Long range interactions in quantum gases
a tutorial

Tilman Pfau
5. Physikalisches Institut – Universität Stuttgart

5. Physikalisches Institut

STUTTGART

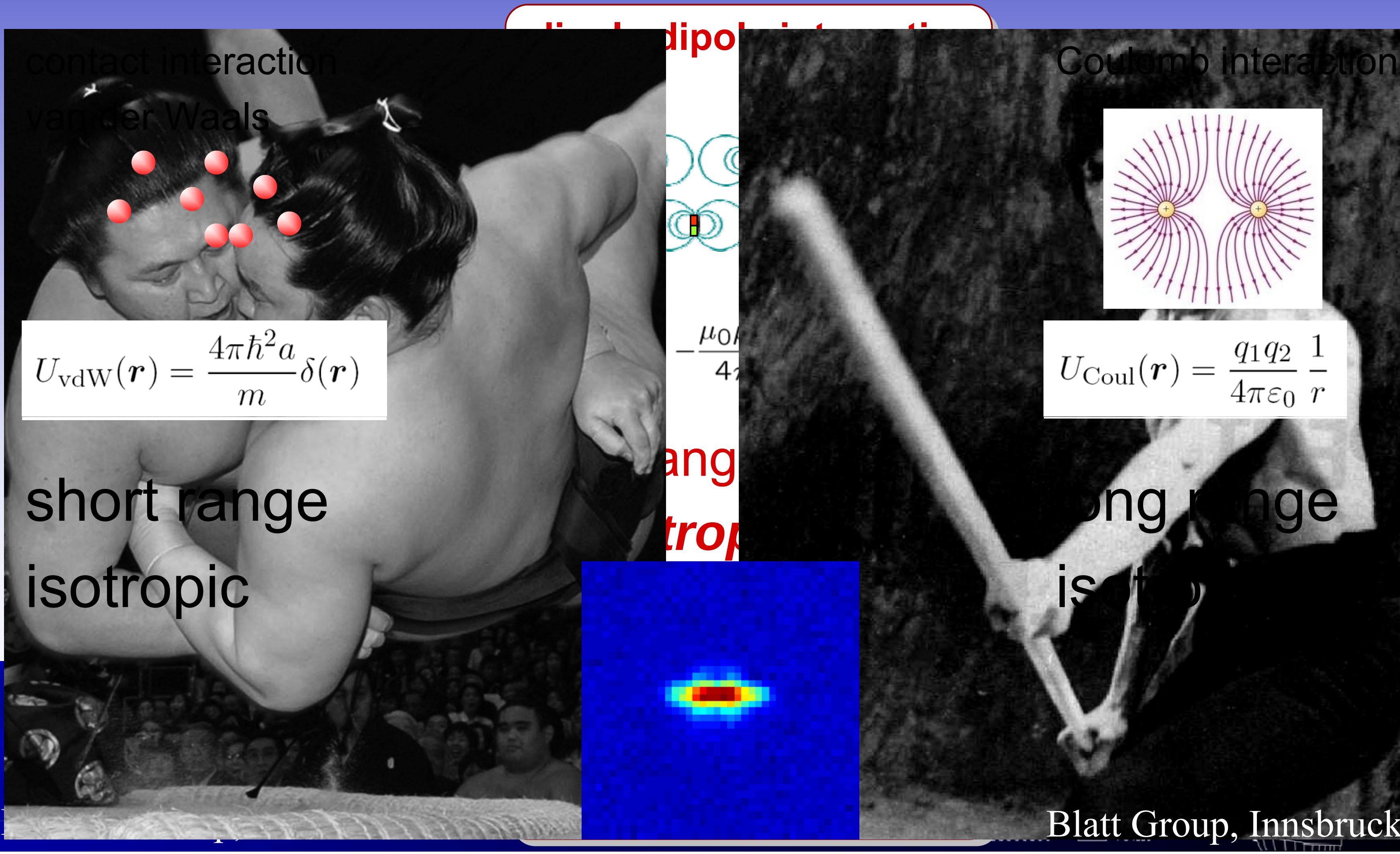
CO.CO.MAT
SFB/TRR 21

IQ^ST Integrated Quantum Science and Technology

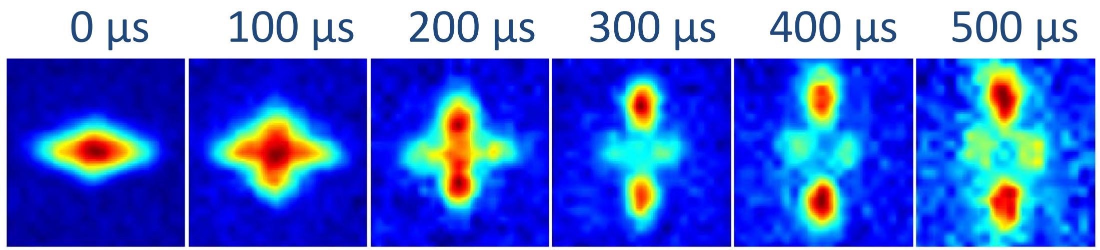
erc

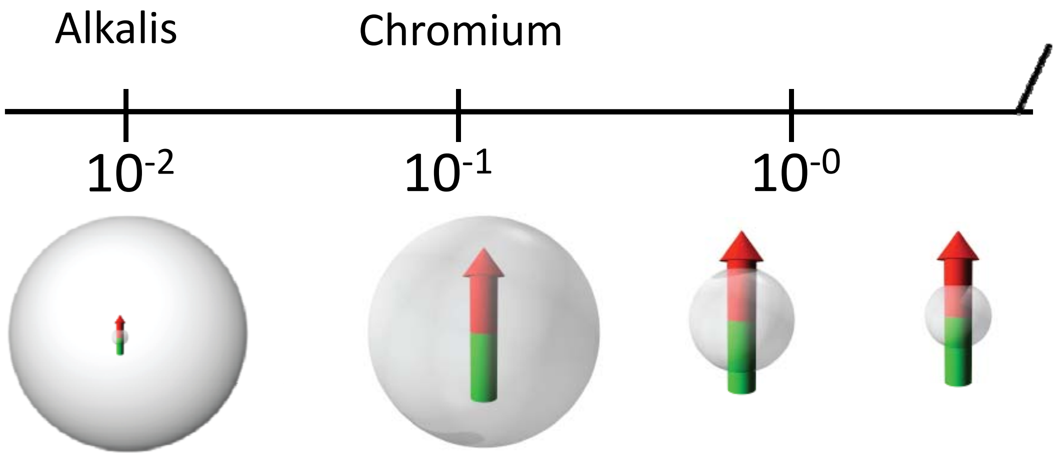
Interactions make life interesting

Short range interactions **Long range interactions**

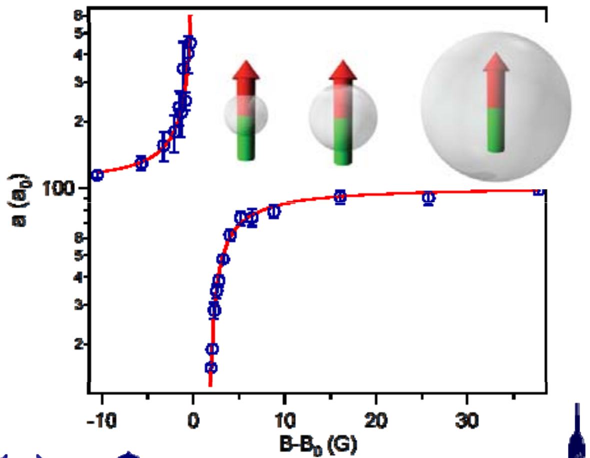


dipolar interaction





Feshbach resonance

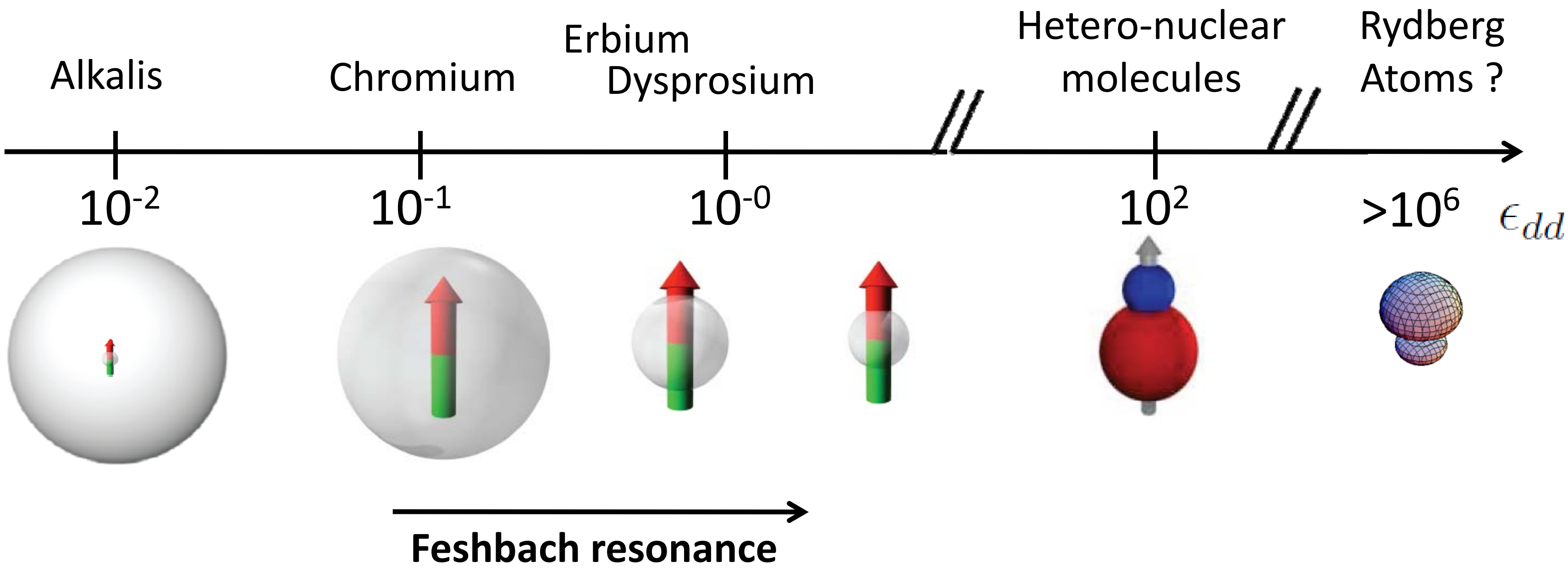


→ *Realization of a purely dipolar condensate*

T. Lahaye *et al.*, Nature **448**, 672 (2007)

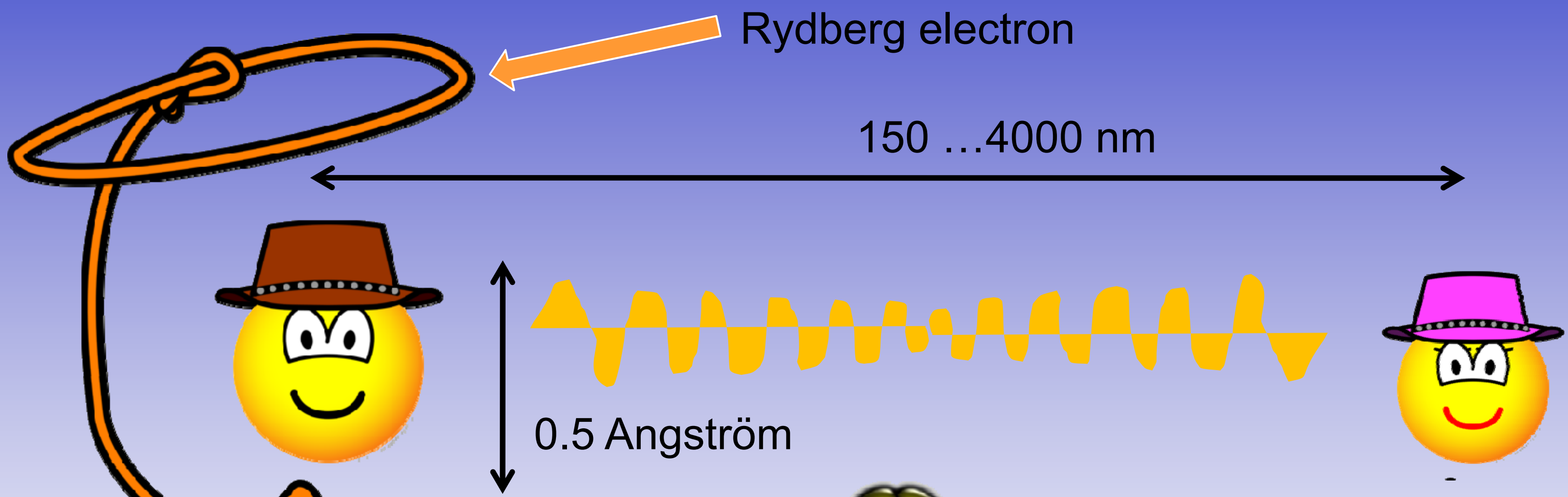
T. Koch, *et al.*, Nature Physics **4**, 218 (2008)

dipolar interaction



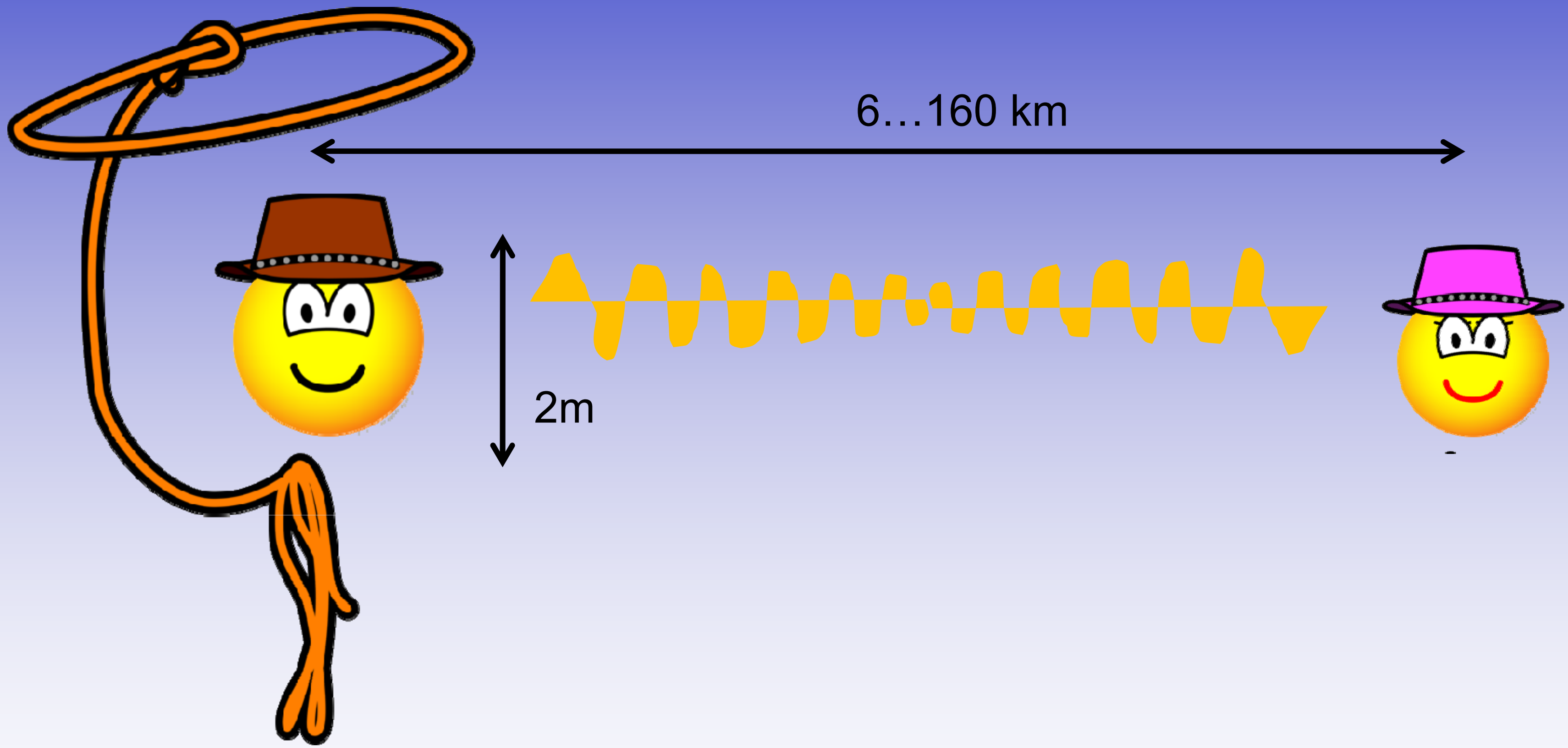
Long range bound states

in the atomic world

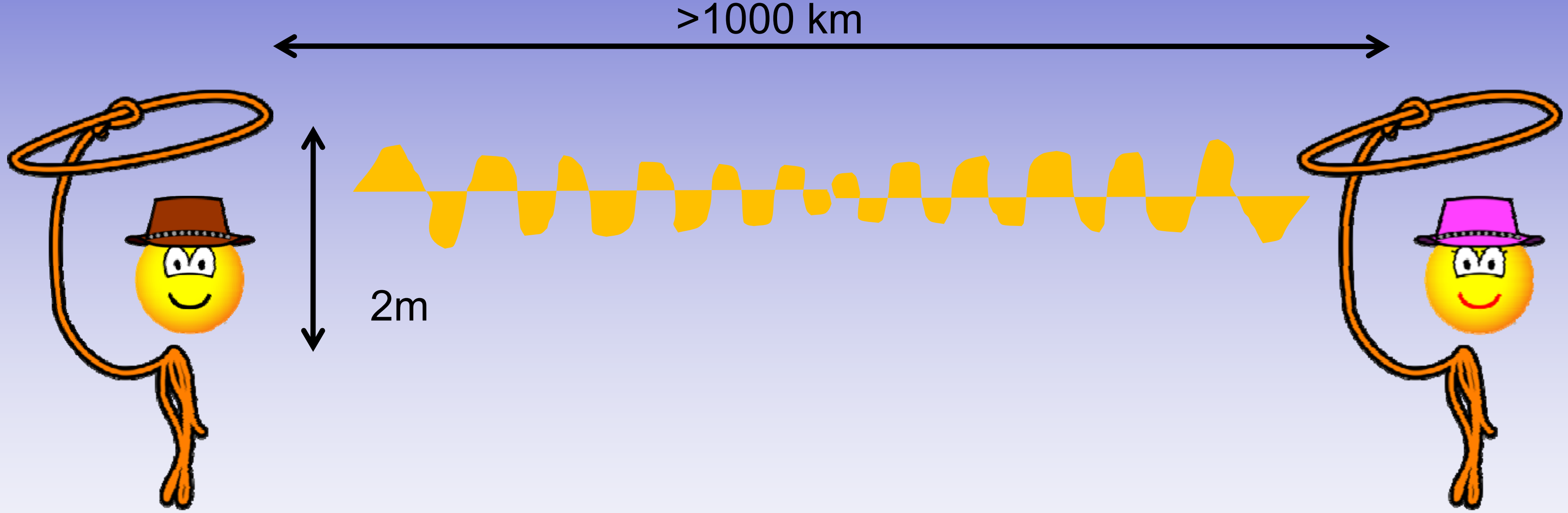


Nature ***458****, 1005 (2009)*
PRL ***105****, 163201 (2010)*
Science ***334****, 1110 (2011)*
Nature ***502****, 664 (2013)*

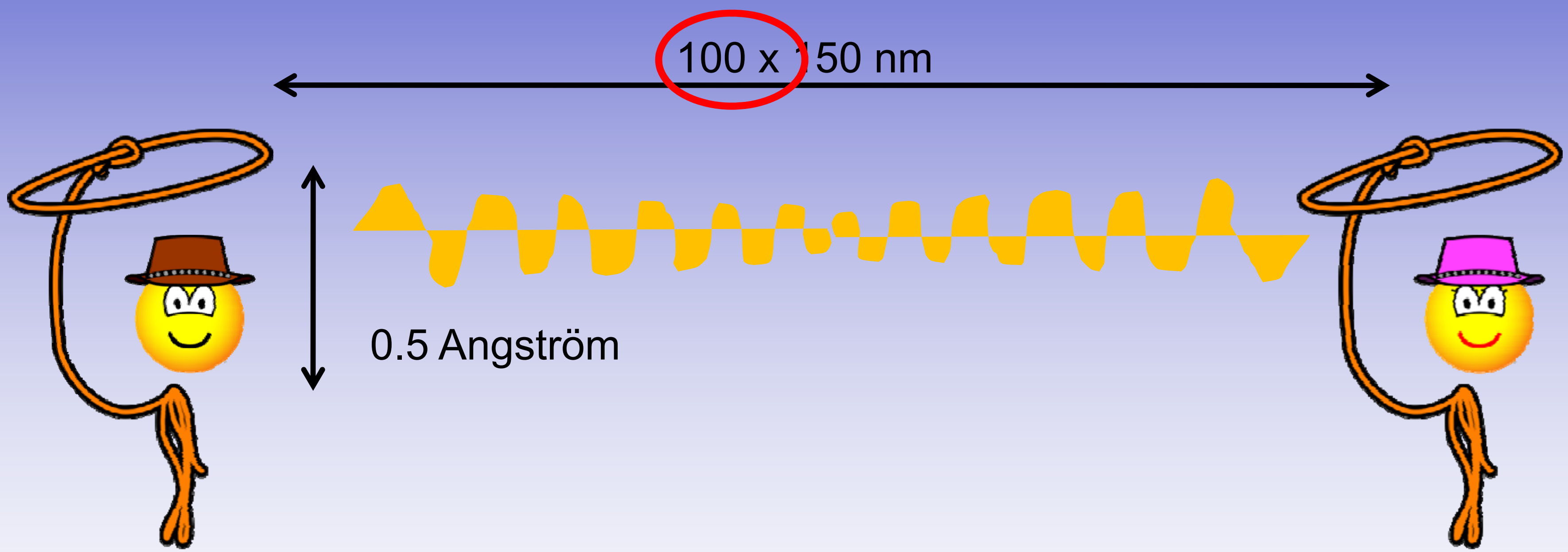
Bound states



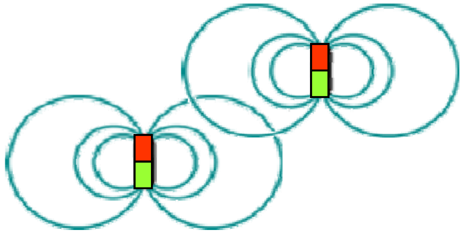
Long range interaction

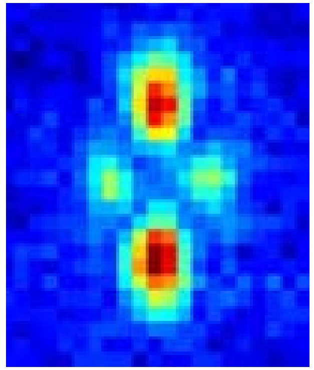


Long range interaction between Rydberg atoms

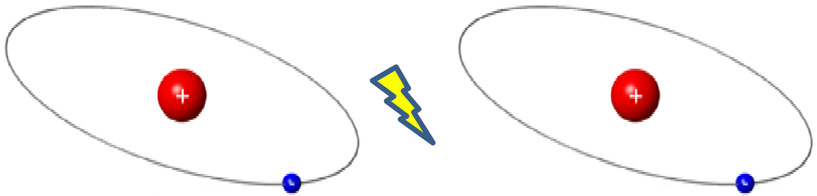


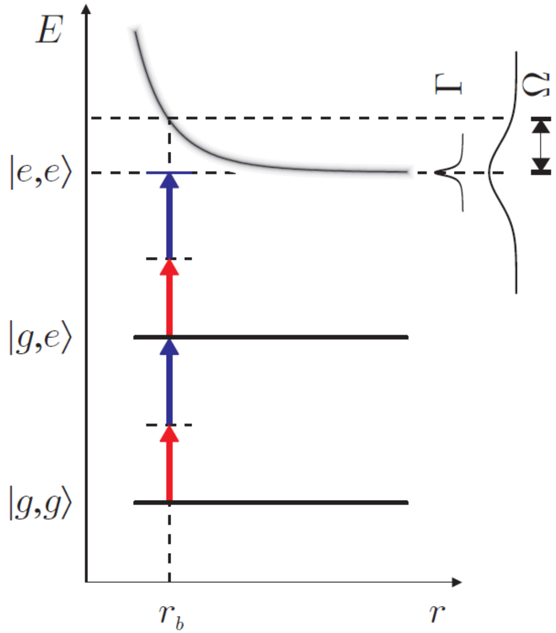
Lecture I: (magnetic) dipolar gases



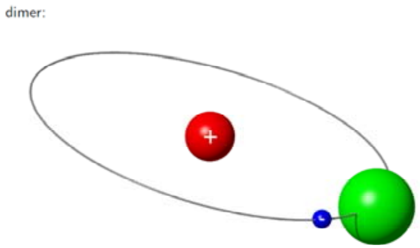


Lecture II: Rydberg Rydberg interaction

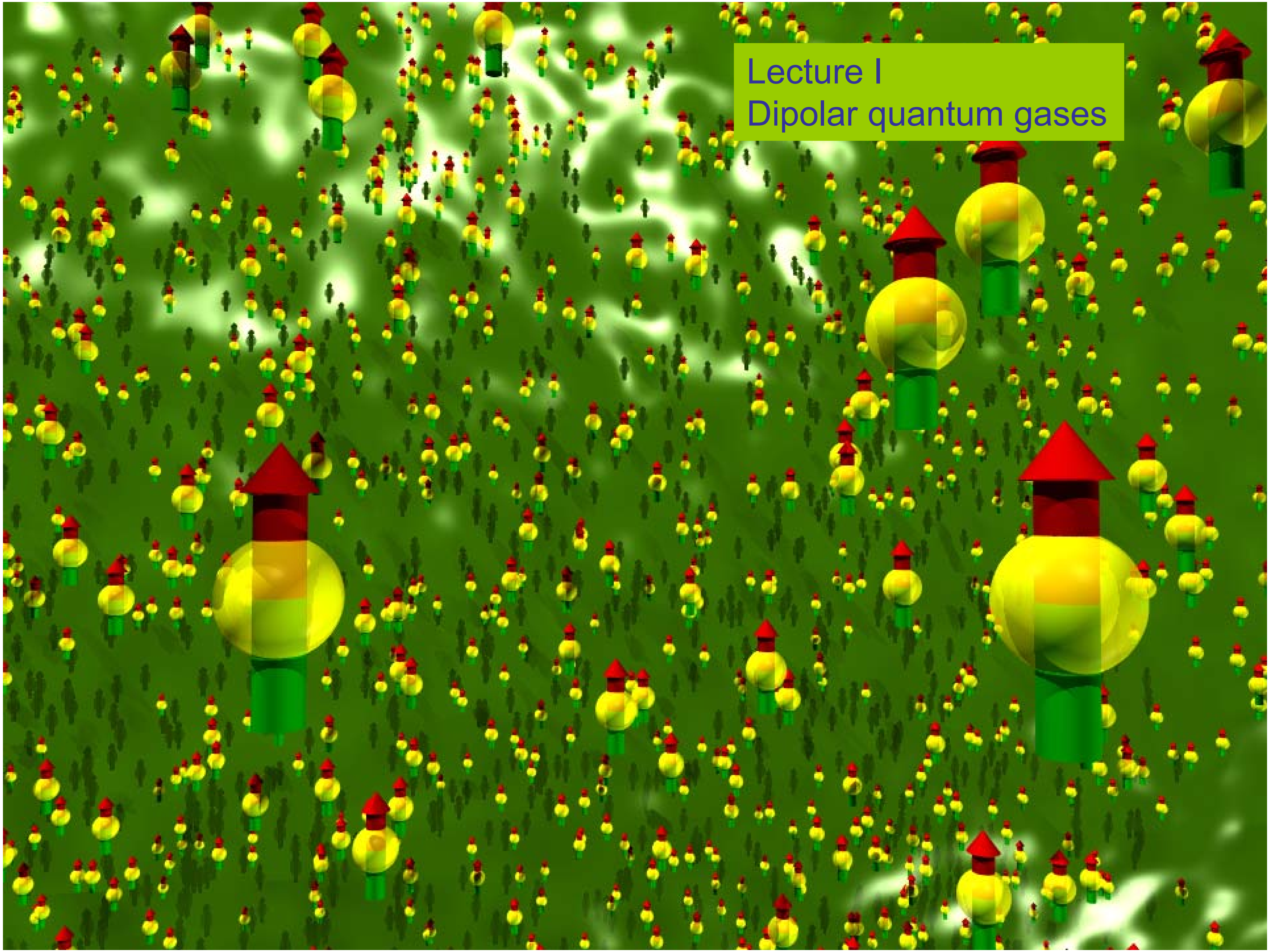


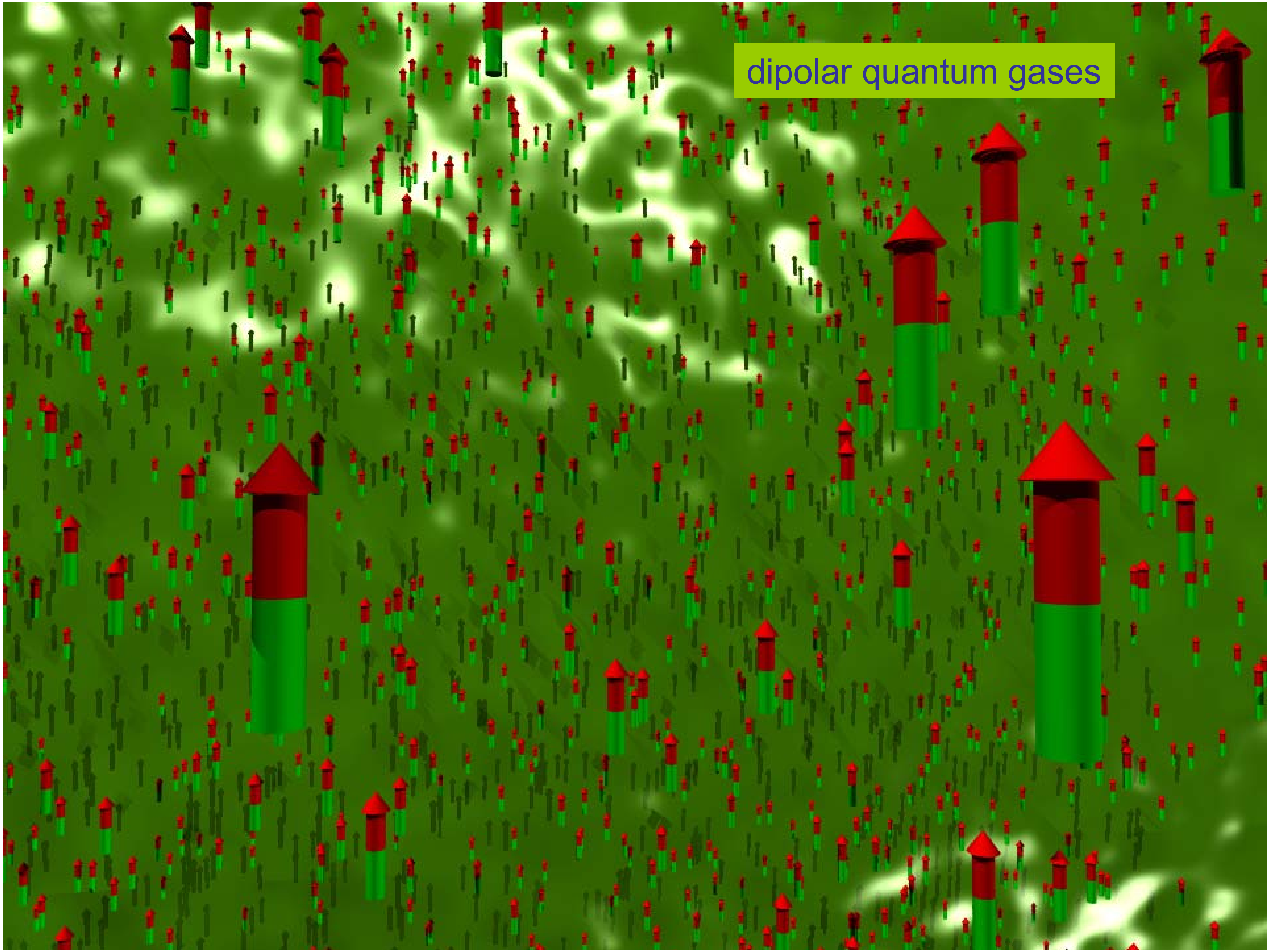


Lecture III : Rydberg ground state interaction



Lecture I
Dipolar quantum gases




dipolar quantum gases

Early interest in dipoles

- Compass needles
- 1970 DeGennes:

