Exploring dynamics of quantum impurities in ultracold atoms

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Harvard-MIT CUA

$$ NSF, AFOSR MURI ON POLAR MOLECULES,
AFOSR NEW QUANTUM PHASES MURI,
DARPA OLE, ARO MURI ATOMTRONICS

Outline

Exploring orthogonality catastrophe with cold atoms
M. Knap et al., PRX (2012)

Rabi oscillations of impurity spin
M. Knap, D. Abanin, E. Demler, PRL (2013)

Mobile magnetic impurities in a Fermi superfluid:

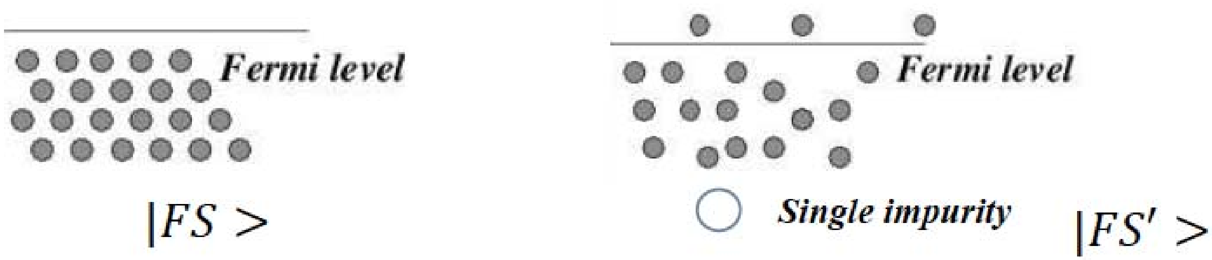
E. Vernier, D. Pekker, M. Zwierlein, E. Demler, PRA (2011)

S. Gopalakrishnan, E. Demler, arXiv (2014)

Quantum flutter and Bloch oscillations of impurities in 1d
C. Mathy, M. Zvonarev, E. Demler, Nature Physics (2012)
M. Knap et al., PRL (2013)

Exploring orthogonality catastrophe with ultracold atoms

Anderson orthogonality catastrophe



-Overlap $S = \langle FS \,|\, FS' \rangle$

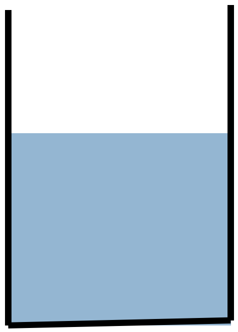
- $S \to 0$ as system size $L \to \infty$, “orthogonality catastrophe”

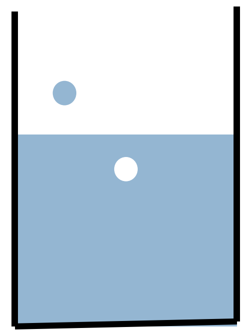
-Infinitely many low-energy electron-hole pairs produced

Fundamental property of the Fermi gas

Orthogonality catastrophe: perturbative approach

Consider a wavefunction with a single excited electron-hole pair





$$|\,\Psi_0\,\rangle = \prod_{E_k \leq E_F} c_k^\dagger\,|\,0\,\rangle$$

$$|\,\Psi_1\,\rangle = \frac{1 + \sum_{mn} \frac{V_{mn}}{(E_n - E_m)}\, c_m^\dagger\, c_n\,|\,\Psi_0\,\rangle}{\left(1 + \sum_{mn} \left(\frac{V_{mn}}{E_n - E_m}\right)^2\right)^{1/2}} \qquad \sum_{mn} = \sum_{\substack{E_m > E_F \\ E_n \leq E_F}}$$

Wavefunction overlap

$$|\,\langle\,\Psi_0\,|\,\Psi_1\,\rangle\,|^2 \approx 1 - \frac{1}{2}\sum_{mn}\left(\frac{V_{mn}}{E_n - E_m}\right)^2 = 1 - \frac{1}{2}V^2\,\nu(0)^2\int_{\Delta}^{\mathrm{W}} dE_m \int_{-E_F}^{0} dE_n \frac{1}{(E_m + |E_n|)^2}$$

$$= 1 - \frac{1}{2}V^2\,\nu(0)^2\,\mathrm{Log}\left(\frac{\mathrm{W}}{\Delta}\right)$$

Level spacing $\Delta = \dfrac{v_F}{L}$

Orthogonality catastrophe

Resummation of Logs gives a power law. Anderson (1967)

$$|\langle \Psi_0 | \Psi_{\rm IMP} \rangle|^2 = \left(\frac{N}{2}\right)^{-\left(\frac{\delta}{\pi}\right)^2}$$

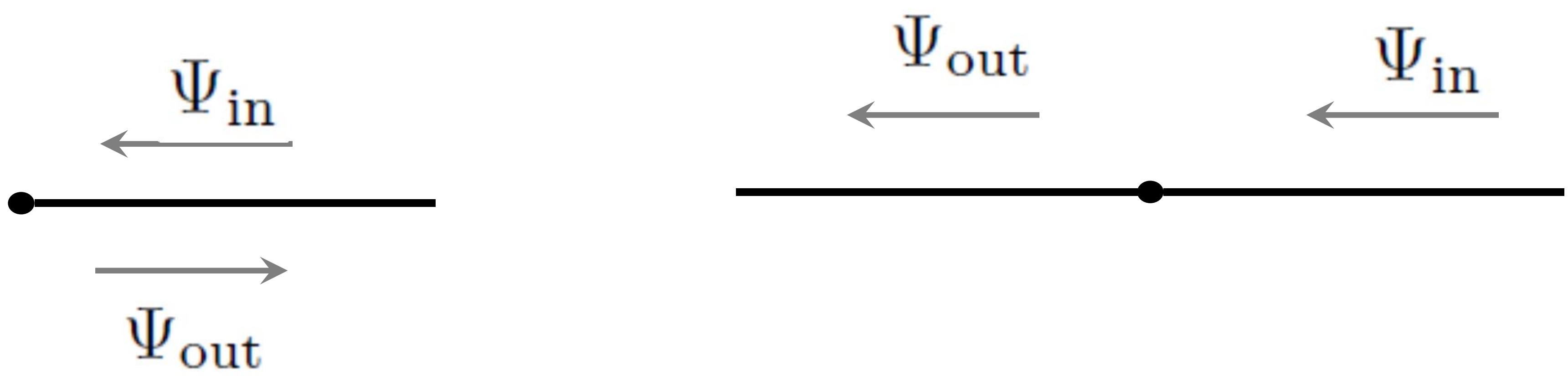
scattering phase shift at the Fermi energy $\tan\delta = k_F\, a$

Quantum impurities: 1d physics in disguise

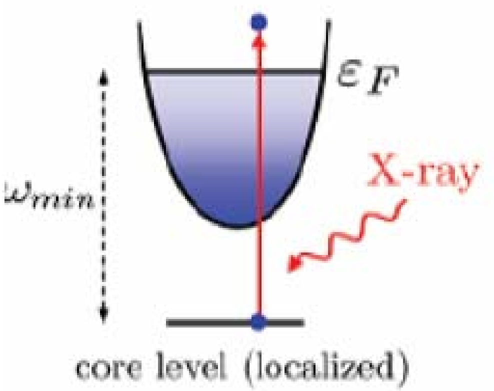
For a local scattering

$$\mathcal{H}_{\rm scat}(t) = V_{\rm sc}(t)\,\Psi^\dagger_{r=0}\,\Psi_{r=0}$$

Only fermions with angular momentum *l=0* interact with the scattering potential. Incoming and outgoing particles can be combined to make a 1d system.



Orthogonality catastrophe in X-ray absorption spectra

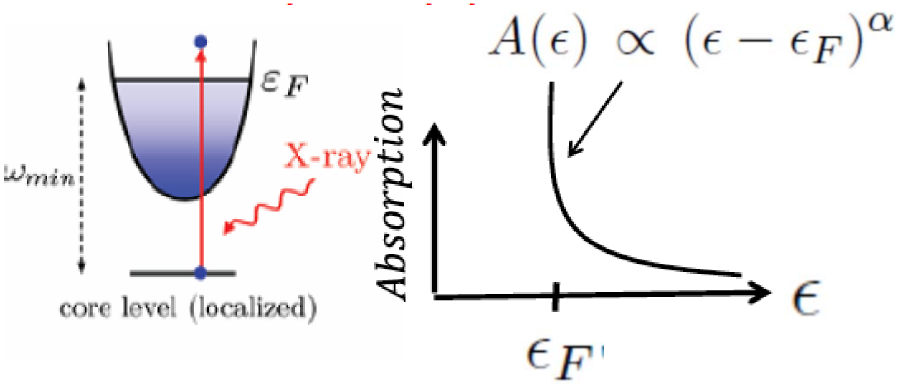


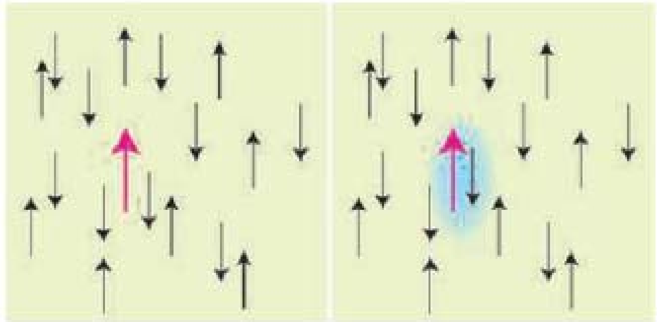
Without impurity   *With impurity*

-Relevant overlap: $S(t) = \langle FS \,|\, e^{iH_0 t} e^{-iH_f t} \,|\, FS \rangle \propto t^{-\delta^2/\pi^2}$

δ -- scattering phase shift at Fermi energy

Orthogonality catastrophe: paradigm of impurity problem in condensed matter





-Edge singularities in the X-ray absorption spectra (exact solution of non-equilibrium many-body problem)

-Kondo effect: entangled state of impurity spin and fermions

Influential area, both for methods (renormalization group) and for strongly correlated materials

Xray absorption in Na

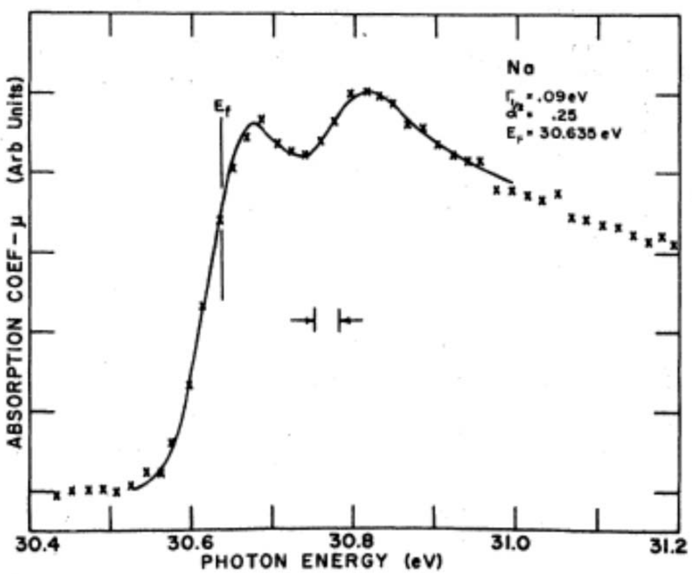


FIG. 6. SXA spectrum for Na at 100 K.

