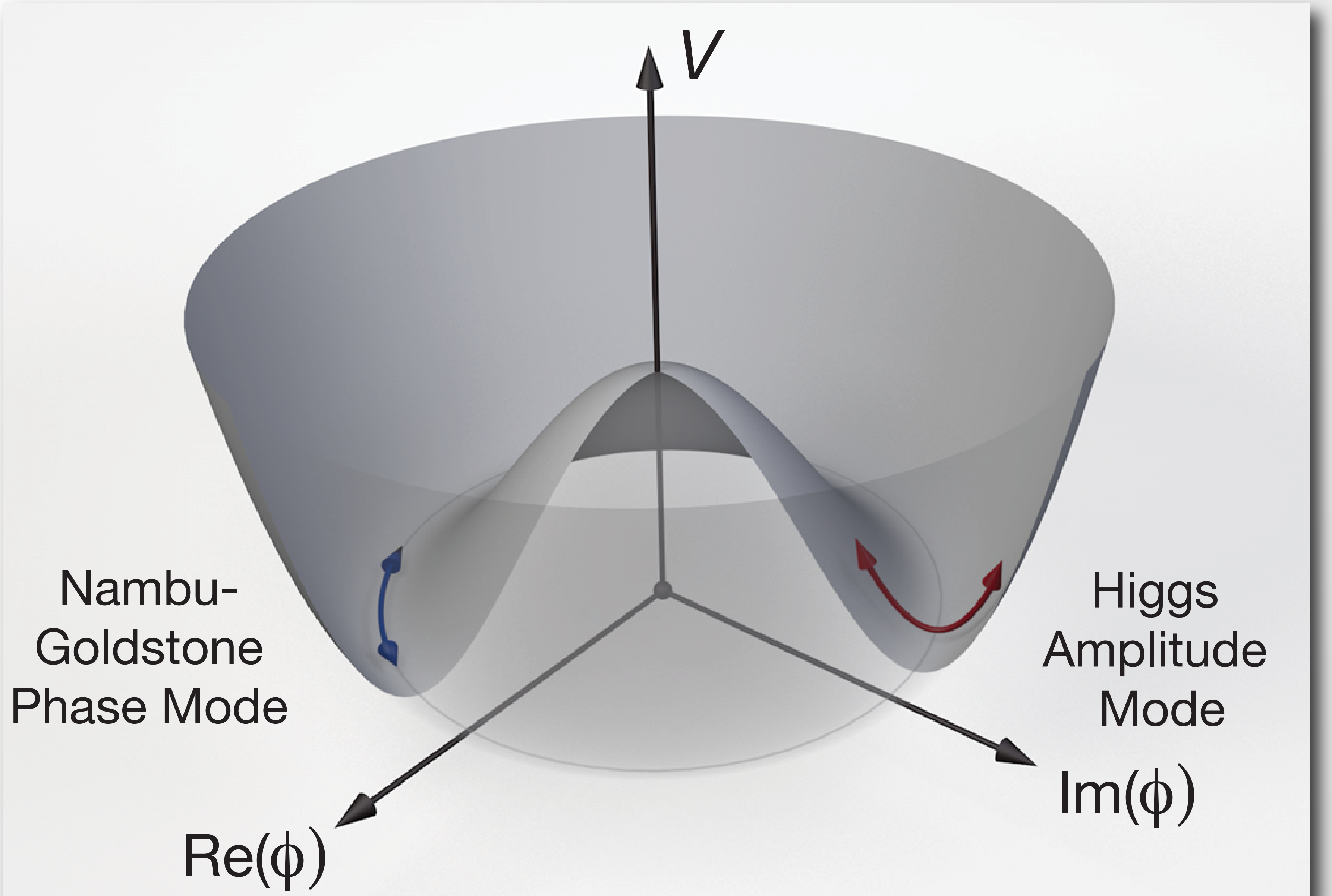
Massless Nambu-Goldstone mode

Massive Higgs mode

φ_1, φ_2 real scalar fields

Spontaneous Symmetry Breaking - Modes



Excitations in radial direction are gapped due to 'Higgs mass'!

Anderson-Higgs Mechanism

$\theta \to \theta(x)$ Extend to local U(1) gauge symmetry.

$A_\mu \to A_\mu(x) - \frac{1}{e}\partial_\mu \theta(x)$ introduces vector potential

$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi$ minimal coupling

$$L = D_\mu \phi^* D^\mu \phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)$$

Breaking symmetry leads to:

$$L = \frac{1}{2}(\partial \varphi_1)^2 + \frac{1}{2}(\partial_\mu \varphi_2 + evA_\mu)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\lambda v^2 \varphi_1^2 + \ldots$$

Photons have become massive (m^2=ev)! ⇒ Meissner effect Anderson 1963

Similiar for non-Abelian gauge theory U(1)xSU(2) ⇒ W,Z bosons acquire mass

Englert, Brout, Higgs, Guralnik, Hagen, Kibble, Weinberg ~1964

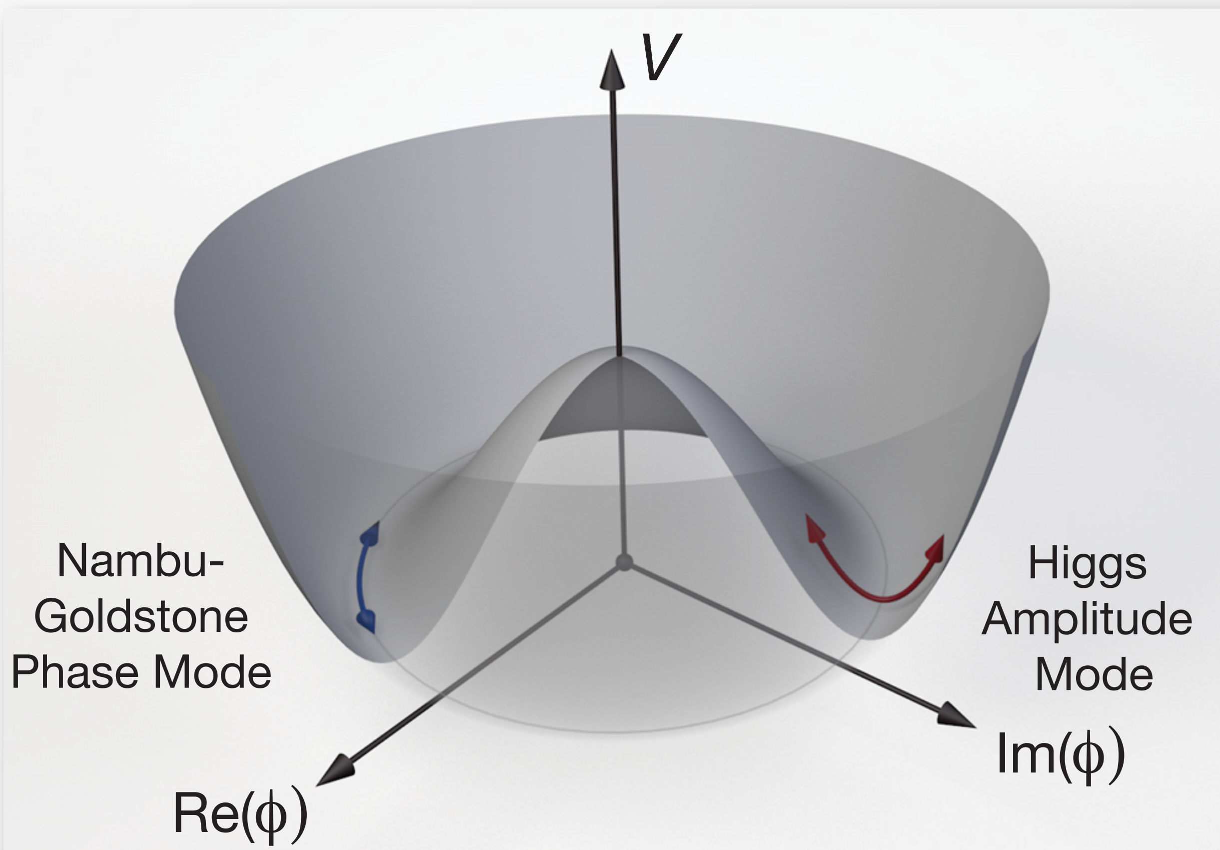
LMU

Higgs Mode in CM Physics

Close to a quantum critical point, effectively relativistic field theory!

see e.g.: Subir Sachdev, Quantum Phase Transitions

Here: SF-MI transition for n=1, O(2) field theory in 2+1 dimension



Fundamental question in 2D:

is mode observable or overdamped?

Chubukov & Sachdev, PRB 1993
Sachdev, PRB 1999; Zwerger, PRL 2004;
Altman, Blatter, Huber, PRB 2007, PRL 2008;
Menotti & Trivedi, PRB 2008; Podolsky, Auerbach, Arovas, PRB 2011; Pollet & Prokof'ev PRL 2012; Sachdev & Podolsky, PRB 2012; ...

Other systems: Quantum spin systems O(3) in 3+1 dimensions

Ch. Rüegg et al. Physical Review Letters (2008)

in superconductors coupled to CDW:

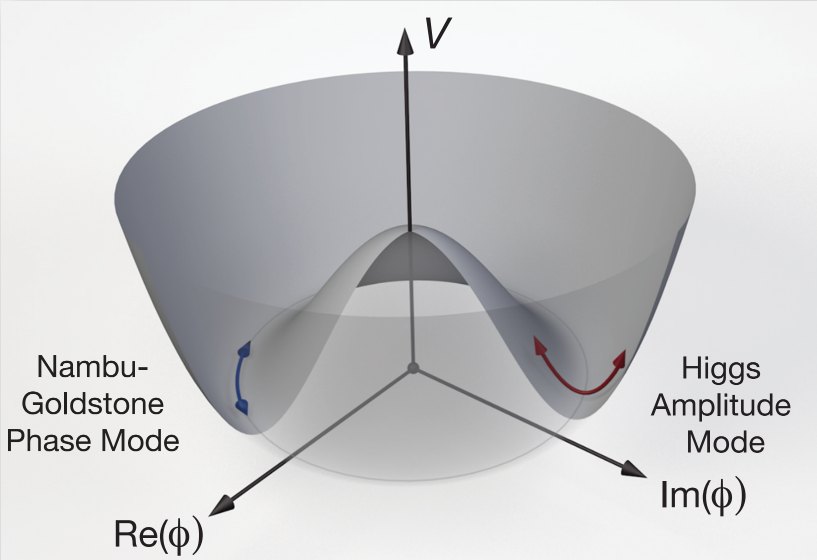
C. Varma & P. Littlewood PRL, PRB (1981,1982)

LMU

Higgs Mode in CM Physics

Theoretically difficult/debated problem!

