Lecture 2


Seminar
BINGO!
To play, simply print out this bingo sheet and attend a departmental seminar.
Mark over each square that occurs throughout the course of the lecture.
The first one to form a straight line (or all four corners) must yell out
BINGO!! to win!
SEMINAR
B I N G O
Speaker bashes previous work
Repeated use of "um…"
Speaker sucks up to host professor
Host Professor falls asleep
Speaker wastes 5 minutes explaining outline
Laptop malfunction
Work ties in to Cancer/HIV or War on Terror
"…et al."
You're the only one in your lab that bothered to show up
Blatant typo
Entire slide filled with equations
"The data clearly shows…"
FREE
Speaker runs out of time
Use of Powerpoint template with blue background
References Advisor (past or present)
There's a Grad Student wearing same clothes as yesterday
Bitter Post-doc asks question
"That's an interesting question"
"Beyond the scope of this work"
Master's student bobs head fighting sleep
Speaker forgets to thank collaborators
Cell phone goes off
You've no idea what's going on
"Future work will…"
Results conveniently show improvement
JORGE CHAM © 2007
WWW.PHDCOMICS.COM

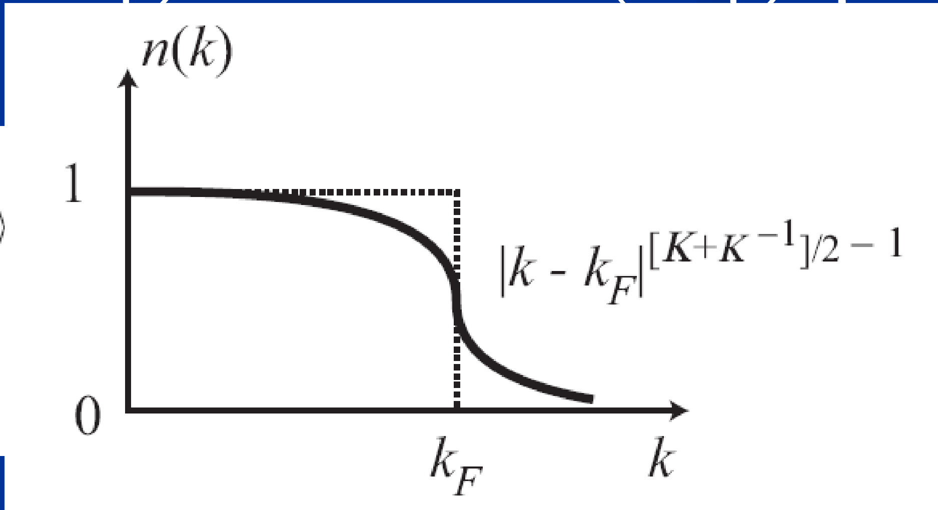
Other 1D systems

Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$) particles



$\langle\rho(x,\tau)\rho(0)\rangle$ … $\rho_0 x) \left(\frac{\alpha}{r}\right)^{2K}$

… $\pi\rho_0 x) \left(\frac{\alpha}{r}\right)^{8K} + \cdots$

Spins

Use boson or fermions mapping

$$S^+ = (-1)^i e^{i\theta} + e^{i\theta}\cos(2\phi)$$

$$S^z = \frac{-1}{\pi}\nabla\phi + (-1)^i \cos(2\phi)$$

Powerlaw correlation functions

$$\langle S^z(x,0)S^z(0,0)\rangle = C_1\frac{1}{x^2} + C_2(-1)^x\left(\frac{1}{x}\right)^{2K}$$

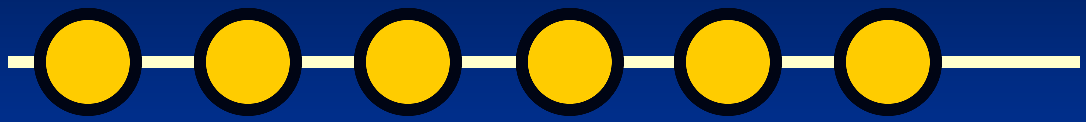
$$\langle S^+(x,0)S^-(0,0)\rangle = C_3\left(\frac{1}{x}\right)^{2K+\frac{1}{2K}} + C_4(-1)^x\left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Non universal exponents K(h,J)

Calculation of Luttinger parameters

- Trick: use thermodynamics and BA or numerics
- Compressibility: u/K
- Response to a twist in boundary: u K
- Specific heat : T/u
- Etc.

Tonks limit



U = 1 : spinless fermions

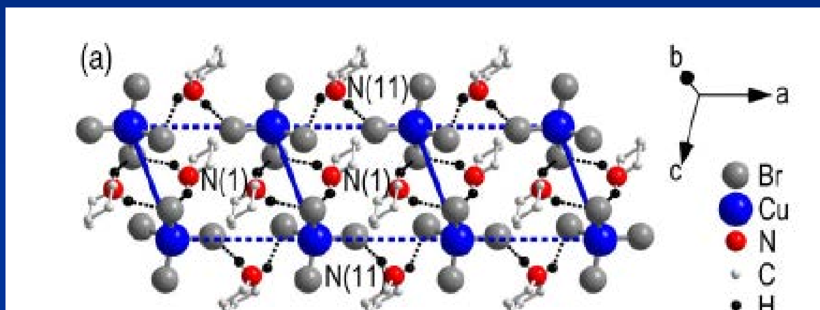
Not for n(k) : $\psi_F \neq \psi_B$

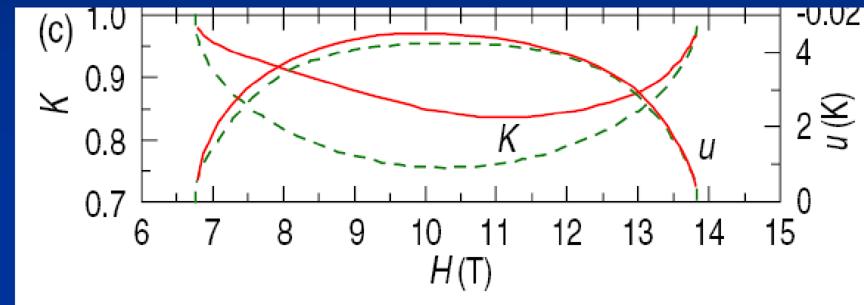
Free fermions: $$\langle \rho(x)\rho(0)\rangle \propto \cos(2k_F x)\left(\frac{1}{x}\right)^2$$

K=1

Note: $$\langle \psi_B(x)\psi_B(0)^\dagger\rangle \propto \left(\frac{1}{x}\right)^{1/2}$$

Luttinger parameters

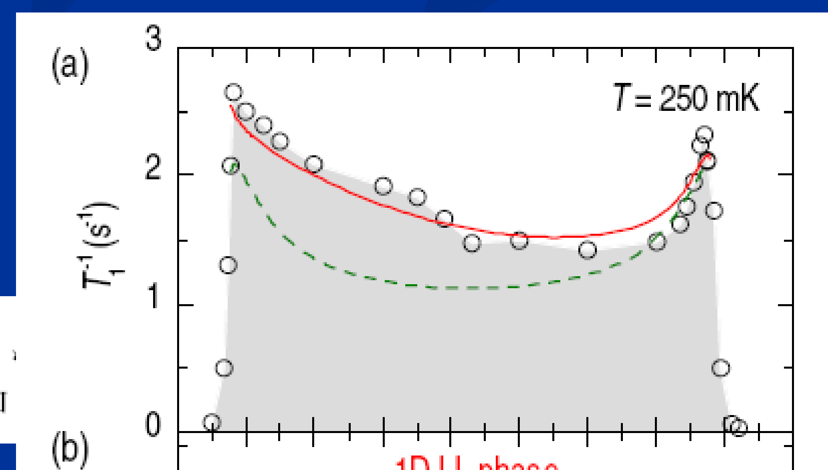




M. Klanjsek et al., PRL 101 137207 (2008)

- NMR relaxation rate:

$$T_1^{-1} = \frac{\hbar\gamma^2 A_\perp^2 A_x^0}{k_B u}\cos\left(\frac{\pi}{4K}\right)B\left(\frac{1}{4K}, 1-\frac{1}{2K}\right)\left(\frac{2\pi T}{u}\right)^{(1/2K)-1}$$



Tests of 1d physics/ TLL in cold atoms

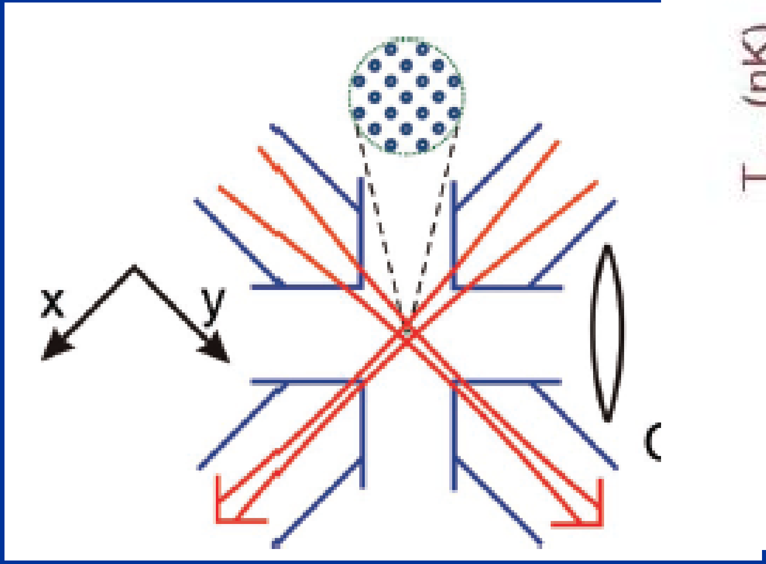
Bosons (continuum)

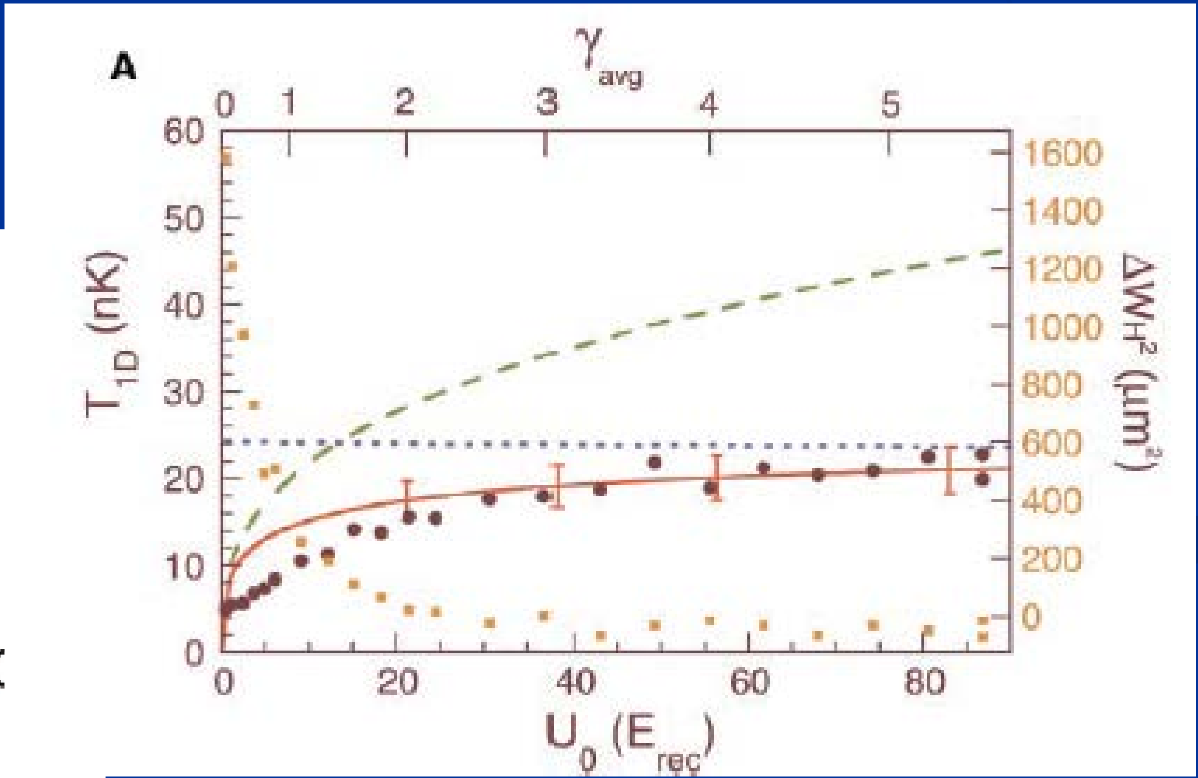
Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

