



International School of Physics "E. Fermi"

Light in a Twist: Orbital Angular Momentum

Miles Padgett

Kelvin Chair of Natural Philosophy



THE ROYAL SOCIETY



The Leverhulme Trust

The talk today

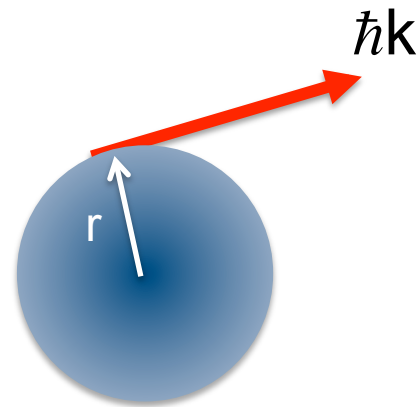
- Orbital Angular Momentum, what is it?
- What has been done with OAM
- A couple of example of what we have done and doing!



A question

- A photon carries a spin angular momentum of \hbar
- So how does a multi-pole transition ($\Delta J > \hbar$) conserve angular momentum?

Linear momentum at a radius exerts a torque



$\hbar k \times r \rightarrow$ multipole transition

PROCEEDINGS OF THE ROYAL SOCIETY A | MATHEMATICAL,
PHYSICAL & ENGINEERING
SCIENCES

Notes on the Theory of Radiation

C. G. Darwin

Proc. R. Soc. Lond. A 1932 **136**, 36-52

Providing the lever is long enough, a fixed linear momentum can exert an arbitrary high torque

Getting started on Orbital Angular Momentum of Light

- 1992, Allen, Beijersbergen, Spreeuw and Woerdman

PHYSICAL REVIEW A

VOLUME 45, NUMBER 11

1 JUNE 1992

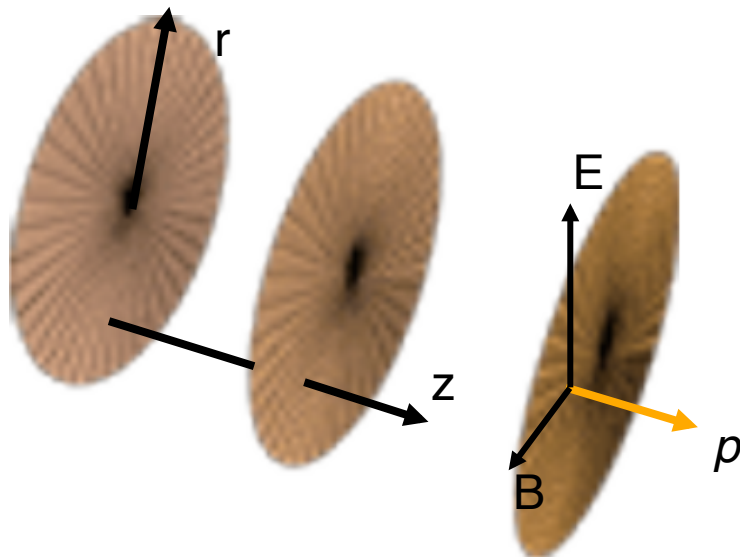
Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes

L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman
Huygens Laboratory, Leiden University, P.O. Box 9504, 2300 RA Leiden, The Netherlands
(Received 6 January 1992)

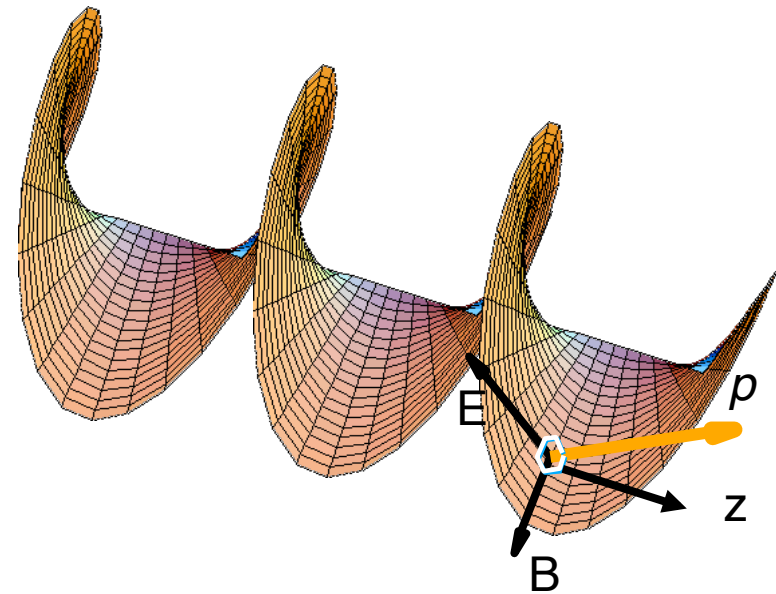
- 1994, Les meets Miles at dinner.....



Orbital Angular Momentum from helical phase fronts



$$p_{\theta} = 0$$



$$p_{\theta} \neq 0$$

Angular momentum in terms of photons

- Spin angular momentum
 - Circular polarisation
 - $\sigma\hbar$ per photon
- Orbital angular momentum
 - Helical phasefronts
 - $\ell\hbar$ per photon

$$\sigma = +1$$



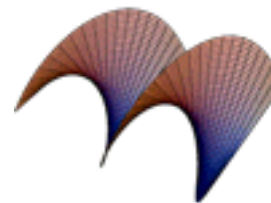
$$\sigma = -1$$



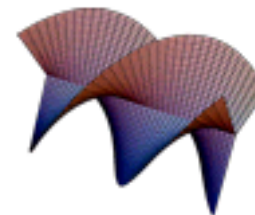
$$\ell = 0$$



$$\ell = 1$$



$$\ell = 2$$

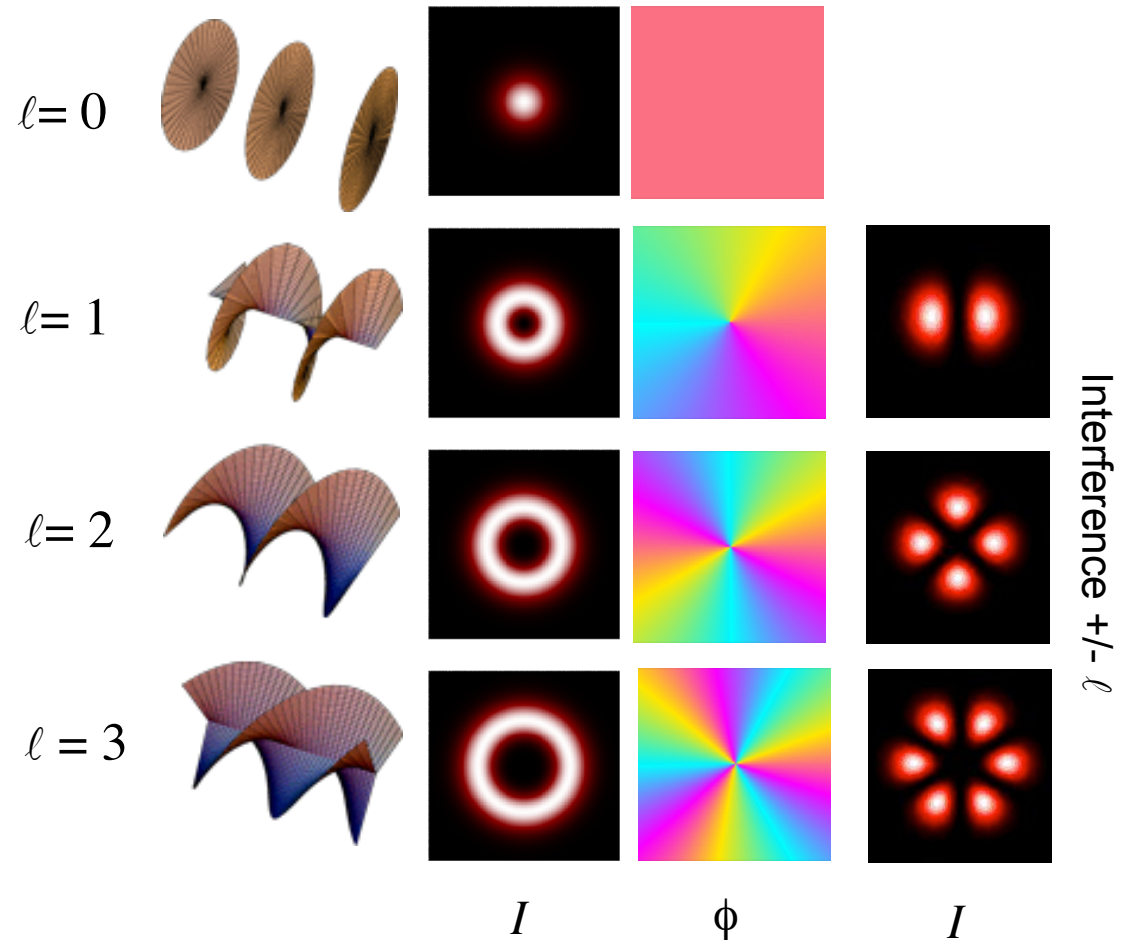


$$\ell = 3 \quad \text{etc}$$

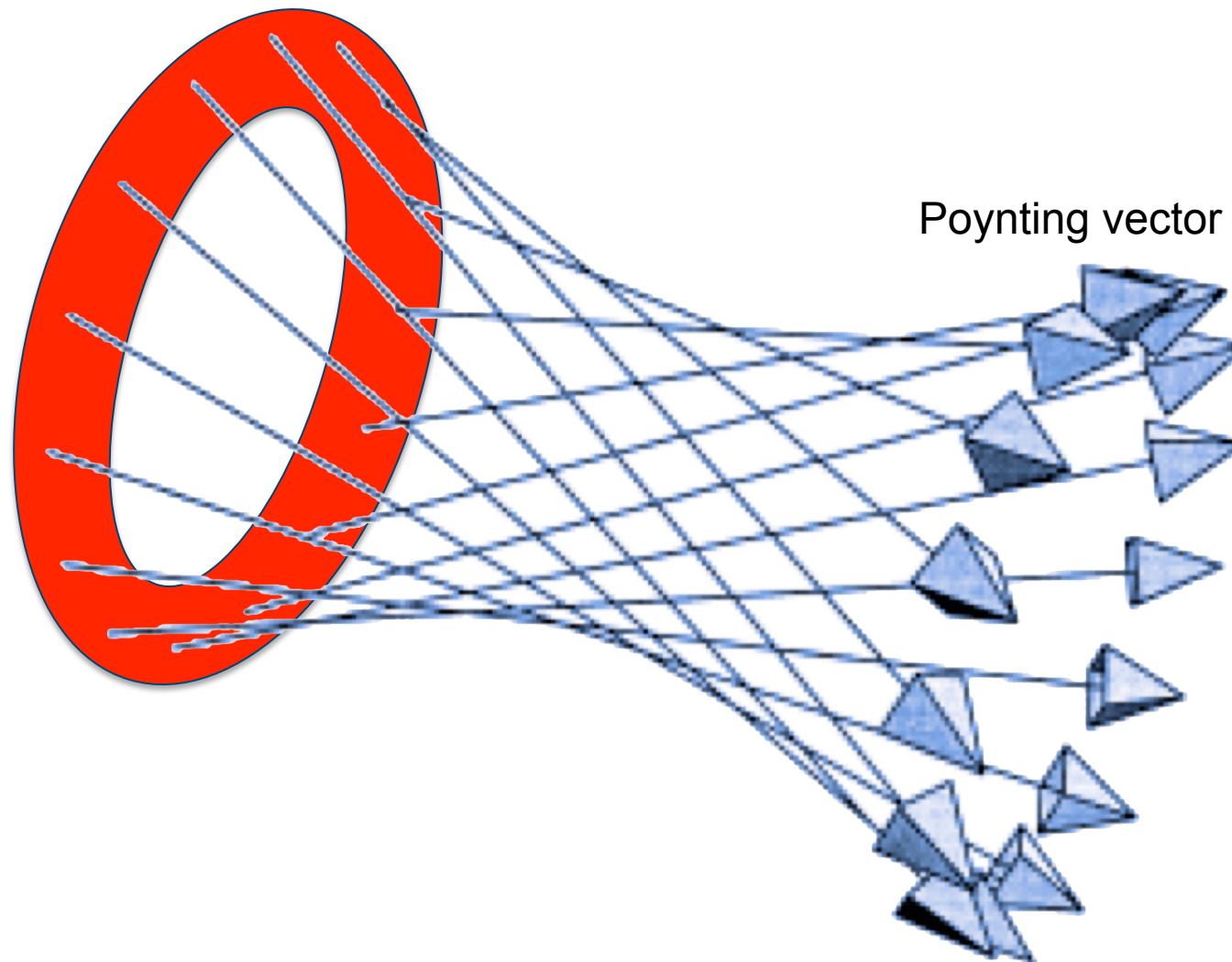
Optical vortices, Helical phasefronts , Angular momentum

- Intensity, $I \geq 0$
- Phase, $2\pi \geq \phi \geq 0$
 - $\ell = 0$, plane wave
 - $\ell = 1$, helical wave
 - $\ell = 2$, double helix
 - $\ell = 3$, pasta fusilli
 - etc.

ℓ = vortex charge



Orbital angular momentum from Skew rays



Making helical phasefronts with holograms

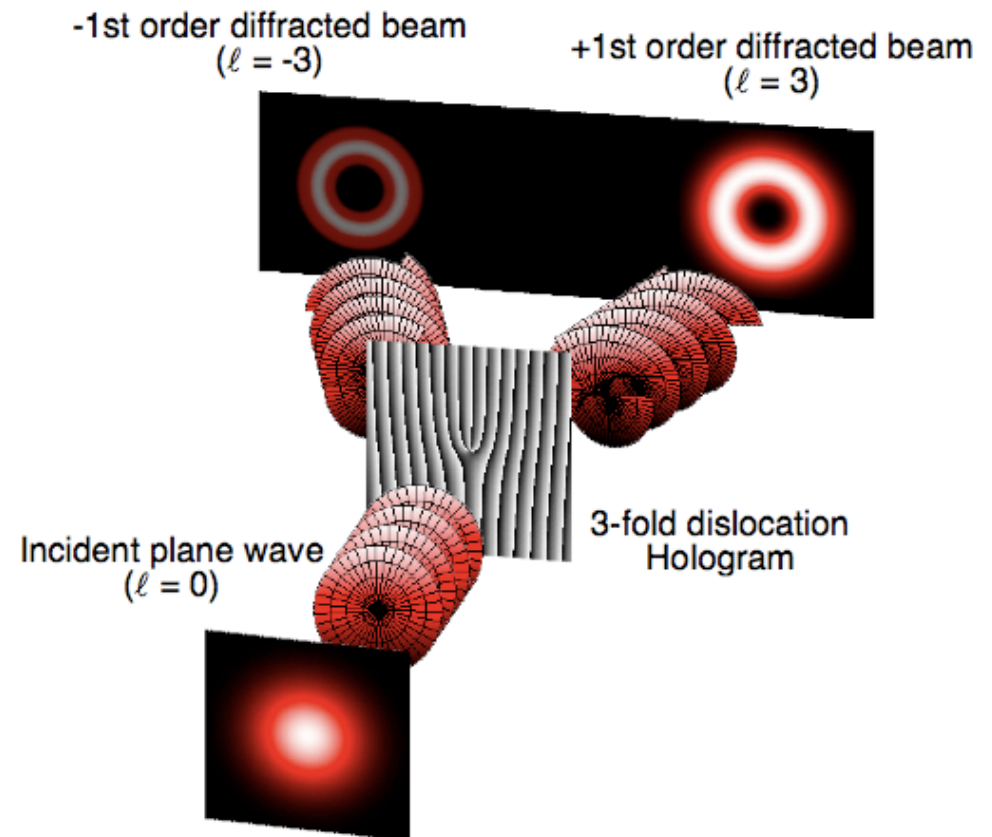
Screw dislocations in light wavefronts

V. YU. BAZHENOV, M. S. SOSKIN and M. V. VASNETSOV

Institute of Physics, Academy of Sciences of Ukraine,
252650 Kiev, Prospect Nauki 46, Ukraine

(Received 14 June 1991; revision received 8 January 1992)

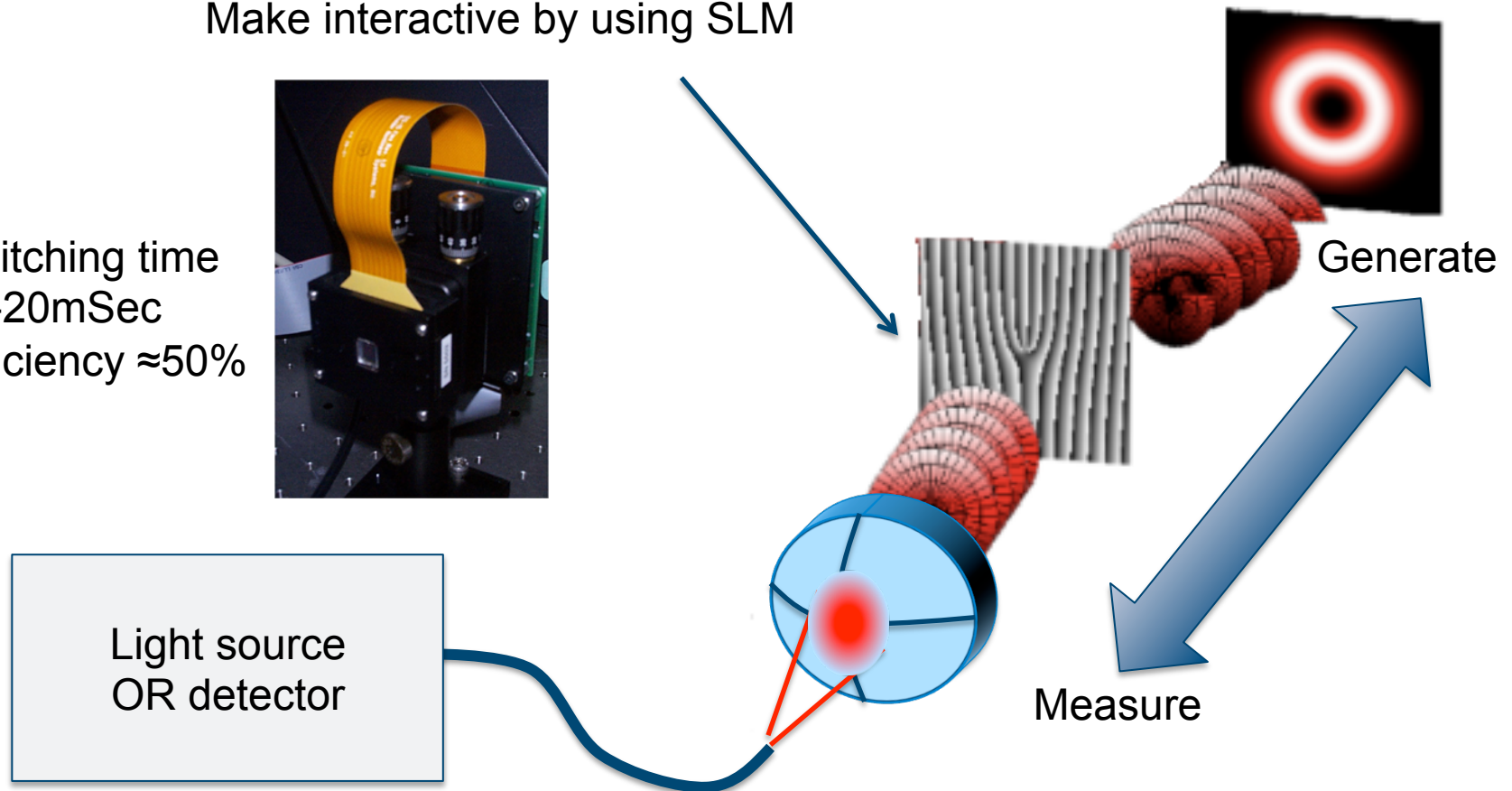
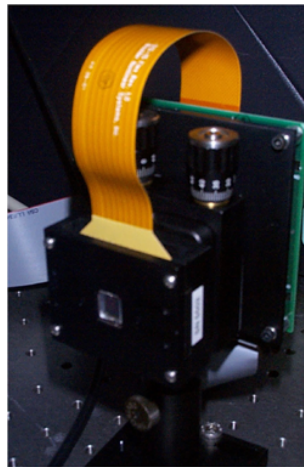
JOURNAL OF MODERN OPTICS, 1992, VOL. 39, NO. 5, 985–990



Making OR measuring phasefronts with holograms

Make interactive by using SLM

Switching time
 $\approx 5\text{-}20\text{mSec}$
Efficiency $\approx 50\%$





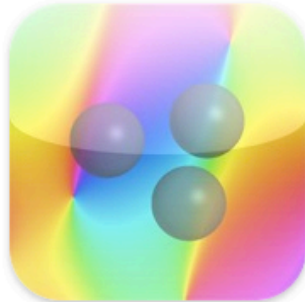
University
of Glasgow

Richard Bowman



A gift for all the family.....

