Measurement of the mobility edge for 3D Anderson localization

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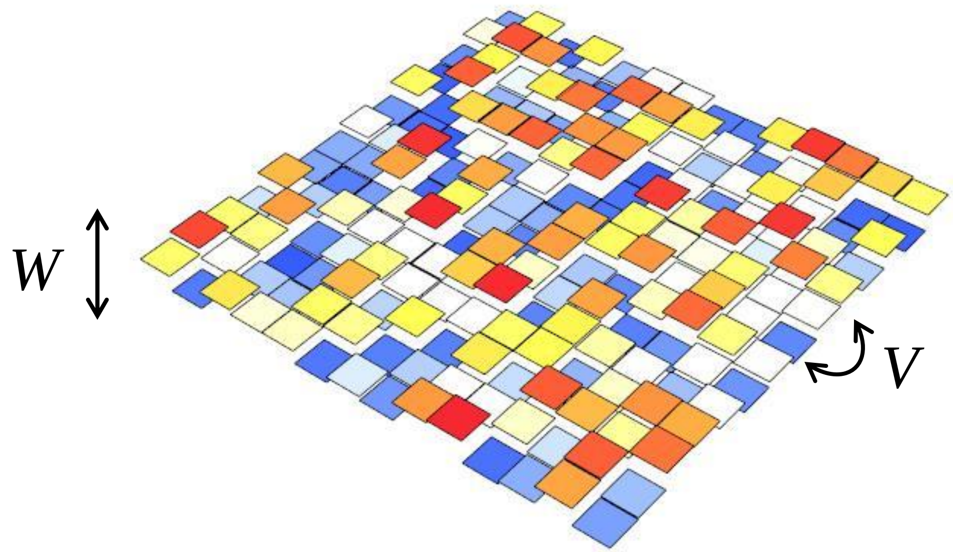
Giacomo Spagnolli

Anderson localization

Individual electrons hopping on a disordered lattice:



$$\hat{H} = -V\sum_{\langle i,j\rangle} \hat{b}_i^{+} \hat{b}_j + \sum_i \varepsilon_i \hat{n}_i \qquad \varepsilon \in [-W/2, W2]$$

Is it possible to decide whether they spread or they are localized?

Anderson, Phys. Rev. 109, 1492 (1958); G. Feher, and E. A. Gere, Phys. Rev. 114, 1245 (1959).

Anderson localization

Individual electrons hopping on a disordered lattice:

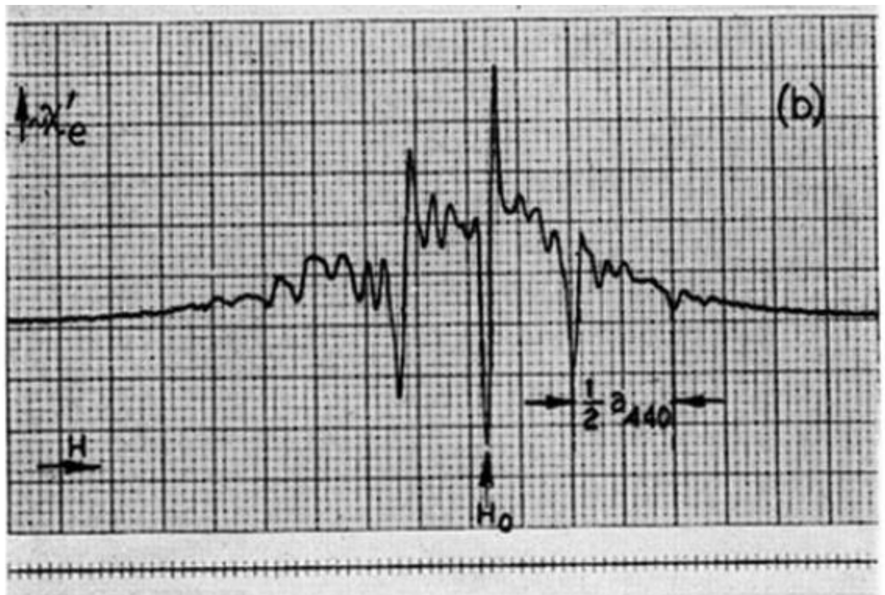


Fig. 11. "Discrete" spin diffusion in *As*-doped silicon ($\sim 2\times10^{16}$ As/cm^3, $T=1.3°$K, $H\simeq3200$ oersteds). The line was saturated by setting $H=H_0$ and then turning the microwave power on.

Is it possible to decide whether they spread or they are localized?

Anderson, Phys. Rev. 109, 1492 (1958); G. Feher, and E. A. Gere, Phys. Rev. 114, 1245 (1959).

Anderson localization

$$\frac{2K\ln(W/2V)_c}{1-(4V^2/W^2)_c}=(W/2V)_c.$$

An elaborate perturbation theory: the occupation of the initial site stays finite for strong disorder:

Exponential localization.

A 3D result; things may change in lower dimensions.

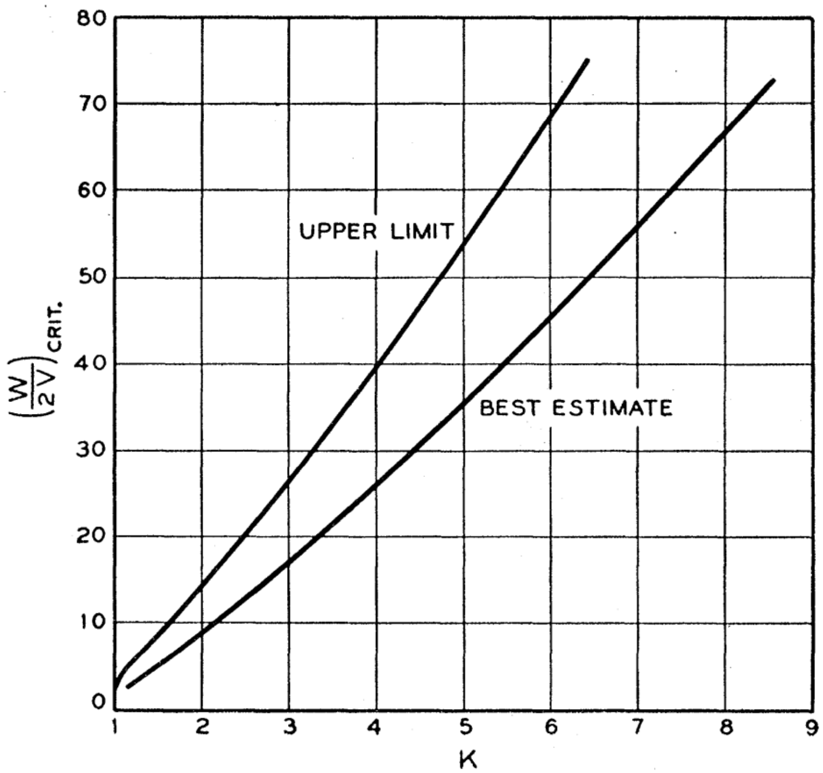
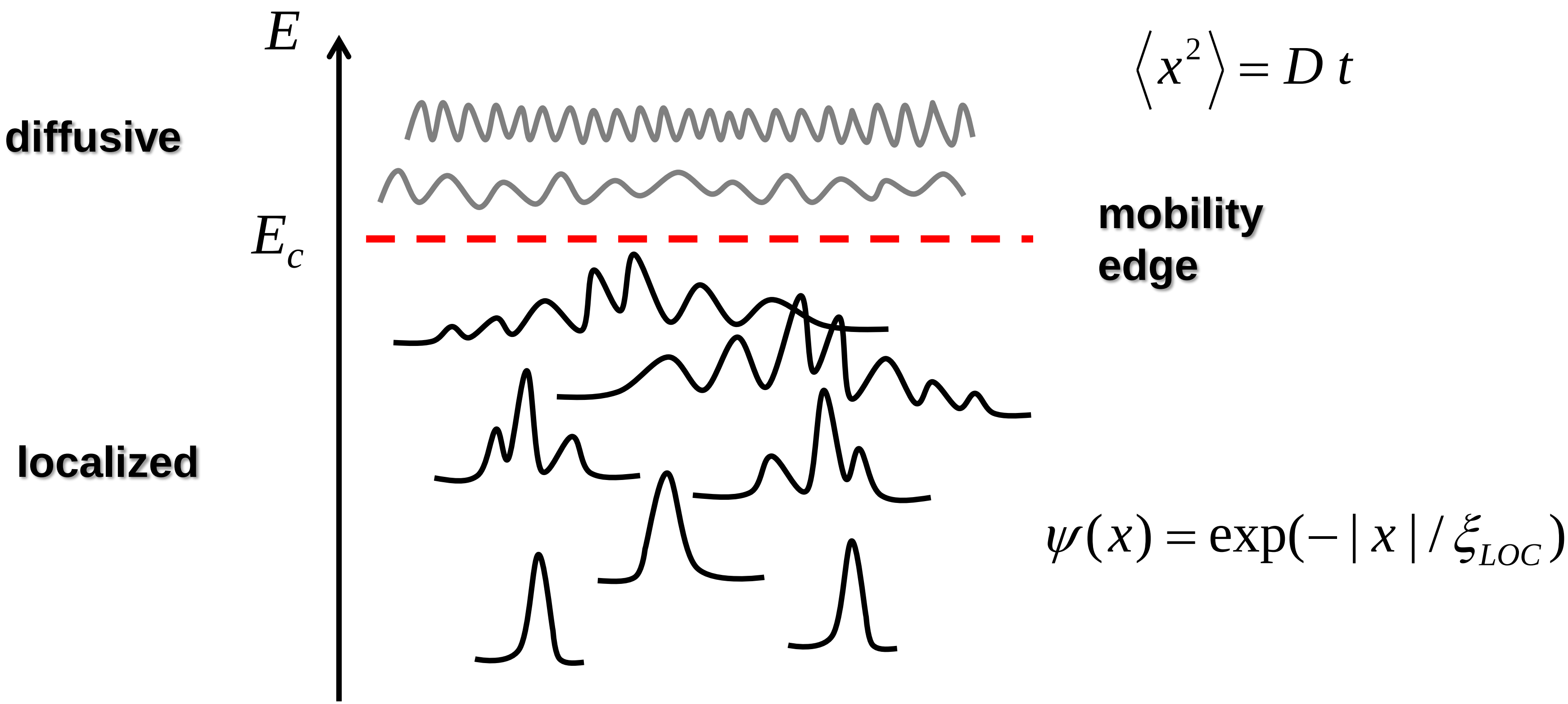


FIG. 3. Numerical estimates for the critical $W/2V$, the ratio of line width to interaction, for transport, plotted against connectivity K. The upper curve is a quasi-exact upper limit; the lower one is our best estimate.

Anderson, Phys. Rev. 109, 1492 (1958)

3D Anderson localization



Only a single mobility edge is possible, since any isolated extended state would be unstable

Anderson localization

E_c depends on the disorder distribution and the particles dispersion.

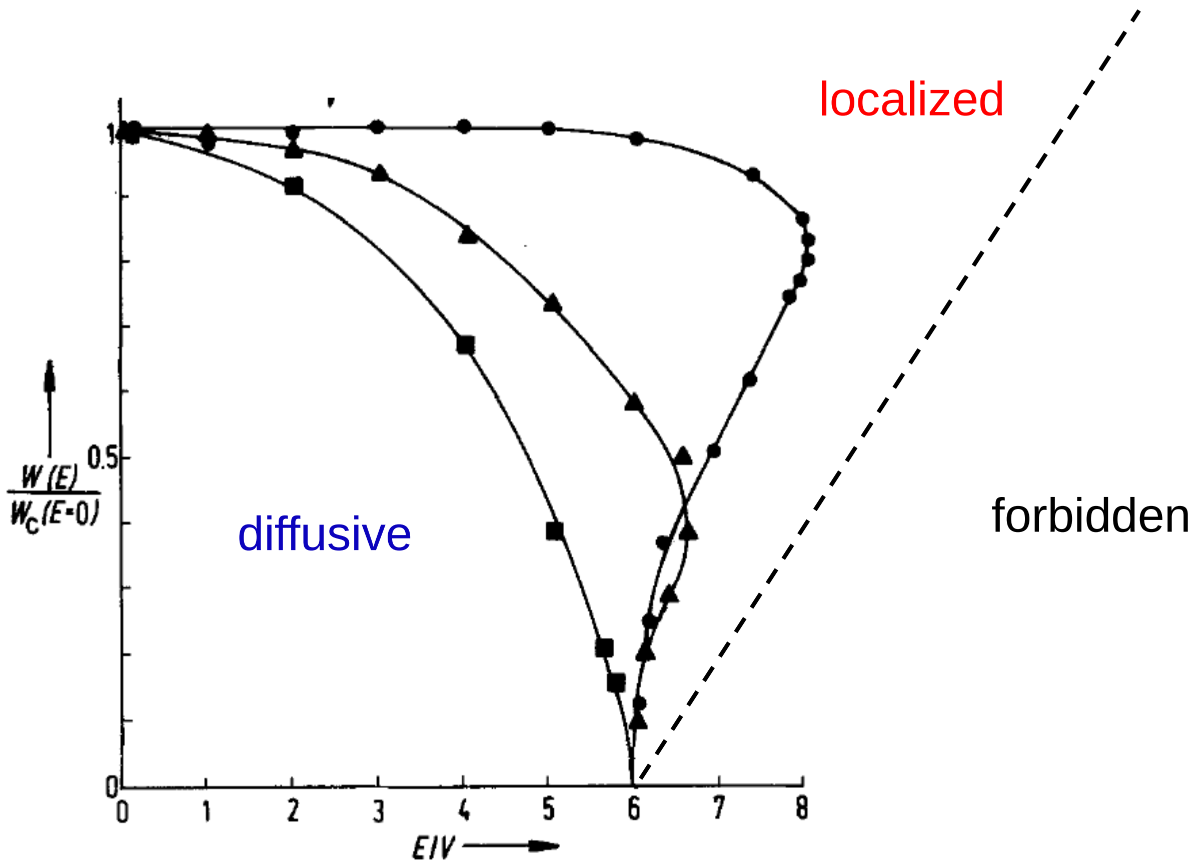


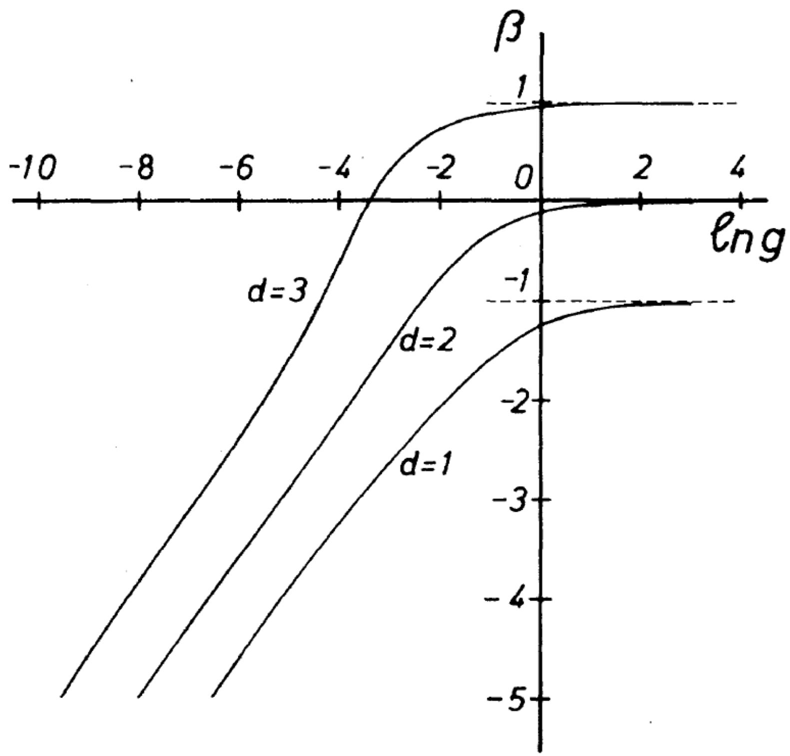
Fig. 1. Mobility edge trajectories $W_c(E)$ for the box (● $W_c(0)/V = 16.3 \pm 0.5$), Gaussian (▲ $W_c(0)/V = 20.9 \pm 0.5$), and the Lorentzian (■ $W_c(0)/V = 3.8 \pm 0.5$) distribution

Bulka, Schreiber, Kramer, Z. Phys. B 66, 21 (1987).

Scaling theory of localization

in a metal, the conductance of a cube of length L is $g(L)=\sigma L^{d-2}$

one-parameter scaling law $g=g(L/\xi) \Rightarrow \partial \ln(g)/\partial \ln(L)= \beta(g)$

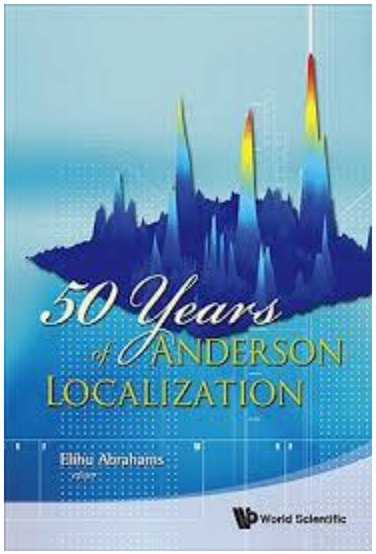


weak disorder
$g(L) \sim L^{d-2}$

strong disorder
$g(L) \sim \exp(-L/\xi)$

Abrahams, Anderson, Licciardello, Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).