SCIENCE VOL 305 20 AUGUST 2004

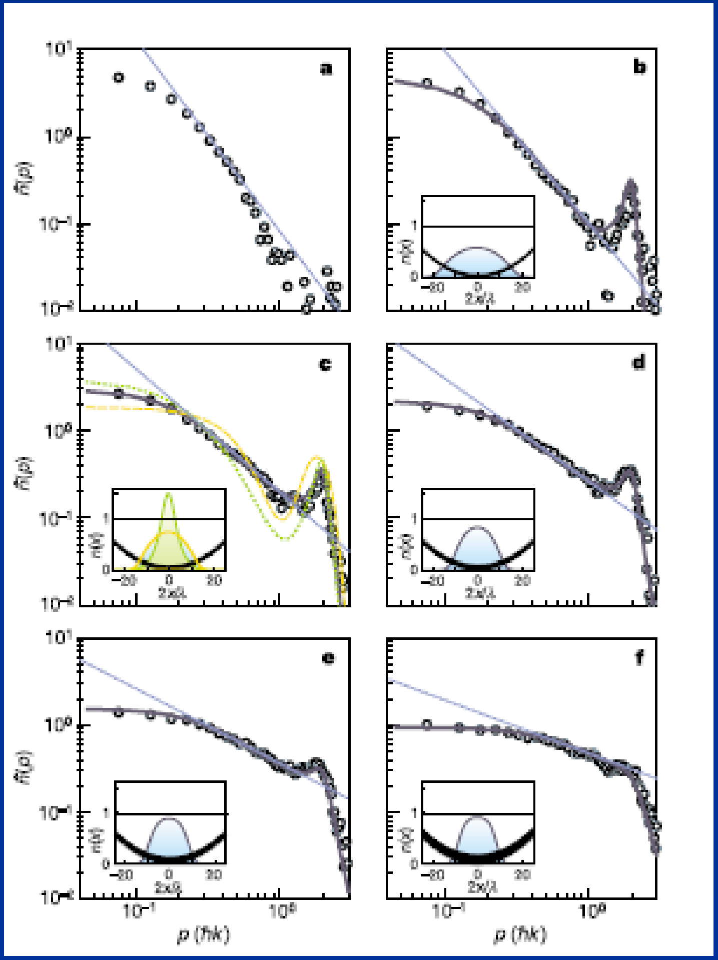
1125

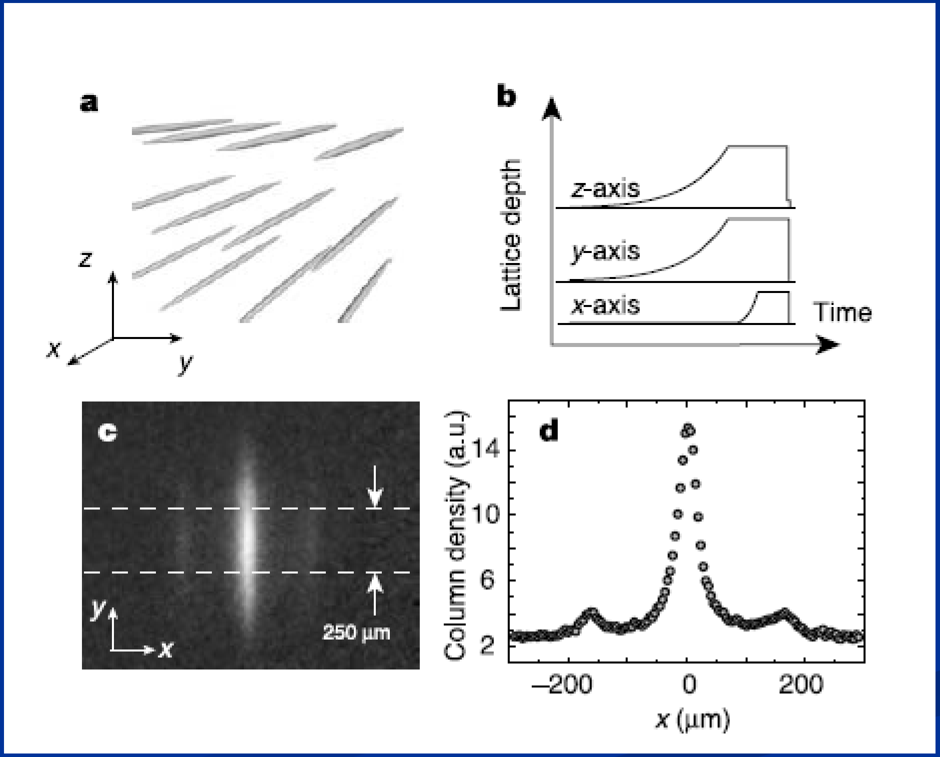




Optical lattices (dilute)

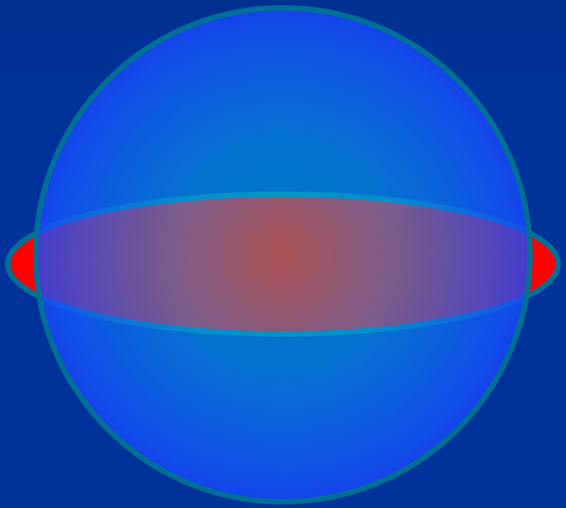
B. Paredes et al., Nature 429 277 (2004)





$$n(k) = \int dx e^{ikx} \langle \psi^\dagger(x)\psi(0)\rangle$$

Confining potential / Trap

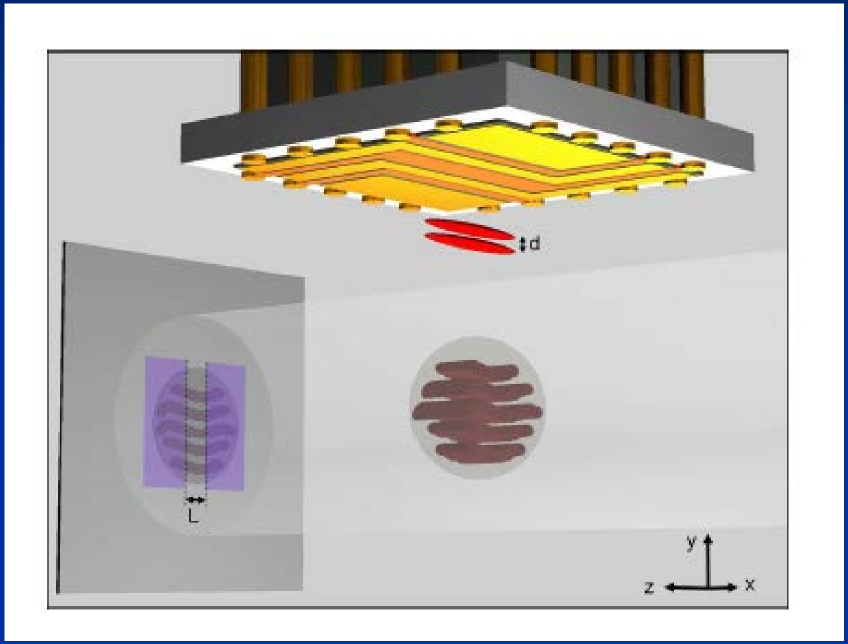


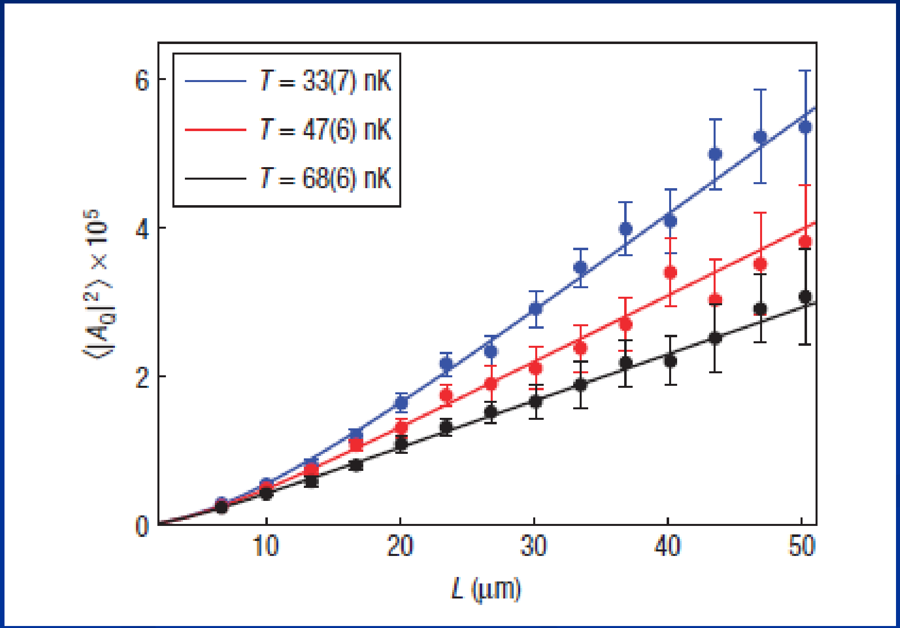
- No homogeneous phase !

$$H = \int d^3 r \frac{1}{2} \omega_0^2 r^2 \rho(r)$$

I am your worst nightmare

Atom chips

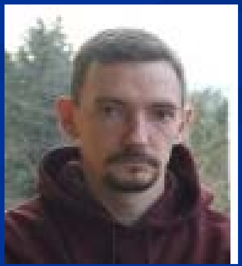




$$\int_0^L dr \langle \psi(r) \psi^\dagger(0) \rangle$$

K large (42)

S. Hofferberth et al. Nat. Phys 4 489 (2008)

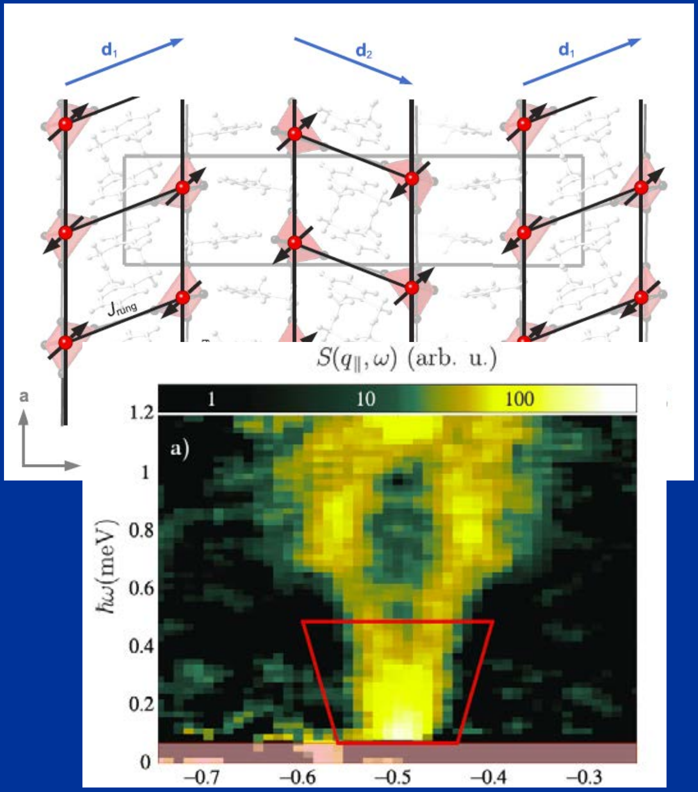


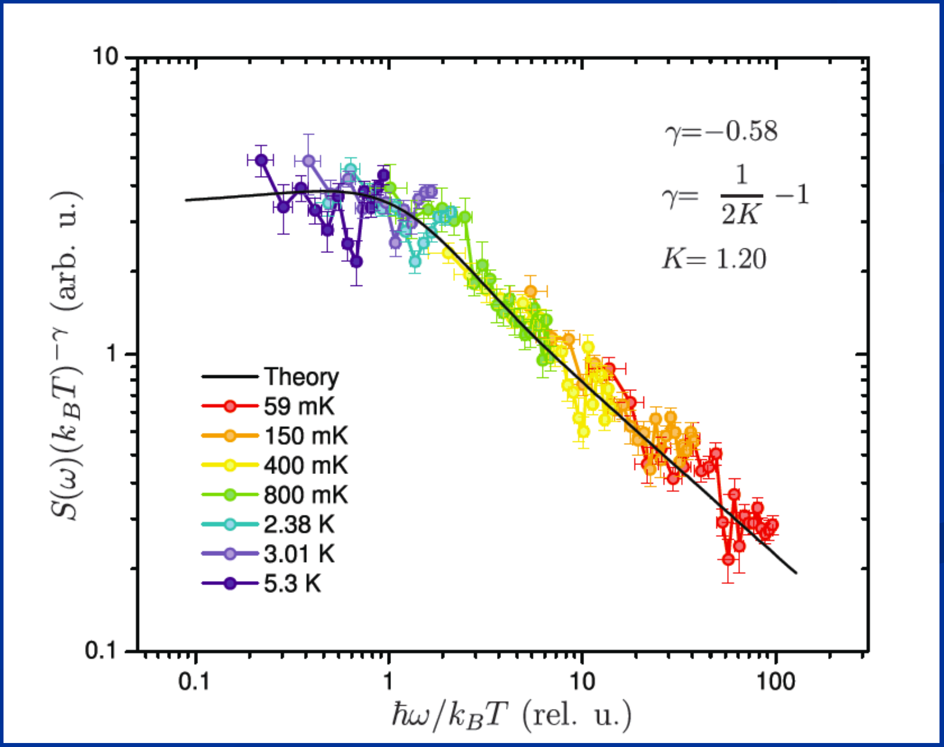
Spin ladders

D. Schmidiger et al. PRL 108 167201 (12):

K. Yu et al. arxiv/1406.6876 (14)

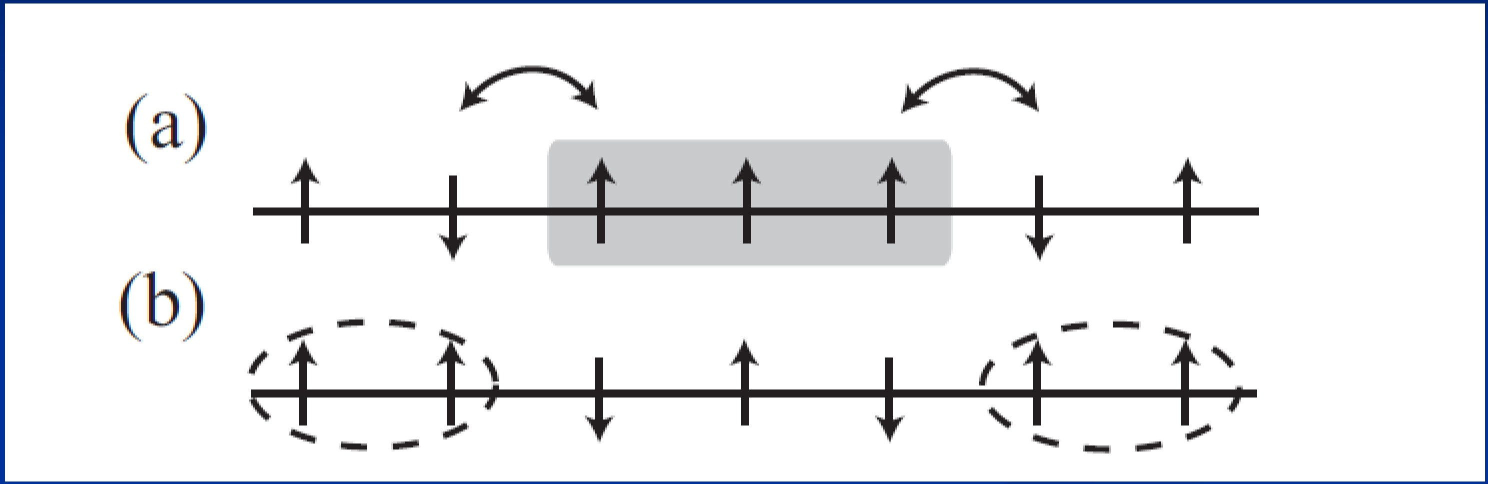
$$\langle S^- S^+ \rangle_{q,\omega} = \langle \psi \psi^\dagger \rangle_{q,\omega}$$

