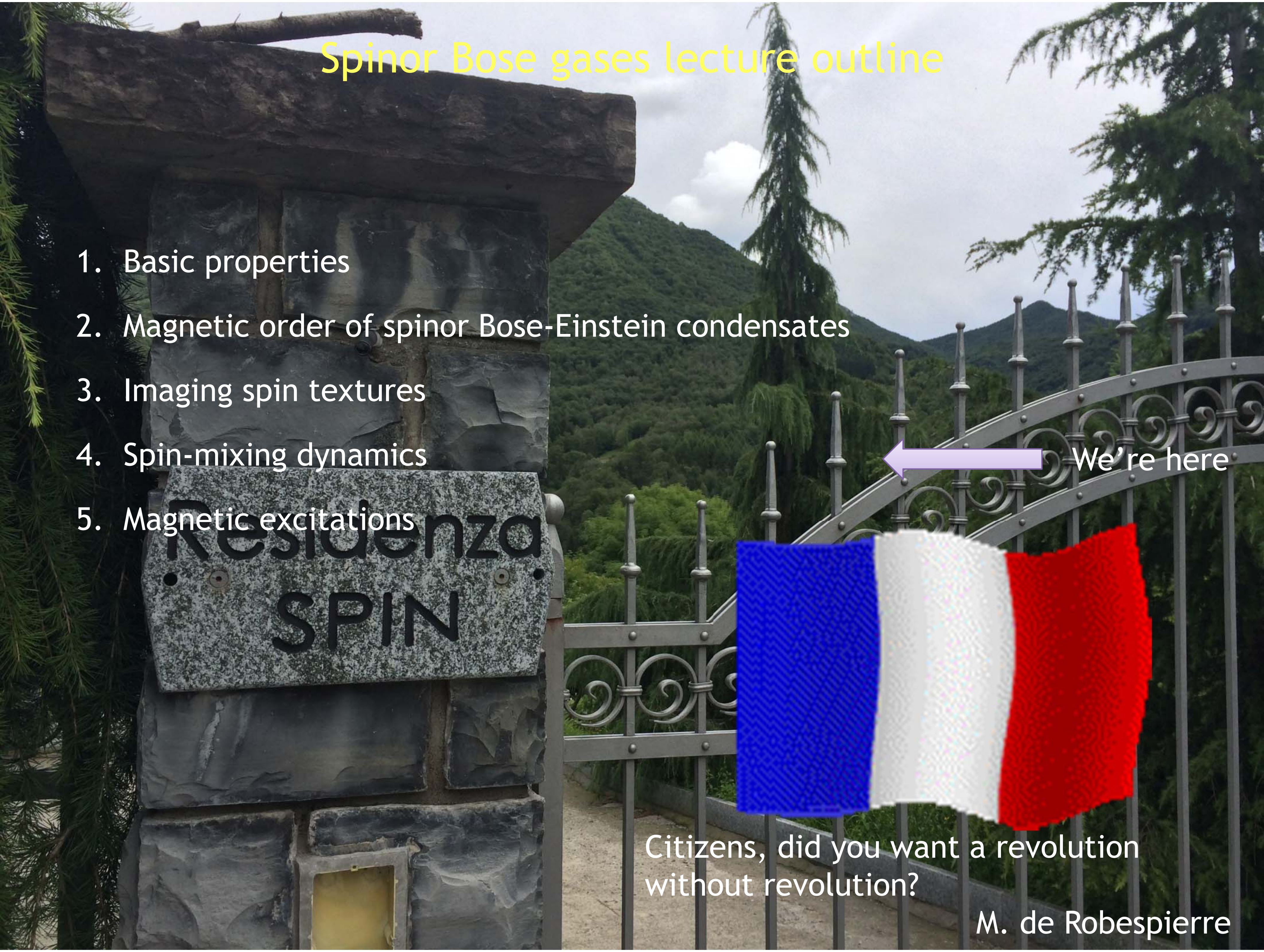
Spinor Bose gases lecture outline

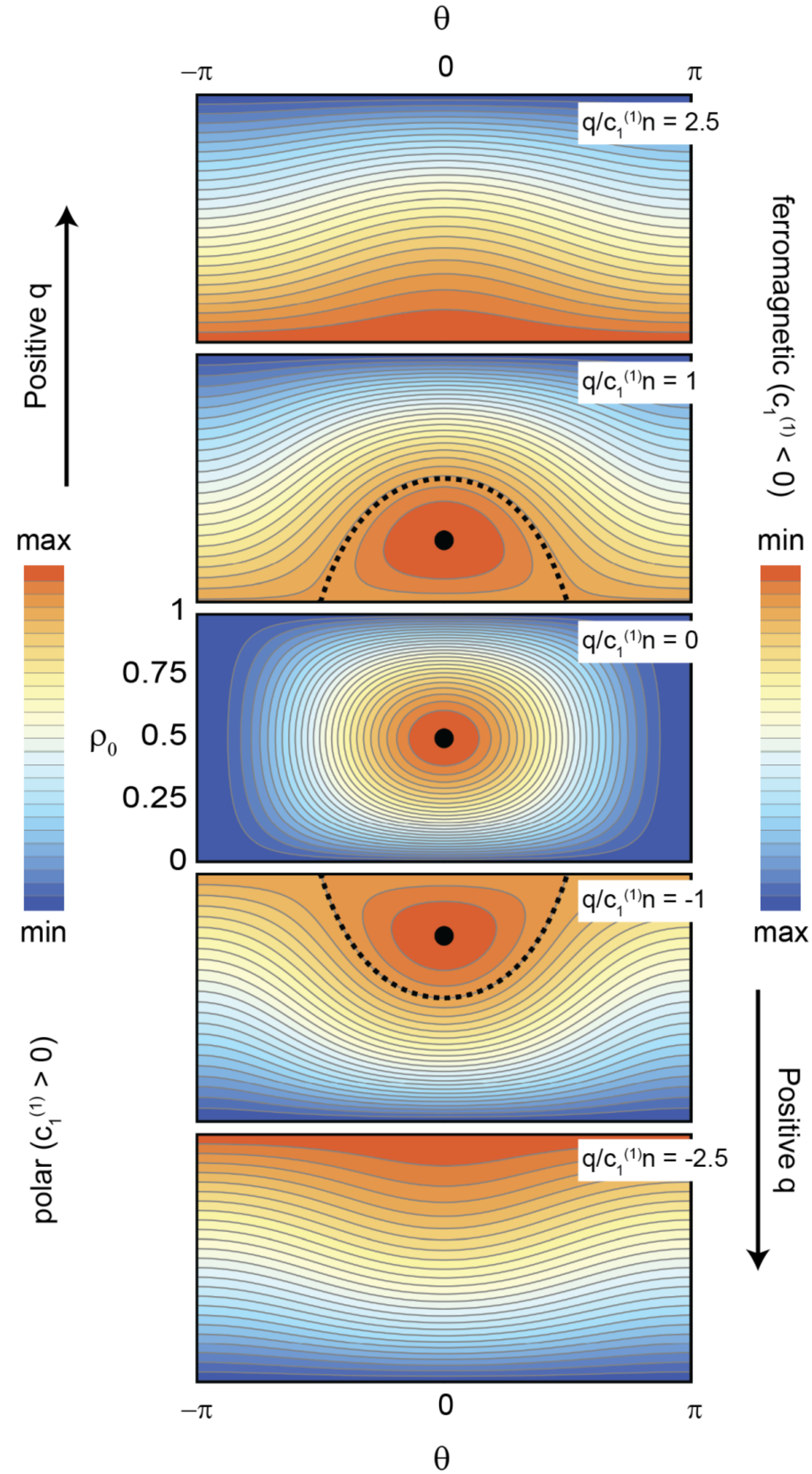
1. Basic properties
2. Magnetic order of spinor Bose-Einstein condensates
3. Imaging spin textures
4. Spin-mixing dynamics
5. Magnetic excitations

We're here



Citizens, did you want a revolution without revolution?

M. de Robespierre



θ
−π
0
π
q/c_1^(1)n = 2.5
q/c_1^(1)n = 1
q/c_1^(1)n = 0
q/c_1^(1)n = -1
q/c_1^(1)n = -2.5
Positive q
ferromagnetic (c_1^(1) < 0)
polar (c_1^(1) > 0)
max
min
ρ_0
1
0.75
0.5
0.25
0

Spin mixing instability

- Perturb stationary state

$$\Psi = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \psi_x \begin{pmatrix} 1/\sqrt{2} \\ 1 \\ -1/\sqrt{2} \end{pmatrix} + \psi_y \begin{pmatrix} i/\sqrt{2} \\ 1 \\ i/\sqrt{2} \end{pmatrix}$$

$$\hat{\Psi} = \psi_z + \underbrace{\hat{\psi}_x + \hat{\psi}_x}_{\text{spin fluctuations}}$$

- Bogoliubov linear stability analysis

$$H = \sum_{\substack{modes,\\ x,y}} \left(\epsilon + q + c_1^{(1)} n\right) \hat{\psi}_\beta^\dagger \hat{\psi}_\beta - \frac{c_1^{(1)} n}{2} \left(\hat{\psi}_\beta^\dagger \hat{\psi}_\beta^\dagger + \hat{\psi}_\beta \hat{\psi}_\beta\right)$$

- Map onto harmonic oscillator per mode $\hat{Z} = \frac{\hat{\psi}_\beta^\dagger + \hat{\psi}_\beta}{2}$ $\hat{P} = i\,\frac{\hat{\psi}_\beta^\dagger - \hat{\psi}_\beta}{2}$

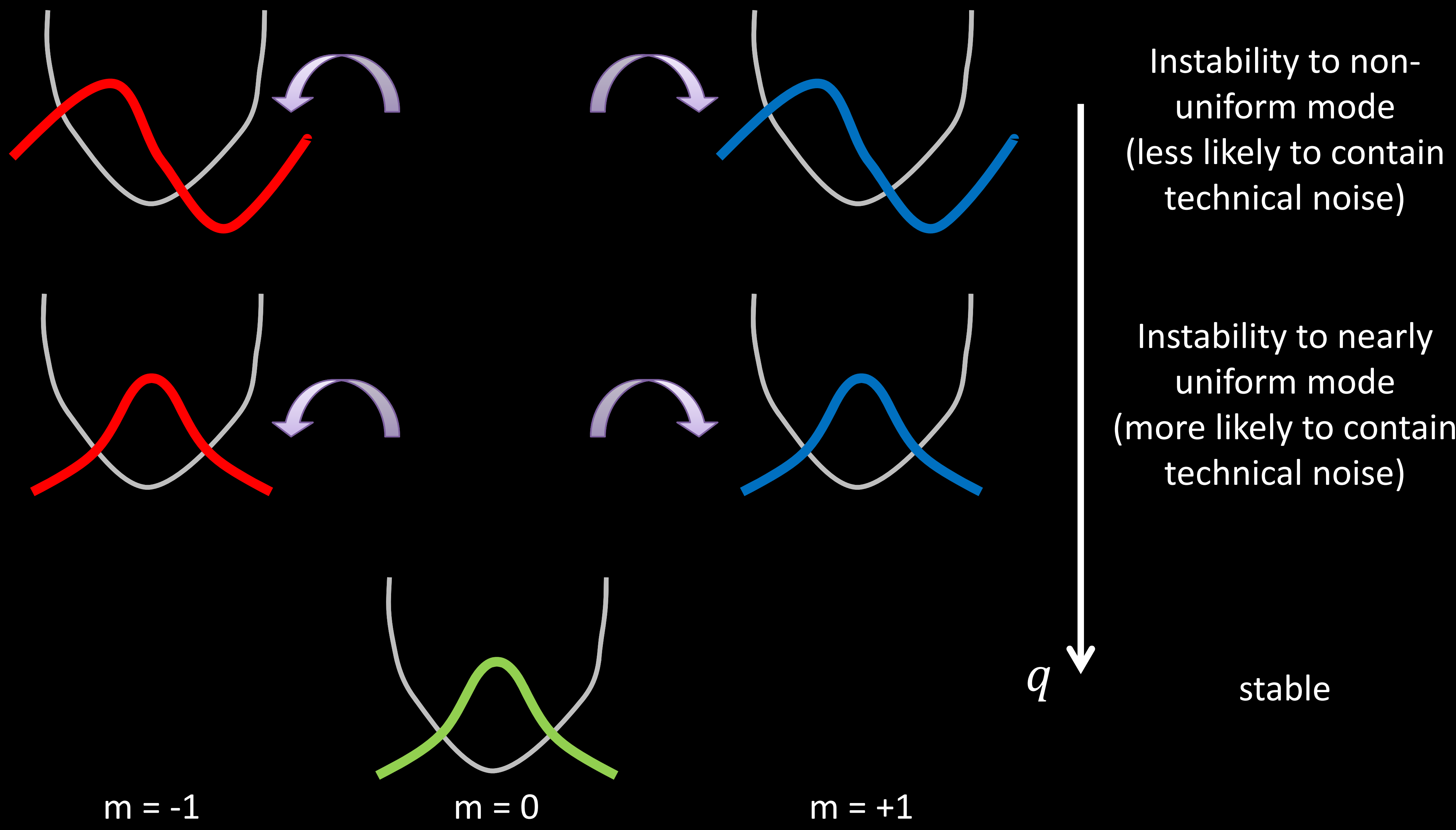
$$H = \sum_{\substack{modes,\\ x,y}} (\epsilon + q)\, \hat{Z}^2 + \left(\epsilon + q + 2\, c_1^{(1)} n\right) \hat{P}^2$$

Spin mixing instability

$$H = \sum_{\substack{modes,\\ x,y}} (\epsilon + q)\,\hat{Z}^2 + \left(\epsilon + q + 2\,c_1^{(1)} n\right)\,\hat{P}^2$$

	ferromagnetc	antiferromagnetic
Stable (H.O. like)	$\epsilon + q > \left\|2\,c_1^{(1)} n\right\| = q_0$	$\epsilon + q > 0$
Stable (H.O. rotating in opposite sense)	$\epsilon + q < 0$	$\epsilon + q < -2\,c_1^{(1)} n$
Unstable	middle range	middle range

Hannover experiments: single-mode quench



Hannover experiments: single-mode quench

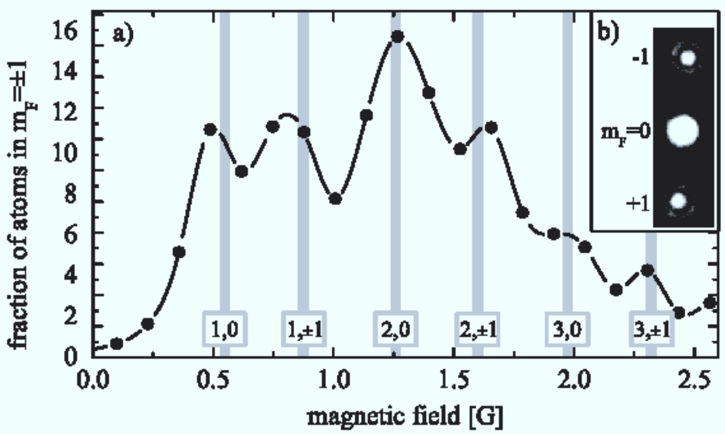


FIG. 1. (a) The fraction of atoms transferred into the $|\pm 1\rangle$ state within 18.5 ms as a function of the applied magnetic field. Each data point is an average over 30 realizations. The vertical gray lines indicate the resonance positions obtained from a 2D circular box model, and the labels indicate the corresponding Bessel modes. (b) Absorption image of a $|0\rangle$ BEC and the $|\pm 1\rangle$ clouds recorded at 1.29 G.