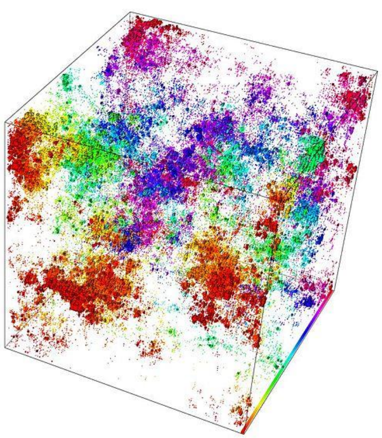
50 years of theory



- The metal-insulator transition is a quantum phase transition (localization length, diffusion coefficient).

- Powerful analytical methods (self-consistent theory), but require approximations, which depend on the disorder type.

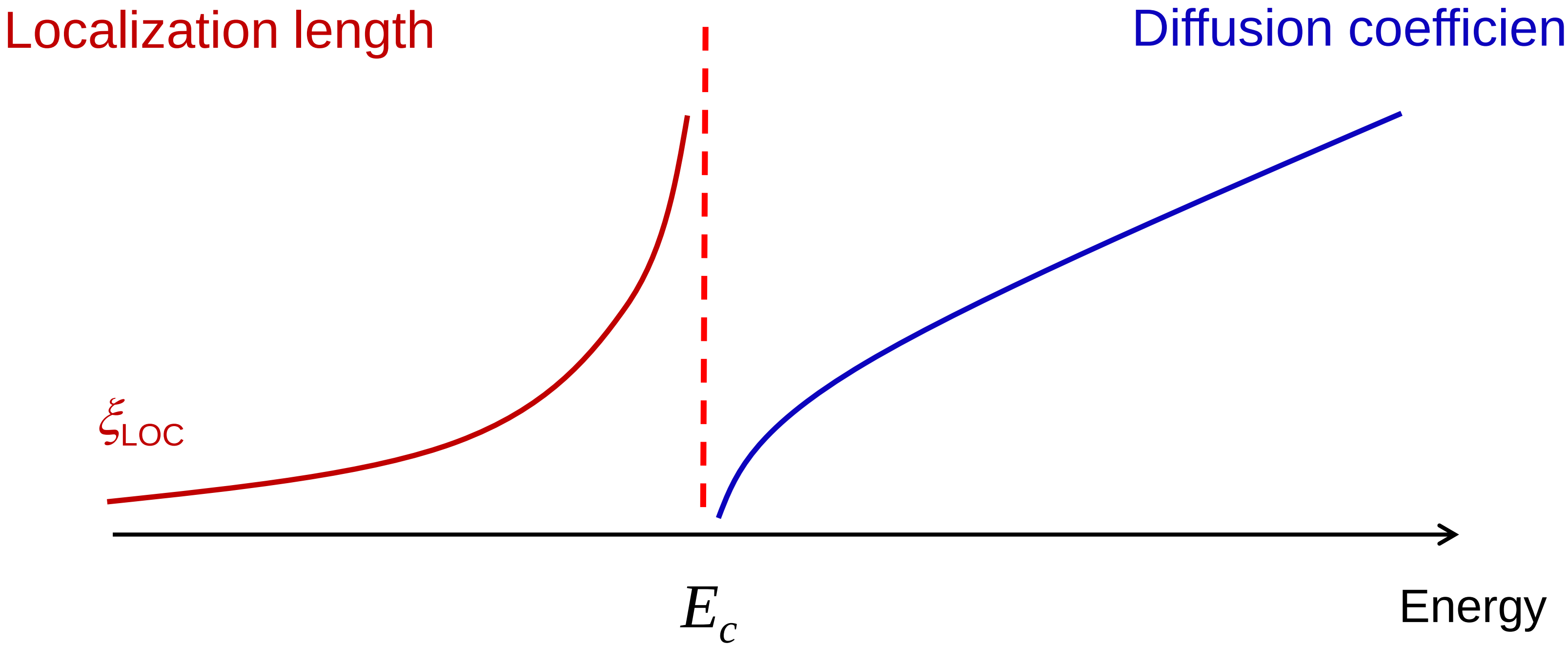
- Numerical calculations are possible but hard.



- **Interactions change completely the problem**:

it is not possible to study Anderson localization with electrons

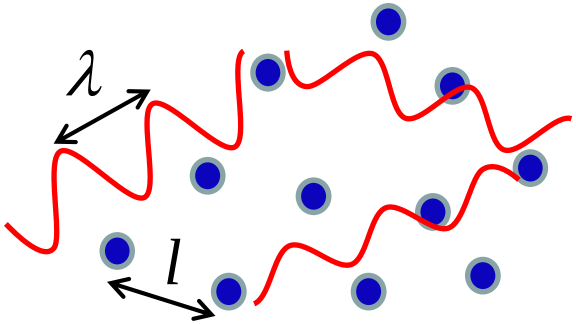
Simple picture of the mobility edge



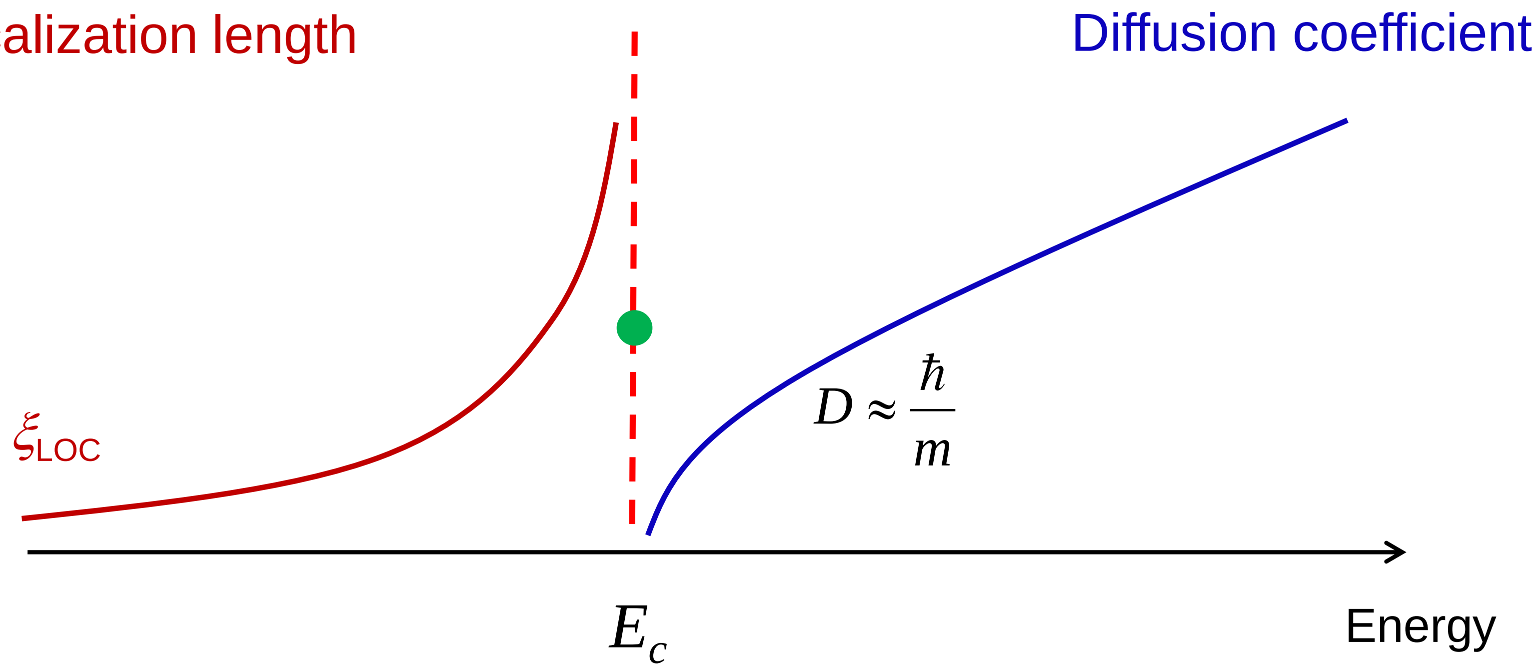
Critical behavior: $\xi_{LOC} \propto |E - E_c|^{-\nu}$ $D \propto |E - E_c|^{\nu}$ $\nu \approx 1.46$

Critical energy: $E_c \approx V_R$

Ioffe-Regel criterion: $kl \approx 1$



Simple picture of the mobility edge

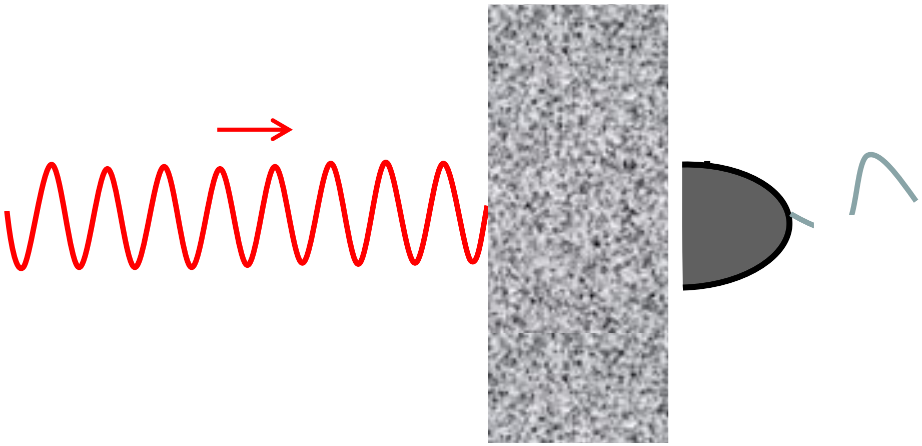


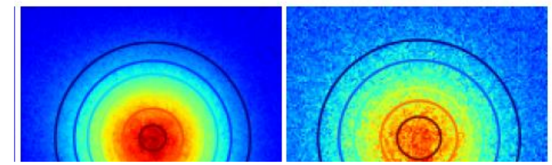
Diffusion coefficient: $D \approx \frac{l^2}{\tau} = \frac{\hbar}{m}$ if $\tau = l / v$ and $kl \approx 1$

Anomalous diffusion at criticality: $\langle x^2 \rangle = \tilde{D}\, t^{2/3}$

Experiments: waves

Light or sound waves with well defined momentum

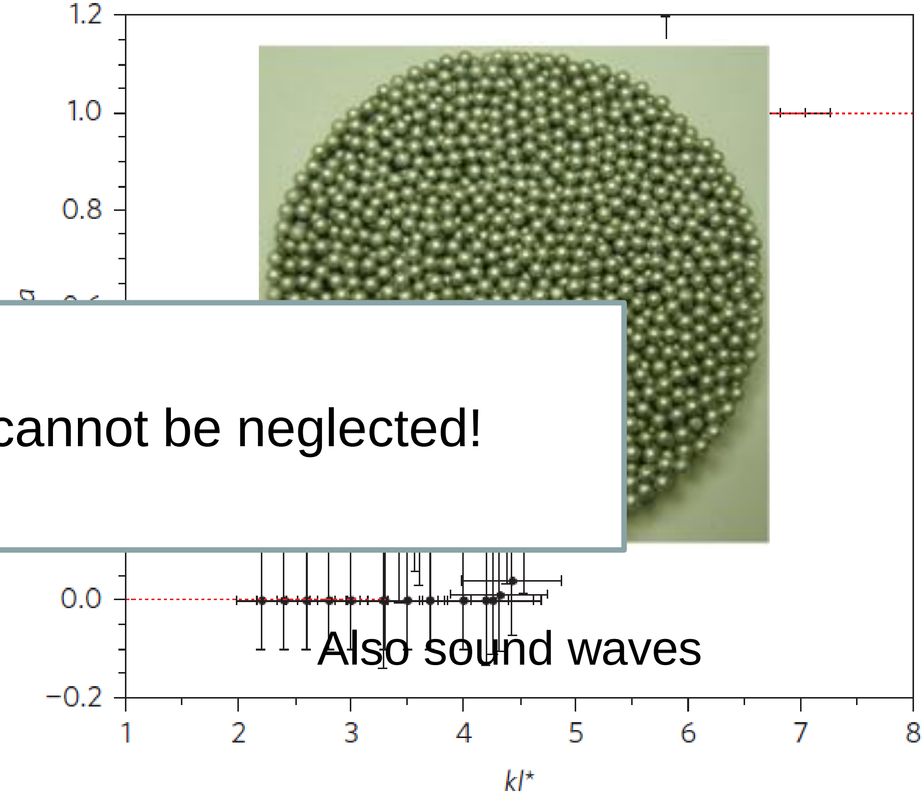




A test of the

Absorption effects cannot be neglected!

$$k\ell \approx 1$$

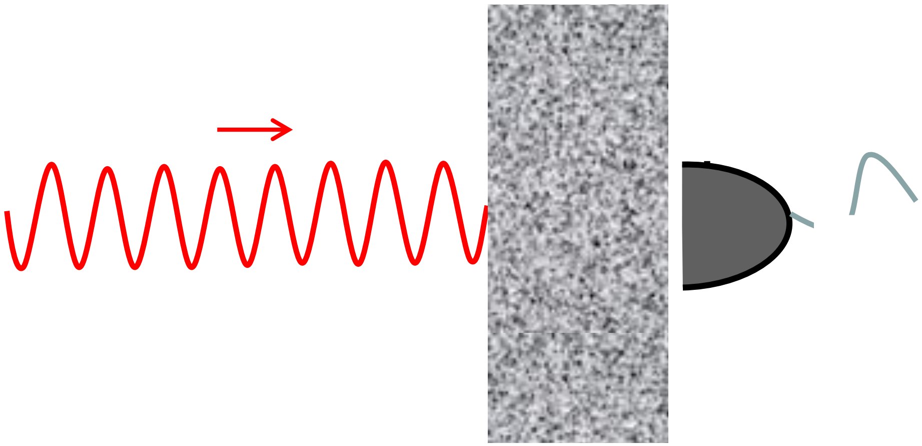


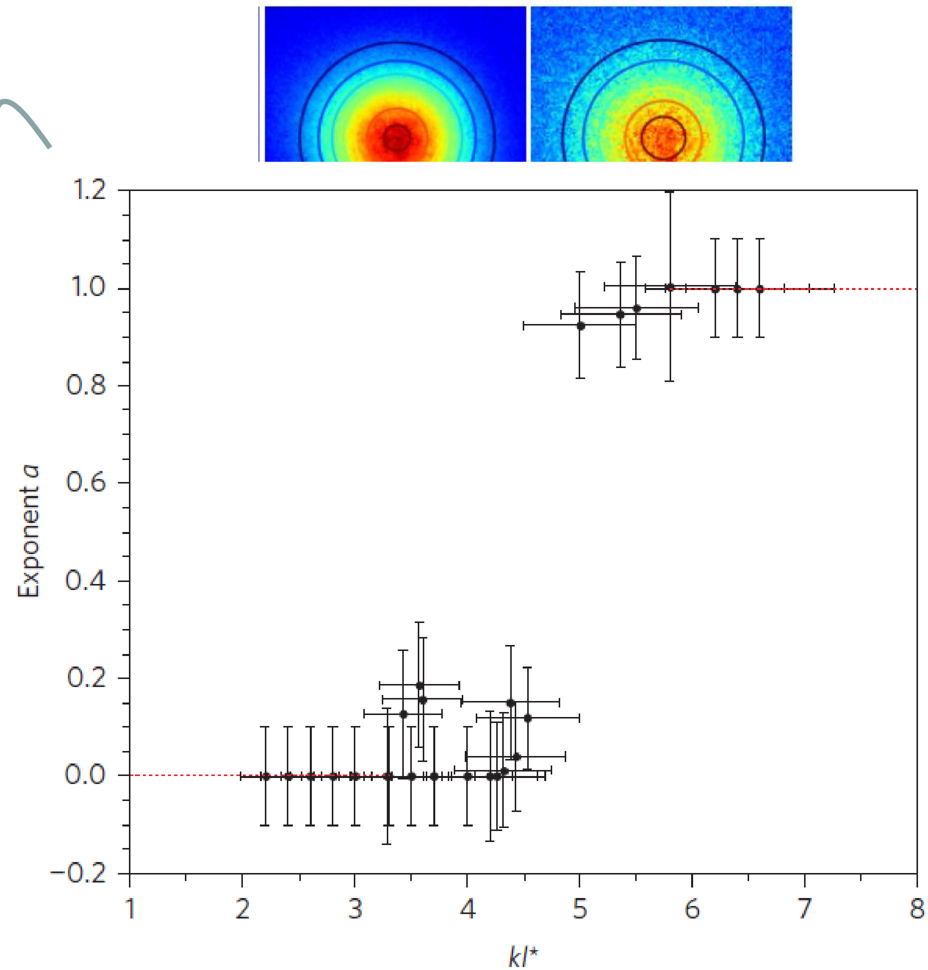
Also sound waves

Sperling, et al. Nat. Photonics (2012); Hu et al, Nat. Physics 4, 945 (2008);
Wiersma et al (3D), Segev et al (2D); Lahini, Silberberg et al (1D)...