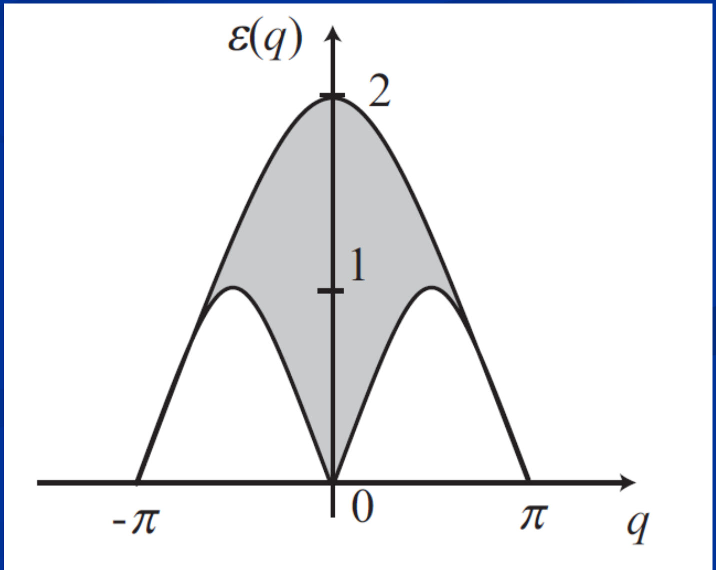




Fractionalization of excitations

1 = ½ + ½

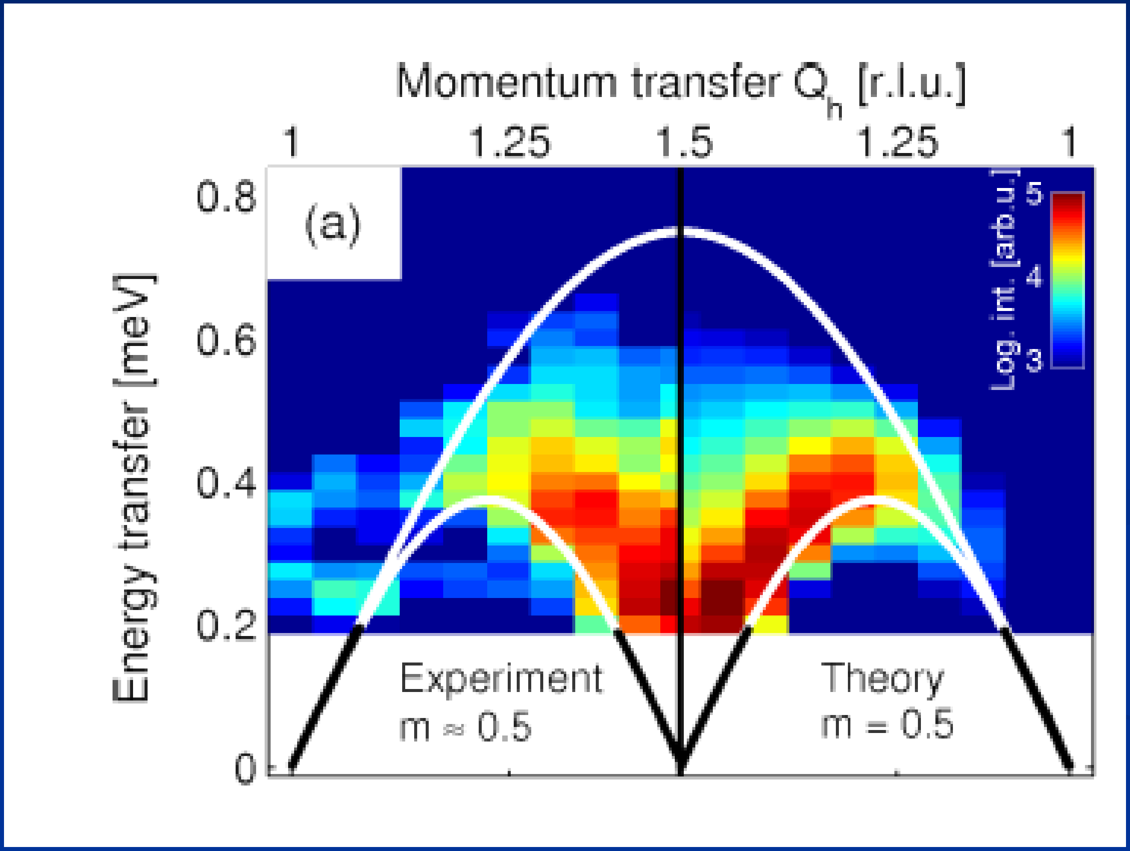




$E(k) = \cos(k_1) + \cos(k_2)$

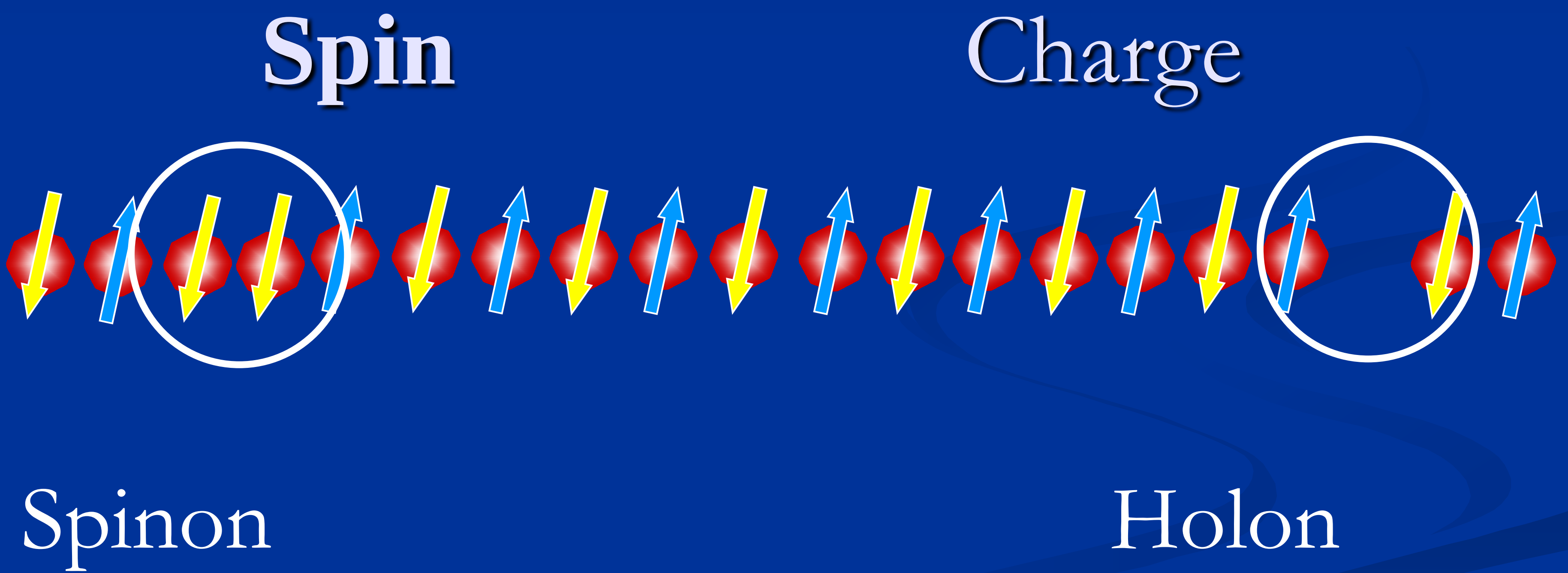
$k = k_1 + k_2$

Neutron scattering

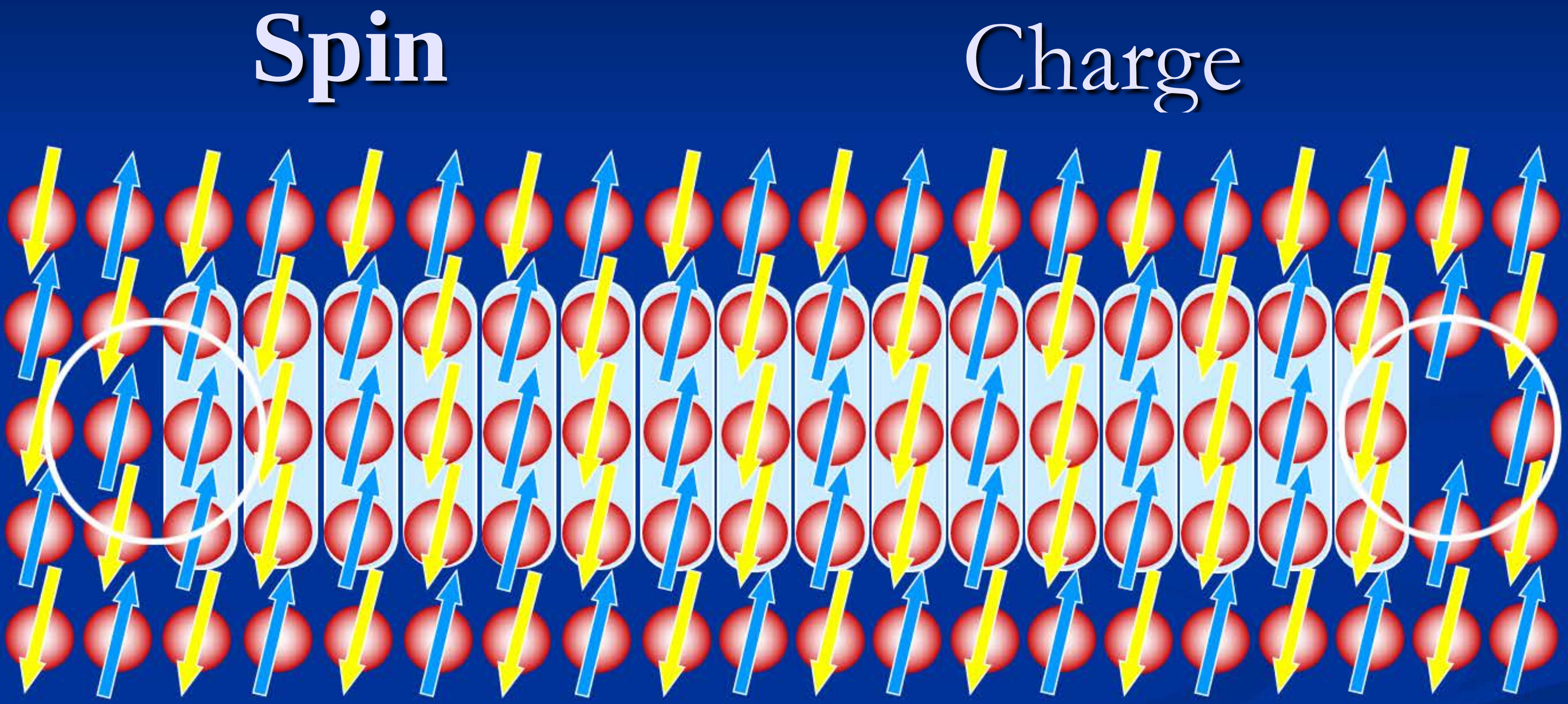


B. Thieleman et al. PRL 102, 107204 (2009)

Spin-Charge Separation



Spin-Charge Separation higher D ?