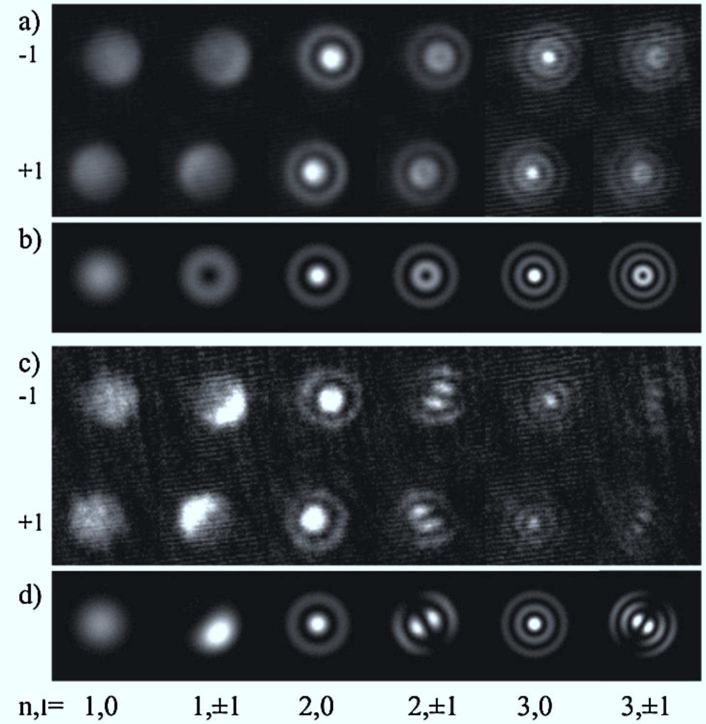
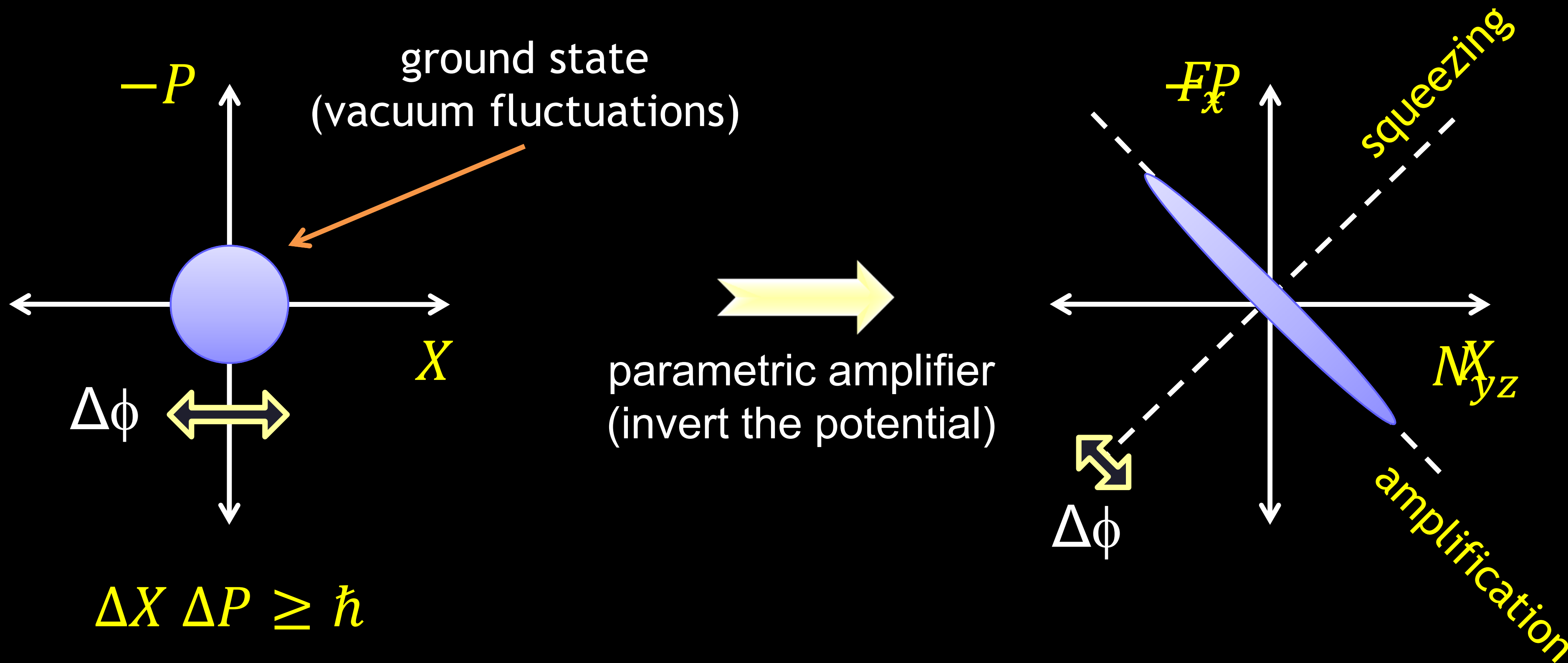


FIG. 2. The experimental and theoretical density distributions on the resonance positions after time-of-flight expansion. (a) Averaged experimental density profiles. (b) Calculated pure Bessel distributions corresponding to the experimental situation. (c) Individual experimental density profiles. (d) Calculated superpositions of Bessel distributions (see text). The $|0\rangle$ BEC was omitted in (a) and (c) for clarity.

PRL **103**, 195302 (2009)
PRL **104**, 195303 (2010)
PRL **105**, 135302 (2010)

Noiseless spin amplification by a quantum amplifier

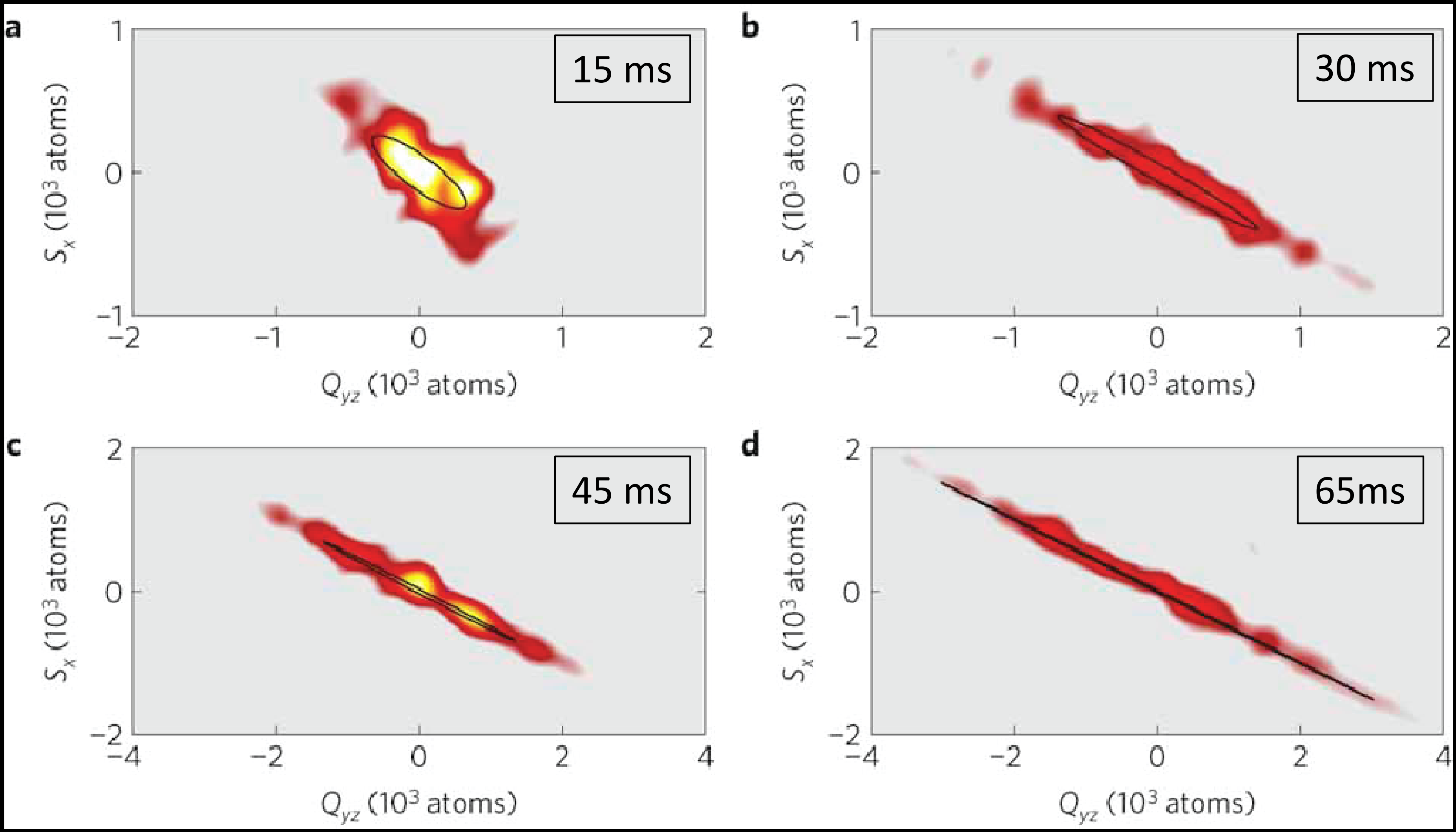
- Dynamics of a single spatial mode are like those of a one-dimensional harmonic oscillator



- For quenched F=1 spinor gas:
  - two phase-space planes
  - quadrature operators are spin-vector and spin-quadrupole moments

“Spin squeezing of high-spin, spatially extended quantum fields,” NJP 12, 085011 (2010)

Quantum spin-nematicity squeezing
(Chapman group, Georgia Tech)



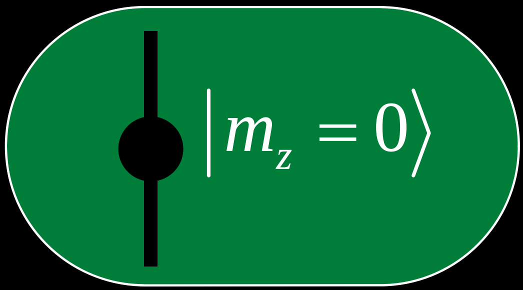
observed squeezing 8.6 dB below standard quantum limit!

Hamley et al., Nature Physics **8**, 305 (2012).

see also Gross et al., Nature 480, 219 (2011) [Oberthaler group], and

Lücke, et al., Science 334, 773 (2011) [Klempt group]

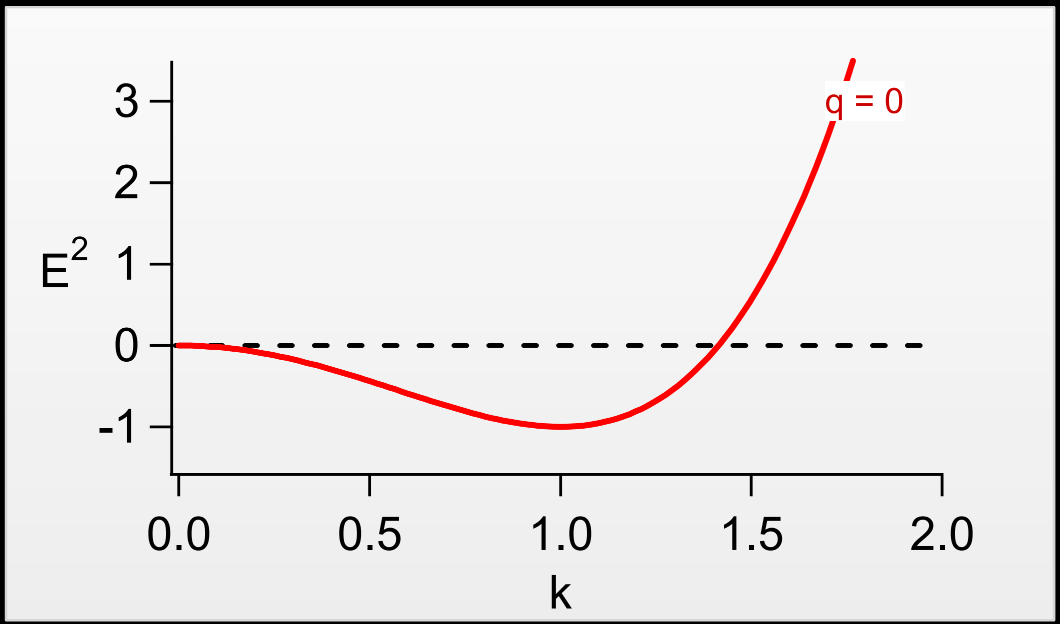
Spectrum of stable and unstable modes



- Bogoliubov spectrum
  - Gapless phonon (m=0 phase/density excitation)
  - Spin excitations

