

# *Characterization of ultrashort pulses*

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Max-Born-Institute

***Enrico Fermi Summer School***

**Varenna, Italy**  
***Saturday, July 5, 2014***

# Overview

- **Fundamental Definitions, electric field, SVEA**
- **Classical methods: autocorrelation and Decorrelation**
- **Frequency-resolved optical gating (FROG)**
- **Spectral phase interferometry for direct electric field reconstruction (SPIDER)**
- **MIIPS, d-scan, etc.**
- **The coherent artifact**
- **Carrier-envelope phase**



# Characterization of short light pulses

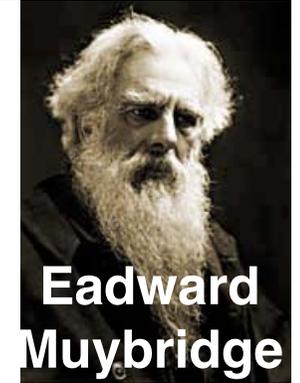
- ◆ **generally:** How can I measure events on femtosecond time scales?
- ◆ How can I characterize the shortest event ever?
- ◆ **Methods:**
  - Autocorrelation
  - Tomography-like methods (FROG)
  - Interferometry-based methods (SPIDER)
- ◆ **Limits:** The coherent artifact
- ◆ **Limits:** The carrier-envelope phase

# Measuring on short time scales

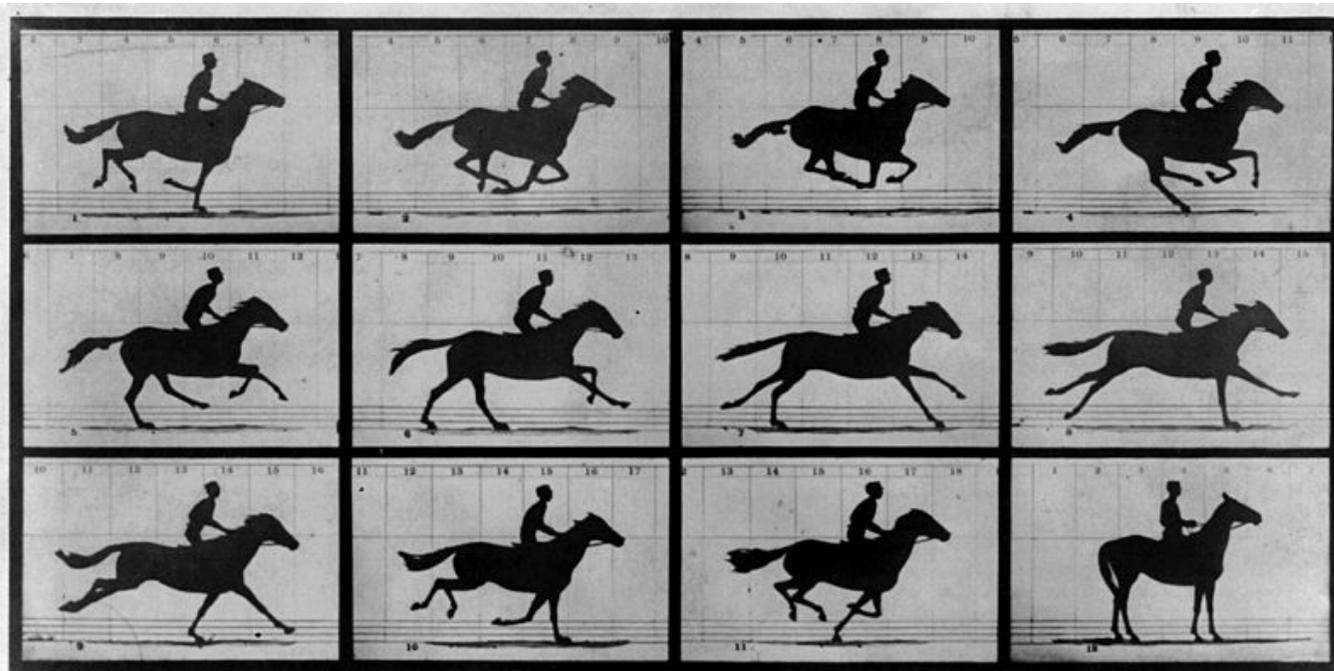
## Electro-mechanical methods

⇒ electrically triggered mechanical shutters

⇒ 1/1000s temporal shutters



**Eadward  
Muybridge**  
(1830-1904)



Copyright, 1878, by MUYBRIDGE.

MORSE'S Gallery, 417 Montgomery St., San Francisco.

## THE HORSE IN MOTION.

Illustrated by  
MUYBRIDGE.

AUTOMATIC ELECTRO-PHOTOGRAPH.

"SALLIE GARDNER," owned by LELAND STANFORD; running at a 1.40 gait over the Palo Alto track, 19th June, 1878.

The negatives of these photographs were made at intervals of twenty-seven inches of distance, and about the twenty-fifth part of a second of time; they illustrate consecutive positions assumed in each twenty-seven inches of progress during a single stride of the mare. The vertical lines were twenty-seven inches apart; the horizontal lines represent elevations of four inches each. The exposure of each negative was less than the two-thousandth part of a second.

**First movie ever  
Horse: Sallie Gardner  
June 19, 1878**

**12 trip wires and  
cameras**

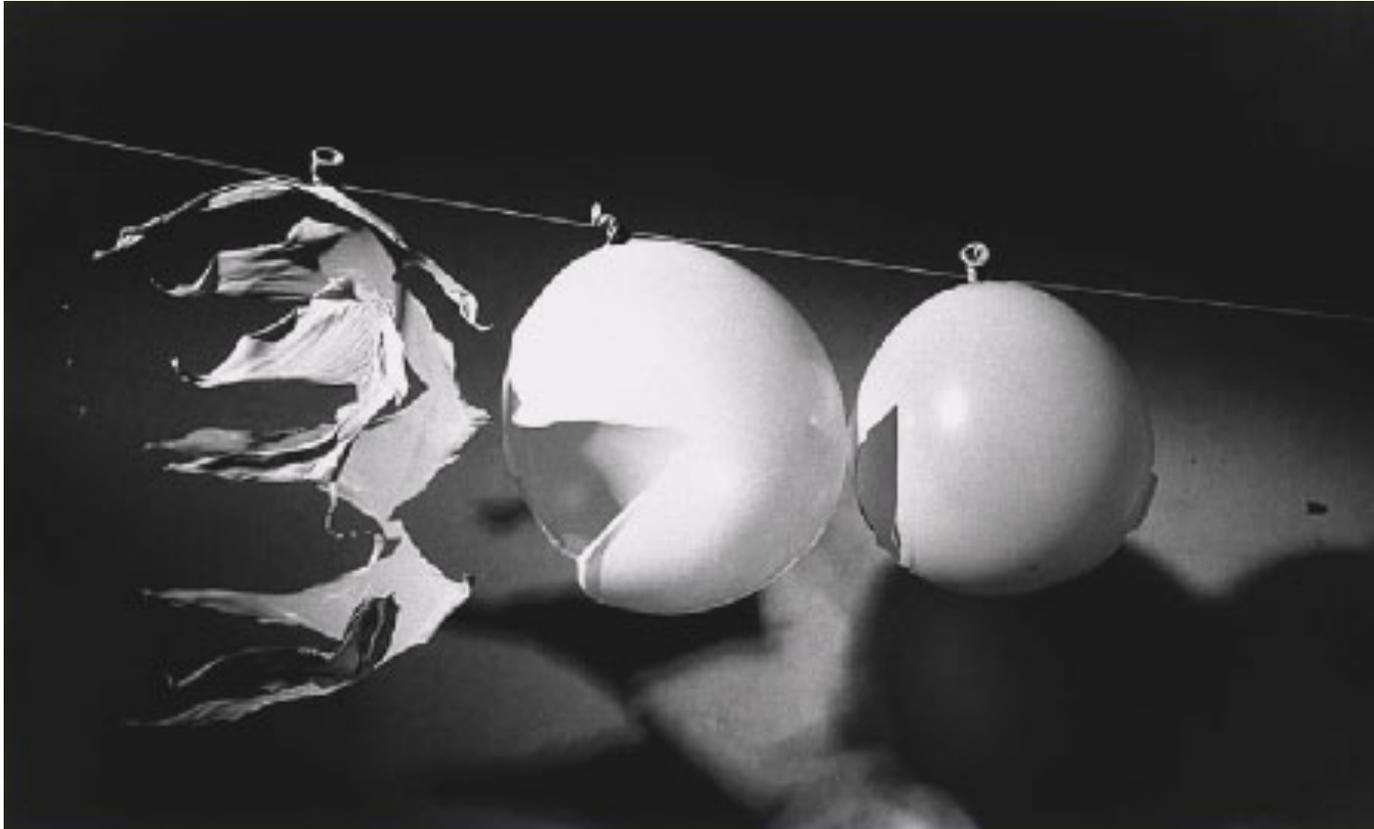
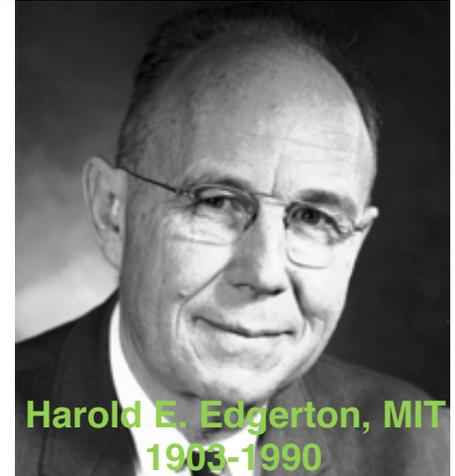


# Microsecond time scales

## Elektronics

⇒ triggered flash lamps

⇒  $< \mu\text{s}$  temporal resolution

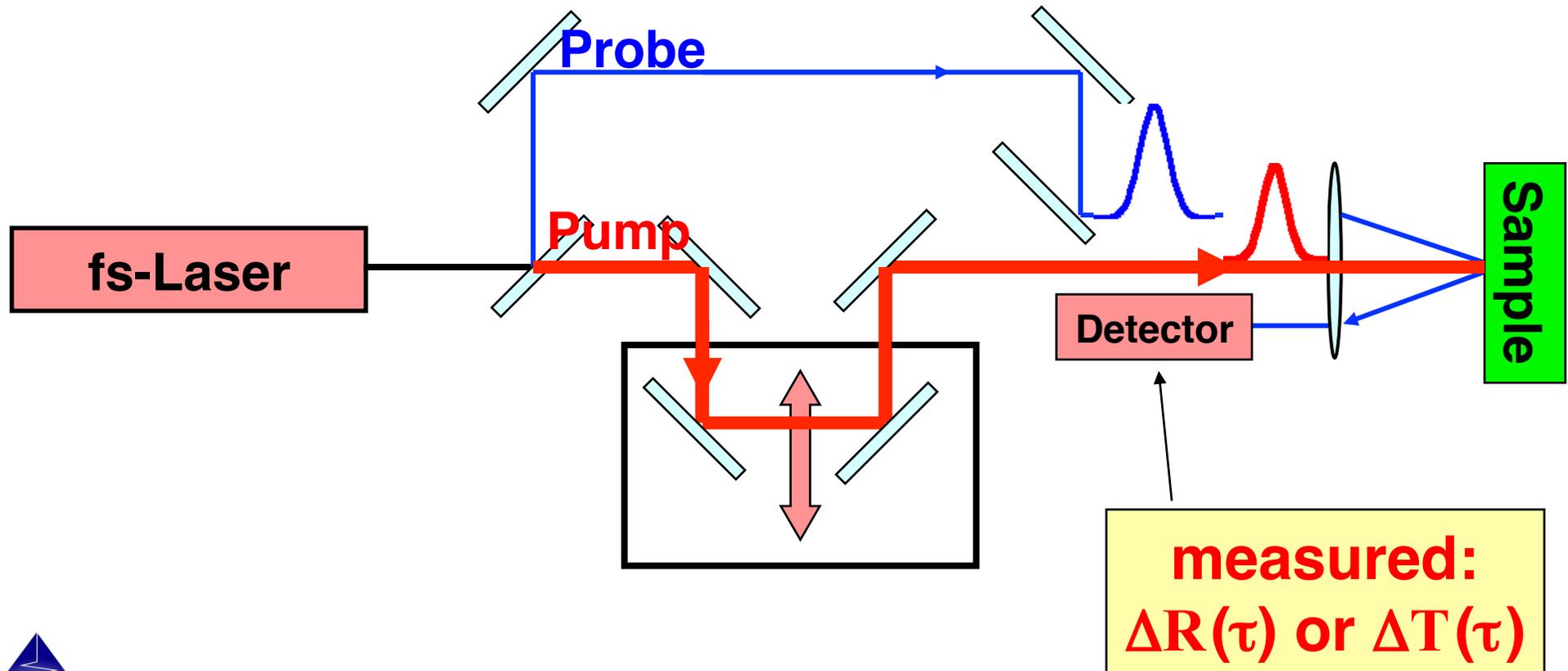


# Stroboscopy with femtosecond lasers

## Photonics

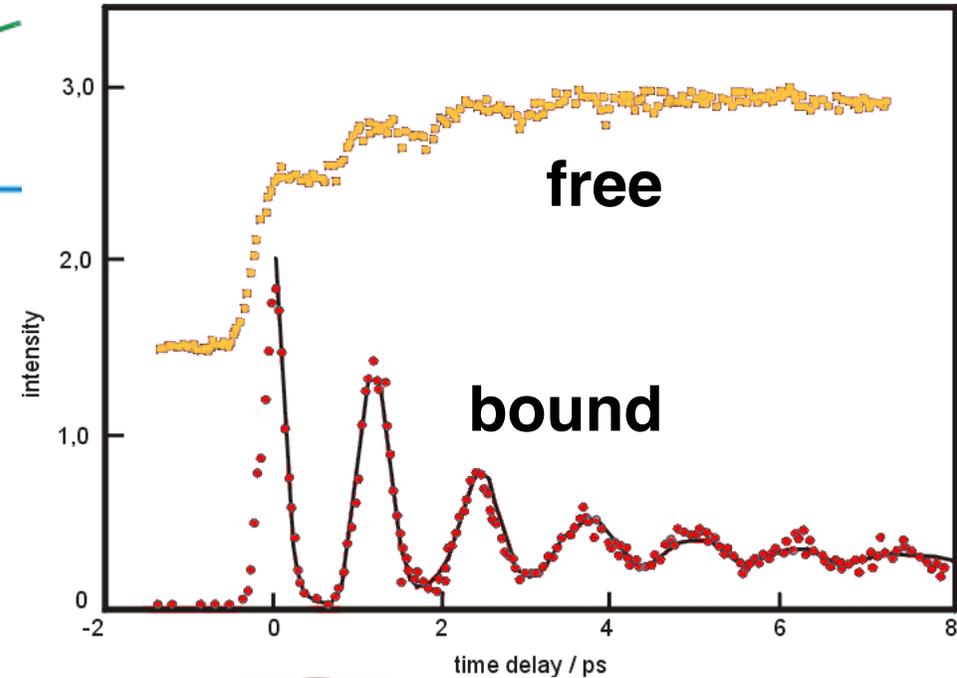
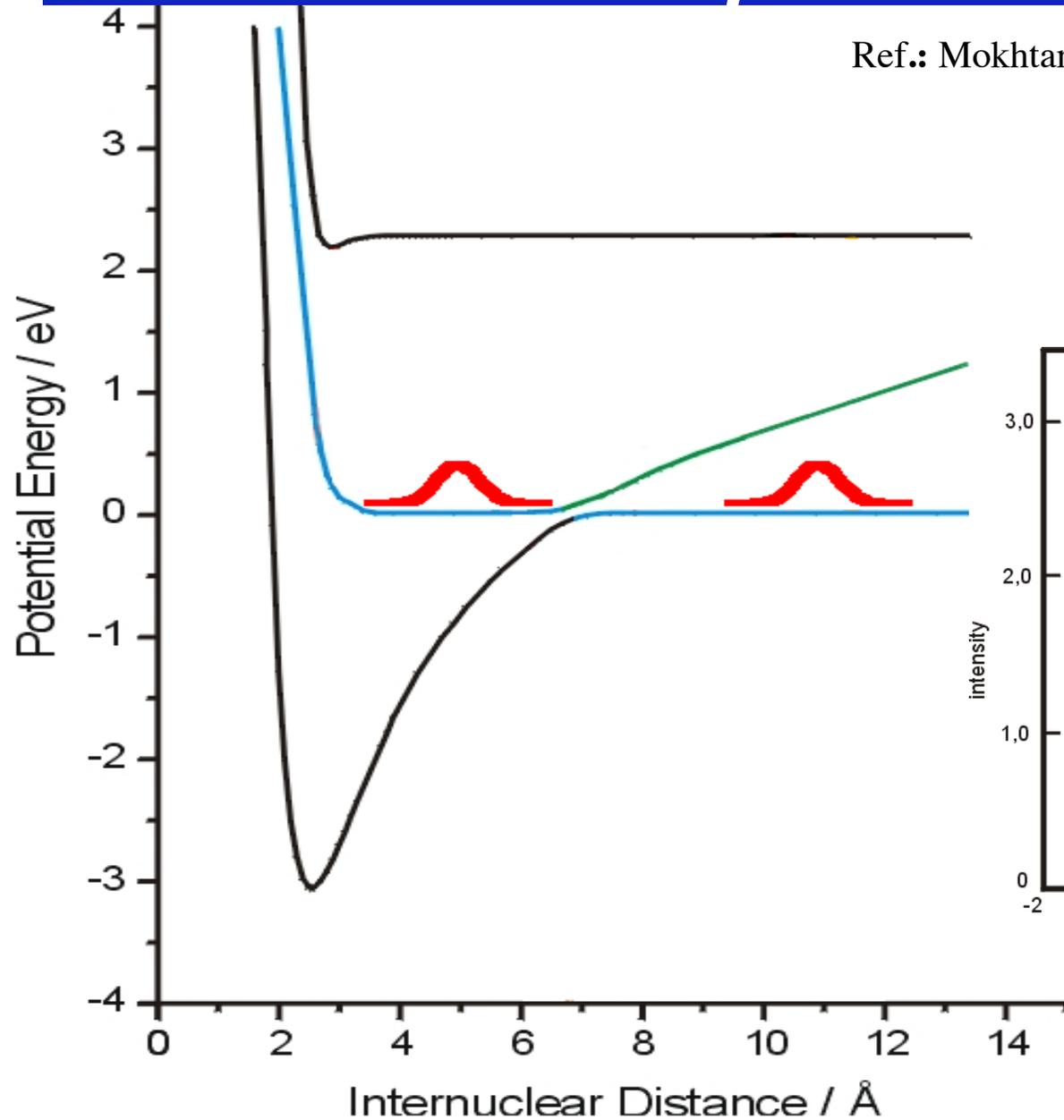
⇒ mode-locked lasers

⇒ a few  $10^{-15}$  s temporal resolution



# Ultrafast Pump-Probe Spectroscopy

Ref.: Mokhtari, Cong, Herek, Zewail, Nature **348**, 225 (1990)



**Nobel Prize in  
Physics 1999**

## **Fundamental definitions and concepts**



***An ultrashort laser pulse has an intensity and phase vs. time.***

**Neglecting the spatial dependence for now, the pulse electric field is given by:**

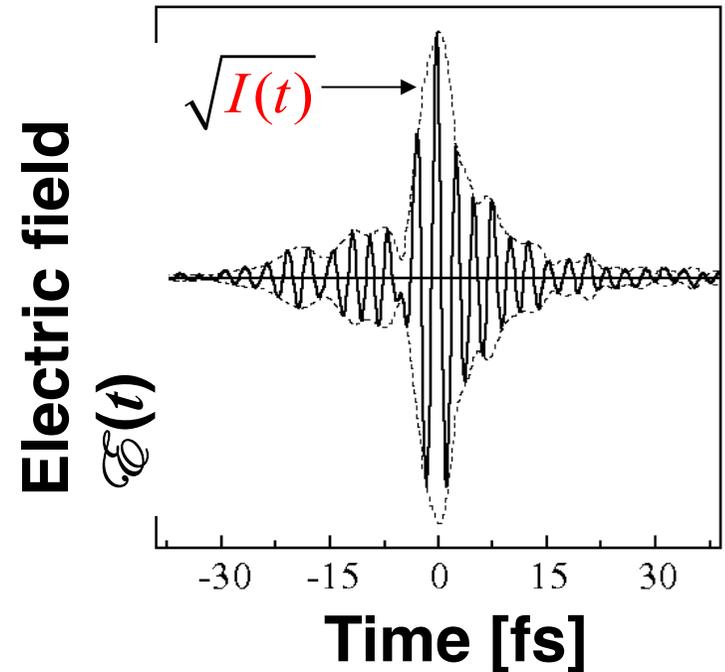
$$\mathcal{E}(t) = \frac{1}{2} \sqrt{I(t)} \exp\{i[\omega_0 t - \phi(t)]\} + c.c.$$

**Intensity**

**Carrier**

**Phase**

**frequency**



**A sharply peaked function for the intensity yields an ultrashort pulse.**

**The phase tells us the color evolution of the pulse in time.**

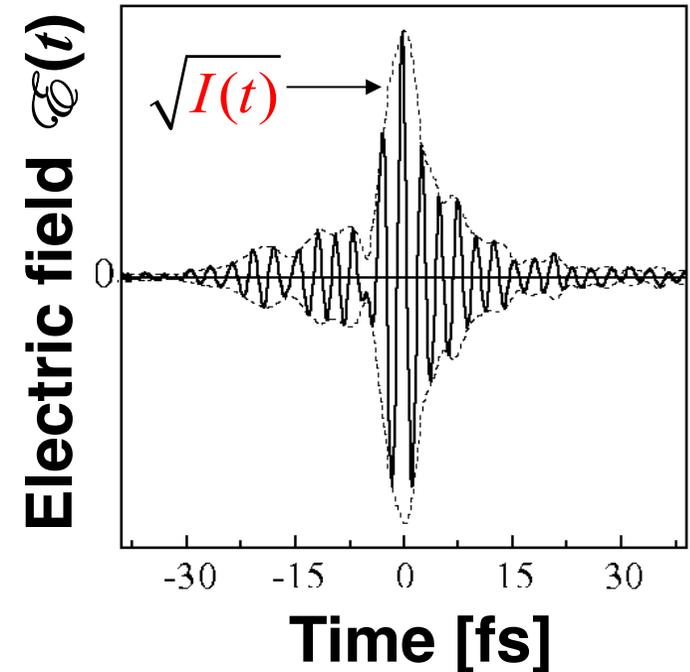
# The real and complex pulse amplitudes

Removing the  $1/2$ , the c.c., and the exponential factor with the carrier frequency yields the **complex amplitude,  $E(t)$** , of the pulse:

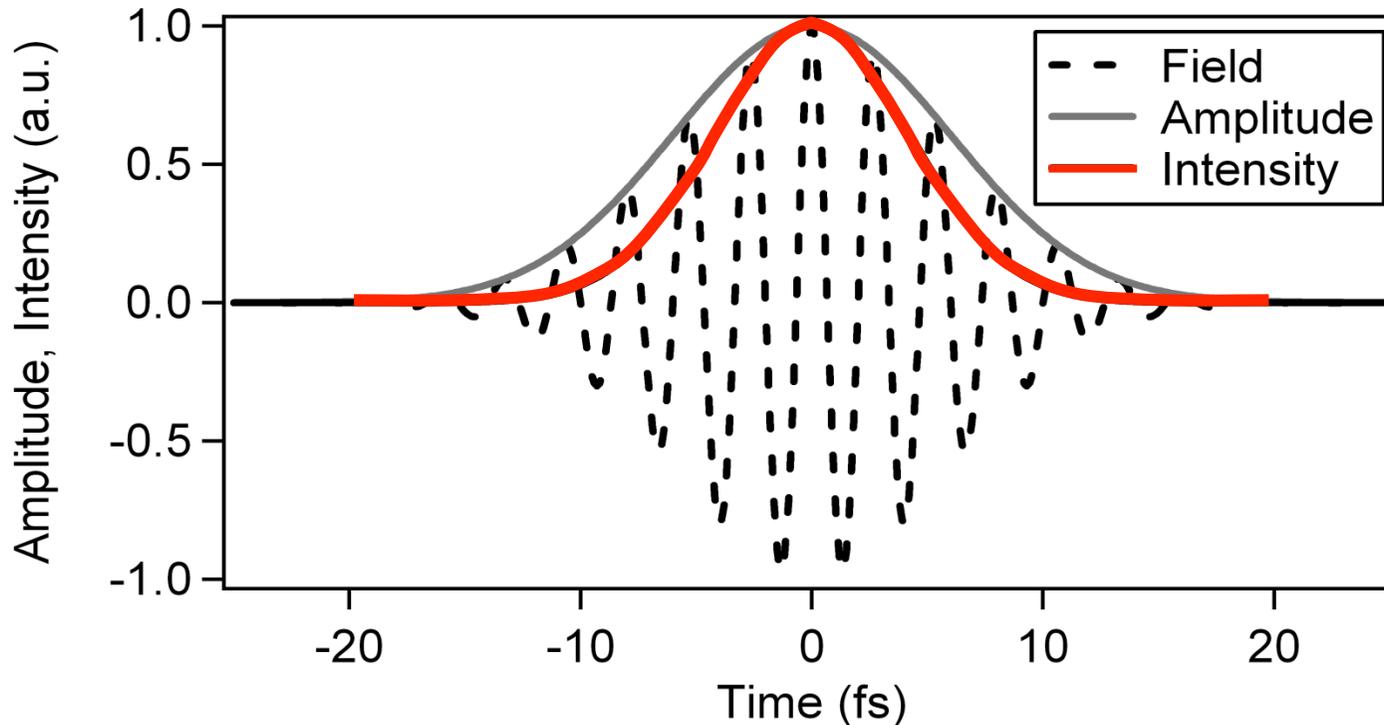
$$E(t) = \sqrt{I(t)} \exp\{-i\phi(t)\}$$

This removes the rapidly varying part of the pulse electric field and yields a complex quantity, which is actually easier to calculate with.

$\sqrt{I(t)}$  is often called the **real amplitude,  $A(t)$** , of the pulse.



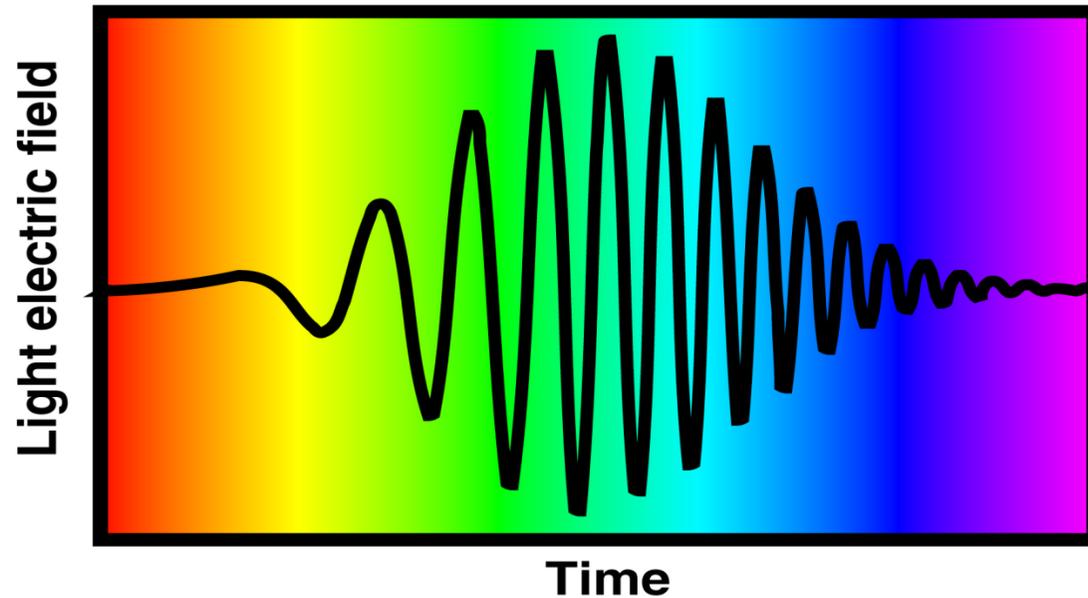
# Intensity vs. amplitude



The phase of this pulse is constant,  $\phi(t) = 0$ , and is not plotted.

The intensity of a Gaussian pulse is  $\sqrt{2}$  shorter than its real amplitude. This factor varies from pulse shape to pulse shape.

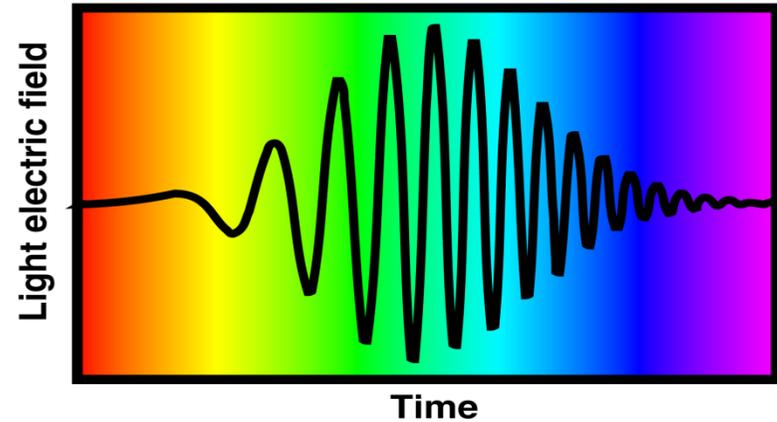
## *Second-order phase: the linearly chirped pulse*



**This pulse increases its frequency linearly in time (from red to blue).**

**In analogy to bird sounds, this pulse is called a "chirped" pulse.**

# *The linearly chirped Gaussian pulse*



We can write a linearly chirped Gaussian pulse mathematically as:

$$E(t) = E_0 \underbrace{\exp \left[ -(t / \tau_G)^2 \right]}_{\substack{\text{Gaussian} \\ \text{amplitude}}} \exp \left[ i \left( \underbrace{\omega_0 t}_{\substack{\text{Carrier} \\ \text{wave}}} + \underbrace{\beta t^2}_{\text{Chirp}} \right) \right]$$

Note that for  $\beta > 0$ , when  $t < 0$ , the two terms partially cancel, so the phase changes slowly with time (so the frequency is low). And when  $t > 0$ , the terms add, and the phase changes more rapidly (so the frequency is larger).

# The Fourier Transform

$$\mathcal{E}(\omega) = \int_{-\infty}^{\infty} \mathcal{E}(t) \exp(-i\omega t) dt$$

$$\mathcal{E}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{E}(\omega) \exp(i\omega t) d\omega$$

**We always perform Fourier transforms on the real or complex pulse electric field, and not the intensity, unless otherwise specified.**

# *The complex frequency-domain pulse field*

Since the negative-frequency component contains the same information as the positive-frequency component, we usually neglect it.

We also center the pulse on its actual frequency, not zero. So the most commonly used complex frequency-domain pulse field is:

$$\mathcal{E}(\omega) \equiv \sqrt{S(\omega)} \exp\{-i\varphi(\omega)\}$$

Thus, the frequency-domain electric field also has an intensity and phase.

$S$  is the spectrum, and  $\varphi$  is the spectral phase.

# *Method I*

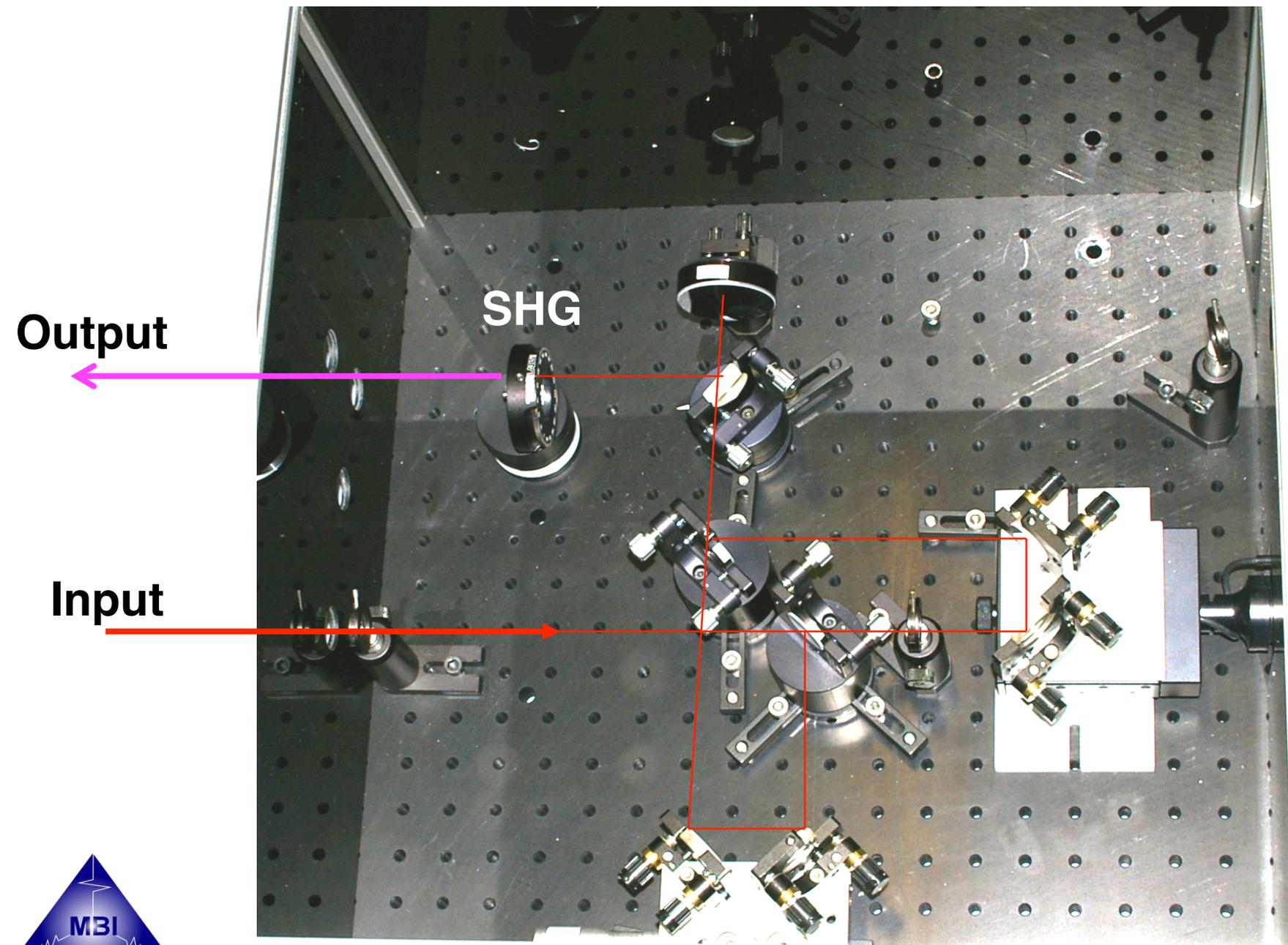
**Characterizing the pulse by  
using one and the same pulse  
for the gate function**

**aka**

***Autocorrelation***



# Autocorrelator





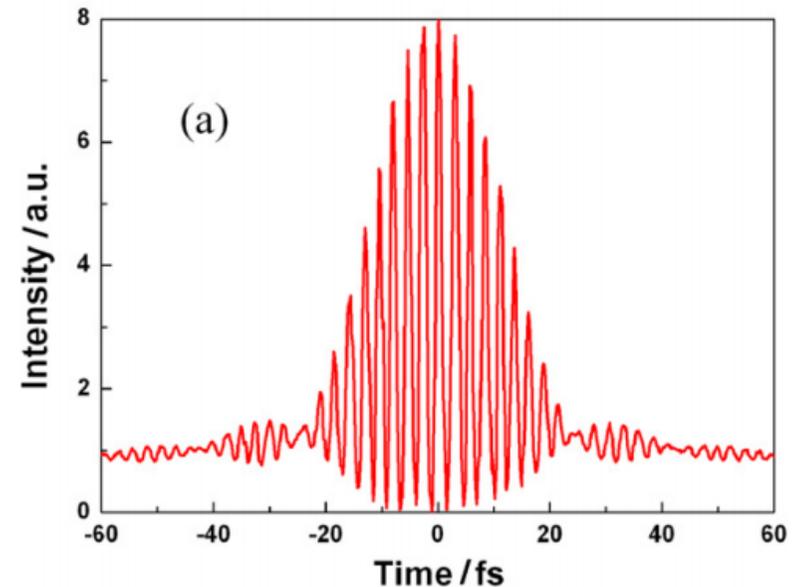
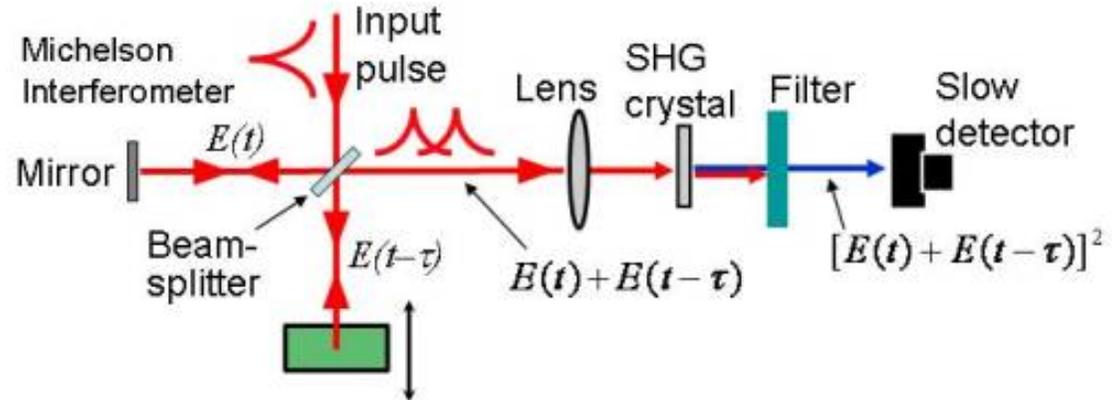
# Interferometric Autocorrelation

**Interferometric  
autocorrelation**

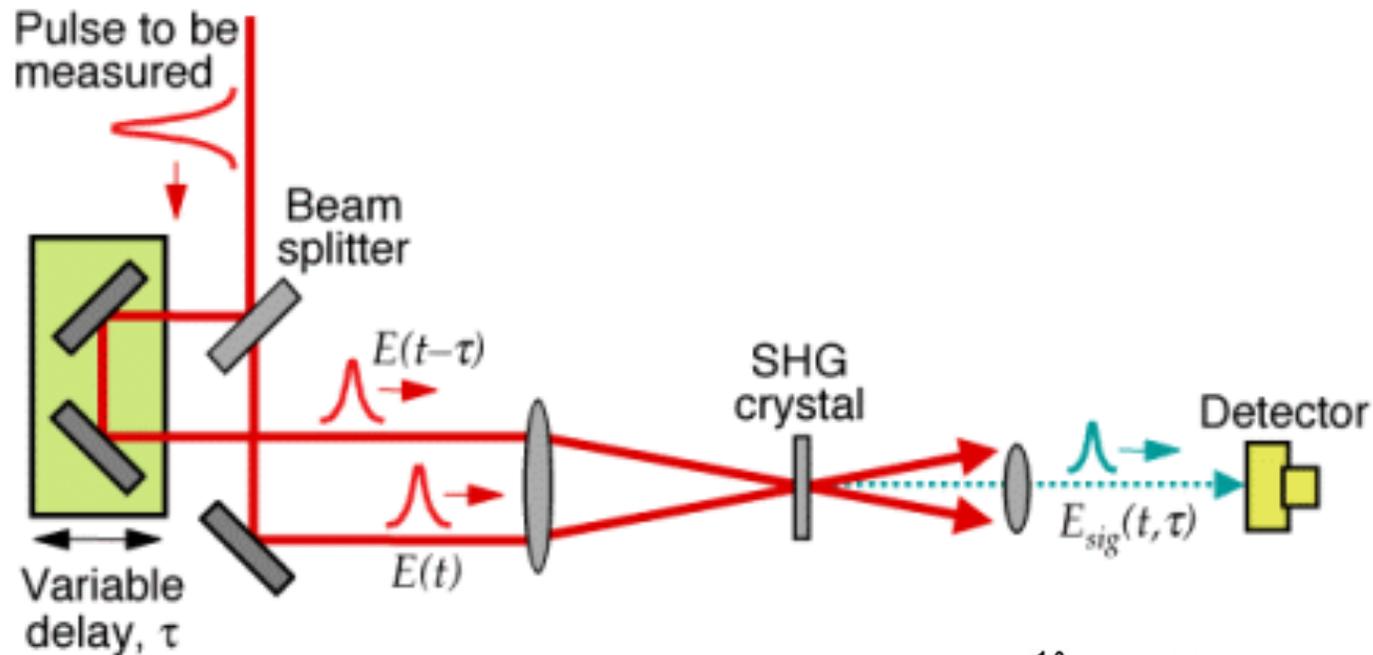
**OR**

**Fringe-resolved  
autocorrelation**

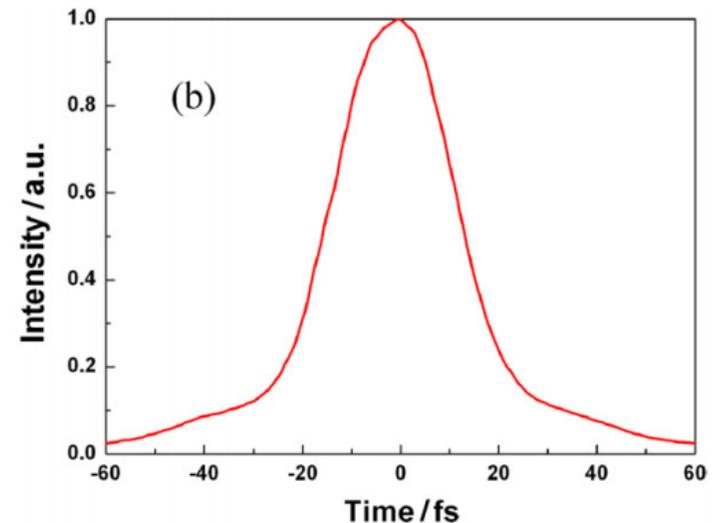
**Suitable for few-cycle pulses  
Maximum information**



