Before starting lecture 3:

Theory:

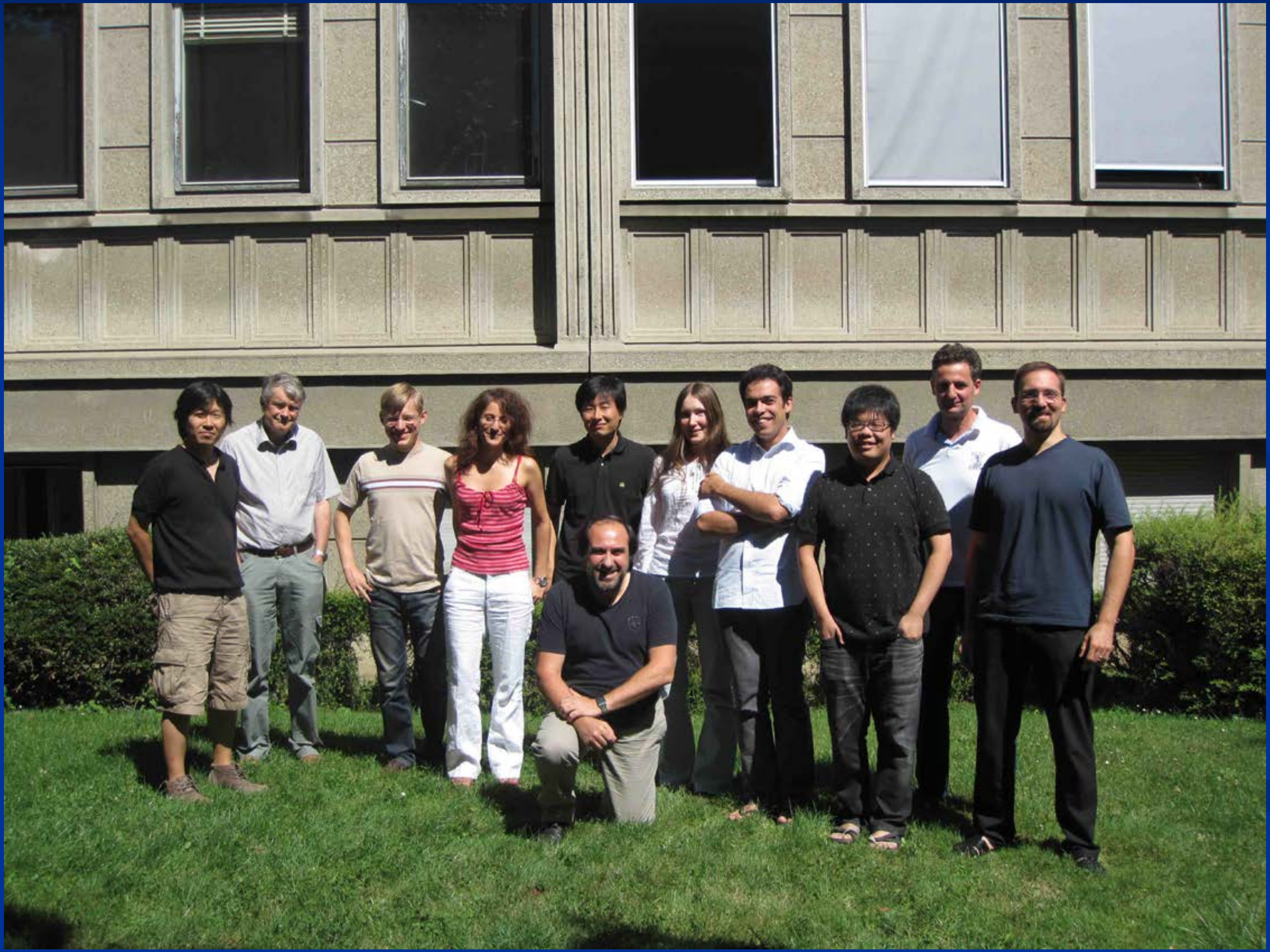
A. Kantian (Geneva U.)
C. Kollath (Bonn U.)
U. Schollwoeck (Munich U.)
E. Orignac (ENS Lyon)
R Citro (Salerno U.)
G. Orso (Paris-Diderot)
A. Iucci (la Plata)
M. Cazalilla (Taiwan)
Z. Ristivojevic (ENS, Paris)
A. Petkovic (ENS, Paris)
P. Le Doussal (ENS, Paris)

M. Zvonarev (LPTMS)
V. Cheianov (Lancaster)
G. Roux (LPTMS)
I. McCulloch (Queensland U.)
…………

Experiments :

Florence: G. Modugno, F. Minardi, M. Inguscio groups
Munich : S. Kuhr, I. Bloch groups
ETH: A. Zheludev's group

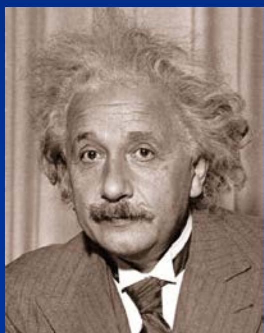
Geneva team:
dqmp.unige.ch/gr_giamarchi



Lecture 3

Disorder: ubiquitous

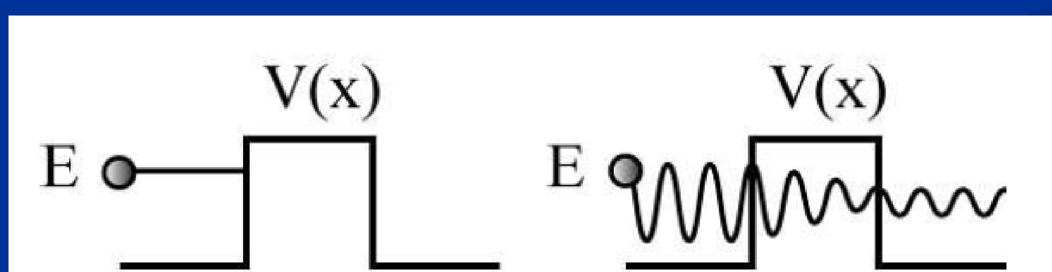
- Classical particles: diffusion $r^2 = Dt$



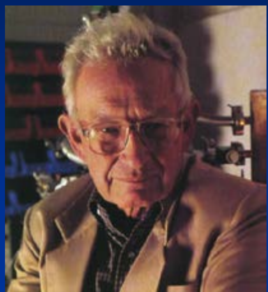

$$D = \mu k_B T$$

Einstein, A., *Z. Elektrochem.*, **14**, 235 (1908).
Sutherland, W., *Phil. Mag.*, **9**, 781 (1905).

- particles are waves: what does it change ?
- Nothing !!!! $\sigma = \dfrac{ne^2\tau}{m}$



Anderson Localization



PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

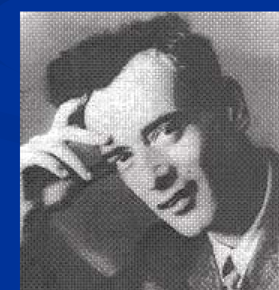
P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

Light, sound, electrons, etc……. waves

www.andersonlocalization.com

What about interactions :

- U>0 Landau Fermi liquid $m \to m^*$



Non interacting problem?

Disorder and Interactions

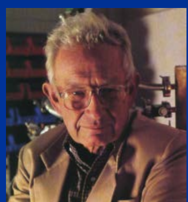
- Fermions: reinforcement of interactions by disorder
  perturbative: Altshuler-Aronov-Lee (80)
  RG: Finkelstein (84); TG+Schulz (88)

Localization ? Phases (electron glass) ? Transport ? …..

Phase diagram

TG arXiv/0403531, Varenna lectures (2002)

- d=3



D

Ins.

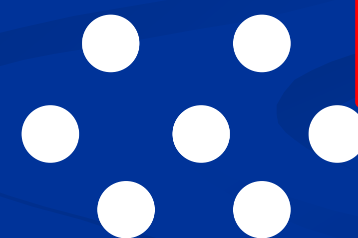
Interactions + disorder (weak localization

Metal

Melting

PinnedSolid

Glass !



Fermi liquid

Wigner Crystal

U

Disorder and Interactions

- Fermions: reinforcement of interactions by disorder

  perturbative: Altshuler-Aronov-Lee (80)

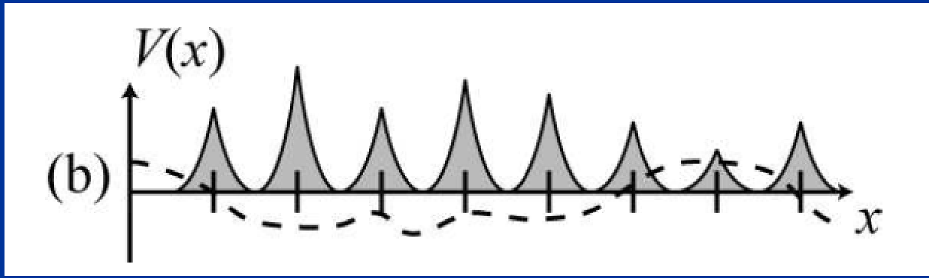
  RG: Finkelstein (84); TG+Schulz (88)

Localization ? Phases (electron glass) ? Transport ? …..