$$E_S^2 = (k^2 + q)(k^2 + q - 2)$$

Energies scaled by $c_2 n$



q>2: spin excitations are gapped by $\sqrt{q(q-2)}$

1>q>2: broad, "white" instability

0>q>1: broad, "colored" instability

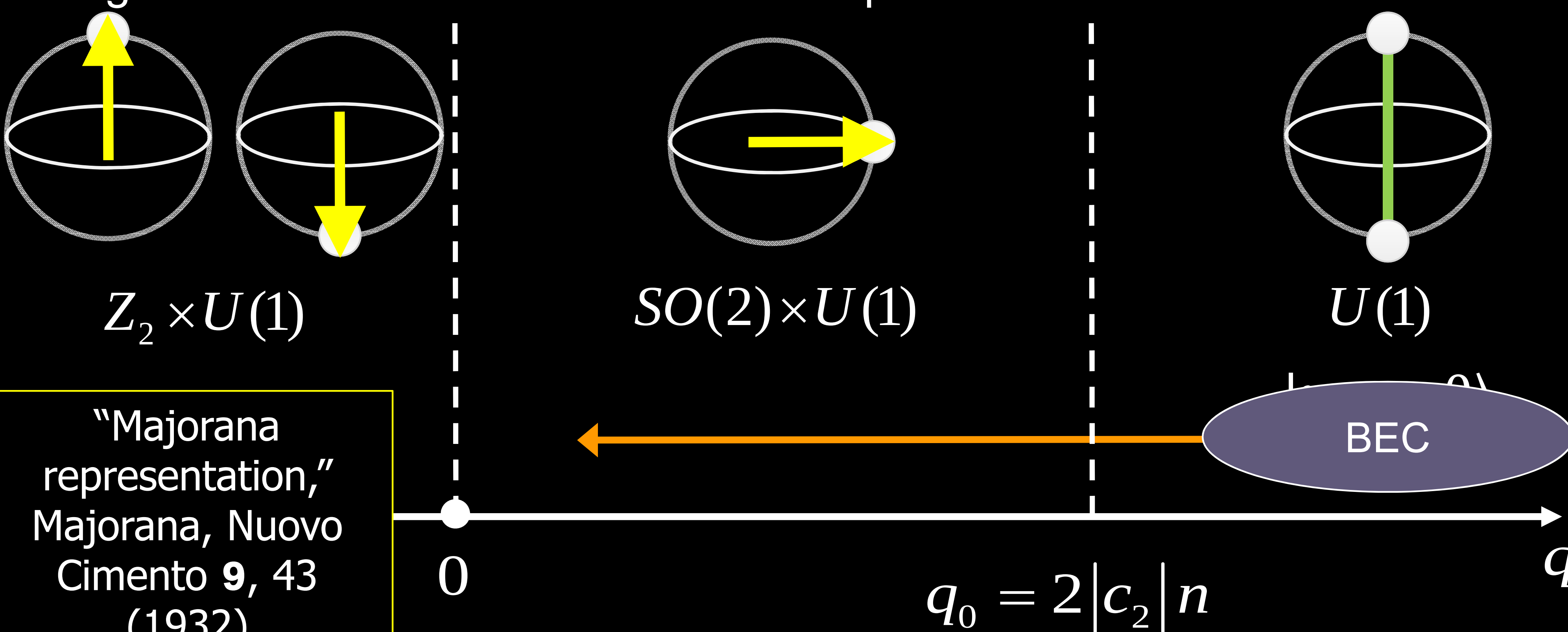
q<0: sharp instability at specific q≠0

Quantum quench

$$E = -|c_2| n \langle \vec{F} \rangle^2 + q \langle F_z^2 \rangle$$

<u>ferromagnetic states</u> <u>unmagnetized state</u>

longitudinal axis transverse plane

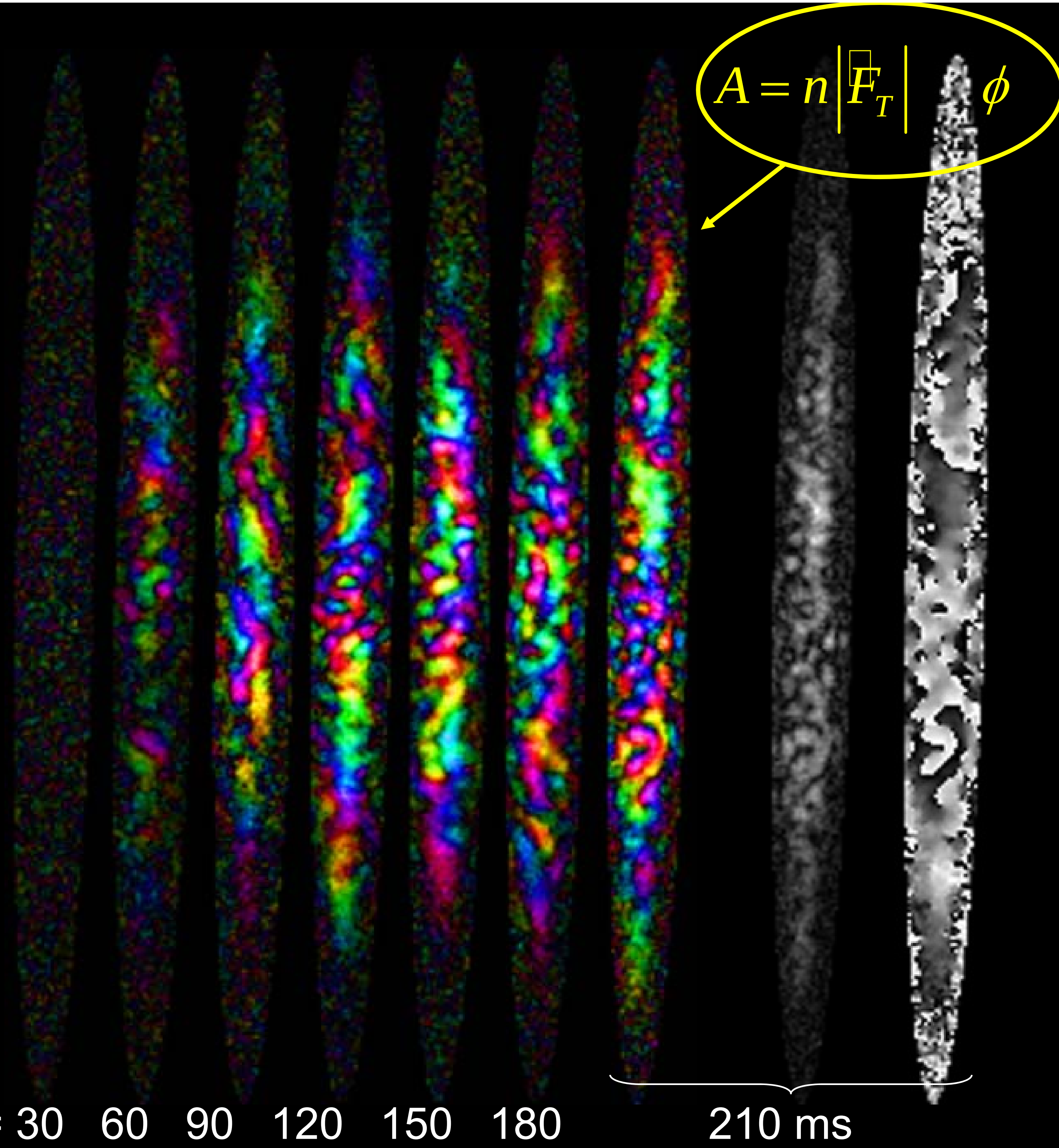


"Majorana representation," Majorana, Nuovo Cimento **9**, 43 (1932)

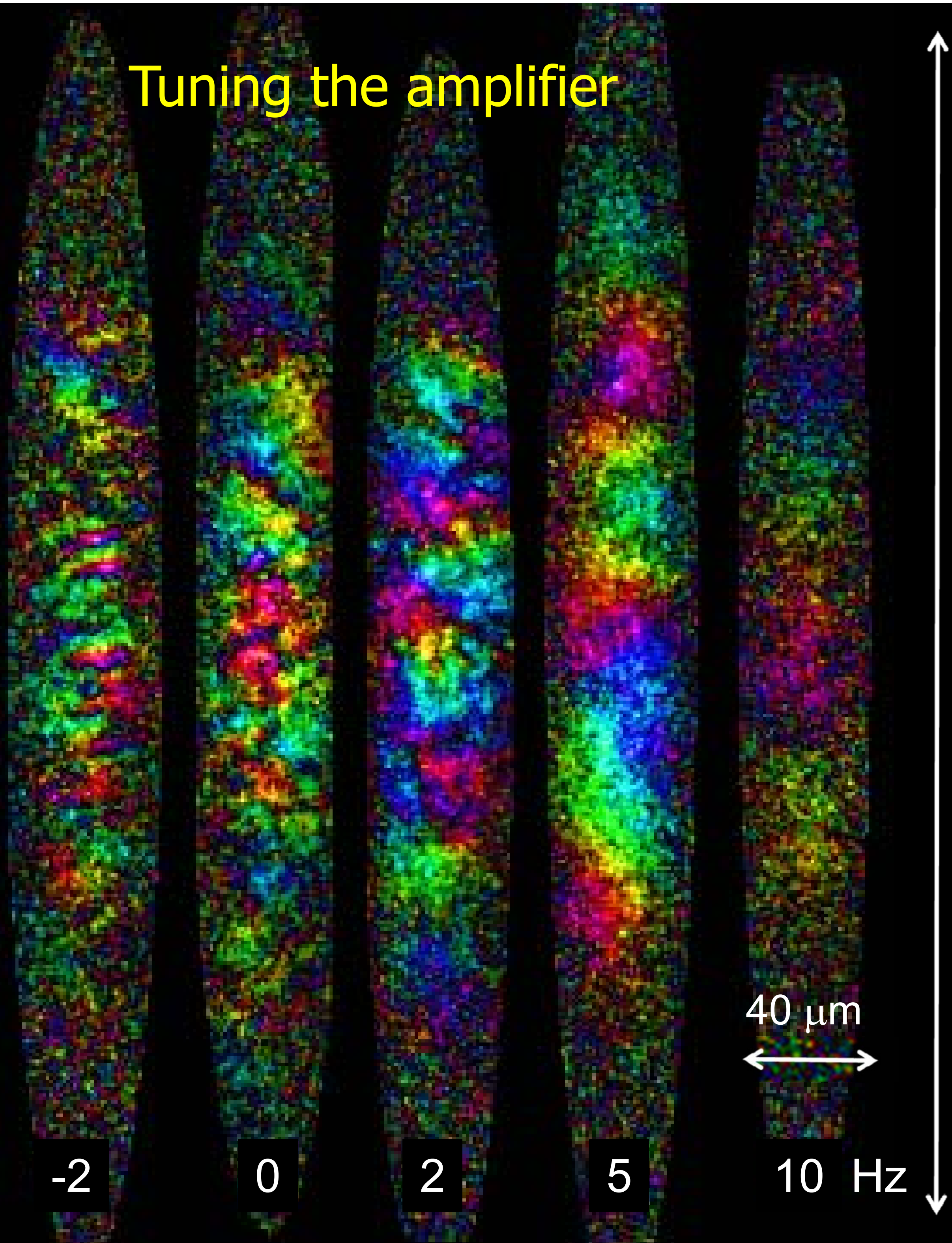
Non-equilibrium (quantum) dynamics at a (quantum) phase transition

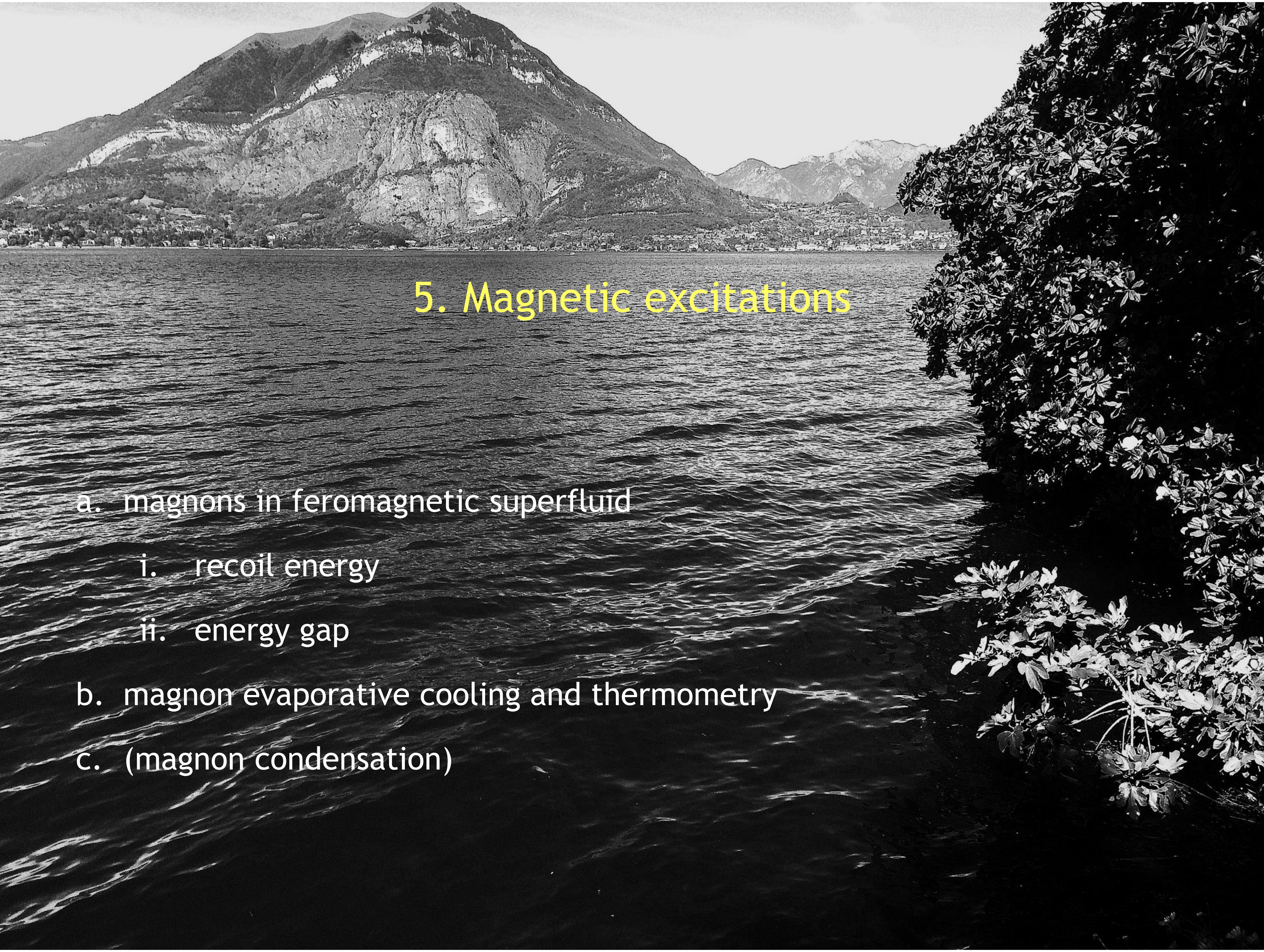
Spontaneously formed ferromagnetism

- inhomogeneously broken symmetry
- ferromagnetic domains, large and small
- unmagnetized domain walls marking rapid reorientation



Tuning the amplifier





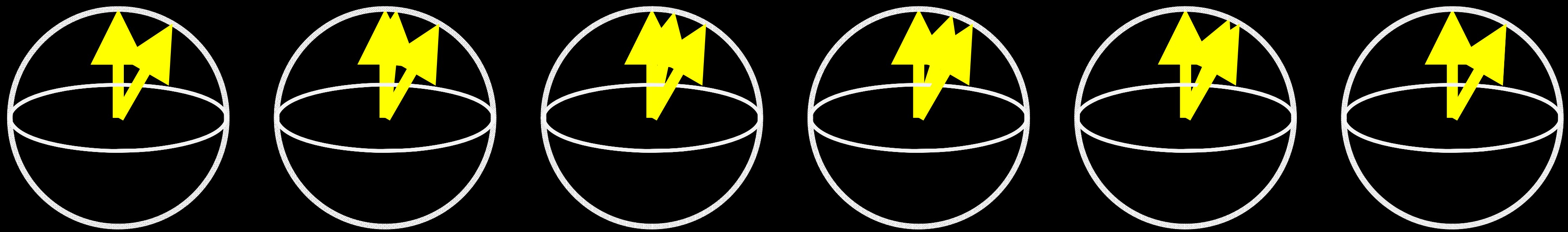
5. Magnetic excitations

a. magnons in feromagnetic superfluid
   i. recoil energy
   ii. energy gap
b. magnon evaporative cooling and thermometry
c. (magnon condensation)

Collective excitations of a ferromagnetic superfluid

- Symmetries of the order parameter space tell us something about excitations

e.g. ferromagnet:

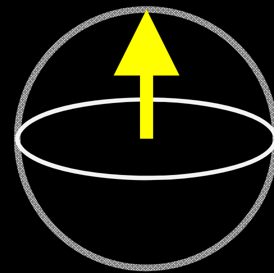


- uniform rotation costs no energy
- nearly uniform rotation costs nearly no energy

→ Gapless modes (Nambu-Goldstone) associated with each broken symmetry

Collective excitations of a ferromagnetic superfluid

→ Gapless modes associated with each broken symmetry



Generators of broken symmetries		Necessary mode
I, F_z	change of the condensate phase	phonon (linear disp.; "massless")
F_x	rotation of magnetization about x	magnon (quadratic disp.; "massive")
F_Y	rotation of magnetization about y	

non-zero expectation value of commutator gives one less mode + turns it quadratic, Watanabe and Murayama, PRL **108**, 251602 (2012)

Collective excitations of a ferromagnetic superfluid

- Is the magnetic excitation of a spinor Bose-Einstein condensate gapless?
- Is it quadratically dispersing?