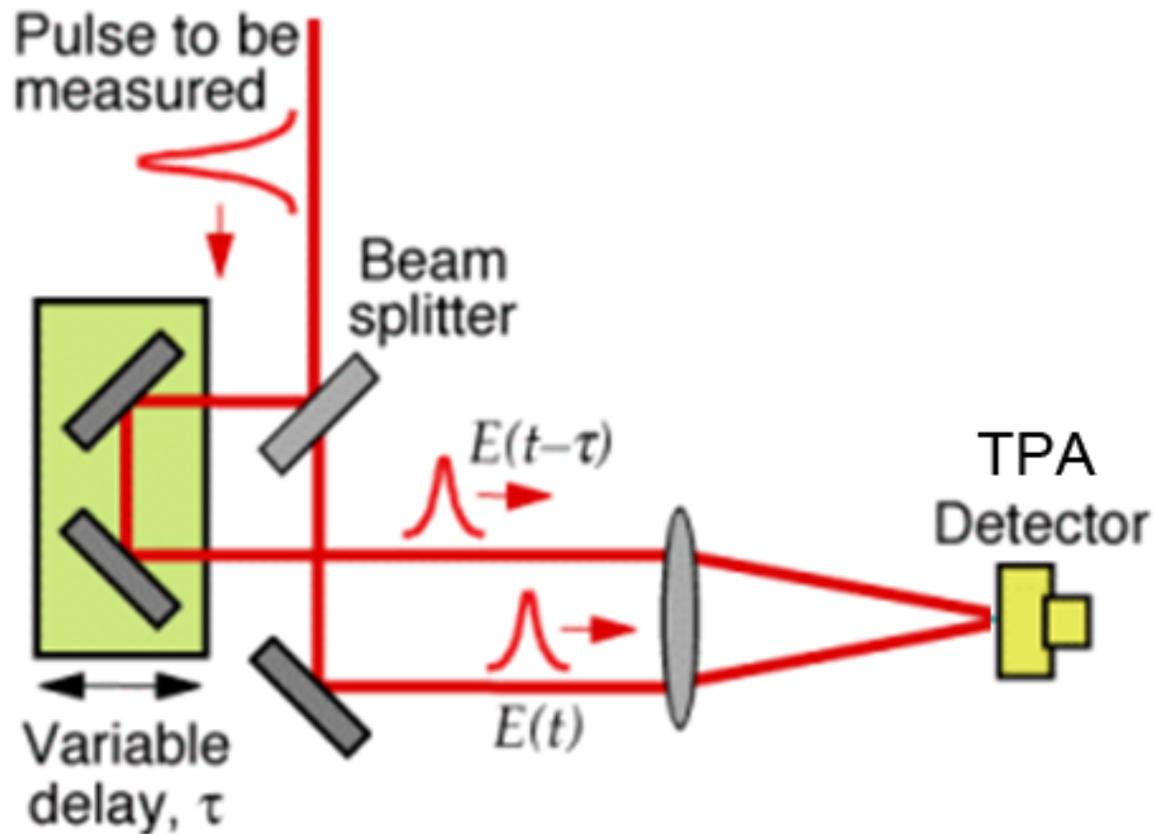
# Background-free autocorrelation



**High dynamic range**  
**Pulse contrast estimation**  
**Not suitable for shortest pulses**



# *Not background free but not necessarily interferometric...*



**Basic Idea: Use a silicon photo diode at  $1.5 \mu\text{m}$  wavelength**

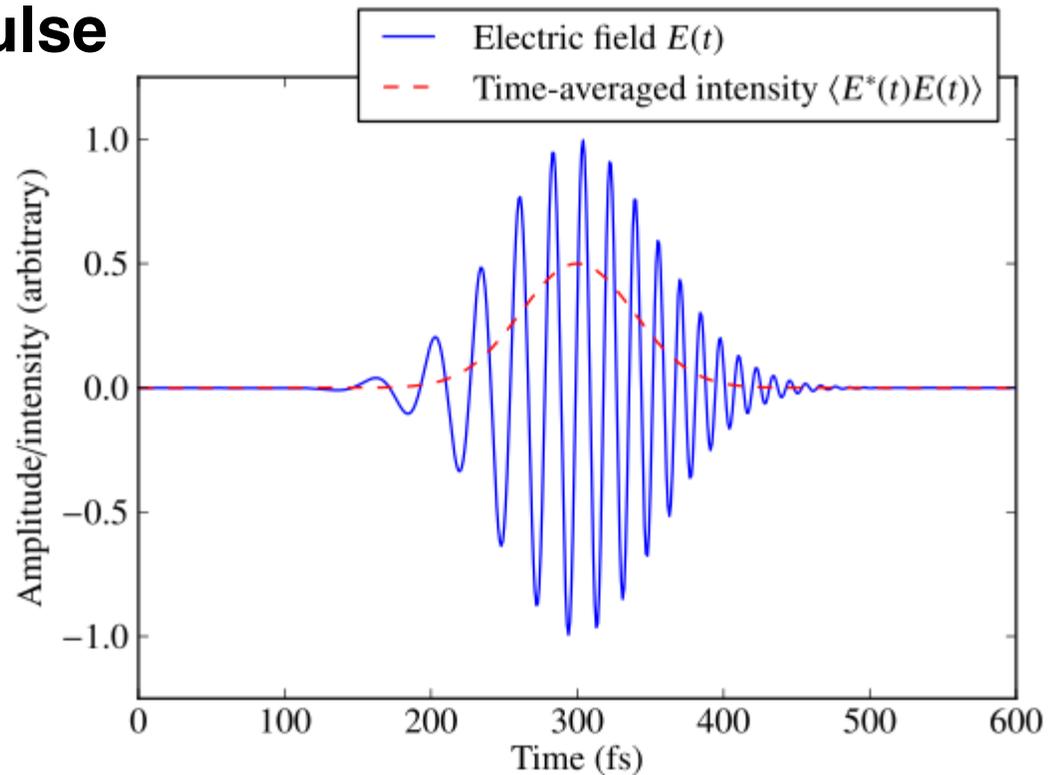
J. K. Ranka et al., Opt. Lett. **22**, 1344-1346 (1997)

D.T. Reid et al., Appl. Opt. **37**, 8142-8144 (1998)



# Mathematical form of autocorrelation

**I(t)=Intensity Envelope of Pulse**



**Intensity autocorrelation:**

$$I(t) \otimes I(t) =$$

$$\int I(t') I(t' - t) dt' = : \mathcal{AC}(t)$$

**Loss of symmetry information on I(t)**

**Impossible to retrieve I(t)**

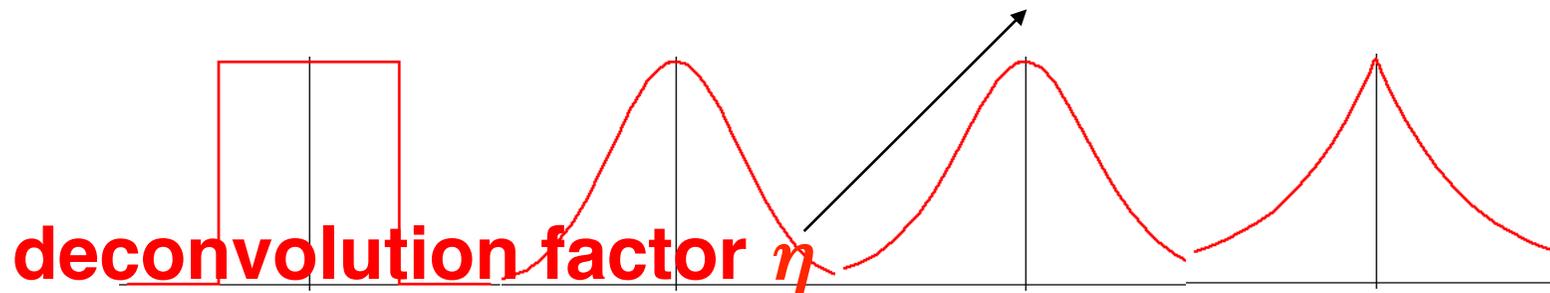


# Evaluating autocorrelation traces

traditional: assumption of a pulse shape

$$I(t) := \text{sech}^2(t / t_0)$$

$$\Rightarrow \text{FWHM}[\mathcal{AC}(t)] = 1.763 \text{ FWHM}[I(t)]$$



$\eta$	rect	gaussian	sech	dbl exp
	1.0	1.177	1.763	2.0



# Extensive tables for pulse shape guessing

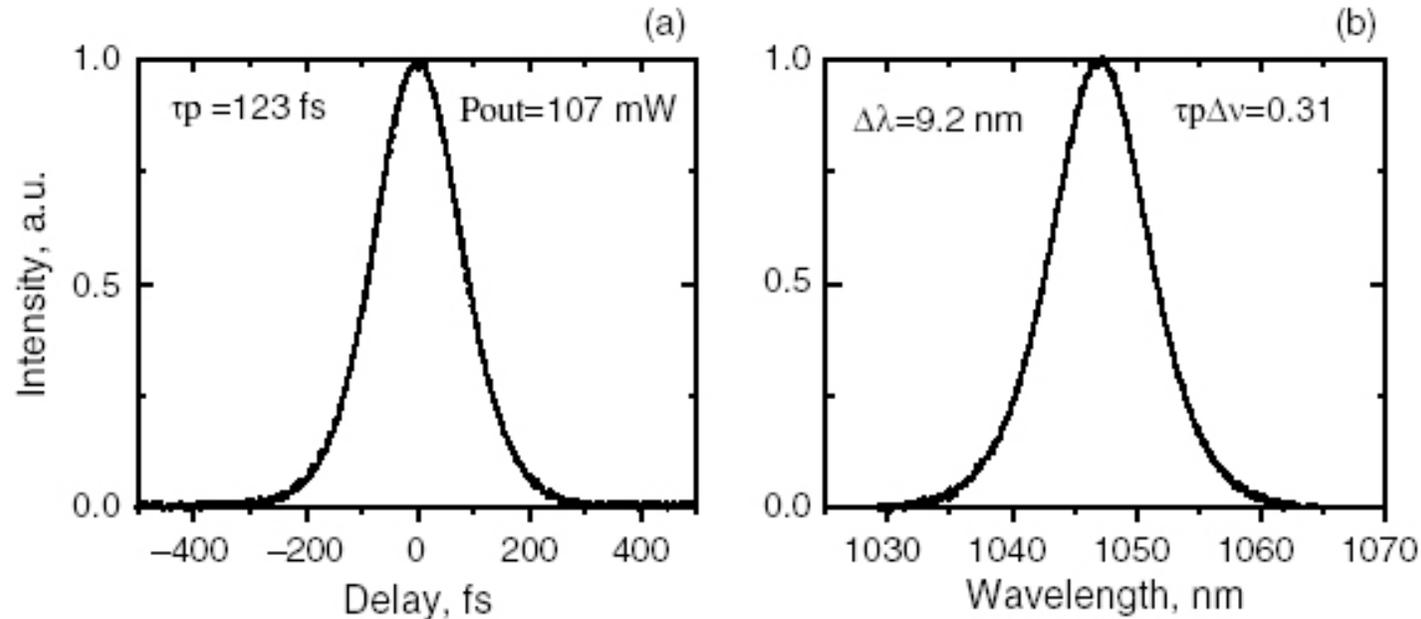
Table I. Diagnostic Functions Corresponding to Various Pulse Shapes

$I(t)$	$\Delta t$	$I(\omega)$	$\Delta\omega$	$\Delta\omega/\Delta t$	$g_1(\tau)$	$\Delta\tau$	$\Delta\tau/\Delta t$	$G_2(\tau)$	$\Delta\tau$	$\Delta\tau/\Delta t$	$g_2(\tau)$
$e^{-t^2}$	1.665	$e^{-\omega^2}$	1.665	2.772	$1 + e^{-\frac{\tau^2}{4}}$	3.330	2	$e^{-\frac{\tau^2}{2}}$	2.355	1.414	$1 + 3G_2(\tau) \pm 4e^{-\frac{3}{8}\tau^2}$
$\text{sech}^2 t$	1.763	$\text{sech}^2 \frac{\pi\omega}{2}$	1.122	1.978	$1 \pm \frac{\tau}{\sinh\tau}$	4.355	2.470	$\frac{3(\tau \cosh\tau - \sinh\tau)}{\sinh^3\tau}$	2.720	1.543	$1 + 3G_2(\tau) \pm \frac{3(\sinh 2\tau - 2\tau)}{\sinh^3\tau}$
$\frac{1}{(e^{\frac{t}{A} + 1} + e^{-\frac{t}{A}})^2}$ $A = \frac{1}{2}$	1.715	$\frac{1 + 1/\sqrt{2}}{\cosh \frac{15\pi\omega}{16} + 1/\sqrt{2}}$	1.123	1.925	$1 \pm 4 \frac{\sinh \frac{1}{3}\tau}{\sinh \frac{4}{3}\tau}$	3.405	1.985	$\frac{1}{\cosh^3 \frac{8}{15}\tau}$	2.648	1.544	$1 + 3G_2(\tau) \pm 4 \frac{\cosh^3 \frac{4}{15}\tau}{\cosh^3 \frac{8}{15}\tau}$
$A = \frac{1}{2}$	1.565	$\text{sech} \frac{3\pi}{4}\omega$	1.118	1.749	$1 \pm 2 \frac{\sinh\tau}{\sinh 2\tau}$	2.634	1.683	$\frac{3\sinh \frac{8}{3}\tau - 8\tau}{4\sinh^3 \frac{4}{3}\tau}$	2.424	1.549	$1 + 3G_2(\tau) \pm 4 \frac{\tau \cosh 2\tau - \frac{3}{2} \cosh^2 \frac{2}{3}\tau \sinh \frac{2}{3}\tau (2 - \cosh \frac{4}{3}\tau)}{\sinh^3 \frac{4}{3}\tau}$
$A = \frac{1}{4}$	1.278	$\frac{1 - 1/\sqrt{2}}{\cosh \frac{7\pi}{16}\omega - 1/\sqrt{2}}$	1.088	1.391	$1 \pm \frac{4\sinh 3\tau}{3\sinh 4\tau}$	1.957	1.531	$\frac{2\cosh \frac{16}{7}\tau + 3}{5\cosh^3 \frac{8}{7}\tau}$	2.007	1.570	$1 + 3G_2(\tau) \pm 4 \frac{\cosh^3 \frac{4}{7}\tau (6\cosh \frac{8}{7}\tau - 1)}{5\cosh^3 \frac{8}{7}\tau}$
$\frac{1}{(e^t + e^{-rt})^2}$		$\frac{2(1-y)x}{x^2 - 2yx + 1}$ where, $x = \exp(\frac{2\pi\omega}{1+r})$ $y = \cos(\frac{2\pi r}{1+r})$			$1 \pm \frac{r+1}{r-1} \frac{\sinh(\frac{r-1}{2}\tau)}{\sinh(\frac{r+1}{2}\tau)}$						

$I(t)$  and  $I(\omega)$  are the intensities in the time and (angular) frequency domains, respectively.  $g_1(\tau)$  is the first-order autocorrelation of the field envelope.  $G_2(\tau)$  is the intensity autocorrelation, and  $g_2(\tau)$  the envelope of the interferometric correlation. The FWHM is indicated in the next column to the right for each function.



# Time-bandwidth product



**TBP = FWHM(Spectrum) x FWHM(ACF) x deconv. factor**

**Ideal sech<sup>2</sup> pulse: TBP = 0.32**

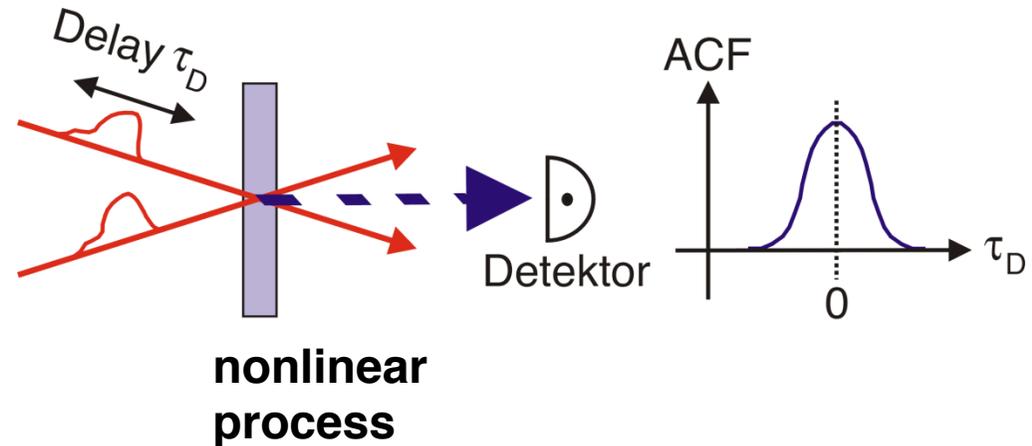
**Careful: TBP can be as small as 0.2 for asymmetric pulses**

Example taken from Brown et al., New J. Phys. **6** (2004) 175



# The autocorrelation paradox

*It's simply impossible to measure the shortest pulse without having an even shorter event at hand...*



*...therefore one has to escape the swamp by pulling oneself up by one's own hair...*

**Can we fix the problem of  
autocorrelation?**

***Decorrelation***



# The decorrelation problem

## Wiener-Khinchin theorem

$$\text{AC}[f(t)] = \mathcal{F}^{-1} [ |\mathcal{F}[f(t)]|^2 ]$$

$$f \otimes f \xleftrightarrow{\mathcal{F}} f \cdot f$$

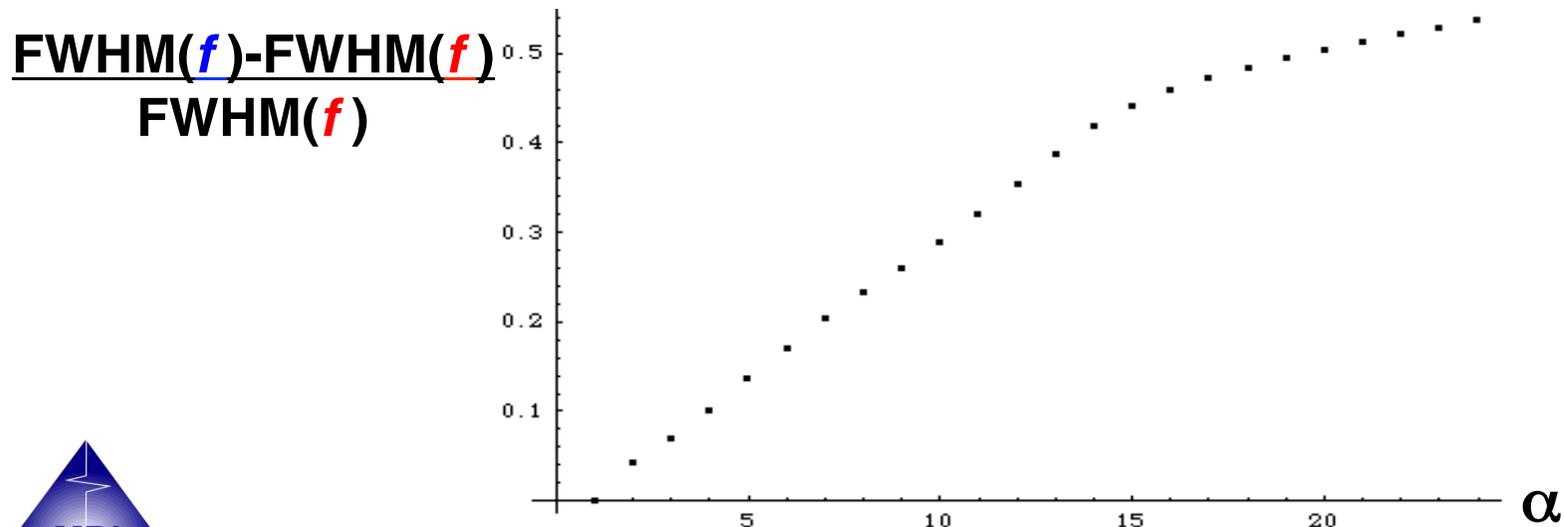
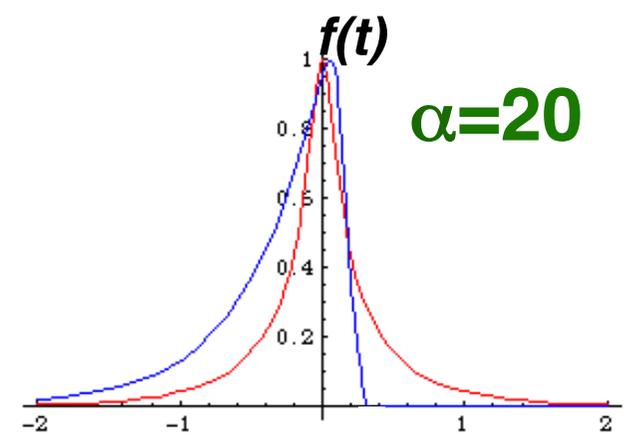
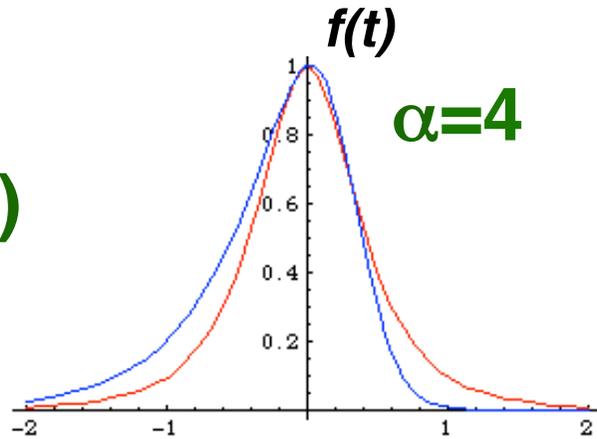
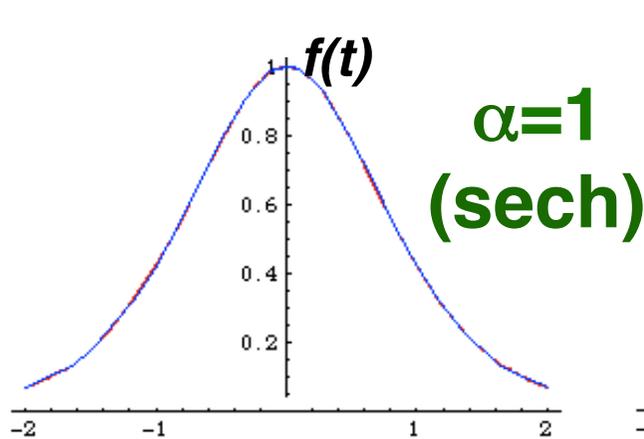
“Convolution is multiplication in the Fourier domain”

$$f(t) = \mathcal{F}^{-1} \left[ \sqrt{\mathcal{F}[\text{AC}[f(t)]]} \right] ??$$

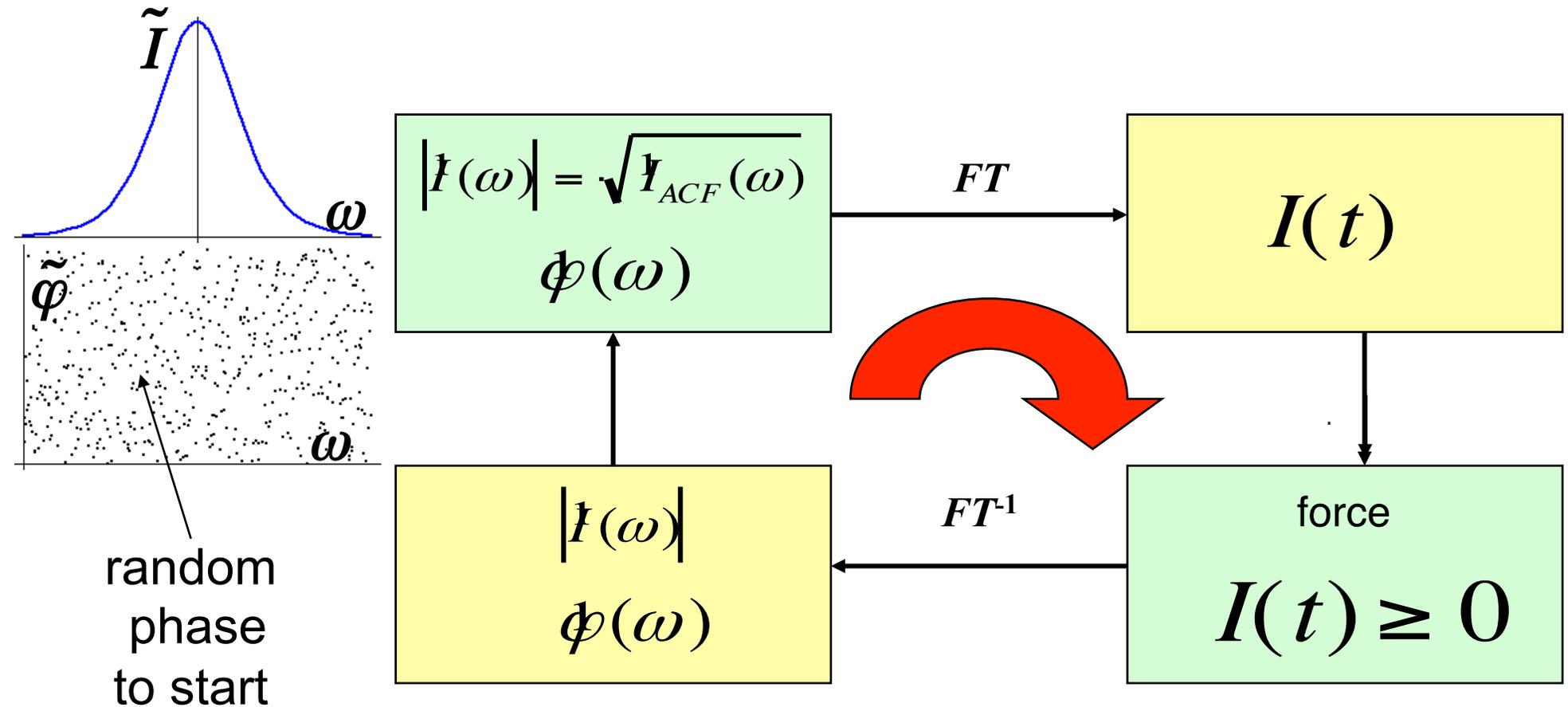


# The Decorrelation dilemma

$$f(t) = (\exp(t) + \exp(-\alpha t))^{-1} \quad f(t) = \mathcal{F}^{-1}[\sqrt{|\mathcal{F}[\text{AC}[f(t)]]|}]$$



# Decorrelation from ACF+spectrum

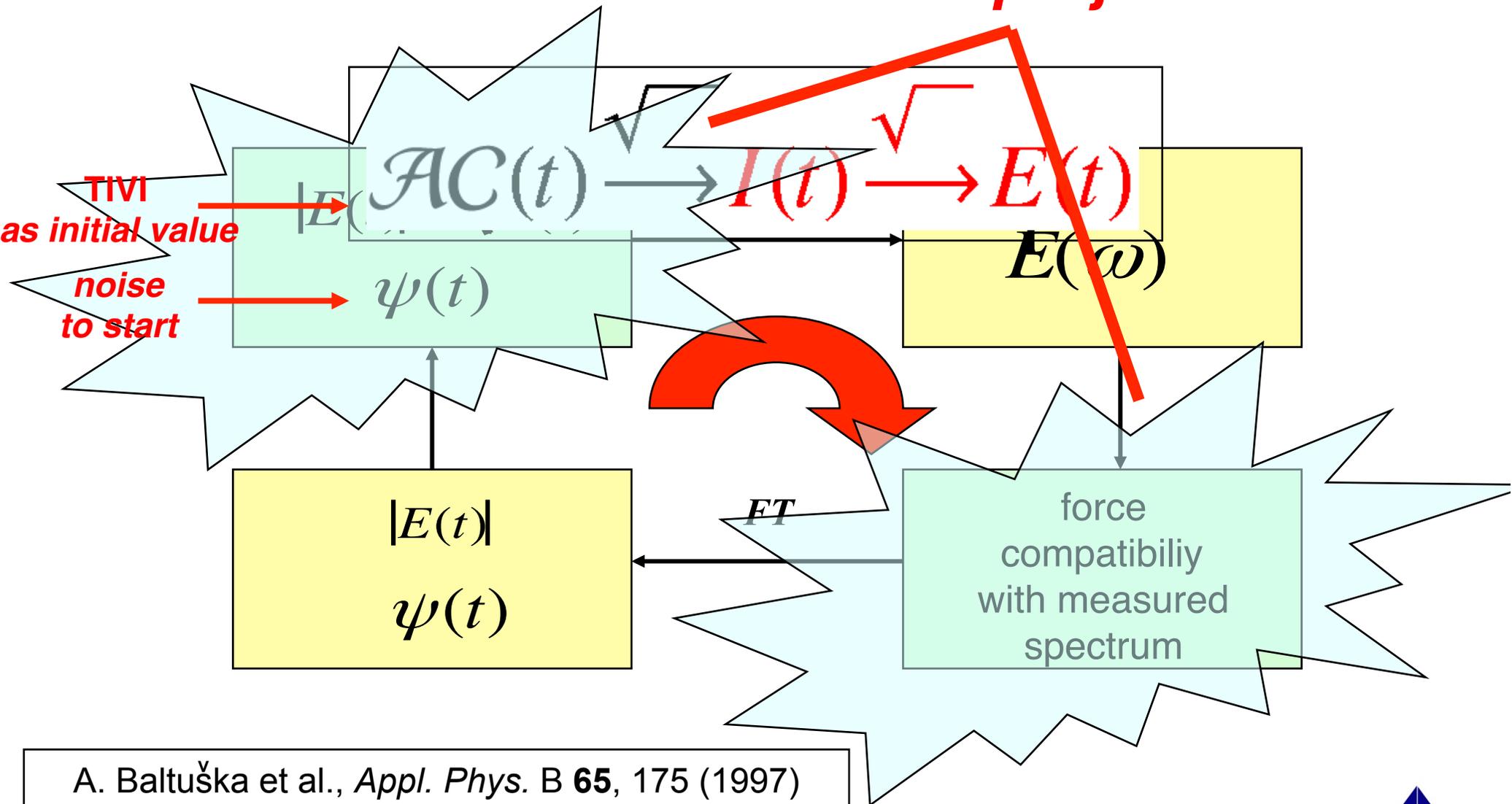


**TIVI**-algorithm, Peatross et al., JOSA B **15**, 216 (1998)  
R.W. Gerchberg & W.O. Saxton, Optik **35**, 237 (1972)



# decorrelation from ACF+Spectrum

## Generalized projections

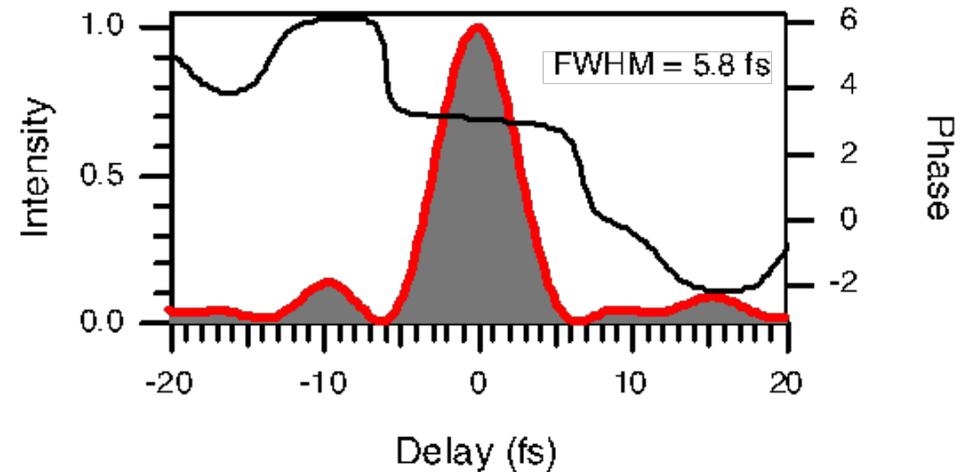
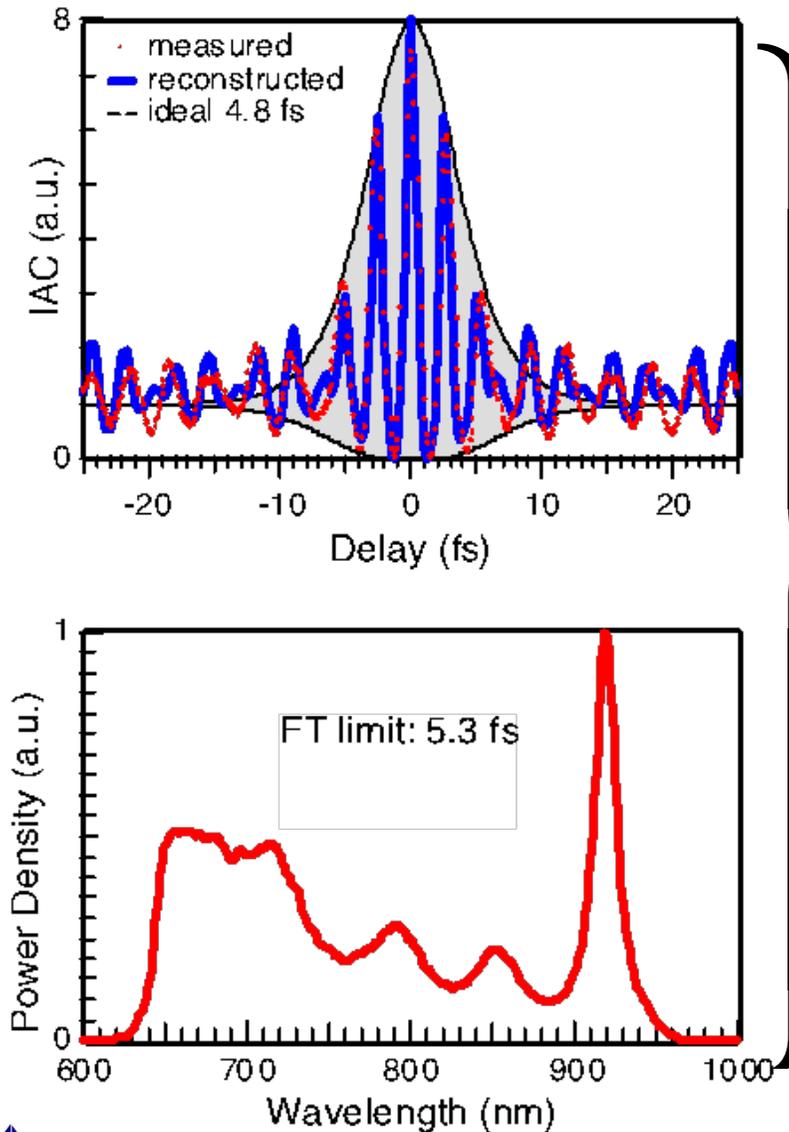


A. Baltuška et al., *Appl. Phys. B* **65**, 175 (1997)

<http://www.chem.rug.nl/spectro/Projects/decorrelation.htm>



# decorrelated measurements



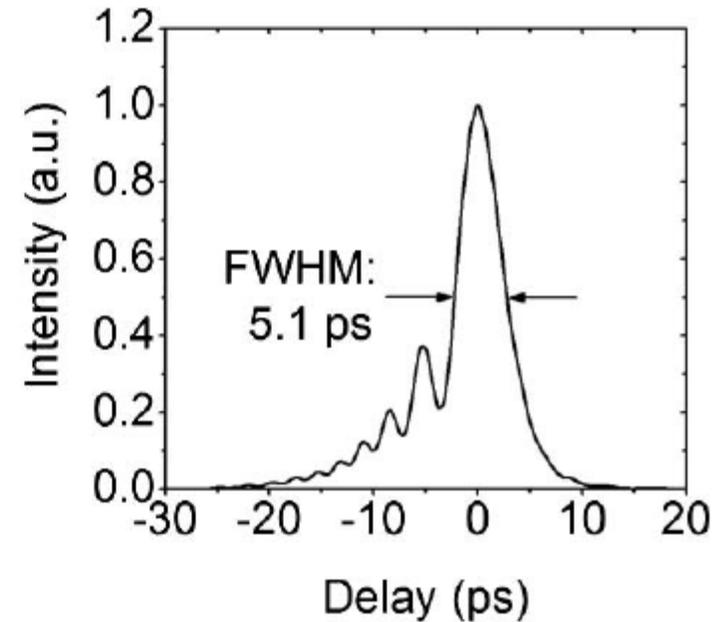
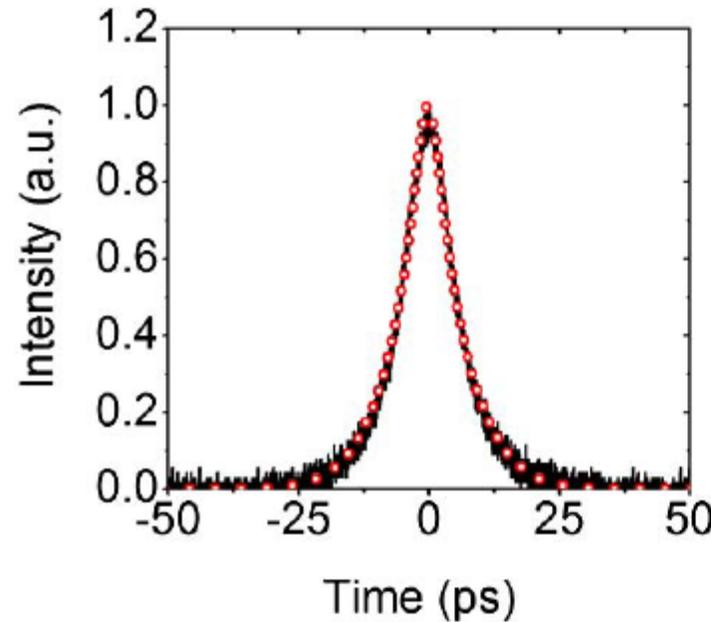
**Certainly better than assumption of some arbitrary deconvolution factor, yet very susceptible to experimental noise and positively not unambiguous...**

D.Sutter et al., *Opt. Lett.* **24**, 631 (1999)

Chung & Weiner, *IEEE J. Sel. Top. QE* **7**, 656 (2001)



# PICASO method



**Iteratively fit phase to  $E(\omega)$  until autocorrelation is retrieved**

J. W. Nicholson and W. Rudolph, J. Opt. Soc. Am. B 19, 330 (2002).  
S. Ranta et al., Opt. Lett. **38**, 2289-2291 (2013).