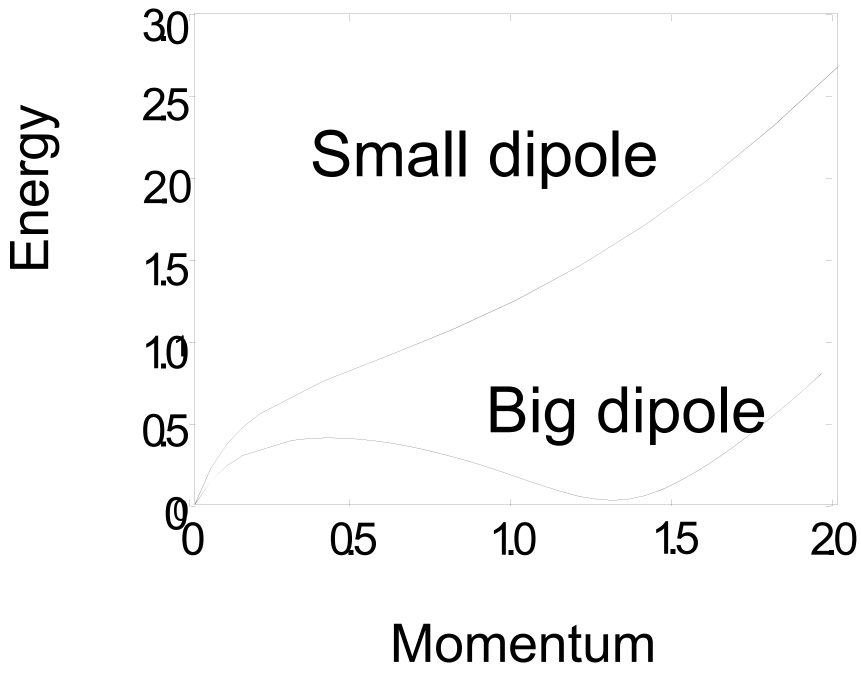
  anisotropic gas; chains
- 1980's ferrofluids

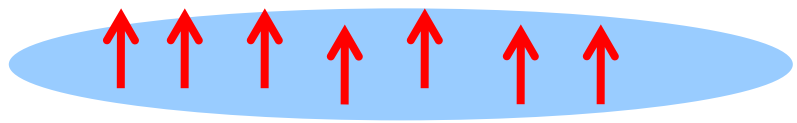
  Rosensweig instability

  M. D. Cowley and R. E. Rosensweig, J. Fluid Mech. **30**, 671 (1967)

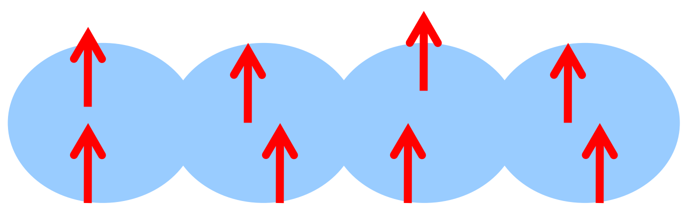
- 21^{st} century : add quantum mechanics

Anisotropy: the roton in dipolar BEC





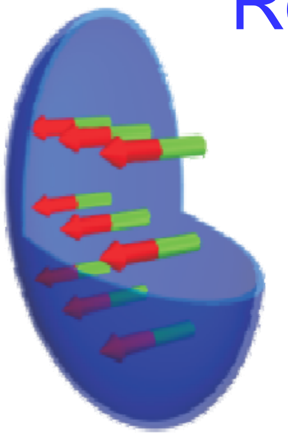
Stable at long wavelengths

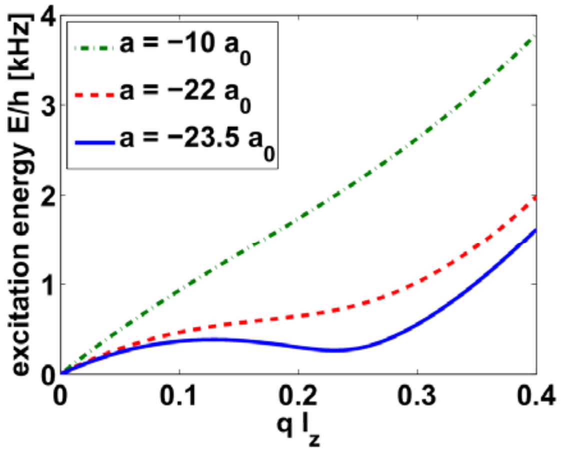


Roton instability at shorter wavelengths

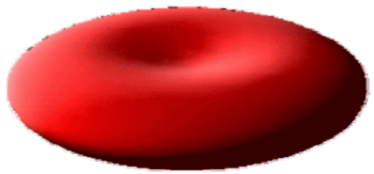
Santos, Shlyapnikov, and Lewenstein PRL 90, 250403 (2003)

Roton – Maxon type excitation spectrum





Structured groundstates



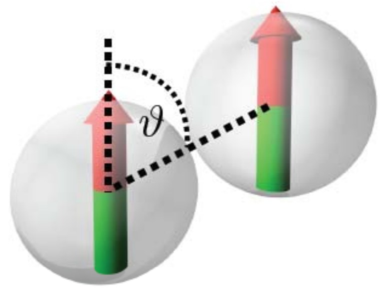
checkerboard supersolid

*R. Richter et al., PRL **94**, 184503 (2005)*

*S. Ronen et al., PRL **98**, 030406 (2007)*

*K. Góral et al., PRL **88**, 170406 (2002)*

Dipolar gases



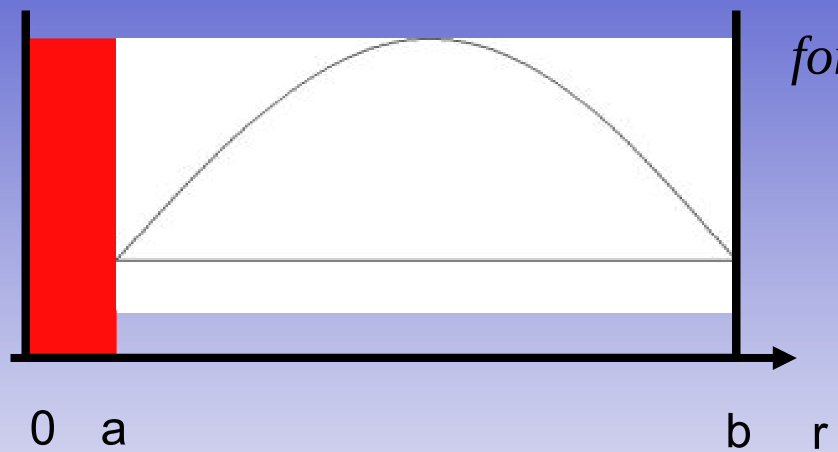
$$\varepsilon_{dd} \propto \frac{m\mu^2}{a}$$

dipolar interaction

contact interaction

Effect of contact interaction

Two particles in a box potential (s-wave)



$$for \quad a << b$$

$$E_{n=1} = \frac{\hbar^2 k^2}{2m_\mu} = \frac{\hbar^2}{2m_\mu}\frac{\pi^2}{(b-a)^2} = \frac{\hbar^2}{2m_\mu}\frac{\pi^2}{b^2}(1-\frac{a}{b})^{-2}$$

$$\approx E(a=0) + \frac{\hbar^2}{m_\mu}\frac{\pi^2}{b^3}a$$

$$E_{contact} = \frac{4\pi\hbar^2}{m}|\psi|^2 a$$

Periodic table
of magnetic moments

5. Physikalisches Institut

H 1																	He 0
Li 1	Be 0											B 0.3	C 0	N 3	O 3	F 2	Ne 0
Na 1	Mg 0											Al 0.3	Si 0	P 3	S 3	Cl 2	Ar 0
K 1	Ca 0	Sc 1.2	Ti 1.3	V 0.6	Cr 6 (2004)	Mn 5	Fe 6	Co 6	Ni 5	Cu 1	Zn 0	Ga 0.3	Ge 0	As 3	Se 3	Br 2	Kr 0
Rb 1	Sr 0	Y 1.2	Zr 1.3	Nb 1.7	Mo 6	Tc 5	Ru 7	Rh 6	Pd 0	Ag 1	Cd 0	In 0.3	Sn 0	Sb 3	Te 3	I 2	Xe 0
Cs 1	Ba 0		Hf 1.3	Ta 0.6	W 0	Re 5	Os 6	Ir 6	Pt 4	Au 1	Hg 0	Tl 0.3	Pb 0	Bi 3	Po 3	At 2	Rn 0
Fr 1	Ra 0		Rf 1.3	Db 0.6	Sg 0	Bh 5	Hs 6	Mt 6	Ds 4	Rg 1	Cn 0	Uut 0.3	Uuq 0	Uup 3	Uuh 3	Uus 2	Uuo 0

La 1.2	Ce 4	Pr 3.3	Nd 2.4	Pm 0.7	Sm 0	Eu 7	Gd 5.3	Tb 10	Dy 10 (2011)	Ho 9	Er 7 (2012)	Tm 4	Yb 0	Lu 1.2
Ac 1.2	Th 1.3	Pa 4.2	U 4.3	Np 3.4	Pu 0	Am 7	Cm 5.3	Bk 10	Cf 10	Es 9.1	Fm 7	Md 4	No 0	Lr 0.3

Periodic table
of magnetic moments

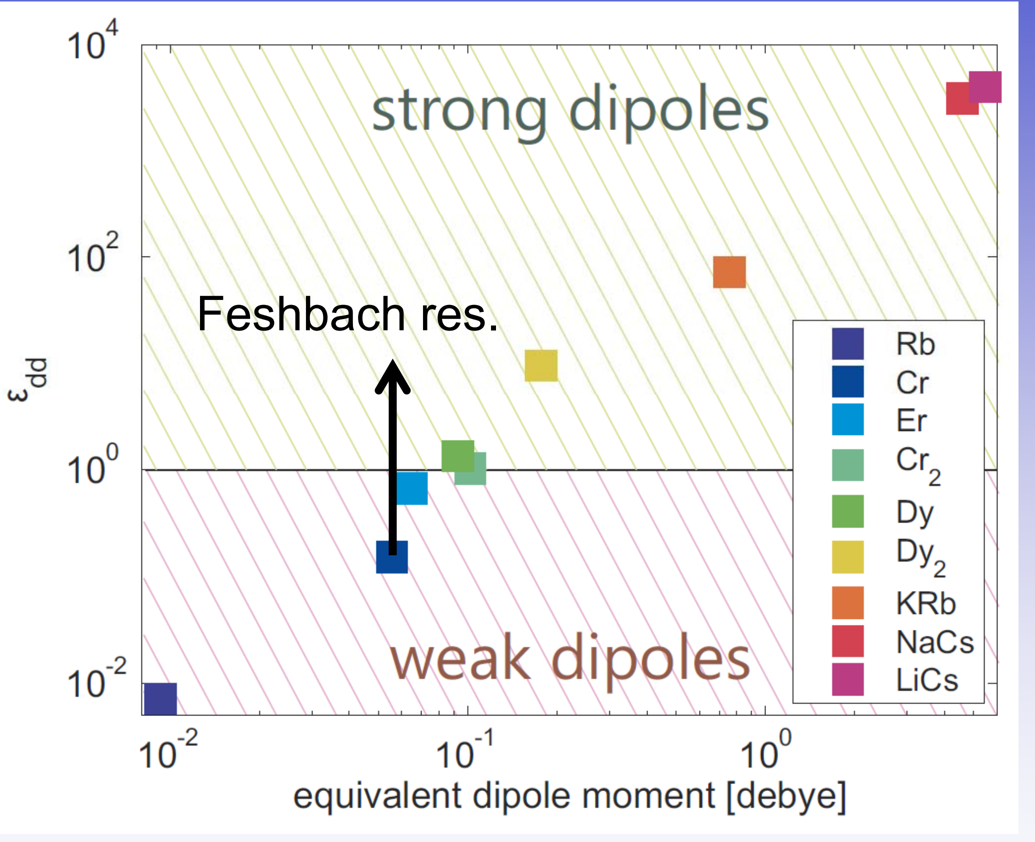
$$\varepsilon_{dd} = \frac{\mu_0 \mu^2 m}{12\pi\hbar^2 a_{bg}}$$

2004 (Cr)

H 1																	He 0
Li 7	Be 0											B 1	C 0	N 126	O 144	F 76	Ne 0
Na 23	Mg 0											Al 3	Si 0	P 279	S 289	Cl 142	Ar 0
K 39	Ca 0	Sc 65	Ti 85	V 18	Cr 1872	Mn 1373	Fe 2010	Co 2122	Ni 1467	Cu 64	Zn 0	Ga 8	Ge 0	As 674	Se 711	Br 320	Kr 0
Rb 85	Sr 0	Y 128	Zr 162	Nb 258	Mo 3455	Tc 2450	Ru 4952	Rh 3705	Pd 0	Ag 108	Cd 0	In 13	Sn 0	Sb 1096	Te 1148	I 508	Xe 0
Cs 133	Ba 0		Hf 317	Ta 65	W 0	Re 4655	Os 6848	Ir 6920	Pt 3121	Au 197	Hg 0	Tl 23	Pb 0	Bi 1881	Po 1881	At 840	Rn 0
Fr 223	Ra 0		Rf 471	Db 96	Sg 0	Bh 6800	Hs 9720	Mt 9936	Ds 4496	Rg 280	Cn 0	Uut 32	Uuq 0	Uup 2592	Uuh 2637	Uus 1176	Uuo 0

2011 (Dy) 2012 (Er)

La 200	Ce 2242	Pr 1509	Nd 831	Pm 74	Sm 0	Eu 7446	Gd 4473	Tb 15893	Dy 16250	Ho 13359	Er 8196	Tm 2703	Yb 0	Lu 252
Ac 327	Th 413	Pa 4135	U 4372	Np 2715	Pu 0	Am 11907	Cm 7026	Bk 24700	Cf 25100	Es 21017	Fm 12593	Md 4128	No 0	Lr 29



strong dipoles
Feshbach res.
weak dipoles
ε_{dd}
10^4
10^2
10^0
10^{-2}
10^{-2}
10^{-1}
10^0
equivalent dipole moment [debye]
Rb
Cr
Er
Cr_2
Dy
Dy_2
KRb
NaCs
LiCs

How to describe an interacting quantum gas

Gross-Pitaevskii equation for the order parameter:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + \left(V_{\mathrm{ext}} + g|\psi|^2 + \Phi_{\mathrm{dd}}(\boldsymbol{r},t)\right)\psi$$

$$g \equiv \frac{4\pi\hbar^2 a}{m}$$

Contact interaction

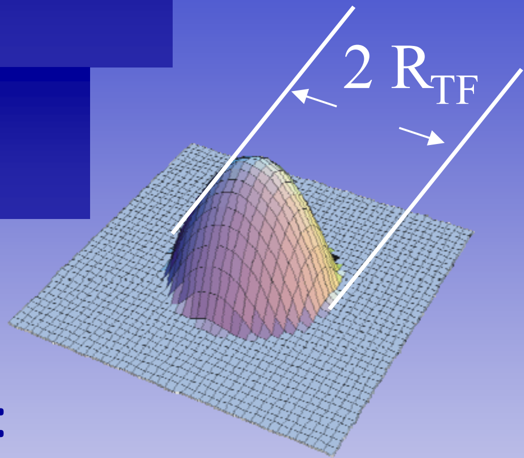
$$\Phi_{\mathrm{dd}}(\boldsymbol{r},t) = \int |\psi(\boldsymbol{r}',t)|^2\, U_{\mathrm{dd}}(\boldsymbol{r}-\boldsymbol{r}')\, \mathrm{d}^3r'$$

Dipolar interaction
NON-LOCAL term

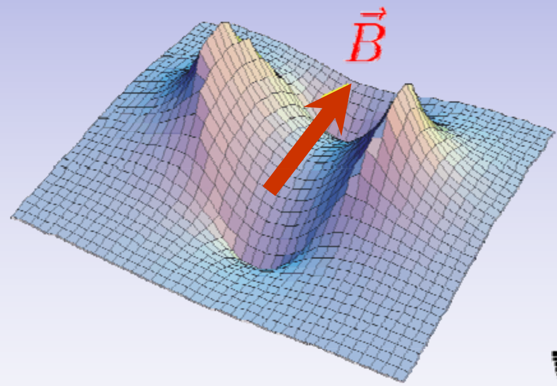
$$U_{\mathrm{dd}}(\boldsymbol{r}) = \frac{\mu_0\mu^2}{4\pi}\frac{1-3\cos^2\theta}{r^3}$$

Elongation of the condensate along ***B***

$\varepsilon_{dd} << 1$, spherical trap:



Mean-field potential due to the dipolar interaction:



Saddle potential.

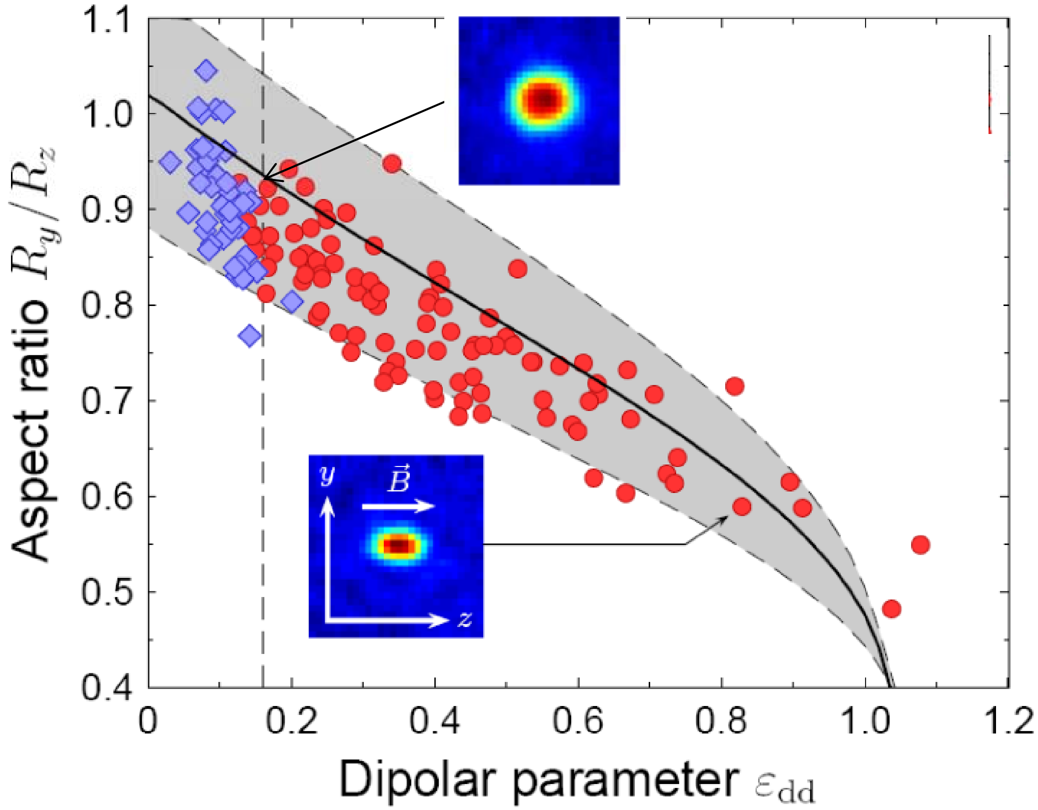
→ The atoms are accommodated **close to the *z* axis.**

These conclusions remain valid:

- for anisotropic traps,
- for arbitrary ε_{dd},
- during the time of flight.

S. Giovanazzi
D. O'Dell
C. Eberlein

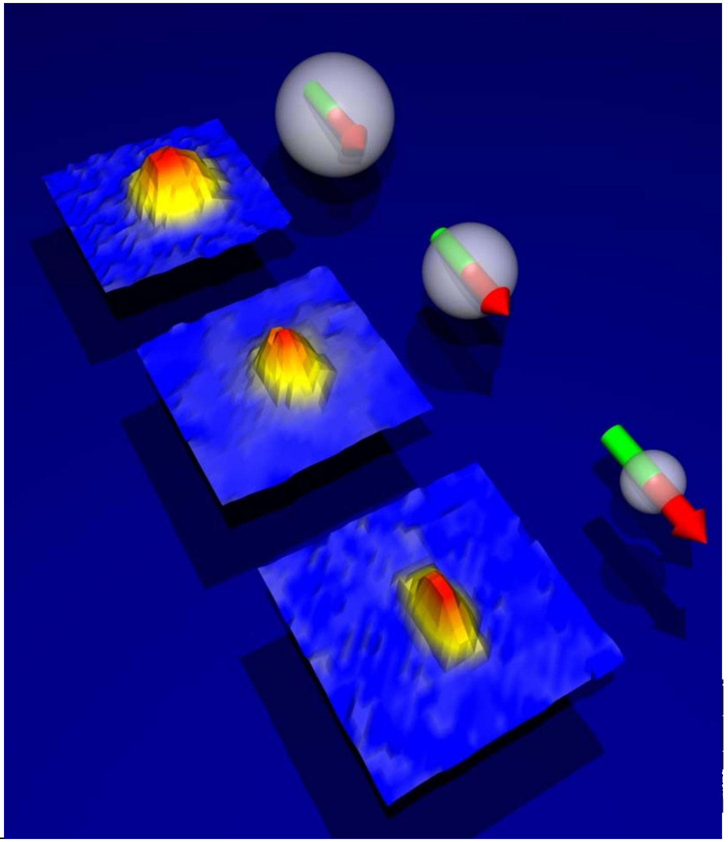
A quantum ferrofluid



dipolar

$$\varepsilon_{dd} = \frac{\mu_0 \mu^2 M}{12\pi\hbar^2 a}$$

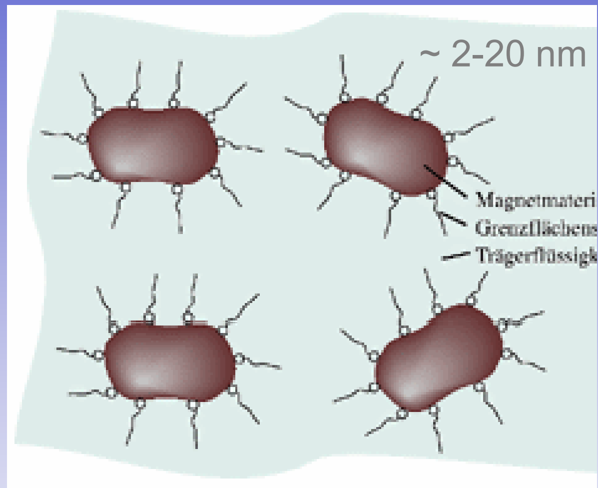
contac

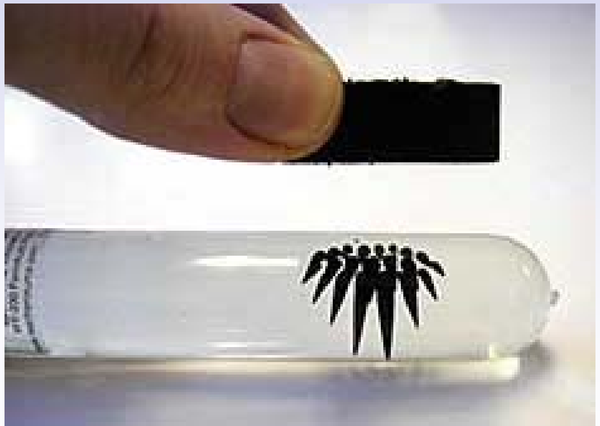


T. Lahaye, T. Koch, B. Fröhlich,
M. Fattori, J. Metz, A. Griesmaier,
S. Giovanazzi, T. Pfau;
Nature **448**, 672 (2007)

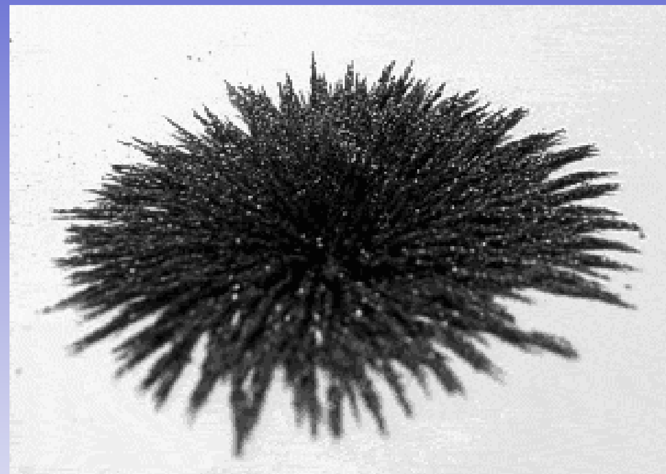
dipolar coupling in fluids

Ferrofluids





Iron particles



RAPID COMMUNICA

PHYSICAL REVIEW A, VOLUME 61, 051601(R)

Bose-Einstein condensation with magnetic dipole-dipole forces

Krzysztof Góral,[1] Kazimierz Rzążewski,[1] and Tilman Pfau,[2,*]
[1] *Center for Theoretical Physics and College of Science, Polish Academy of Sciences, Aleja Lotników 32/46, 02-668 Warsaw, Poland*
[2] *Faculty of Physics, University of Konstanz, 78457 Konstanz, Germany*

(Received 20 July 1999; revised manuscript received 1 October 1999; published 24 March 2000)

Ground-state solutions in a dilute gas interacting via contact and magnetic dipole-dipole forces are investigated. To the best of our knowledge, it is the first example of studies of Bose-Einstein condensation in a system with realistic long-range interactions. We find that for the magnetic moment of, e.g., chromium ($6\mu_B$), and a typical value of the scattering length, all solutions are stable and only differ in size from condensates without long-range interactions. By lowering the value of the scattering length we find a region of unstable solutions. In the neighborhood of this region, the ground-state wave functions show internal structures that we believe have not been seen before in condensates. Finally, we find an analytic estimate for the characteristic length appearing in these solutions.

PACS number(s): 03.75.Fi, 05.30.Jp

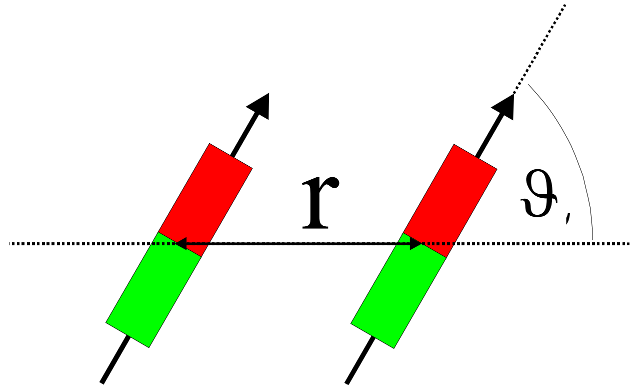
L. Santos, G. Shlyapnikov, P. Zoller, M. Lewenstein, PRL **85**, 1791 (2000).

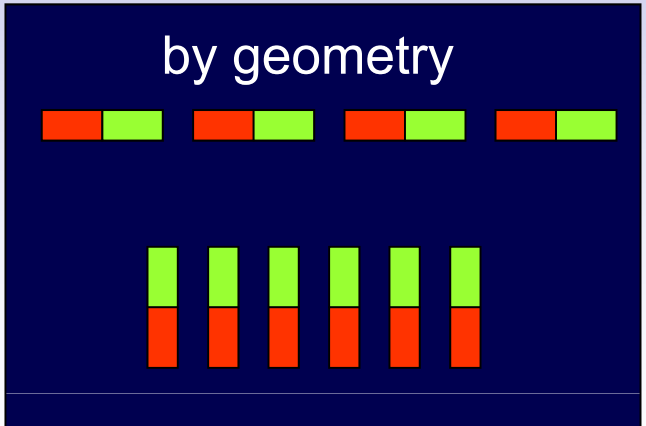
Stabilization of a dipolar gas

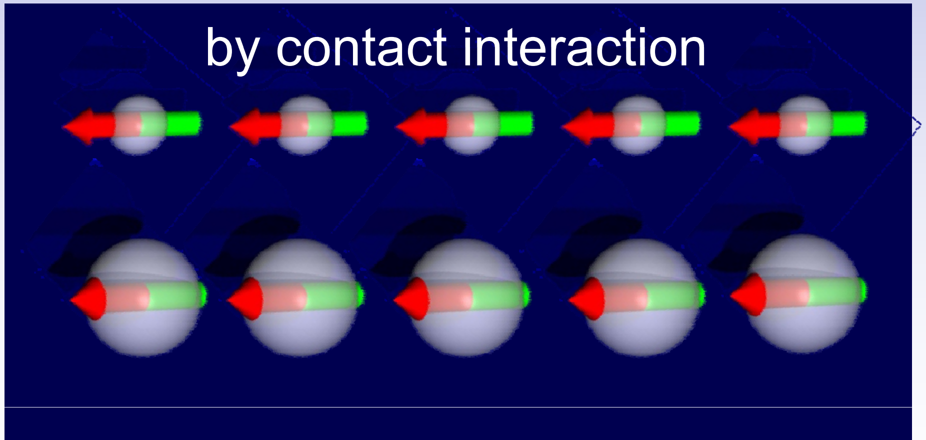
polarized sample

$$V_{dd}(r) \propto \frac{d^2}{r^3}(1 - 3\cos^2\vartheta)$$

long range (r^3) — anisotropic (ϑ)







The stability of a dipolar condensate…

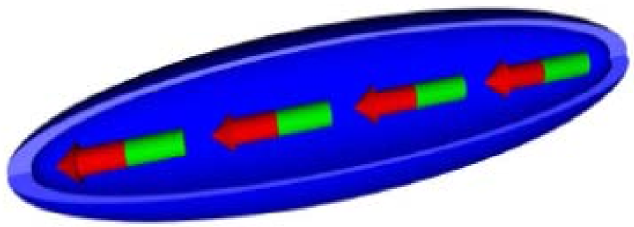
…depends *strongly* on the trap geometry:

$$V(x,y,z) = \frac{m}{2}\left[\omega_\rho^2(x^2+y^2)+\omega_z^2 z^2\right]$$

Aspect ratio: $\lambda \equiv \dfrac{\omega_z}{\omega_\rho}$

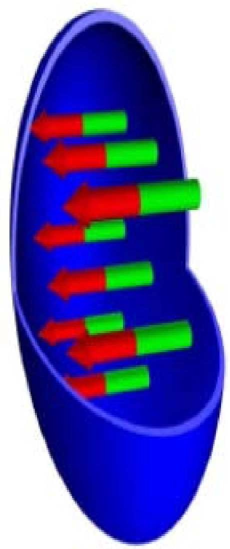
Cigar-shaped

$\lambda < 1$



Pancake

$\lambda > 1$



Attractive: unstable

Repulsive: stable

Stability criterion: a simple model

How to get a simple estimate for the critical value a_{crit} (λ) ?