Experiments: waves

Light or sound waves with well defined momentum





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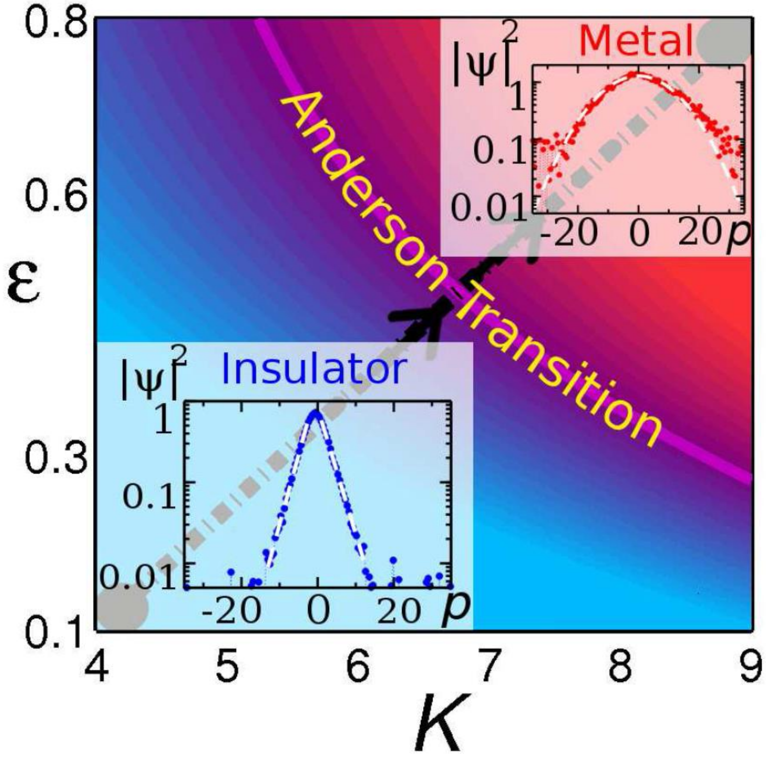
Experiments: waves

The kicked rotor: a momentum space version of the 3D Anderson model

$$H = p^2/2 + K\cos(x)\,[1+\varepsilon \cos(\omega_2 t)\cos(\omega_3 t)]\,\Sigma_n \delta(t-n)$$

free atoms pulsed lattice



Very good for measuring critical properties,

but condensed-matter studies are hard.

Chabé et al. Phys. Rev. Lett. 101, 255702 (2008). Theory by Fishman, Delande, ...

Experiments: ultracold atoms in 1D

Bose-Einstein Condensate in a Random Potential

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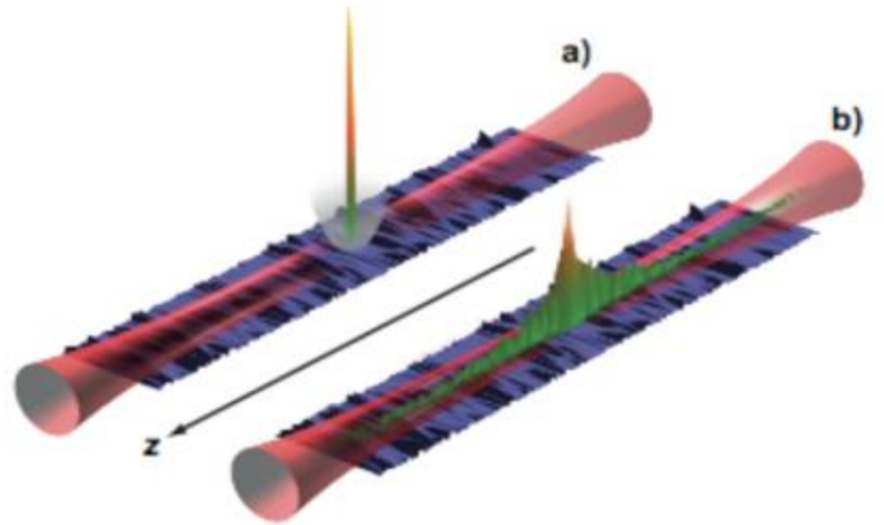
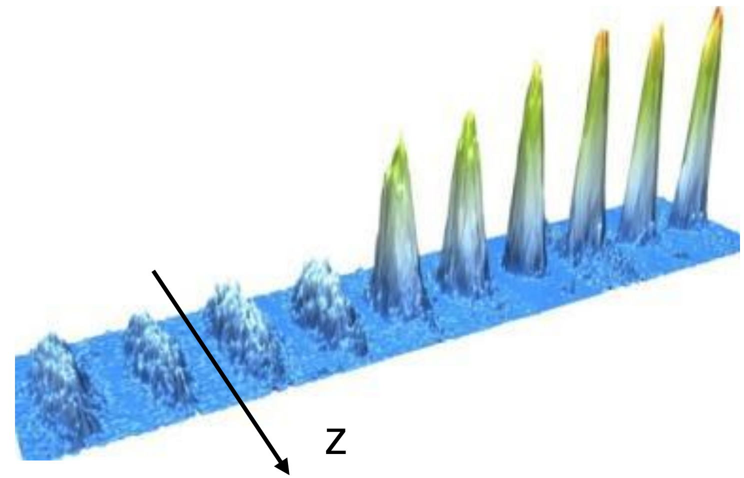
[3]*INFM-MATIS, Catania, Italy*
(Received 7 December 2004; published 9 August 2005)





Laser speckles

Quasiperiodic lattices

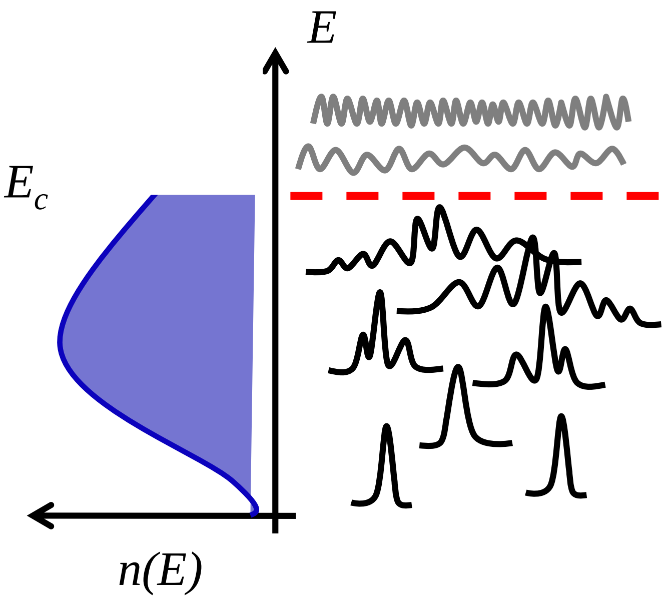
J. Billy et al., Nature 453, 891 (2008)

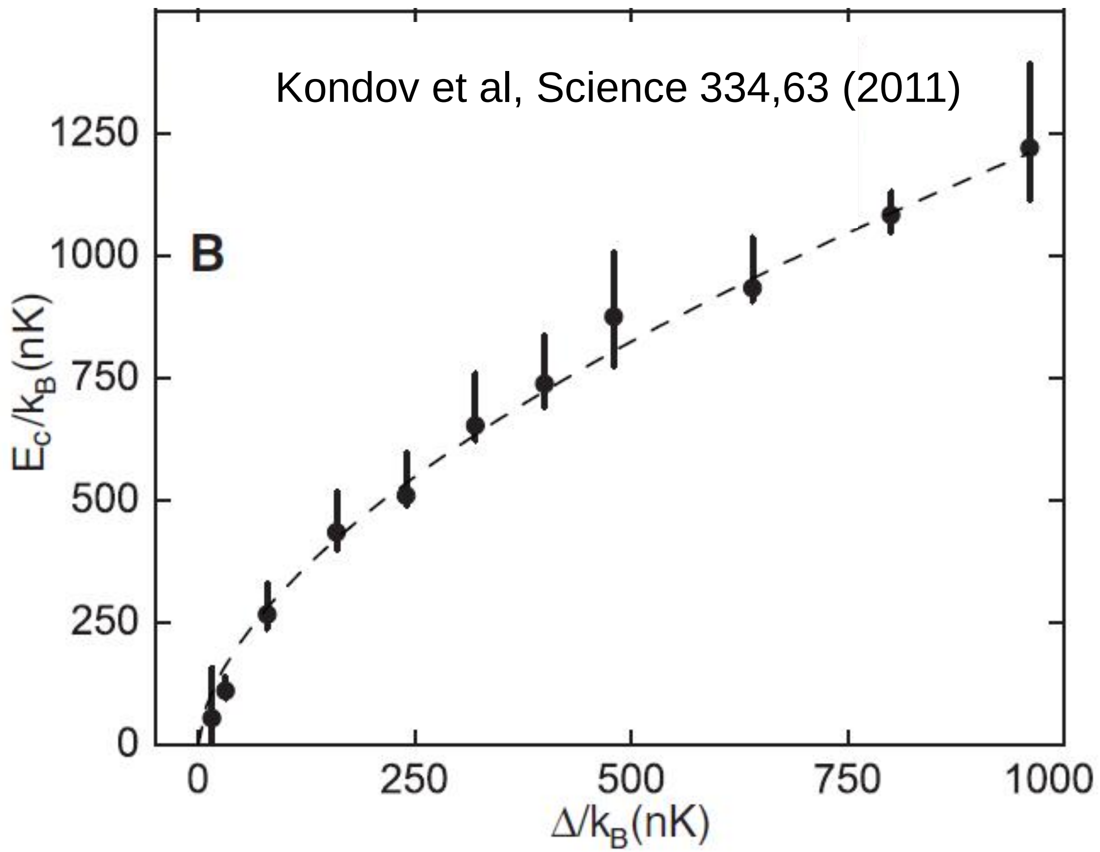
G. Roati et al., Nature 453, 895 (2008)

Reviews: A. Aspect and M. Inguscio, Physics Today 62, 30 (2009); L. Sanchez-Palencia and M. Lewenstein, Nat. Phys. 6, 87 (2010); G. Modugno, Rep. Progr. Phys. 73, 102401 (2010); B. Shapiro, arXiv:1112.5736.

Ultracold atoms: Urbana-Champaign

Non-interacting fermions loaded «adiabatically» into speckles



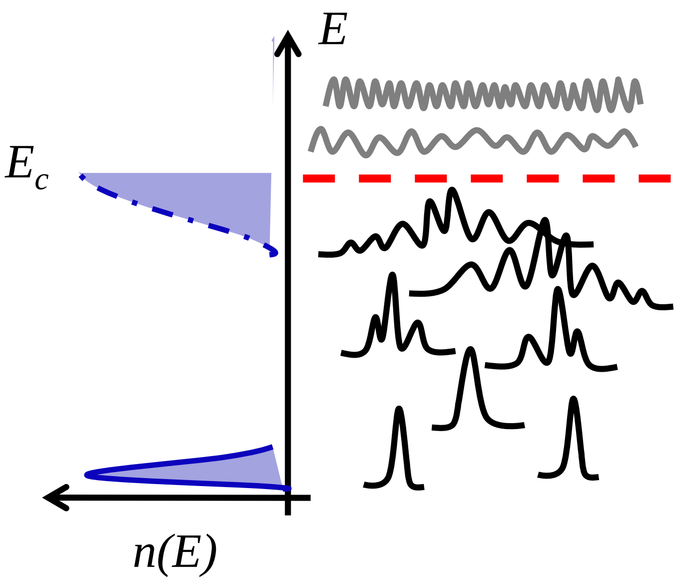


Very short timescale: 100 microns in 100 ms $\Rightarrow$ $D \approx 50 \frac{\hbar}{m}$

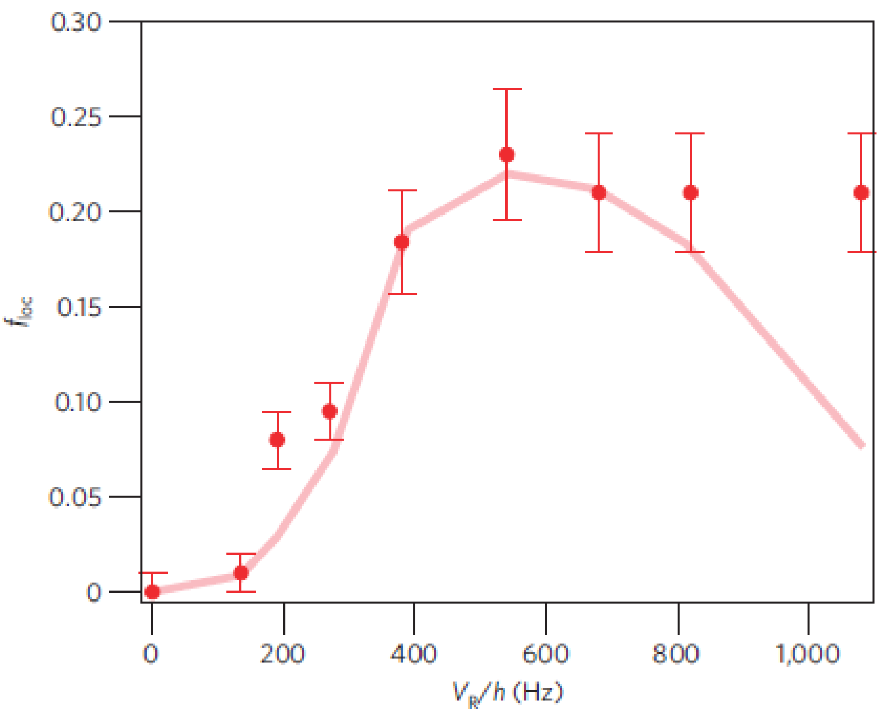
No effects of the disorder on the momentum distribution?
Too large mobility edge ($E_c \sim 2V_R$)?

Ultracold atoms: Palaiseau

Speckles are abruptly turned on an expanding Bose-Einstein condensate



F. Jendrzejewski et al, Nat. Physics 8, 398 (2012)

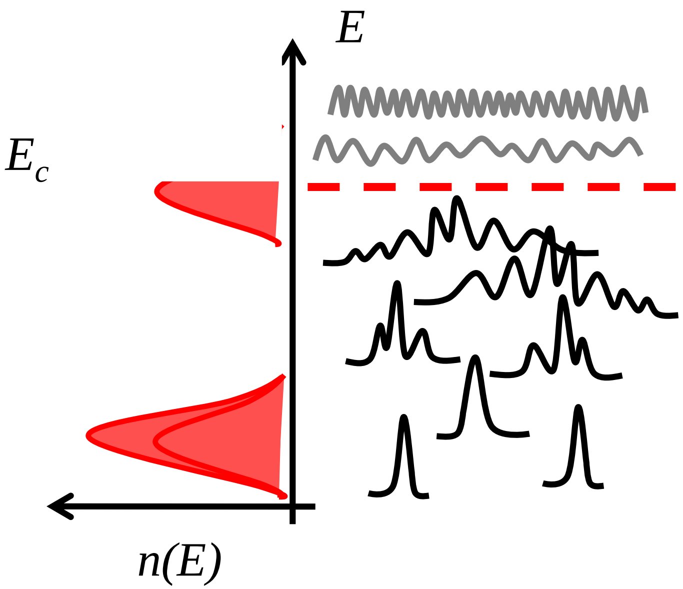


Timescale = 100 microns in 1-10 seconds $\Rightarrow$ $D \leq \frac{\hbar}{m}$

The observations are consistent with calculations based on the self-consistent theory.

Ultracold atoms: Florence

Using a BEC with controllable interaction (^{39}K atoms), it is possible to:



1) Prepare a narrow energy distribution

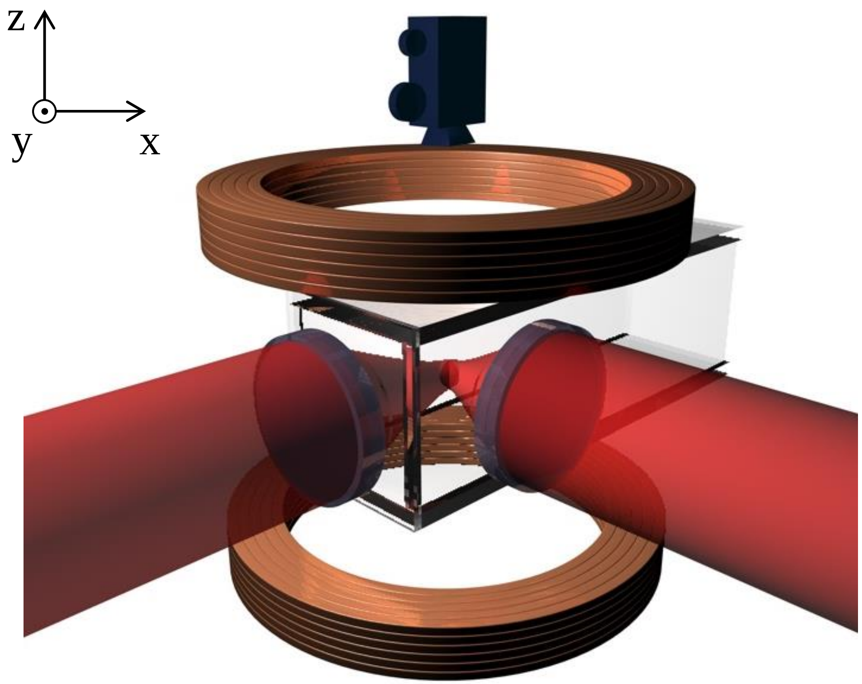
2) Measure it

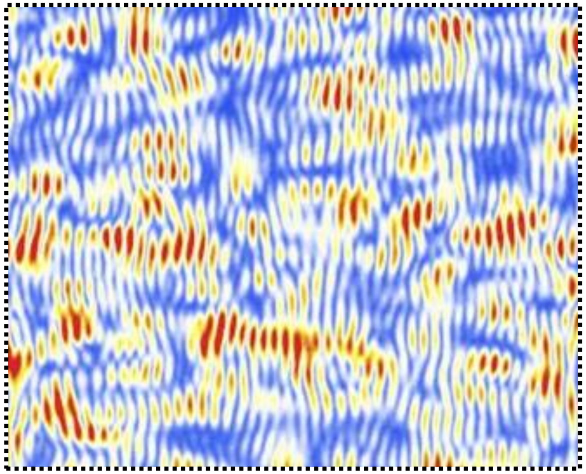
3) Excite a part of it in a controlled way towards E_c

4) Deduce E_c from a modelling of the losses

Semeghini, Landini et al., arXiv:1404.3528

3D speckles disorder



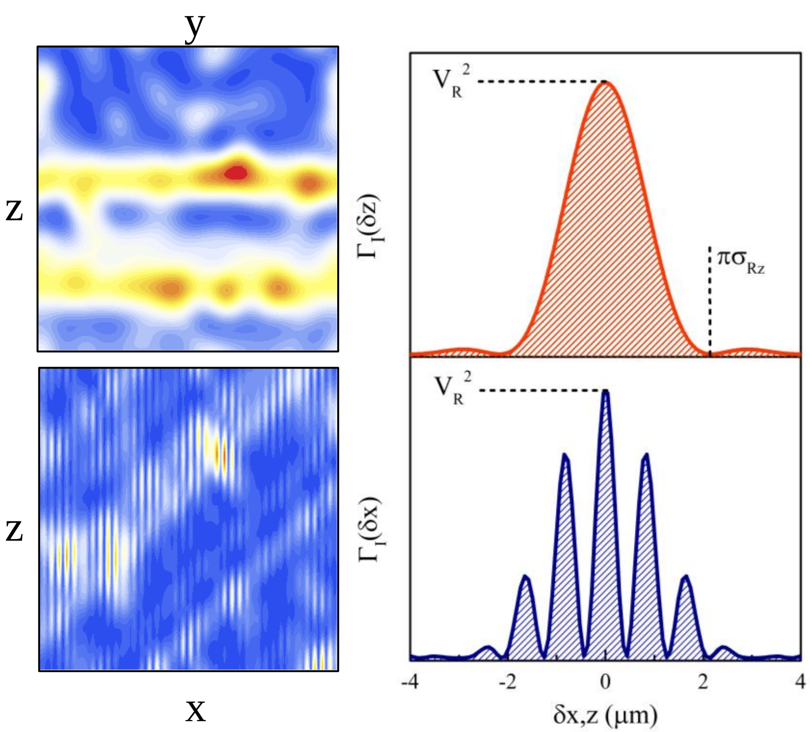


Two crossing coherent speckle fields: the most isotropic disorder so far.

