COHERENTLY COUPLED BOSE GASES

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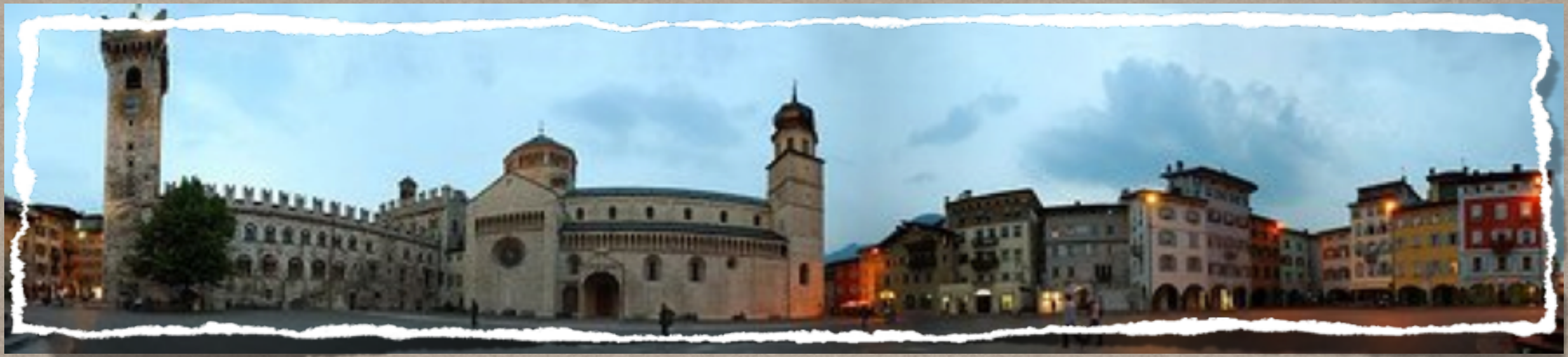
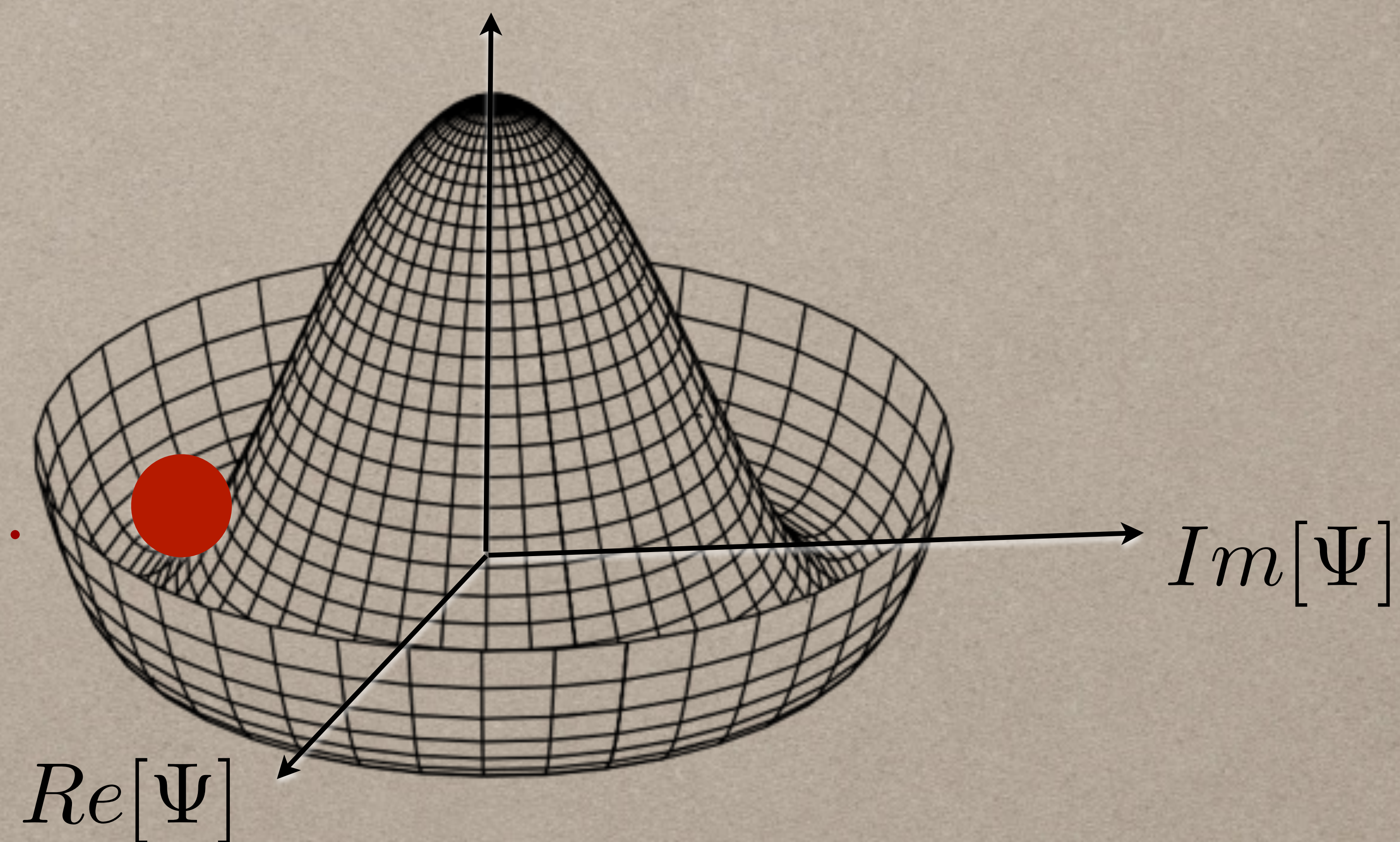
Physics Dep., Trento University



T=0 Bose gases: Elementary excitations

Ground state breaks U(1) symmetry: Goldstone mode - no cost to change the global phase of the wave function

$$\hat{\psi} = \Psi + \ldots$$



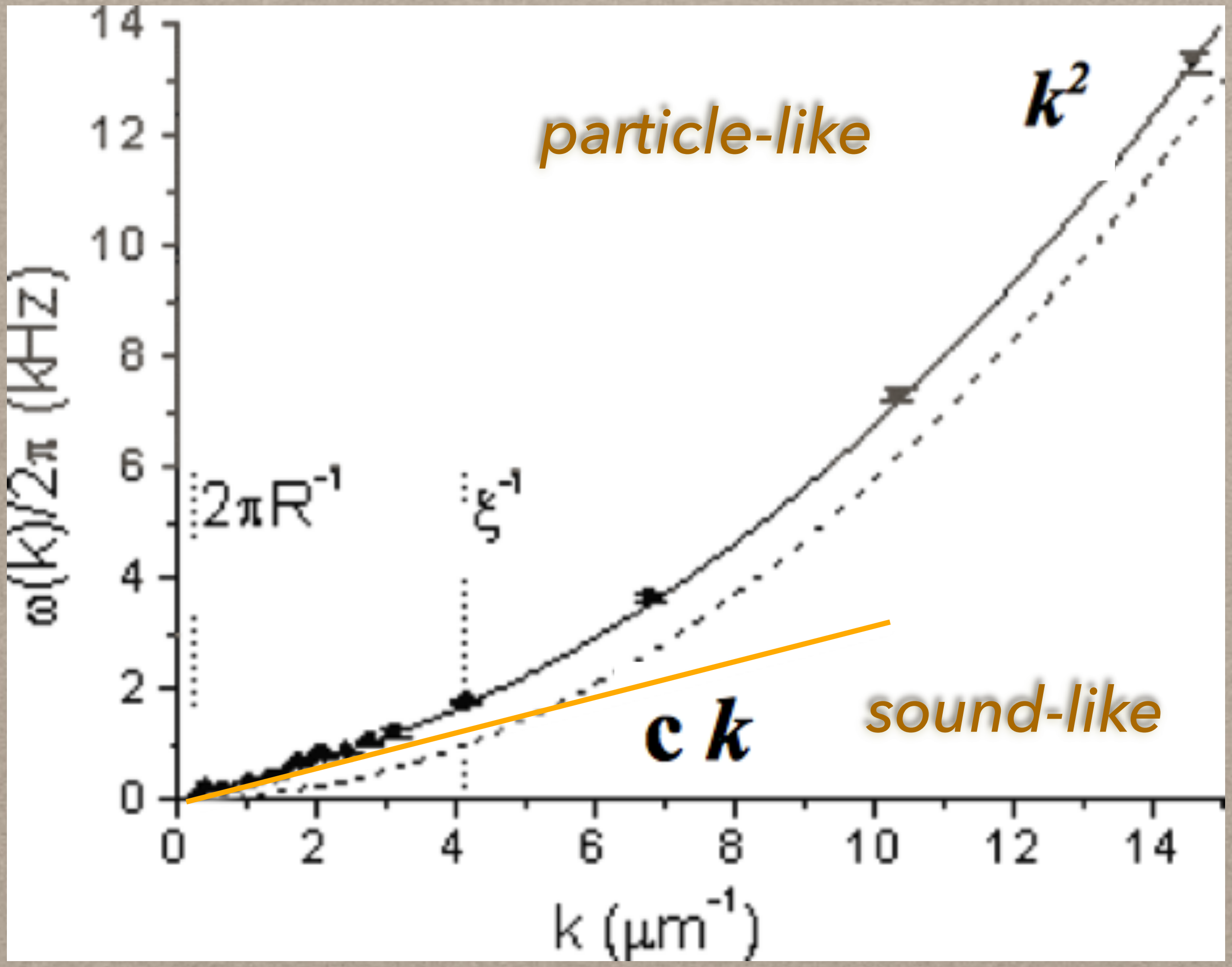
Gross-Pitaevskii equation:

$$i\hbar \frac{\partial}{\partial t} \Psi = \left[-\frac{\hbar^2 \nabla^2}{2m} + V(x) + g|\Psi|^2 \right] \Psi$$

T=0 Bose gases: Elementary excitations

Ground state breaks U(1) symmetry: Goldstone mode - no cost to change the global phase of the wave function

Bogoliubov Spectrum



MIT '99

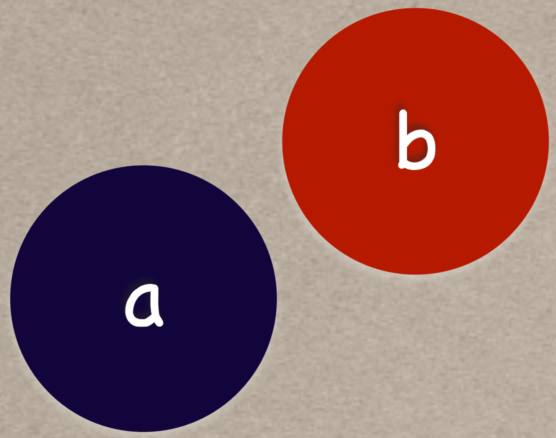
$$\omega_k = \sqrt{\frac{q^2}{2m}\left(2mc^2 + \frac{q^2}{2m}\right)}$$

where the speed of sound is:

$$c^2 = gn/m$$

and the healing length

$$\xi = \frac{\hbar}{\sqrt{2}mc}$$



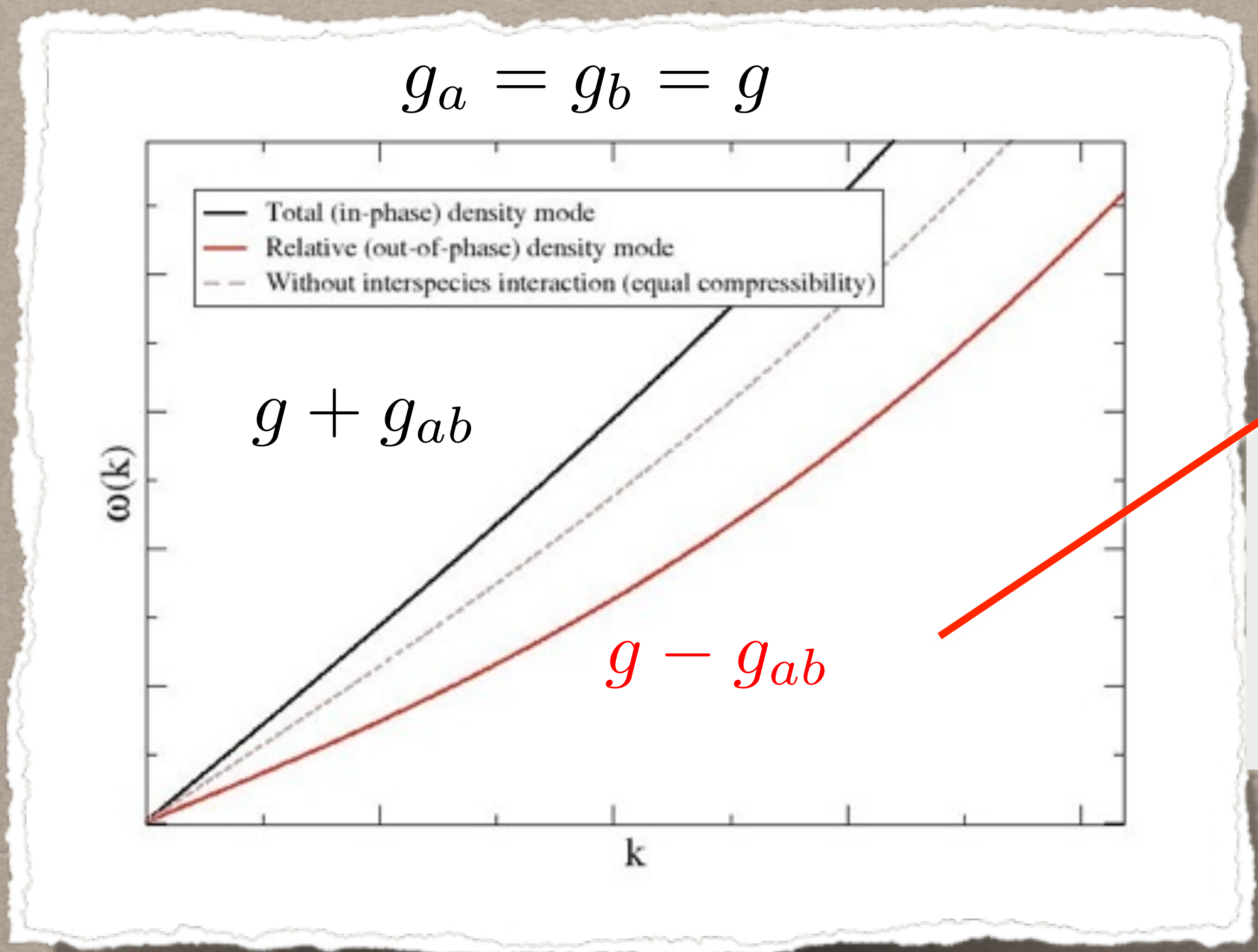
T=0 Bose mixtures

Energy/Volume $e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$

Both Na and Nb are conserved

Elementary excitations

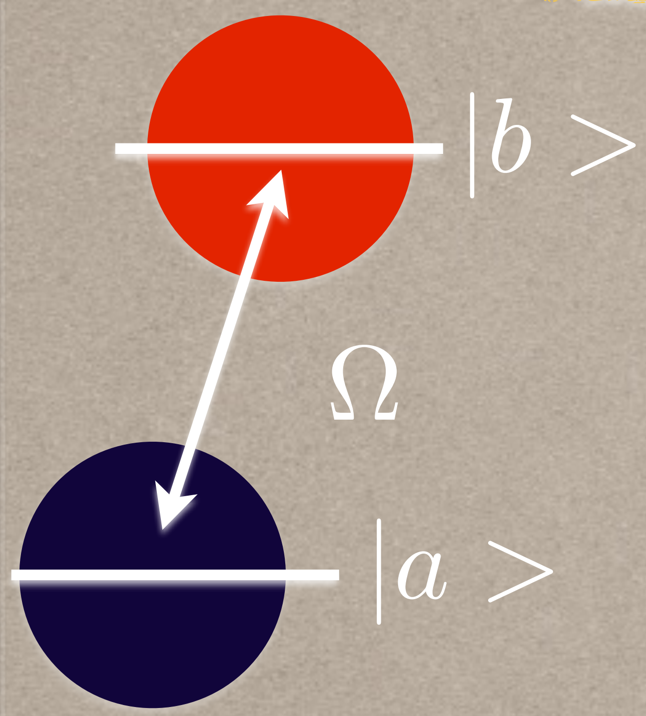
Ground state breaks U(1)xU(1) symmetry: 2 Goldstone modes - coming from no cost to change the global phase of the 2 wave functions