→ Gaussian Ansatz

- Gross-Pitaevskii energy functional:

$$E[\Phi] = \int \left[\frac{\hbar^2}{2m} |\nabla \Phi|^2 + V_{\mathrm{trap}} |\Phi|^2 + \frac{g}{2} |\Phi|^4 + \frac{1}{2} |\Phi|^2 \int U_{\mathrm{dd}}(\boldsymbol{r} - \boldsymbol{r}') |\Phi(\boldsymbol{r}')|^2 \mathrm{d}\boldsymbol{r}' \right] \mathrm{d}\boldsymbol{r}$$

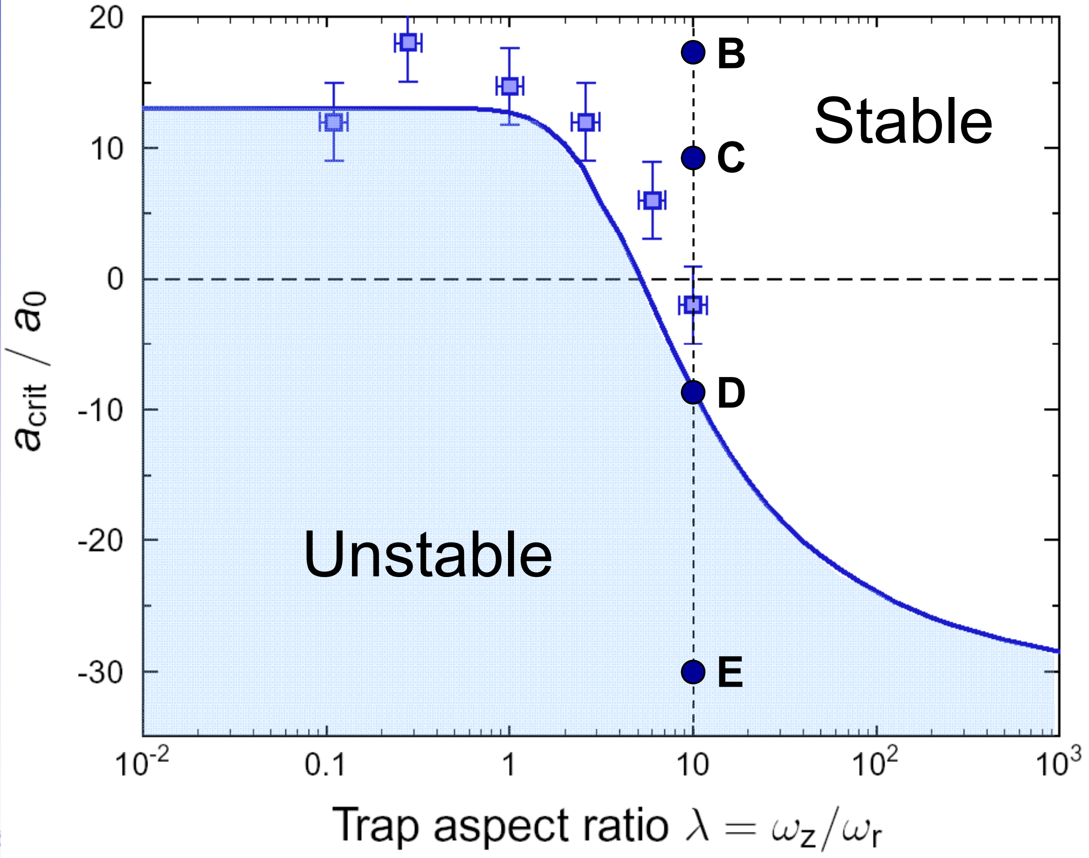
- Gaussian Ansatz (sizes σ_r and σ_z as variational parameters)

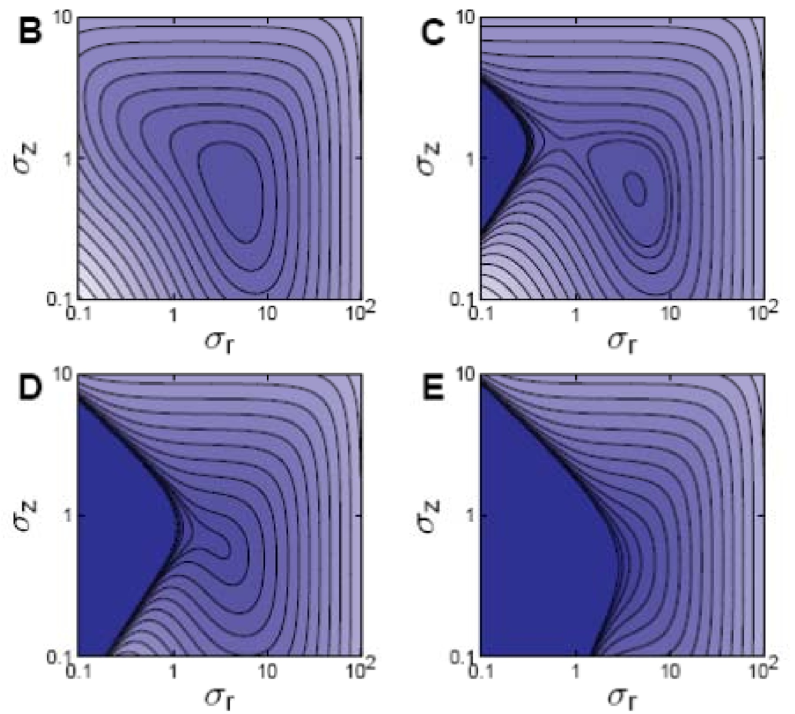
$$\Phi(r, z) = \left(\frac{N}{\pi^{3/2} \sigma_r^2 \sigma_z a_{\mathrm{ho}}^3} \right)^{1/2} \exp\left(-\frac{1}{2a_{\mathrm{ho}}^2} \left(\frac{r^2}{\sigma_r^2} + \frac{z^2}{\sigma_z^2} \right) \right)$$

- If a is too small, there is no more local minimum for $E[\Phi]$: this gives a_{crit}.

Stability diagram

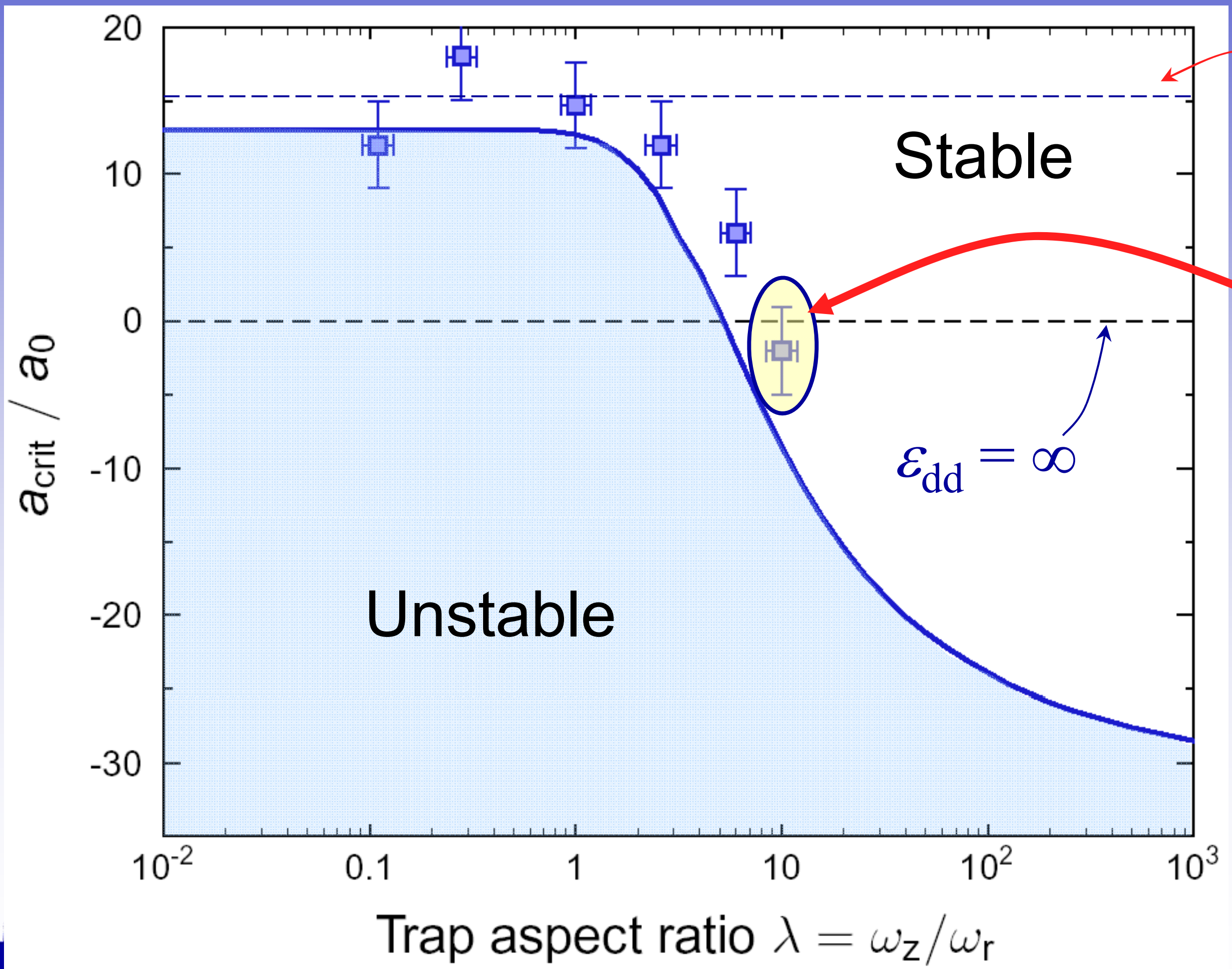
a_{crit} as a function of the trap aspect ratio λ
(N = 20,000 atoms; $\bar{\omega} \simeq 2\pi \times 800\,\mathrm{Hz}$)





T. Koch, T. Lahaye, J. Metz,
B. Fröhlich, A. Griesmaier, T. Pfau
Nature Physics ***4**, 218 (2008)*

Stability diagram



$\varepsilon_{dd} = 1$

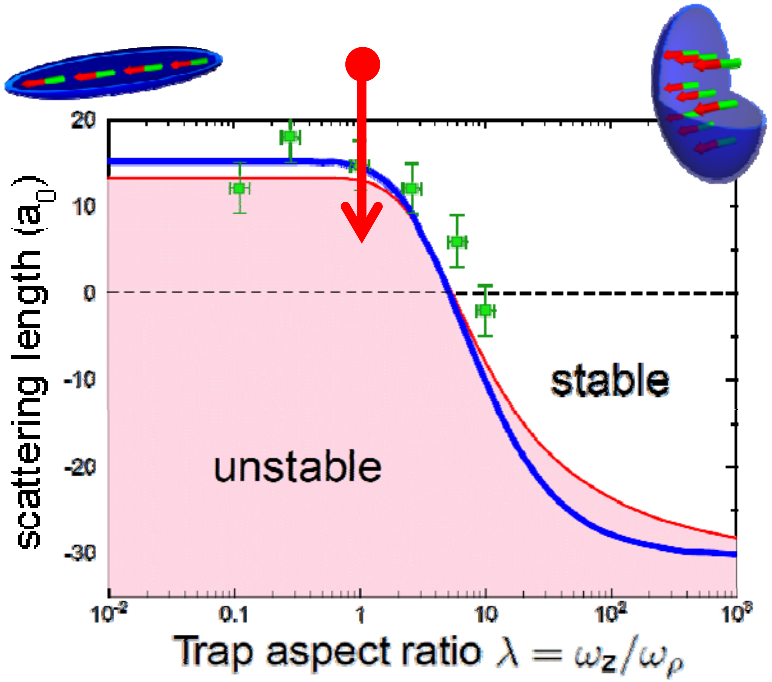
Stabilization of a purely dipolar condensate!

T. Koch *et al.*, arXiv:0710

Stability & collapse of a dipolar BEC

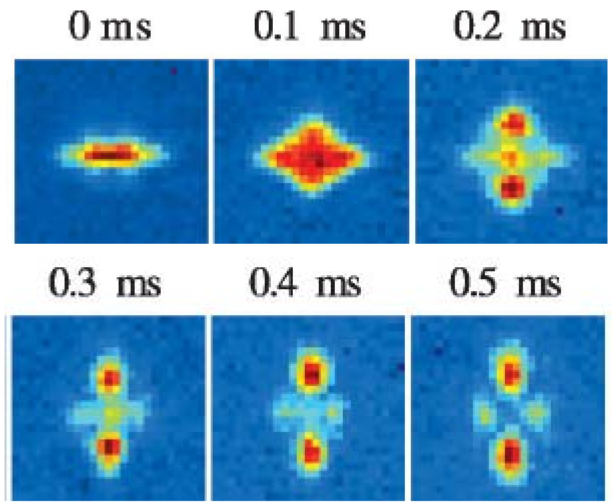
dipole-dipole interaction:
long-range and anisotropic

$\rightarrow$ *geometry-dependent stability / collapse*



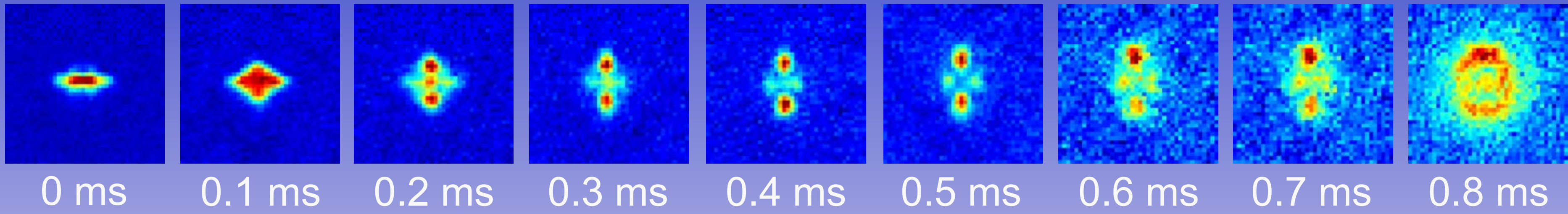
T. Koch *et al., Nat. Phys.* **4** (2008)

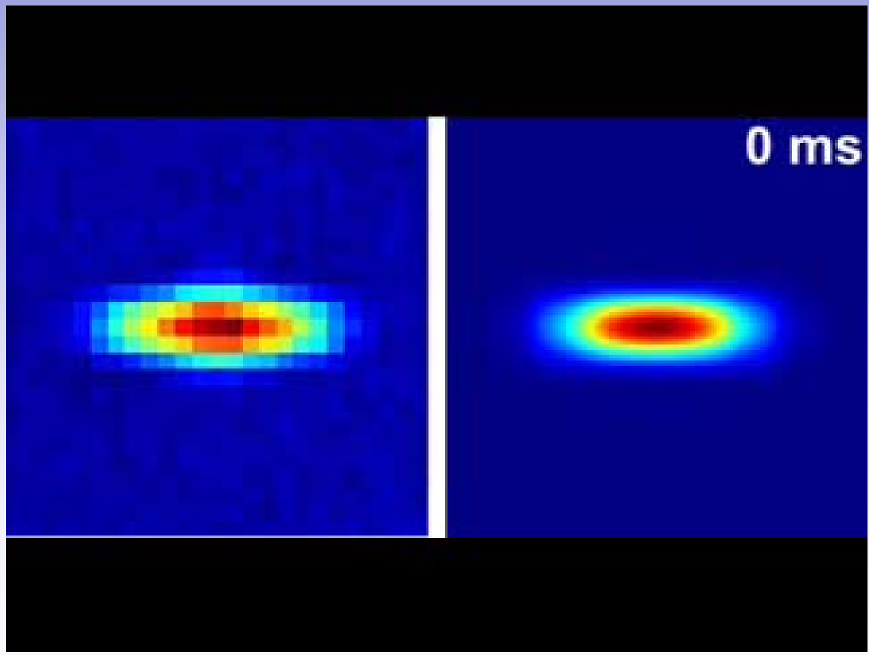
d-wave collapse



T. Lahaye et al., PRL ***101*** *(2008)*
J. Metz et al., New J. Phys. ***11*** *(2009)*

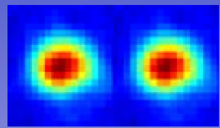
Collapse dynamics

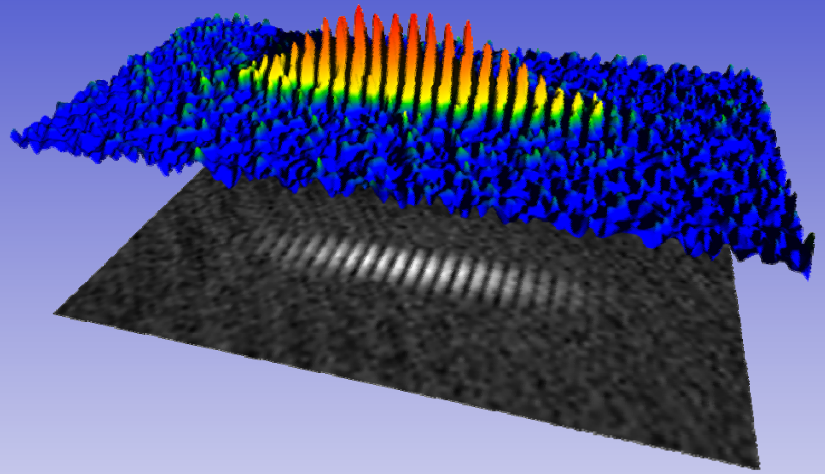


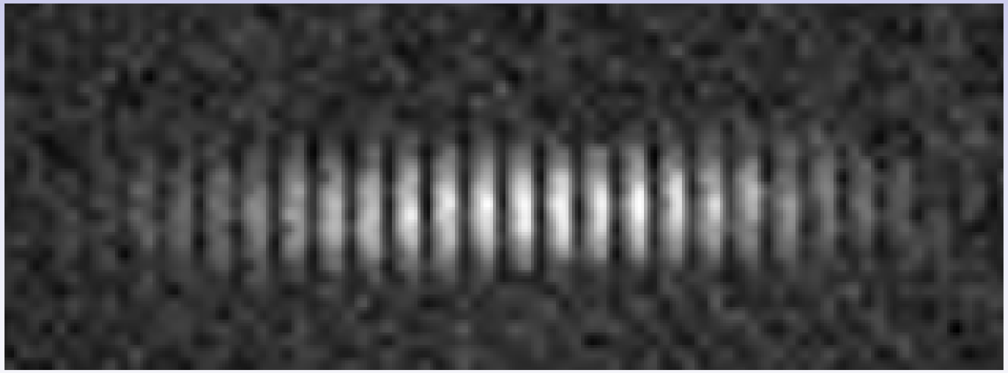


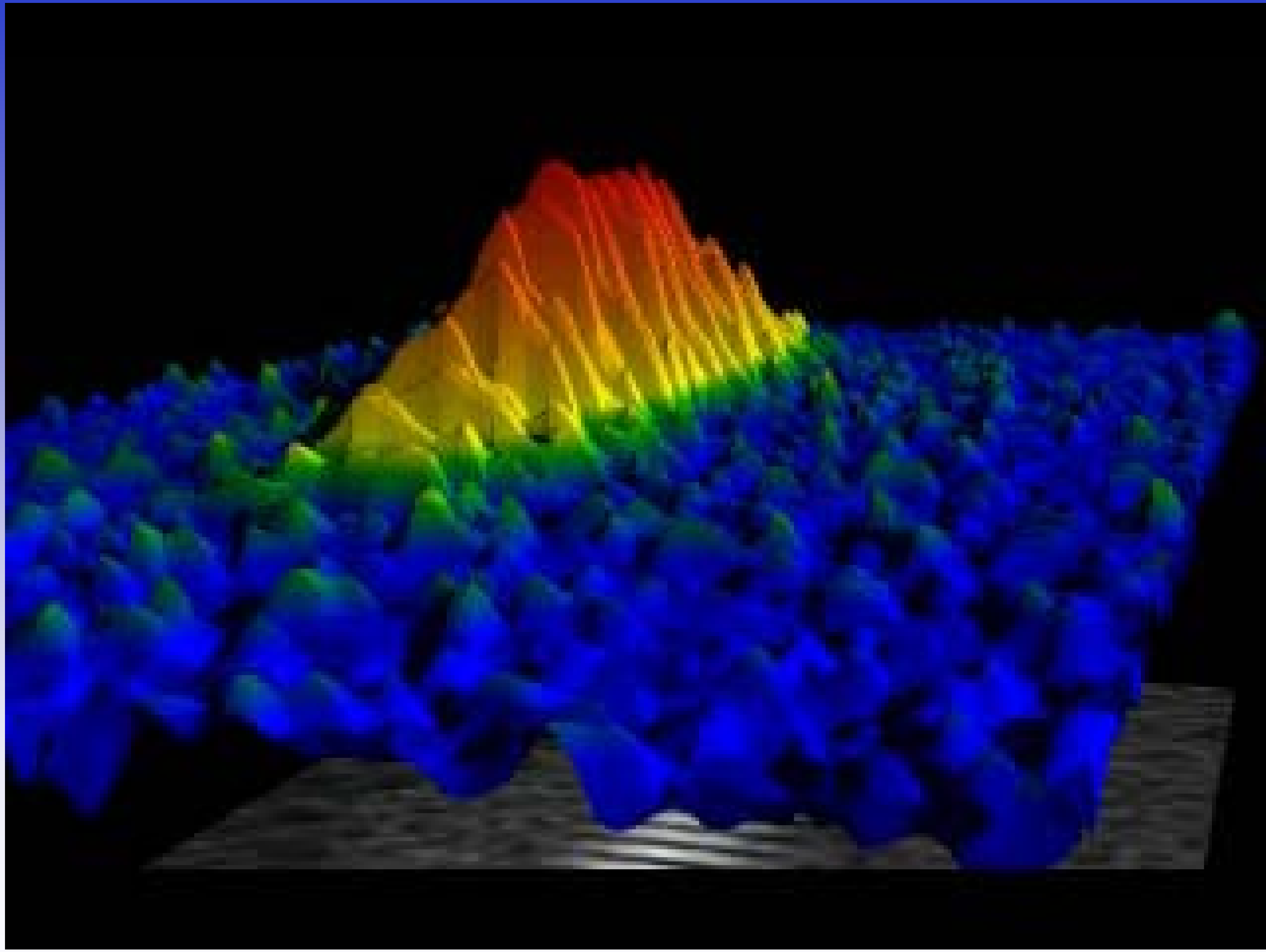
T. Lahaye, M. Ueda et al., PRL ***101*** *(2008)*

Is the collapse coherent?

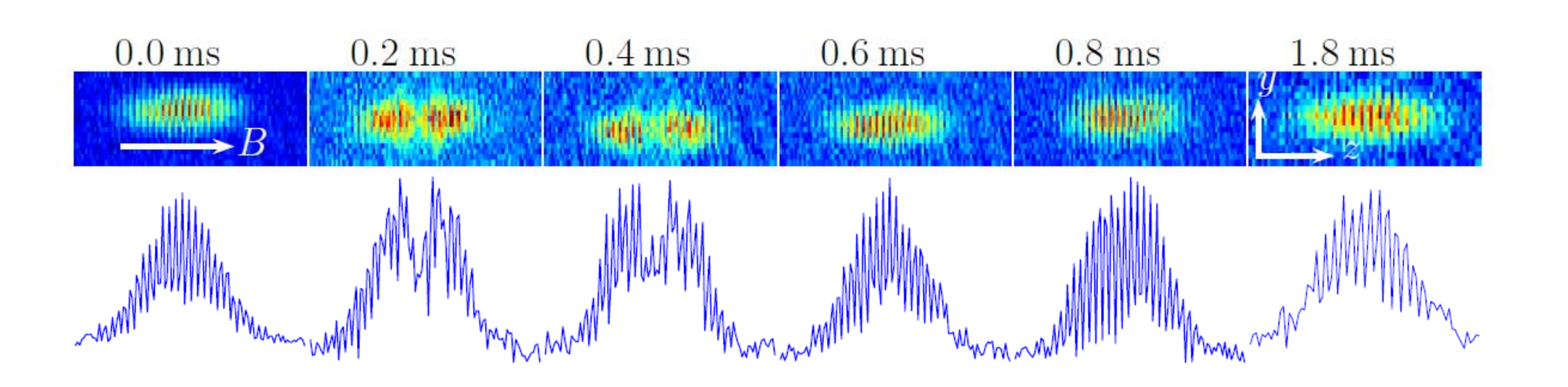






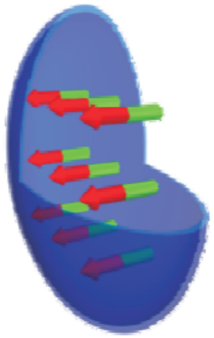


Coherent collapse dynamics



J. Metz, T. Lahaye, B. Fröhlich, A. Griesmaier, T. Pfau, H. Saito, Y. Kawaguchi, and M. Ueda
New J. Phys. ***11****, 055032 (2009)*

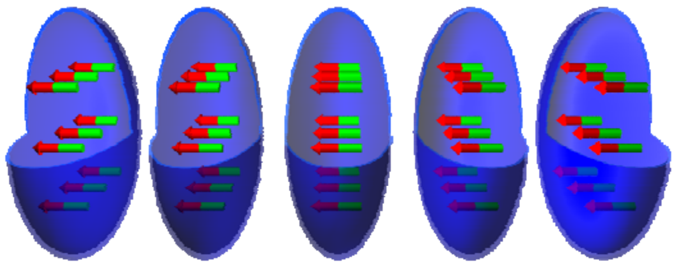
Stability of a dipolar BEC



Interactions:

- contact interaction (scattering length *a):*
  tuned via Feshbach resonance
  isotropic and short-range

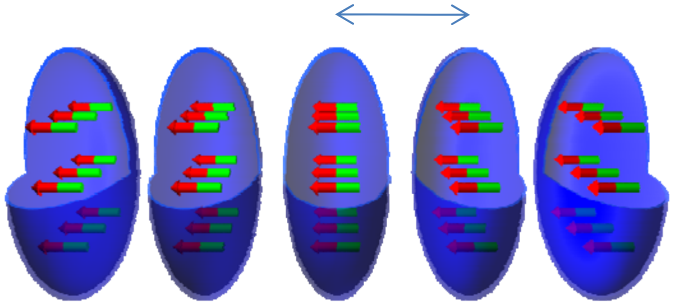
- dipole-dipole interaction (DDI):
  anisotropic and long-range



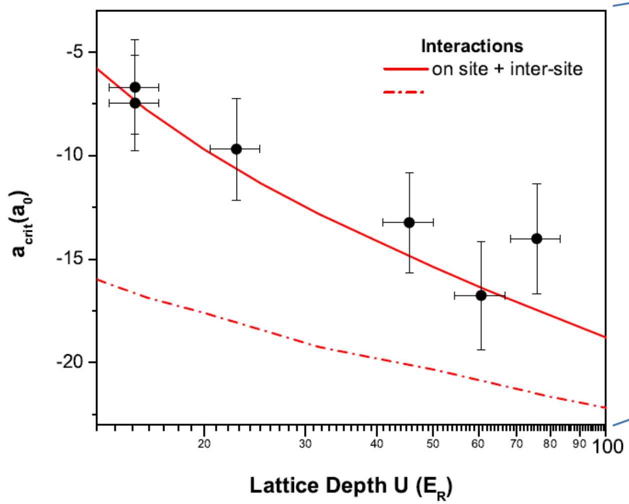
Multi-well potentials:
inter-site interaction mediated by DDI

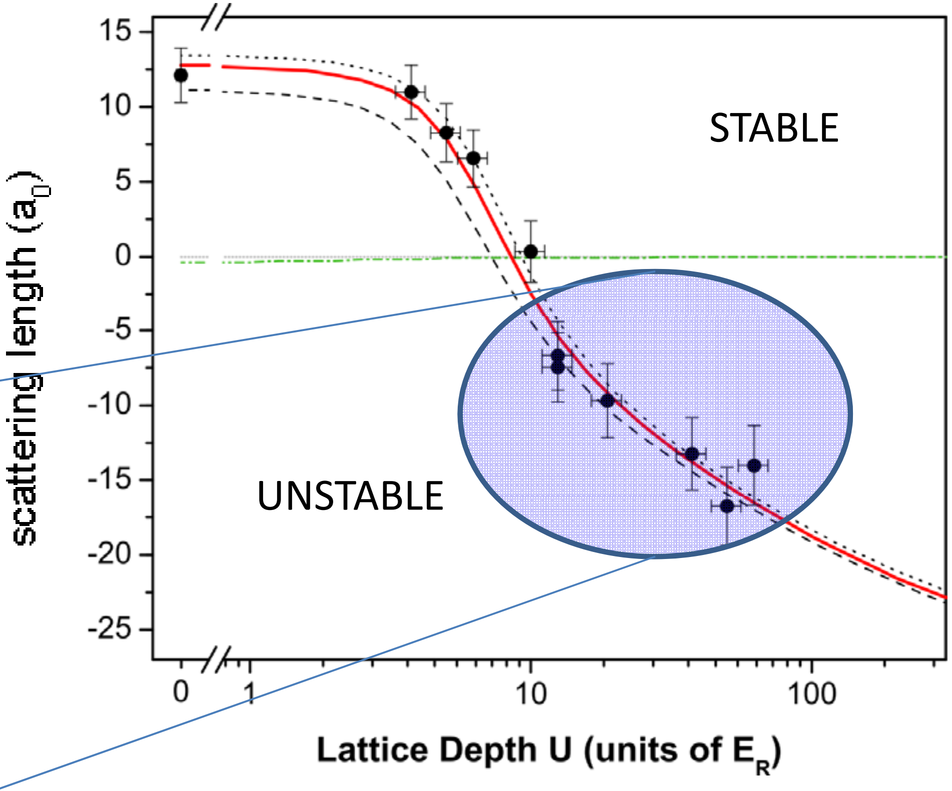
Stability given by energy balance between
- on-site interaction (contact + DDI)
- inter-site interaction (DDI)

A dipolar BEC in a 1D optical lattice



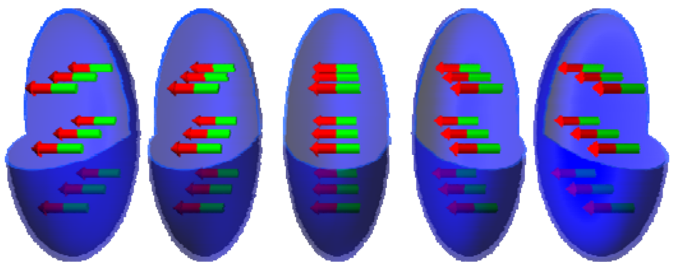
Confinement: lattice + optical trap





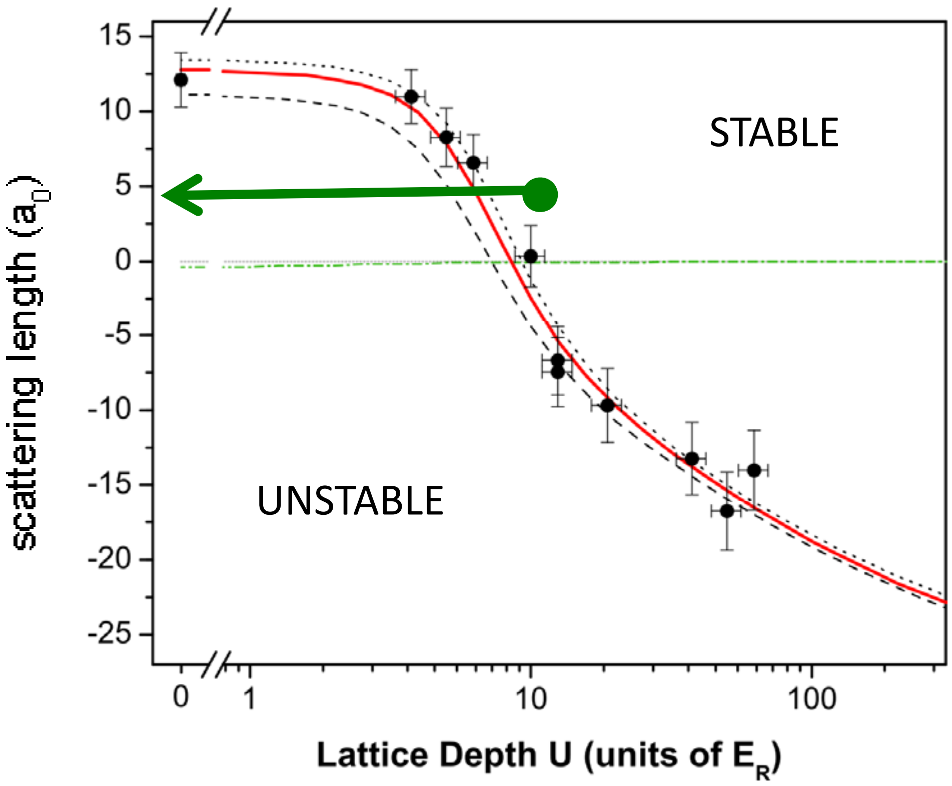
S. Müller et al., PRA 84, 053601 (2011)

A dipolar BEC in a 1D optical lattice



Confinement: lattice + optical trap

Deconfinement-induced collapse

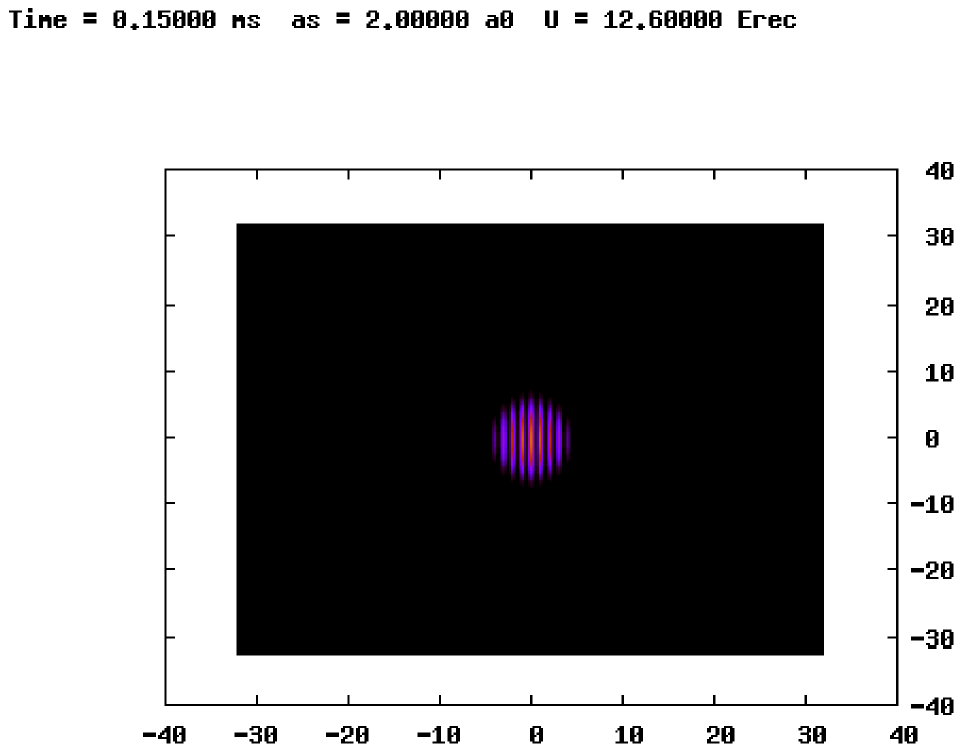


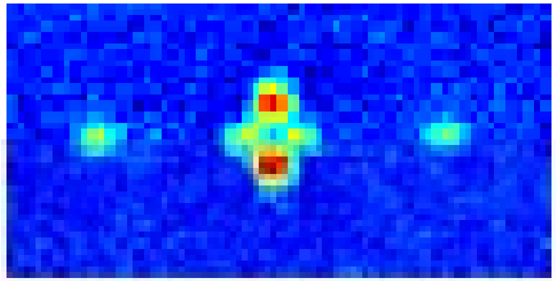
New method to induce the collapse !