Sachdev, QPT book, 2011:
„...., for N>1, d<3, there **is no Higgs particle**...“
Bose-Hubbard in 2d?



Higgs Mode in CM Physics

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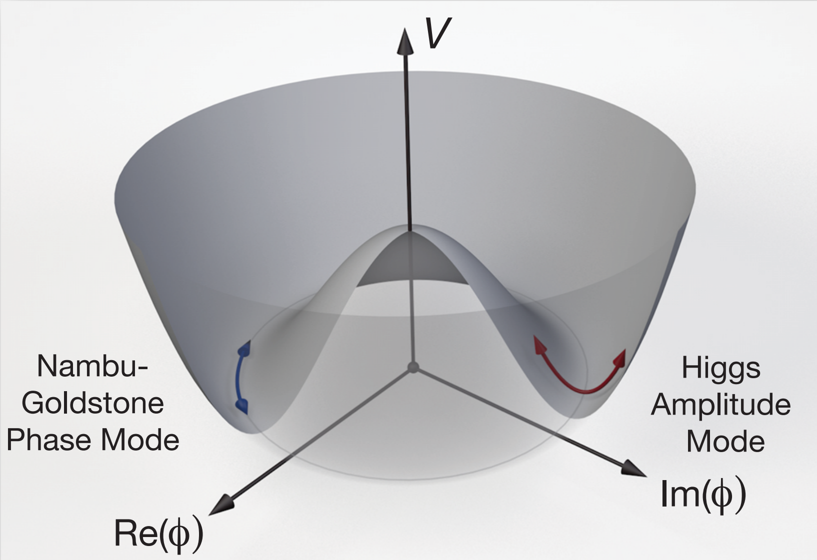


Podolsky, 2011: **It‘s there!**
- use „correct“ response function
- Lowest-order analysis
- Some weird properties

Experiment, 2012: **It‘s there**!
- Measure „correct“ response function
- Finite temp.
- Trap

Pollet, 2012: **It‘s there!**
- Quantum Monte Carlo
- Reliable?
- Calculation in im. time

Podolsky, Sachdev, 2012:
It‘s there!
Higher order analysis
Very cumbersome
Not reliable for N=2

Dynamics in the Superfluid Phase

Far from the Mott lobe, SF described by Gross-Pitaevskii action:

$$S = \int d^3r dt \left(-i\psi^* \partial_t \psi - \frac{1}{2m^*} |\nabla \psi|^2 + \mu |\psi|^2 - g|\psi|^4 \right)$$

Galilean invariant. Predicts massless Goldstone mode, but no Higgs mode.

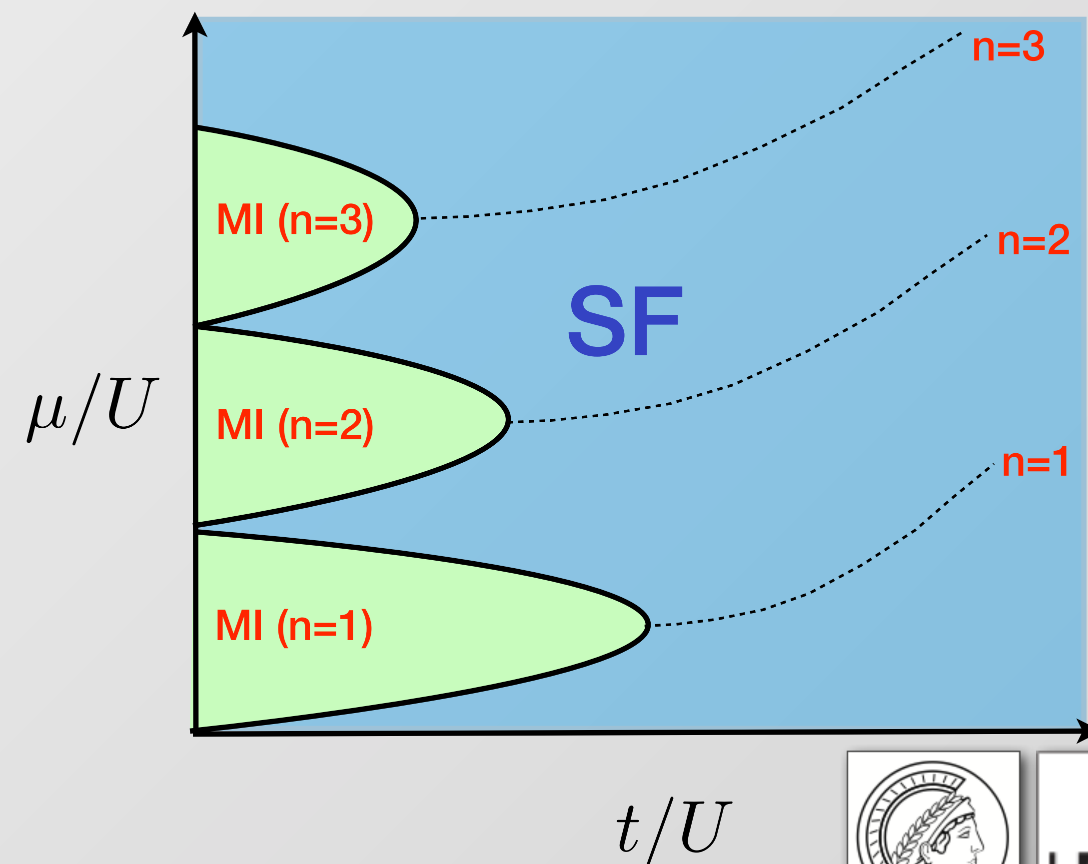
Imported Author 23 Oct 2013, 7:24
GPE: Phase and amplitude mode are c.c. variables! Therefore only one mode!

Close to QCP: Phase and ampltiude of order parameter commute: two excitation degrees of freedom.

Near the Mott lobe at integer filling, particle-hole symmetry leads to relativistic dynamics:

$$S = \int d^3r\, dt \left(|\partial_t \psi|^2 - c^2 |\nabla \psi|^2 + r|\psi|^2 - u|\psi|^4 \right)$$

*Lorentz invariant. Predicts Goldstone mode **and** Higgs mode.*



Courtesey: Danny Podolsky (Technion)

Relativistic vs Non-Relativistic Dynamics

Density

$$n = |\Psi|^2$$

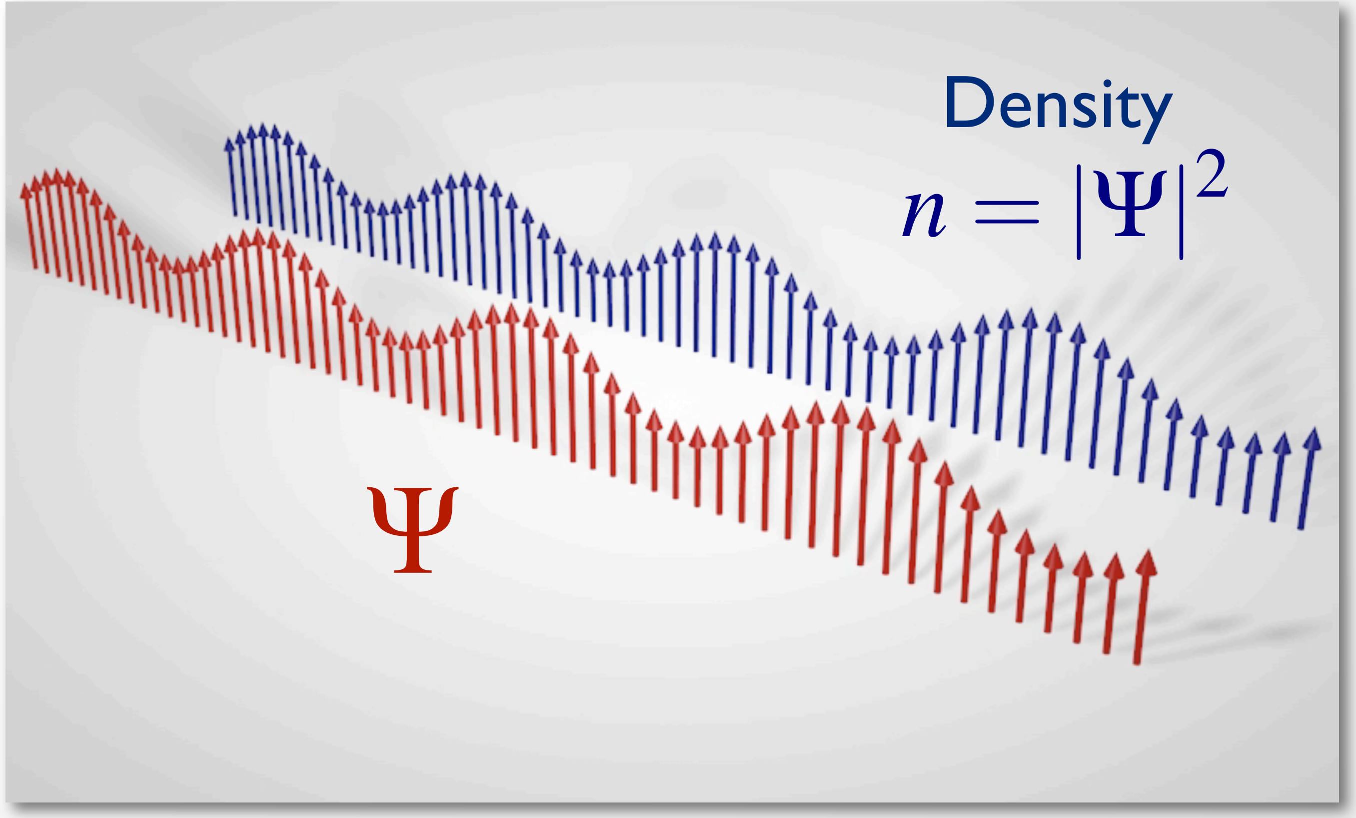
Ψ

Weakly Interacting BEC

(non-relativistic)