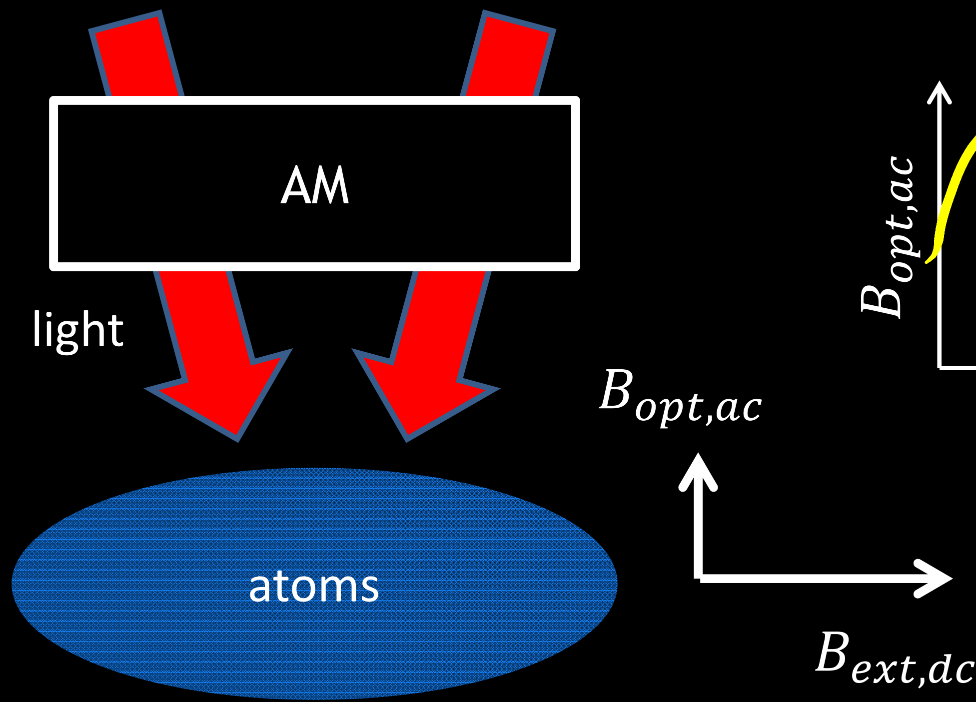
For atomic superfluid with weak, local interactions, expect:

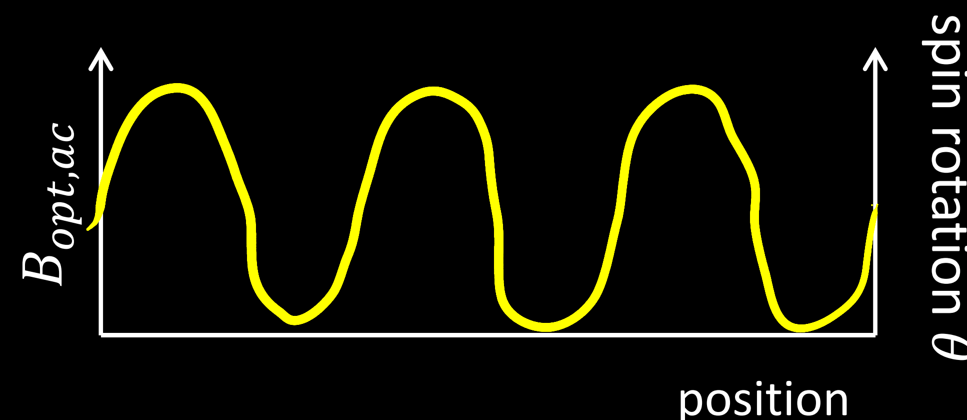
magnon mass = atomic mass $\times (1 + 0.6\sqrt{na^3}) = 1.003$

Phuc and Ueda, Annals of Physics **328**, 158 (2013)
(Beliaev theory)

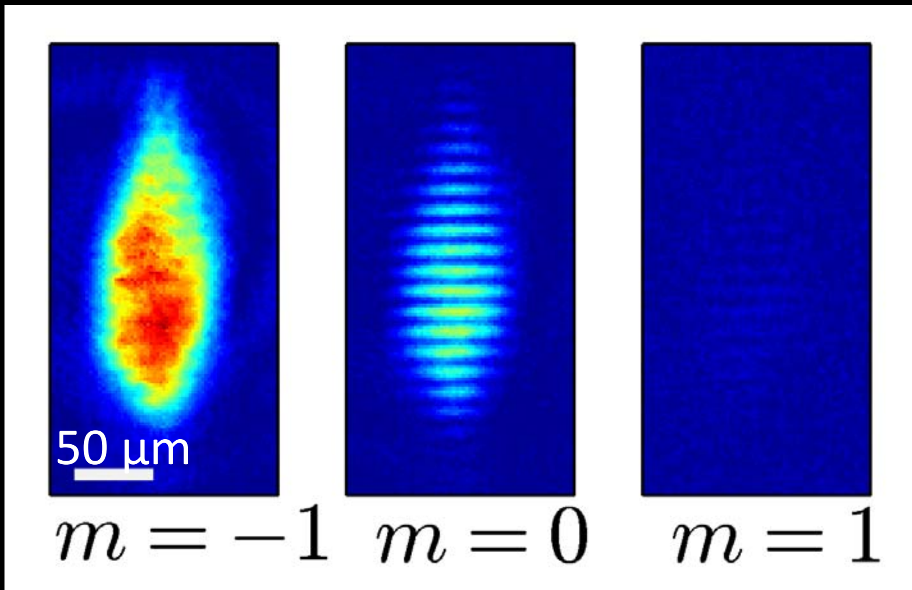
Magnon interferometry

- [87]Rb, F=1, optically trapped, spinor Bose-Einstein condensate, prepared in longitudinally polarized ($m_F = -1$), ferromagnetic state
- Circular polarized light at correct wavelength produces fictitious magnetic field [see Cohen-Tannoudji, Dupont-Roc, PRA **5**, 968 (1972)]
- Rabi-pulse produces wavepacket of magnons

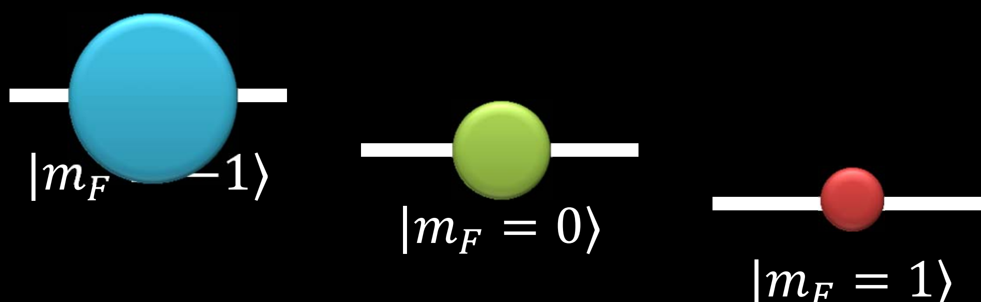




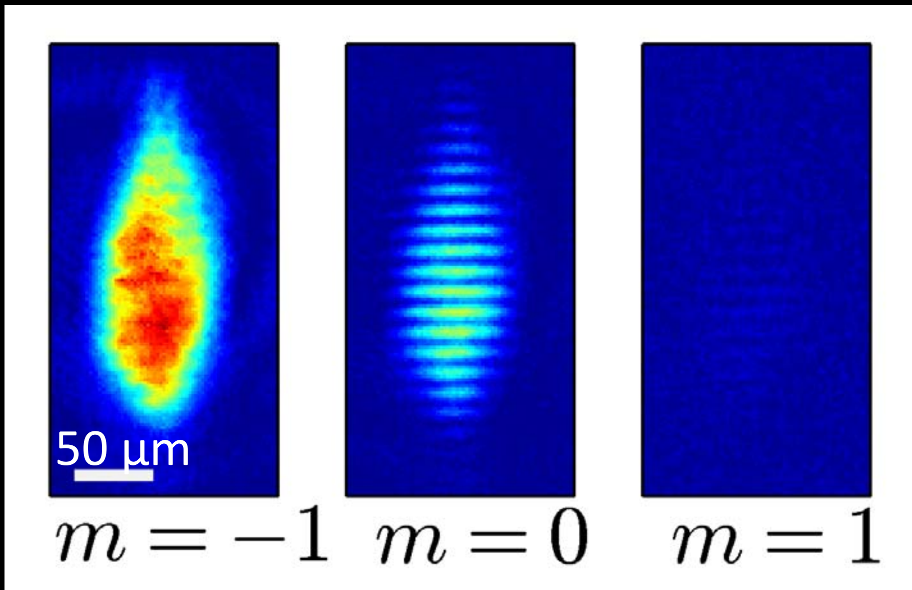
Imaging the magnon distribution (ASSISI)



$$\hat{R}(\theta(r), \phi(r)) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \simeq \begin{pmatrix} 0 \\ \theta e^{i\phi}/\sqrt{2} \\ 1 \end{pmatrix} + \mathcal{O}(\theta^2)$$