Repulsive potential, to avoid deep minima.

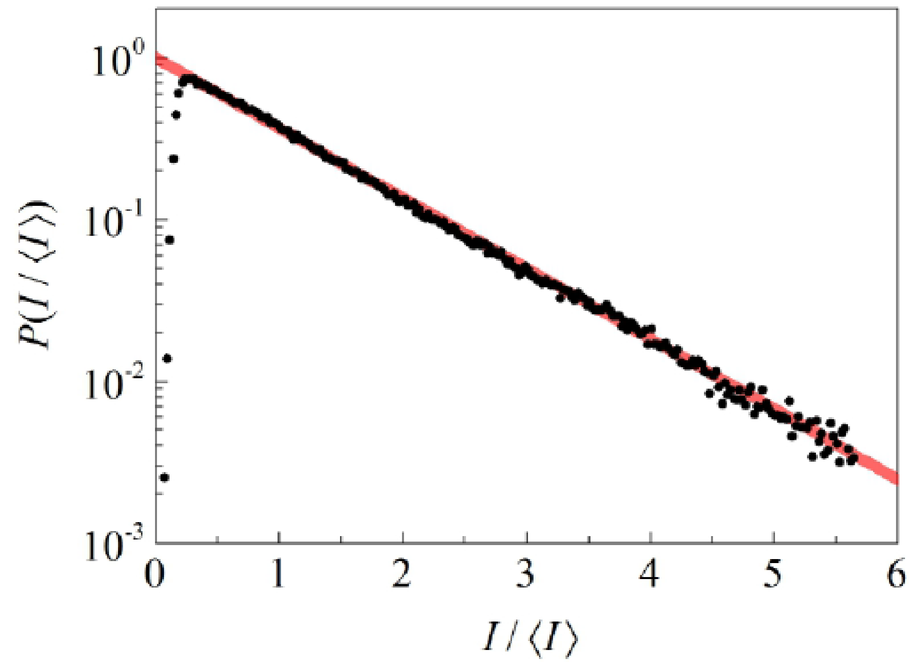
3D speckles disorder

Correlations

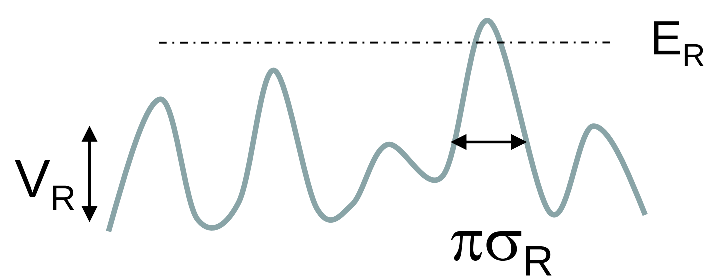


$g(x) = \langle V(r)V(r+x)\rangle$

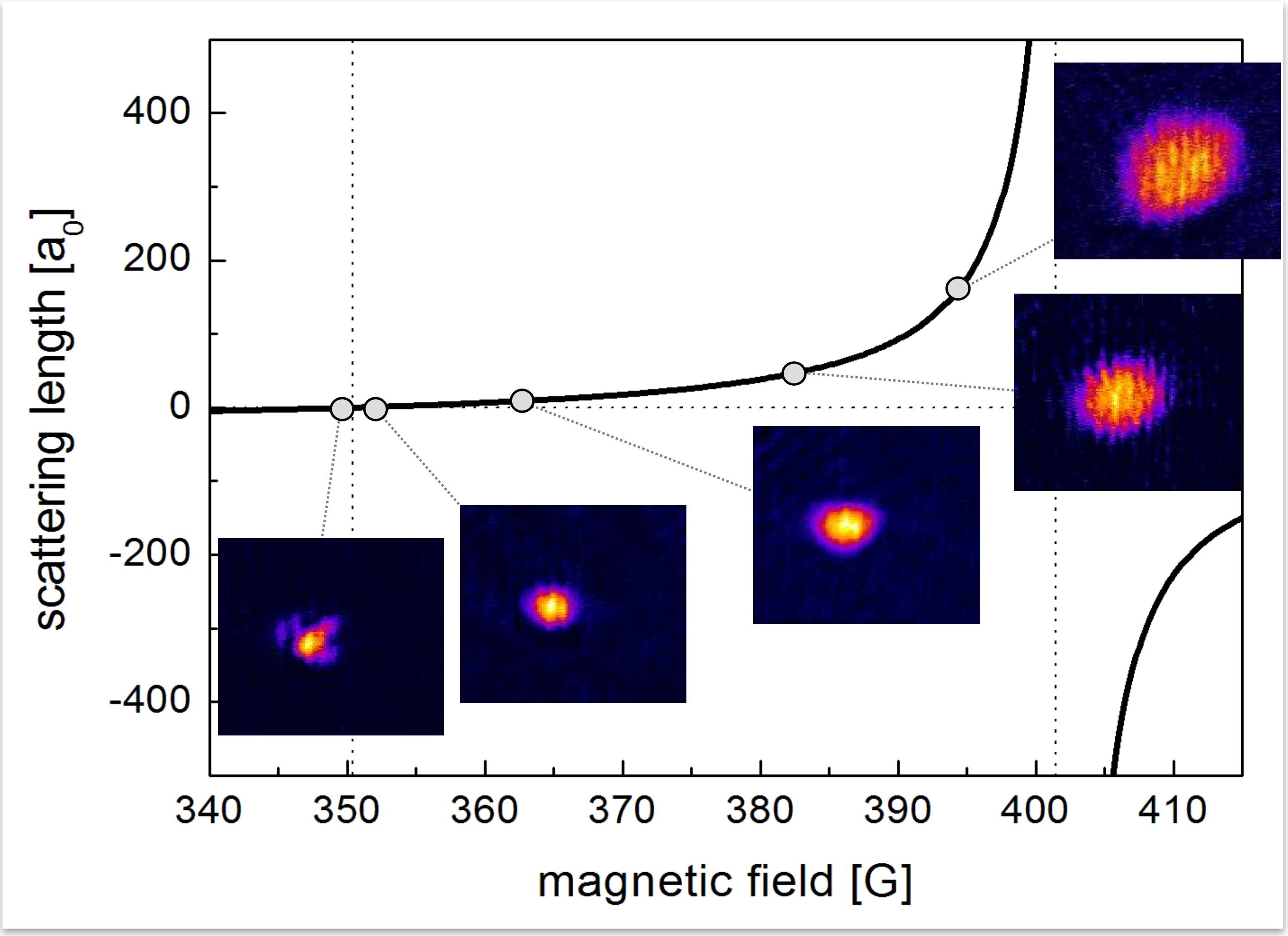
Distribution



correlation energy: $E_R \leq \hbar^2/m\sigma_R^2 \approx 70\text{nK}$

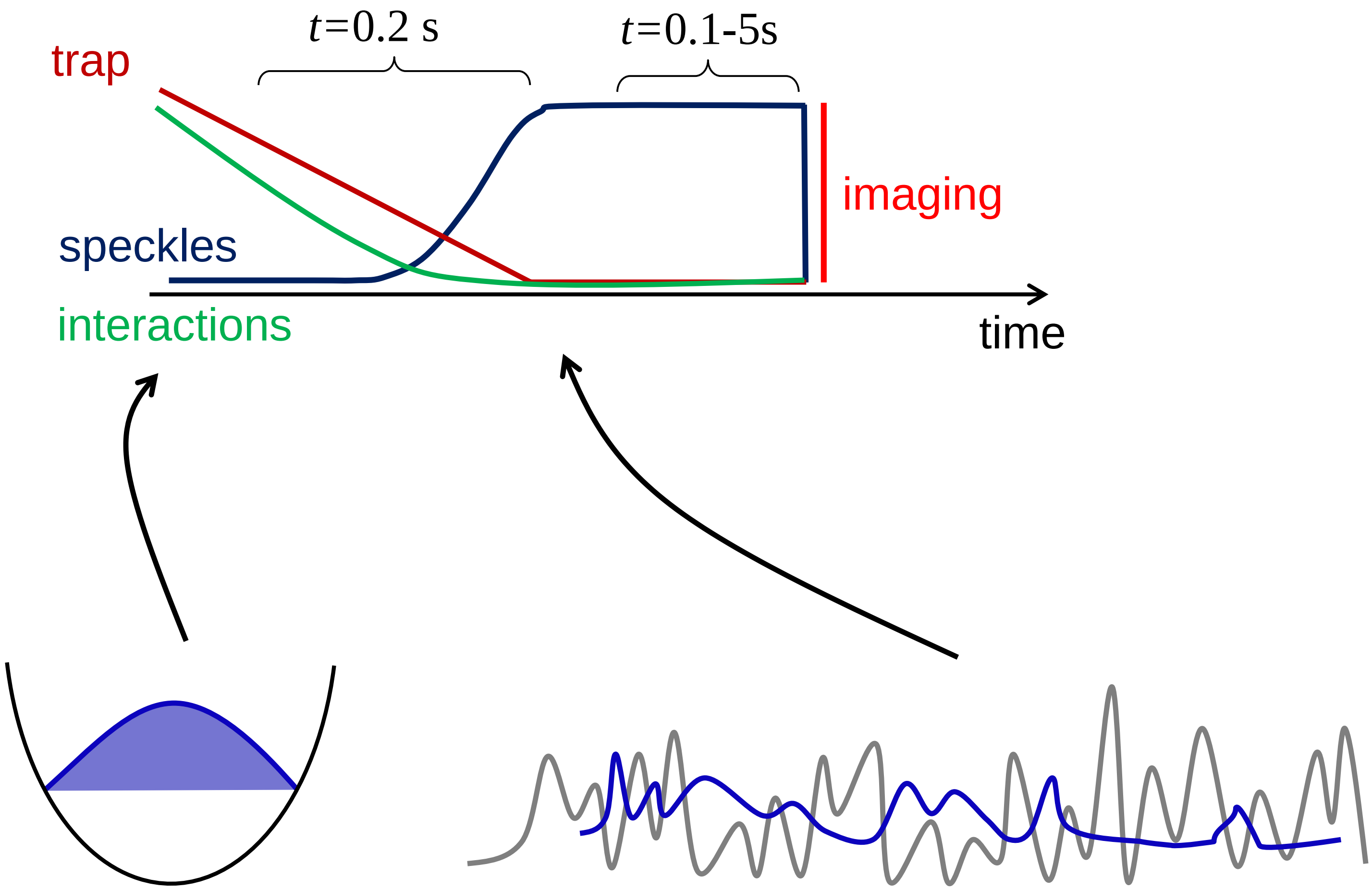


Tunable contact interactions via Feshbach resonances



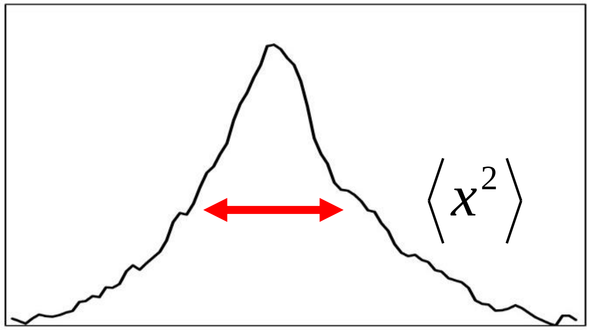
Roati et al. Phys. Rev. Lett. 99, 010403 (2007).

