Introduction to the physics of artificial gauge fields

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Why artificial magnetism?

Orbital and/or spin magnetisms are at the origin of many fundamental phenomena and practical devices in condensed matter physics:

Ferromagnetism or anti-ferromagnetism

Aharonov-Bohm effect

Superconductivity and Meissner effect

Quantum Hall effect (integer or fractional)

Spintronics

Topological insulators and superconductors

Can one address some of them, at least partially, with cold atomic gases?

Orbital magnetism

Hamiltonian : $\hat{H} = \dfrac{(\hat{\boldsymbol{p}} - q\boldsymbol{A}(\hat{\boldsymbol{r}}))^2}{2M}$ $\qquad\qquad \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$

Lorentz force : $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$

Program for the three lectures:

How can one generate such a Hamiltonian and its non-Abelian generalizations?

What are its key features, either for a homogeneous system or a lattice gas?

spectrum, eigenstates,...

What are the combined effects of orbital magnetism and interactions?

Introductory lectures, to themes that will appear
in several other presentations during the school

Outline of the first lecture

1. The essential aspects of orbital magnetism

    Length and energy scales, spectrum, Aharonov-Bohm effect

2. A first approach: rotation of the fluid

    Analogy between Coriolis and Lorentz force, critical rotation

3. A second approach: use of geometric phases

    Berry's phase, adiabatic following of a dressed state

4. Generalisation to non-Abelian potentials

    Spin-orbit coupling

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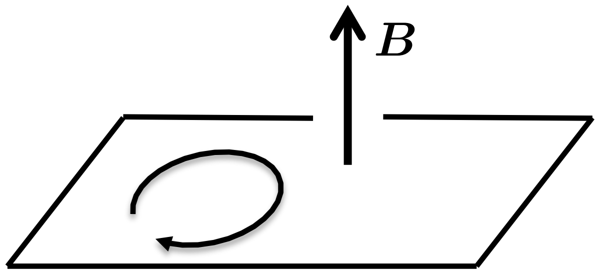
Natural scales for energy and length



cyclotron motion $r_0 = \dfrac{v_0}{\omega_c}$

Natural frequency scale: $\omega_c = \dfrac{qB}{M}$

Energy scale: $\hbar\omega_c$

Length scale: $\ell_m = \sqrt{\dfrac{\hbar}{M\omega_c}} = \sqrt{\dfrac{\hbar}{qB}}$ *magnetic length*

electron, $B = 1\,\mathrm{T}$: $\omega_c/2\pi = 28\,\mathrm{GHz}$ $\ell_m \approx 26\,\mathrm{nm}$

How does this length scale appear:

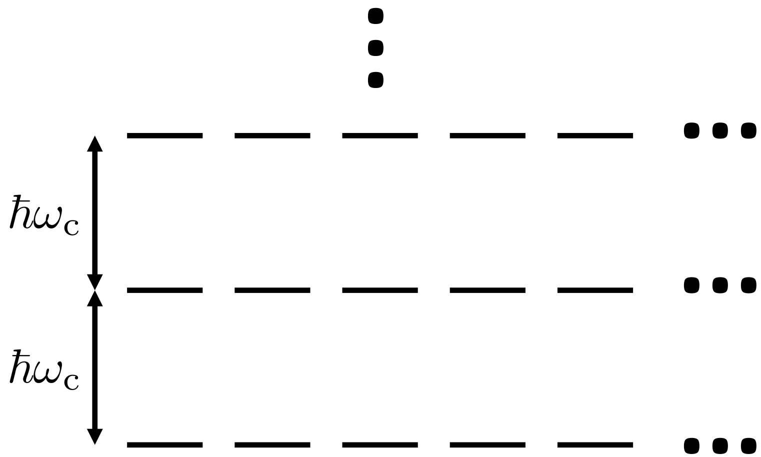
Classically: $r_0 = \dfrac{v_0}{\omega_c}$ The extension r_0 decreases with the velocity v_0

Quantum mechanics : $\Delta r_i \Delta p_i \geq \hbar/2$

The magnetic length represents the minimal size of a cyclotron orbit that is compatible with Heisenberg inequality

The energy spectrum for a single particle

Equidistant levels (like a harmonic oscillator): Landau levels



$$E_n = (n + 1/2)\hbar\omega_{\mathrm{c}}$$

Lowest Landau level = LLL

Macroscopic degeneracy for each level. Number of independent states in an area $\mathcal{A}$:

$$\mathcal{N} = \frac{\mathcal{A}}{2\pi\ell_m^2} = \frac{\Phi}{\Phi_0}$$

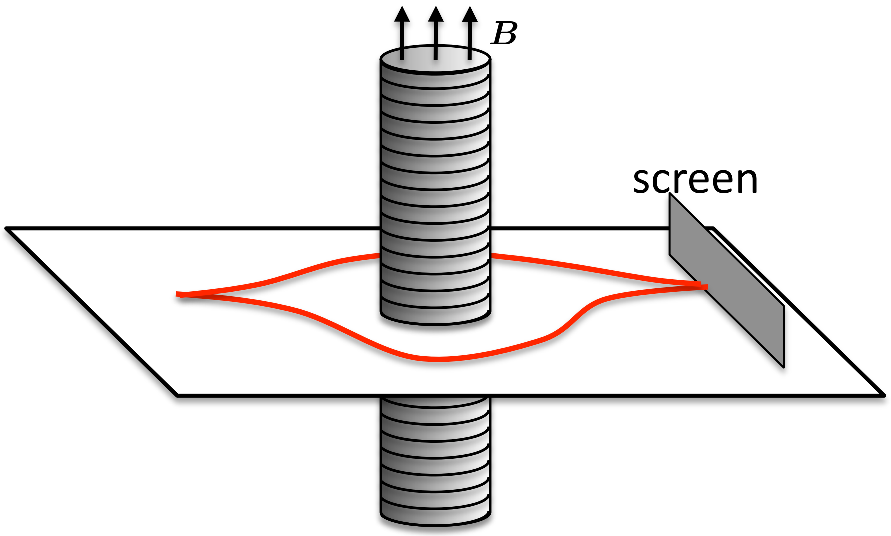
$\Phi = \mathcal{A}B$ flux of B across the area $\mathcal{A}$

$\Phi_0 = h/q$ flux quantum

Electron in a 1 Tesla field: 240 states in 1 μm^2 area

The Aharonov-Bohm effect

Aharonov-Bohm (1959)
Ehrenberg-Siday (1949)



Can one detect that a current runs in the solenoid even though the particles do not enter the zone $\boldsymbol{B} \neq 0$?