$g = g_{ab}$

Spin mode soft: unstable with respect to phase separation

T=0 coherently coupled Bose gases



$$\Omega(|a\rangle\langle a| + |b\rangle\langle b|) = \Omega\sigma_x$$
$$|\xi\rangle = (|a\rangle - |b\rangle)/\sqrt{2}$$

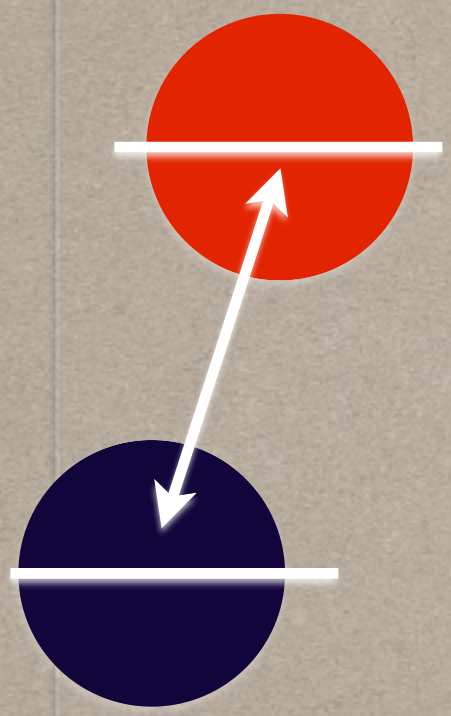
Assuming the gas condense in a ground state :

$$(\Psi_a = \sqrt{n_a}e^{i\phi_a}, \Psi_b = \sqrt{n_b}e^{i\phi_b})$$

$$i\hbar\frac{\partial}{\partial t}\Psi_a = \left[-\frac{\hbar^2\nabla^2}{2m} + V_a + g_a|\Psi_a|^2 + g_{ab}|\Psi_b|^2\right]\Psi_a + \Omega\Psi_b \tag{1}$$

$$i\hbar\frac{\partial}{\partial t}\Psi_b = \left[-\frac{\hbar^2\nabla^2}{2m} + V_b + g_b|\Psi_b|^2 + g_{ab}|\Psi_a|^2\right]\Psi_b + \Omega^*\Psi_a, \tag{2}$$

T=0 coherently coupled Bose gases



Energy/Volume

$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$$
$$+2|\Omega| \cos(\phi_a - \phi_b)\sqrt{n_a n_b}$$

Only Na+Nb is conserved

Indeed the system is a single condensate
with a 2-component wave function: polarized states

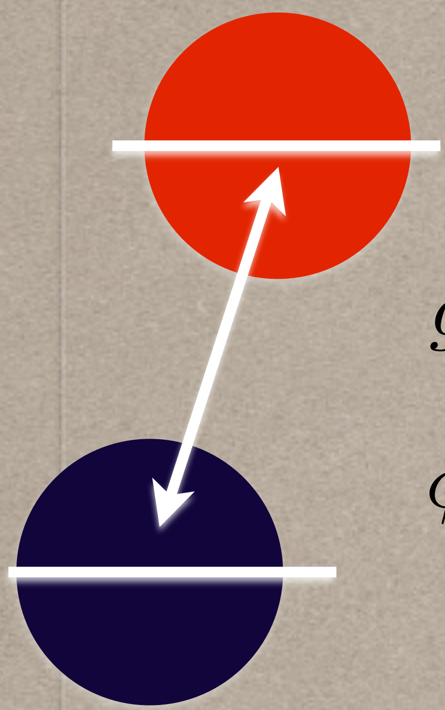
Elementary excitations

Ground state breaks U(1) symmetry:

1. Goldstone mode - coming from no cost to change the global total phase.

2. A gapped mode - due to the cost of changing the relative phase

Ground State



$$g_a = g_b = g$$

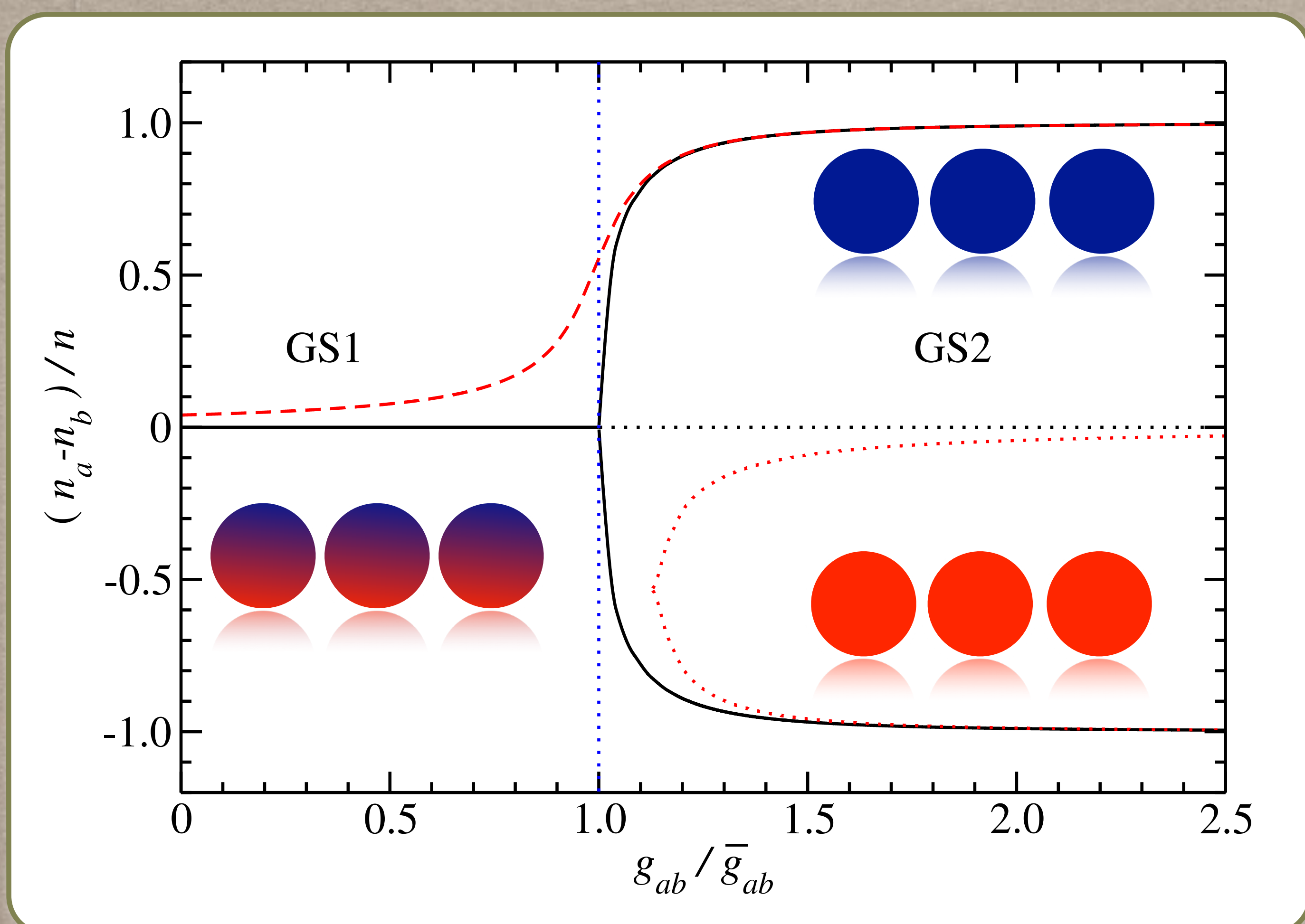
$$\phi_a - \phi_b = \pi$$

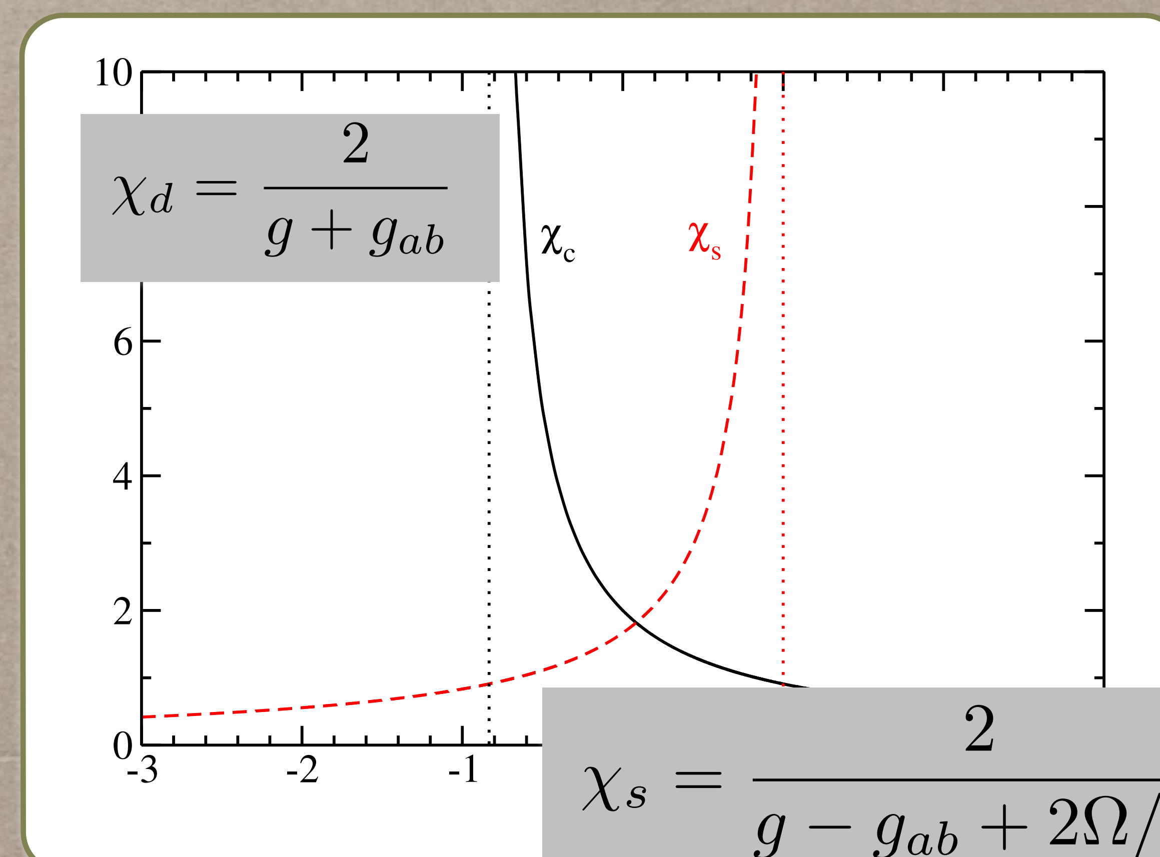
GS1 $\quad n_a - n_b = 0$

GS2 $\quad (n_a - n_b)_\pm = \pm n\sqrt{1 - \left(\frac{2|\Omega|}{(g-g_{ab})n}\right)^2}$

Critical condition for the II order phase transition

$$g - g_{ab} + \frac{2\Omega}{n} = 0$$

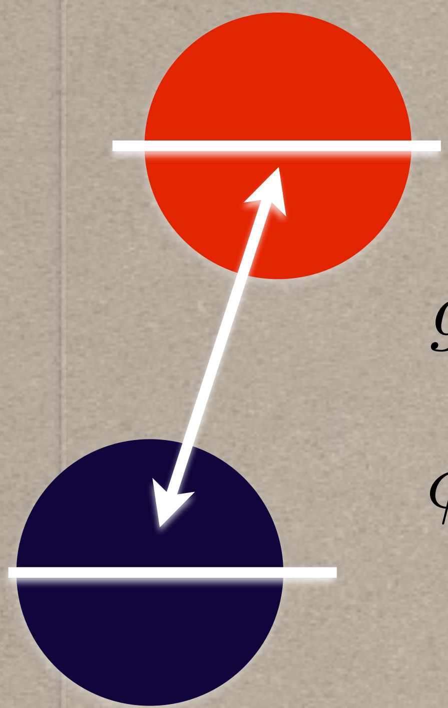




$$\chi_d = \frac{2}{g + g_{ab}}$$

$$\chi_s = \frac{2}{g - g_{ab} + 2\Omega/n}$$

Ground State



$$g_a = g_b = g$$

$$\phi_a - \phi_b = \pi$$

GS1 $\quad n_a - n_b = 0$

GS2 $\quad (n_a - n_b)_{\pm} = \pm n \sqrt{1 - \left(\frac{2|\Omega|}{(g - g_{ab})n} \right)^2}$