→ *keep interaction strength constant*
→ *change external degree of freedom*

S. Müller et al., PRA 84, 053601 (2011)

Time-of-flight induced collapse





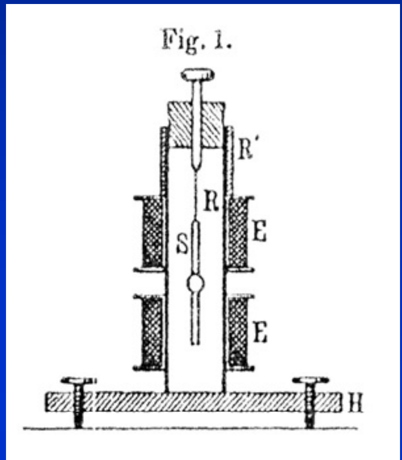
Novel collapse mechanism !

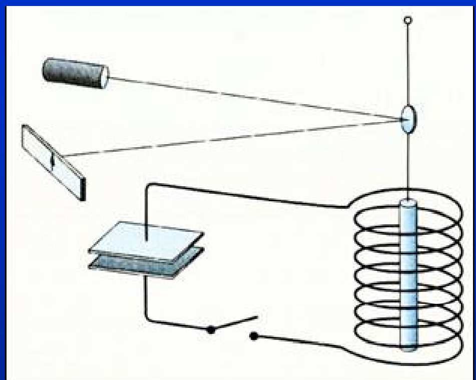
2-step process:

1. *High momentum peaks 2ħk leave*
2. *The 0ħk component collapses!*

Movie by
Mattia Jona-Lasinio, Luis Santos

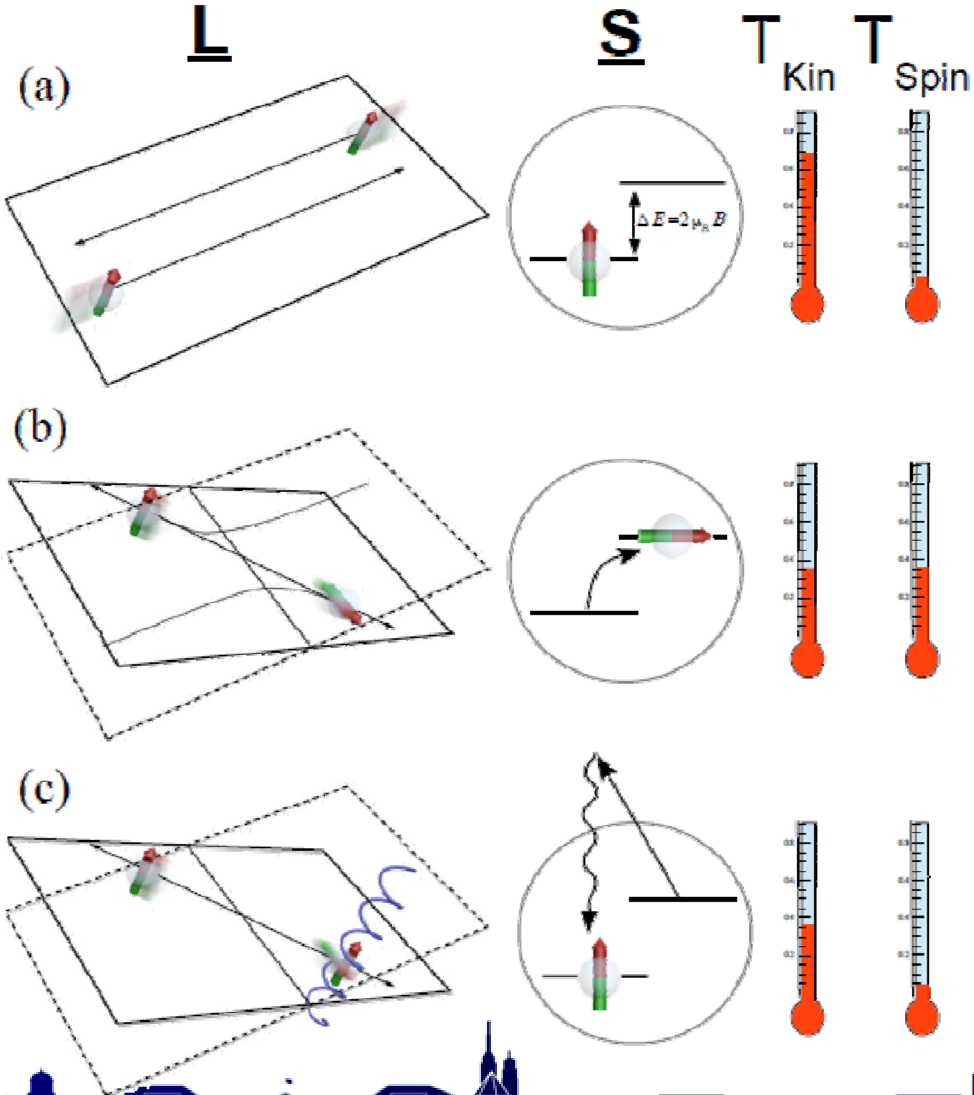
Another dipolar effect: Coupling spin and motion





A.Einstein, W.J. de Haas
Experimenteller Nachweis der Ampéreschen Molekularströme
Verhandlungen der DPG 17, 152 (1915)

Principle of demagnetization cooling of dipolar atoms



Appl. Phys. B 77, 765–772 (2003)
DOI: 10.1007/s00340-003-1334-0

Applied Physics B
Lasers and Optics

S. HENSLER[1,✉]
J. WERNER[1]
A. GRIESMAIER[1]
P.O. SCHMIDT[1]
A. GÖRLITZ[1]
T. PFAU[1]
S. GIOVANAZZI[2]
K. RZĄŻEWSKI[3]

Dipolar relaxation in an ultra-cold gas of magnetically trapped chromium atoms

[1] 5. Physikalisches Institut, Universität Stuttgart, Pfaffenwaldring 57, 70550 Stuttgart, Germany
[2] School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews, Fife, KY 16 9SS, Scotland
[3] Center for Theoretical Physics and College of Science, Polish Academy of Science, Aleja Lotników 32/46, 02-668 Warsaw, Poland

$$U_{\mathrm{dd}}(\boldsymbol{r}) = \mu_0 (g_S \mu_{\mathrm{B}})^2 \frac{(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2) - 3\left(\boldsymbol{S}_1 \cdot \hat{\boldsymbol{r}}\right)\left(\boldsymbol{S}_2 \cdot \hat{\boldsymbol{r}}\right)}{4\pi r^3} . \qquad (1)$$

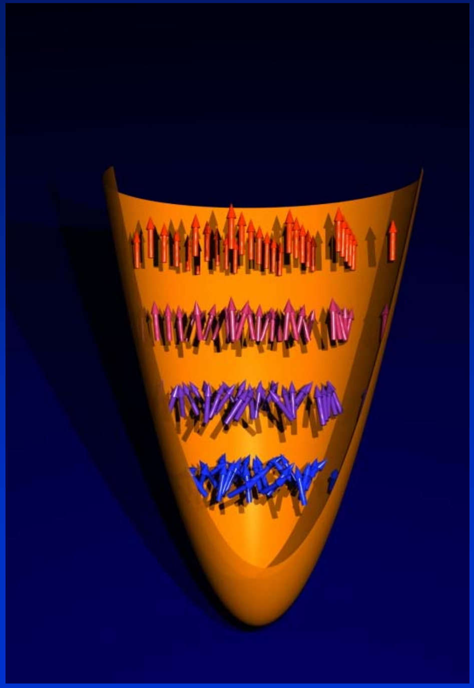
Here we have introduced the interatomic separation $\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$ with $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$ and the magnetic permeability of the vacuum μ_0. The tensorial part of the dipolar interaction (1), namely $(\boldsymbol{S}_1 \boldsymbol{S}_2) - 3(\boldsymbol{S}_1 \hat{\boldsymbol{r}})(\boldsymbol{S}_2 \hat{\boldsymbol{r}})$, can be rewritten in terms of spin-flip operators as

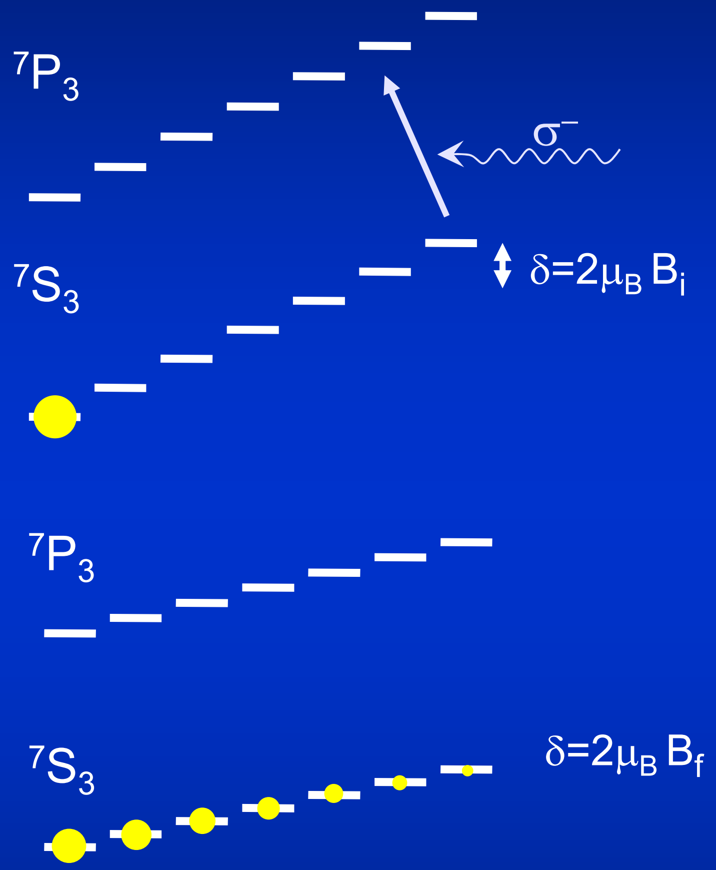
$$\begin{aligned} S_{1z} S_{2z} &+ \frac{1}{2}\left(S_{1+} S_{2-} + S_{1-} S_{2+}\right) \\ &- \frac{3}{4}\left(2\hat{z} S_{1z} + \hat{r}_- S_{1+} + \hat{r}_+ S_{1-}\right) \\ &\times \left(2\hat{z} S_{2z} + \hat{r}_- S_{2+} + \hat{r}_+ S_{2-}\right) , \end{aligned}$$

$$\sigma_0 = \frac{16\pi}{45} S^4 \left(\frac{\mu_0 (g_S \mu_{\mathrm{B}})^2 m}{4\pi\hbar^2}\right)^2 [1 + h(1)] ,$$

$$\sigma_1 = \frac{8\pi}{15} S^3 \left(\frac{\mu_0 (g_S \mu_{\mathrm{B}})^2 m}{4\pi\hbar^2}\right)^2 [1 + h(k_f/k_i)] \frac{k_f}{k_i} ,$$

$$\sigma_2 = \frac{8\pi}{15} S^2 \left(\frac{\mu_0 (g_S \mu_{\mathrm{B}})^2 m}{4\pi\hbar^2}\right)^2 [1 + h(k_f/k_i)] \frac{k_f}{k_i} ,$$





Coupling spin and motion

Demagnetization cooling

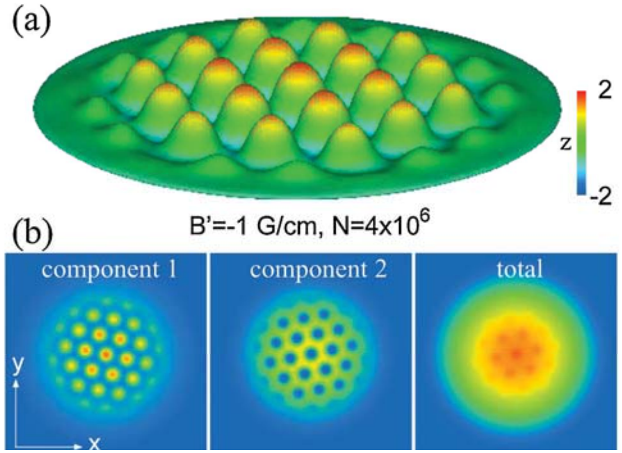
1950 A. Kastler: lumino-refridgeration

S. Hensler, A. Greiner, J. Stuhler and T. Pfau
Europhys. Lett., **71**, 918 (2005)
M. Fattori, T. Koch, S. Goetz, A. Griesmaier, S. Hensler, J. Stuhler, and T. Pfau
*Nature Physics **2**, 765 (2006)*
V. V. Volchkov, J. Rührig, T. Pfau, A. Griesmaier
*Phys. Rev. A **89** (2013)*

Outlook: Stronger dipoles - ferrofluid

Classical

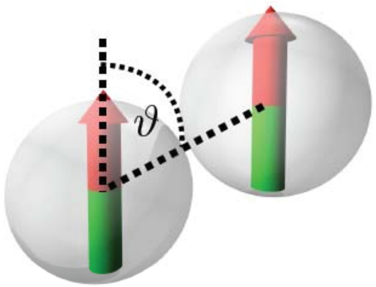
Quantum



H. Saito, Y. Kawaguchi, and M. Ueda
Phys. Rev. Lett. **102**, 230403 (2009)

Interactions in ultracold gases

Dipolar gases



$$\varepsilon_{dd} \propto \frac{m\mu^2}{a}$$

dipolar interaction (μ^2)

contact interaction (a)

	Chromium	Erbium	Dysprosium
Magnetic moment μ	6 μ_B	7 μ_B	10 μ_B
$\varepsilon_{dd} = \frac{\mu_0 \mu^2 m}{12\pi\hbar^2 a_{bg}}$	0.16	0.33-0.45	≈ 1

Degenerate Erbium

K. Aikawa et al., PRL **108**, 210401 (2012)

K. Aikawa et al., PRL **112**, 010404 (2014)

Degenerate Dysprosium

M. Lu et al., PRL **107**, 190401 (2011)

M. Lu et al., PRL **108**, 215301 (2012)

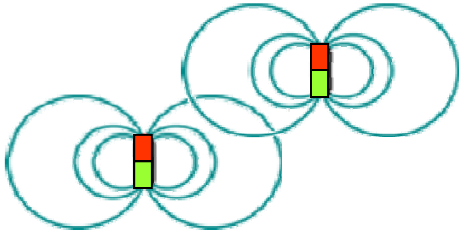
Properties of Dysprosium

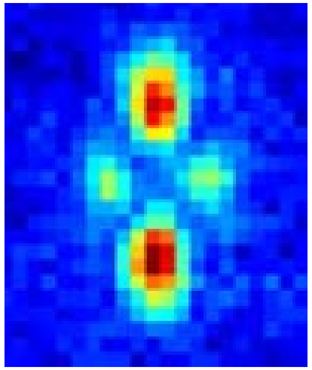
Stable Isotopes	^{161}Dy (19%), ^{162}Dy (26%), ^{163}Dy (25%), ^{164}Dy (28%)
Electronic structure	[Xe] $4f^{10}\,6s^2$ → $^5I_8$
Nuclear spin	5/2 (for fermions)
Magnetic moment μ	10 μ_B (highest of all atomic elements)

The (current) Team

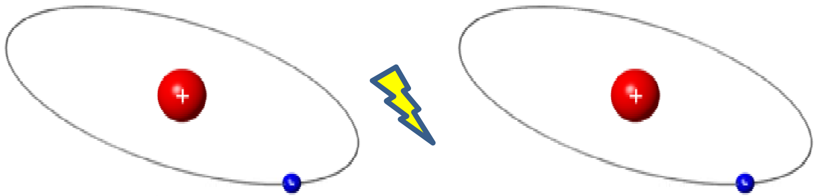
Matthias Wenzel

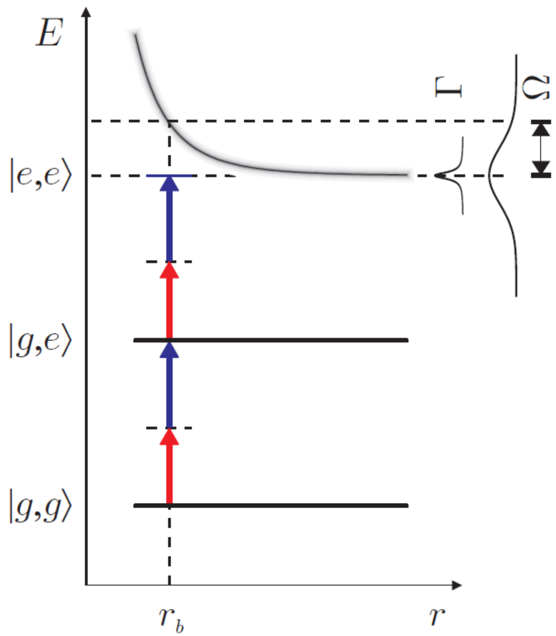
Lecture I: (magnetic) dipolar gases



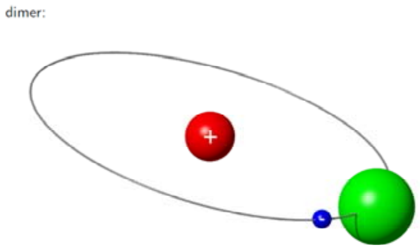


Lecture II: Rydberg Rydberg interaction

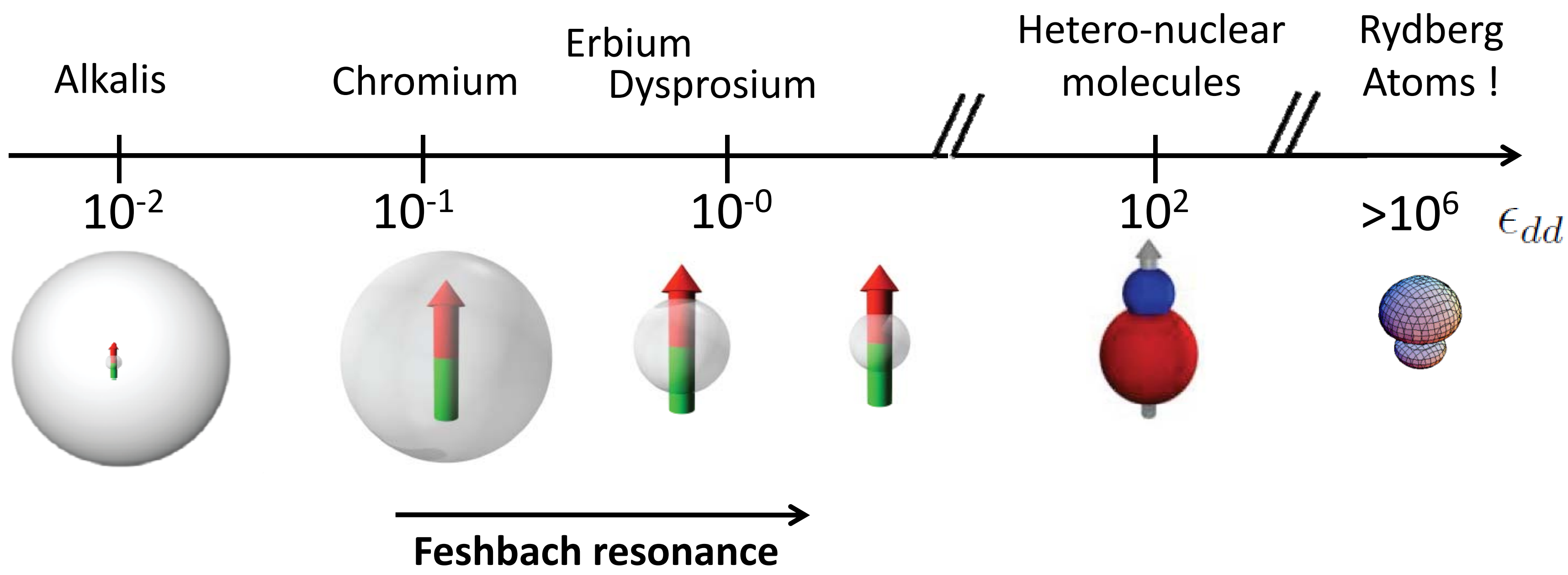




Lecture III : Rydberg ground state interaction



dipolar interaction

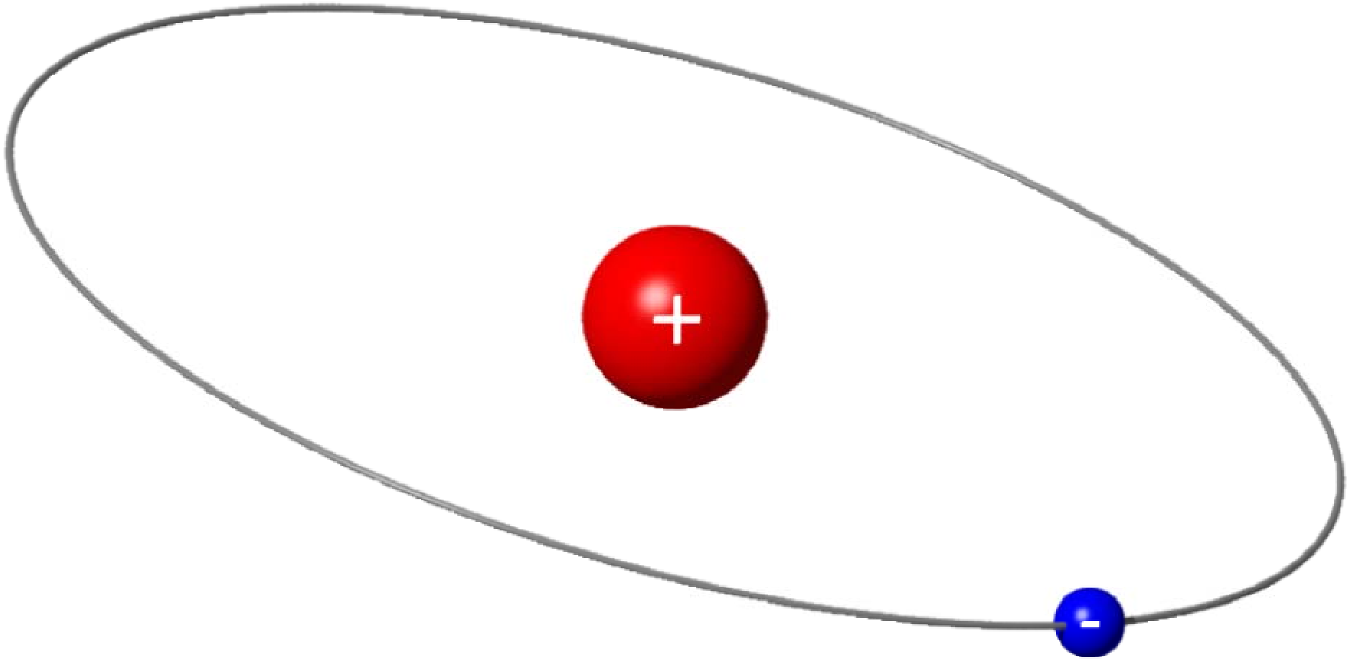


$\rightarrow$ *Realization of a purely dipolar condensate*

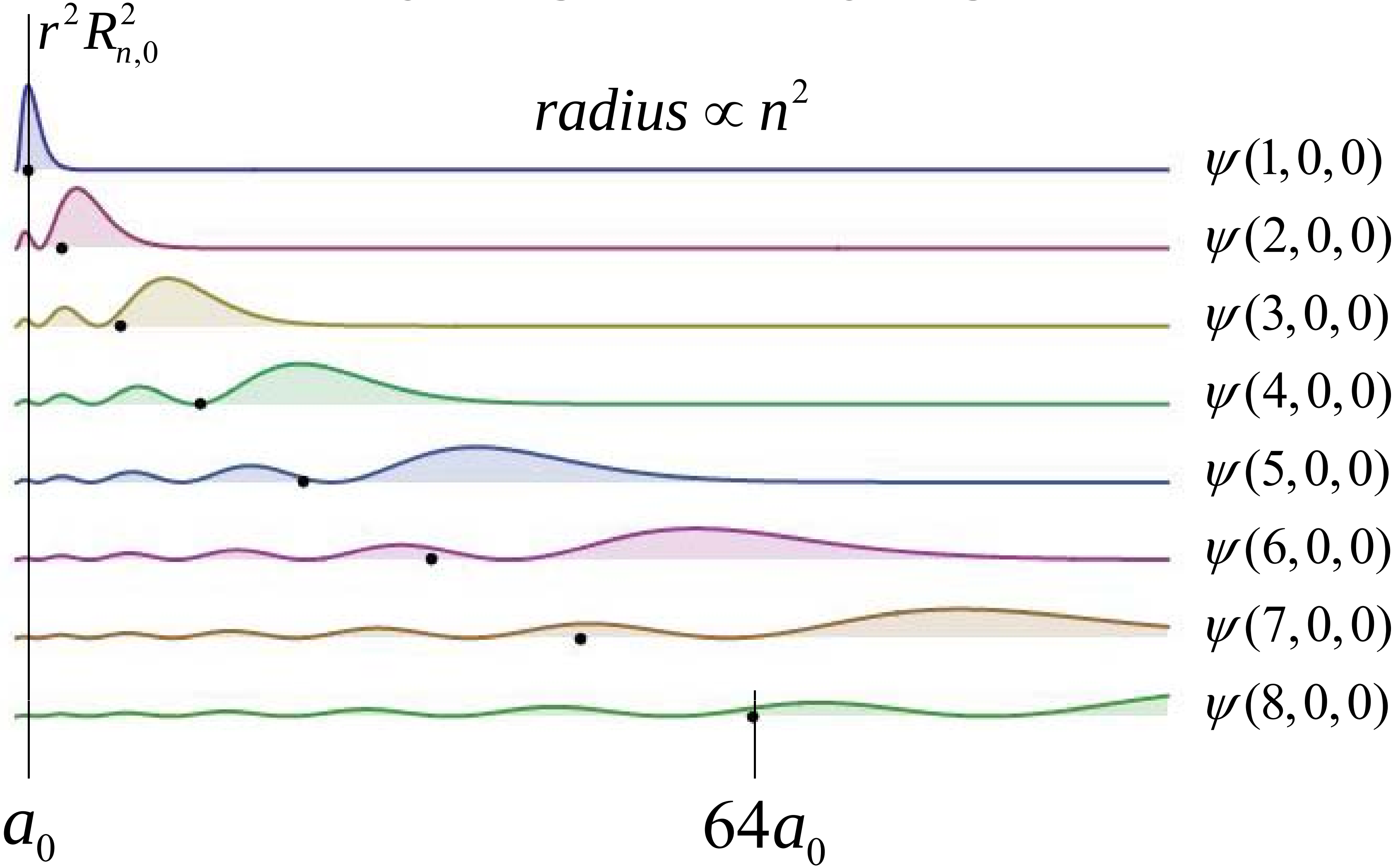
T. Lahaye *et al.*, Nature **448**, 672 (2007)