- Bosons: competition between superfluidity/localization

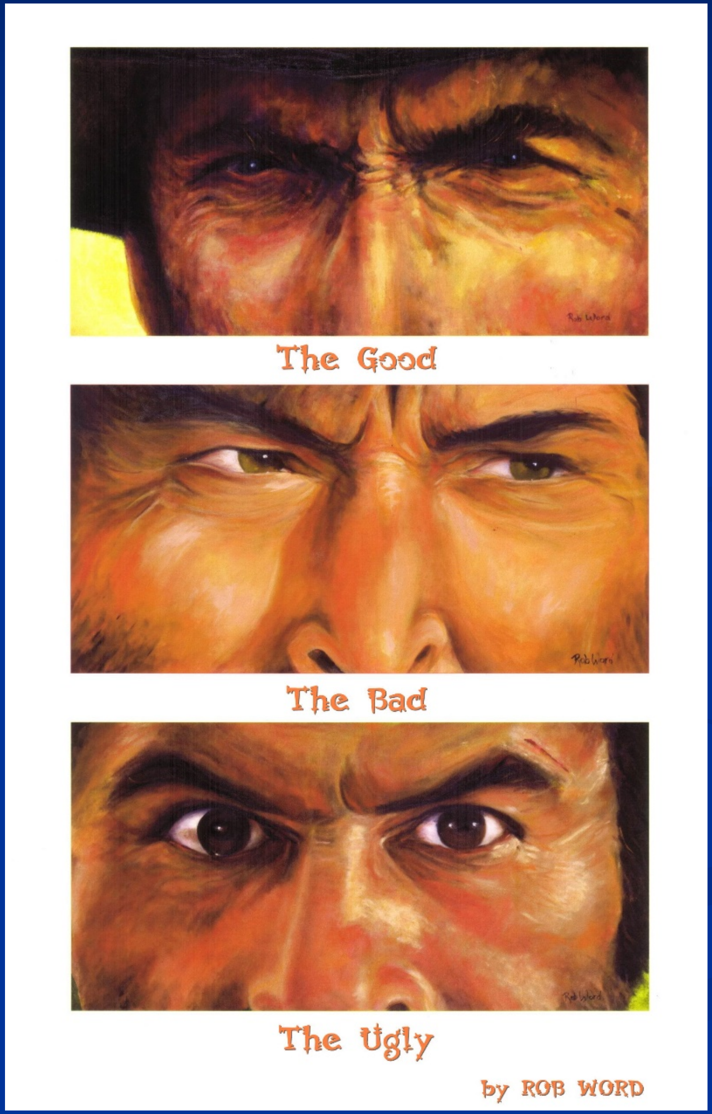
Free Bosons: pathological

$$H = \frac{1}{2m}\left(\frac{1}{L}\right)^2 - V_0$$



Interactions needed from the start

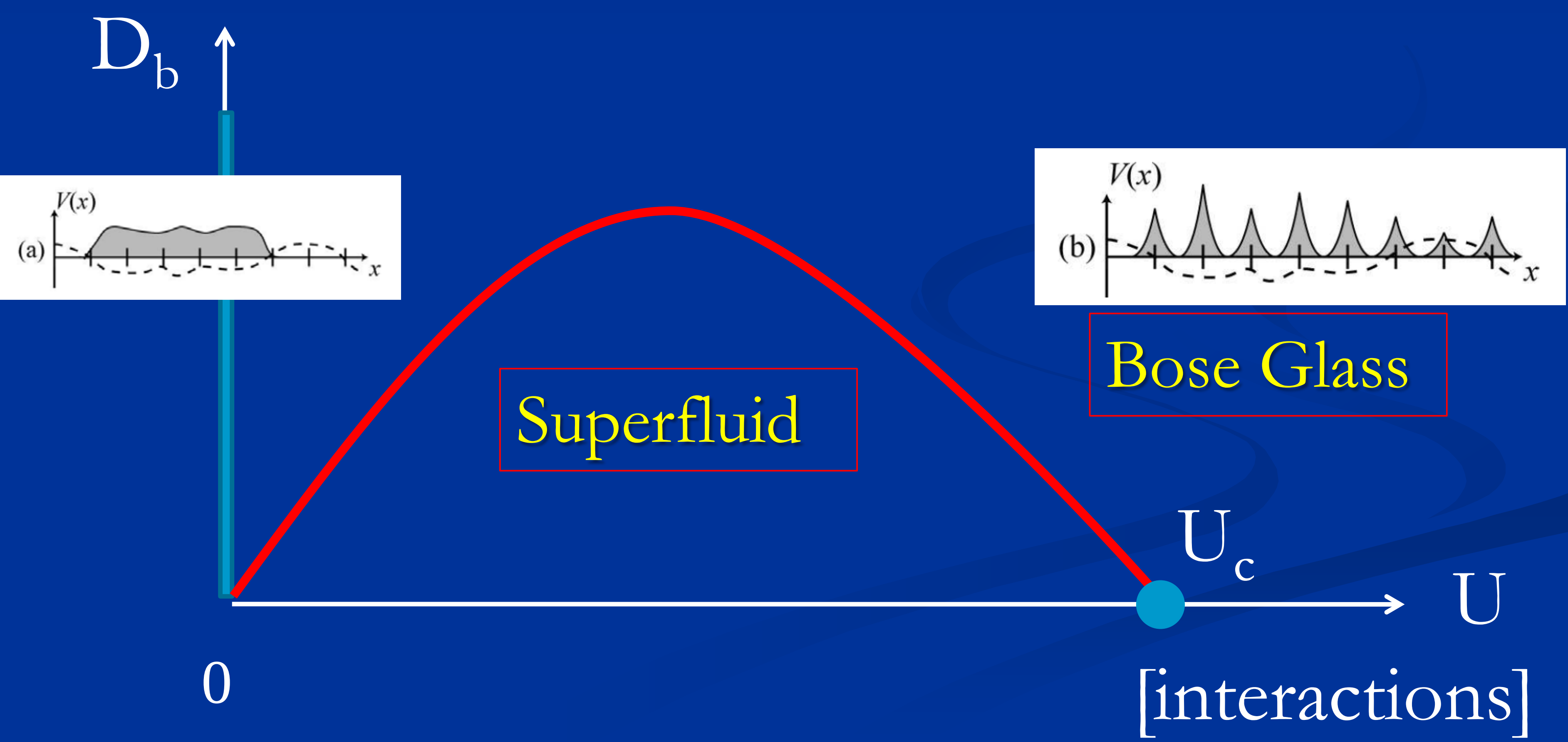
So, what is the physics ?



Bose glass phase

TG + H. J. Schulz EPL 3 1287 (87); PRB 37 325 (1988);

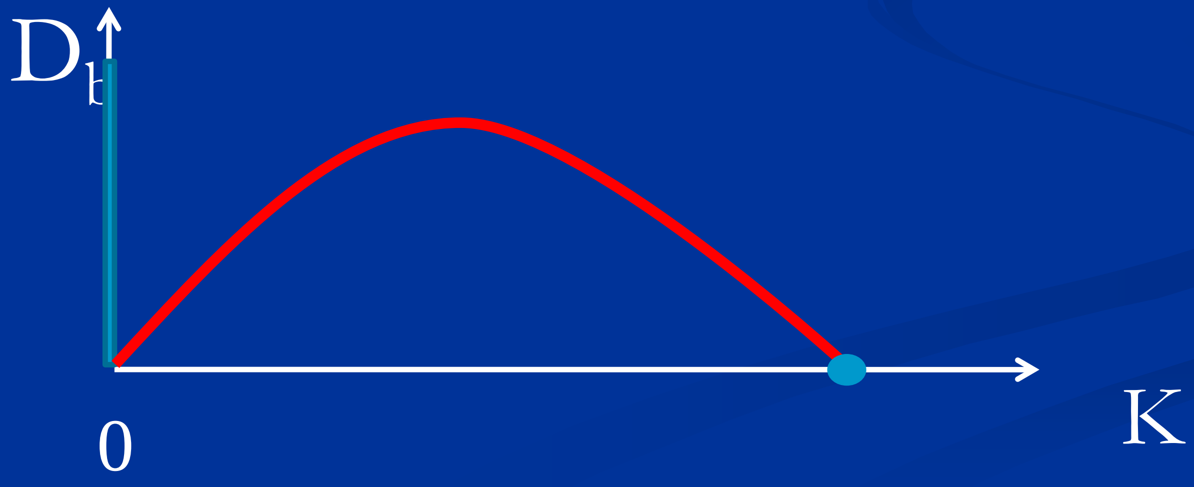
M.P.A. Fisher et al. PRB 40 546 (1989)



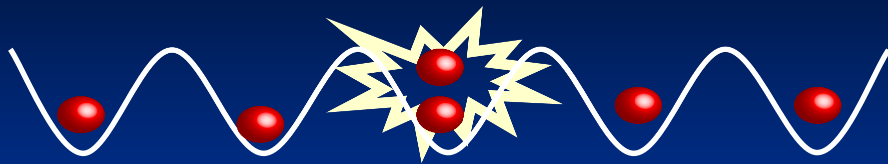
SU-BG transition in d=1

$$\langle \psi(r)\psi^\dagger(0)\rangle \sim \left(\frac{1}{r}\right)^{\frac{1}{2K}} \qquad K \to 3/2$$

Universal exponent at the SU-BG transition !

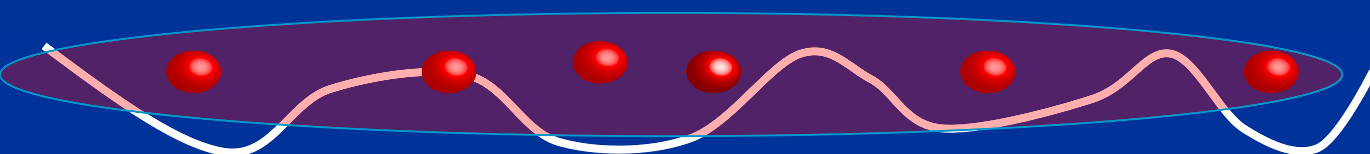


Various phases



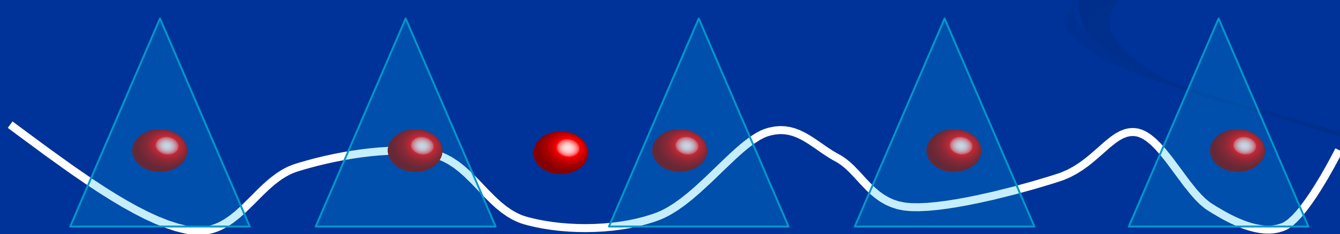
$$\frac{dN}{d\mu} = 0$$

- Mott insulator: incompressible; $\langle \psi \rangle = 0$



$$\frac{dN}{d\mu} \neq 0$$

- Superfluid: compressible; $\langle \psi \rangle \neq 0$



$$\frac{dN}{d\mu} \neq 0$$

- Bose Glass : compressible; $\langle \psi \rangle = 0$

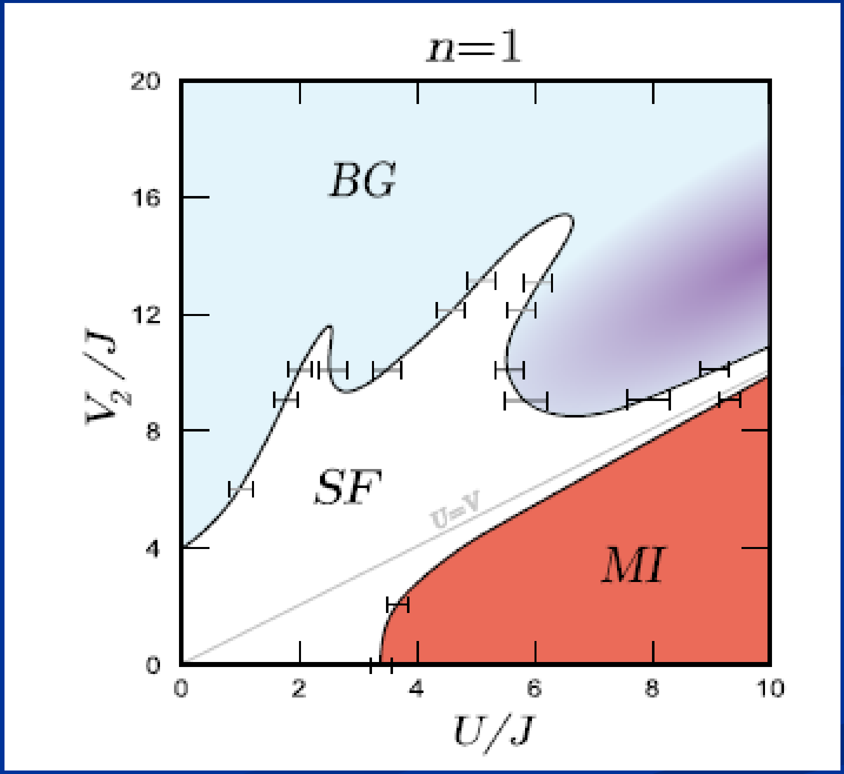
TG, P. le Doussal PRB 53 15206 (96); T. Nattermann et al. PRL 91 056603 (03); E. Altman et al PRB 81 174528 (10),…….