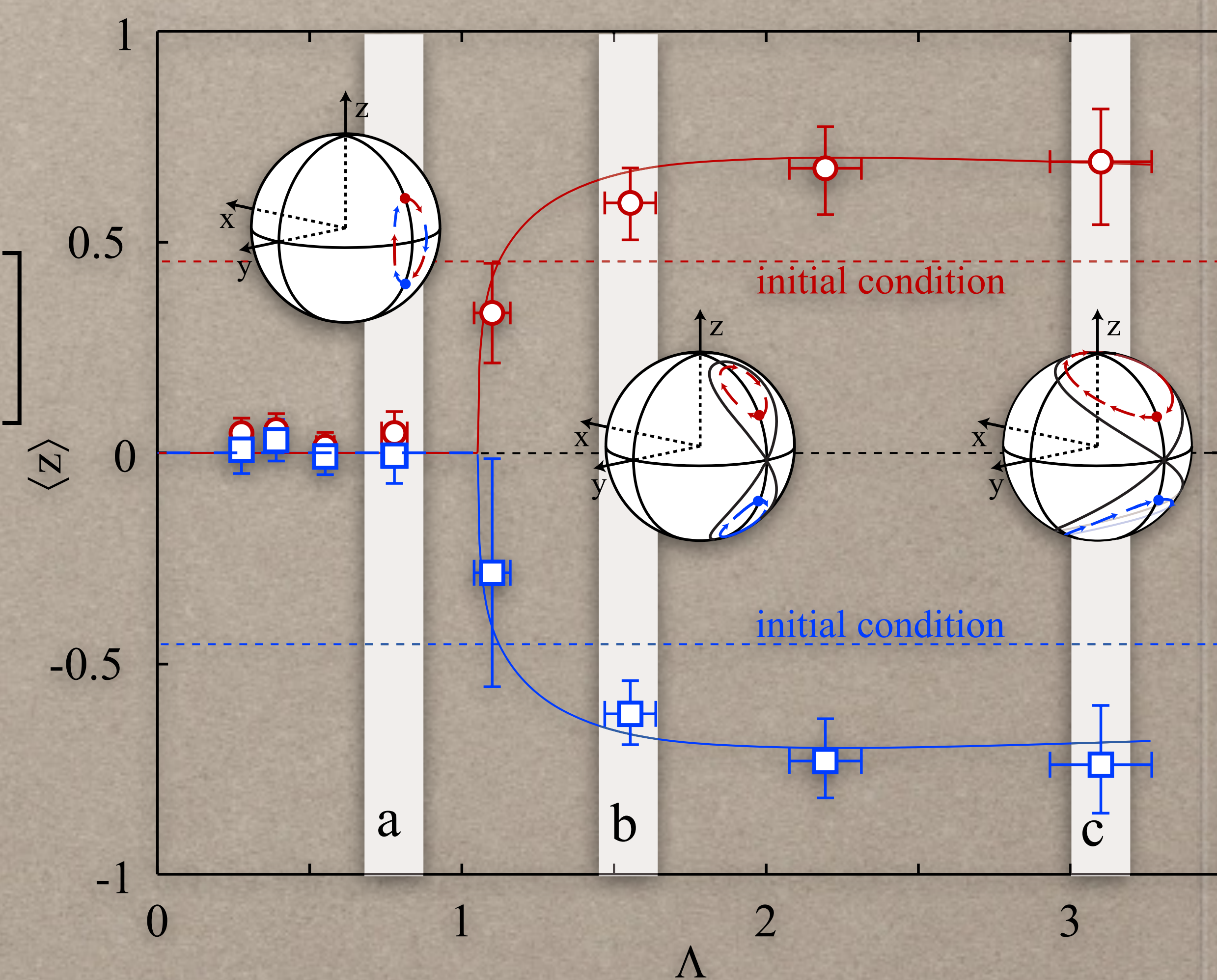
Internal Josephson effect:

$$H(Z, \phi) = -\frac{2\Omega}{\hbar} \left[\frac{1}{2} \Lambda Z^2 - \sqrt{1 - Z^2} \cos \phi \right]$$

phase difference

polarization

$$\Lambda = -\frac{(g - g_{ab})n}{2\Omega}$$



T. Zibold et al. PRL (2010)

Excitations: Hydrodynamics

The (low) energy functional for the fluctuations of the field operators above the ground state (GS1) can be written (to II order in the density and phase fields) as

$$E_0 = \sum_{\sigma=a,b} \int \left[\frac{\hbar^2 n_\sigma}{2m} (\nabla \phi_\sigma)^2 + \frac{m c_\sigma^2}{2 n_\sigma} \Pi_\sigma^2 \right]$$

$$V_{ab} = \left(g_{ab} - \frac{\Omega}{2\bar{n}} \right) \int \Pi_a \Pi_b - \frac{\Omega \bar{n}}{4} \int \left[\left(\frac{\Pi_a}{n_a} \right)^2 + \left(\frac{\Pi_b}{n_b} \right)^2 \right] + \Omega \bar{n} \int (\phi_a - \phi_b)^2$$

Density fields

$$\Pi_d = \frac{\Pi_a + \Pi_b}{2} \qquad \phi_d = \frac{\phi_a + \phi_b}{2}$$

Spin fields

$$\Pi_s = \frac{\Pi_a - \Pi_b}{2} \qquad \phi_s = \frac{\phi_a - \phi_b}{2}$$

$$E_{HD} = \int \left[\frac{\hbar^2 n}{4m} (\nabla \phi_d)^2 + \frac{m c_d^2}{n} \Pi_d^2 \right] + \int \left[\frac{\hbar^2 n}{4m} (\nabla \phi_s)^2 + \frac{m c_s^2}{n} \Pi_s^2 + \Omega n \phi_s^2 \right]$$

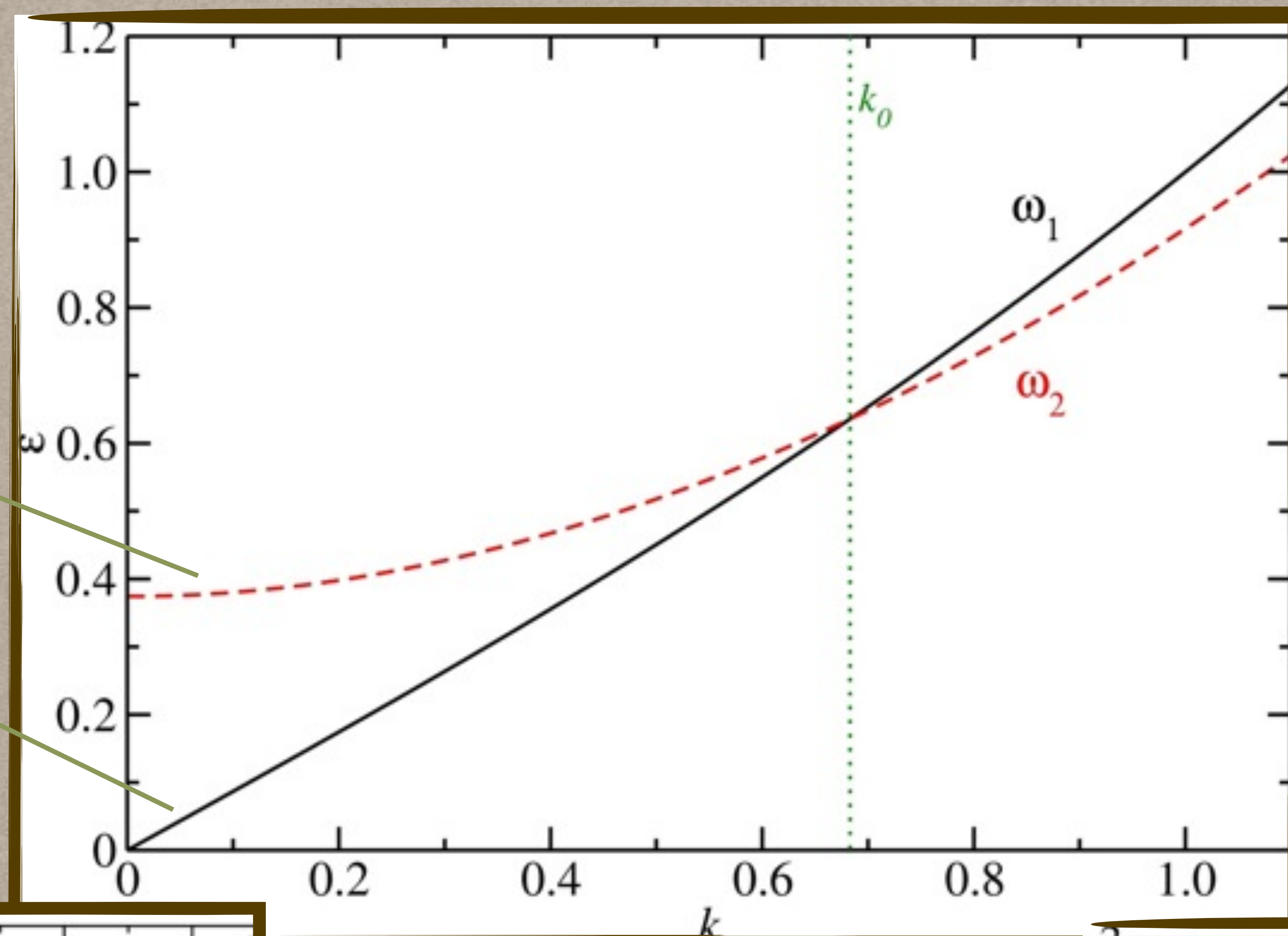
Gapless density sector

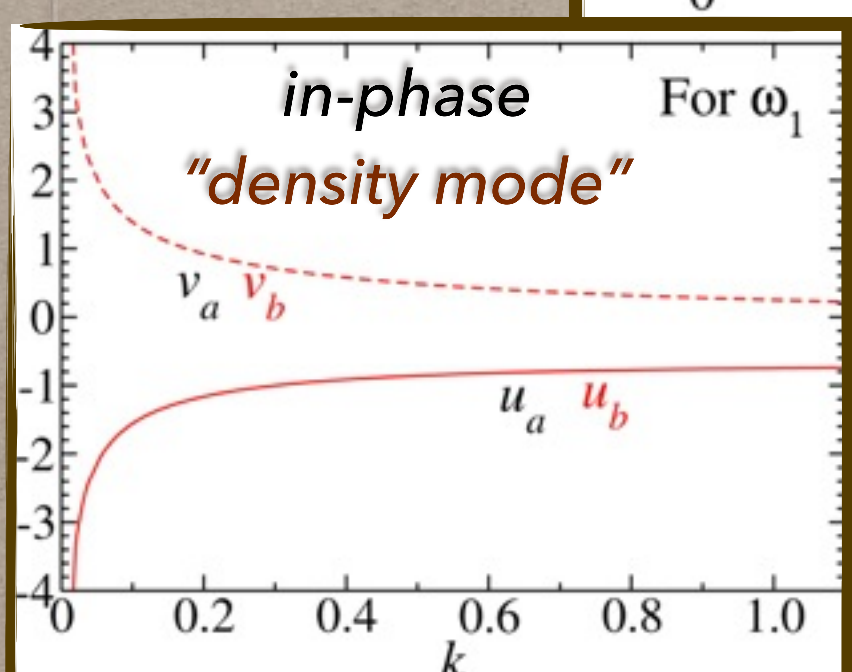
Gapped spin sector

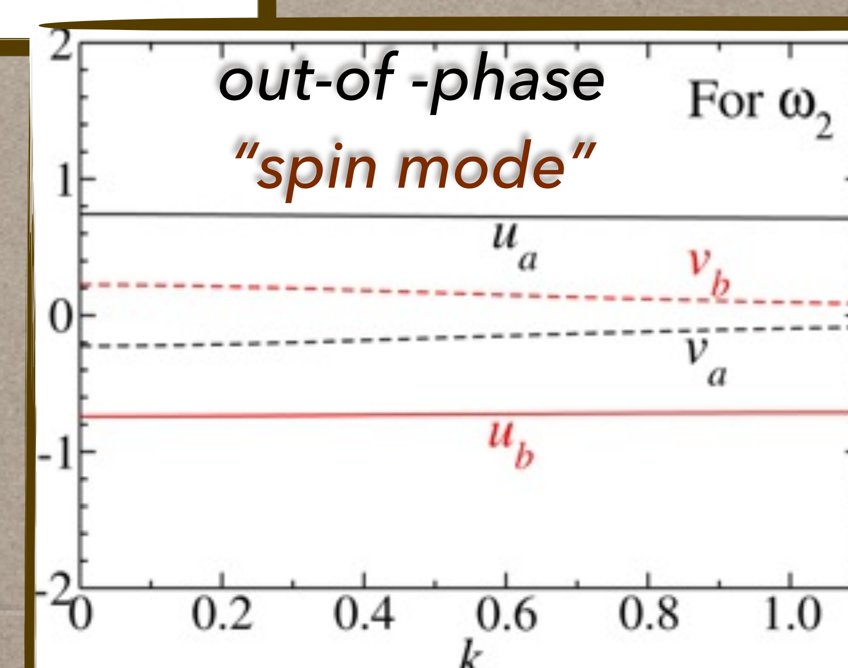
Excitations: Bogoliubov modes (GS1 symmetric)

gap + parabolic

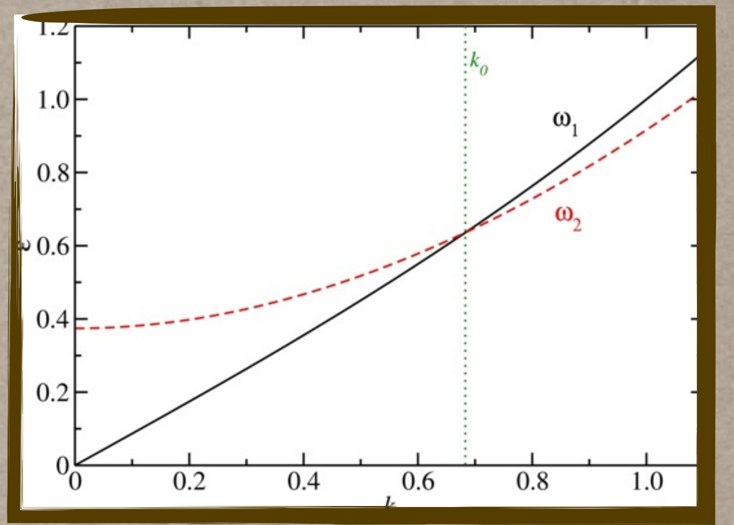
linear "sound"







Excitations: Bogoliubov modes



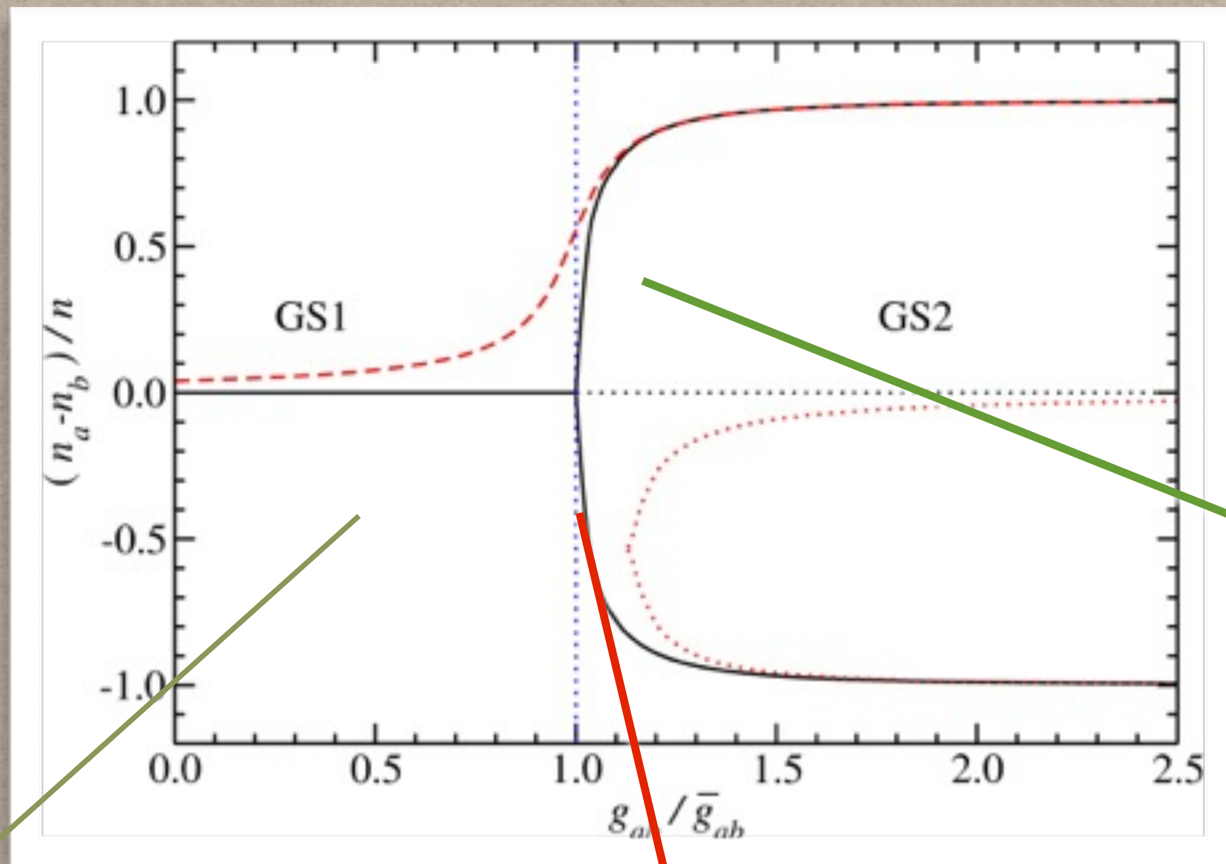
$$(\hbar\omega_1)^2 = \frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + (g + g_{ab})n\right)$$

$$(\hbar\omega_2)^2 = \frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + (g - g_{ab})n + 4|\Omega|\right) + 2|\Omega|\left[(g - g_{ab})n + 2|\Omega|\right]$$