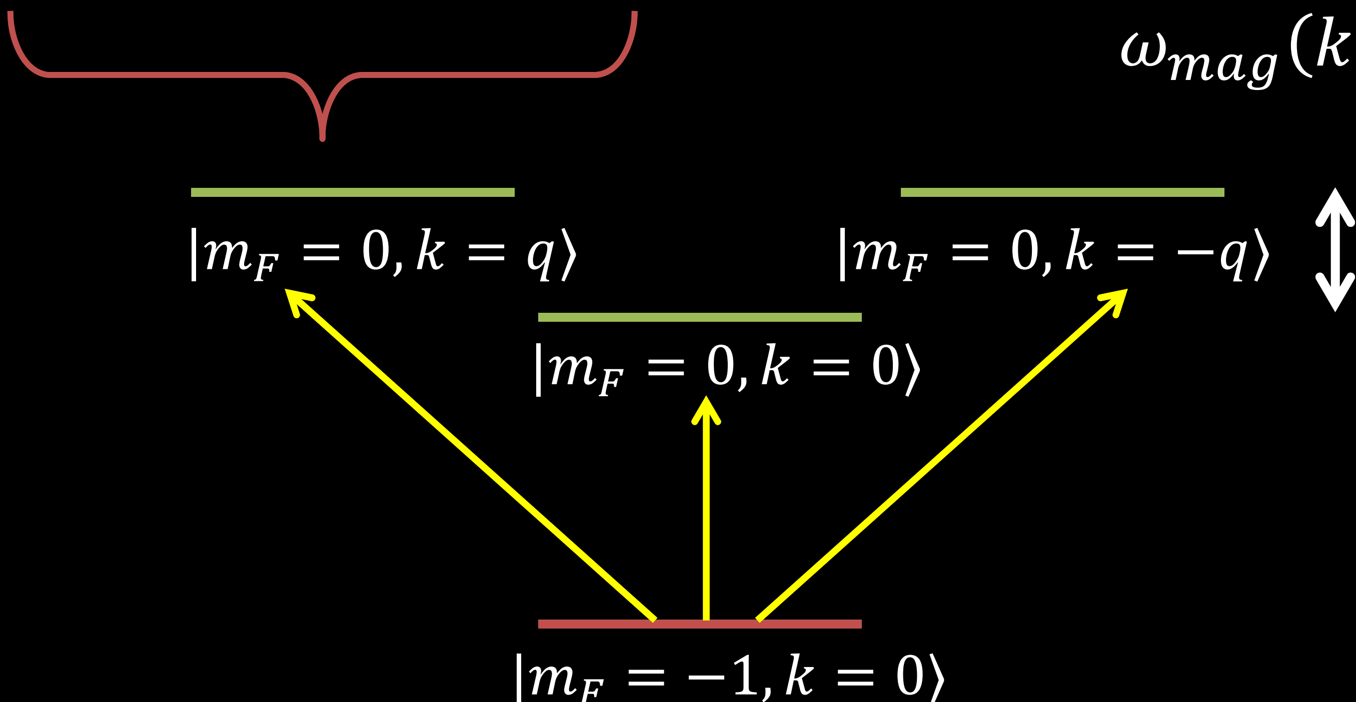


Magnon contrast interferometry

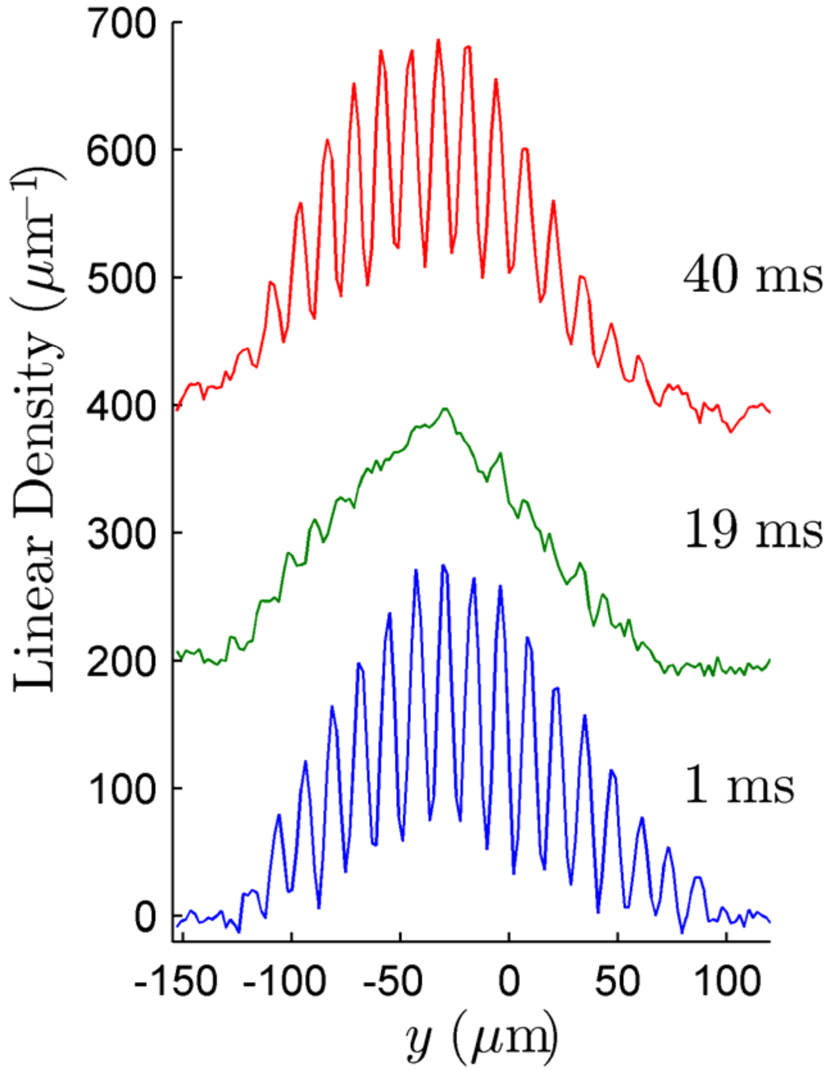


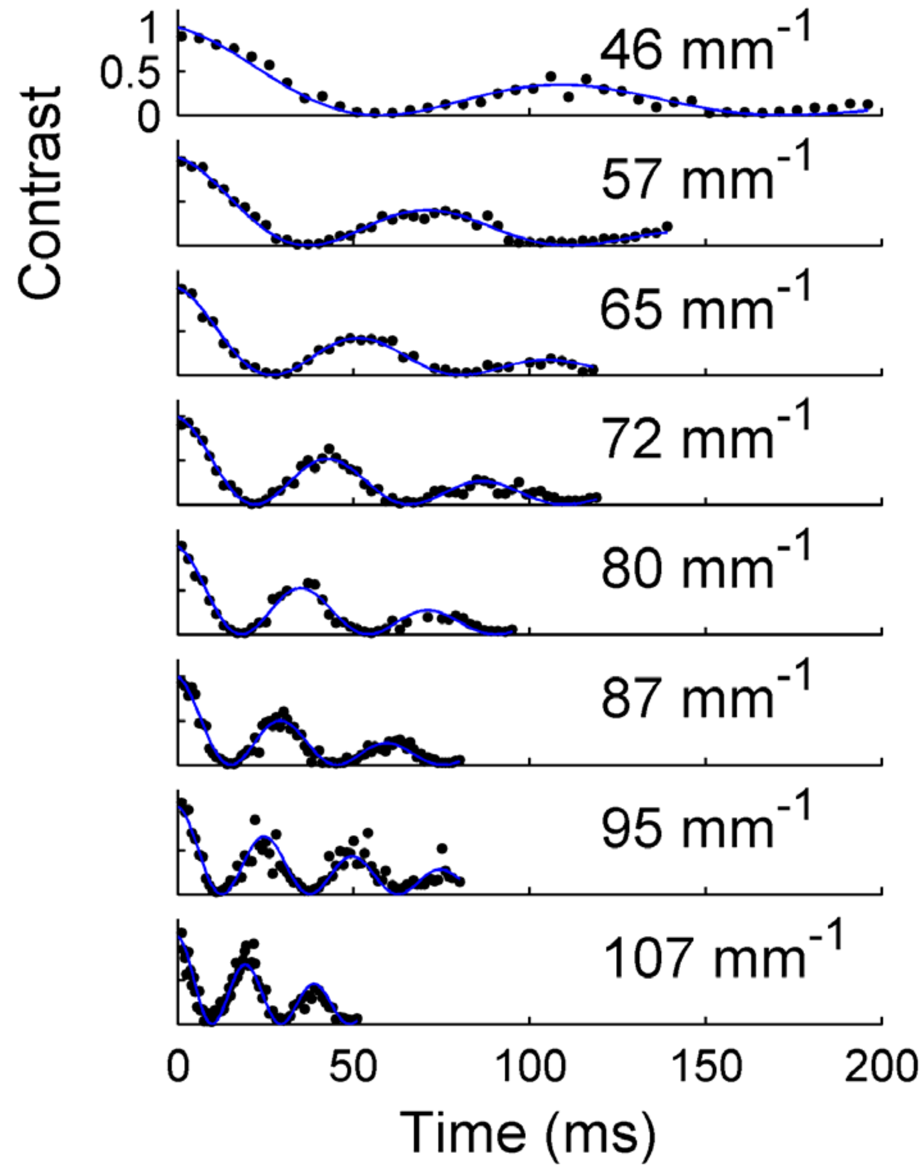
Interference contrast evolves
at the frequency difference

$$\omega_{mag}(k = \pm q) - \omega_{mag}(k = 0)$$



Magnon contrast interferometry



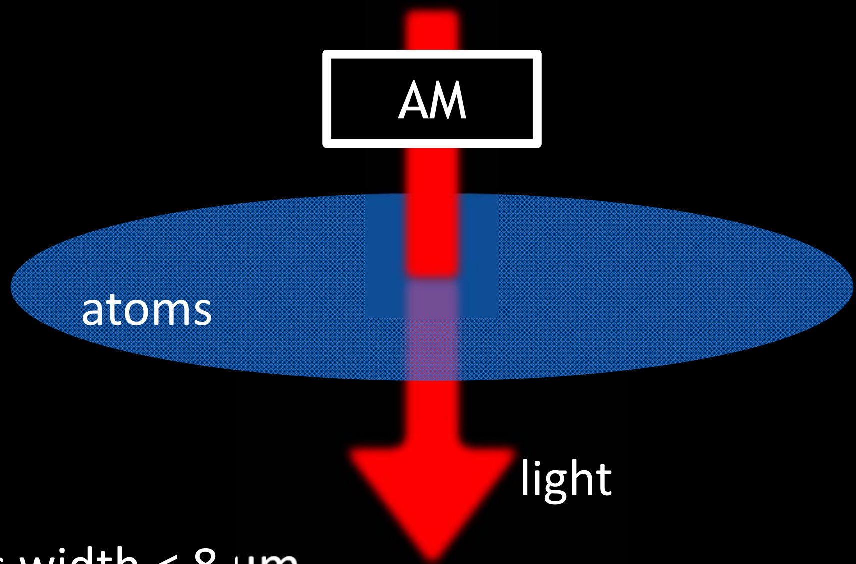


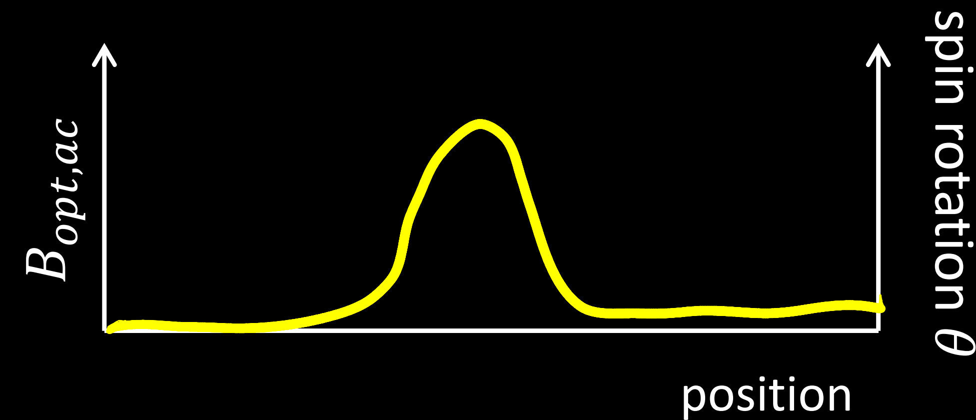
related works:

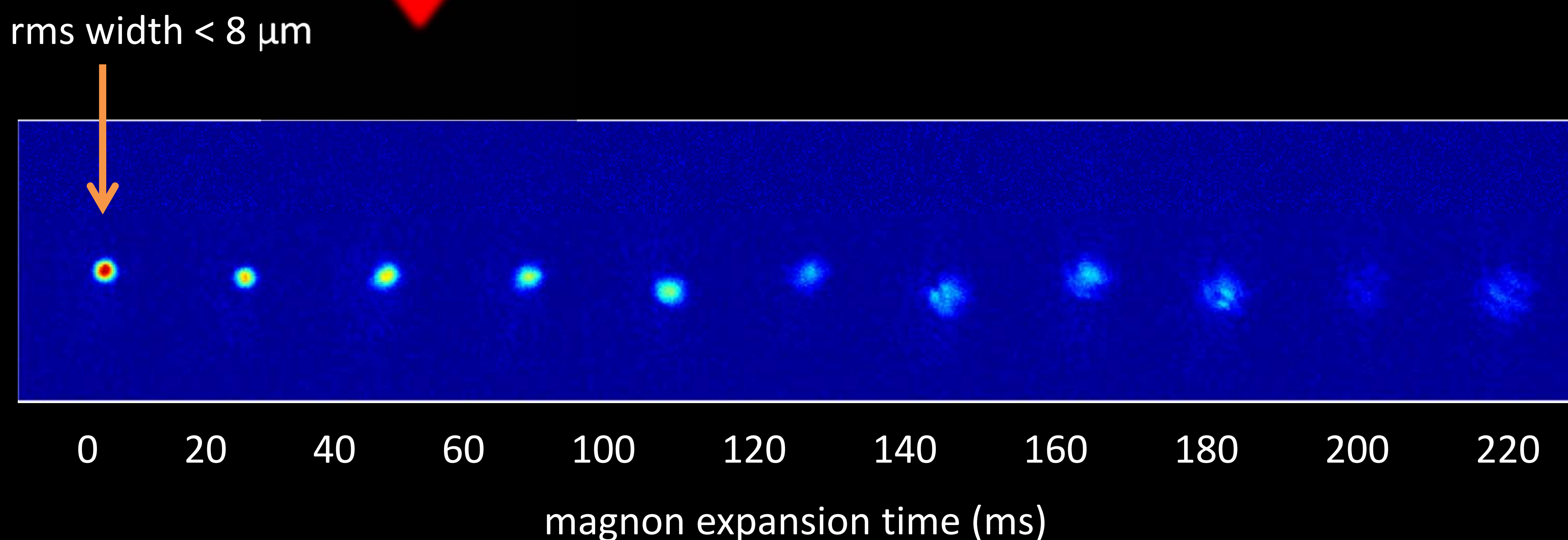
Gupta et al., PRL **89**, 140401 (2002): contrast interferometry in dilute scalar gas (interaction shift)

Gedik et al., Science **300**, 1410 (2003): grating interferometry of quasiparticles in cuprate SC

Freely expanding magnon pulse

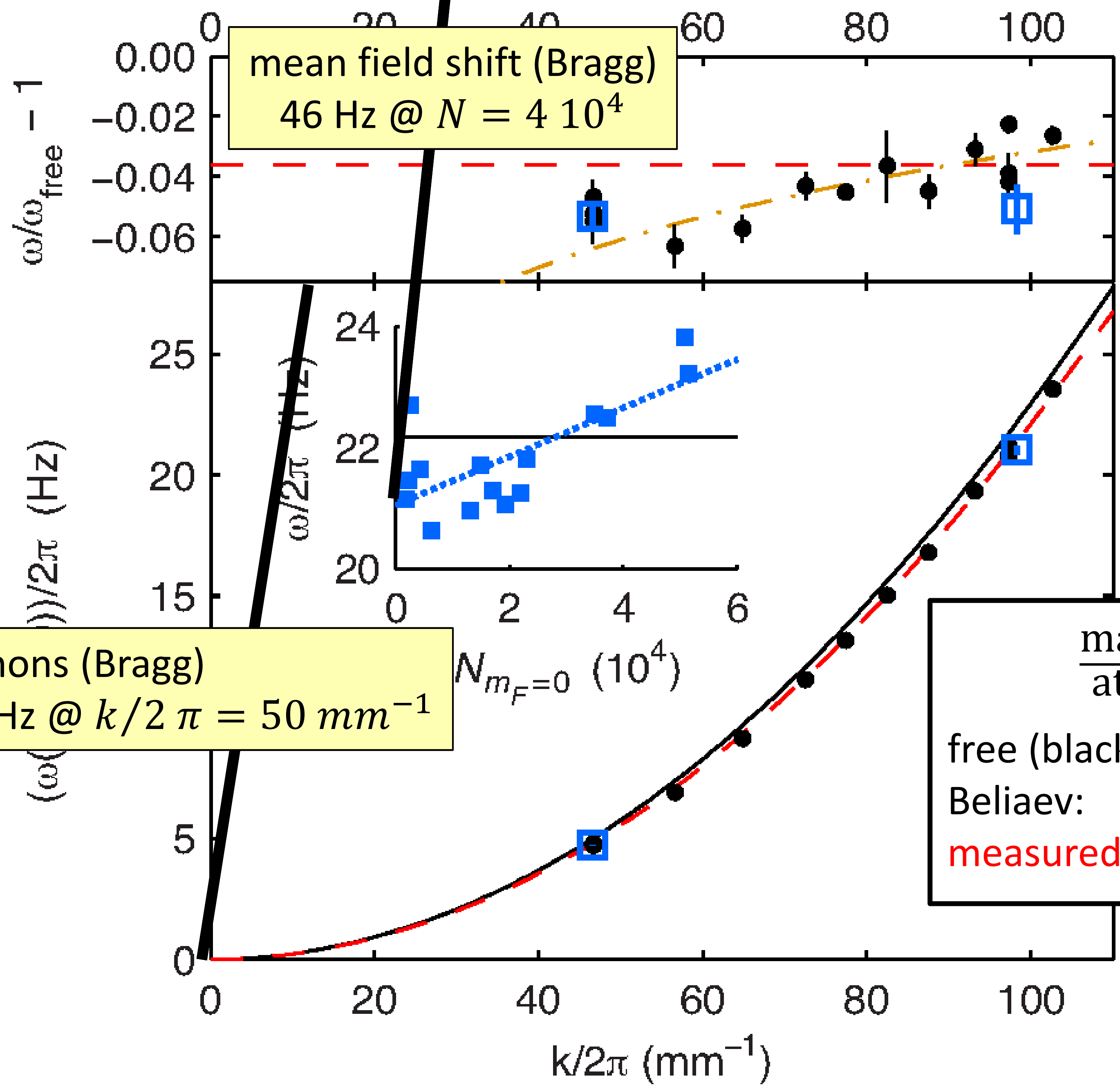






Magnons in dense ferromagnetic superfluid $\simeq$ free particles in potential free space

Dispersion of magnons in ferromagnetic superfluid



mean field shift (Bragg)
46 Hz @ $N = 4\ 10^4$

phonons (Bragg)
115 Hz @ $k/2\pi = 50\ mm^{-1}$

$\frac{\text{magnon mass}}{\text{atomic mass}}$	
free (black):	1
Beliaev:	1.003
measured (red):	1.038(2)(8)

skip to gap

Is the magnon a gapless excitation?

- Conditions of Nambu-Goldstone theorem:
  - Short-ranged interactions
  - Broken continuous symmetry

Clearly not!

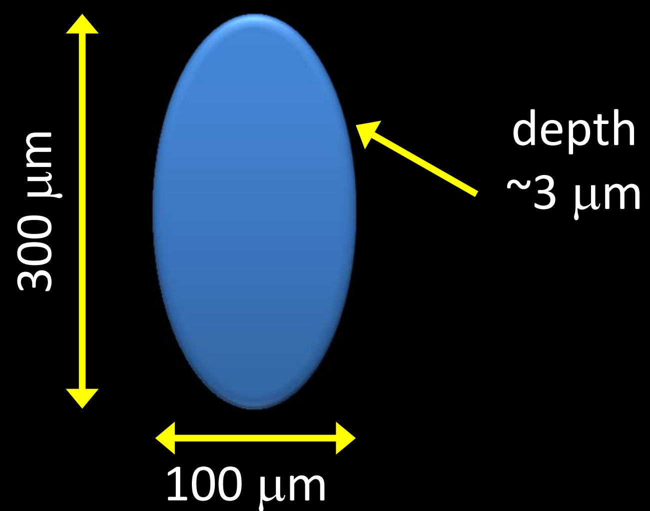
1. Applied magnetic bias field

$$\omega_{mag}(k=0) = \omega_{Larmor}$$

(perhaps gauged away in rotating frame?)