**PICASO** = Phase and Intensity from  
Correlation and Spectrum Only



# Summary decorrelation

## 1. Assumption of a particular pulse shape ("sech<sup>2</sup>-deconvolution")

- simple
- large **error**
- not really characterization of the pulse shape

## 2. Decorrelation via generalized projection

- simple application of Wiener-Khinchin theorem ignores pulse asymmetry
- may cause **unphysical pulse shapes** ( $I(t) < 0$ )
- TIVI algorithm forces  $I(t)$  to be  $> 0$
- has to be used with great care in the interpretation

### **In general:**

- too little information for reconstructing the complete pulse shape
- pulse shape not unambiguously determined by measured data
- methods are helpful if FROG or SPIDER cannot be done

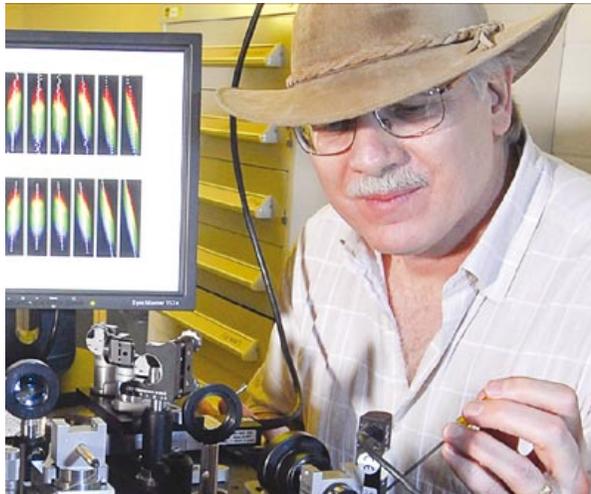


# Method II

**Spectrally resolved autocorrelation**

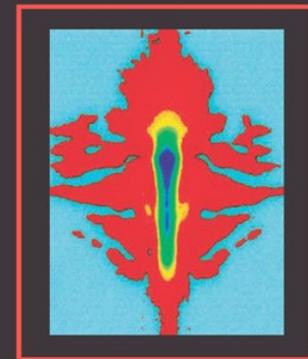
**aka**

***Frequency-resolved optical gating***



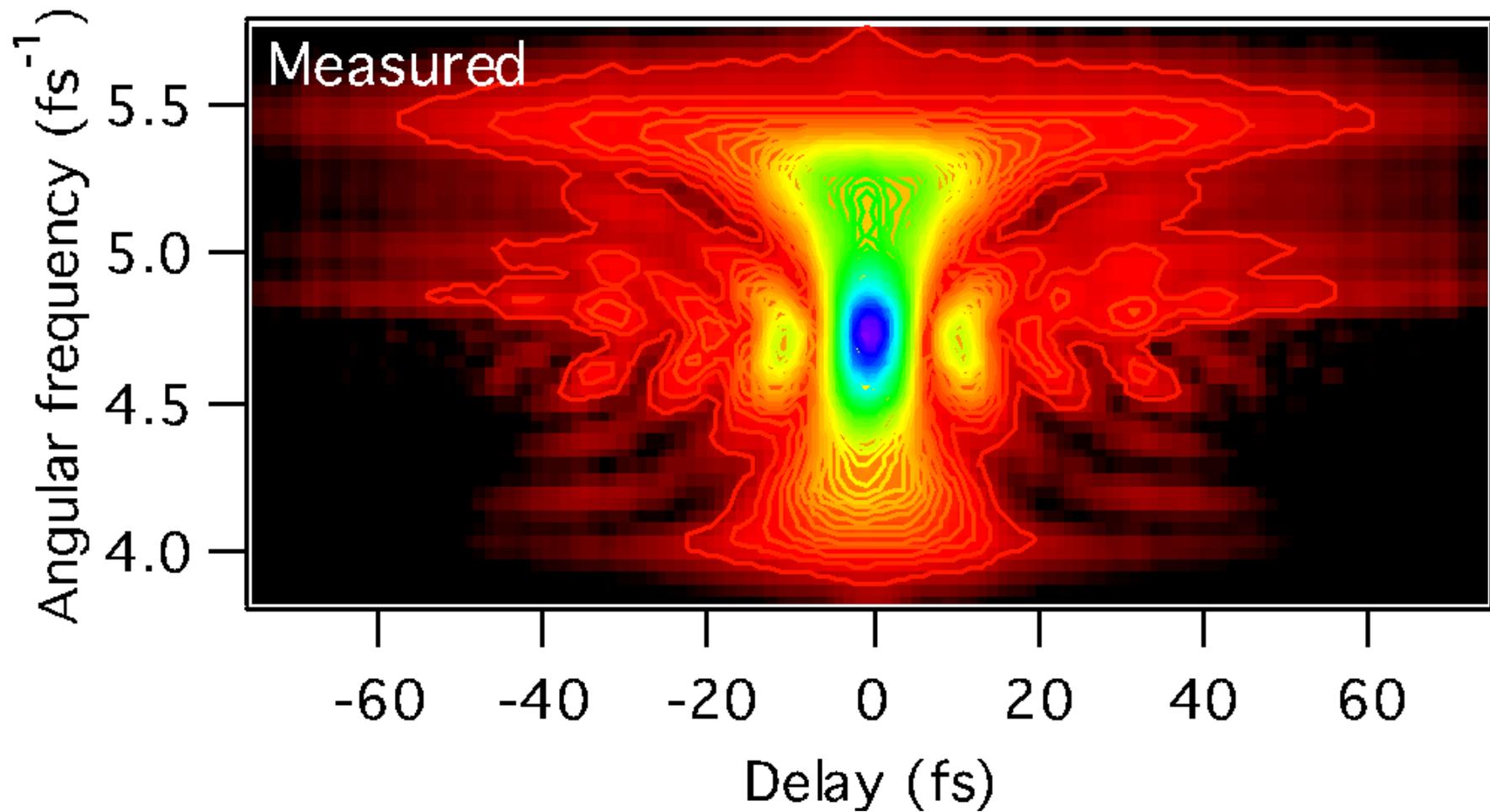
***FROG***

**Frequency-Resolved  
Optical Gating:  
The Measurement of  
Ultrashort Laser Pulses**



**Rick Trebino**

# *Spectrally resolved autocorrelation*



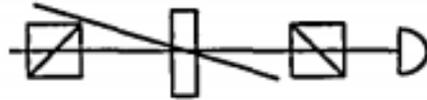
# *Core idea of FROG*

- 1. Simple autocorrelation data  $ACF(t)$  is not sufficient to unambiguously define the pulse shape.**
- 2. Simply adding the spectrum does not suffice either.**
- 3. Spectrally resolved autocorrelation data  $ACF(\omega, t)$ , however, essentially unambiguously defines the pulse shape**
- 4. FROG trace retrieval is an inverse problem. You have the answer, but have to find the one matching question that yields this very answer....**



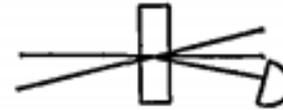
# Various nonlinearities can be employed:

## Polarization Gate



$$E_{sig}(t, \tau) = E(t)|E(t - \tau)|^2$$

## Self-Diffraction



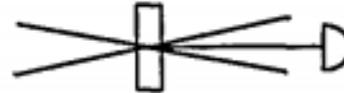
$$E_{sig}(t, \tau) = E^2(t)E^*(t - \tau)$$

## Third-Harmonic Generation



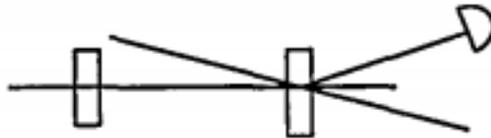
$$E_{sig}(t, \tau) = E^2(t)E(t - \tau)$$

## Second-Harmonic Generation



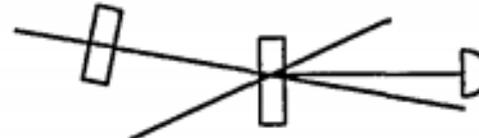
$$E_{sig}(t, \tau) = E(t)E(t - \tau)$$

## Parametric Downconversion



$$E_{sig}(t, \tau) = E^2(t)E^*(t - \tau)$$

## Parametric Upconversion



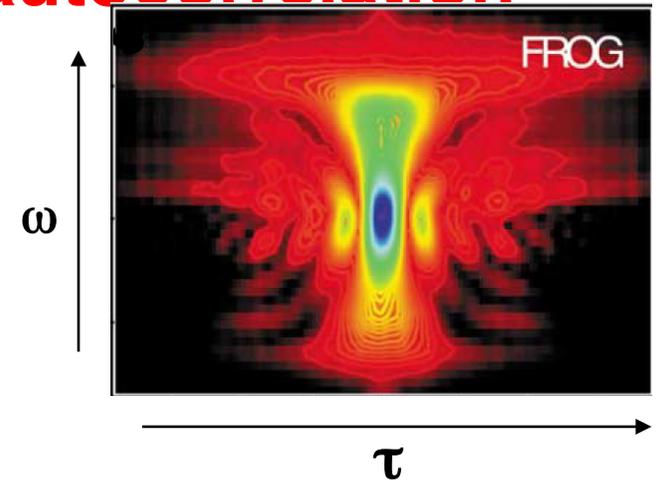
$$E_{sig}(t, \tau) = E^2(t)E(t - \tau)$$

Fig. 1. Schematic of the various experimental geometries for generating FROG traces. The nonlinear mixing signal is spectrally resolved as a function of delay time between the two replicas of the beam to be measured. The parametric conversion geometries use two crystals with a second-order nonlinearity, cascaded to produce an effective third-order nonlinearity.

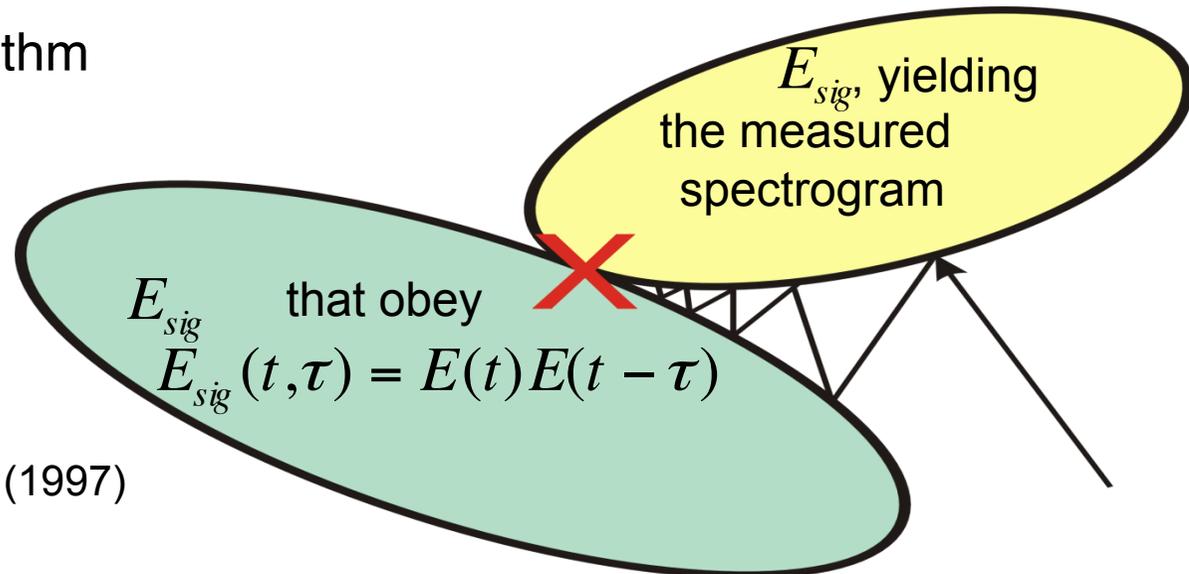
# Frequency-resolved optical gating (FROG)

## Measuring the spectrogram of the autocorrelation

$$I_{FROG}^{SHG}(\omega, \tau) = \left| \int E(t) E(t - \tau) \exp(-i\omega t) dt \right|^2 = E_{sig}(t, \tau)$$



- ✱ Phase information of signal field  $E_{sig}(t, \tau)$  gone
- ✱ However: phase is redundant and **unambiguously** defined by FROG trace
- ✱ Generalized Projections Algorithm



# *FROG software*

**It is very instructive to write your own FROG software...**

**but there are much quicker alternatives:**



**Freely available Matlab code**

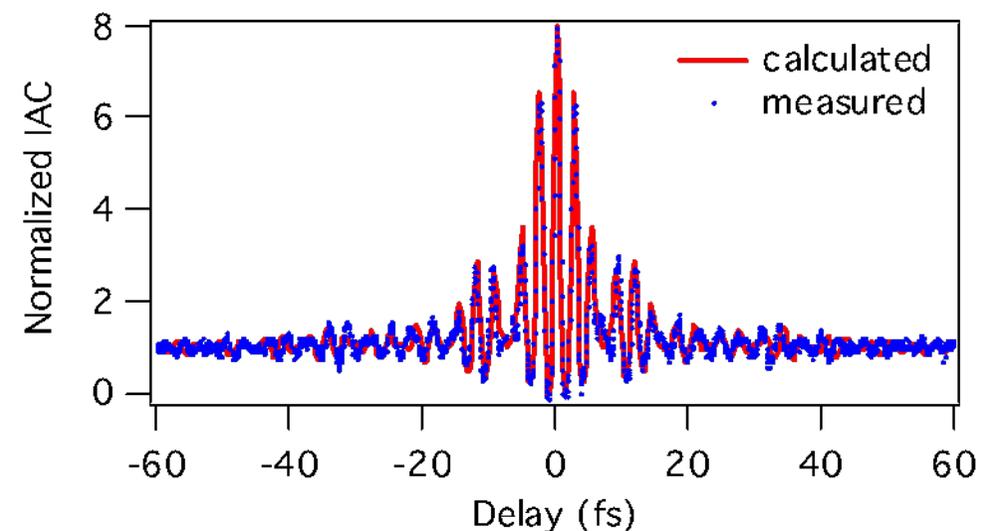
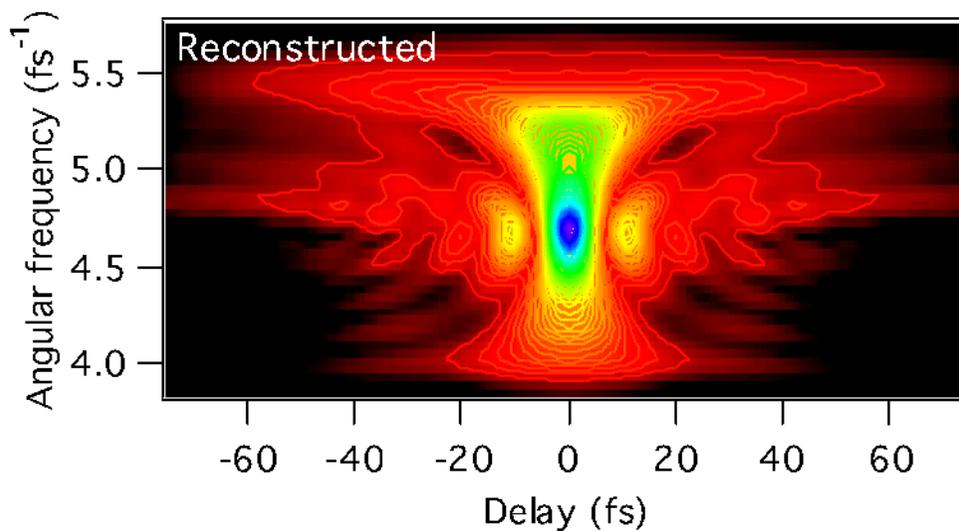
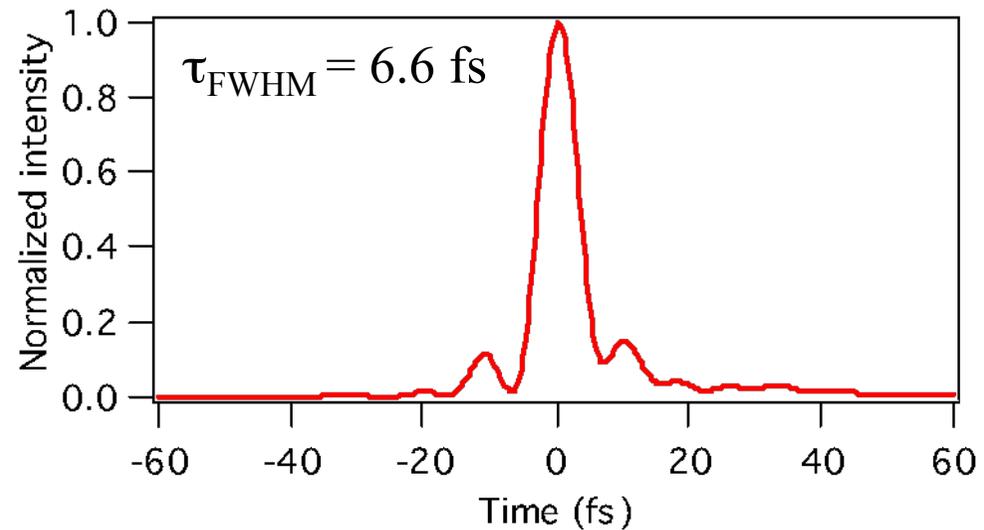
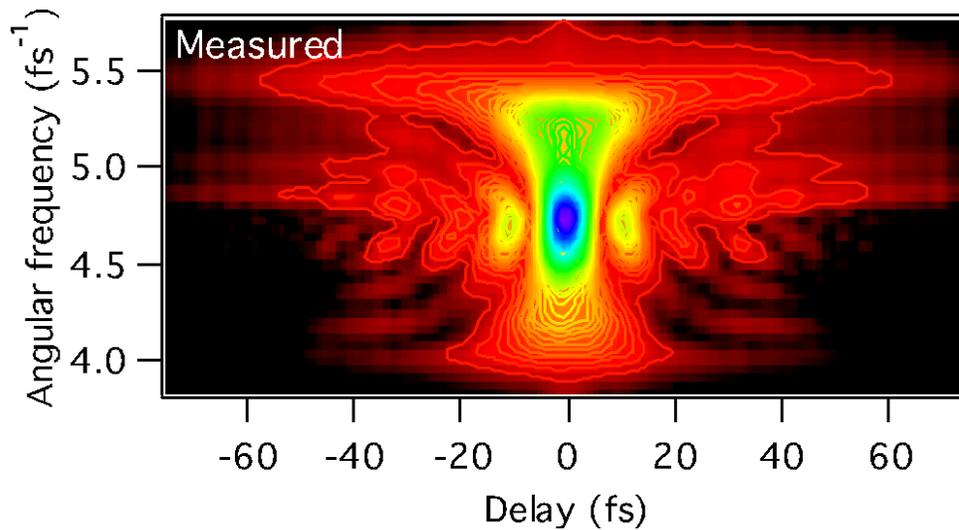
**<http://frog.gatech.edu/code.html>**

**Commercial retrieval code**

**<http://www.swsciences.com/>**



# Example FROG measurements



transform limit: 5.3 fs

sech<sup>2</sup>-Fit: 4.5 fs!!



# Marginal tests in FROG

**Delay marginal**

$$M_{\tau}(\tau) \equiv \int_{-\infty}^{\infty} d\omega I_{\text{FROG}}(\omega, \tau)$$

(to be compared w/ independently measured autocorrelation)

**Frequency marginal**

$$M_{\omega}(\omega) \equiv \int_{-\infty}^{\infty} d\tau I_{\text{FROG}}(\omega, \tau)$$

to be compared w/ independently measured spectrum via

$$M_{\omega}^{\text{SHG}}(\omega) = 2I(\omega) * I(\omega)$$

**autoconvolution**

Ref.: K. De Long et al., JOSA B 11, 1595 (1994).



# Marginals can be used to compensate for phase matching bandwidth...

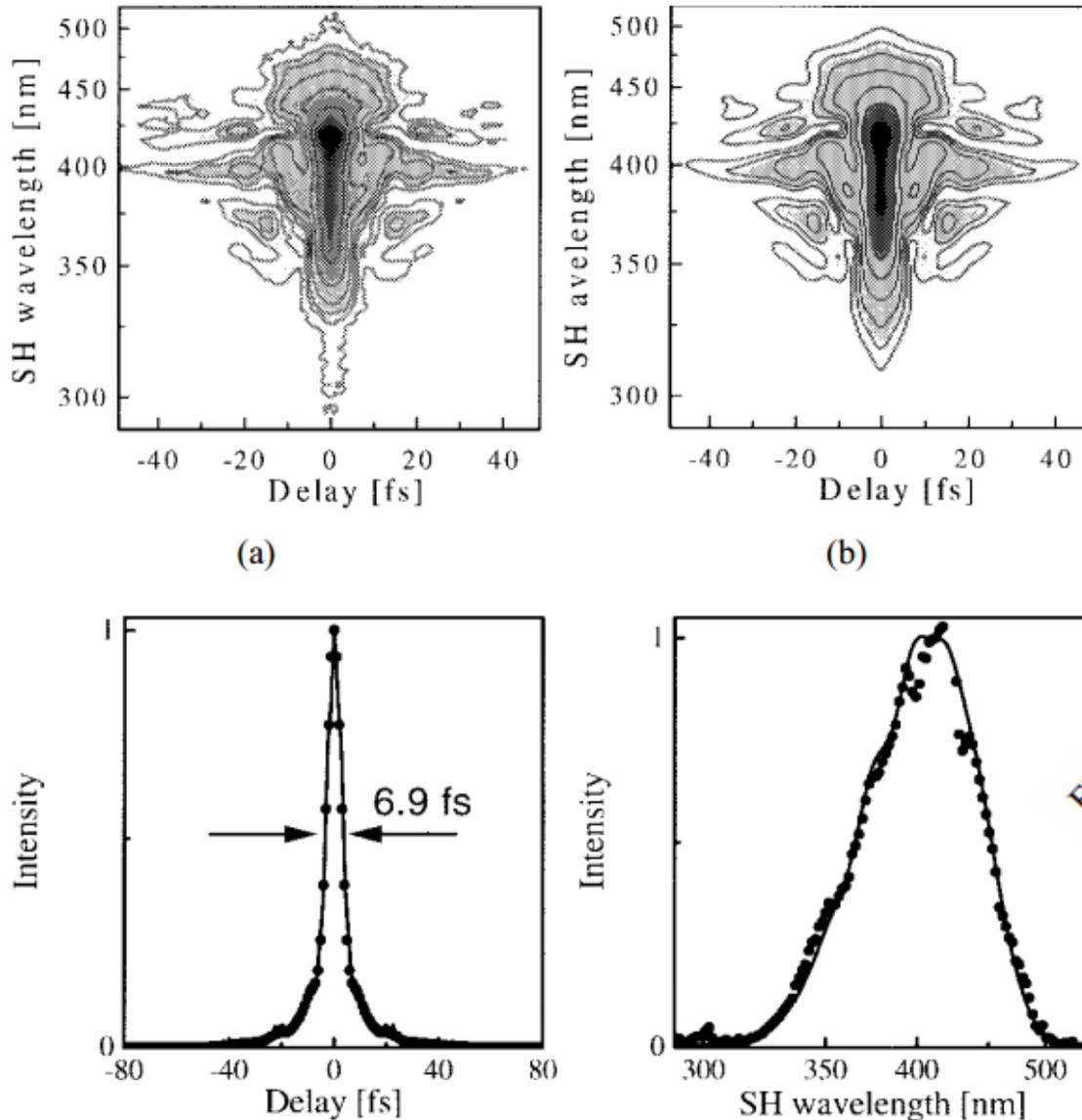


Fig. 17. The results of SHG FROG characterization of compressed pulses. (a) Experimental and (b) retrieved traces. (c) Temporal marginal (filled circles) and independently measured autocorrelation of 4.5-fs pulses (solid curve). (d) Frequency marginal (filled circles) and autoconvolution of the fundamentals (solid curve).

Ref.: A. Baltuska,  
IEEE JQE 35, 459 (1995)

# $\lambda^2$ correction

Spectral power density

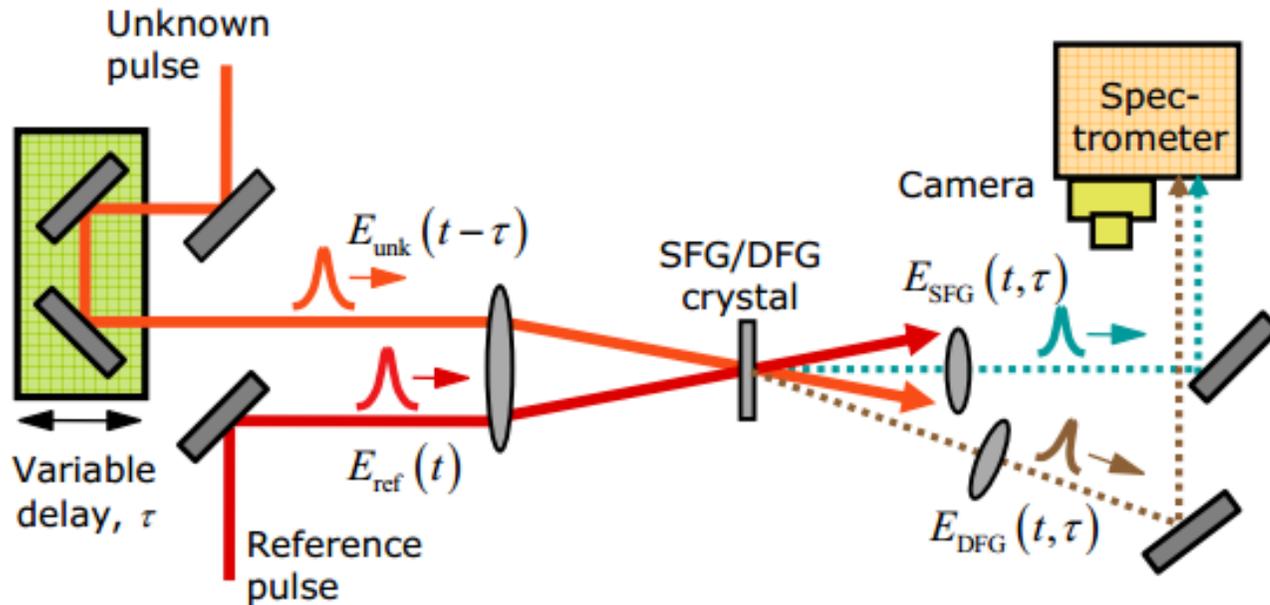
$$\frac{\partial P}{\partial \lambda} = \frac{\partial \nu}{\partial \lambda} \frac{\partial P}{\partial \nu} = -\frac{c}{\lambda^2} \frac{\partial P}{\partial \nu}$$

this is what your  
(calibrated) spectrograph  
gives you

but this is what you  
need for the marginal  
test



# XFROG



(picture from thesis Xun Gu,  
Georgiatech)

**Crosscorrelation**

**DFG or SFG**

**Well-defined  
reference pulse**

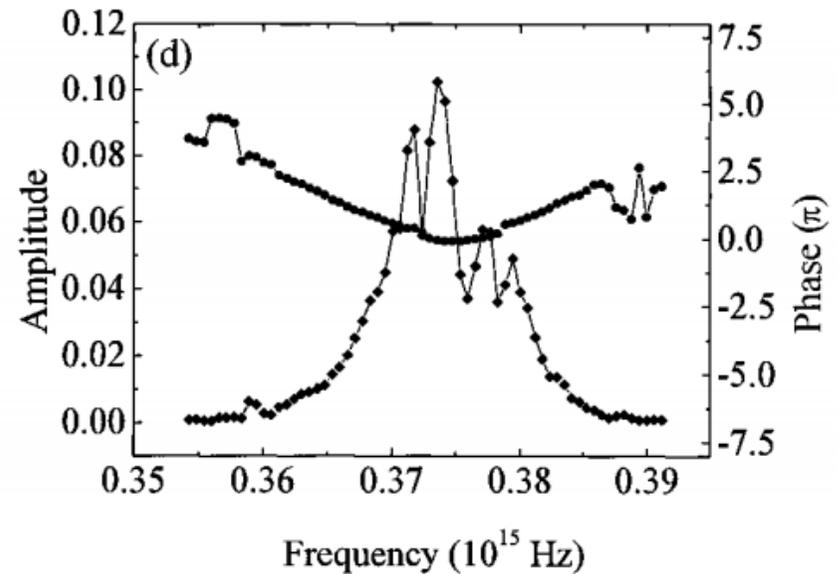
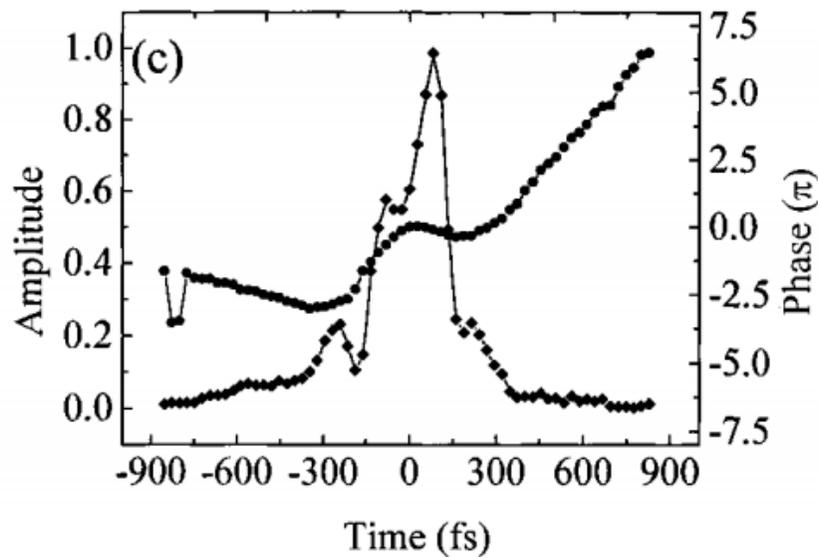
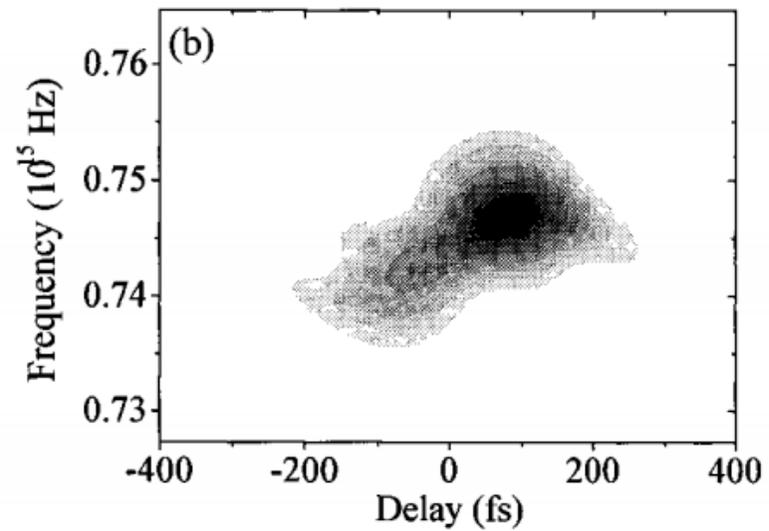
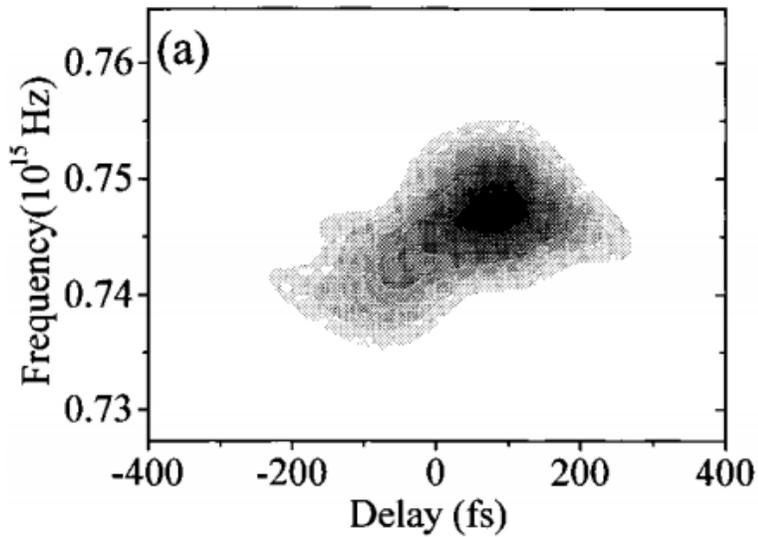
**Most powerful  
FROG method**

**Can resolve  
very complex  
pulses**

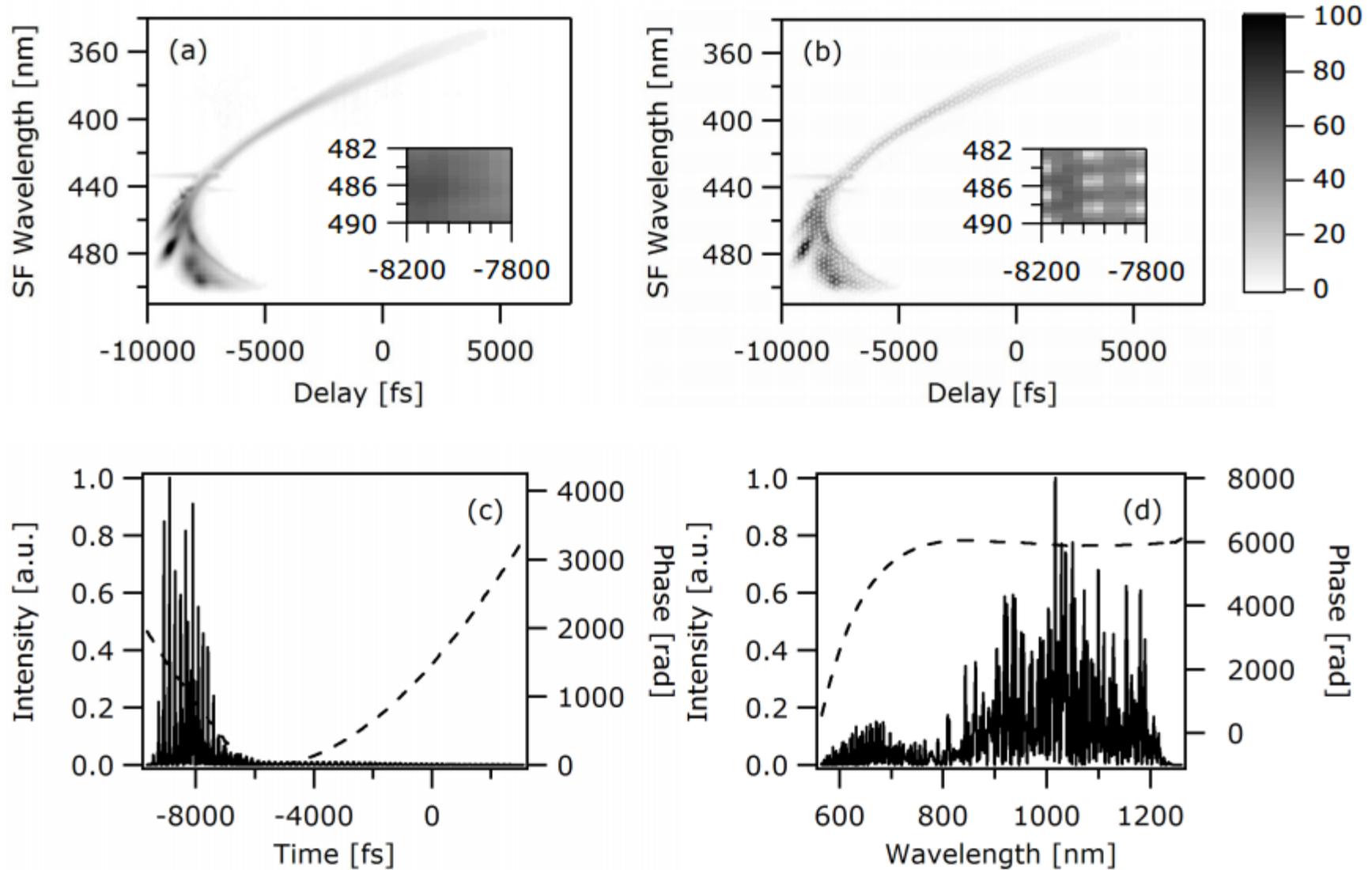
S. Linden et al., phys. stat. sol. (b) 206, 119 (1998)