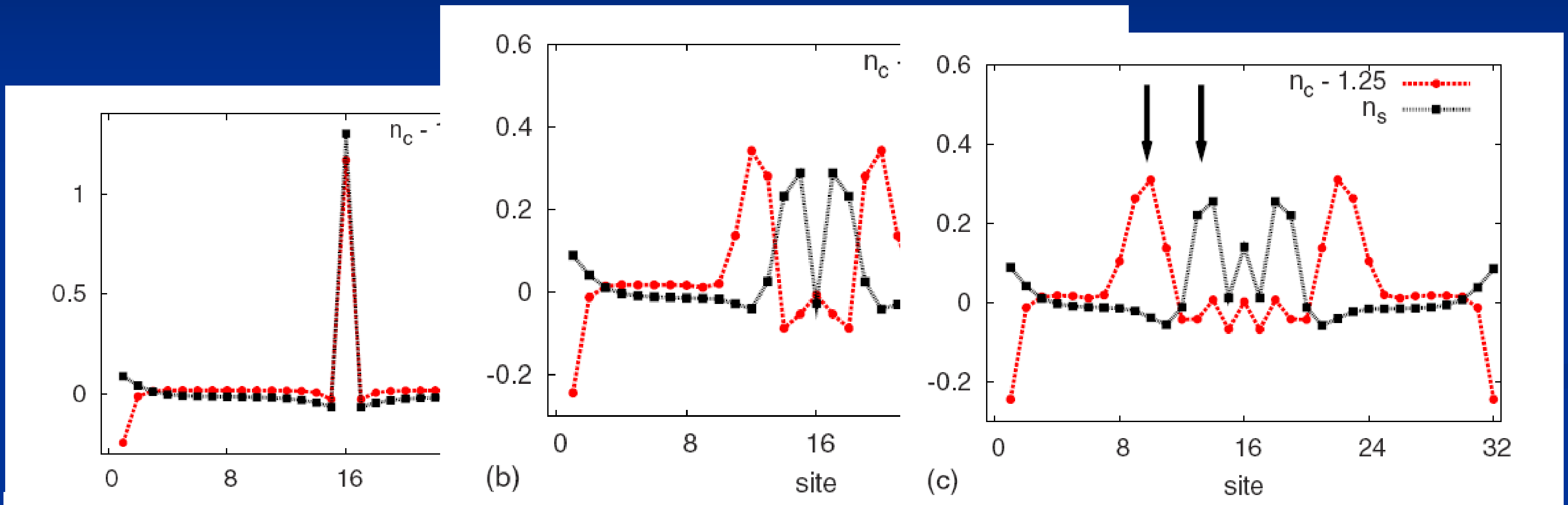


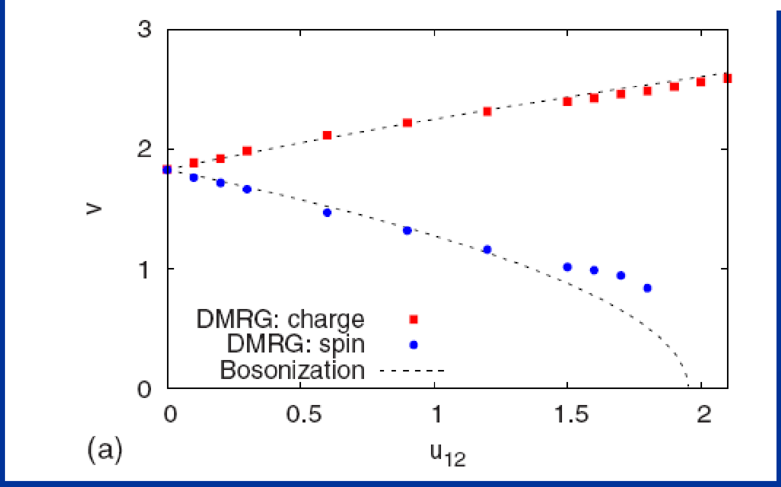
Energy increases with spin-charge separation

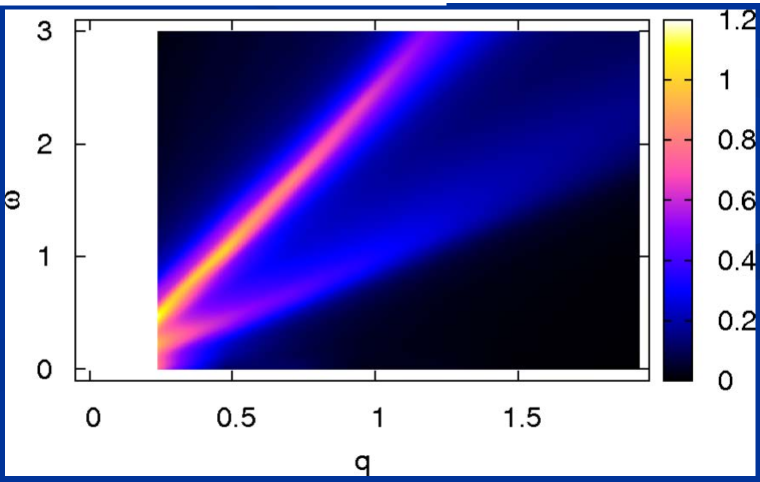
Confinement of spin-charge: « quasiparticle »

Proposal for cold atoms (Rb)

A. Kleine, C. Kollath et al. PRA 77 013607 (2008); NJP 10 045025 (2008)







Quantitative agreement with the predictions of the Luttinger liquid

End of the story …… ??????

NO! Luttinger liquid plays the same role than Fermi liquid did

To boldly go where no theorist has gone before….

Beyond Luttinger liquids

- 1D additional perturbation:

Lattice (Mott transition), disorder (Bose glass) etc.

Multicomponents, mixtures, …..

- Out of equilibrium situations

  A. Mitra, TG, Phys. Rev. Lett. 107, 150602 (2011)

  E. Dalla Torre, E. Demler,TG, E. Altman, Nat. Phys. 6 806 (2010); PRB 85 184302 (2012)

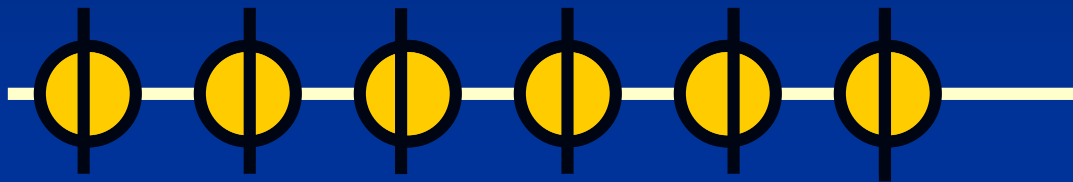
- Ladders and magnetic fields:
  - E. Orignac, TG Phys. Rev. B **64**, 144515 (2001)
  - M. Atala et al., arxiv/1402.0819

- Impurities, polarons in 1D systems:

- Dimensional crossover 1d – 2d/3d:
  - AF Ho, M.A. Cazalilla, TG, PRL **92** 130403 (2004);NJP 8 158 (2006).

Mott transition

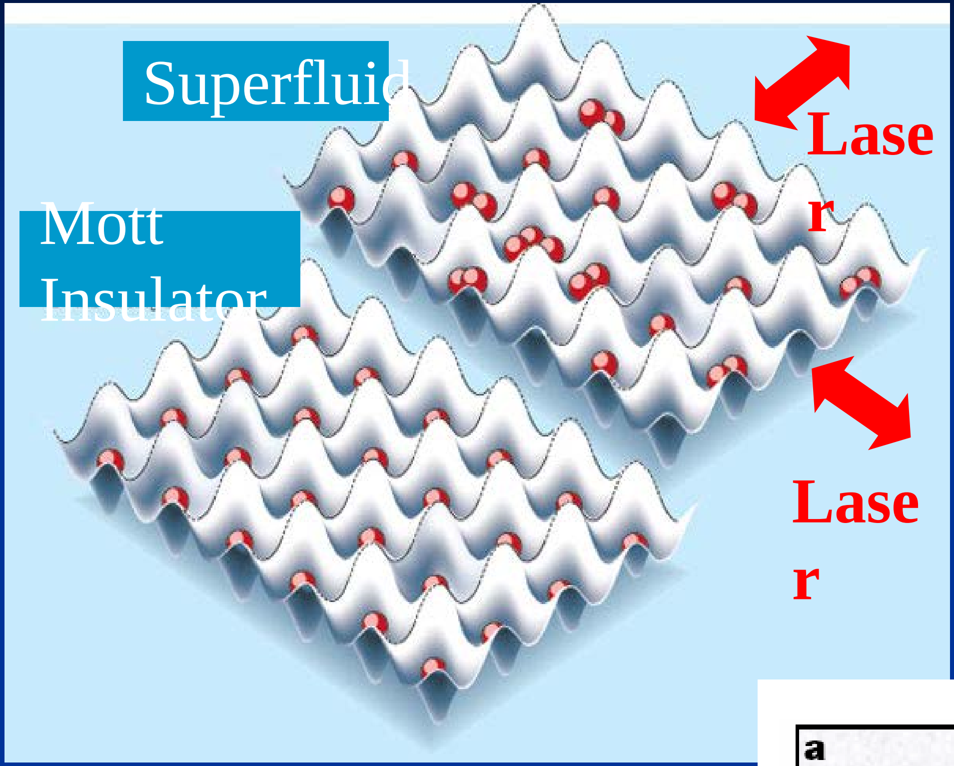
Lattice: Mott transition

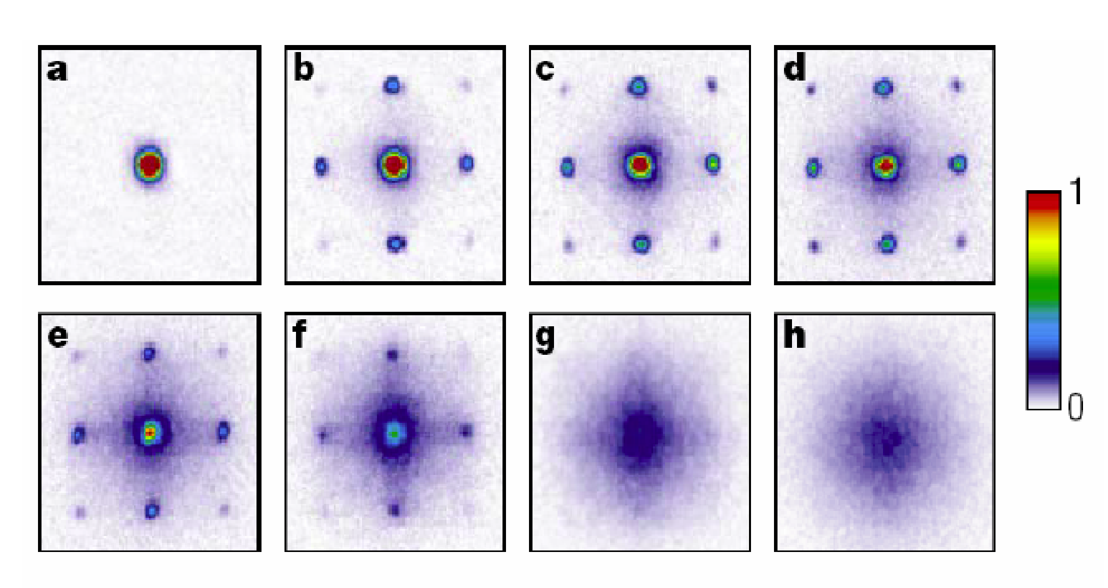


Costs U

Quantum phase transition

Mott transition and cold atoms



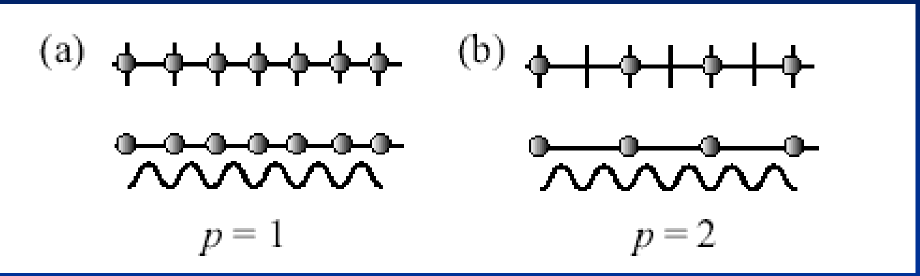


Superfluid to Mott insulator transition in a 3D optical lattice

[D. Jaksch et al. PRL 81 (1998)]

[M Greiner *et al.* Nature, 415 (2002

How to treat?



$$H = \int dx V_0 \cos(Qx)\rho(x)$$

$$H = \int dx V_0 \cos(Qx)\rho_0 e^{i(2\pi\rho_0 x - 2\phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x) + \delta x)$$

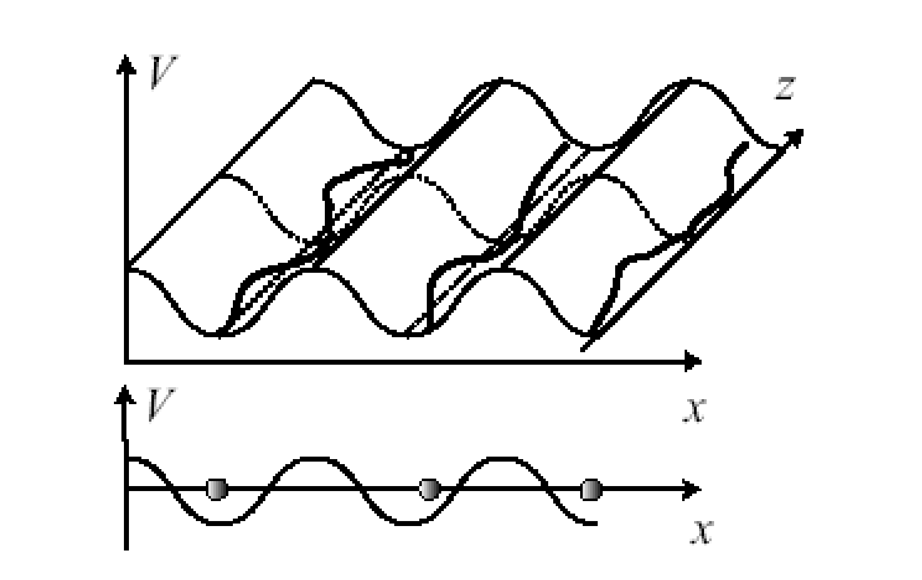
- Commensutate: $Q = 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x))$$

Competition

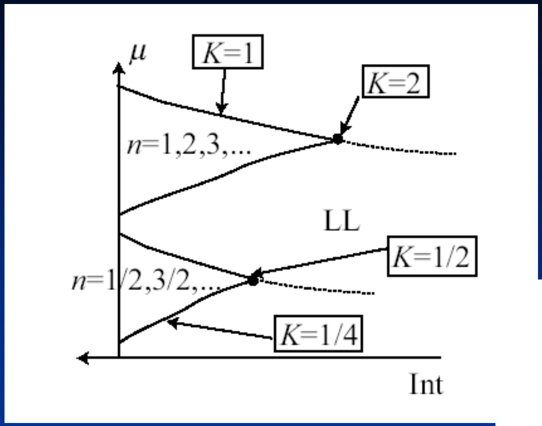
$$S_0 = \int \frac{dxd\tau}{2\pi K}[\tfrac{1}{u}(\partial_\tau \varphi(x,\tau))^2 + u(\partial_x \varphi(x,\tau))^2]$$