T. Koch, *et al*., Nature Physics **4**, 218 (2008)

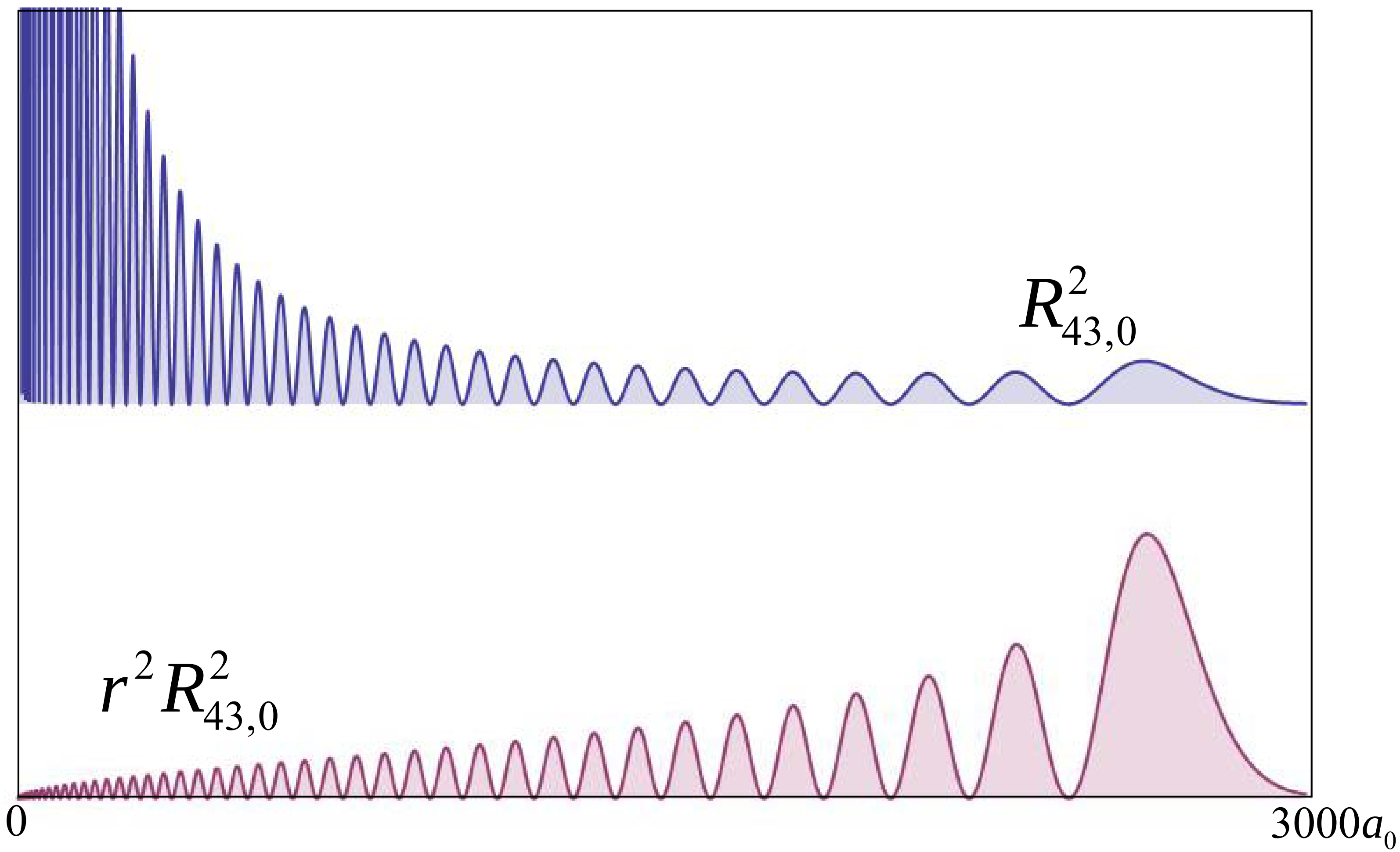
Rydberg atoms

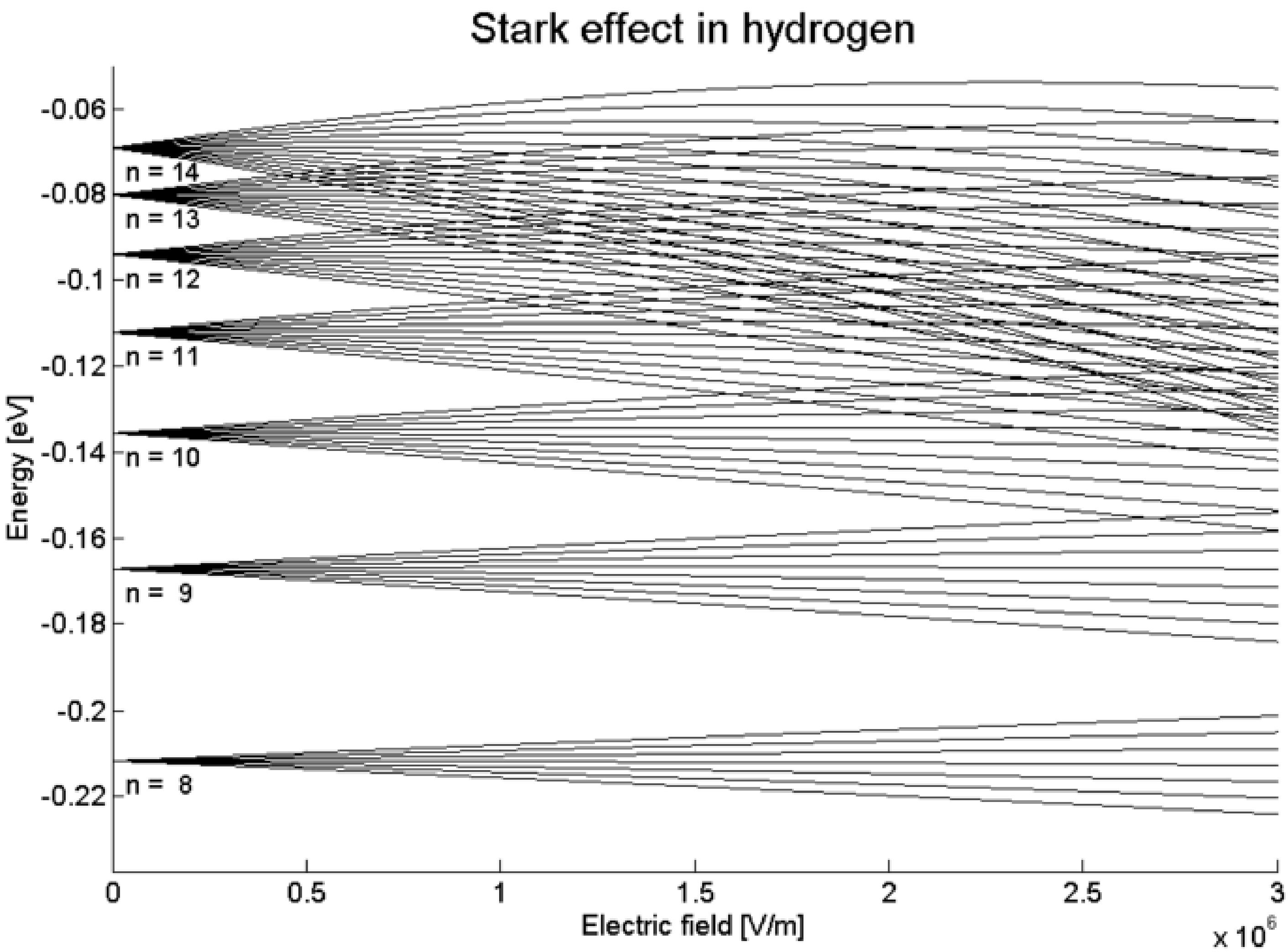


Rydberg basics - Hydrogen

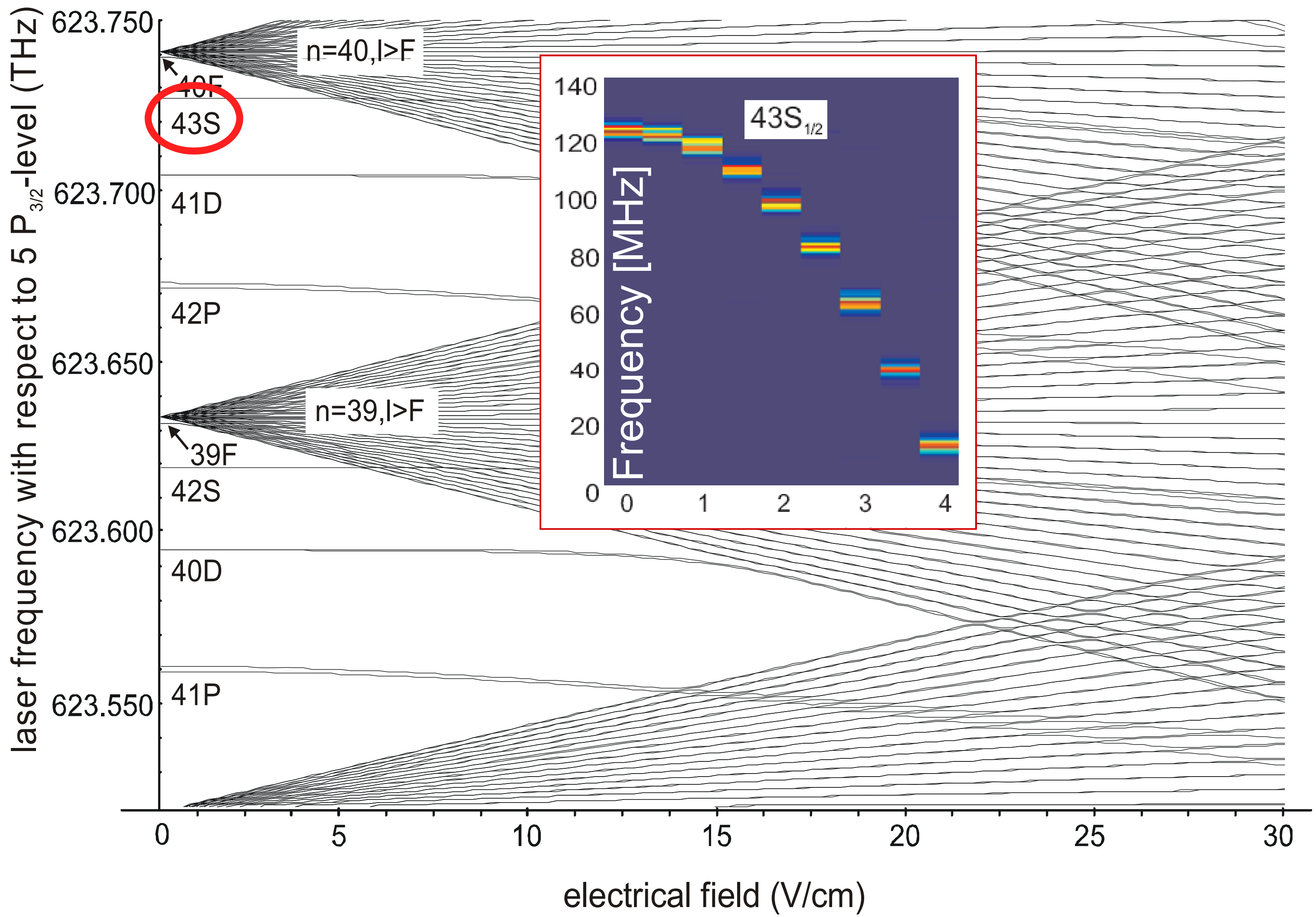


One typical example: 43S

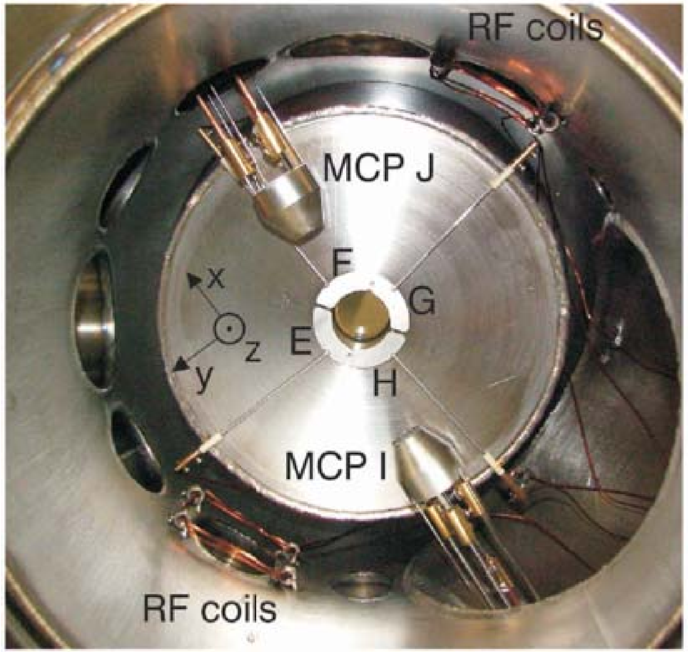



Stark effect in hydrogen
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
-0.22
n = 14
n = 13
n = 12
n = 11
n = 10
n = 9
n = 8
Energy [eV]
0
0.5
1
1.5
2
2.5
3
Electric field [V/m]
x 10^6

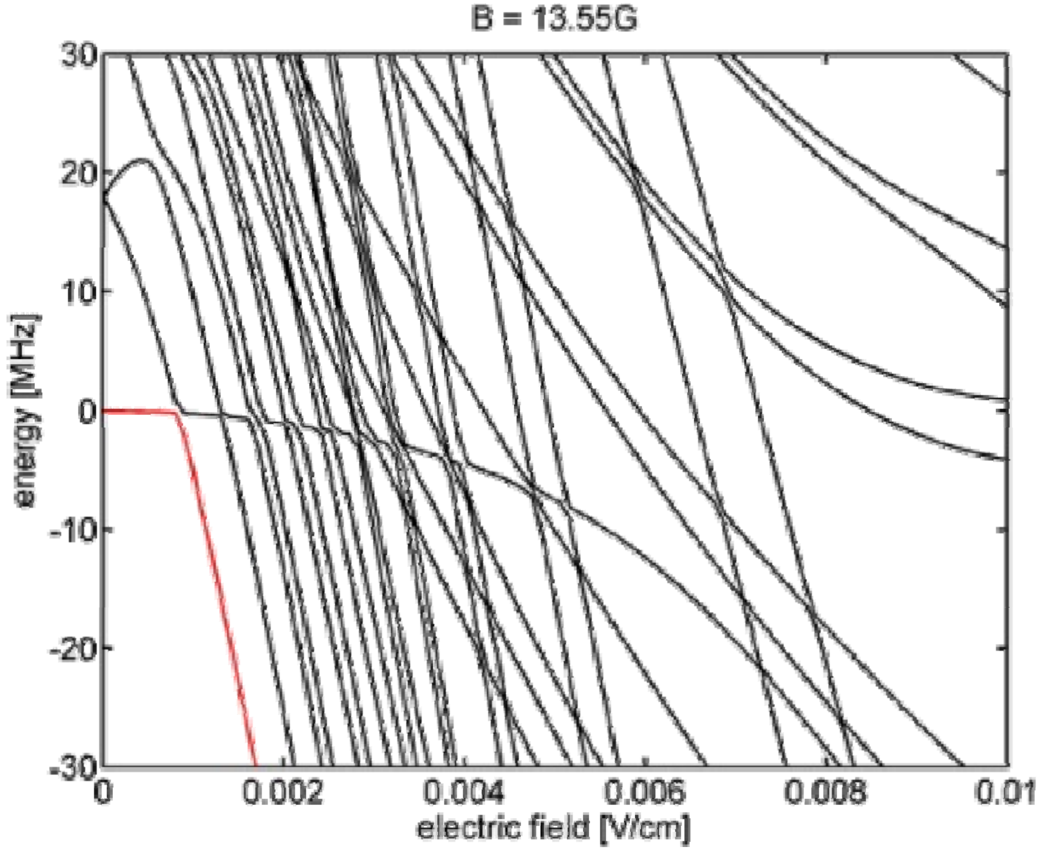
Reminder: Stark map of Rubidium



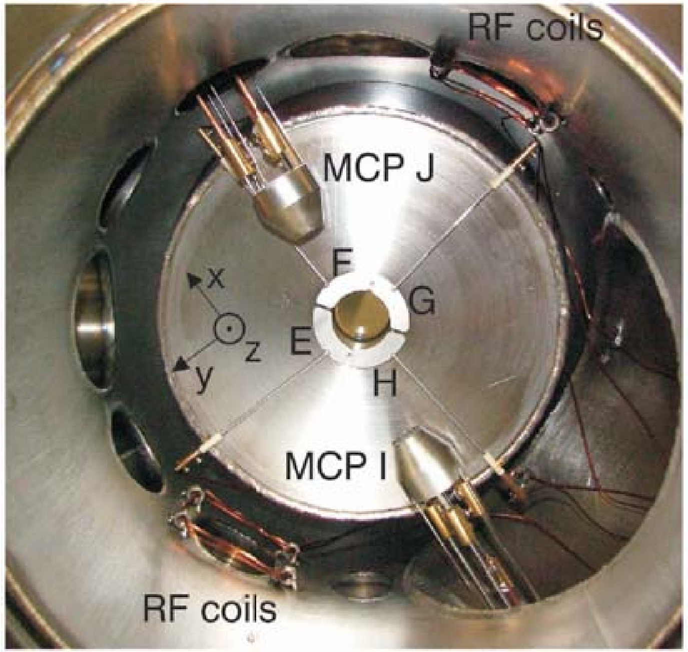
Electric field control

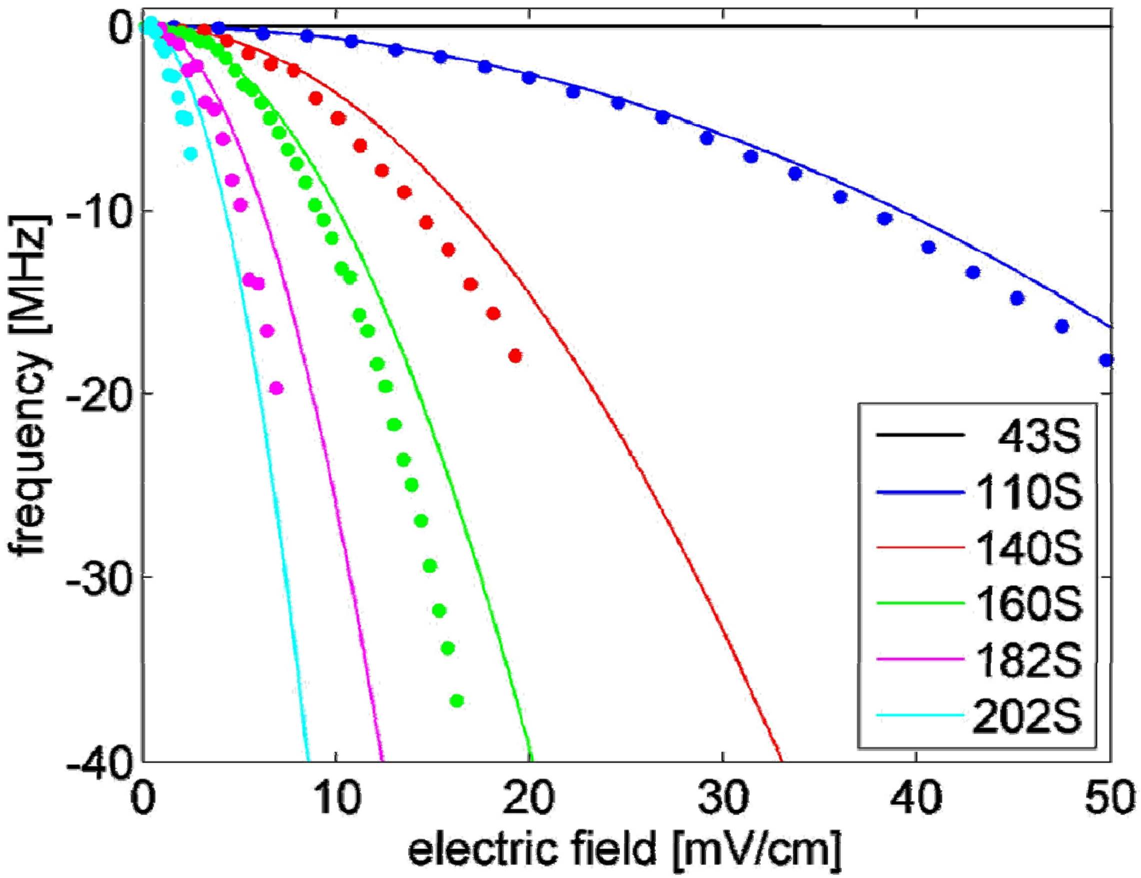


Stark map 182S

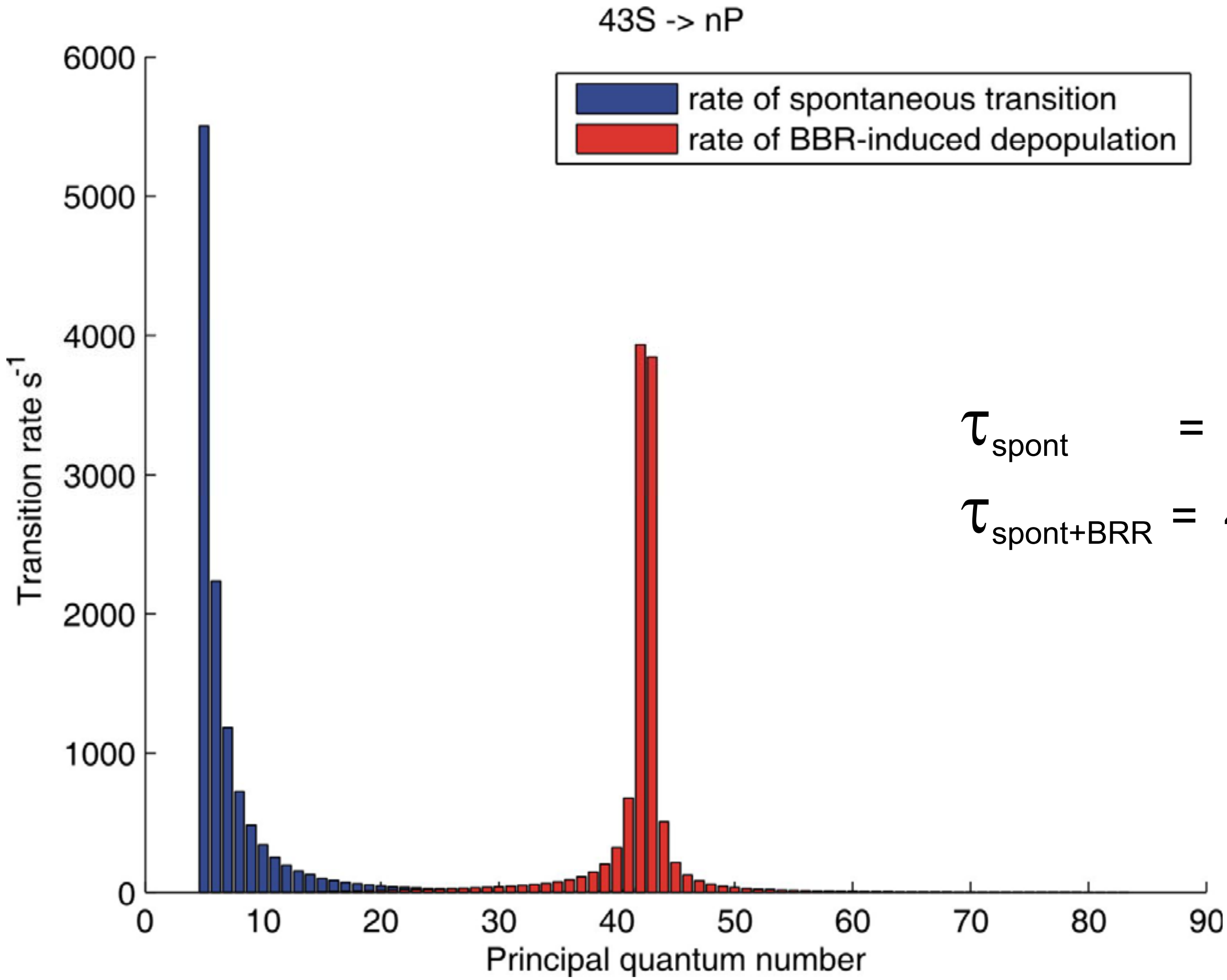


Electric field control





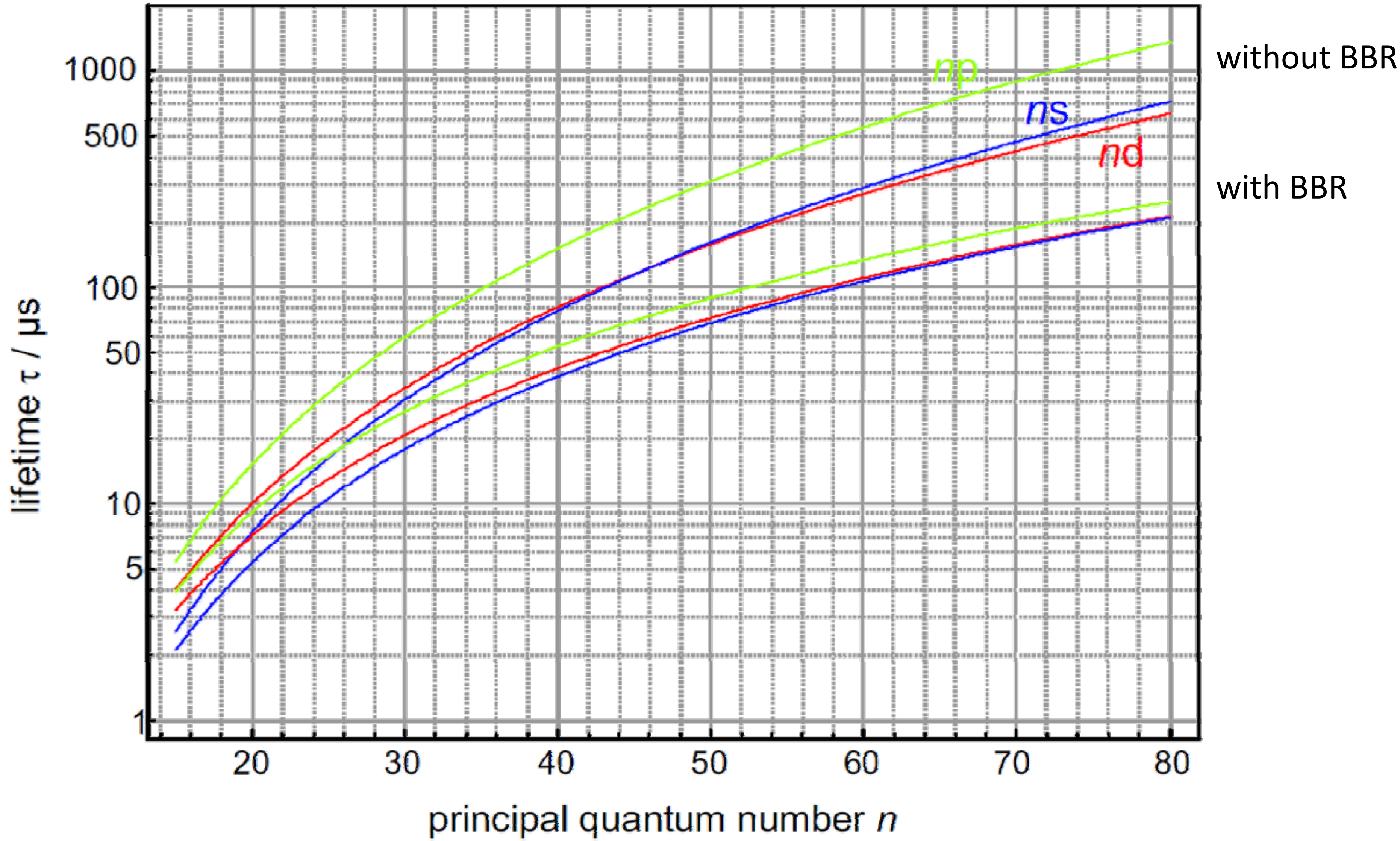
Lifetime of the 43s state with Blackbody Radiation

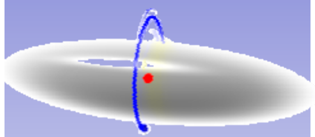


$\tau_{spont} = 99\mu s$

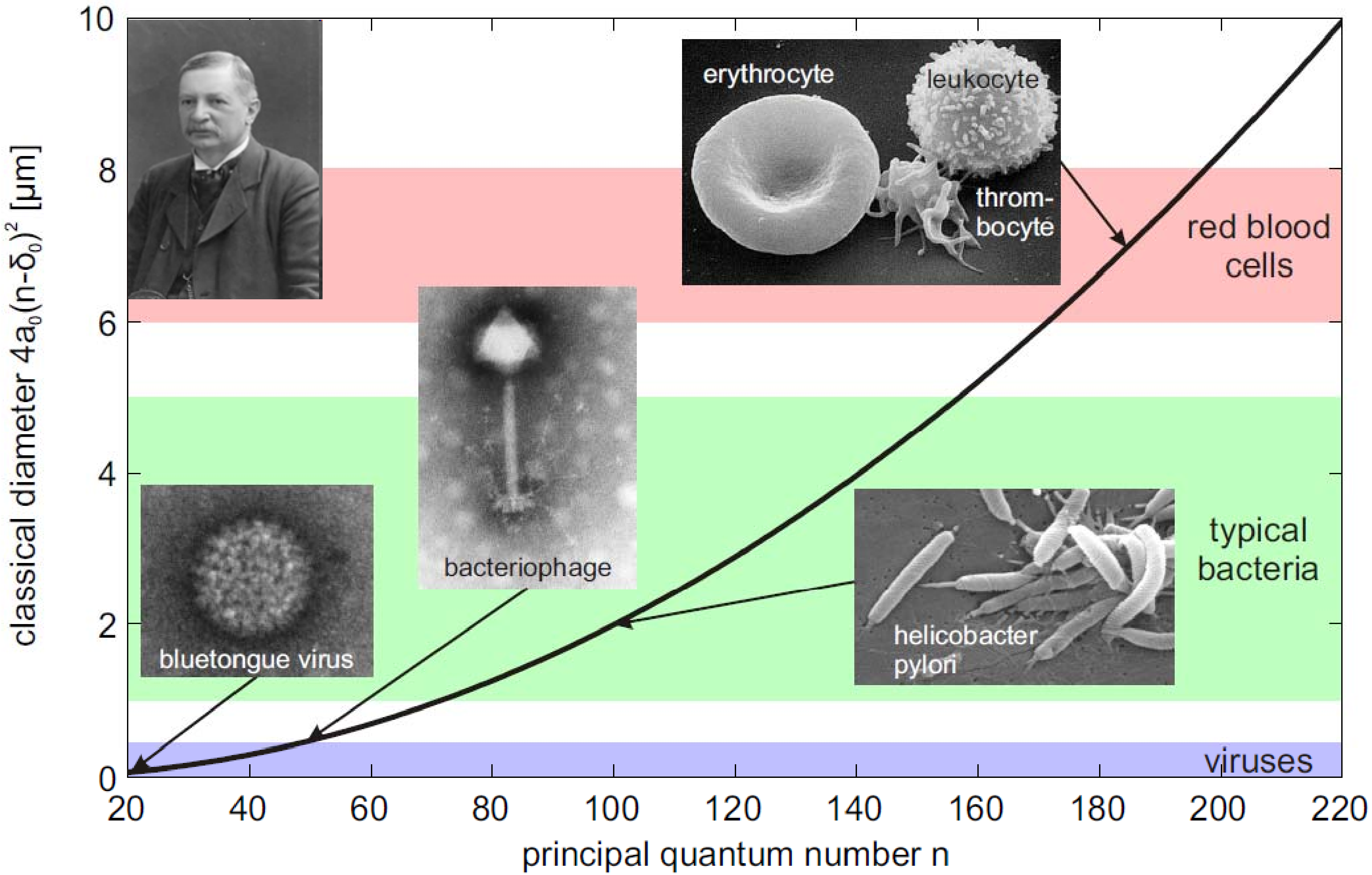
$\tau_{spont+BRR} = 46\mu s$

Rb lifetimes



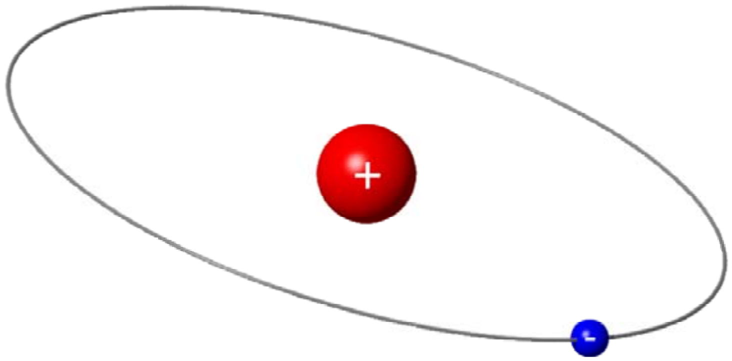


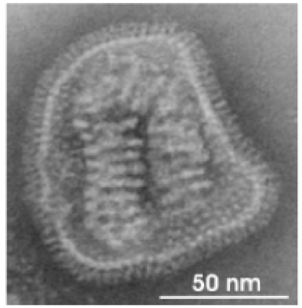
BEC



classical diameter $4a_0(n-\delta_0)^2$ [µm]
principal quantum number n
10
8
6
4
2
0
20
40
60
80
100
120
140
160
180
200
220
erythrocyte
leukocyte
throm-
bocyte
red blood
cells
bacteriophage
typical
bacteria
helicobacter
pylori
bluetongue virus
viruses

Properties of Rydberg Atoms

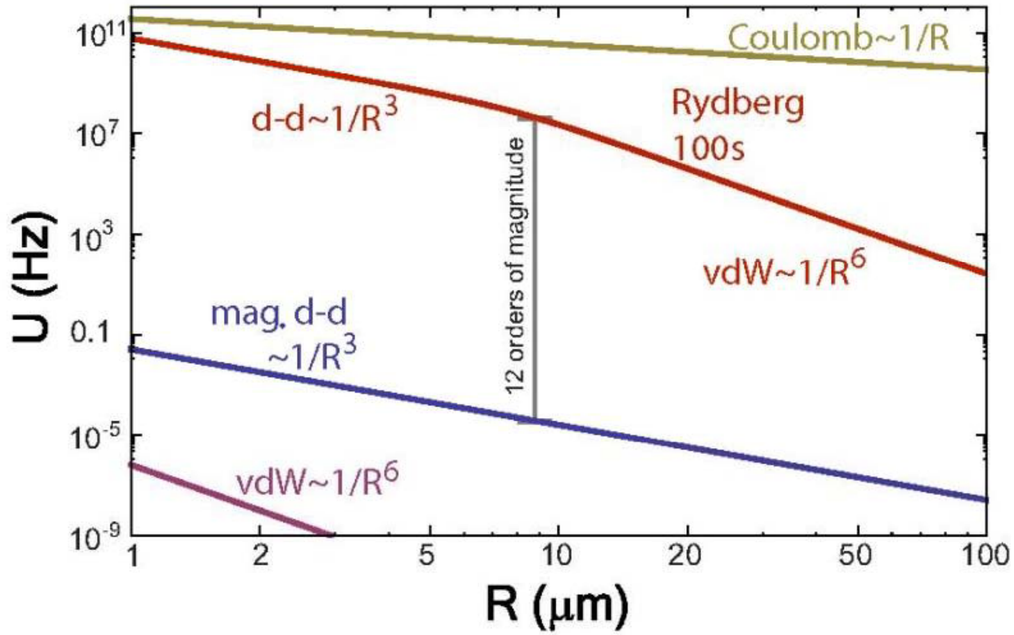




quantity	scaling	43S-state of ^{87}Rb
radius	$\propto n^2$	2384 a_0
lifetime	$\propto n^3$	50µs
Polarizability	$\propto n^7$	8 MHz $(V/cm)^{-2}$
Van der Waals C_6	$\propto n^{11}$	-1.7×10^{19} a.u.