# XFROG



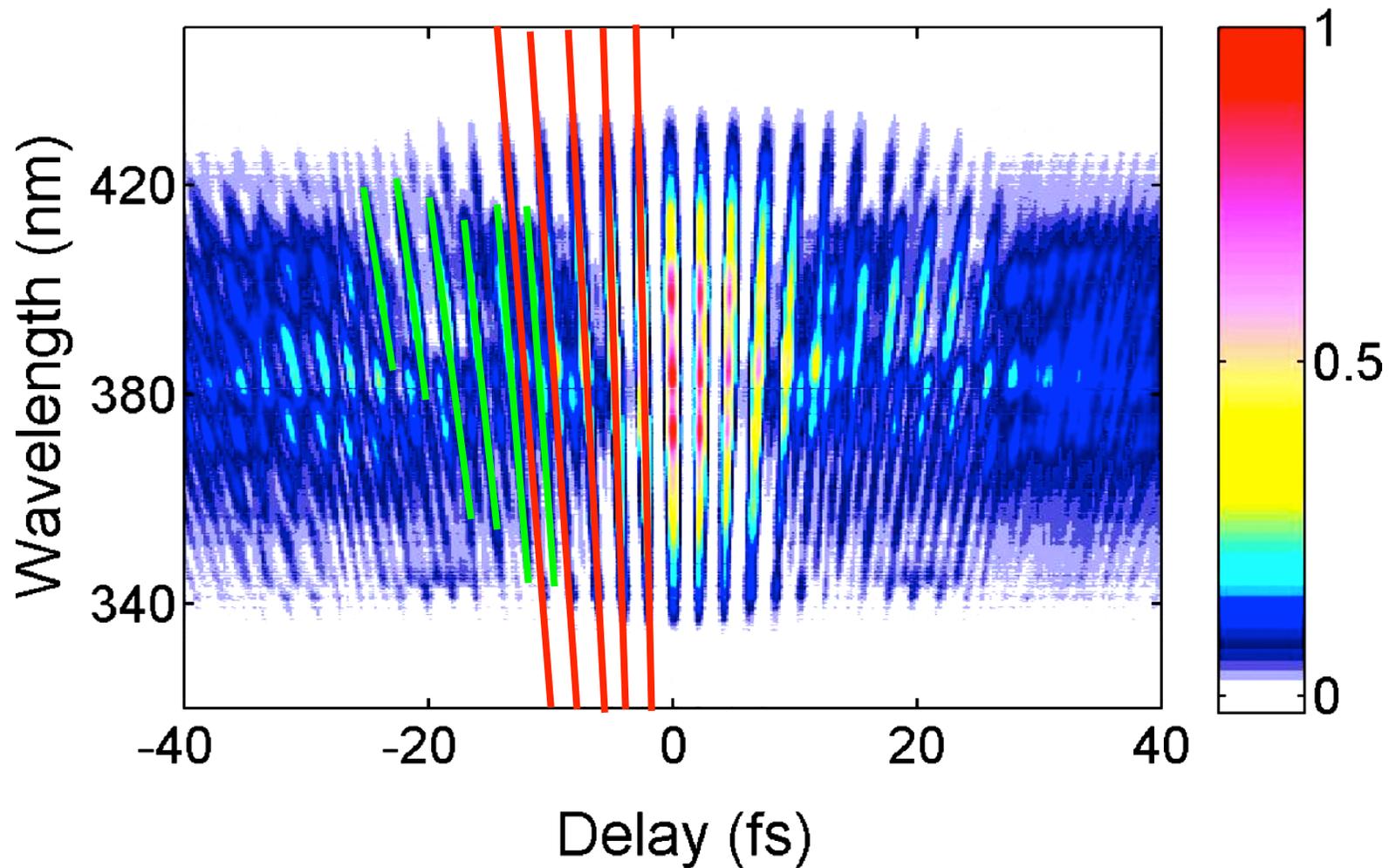
# *XFROG can measure extremely complex pulses*



thesis Xun Gu, Georgiatech



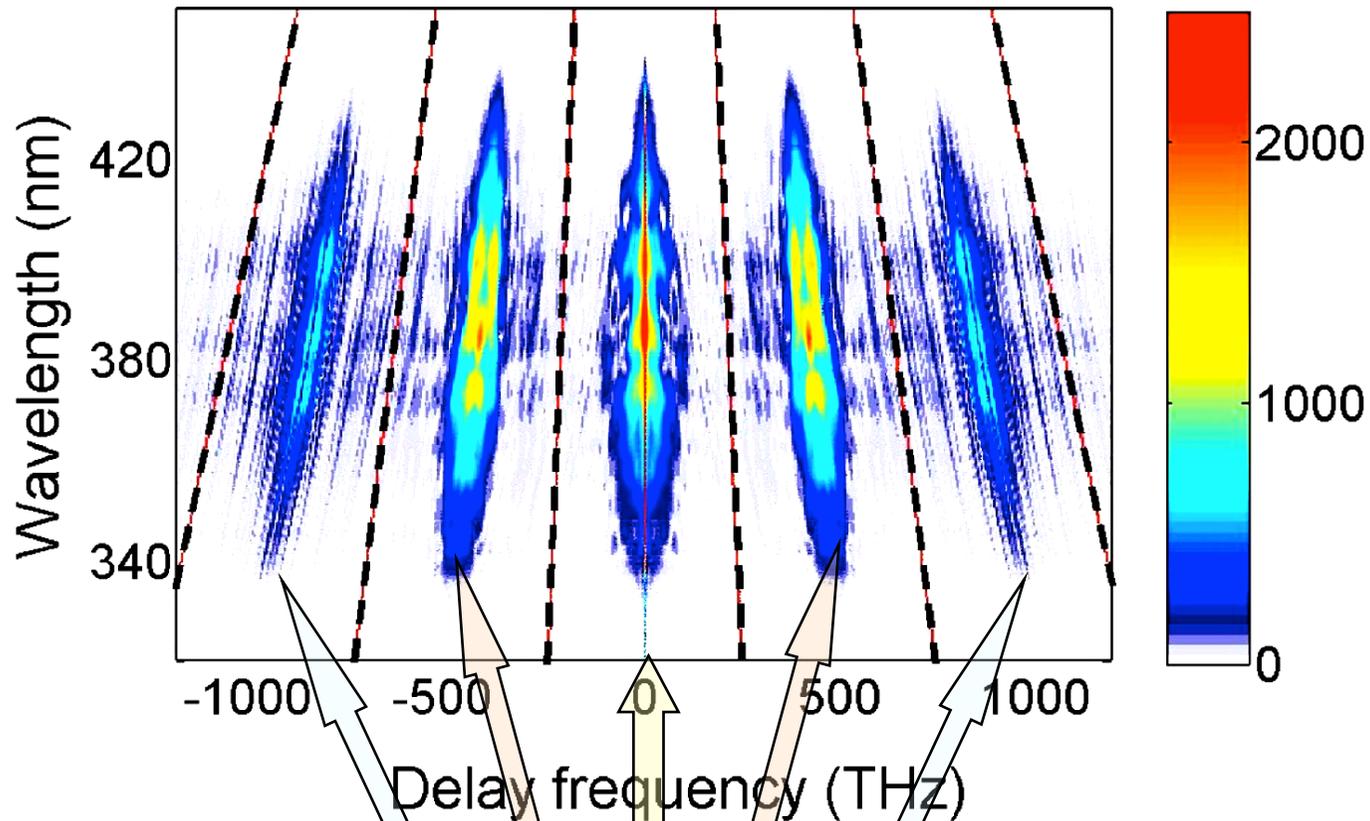
# Interferometric FROG



**Source: Hollow fiber continuum, compressed w/ chirped mirrors**



# Interferometric FROG



**3 components:**

**1.) DC**

**2.) fund. mod.**

**3.) SH mod.**

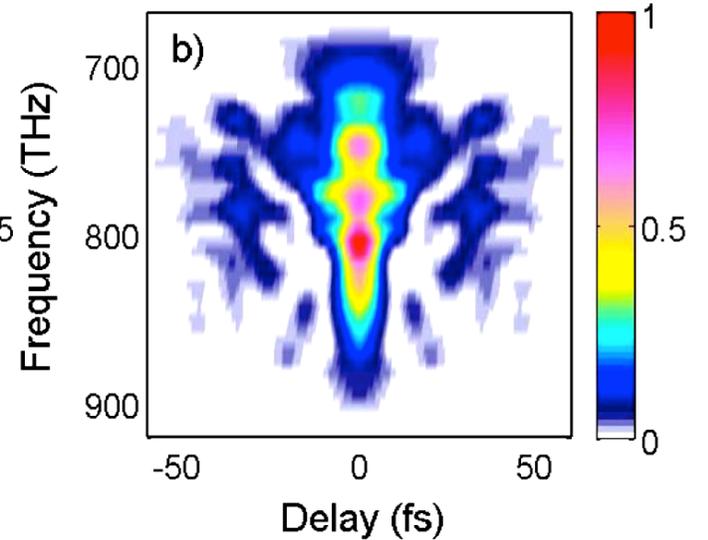
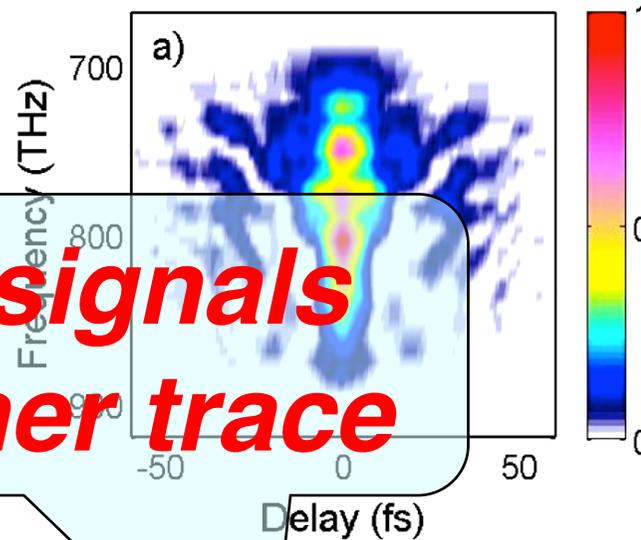
$I_{\text{IFROG}}(\omega, \tau)$

$$\begin{aligned}
 I_{\text{IFROG}}(\omega, \tau) = & \quad 2 |E_{\text{SH}}(\Delta\omega)|^2 + 4 |E_{\text{FROG}}(\Delta\omega, \tau)|^2 \\
 & + 8 \cos \left[ \left( \omega_0 + \frac{\Delta\omega}{2} \right) \tau \right] \text{Re} \left[ E_{\text{FROG}}(\Delta\omega, \tau) E_{\text{SH}}^*(\Delta\omega) \exp \left( i \frac{\Delta\omega}{2} \tau \right) \right] \\
 & + 2 \cos [(2\omega_0 + \Delta\omega) \tau] |E_{\text{SH}}(\Delta\omega)|^2 .
 \end{aligned}$$

# Interferometric FROG

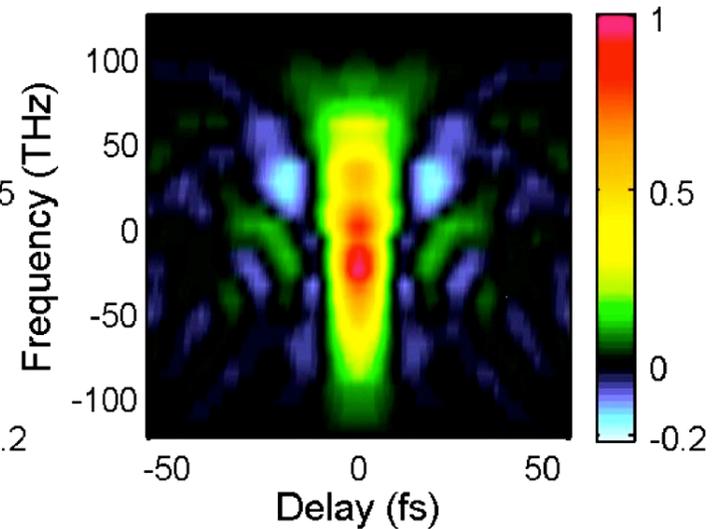
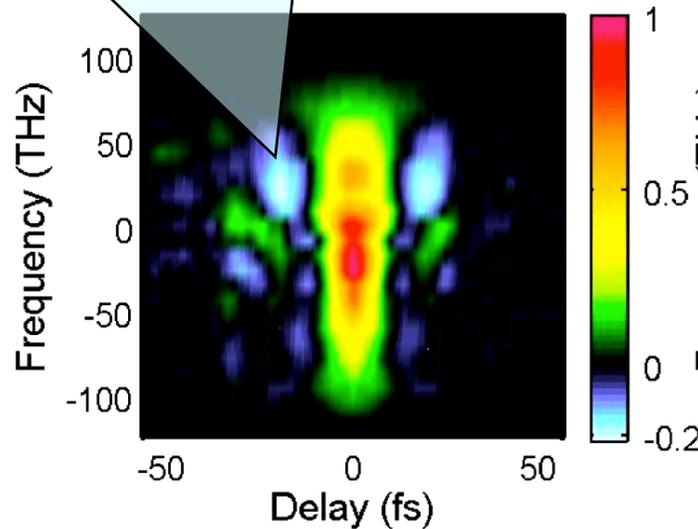
measured:

retrieved:



standard SH-FROG  
extracted from IFROG

*negative signals  
as in Wigner trace*



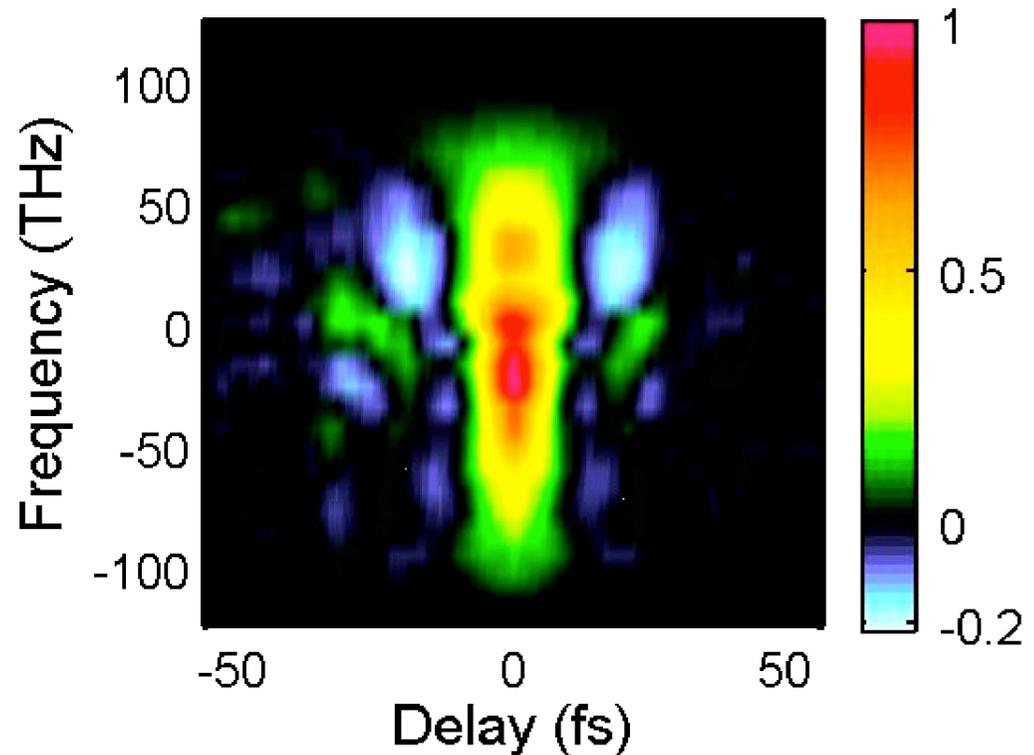
new kind of FROG-trace  
from modulation



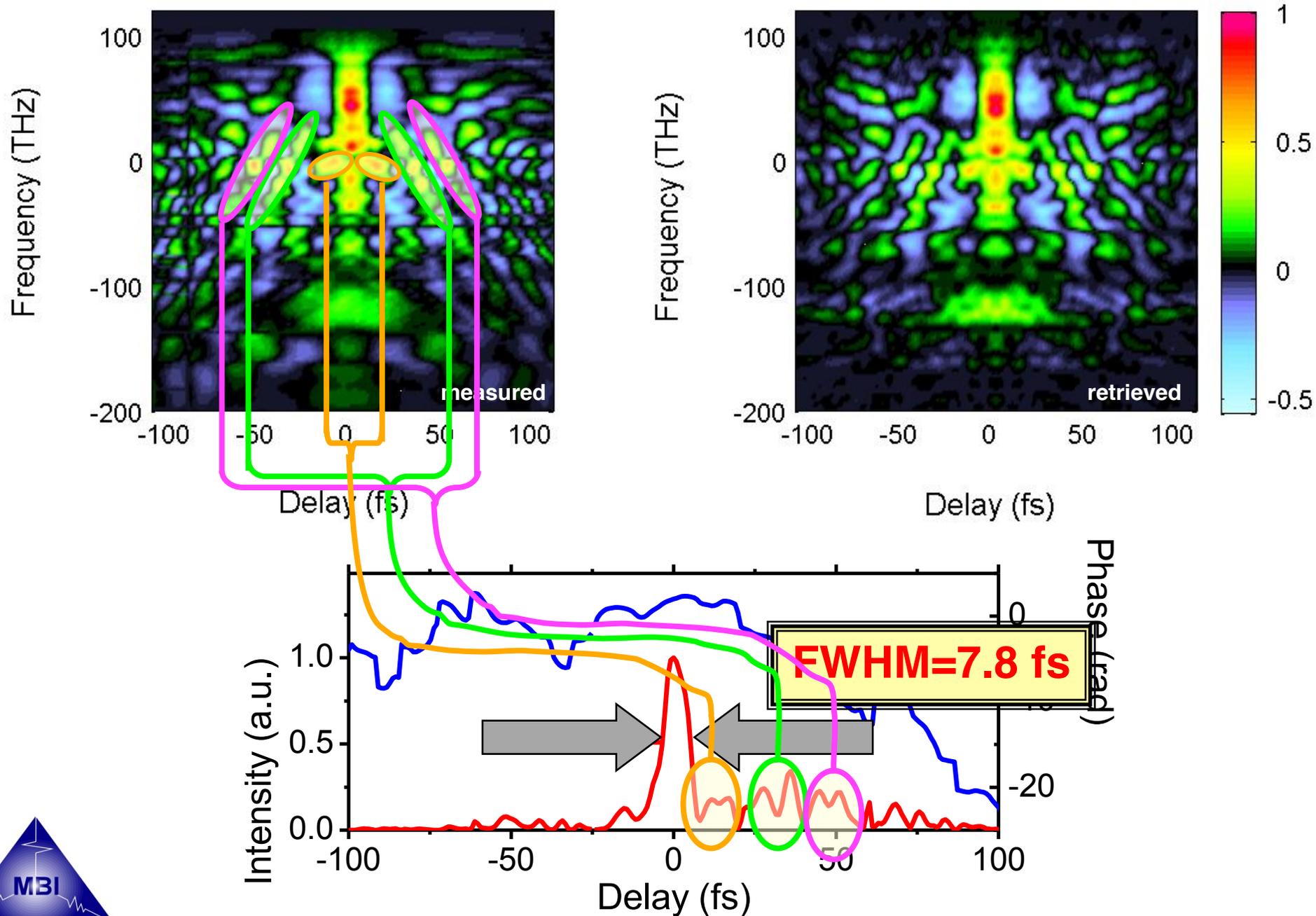
# Retrieval

- Don't use the full IFROG trace
- Extract the **F**undamental **M**odulation part
- Resample on smaller grid

FMFROG



# A complicated pulse



# Spectral phase interferometry for direct electric-field reconstruction

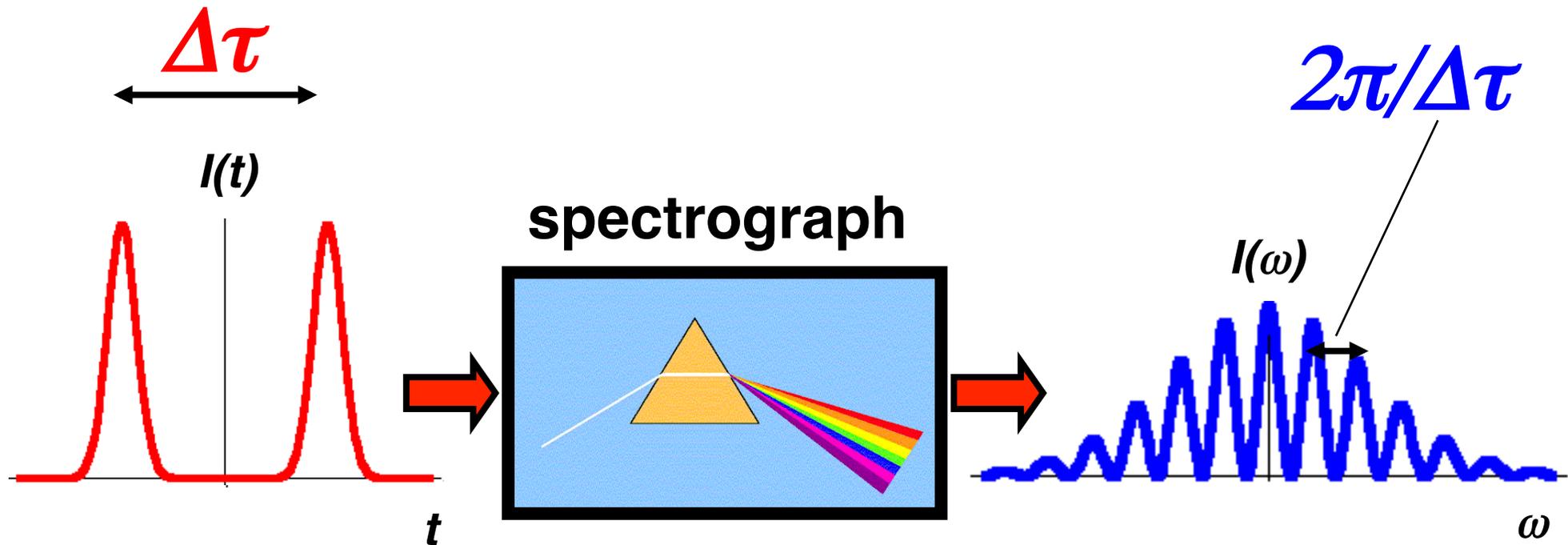
***SPIDER***



**Ian Walmsley**



# Characterization via spectral Interferometry

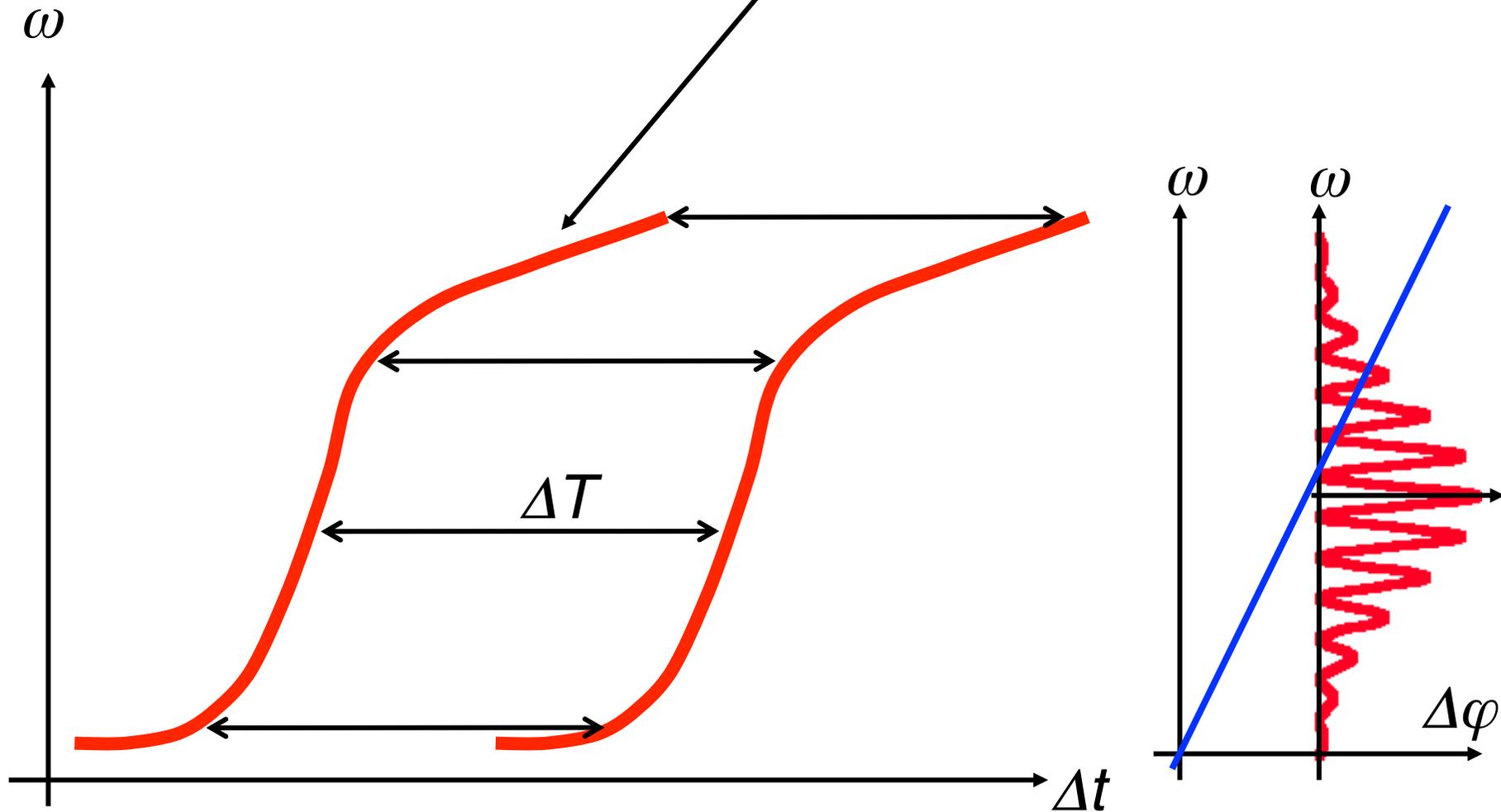


**Important:** Modulation period independent of  $\omega$  !

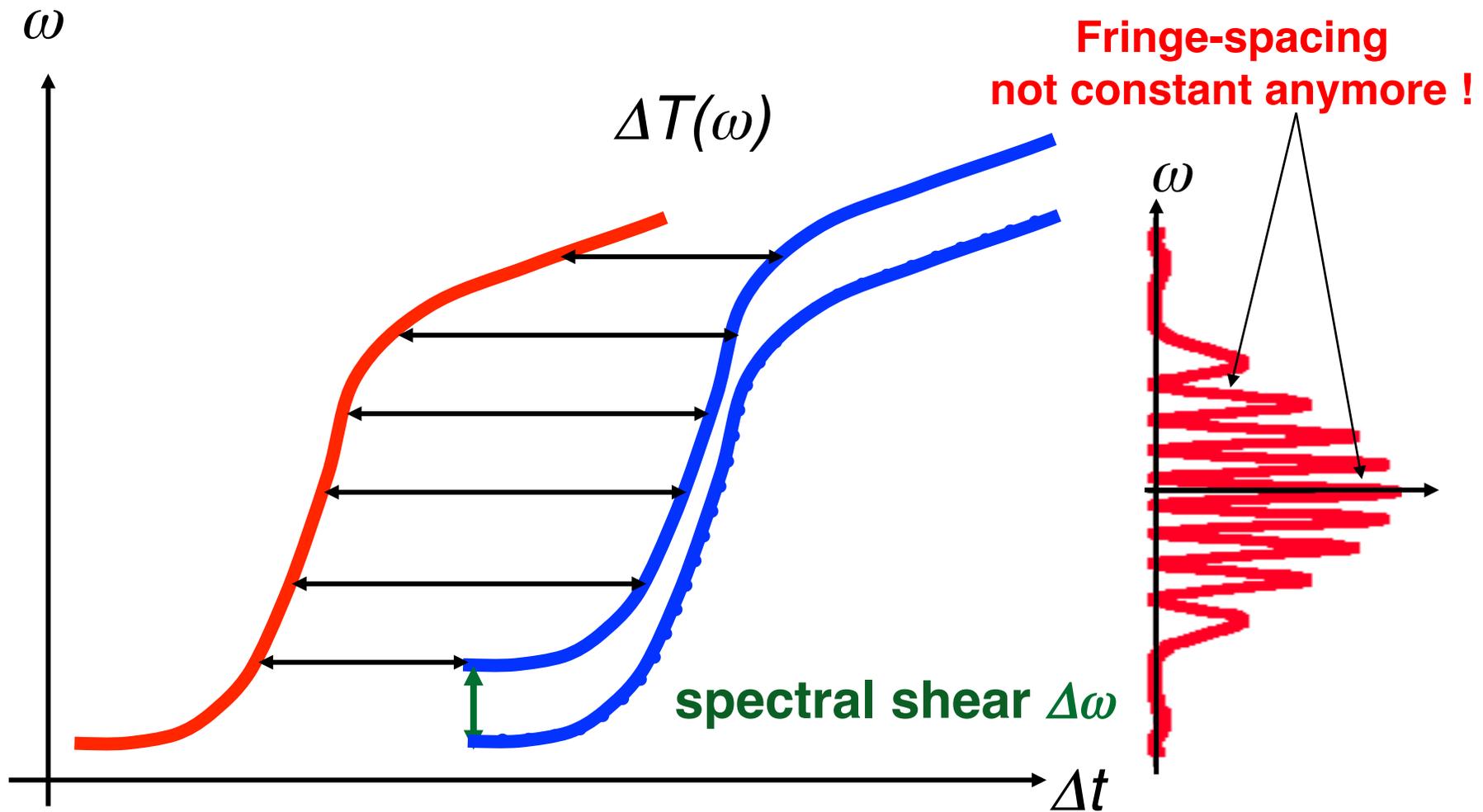


# Spectral Interference of phase fronts

$$E(\omega) = |A(\omega)| \cdot \exp(i\varphi(\omega)), \quad \varphi = \omega \cdot \Delta t$$

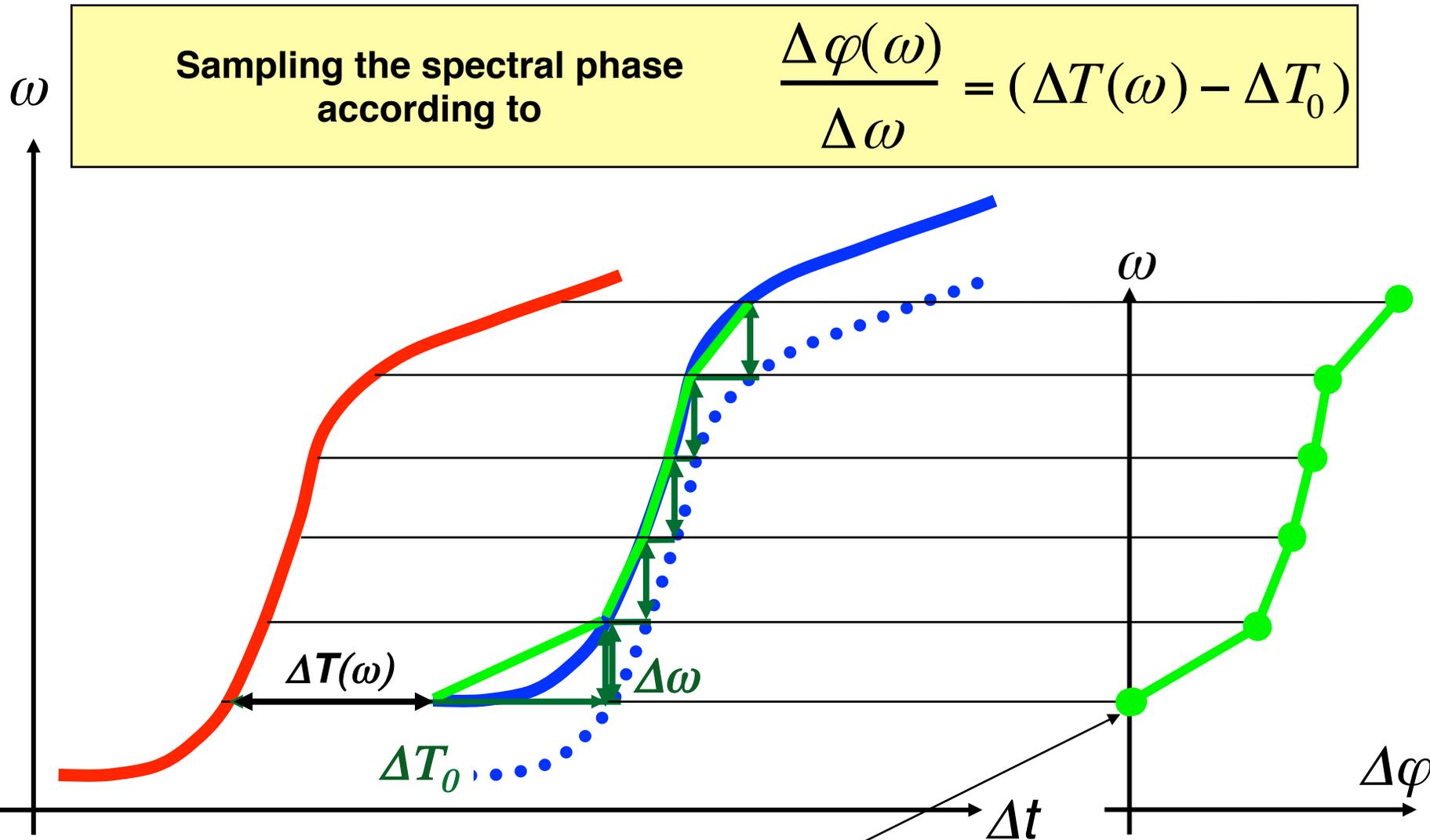


# Introducing the spectral shear



# Reconstruction of the spectral phase

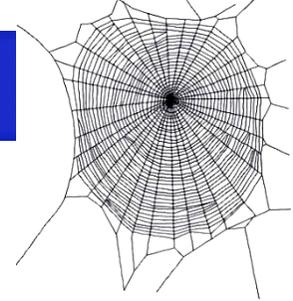
$$\varphi(\omega) = \omega \cdot \Delta T(\omega)$$



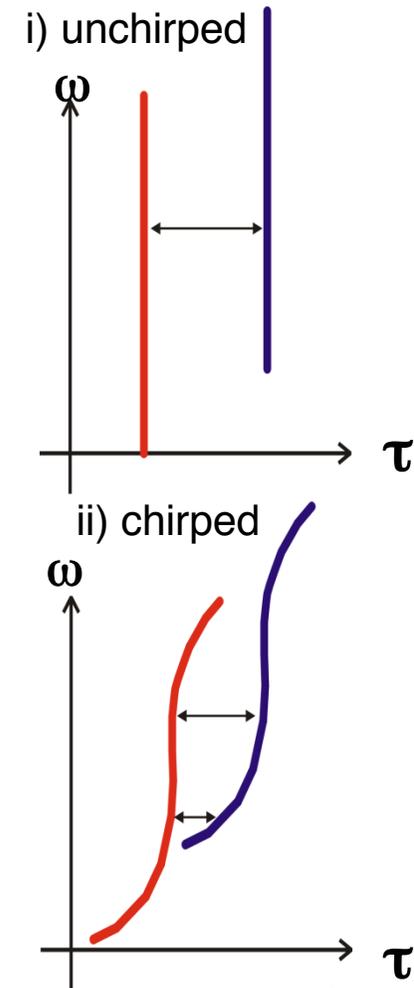
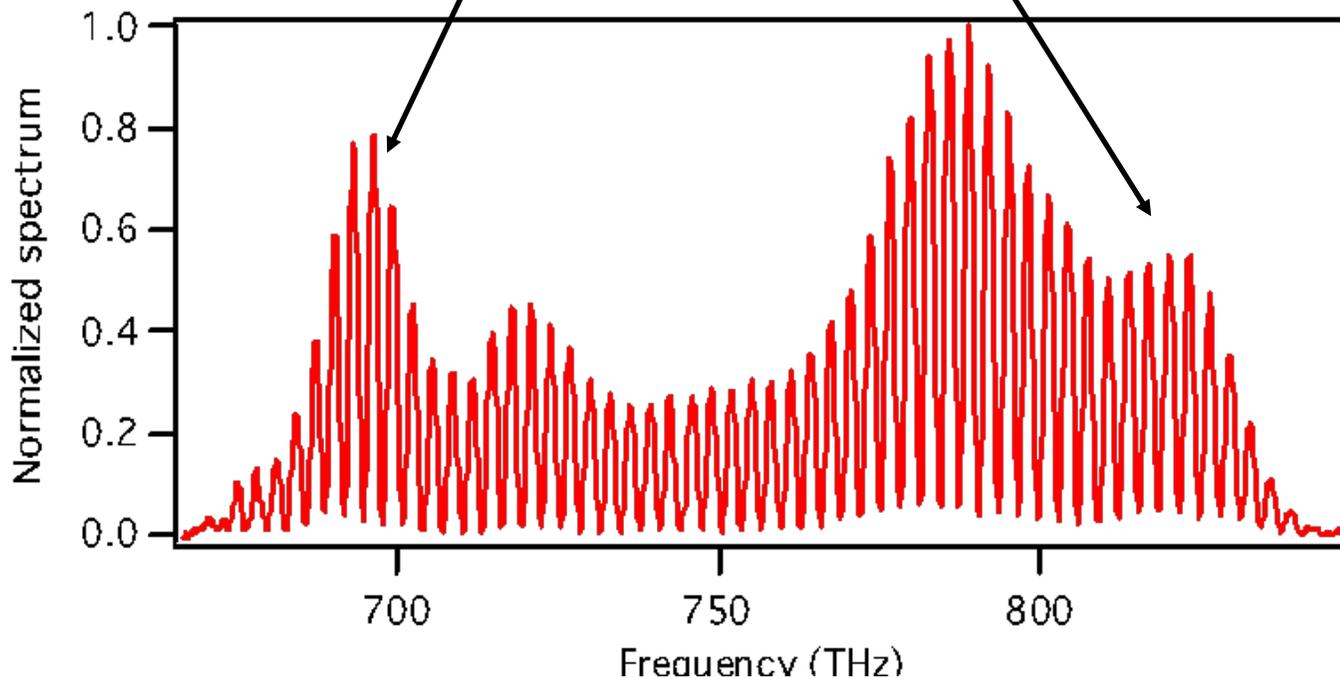
Choose phase start value arbitrarily



# Spectral interference pattern



➔ **Chirp causes spectrally dependent fringe spacing**

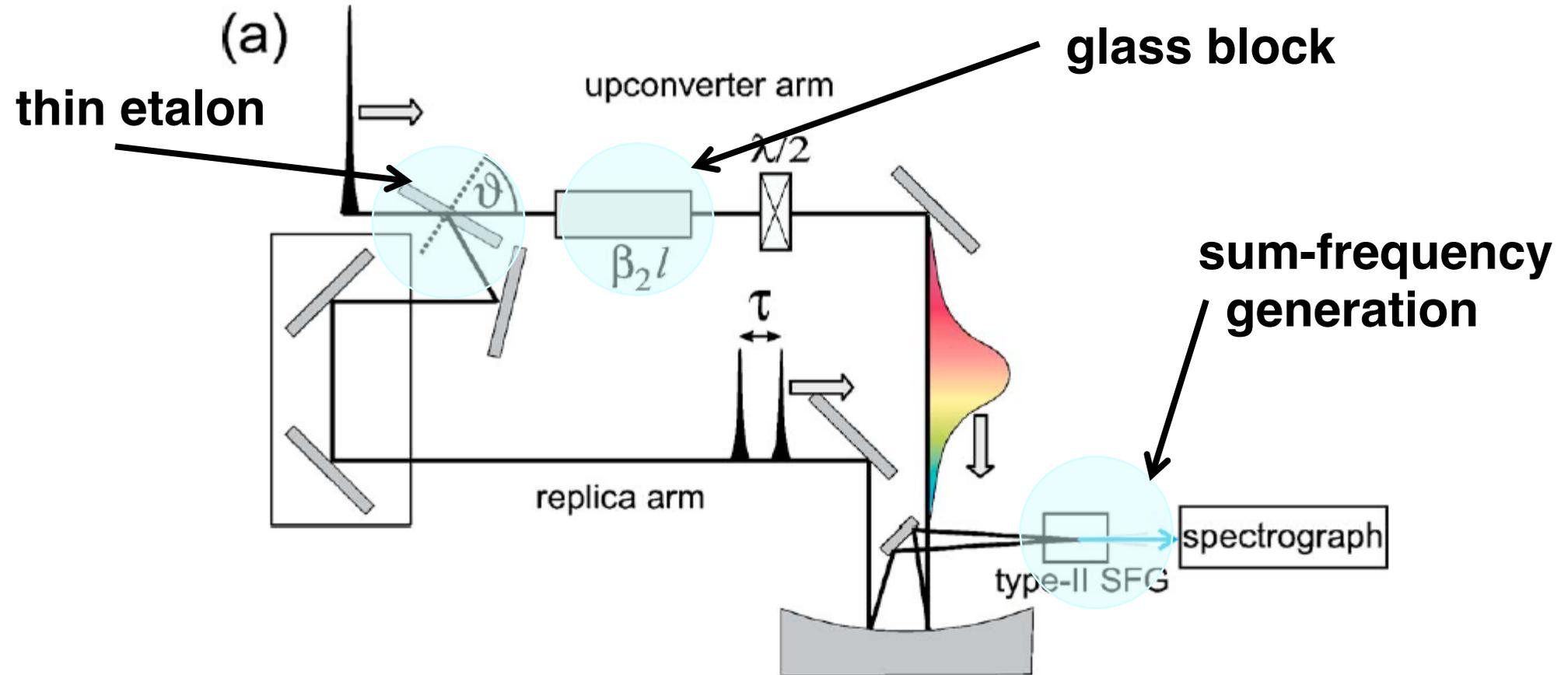
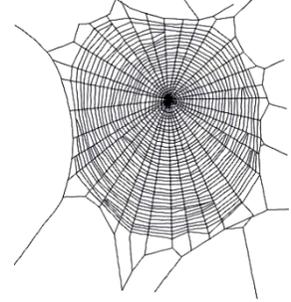


➔ **Phase information of spectral modulation**

➔ **Phase extraction does not require amplitude information**

# SPIDER

(Spectral Phase Interferometry for Direct E-field Reconstruction)