$$\omega(\tilde{k}) = \sqrt{\mu^2 \tilde{k}^2 (\tilde{k}^2 + 2)}$$

Relativistic vs Non-Relativistic Dynamics

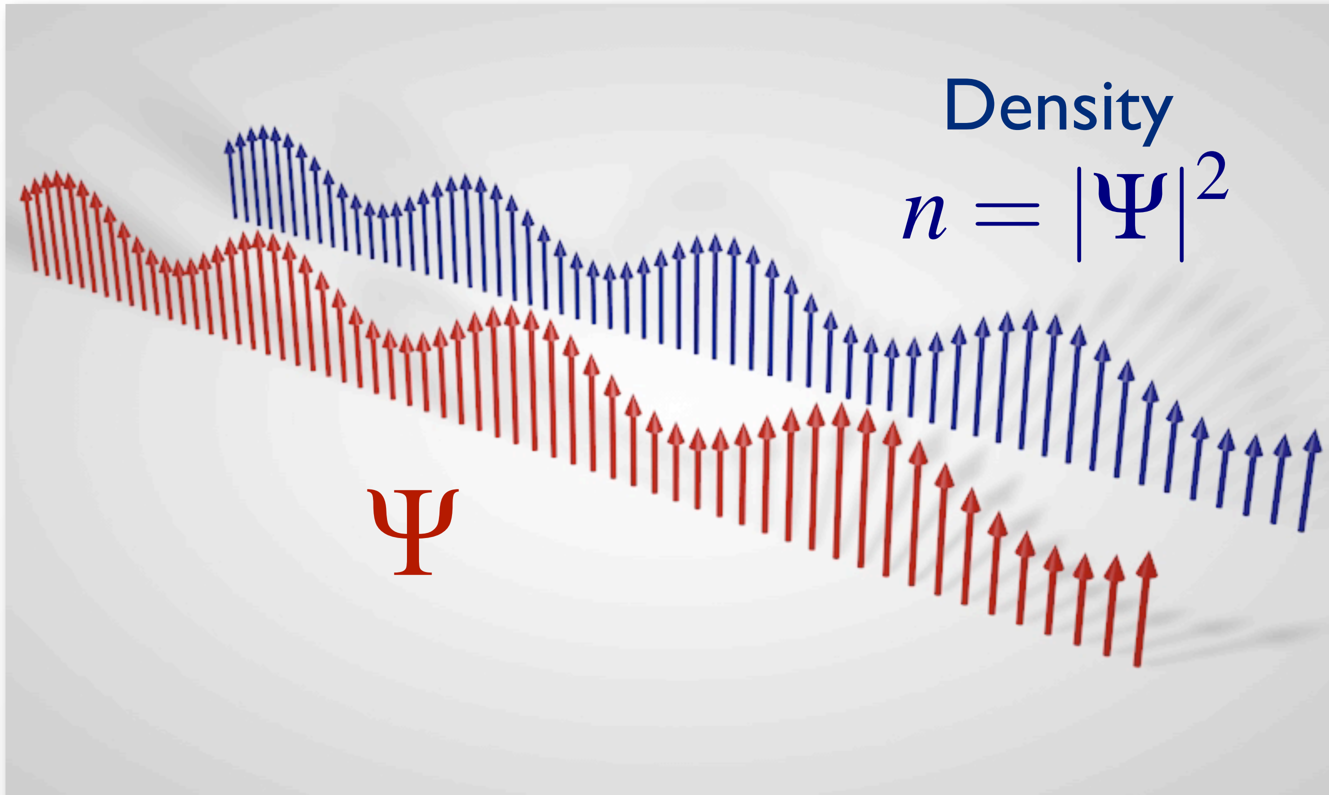


Weakly Interacting BEC

(non-relativistic)

$$\omega(\tilde{k}) = \sqrt{\mu^2 \tilde{k}^2 (\tilde{k}^2 + 2)}$$

Relativistic vs Non-Relativistic Dynamics



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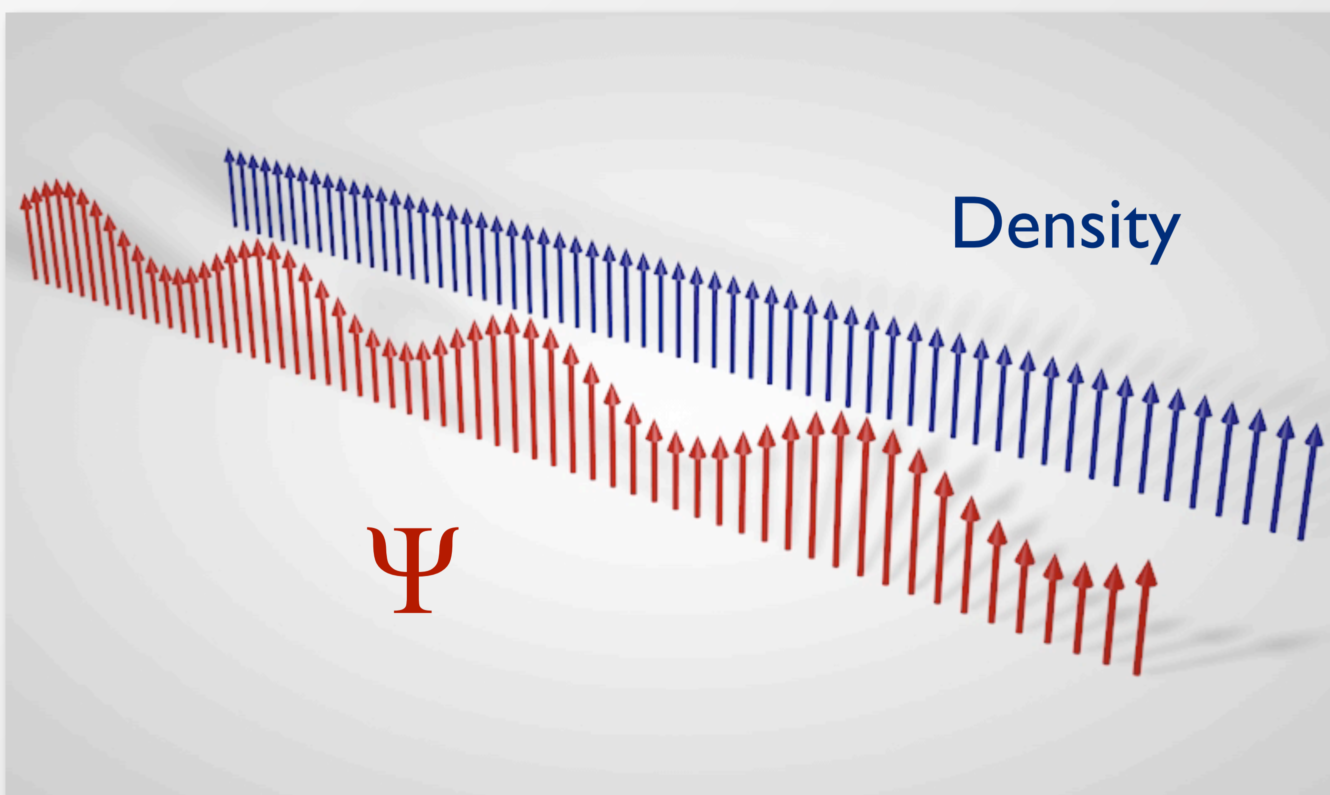


SF @ Quantum Critical Point

(relativistic)

$$\omega_1(k) = \sqrt{\Delta_0^2 + c_s^2 k_1^2}$$

$$\omega_2(k) = c_s k$$

Relativistic vs Gross-Pitaevskii Dynamics

From Euler-Lagrange equation, we obtain:

Lorentz invariant action

$$\ddot{\varphi}_1 = c_s^2 \nabla^2 \varphi_1 - \Delta_0^2 \varphi_1$$
$$\ddot{\varphi}_2 = c_s^2 \nabla^2 \varphi_2$$

$$\omega_1(k) = \sqrt{\Delta_0^2 + c_s^2 k_1^2}$$ Relativistic Mode — Amplitude!

$$\omega_2(k) = c_s k$$ Sound Mode — Density!