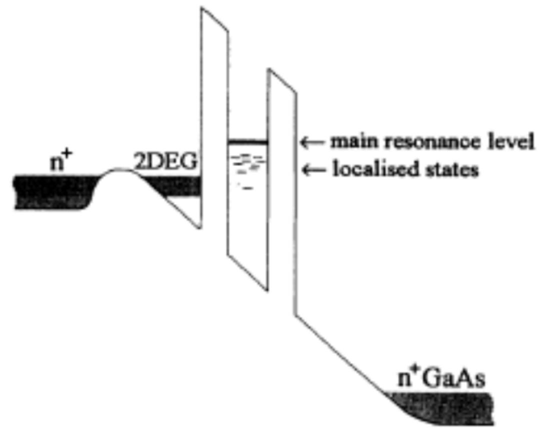
Expect $A(\epsilon) \propto (\epsilon - \epsilon_F)^\alpha$

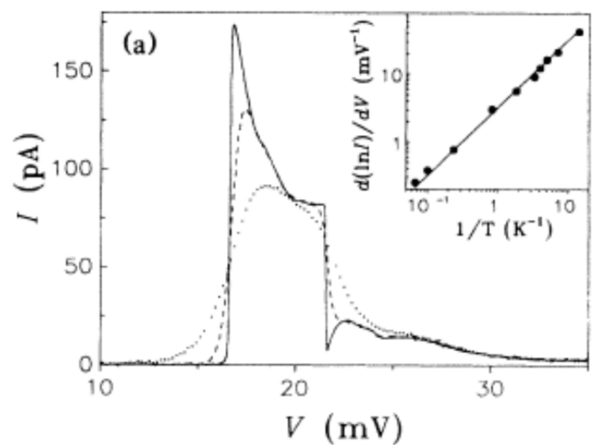
$$\alpha = \frac{2\delta}{\pi} - \left(\frac{\delta}{\pi}\right)^2$$

δ- phase shift

Tunneling through a resonant level

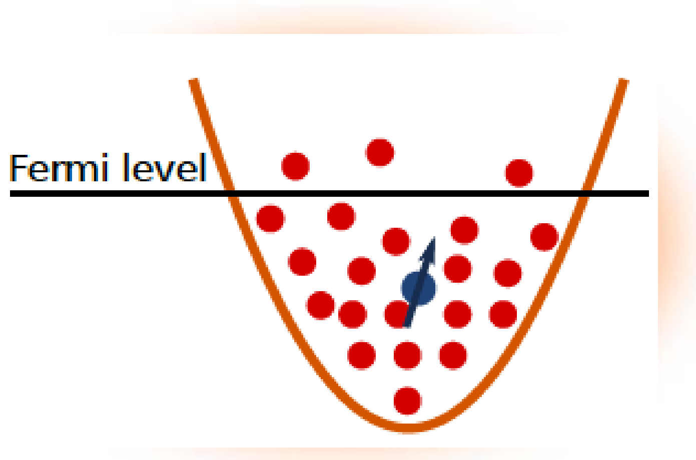
A. Geim et al., PRL (1994)





Orthogonality catastrophe with cold atoms: Setup

M. Knap. A. Shashi, et al., PRX (2012)



-Fermi gas+single impurity

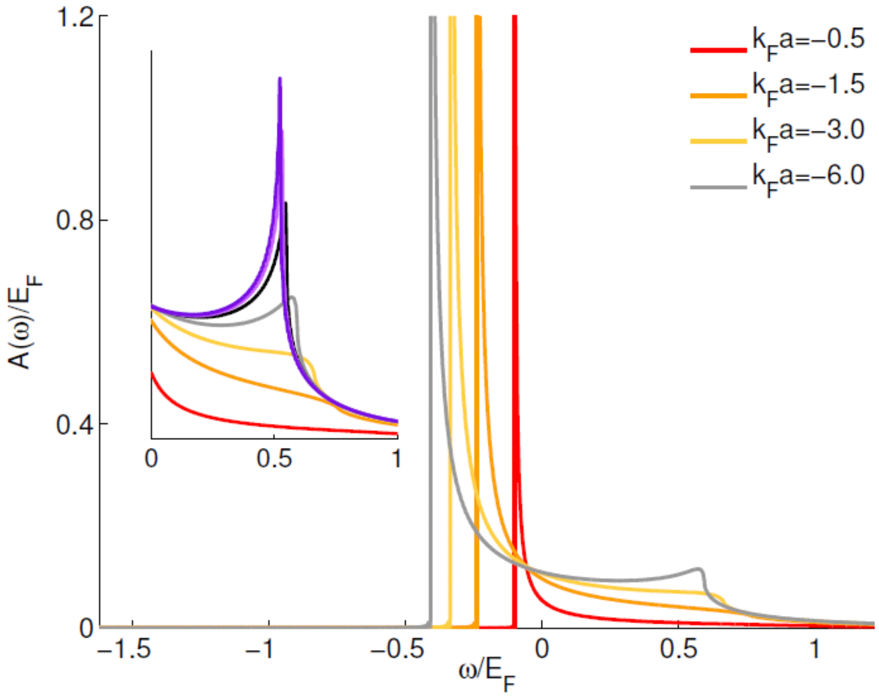
-Two pseudospin states of impurity, $|\uparrow\rangle$ and $|\downarrow\rangle$

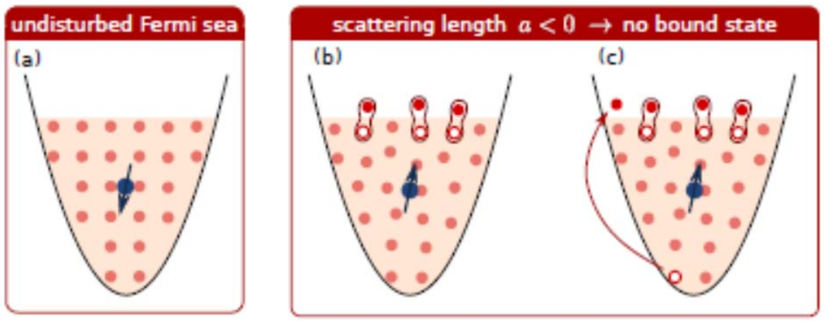
- $|\uparrow\rangle$ -state scatters fermions
  $|\downarrow\rangle$ -state does not

-Scattering length a

-Fermion Hamiltonian for pseudospin $|\uparrow\rangle, |\downarrow\rangle$ -- H_0, H_f

RF spectra



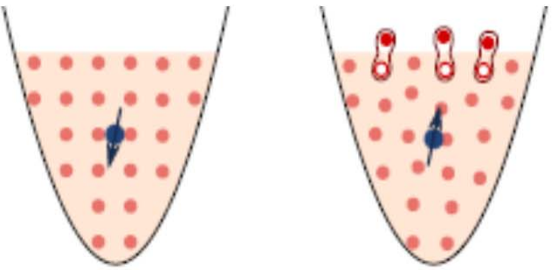


a < 0; no impurity bound state

Cusp at E_F

Single threshold in absorption

Functional determinant method



$$S(t) = \frac{\sum_n e^{-\beta E_n} \langle n | e^{i\mathcal{H}_0 t} e^{-i\mathcal{H}_f t} | n \rangle}{\sum_n e^{-\beta E_n}} = \frac{\mathrm{Tr}\{ e^{-\beta\mathcal{H}_0} e^{i\mathcal{H}_0 t} e^{-i\mathcal{H}_f t}\}}{\mathrm{Z}}$$

Both $\mathcal{H}_0$ and $\mathcal{H}_f$ are quadratic Hamiltonians. Electrons are non-interacting. All initial states are states are Slater determinants. During time evolution they remain Slater determinantns. We only need to find evolution of single particle eigenstates.

Useful properties of Slater determinants

Consider $\langle e^X \rangle = \mathrm{tr}\left[e^X e^{-\beta H}\right] / \mathrm{tr}\left[e^{-\beta H}\right]$ for quadratic X, H.

- In basis where $X = \sum_\alpha \omega_\alpha \hat{n}_\alpha$

  $$\mathrm{tr}\left[e^X\right] = \prod_\alpha \sum_{n_\alpha=0,1} e^{n_\alpha \omega_\alpha} = \prod_\alpha (1 + e^{\omega_\alpha}) = \det\left(1 + e^X\right)$$

- BCH: $e^X e^Y = e^Z$, Z quadratic, $\mathrm{tr}\left[e^X e^Y\right] = \det\left(1 + e^X e^Y\right)$

Ramsey fringes – new manifestation of OC

-Utilize control over spin

-Access coherent coupled dynamics of spin and Fermi gas

-Ramsey interferometry

1) π/2 pulse $\quad |\downarrow\rangle|FS\rangle \rightarrow \frac{1}{\sqrt{2}}|\downarrow\rangle|FS\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle|FS\rangle$