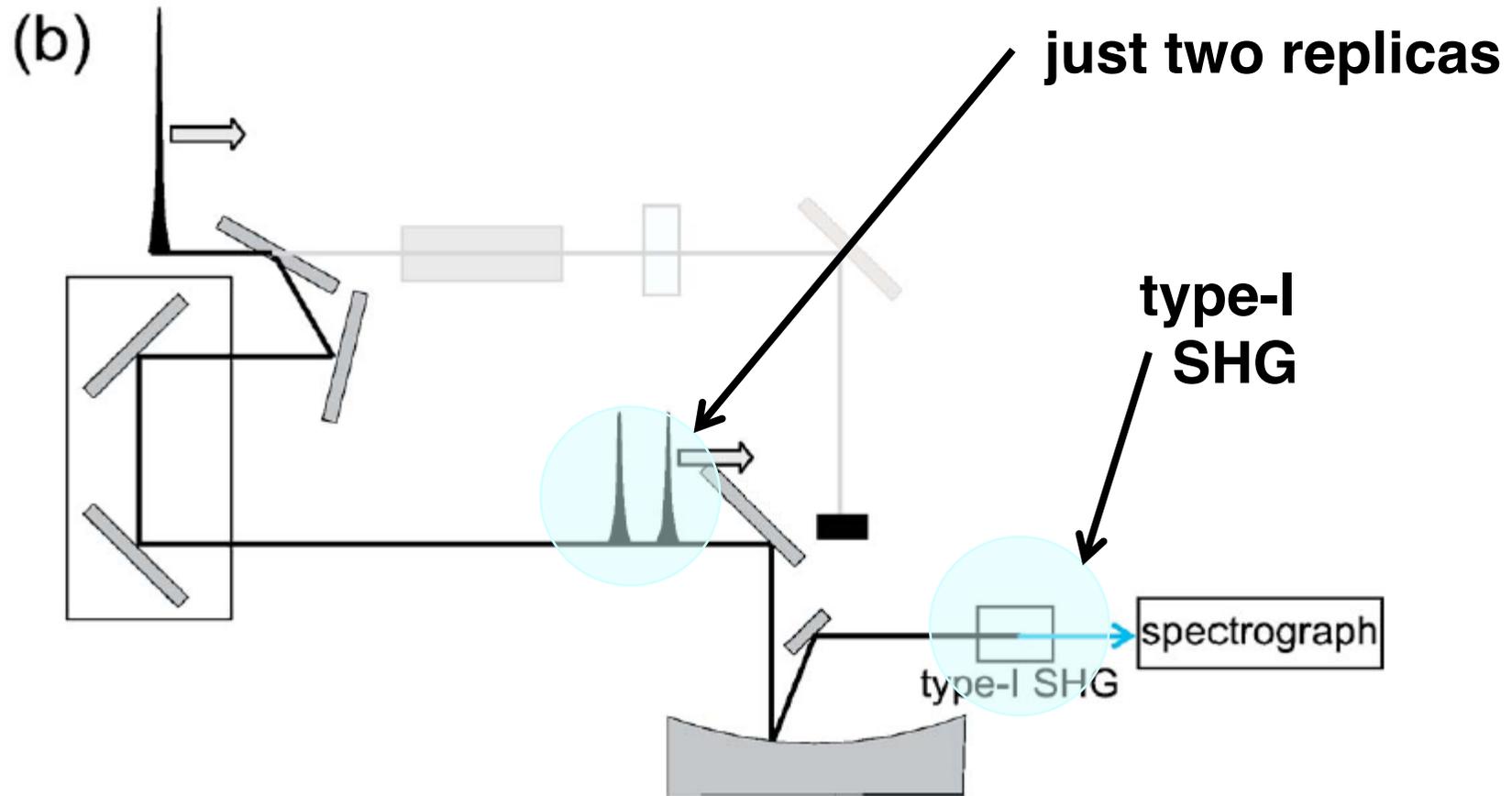
C. Iaconis, I.A. Walmsley, IEEE JQE **35**, 501 (1999)

Gallmann et al.: Opt. Lett. **24**, 1314 (1999)

Stibenz & Steinmeyer, Rev. Sci. Instrum. **77**, 073105 (2006)



# Calibration step



C. Iaconis, I.A. Walmsley, IEEE JQE **35**, 501 (1999)

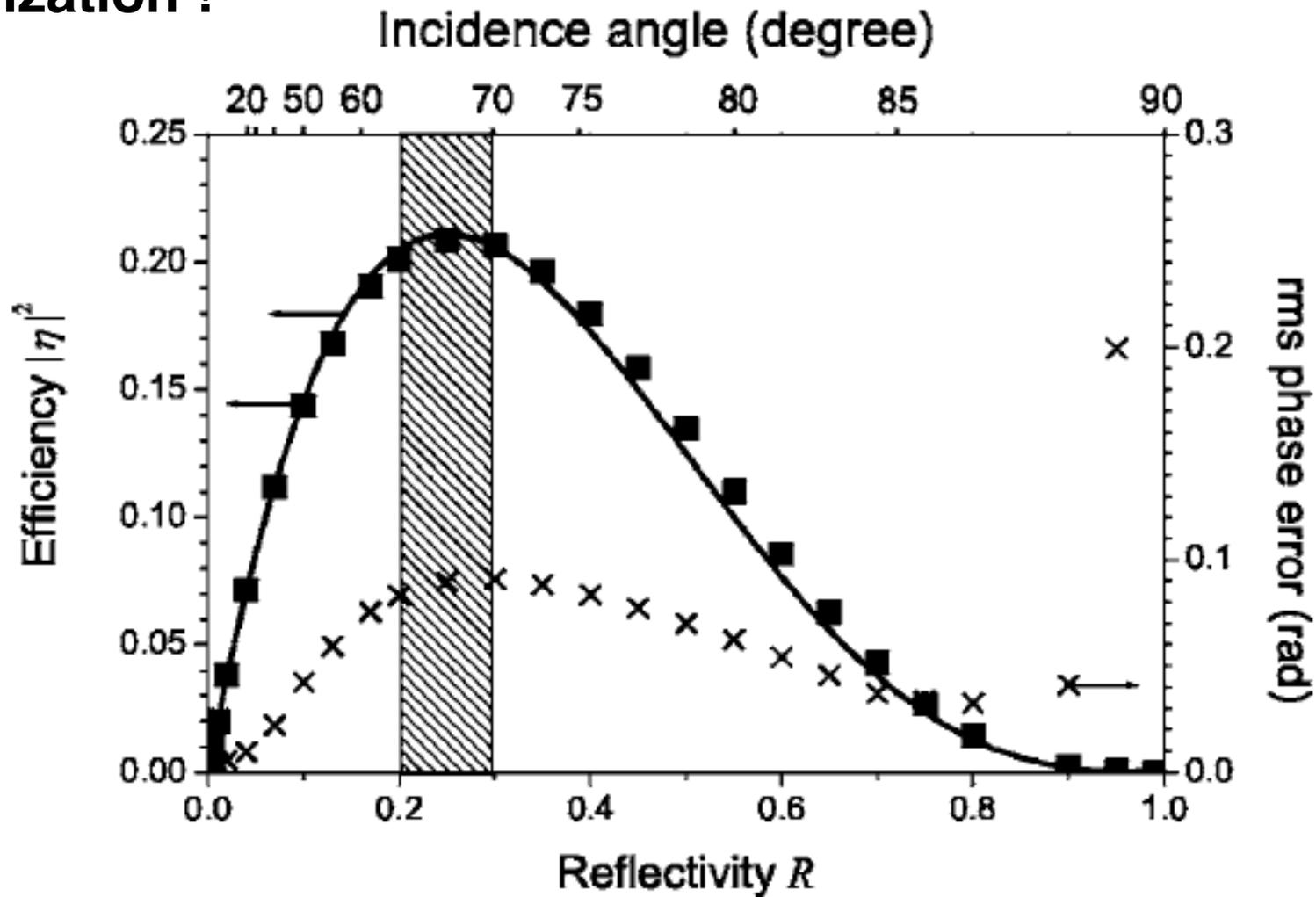
Gallmann et al.: Opt. Lett. **24**, 1314 (1999)

Stibenz & Steinmeyer, Rev. Sci. Instrum. **77**, 073105 (2006)

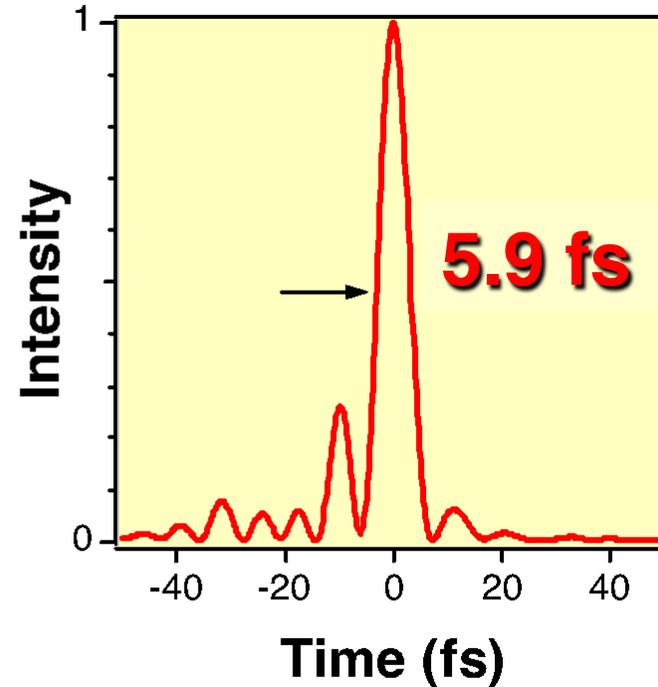
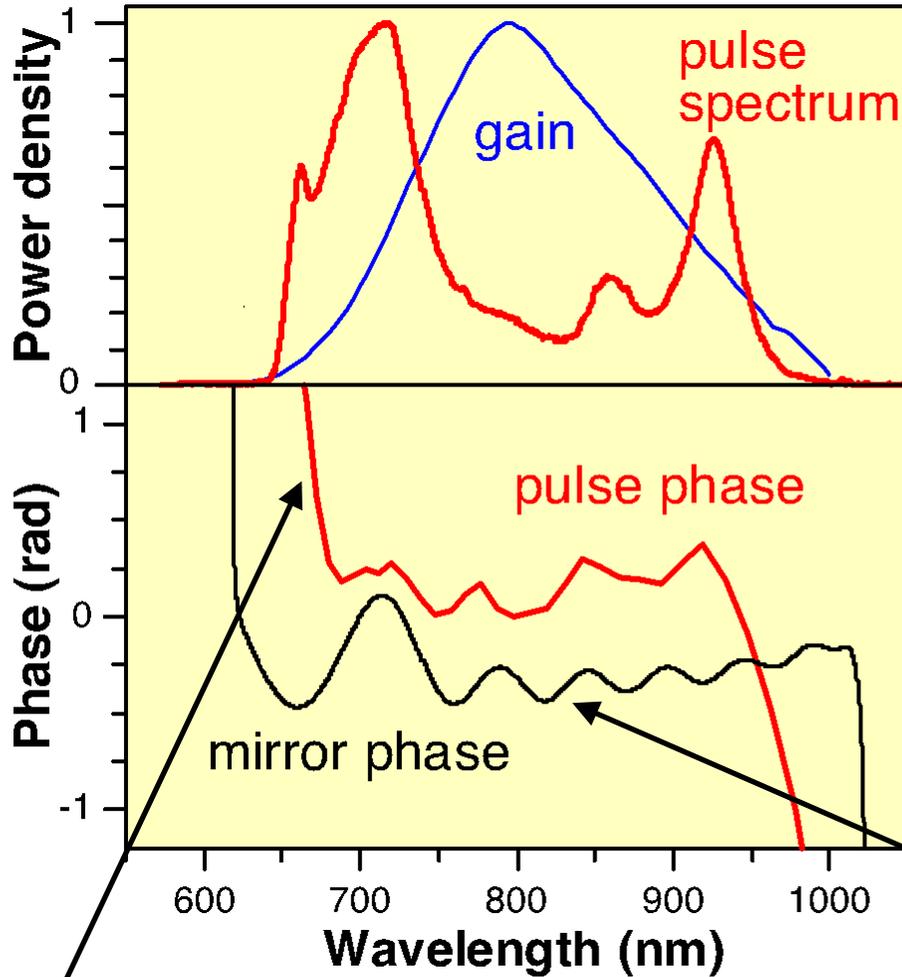
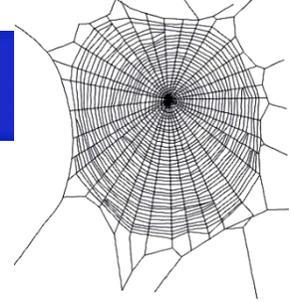


# *Etalon for optimum beam splitting*

**s polarization !**



# SPIDER-Results

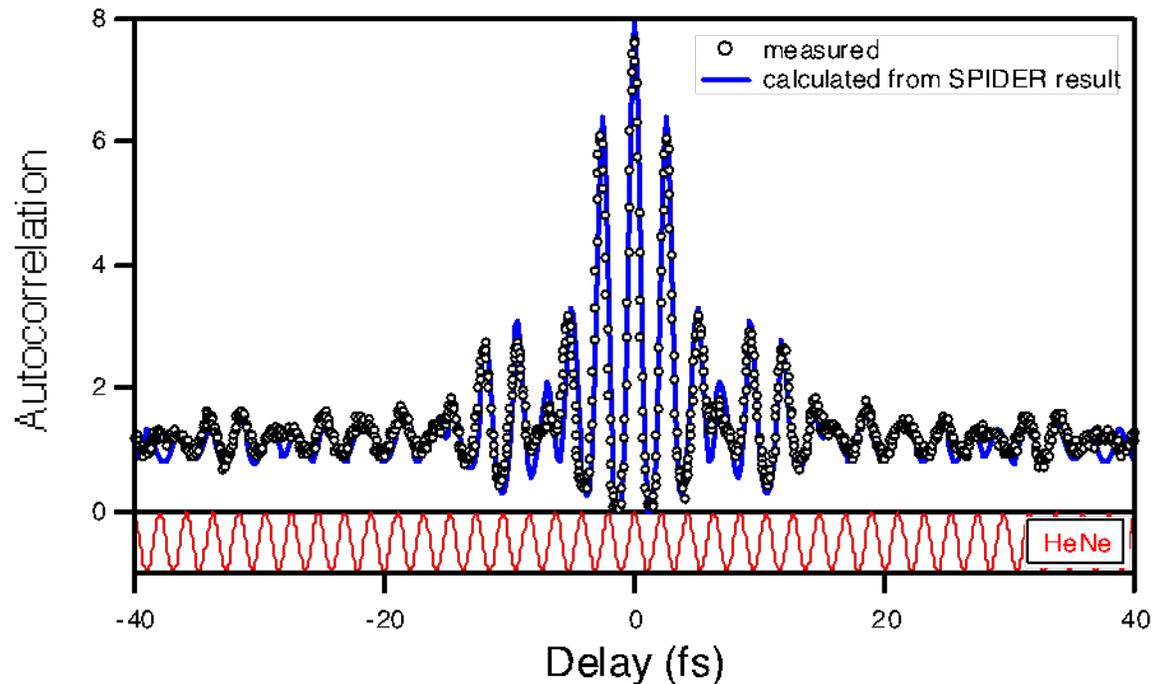
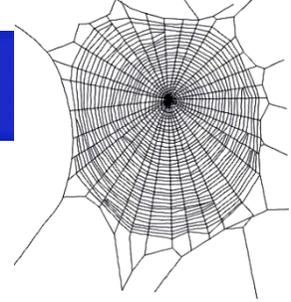


**Dispersion oscillations**

**Output coupler phase response ( $\lambda/4$  single stack)**



# SPIDER-Measurements



**SPIDER yields excellent agreement  
with independently measured IAC**



**SPIDER enables measurement of a  $GDD < 2 \text{ fs}^2$   
( $\approx 10 \text{ cm air path...}$ )**



.. Gallmann et al., Appl. Phys. B 70 [Suppl.], S67–S75 (2000)

# Advanced SPIDER

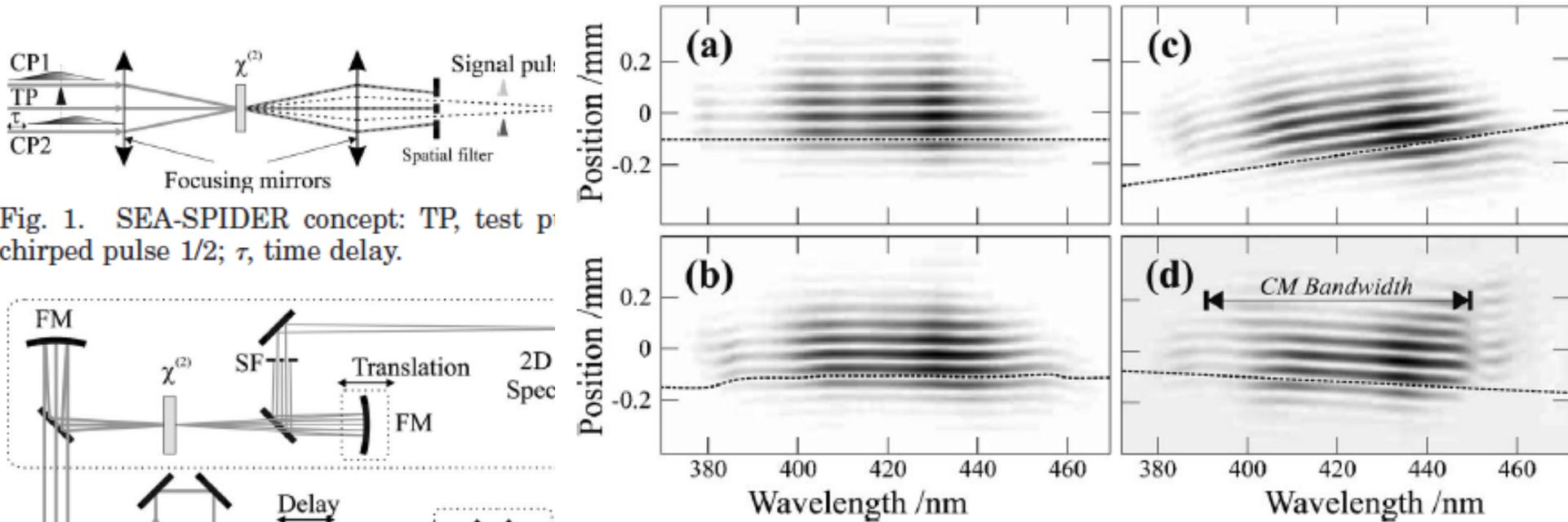


Fig. 1. SEA-SPIDER concept: TP, test pulse; CP1, CP2, chirped pulses; Focusing mirrors; Signal pulse; Spatial filter; Position /mm; Wavelength /nm.

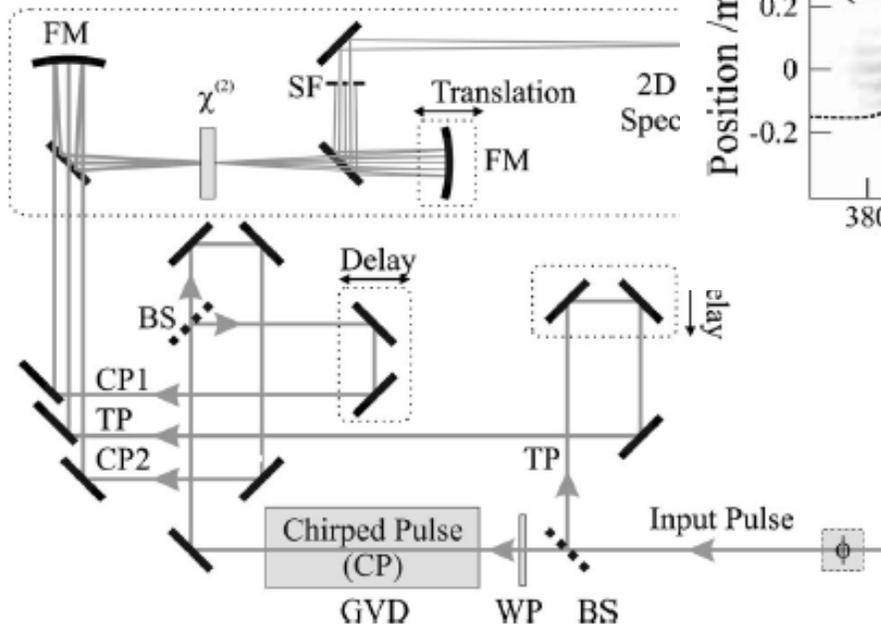


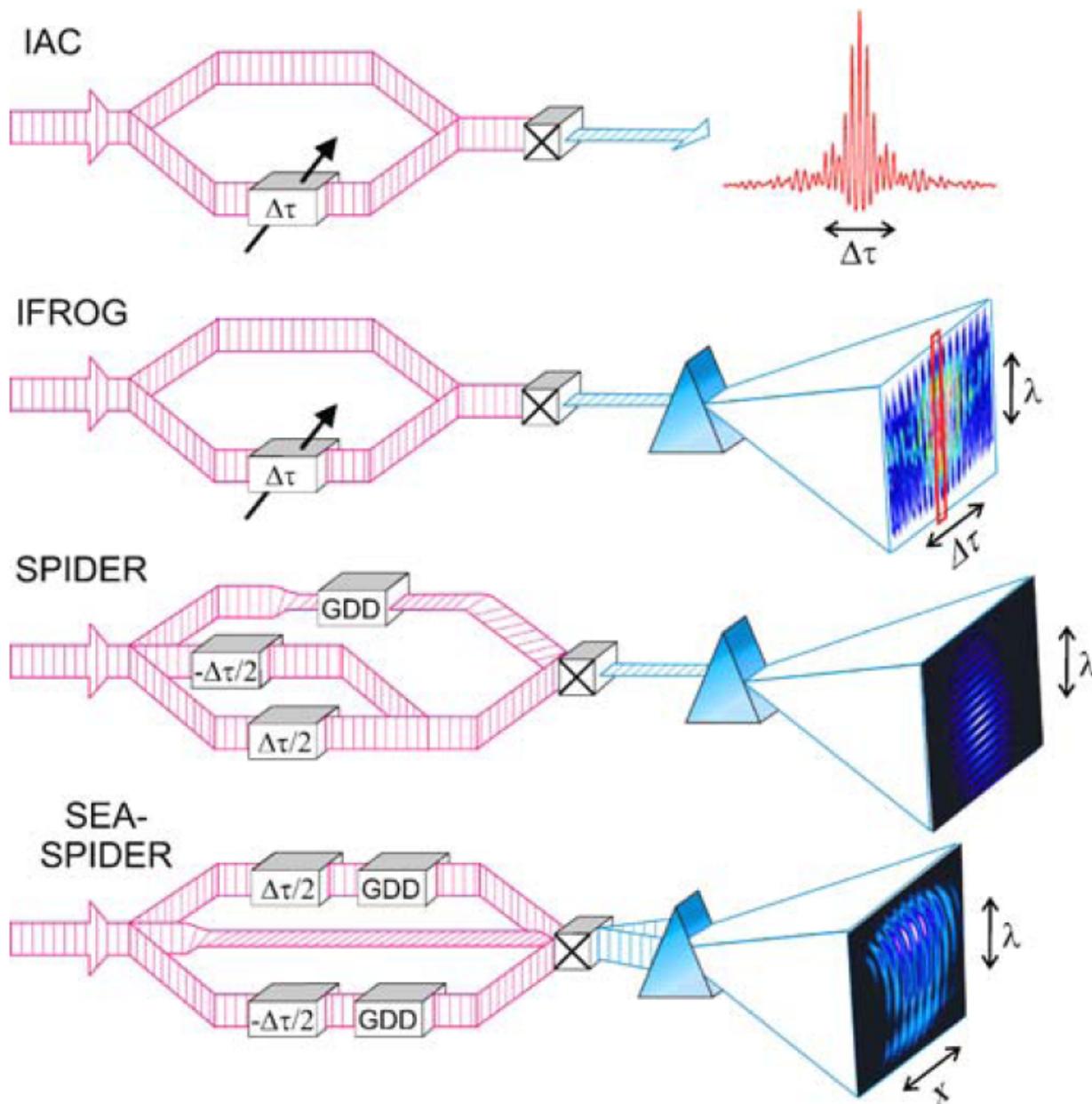
Fig. 2. SEA-SPIDER setup: BS, beam splitter; WP,  $\lambda/2$  wave plate; GVD, dispersive glass block (10 cm SF10); FM, focusing mirror ( $f=100$  mm);  $\chi^{(2)}$ , nonlinear crystal ( $30 \mu\text{m}$   $\beta$ -barium borate, type II); SF, spatial filter;  $\phi$ , optional additional phase.

**SEA-SPIDER = spatially encoded arrangement for SPIDER**

A. S. Wyatt, et al. Opt. Lett. **31**, 1914-1916 (2006)



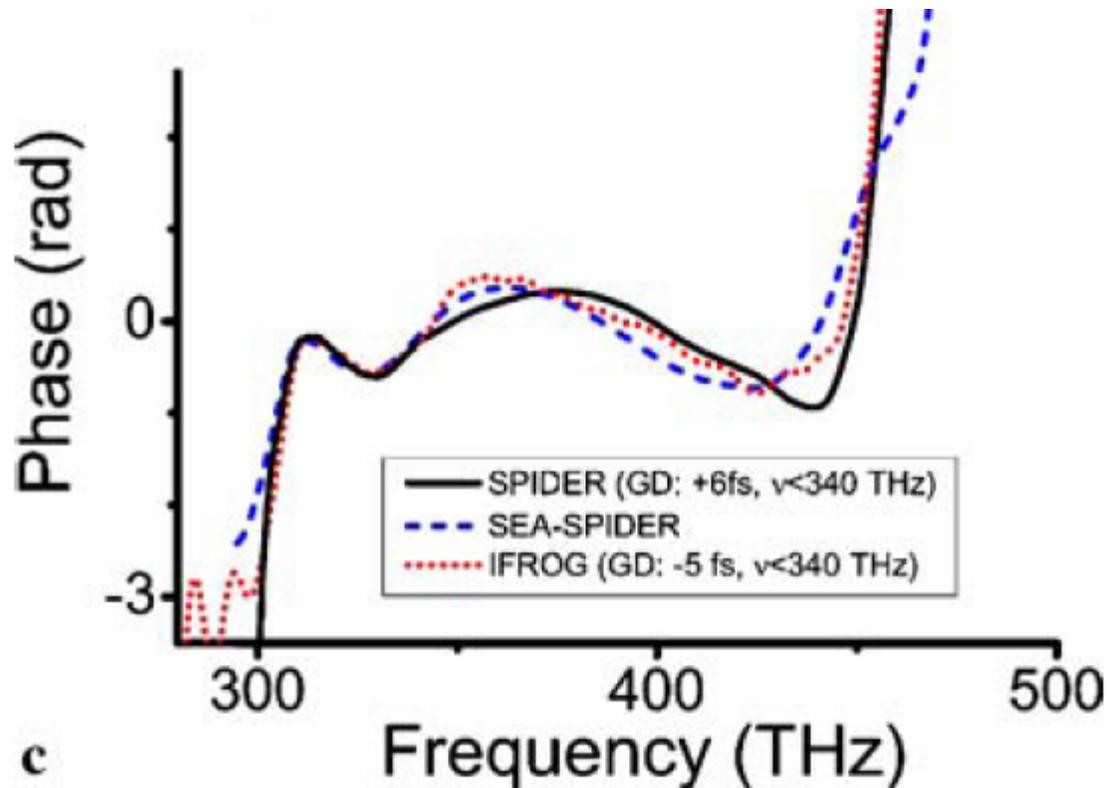
# Comparison of characterization architectures



G. Stibenz et al., Appl. Phys. B 83, 511–519 (2006)



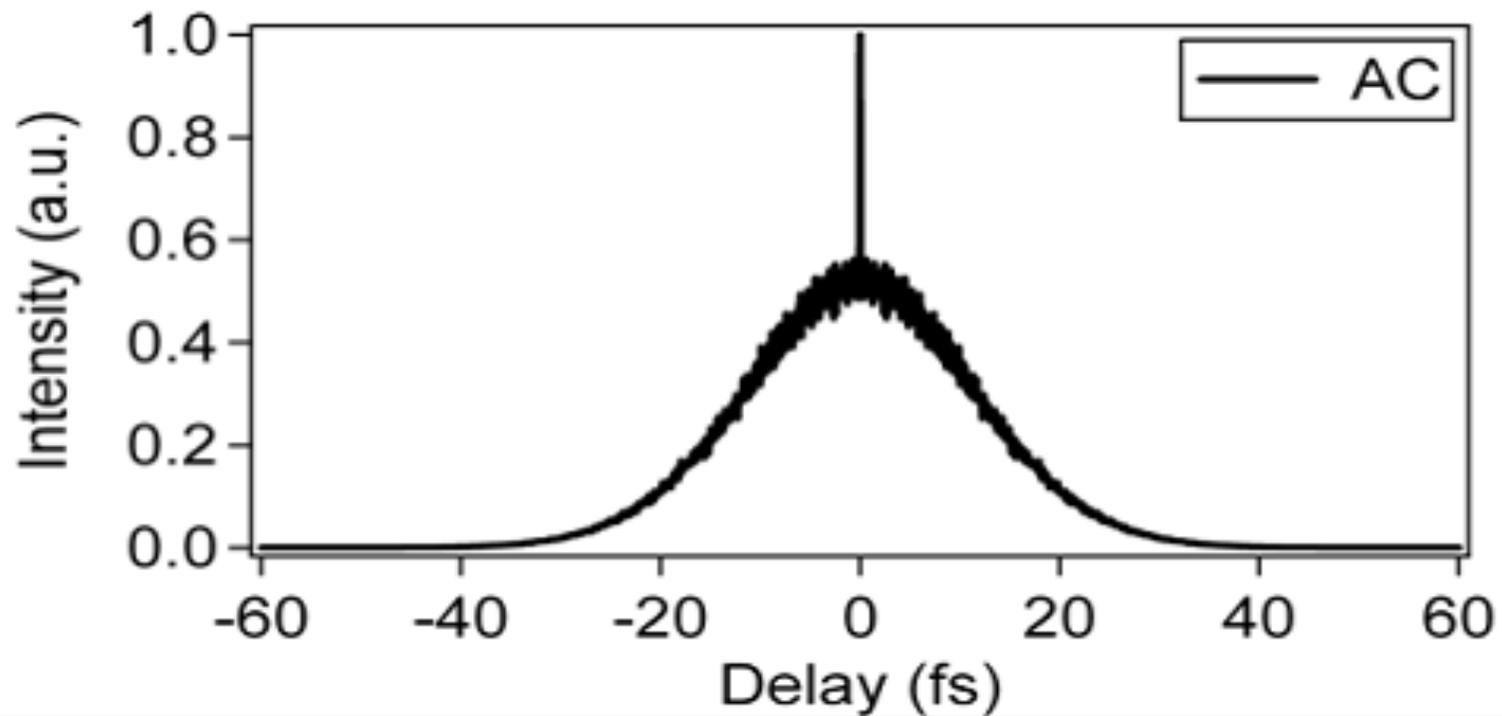
# Comparison



G. Stibenz et al., Appl. Phys. B 83, 511–519 (2006)

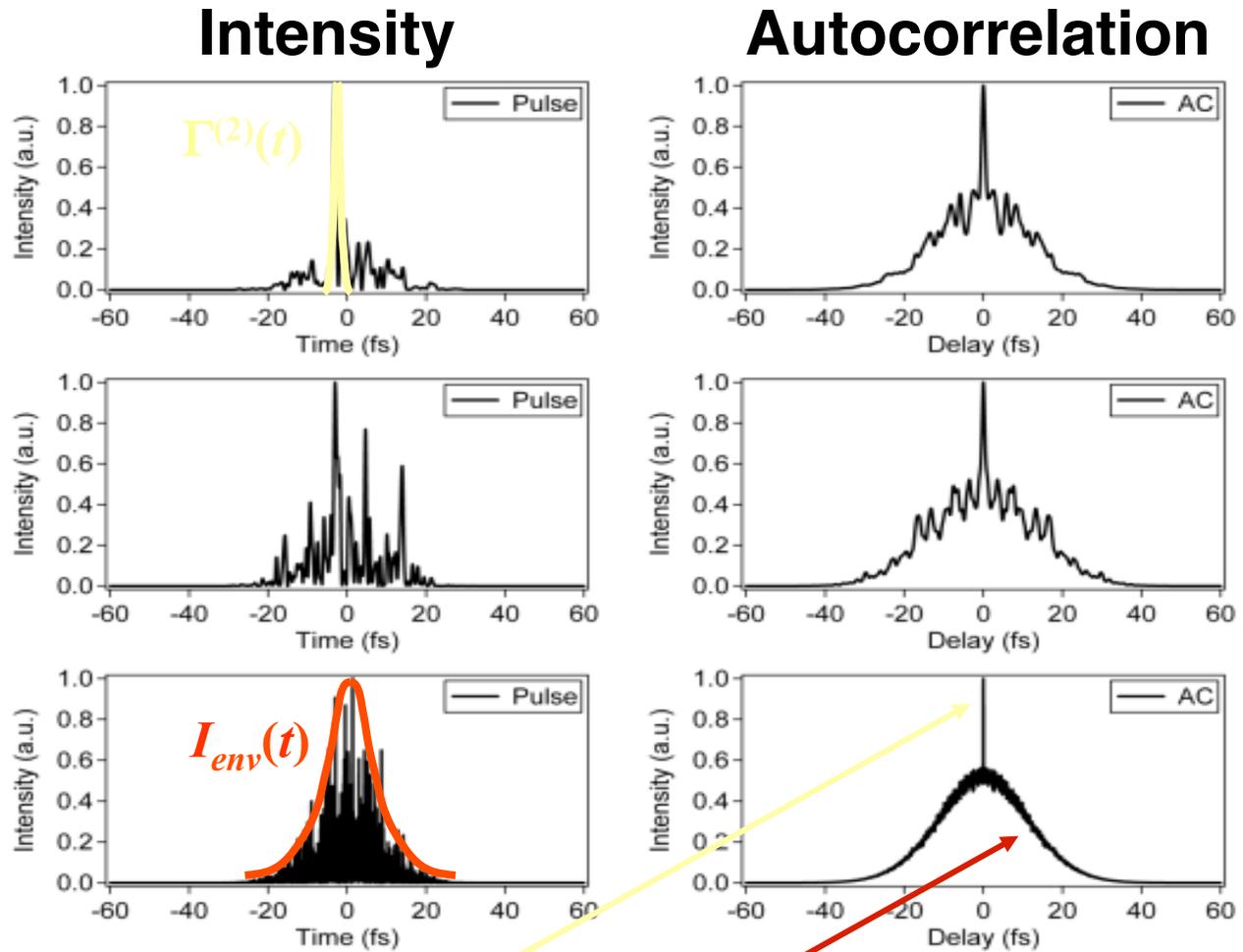


## The coherent artifact



As the intensity increases in complexity, its autocorrelation approaches a broad smooth background and a coherence spike.

This shows why retrieving the intensity from the autocorrelation is fundamentally impossible!



$$A^{(2)}(\tau) = \left| \Gamma^{(2)}(\tau) \right|^2 + \int_{-\infty}^{\infty} I_{env}(t) I_{env}(t - \tau) dt$$



# The coherent artifact

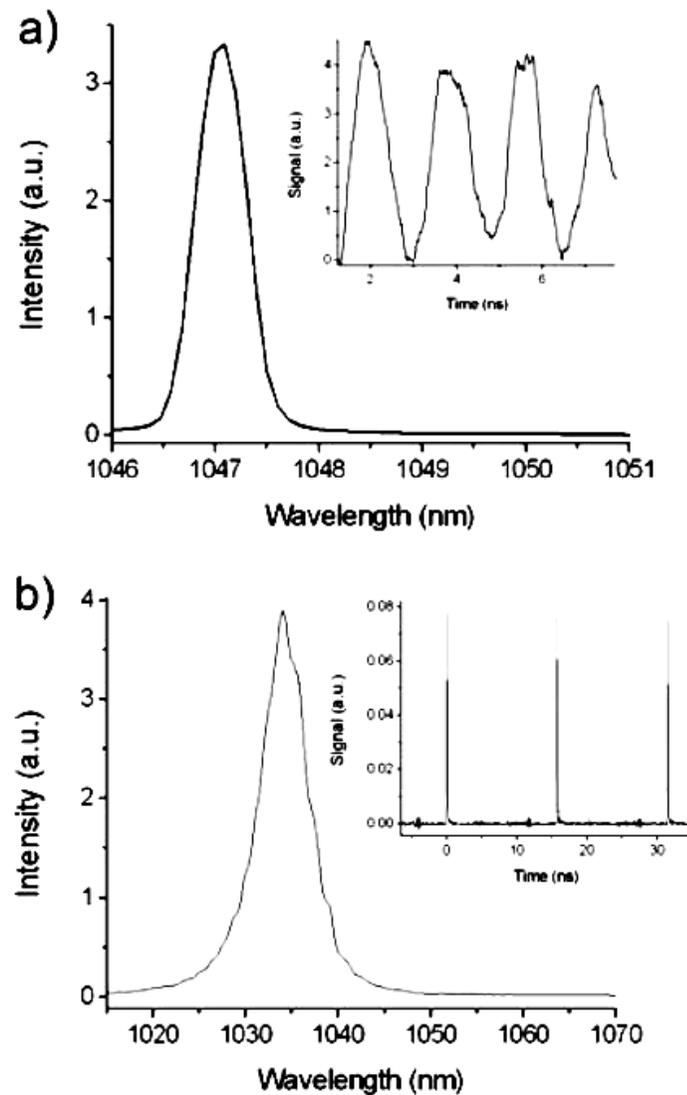
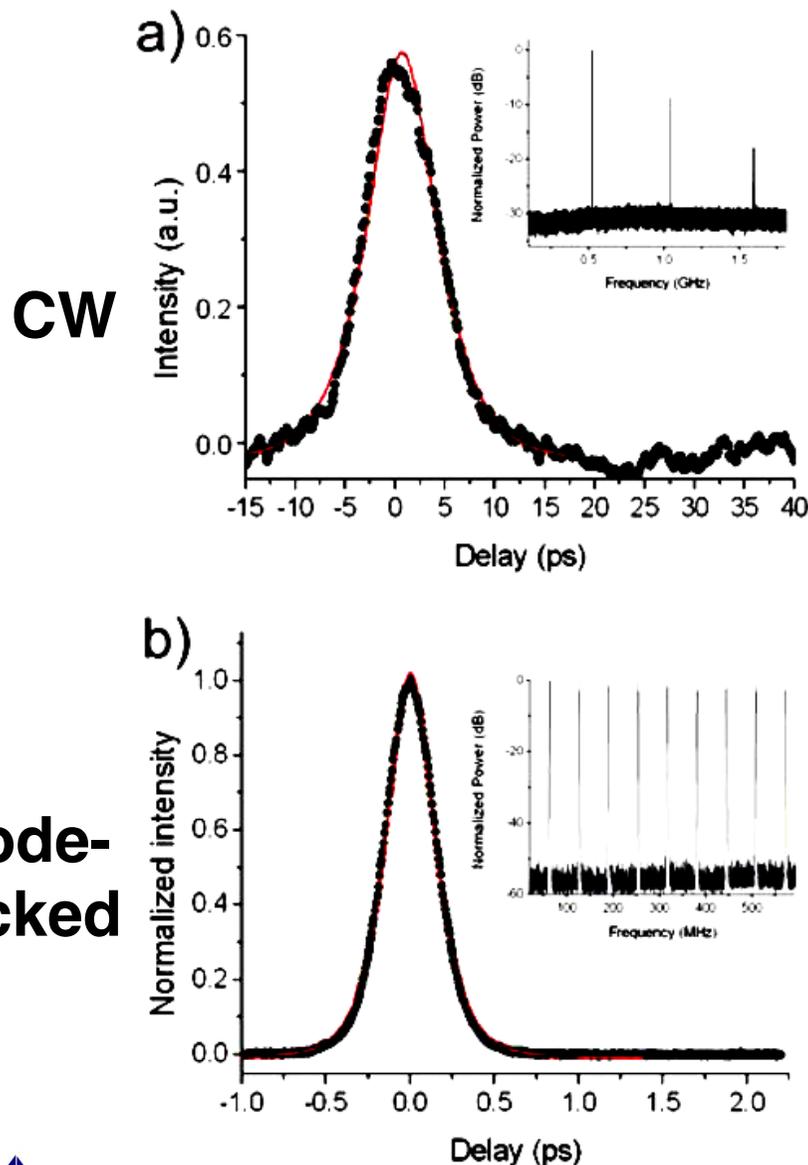
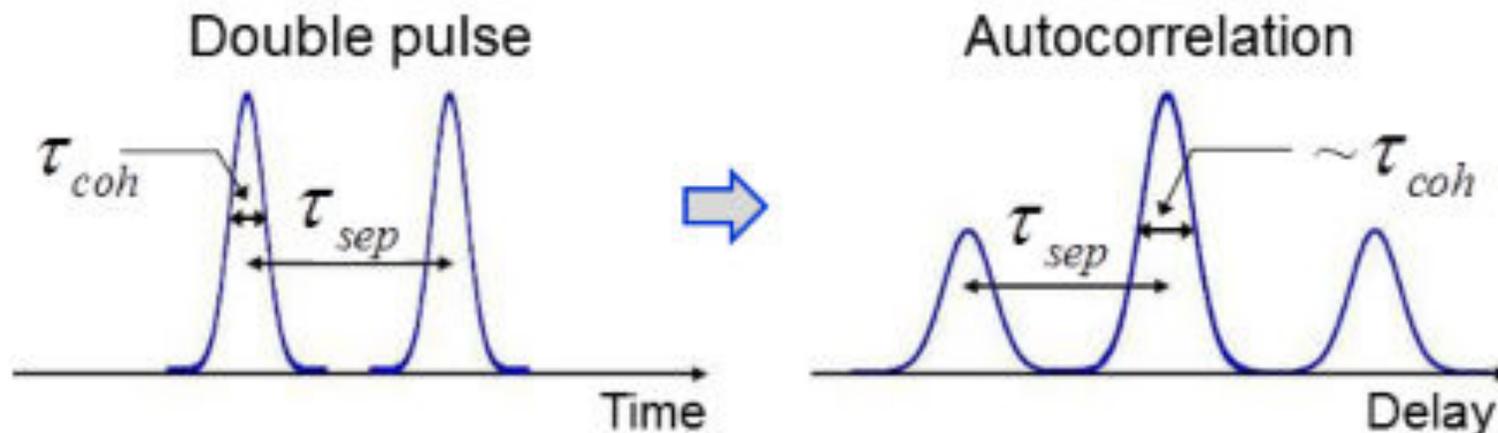


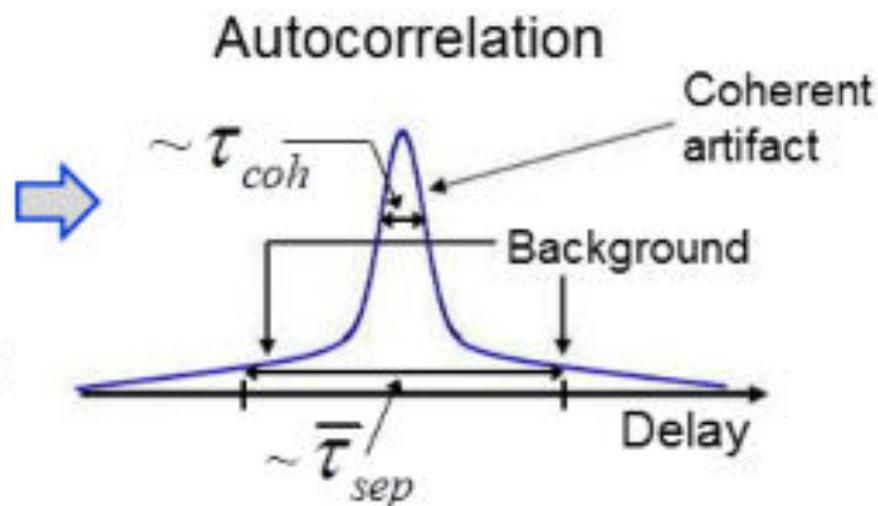
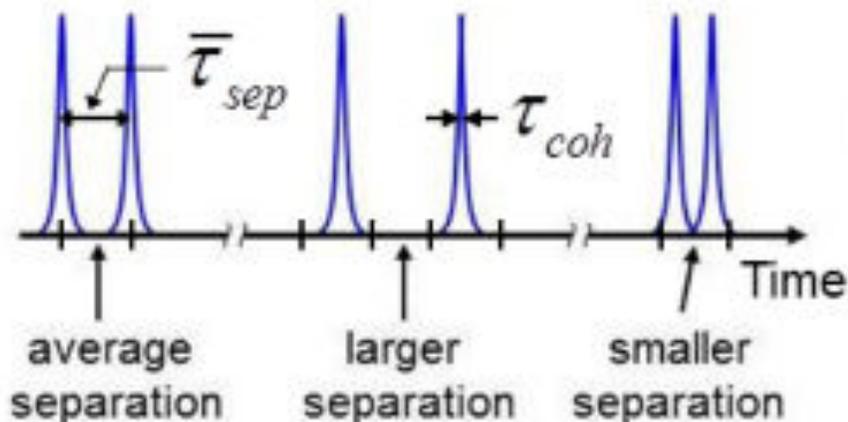
Figure 2 Optical spectrum and fast oscilloscope trace (inset) for a) a Nd:YLF CW laser and b) a mode-locked Yb:glass laser.