Yes, in a two-wave interference. The magnetic field creates a phase difference $\gamma(\mathcal{C})$:

$$\frac{\gamma(\mathcal{C})}{2\pi} = \frac{q}{h} \oint_{\mathcal{C}} \boldsymbol{A}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{r} = \frac{\Phi}{\Phi_0}$$

Φ : flux of B across a closed contour encircling the solenoid

$\Phi_0 = h/q$: flux quantum

$\mathrm{e}^{\mathrm{i}\gamma(\mathcal{C})}$ **: gauge-invariant, measurable quantity**

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Analogy between Coriolis and Lorentz force

A.L. Fetter, Rev. Mod. Phys. 81, p. 647 (2009)
N.R. Cooper, Advances in Physics, 57, p. 539 (2008)

3. A second approach: use of geometric phases

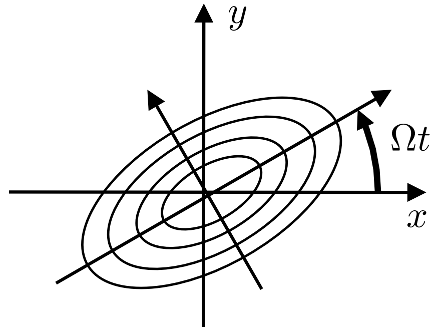
Berry's phase, adiabatic following of a dressed state

4. Generalisation to non-Abelian potentials

Spin-orbit coupling

A first approach: orbital magnetism and rotation

Hamiltonian in a rotating frame



$$\hat{\hat{H}} = \frac{\hat{\boldsymbol{p}}^2}{2M} + V(\hat{\boldsymbol{r}}) - \Omega \hat{L}_z \qquad L_z = x p_y - y p_x$$

$$= \frac{(\hat{\boldsymbol{p}} - q\boldsymbol{A}(\hat{\boldsymbol{r}}))^2}{2M} + V(\boldsymbol{r}) + V_{\text{centrif.}}(r)$$

$$q\boldsymbol{A} = M\Omega(-y\boldsymbol{u}_x + x\boldsymbol{u}_y) \qquad V_{\text{centrif.}}(r) = -\frac{1}{2}M\Omega^2 r^2$$

Vector potential (symmetric gauge)
for a uniform field $qB = 2M\Omega$

$$\boldsymbol{F}_{\text{Lorentz}} = q\,\boldsymbol{v} \times \boldsymbol{B} \quad \longleftrightarrow \quad \boldsymbol{F}_{\text{Coriolis}} = 2M\,\boldsymbol{v} \times \boldsymbol{\Omega}$$

$$\boldsymbol{F}_{\text{centrif.}} = M\Omega^2 \boldsymbol{r}$$

Rotation : uniform magnetic field + quadratic deconfining potential

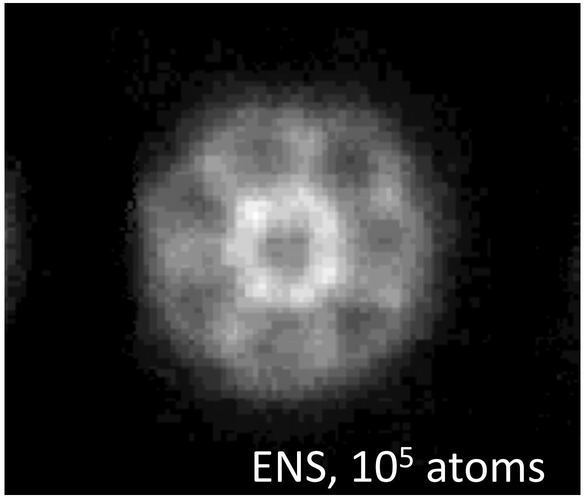
Rotating a cold atom gas

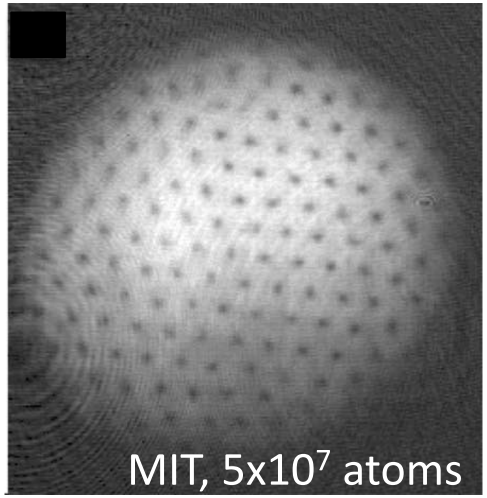
Start from a trap as round as possible in the lab frame: $\frac{1}{2}M\left(\omega_x^2 x^2 + \omega_y^2 y^2\right)$ $\qquad \omega_x \approx \omega_y$

Add a stirrer with controlled amplitude and frequency

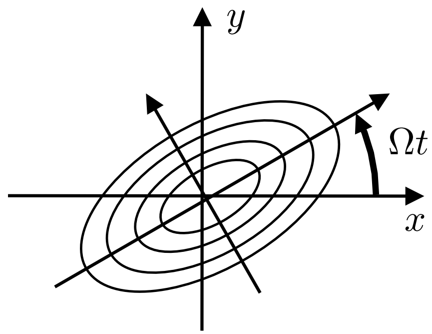
- Auxiliary laser beams
- Magnetic potential

It works, as shown by the nucleation of vortices in a superfluid ...





Critical rotation



Choose a quadratic stirring potential

$$V_0(r) = \frac{1}{2} M\omega^2 (x^2 + y^2)$$

isotropic

$$\delta V(x,y) = \frac{\epsilon}{2} M\omega^2 \left(y^2 - x^2\right)$$

In the rotating frame

Can one reach the Hamiltonian of a free particle in a uniform magnetic field?

$$\hat{H} = \frac{\left(\hat{\boldsymbol{p}} - q\boldsymbol{A}(\hat{\boldsymbol{r}})\right)^2}{2M} + V_0(\hat{r}) + \delta V(\hat{\boldsymbol{r}}) + V_{\text{centrif.}}(\hat{r}) \quad \overset{?}{=} \quad \frac{\left(\hat{\boldsymbol{p}} - q\boldsymbol{A}(\hat{\boldsymbol{r}})\right)^2}{2M}$$

$$V_{\text{centrif.}}(r) = -\frac{1}{2} M\Omega^2 (x^2 + y^2)$$

We must choose simultaneously

$$\begin{cases} \Omega \to \omega & \text{(vanishing confinement)} \\ \epsilon \to 0 & \text{(vanishing stirring)} \end{cases}$$

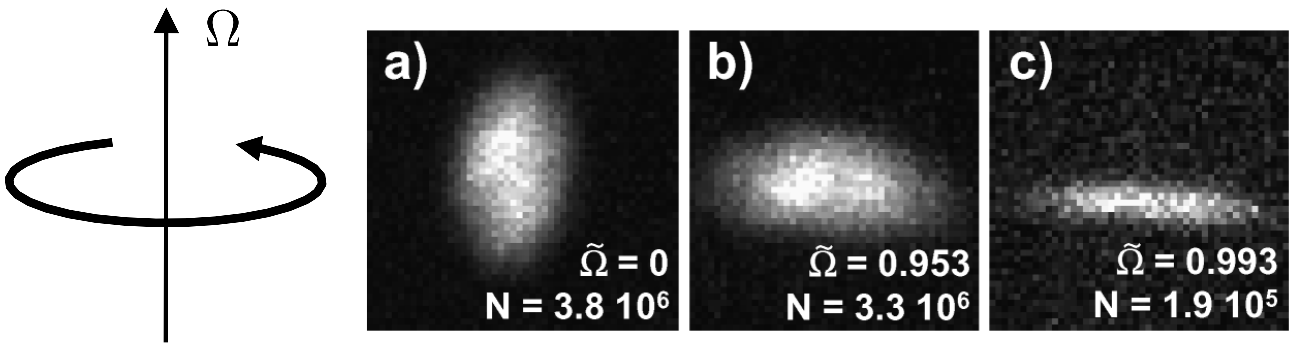
Possible paths towards the critical rotation

ENS: add a positive quartic potential to ensure confinement even when $\Omega = \omega$

Boulder: "evaporative spin-up"

- Prepare a cloud rotating at a "moderate" velocity with a stirrer at $\Omega \sim 0.7\,\omega$
- Switch off the stirrer and evaporate the particles along the trap axis

Evaporated particles have less angular momentum than average



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Berry's phase, adiabatic following of a dressed state