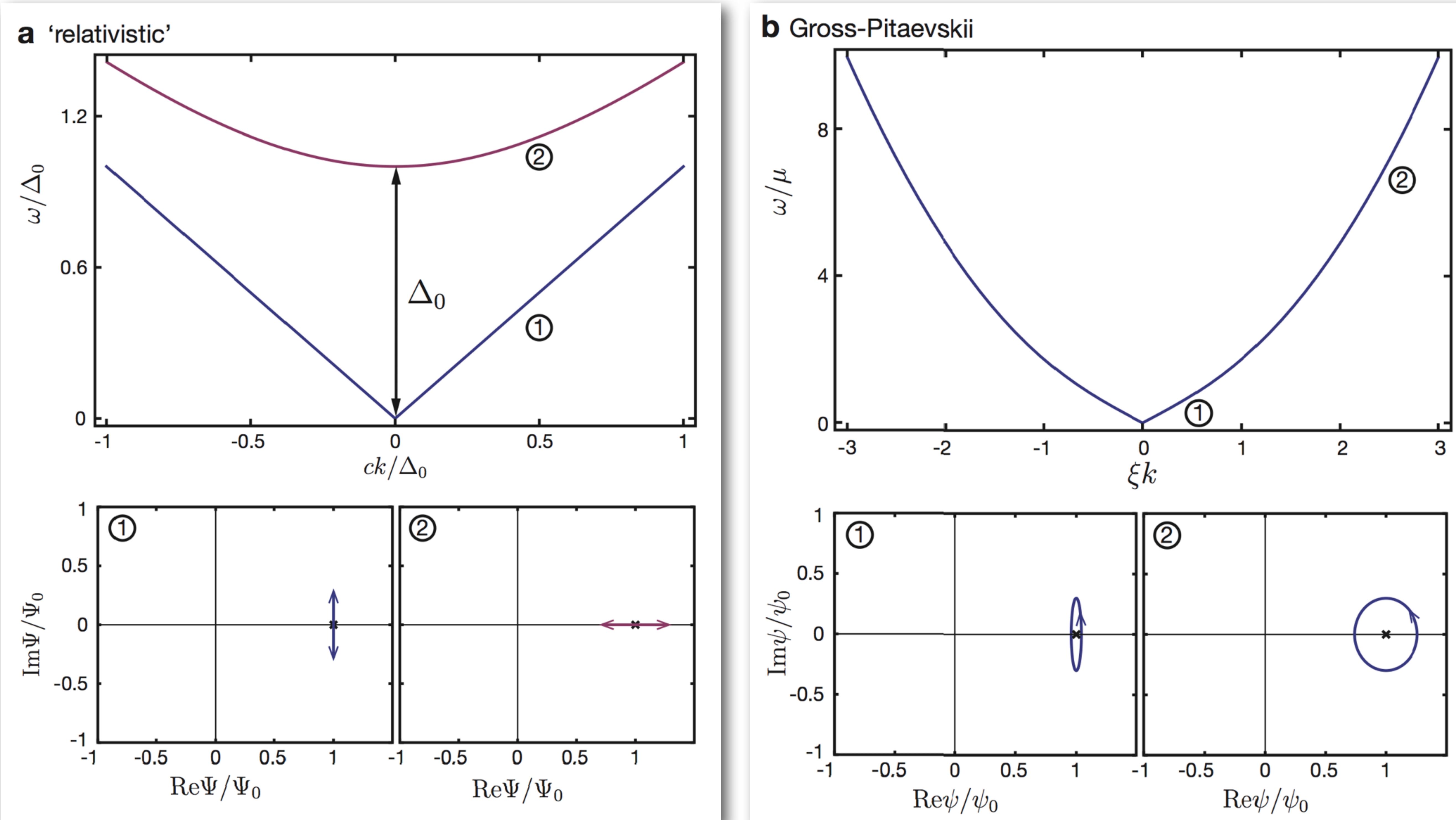
Galilean invariant action

$$-\dot{\varphi}_1 = \frac{\hbar^2}{2m} \nabla^2 \varphi_2$$
$$\dot{\varphi}_2 = \frac{\hbar^2}{2m} \nabla^2 \varphi_1 - 2\mu \varphi_1$$

$$\omega(\tilde{k}) = \sqrt{\mu^2 \tilde{k}^2 (\tilde{k}^2 + 2)}$$ Bogoliubov Mode

Amplitude-Density Coupled!

Relativistic vs Gross-Pitaevskii Dynamics



'Relativistic'
Lorentz Invariant

'Classical
Galilean Invariant

Broken Symmetry and Collective Modes

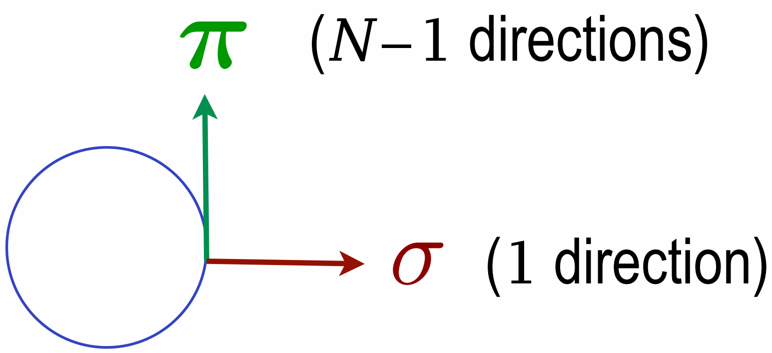
Courtesey: Danny Podolsky (Technion)

Broken Symmetry and Collective Modes

Two ways to parameterize deviations from the ordered state :

I) Cartesian : $\phi = (\sqrt{N} + \sigma, \boldsymbol{\pi})$

$$\mathcal{L}_0 = \frac{1}{2g}\left[(\partial_\mu \sigma)^2 - m^2\sigma^2 + (\partial_\mu \boldsymbol{\pi})^2\right]$$



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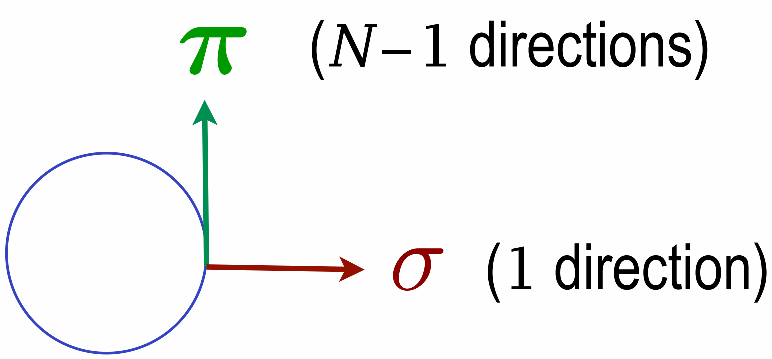
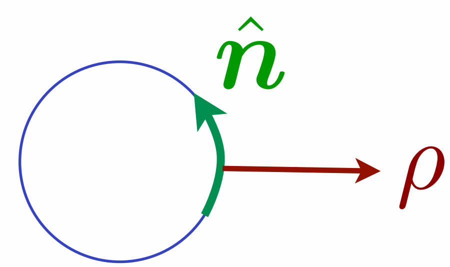
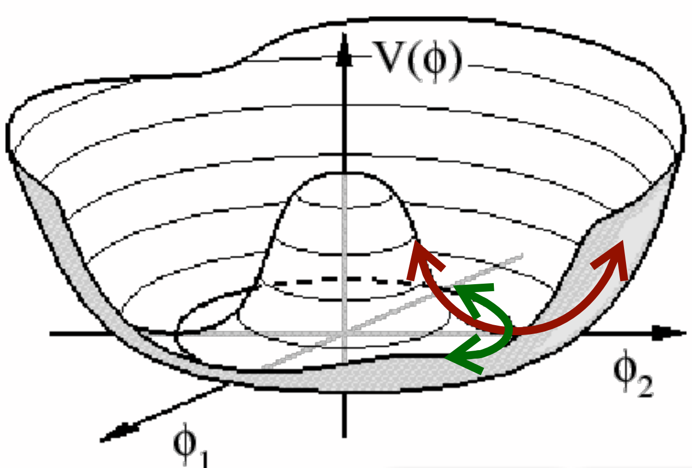


2) Polar : $\phi = \sqrt{N}\,(1+\rho)^{1/2}\,\hat{\boldsymbol{n}}$





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Broken Symmetry and Collective Modes

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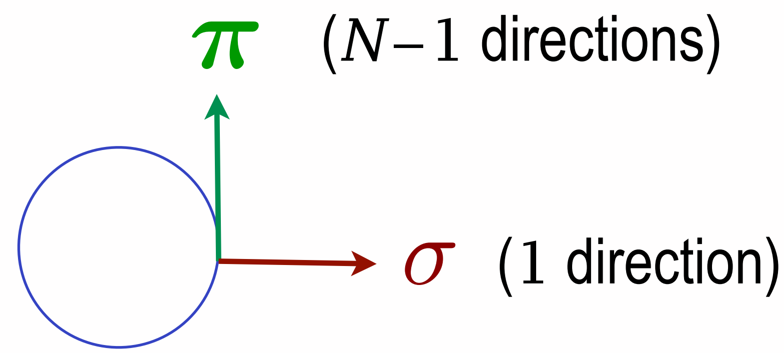
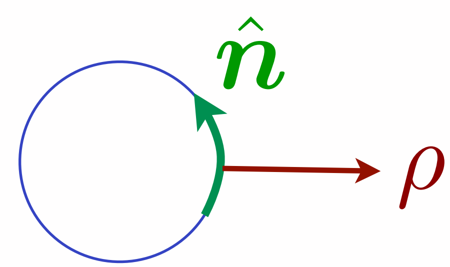


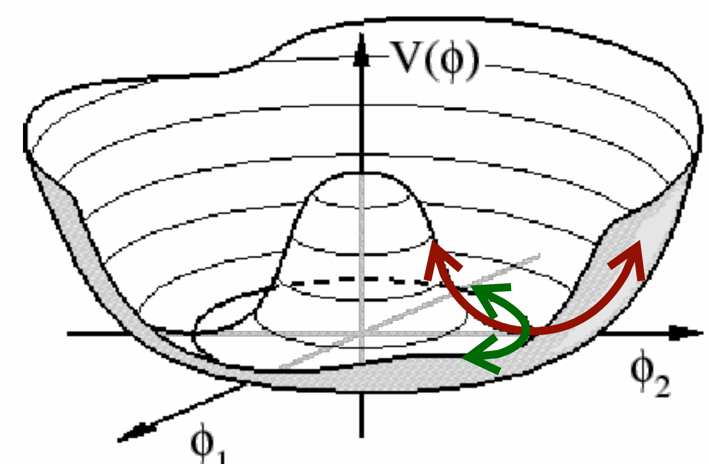
$$\mathcal{L}_0 = \frac{1}{2g}\left[(\partial_\mu \sigma)^2 - m^2\sigma^2 + (\partial_\mu \boldsymbol{\pi})^2\right]$$

$$\mathcal{L}_1 = \frac{m^2}{2g}\left[\frac{1}{\sqrt{N}}\,\sigma\boldsymbol{\pi}^2 + \frac{1}{\sqrt{N}}\,\sigma^3 + \frac{1}{4N}\,\sigma^4 + \frac{2}{N}\,\sigma^2\boldsymbol{\pi}^2 + \frac{1}{4N}(\boldsymbol{\pi}^2)^2\right]$$