App Store > Education > Richard Bowman



iHologram

Description

iHologram creates beautiful patterns by rendering the Fraunhofer holograms used in Holographic Optical Tweezers iPhone/iPad graphics chip. Use it to learn about diffraction and holography, or just to make pretty pictures!

[Richard Bowman Web Site >](#) [iHologram Support >](#)

Free App

+ This app is designed for both
iPhone and iPad

Category: Education

Released: 14 October 2010

Version: 1.0

1.0

0.2 MB

Language: English

Developer: Richard Bowman

© Richard Bowman

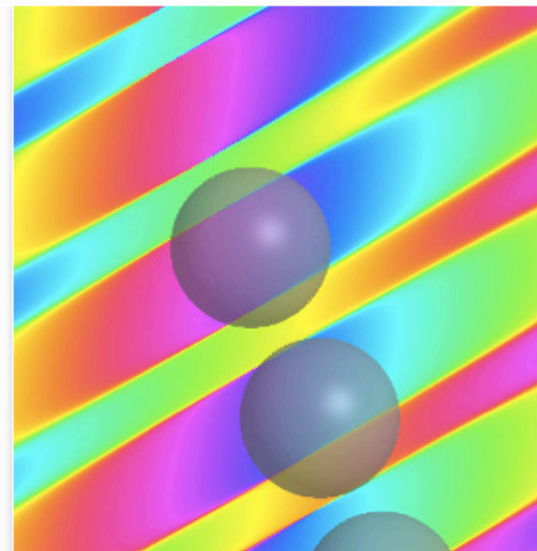
Rated 4+

Requirements: Compatible with
iPhone 3GS, iPhone 4, iPod touch
(3rd generation), iPod touch (4th
generation) and iPad. Requires iOS
3.2 or later.

Screenshots

iPhone

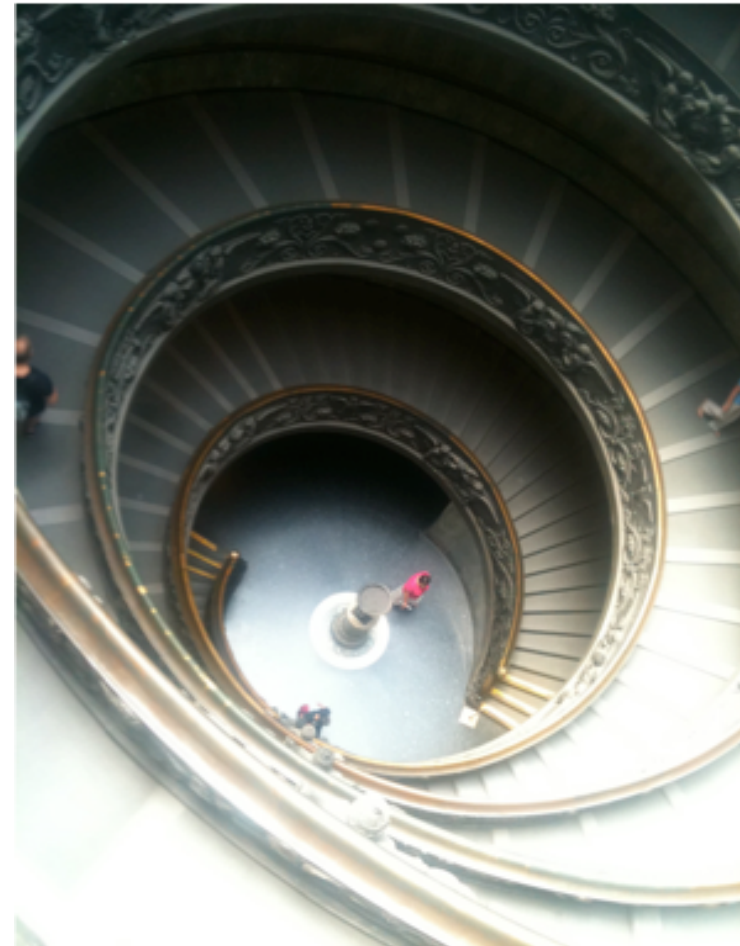
iPad



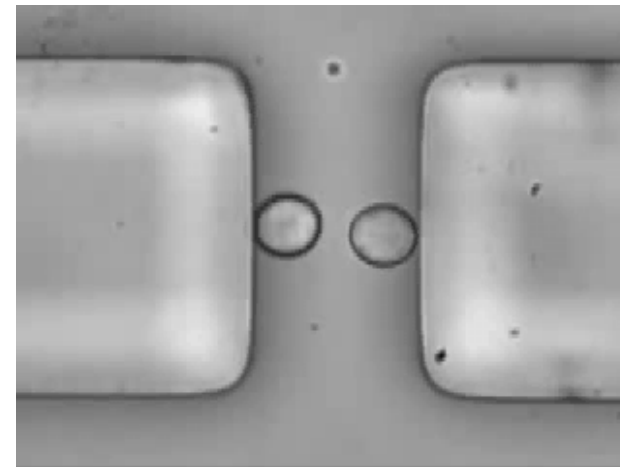
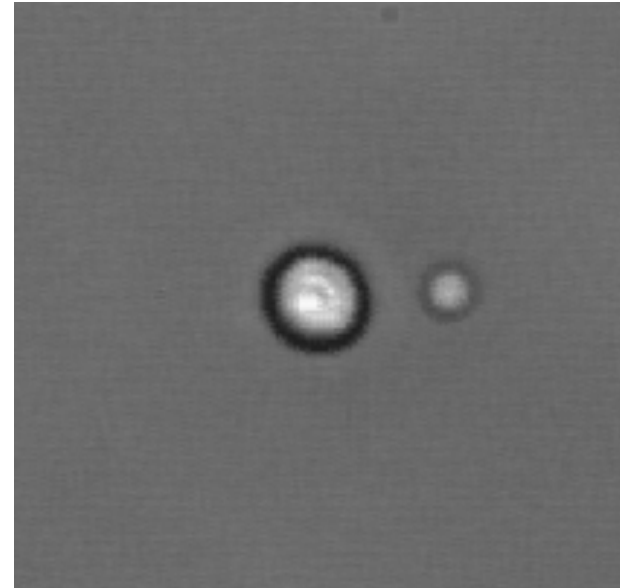
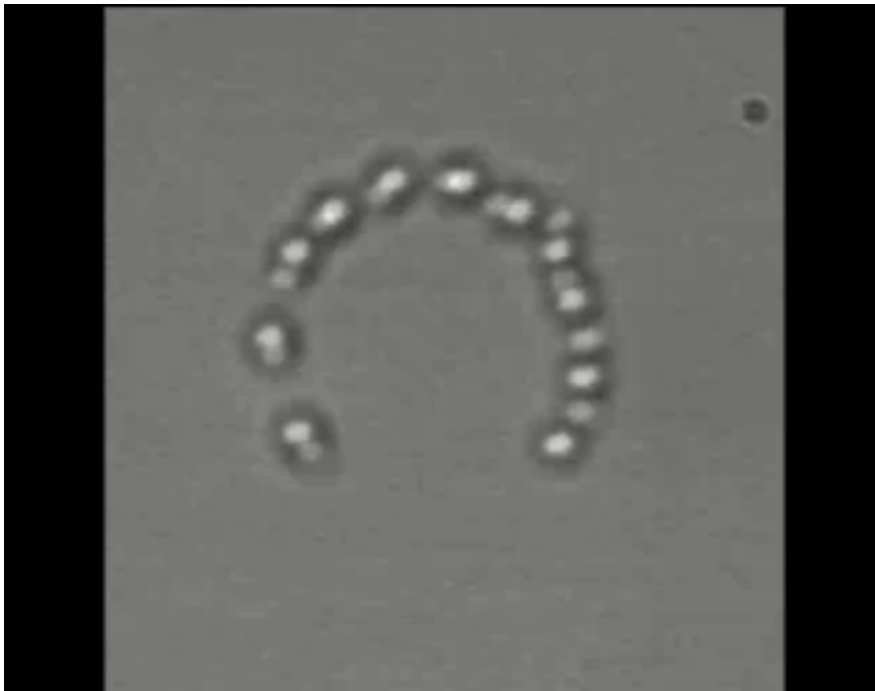
A double-start helix ($\ell=2$)



Chambord castle (chateaux de la Loire)



And the point of shaping the spot is.....



OAM in optical manipulation

VOLUME 75, NUMBER 5

PHYSICAL REVIEW LETTERS

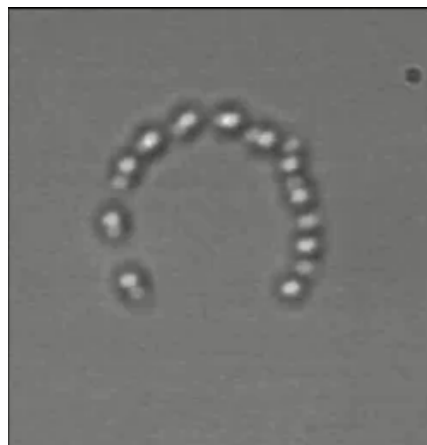
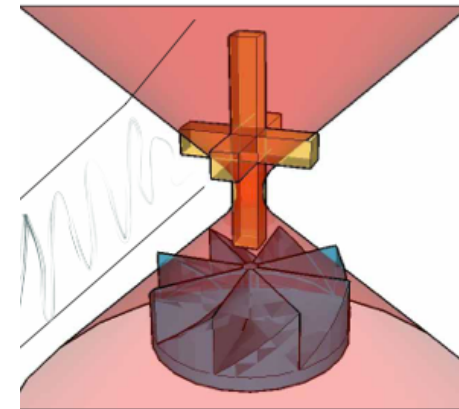
31 JULY 1995

Direct Observation of Transfer of Angular Momentum to Absorptive Particles from a Laser Beam with a Phase Singularity

H. He, M. E. J. Friese, N. R. Heckenberg, and H. Rubinsztein-Dunlop

Department of Physics, The University of Queensland, Brisbane, Queensland, Australia Q4072
(Received 28 November 1994; revised manuscript received 4 April 1995)

He et al. PRL 1995



15 June 2002

Optics Communications 207 (2002) 169–175

OPTICS
COMMUNICATIONS

www.elsevier.com/locate/optcom

Dynamic holographic optical tweezers

Jennifer E. Curtis, Brian A. Koss, David G. Grier*

Curtis et al. Opt Commun. 2002

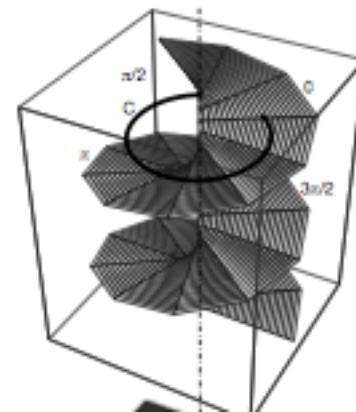
OAM in quantum optics

Entanglement of the orbital angular momentum states of photons

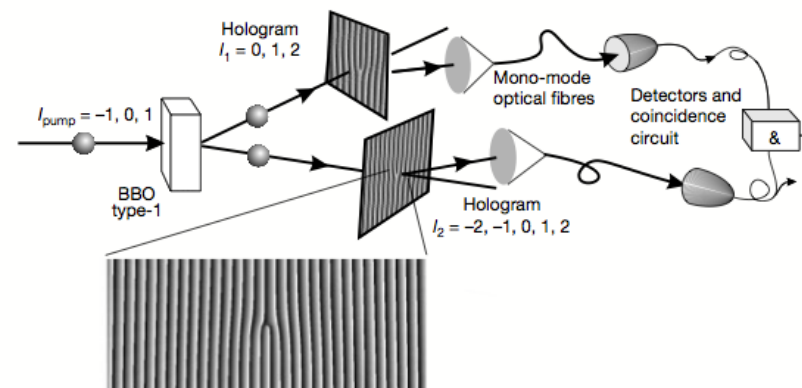
Alois Mair*, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger

Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria

Entangled quantum states are not separable, regardless of the spatial separation of their components. This is a manifestation of an aspect of quantum mechanics known as quantum non-locality^{1,2}. An important consequence of this is that the measurement of the state of one particle in a two-particle entangled state defines the state of the second particle instantaneously, whereas neither particle possesses its own well-defined state before the



Mair et al. Nature 2001



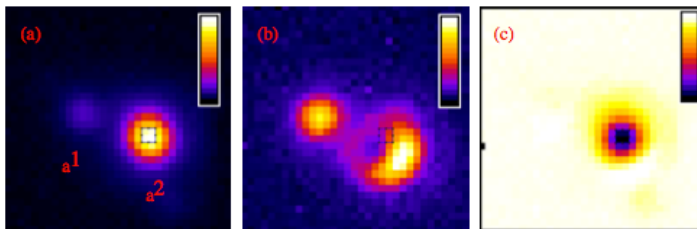
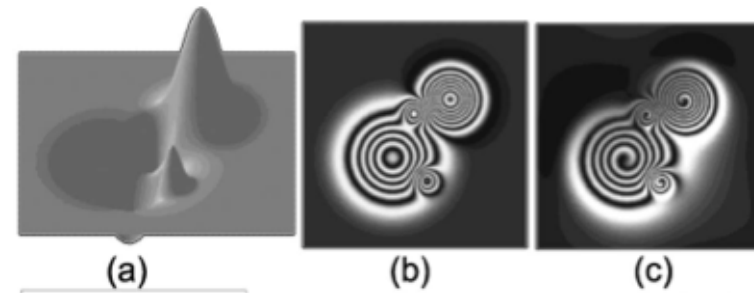
OAM in imaging

Spiral interferometry

Severin Fürhapter, Alexander Jesacher, Stefan Bernet, and Monika Ritsch-Marte

Division of Biomedical Physics, Innsbruck Medical University, Müllerstrasse 44, A-6020 Innsbruck, Austria

Fürhapter et al. Opt. Lett. 2005



Astronomical demonstration of an optical vortex coronagraph

Grover A. Swartzlander, Jr.,^{1,2} Erin L. Ford,¹ Rukiah S. Abdul-Malik,¹
Laird M. Close,³ Mary Anne Peters,² David M. Palacios,³ and Daniel W. Wilson³

Swartzlander et al. Opt. Express 2008

OAM in communication

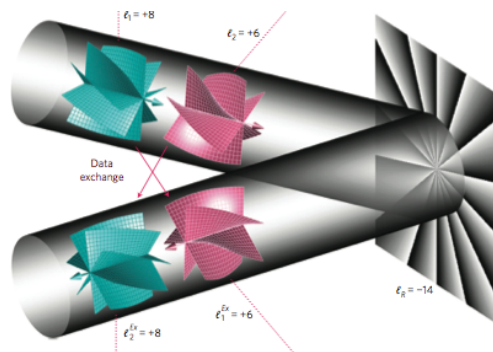
New Journal of Physics

The open-access journal for physics

**Encoding many channels on the same frequency
through radio vorticity: first experimental test**

Fabrizio Tamburini^{1,2,8}, Elettra Mari³, Anna Sponselli¹,
Bo Thidé^{4,5}, Antonio Bianchini¹ and Filippo Romanato^{6,7}

Tamburini et al. New J Phys. 2012



ARTICLES

PUBLISHED ONLINE: 24 JUNE 2012 | DOI: 10.1038/NPHOTON.2012.138

nature
photonics

Terabit free-space data transmission employing orbital angular momentum multiplexing

Jian Wang^{1,2,*}, Jeng-Yuan Yang¹, Irfan M. Fazal¹, Nisar Ahmed¹, Yan Yan¹, Hao Huang¹, Yongxiong Ren¹,
Yang Yue¹, Samuel Dolinar³, Moshe Tur⁴ and Alan E. Willner^{1,*}

Wang et al. Nature Photon 2012

OAM in not just light

PRL 100, 024302 (2008)