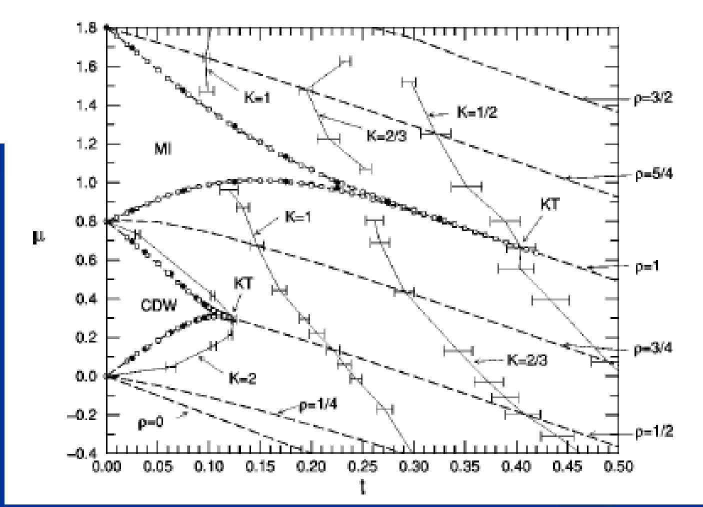
$$S_L = -V_0 \rho_0 \int dxd\tau \cos(2\phi(x))$$



Beresinskii-Kosterlitz-Thouless transition at K=2

String order parameter





Mott insulator:

ϕ is locked

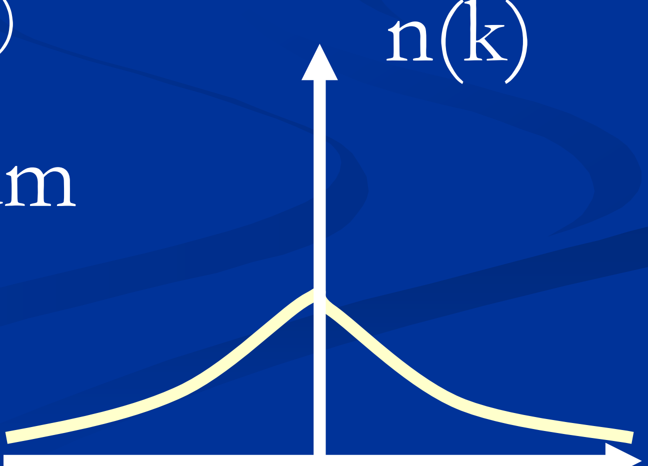
Density is fixed

TG, Physica B 230 975 (97)

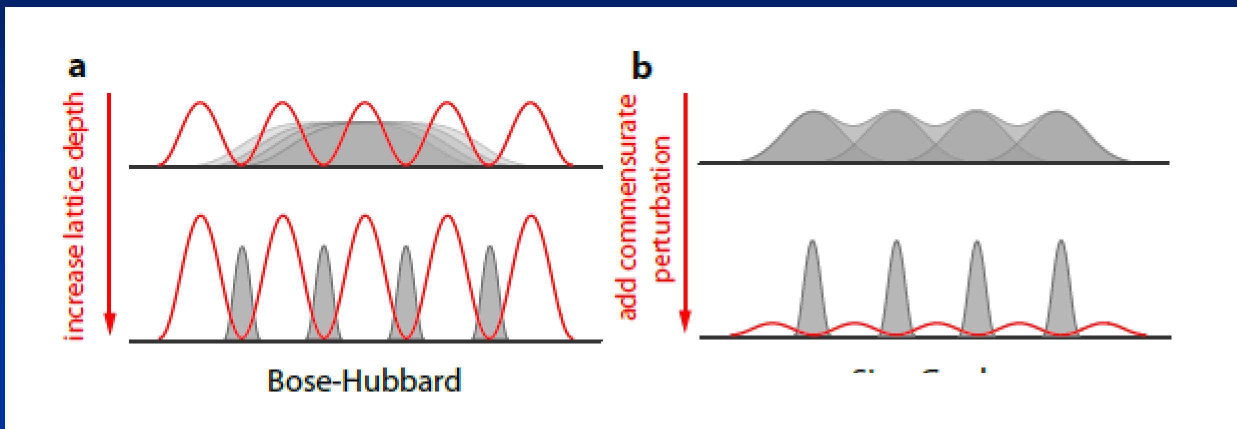
T. Kuhner et al. PRB 61 12474 (2000)

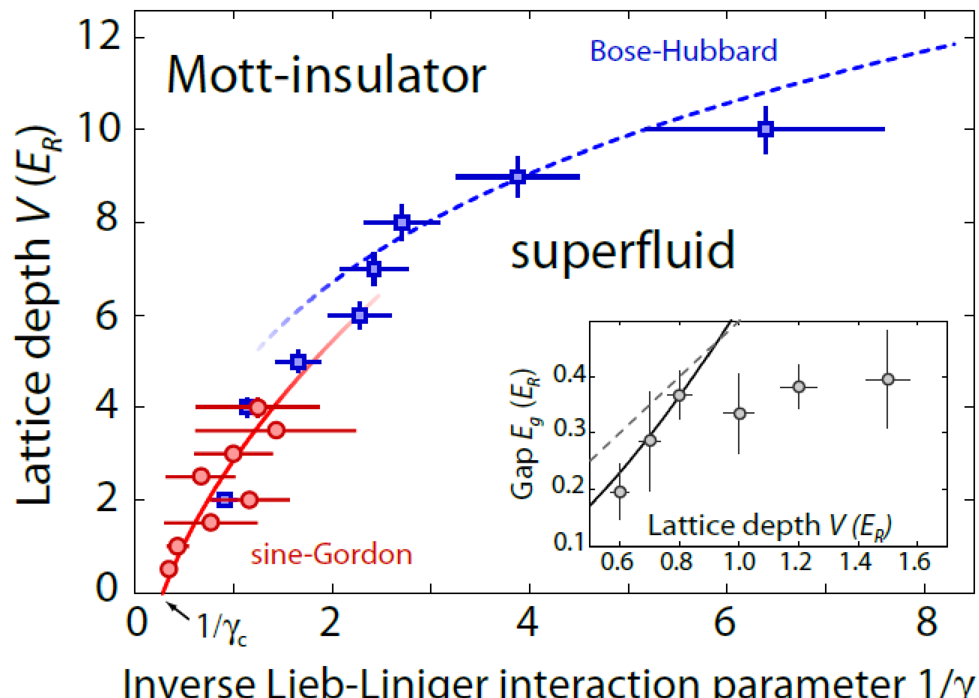
Gap in the excitation spectrum

$$G(r) \propto e^{-r/\xi}$$



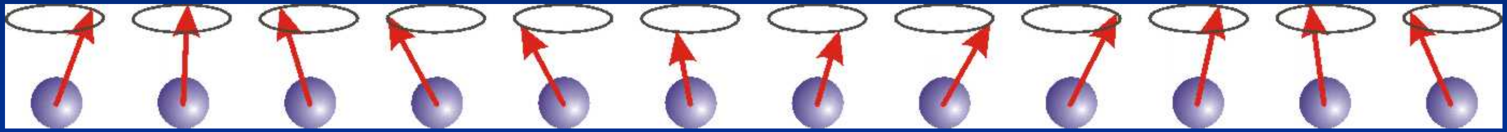
E. Haller et al. Nature 466 597 (2010)





Non-LL and impurity problems

Itinerant ferromagnet (two component bosons)



$$G_T(x,t) = \langle S^+(x,t) S^-(0,0) \rangle$$

Localized magnet: simple

spin waves $\epsilon(q) = 2J(1-\cos(q)) = \frac{q^2}{2m}$

$$G_H(x,t) \propto \frac{1}{\sqrt{t}} e^{\frac{imx^2}{2t}}$$

Itinerant magnet: non trivial !

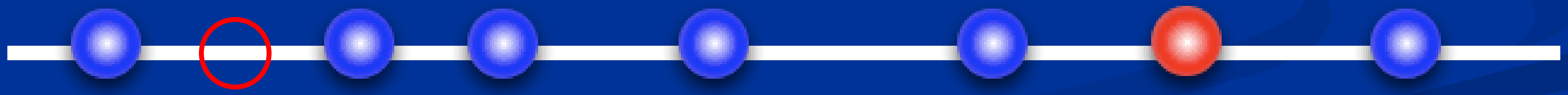
- Interplay of density and spin fluctuations

- Itinerant: still a quadratic dispersion

$$\epsilon(q) = \frac{q^2}{2m^*}$$

[Fuchs et. al.,PRL (2005)]

- Standard 1D theory (TLL) not applicable !



- Mobile Impurity in a LL

Mobile impurity

M. B. Zvonarev, V. V. Cheianov, TG, PRL 99 240404 (2007);

$$H = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \sum_{i<j} [g\delta(x_i - x_j) + U(x_i - x_j)]$$

$$\gamma = mg/\hbar^2 \rho_0$$

$$\frac{m^*}{m} = \frac{3\gamma}{2\pi^2}$$

Single spin down particle

$$G_\perp(x, t) = \langle \Uparrow | s_+(x, t) s_-(0, 0) | \Uparrow \rangle$$

Link with other systems

Feynman polaron

$$\sum_q \epsilon(q) c_q^\dagger c_q + \sum_k u|k| b_k^\dagger b_k + g \sum_k A_k (b_k + b^\dagger_{-k}) \rho_\downarrow(-k)$$

- Quantum particle dressed with phonons
- Polaron problem
- Renormalization of the mass m -> m*
- Dissipation ??

Quantum dissipative systems

- Caldeira Leggett model

$$H_{LL} = \sum_k u|k| b_k^\dagger b_k$$

$$\rho_\uparrow(x) \sum_k e^{ikx} A_k (b_k + b_{-k}^\dagger)$$

$$H_{LL} + \frac{P^2}{2M} + g \int dx \rho_\uparrow(x) \delta(x - X)$$

$$\frac{P^2}{2M} + \sum_k u|k| b_k^\dagger b_k + g \sum_k A_k (b_k + b_{-k}^\dagger) e^{ikX}$$

- Quantum particle + bath of harmonic osc.

- Note: non-linear coupling

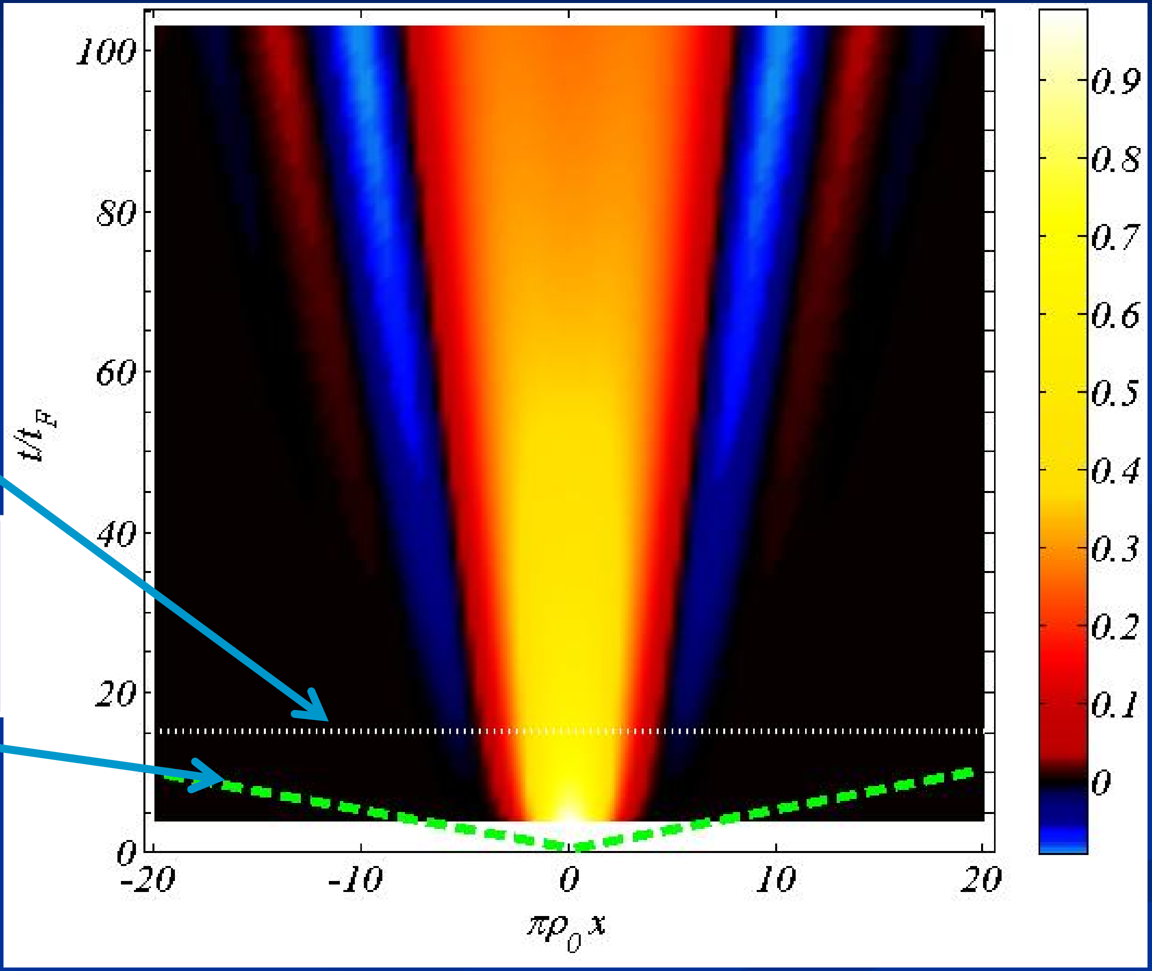
$$\frac{P^2}{2M} + \sum_k u|k| b_k^\dagger b_k + g \sum_k A_k (b_k + b_{-k}^\dagger) ikX$$