2. Anisotropic trap

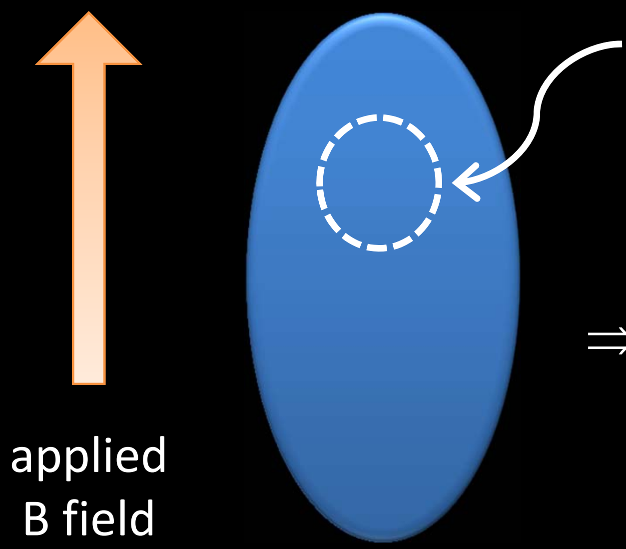
geometry
≈ surfboard



× Magnetic dipolar interactions

Measurement of the magnon gap energy

theoretical suggestion: Kawaguchi, Saito, Ueda, PRL 98, 110406 (2007)



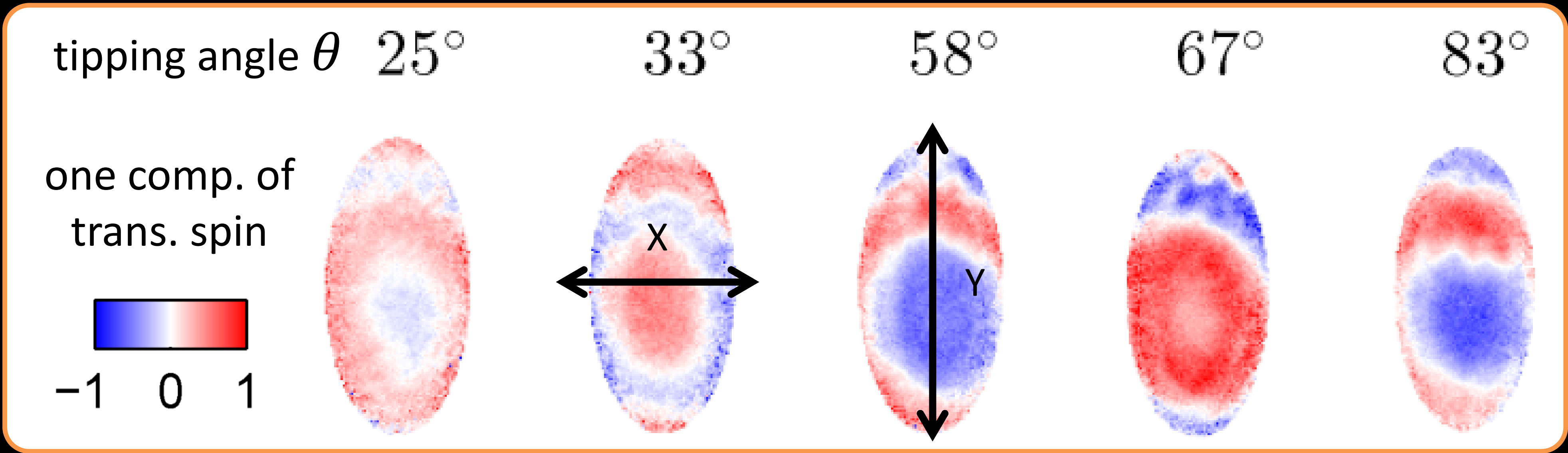
$\approx$ infinite 2D slab at local density $n(x, y)$

Dipolar field isn't proportional to $\vec{\mu}$

$\Rightarrow$ Dipolar Larmor frequency shift $\propto n(x, y) \cos\theta$

- detect on top of constant B-field inhomogeneity by spatially resolved, spin-sensitive imaging
  - Only dipolar shift varies with tipping angle θ

Measurement of the magnon gap energy



Larmor phase:

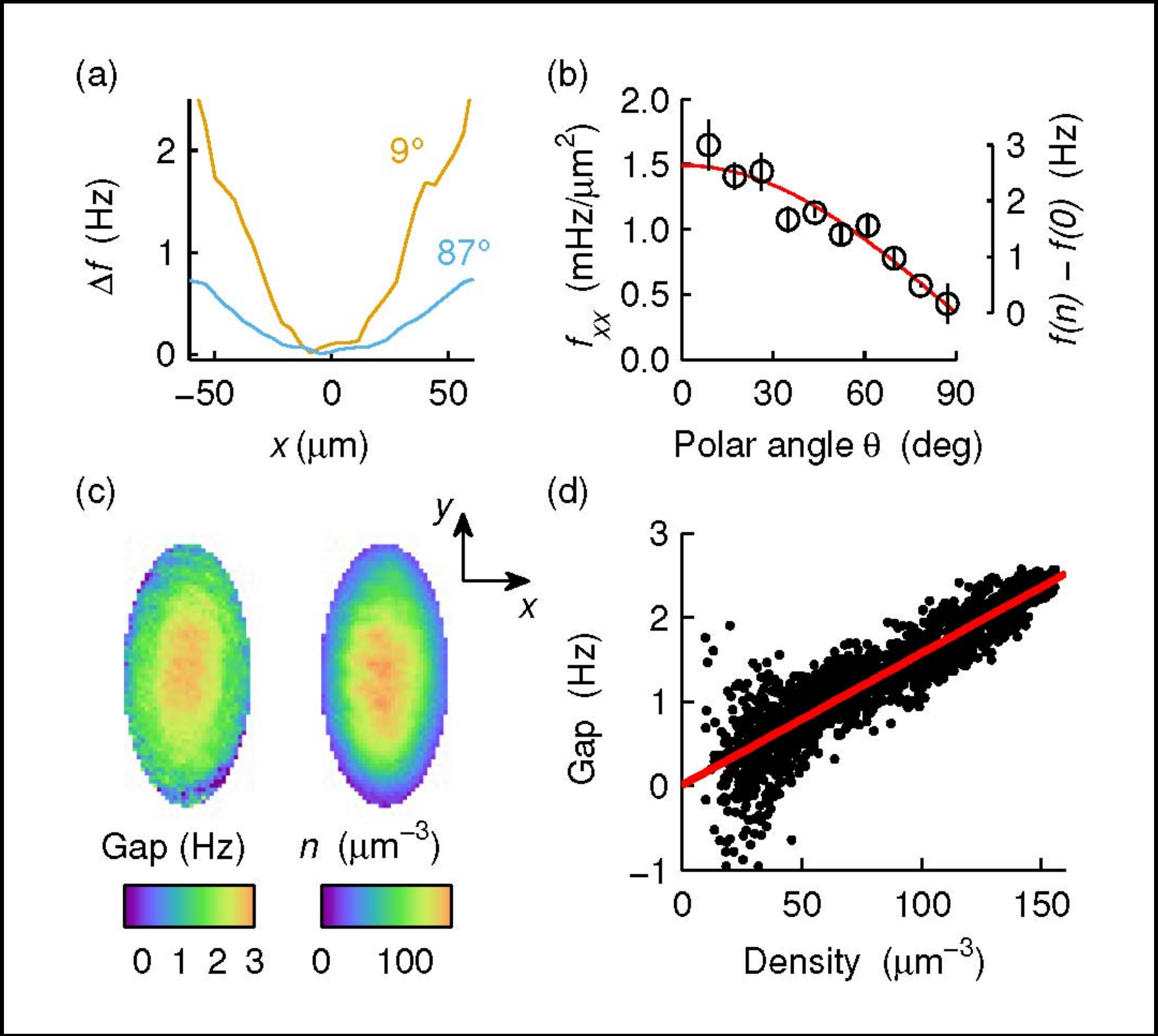
$$\Phi(x,y) = \Phi_i + \Phi_{ext}(x,y) + \Phi_{dipole}(x,y)\cos\theta$$

random from shot to shot due to field fluctuations

from static B-field inhomogeneity (gradient and curvature)

magnon gap: isolate from dependence on tipping angle

Magnon gap map



“Coherent magnon optics in a ferromagnetic spinor Bose-Einstein condensate,” arXiv: 1404.5631

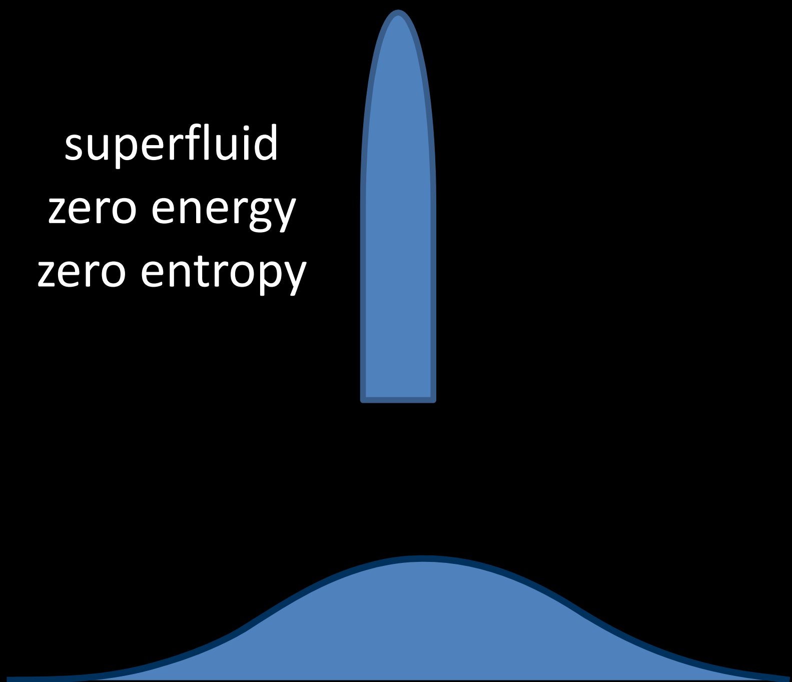
Magnon evaporative cooling of a ferromagnet

1) start with fully magnetized degenerate Bose gas

(ignore for clarity)