The interactions between Rydberg states are …

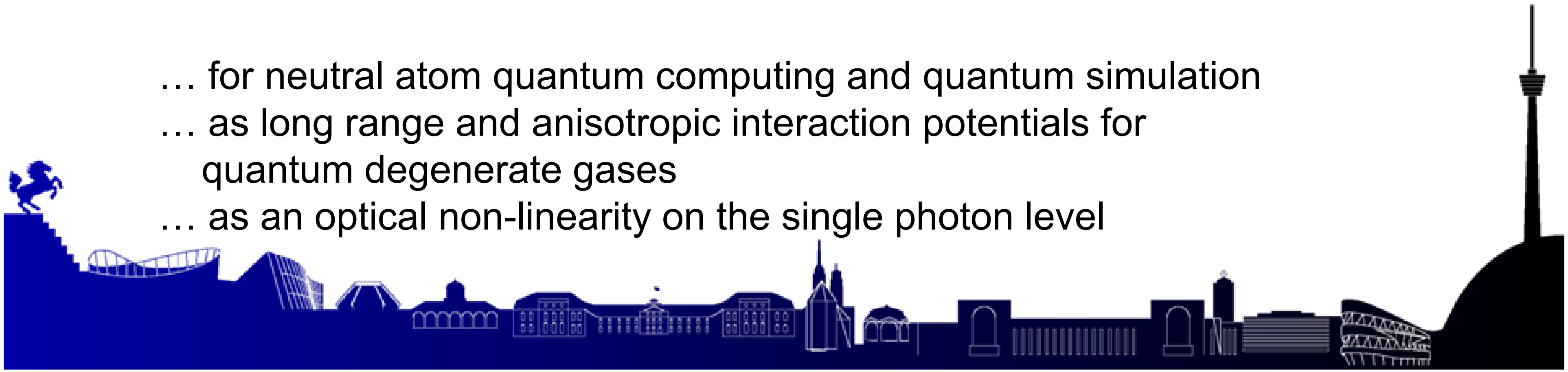
… strong
… long-range
… tunable
… switchable
… anisotropic



M. Saffman et al., Rev. Mod. Phys. 82, 2313 (2010)

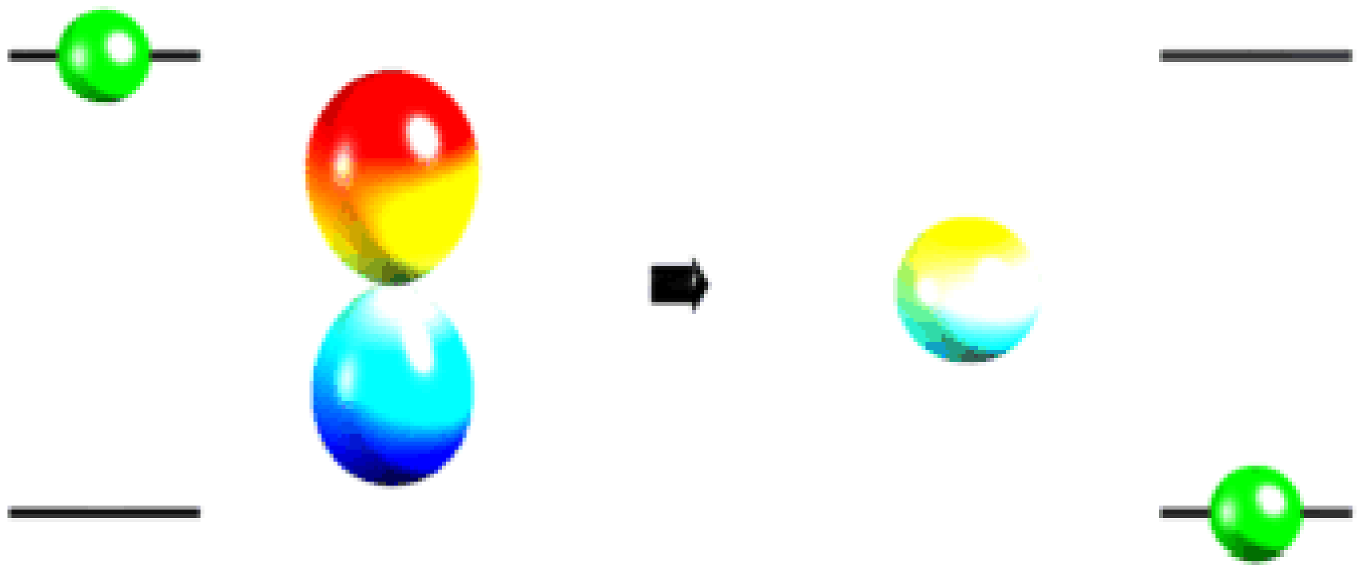
and can be used

… for neutral atom quantum computing and quantum simulation
… as long range and anisotropic interaction potentials for quantum degenerate gases
… as an optical non-linearity on the single photon level



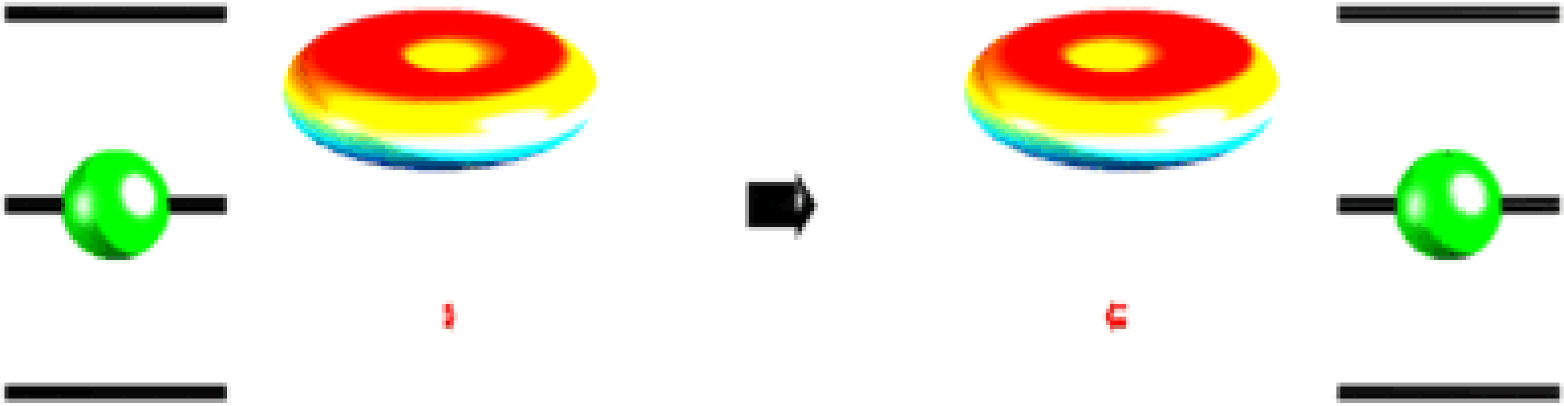
T. Förster, Z. Naturforsch 4a, 321 (1949)

Förster energy transfer



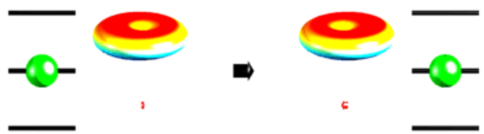
T. Förster, Z. Naturforsch 4a, 321 (1949)

Förster Resonance

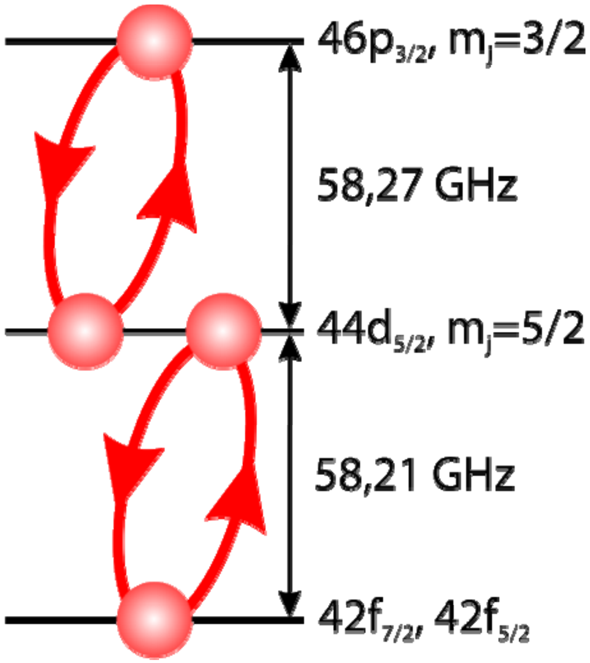


Dipolar interactions: Förster resonances

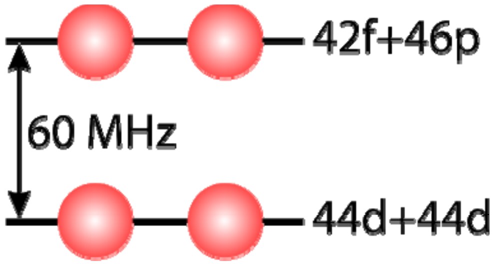
T. Förster, Z. Naturforsch 4a, 321 (1949)



Bare states



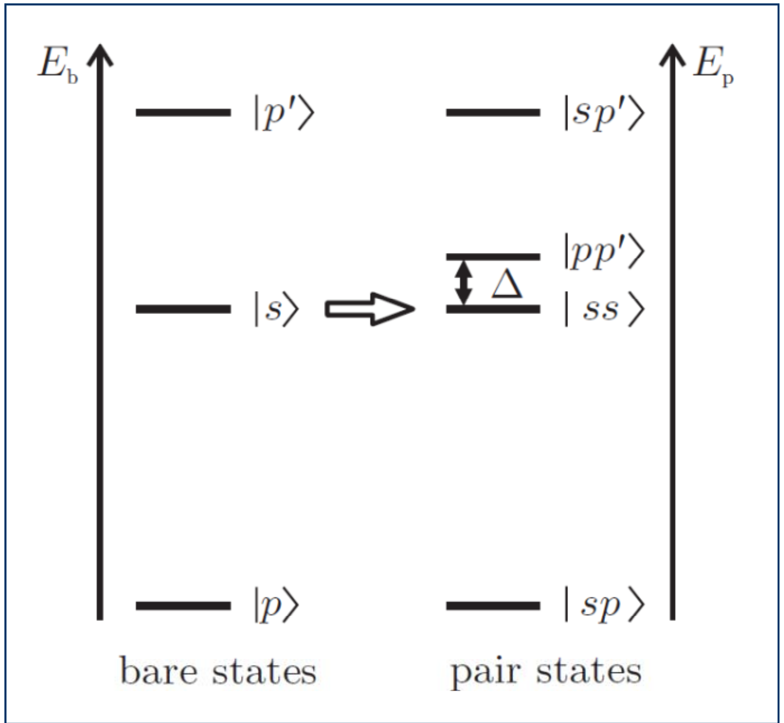
Pair states



finite Förster defect Δ:
van-der-Waals interaction (~ $1/R^6$)

no Förster defect Δ = 0:
resonant dipole-dipole interaction (~ $1/R^3$)

Interaction between Rydberg atoms



Förster resonance: tune Δ to zero

$$\mathcal{H}_{\mathrm{dd}} = \begin{pmatrix} 0 & \frac{d_1 d_2}{R^3} \\ \frac{d_1 d_2}{R^3} & \Delta \end{pmatrix}$$

$$E_{\pm} = \frac{\Delta}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + \left(\frac{d_1 d_2}{R^3}\right)^2}$$

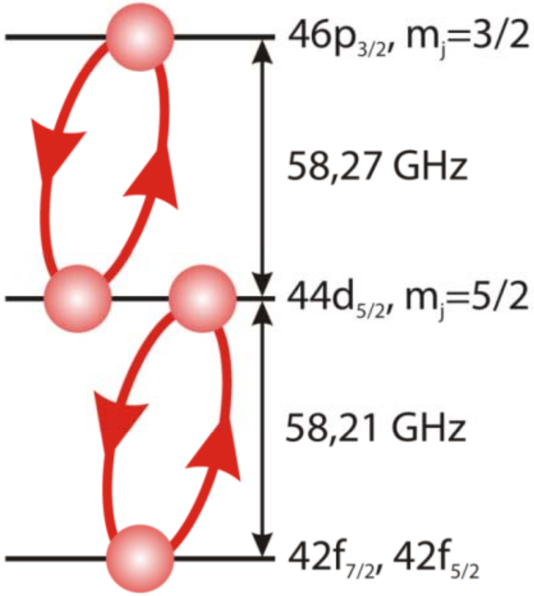
$$\Delta \gg d_1 d_2 / R^3$$

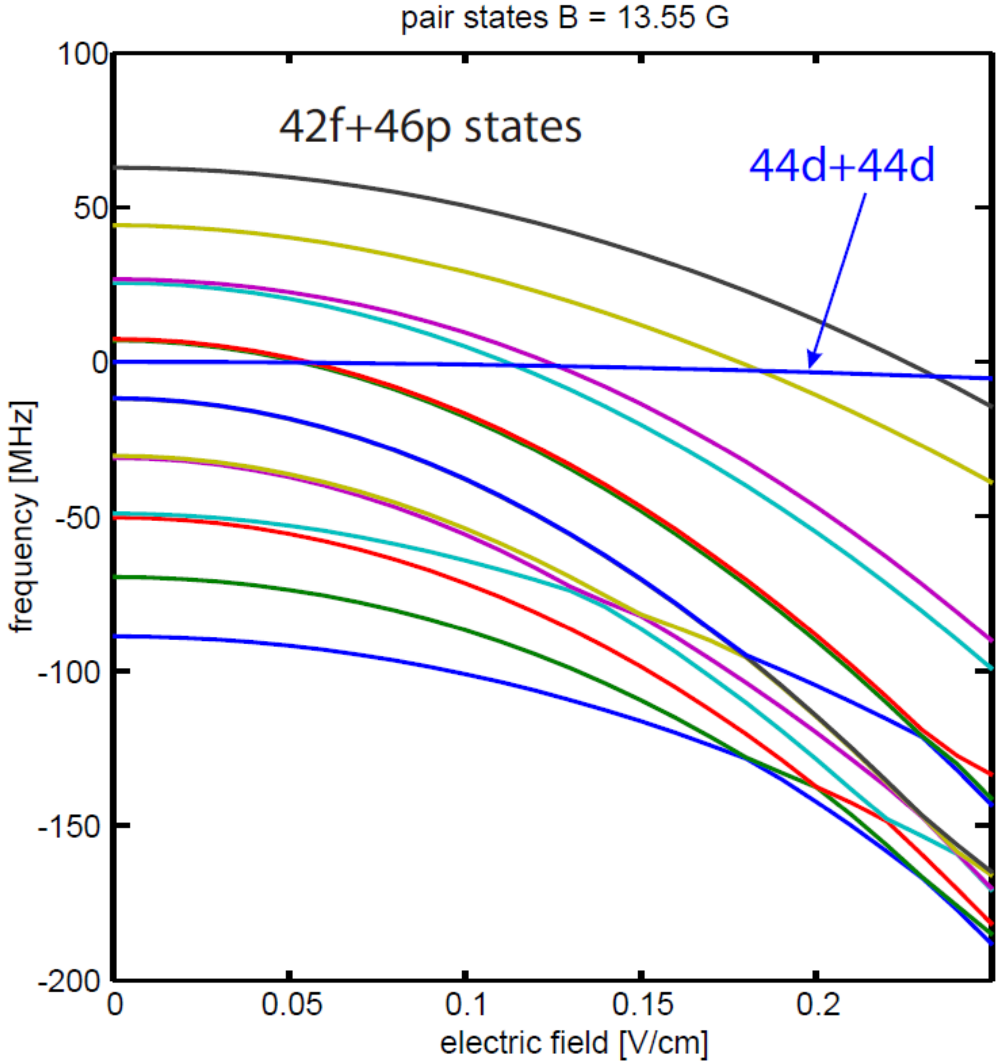
$$E_{\mathrm{vdW}} = E_- = -\frac{1}{\Delta}\frac{(d_1 d_2)^2}{R^6} \equiv \frac{C_6}{R^6}$$

sign depends on Δ !

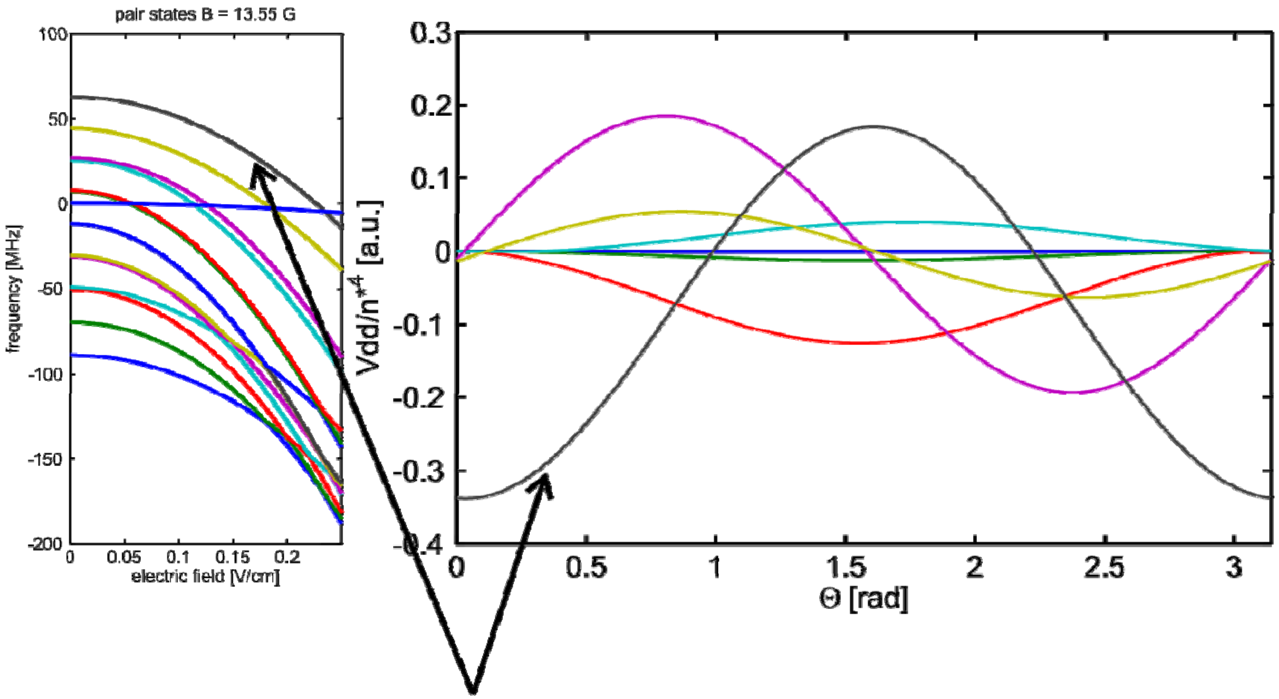
Dipolar interaction for $R << \sqrt[3]{\frac{d_1 d_2}{\Delta}}$

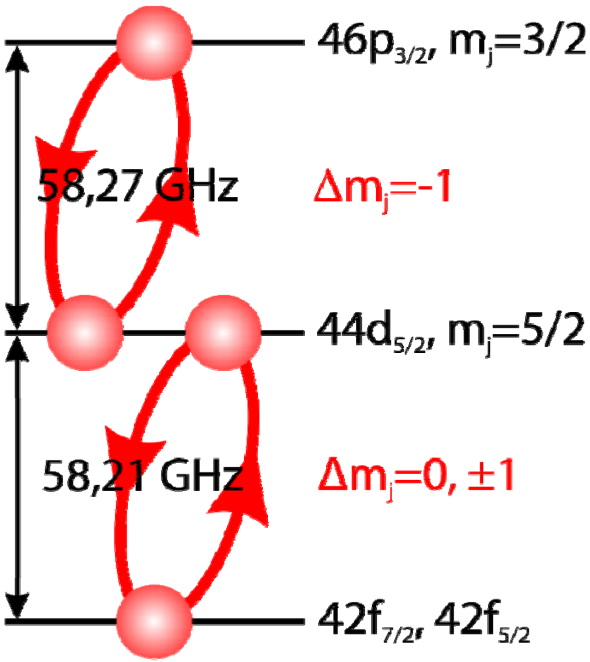
Stark tuned Förster resonances





Förster resonances





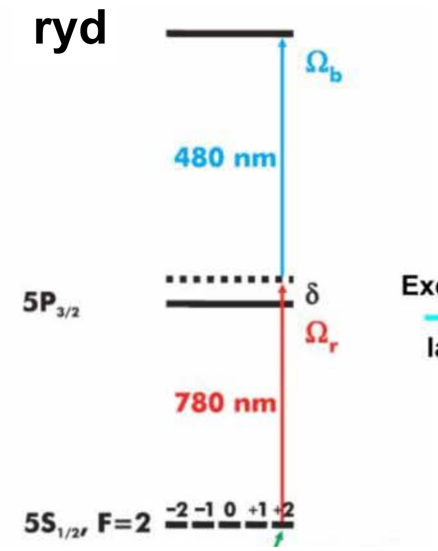
$|46p_{3/2}, m_j{=}3/2; 42f_{7/2}, m_j{=}7/2\rangle$

$\leftrightarrow |44d_{5/2}, m_j{=}5/2; 44d_{5/2}, m_j{=}5/2\rangle$

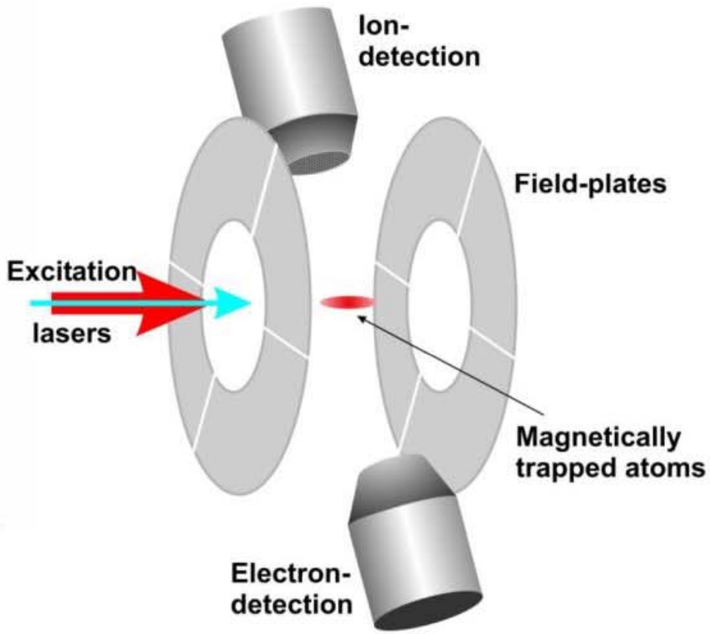
Förster resonances

Is this all coherent in a dense gas??

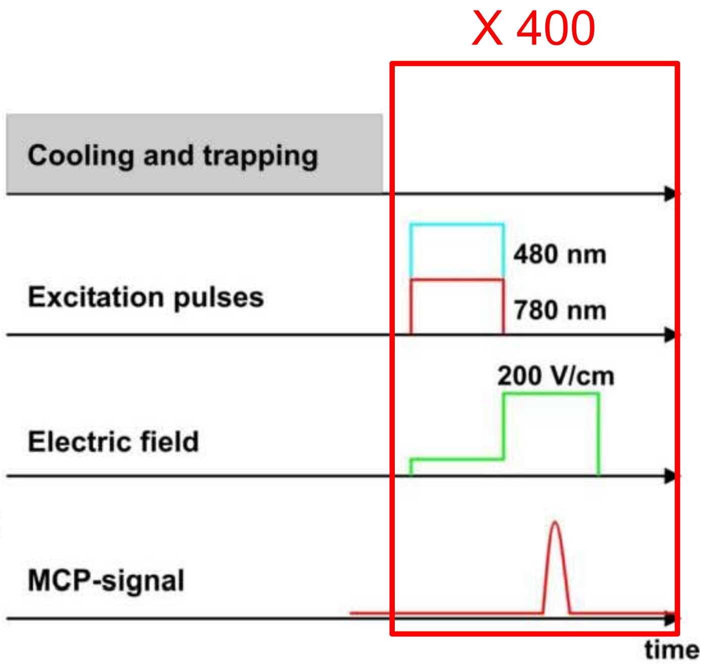
Some experimental details



Excitation scheme

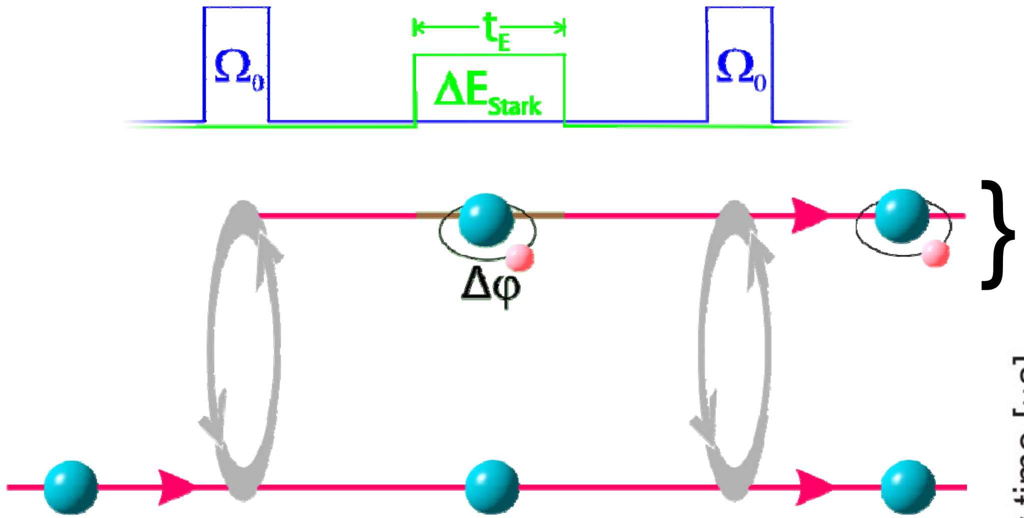


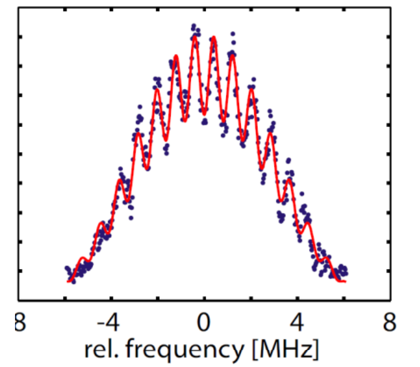
Experimental setup

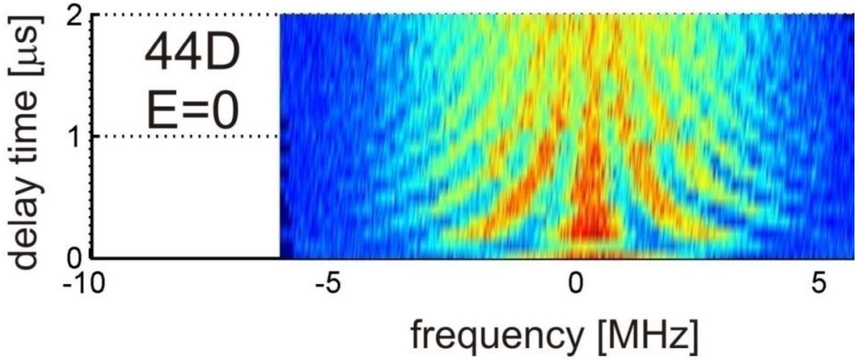


Experimental sequence

Ramsey interferometer



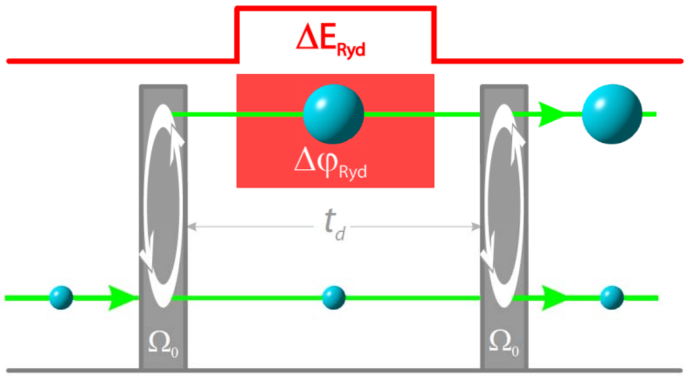


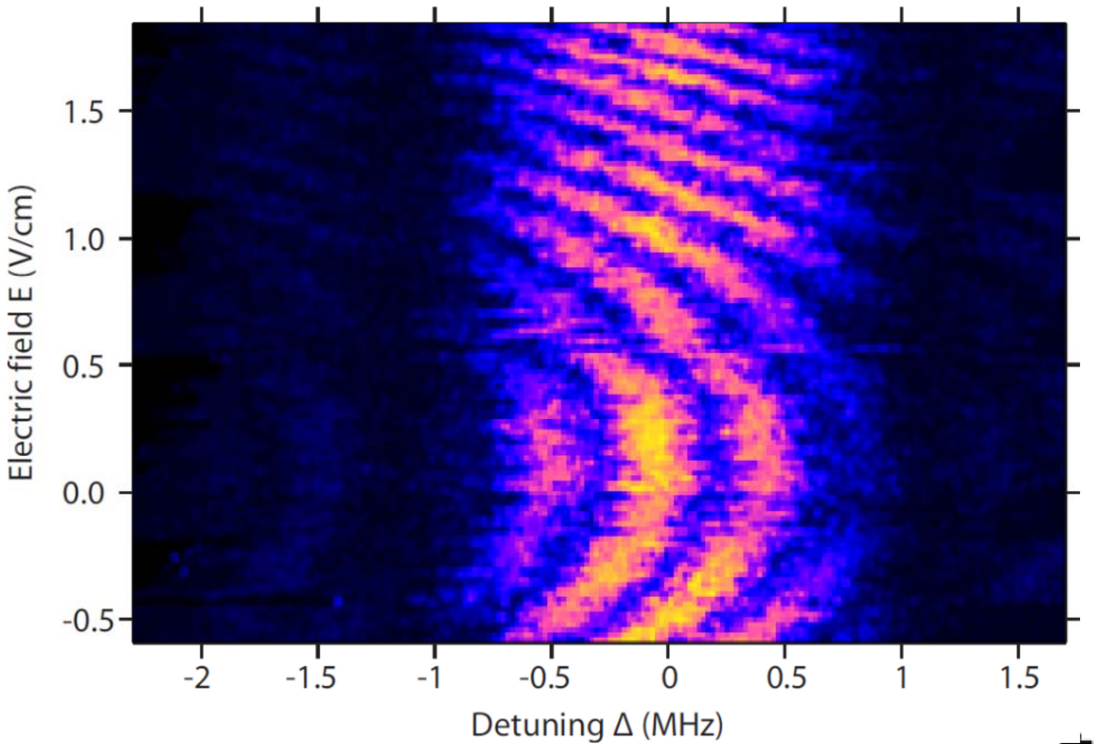


- visibility:
  - ➔ decoherence
  - ➔ population transfer
- phase:
  - ➔ energy shift (e.g. Stark effect $\Delta \Phi = 1/2 \alpha E^2/h\ t$)