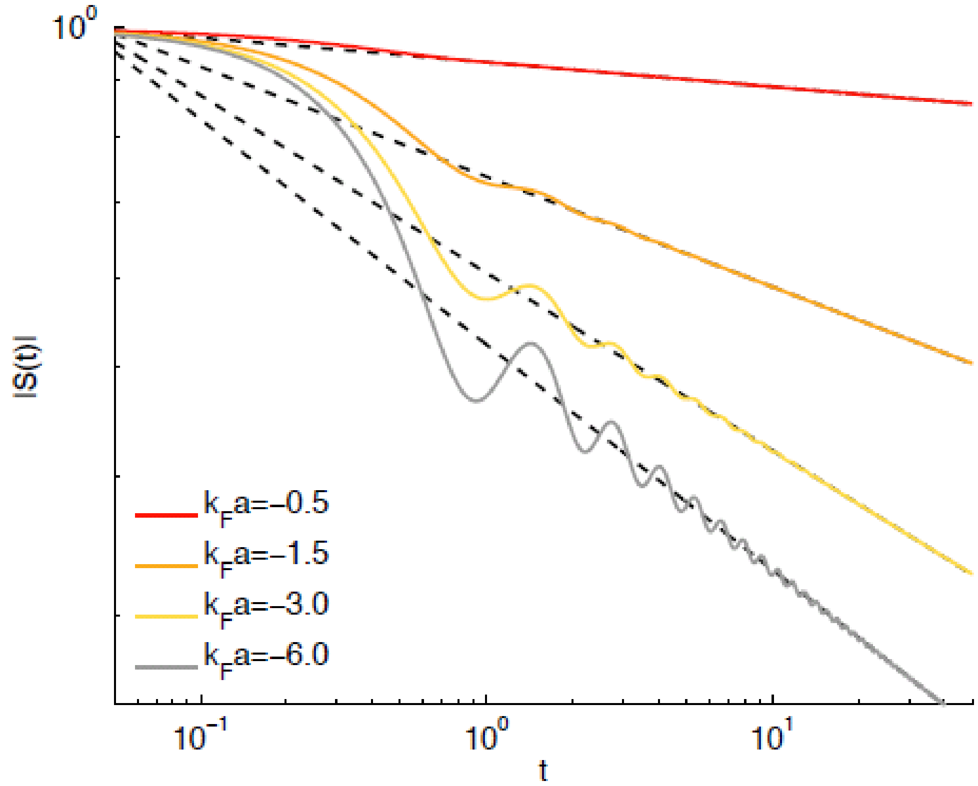
2) Evolution $\quad \frac{1}{\sqrt{2}}|\downarrow\rangle e^{-iH_0 t}|FS\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle e^{-iH_f t}|FS\rangle$

3) Use π/2 pulse to measure $\quad < S_x >= \mathrm{Re}[S(t)]$

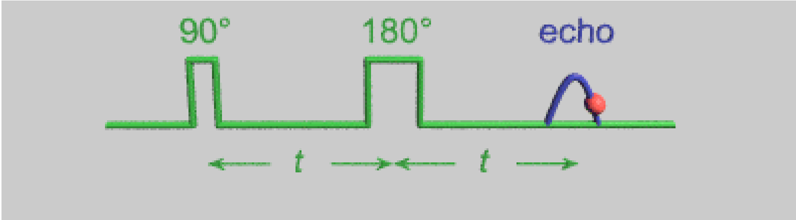
$$S(t) = \left\langle FS \,|\, e^{iH_0 t} e^{-iH_f t} \,|\, FS \right\rangle$$

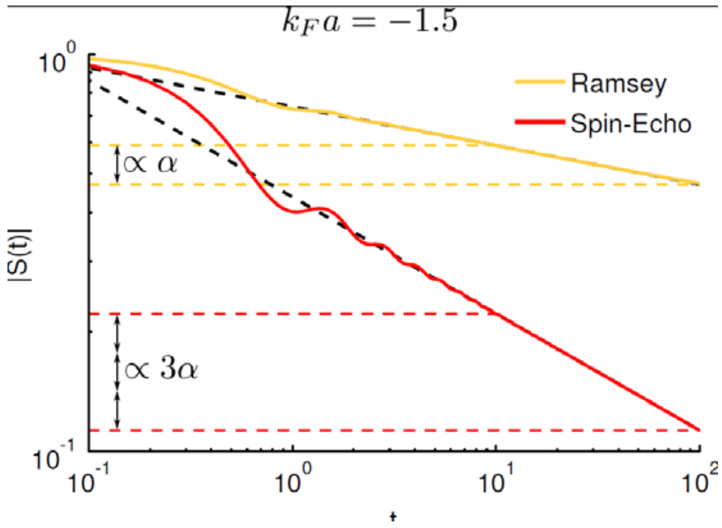
Direct measurement of OC in the time domain

Ramsey fringes as a probe of OC



Spin echo: probing non-trivial dynamics of the Fermi gas





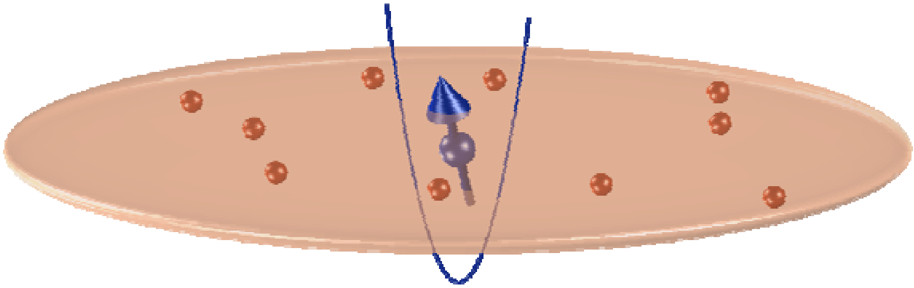
-Unlike the usual situation (spin-echo decays slower than Ramsey)

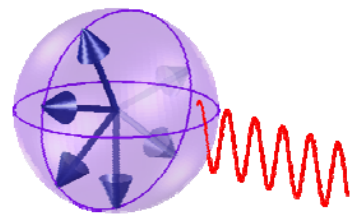
-Cancels magnetic field fluctuations

-Universal

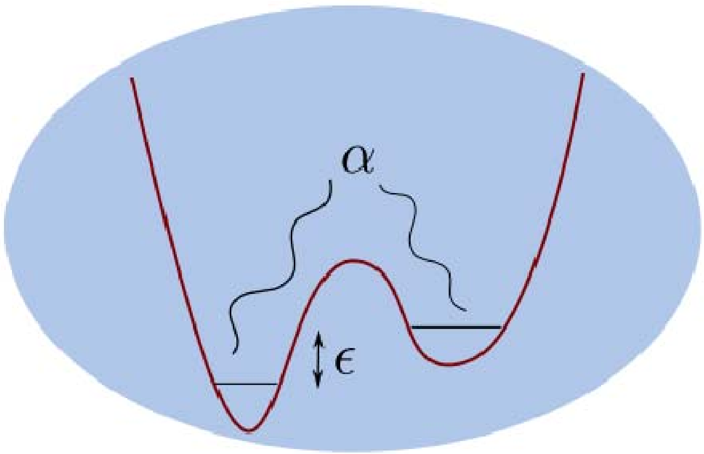
-Generalize to n pi-pulses to study even more complex response functions

Driven impurity





Low energy description: spin-bath problem



Leggett, Zwerger, et al., RMP (1987).

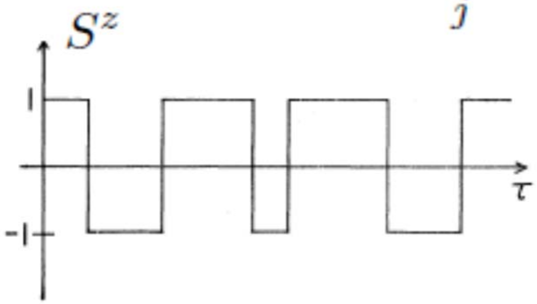
Relies on

- Mapping to 1d fermions
- Bosonization of fermions

$$\hat{H} = \sum_q v_F |q| b_q^\dagger b_q + \sqrt{2\alpha_1}\pi v_F \sum_{q>0} \left(\frac{q}{2\pi L}\right)^{1/2} (b_q^\dagger + b_q)\hat{\sigma}_z + \Omega_0 \hat{\sigma}_x + \epsilon \hat{\sigma}_z$$

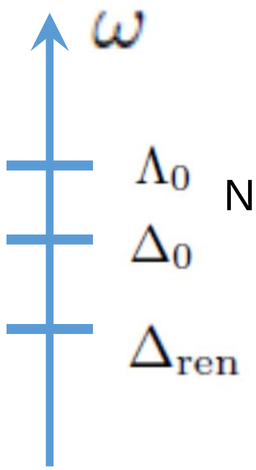
Power laws in the spin bath problem

$$\mathcal{H} = \Delta_0 S_x + \sum_j \omega_j b_j^\dagger b_j + S^z \sum_j g_j (b_j + b_j^\dagger)$$



Integrate out phonons $\Delta_0 < \omega_j < \Lambda_0$

Each phonon reduces tunneling by $\langle \Psi_j(\uparrow) | \Psi_j(\downarrow) \rangle = e^{-\frac{g_j^2}{\omega_j^2}}$

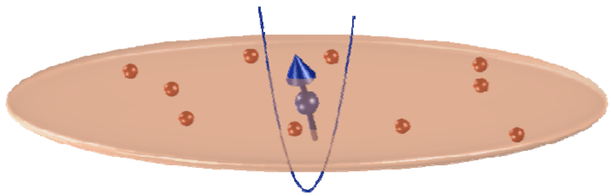


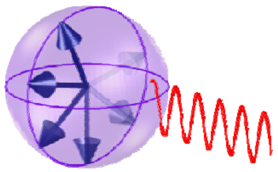
New effective tunneling

$$\Delta_{\rm ren} = \Delta_0 \prod_{\Delta_0 < \omega_j < \Lambda_0} e^{-\frac{g_j^2}{\omega_j^2}} = \exp\{-\int_{\Delta_0}^{\Lambda_0} \frac{\rho(\omega)\, g^2(\omega)\, d\omega}{\omega^2}\}$$

$$= \exp\{-\alpha \int_{\Delta_0}^{\Lambda_0} \frac{d\omega}{\omega}\} = \Delta_0 \left(\frac{\Delta_0}{\Lambda_0}\right)^\alpha$$

Reduction of Rabi oscillation frequency





Orthogonality catastrophe cut-off by the Rabi frequency itself

$$S(t) = \left\langle FS \,|\, e^{iH_0 t} e^{-iH_f t} \,|\, FS \right\rangle \propto t^{-\delta^2/2\pi^2}$$

$$t \sim \frac{1}{\Omega} \qquad \eta = \frac{\delta^2}{2\pi^2}$$

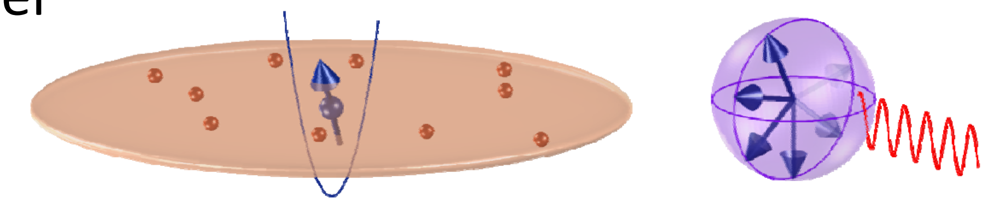
$$\Omega \sim \Omega_0 \left(\frac{\Omega}{\omega_c}\right)^{\eta} \qquad \Omega^{1-\eta} \sim \Omega_0 \qquad \frac{\Omega}{\Omega_0} \sim \left(\frac{\Omega_0}{\omega_c}\right)^{\frac{\eta}{1-\eta}}$$