$|m_z = -1\rangle$ $|m_z = 0\rangle$ $|m_z = 1\rangle$

superfluid
zero energy
zero entropy



normal

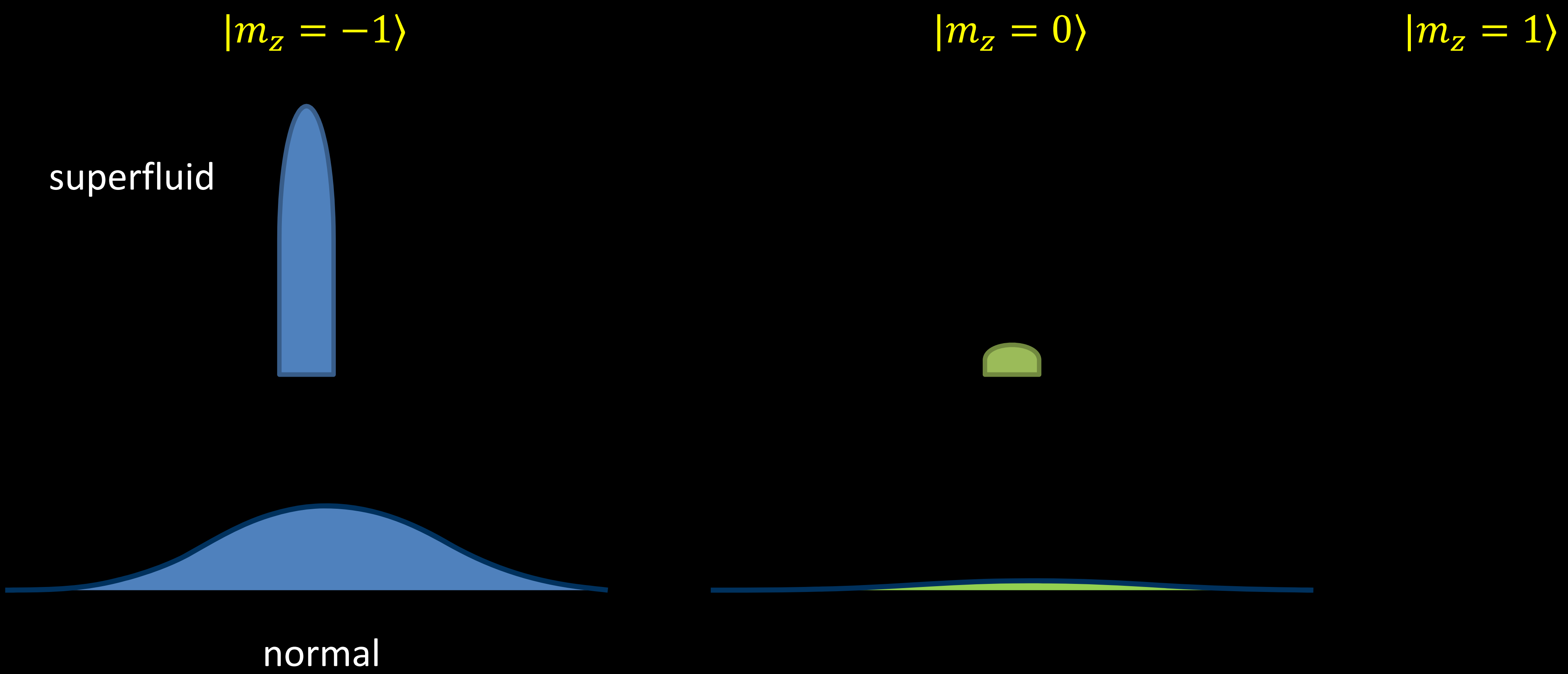
$E \sim N_{th}(T_i) \times 3\, k_B T_i$

$S \sim N_{th}(T_i) \times k_B$

Magnon evaporative cooling of a ferromagnet

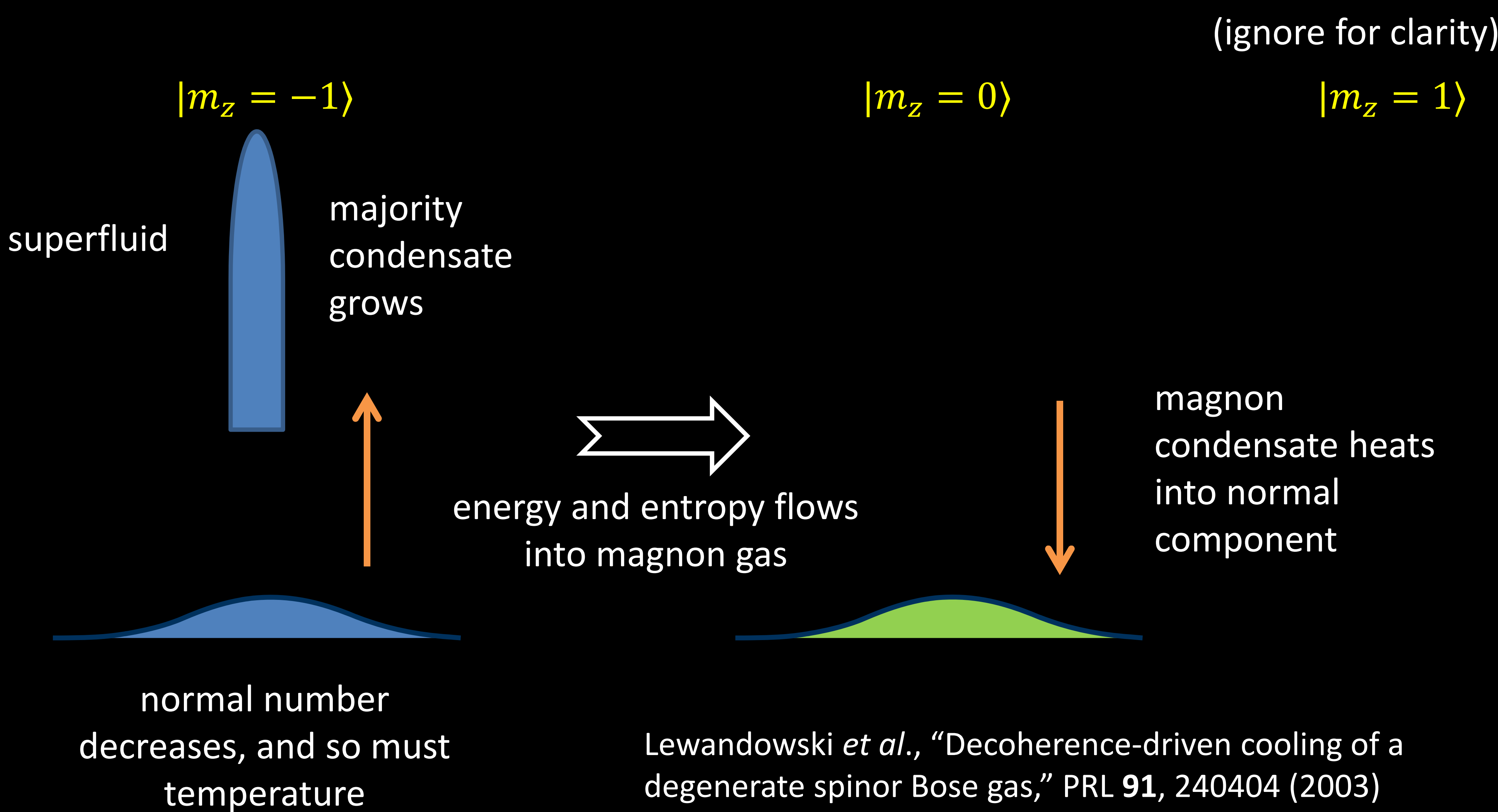
1) start with fully magnetized degenerate Bose gas
2) tip magnetization by small angle

(ignore for clarity)



Magnon evaporative cooling of a ferromagnet

1) start with fully magnetized degenerate Bose gas
2) tip magnetization by small angle
3) thermalize at constant energy, magnetization



Lewandowski *et al.*, "Decoherence-driven cooling of a degenerate spinor Bose gas," PRL **91**, 240404 (2003)

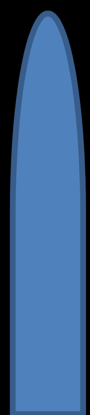
Magnon evaporative cooling of a ferromagnet

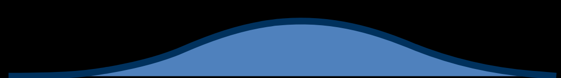
1) start with fully magnetized degenerate Bose gas
2) tip magnetization by small angle
3) thermalize at constant energy, magnetization
4) eject magnons

(ignore for clarity)

$|m_z = -1\rangle$ $|m_z = 0\rangle$ $|m_z = 1\rangle$

superfluid





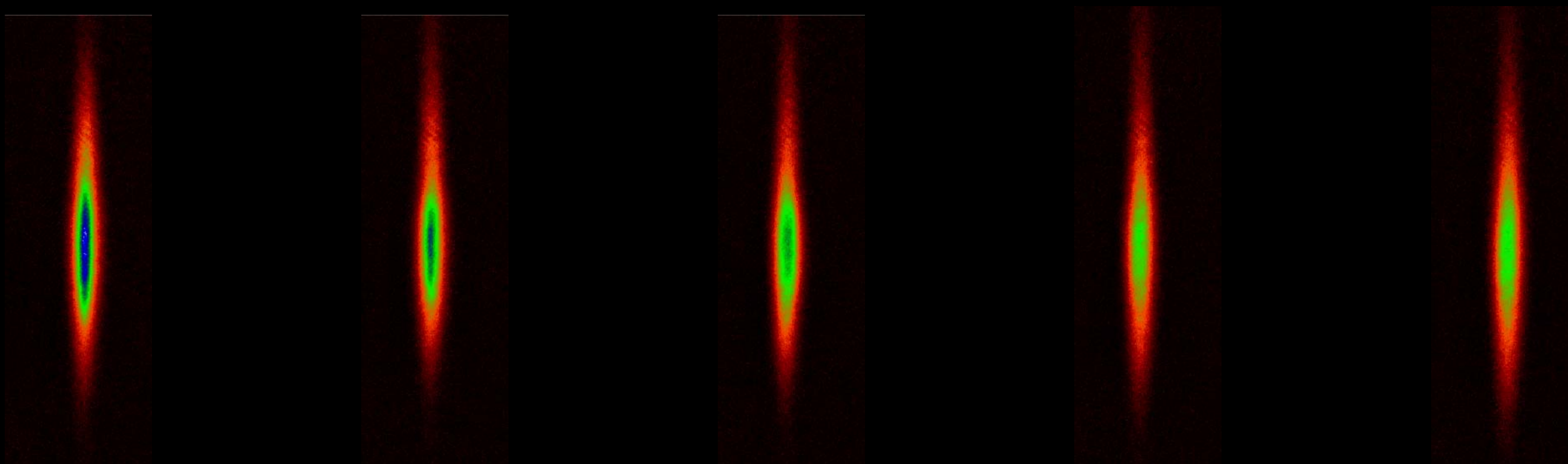
temperature reduced, entropy per particle reduced

"evaporated" atoms carry $E/N = 3\ k_B T_f$, greater than average

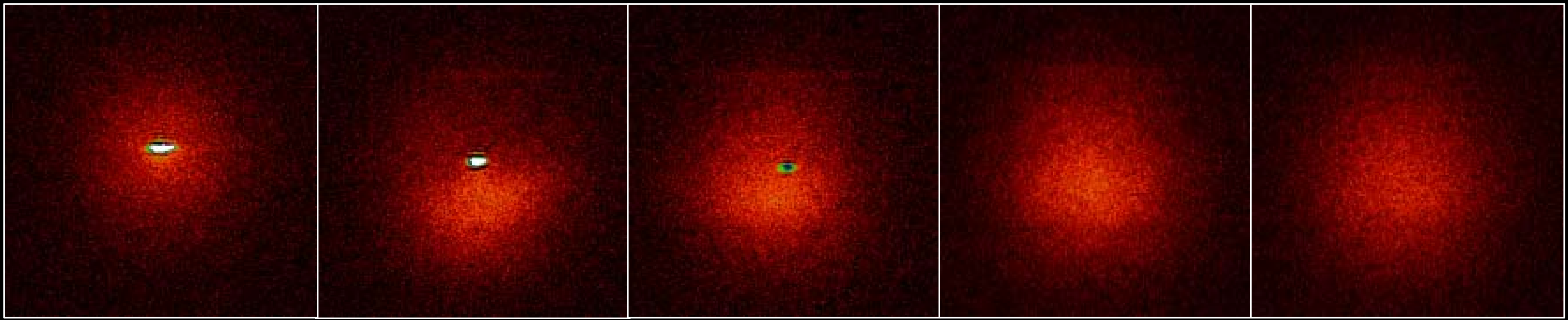
Magnon thermalization and thermometry

Create small fraction of magnons, apply weak field gradient, wait

in-situ magnon distribution

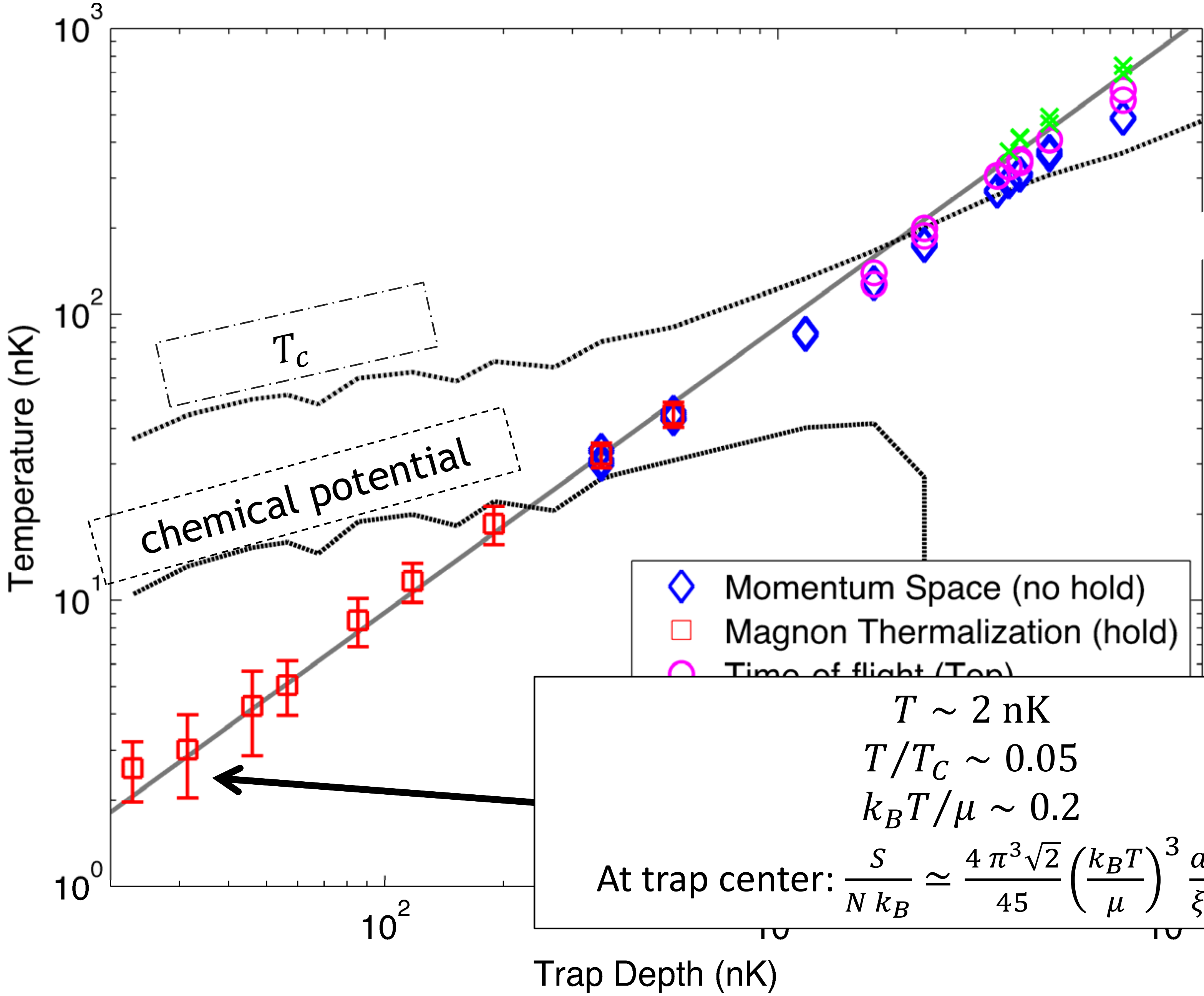


momentum space distribution (magnetic focusing)



0 ms wait 40 ms wait

Magnon thermometry of normal evaporative cooling



$$T \sim 2 \text{ nK}$$
$$T/T_C \sim 0.05$$
$$k_B T/\mu \sim 0.2$$

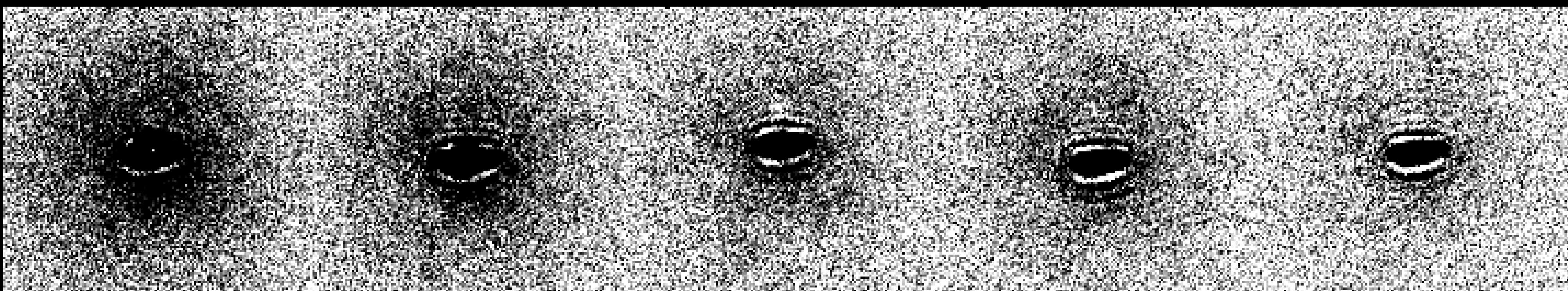
At trap center: $\frac{S}{N\,k_B} \simeq \frac{4\,\pi^3\sqrt{2}}{45}\left(\frac{k_B T}{\mu}\right)^3 \frac{a}{\xi} \simeq 10^{-4}$