- ✓ For Ω → 0, results for mixture of 2 BECs
- ✓ Gap in spin mode (Josephson) $\hbar\omega_J = \sqrt{2|\Omega|\left[(g - g_{ab})n + 2|\Omega|\right]}$
- ✓ Crossing of the modes at k0

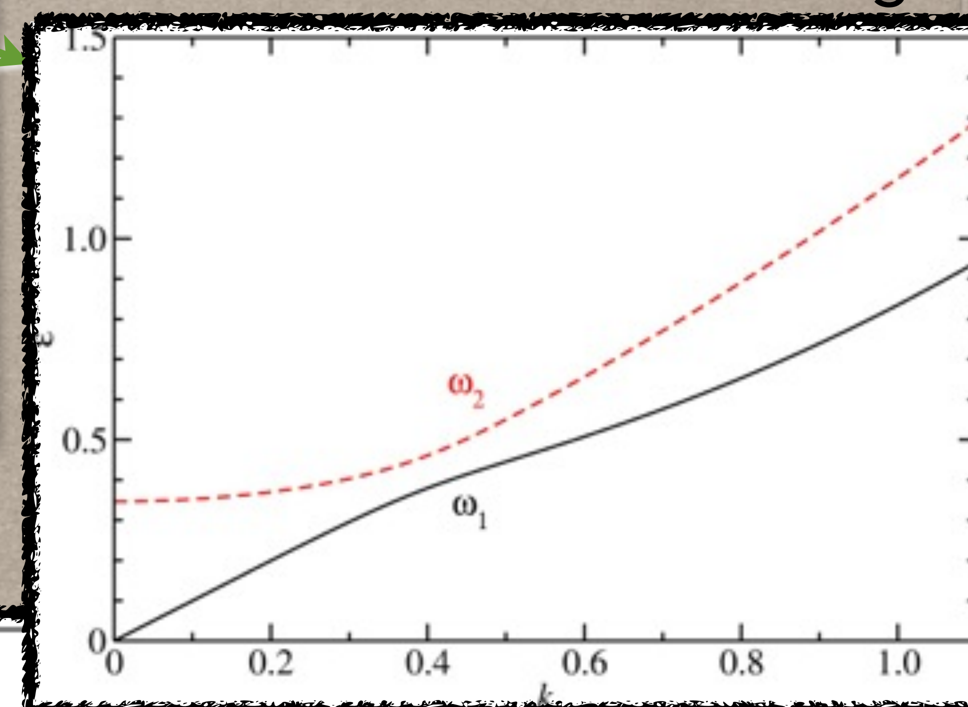
Goldstein & Meystre, PRA (1997); Search et al., PRA (2001); Tommasini et al., PRA (2003)...

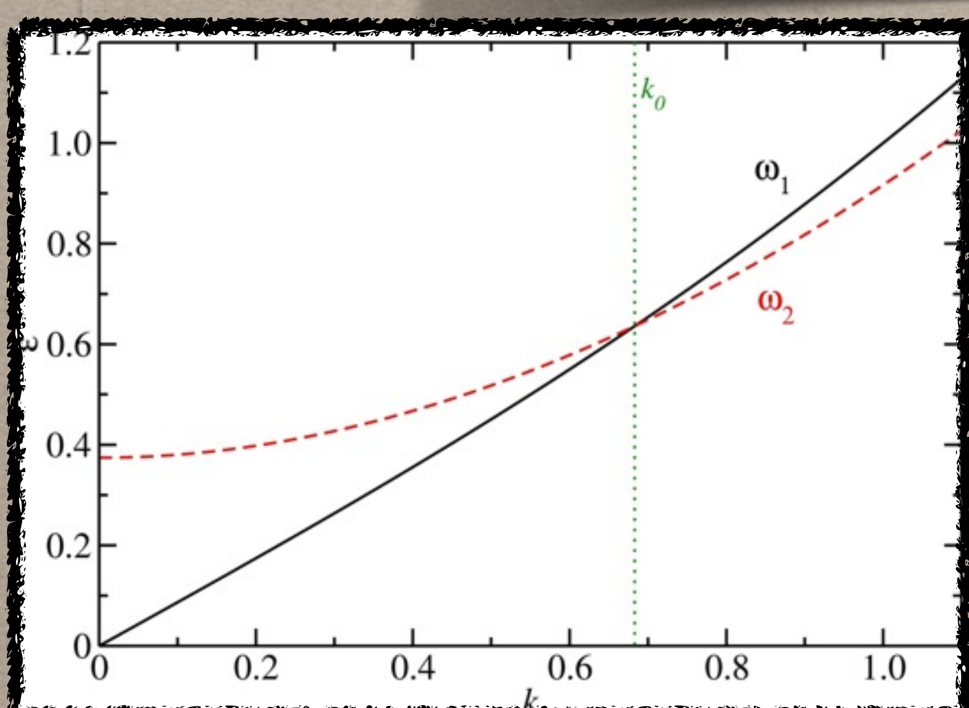
Excitations: Bogoliubov modes across the transition

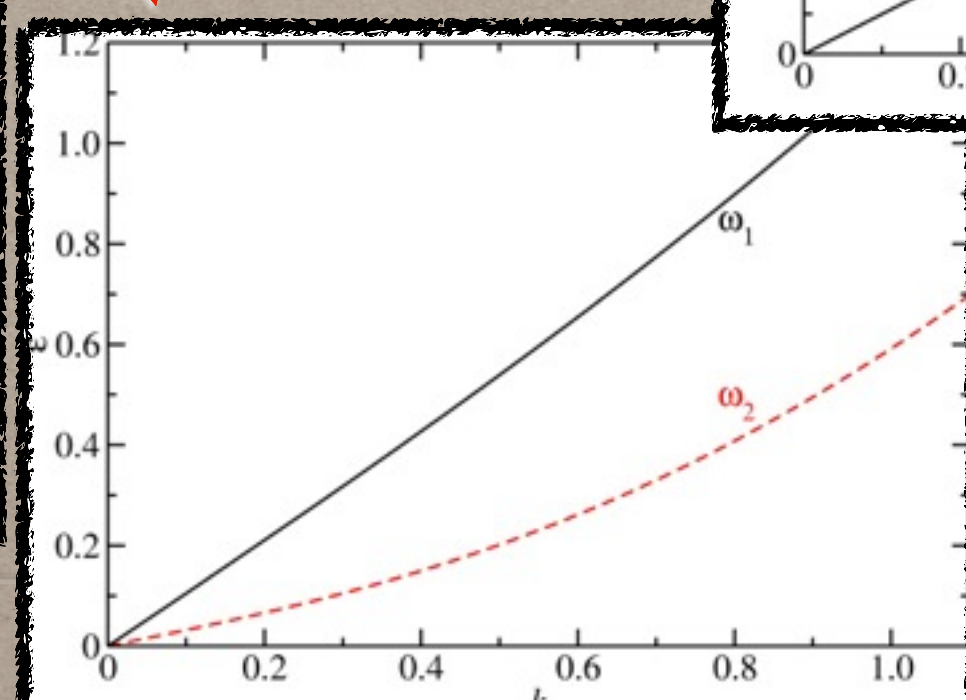


hybridization ⇒

avoided crossing





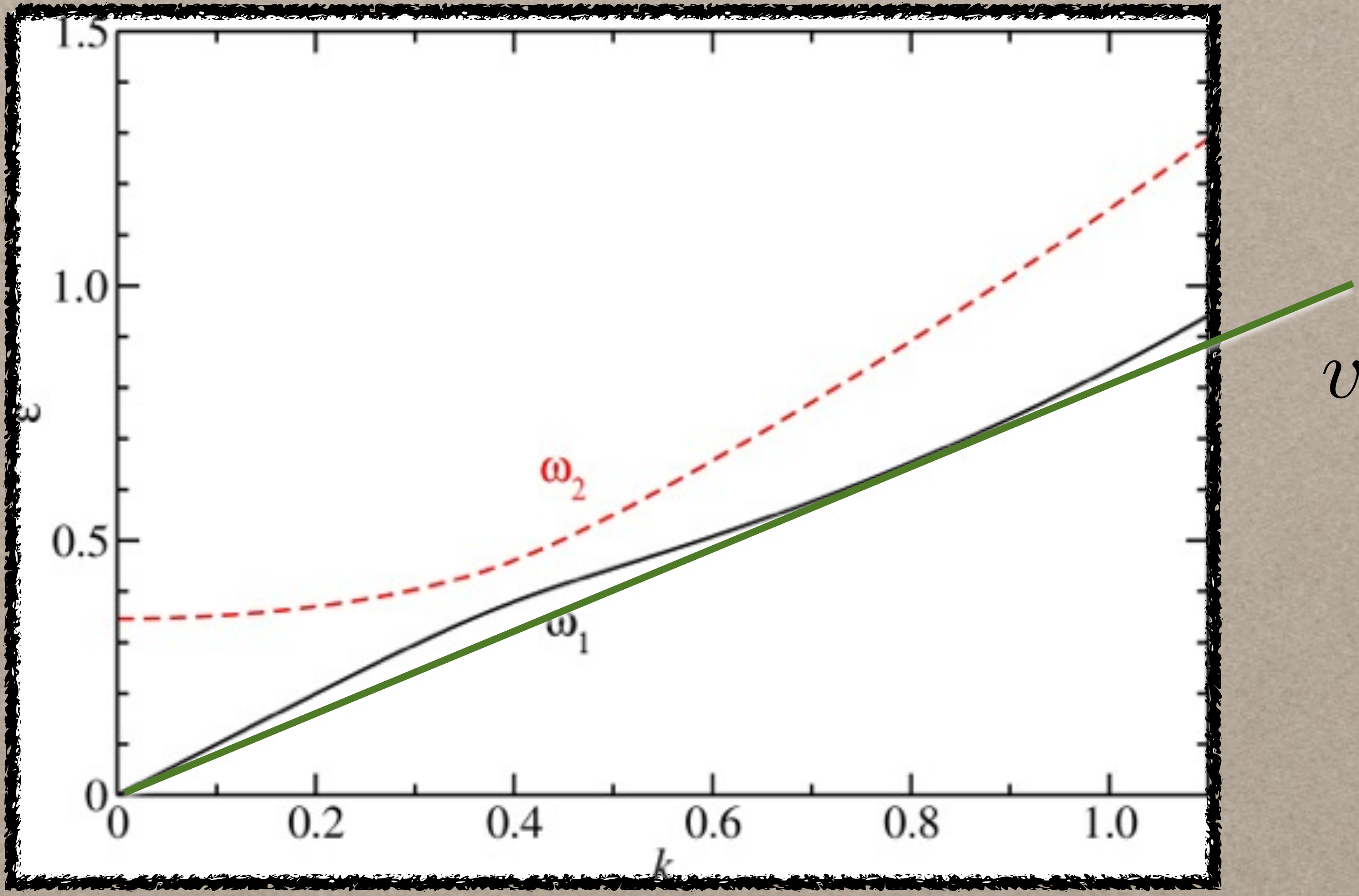


linear gapless modes (different behaviour in mixtures, HD)