Dalibard, Gerbier, Juzeliunas, Ohberg, Rev. Mod. Phys. 83, p.1523 (2011)
Goldman, Juzeliunas, Ohberg, Spielman, arXiv:1308.6533

4. Generalisation to non-Abelian potentials

Spin-orbit coupling

Adiabatic approximation and geometric phase

Berry, 1984

Hamiltonian $\hat{H}(\boldsymbol{\lambda})$ $\boldsymbol{\lambda}$: continuous external parameter

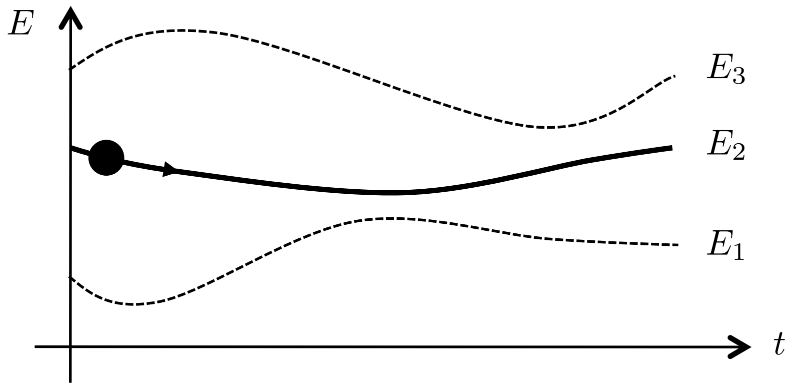
For each $\boldsymbol{\lambda}$, the eigenstates $|\psi_n(\boldsymbol{\lambda})\rangle$ and their energies $E_n(\boldsymbol{\lambda})$ are known

$$\hat{H}(\boldsymbol{\lambda})|\psi_n(\boldsymbol{\lambda})\rangle = E_n(\boldsymbol{\lambda})\,|\psi_n(\boldsymbol{\lambda})\rangle$$

Start at $t = 0$ in a given eigenstate : $|\psi(0)\rangle = |\psi_\ell[\lambda_0]\rangle$

Suppose that the parameter $\boldsymbol{\lambda}$, controlled from outside (for the moment) slowly varies in time

State of the system: $|\psi\rangle \;=\; \sum_n c_n\,|\psi_n[\boldsymbol{\lambda}]\rangle \;\approx\; c_\ell\,|\psi_\ell[\lambda]\rangle$ $\qquad |c_\ell(t)| = 1 \quad \forall t$



Equation for the coefficient c_ℓ :

$$\mathrm{i}\hbar\,\dot{c}_\ell = \left[E_\ell(t) - \mathrm{i}\hbar\dot{\boldsymbol{\lambda}}\cdot\langle\psi_\ell|\boldsymbol{\nabla}\psi_\ell\rangle\right]\,c_\ell$$
$$= \left[E_\ell(t) - \dot{\boldsymbol{\lambda}}\cdot\boldsymbol{\mathcal{A}}_\ell(\boldsymbol{\lambda})\right]\,c_\ell$$

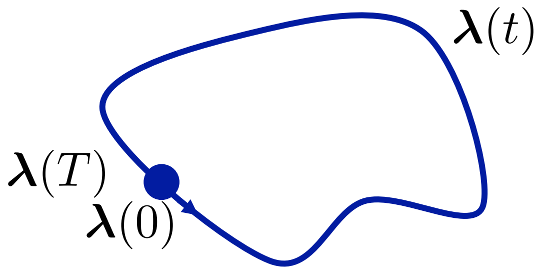
Berry's connection (real) : $\boldsymbol{\mathcal{A}}_\ell(\boldsymbol{\lambda}) = \mathrm{i}\hbar\,\langle\psi_\ell|\boldsymbol{\nabla}\psi_\ell\rangle$

Berry's phase and Berry's curvature

$$\boldsymbol{\mathcal{A}}_\ell(\boldsymbol{\lambda}) = \mathrm{i}\hbar\,\langle \psi_\ell | \boldsymbol{\nabla}\psi_\ell\rangle$$

At any time: $|\psi(t)\rangle \;\approx\; c_\ell(t)\;|\psi_\ell[\boldsymbol{\lambda}(t)]\rangle$ $\qquad \mathrm{i}\hbar\,\dot{c}_\ell = \left[E_\ell(t) - \dot{\boldsymbol{\lambda}}\cdot\boldsymbol{\mathcal{A}}_\ell(\lambda)\right]\,c_\ell$

Closed contour $\mathcal{C}$



$$c_\ell(T) = \mathrm{e}^{\mathrm{i}\Phi^{\mathrm{dyn.}}(T)}\;\mathrm{e}^{\mathrm{i}\Phi^{\mathrm{geom.}}(T)}\;c_\ell(0)$$

$$\Phi^{\mathrm{dyn.}}(T) = -\frac{1}{\hbar}\int_0^T E_\ell(t)\;\mathrm{d}t$$

$$\Phi^{\mathrm{geom.}}(T) = \frac{1}{\hbar}\int_0^T \dot{\boldsymbol{\lambda}}\cdot\boldsymbol{\mathcal{A}}_\ell(\boldsymbol{\lambda})\;\mathrm{d}t = \frac{1}{\hbar}\oint \boldsymbol{\mathcal{A}}_\ell(\boldsymbol{\lambda})\;\cdot\mathrm{d}\boldsymbol{\lambda}$$

The two phases $\Phi^{\mathrm{dyn.}}$ et $\Phi^{\mathrm{geom.}}$ are gauge invariant: physical quantities

Restrict to the case where $\boldsymbol{\lambda}$ evolves in a 2D or 3D space

- position of a particle
- quasi-momentum in the Brillouin zone

$$\boldsymbol{\mathcal{B}}_\ell = \boldsymbol{\nabla}\times\boldsymbol{\mathcal{A}}_\ell$$

Berry curvature:
real, gauge-invariant

$$\Phi^{\mathrm{geom.}}(\mathcal{C}) \;=\; \frac{1}{\hbar}\oint_{\mathcal{C}} \boldsymbol{\mathcal{A}}_\ell(\boldsymbol{\lambda})\cdot\mathrm{d}\boldsymbol{\lambda} \;=\; \frac{1}{\hbar}\iint_{\mathcal{S}} \boldsymbol{\mathcal{B}}_\ell\cdot\mathrm{d}^2\boldsymbol{S}$$

Full analogy with Aharonov-Bohm phase

Full quantum treatment for an atom moving in a light field

External degrees of freedom: center-of-mass position and momentum $\hat{\boldsymbol{r}}, \quad \hat{\boldsymbol{p}}$

Internal degrees of freedom: electronic energy levels

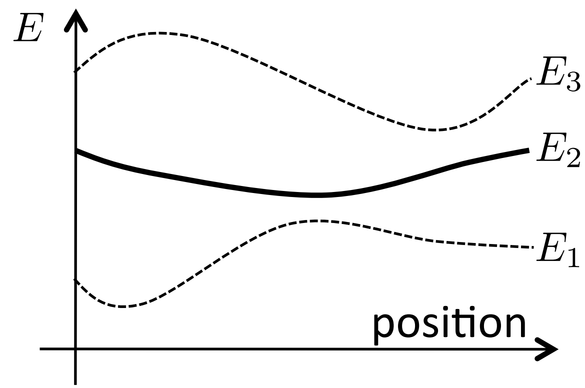
Hamiltonian :

$$\hat{H}_{\rm tot} = \underbrace{\frac{\hat{\boldsymbol{p}}^2}{2M} \otimes \hat{1}_{\rm int}}_{\text{CoM kinetic energy}} + \underbrace{\hat{H}_{\rm int}(\hat{\boldsymbol{r}})}_{\text{Coupling between the atomic dipole and the field}}$$

Dressed states :

$$\hat{H}_{\rm int}(\boldsymbol{r})|\psi_n(\boldsymbol{r})\rangle = E_n(\boldsymbol{r})\ |\psi_n(\boldsymbol{r})\rangle$$

What happens if the atom moves slowly enough to follow adiabatically a given dressed state ?