Other potentials: Biperiodics

$$V(x) = V_0 \cos(Q_0 x) + V_1 \cos(Q_1 x)$$

- U= 0
  Aubry-André model
- Localization transition

Effect of interactions?
Same as ``true” disorder ?

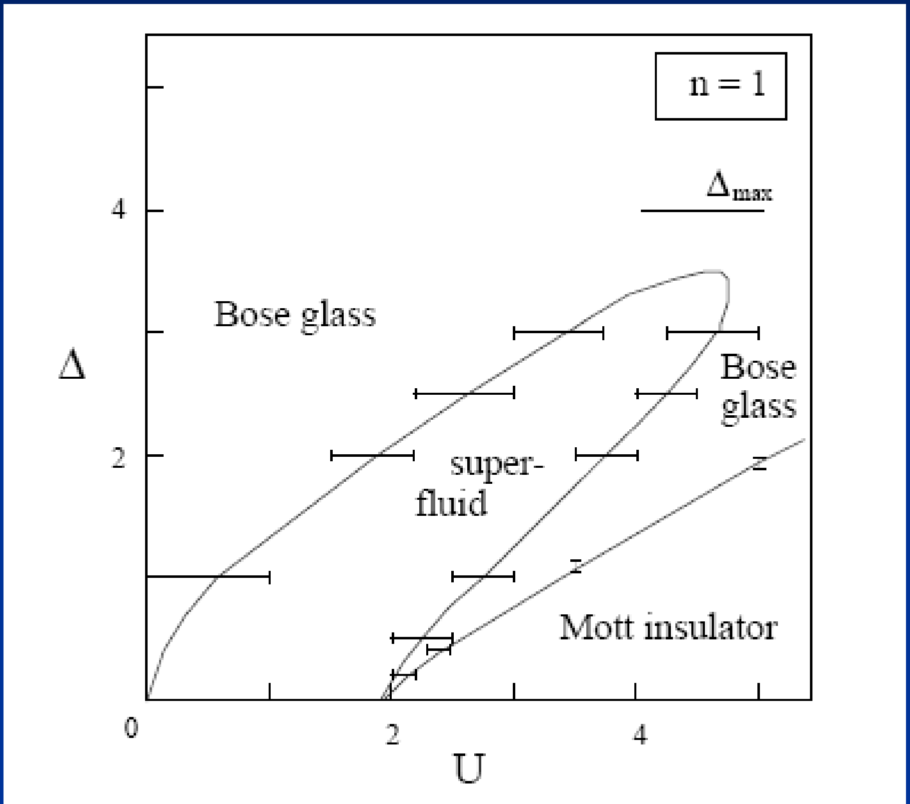


J. Vidal, D. Mouhanna, TG PRL 83 3908 (1999);
PRB 65 014201 (2001)

G. Roux et al. PRA 78 023628 (2008);
T. Roscilde, Phys. Rev. A 77, 063605 2008;
X. Deng et al PRA 78, 013625 (2008);

The hunt for the Bose glass

Numerics (disorder)

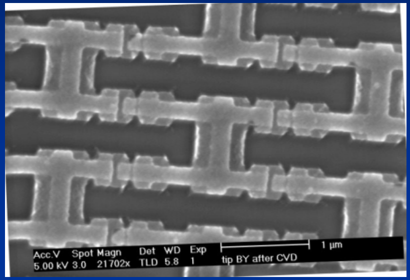


S. Rapsch, U. Schollwoeck, W. Zwerger EPL 46 559 (1999);

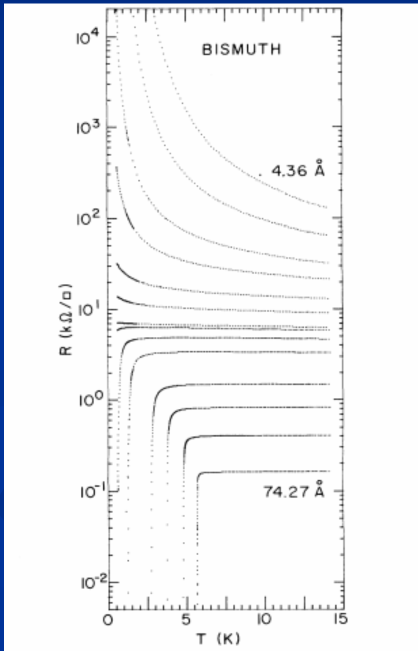
G. Batrouni et al. PRL 65 1765 (90);
N. V. Prokof'ev and B. V. Svistunov, PRL 80 4355 (96);
N. Prokofev et al. PRL 92 015703 (04);
O. Nohadani et al. PRL 95, 227201 (05)
K. G. Balabanyan et al. PRL 95, 055701 (05);
L. Pollet et al. PRL 103, 140402 (2009)
…..

Old Experiments

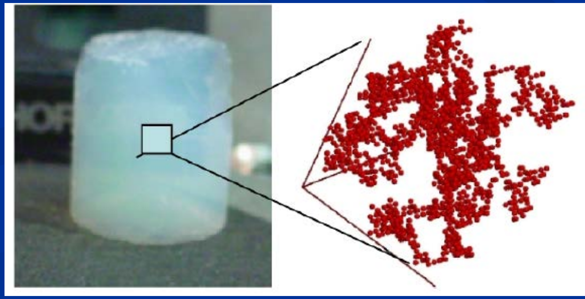
Josephson junction arrays



Disordered superconducting films
(D.B. Haviland et al, PRL 62 2180 (89))

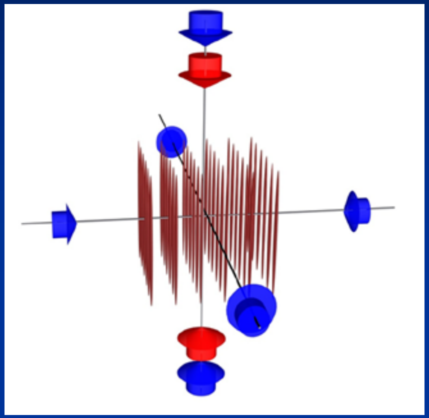


Helium in porous media

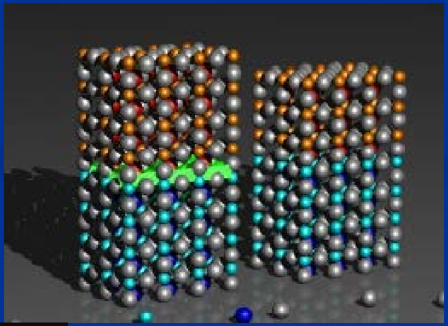


New remarkable systems

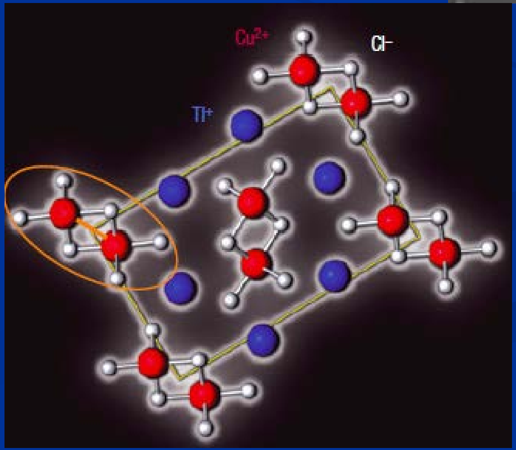
Cold atoms



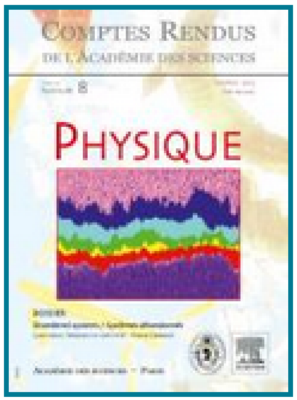
New superconducting films



Spin dimers



Comptes Rendus AS



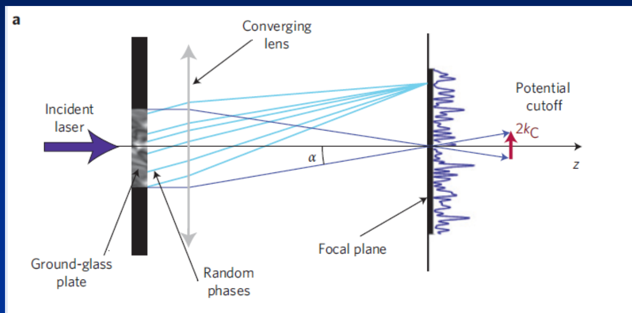
Vol 14 - N° 8 - octobre 2013

P. 637-756

Académie des sciences

Disordered systems / Systèmes désordonnés

Cold atomic gases

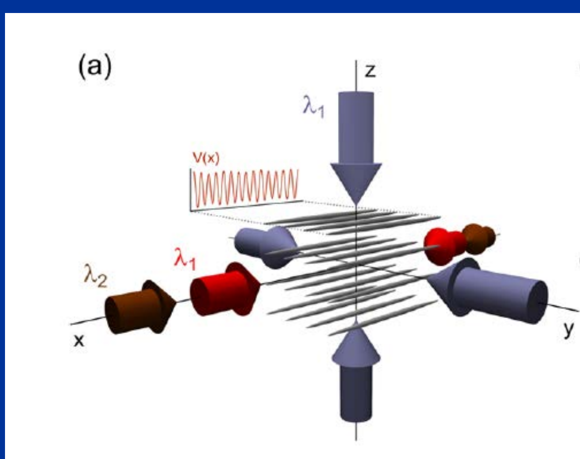


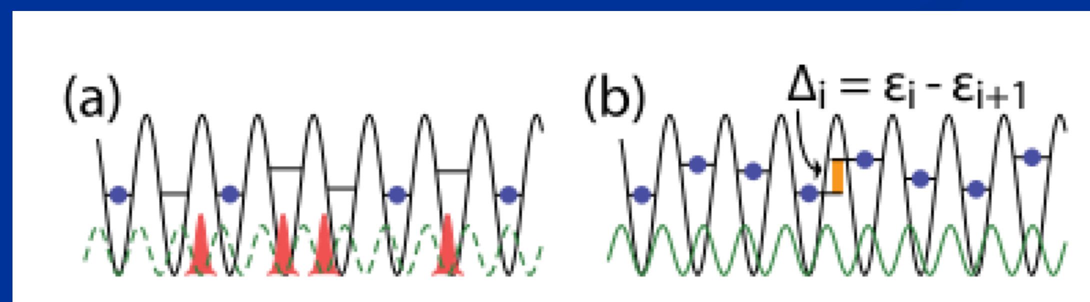
Speckle (Palaiseau, Florence, Urbana)

nature physics | PROGRESS ARTICLE
PUBLISHED ONLINE: 1 FEBRUARY 2010 | DOI: 10.1038/NPHYS1507