Rydberg atom interferometry

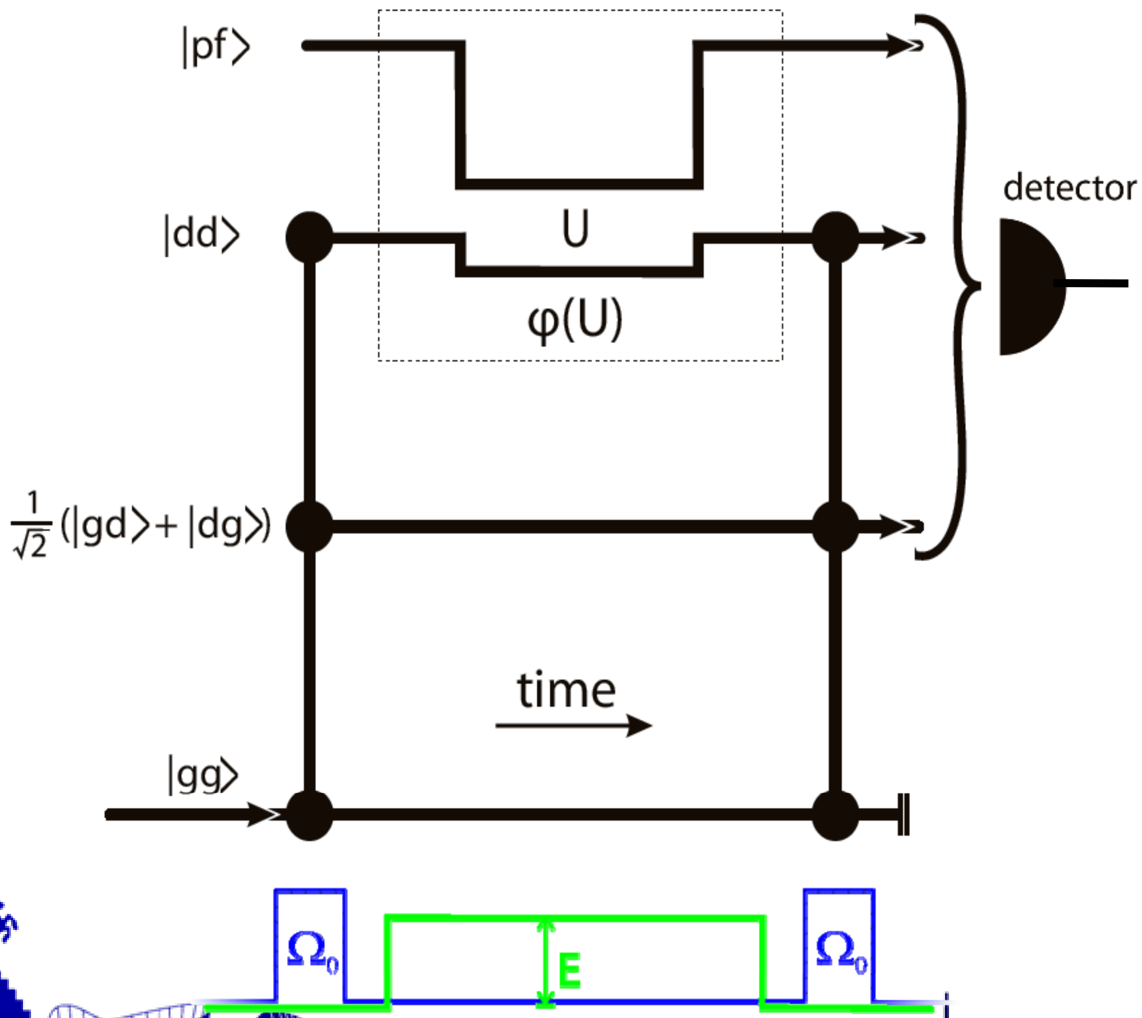
$$\Delta\varphi_{Ryd} = \frac{1}{\hbar}\int \alpha \underline{E}^2(t)dt$$



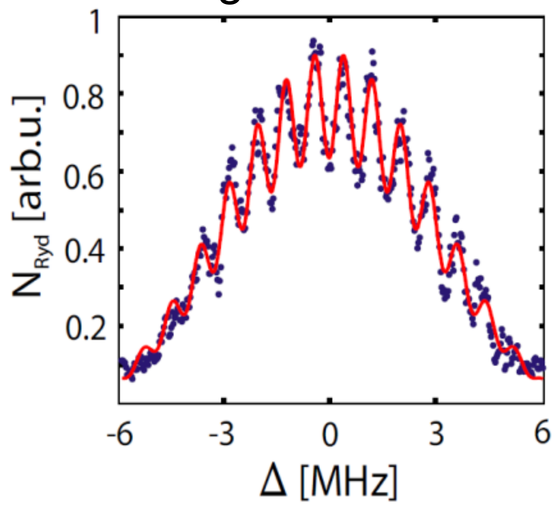


43S high density

A pair state interferometer

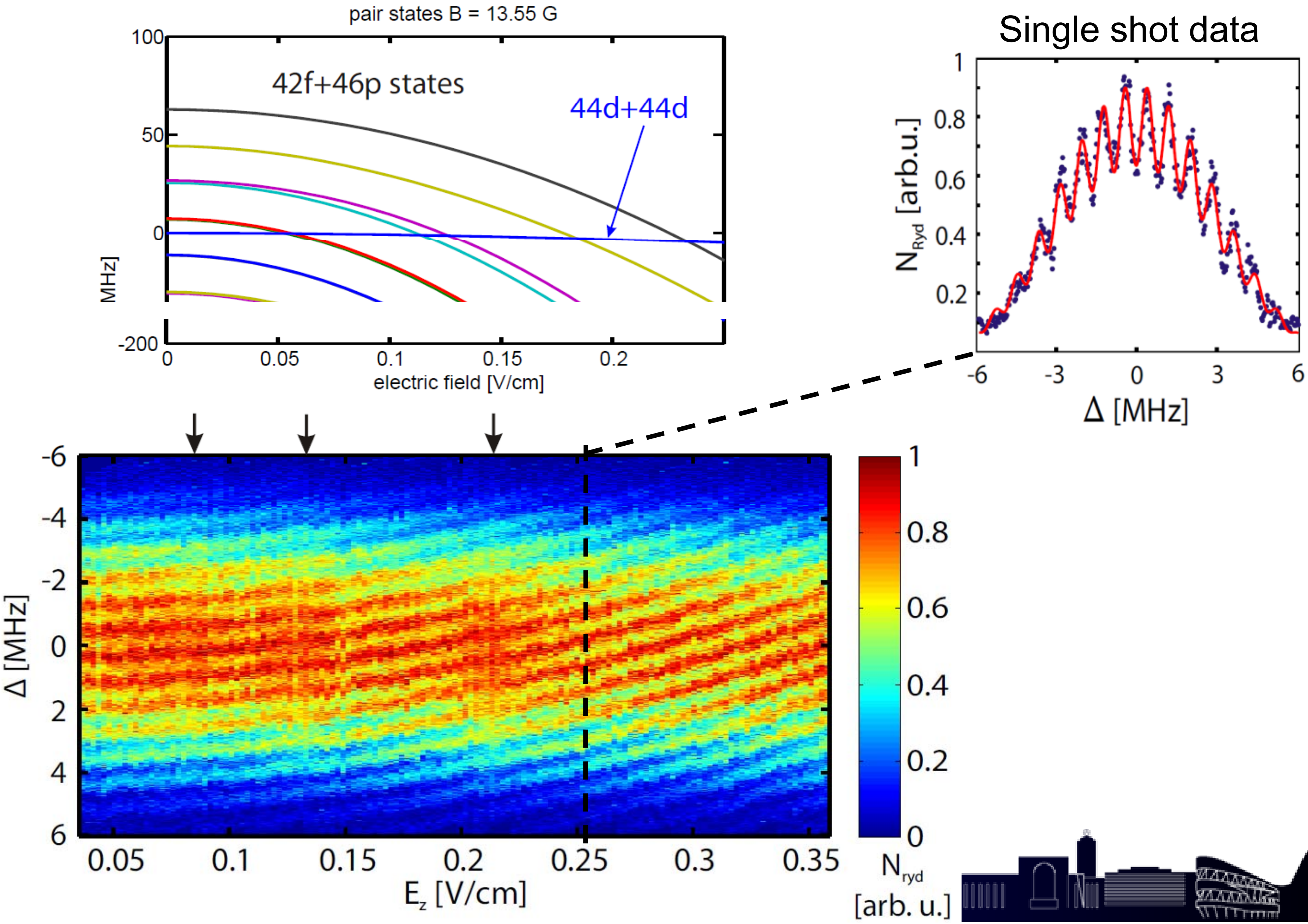


Single shot data

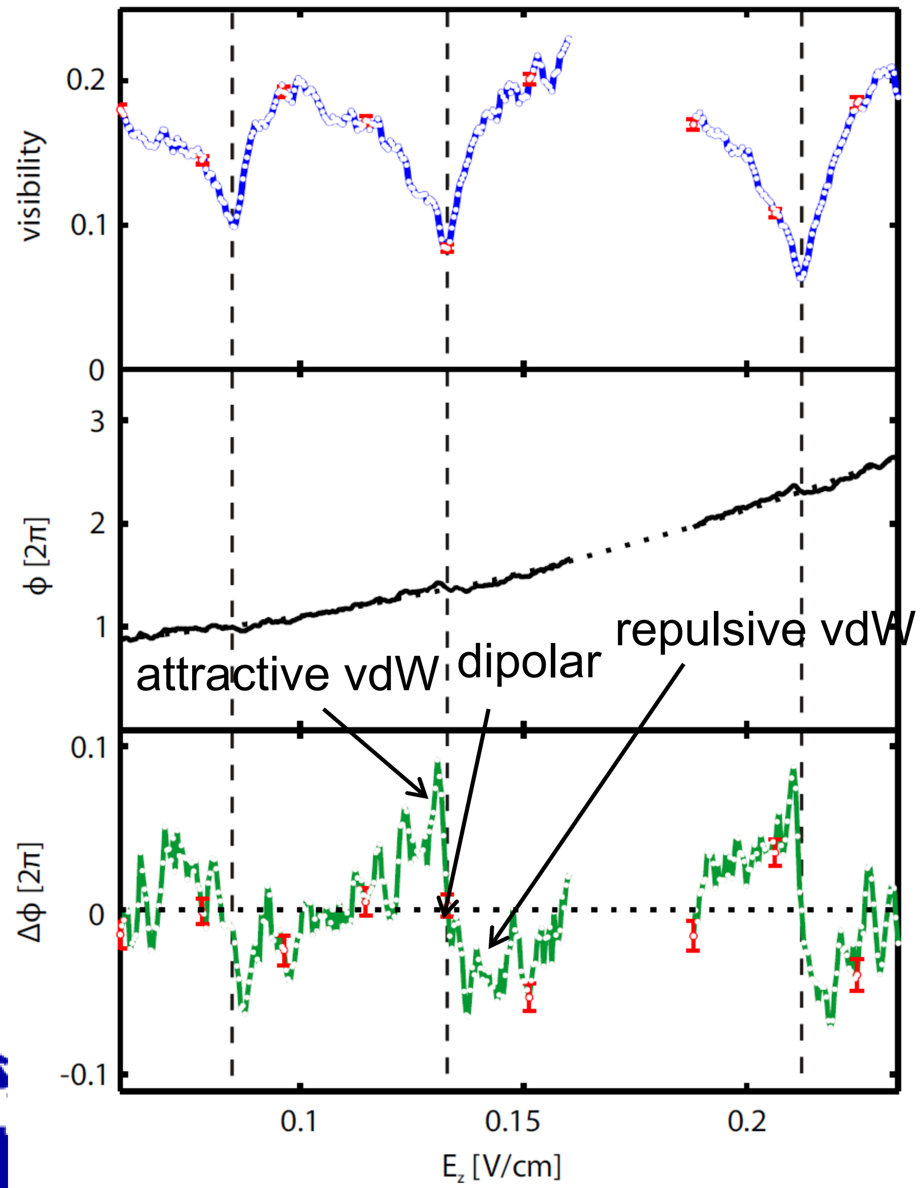


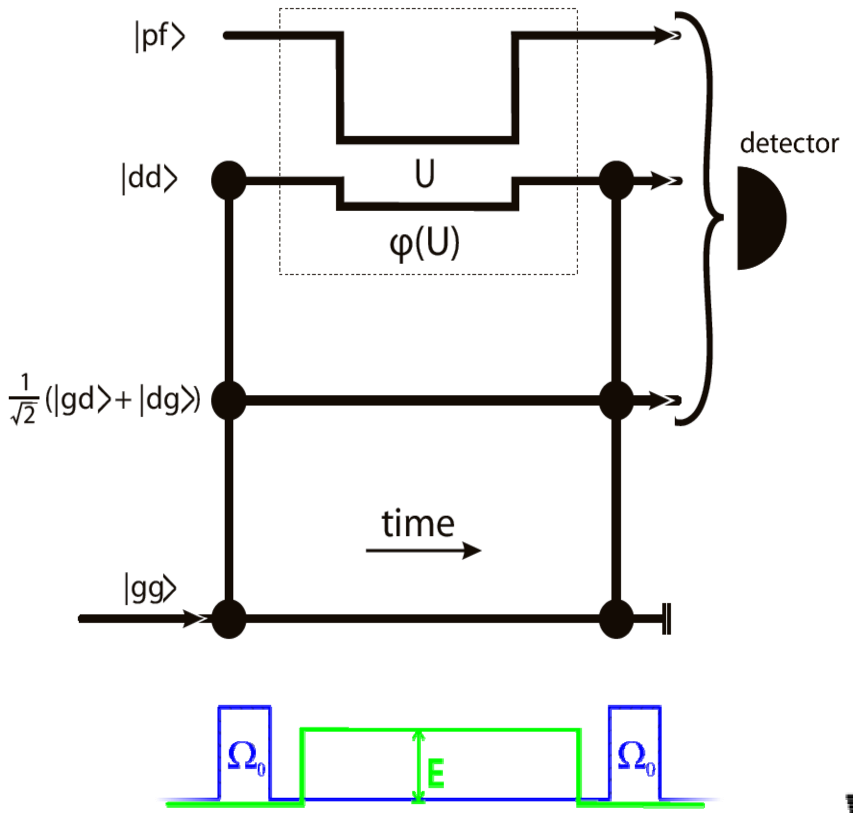
→ see Nipper et al.
PRL 2012
PRX 2012

A pair state interferometer

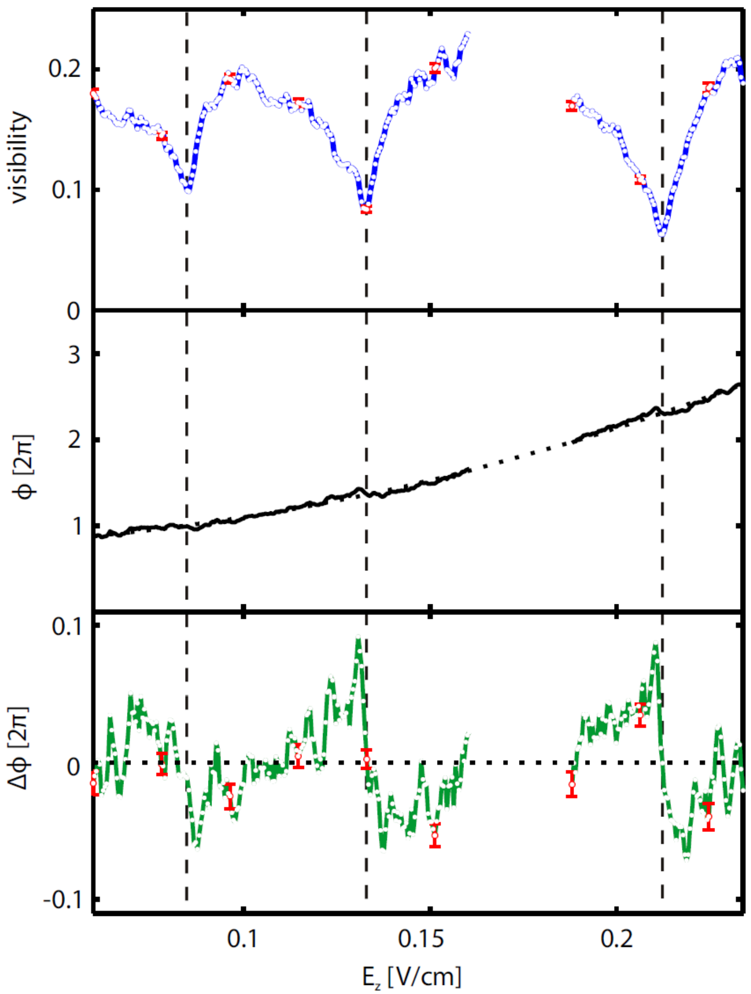


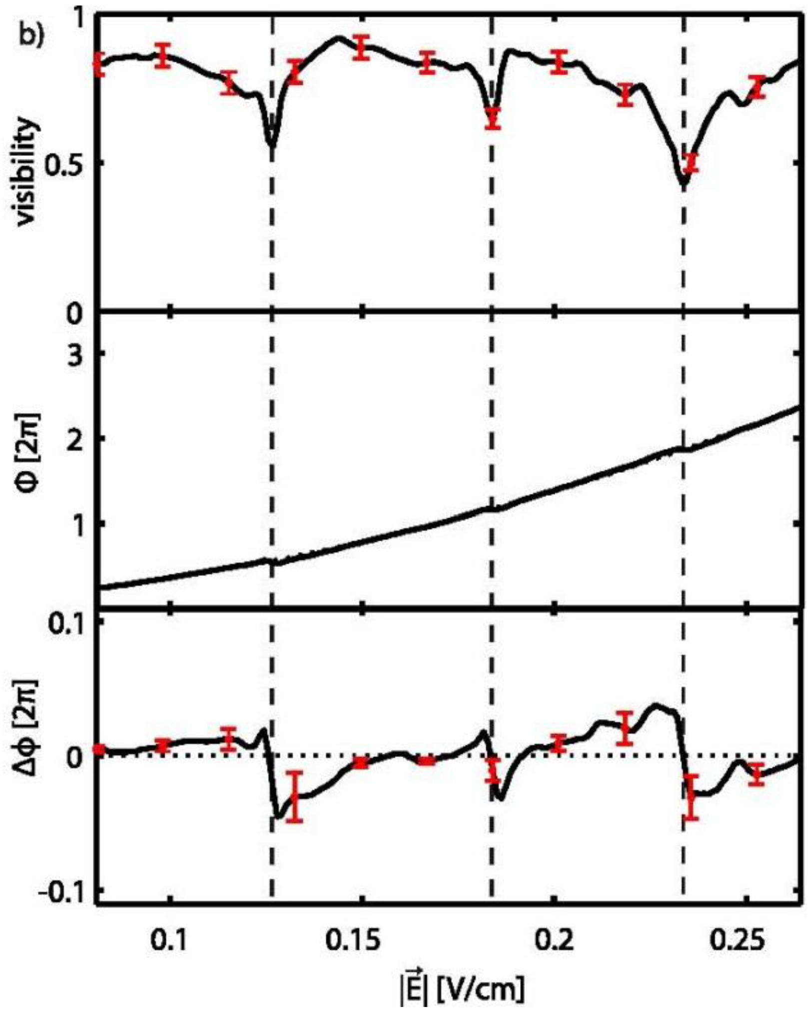
Interaction induced dephasing and phase shift



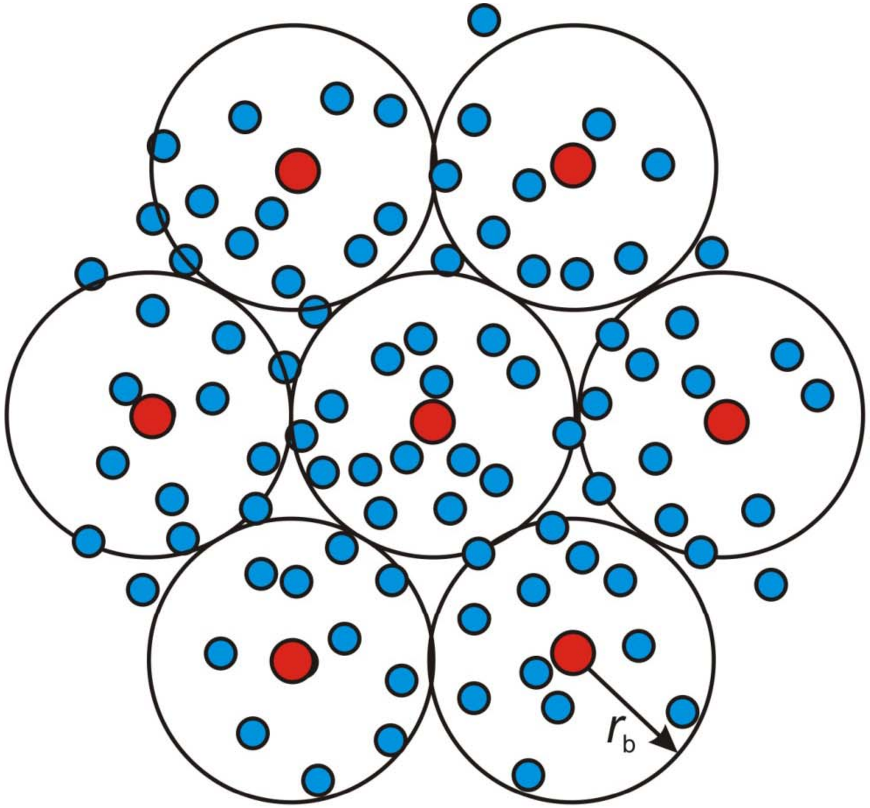


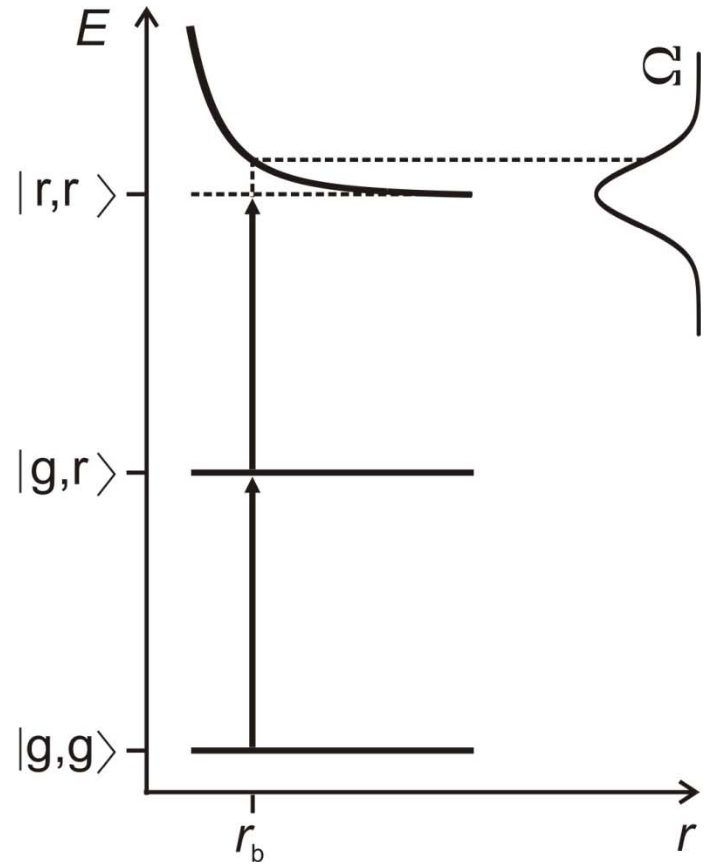
Interaction induced dephasing and phase shift





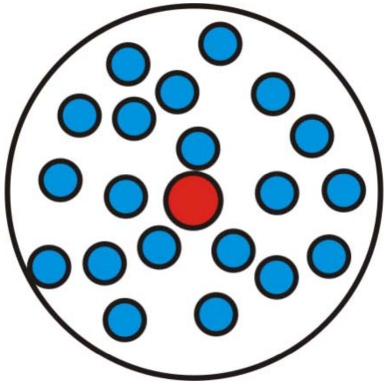
Excitation blockade by van der Waals interaction

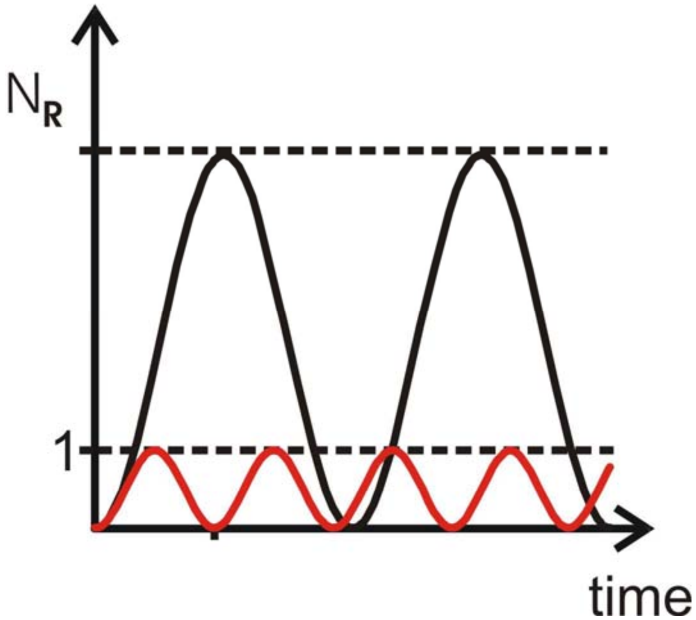




Distance between to atoms

Collective state



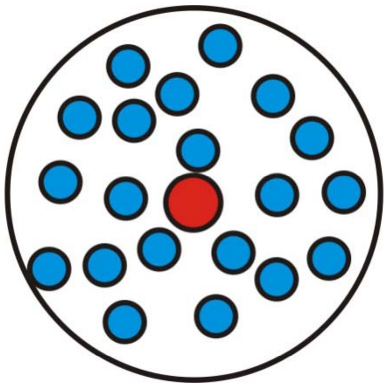


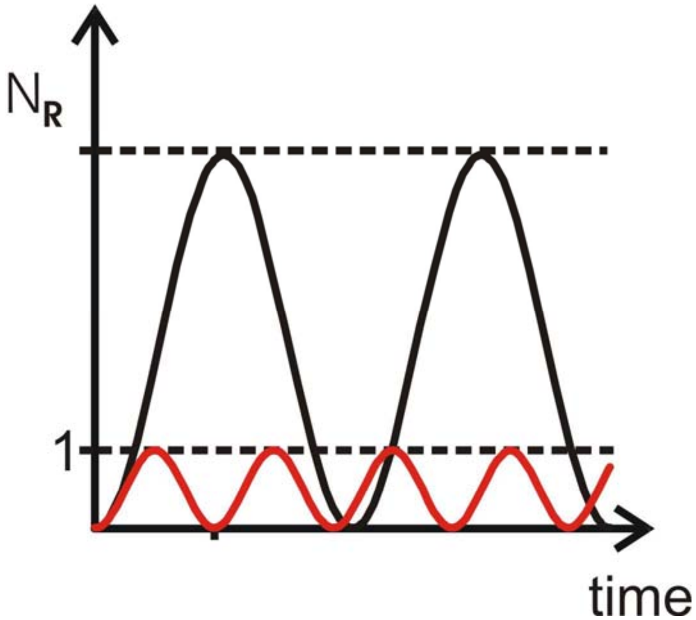
$$|E\rangle = \frac{1}{\sqrt{N}}\{|ryd, g, g, ..., g\rangle + |g, ryd, g, ..., g\rangle + ... + |g, g, ..., g, ryd\rangle\}$$

$$\Omega = \sqrt{N}\Omega_0$$

$$|G\rangle = |g, g, g, ..., g\rangle$$

Collective state



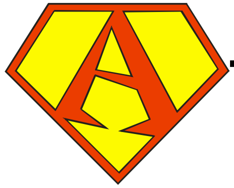
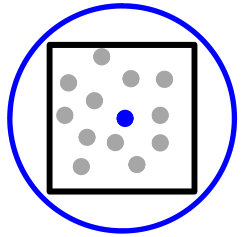


$$|E\rangle = \frac{1}{\sqrt{N}}\{|\color{red}{ryd}, g, g, ..., g\rangle + |g, \color{red}{ryd}, g, ..., g\rangle + ... + |g, g, ..., g, \color{red}{ryd}\rangle\}$$

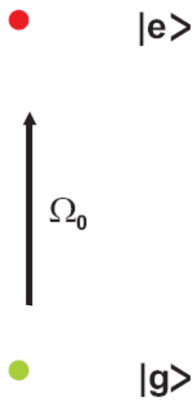
$$\Omega = \sqrt{N}\Omega_0$$

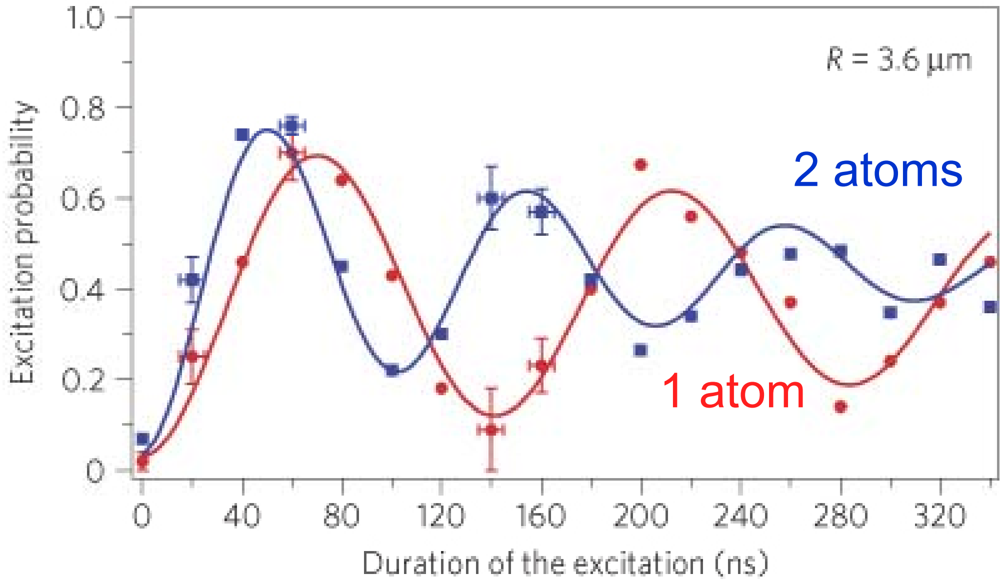
$$|G\rangle = |g, g, g, ..., g\rangle$$

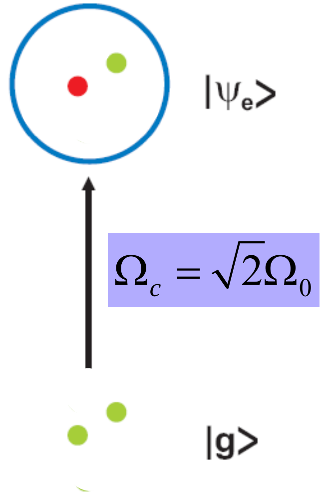
SuperAtom

- Supertom made of 2-100000 atoms 

Ultracold samples:

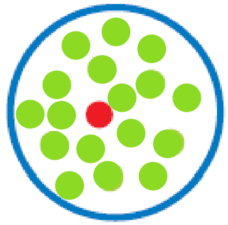






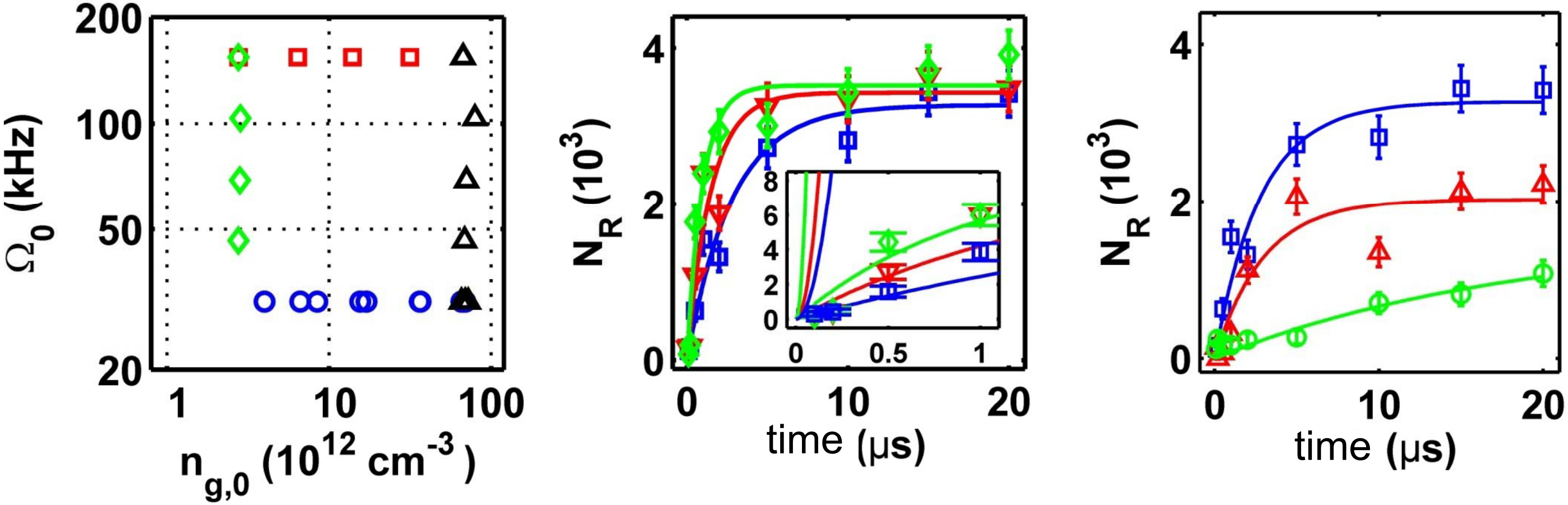
N=2: Gaetan et al., *Nature Phys.* ***5****, 115 (2009)*

N ~ 1000: Heidemann et al., *PRL* ***99****, 163601 (2007)*
see also Kuzmich group 2012
see also Walker/Saffman 2013
see J. Balewski phD thesis 2013 (N>1000



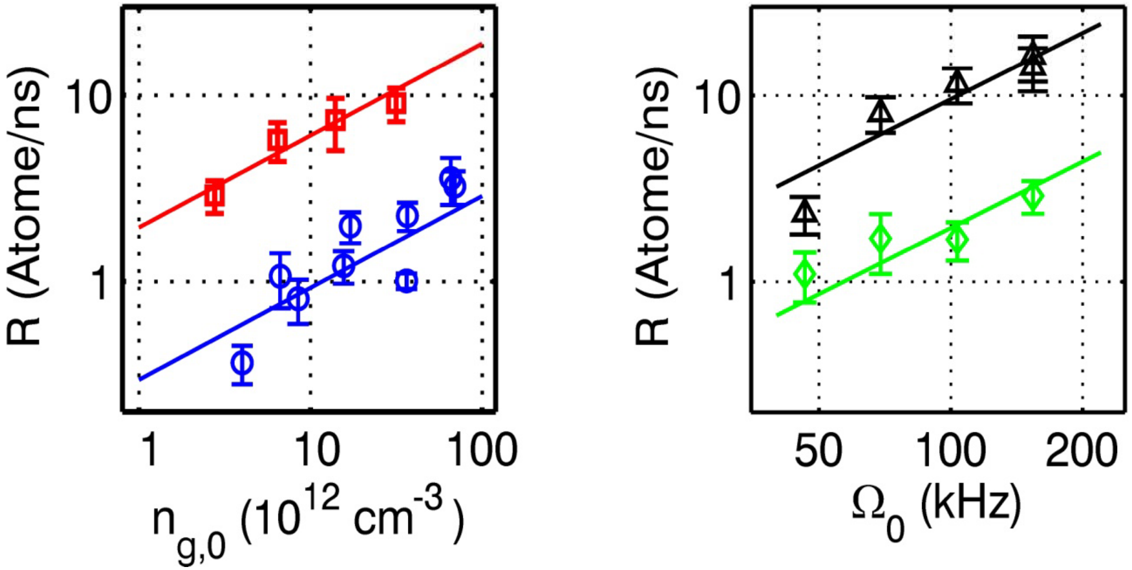
Blockade measurements

change density by microwave   change Ω_0



Heidemann et al., *PRL **99**, 163601 (2007)*

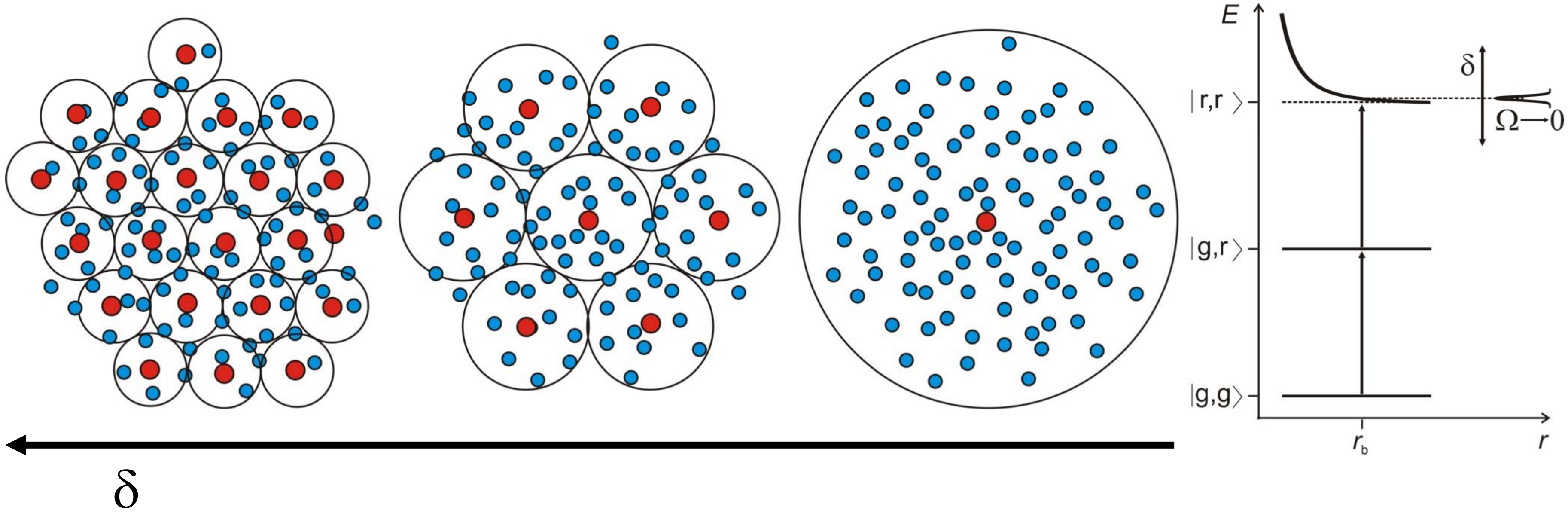
Scaling of excitation rate R



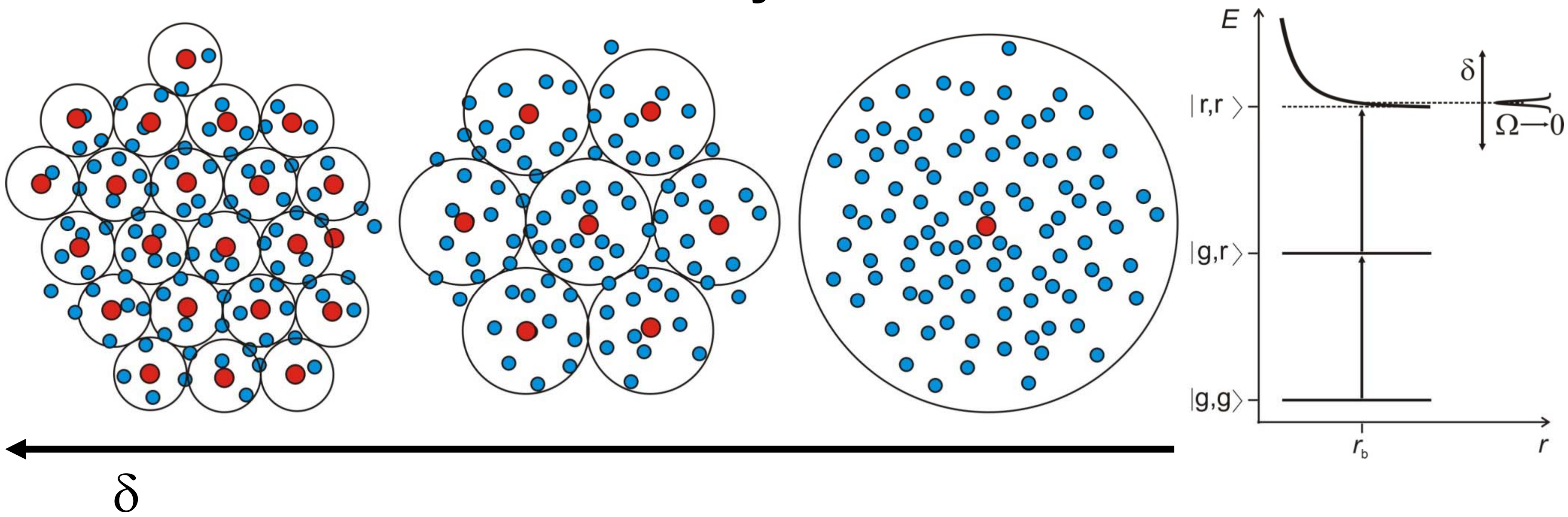
$$R \propto n_{g,0}^{0.49(7)} \Omega_0^{1.2(1)}$$

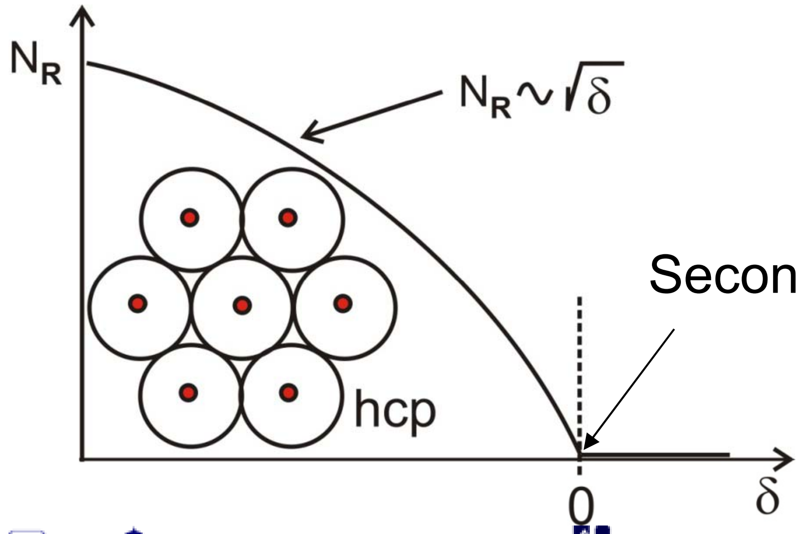
Heidemann et al., *PRL **99**, 163601 (2007)*

Is it Crystaline?



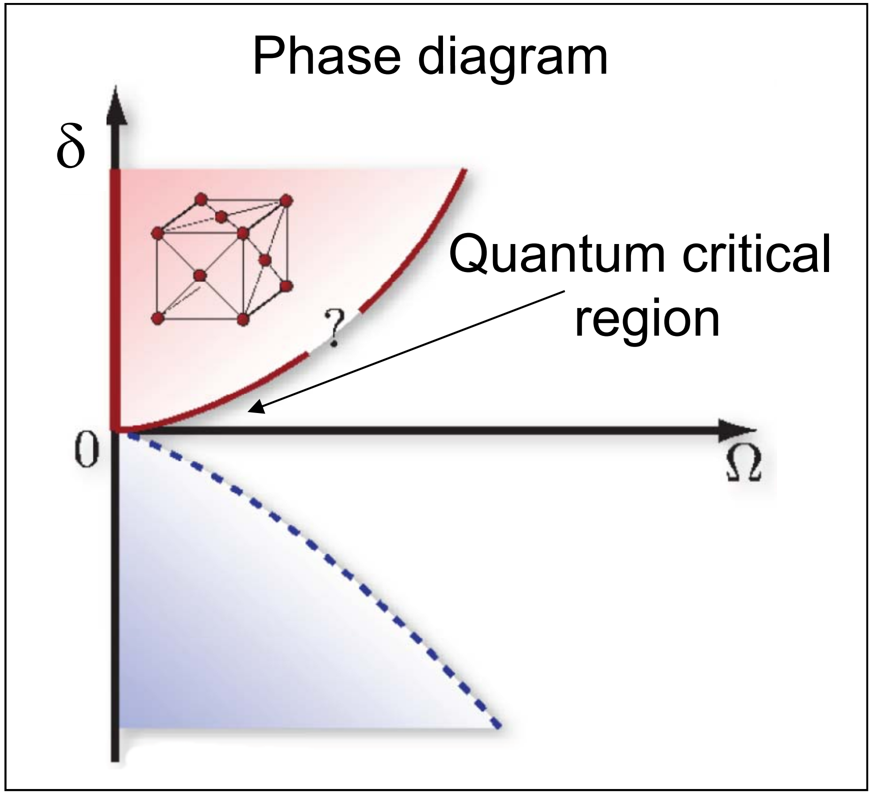
Is it Crystaline?





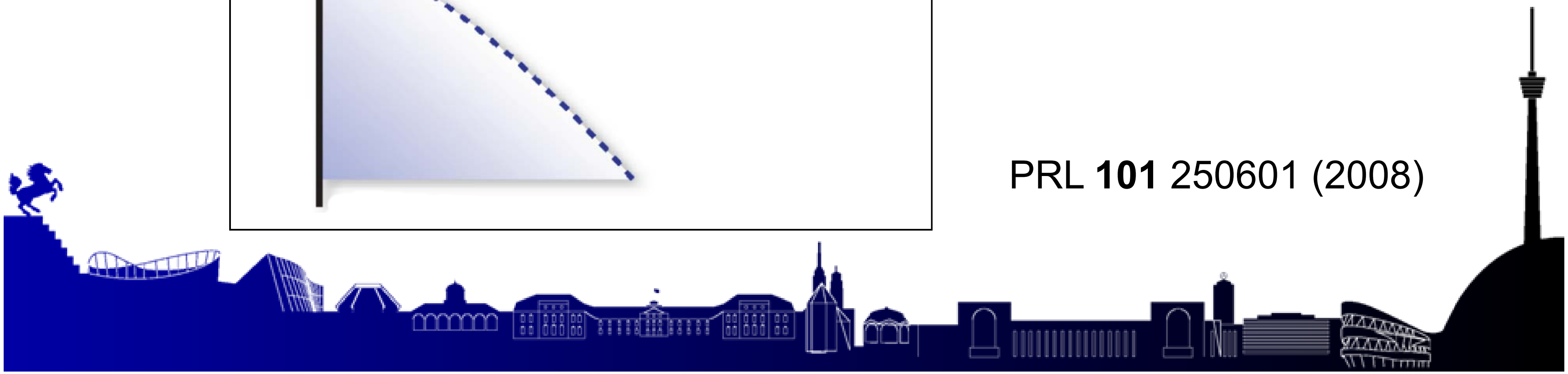
Effective Spin Hamiltonian

$$H = -\frac{\hbar\delta}{2}\sum_i \sigma_z^{(i)} + \frac{\hbar\Omega}{2}\sum_i \sigma_x^{(i)} + C_6 \sum_{j<i} \frac{P_{ee}^{(i)} P_{ee}^{(j)}}{\left|r_i - r_j\right|^6}$$



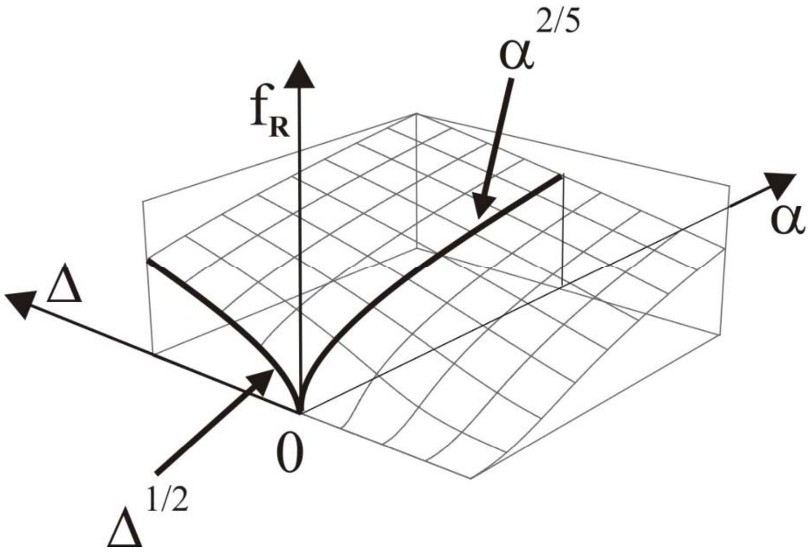
Projector: $P_{ee}^{(i)} = (1+\sigma_z^{(i)})/2$

PRL **101** 250601 (2008)



Universal scaling close to a quantum critical point

Strongly interacting Rydberg gas



Mean-field result:

$$\alpha = f_R^{5/2}\left|1-\frac{\Delta}{f_R^2}\right|$$

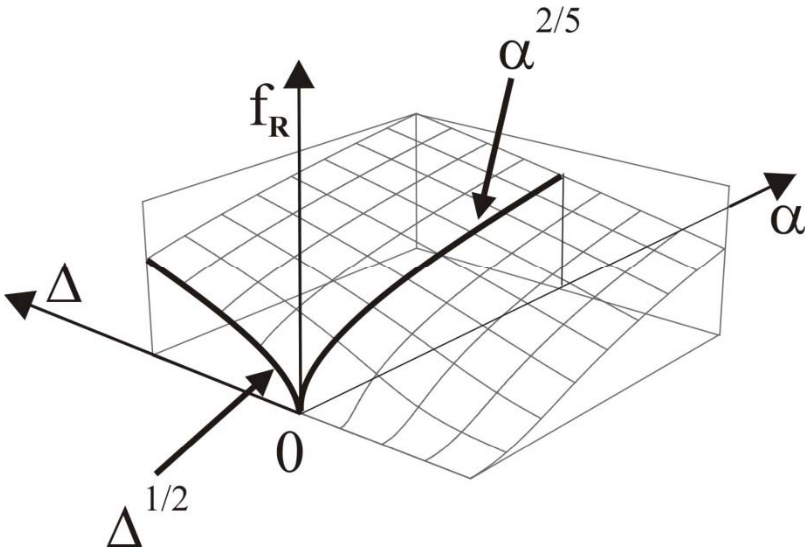
Dimensionless parameters

$$\alpha = \frac{\hbar\Omega}{C_6 n^2} \qquad \Delta = \frac{\hbar\delta}{C_6 n^2}$$

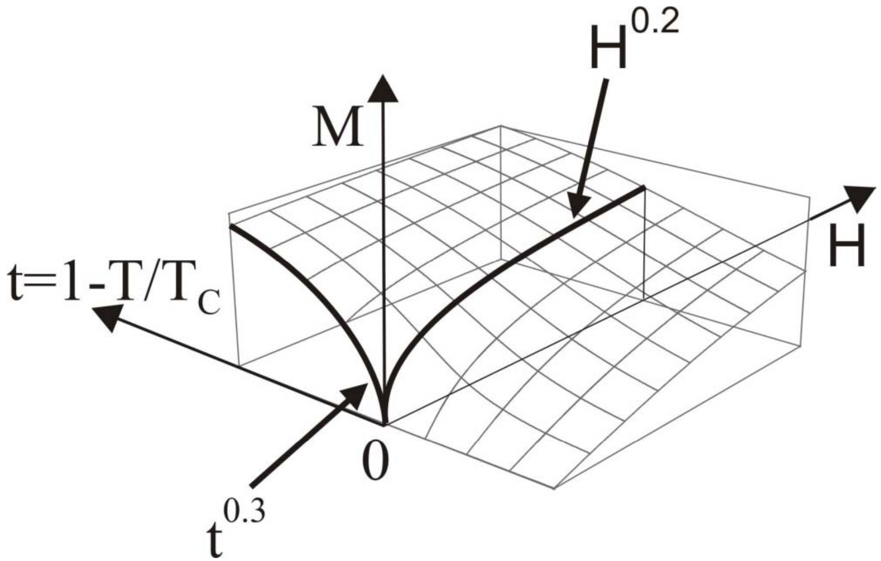
$$\alpha = \left(\frac{\text{Mean atomic distance}}{\text{blockade radius } r_b}\right)^6$$

Universal scaling close to a quantum critical point

Strongly interacting Rydberg gas



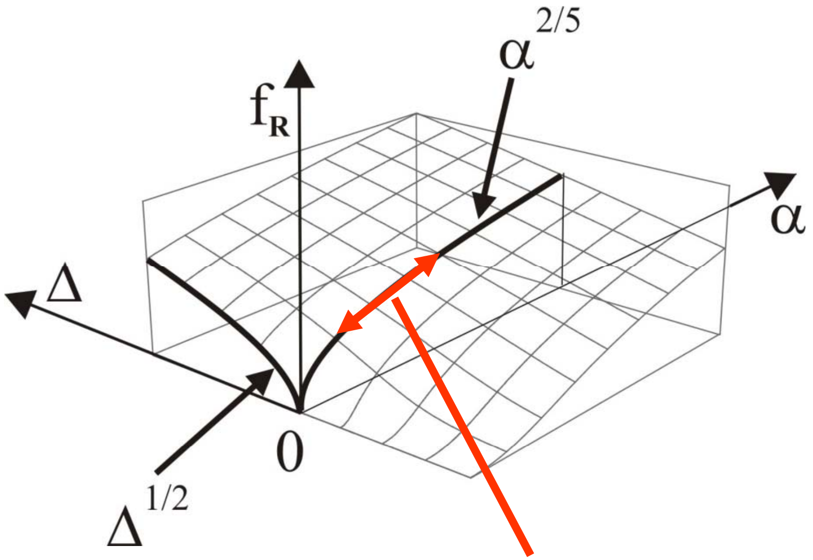
Ferromagnet - Ising model

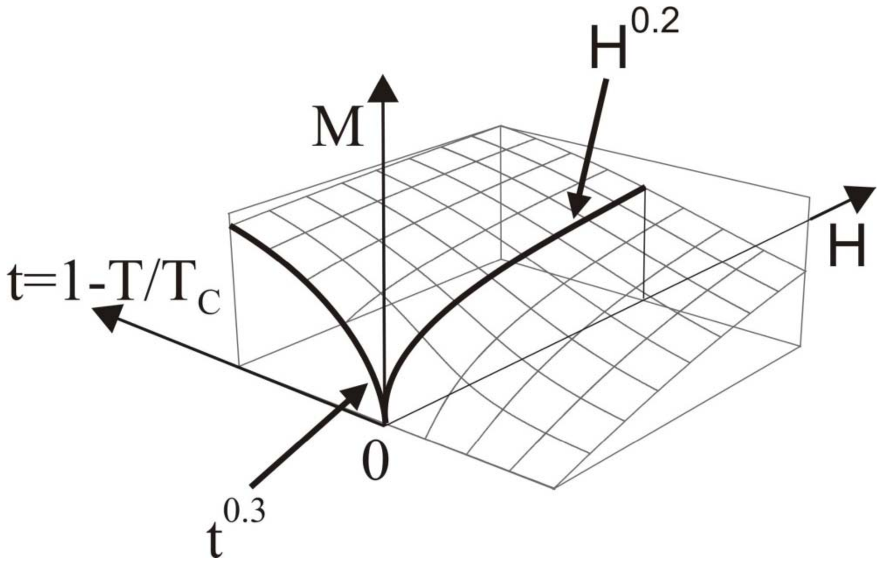


Universal scaling close to a quantum critical point

Strongly interacting Rydberg gas

Ferromagnet - Ising model

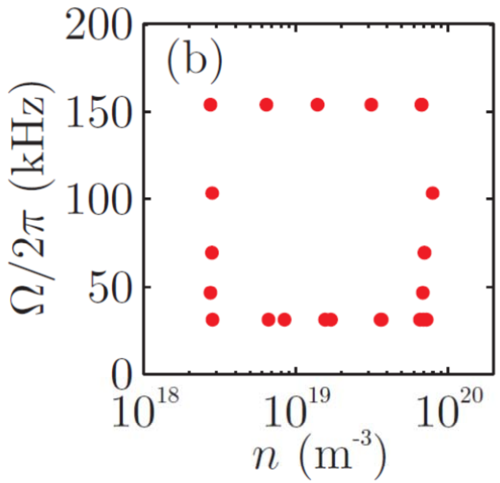


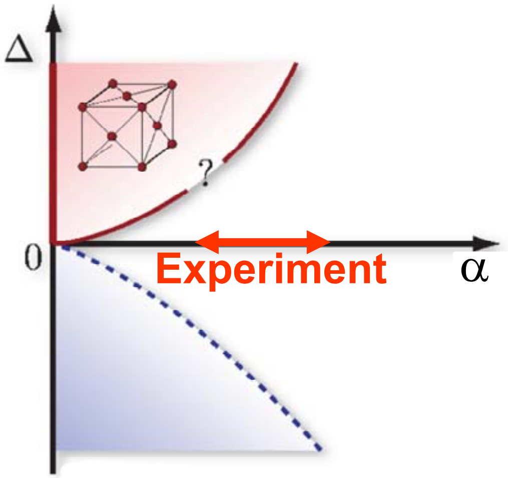


PRA 80, 033422 (20

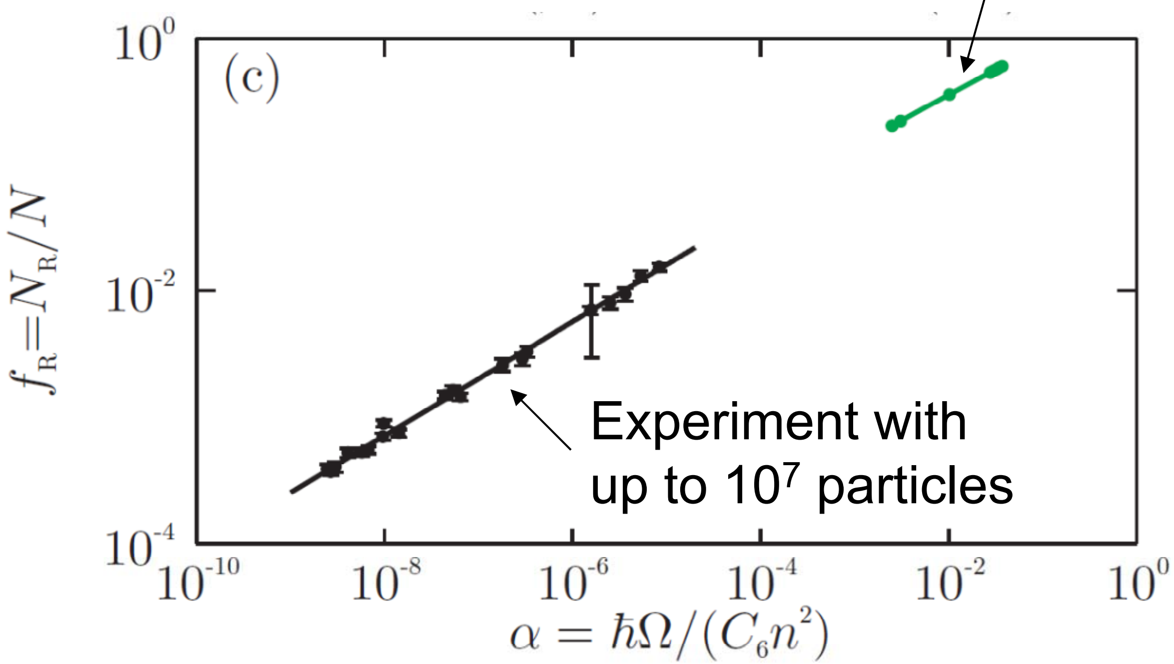
Data collapse on a simple power law – Universal scaling

Scanned parameter space





Numerical simulations with up to 100 particles

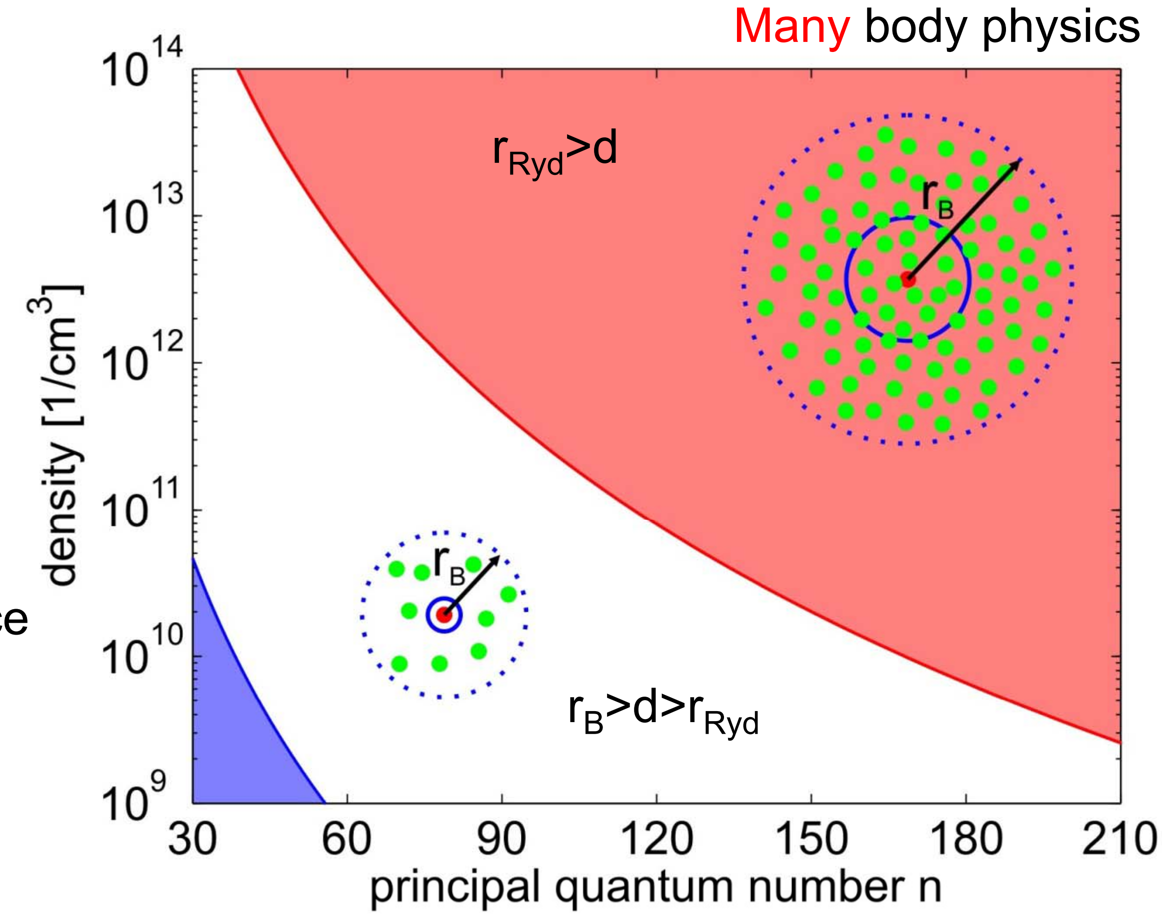


Fit result: $f_R \propto \alpha^{0.45\pm0.01}$

Theory: $f_R \propto \alpha^{2/5} = \alpha^{0.4}$

Heidemann et al., *PRL* ***99****, 163601 (2007)*
R. Löw, et al., *PRA* ***80****, 033422 (2009)*

Rydberg atoms in dense gases



d: mean particle distance

r_B: blockade radius

r_{Ryd}: size of electron orbit

The COLD RYDBERG team

W Li, T Pohl, JM Rost
ST Rittenhouse, HR Sadeghpour,
D Peter, HP Büchler,
K Rzążewski, M Brewczyk
M. Kurz, P. Schmelcher

erc

Sebastian Hofferberth: Rydberg quantum optics

CO.CO.MAT

SFB/TRR 21