Landau critical velocity

hybridization ⇒

avoided crossing

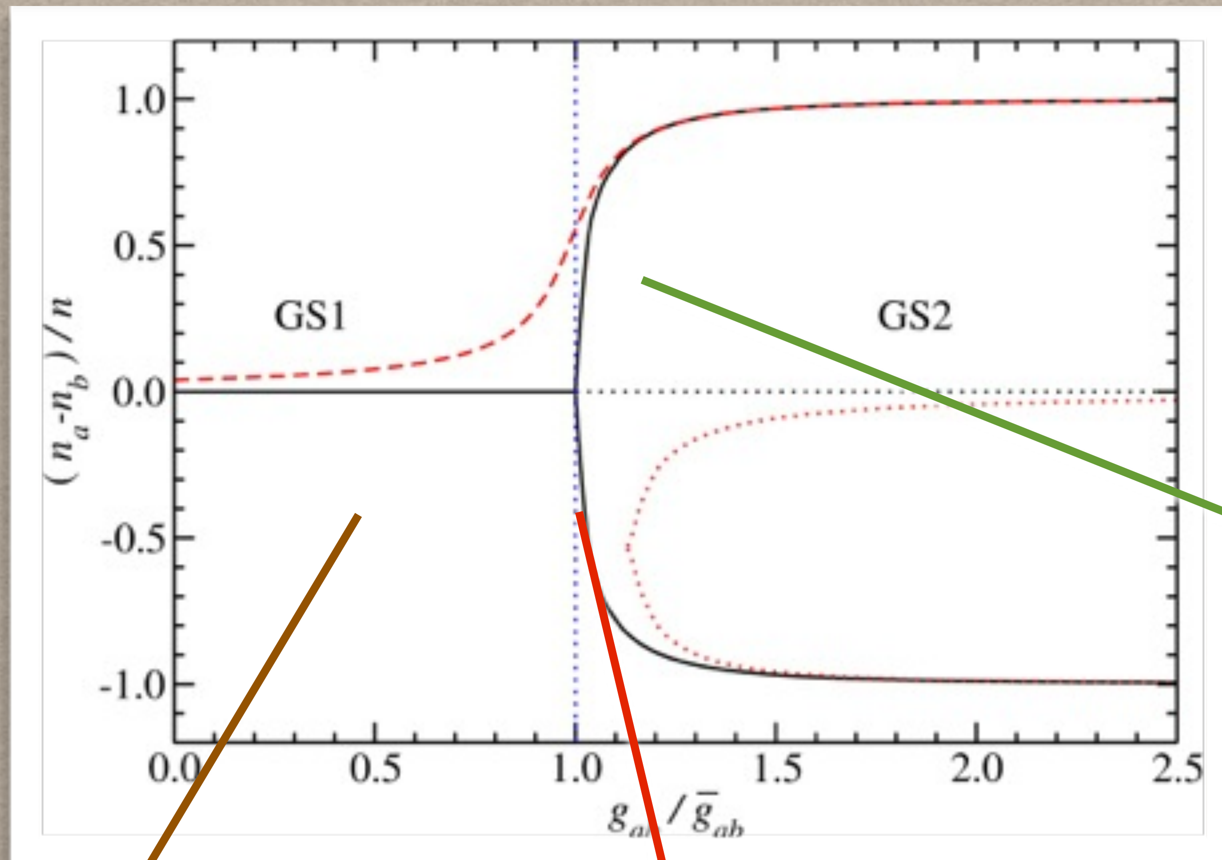


$$v_L = \min \frac{\omega(k)}{k}$$

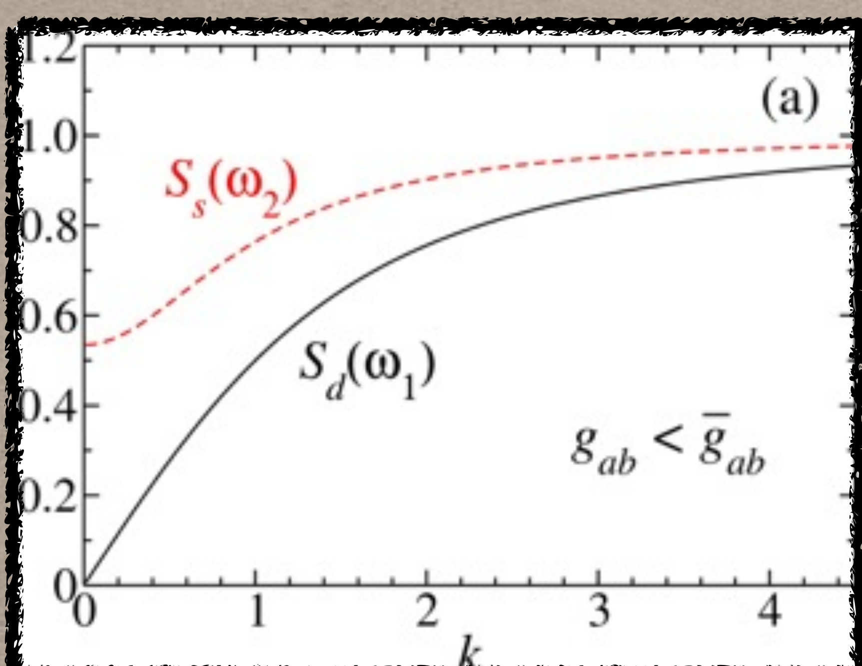
hybridization ⇒

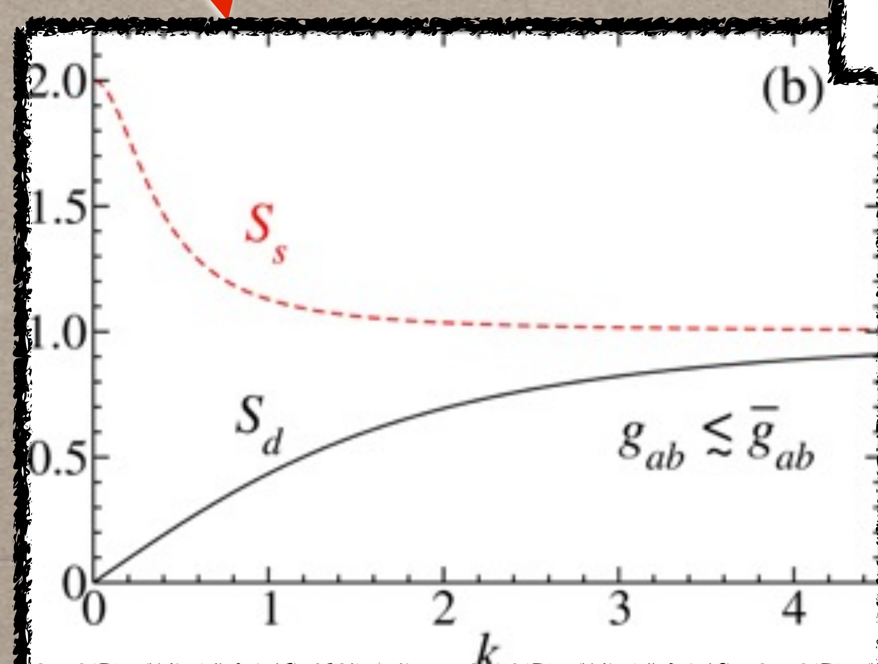
smaller Landau critical velocity

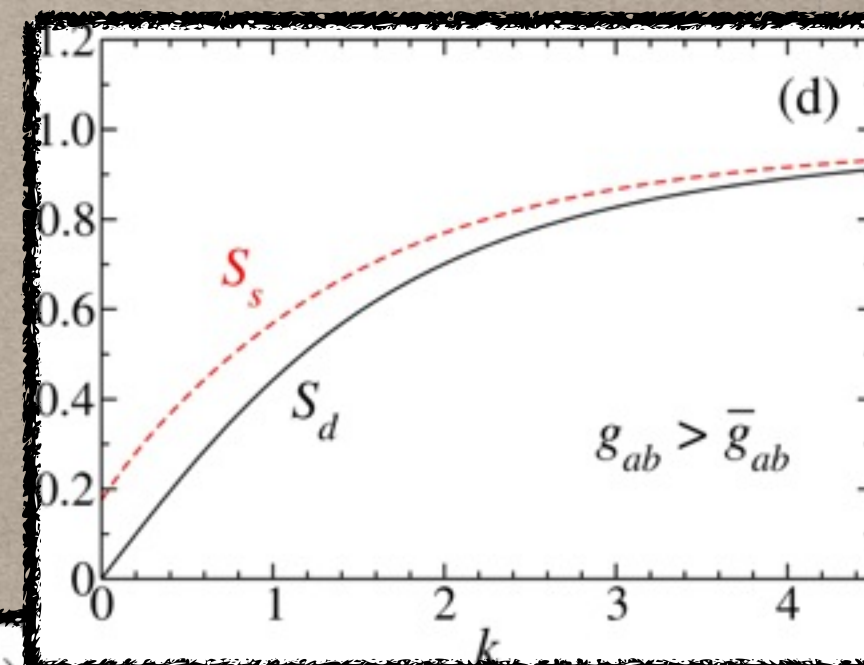
Static structure factor across the transition



gap due to single-particle

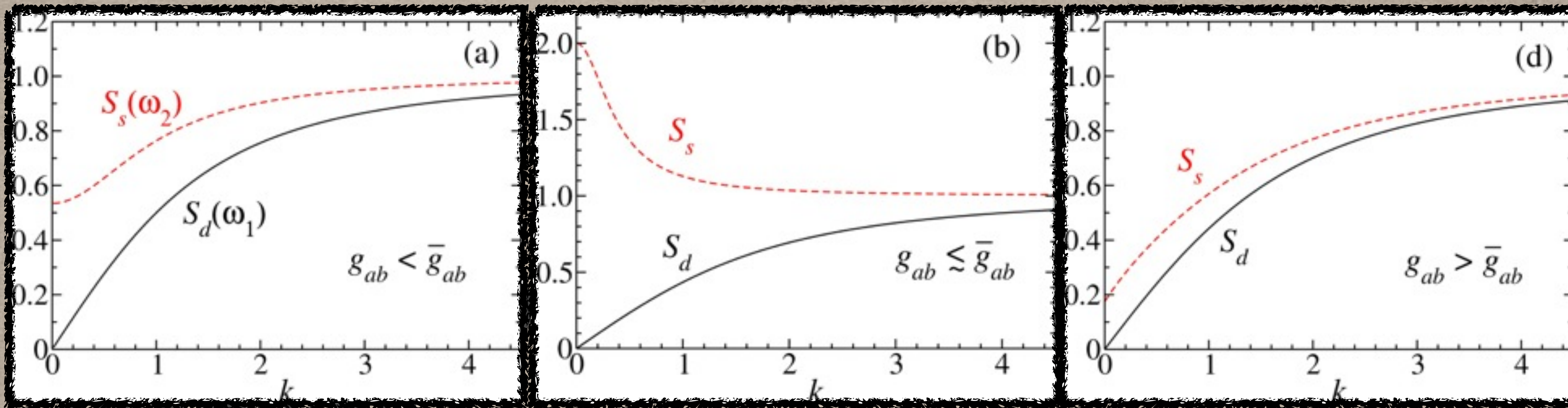






Feynman criterion for the density response

Static structure factor and density/spin fluctuations



S(k) is the Fourier Transform of the density-density correlation function and one can write in particular the FLUCTUATIONS IN A REGION as:

$$\Delta N^2 = n \int S_d(\mathbf{k},T) H(\mathbf{k}) \frac{d\mathbf{k}}{(2\pi)^D} \simeq N S_d(1/R,T)$$

geometrical factor

$$\Delta M^2 = n \int S_s(\mathbf{k},T) H(\mathbf{k}) \frac{d\mathbf{k}}{(2\pi)^D} \simeq N S_s(1/R,T)$$

Close to the phase transition the fluctuations in the polarization grow ⇒ structure factor at k=0 grows (diverges for infinite system)

Trapped Gases

What happens if we consider the external trapping potential?

Local Density Approximation

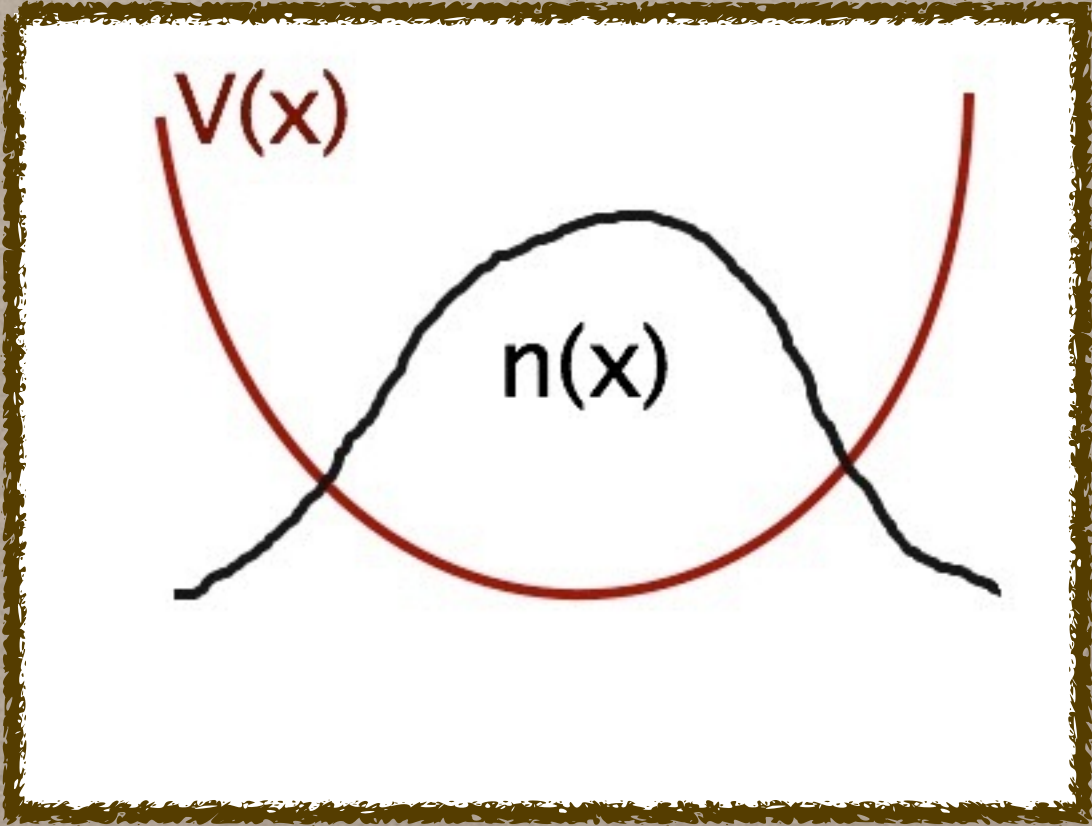
$$\left(g - g_{ab} + \frac{|\Omega|}{\sqrt{n_a n_b}}\right)(n_a - n_b) = V_b - V_a,$$

$$\left(g + g_{ab} - \frac{|\Omega|}{\sqrt{n_a n_b}}\right)(n_a + n_b) = 2\mu - (V_b + V_a).$$

In most of the experiments on cold-gases the atoms are trapped in harmonically-shaped potential. Suppose the potential is spin-independent.

Critical condition for the phase transition is spatial dependent as well as the (local) polarization

$$g - g_{ab} + \frac{2\Omega}{n} = 0$$



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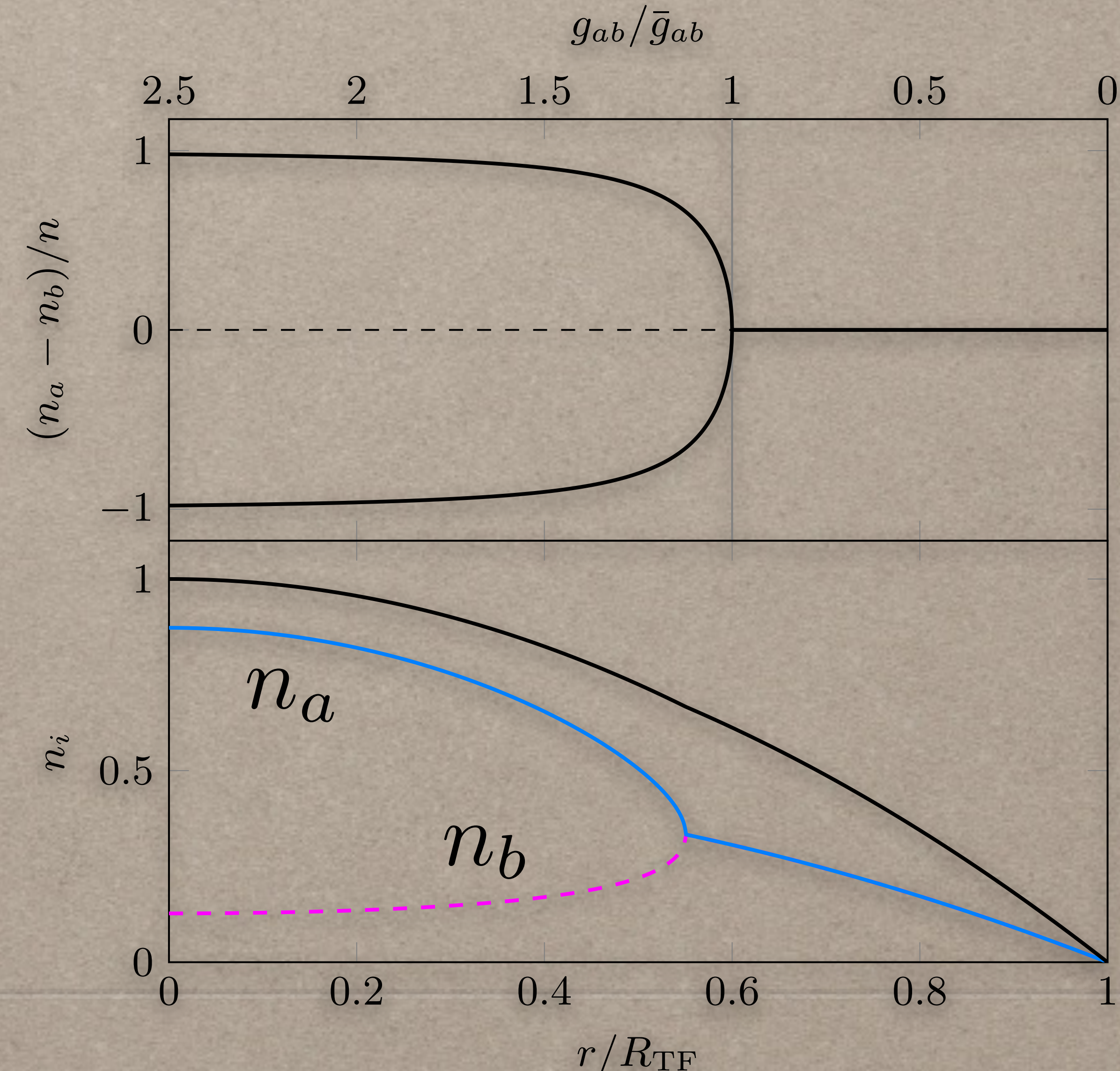
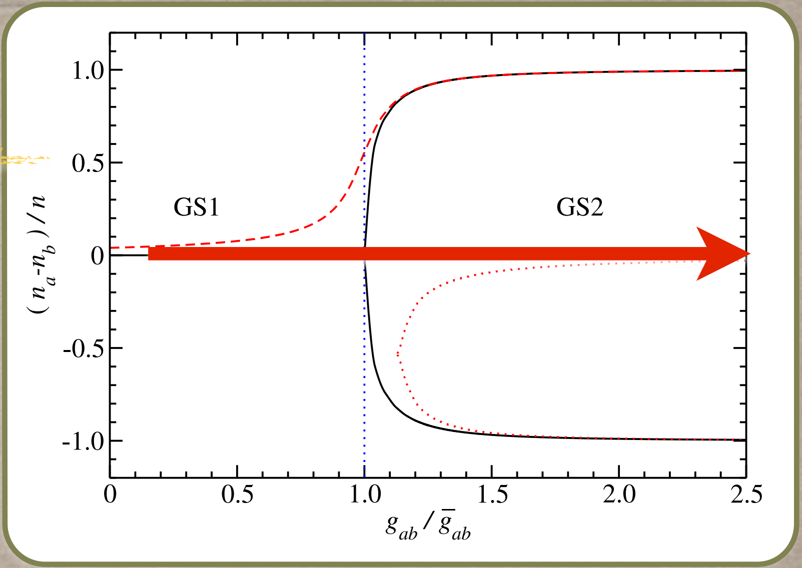