Figure 1 Autocorrelation and RF spectrum (inset) for a) a Nd:YLF CW laser and b) a mode-locked Yb:glass laser. Both autocorrelations include a sech<sup>2</sup> fit to the data (red lines).

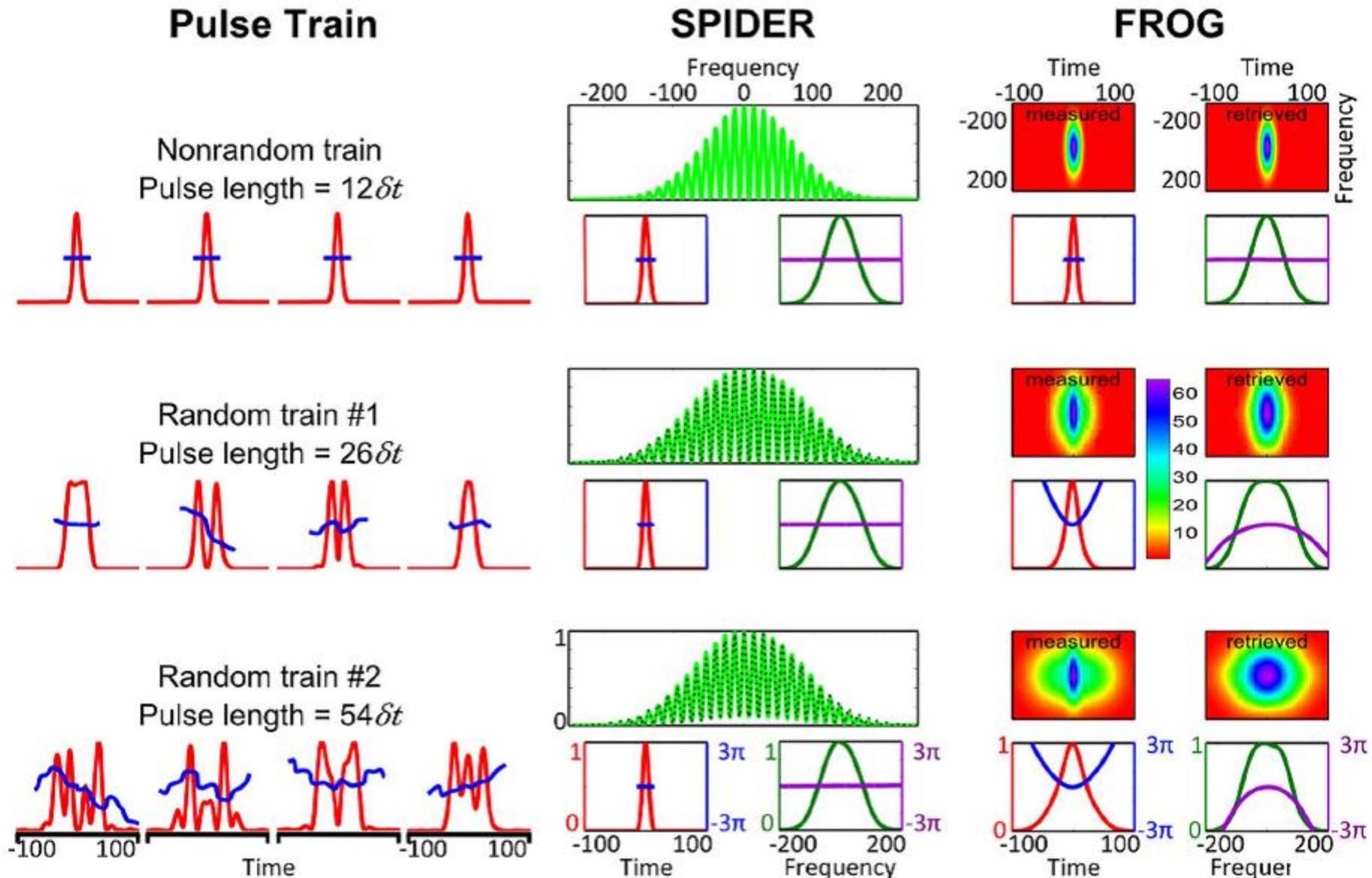
# Coherent artifact in autocorrelation



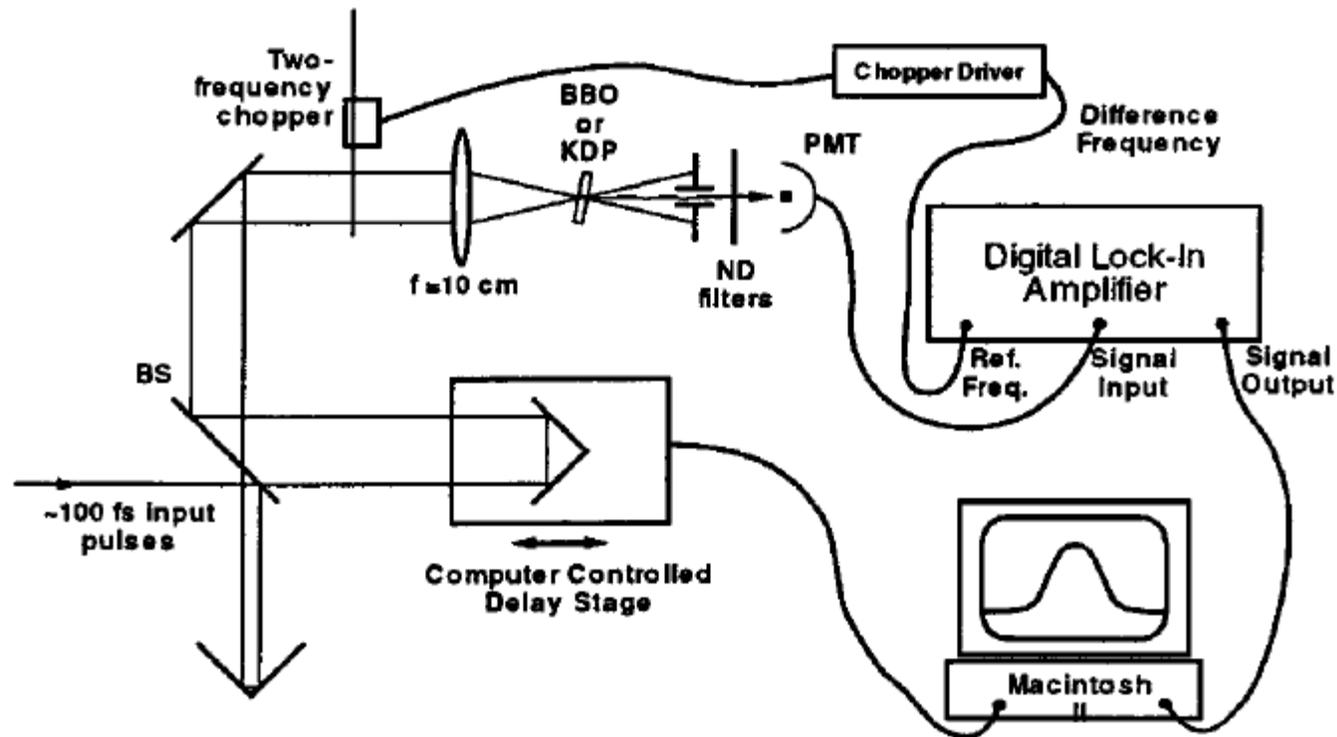
Unstable train of double pulses



# Coherent artifact in SPIDER and FROG



# High dynamic range autocorrelation



Ref: A. Braun et al., Opt. Lett. **20**, 1889-91 (1995)



# Example for high dynamic range AC

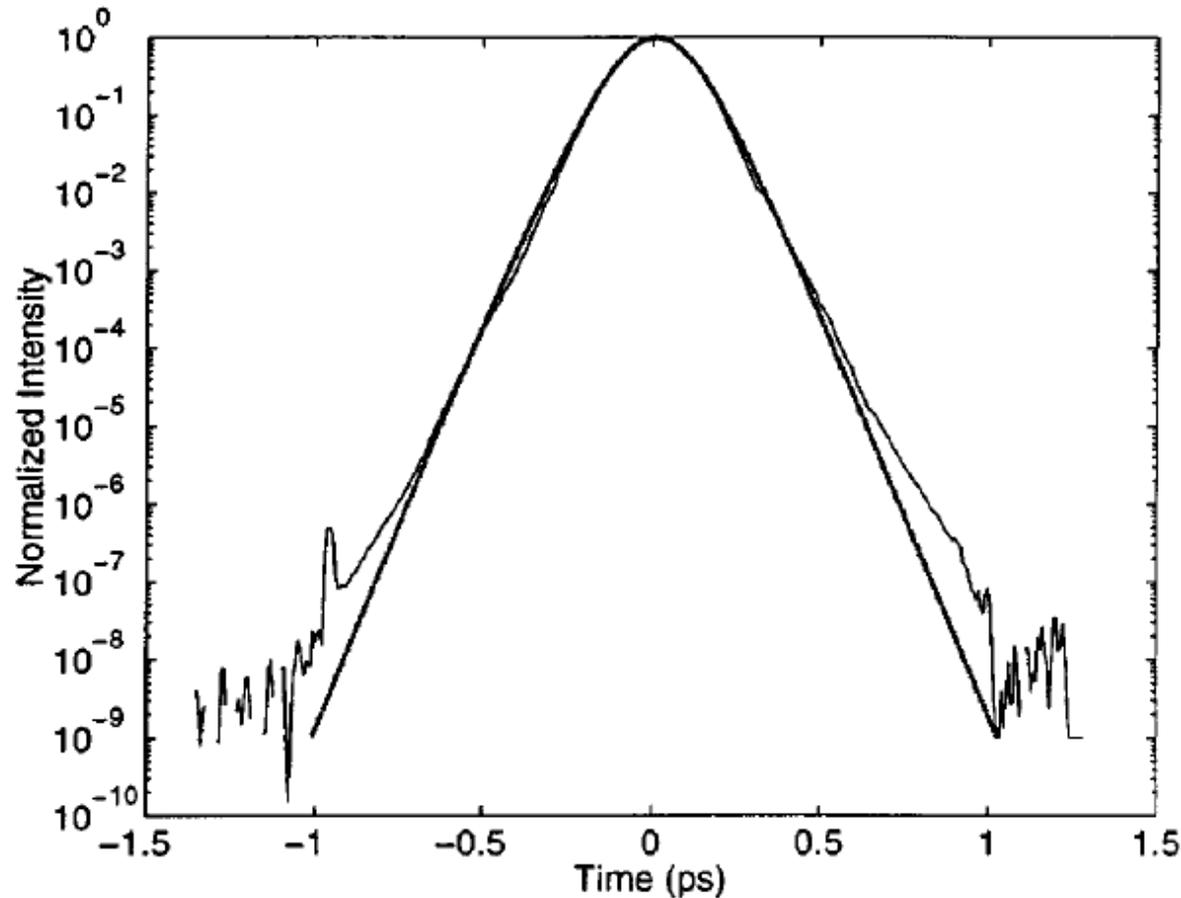


Fig. 4. Autocorrelation trace of a Nd:glass oscillator mode-locked by an A-FPSA sample fit to a 140 fs (FWHM)  $\text{sech}^2$  pulse shape. The spectrum is centered at  $1.058 \mu\text{m}$ .

Ref: A. Braun et al., Opt. Lett. **20**, 1889-91 (1995)



# *Method IV*

## **Multiphoton intrapulse interference phase scan**

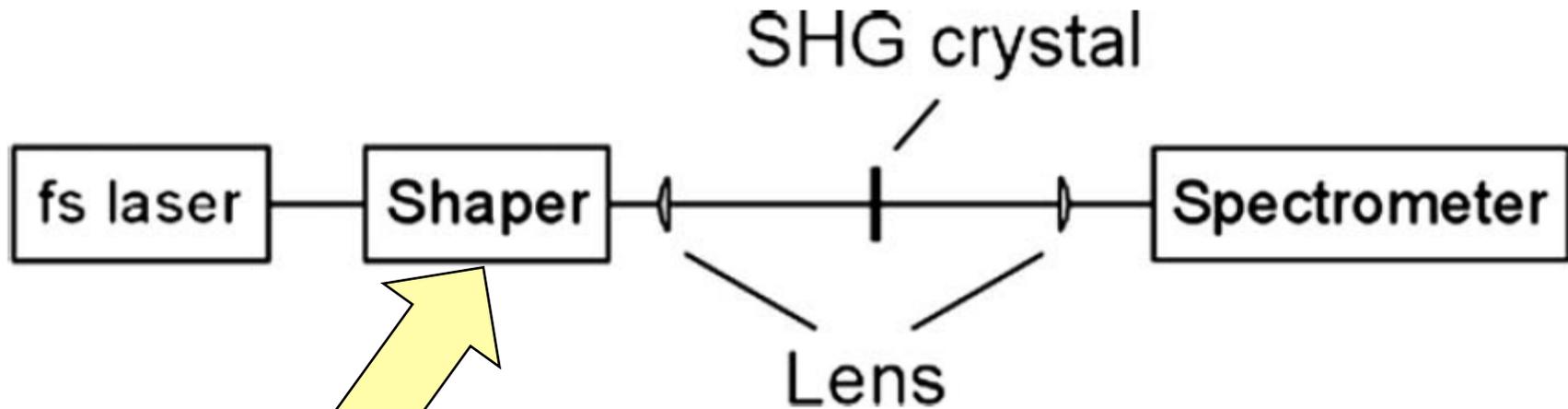
# ***MIIPS***



**Marcos Dantus**



# MIIPS setup



$$f(\delta, \omega) = \alpha \sin(\gamma\omega - \delta)$$

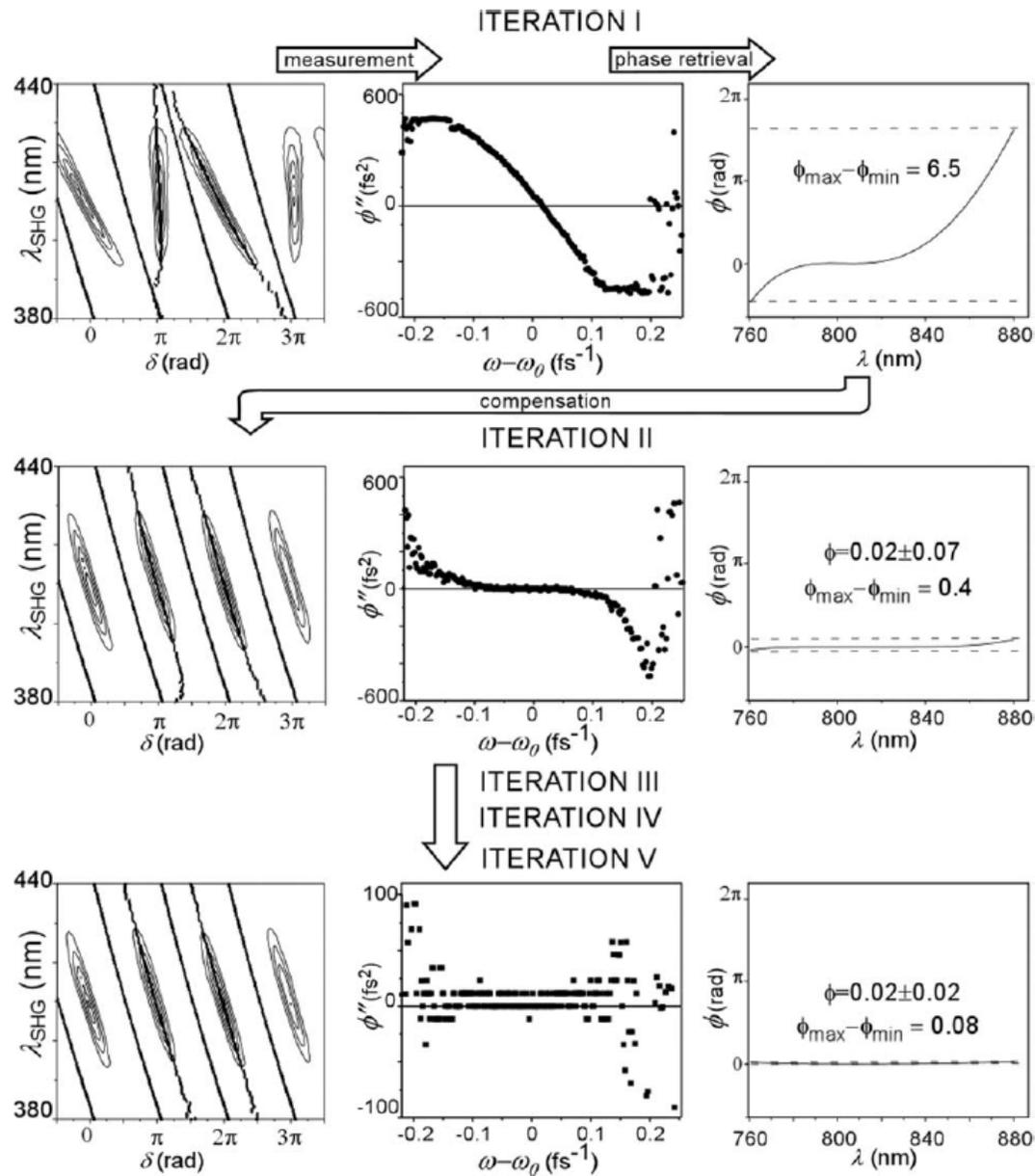
$\delta$ : phase scanned from 0 to  $4\pi$

$\alpha$ : typically  $1.5\pi$

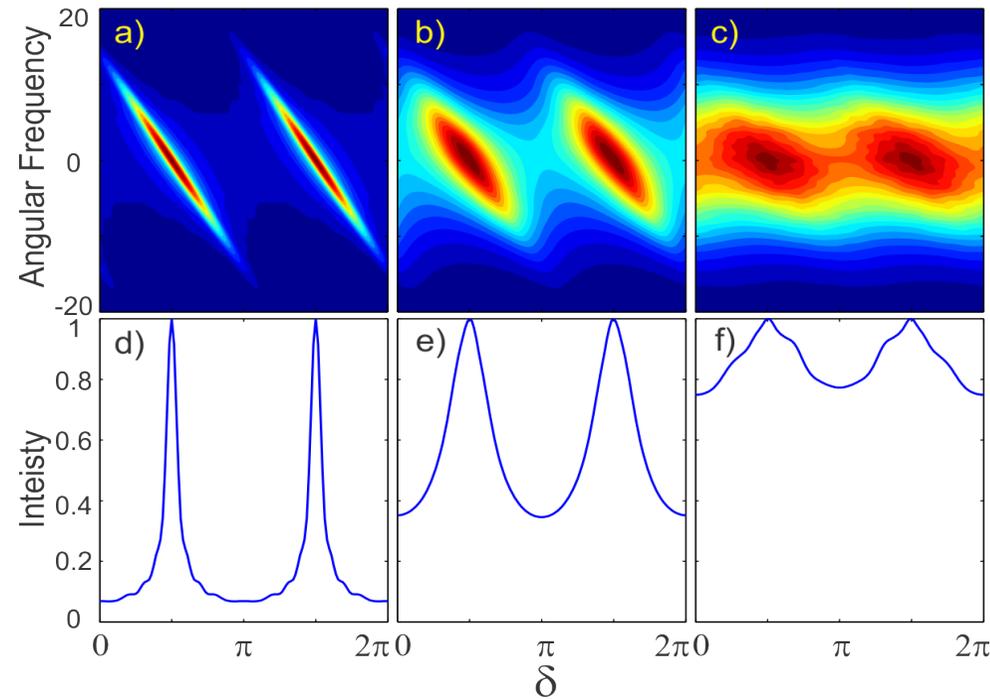
$\gamma$ : estimated pulse duration



# Three MIIPS iterations



# *MIIPS may even master the coherent artifact...*



**Loss of coherence, but constant pulse duration**

Dantus & Trebino & Steinmeyer, to be submitted



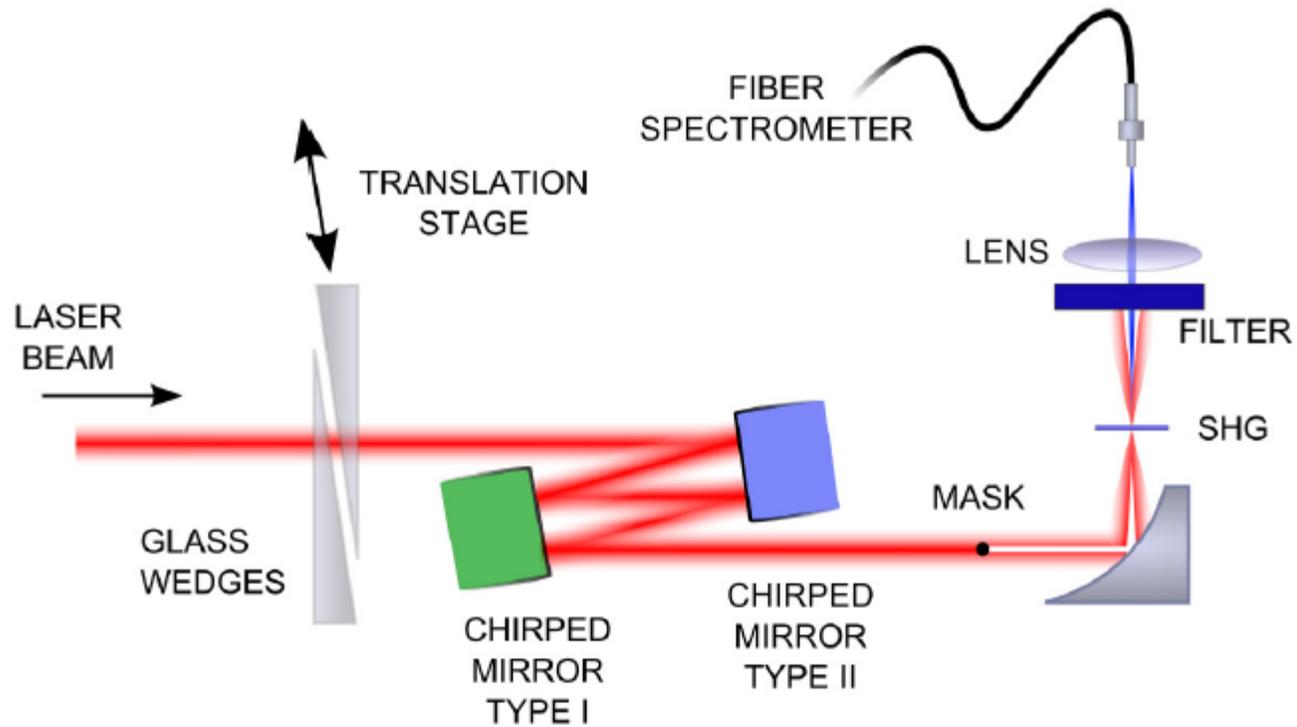
## **Dispersion-scan**

***Dscan***



**Helder Crespo**

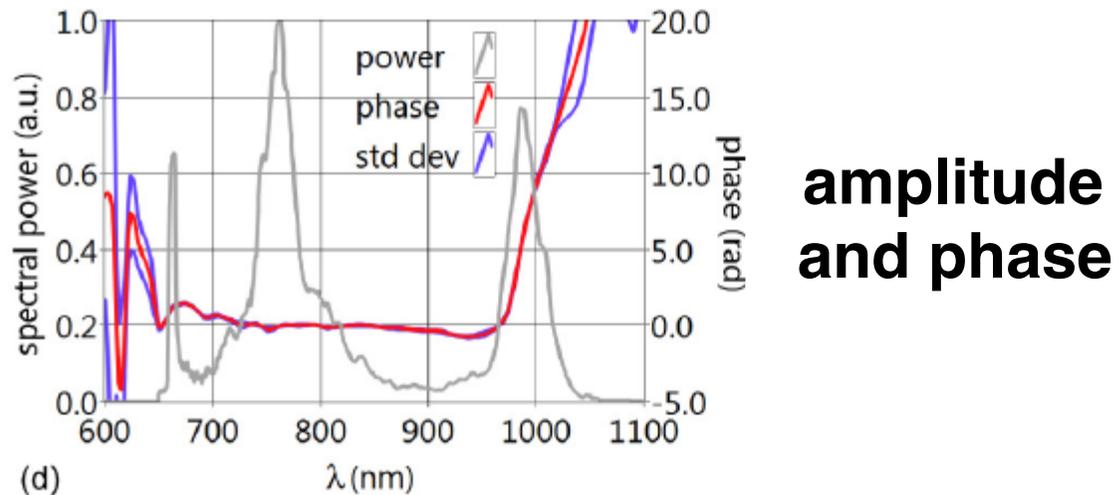
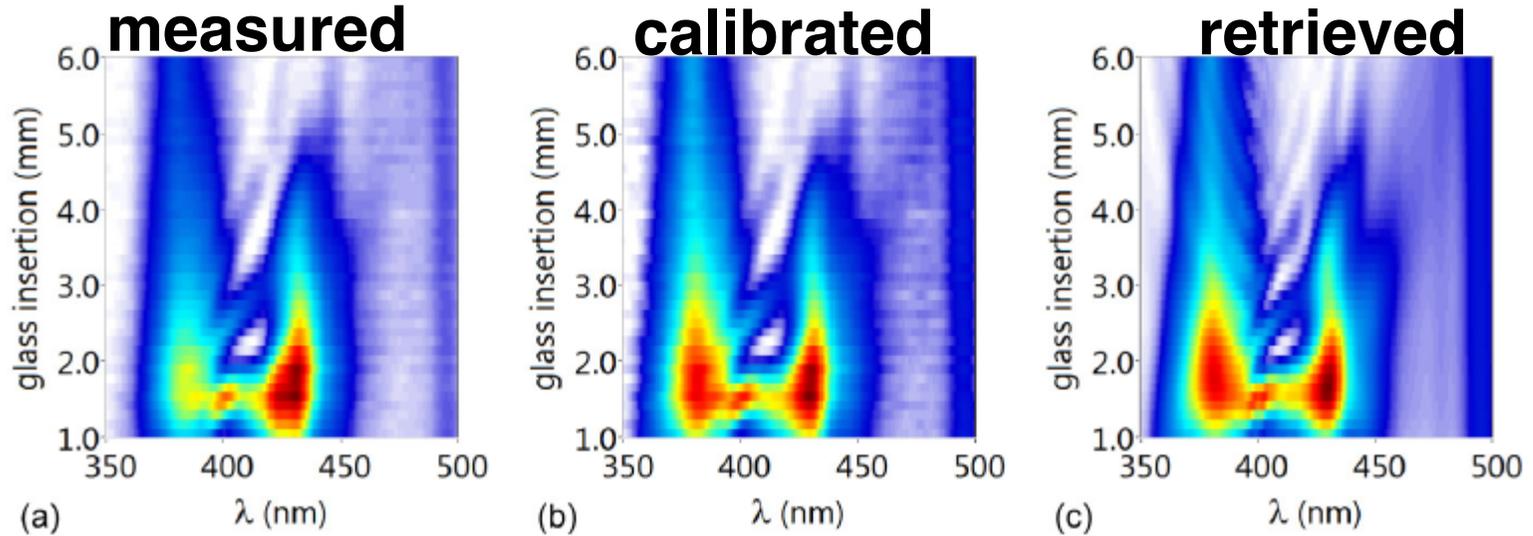
# D-scan



M. Miranda et al., Opt. Express **20**, 18732-18743 (2012)



# D-scan



M. Miranda et al., Opt. Express **20**, 18732-18743 (2012)



# *Problem*

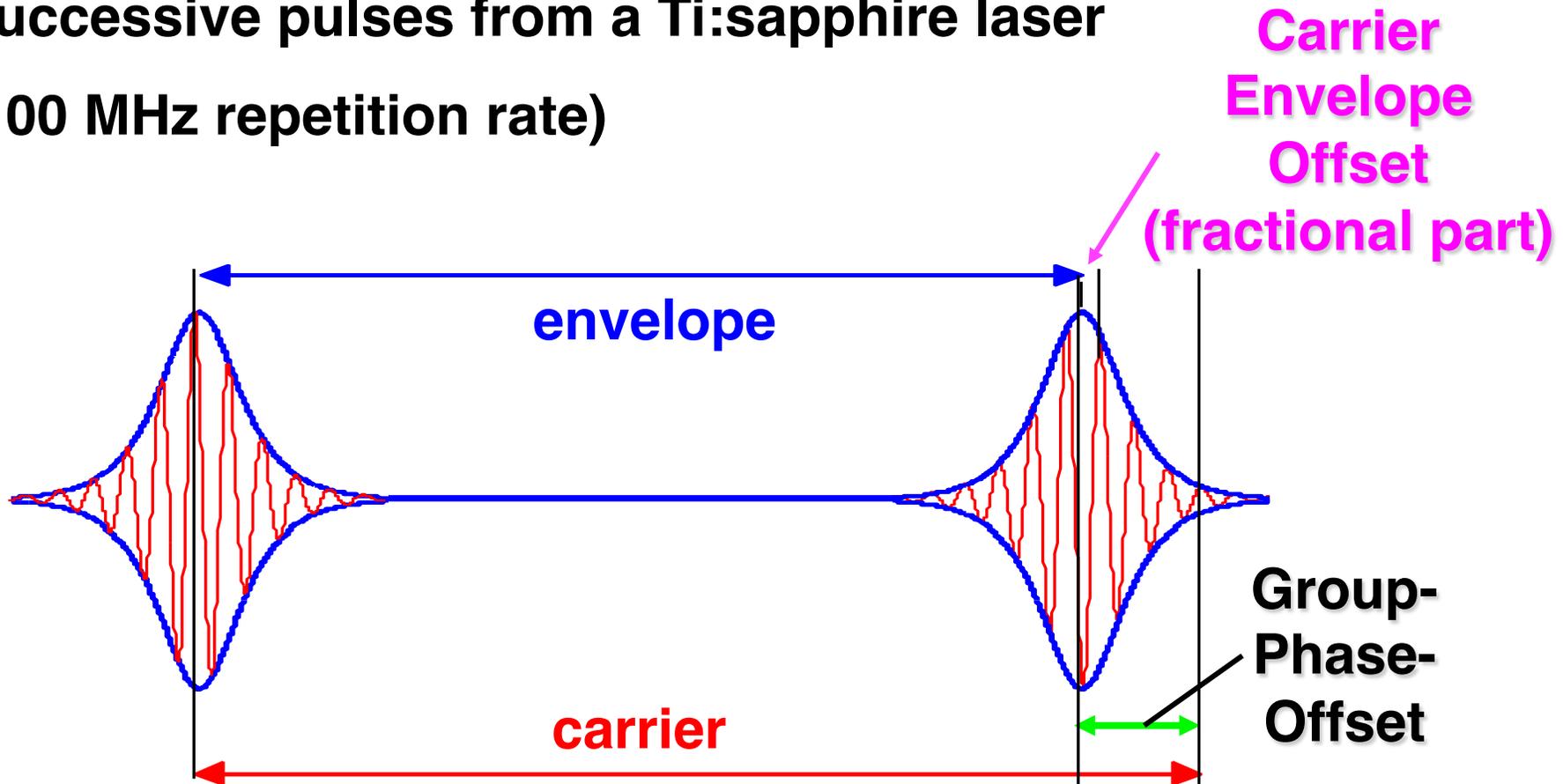
## **The carrier-envelope phase**

***CEP***



# Carrier Envelope Offset (CEO)

Successive pulses from a Ti:sapphire laser  
(100 MHz repetition rate)

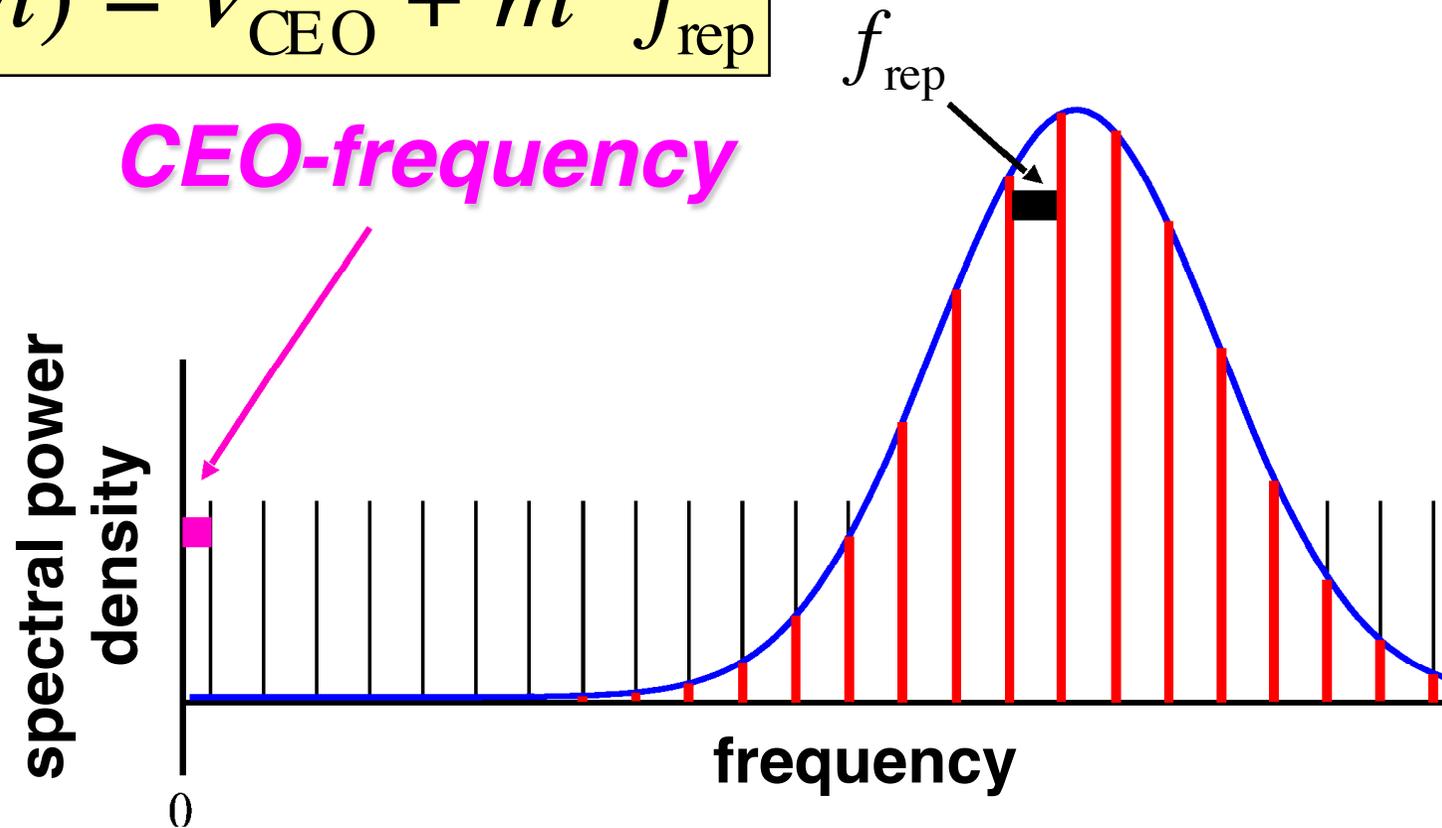


4.6 mm Ti:sapphire:	120.X cycles
3 m air:	20.X cycles
Prism compressor:	140.X cycles
	<hr/>
	260.X cycles