2) Polar : $\phi = \sqrt{N}\,(1+\rho)^{1/2}\,\hat{\boldsymbol{n}}$



$$\mathcal{L} = \frac{1}{2g}\left[N(1+\rho)(\partial_\mu \hat{\boldsymbol{n}})^2 + \frac{(\partial_\mu \rho)^2}{4(N+\rho)} + \frac{m^2\rho^2}{4N}\right]$$

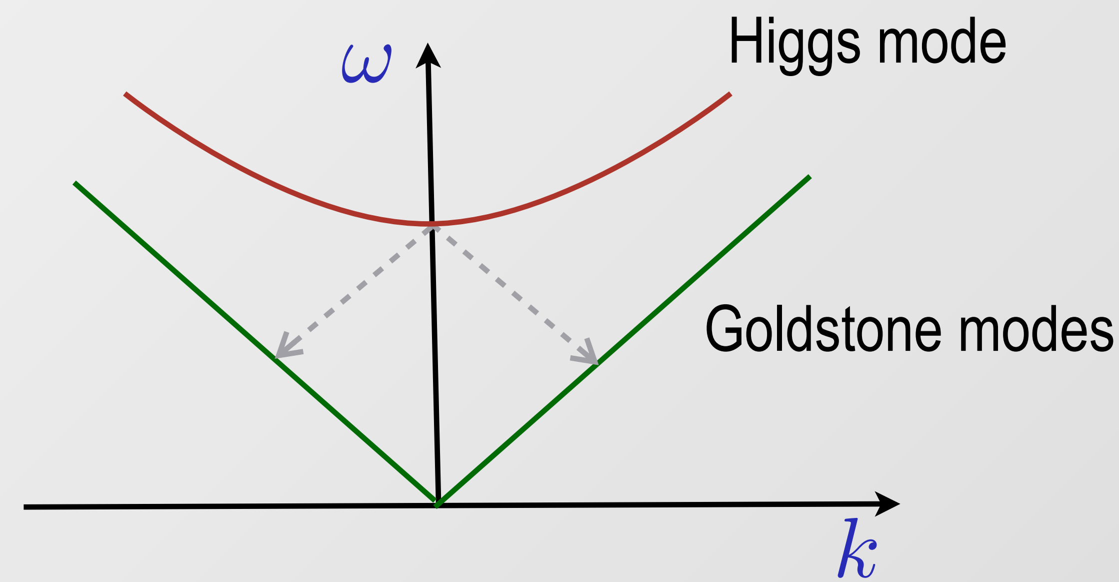


Courtesey: Danny Podolsky (Technion)

Lifetime of Higgs Excitation

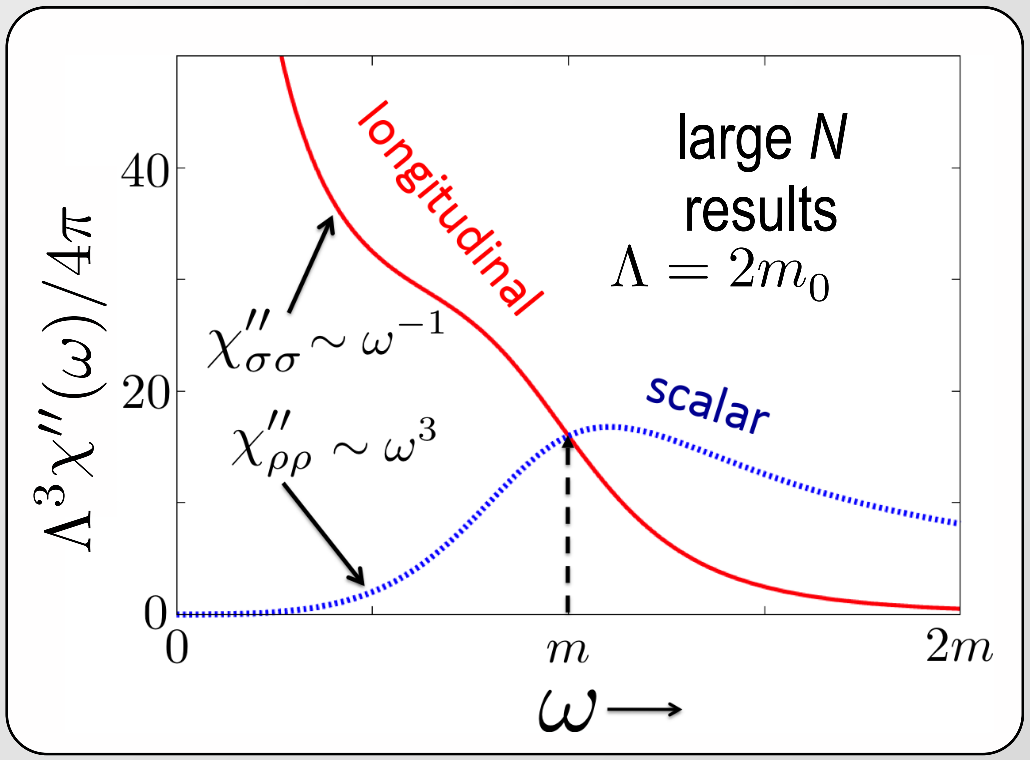
It can decay into a pair of Goldstone bosons :

$$\mathcal{L}_{\text{int}} \propto \begin{cases} \sigma \boldsymbol{\pi}^2 & \text{(Cartesian)} \\ \rho \left(\partial_\mu \hat{\boldsymbol{n}}\right)^2 & \text{(polar)} \end{cases}$$



Cartesian and polar calculations correspond to *different correlation functions.*

Depends on the type of experiment performed.



Courtesey: Danny Podolsky (Technion)

D. Podolsky, D., Auerbach, A. & Arovas, Phys. Rev. B 84, 174522 (2011)
L. Pollet, N. Prokof'ev, Phys. Rev. Lett. 109, 010401 (2012)
S. Gazit, D. Podolsky, A. Auerbach, Phys. Rev. Lett. 110, 140401 (2013)
D. Podolsky and S. Sachdev, Phys. Rev. B 86, 054508 (2012)

LMU

Higgs Response Function

The longitudinal response function is measured by an experiment where the probe couples directly to the order parameter field:

$$S_{\text{probe}} = \int d^d x \int dt \, \boldsymbol{h}(\boldsymbol{x}, t) \cdot \boldsymbol{\phi}(\boldsymbol{x}, t)$$

Example : neutron scattering in an antiferromagnet.

Courtesey: Danny Podolsky (Technion)

Higgs Response Function

The longitudinal response function is measured by an experiment where the probe couples directly to the order parameter field:

$$S_{\mathrm{probe}} = \int d^d x \int dt\, \boldsymbol{h}(\boldsymbol{x},t) \cdot \boldsymbol{\phi}(\boldsymbol{x},t)$$

Example : neutron scattering in an antiferromagnet.

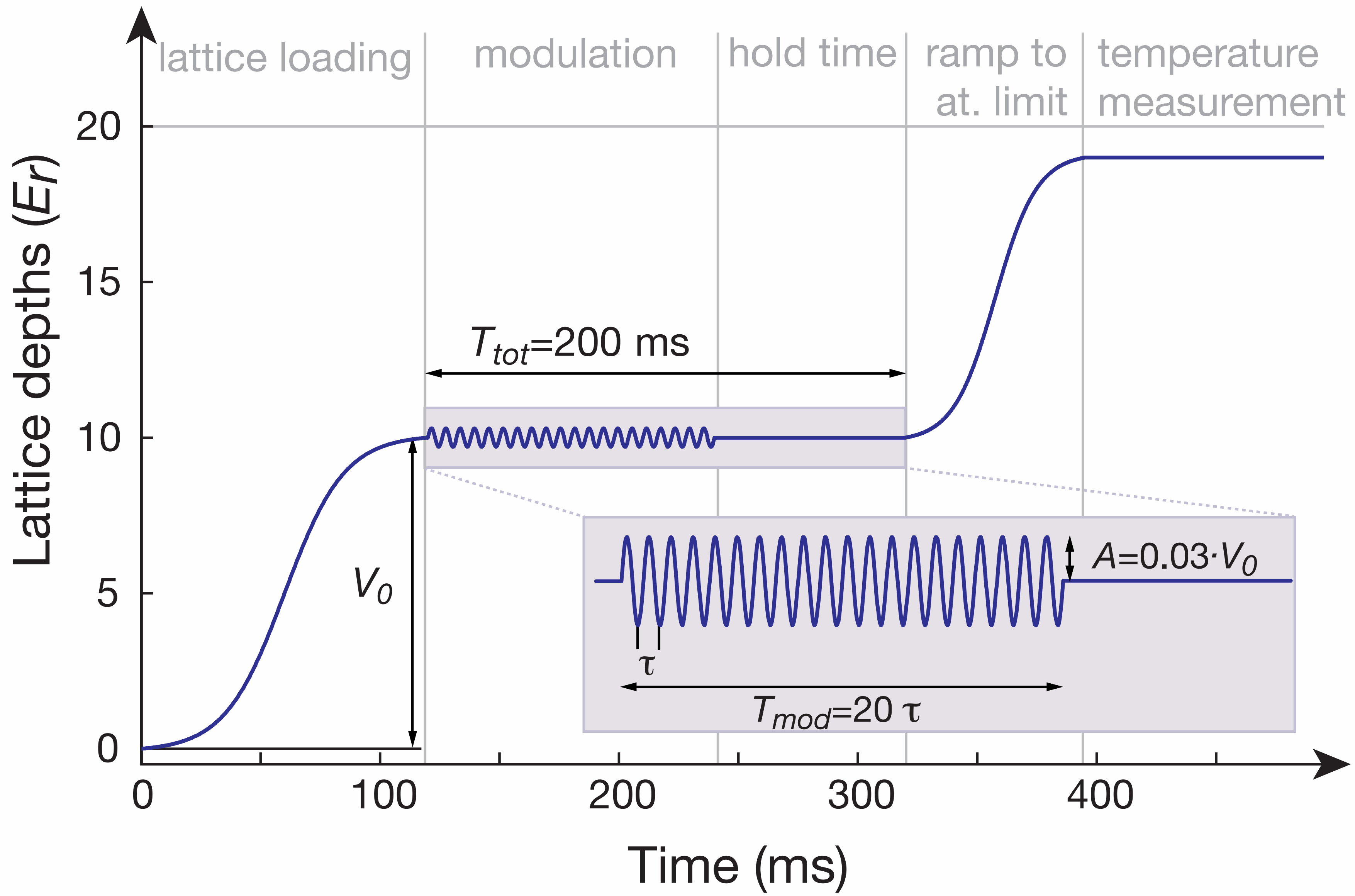
The scalar response function is measured by an experiment where the probe couples directly to the magnitude of the order parameter field:

$$S_{\mathrm{probe}} = \int d^d x \int dt\, u(\boldsymbol{x},t) \left|\boldsymbol{\phi}(\boldsymbol{x},t)\right|^2 \qquad |\phi|^2 = N(1+\rho)$$

Examples : lattice modulation spectroscopy

Courtesey: Danny Podolsky (Technion)

Exciting the Amplitude Mode



Absorbed energy

$$E = 2\pi(\delta J)^2 S(\omega)\omega T_{\text{mod}}$$

Very low modulation amplitude!

Very sensitive temperature measurement!

Exciting the Amplitude Mode

V

Modulate coupling strength close to Quantum Phase Transition!

$$j = j + \delta j \sin(\omega t)$$

$$j = J/U$$

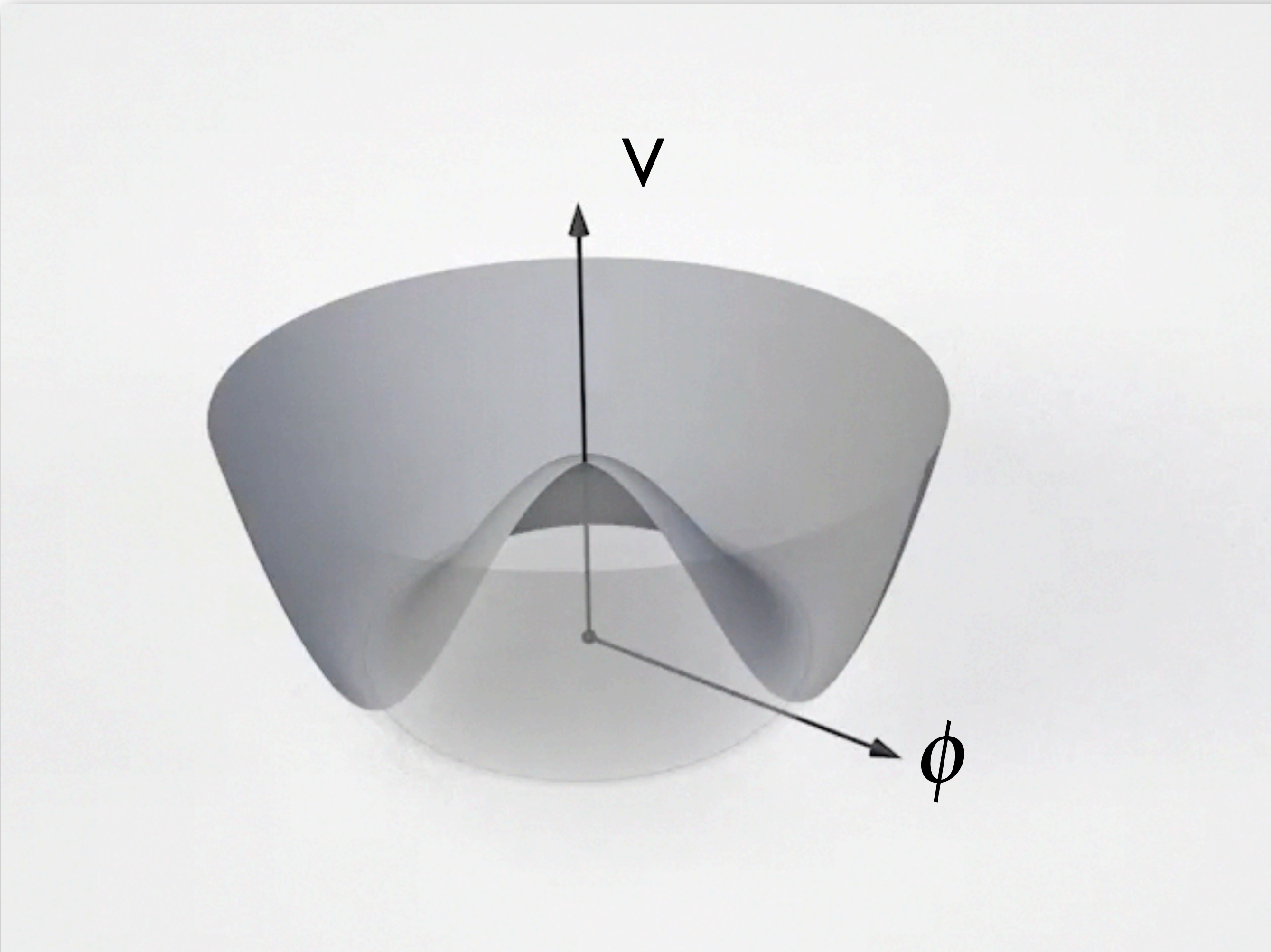
ϕ

Amplitude Modulation of Lattice

Bragg spectroscopy: couples mainly to phonons

Exp.: Ch. Schori et al. Phys. Rev. Lett. (2004) (ETHZ), Theory: C. Kollath et al., Phys. Rev. Lett (2006)
U. Bissbort et al. Phys. Rev. Lett. (2011) (Frankfurt, Hamburg)