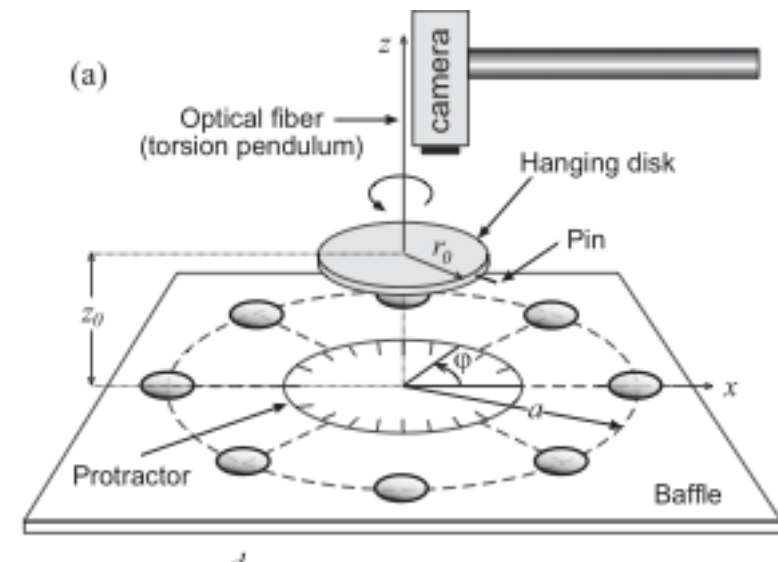
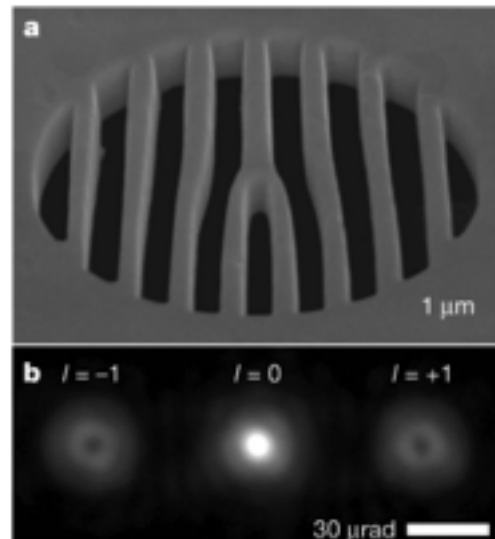
PHYSICAL REVIEW LETTERS

week ending
18 JANUARY 2008

Transfer of Angular Momentum to Matter from Acoustical Vortices in Free Space

Karen Volke-Sepúlveda,¹ Arturo O. Santillán,^{2,*} and Ricardo R. Boulosa²

Volke-Sepulveda et al. PRL 2008



Vol 467 | 16 September 2010 | doi:10.1038/nature09366

nature

LETTERS

Production and application of electron vortex beams

J. Verbeeck¹, H. Tian¹ & P. Schattschneider²

Verbeeck et al. Nature 2010

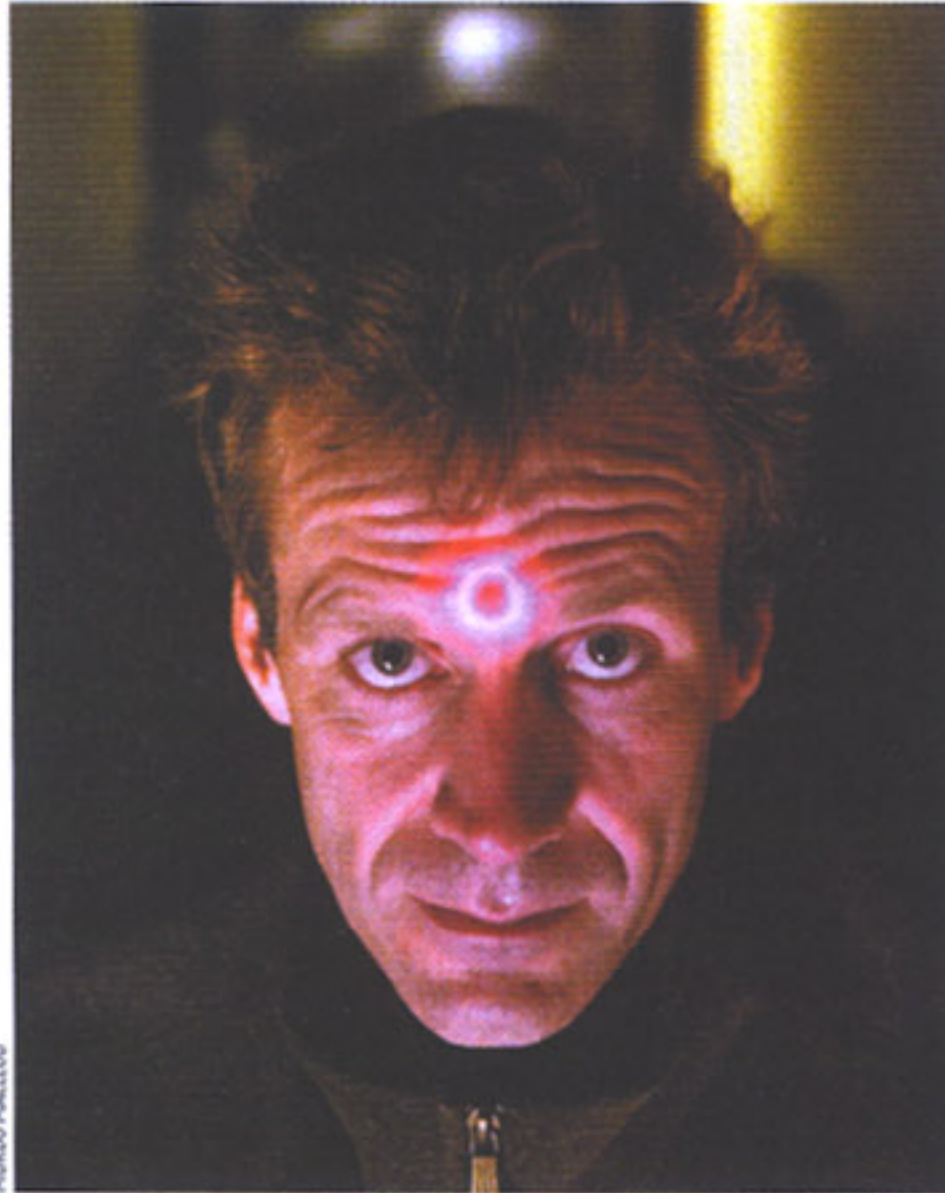
The OAM communicator





University
of Glasgow

Miles Padgett's corkscrew laser beam
creates a ring of light with a dark centre



Optical Vortices before Angular Momentum

Proc. R. Soc. Lond. A. **336**, 165–190 (1974)

Printed in Great Britain

Dislocations in wave trains

BY J. F. NYE AND M. V. BERRY

H. H. Wills Physics Laboratory, University of Bristol

Quantised Singularities in the Electromagnetic Field

P. A. M. Dirac

Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, Vol. 133, No. 821. (Sep. 1, 1931), pp. 60-72.



Fractality and Topology of Light's darkness

Kevin O'Holleran
Florian Flossmann

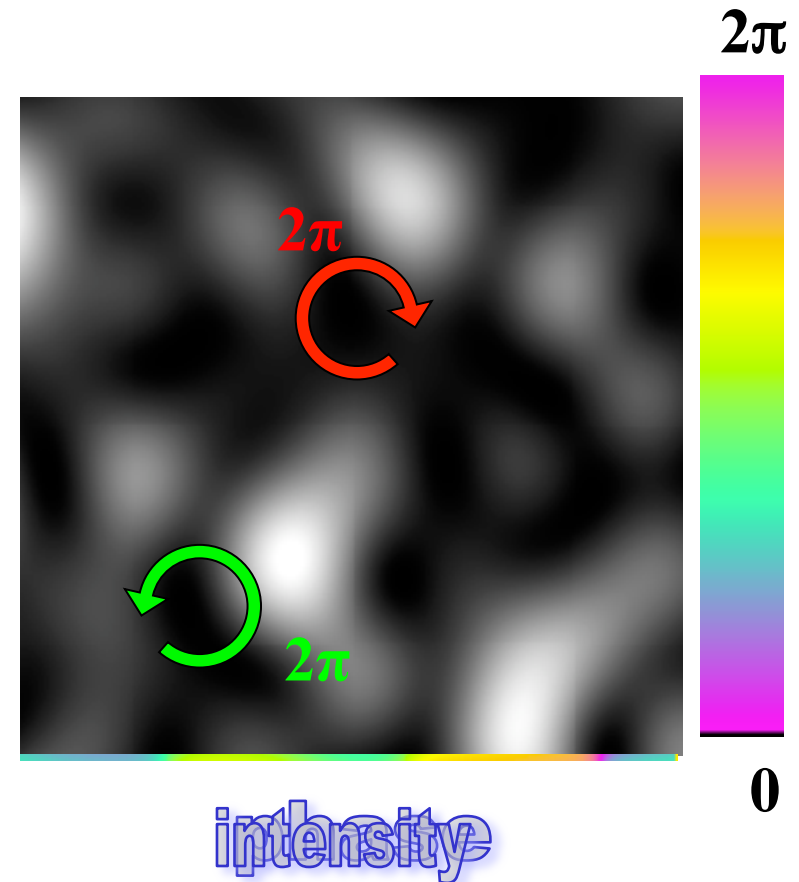


Mark Dennis (Bristol)



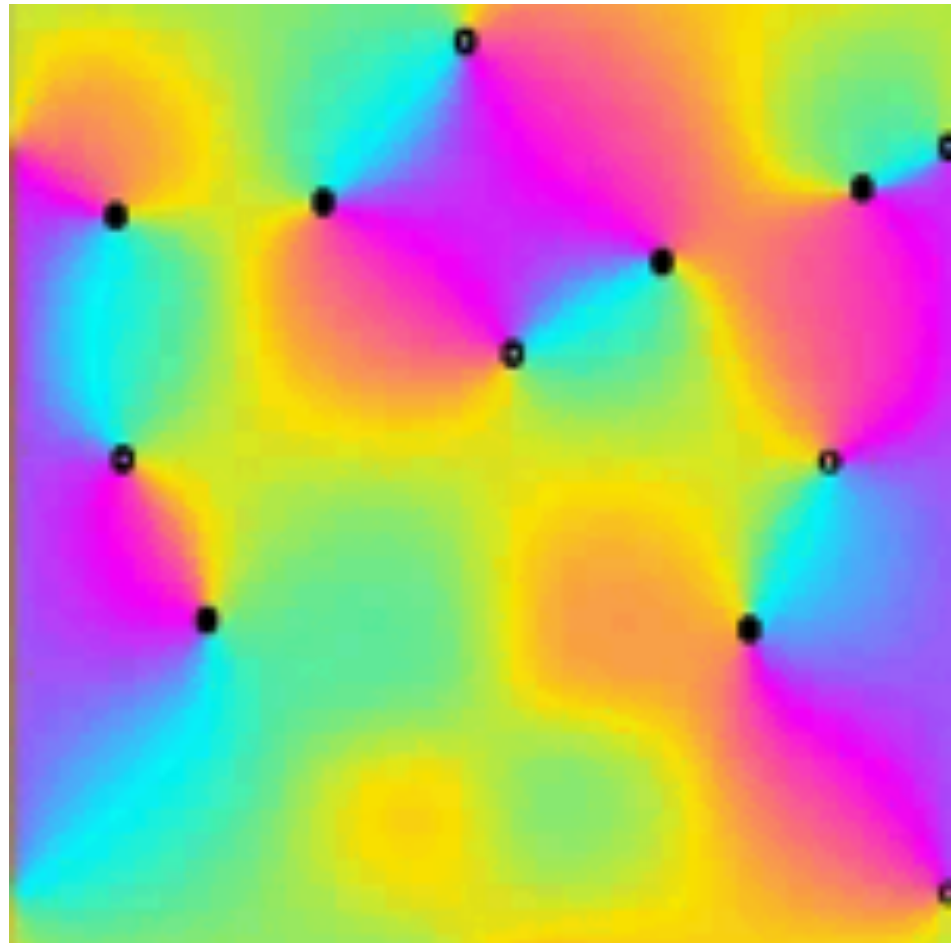
Vortices are ubiquitous in nature

- Whenever **three** (or more) plane waves interfere optical vortices are formed
 - Charge one vortices occur wherever there is diffraction or scattering

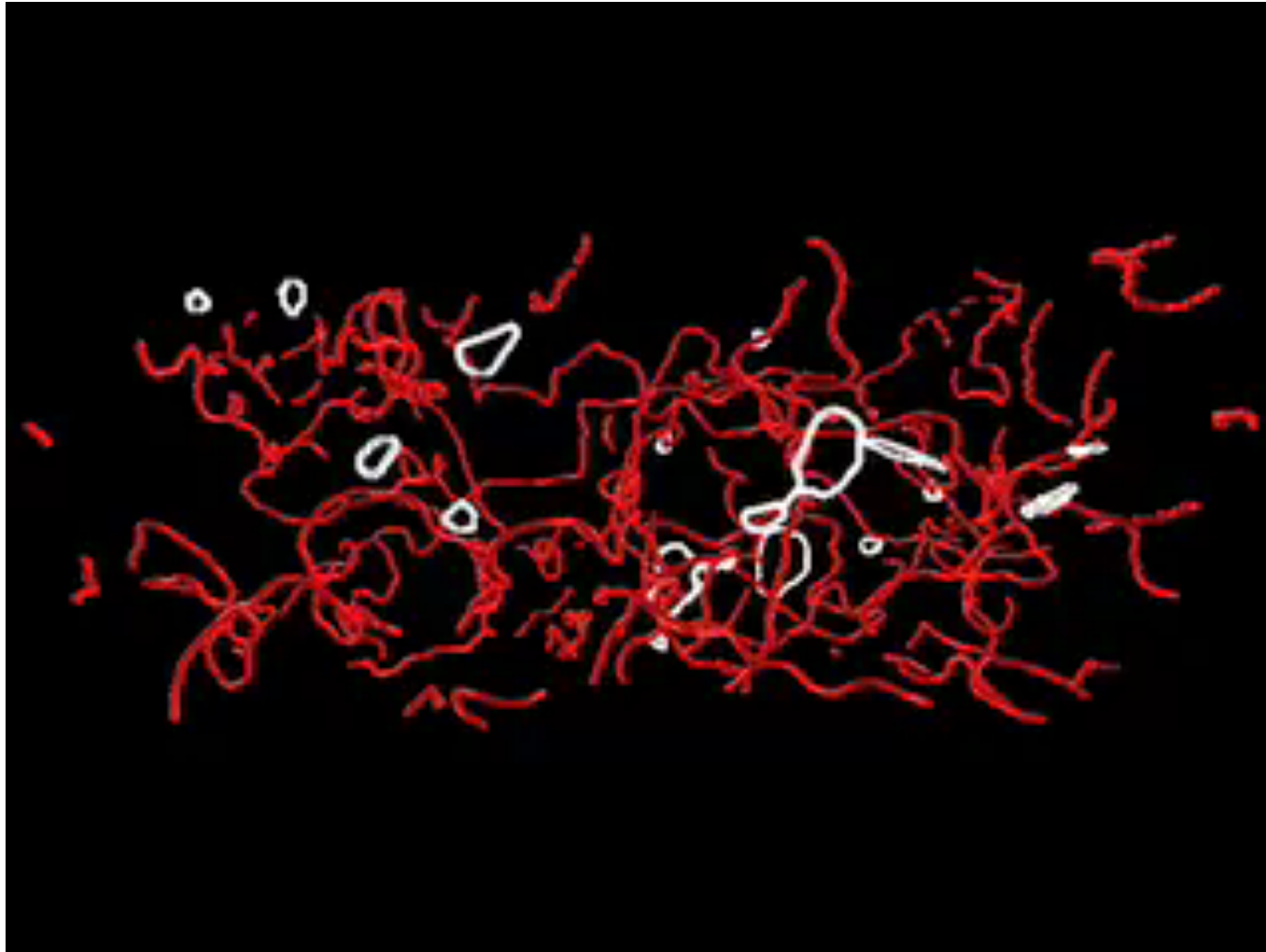


Map out the vortex position in different planes

- Either numerically or experimentally one can map the vortex positions in different planes



The tangled web of speckle



Entanglement of OAM states

.....

Entanglement of the orbital angular momentum states of photons

NATURE | VOL 412 | 19 JULY 2001 |

Alois Mair*, Alpasha Vaziri, Gregor Weihs & Anton Zeilinger

VOLUME 93, NUMBER 5

PHYSICAL REVIEW LETTERS

week ending
30 JULY 2004

Measuring Entangled Qutrits and Their Use for Quantum Bit Commitment

N. K. Langford,* R. B. Dalton, M. D. Harvey, J. L. O'Brien, G. J. Pryde, A. Gilchrist, S. D. Bartlett, and A. G. White



Quantum entanglement with spatial light modulators

Jonathan Leach
Barry Jack
Sonja Franke-Arnold
(Glasgow)



Steve Barnett
and Alison Yao (Strathclyde)



Bob Boyd
Anand Jha (Rochester)



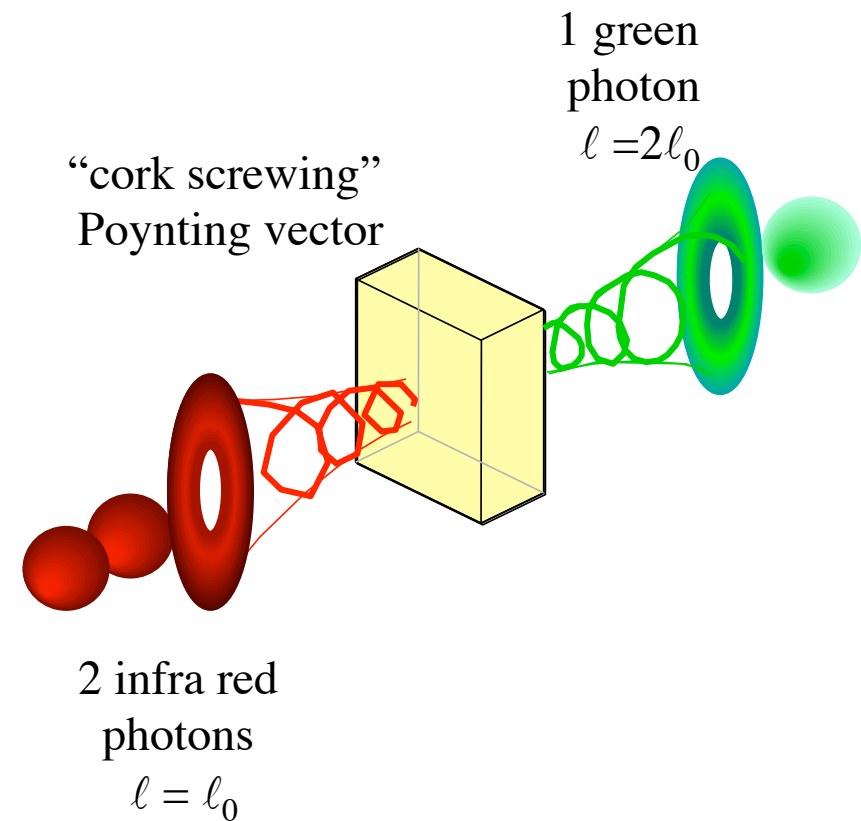


Second-harmonic generation and the conservation of orbital angular momentum with high-order Laguerre-Gaussian modes

J. Courtial, K. Dholakia, L. Allen, and M. J. Padgett

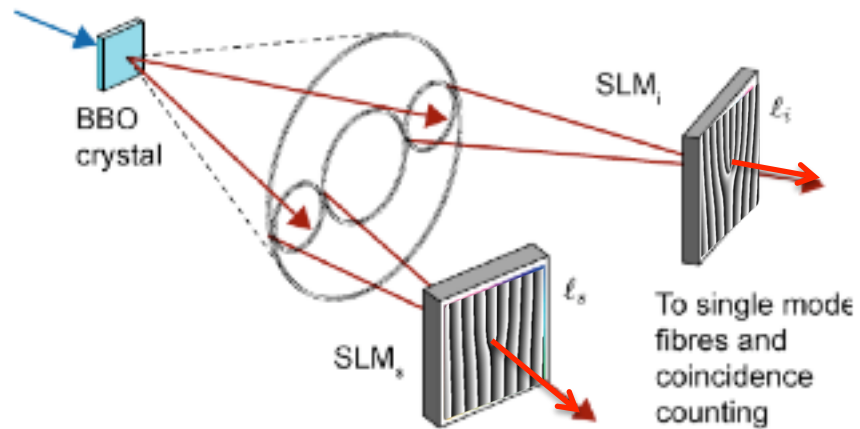
OAM in second harmonic generation

- Poynting vector “cork screws”, azimuthal skew angle is
 - $\theta = \ell/kr$
- Does this upset a co-linear phase match? -No
- Frequency & ℓ -index both double
- “Path” of Poynting vector stays the same
 - phase matching maintained