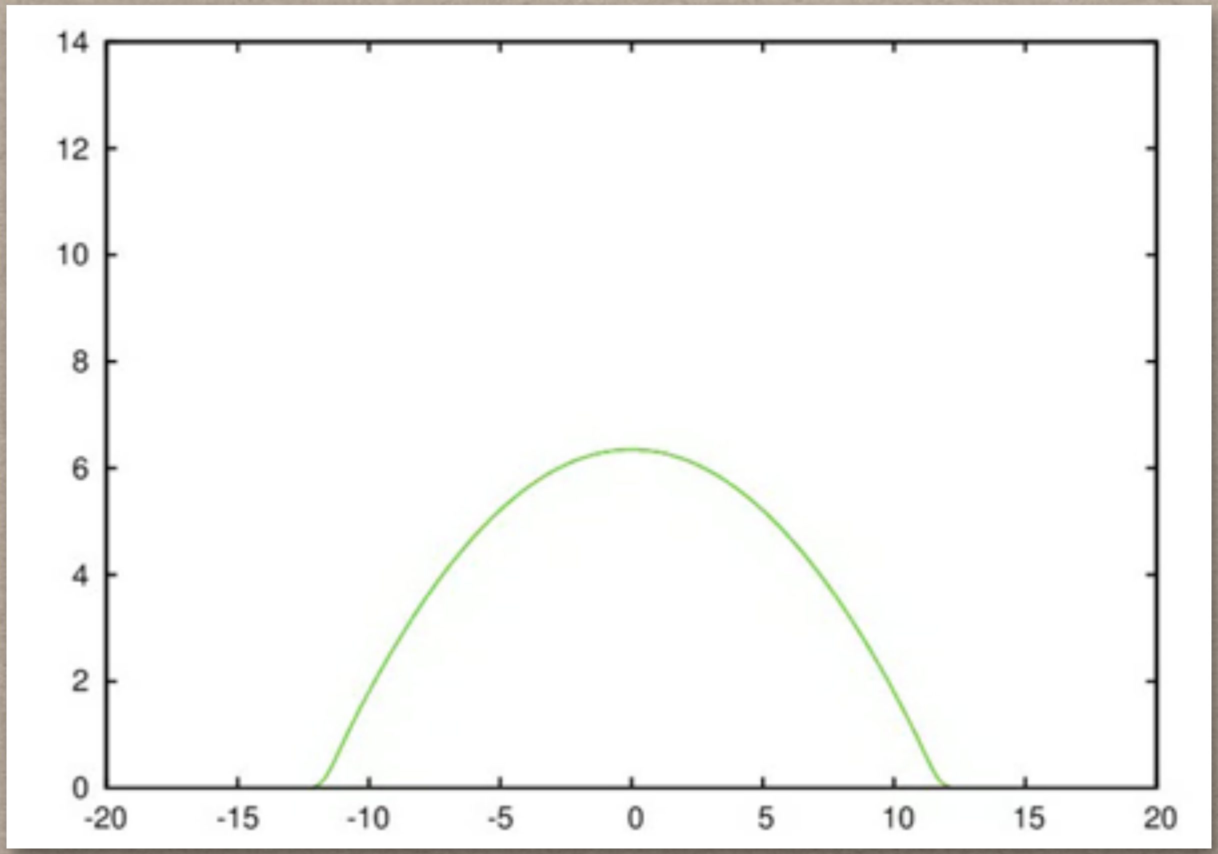
PHASE DIAGRAM
IN A

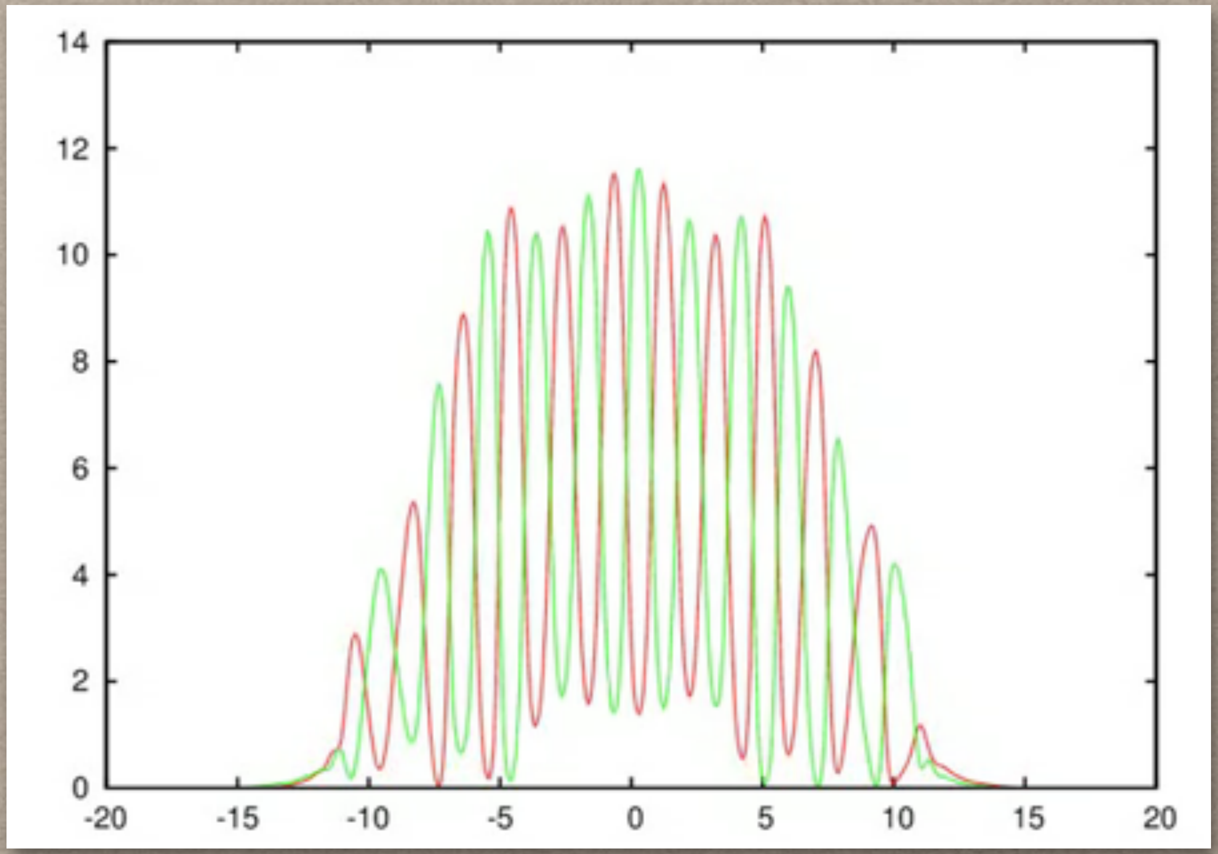
(SUDDEN) Quenching



Since we are dealing with a second order phase transition a natural question is how the system reacts to a quenching

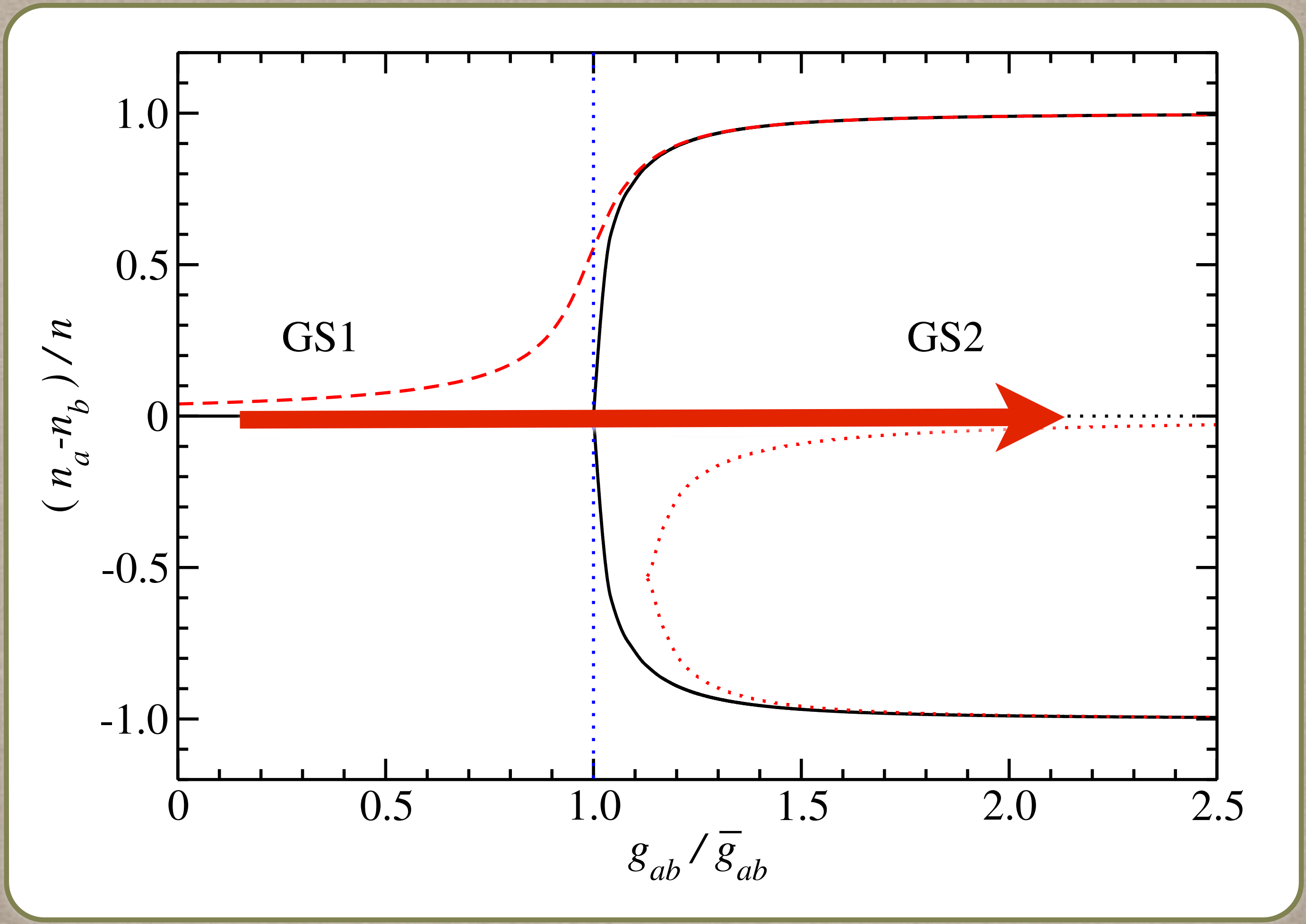
Domain formation





(KZM) Quenching

Since we are dealing with a second order (quantum) phase transition a natural question is how the system reacts to a quenching



For a linear quenching the number of defect should follow Kibble-Zurek mechanism

Sabbatini, Zurek, Davis, PRL 2011, NJP 2012

First measurements in Oberthaler's group

Vortices in coherently coupled BECs

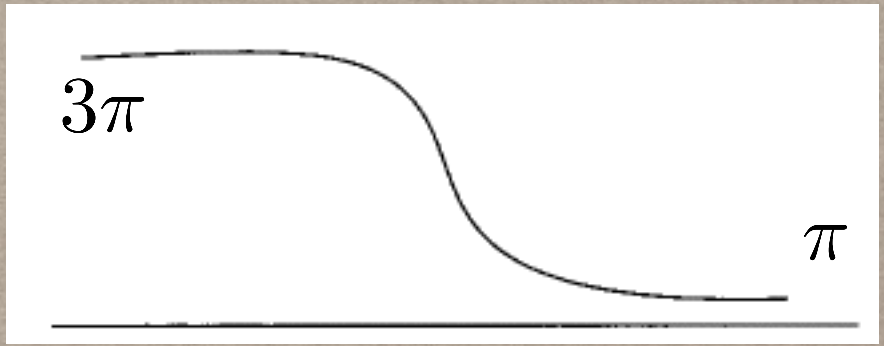
Phase domain walls: simple picture [Son & Stephanov PRA '02]

For fixed (equal) densities the functional energy of the relative phase reads:

$$E_{spin} = \int \left[\frac{\hbar^2 n}{4m} (\nabla \phi_s)^2 + 2\Omega n \cos(\phi_s) \right]$$

Global minimum for $\phi_s = (2n+1)\pi$

Domain wall or kink is a local minimum solution which connects 2 global minima



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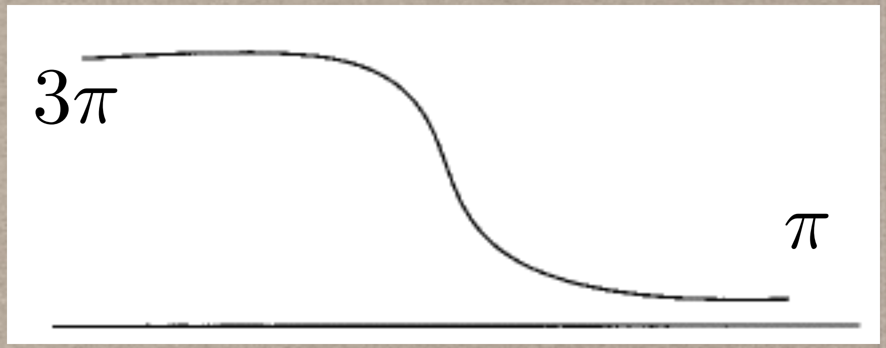
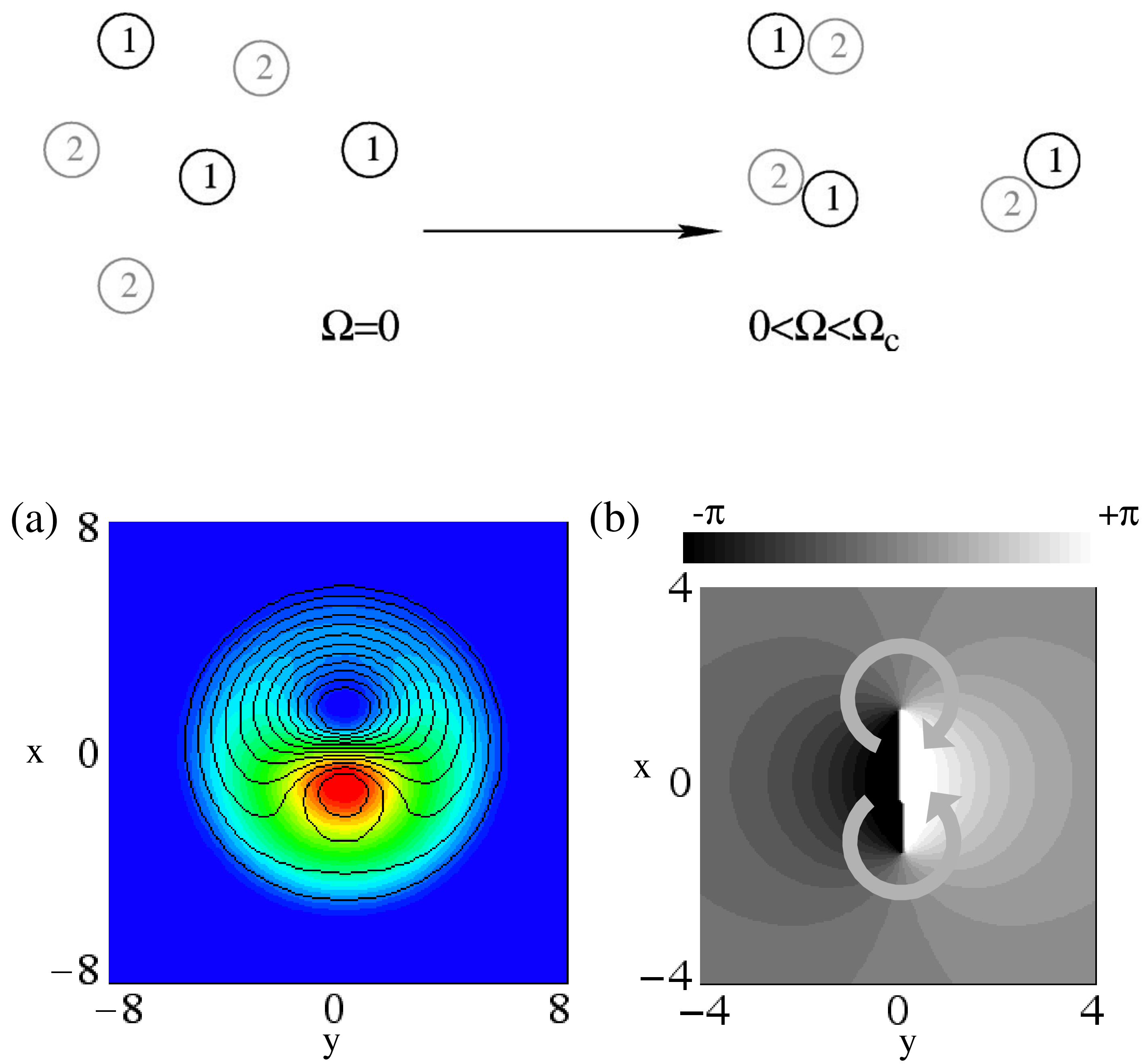