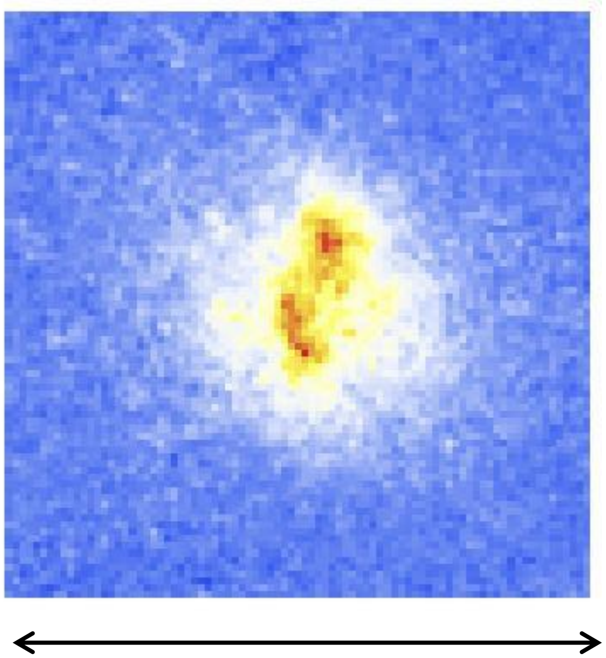
Quasi-adiabatic preparation



Optimized by minimizing the kinetic energy

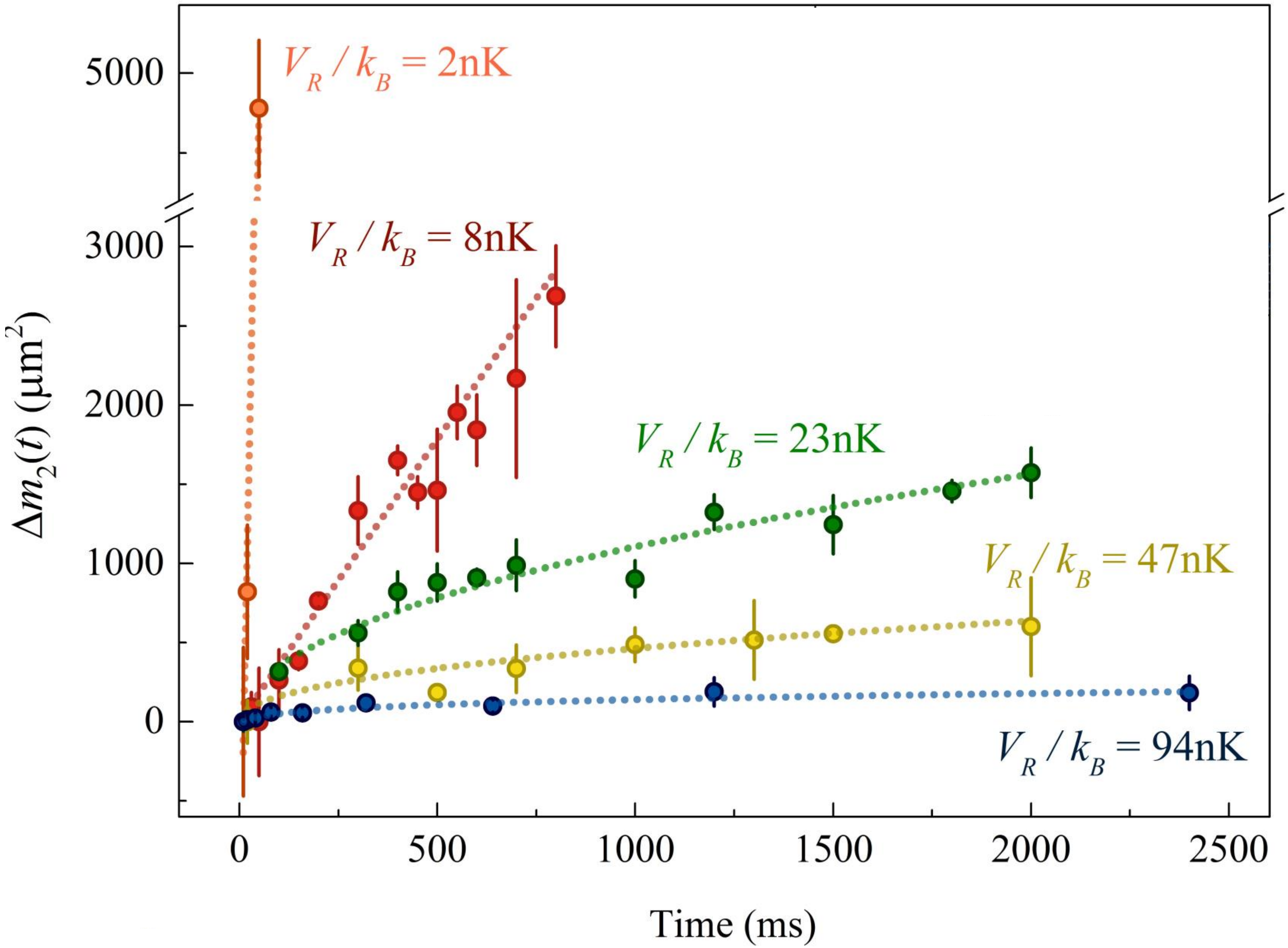
Dynamics in the disorder



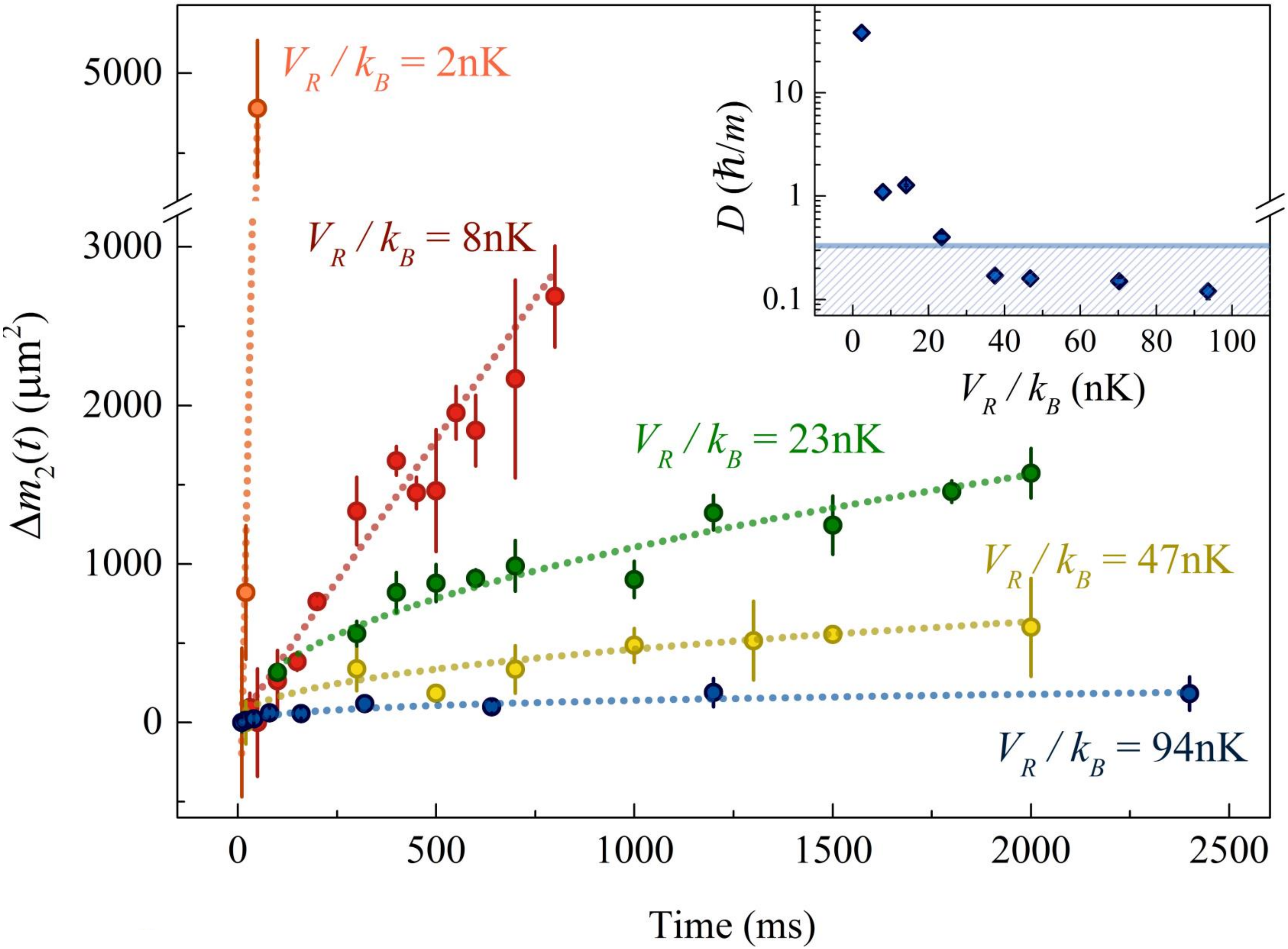


~ 300 μm

From diffusion to localization



From diffusion to localization



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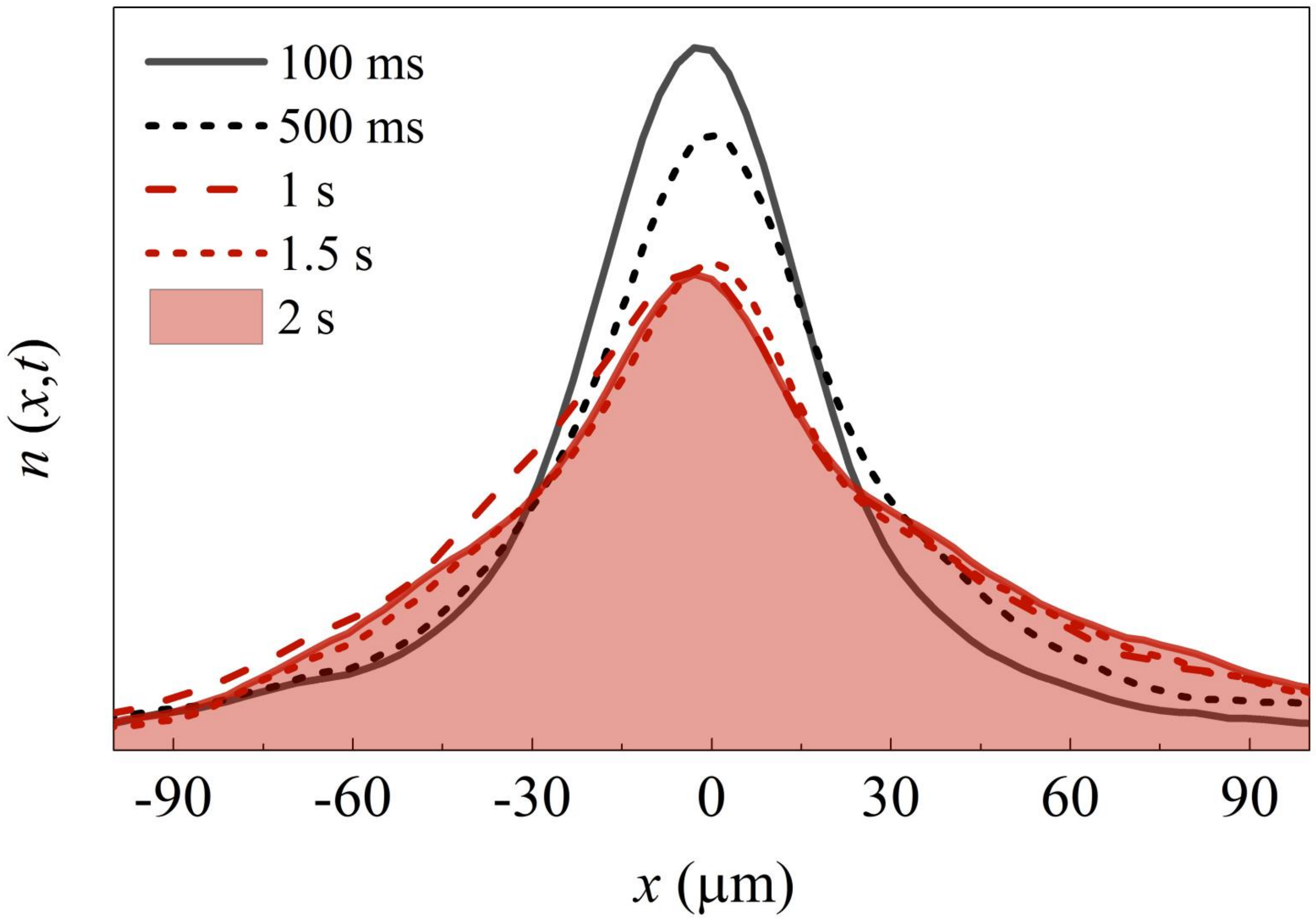
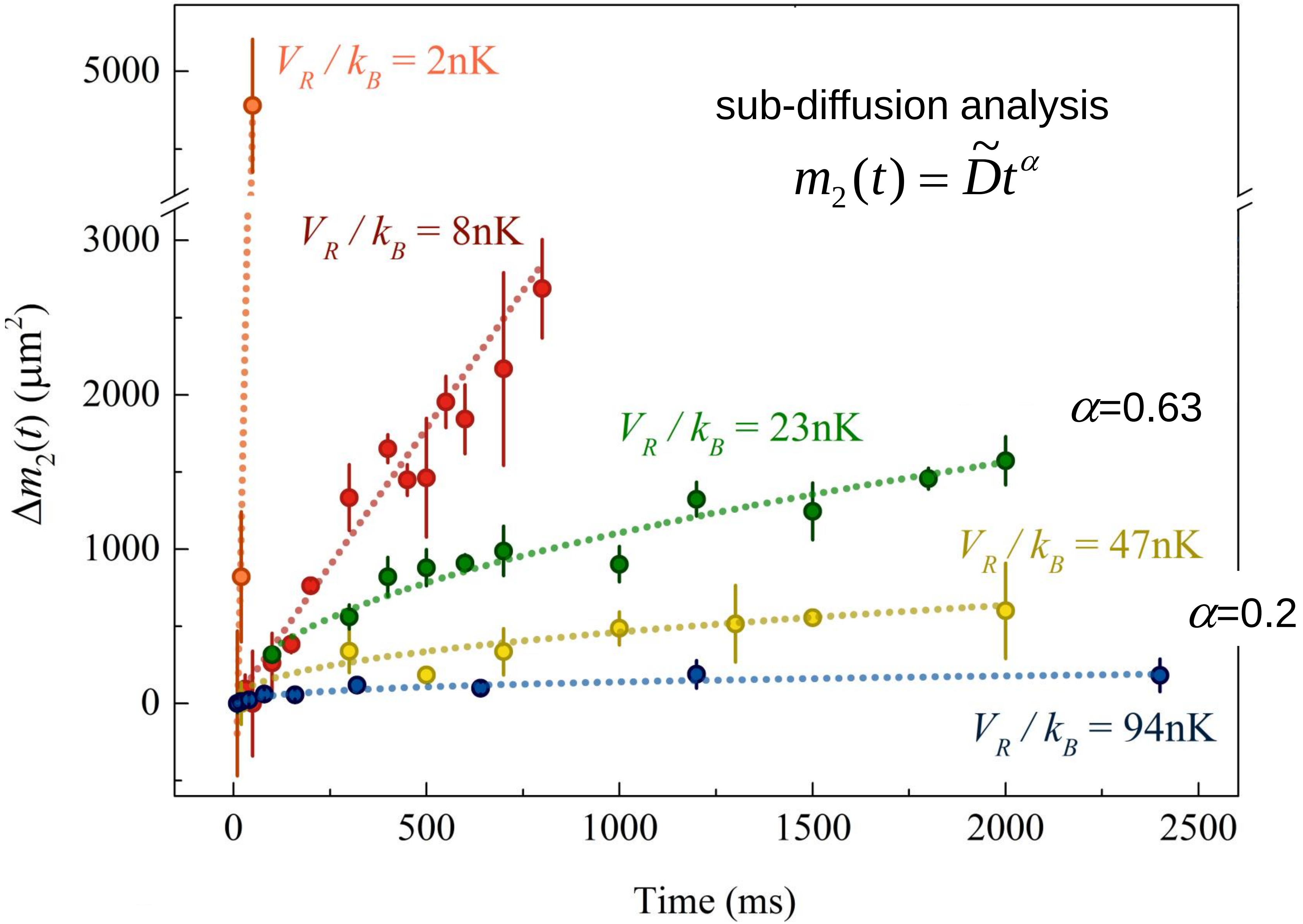
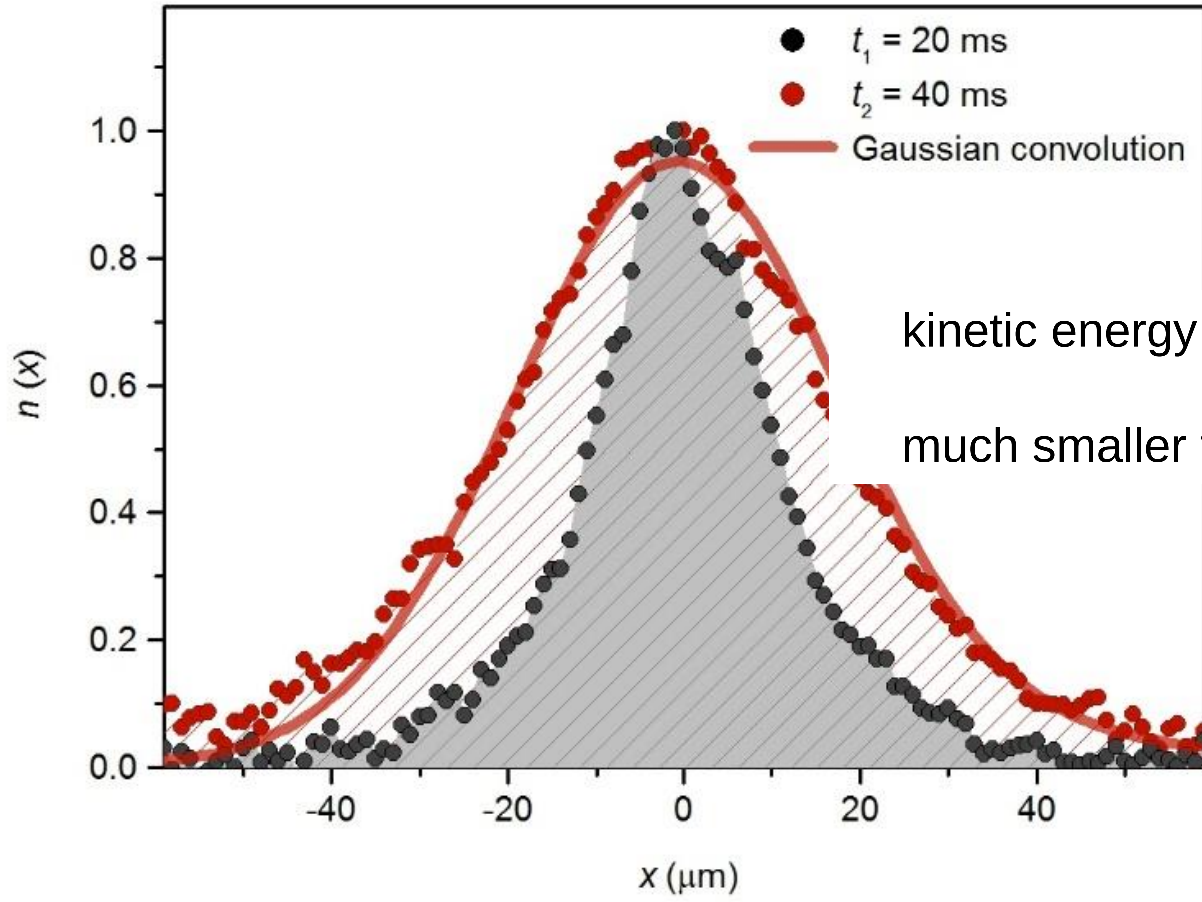


From diffusion to localization



Momentum distribution



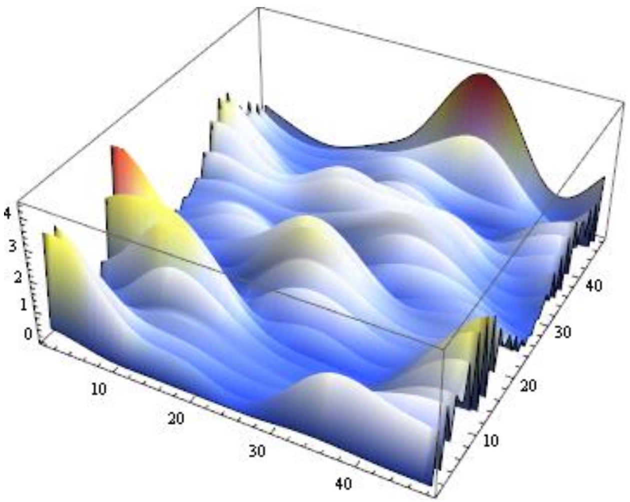
kinetic energy ~10 nK

much smaller than V_R=47 nK

$$n(k) = \int \rho(E,k) f(E) dE \qquad n(E) = \int \rho(E,k) f(E) dk$$

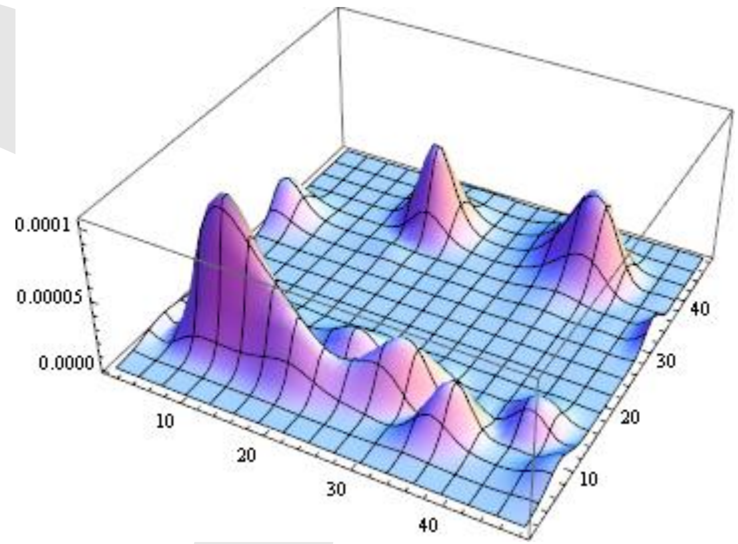
Spectral function: probability of having a momentum k at an energy E