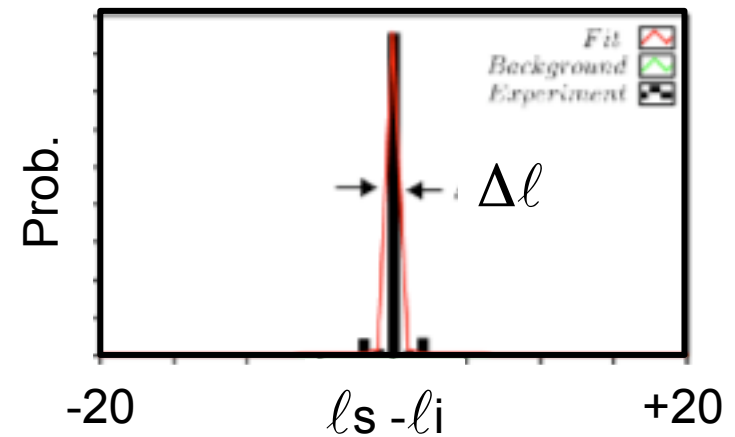
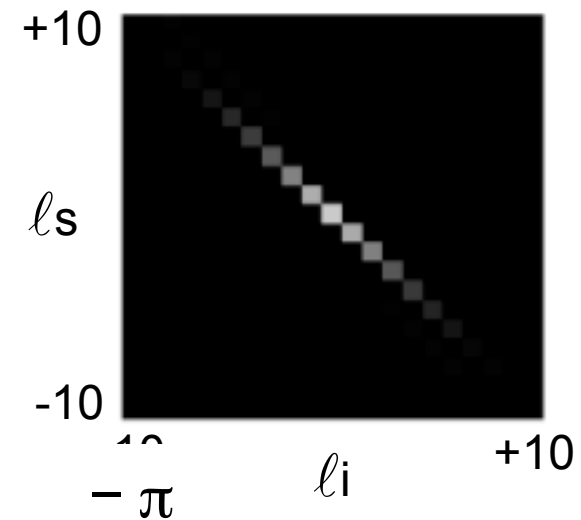


Correlations in angular momentum

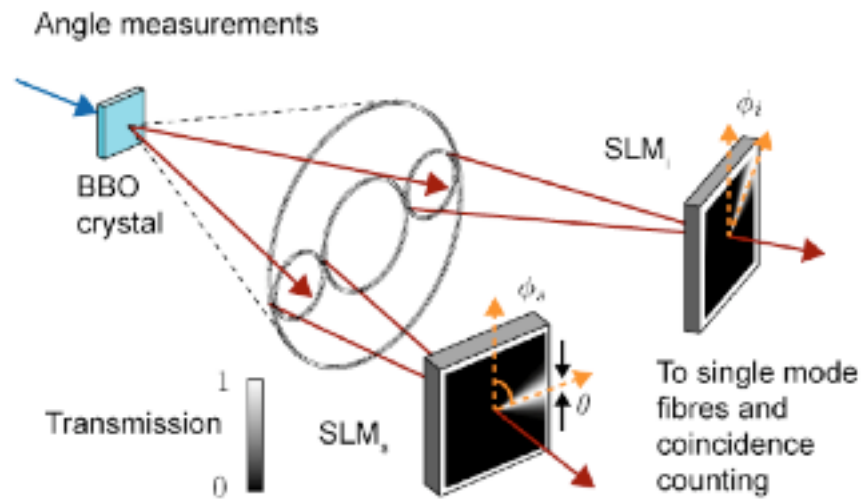
Orbital angular momentum measurements



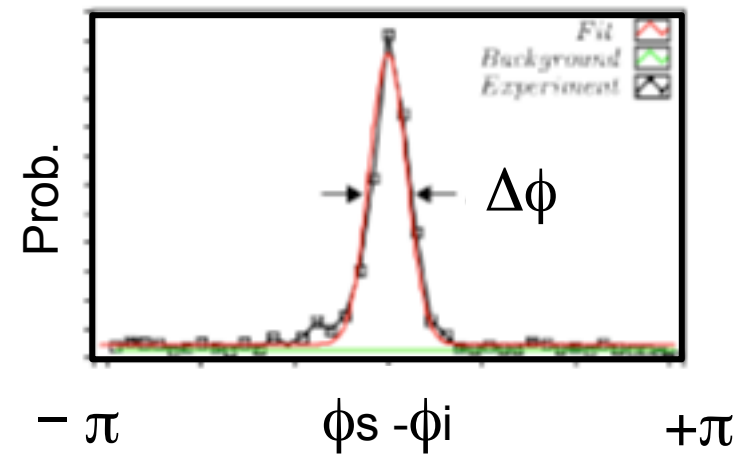
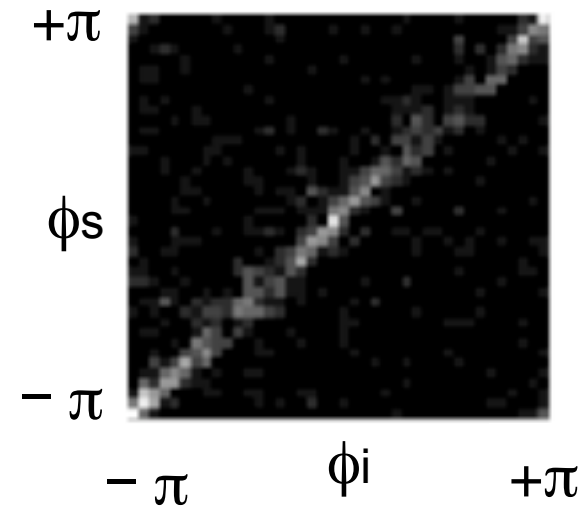
Near perfect (anti)
Correlations in angular
momentum



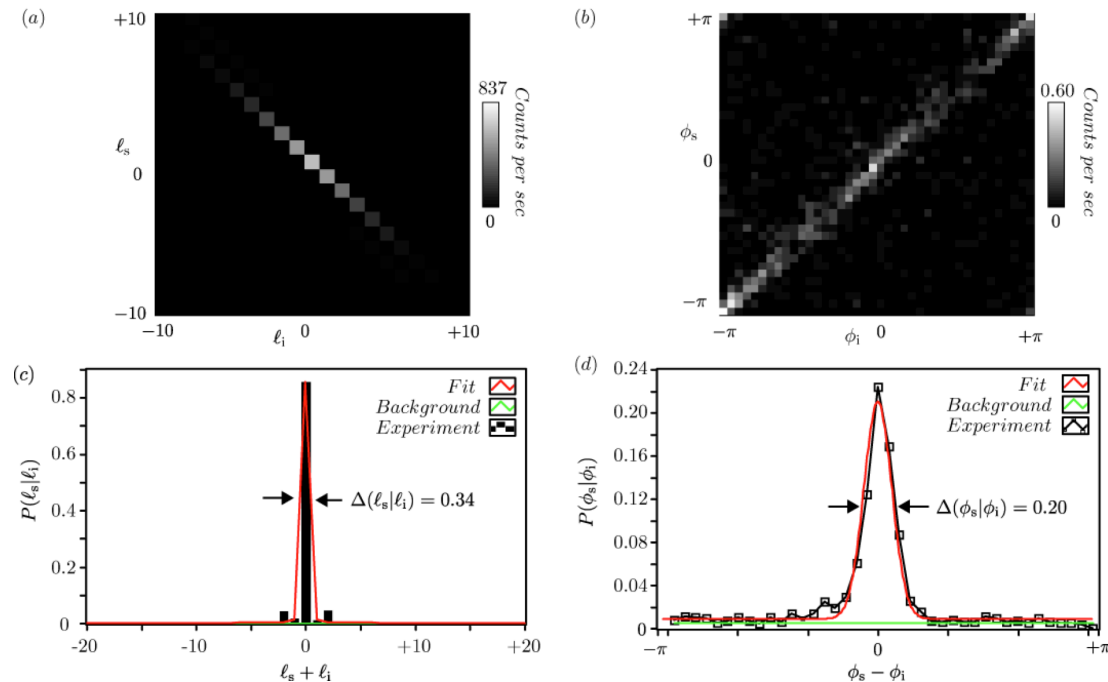
Correlations in angle



Near perfect
Correlations in angle



Angular EPR



Correlations in complimentary basis sets
 -> demonstrates EPR for Angle and Angular momentum

$$\left[\Delta(\ell_s|\ell_i) \hbar \right]^2 \left[\Delta(\phi_s|\phi_i) \right]^2 = 0.00475 \hbar^2 \ll 0.25 \hbar^2$$

Entanglement of OAM states

Proc. R. Soc. Lond. A. **349**, 423–439 (1976)

Printed in Great Britain

Rotary ‘aether drag’

BY R. V. JONES, F.R.S.

Department of Natural Philosophy, University of Aberdeen, Scotland



Optical Activity /Faraday effect for OAM

Sonja Franke-Arnold
Graham Gibson
Emma Wisniewski-Barker

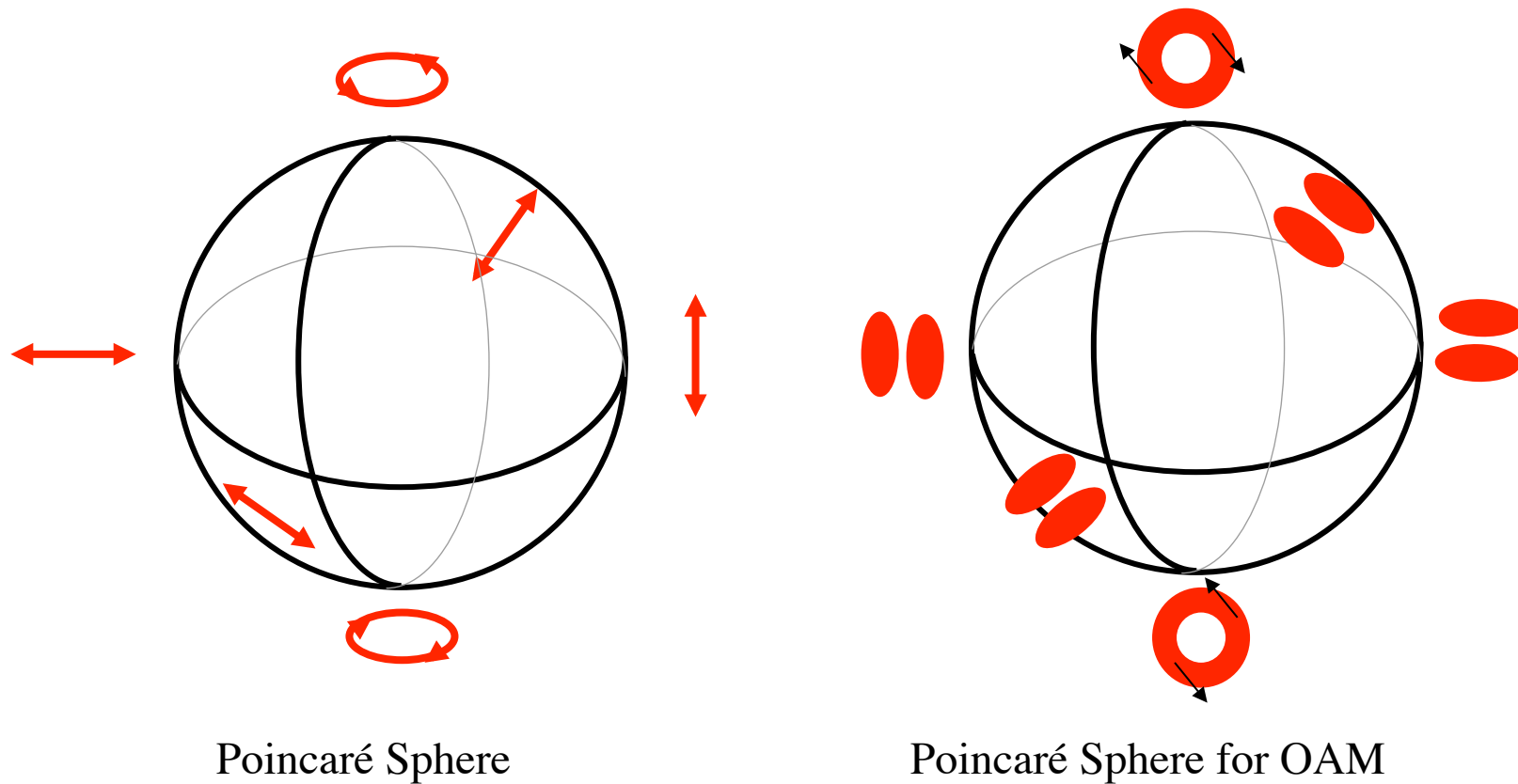


Bob Boyd



Poincaré-sphere equivalent for light beams containing orbital angular momentum

M. J. Padgett and J. Courtial



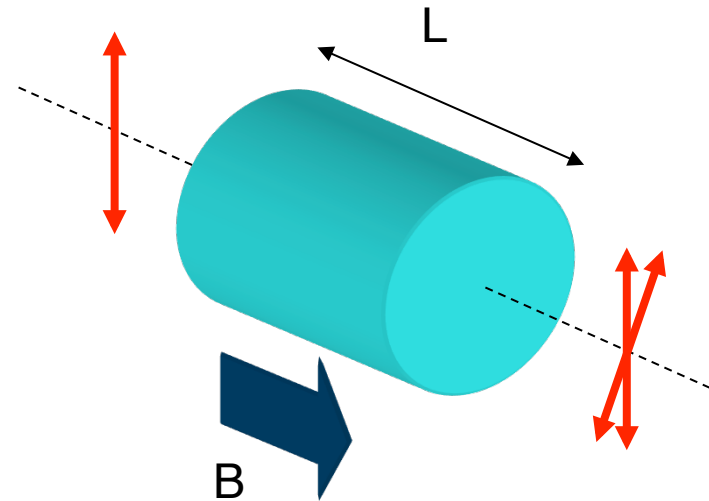
The (Magnetic) Faraday Effect

- Rotation of plane polarised light

$$\Delta\theta_{pol} = BLV$$

- V Verdet constant
- OR treat as phase delay of circularly polarised light

$$\Delta\phi = \sigma BLV$$



$$\Delta\theta = \Delta\phi_{+\sigma, -\sigma} / \Delta\sigma$$

But the magnetic Faraday effect does NOT rotate an Image

Rotary ‘aether drag’

BY R. V. JONES, F.R.S.

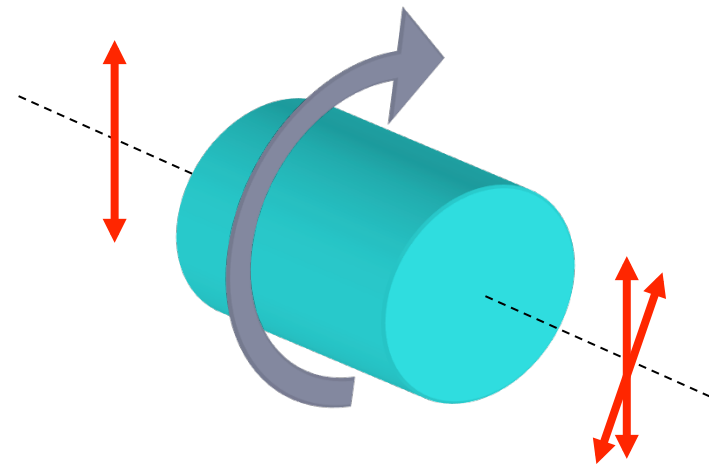
Department of Natural Philosophy, University of Aberdeen, Scotland

- Photon drag, gives Polarisation rotation

$$\Delta\theta = \frac{\Omega L}{c} \left(n_g - 1/n_\phi \right)$$

$$\Delta\phi = \frac{\sigma\Omega L}{c} \left(n_g - 1/n_\phi \right)$$

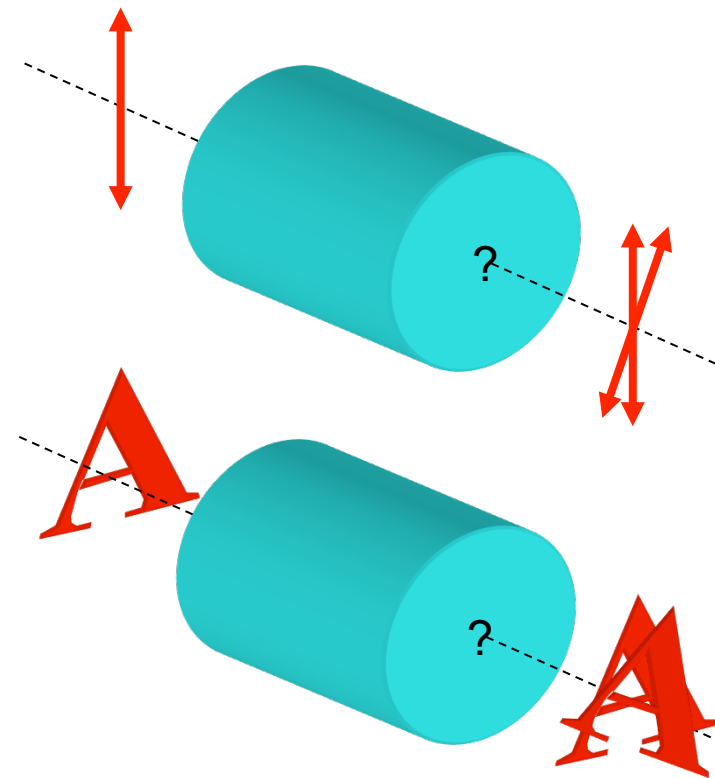
- Mechanical Faraday Effect



Equivalent geometric transformations for spin and orbital angular momentum of light

L. ALLEN^{*†‡} and MILES PADGETT[†]

- SAM \rightarrow Polarisation rotation
- OAM \rightarrow Image rotation
- Look through a Faraday isolator ($\Delta\theta \approx 45^\circ$), is the “world” rotated - NO
 - SAM and OAM are not equivalent in the Magnetic Faraday effect
 - SAM and OAM are not equivalent in the optical activity



Enhancing the effect.....