Reduction of Rabi oscillation frequency

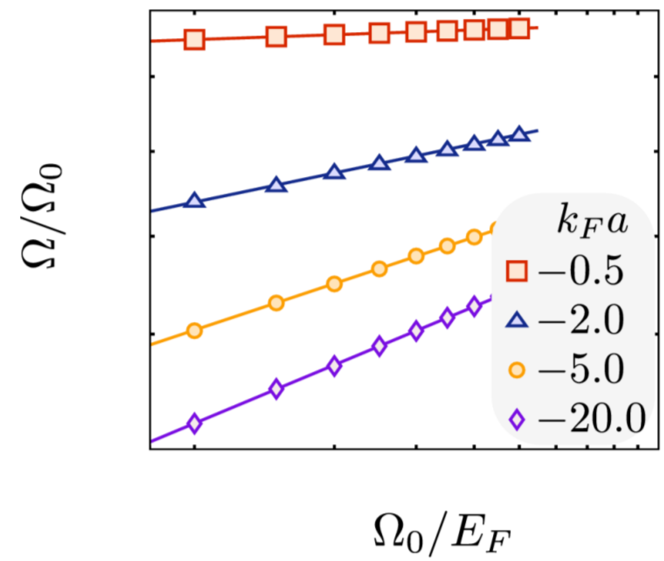
T. Leggett et al. RMP(1987)

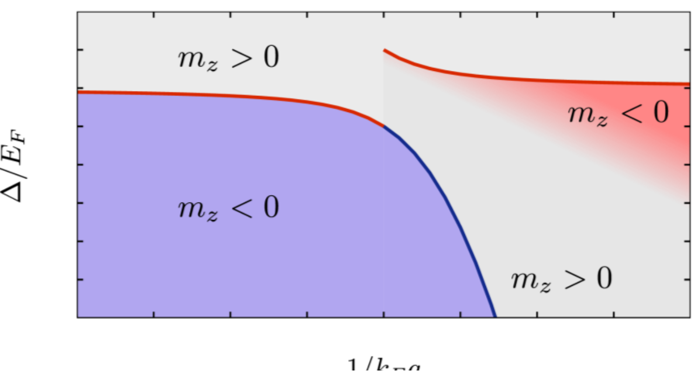
- NIBA and predictions for SB model

→ power-law scaling



$$\frac{\Omega}{\Omega_0} = F(\eta)\left(\frac{\Omega_0}{\omega_c}\right)^{\frac{\eta}{1-\eta}}$$



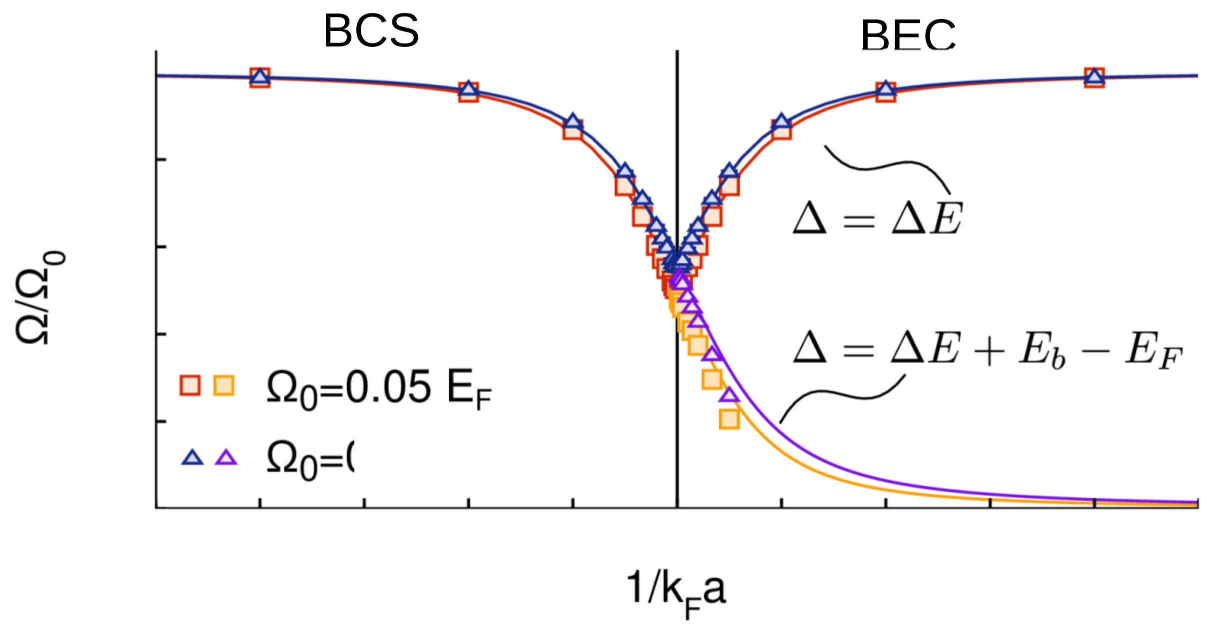


$$\eta_1 = \frac{\delta_F^2}{2\pi^2}$$

$$\eta_2 = \frac{(\delta_F/\pi + 1)^2}{2}$$

Reduction of Rabi frequency

- Ω as a function of interaction strength:

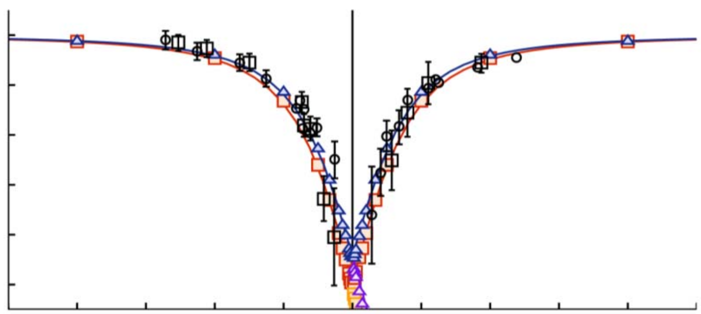


Comparison to experiment

- polaron experiment from Innsbruck
- mixture of $^{6}Li^{40}K$, short pulse duration

→ essentially static

- our model provides a possible explanation

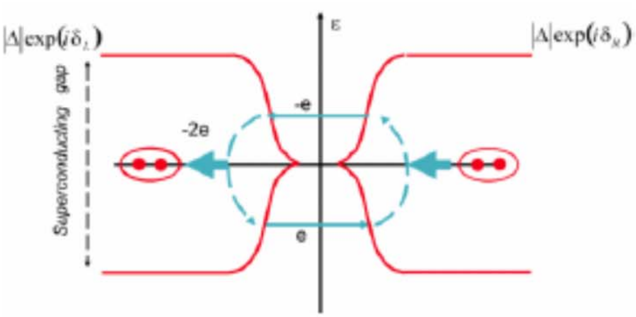


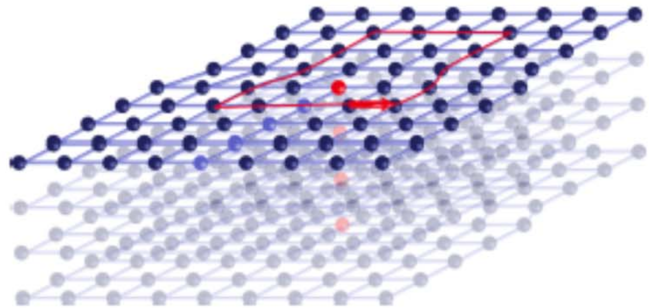
C. Kohstall, et al., Nature 485, 615 (2012).

Mobile magnetic impurities in Fermi superfluids

Impurities in solid state systems

Static impurities / defects
deform electronic structure, create bound states

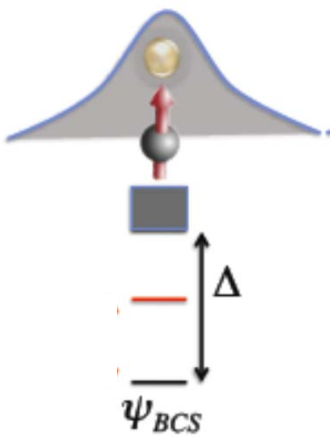




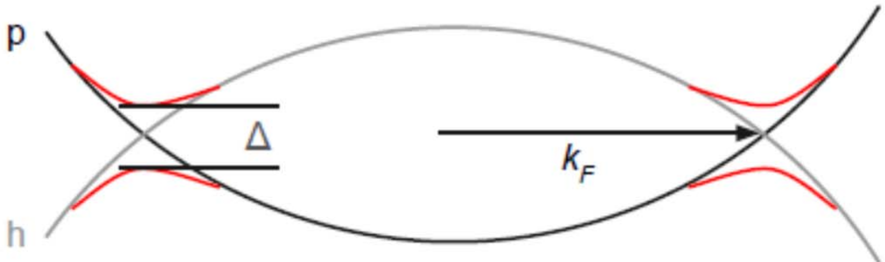
What if we let the impurity/defect move?

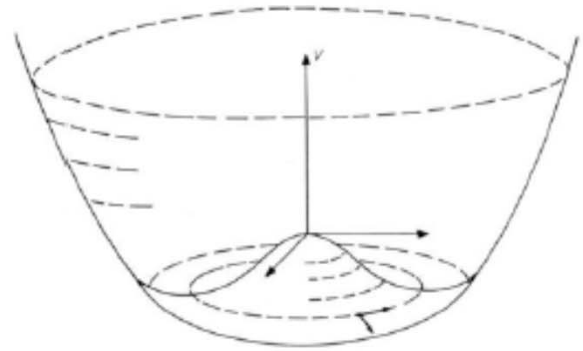
- Does the bound state survive?
- How does bound state affect impurity/defect dispersion?
- Can interesting defect/bound-state "molecules" be engineered?