Disordered quantum gases under control

Laurent Sanchez-Palencia[1]* and Maciej Lewenstein[2]*





Biperiodic lattices (Florence)

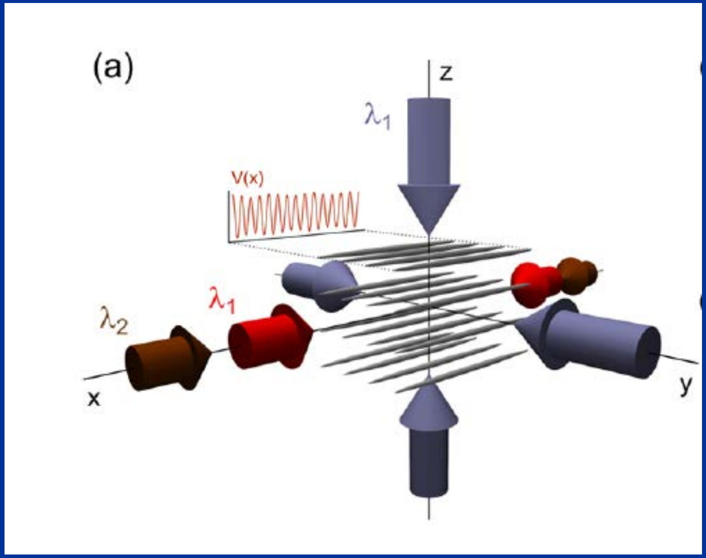
Mixtures (Stony Brook)

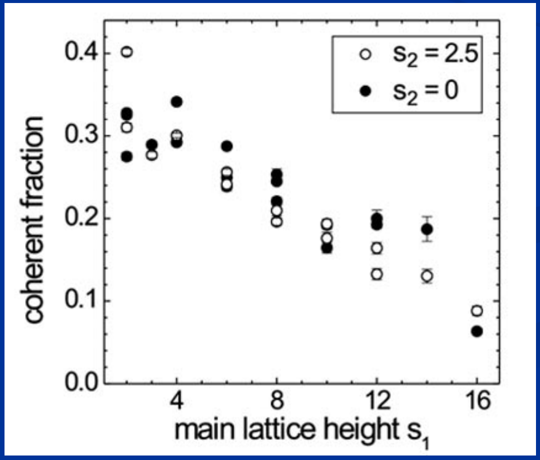
Quasi-periodic

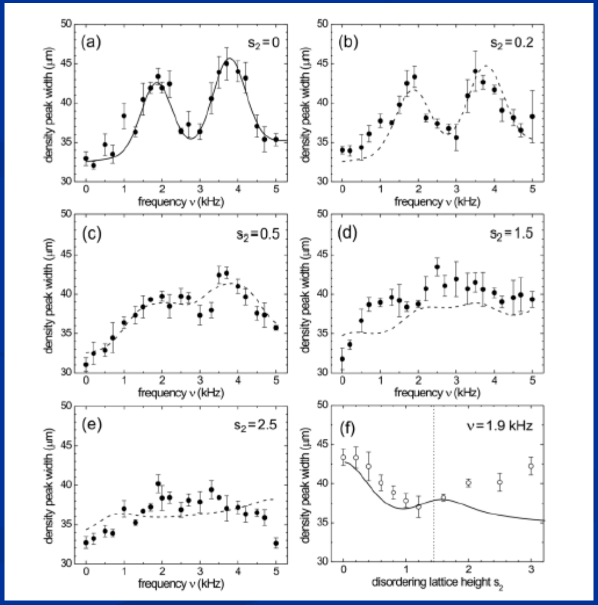
Quasiperiodic (1D):

J. E. Lye, et al., PRA **75, 061603R** 2007.

L. Fallani, et al. PRL **98, 130404 2007**

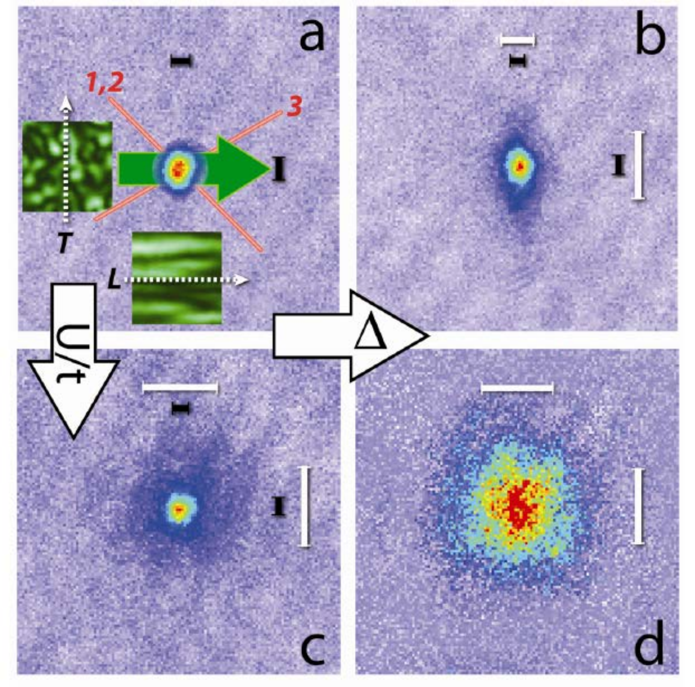


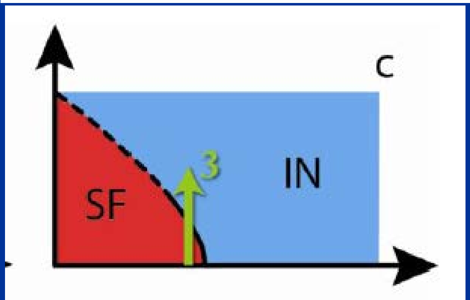


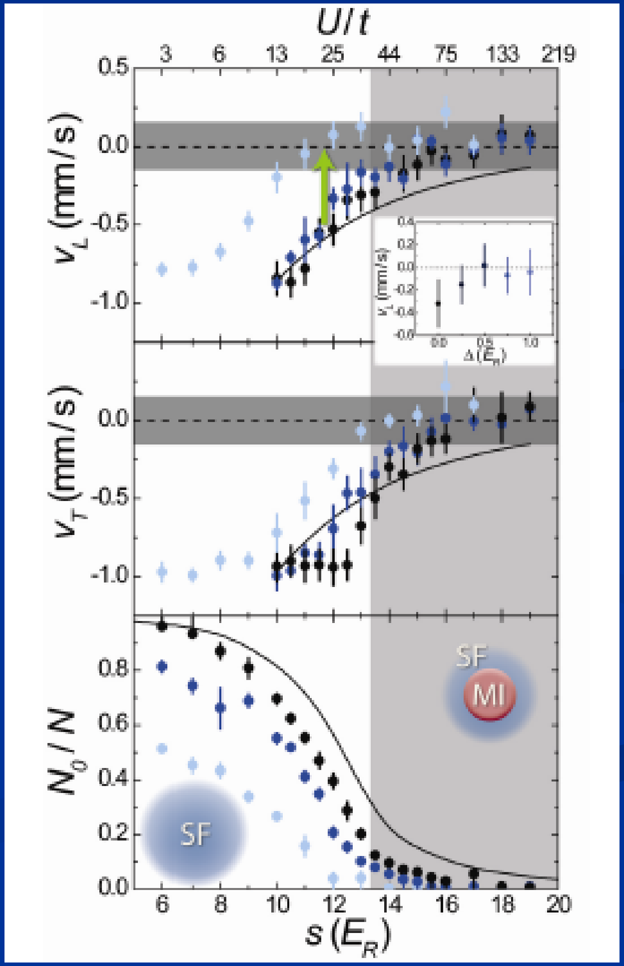


Speckle

Pasienski et al. Nat. Phys 6 677 (2010)

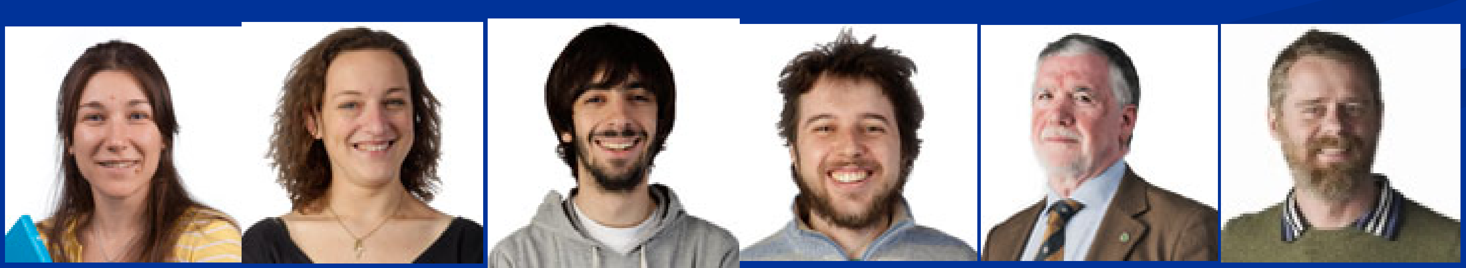


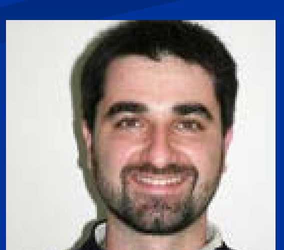




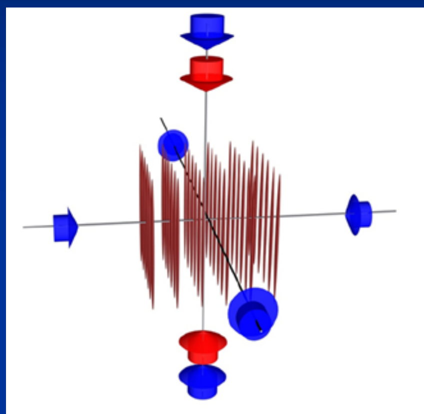
Control of Interactions and Disorder

Chiara D Errico, Eleonora Lucioni, Luca Tanzi, Lorenzo Gori, Guillaume Roux, Ian P. McCulloch, TG, Massimo Inguscio, Giovanni Modugno, arxiv/1405.1210