- Plug in “sensible numbers” and get a micro-radian rotation...
- Increase the group index to enhance the effect

$$\Delta\theta_{image} = \frac{\Omega L}{c} \left(n_g - 1/n_\phi \right)$$

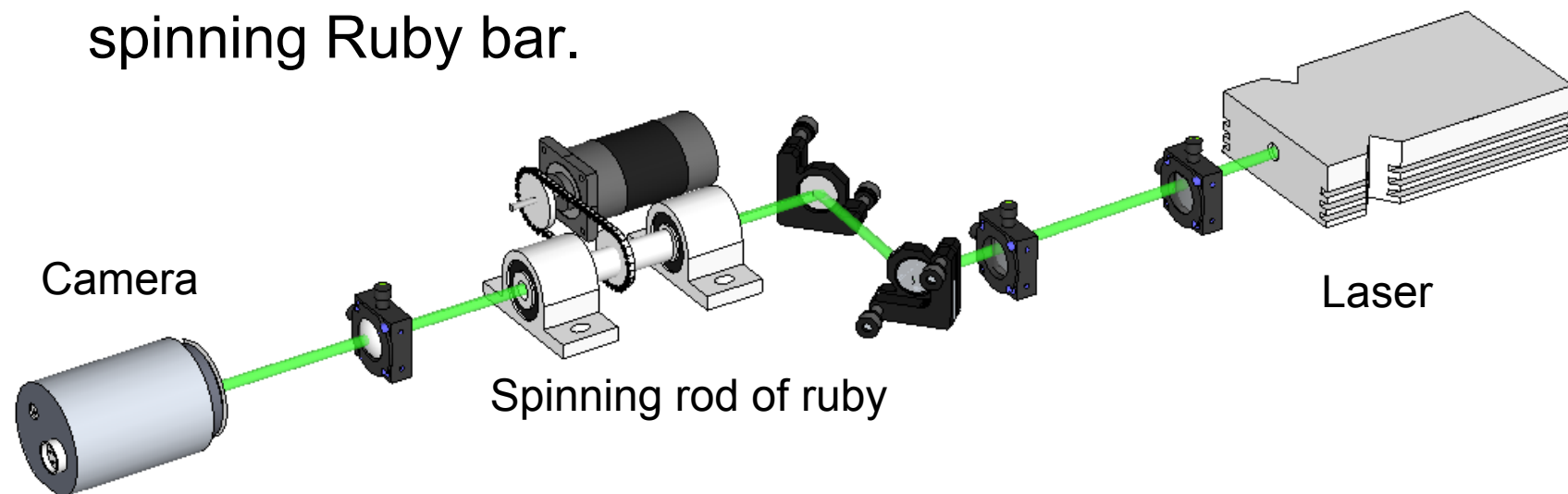
Rotary Photon Drag Enhanced by a Slow-Light Medium

Sonja Franke-Arnold,^{1*} Graham Gibson,¹ Robert W. Boyd,^{2,3} Miles J. Padgett¹

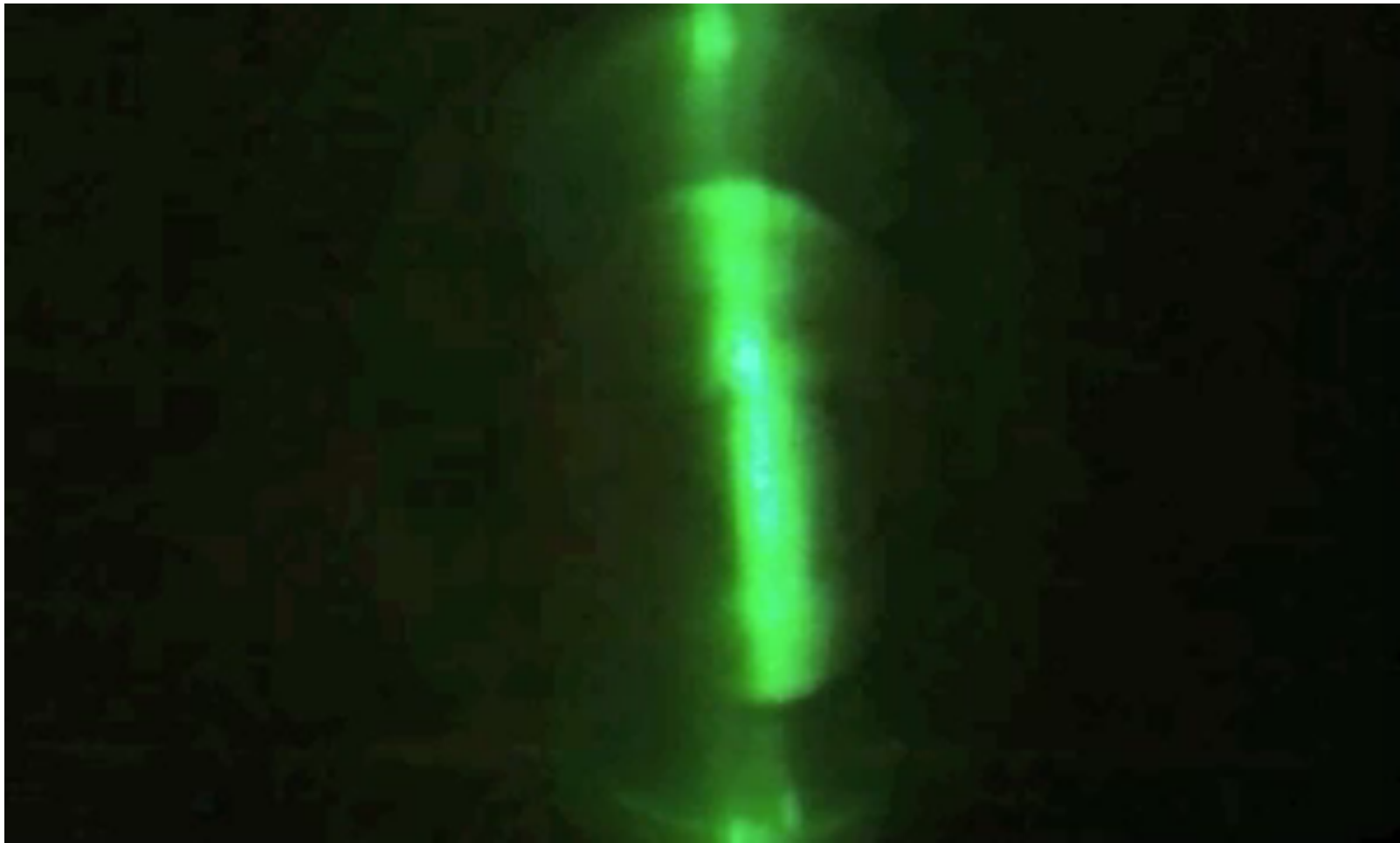
SCIENCE VOL 333 1 JULY 2011



- Shine an elliptical laser beam (\approx LG, $\Delta\ell=2$) @ 532nm through a spinning Ruby bar.



$\approx 25\text{Hz}$ clockwise \leftrightarrow anticlockwise



A beam splitter for OAM

Martin Lavery



Gregorius Berkhout (Leiden)



Johannes Courtial



- Spin angular momentum
 - Circular polarisation
 - $\sigma\hbar$ per photon
- Orbital angular momentum
 - Helical phasefronts
 - $\ell\hbar$ per photon

$$\sigma = +1$$



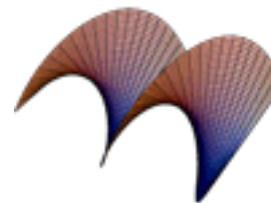
$$\sigma = -1$$



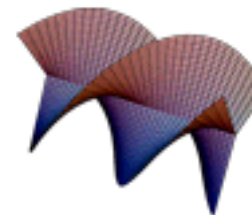
$$\ell = 0$$



$$\ell = 1$$



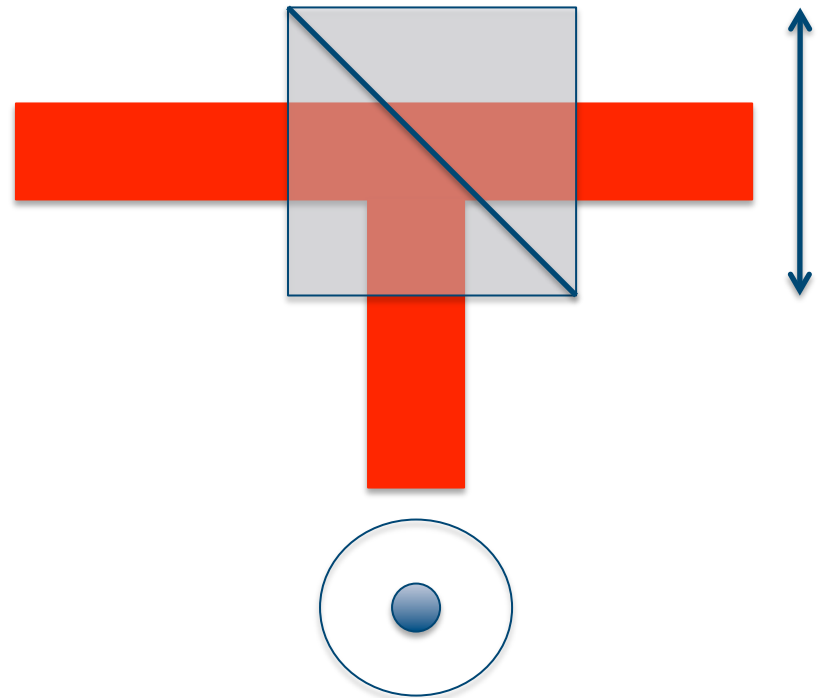
$$\ell = 2$$



$$\ell = 3 \quad \text{etc}$$

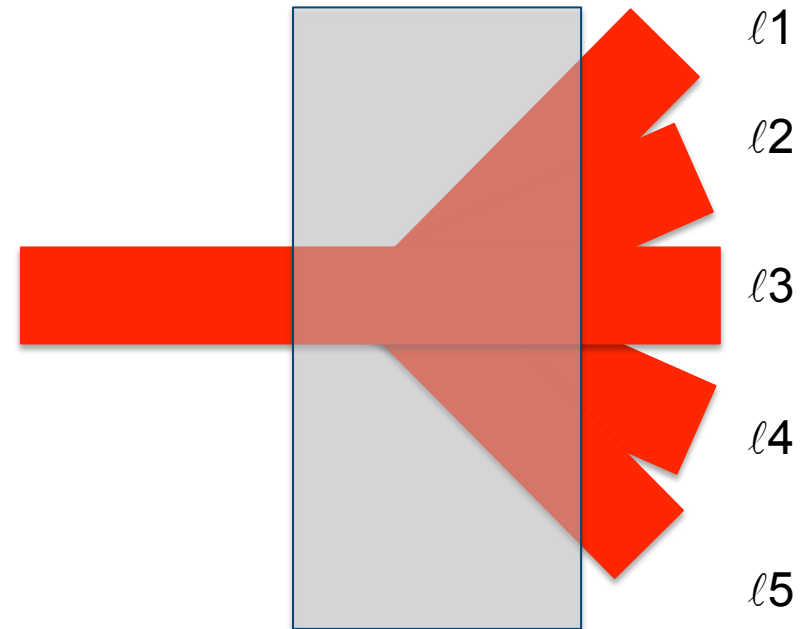
Measuring spin AM

- Polarising beam splitter give the “perfect” separation of orthogonal (linear) states
 - Use quarter waveplate to separate circular states
 - Works for classical beams AND single photons



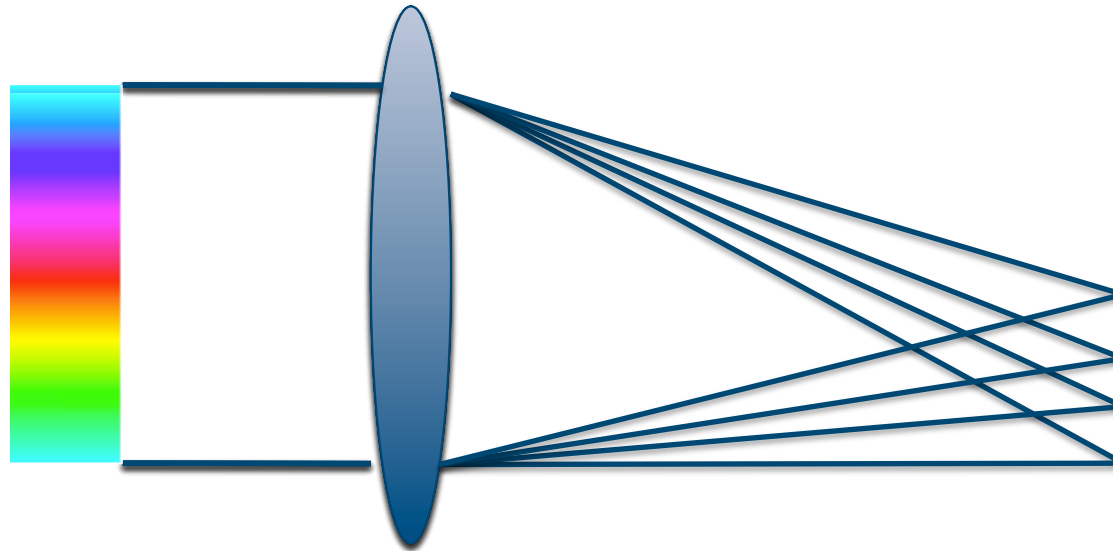
Measuring Orbital AM

- OAM beam splitter give the “perfect” separation of orthogonal states
 - But how?



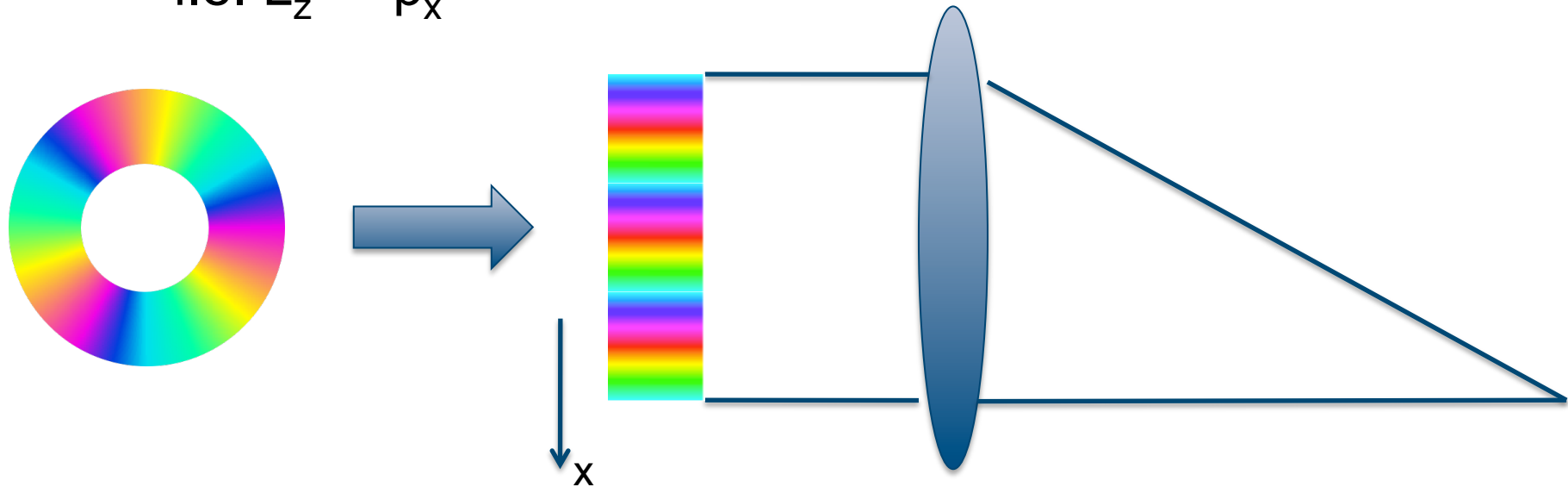
It works for plane waves

- A “plane-wave” is focused by a lens
- A phase ramp of 2π displaces the spot



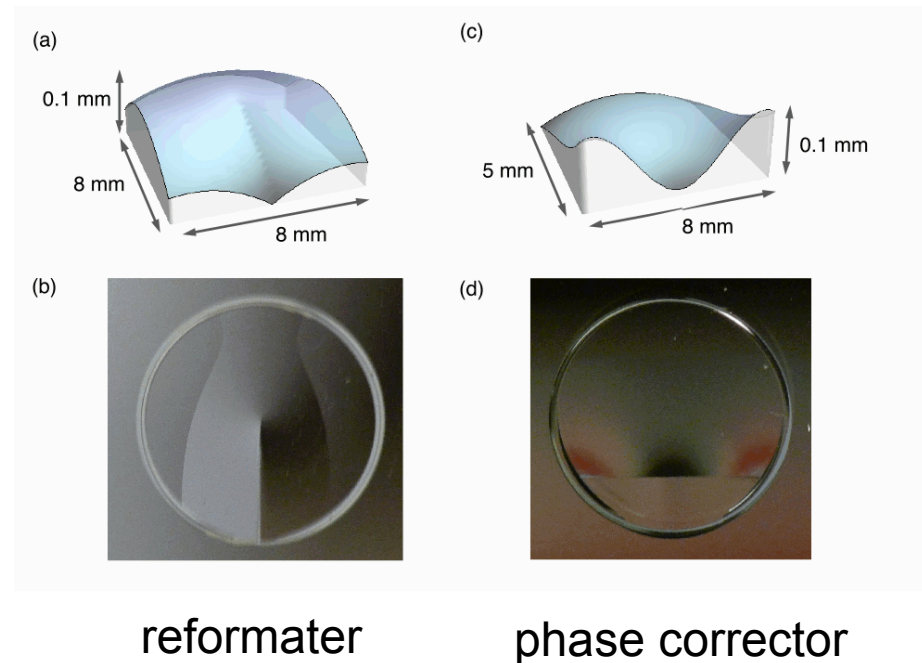
It works for plane waves

- Image transformation
 - $\phi \rightarrow x$ and $r \rightarrow y$
 - i.e. $L_z \rightarrow p_x$



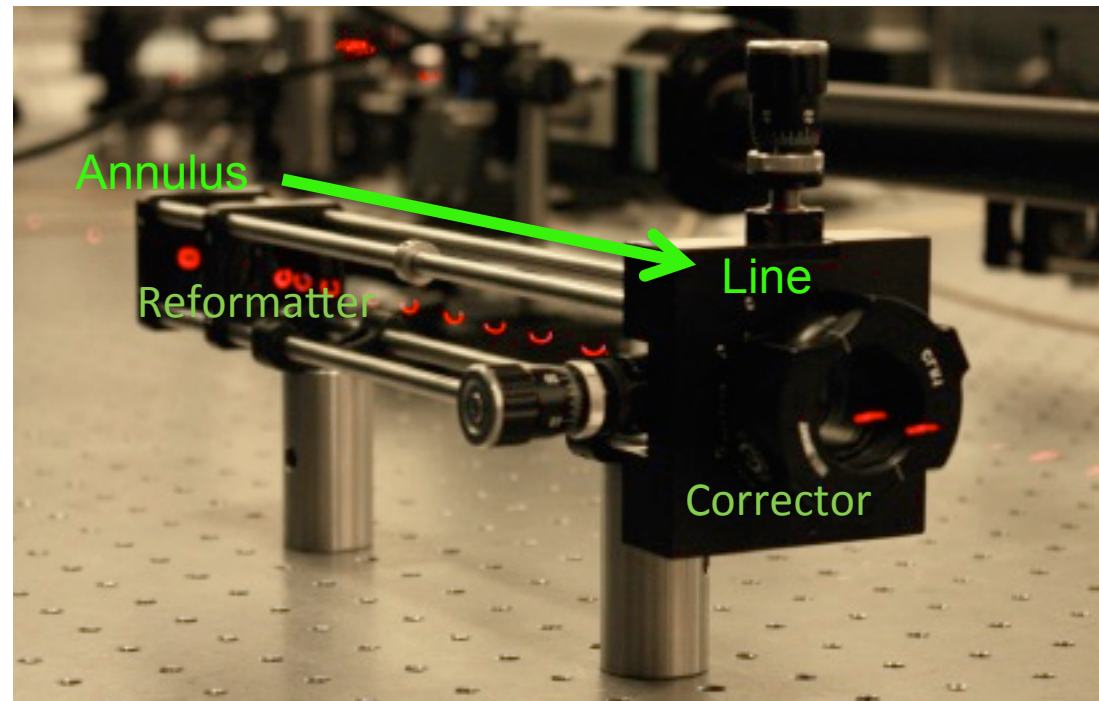
Replacing the SLMs

- The principle works
- But the SLMs are inefficient ($\approx 50\% \times 2$)
- Use bespoke optical elements (plastic)
 - Prof. David J Robertson
 - Prof. Gordon Love