Bound states on magnetic impurities in superconductors



BCS pairing → Mexican hat





DOS singularity ($1/\sqrt{E}$, “1D-like”) at bottom of quasiparticle band, weak-coupling bound state with polynomial energy

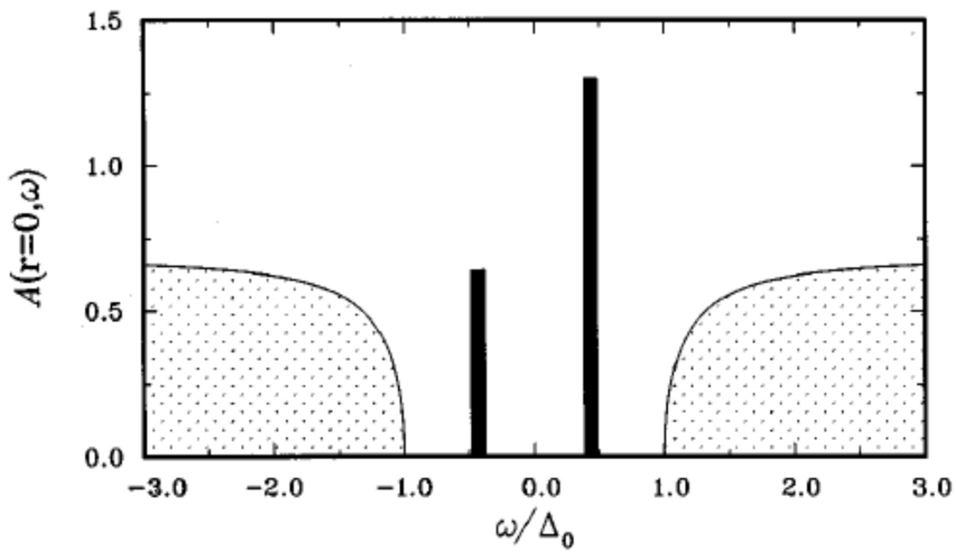
$$E \sim \Delta \frac{1-(\pi N(E_F)J)^2}{1+(\pi N(E_F)J)^2}$$

interspecies coupling

Impurity must be magnetic, otherwise quasiparticles decouple from it at low energies

Bound states on magnetic impurities in superconductors

Yu, Acta Phys. Sin. 21, 75 (1965)
Shiba, Prof. Theor. Phys. 40, 435 (1968).
Rusinov, Sov. Phys. JETP Lett. 9, 85 (1969)



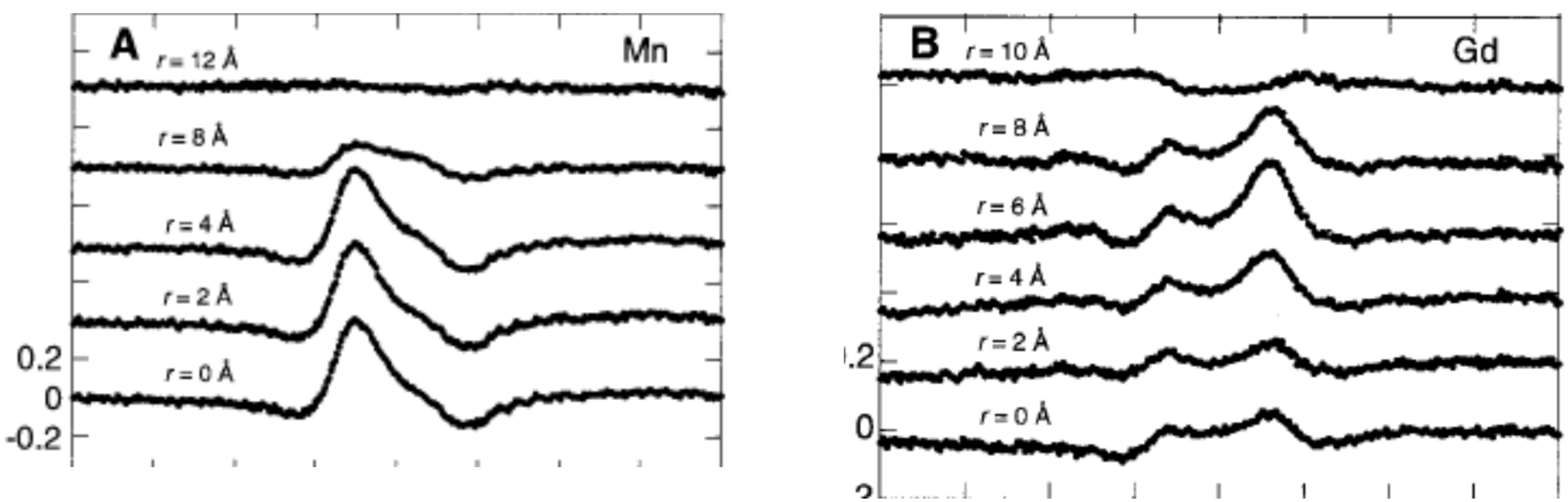
Salkola, Balatsky, Schrieffer, PRB 55:12648 (1997)

Probing the Local Effects of Magnetic Impurities on Superconductivity

Ali Yazdani,* B. A. Jones, C. P. Lutz, M. F. Crommie,†
D. M. Eigler

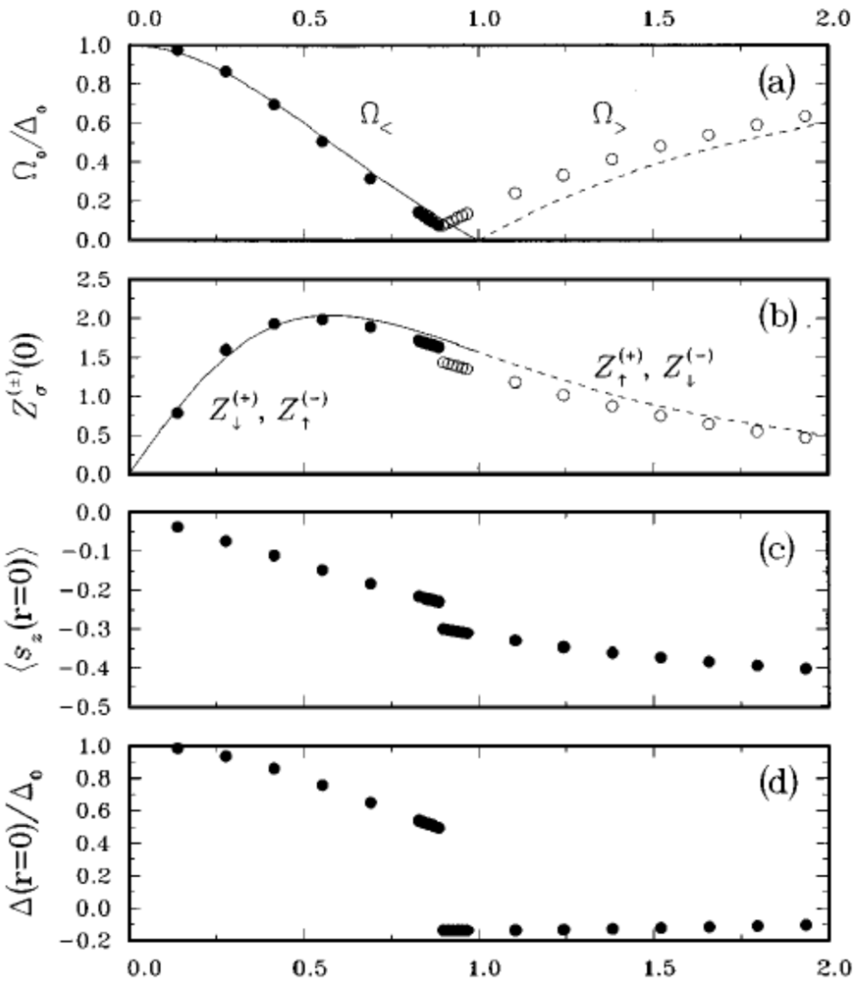
Science 275:1767 (1997)

Local density of states near



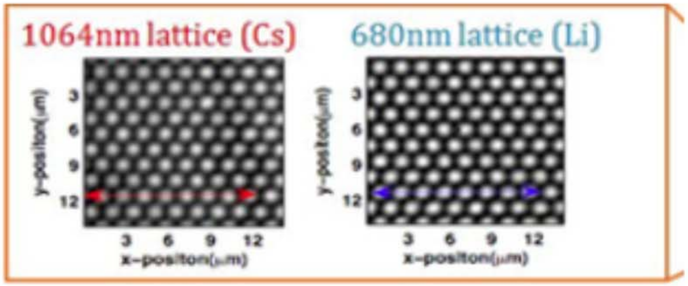
Parity changing transition

Salkola, Balatsky, Schrieffer, PRB 55:12648 (1997)



Analogous to Kondo singlet formation

Possible realization: Cs impurities in Li fermionic condensate



e.g. Chen Chin's group

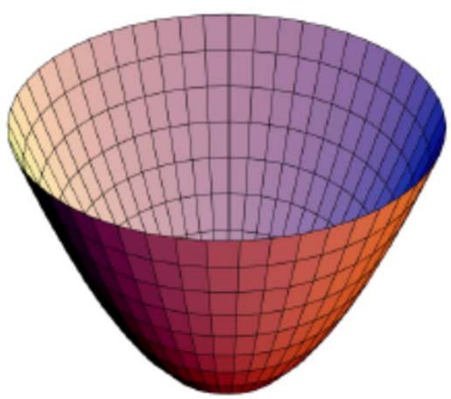
- Heavy Cs impurities in light Li fermionic condensate
- Interactions between Cs and the two hyperfine states of Li can be tuned independently via heteronuclear Feshbach resonance: impurities are "magnetic"
- Infinite-mass limit: Shiba states bound to each Cs atom
  - Our question: **fate of a mobile Shiba state**

Bound state of Bogoliubov quasiparticle and impurity atom

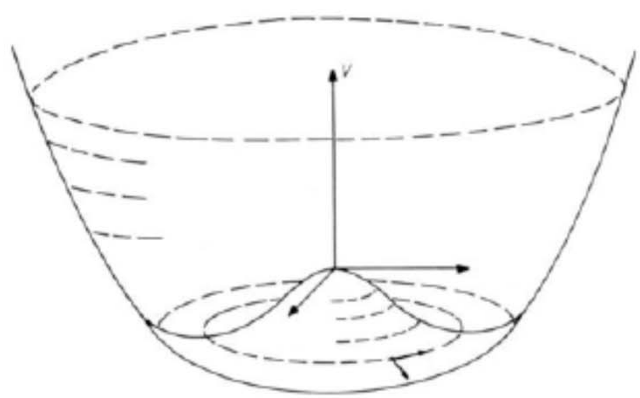
Overall Hamiltonian $H_0 = \sum_{k\sigma} \xi_k \gamma^\dagger_{k\sigma}\gamma_{k\sigma} + p^2/2M$

$$H_1 = J \sum_{kq\sigma} e^{iqr}(\gamma^\dagger_{k+q,\sigma}\gamma_{k,\sigma} + \gamma^\dagger\gamma^\dagger + \text{h.c.})$$