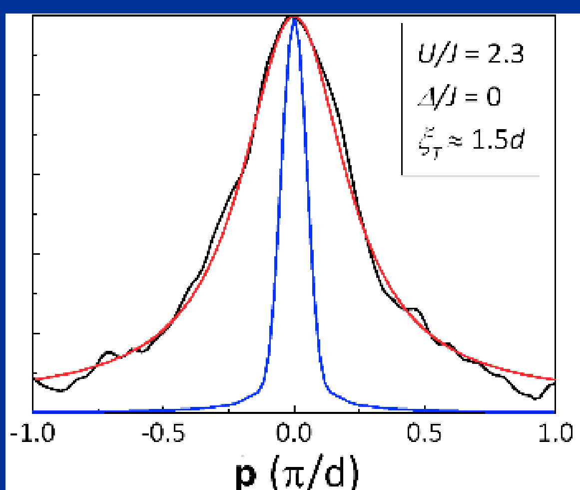




Bosons in bichromatic lattice

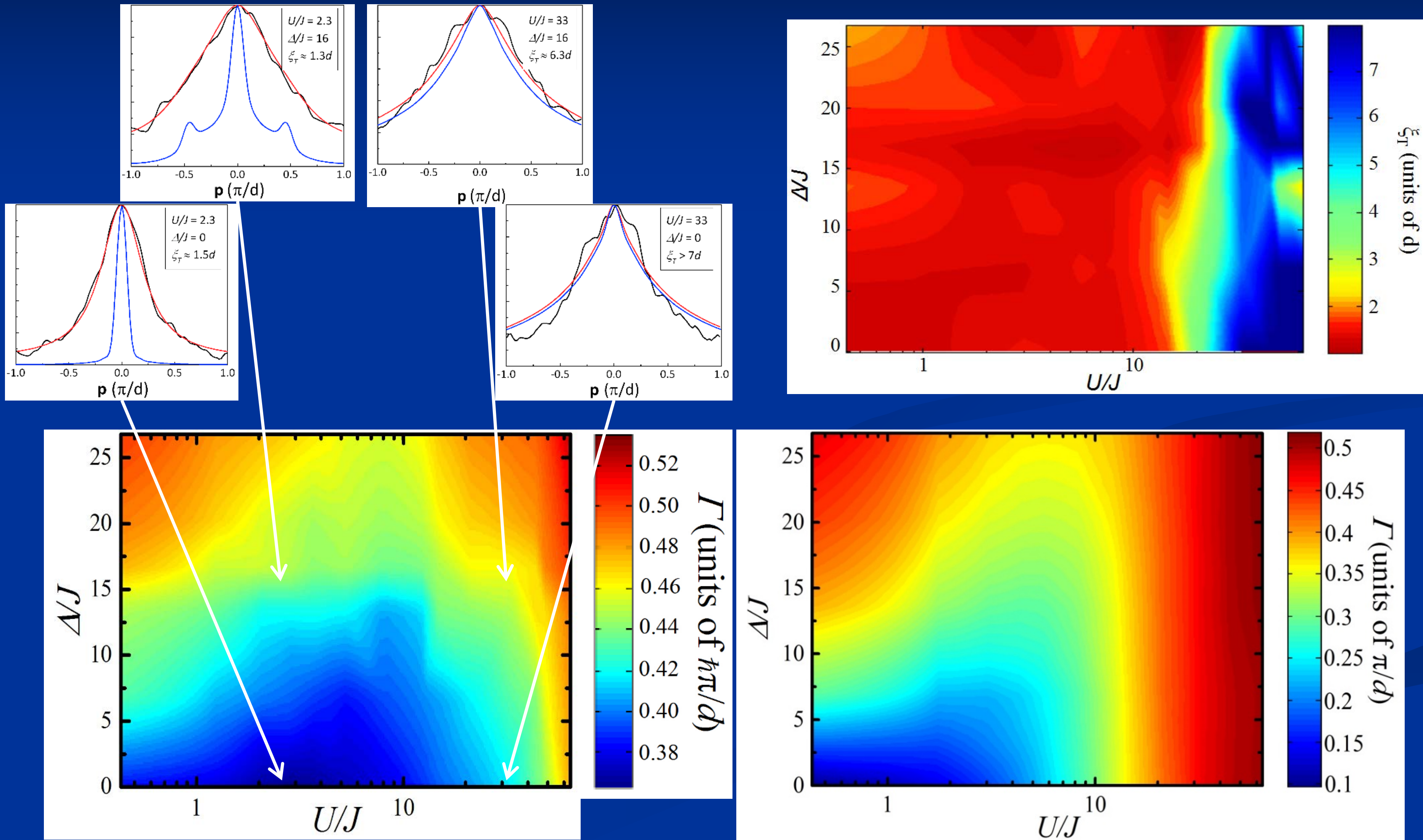


39K atoms
Bichromatic r = 1.243
J = 110 Hz

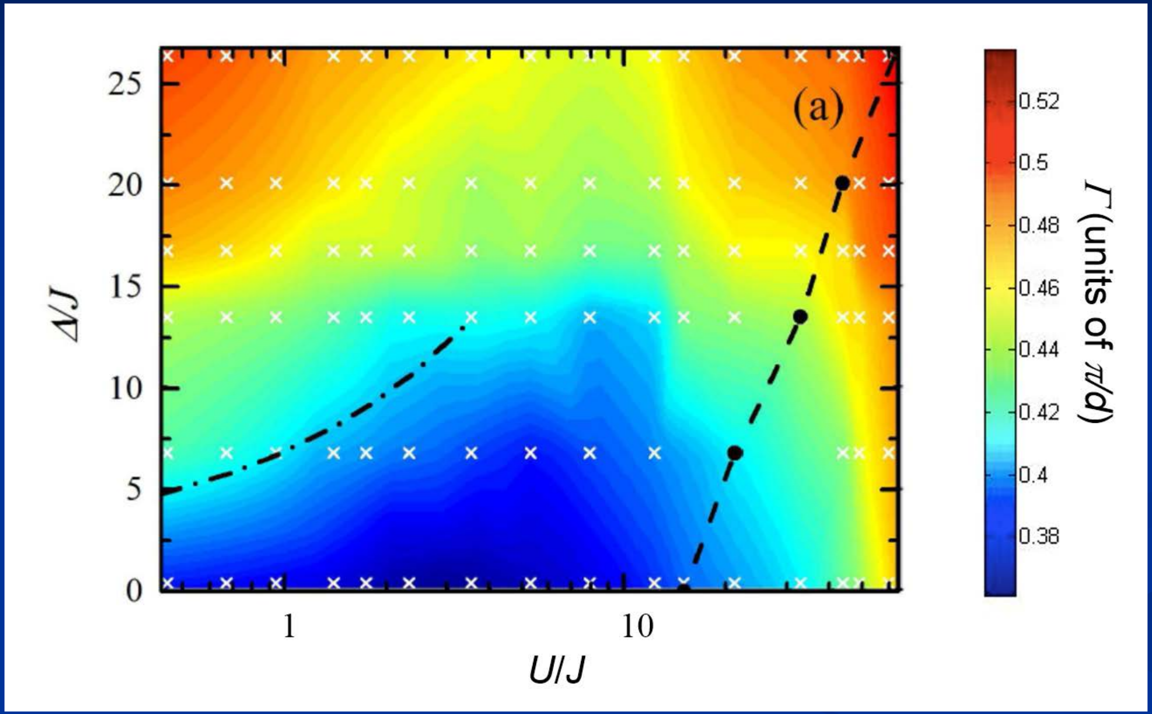


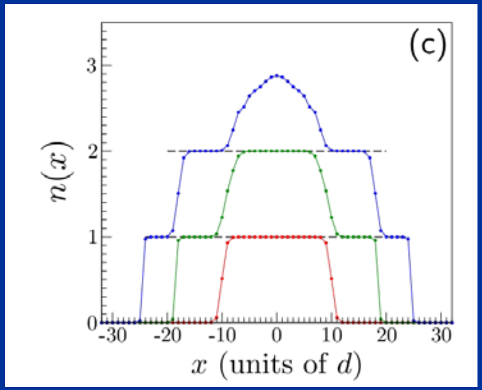
Measure of n(k) (TOF)
Comparison with LL and DMRG calculations