# Carrier-Envelope Offset (CEO)

$$\nu(m) = \nu_{\text{CEO}} + m \cdot f_{\text{rep}}$$



## ● mode-locked laser = optical frequency ruler

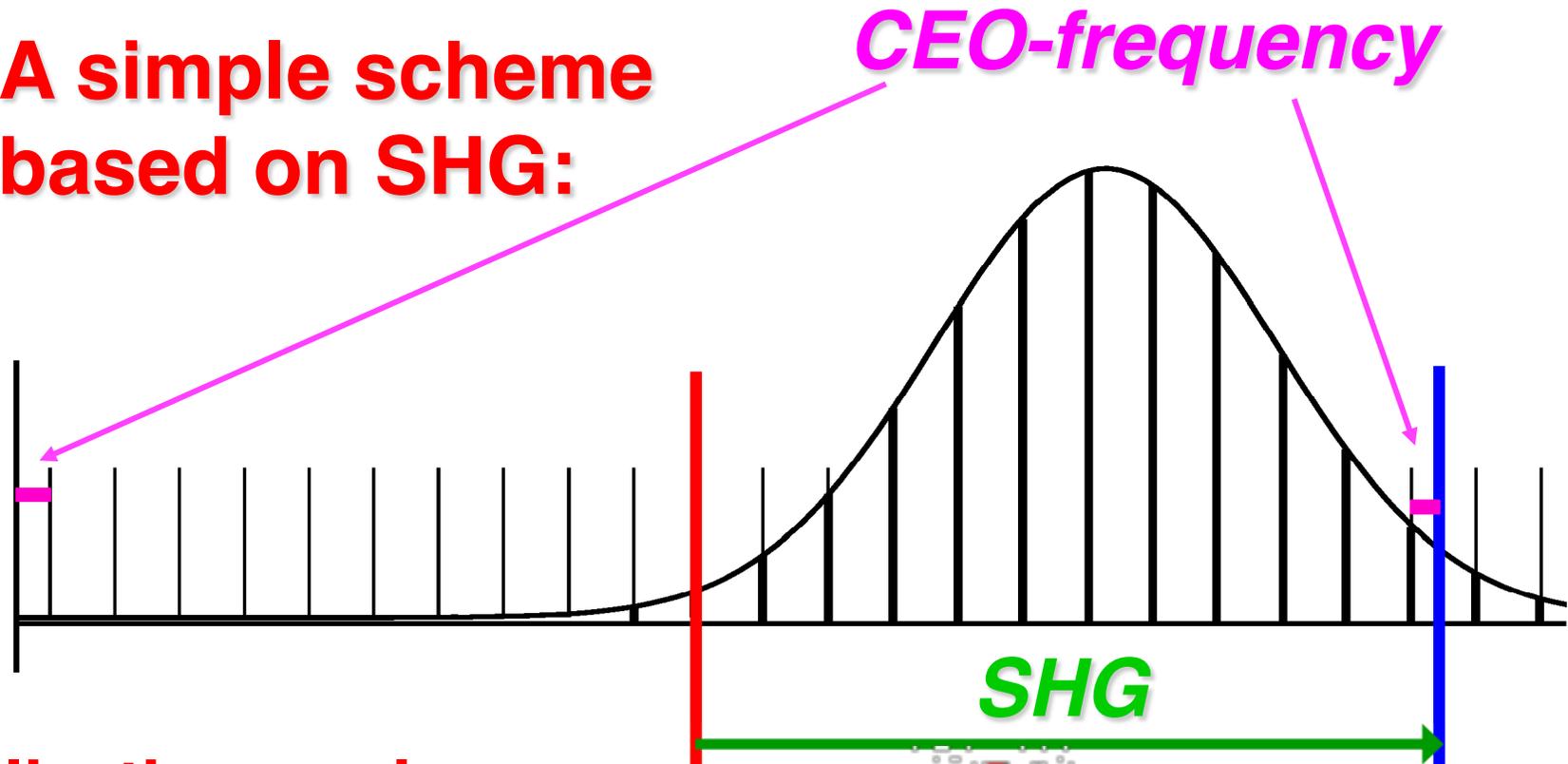
- mode comb uniformity better than  $10^{-15}$
- otherwise rep-rate would be function of wavelength
- **2 degrees of freedom**: "translation" and "breathing"

T.Udem et al., *Opt. Lett.* 24, 881 (1999)



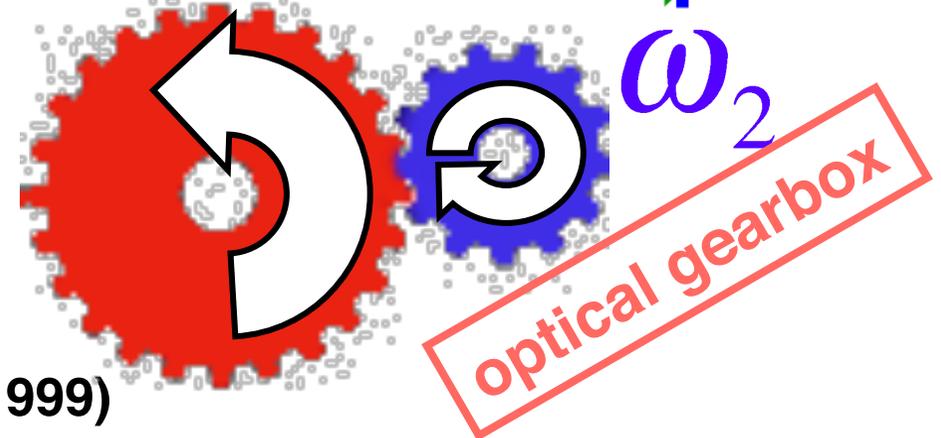
# Measuring the CEO: f-2f interferometer

**A simple scheme  
based on SHG:**

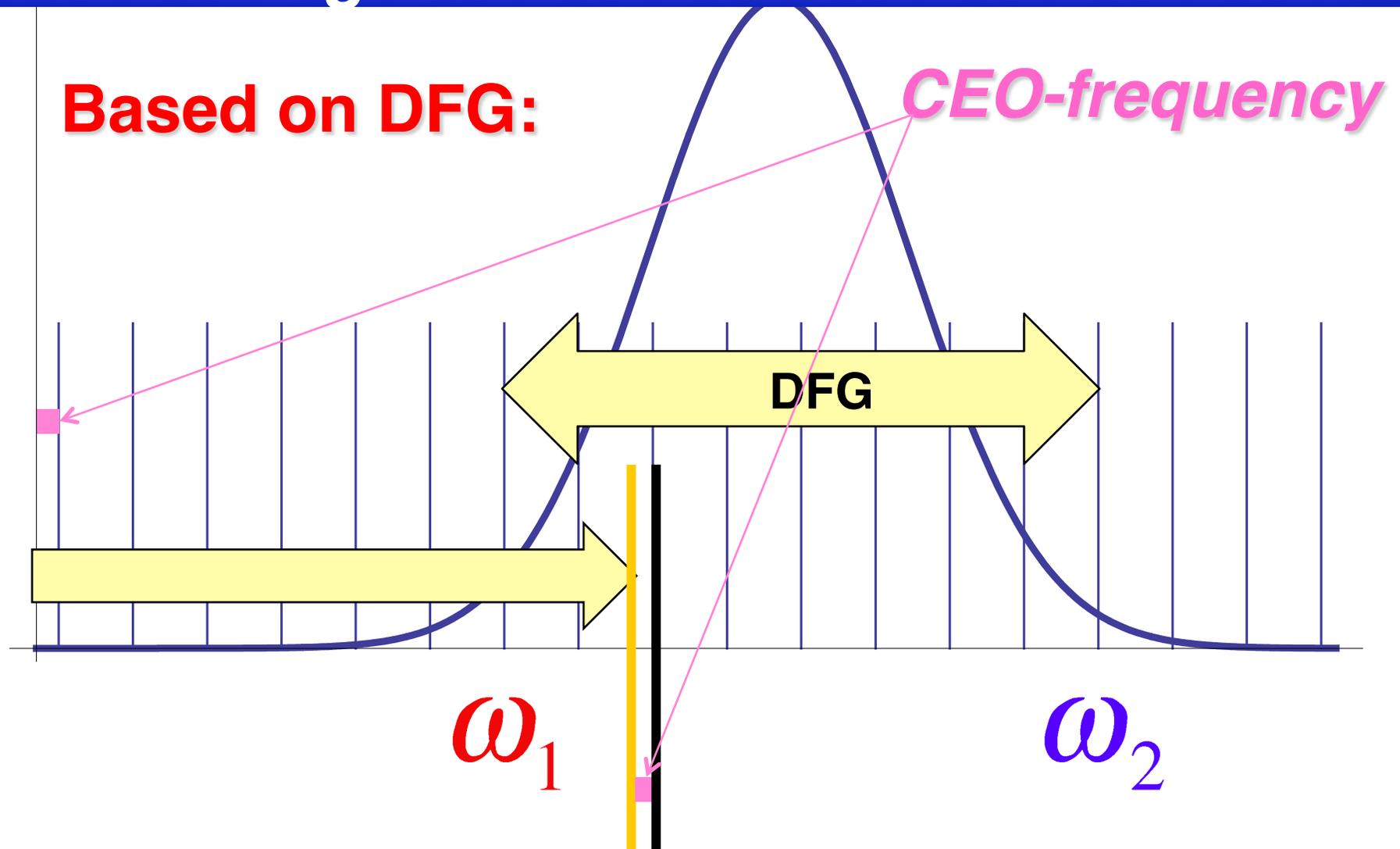


**Stabilization goal:**  
provide phase lock  
between  $\omega_1$  and  $\omega_2$

first proposed:  
H.R. Telle et al., *Appl. Phys. B* 69, 327 (1999)



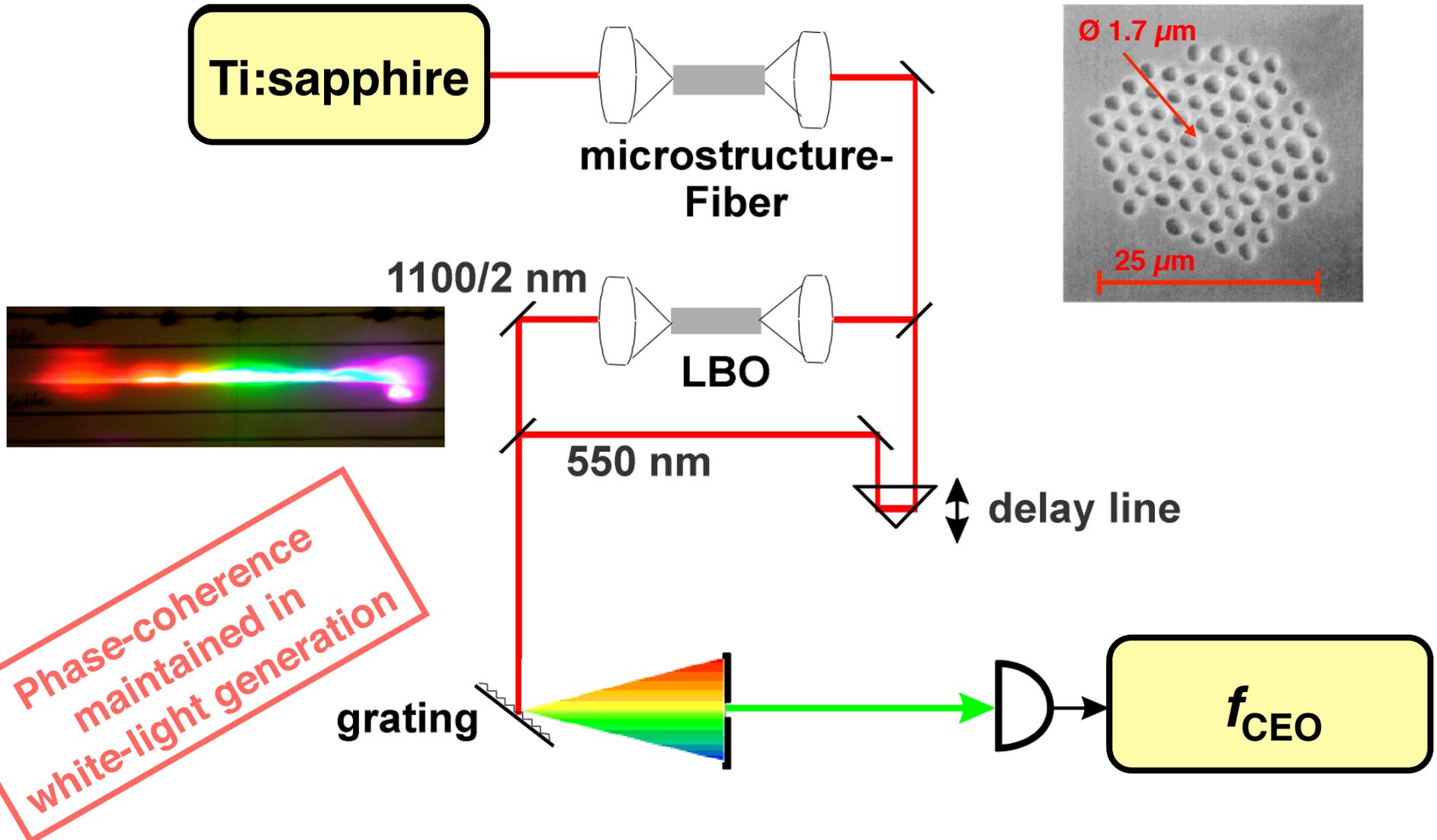
# Measuring the CEO: 0-f interferometer



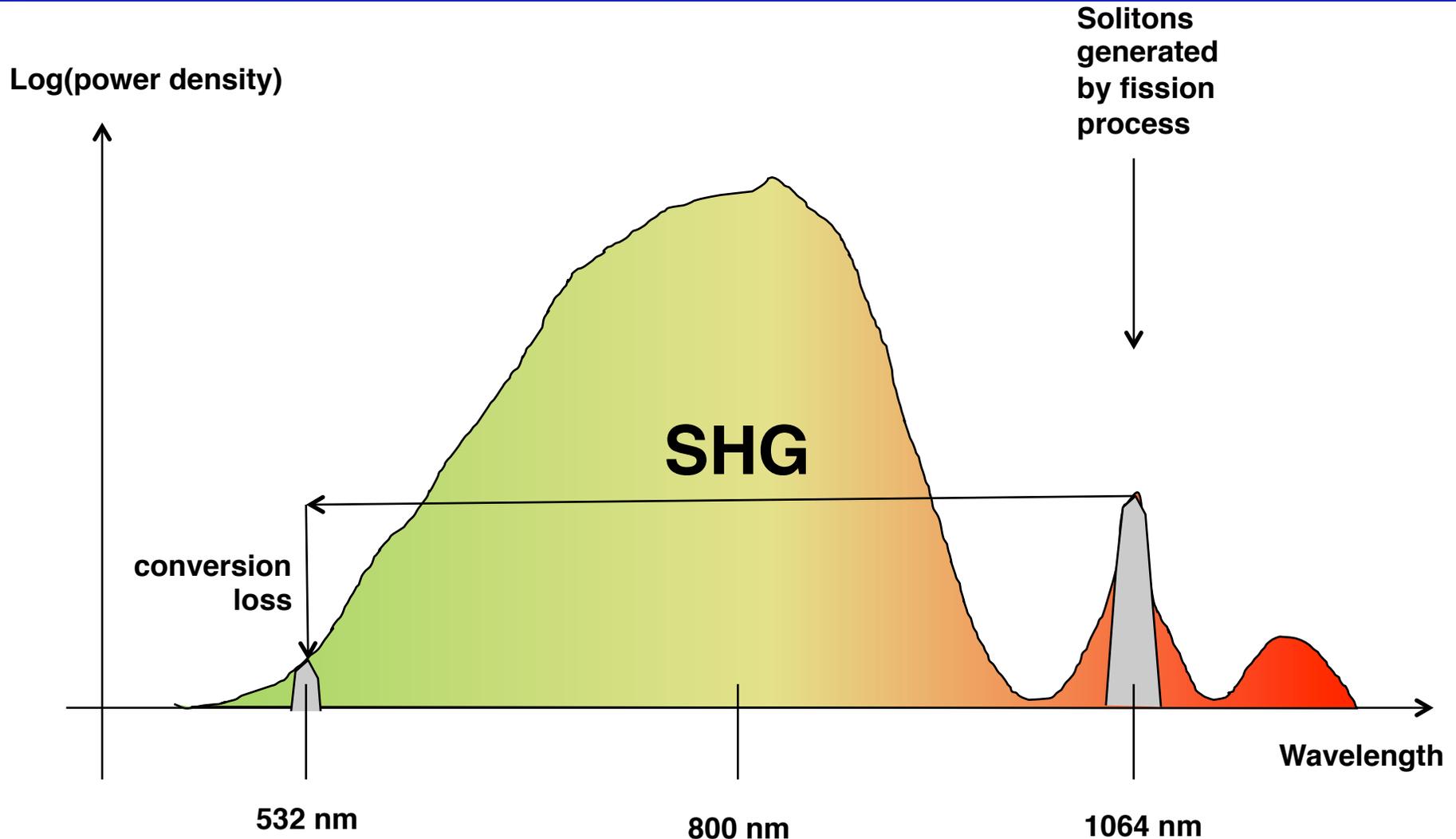
proposed: T. Fuji et al., Opt. Lett. 30, 332 (2005).



# Setup Schematic f-2f



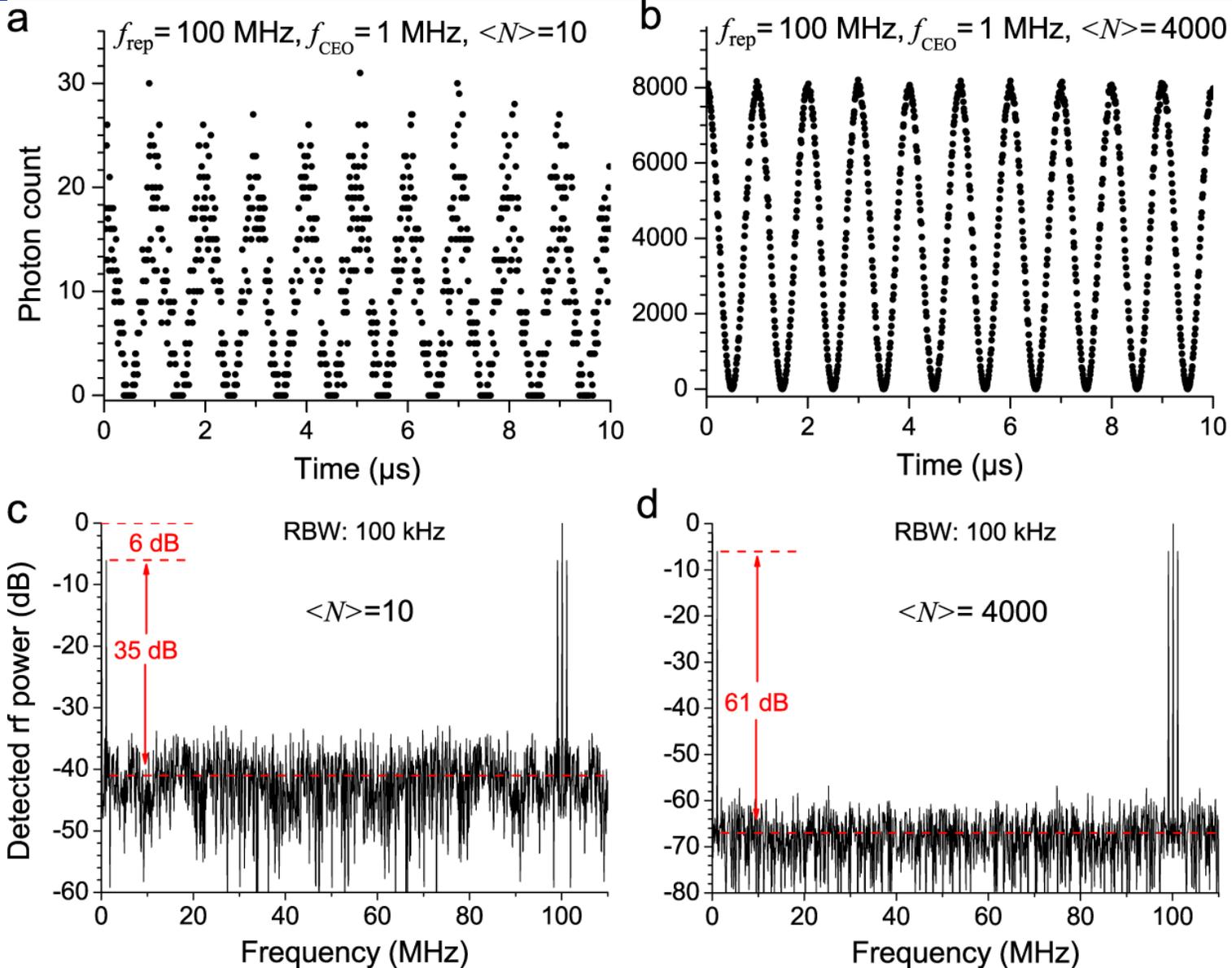
# SHG bottleneck



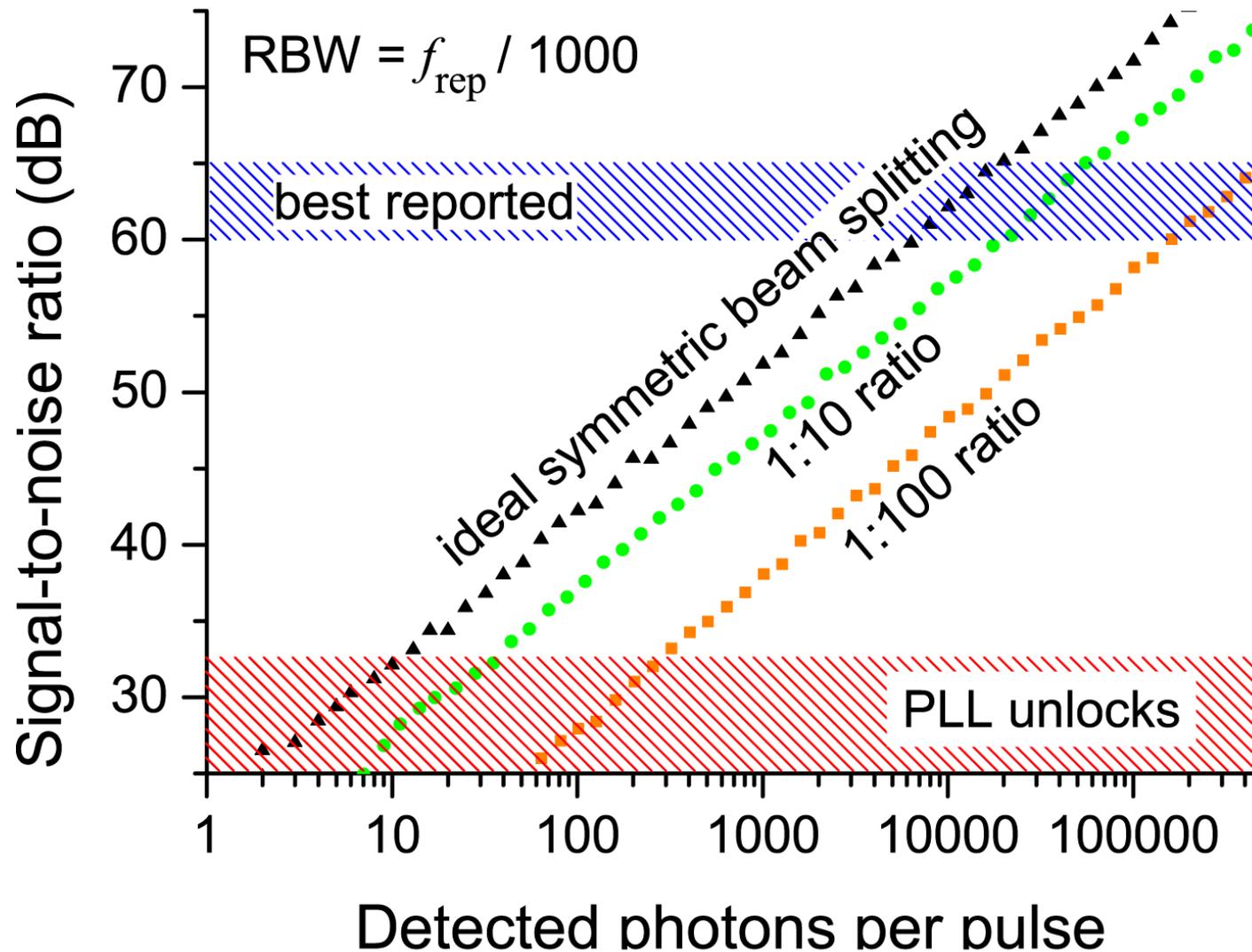
## Lessons learned:

- Use material w/ highest available nonlinearity for SHG
- Match spectral broadening process, shorter fibers are better

# Shot noise



# Shot noise



# Optimizing the $f$ - $2f$ interferometer

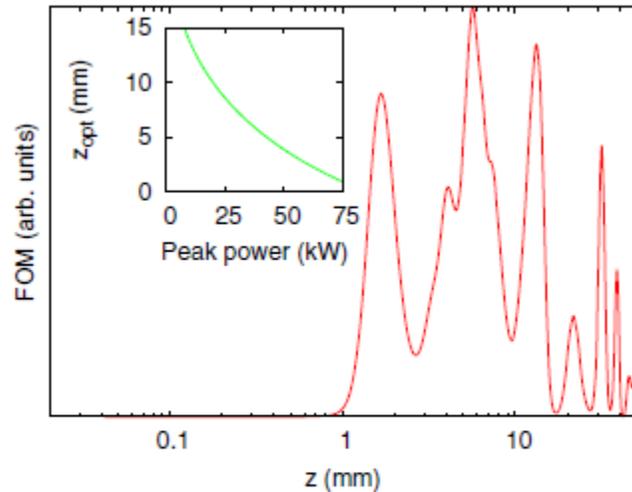
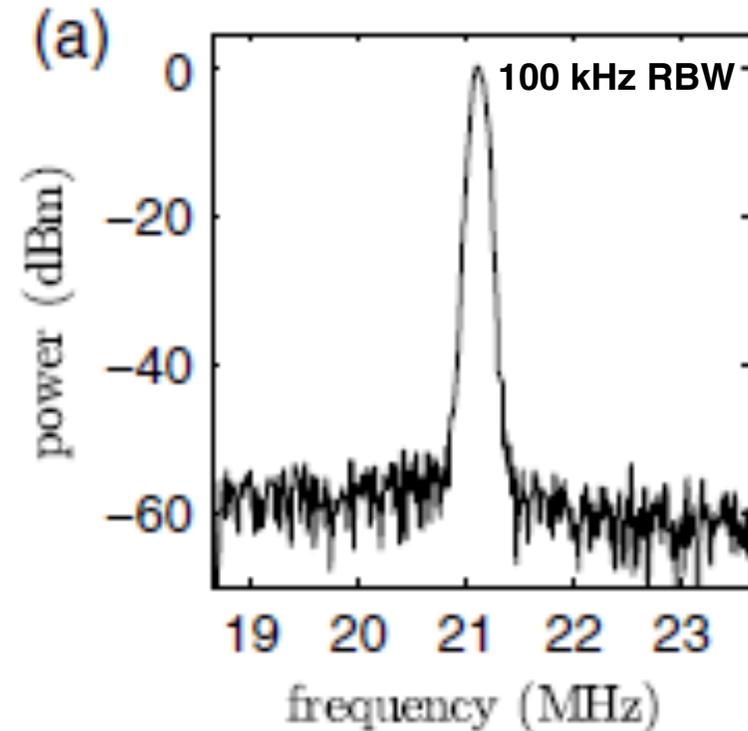


Fig. 1. (Color online) Dependence of the FOM on the propagation length for 15 fs input pulse at 770 nm with peak power of 22 kW. The inset illustrates the dependence of the optimum fiber length on the input power.

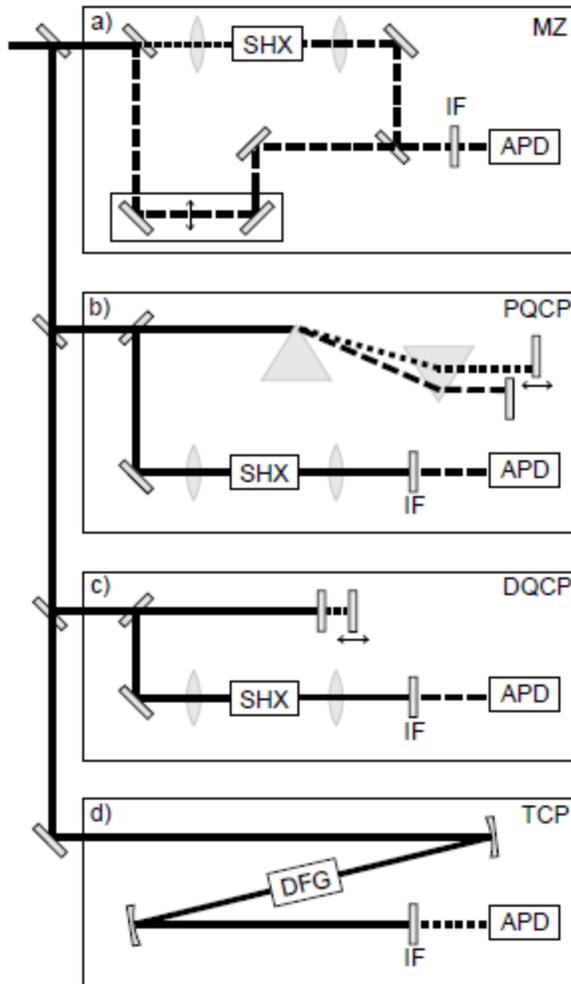
Simulation by A. Husakou



B. Borchers, S. Koke, A. Husakou, J. Herrmann, and G. Steinmeyer, "Carrier-envelope phase stabilization with sub-10 as residual timing jitter," *Opt. Lett.* 36, 4146-4148 (2011)



# Interferometer topology - drift



**Traditional Mach-Zehnder**

**Prism based quasi-common path**

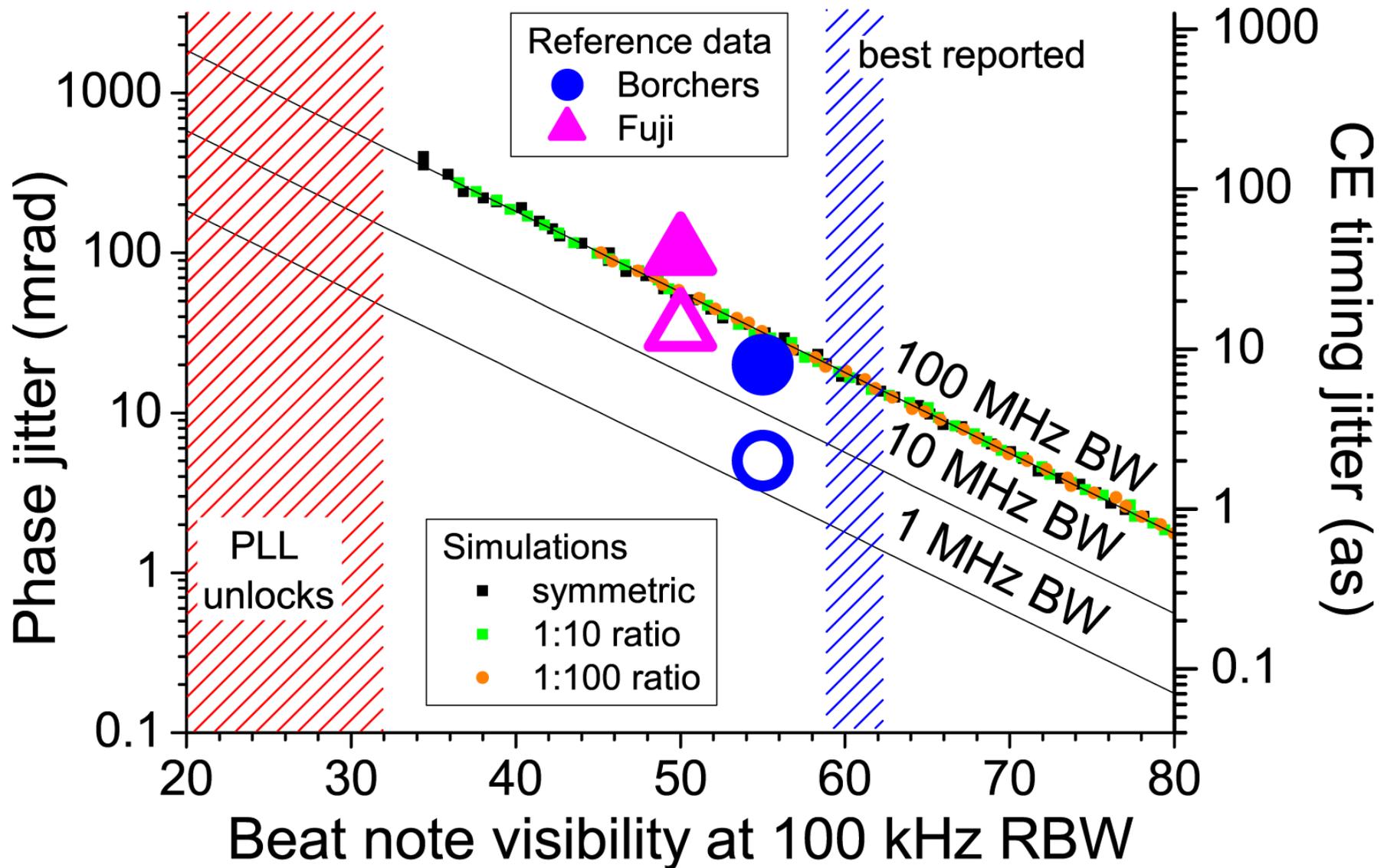
**Dichroic quasi-common path**

**True common path**

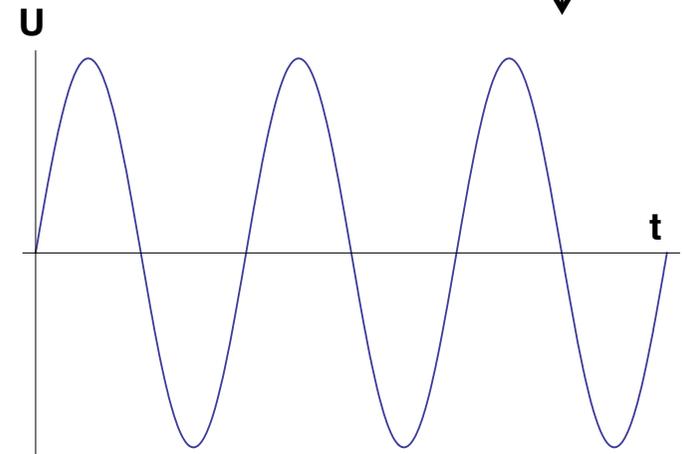
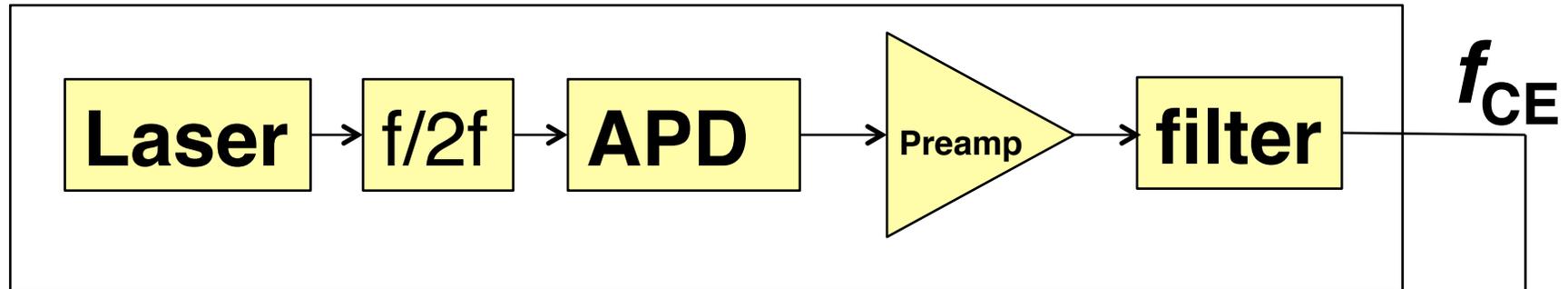
**Susceptibility to drift**

C. Grebing, et al., *Performance comparison of interferometer topologies for carrier-envelope phase detection*, Appl. Phys. B 95, 81 (2009).

# Ultimate jitters depend on beat note visibility



# Phase stabilization via feedback



How can we change the CE frequency?