- Ambegaokar-Hakim model

Propagation of the impurity

Trapped/open regimes

Light cone of spinless bosons

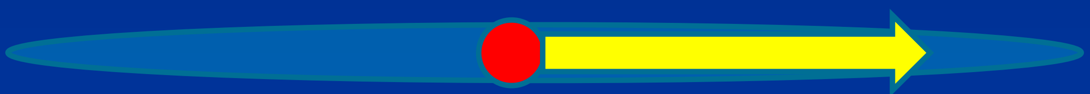


$$G_\perp(x,t) \simeq \frac{1}{\sqrt{\ln(t/t_F)}} \exp\left\{-\frac{1}{K}\frac{(\pi\rho_0 x)^2}{2\ln(t/t_F)}\right\}.$$

$$G_\perp \simeq e^{-(x^2/2\ell^2)} t^{-\alpha} G_\perp^H, \qquad \ell(t) = \frac{2K^{-(1/2)}}{\pi\rho_0}\frac{t/t_F}{\sqrt{\ln t/t_F}}\frac{m}{m_*}.$$

Driven impurity vs diffusion

- Normal transport



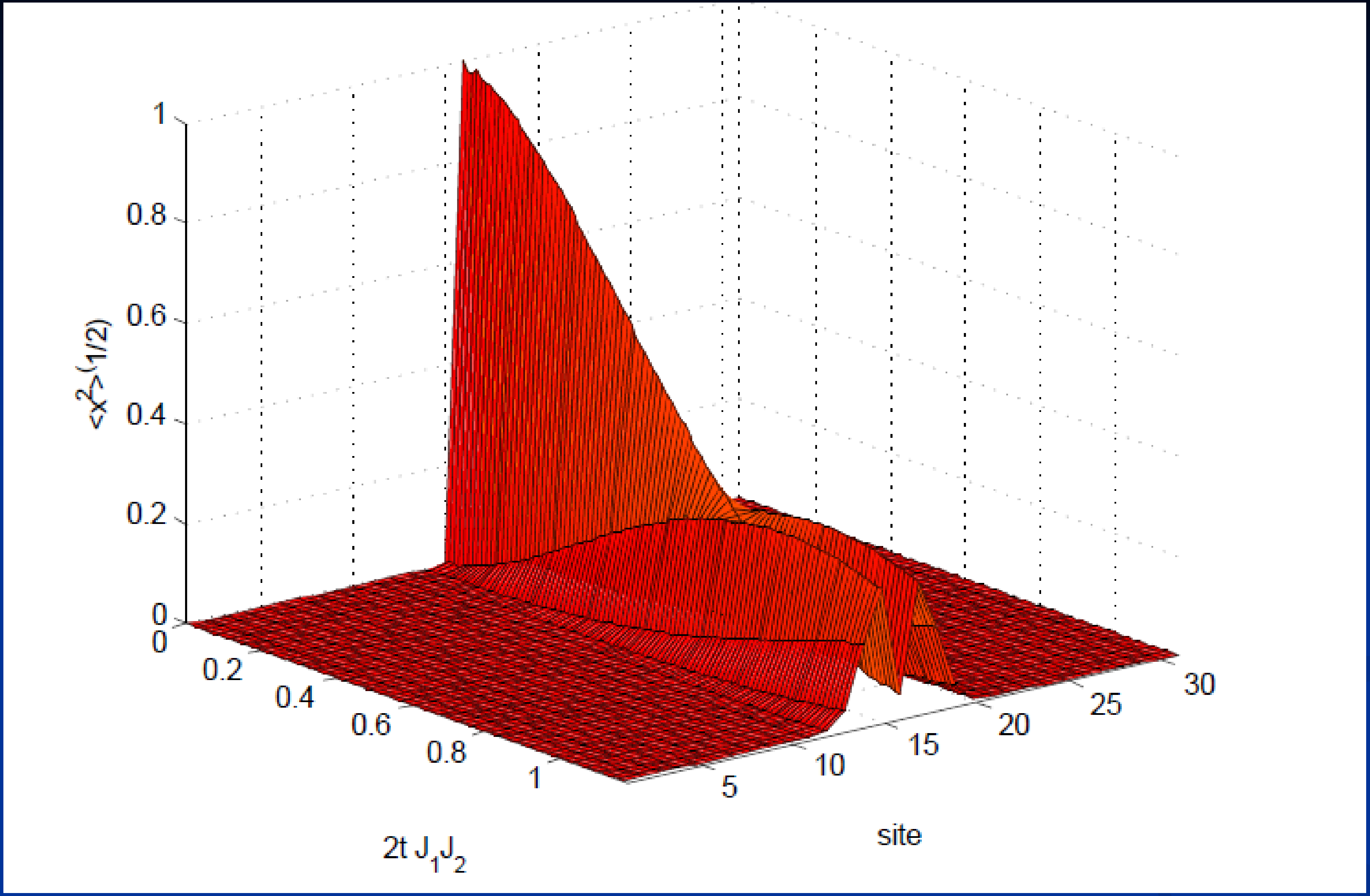
$$v = \mu F \qquad\qquad v = f(F)$$

$$\langle x^2 \rangle \sim Dt \qquad\qquad \langle x^2 \rangle \sim \log(t)$$

- Einstein relation: μ = D

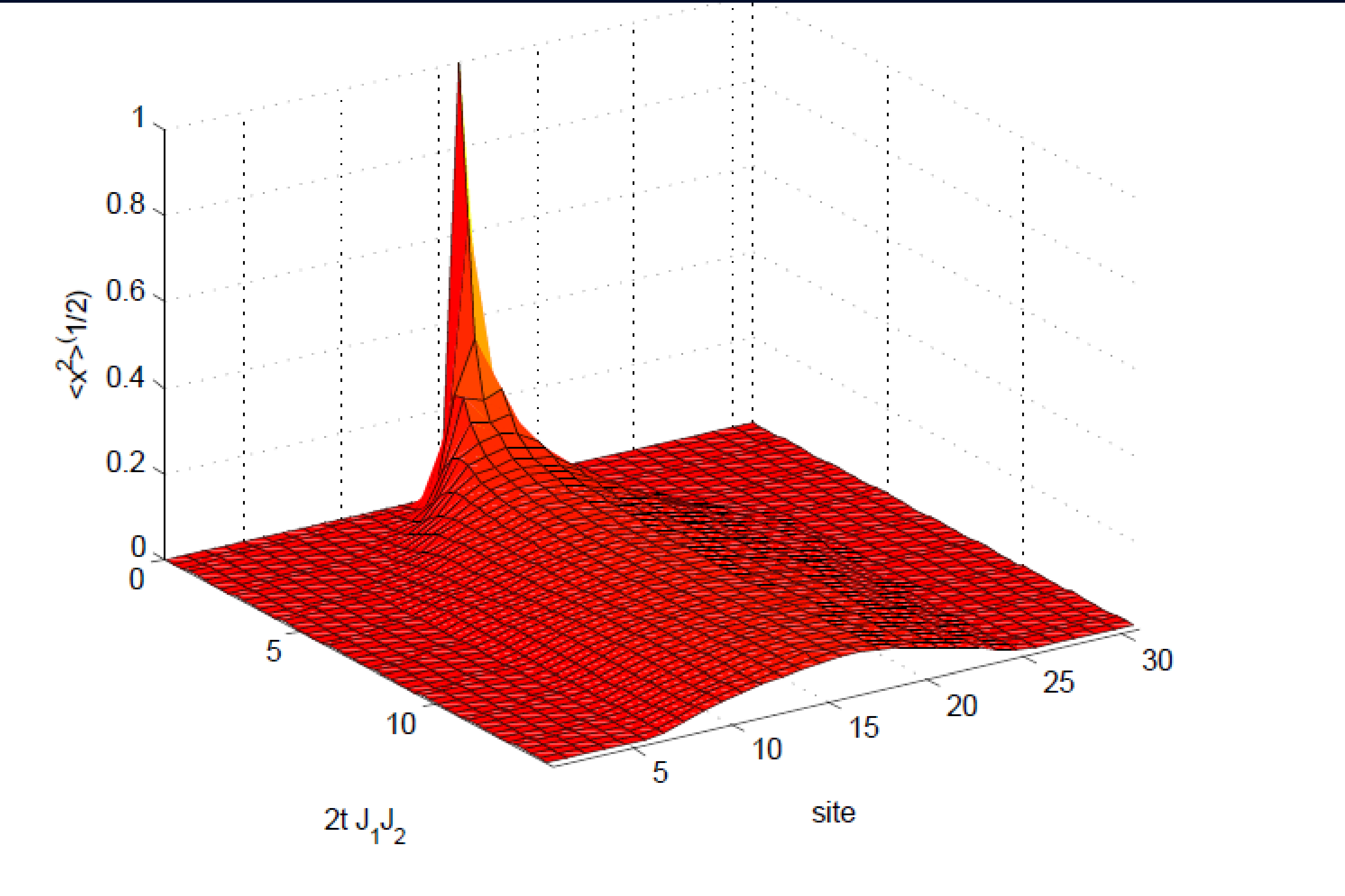
Simulations

A. Kantian, U. Schollwoeck, TG arxiv/1311.1825



$$U = \infty \to U/J = 20$$

Coherent motion



$$U = \infty \rightarrow U/J = 4$$

``Diffusion”

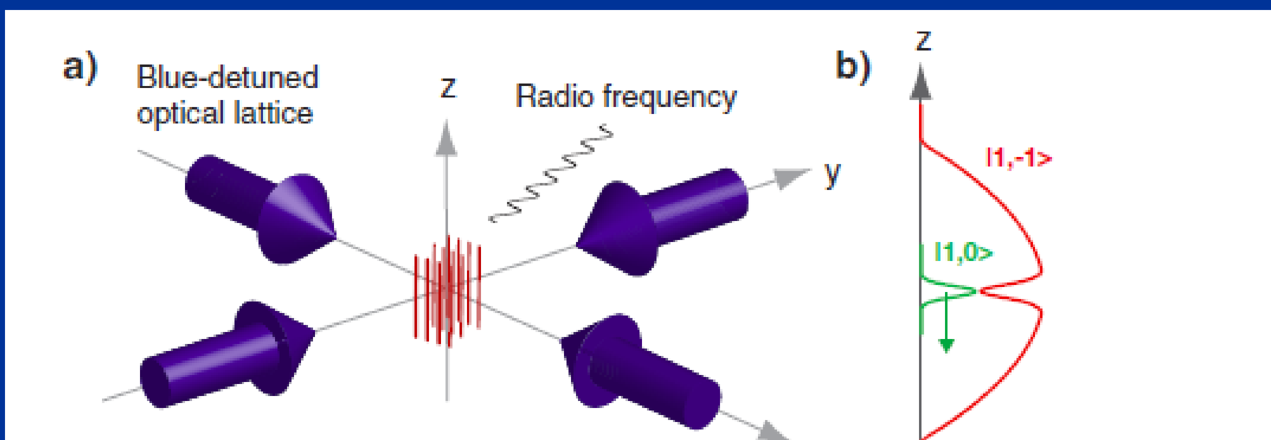
Experiments

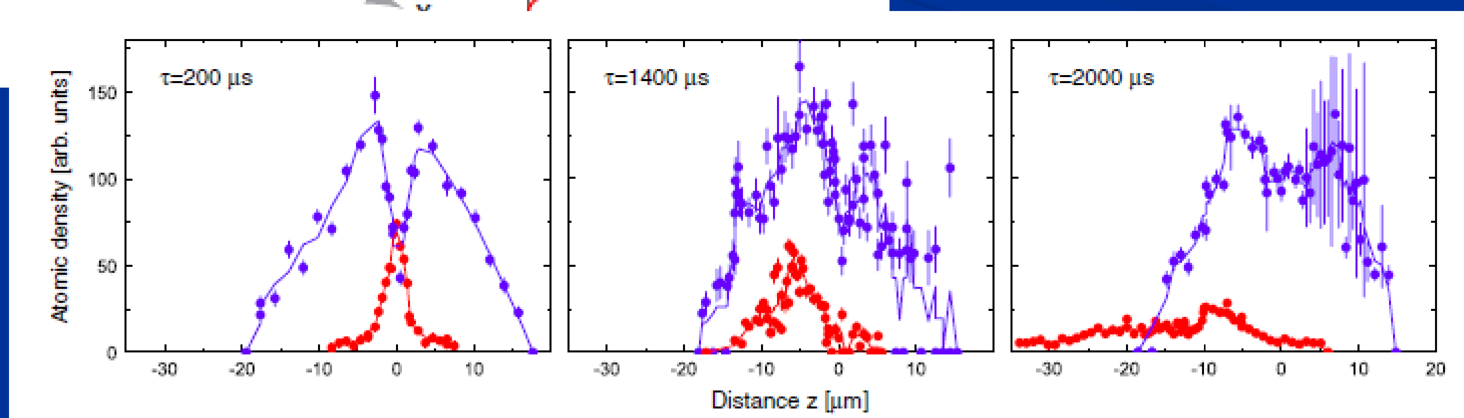
Driven impurity

PRL **103,** 150601 (2009) PHYSICAL REVIEW LETTERS we 9 OCTOBER 2009

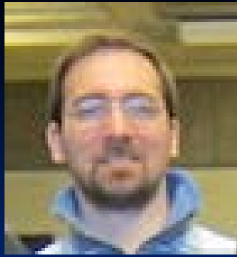
Quantum Transport through a Tonks-Girardeau Gas