Fitted thermal length



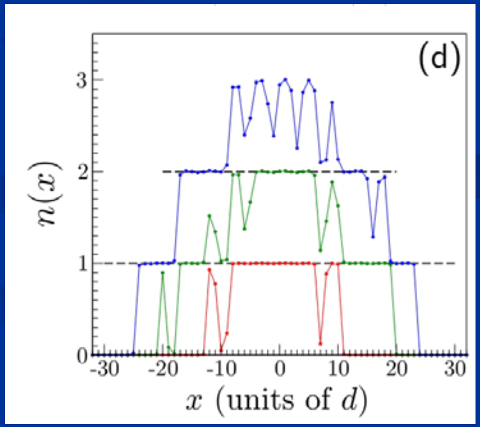
Loss of coherence





$\Delta = 6.3\ J$

$\Delta = 9.5\ J$

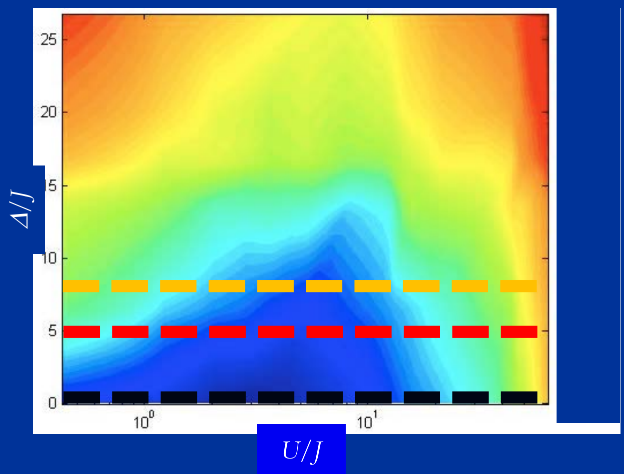


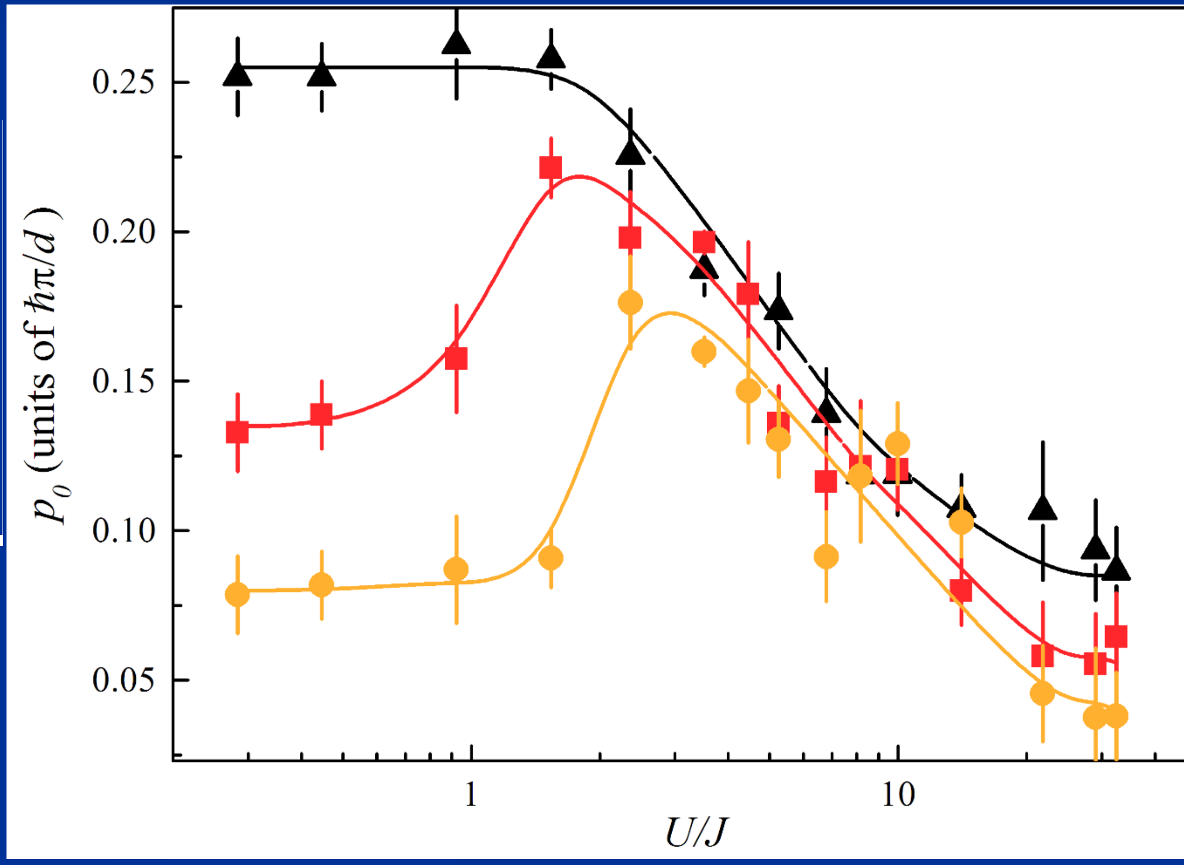
Insulating phase(s)

prepare in equilibrium

shift, wait 0.8ms

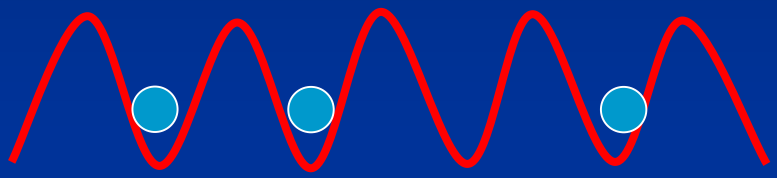
free expansion





Incoherent regimes are also insulating

Shaking of the lattice

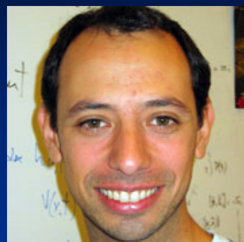


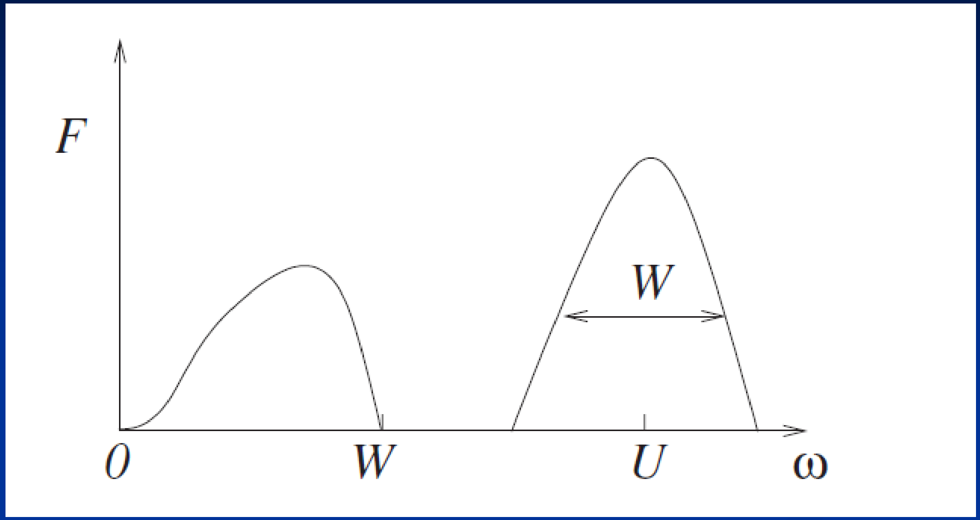
- Modulate amplitude of optical lattice at frequency ω

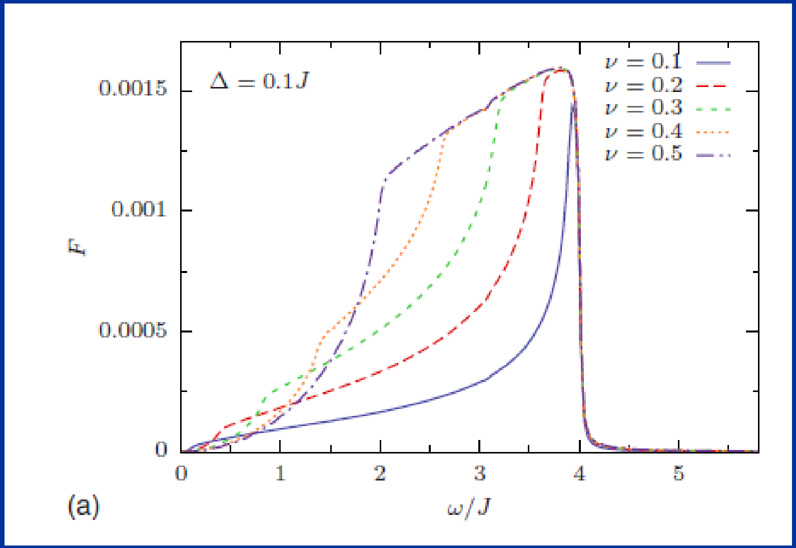
- Mesure the absorbed energy

$$\frac{dE}{dt} \propto \langle [H_K(t), H_K(0)] \rangle$$

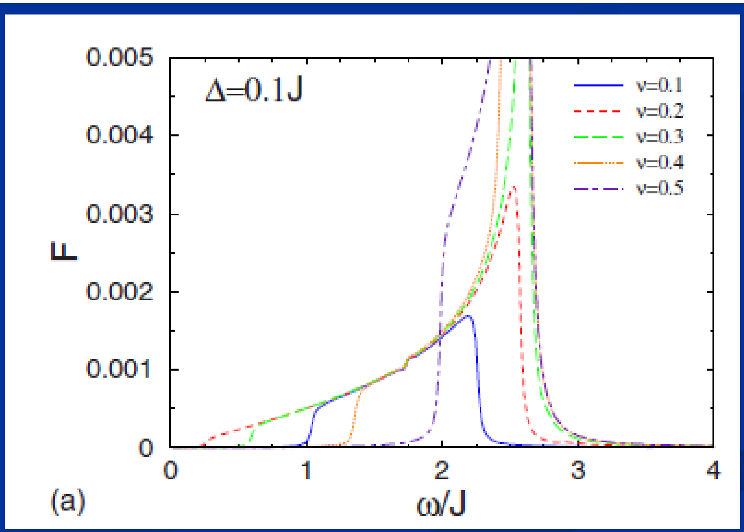
G. Orso, A. Iucci, M. Cazalilla, TG PRA 80 033625 (2009)



Disorder

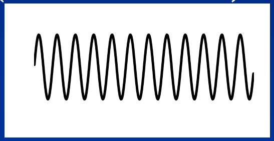


Quasi

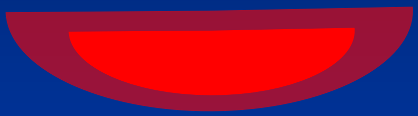
Experiment

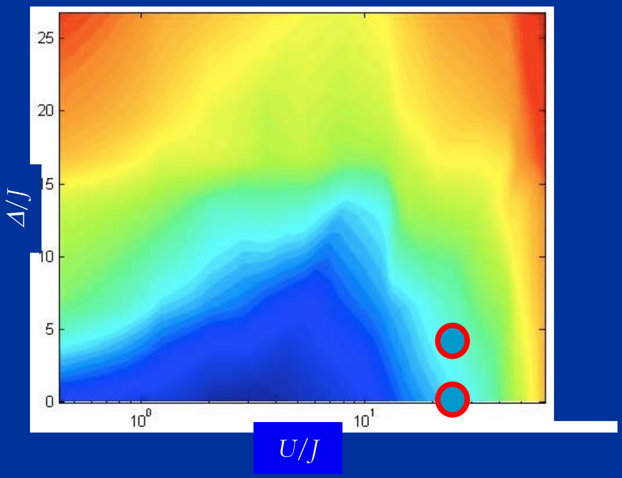
prepare in equilibrium

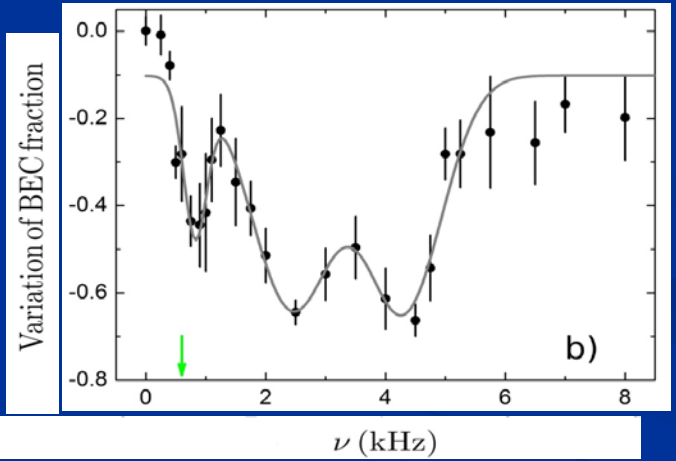
main lattice modulation
(15%, 200ms)