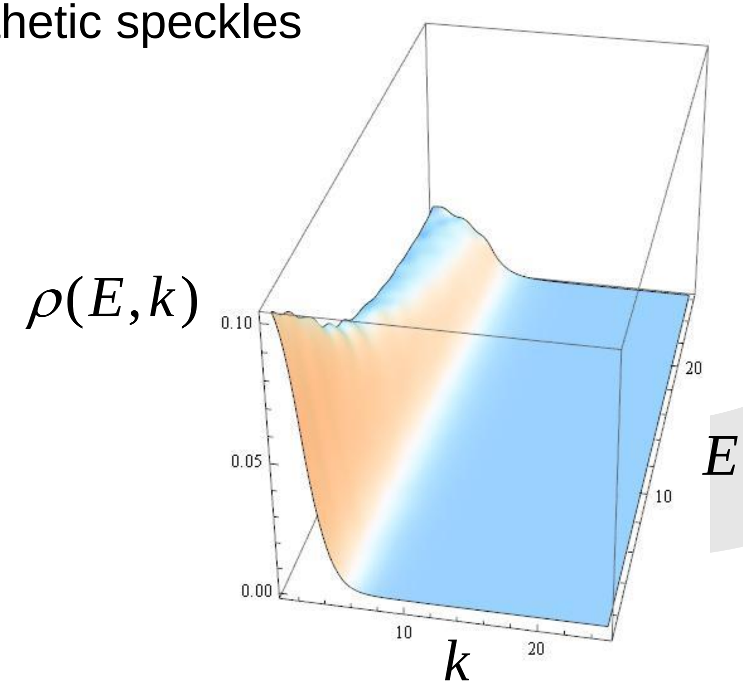
Numerical diagonalization of small systems



Synthetic speckles

Exact diagonalization of $H = \frac{p^2}{2m} + V_R(\mathbf{r})$

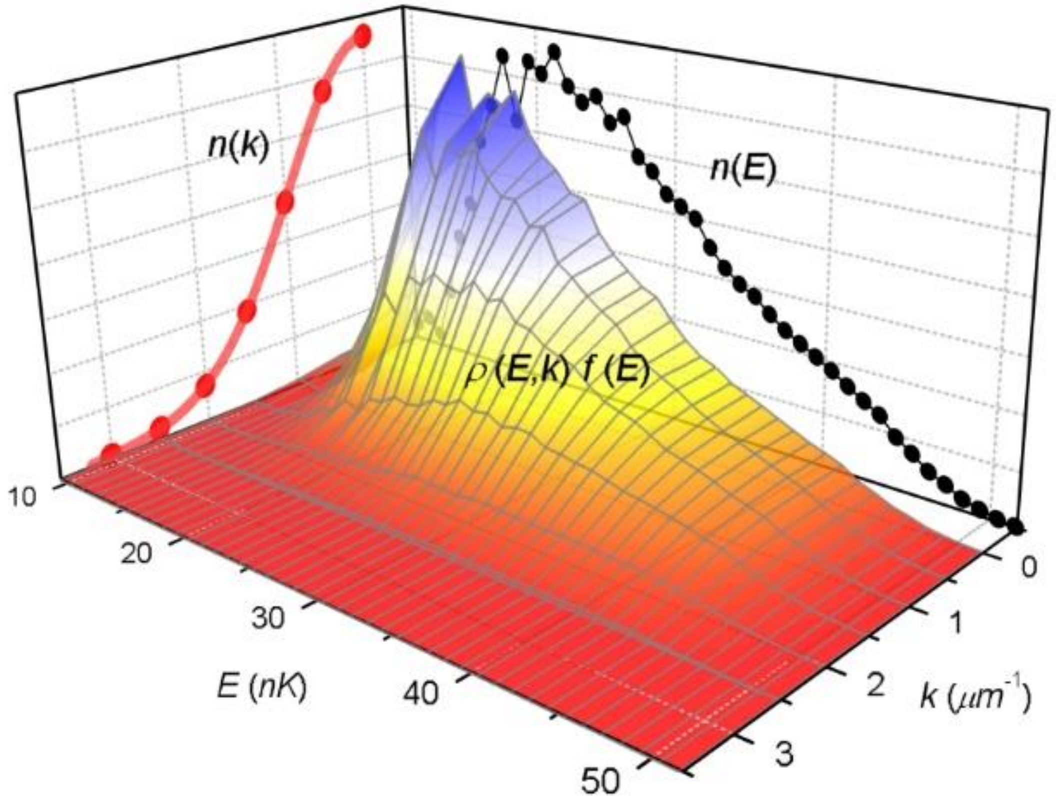




Fourier transform: $\rho(E_i, k)$

Energy and disorder average

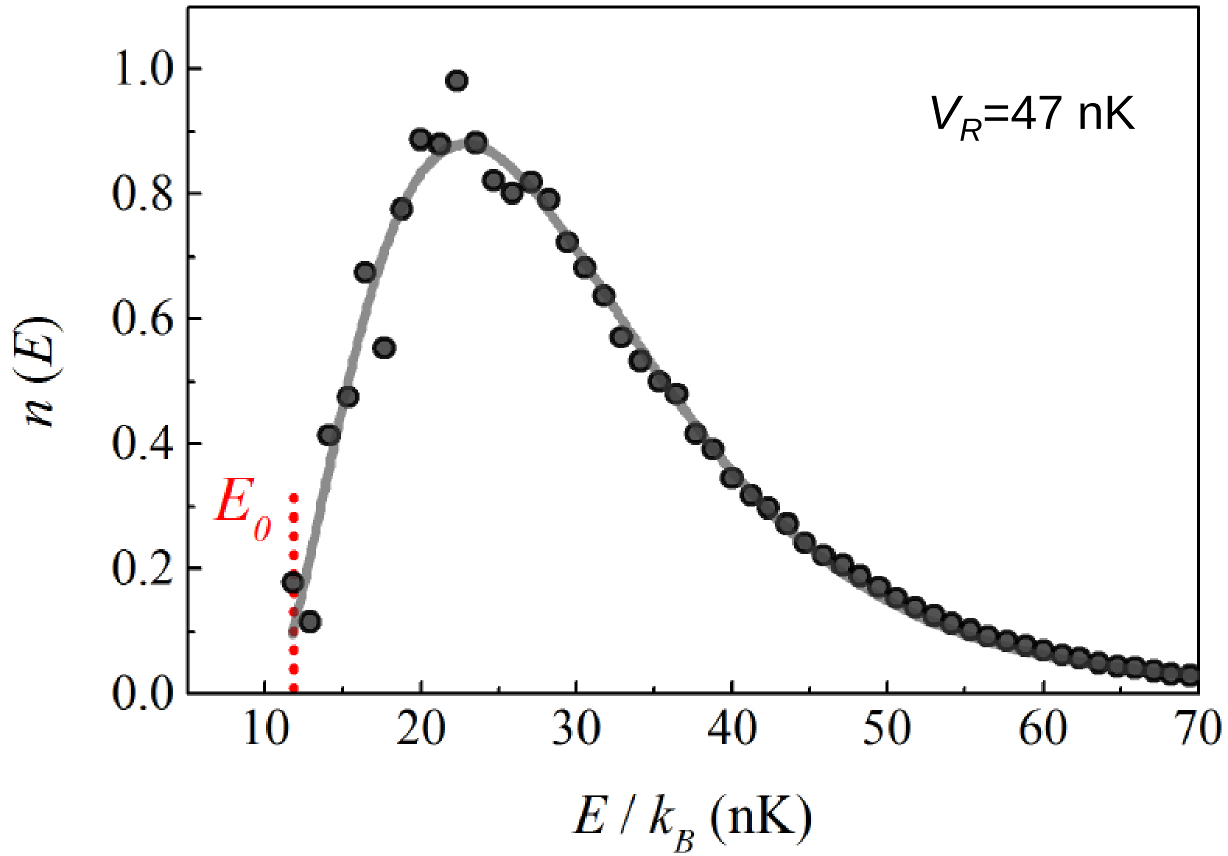
Energy distribution from momentum distribution



$$n(k) = \int \rho(E,k) f(E) dE \qquad n(E) = \int \rho(E,k) f(E) dk$$

$$f(E) = \exp(-(E - E_0)/E_m)$$

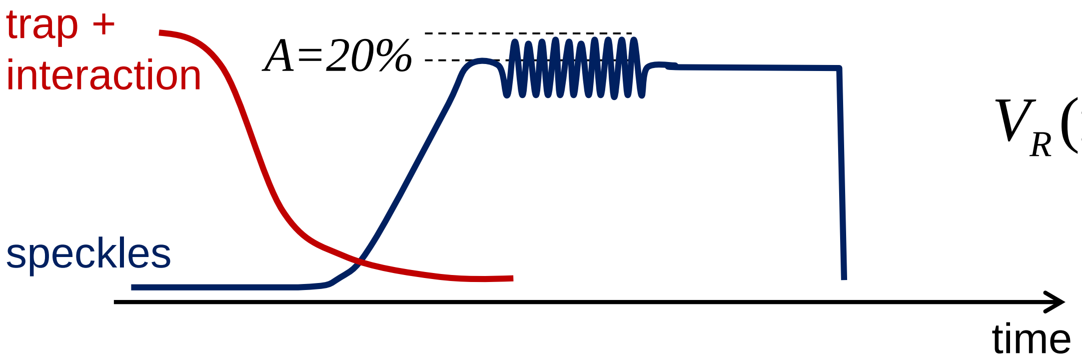
Energy distribution



$$n(E) = g(E)\,f(E) = E^{\beta} e^{\frac{-(E-E_0)}{E_m}} \qquad \beta \approx 1$$

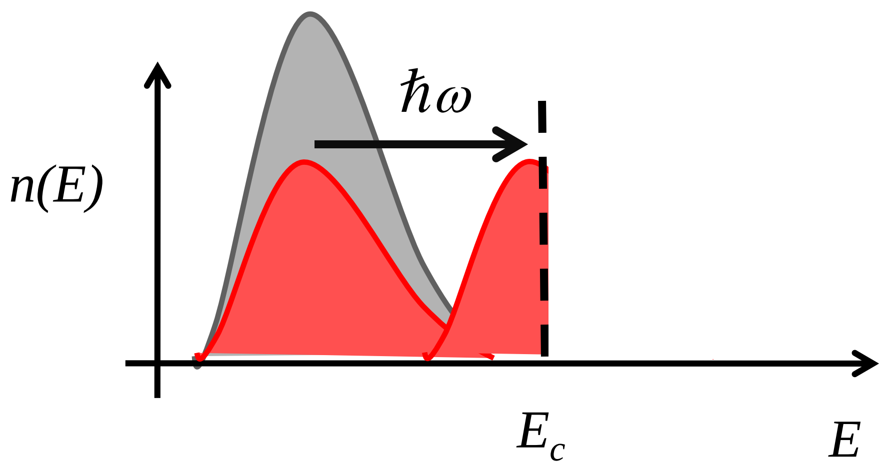
$$g(E) = \int \rho(E,k)\,dk$$

Excitation spectroscopy

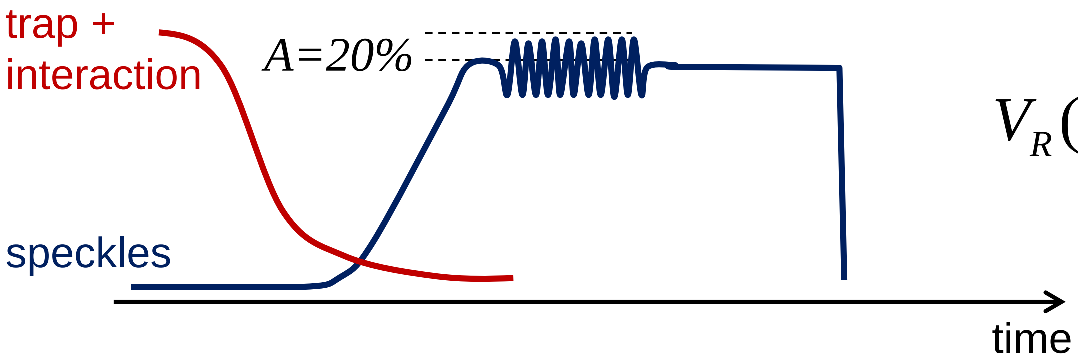


$$V_R(\mathbf{r},t) = V_R(\mathbf{r})(1 + A\cos(\omega t))$$

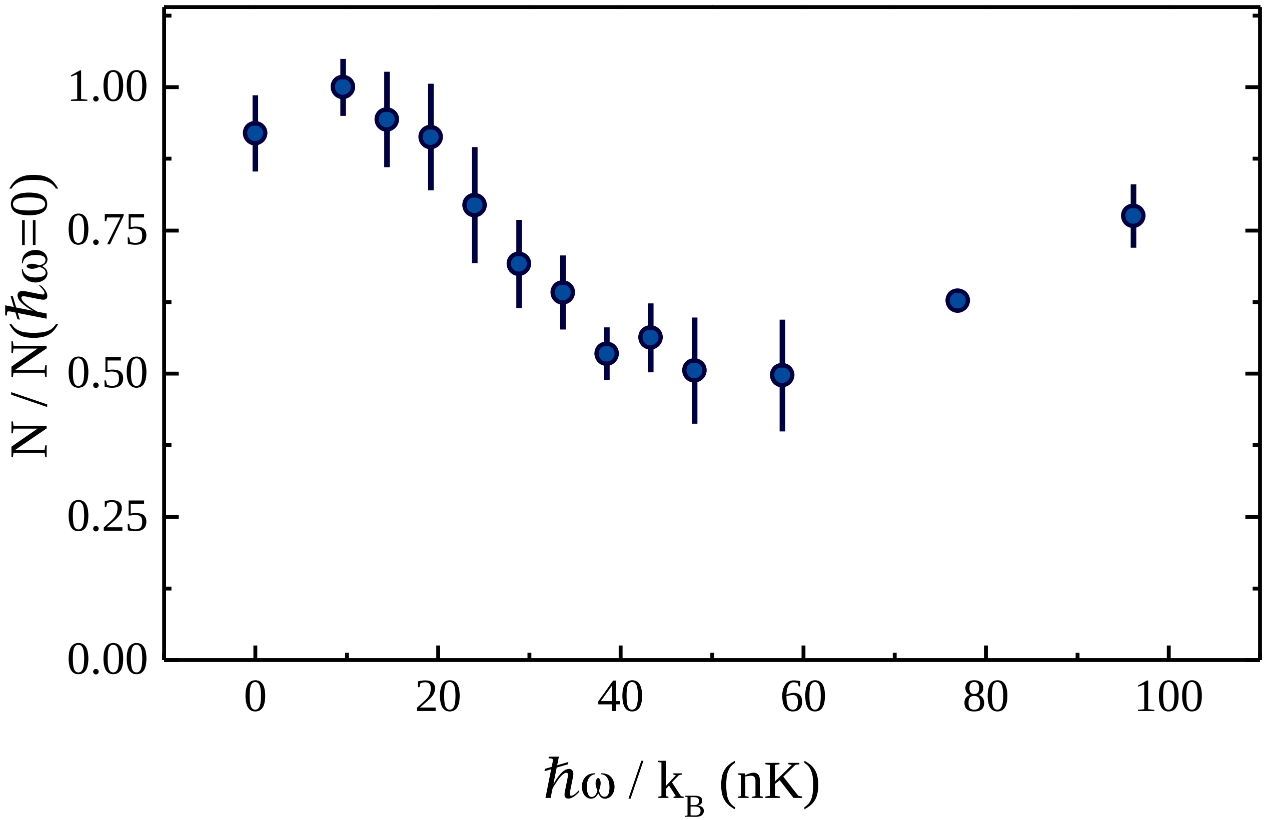
In the linear regime: $$P(\omega) \approx \sum_{i,f} f(E_i) \left\langle f \left| V_R(x) \right| i \right\rangle^2 \delta(E_f - E_i - \hbar\omega)$$