Geometrical gauge fields

Dum & Olshanii, 1996

center of mass
$\hat{\boldsymbol{r}}, \quad \hat{\boldsymbol{p}}$
slow

internal degrees of freedom $|\psi_n(\boldsymbol{r})\rangle$
fast

$$\Psi(\boldsymbol{r},t) = \sum_n \phi_n(\boldsymbol{r},t)|\psi_n(\boldsymbol{r})\rangle$$

$$\approx \phi_\ell(\boldsymbol{r},t)|\psi_\ell(\boldsymbol{r})\rangle$$

Adiabatic approximation:
only one state $|\psi_\ell(\boldsymbol{r})\rangle$
is populated

Schrödinger equation for the probability amplitude $\phi_\ell(\boldsymbol{r},t)$

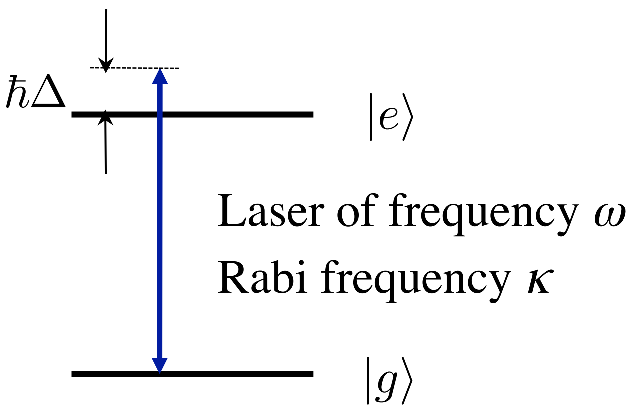
$$\mathrm{i}\hbar\frac{\partial \phi_\ell}{\partial t} = \left[\frac{(\hat{\boldsymbol{p}} - \boldsymbol{\mathcal{A}}_\ell(\boldsymbol{r}))^2}{2M} + E_\ell(\boldsymbol{r}) + \mathcal{V}_\ell(\boldsymbol{r})\right]\phi_\ell(\boldsymbol{r},t)$$

$$\begin{cases} \boldsymbol{\mathcal{A}}_\ell(\boldsymbol{r}) & : \text{ vector potential} \\ \mathcal{V}_\ell(\boldsymbol{r}) & : \text{ scalar potential} \end{cases}$$

$$\boldsymbol{\mathcal{A}}_\ell(\boldsymbol{r}) = \mathrm{i}\hbar\langle\psi_\ell|\boldsymbol{\nabla}\psi_\ell\rangle \qquad \mathcal{V}_\ell(\boldsymbol{r}) = \frac{\hbar^2}{2M}\sum_{n\neq\ell}|\langle\boldsymbol{\nabla}\psi_\ell|\psi_n\rangle|^2$$

The two-level atom model (no spontaneous emission)

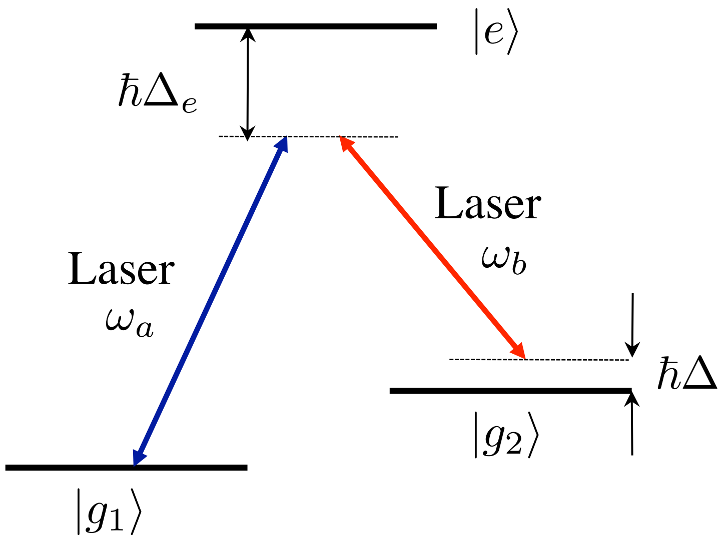
Narrow line: two-electron atoms Mg, Sr, Yb



Dynamics in the subspace

$$\{|g\rangle, |e\rangle\}$$

Raman transition: alcali, Er, Dy



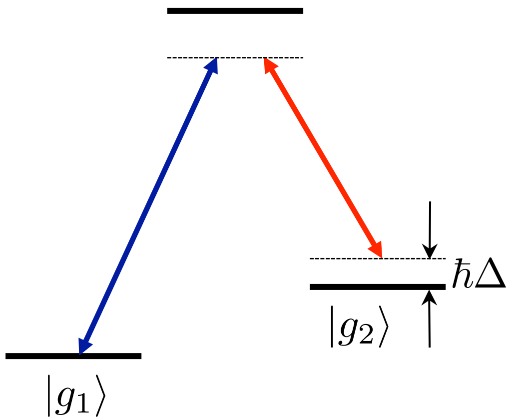
Elimination of the excited state $|e\rangle$ to restrict the dynamics to the subspace $\{|g_1\rangle, |g_2\rangle\}$

$$\kappa = \frac{\kappa_a \kappa_b^*}{2\Delta_e}$$

Atom-laser coupling is characterized by the detuning Δ and the Rabi frequency κ

$$\hat{H}_{\text{int}} = \frac{\hbar}{2} \begin{pmatrix} \Delta & \kappa^* \\ \kappa & -\Delta \end{pmatrix}$$

Dressed states and geometric potentials

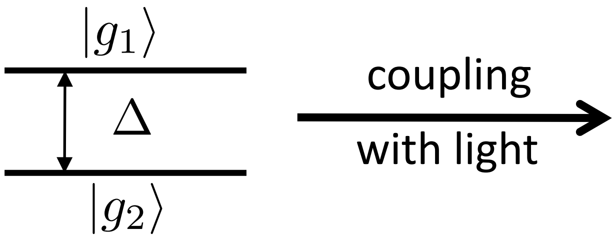


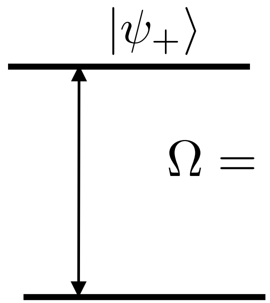
Look for the eigenstates of

$$\hat{H}_{\text{int}} = \frac{\hbar}{2}\begin{pmatrix} \Delta & \kappa^* \\ \kappa & -\Delta \end{pmatrix}$$

$$= \frac{\hbar\Omega}{2}\begin{pmatrix} \cos\theta & \mathrm{e}^{-\mathrm{i}\phi}\sin\theta \\ \mathrm{e}^{\mathrm{i}\phi}\sin\theta & -\cos\theta \end{pmatrix}$$

where we set: $\Omega = \left(\Delta^2 + |\kappa|^2\right)^{1/2}$ $\qquad \tan\theta = |\kappa|/\Delta$ $\qquad \kappa = |\kappa|\,\mathrm{e}^{\mathrm{i}\phi}$





$$\Omega = \sqrt{\Delta^2 + |\kappa|^2} \qquad E_\pm = \pm\frac{\hbar\Omega}{2}$$