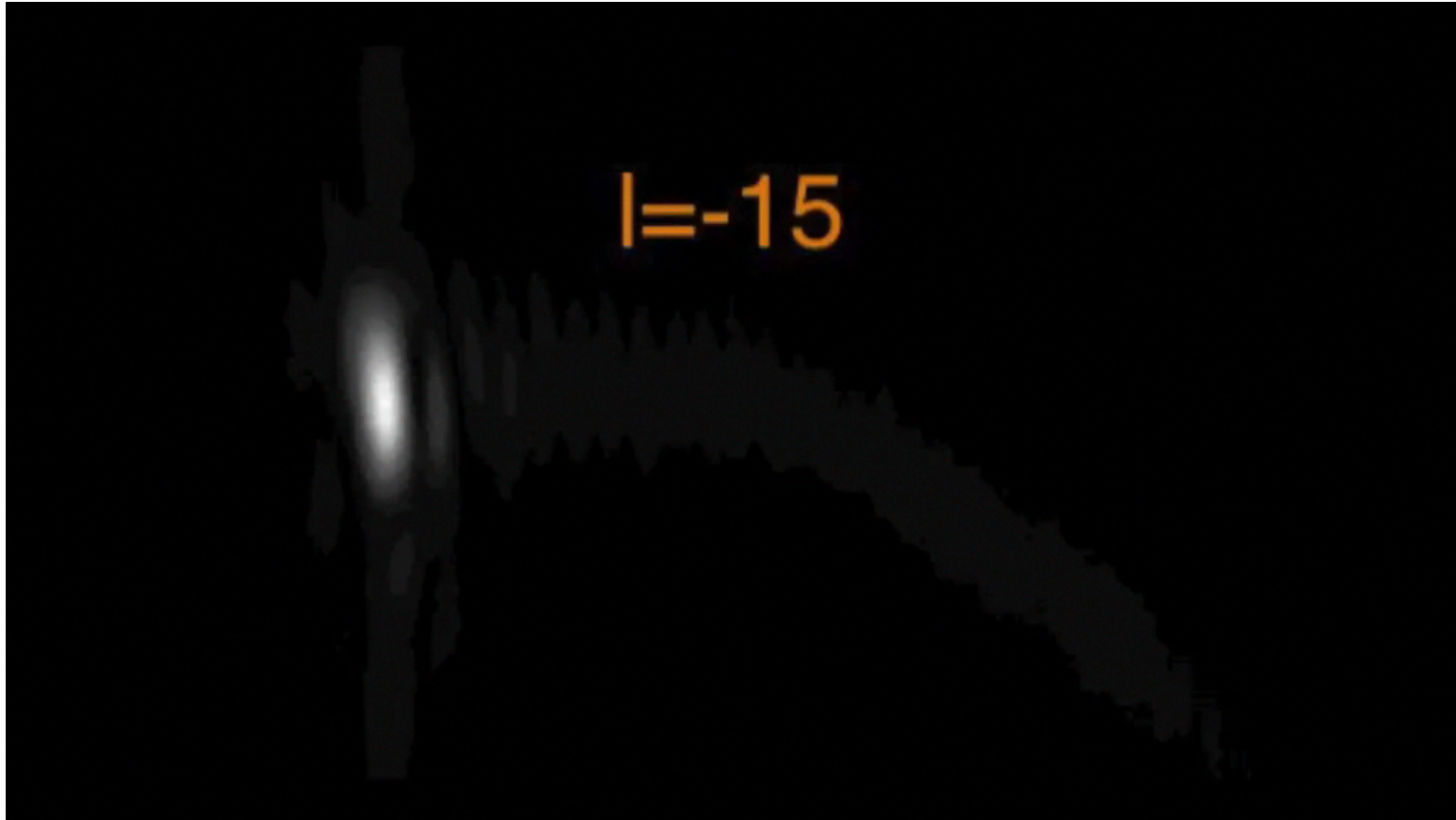


Doughnut to hot-dog

- The principle works
- But the SLMs are inefficient ($\approx 50\% \times 2$)
- Use bespoke optical elements (glass/plastic)
 - Prof. David J Robertson
 - Prof. Gordon Love



The output

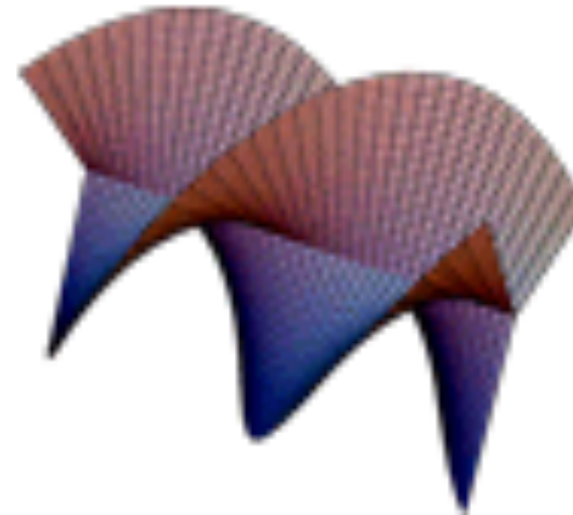


Evolution in time c.f. translation and rotation



$$\Phi = f(kz + \omega t)$$

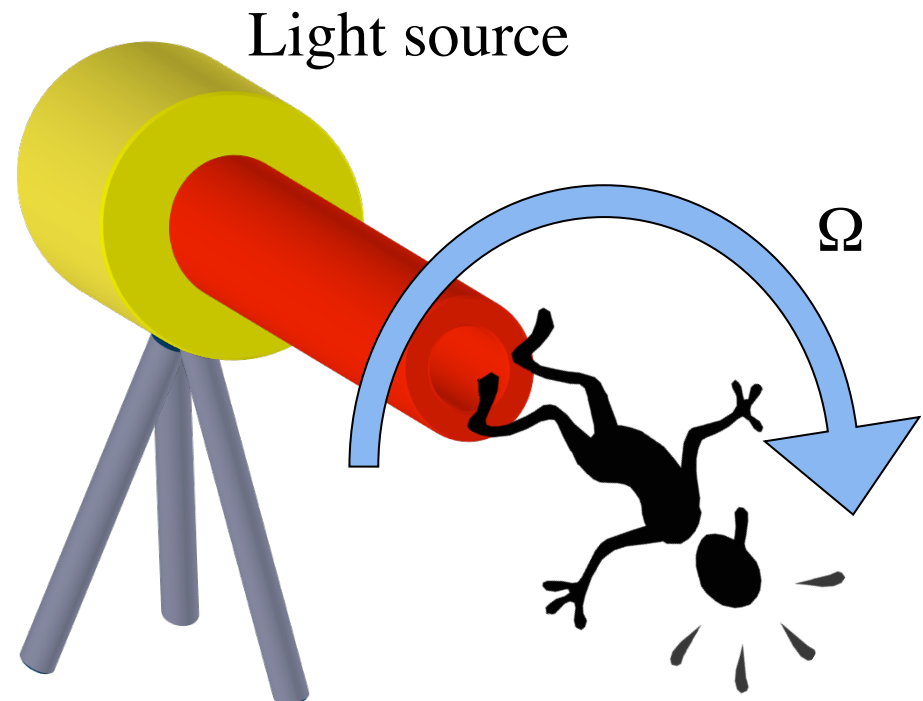
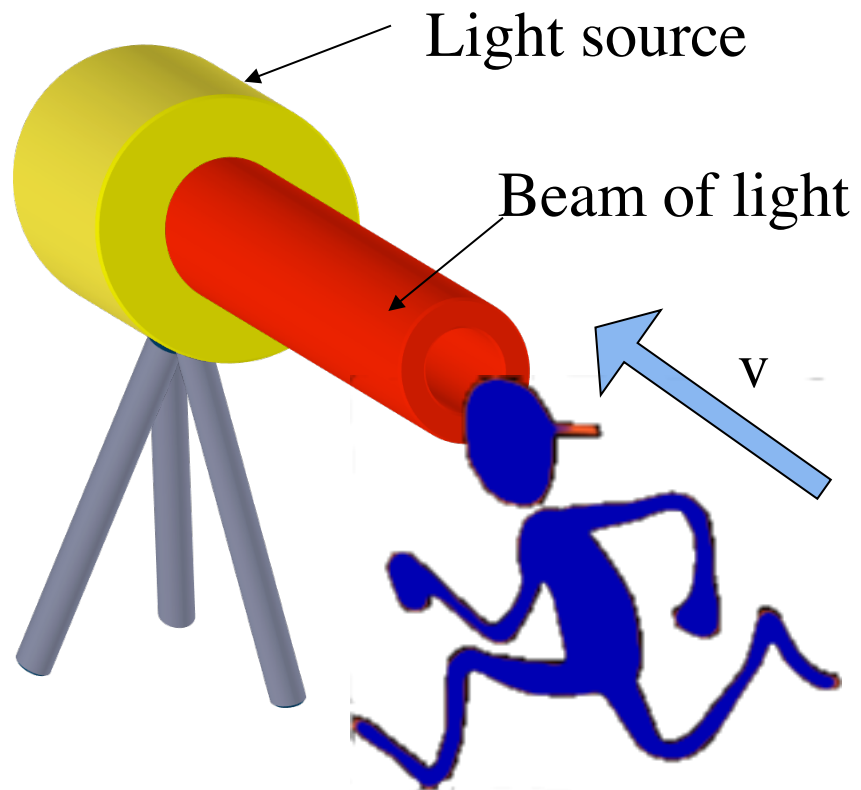
$\ell = 0$ time c.f. translation



$$\Phi = f(kz + \omega t + \ell \theta)$$

$\ell > 0$ time c.f. rotation

Linear vs. Rotational Doppler shifts



Rotational Frequency Shift

Iwo Białynicki-Birula
Center for Theoretical Physics, Lotników 32/46, 02-668 Warsaw, Poland
 Zofia Białynicka-Birula

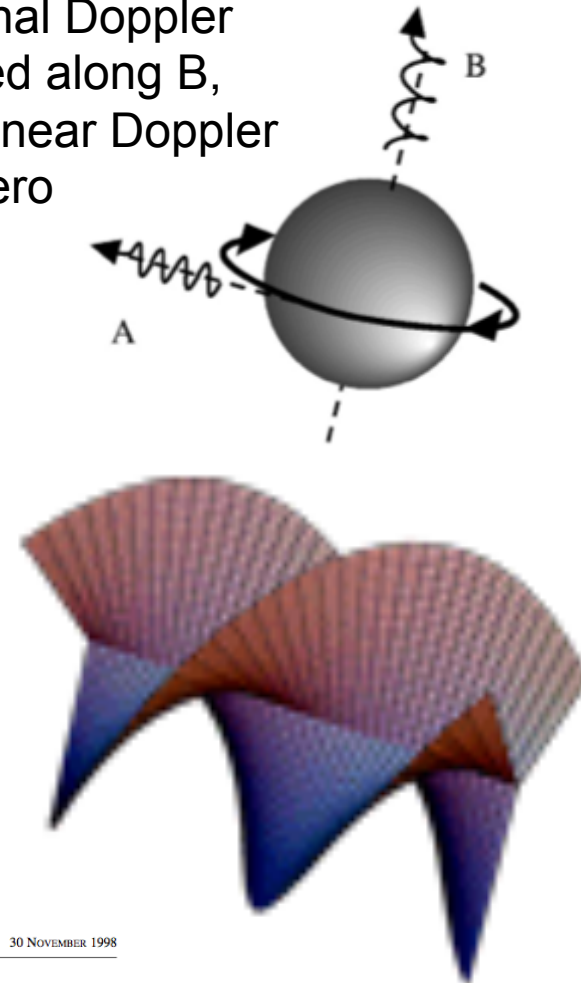
Doppler effect induced by rotating lenses

Gerard Nienhuis
Huygens Laboratorium, Rijksuniversiteit Leiden, Postbus 9504, 2300 RA Leiden, The Netherlands
 Received 8 February 1996; accepted 24 April 1996

Rotational (angular) Doppler shift

- For pure OAM states a rotation of frame between source and observer give a frequency shift
 - $\Delta\omega_\ell = \Omega \ell$
- Rotation of the state “looks-like” an advance in time, but...
- The rotational symmetry stays the same.....

Rotational Doppler observed along B, where linear Doppler (A) is zero



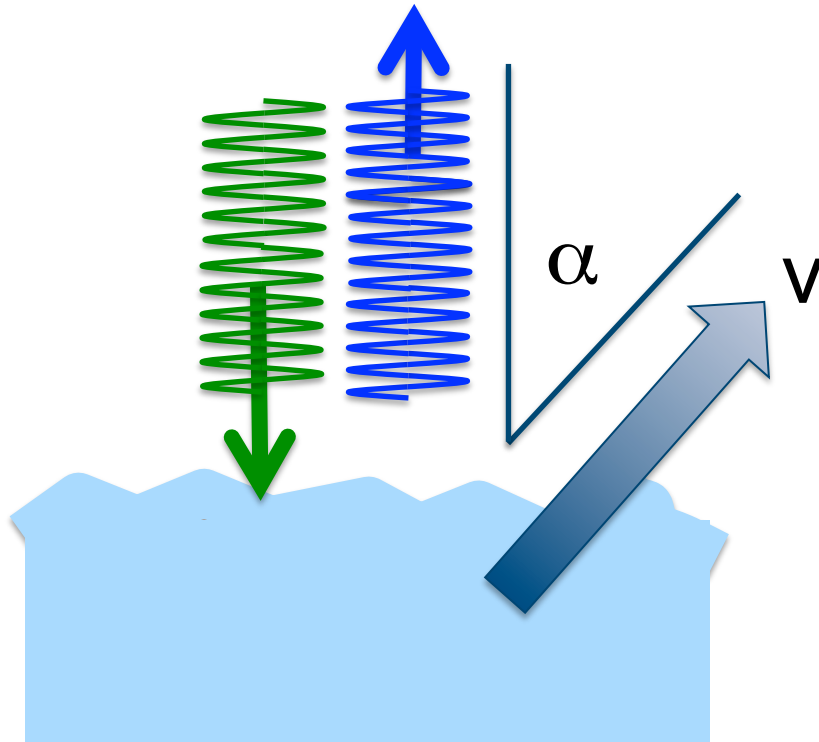
Rotational Frequency Shift of a Light Beam

J. Courtial, D.A. Robertson, K. Dholakia, L. Allen, and M.J. Padgett

Where shall we start?

- Light scattered by a moving body is shifted in both energy ($\hbar\omega$) and linear momentum ($\hbar k$).
 - Doppler Shift
- Doppler shift is used to remotely detect the movement of a distant body
- How might we use the OAM, what might it detect?