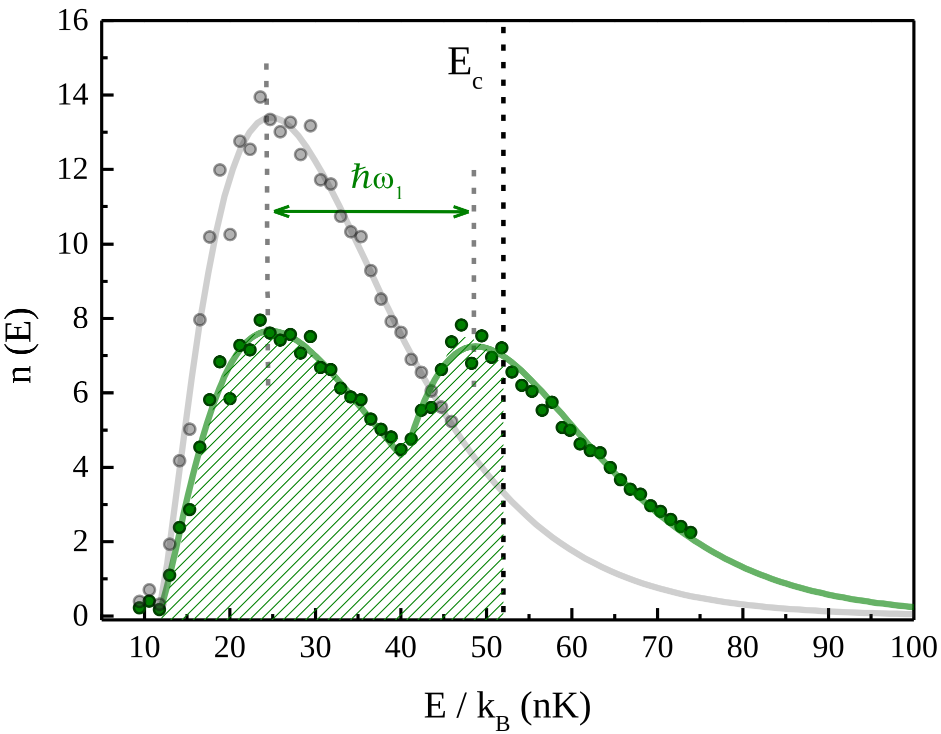
Excitation spectroscopy



$$V_R(\mathbf{r},t) = V_R(\mathbf{r})(1 + A\cos(\omega t))$$



Excitation spectroscopy



Fitting model for the mobility edge:

$$N(\omega) = \int_{E_0}^{E_c} n'(E,\omega)dE$$

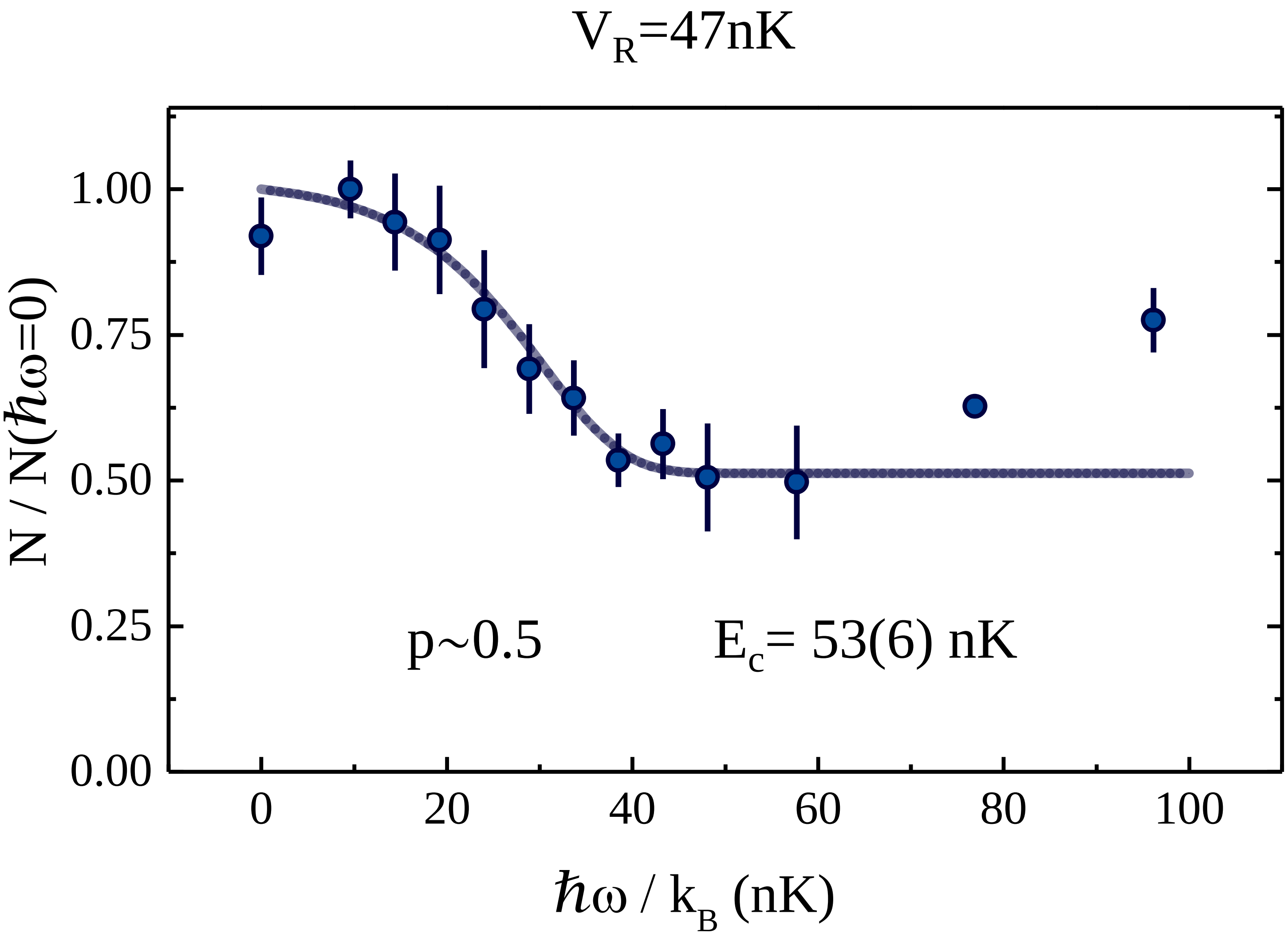
$$n'(E,\omega) = (1-p)n(E) + pn(E-\hbar\omega)$$

p and E_c are fitting parameters

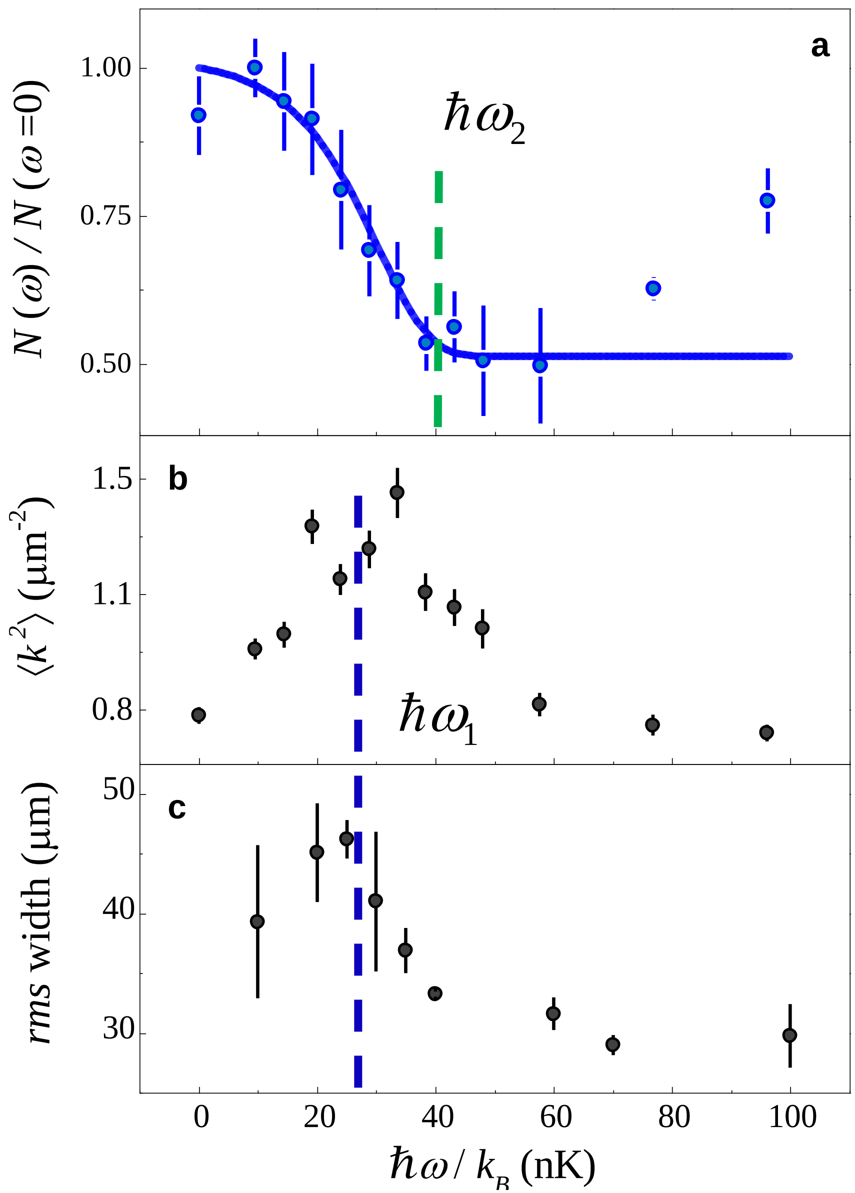
p can in principle be calculated, but its exact form is not crucial

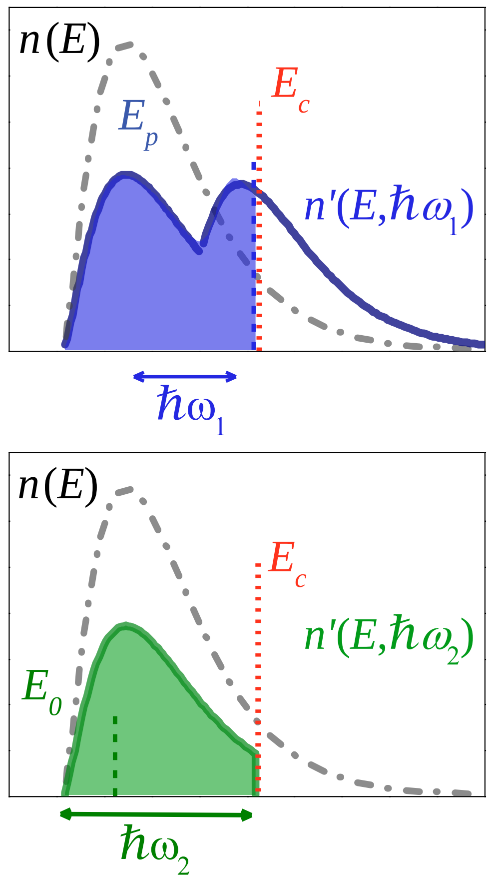
$$p(E,\omega) = A^2 \sum_{i,f} |\langle f | V(\mathbf{r}) | i \rangle|^2 \, \delta(E_i - E)\delta(E_f - (E + \hbar\omega))$$

Excitation spectroscopy

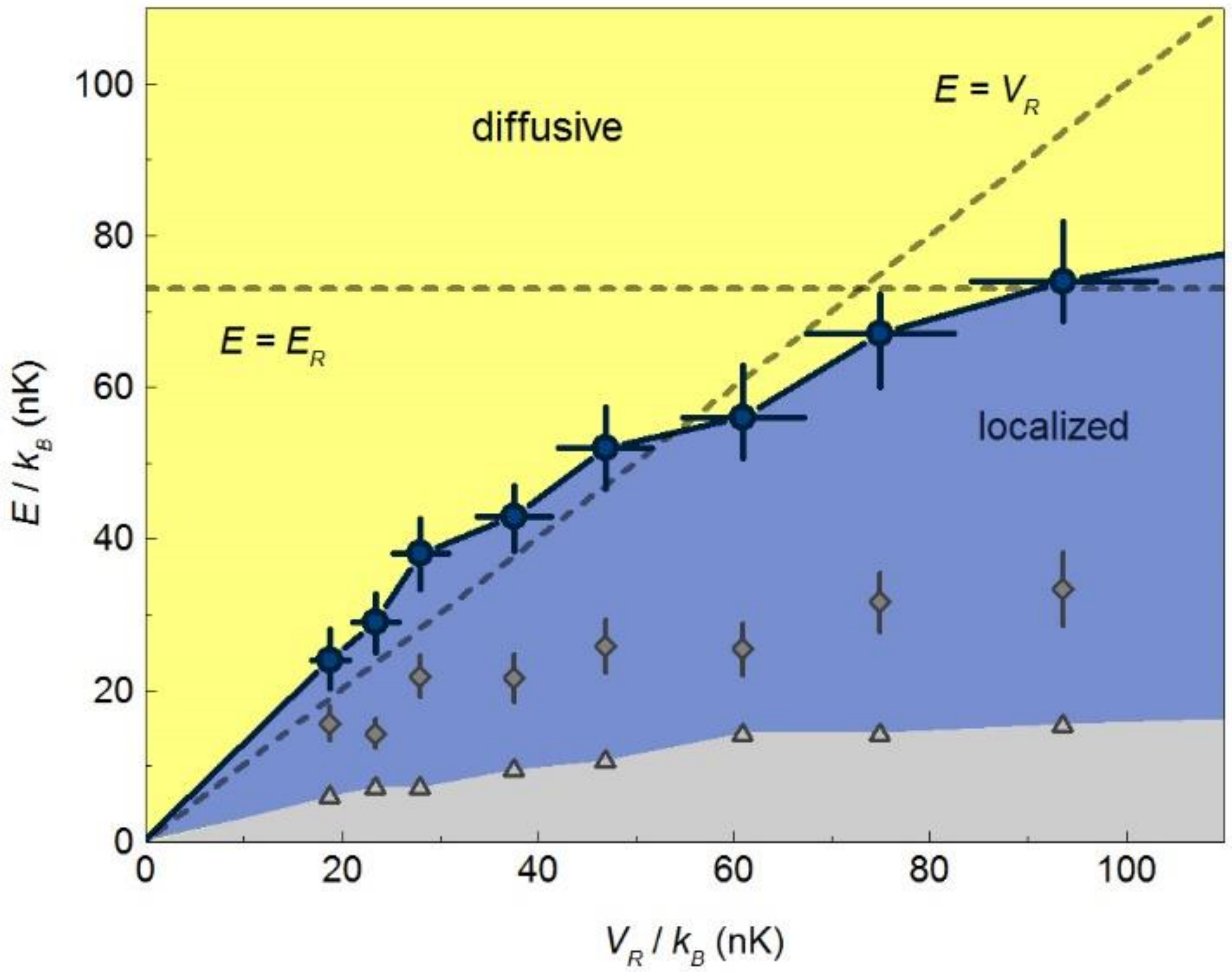


Excitation spectroscopy

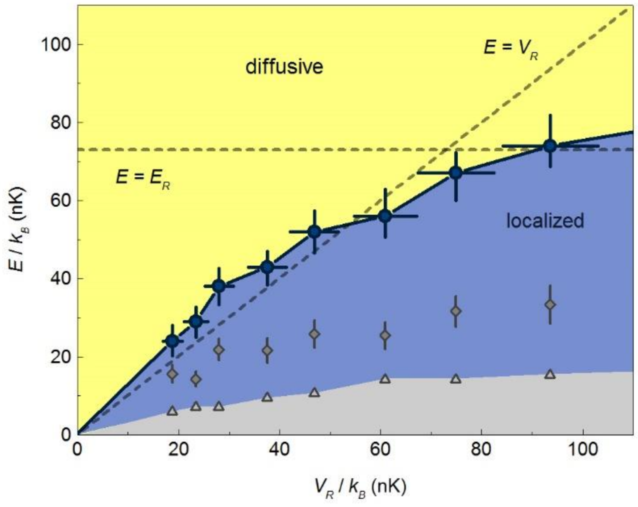




The mobility edge



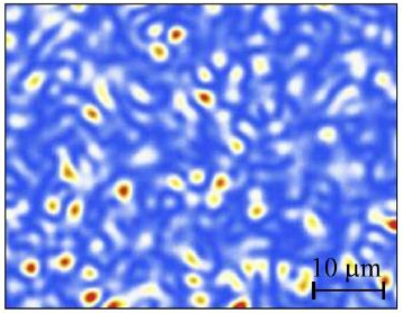
The mobility edge



Arguments for the bending of $E_c(V_R)$

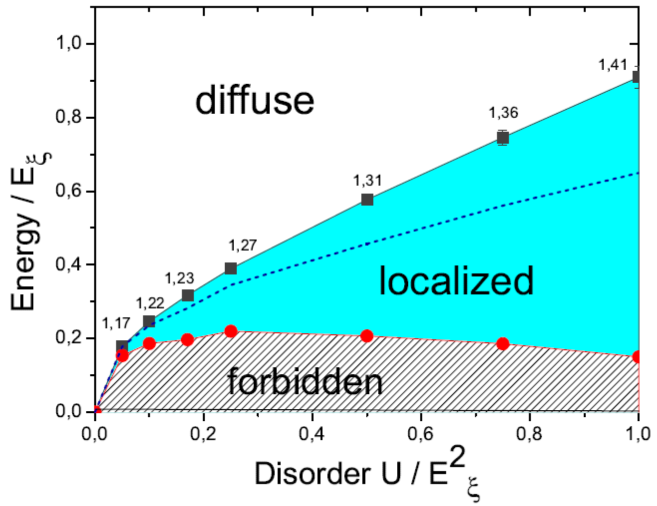
- Ioffe-Regel criterion: in the hard-wall regime $l \approx \sigma_R \Rightarrow k \approx 1/\sigma_R$

- The percolation threshold («mobility edge» for a classical fluid) is very small: $E_{perc} \approx V_R / 100$
  (Pilati et al, New J. Phys. 12 073003 (2010))

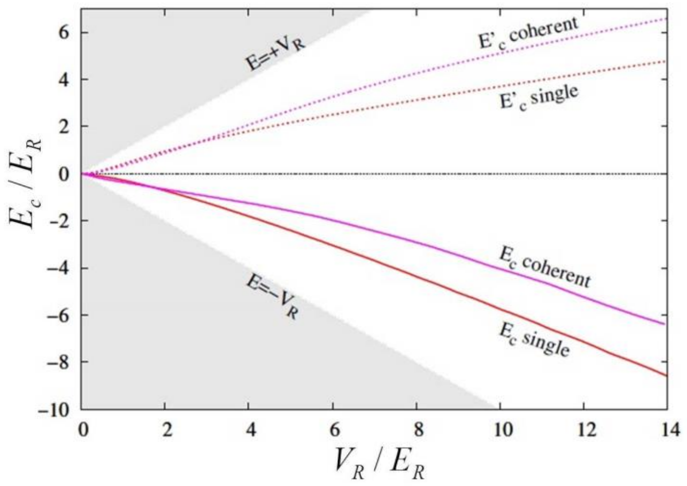


- The optical vortexes lead to a 1D problem with binding energy of the order of E_R
  (A. Scardicchio)

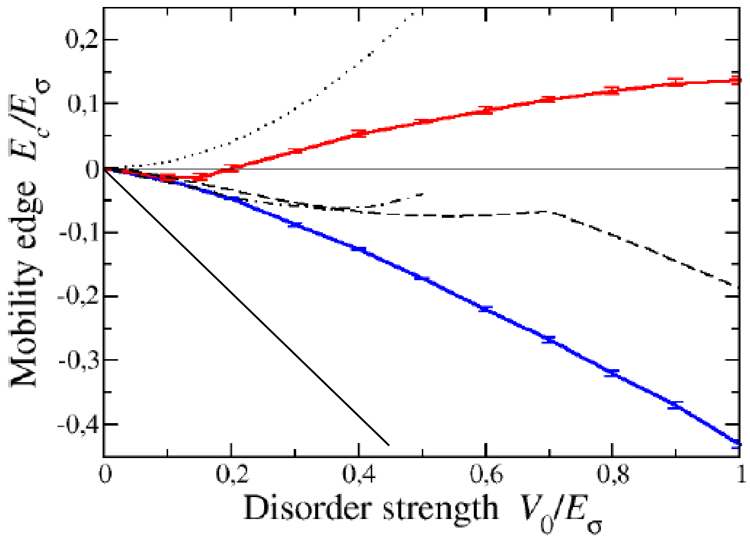
Self-consistent theories and exact calculations



Isotropic: Yedjour & Van Tiggelen, Eur. Phys. J. D (2009).