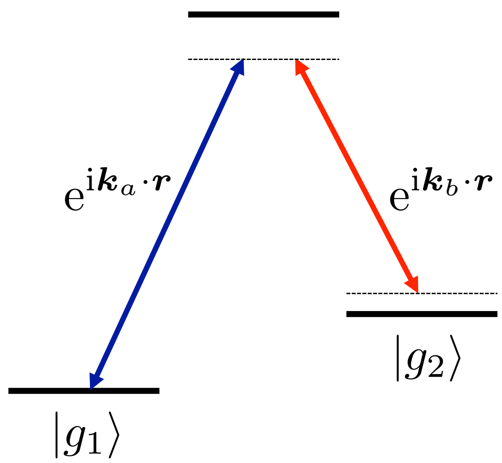
$$|\psi_-\rangle = \cos(\theta/2)|g_2\rangle - \mathrm{e}^{-\mathrm{i}\phi}\sin(\theta/2)|g_1\rangle$$

$$\boldsymbol{\mathcal{A}}_- = \mathrm{i}\hbar\,\langle\psi_-|\boldsymbol{\nabla}\psi_-\rangle = \frac{\hbar}{2}\,\boldsymbol{\nabla}\phi\,\,(1-\cos\theta)$$

$$\boldsymbol{\mathcal{B}}_- = \boldsymbol{\nabla}\times\boldsymbol{\mathcal{A}}_- = -\frac{\hbar}{2}\boldsymbol{\nabla}(\cos\theta)\times\boldsymbol{\nabla}\phi$$

: need for a gradient of phase and mixing angle (detuning or laser intensity)

Use a gradient of detuning

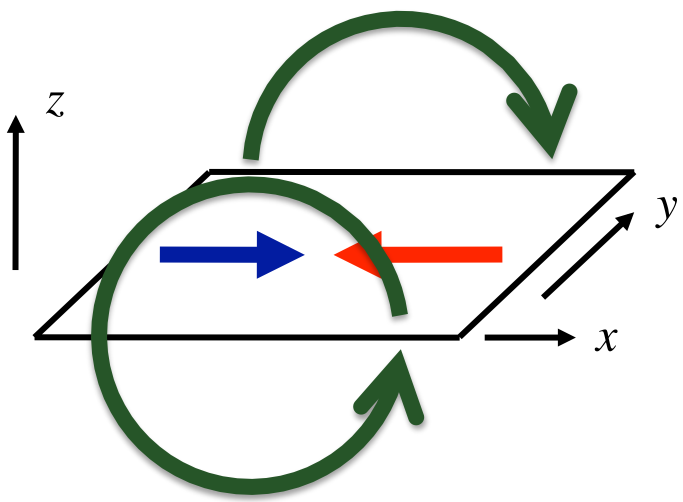


Raman transition with two plane waves along x

$$\kappa(\boldsymbol{r}) = \kappa_0\ e^{2ikx} \qquad\qquad \tan\theta = |\kappa|/\Delta$$

Gradient of detuning Δ along y : $\Delta(\boldsymbol{r}) = \Delta' y$

Characteristic length : $\ell = \kappa_0/\Delta'$



Artificial magnetic field

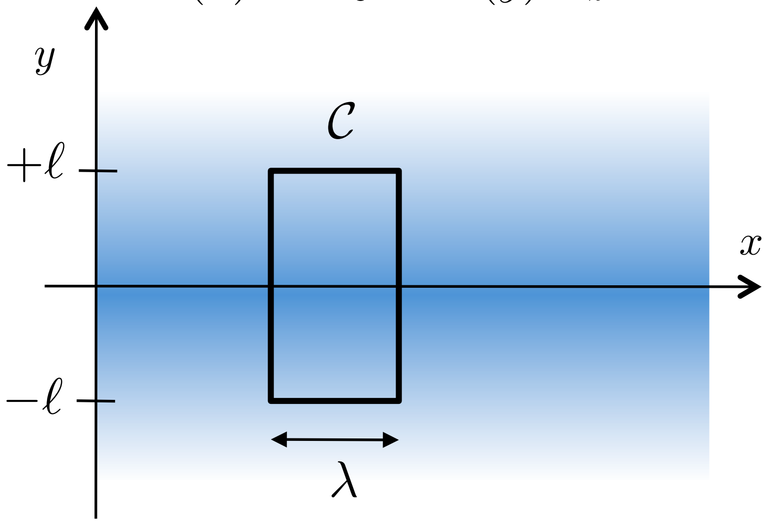
$$\boldsymbol{\mathcal{B}}(\boldsymbol{r}) = B_0\ \mathcal{L}^{3/2}(y)\ \boldsymbol{u}_z$$

$$B_0 = \frac{\hbar k}{\ell}, \qquad \mathcal{L}(y) = \frac{1}{1+y^2/\ell^2}$$

How large is this magnetic field?

How big should a contour *C* be to reach a Aharonov-Bohm-Berry phase of the order of 2π ?

$$\boldsymbol{\mathcal{B}}(\boldsymbol{r}) = B_0\ \mathcal{L}^{3/2}(y)\ \boldsymbol{u}_z$$



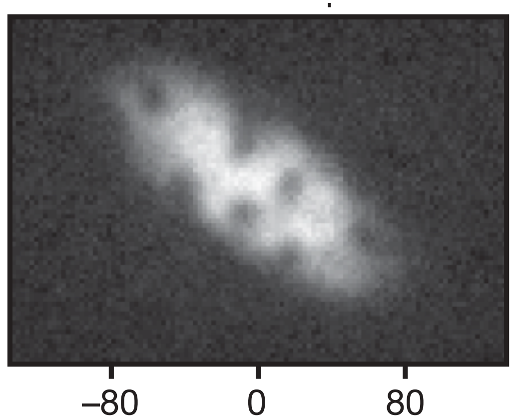
$$B_0 = \frac{\hbar k}{\ell}, \qquad \mathcal{L}(y) = \frac{1}{1 + y^2/\ell^2}$$

$$\frac{\gamma(\mathcal{C})}{2\pi} = \frac{1}{h} \iint_{\mathcal{S}} \boldsymbol{\mathcal{B}} \cdot \boldsymbol{u}\ \mathrm{d}^2 r,$$

criterion reached for a rectangle $2\ell \times \lambda$

$$\frac{\gamma(\mathcal{C})}{2\pi} \approx \frac{1}{h}\ \frac{\hbar k}{\ell}\ 2\ell\lambda \sim 1$$

Spielman’s team, NIST 2009
Raman transition with a ^{87}Rb BEC



limitation due to the residual scattering of photons

Validity of the adiabatic approximation

General criterion: $\dfrac{\text{angular velocity of eigenstate } \psi_\ell}{\text{Bohr frequency for } \psi_\ell} \ll 1$

Here:
- angular velocity of the eigenstate $\left|\langle \psi_- | \dot{\psi}_- \rangle\right| \sim v\, |\langle \psi_- | \boldsymbol{\nabla} \psi_- \rangle| \sim kv$
- Bohr frequency $\Omega = (\kappa_0^2 + \Delta^2)^{1/2}$

The approximation is valid if the velocities are low enough: $k\,\bar{v} \ll \Omega$