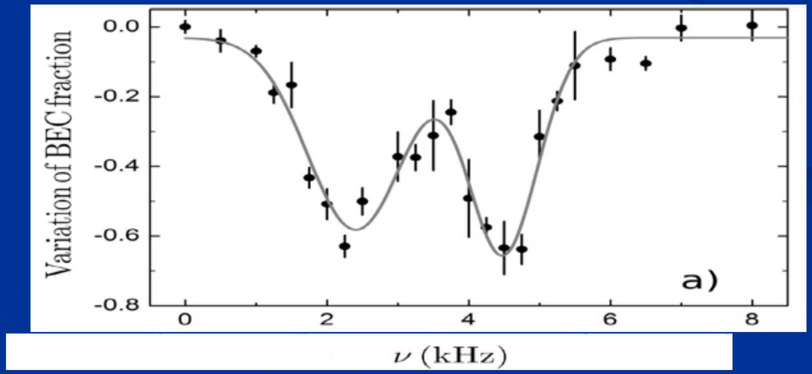


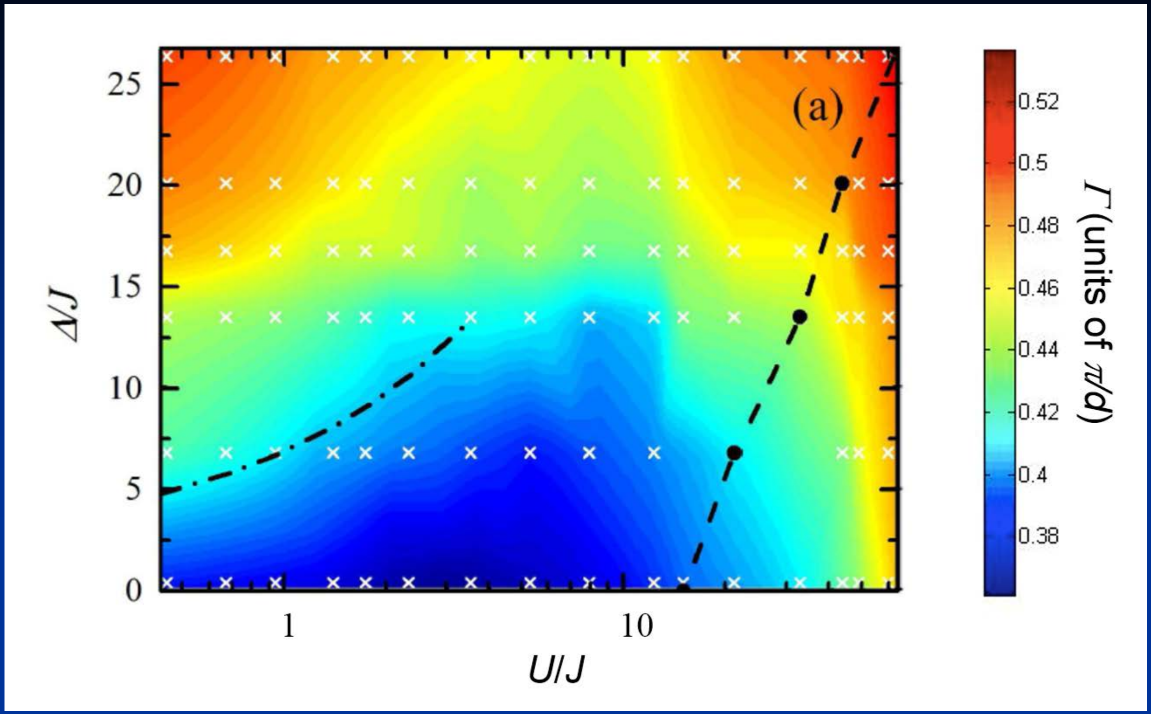
"energy" measurement











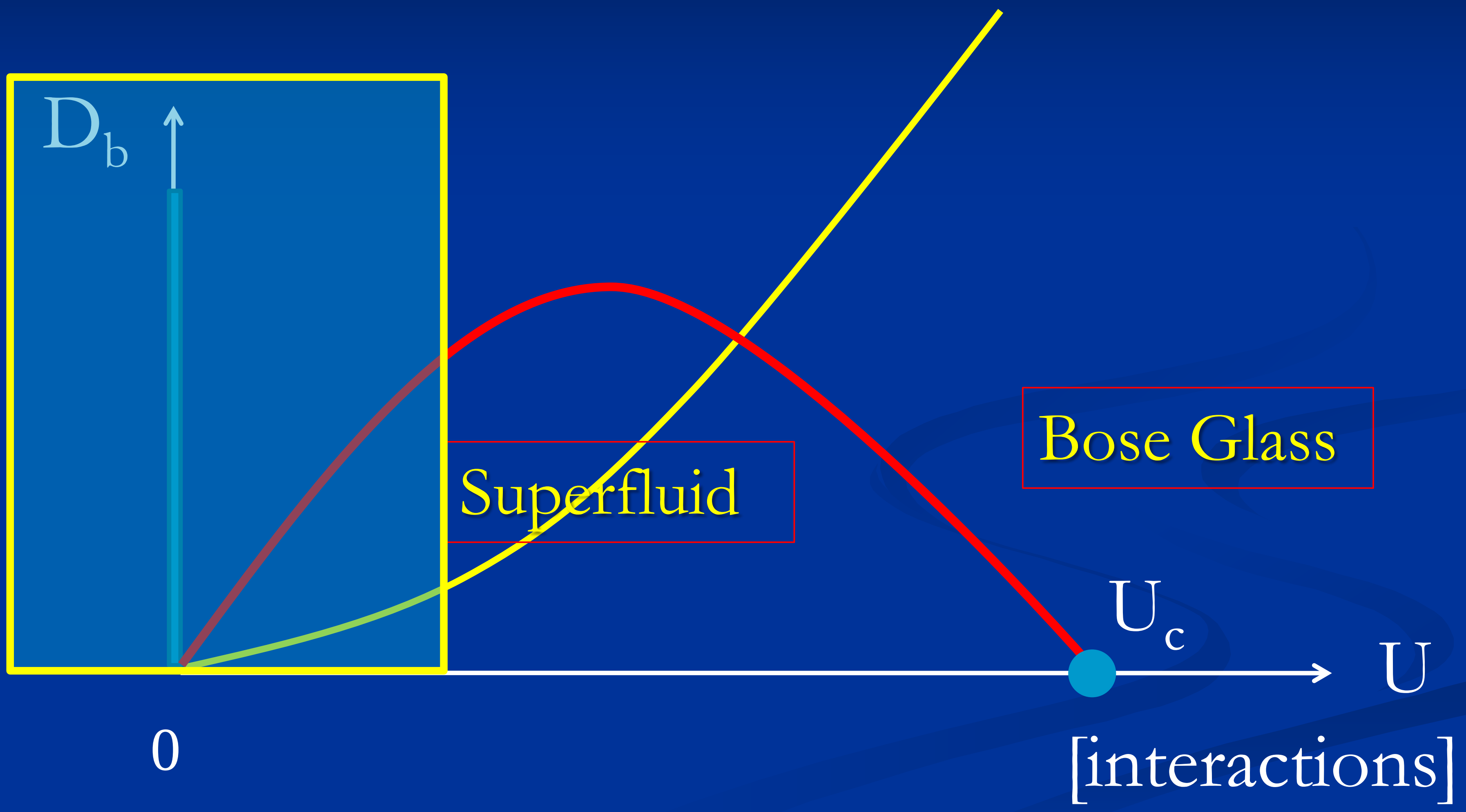
- Reentrant phase diagram (consistent with theory)
- Disordered insulator (``Bose glass”)

- But one can improve on : temperature, Mott insulator, trap

Theoretical questions:

Phase diagram and phases

Strong disorder, weak interactions



Strong disorder, weak interactions

PHYSICAL REVIEW B **81**, 174528 (2010)

Superfluid-insulator transition of disordered bosons in one dimension

Ehud Altman,[1] Yariv Kafri,[2] Anatoli Polkovnikov,[3] and Gil Refael[4]

phase diagram. We show that the superfluid-insulator transition is always Kosterlitz-Thouless like in the way that length and time scales diverge at the critical point. Interestingly however, we find that the transition at strong disorder occurs at a nonuniversal value of the Luttinger parameter, which depends on the disorder strength. This result places the transition in a universality class different from the weak disorder transition first analyzed by Giamarchi and Schulz [Europhys. Lett. **3**, 1287 (1987)]. While the details of the disorder potential

How to connect the two results ?

Not an easy question

PRL **109**, 265303 (2012) PHYSICAL REVIEW LETTERS week ending 28 DECEMBER 2012

Disordered Bosons in One Dimension: From Weak- to Strong-Randomness Criticality

Fawaz Hrahsheh and Thomas Vojta
Department of Physics, Missouri University of Science and Technology, Rolla, Missouri 65409, USA
(Received 24 October 2012; published 27 December 2012)

We investigate the superfluid-insulator quantum phase transition of one-dimensional bosons with off-diagonal disorder by means of large-scale Monte Carlo simulations. For weak disorder, we find the transition to be in the same universality class as the superfluid-Mott insulator transition of the clean system. The nature of the transition changes for stronger disorder. Beyond a critical disorder strength, we find nonuniversal, disorder-dependent critical behavior. We compare our results to recent perturbative and strong-disorder renormalization group predictions. We also discuss experimental implications as well as extensions of our results to other systems.

Classical-Field Renormalization Flow of One-Dimensional Disordered Bosons

Lode Pollet,[1] Nikolay V. Prokof'ev,[2,3] and Boris V. Svistunov[2,3]
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We show that in the regime when strong disorder is more relevant than field quantization the superfluid–to–Bose-glass criticality of one-dimensional bosons is preceded by the prolonged logarithmically slow classical-field renormalization flow of the superfluid stiffness at mesoscopic scales. With the system compressibility remaining constant, the quantum nature of the system manifests itself only in the renormalization of dilute weak links. On the insulating side, the flow ultimately reaches a value of the Luttinger parameter at which the instanton–anti-instanton pairs start to proliferate, in accordance with the universal quantum scenario. This happens first at astronomic system sizes because of the suppressed instanton fugacity. We illustrate our result by first-principles simulations.