Odd sector: two-body problem with different dispersions:



impurity



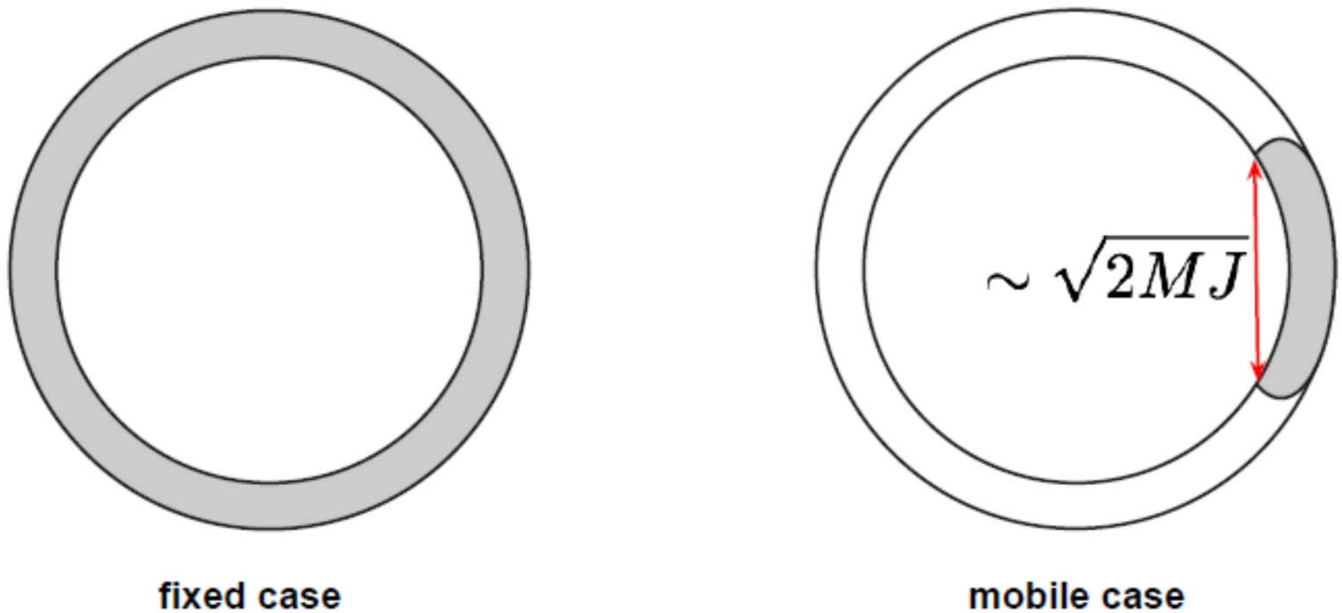
Bogoliubov qp

Sketch of the wavefunction

Total momentum conserved, ~ k_F

Scattering across Mexican hat costs impurity recoil

$$\mathcal{E} \simeq k_F^2/2M \gg J$$



Bound state energy

Label lowest-energy odd states by p as $(k_F, 0)$; $(k_F - p, p)$...

- In directions transverse to k_F, unperturbed energy of state p is $p^2/2M$ (to quadratic order)
- In the longitudinal direction it is $p^2(1/2M + v_F^2/2\Delta)$

Bound state problem with strongly anisotropic mass

$$E_{1D} \simeq -J^2\left(\frac{1}{2M}+\frac{v_F^2}{\Delta}\right)$$

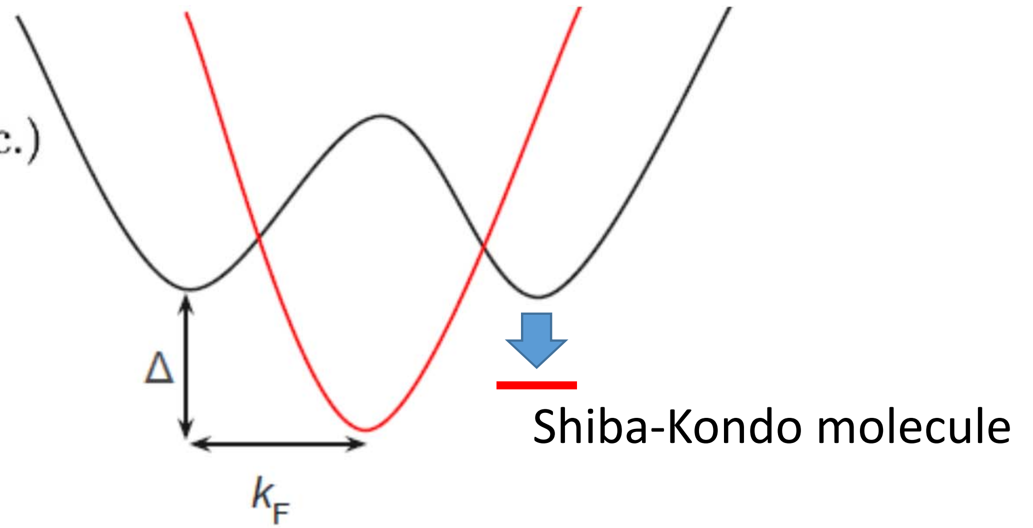
$$E_{2D} \simeq -\Delta\exp\left(-\frac{\sqrt{\frac{1}{M}+\frac{v_F^2}{\Delta}}}{|J|\sqrt{M}}\right)$$

$$E_{3D} \simeq -\Lambda\left[1-\left\{\left(\frac{1}{M}+\frac{v_F^2}{\Delta}\right)^{1/3}\Big/M^{2/3}|J|\right\}\right]$$

reduced mass

Weak interaction: parity transition

$$H_1 = J \sum_{kq\sigma} e^{iqr} (\gamma^\dagger_{k+q,\sigma}\gamma_{k,\sigma} + \gamma^\dagger\gamma^\dagger + \text{h.c.})$$



Quasiparticle number conserved (mod 2)

Min. odd-parity energy ~ Δ (P = k_F)

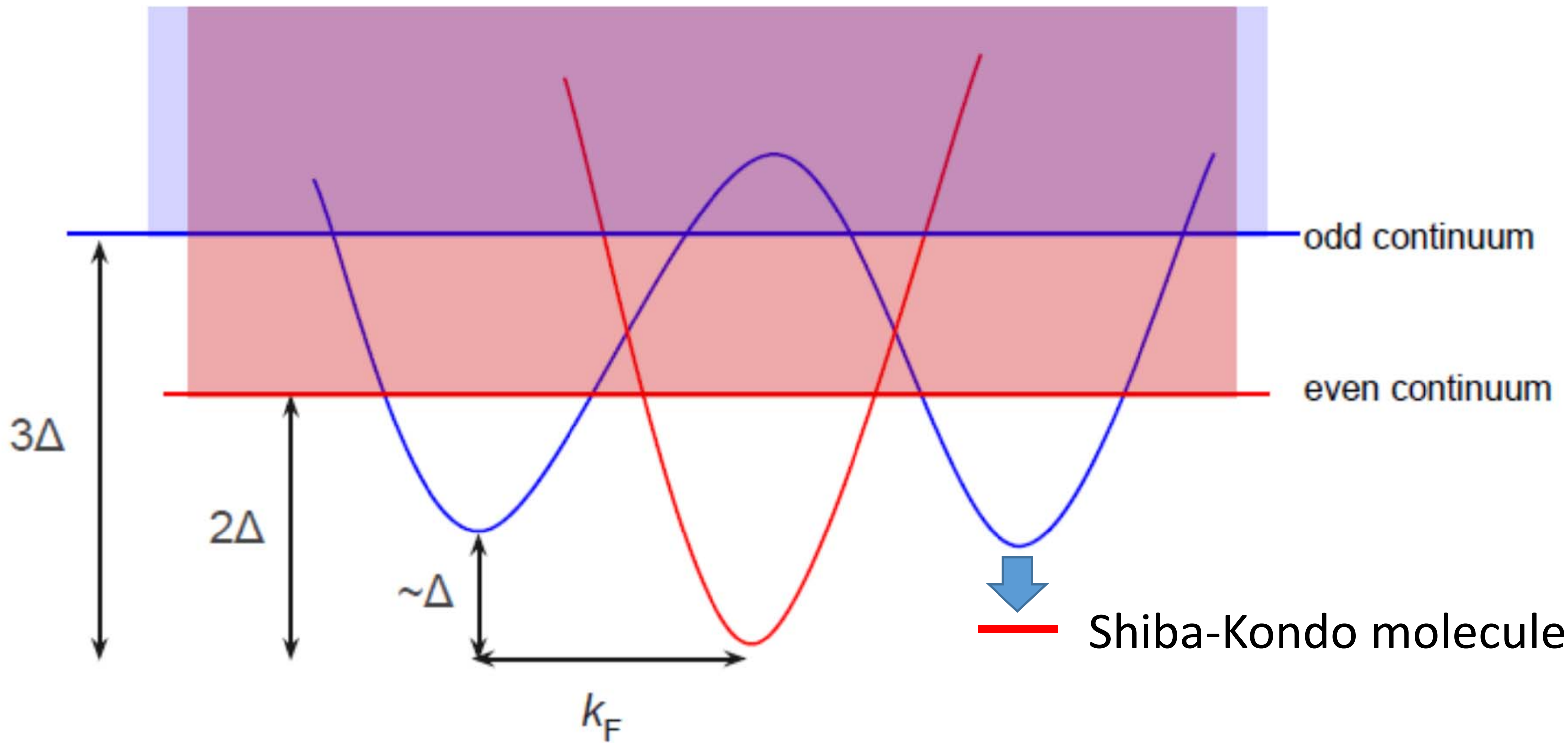
Min. even-parity energy ~ 0 (P = 0)

Which is lower at a given k?

If $k_F^2/2M \gg \Delta$ then parity of lowest-energy state changes with P

Because parity is conserved, this level crossing is sharp to all orders

Strong interaction: global minimum



As J increases, odd branch comes down in energy, eventually becomes global ground state (can be captured via T-matrix approach)

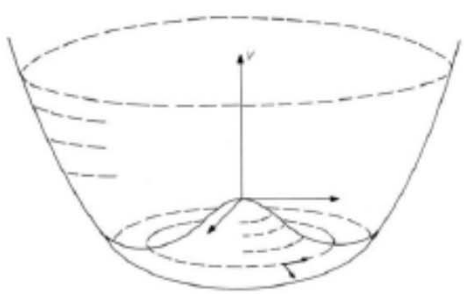
“Exotic” molecule

Mass of the relative coordinate

$$\frac{1}{\mu^{\|}} = \frac{1}{M} + \frac{1}{\infty}; \quad \frac{1}{\mu^{\perp}} = \frac{1}{M} + v_F^2 \Delta$$