Relevant velocities are at least of the order of the recoil velocity: $v_\mathrm{r} = \hbar k/M$

Necessary condition: $E_\mathrm{r} \ll \hbar\Omega$ $\qquad E_\mathrm{r} = \hbar^2 k^2/2M$

Large artificial magnetic fields? $\boldsymbol{B} \sim \hbar \boldsymbol{\nabla}(\cos\theta) \times \boldsymbol{\nabla}\phi$

If both gradients are $|\nabla(\cos\theta)| \sim |\boldsymbol{\nabla}\phi| \sim k$ (typically in an optical lattice), then $B \sim \hbar k^2$

Corresponding cyclotron frequency $\omega_\mathrm{c} = B/M \quad \Longrightarrow \quad \hbar\omega_\mathrm{c} \sim E_\mathrm{r}$

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Spin-orbit coupling

Galitski & Spielman, Nature 494, p. 49 (2013)
Goldman, Juzeliunas, Ohberg, Spielman, arXiv:1308.6533

Non Abelian gauge fields

We are still interested in $\hat{H} = \dfrac{\left(\hat{\boldsymbol{p}} - \hat{\boldsymbol{\mathcal{A}}}(\hat{\boldsymbol{r}})\right)^2}{2M} + \ldots$, but $\hat{\boldsymbol{\mathcal{A}}}(\hat{\boldsymbol{r}})$ is now an operator with respect to the internal degrees of freedom.

In particular two components of $\hat{\boldsymbol{\mathcal{A}}}(\hat{\boldsymbol{r}})$ may not commute: $[\hat{\mathcal{A}}_x(\boldsymbol{r}), \hat{\mathcal{A}}_y(\boldsymbol{r})] \neq 0$

Velocity operator: $\hat{\boldsymbol{v}} = \dfrac{\mathrm{d}\hat{\boldsymbol{r}}}{\mathrm{d}t} = \dfrac{\mathrm{i}}{\hbar}[\hat{H}, \hat{\boldsymbol{r}}] = \dfrac{\hat{\boldsymbol{p}} - \hat{\mathcal{A}}(\hat{\boldsymbol{r}})}{M}$

Force operator: $\hat{\boldsymbol{F}} = M\dfrac{\mathrm{d}\hat{\boldsymbol{v}}}{\mathrm{d}t} = \dfrac{\mathrm{i}}{\hbar}[\hat{H}, M\hat{\boldsymbol{v}}] = \dfrac{1}{2}\left(\hat{\boldsymbol{v}} \times \hat{\boldsymbol{\mathcal{B}}} - \hat{\boldsymbol{\mathcal{B}}} \times \hat{\boldsymbol{v}}\right)$ *Lorentz*

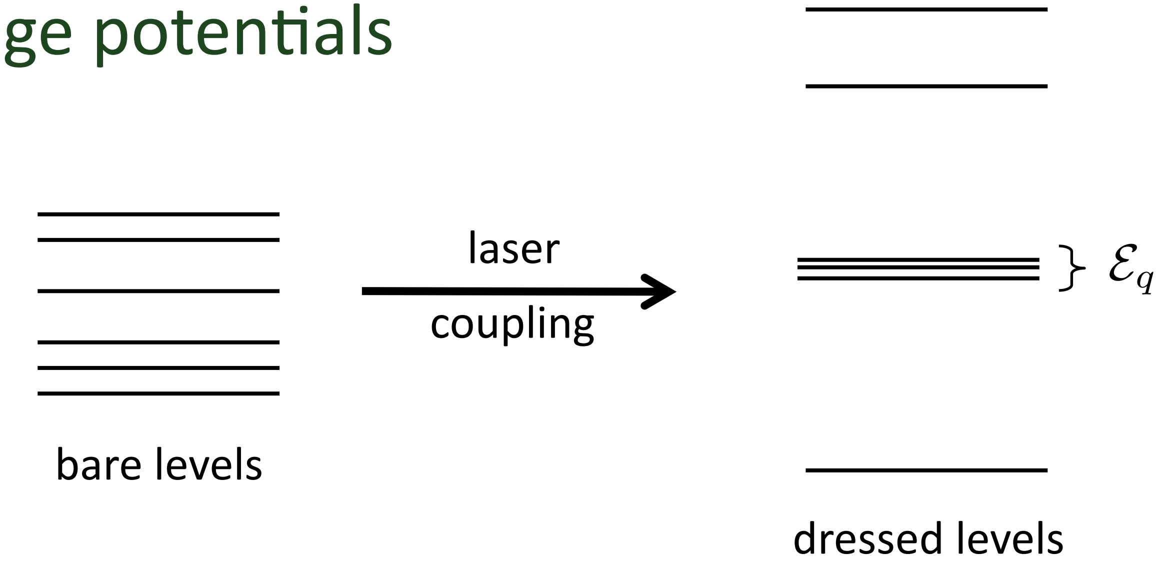
Example: 2D case with $\mathcal{A}_z = 0$ and $\mathcal{A}_{x,y}$ functions of x,y

Then : $\hat{\boldsymbol{\mathcal{B}}} = \hat{\mathcal{B}}_z \boldsymbol{u}_z$ $\qquad \hat{\mathcal{B}}_z = \dfrac{\partial \hat{\mathcal{A}}_x}{\partial y} - \dfrac{\partial \hat{\mathcal{A}}_y}{\partial x} - \dfrac{\mathrm{i}}{\hbar}[\mathcal{A}_x, \mathcal{A}_y]$

$\hat{\mathcal{B}}_z$ **can be non-zero even if** $\hat{\mathcal{A}}_x$ **and** $\hat{\mathcal{A}}_y$ **are uniform**

Adiabatic approximation and non Abelian gauge potentials

Wilczek & Zee, 1984,
in a Quantum Optics version



An atom prepared in the (quasi) degenerate subspace $\mathcal{E}_q$ will stay there if its velocity is small

$$\Psi(\boldsymbol{r},t) = \sum_{n\in\mathcal{E}_q} \phi_n(\boldsymbol{r},t)\,|\psi_n(\boldsymbol{r})\rangle$$

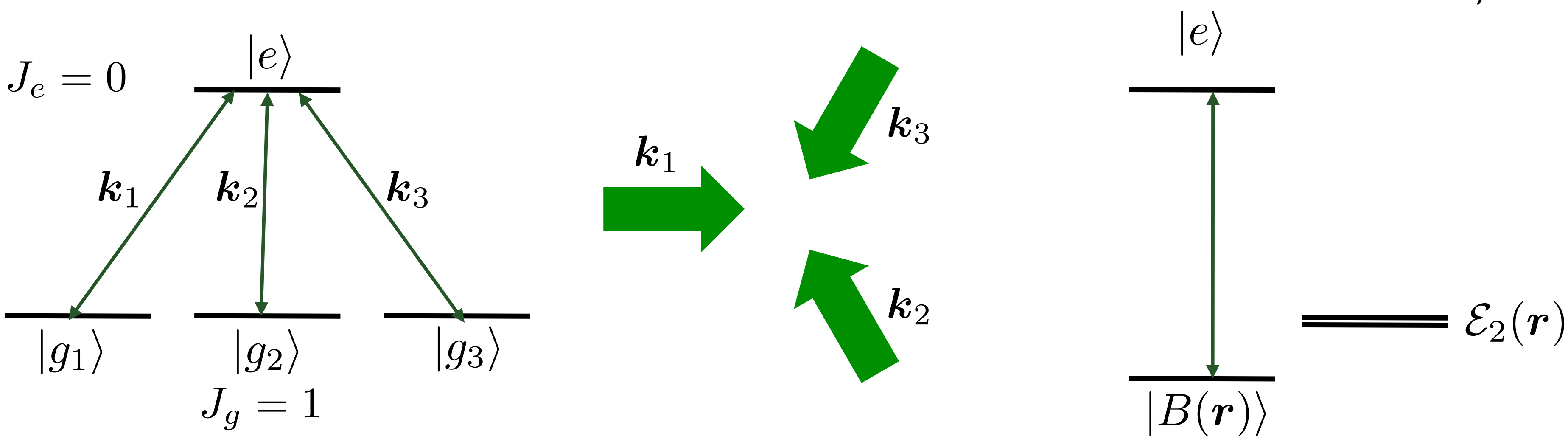
The q coupled equations for the amplitudes $\phi_n(\boldsymbol{r},t)$ contain a matrix vector potential

$$\hat{\boldsymbol{\mathcal{A}}}^{(n,m)} = \mathrm{i}\hbar\langle\psi_n(\boldsymbol{r})|\boldsymbol{\nabla}\psi_m(\boldsymbol{r})\rangle$$