Stefan Palzer, Christoph Zipkes, Carlo Sias,* and Michael Köhl

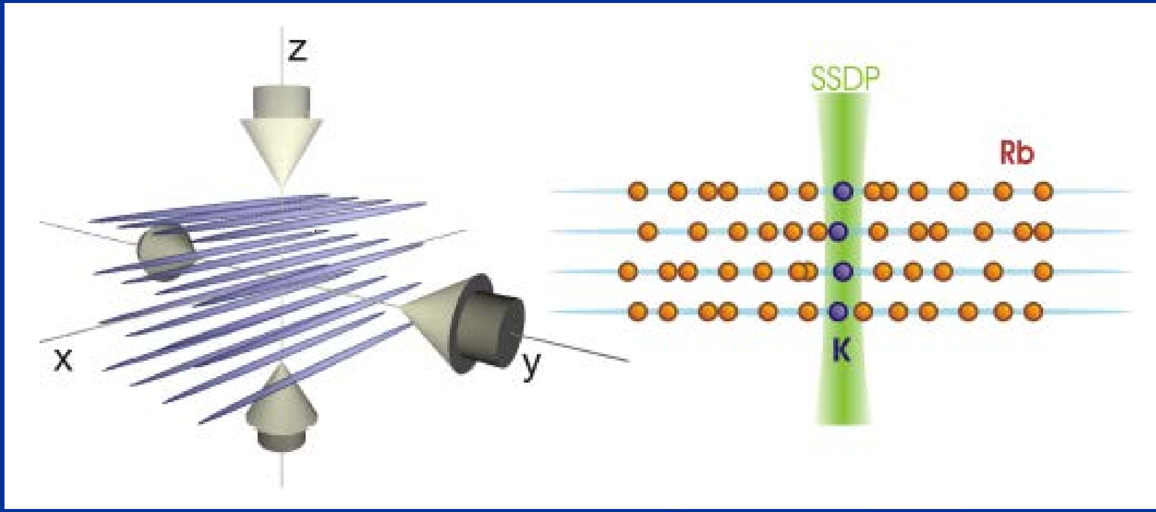


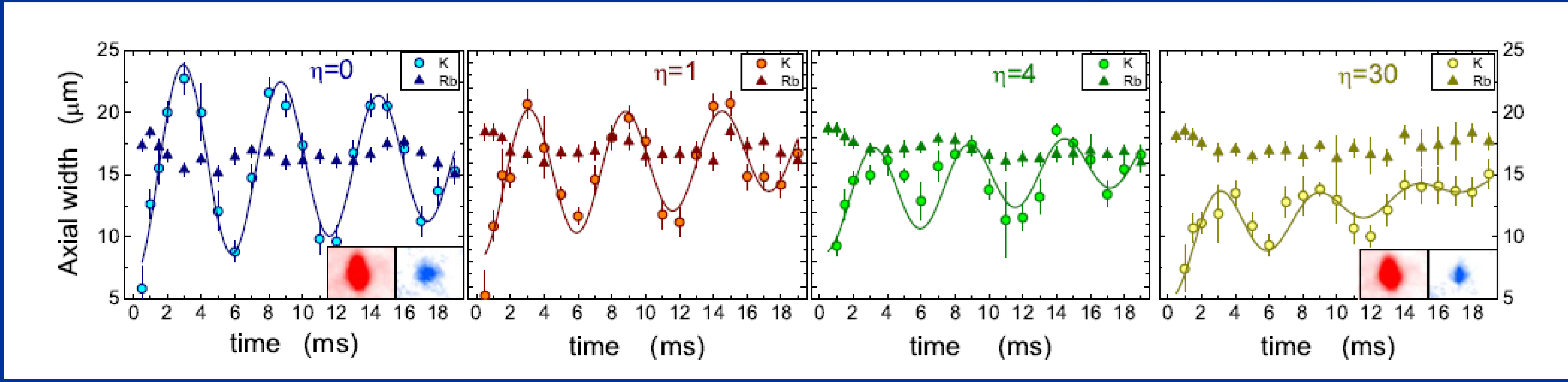


Diffusive impurity



J. Catani, G. Lamporesi, D. Naik, M. Gring, M. Inguscio, F. Minardi, A. Kantian, TG
PRA 85 023623 (2012)

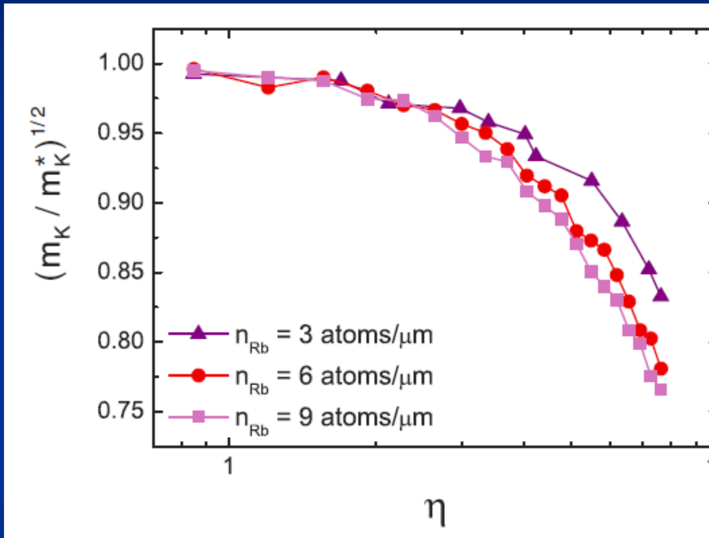




Polaronic effect

$$\hat{H} = \frac{\hat{p}^2}{2m_K} + \sum_{k\neq 0} \epsilon_k \hat{b}_k^\dagger \hat{b}_k + \sum_{k\neq 0} V_k e^{ik\hat{x}} (b_k + b_{-k}^\dagger),$$

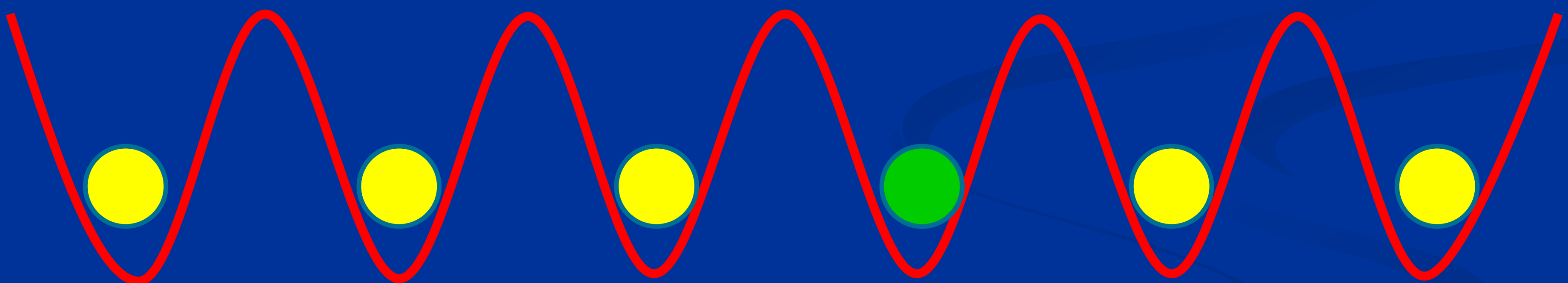
$$S = \int_0^{\beta\hbar} d\tau \frac{m_K}{2} \dot{x}^2(\tau) - \sum_k \frac{V_k^2}{2\hbar} \int_0^{\beta\hbar} d\tau \times \int_0^{\beta\hbar} d\tau' \, G(k, |\tau - \tau'|) e^{ik[x(\tau) - x(\tau')]},$$



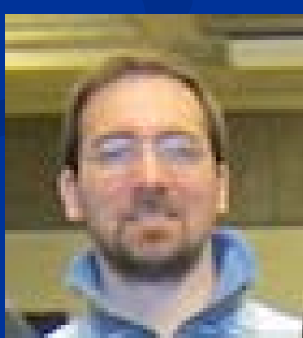
$$S_0 = \int_0^{\beta\hbar} d\tau \frac{m_K}{2} \dot{x}^2(\tau) + \frac{M W^3}{8} \int \int_0^{\beta\hbar} \times d\tau \, d\tau' \frac{\cosh(W|\tau - \tau'| - W\hbar\beta/2)}{\sinh(W\beta\hbar/2)} [x(\tau) - x(\tau')]^2,$$

Coherent spin excitation Heisenberg ferromagnet

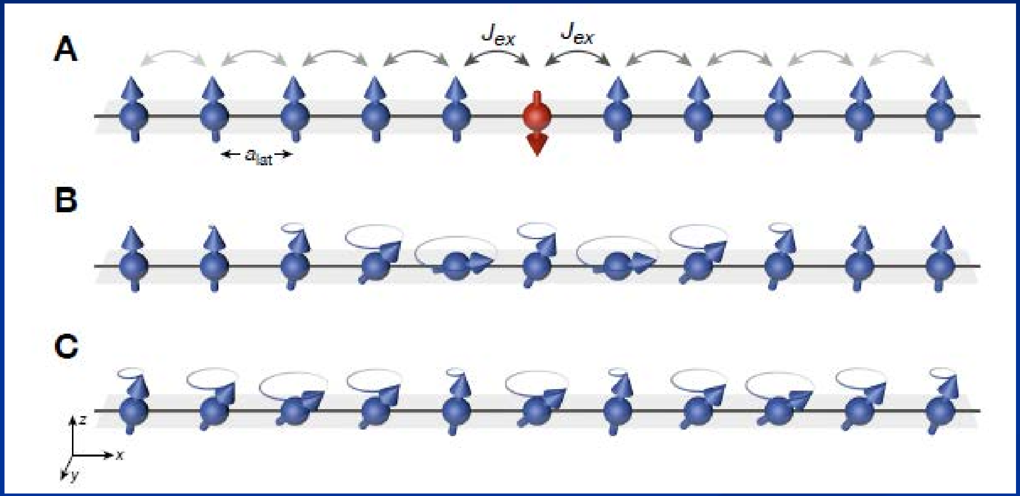
T. Fukuhara, A. Kantian, M. Endres, M. Cheneau, P. Schauss, S. Hild, D. Bellem, U. Schollwock, TG, C. Gross, I. Bloch, S. Kuhr , Nat. Phys. (2013)