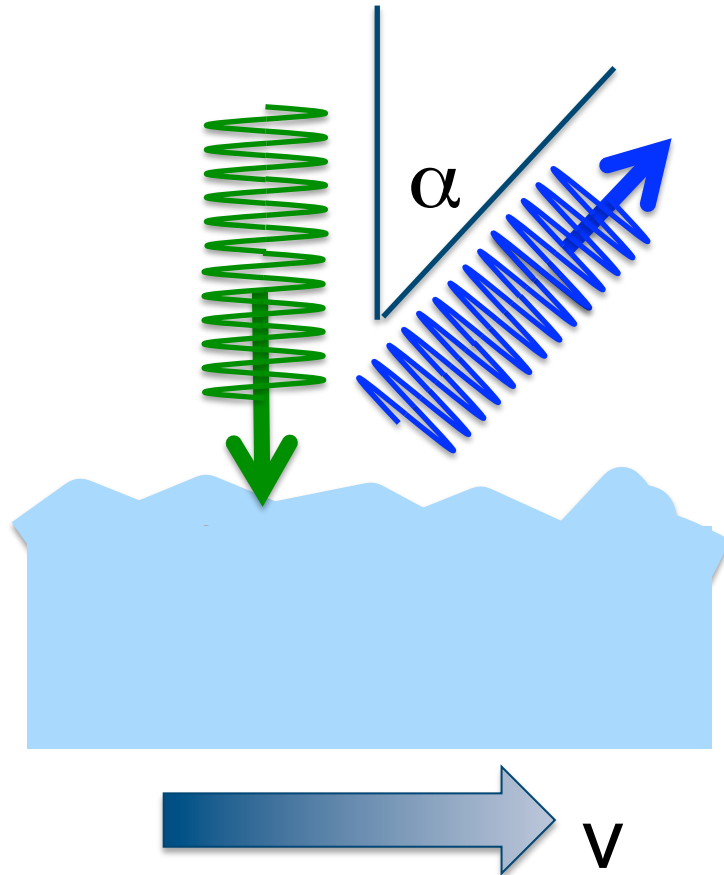
Doppler shift from a moving surface



$$\Delta\omega = 2\cos\alpha \omega_0 v/c$$

when $\alpha = \pi/2$, $\Delta\omega = 0$

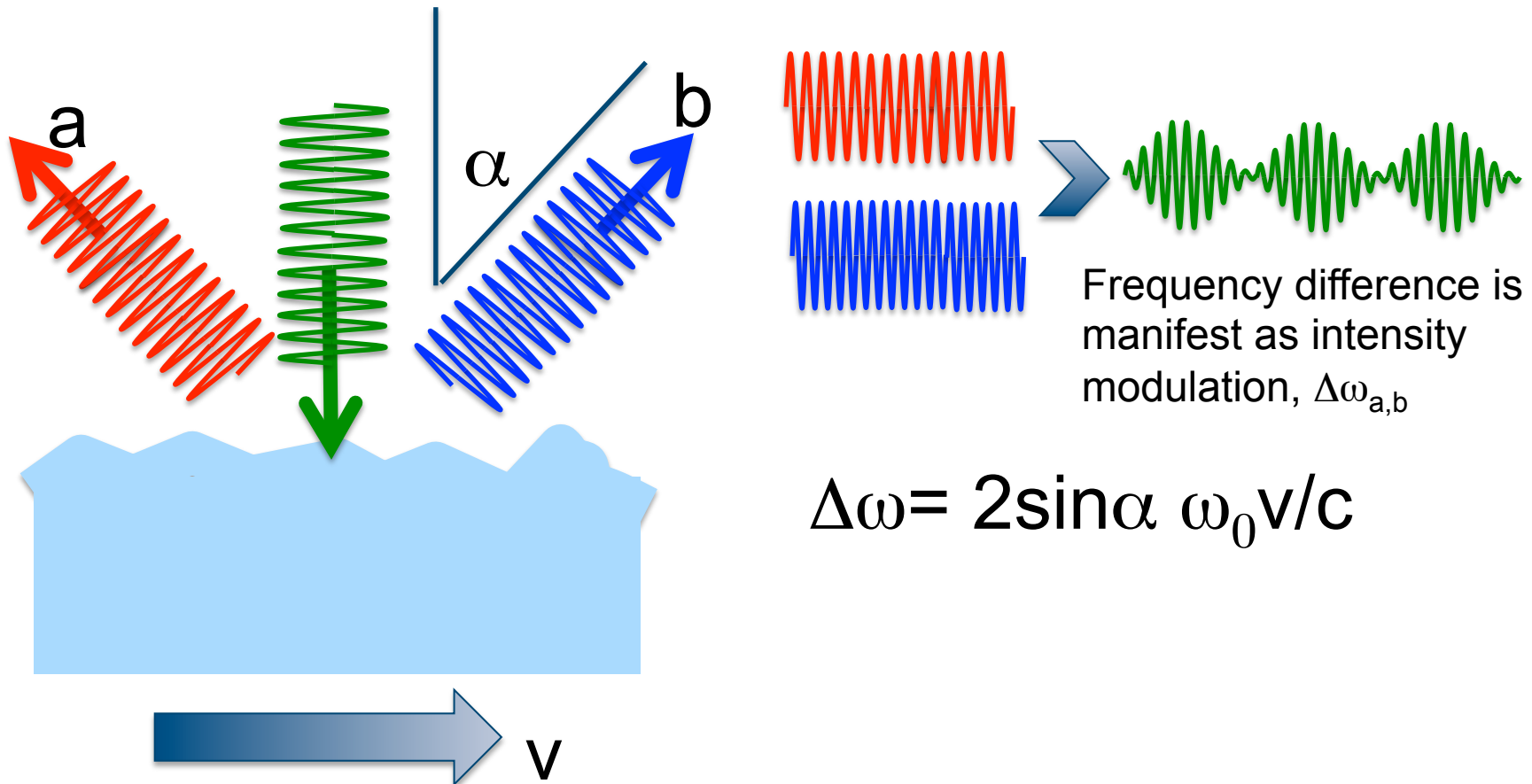
Doppler shift from translating surface



$$\Delta\omega = \sin\alpha \, \omega_0 v/c$$

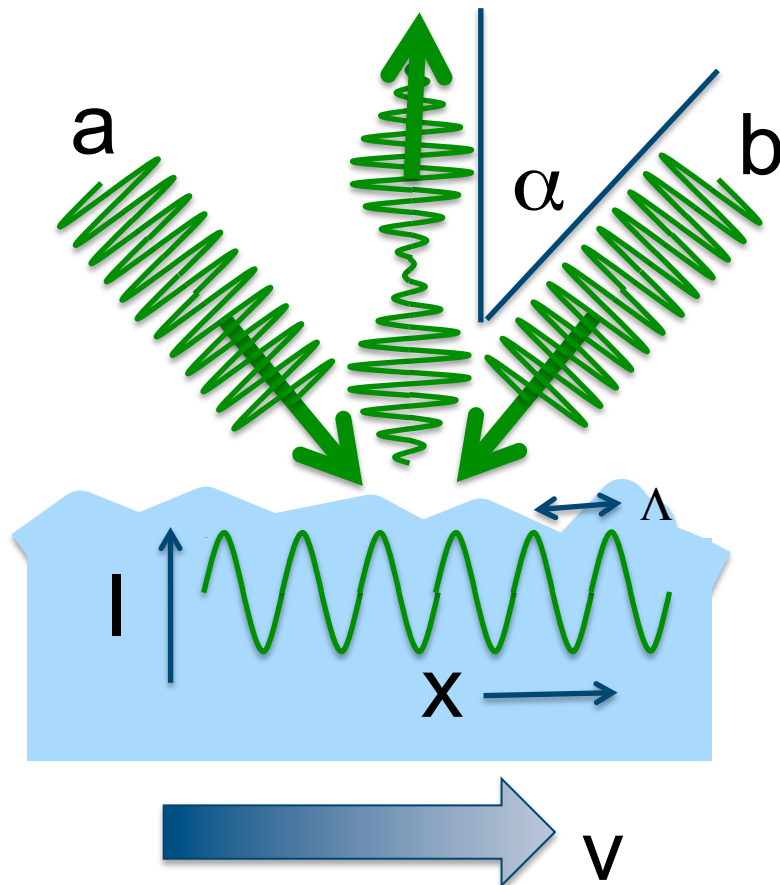
Basis of Doppler velocimetry

Doppler velocimetry (frequency domain)



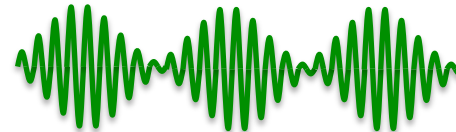
Illuminate on-axis, detect off-axis

Doppler velocimetry (time domain) – projected fringes



$$\Lambda = \lambda / 2 \sin \alpha$$

Scattering centres move across the fringe pattern give intensity modulation of scattered light, $\Delta\omega_{\Lambda}$

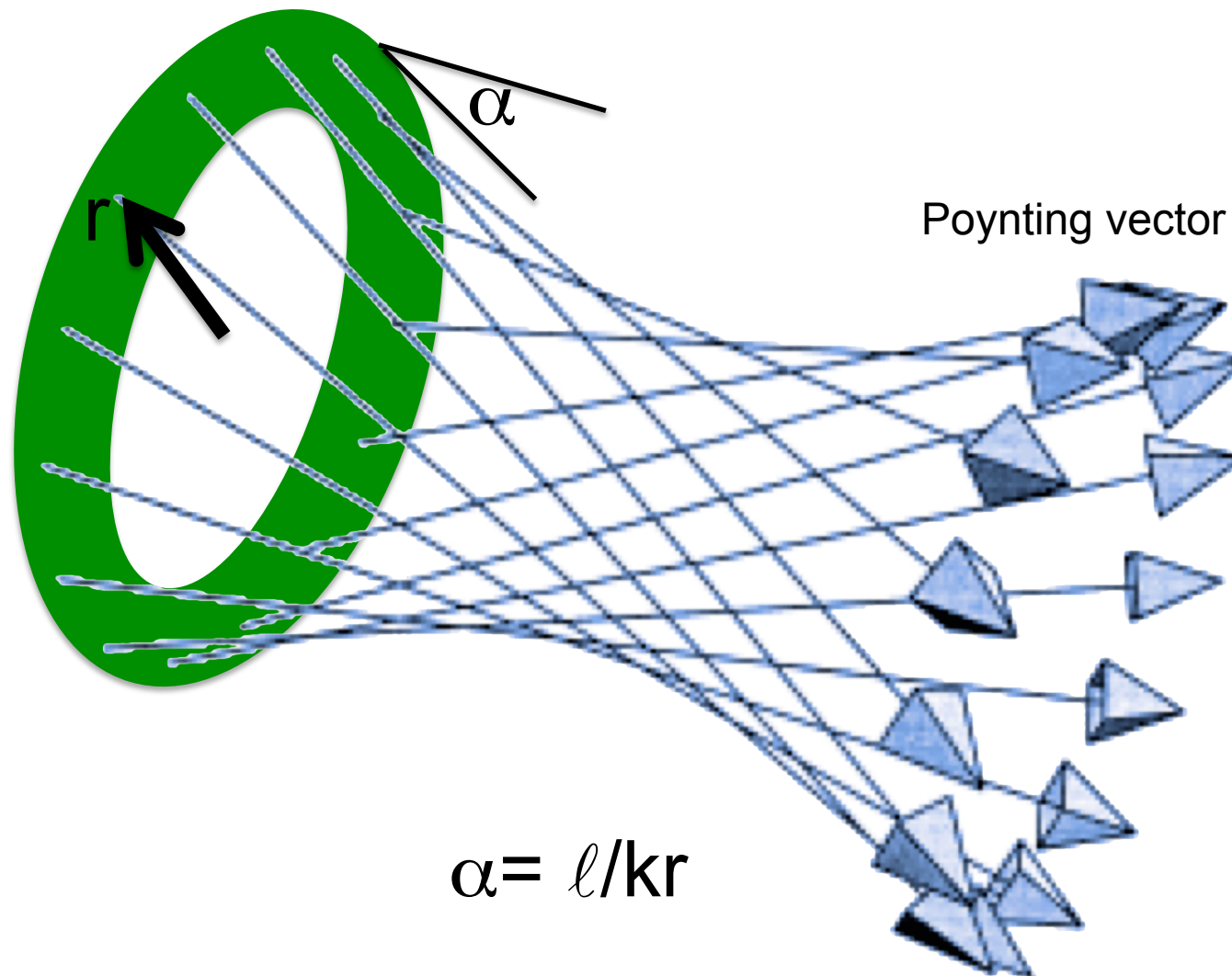


$$\Delta\omega_{\Lambda} = 2\pi v / \Lambda$$

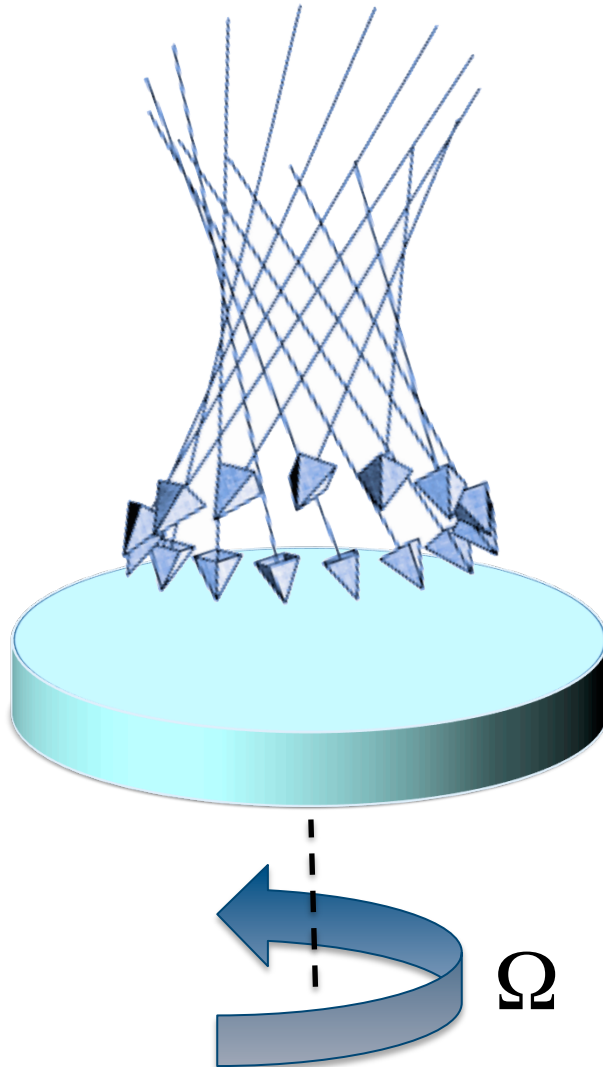
Illuminate off-axis, detect on-axis

$$\Delta\omega_{a,b} = \Delta\omega_{\Lambda}$$

Orbital angular momentum -> Skew rays, $\alpha \neq 0$



Doppler shift from a SPINNING surface



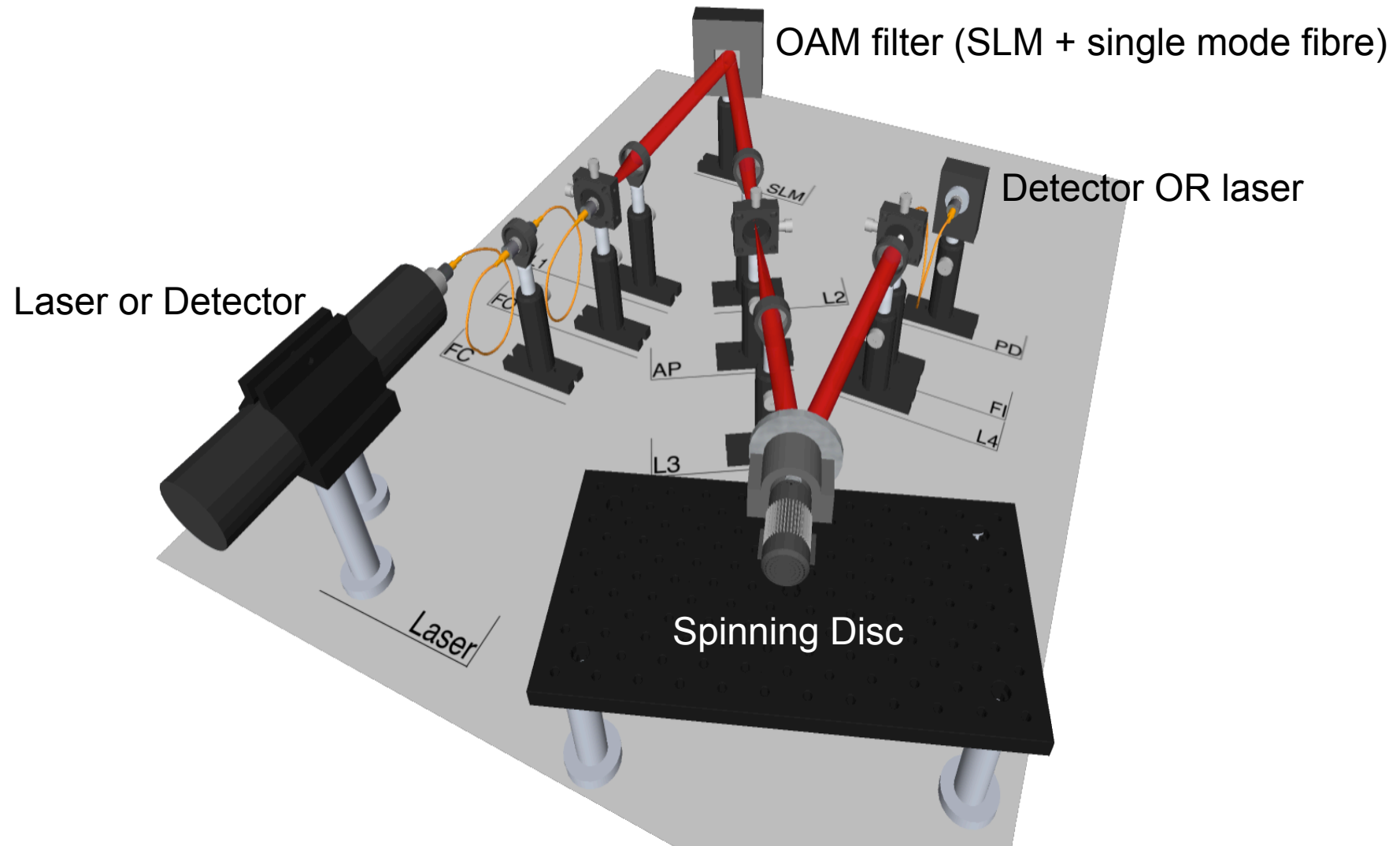
c.f. $\Delta\omega = \sin\alpha \, \omega_0 v/c$

$\sin\alpha \approx \ell/k r$ & $v = \Omega r$

$\Delta\omega_\ell = \Omega \ell$

$\Delta\omega_{\ell, -\ell} = 2\Omega \ell$

Experimental arrangement

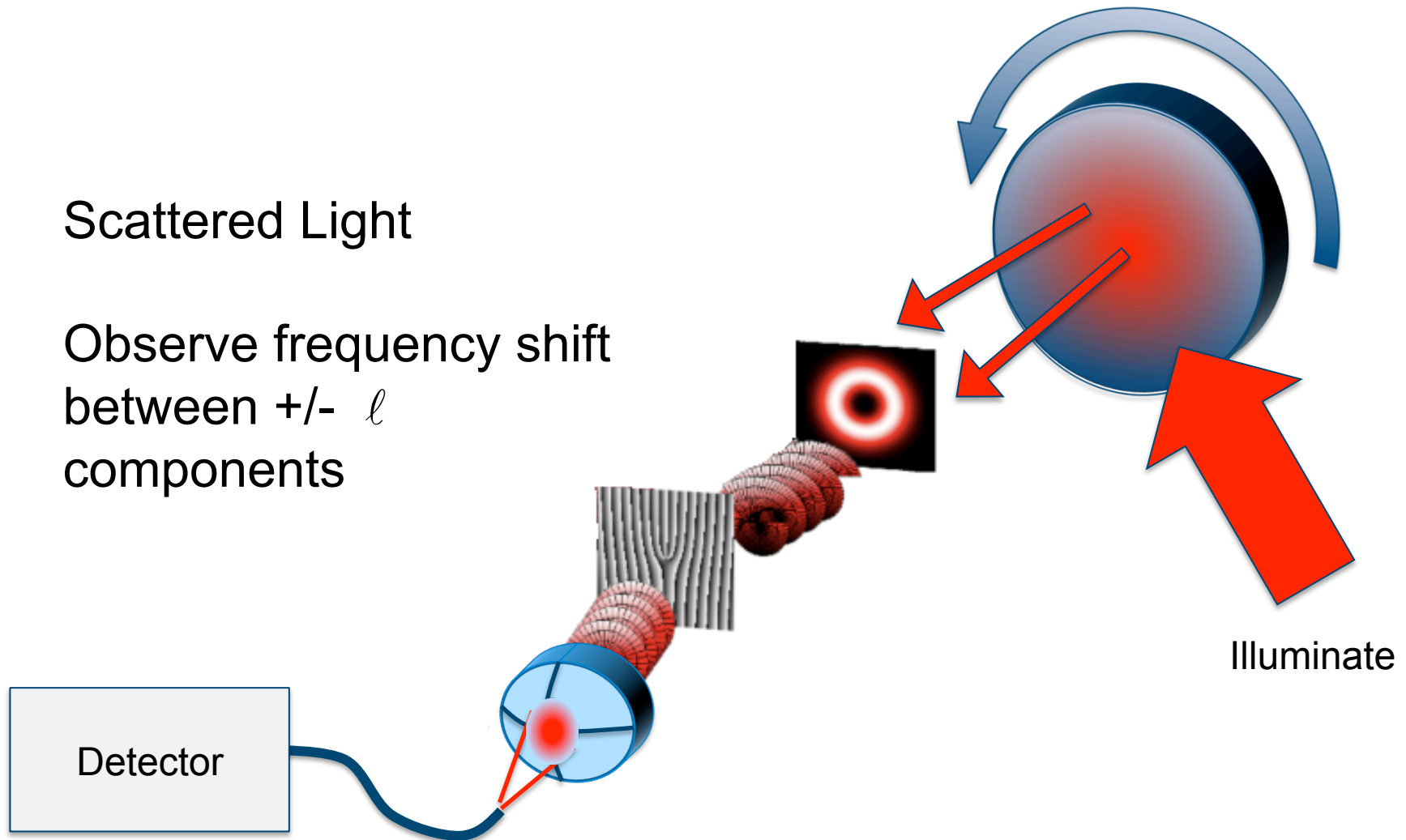




The Rotational Frequency shift of Scattered Light

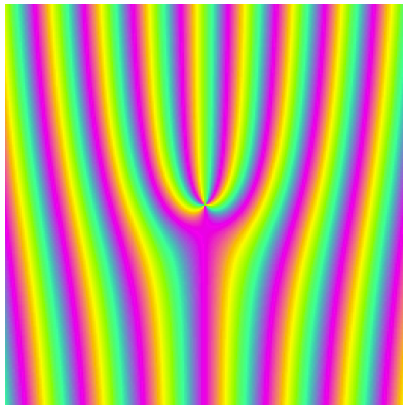
Scattered Light

Observe frequency shift
between $\pm \ell$
components

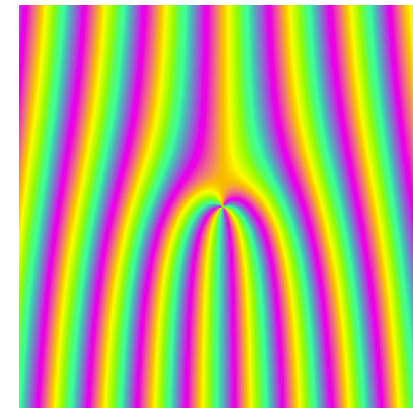


Making/Measuring OAM

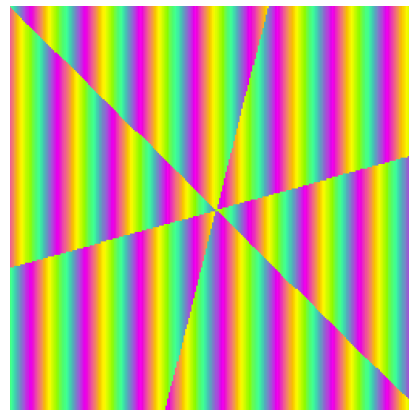
Diffraction grating
(hologram) to make/
measure $\ell=3$