Important in molecular and condensed matter physics

Example: symmetric tripod configuration

Juzeliunas,
Ruseckas,
Dalibard, 2010



Only one linear combination of $\{|g_1\rangle, |g_2\rangle, |g_3\rangle\}$ is coupled at any point to $|e\rangle$

$$|B(\boldsymbol{r})\rangle = \frac{1}{\sqrt{3}}\left(\mathrm{e}^{-\mathrm{i}\boldsymbol{k}_1\cdot\boldsymbol{r}}|g_1\rangle + \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_2\cdot\boldsymbol{r}}|g_2\rangle + \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_3\cdot\boldsymbol{r}}|g_3\rangle\right)$$ **bright state**

Adiabatic evolution in the orthogonal “dark” subspace ($\mathcal{E}_2$ of dimension 2)

$$\hat{\boldsymbol{\mathcal{A}}} = \frac{\hbar k}{2}\left(\hat{\sigma}_x \boldsymbol{u}_x + \hat{\sigma}_y \boldsymbol{u}_y\right)$$ $\hat{\sigma}_j$: Pauli matrices

Corresponding Hamiltonian : $$\hat{H} = \frac{\left(\hat{\boldsymbol{p}} - \hat{\boldsymbol{\mathcal{A}}}(\hat{\boldsymbol{r}})\right)^2}{2M} = \frac{\hat{\boldsymbol{p}}^2}{2M} - \frac{\hbar k}{2M}\left(\hat{p}_x\hat{\sigma}_x + \hat{p}_y\hat{\sigma}_y\right) + \ldots$$

spin-orbit coupling

Physical origin of “usual” spin-orbit coupling

Essentially a relativistic phenomenon

A charged particle (electron) moves with velocity $\boldsymbol{v}$ in a region with an electric field $\boldsymbol{\mathcal{E}}$.
In the frame of the particle, a motional magnetic field appears: $\boldsymbol{\mathcal{B}} \propto \boldsymbol{v} \times \boldsymbol{\mathcal{E}}$

Spin-orbit coupling results from the interaction between the motional magnetic field and the intrinsic magnetic moment of the particle, proportional to its spin

$\boldsymbol{\mu} = \gamma\, \boldsymbol{S}$ Coupling between $\boldsymbol{v}$ and $\boldsymbol{S}$

Atomic physics: $(\boldsymbol{r} \times \boldsymbol{p}) \cdot \boldsymbol{S} = \boldsymbol{L} \cdot \boldsymbol{S}$

Solid materials : $\boldsymbol{\mathcal{E}}$ uniform, coupling $\boldsymbol{p}_i \boldsymbol{S}_j$ *Rashba, Dresselhaus*

Physics of spin-orbit coupling

Galitski & Spielman, 2013

Applications: Spintronics

Control of the interaction between spins and linear momentum with an external electric field

Fundamental physics:

- Topological insulators, analogous to Quantum Hall effect (at least in 2D), but which do not require the breaking of time-reversal symmetry and which should be more robust with respect to thermal excitations.

- For a Fermi gas in contact with a superconductor, it can lead to the creation of Majorana particles

- Single atom physics: massive degeneracy of the ground state of the Hamiltonian

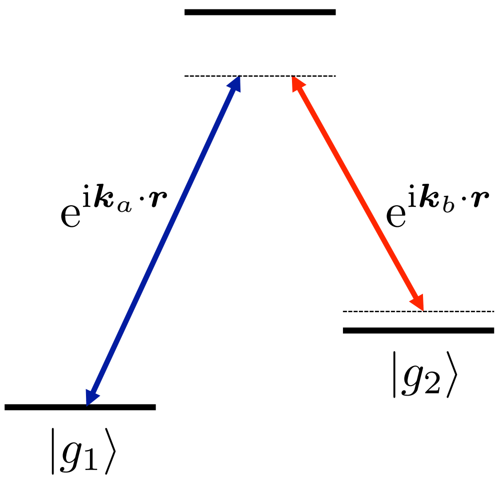
$$\hat{H} = \frac{\left(\hat{\boldsymbol{p}} - \eta\hat{\boldsymbol{S}}\right)^2}{2M}$$

Zero-energy state for all momenta such that $|\boldsymbol{p}| = \eta S$

1D version of spin-orbit coupling

Realised with a Bose gas (NIST, 2011), a Fermi gas (MIT, Tsinghua 2012)
+ lattice version (Munich 2013, MIT 2013)

Raman transition with two plane waves: translation-invariant problem



$$2\boldsymbol{k} = \boldsymbol{k}_a - \boldsymbol{k}_b$$

Family : $\mathcal{F}(\boldsymbol{p}) = \{|g_1, \boldsymbol{p} - \hbar\boldsymbol{k}\rangle,\ |g_2, \boldsymbol{p} + \hbar\boldsymbol{k}\rangle\}$

globally stable under the action of the atom-light coupling

$$\hat{H}(\boldsymbol{p}) = \begin{pmatrix} (\boldsymbol{p} - \hbar\boldsymbol{k})^2/2M + \hbar\Delta/2 & \hbar\kappa_0/2 \\ \hbar\kappa_0/2 & (\boldsymbol{p} + \hbar\boldsymbol{k})^2/2M - \hbar\Delta/2 \end{pmatrix}$$

$$= \frac{1}{2M}\left(\boldsymbol{p} - \hat{\boldsymbol{A}}\right)^2 + \frac{\hbar\Delta}{2}\hat{\sigma}_z + \frac{\hbar\kappa_0}{2}\hat{\sigma}_x$$