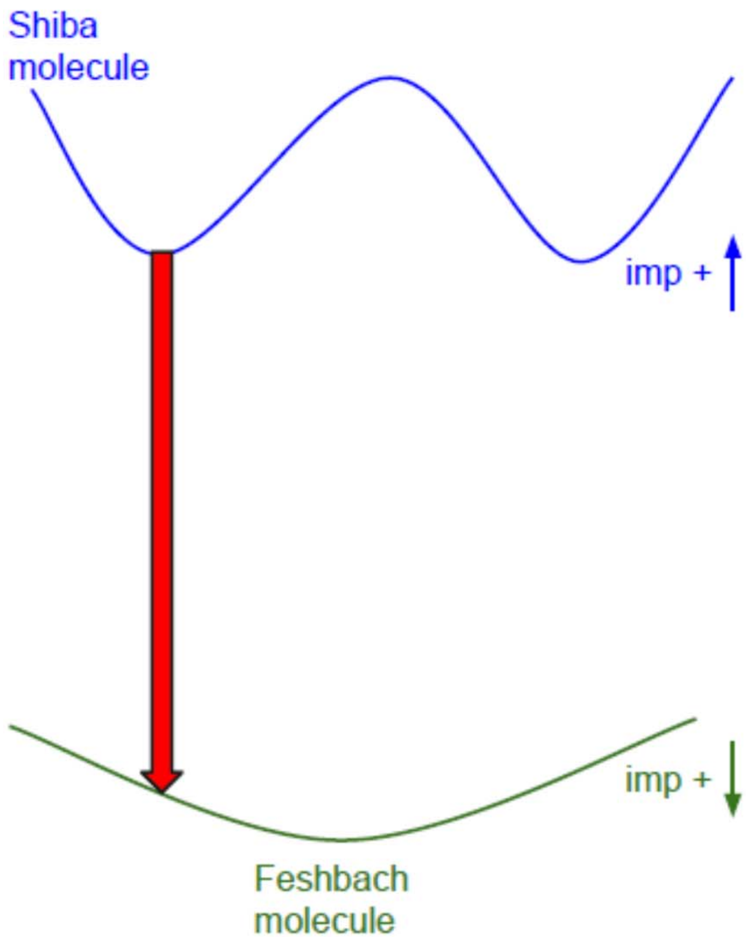
What about the center-of-mass coordinate?

$$\mathbb{M}^{\|} = \infty; \mathbb{M}^{\perp} = M + \Delta / v_F^2$$



Molecule itself has Mexican-hat dispersion: impurity momentum ~ 0, quasiparticle momentum ~ k_F in any direction

Detection via RF



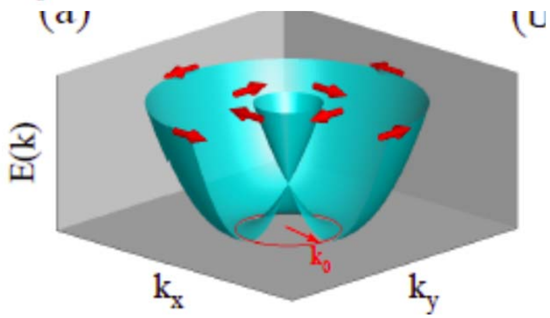
Tune to Cs-Li scattering length attractive for “up spins”, repulsive for “down spins”

Implies existence of Feshbach molecule between impurity and down spins

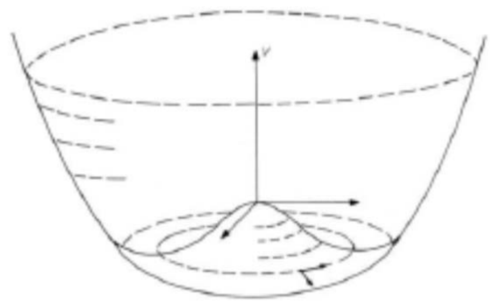
Exotic many-body phases?

Reminiscent of electrons with spin-orbit coupling

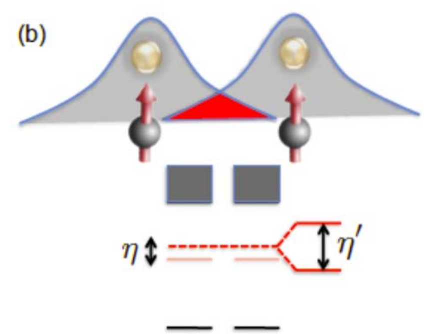
$$H = \sum_j \left\{ \frac{1}{2m} \left[-\nabla_j^2 - \frac{2k_0}{i} (\nabla_j \times \hat{z}) \cdot \vec{\sigma}_j \right] + E_0 \right\}$$



in 2D and contact interaction, ground state: breaks rotational symmetry. Wigner crystal or nematic.
Berg, Rudner, Kivelson (2012)



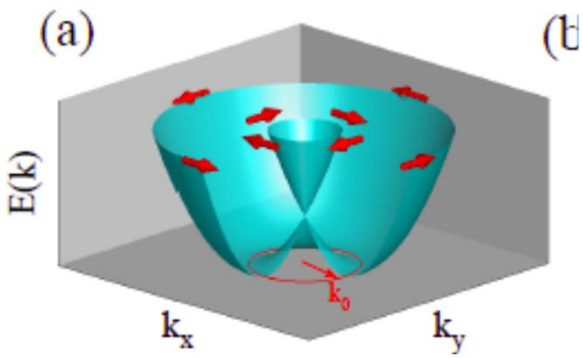
Interaction between two Shiba states: Yao et al., 13092633



$$\eta_{\uparrow\uparrow} = -\frac{\Delta}{2} \frac{\cos(k_f r)}{(k_f r)^2} e^{-\frac{2r}{\xi}}$$

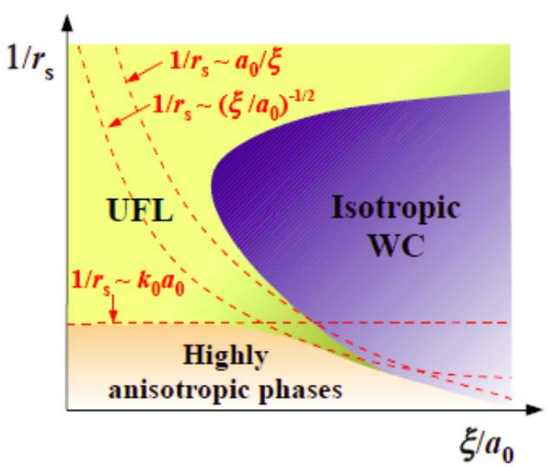
For contact interaction Wigner crystal with aspect ratio $\eta^\star \sim (n/k_0^2)^{-\frac{1}{3}}$
has energy per particle parametrically better than uniform phase

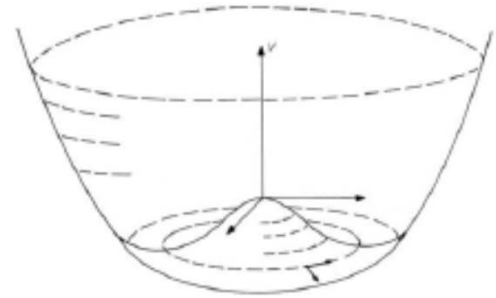
$$\varepsilon^\star(\eta^\star) \sim |E_0| \, (n/k_0^2)^{\frac{4}{3}}$$



$$V(r) \approx e^2/\kappa r \ \text{ for } r \ll \xi$$

$$V(r) \sim e^2\xi^2/\kappa r^3 \text{ for } r \gg \xi$$





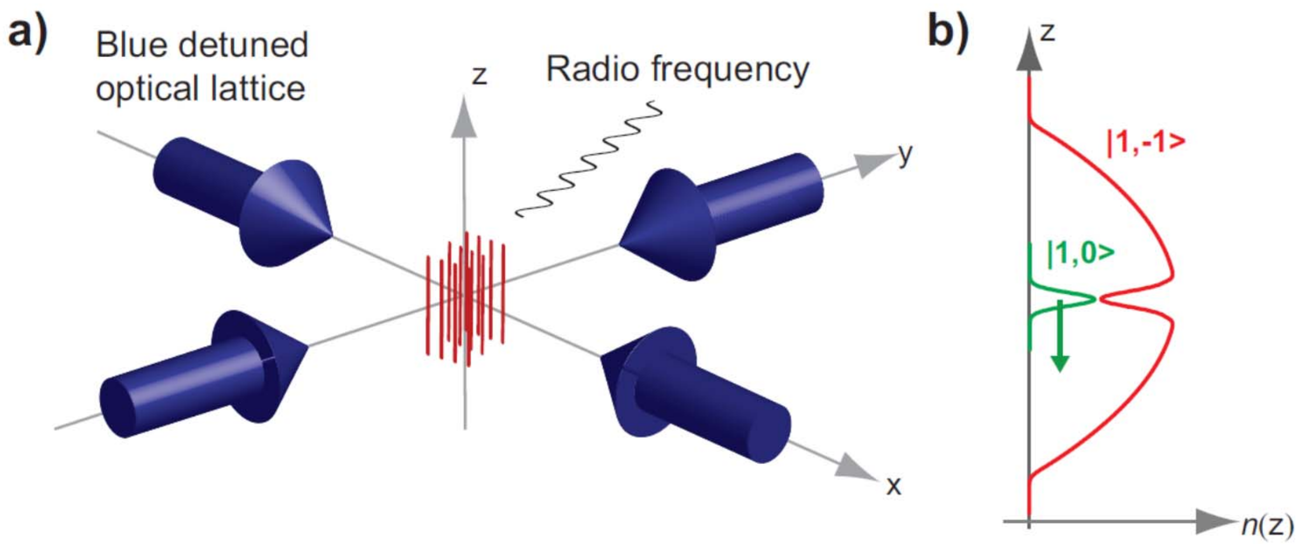
$$V(r) = -\frac{\Delta}{2}\frac{\cos(k_f r)}{(k_f r)^2}e^{-\frac{2r}{\xi}}$$

???