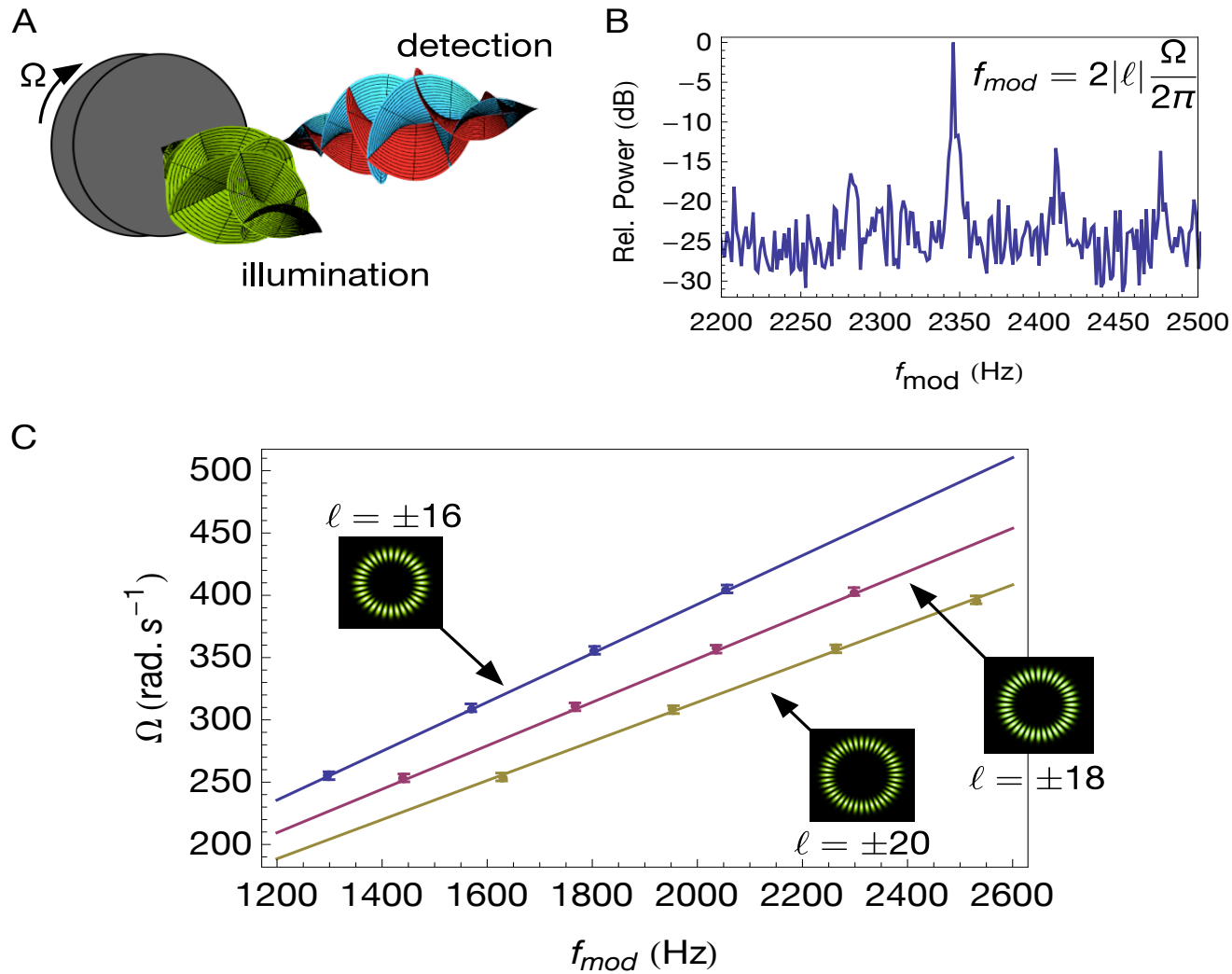
Diffraction grating
(hologram) to make/
measure $\ell=-3$



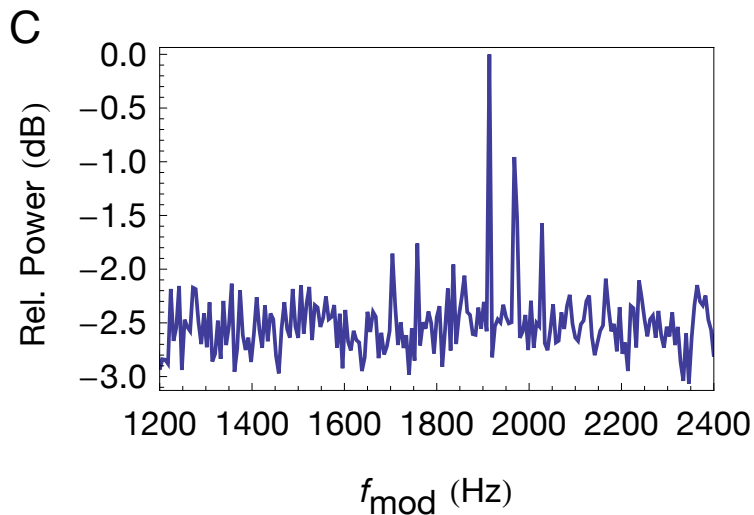
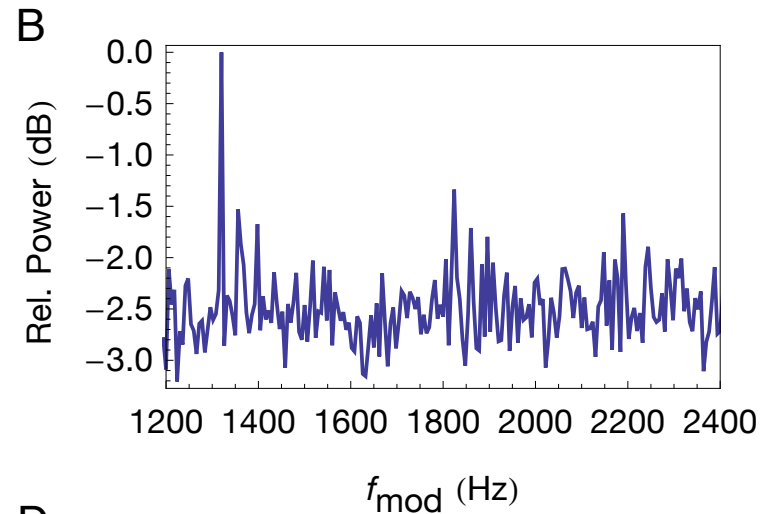
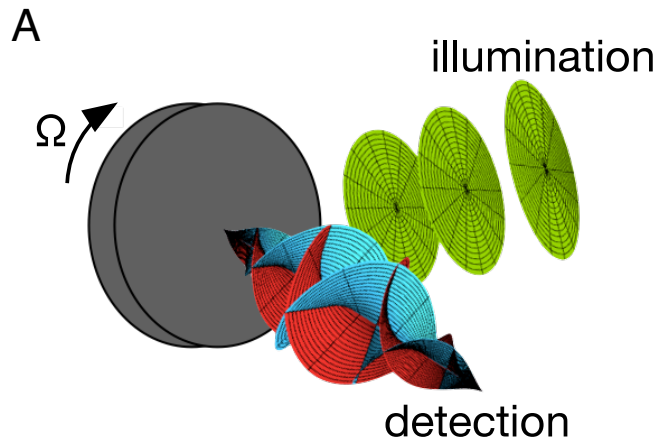
Diffraction grating
(hologram) to make/
measure $\ell=+/-3$



Illuminate with OAM at $\pm \ell$ and measure $\Delta\omega$



Illuminate with $\approx \ell=0$, detect OAM at $\pm \ell$ and measure $\Delta\omega$



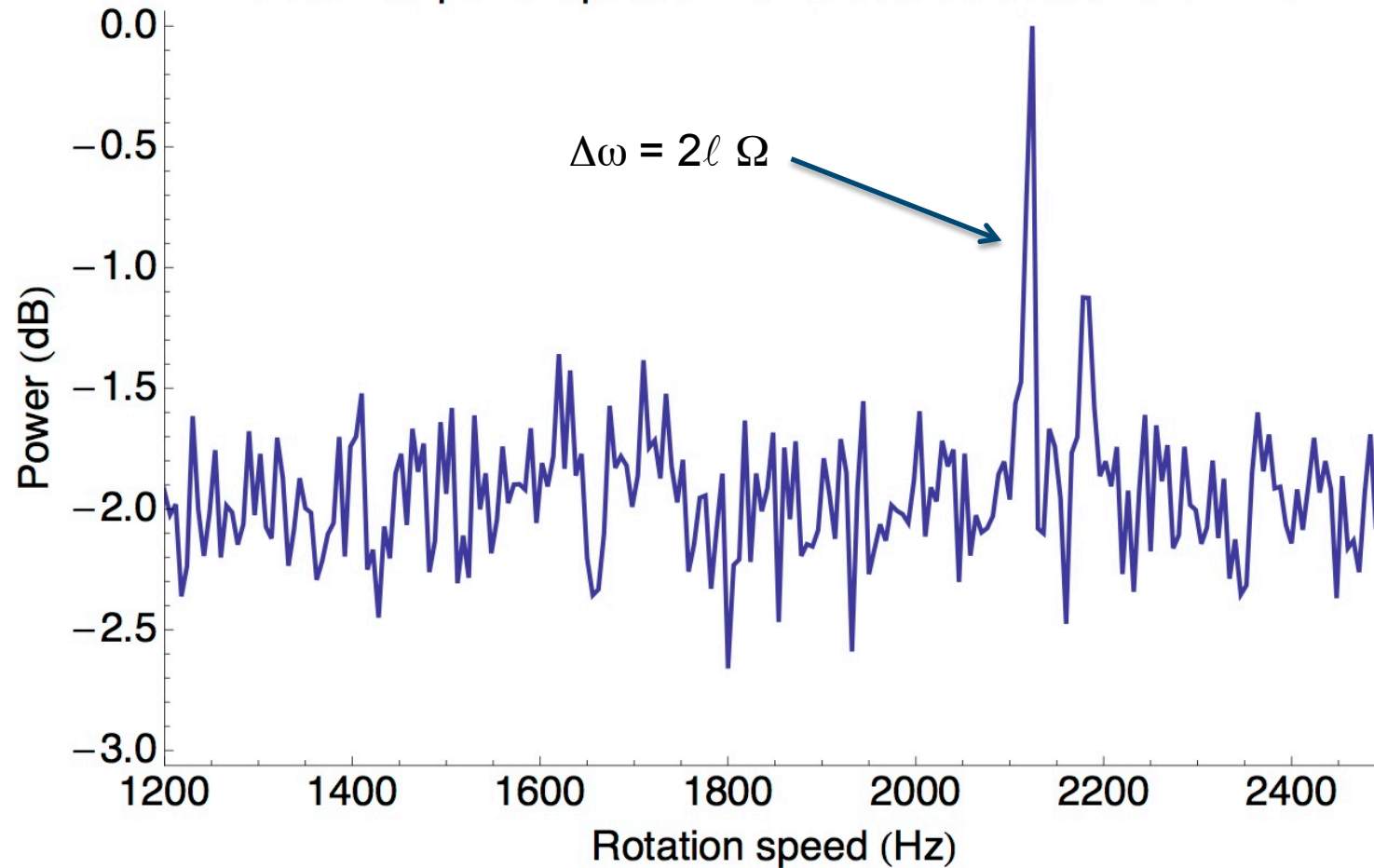
Detection of a Spinning Object Using Light's Orbital Angular Momentum

Martin P. J. Lavery,^{1*} Fiona C. Speirits,² Stephen M. Barnett,² Miles J. Padgett¹

SCIENCE VOL 341 2 AUGUST 2013

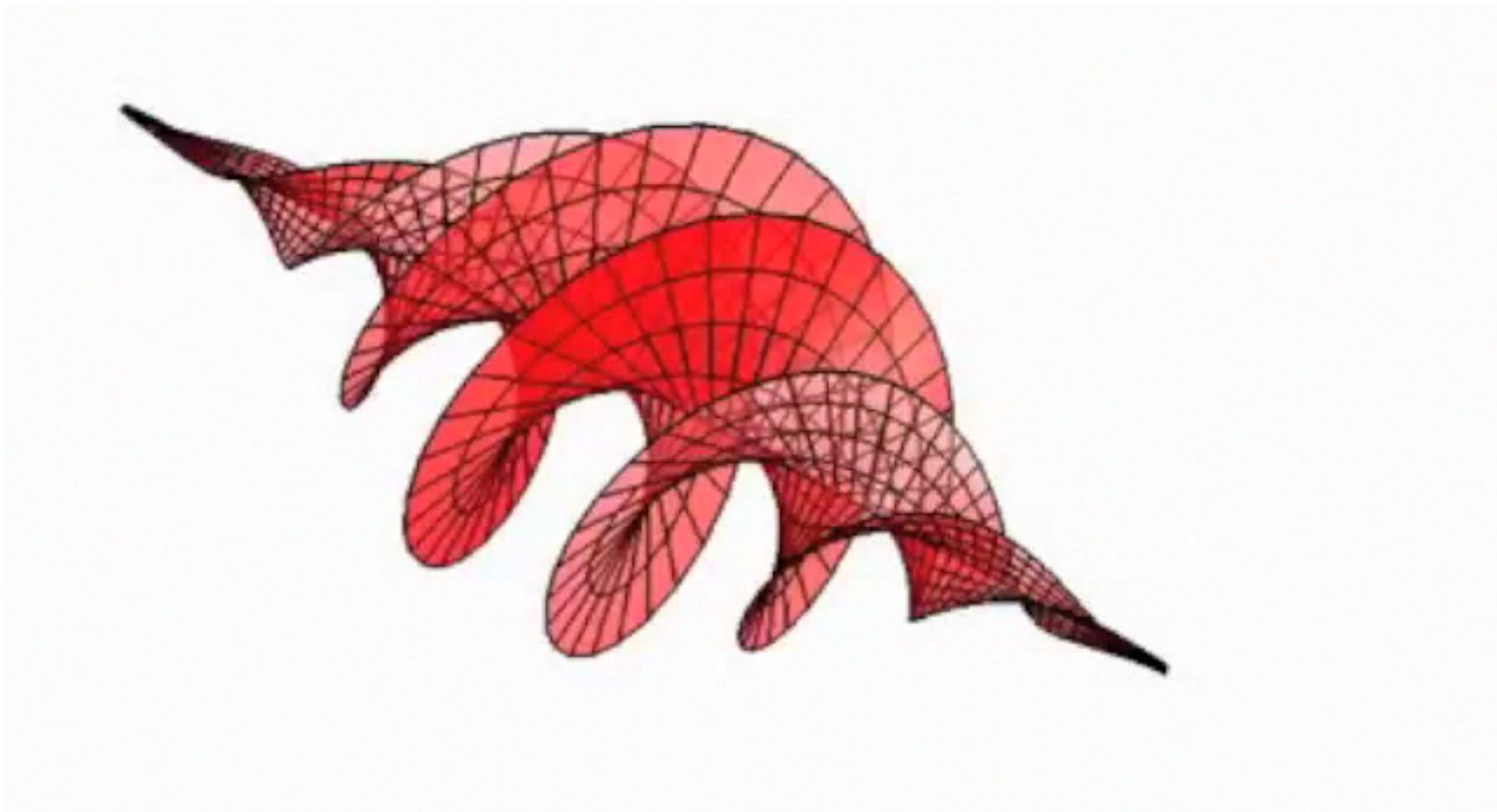
Rotational Doppler shift in scattered light

Observed power spectrum for structured section of $\ell = \pm 18$



c.f. Speckle velocimetry? Albeit, in this case, angular

Thank you to you and my Group



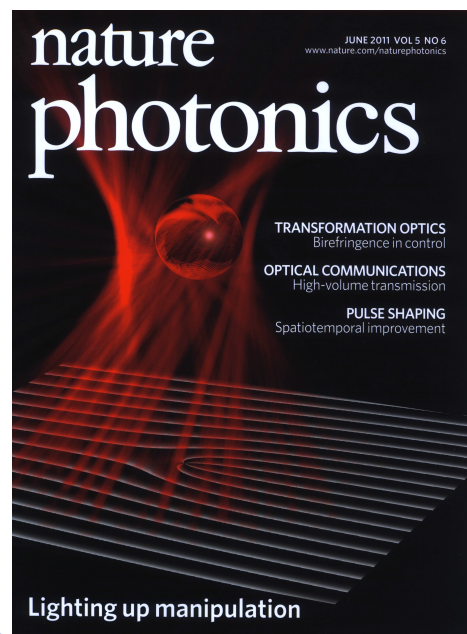
If you would like a copy of this talk please ask me

www.gla.ac.uk/schools/physics/research/groups/optics/

2008



2011



2012