Magnon evaporative cooling in deep trap

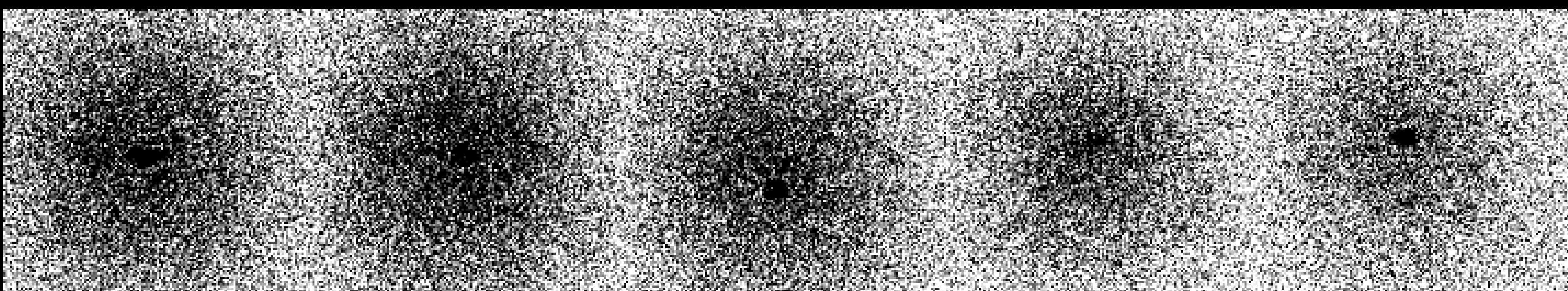
Optical trap depth: 740 nK

Create fraction of magnons, apply weak field gradient, wait, eject, repeat

momentum space distribution of majority atoms



momentum space distribution of magnons



1 cycle
T=54 nK
T/Tc = 0.52

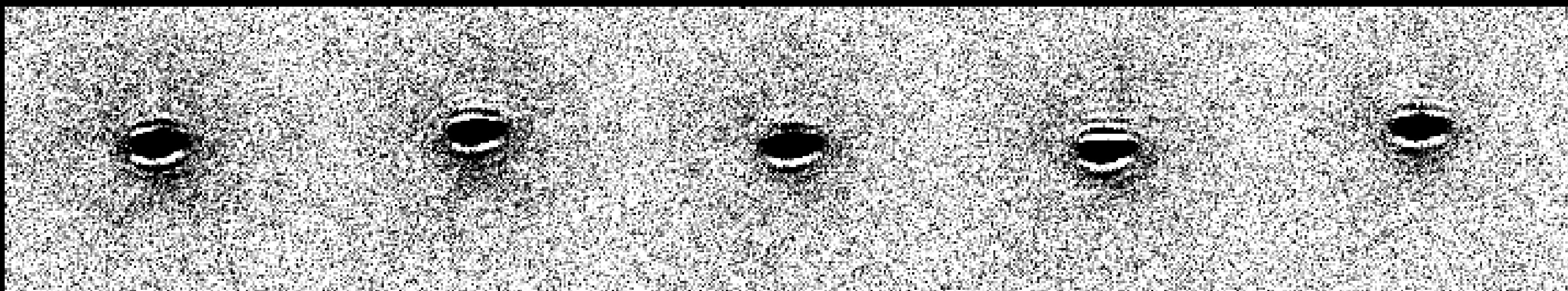
9 cycles
T=38 nK
T/Tc = 0.45

Magnon evaporative cooling in deep trap

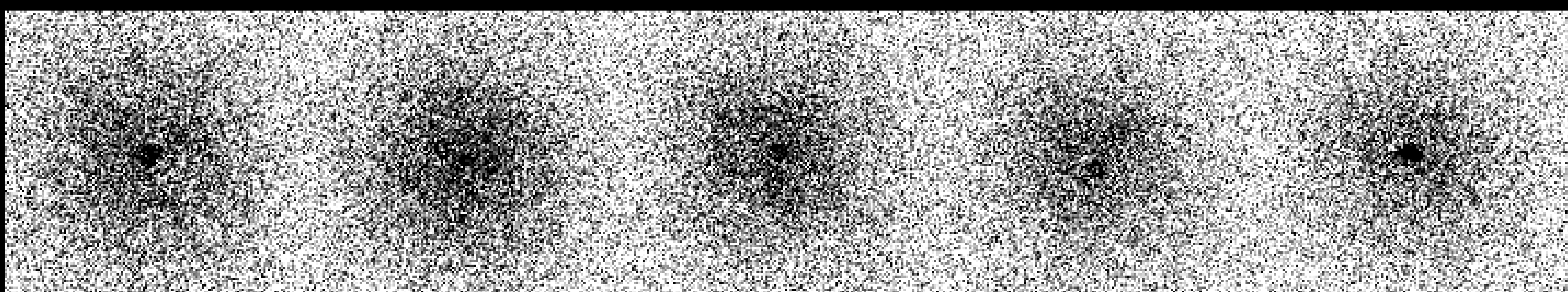
Optical trap depth: 740 nK

Create fraction of magnons, apply weak field gradient, wait, eject, repeat

momentum space distribution of majority atoms



momentum space distribution of magnons

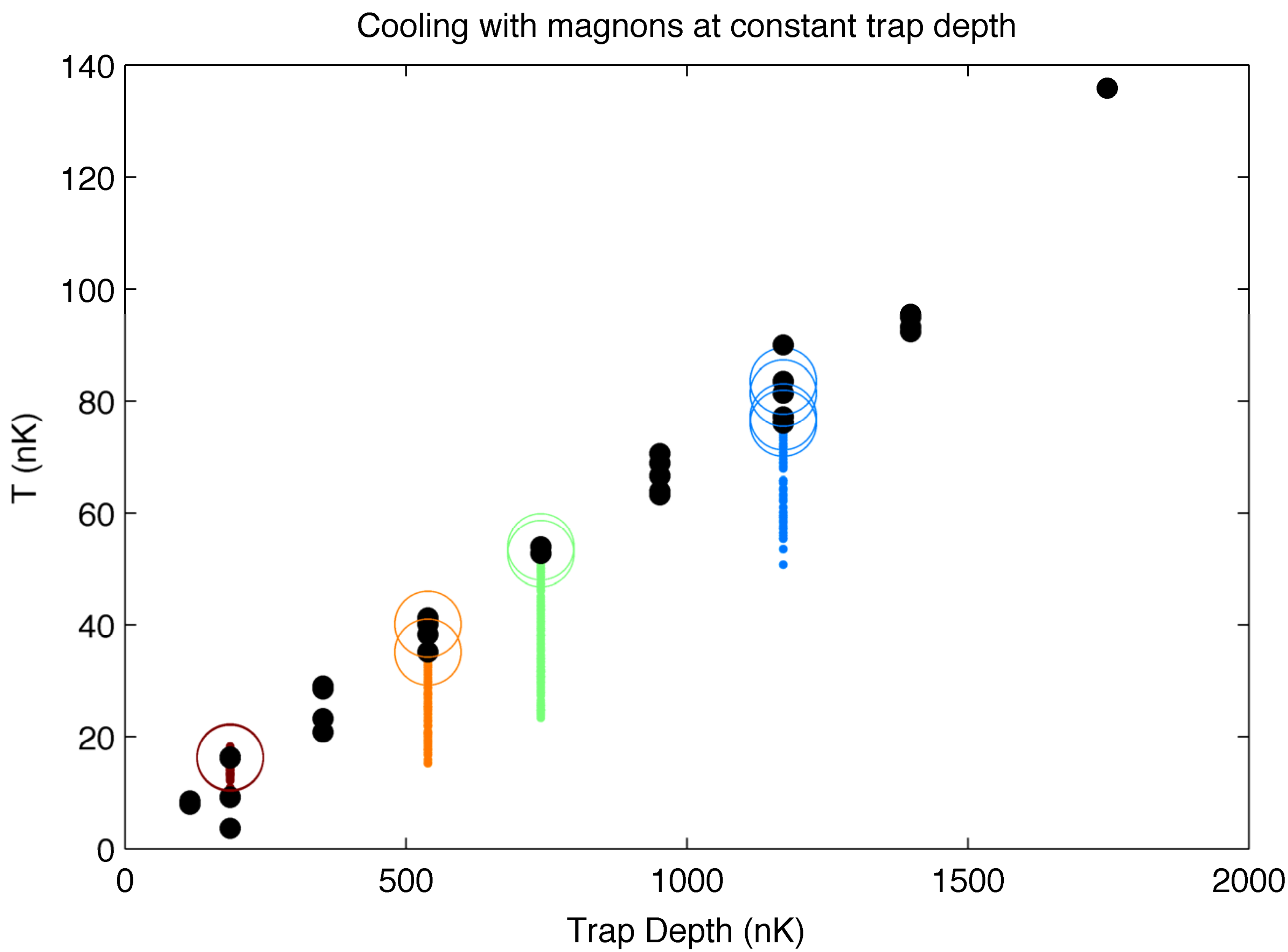


11 cycles
T=36 nK
T/Tc = 0.43

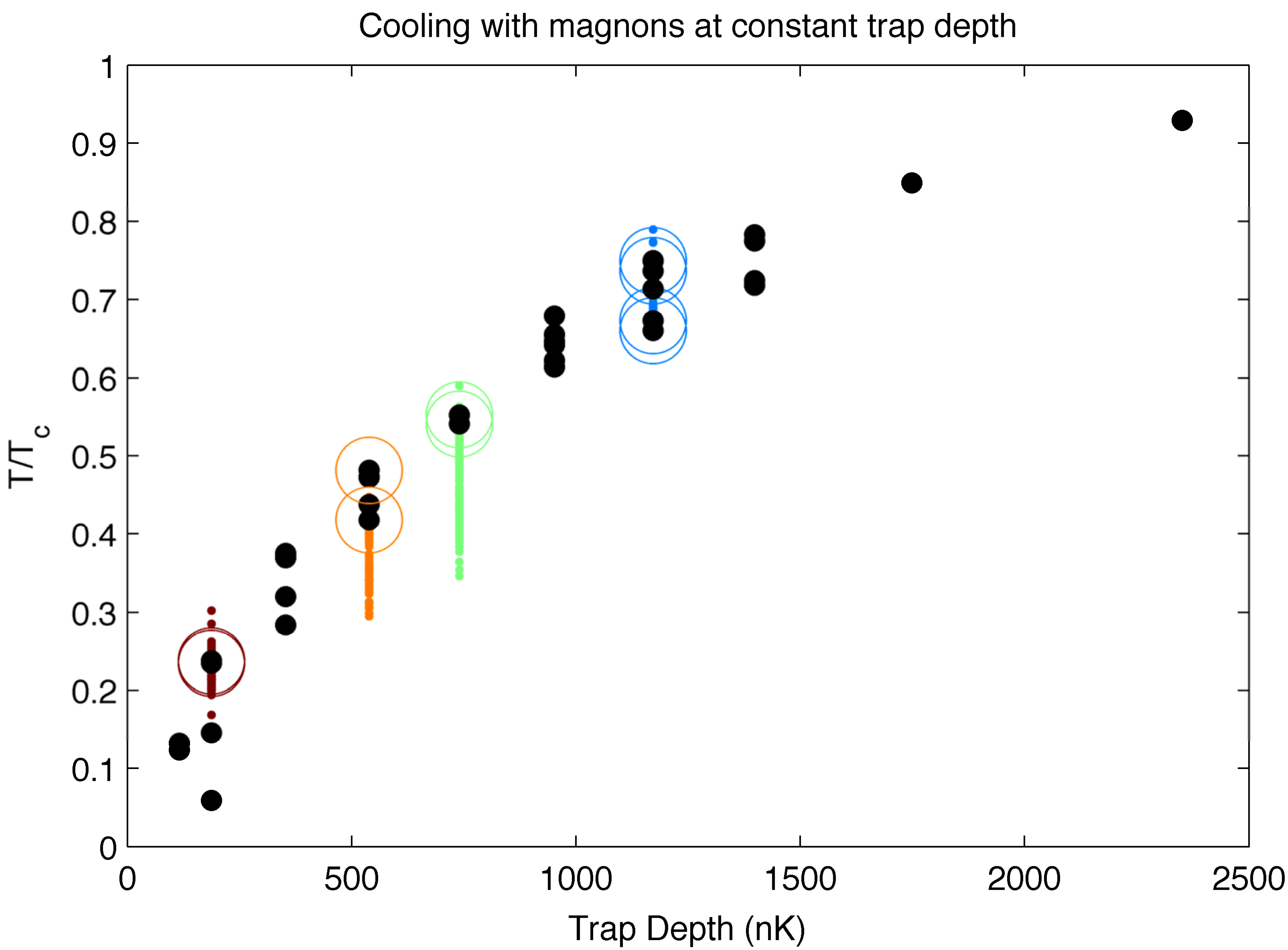
$$\eta = \frac{\text{trap depth}}{k_B T} = 27$$

19 cycles
T=27 nK
T/Tc = 0.40

Magnon cooling results (at present)



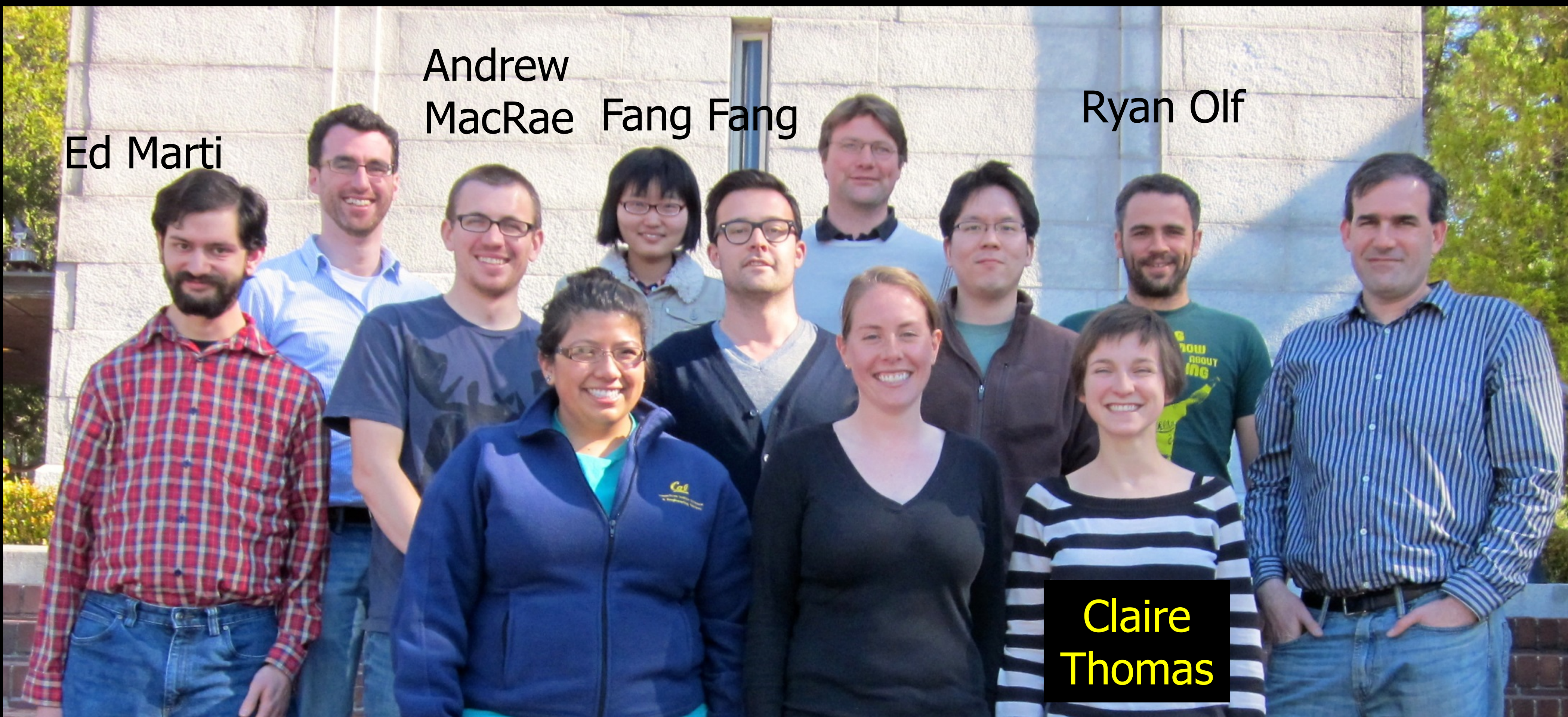
Magnon cooling results (at present)



Other things to ask me about:

a. quantum-limited metrology: cavity optomechanics and photon mediated interactions
b. triangular/kagome lattice experiments (see Claire Thomas)
c. rotation sensing with a ring-shaped Bose-Einstein condensate
d. looking for excellent postdocs and students

$$ Thanks! to NSF, DTRA, DARPA OLE

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