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Asymptotically exact scenario of strong-disorder criticality in one-dimensional superfluids

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We present a controlled rare-weak-link theory of the superfluid-to-Bose/Mott glass transition in one-dimensional disordered systems. The transition has Kosterlitz-Thouless critical properties but may occur at an arbitrary large value of the Luttinger parameter K. The hydrodynamic description is valid under the correlation radius and defines criticality via mutual renormalization of the strength of microscopic weak links and superfluid stiffness. The link strength renormalizes along the lines of Kane and Fisher [Phys. Rev. Lett. **68**, 1220 (1992)], while the renormalization of superfluid stiffness follows the lines of classical-field flow. The hallmark of the theory is the relation $K^{(c)} = 1/\zeta$ between the critical value of the Luttinger parameter at macroscopic scales and the microscopic (irrenormalizable) exponent ζ describing the scaling $\propto 1/N^{1-\zeta}$ for the strength of the weakest link among the $N \gg L$ disorder realizations in a system of fixed mesoscopic size L.

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Disordered systems / Systèmes désordonnés

Universal exponents ?

Off diagonal replica terms ?

[GS] : Lowest order RG ?

$$-D\sum_{ab}\int d\tau d\tau' dx \cos(2\phi_a(x\tau) - 2\phi_b(x\tau'))$$

$$A\sum_{ab}\int dx d\tau \nabla_\tau \phi_a \nabla_\tau \phi_b$$

$$\frac{dA}{dl} = D^2$$

Non universal exponent (cf. Cardy-Ostlund) ?

Need to do next order RG !

Two loops RG for Bose glass

Z. Ristivojevic, A. Petkovic, P. Le Doussal, TG PRL (2012)

$$\frac{\mathrm{d}\mathcal{D}_R}{\mathrm{d}\ell} = -2\mathcal{D}_R\delta_R + A\mathcal{D}_R^2 + \mathcal{O}(\mathcal{D}_R^2\delta_R),$$

$$\frac{\mathrm{d}\delta_R}{\mathrm{d}\ell} = -9\mathcal{D}_R + B\mathcal{D}_R\delta_R + \mathcal{O}(\mathcal{D}_R^2),$$

$$\frac{\mathrm{d}}{\mathrm{d}\ell}(aD_{fR}) = 0, \quad \frac{\mathrm{d}}{\mathrm{d}\ell}\left(\frac{v_R}{K_R}\right) = 0,$$

$$K = \frac{3}{2} + \delta_R$$

Universal exponent at weak disorder/large int.

Strong disorder limit

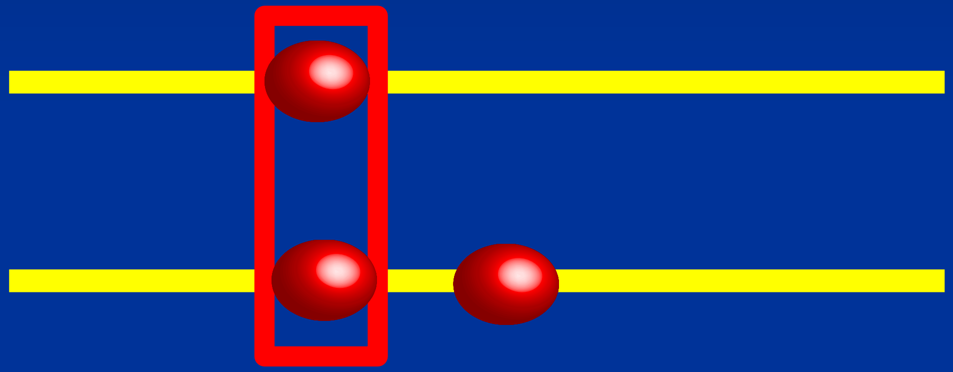
How to "improve" bosonization ?

Density fluctuations limited to << 1

Introduce additional fields
(E. Berg et al., Phys. Rev. B 77, 245119 (08))

Disordered «ladder»

Z. Ristivojevic, A. Petkovic, P. Le Doussal, TG, arxiv/1404.0678



Two fields («up», «down») : larger density fluctuations

Same disorder on the two legs

Non-universal exponents

$$\begin{aligned} H_L =& \frac{\hbar}{2\pi}\int dx \left\{ v_c K_c[\partial_x\theta_c(x)]^2 + \frac{v_c}{K_c}[\partial_x\varphi_c(x)]^2 \right\} \\ &+ \frac{\hbar}{2\pi}\int dx \left\{ v_s K_s[\partial_x\theta_s(x)]^2 + \frac{v_s}{K_s}[\partial_x\varphi_s(x)]^2 \right\} \\ &+ 2V\rho_2^2 \int dx \cos(\sqrt{8}\varphi_s) - \frac{\sqrt{2}}{\pi}\int dx \eta(x)\partial_x\varphi_c \\ &+ 2\rho_2 \int dx \left[\xi^*(x) e^{i\sqrt{2}\varphi_c} \cos(\sqrt{2}\varphi_s) + \text{h.c.} \right] \end{aligned}$$

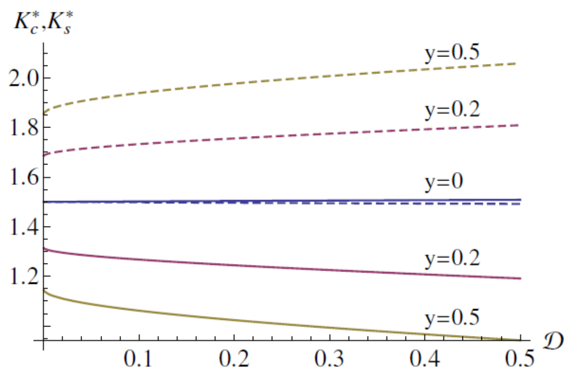
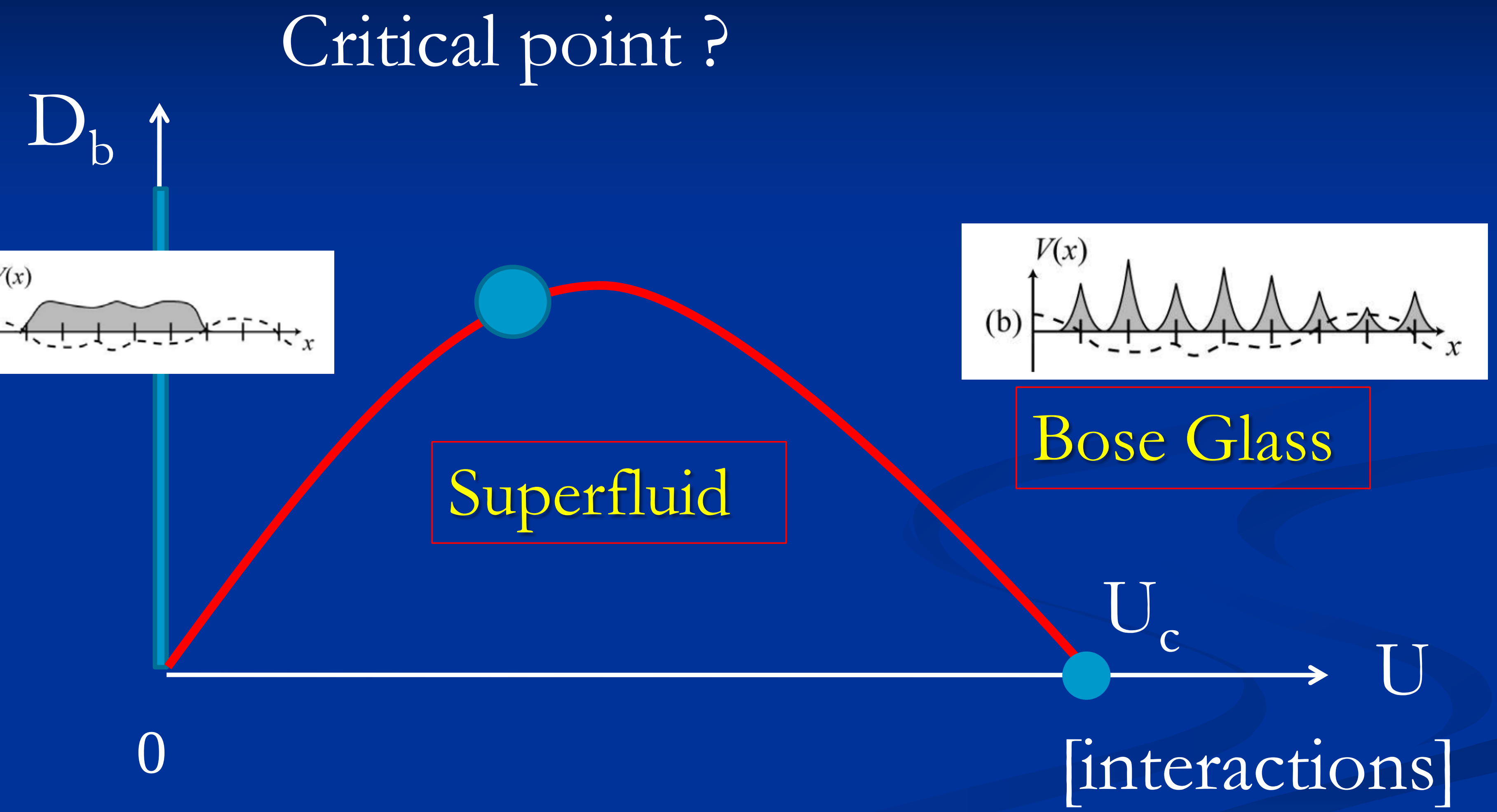


FIG. 2. Solid (dashed) lines represent fixed point value K_c^* (K_s^*) for the parameter K_c (K_s) at the transition as a function of the bare disorder strength $\mathcal{D}$, for different values of the bare coupling y. At the transition we have $K_c^* + K_s^* = 3$, however each of the parameters depends on bare disorder and rung couplings, leading to the nonuniversal exponents in various correlation functions.

Phase diagram



Many other questions: ``glass properties”

Conclusions

- Disorder and interactions: one of the most challenging problem in quantum many-body
- Many new progress both on the theoretical and experimental front
- Theoretical issues still pending on the phase diagram and nature of Bose glass phase(s).
- Many other challenging issues: fermions, glassy properties, aging, etc.
- Cold atoms will be instrumental in helping us understand/make progress on this problem