Anisotropic: Piraud, Pezzè and Sanchez-Palencia, EPL (2012).



Isotropic: Orso & Delande, arXiv:1403.3821

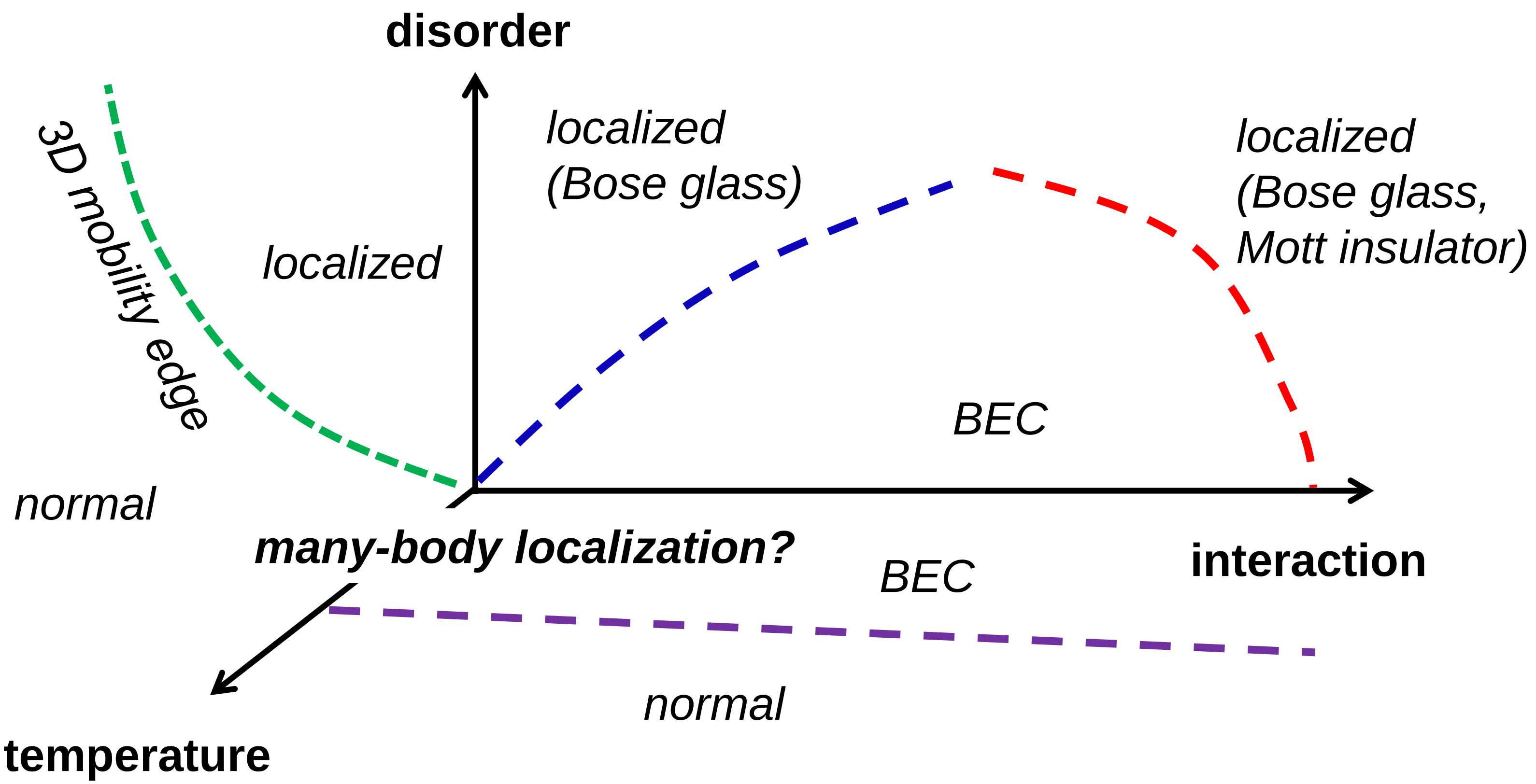
Not yet for our exact spatial correlations.

The experiment stays above the theory.

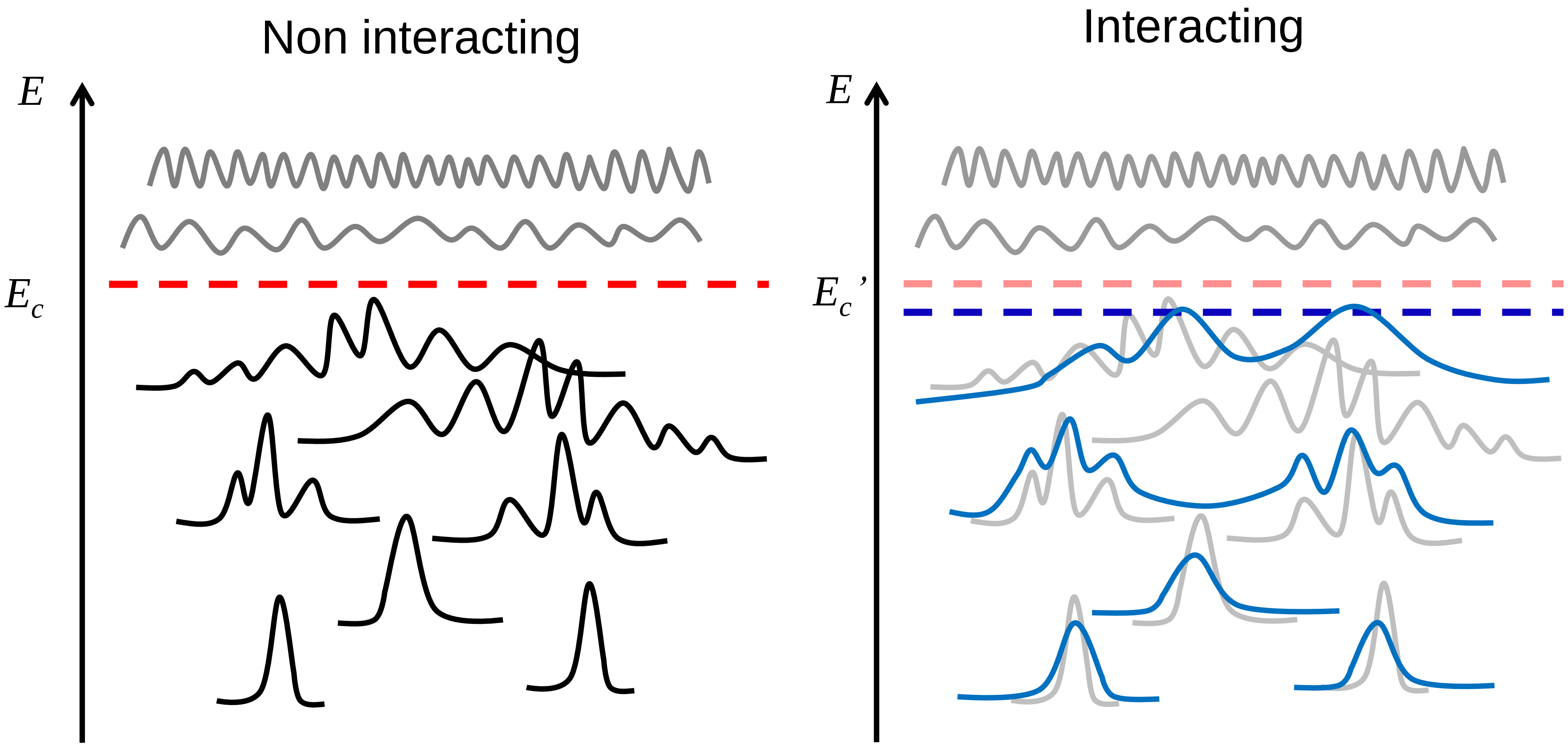
What if we reintroduce interactions?

A very complex problem:

Disorder and interactions can **compete** or **cooperate**, depending on the dimensionality, the presence of a lattice, and on the energy scales (disorder strength and correlations, interaction, kinetic).

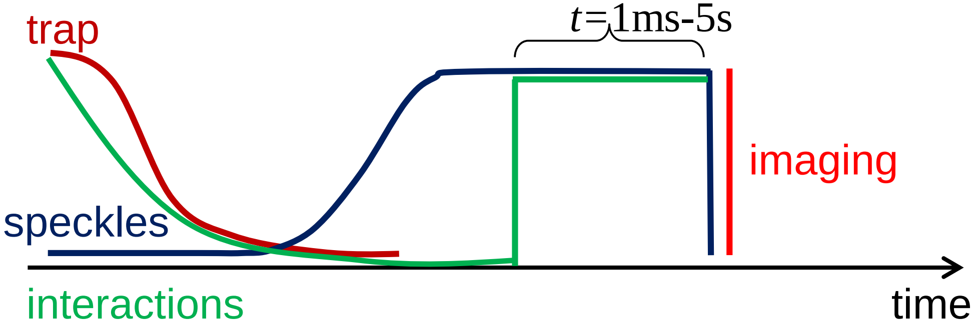


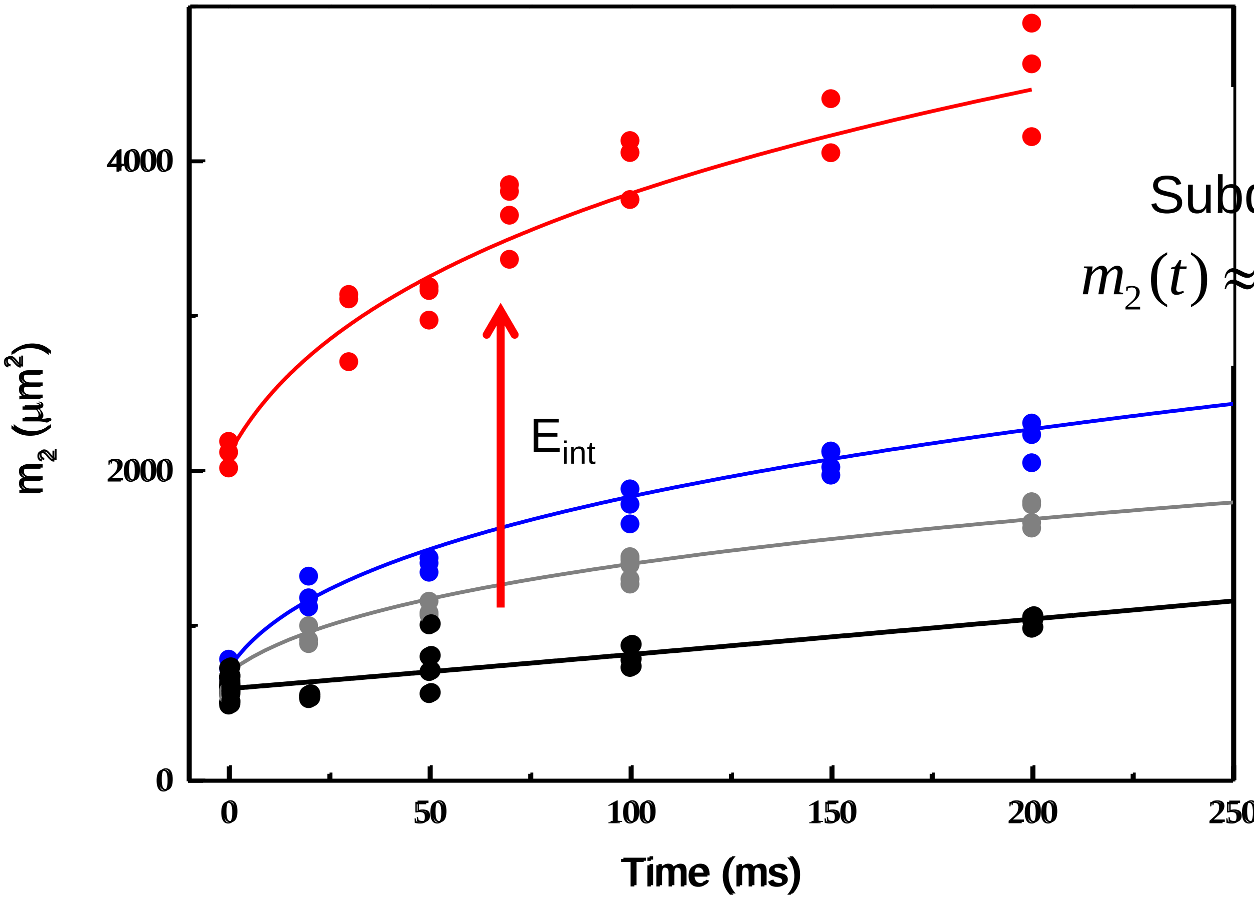
Anderson localization and interactions



- Localized states are hybridized by interactions
- The many-body mobility edge might get shifted

Interaction-induced dynamics

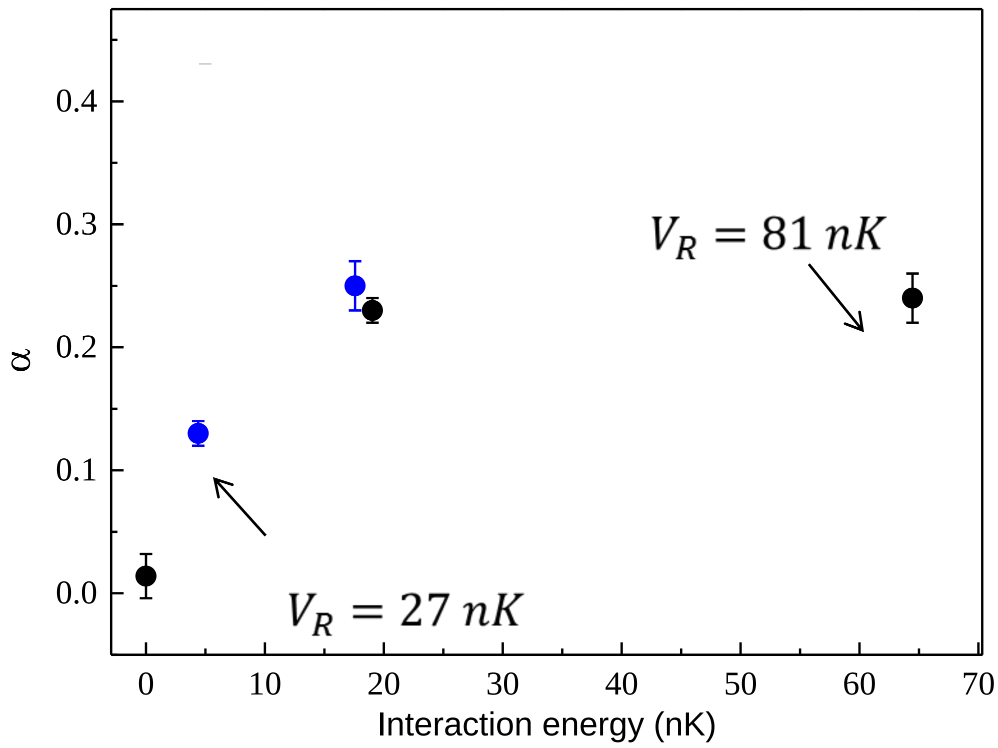




Subdiffusion:

$$m_2(t) \approx D(t)\ t = \tilde{D} t^{\alpha}$$

Interaction-induced dynamics



The exponent and the evolution of the shape can give information on the underlying microscopic physics:

1D: Flach et al., Phys. Rev. Lett.102, 024101 (2009). Lucioni et al. Phys. Rev. Lett. 106, 230403 (2011); Phys. Rev. E (2013).

3D: Flach et al., Phys. Rev. Lett.102, 024101 (2009); Cherroret et al. Phys. Rev. Lett. 112, 170603 (2014).

Outlook

- A physical realization of the Anderson model; the same methods can be applied also to disordered lattices.
- Narrower energy distributions: critical exponents.
- Evolution of the mobility edge with dimensionality at the 3D-2D crossover.
- Quantum simulation of the disordered interacting problem in 3D.