with $\hat{\boldsymbol{A}} = \hbar\boldsymbol{k}\hat{\sigma}_z$

Physical origin: recoil // Doppler effect

1D spin-orbit coupling

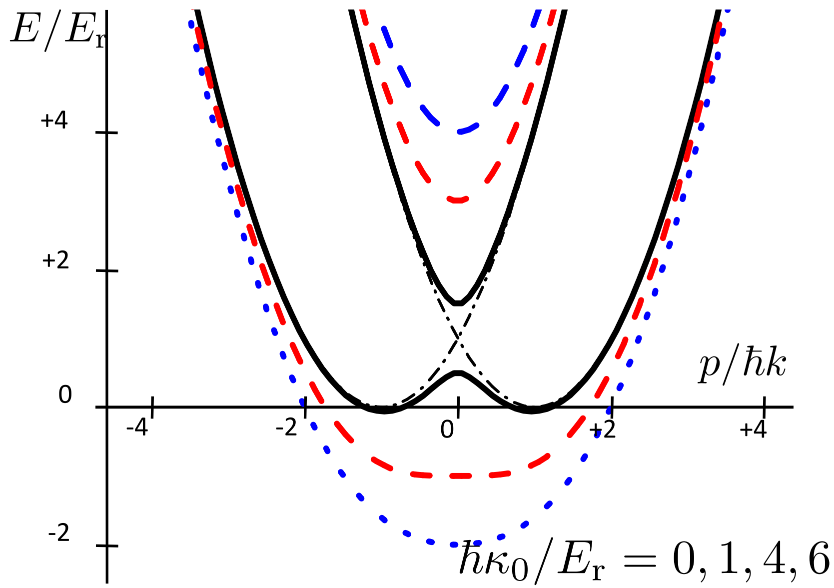
Abelian gauge field: $\hat{\boldsymbol{\mathcal{A}}} = \hbar \boldsymbol{k} \hat{\sigma}_z \longrightarrow [\hat{\mathcal{A}}_x(\boldsymbol{r}), \hat{\mathcal{A}}_y(\boldsymbol{r})] = 0$

However one keeps a non-unique ground state,
at least for small values of the atom-laser coupling

Choose a zero-detuning Δ=0

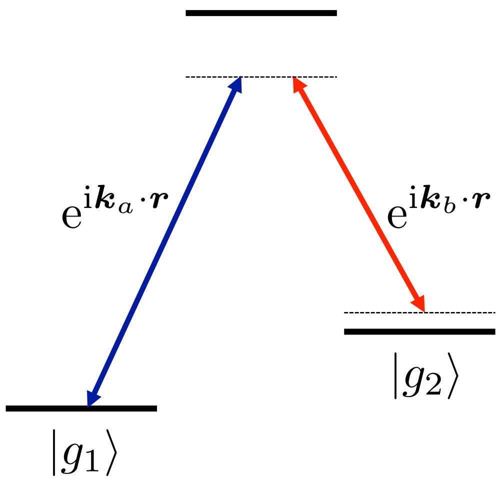
$$\hat{H}(\boldsymbol{p}) = \begin{pmatrix} (\boldsymbol{p} - \hbar\boldsymbol{k})^2/2M & \hbar\kappa_0/2 \\ \hbar\kappa_0/2 & (\boldsymbol{p} + \hbar\boldsymbol{k})^2/2M \end{pmatrix}$$

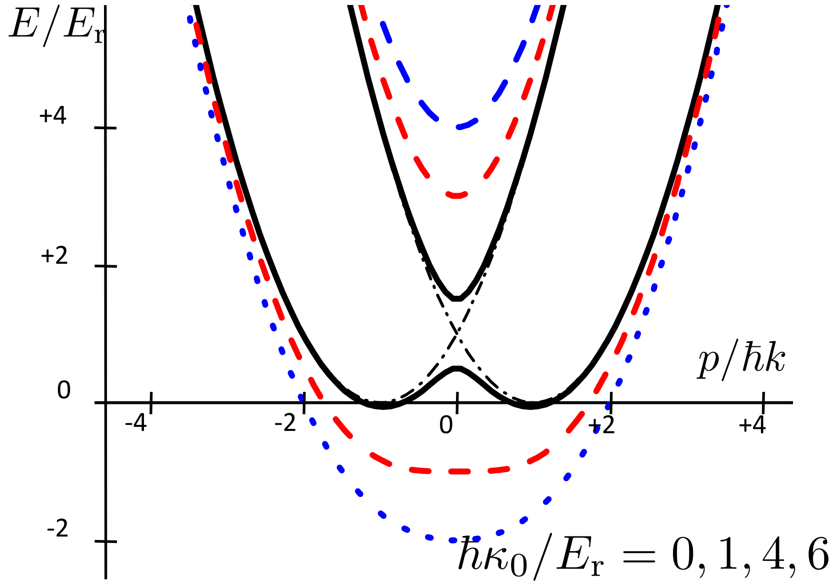
Double minimum if $\hbar\kappa_0 < 4E_{\mathrm{r}}$



Opposite from the situation where the adiabatic approximation is valid

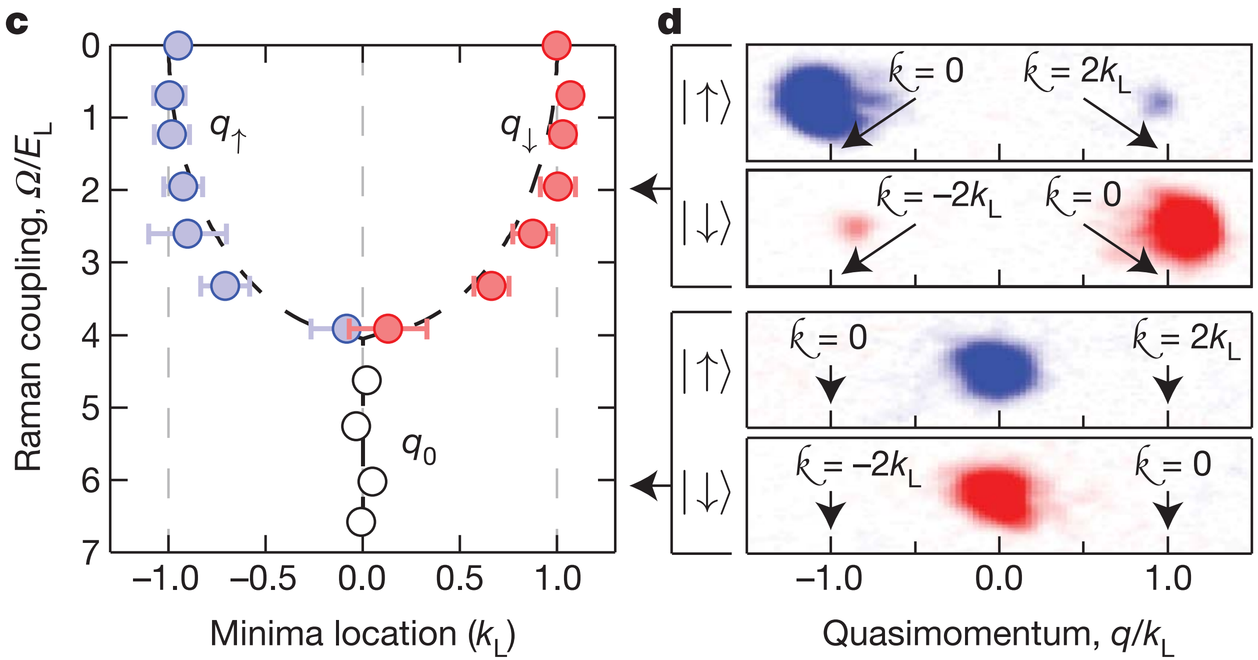
Experimental results for 1D spin-orbit coupling





Spielman, NIST 2012

^{87}Rb condensate



Conclusions

Two routes for the simulation of orbital magnetism in a quasi-uniform system

Time dependent Hamiltonian (rotations)

Geometric phases for laser-dressed states

- Possibility to generate non uniform magnetic field
- Possibility to generate non-Abelian gauge fields, in particular 2D or 3D spin-orbit coupling
- 1D spin-orbit coupling, although quite simple at the single atom level, is promising from the many-body point of view

Stringari et al., Das Sarma et al., Hui Zhai et al., T.-L. Ho et al., ...

Need to control the heating due to spontaneous emission .

Next time: gauge fields in a lattice