Moving quantum impurities in 1d

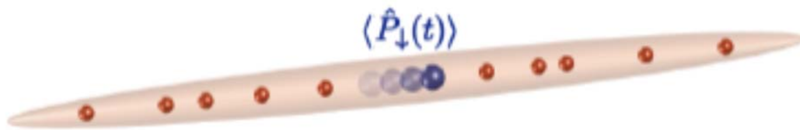
Motion of impurities in interacting Bose gas:

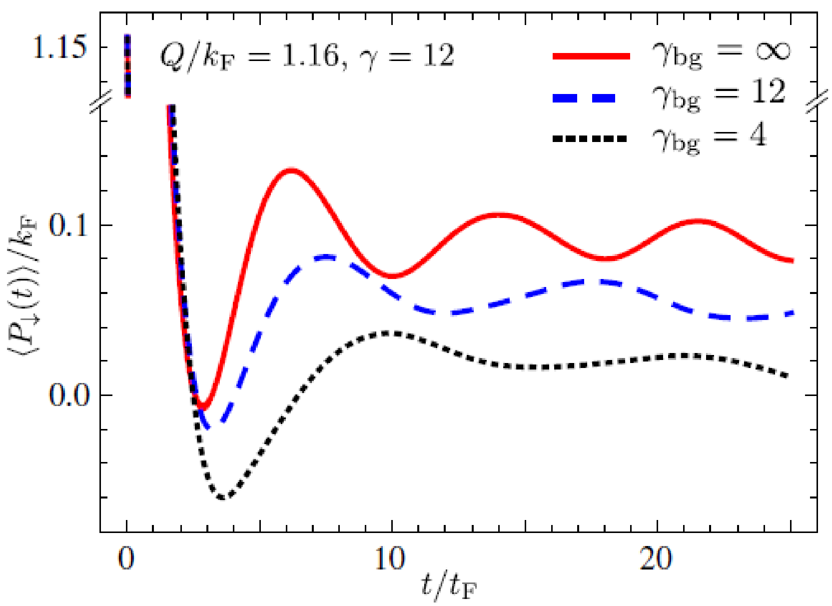
Cambridge/Innsbruck type system

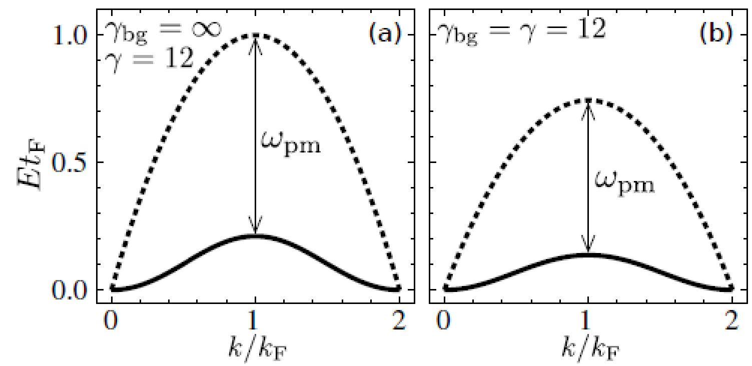


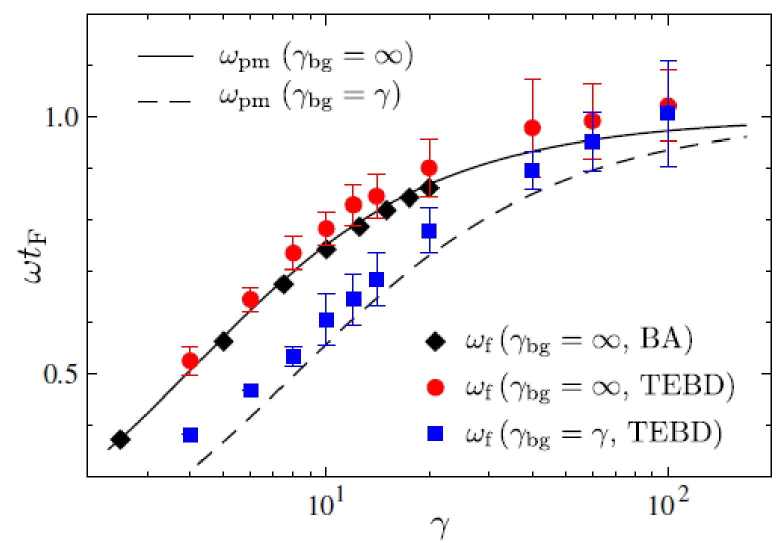
Quantum flutter

C. Mathy, M. Zvonarev, E. Demler, Nature Physics (2012)
M. Knap et al., PRL (2014)



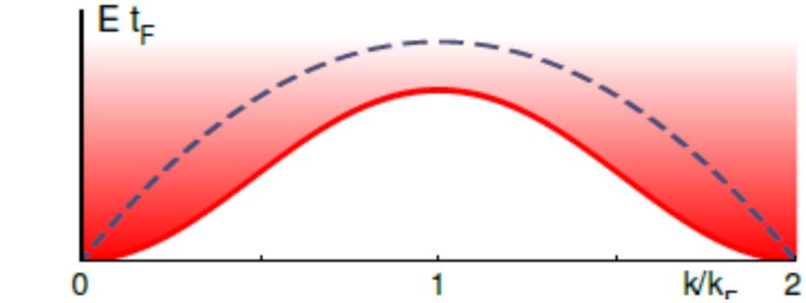


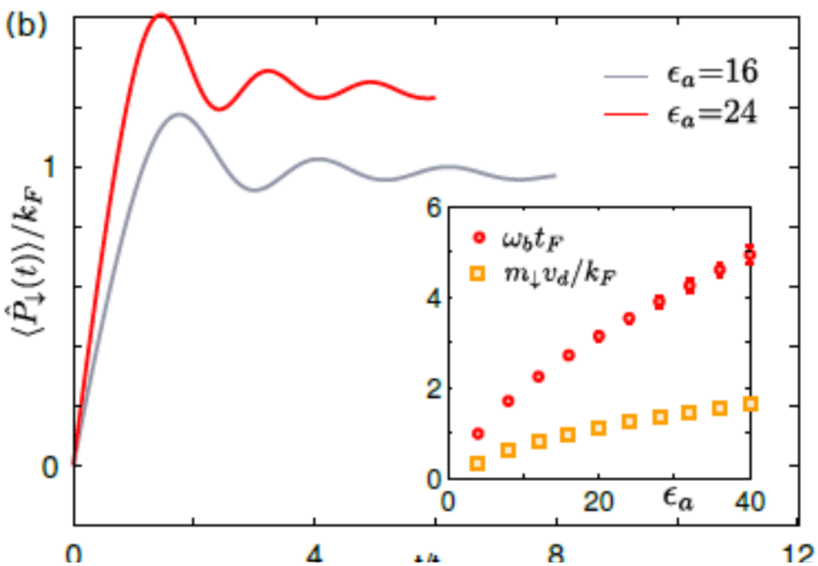


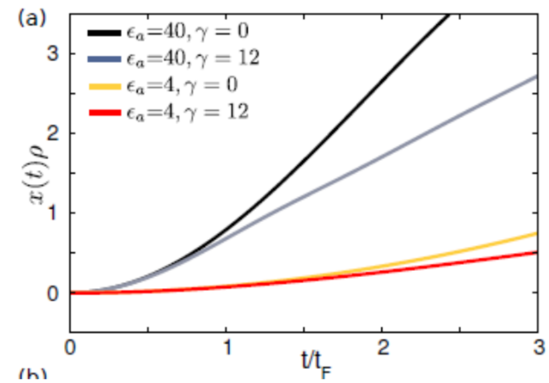


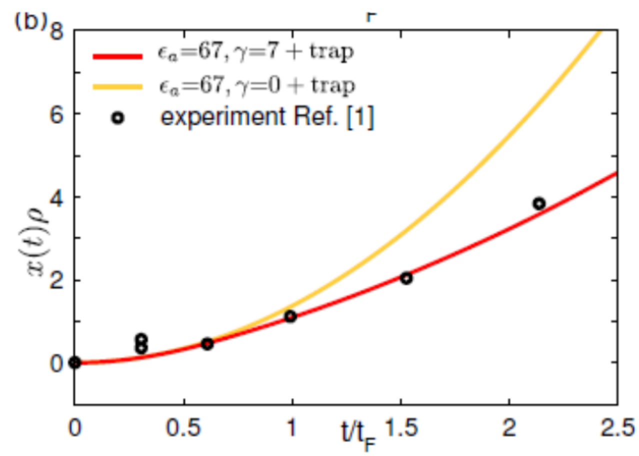
Bloch oscillations of impurity in strongly interacting 1d Bose gas

A. Kamenev et al., Annals of Physics (2012)
M. Knap et. al., arXiv: 1303.3583









Summary

Exploring orthogonality catastrophe with cold atoms
M. Knap et al., PRX (2012)

Rabi oscillations of impurity spin
M. Knap, D. Abanin, E. Demler, PRL (2013)

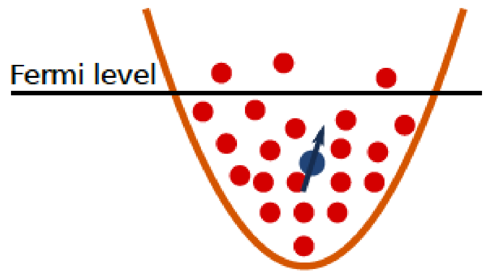
Mobile magnetic impurities in a Fermi superfluid:

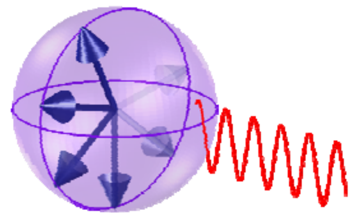
E. Vernier, D. Pekker, M. Zwierlein, E. Demler, PRA (2011)

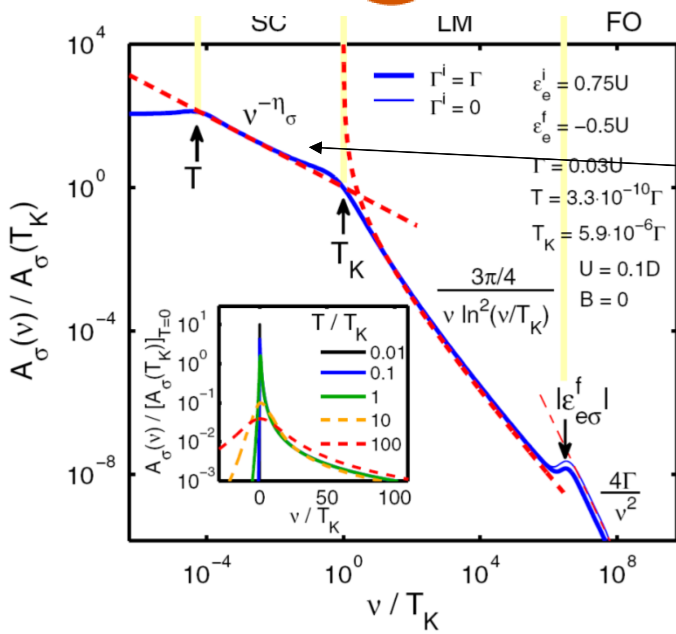
S. Gopalakrishnan, E. Demler, arXiv (2014)

Quantum flutter and Bloch oscillations of impurities in 1d
C. Mathy, M. Zvonarev, E. Demler, Nature Physics (2012)
M. Knap et al., PRL (2013)

Impurity spectroscopy and optical signatures for Kondo physics



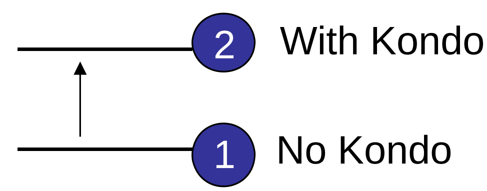


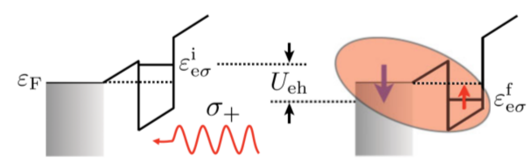


Characteristic power laws

Kondo regime

$$\eta_\sigma = \frac{1}{2}$$





Türeci et al., PRL **106**, 107402 (2011)