Exciting the Amplitude Mode



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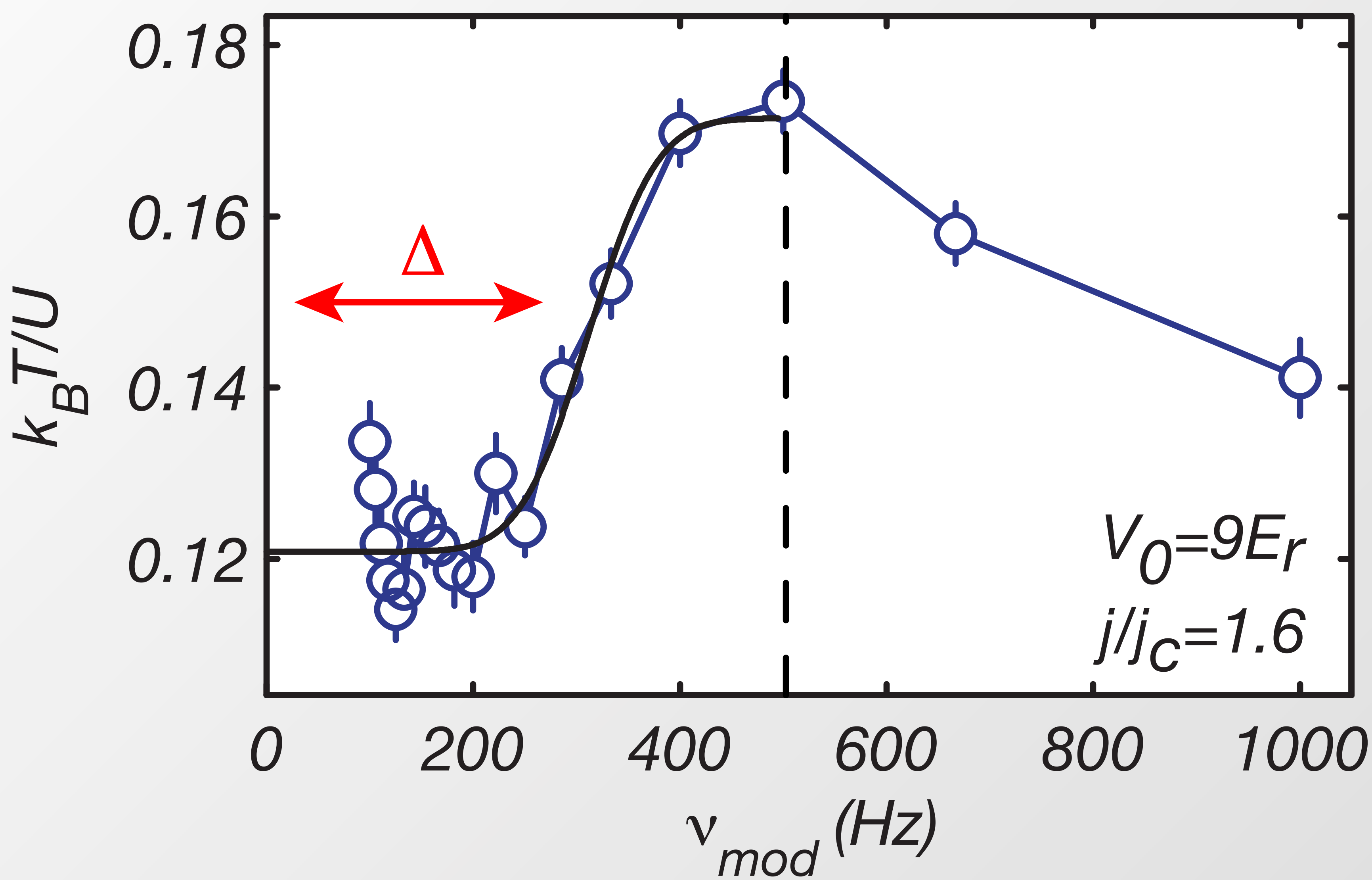
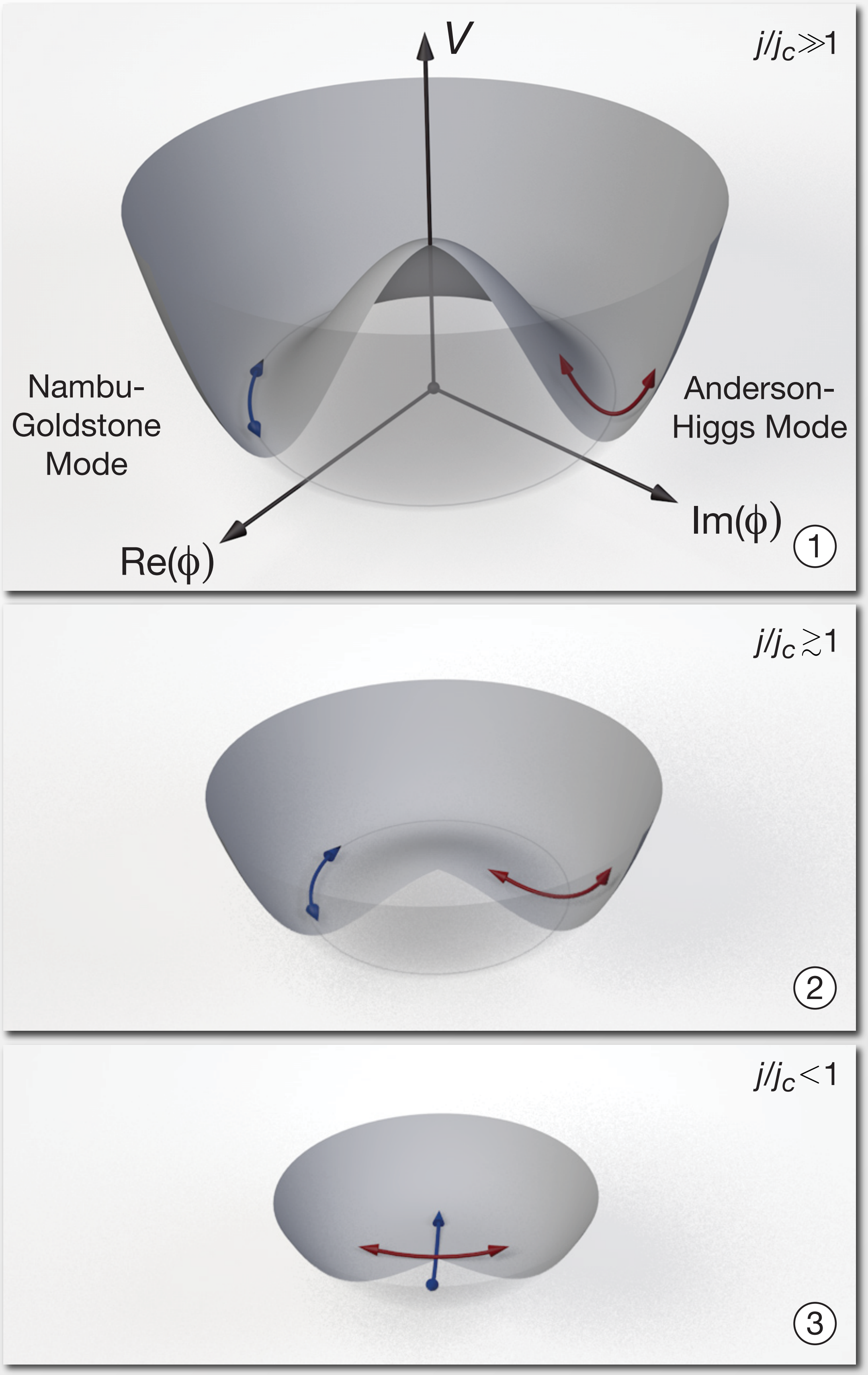
Single Spectrum



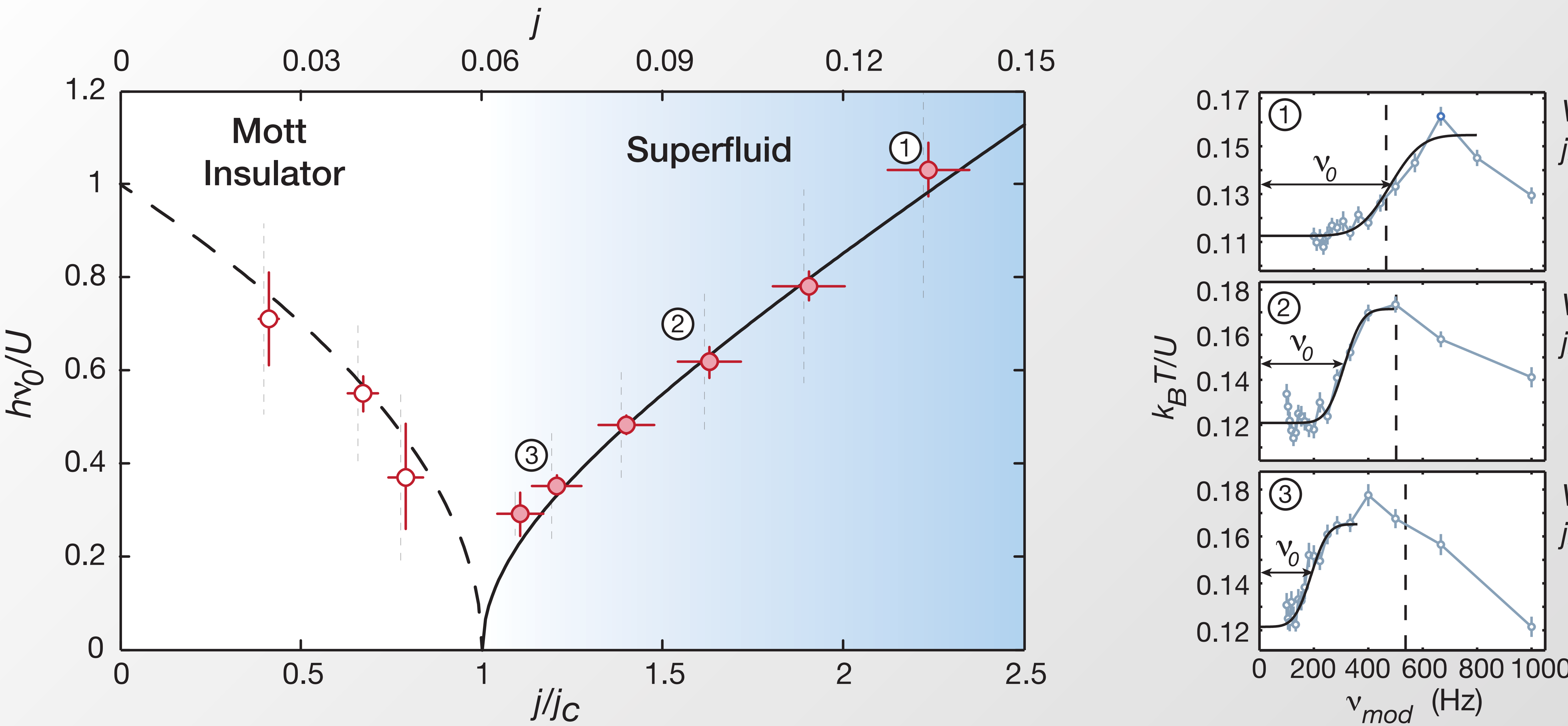
Use fit with error function to find minimum excitation frequency!
(also avoids inhomogeneous trap effects)

Evolution Across Critical Point



Higgs mode softens towards critical point!