$\Delta\phi_s = 2\pi$

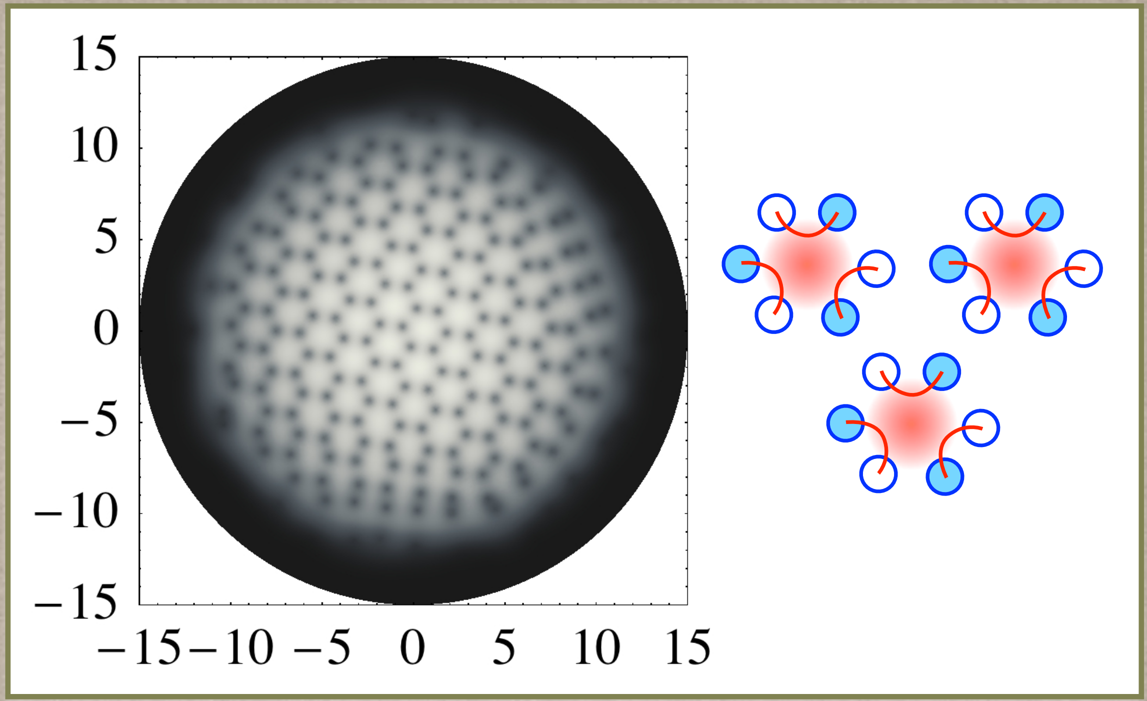
$\Delta\phi_s = -2\pi$

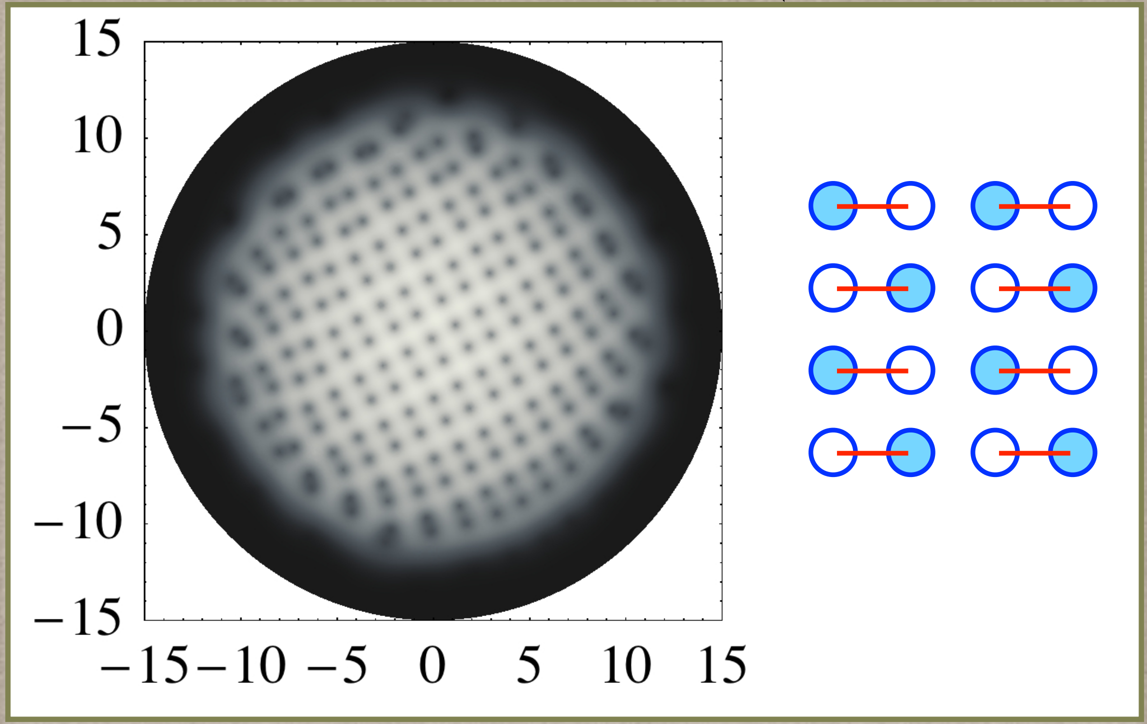
Vortices in coherently coupled BECs: vortex dimers



Kasamatsu, Tsubota, Ueda PRL 2004

Vortex Lattice in coherently coupled BECs





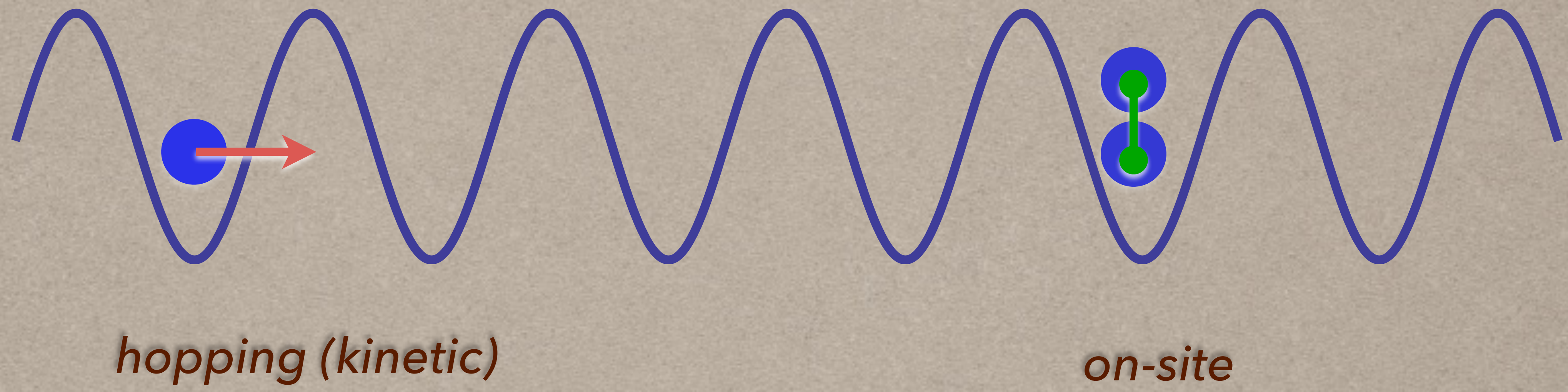
and many more

From 2013 Cipriani's PhD thesis

Bose-Hubbard Model

Barbiero, Abad, AR, arXiv:1403.4185
Zhan, Sabbatini, Davis, McCulloch, arXiv:1403.4823

Bose-Hubbard Model

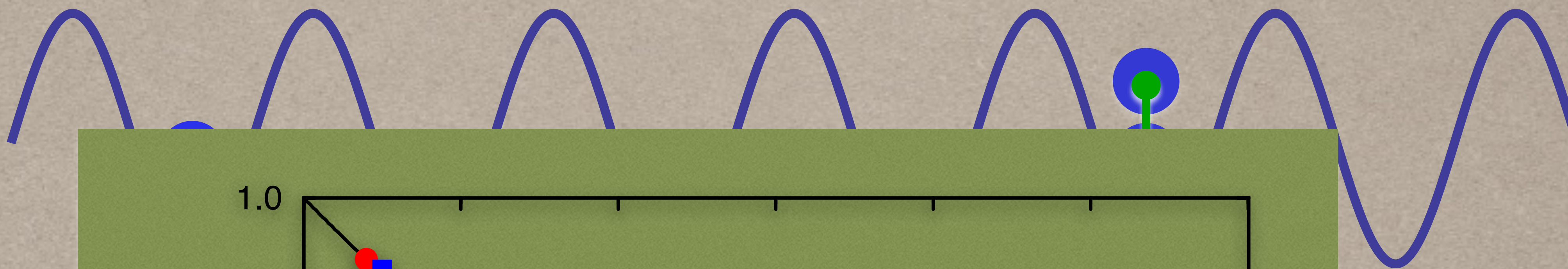


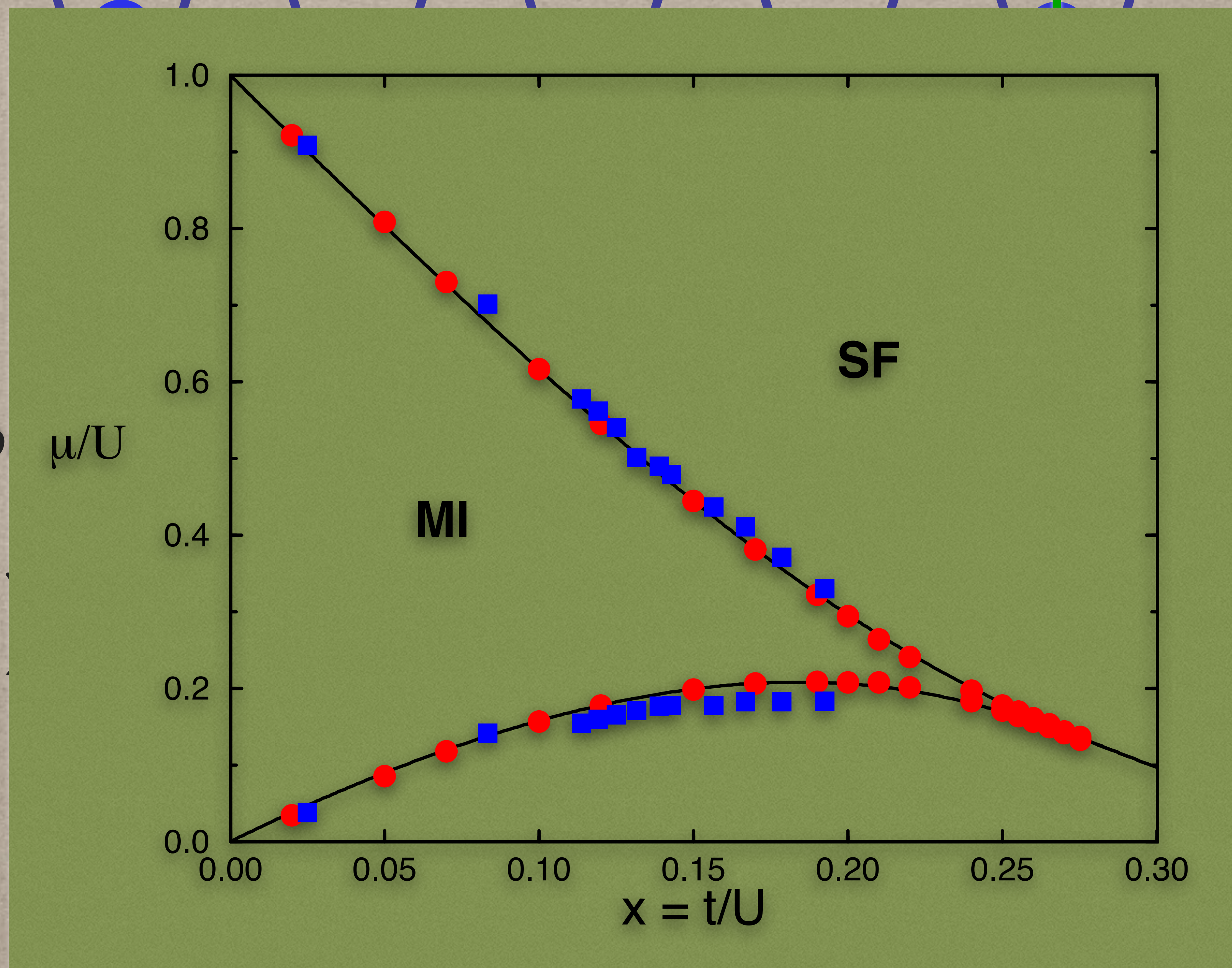
for deep optical lattices and a small number of atoms

$$H = \sum_i \Big[-J(a_i^\dagger a_{i+1} + h.c.) - \mu n_i + \frac{U}{2} n_i (n_i - 1) \Big)$$