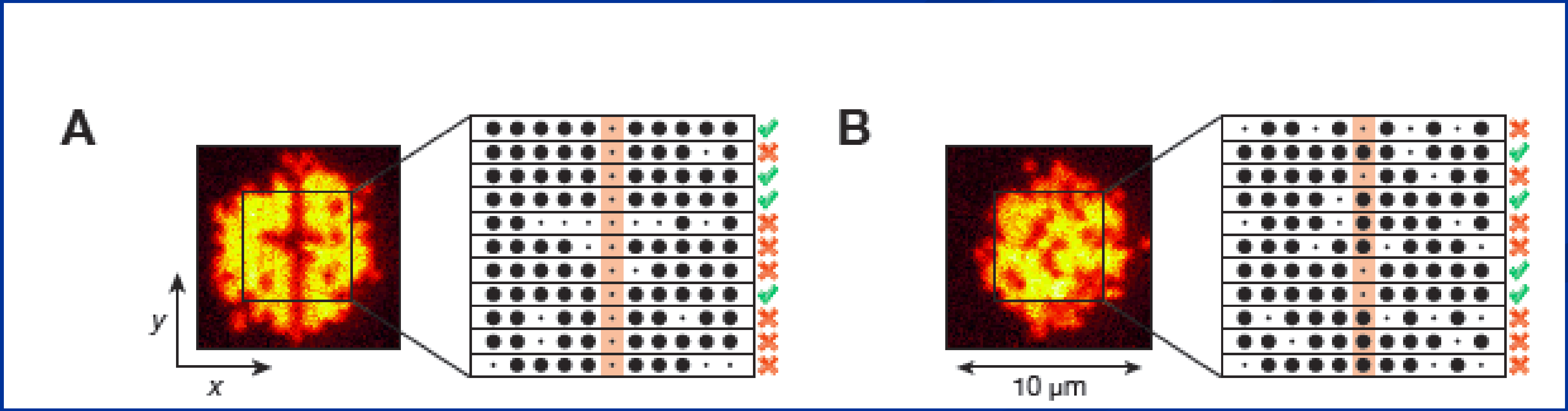




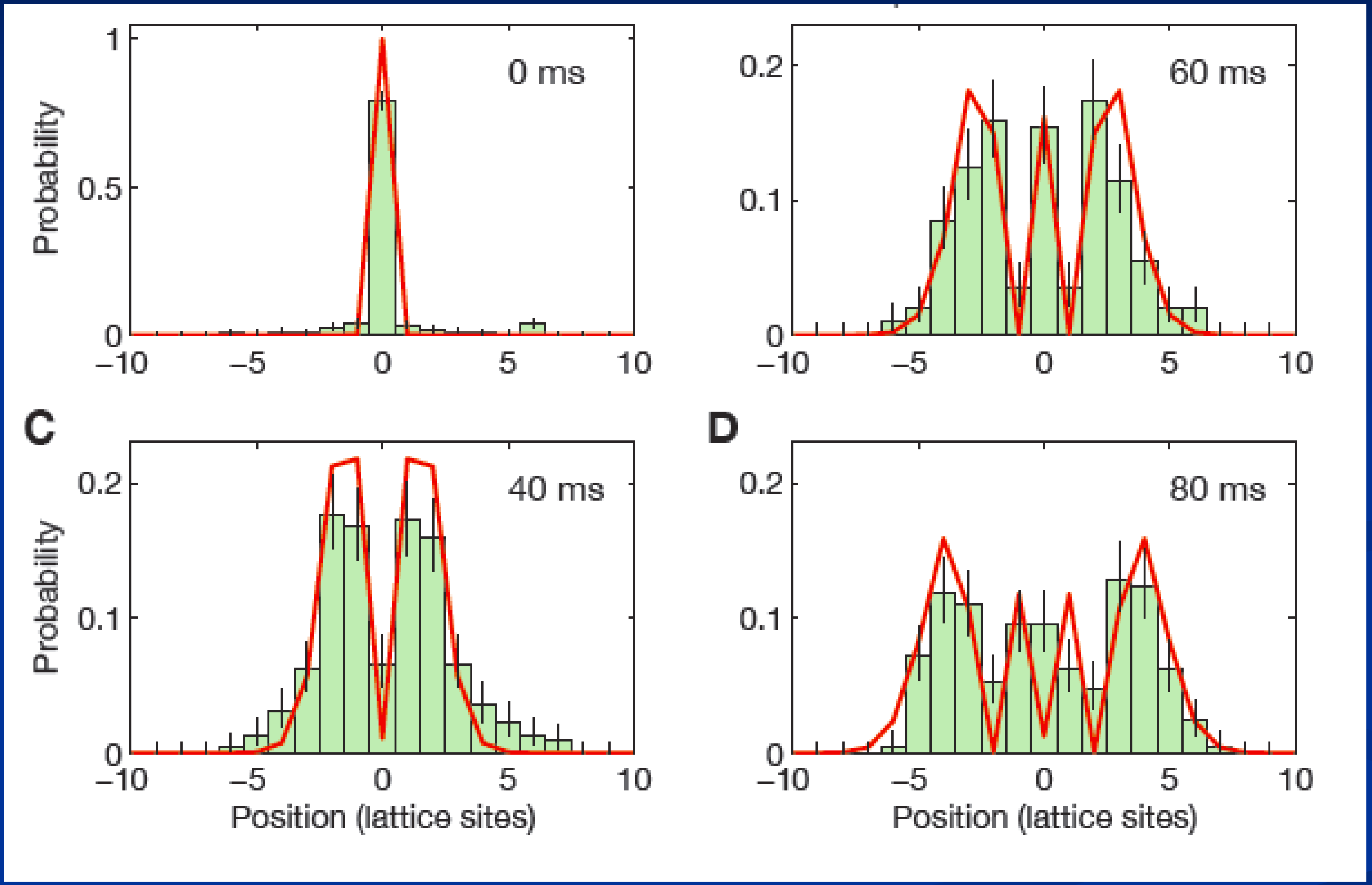



Ferromagnetic Heisenberg



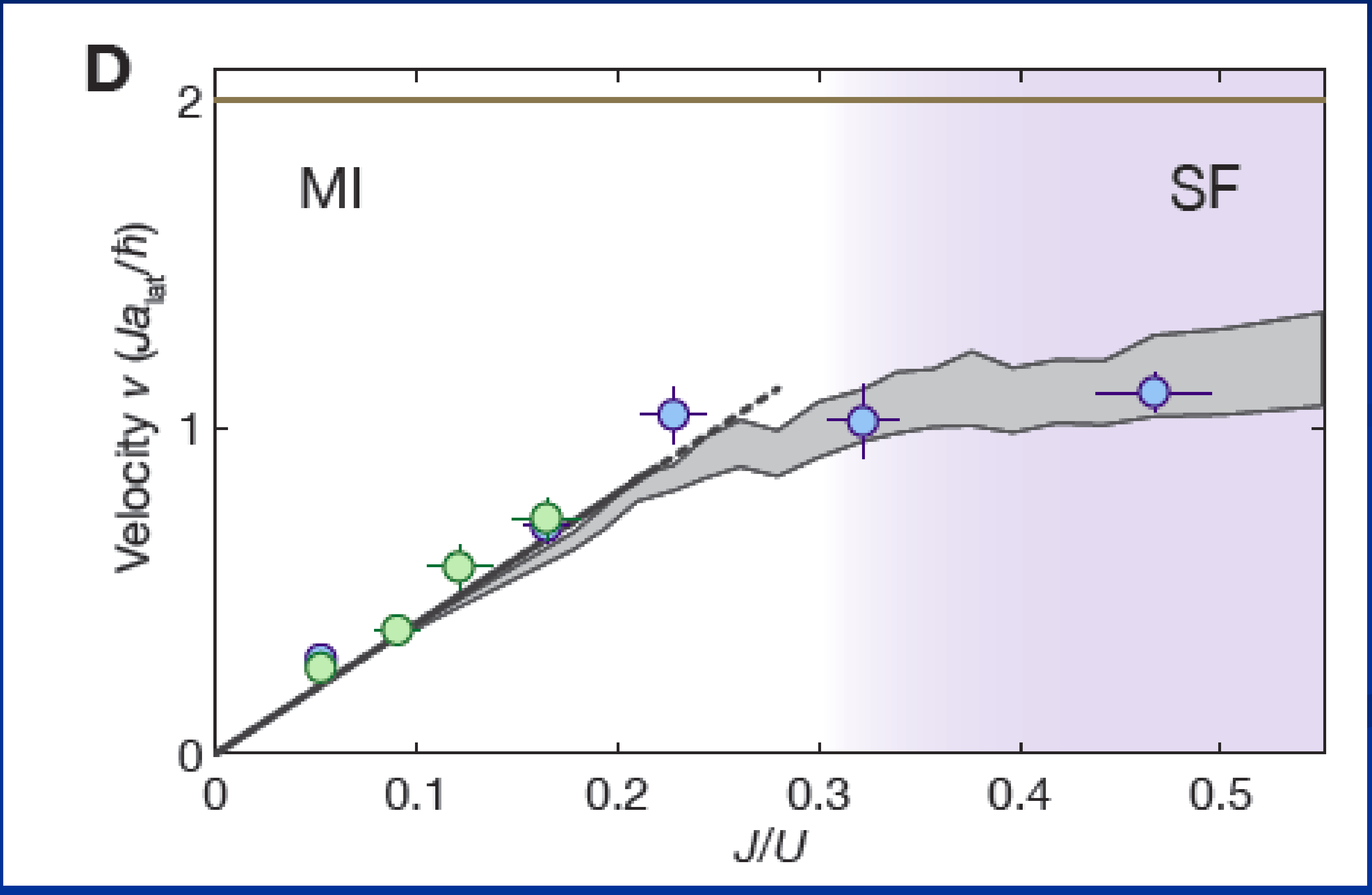


Coherent propagation of a magnon

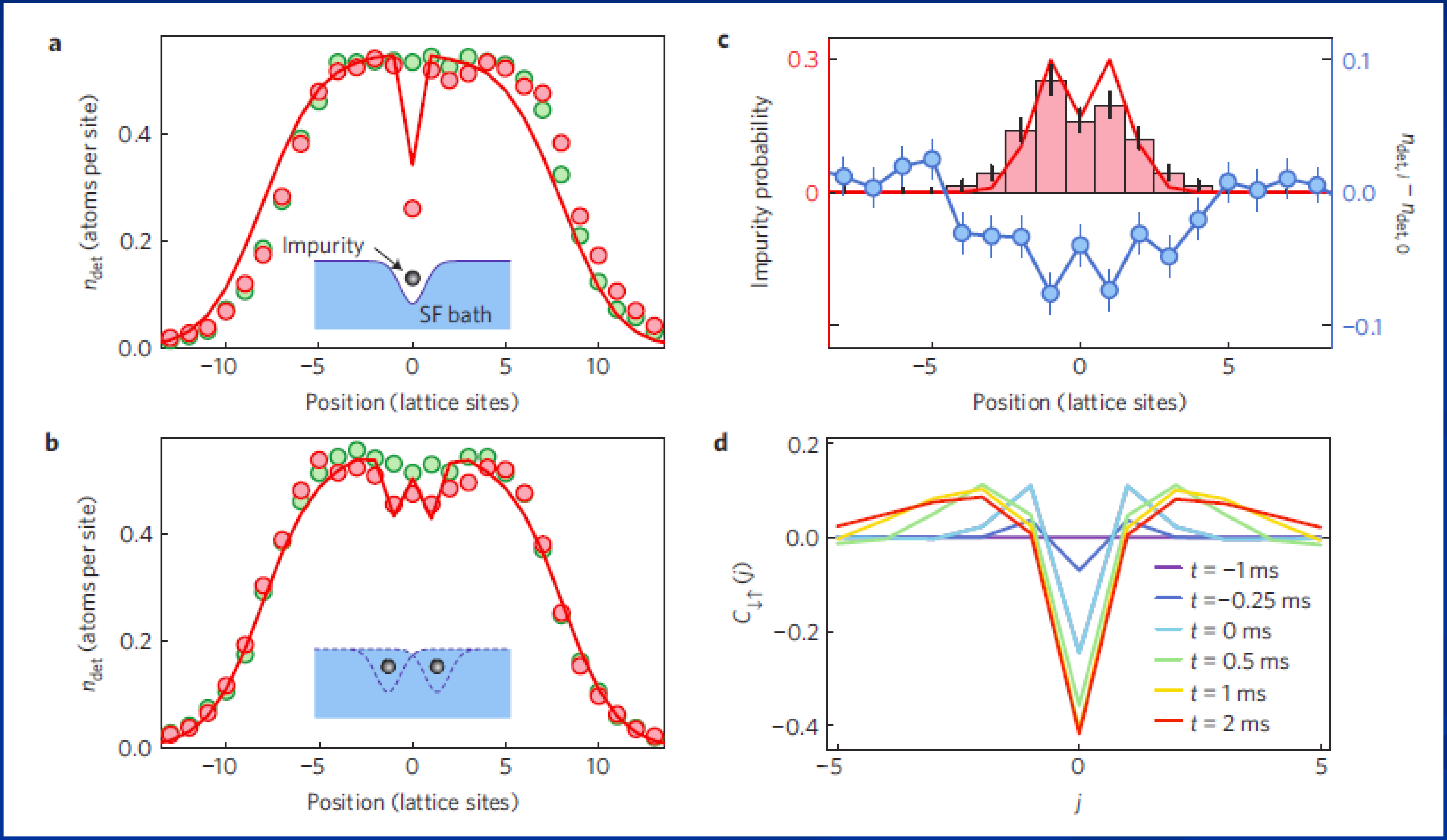


Heisenberg model : $H = J_{ex} \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$ $\qquad J_{ex} = \frac{4J^2}{U}$

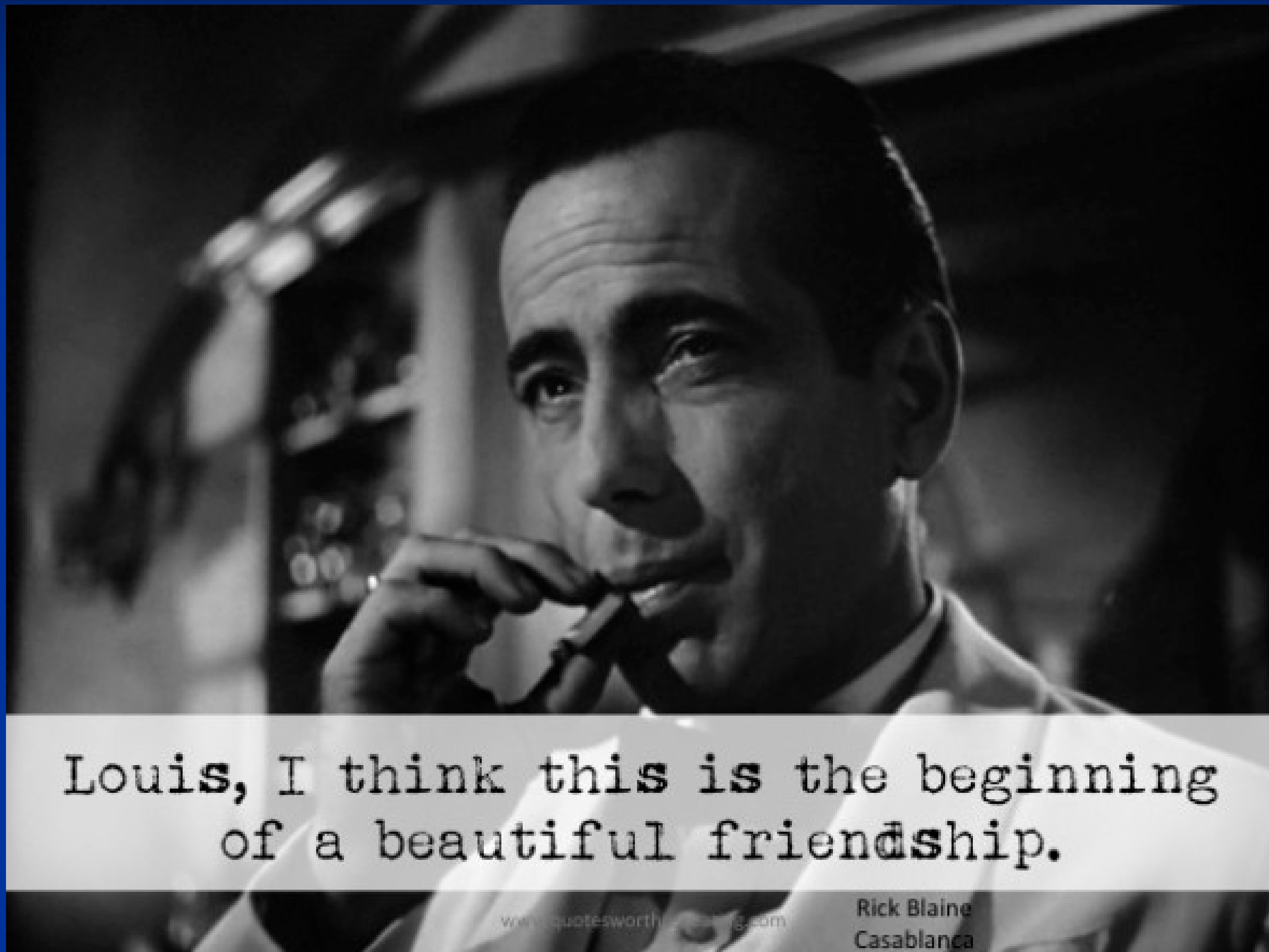
Impurity in a LL (weak lattice)



Polaronic effect



Cold atoms and 1d Physics



Important points / wishes

- Get rid of the trap !!! / Local probes
- Artifical gauge fields
- Fermions / Multicomponents