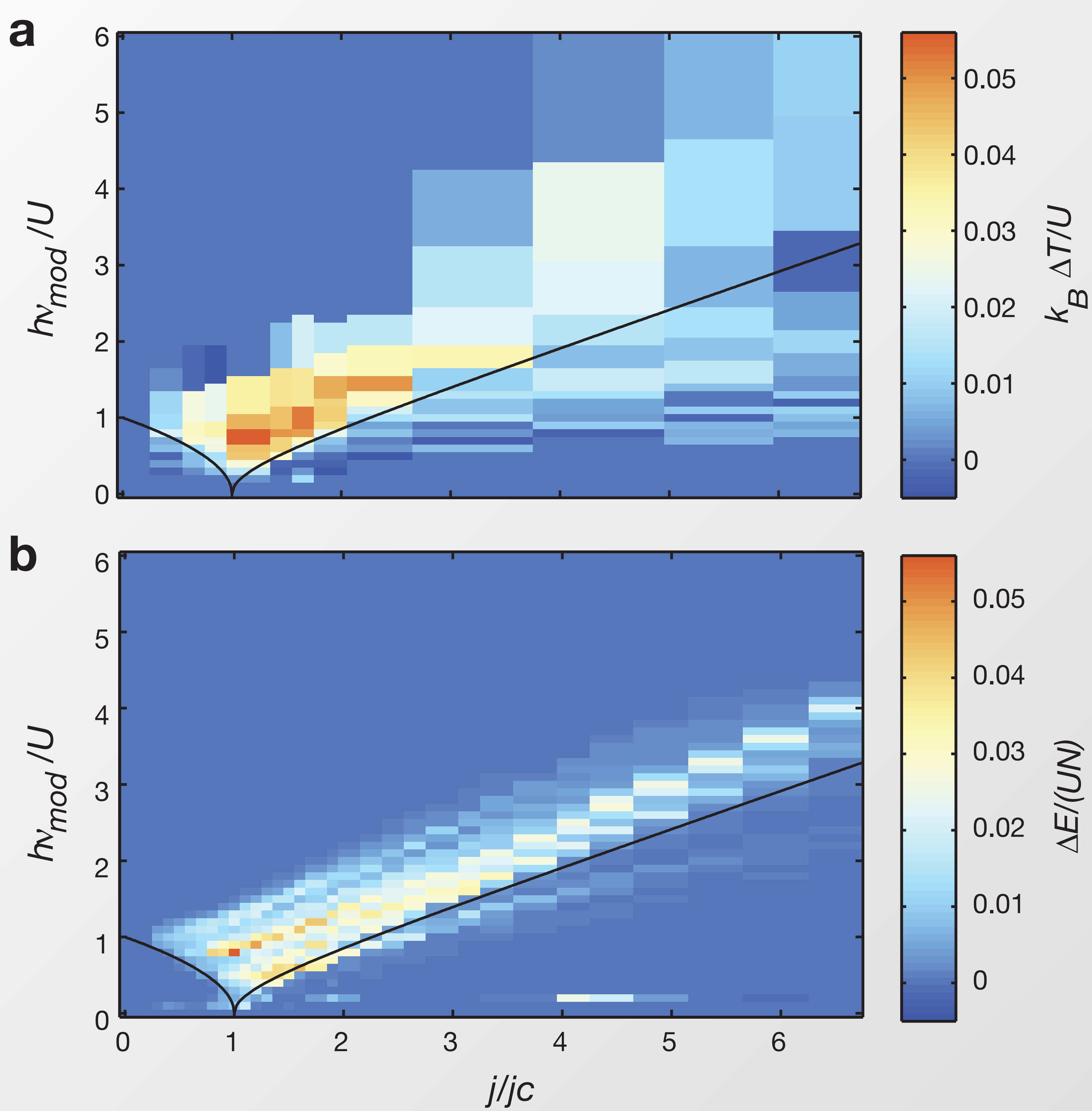
Measuring Across the QCP



'Higgs' mode softens towards critical point and turns into opening gap of Mott Insulator!

Theory in SF (S. Huber et al. PRB 2007) $\Delta_m = \sqrt{3\sqrt{2}-4}\sqrt{(j/j_c)^2-1}$

Full Spectral Response

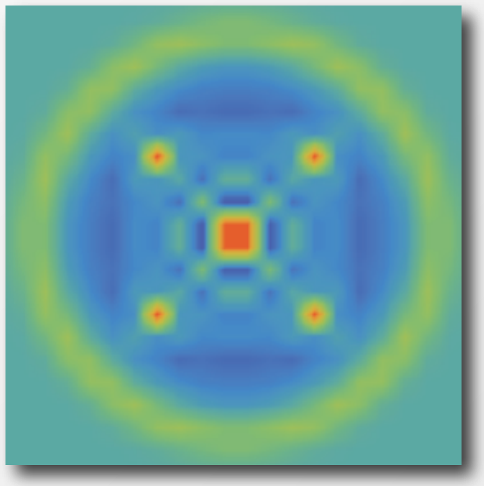


Open theory question: what is the fate of Higgs mode towards weaker interactions?

Outlook

✓ Selectively excite Higgs eigenmodes (larger system, spatial modulation)

✓ Probe Quantum Critical behaviour via Dynamical critical scaling



Higgs drum, spatial eigenmodes!

✓ Fate of mode at weaker interactions (towards GPE)

✓ Ratio of 'Higgs' mass to Mott gap

✓ Well defined mode down to critical point?

✓ Anderson-Higgs Mechanism via Coupling to (Dynamical) Gauge Field

Limits

- Parity projection
- Without parity projection (actual density imaging):

Pure state:

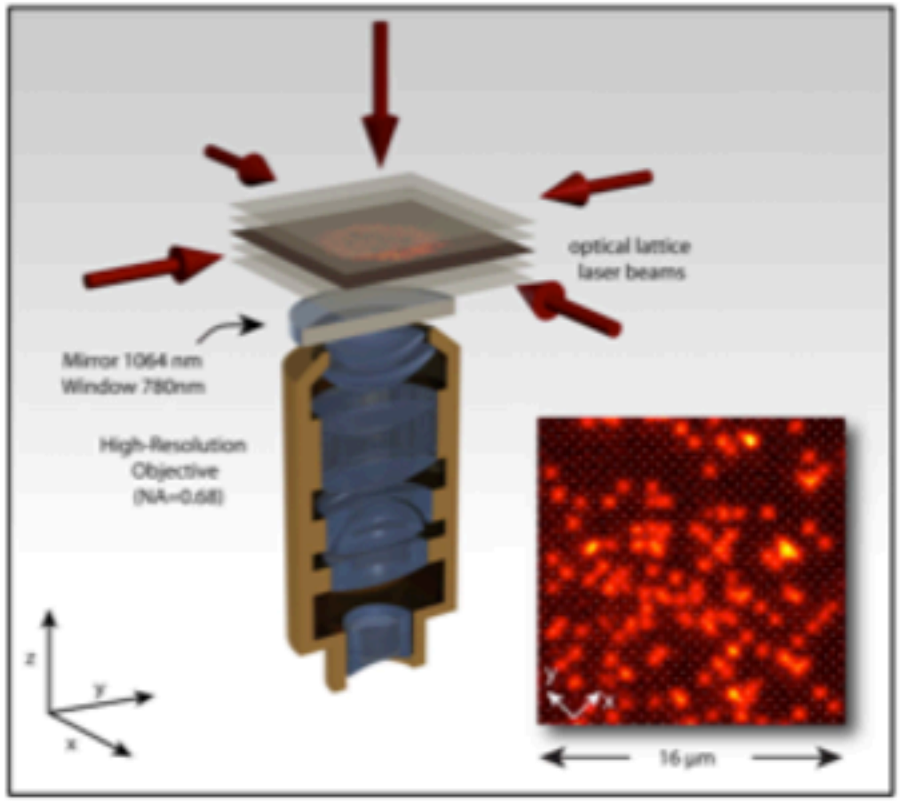
$$|\Psi\rangle = \sum_{\{n_i\}} \alpha_{n_1,\ldots,n_N} |n_1,\ldots,n_N\rangle,$$

→ Most general observable:

$$p(n_1,\ldots,n_N) = |\alpha_{n_1,\ldots,n_N}|^2$$

Mixed state:

$$\hat{\rho} = \sum_{\{n_i\}} |\alpha_{n_1,\ldots,n_N}|^2 |n_1,\ldots,n_N\rangle\langle n_1,\ldots,n_N|$$



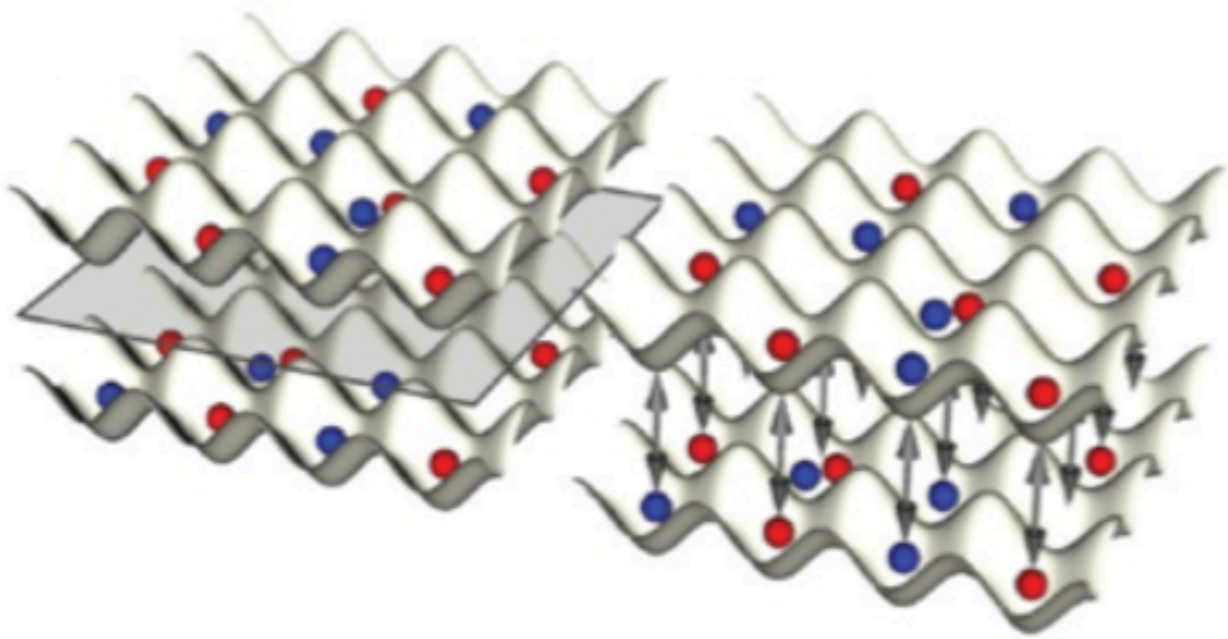
Measurement is limited to diagonal elements
of the density operator in occupation number basis!

M. Endres et al., Appl. Phys. B **113**, 27 (2013)

General entanglement detection

Zoller: Daley et. al, PRL **109**, 020505 (2012), Pichler et. al, New J. Phys. **15** 063003 (2013)
Jacksch: Alves et. al,PRL **93**, 11 (2004)

Measurement of the purity $tr(\hat{\rho}^2)$ of subsystems:



1. Create two copies of the system and interfere them
2. Count atoms in copy A and B
->purity for all possible subsystems

Rényi entropy of order 2: $S_2(\hat{\rho}) = -log(tr(\hat{\rho}^2))$
->Entanglement measure

- Topological entanglement entropy
- Area laws
- Dynamical entanglement generation

Related work: Abanin, Demler, PRL **109**, 020504 (2012)