(Single component) Bose Hubbard Model

Bose-Hubbard Model





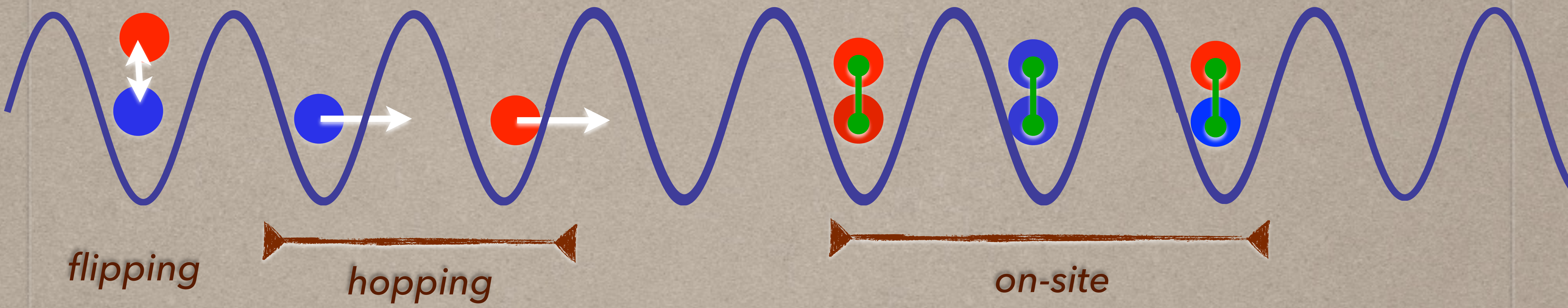
fo … *r of atoms*

$H =$ … $n_i - 1)$

d Model

T. D. Kuehner, and H. Monien PRB 58, 1998.

Coherent coupled Bose-Hubbard model

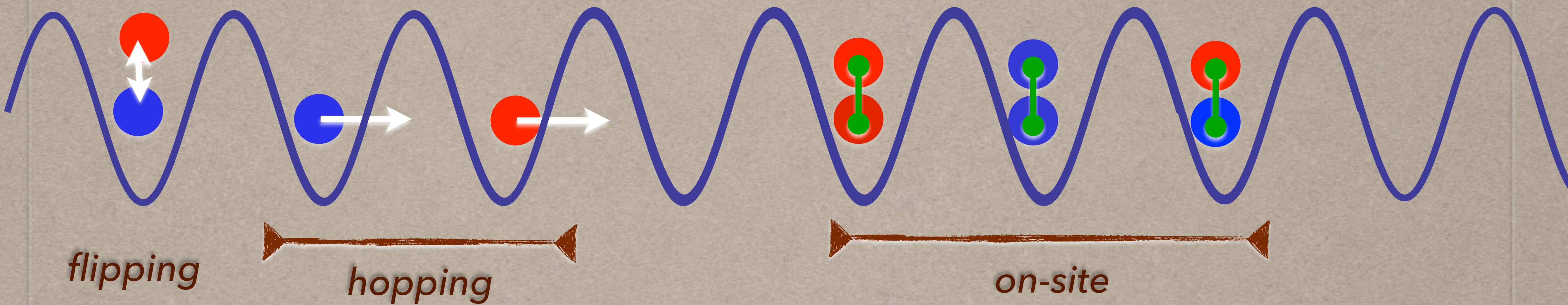


for deep optical lattices and a small number of atoms

$$H = \sum_i \left\{ \sum_\sigma \left[-\mu \hat{n}_{i\sigma} + \frac{U}{2} \hat{n}_{i\sigma}(\hat{n}_{i\sigma} - 1) \right] + U_{ab} \hat{n}_{ia} \hat{n}_{ib} + \right.$$

$$\left. + J_\Omega (\hat{a}_i^\dagger \hat{b}_i + \hat{a}_i \hat{b}_i^\dagger) \right\} - \sum_{<ij>,\sigma} J (\hat{a}_{i\sigma}^\dagger \hat{a}_{j\sigma} + \hat{a}_{i\sigma} \hat{a}_{j\sigma}^\dagger)$$

Coherent coupled Bose-Hubbard model



Insights:

Deep in the SF regime it is essentially the same as for the continuous case.

J=0: single atom per lattice site (Rabi oscillations)

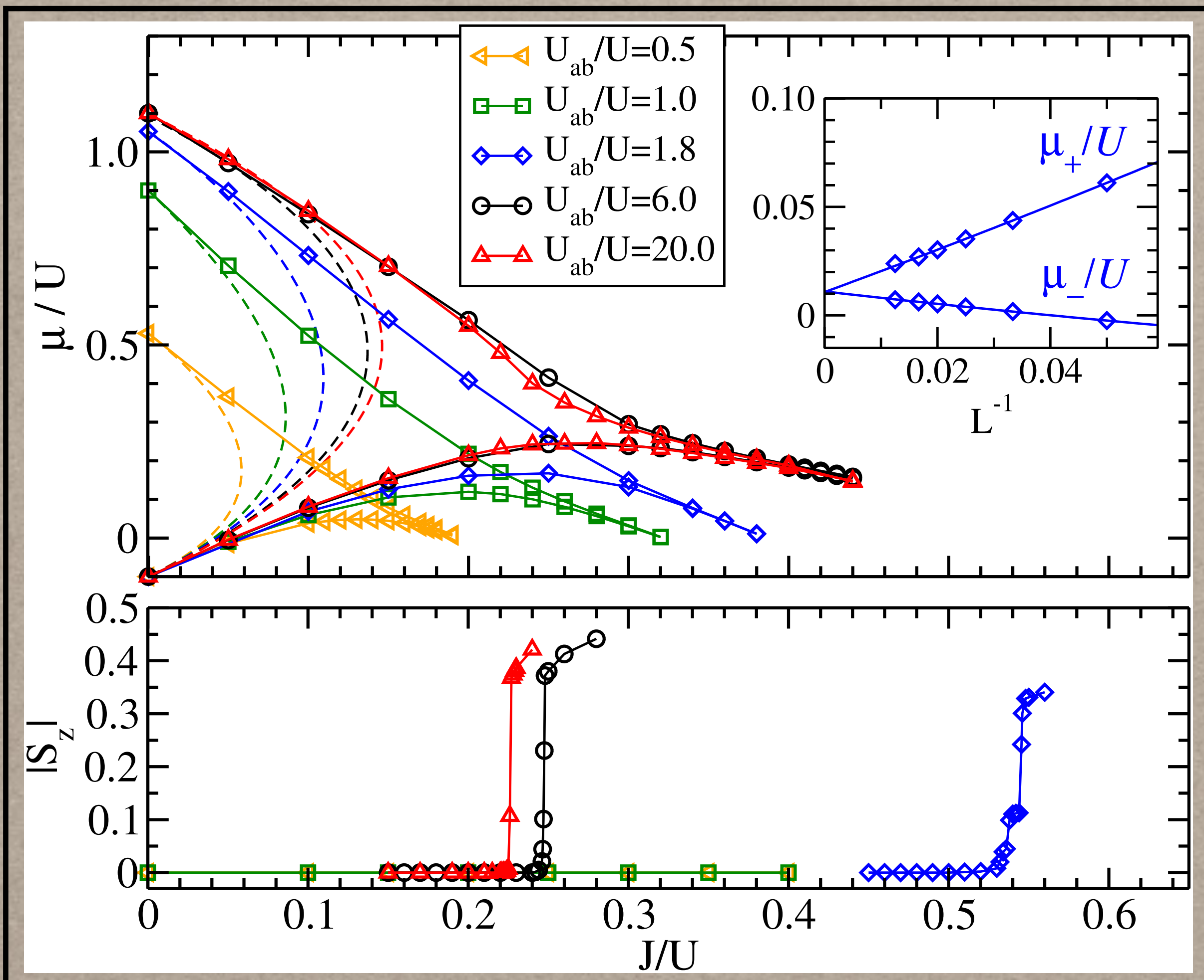
For small J (insulating state) atom exchange dominates -> spin chain:

$$H_{XXZ} = -t \sum_i (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z) + 2J_\Omega \sum_i \hat{S}_i^x$$

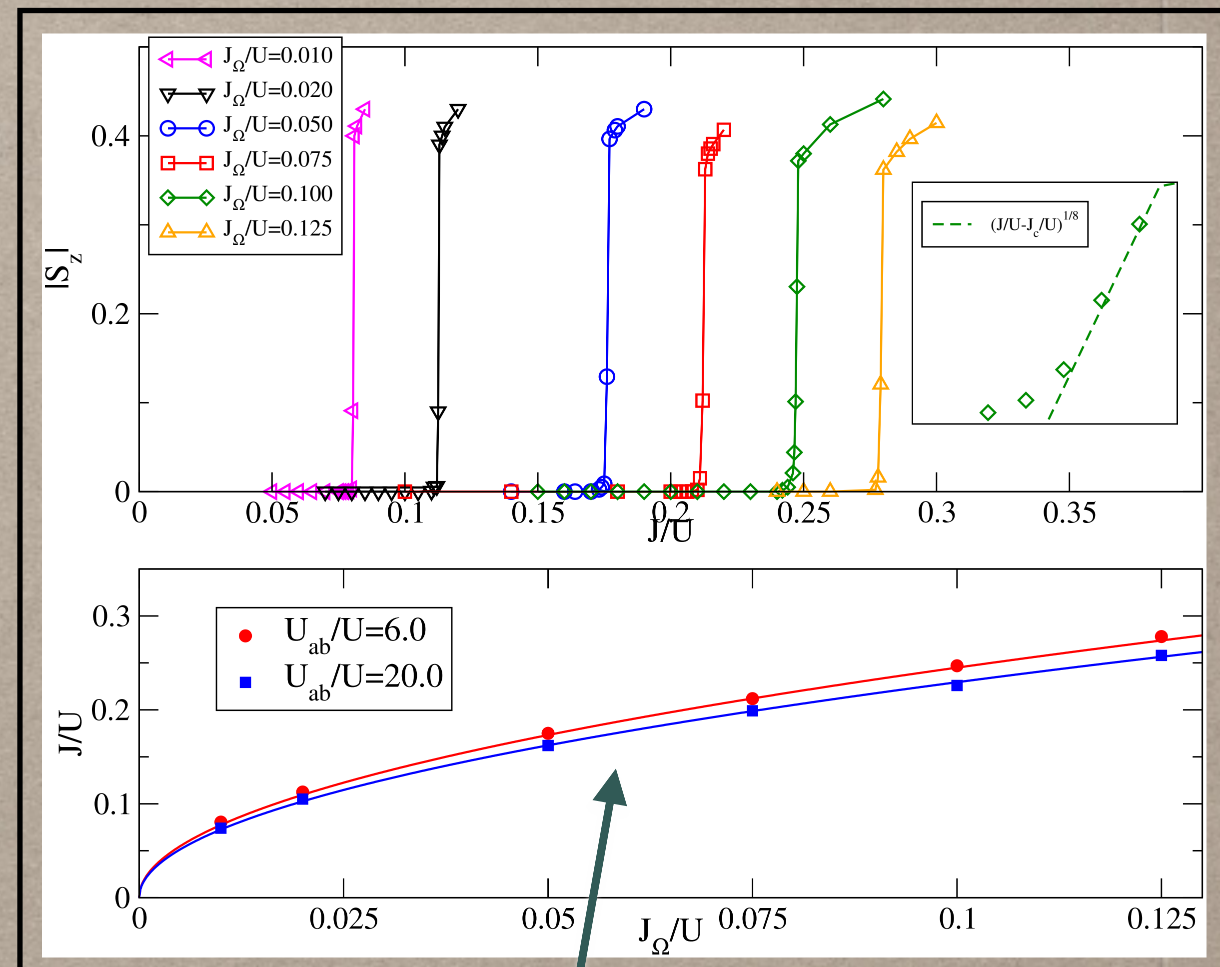
(see e.g., L.-M. Duan, E. Demler, and M. D. Lukin, Phys. Rev. Lett. 91, 090402 (2003))

Coherent coupled Bose-Hubbard model

SF-MI phase diagram for different



MI state magnetization



Magnetization

$U = U_{ab}$ "SU(2)" symmetric case

Solid line:

$$U_{ab} - U = 2J_\Omega \frac{UU_{ab}}{J^2}$$

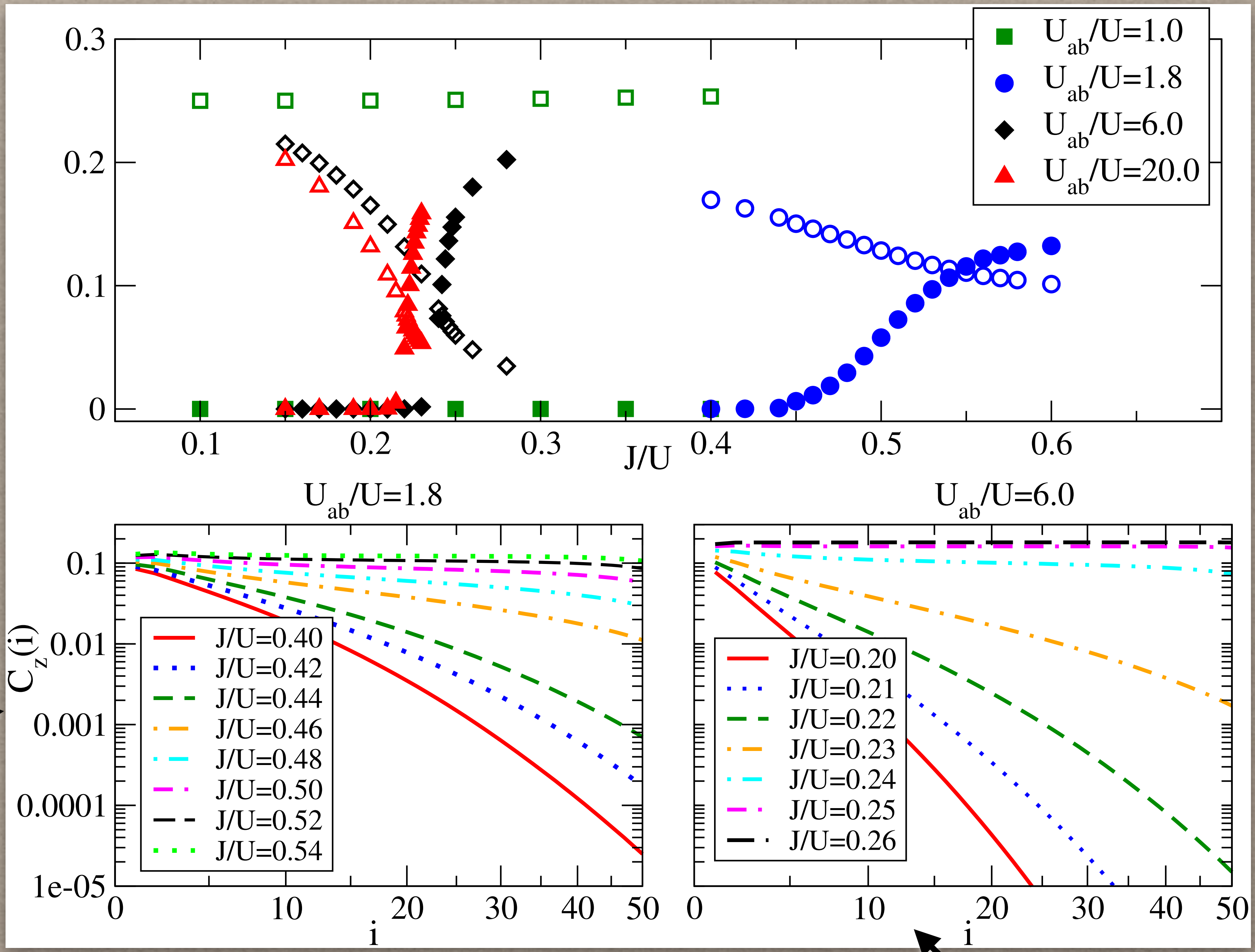
Coherent coupled Bose-Hubbard model

Spin-spin correlation functions $C_\alpha(j) = \langle S_\alpha(i) S_\alpha(i+j) \rangle$

open/filled symbols:

$C_x(50)/C_z(50)$

SF phase



MI phase

Outlook

- *two-body properties*
- *dynamics of phase transition*
- *collective modes and sum-rule modification*

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THANKS!