# *Servo mechanisms*

## **Environmental (temperature, pressure)**

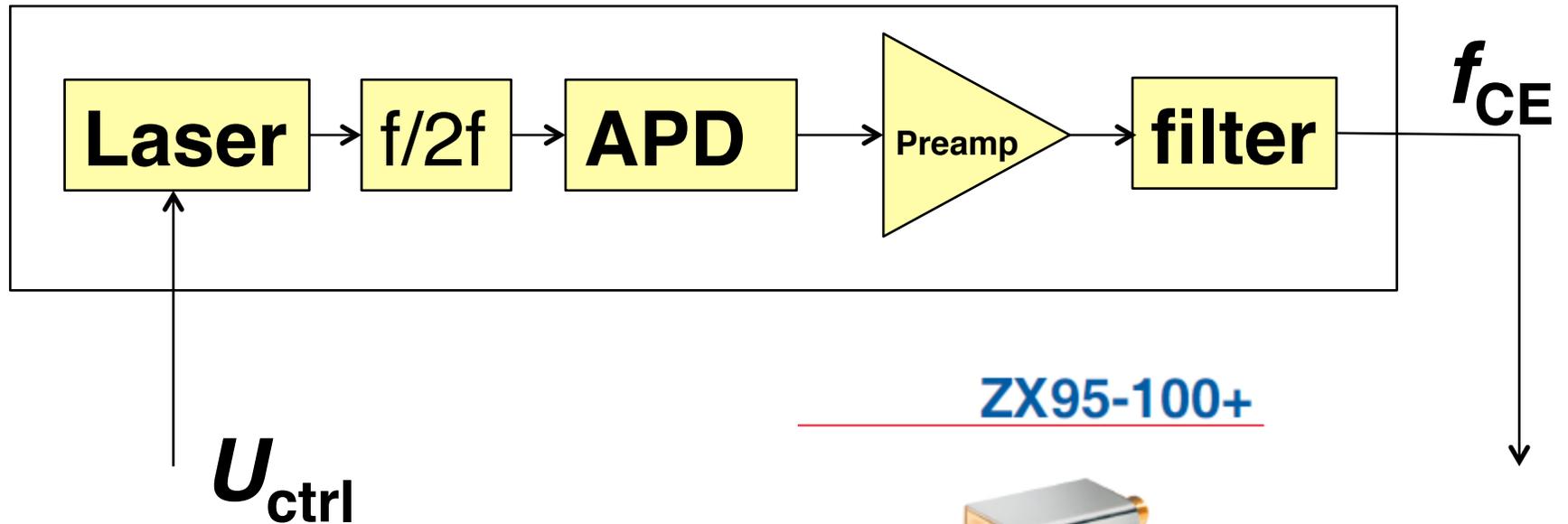
- very slow (a few Hertz at best)
- secondary mechanism for drift compensation

## **Nonlinearity induced (pump power mod.)**

- Kerr effect is dispersive
- can be made very fast ( $>100$  kHz w/ AOM or EOM)
- most established way



# The laser as a VCO

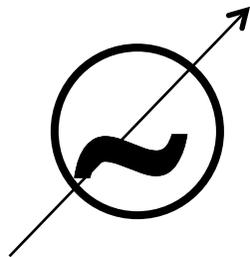


**ZX95-100+**



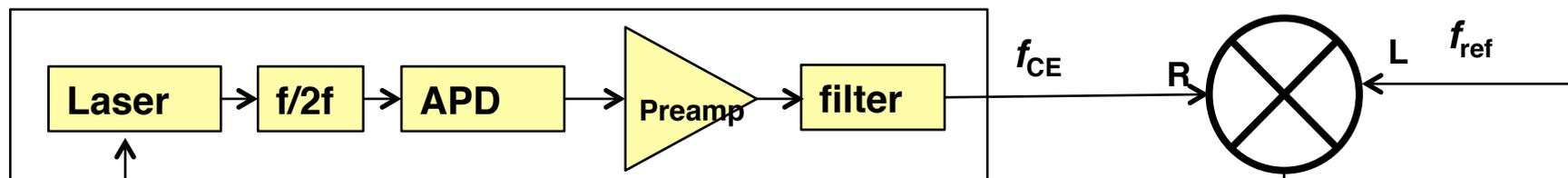
CASE STYLE: GB956

Connectors	Model	Price	Qty.
SMA	ZX95-100-S+	\$37.95 ea.	(1-9)



**VCO = voltage controlled oscillator**

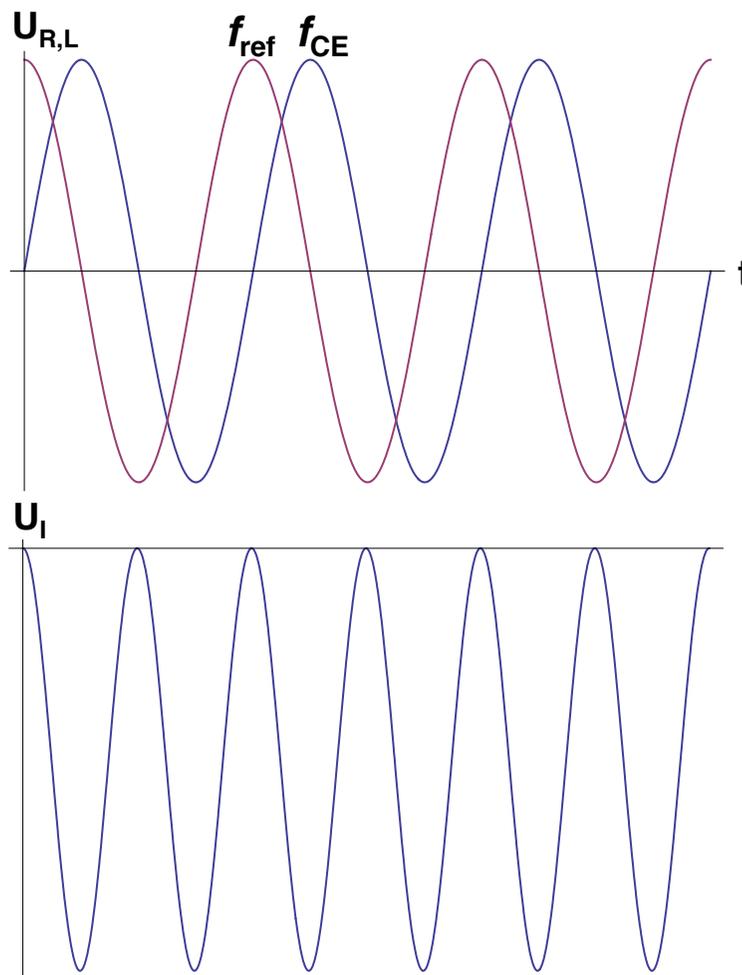
# Closing the loop



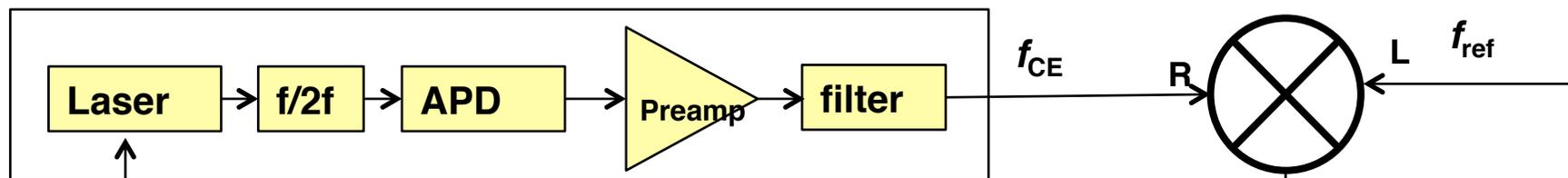
$U_{ctrl}$

$\Delta\varphi = \pi/2$

$\overline{U_I} = -\max$



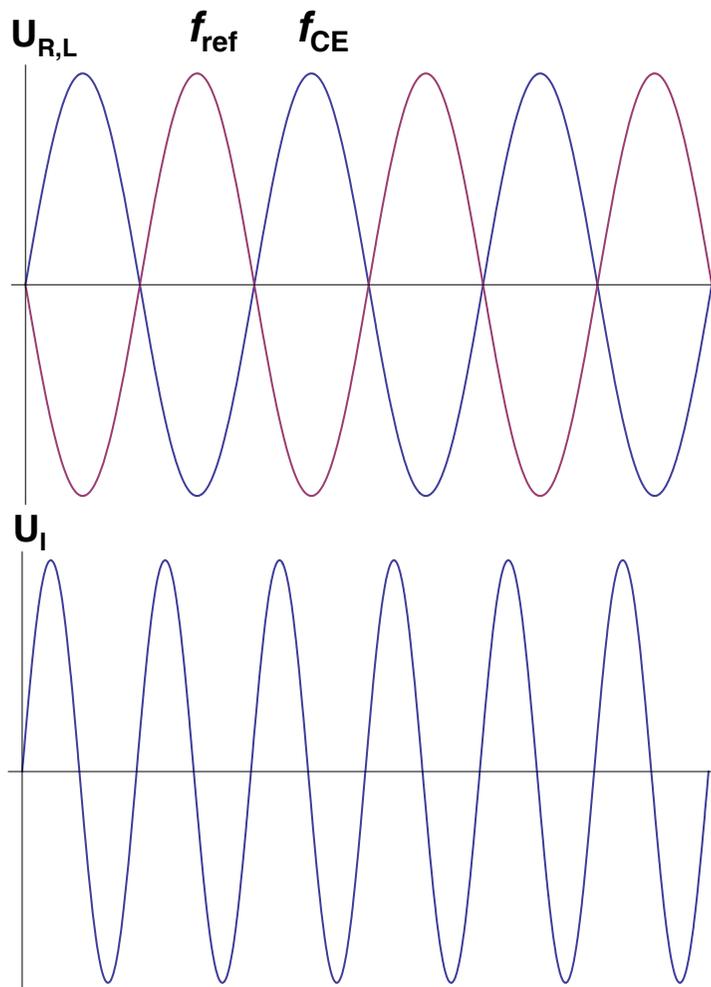
# Closing the loop



$U_{ctrl}$

$\Delta\varphi = \pi/2$

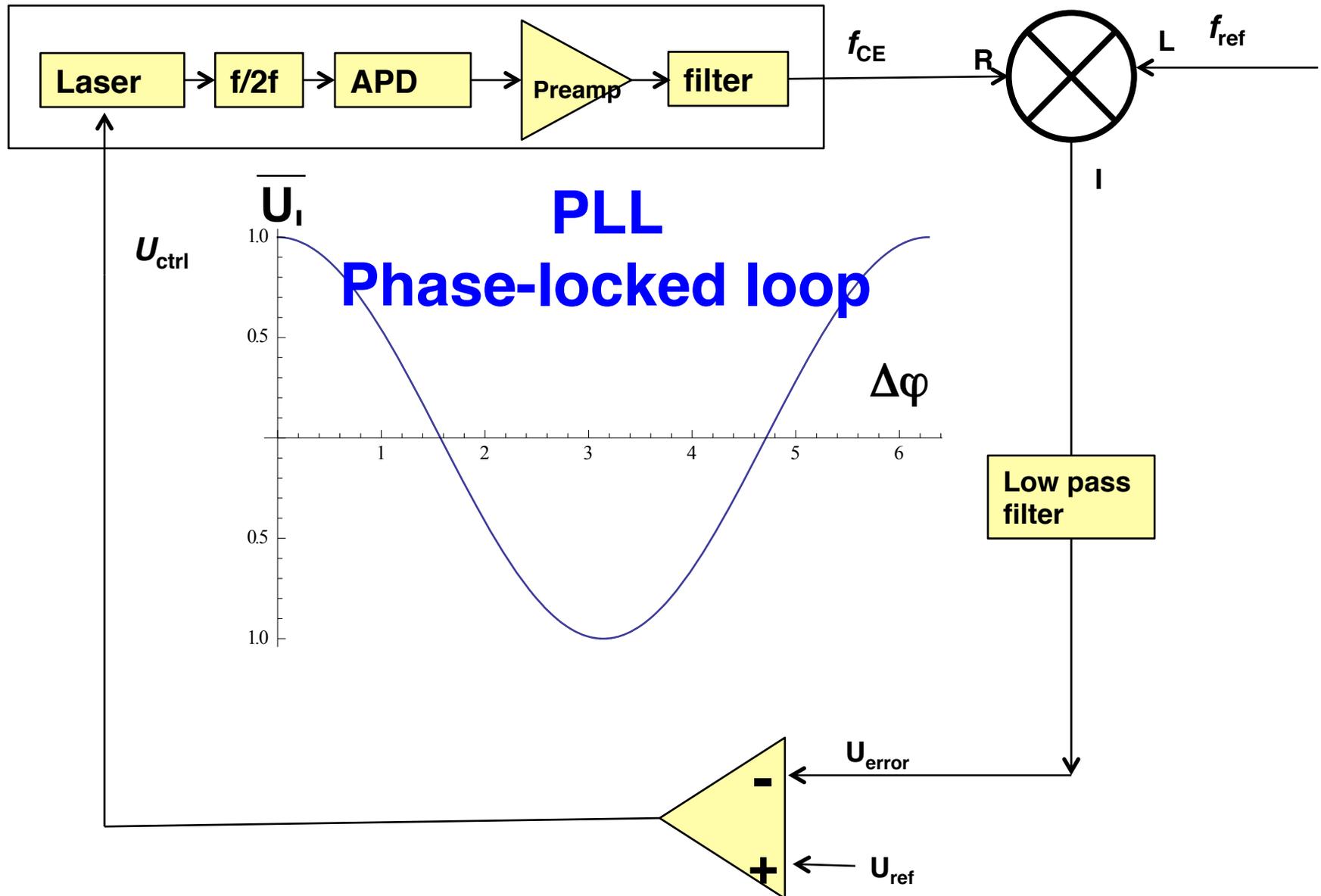
$\overline{U_i} = 0$



I

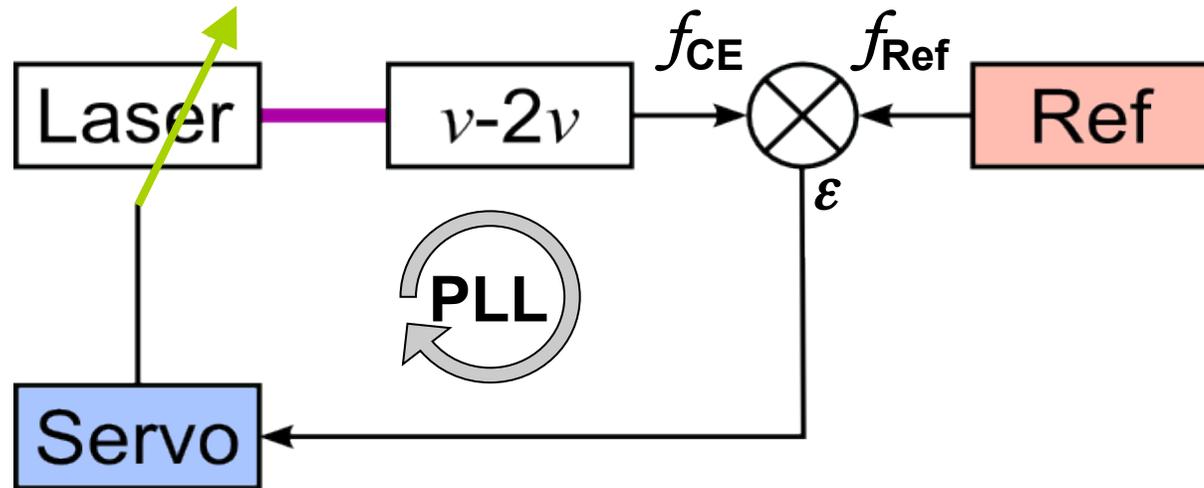


# Closing the loop



# Stabilization with servo loop

PLL= phase-locked loop



**PLL prevents stabilization to zero offset**

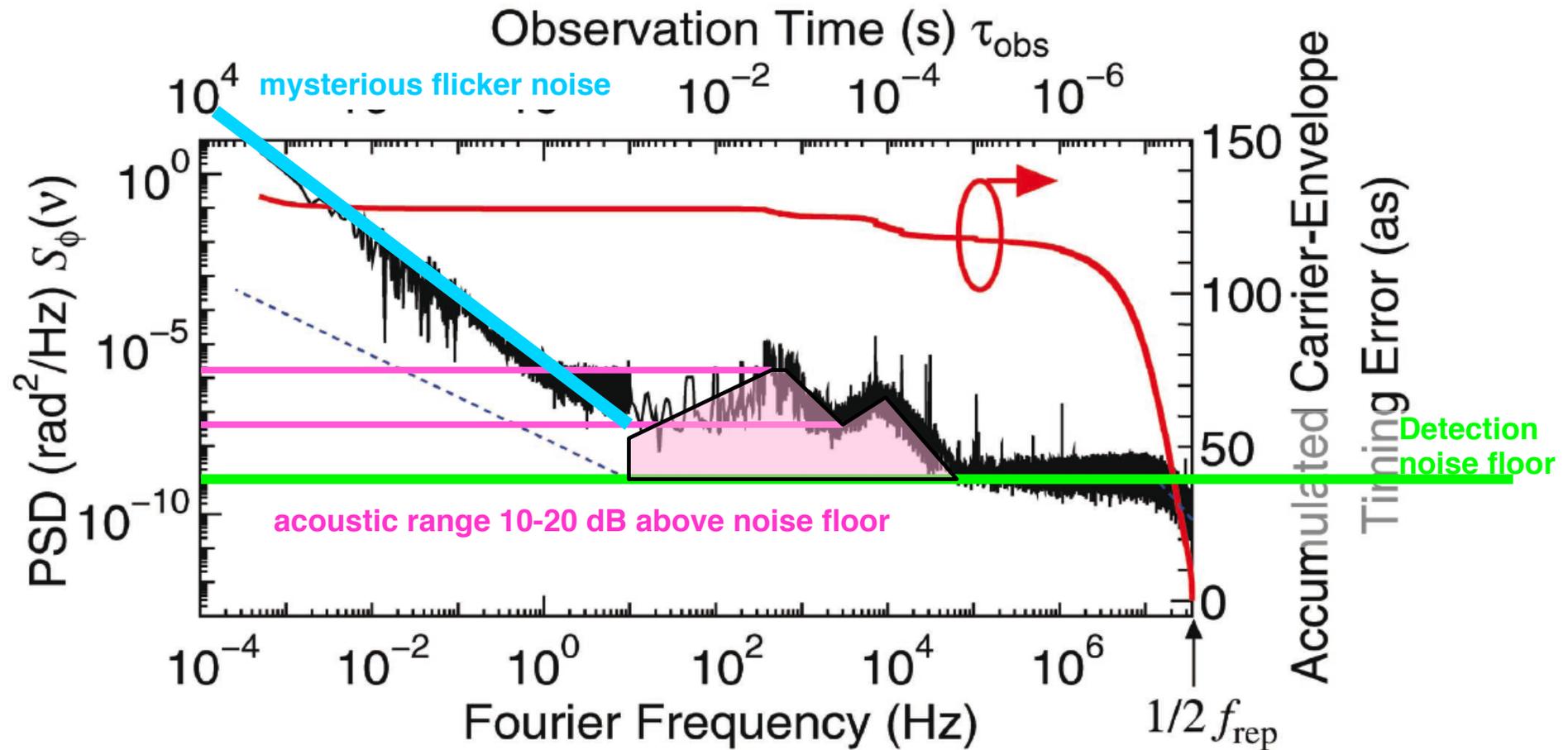
**Complex locking electronics required (Phase margin!)**

**Feedback compromises laser performance**

**Tradeoff: phase capture range vs. precision**



# Best performance achieved with feedback scheme



residual jitter: 180 mrad=70 attoseconds ( $\hat{0.2\text{Hz}-2\text{MHz}}$ )

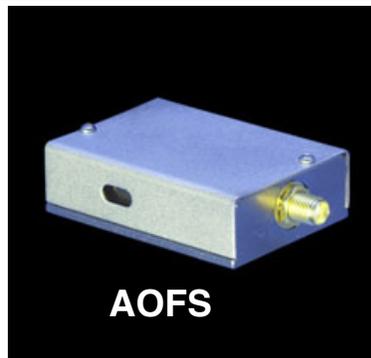
monolithic scheme: Takao Fuji et al., Opt. Lett. 30, 332 (2005).  
 similar excellent results obtained by Tara Fortier, Opt. Lett. 27, 1436 (2002).



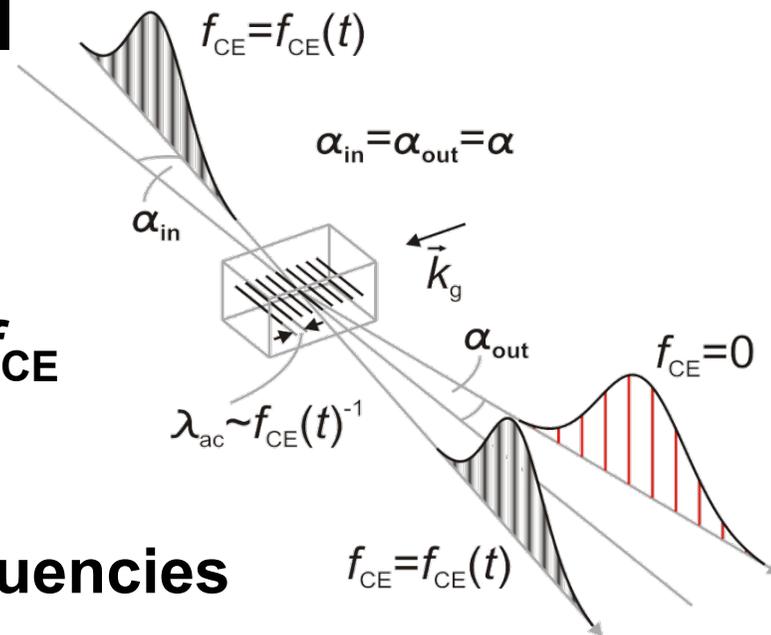
# Direct feed-forward scheme

Koke et al., *Nature Photonics* 4, 462 (2010).  
B. Borchers et al. *Opt. Lett.* 36, 4146-4148 (2011)

**Idea**

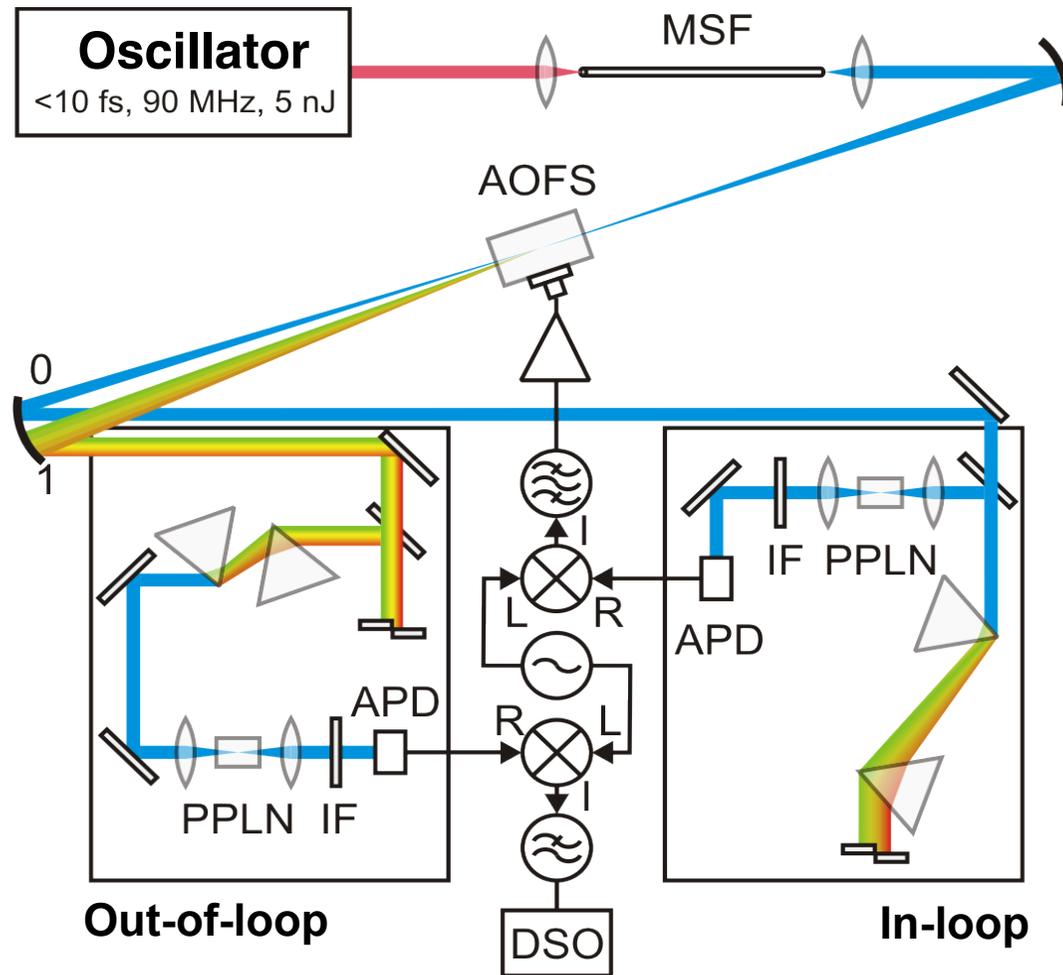


1. We can measure the comb offset  $f_{CE}$
2. There exist devices to shift laser frequencies by radio frequencies



3. Why don't we simply shift the entire comb by  $f_{CE}$

# Experimental set-up



**AOFS center frequency:**  
 $f = 70\text{ MHz}$   
(2.7 cm fused silica @  $1000\text{ fs}^2$ )

**Out-of-loop  
measurement scheme**

MSF...microstructured fiber

IF...interference filter

AOFS...acousto-optical frequency shifter

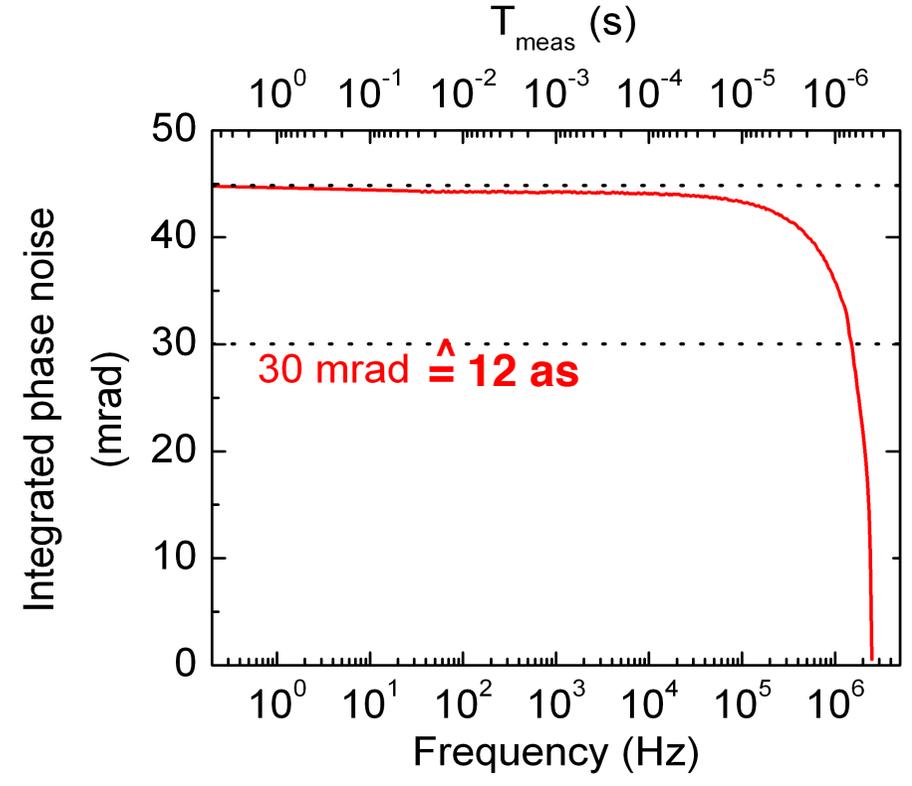
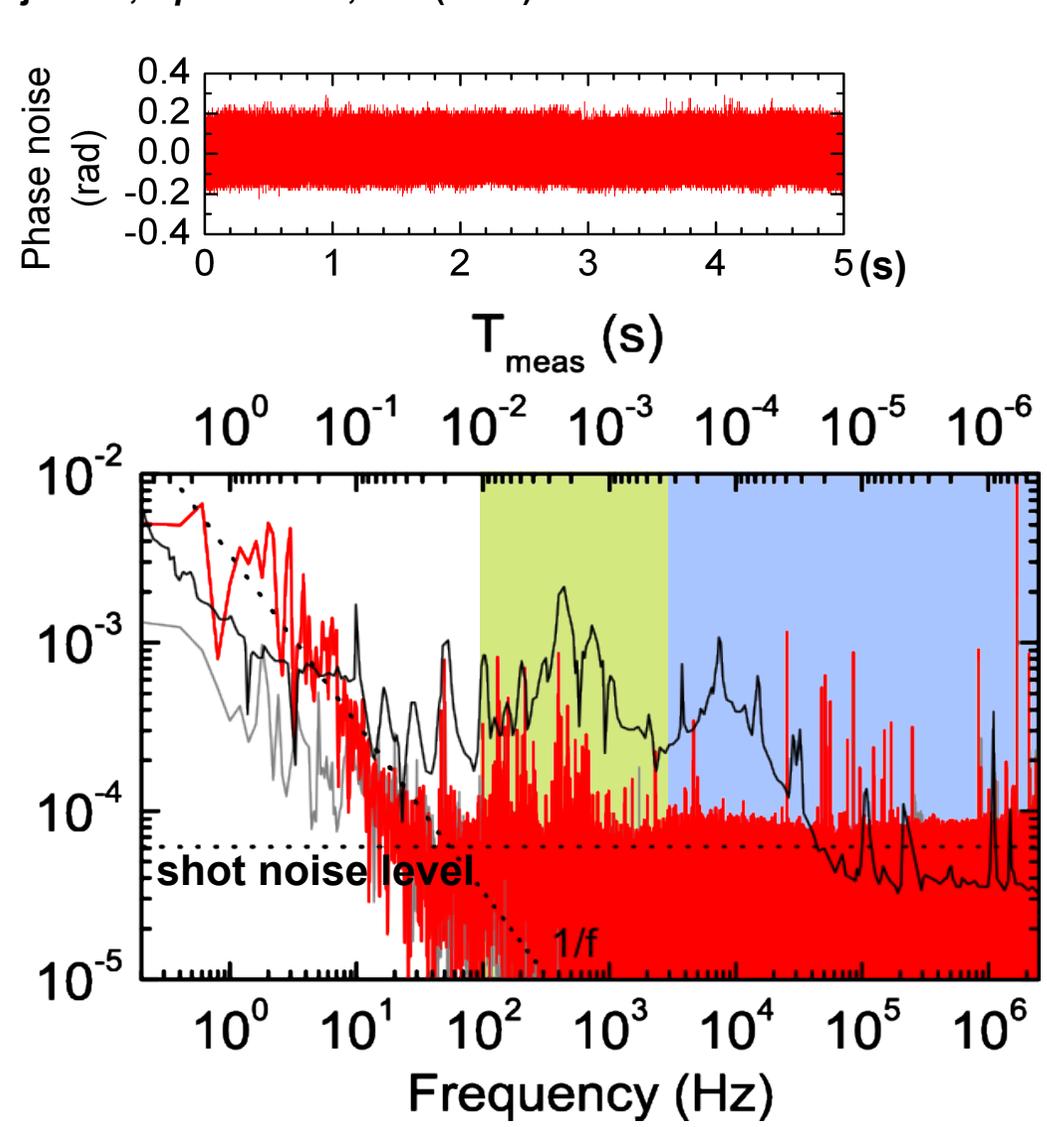
APD...avalanche photo diode

PPLN...periodically poled lithium niobate

DSO...digital sampling oscilloscope

# Results

Koke et al., *Nature Photonics* 4, 462 (2010).  
Fuji et al., *Opt. Lett.* 30, 332 (2005)



**>kHz: technical noise**  
**100 Hz – 3kHz: acoustical range**  
**<100 Hz: flicker noise**

# CEP measurement of amplified systems

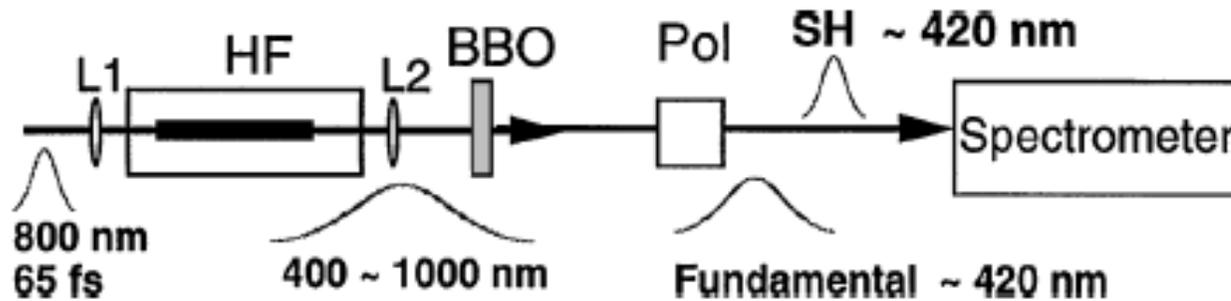
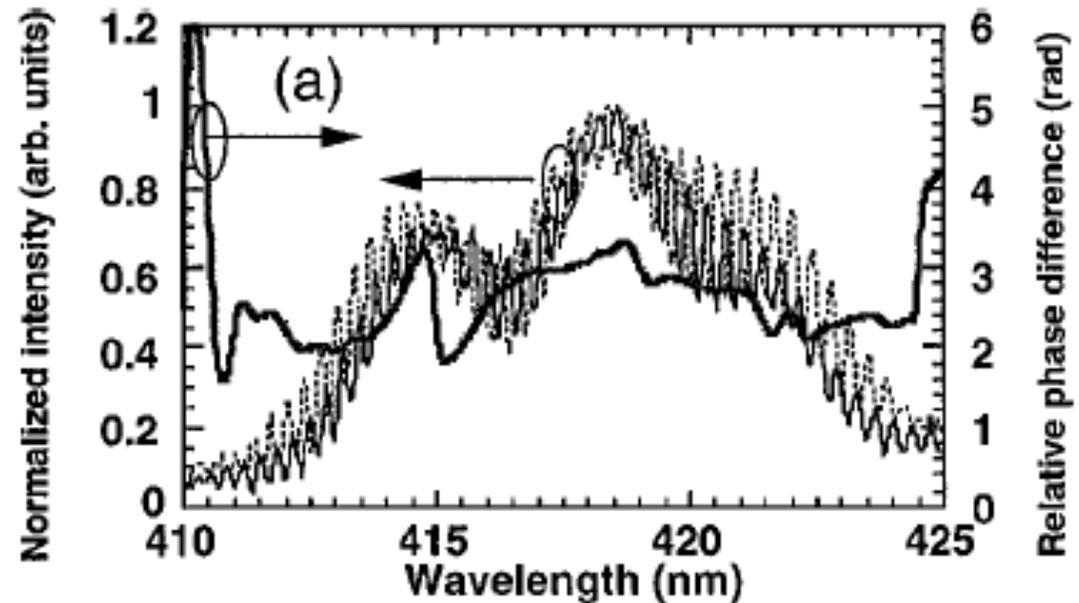


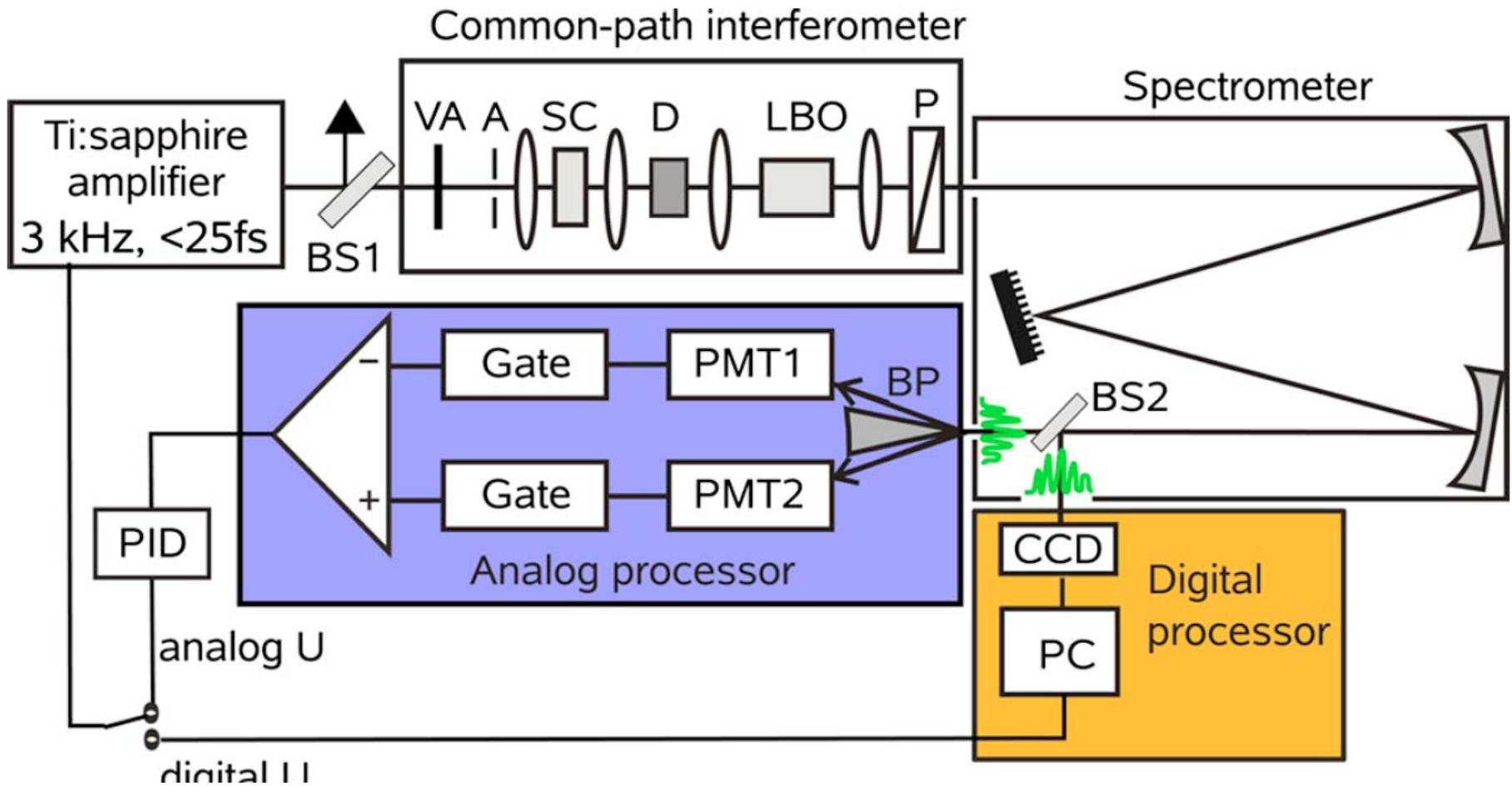
Fig. 2. Experimental setup: L1, 250-mm focal-length lens; L2, 600-mm focal-length lens; HF, hollow fiber; BBO, 300- $\mu$ m-thick BBO crystal; Pol, polarizer.



M. Takehata et al., Opt. Lett. **26**, 1436-1438 (2001)



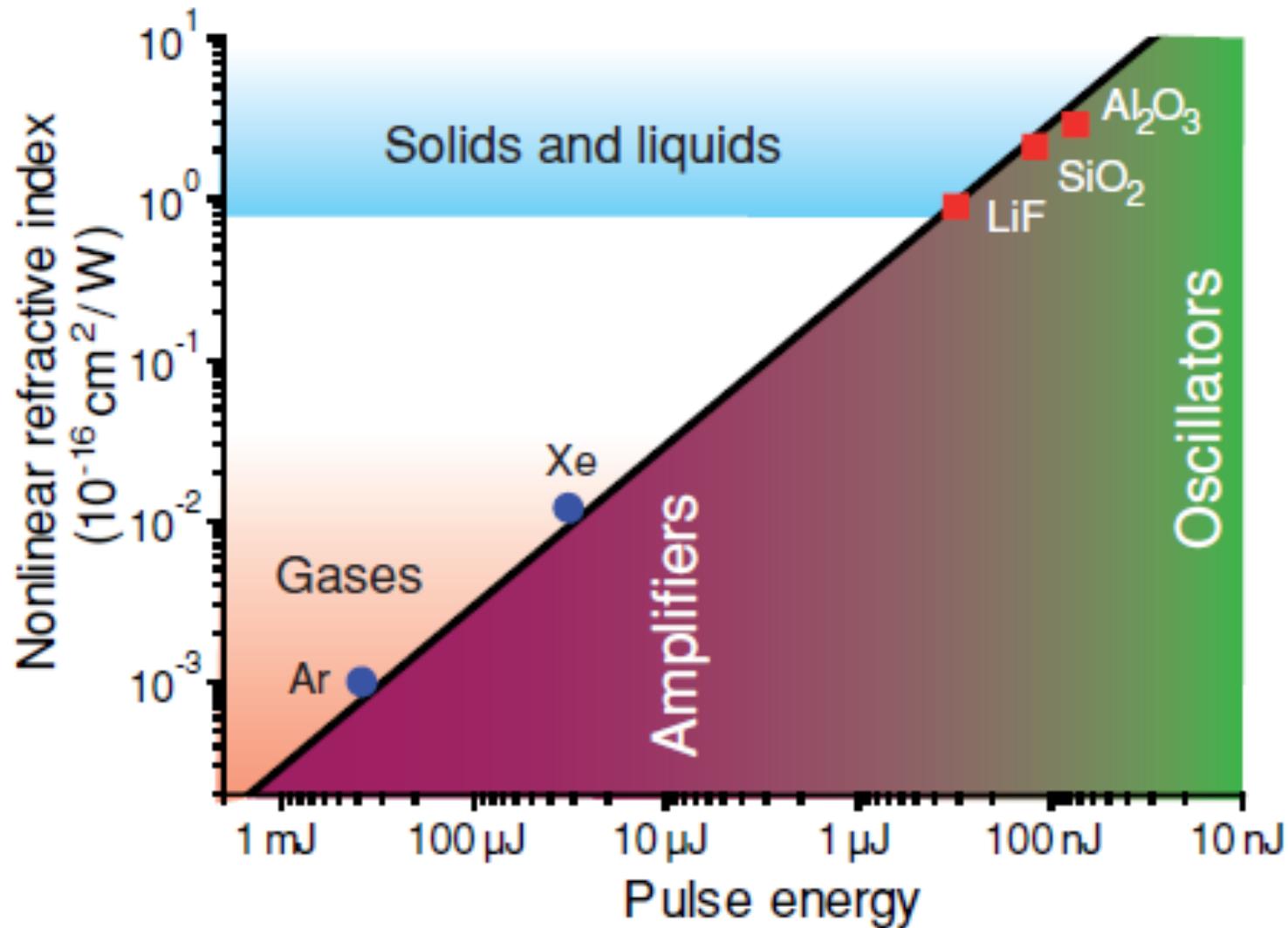
# Fast $f$ -to- $2f$



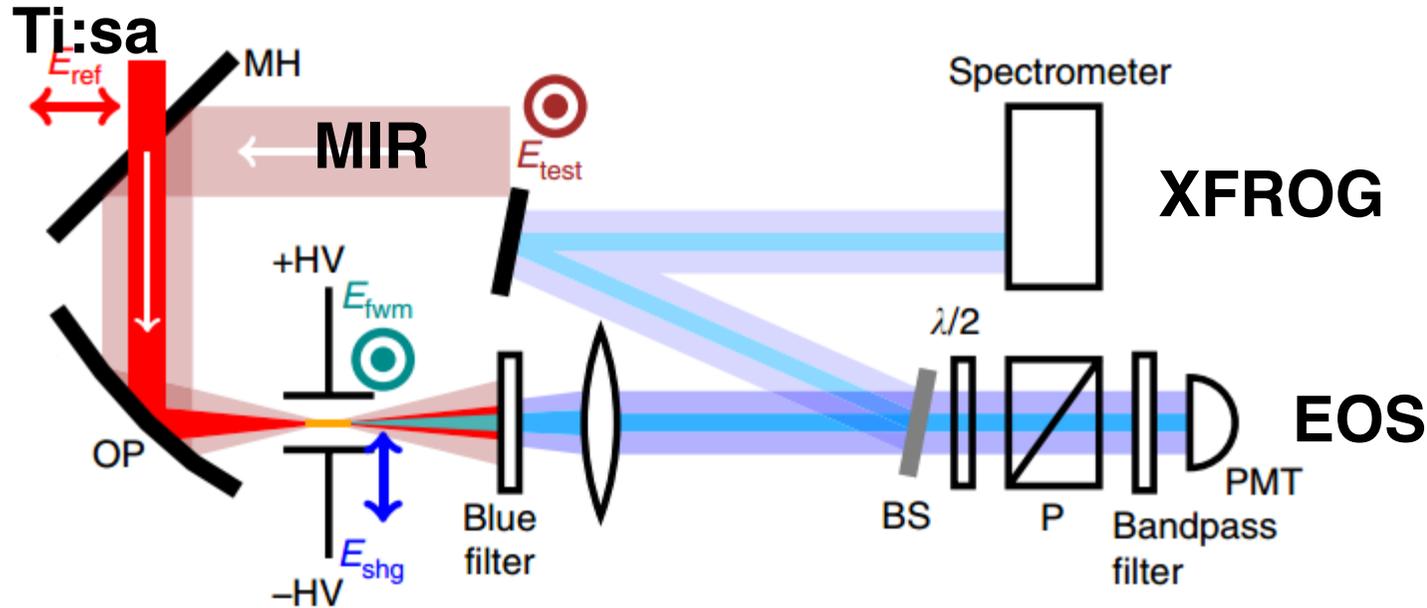
S. Koke et al., Opt. Lett. **33**, 2545-2547 (2008)



# Optimizing single-shot detection



# Combining FROG and CEP measurement



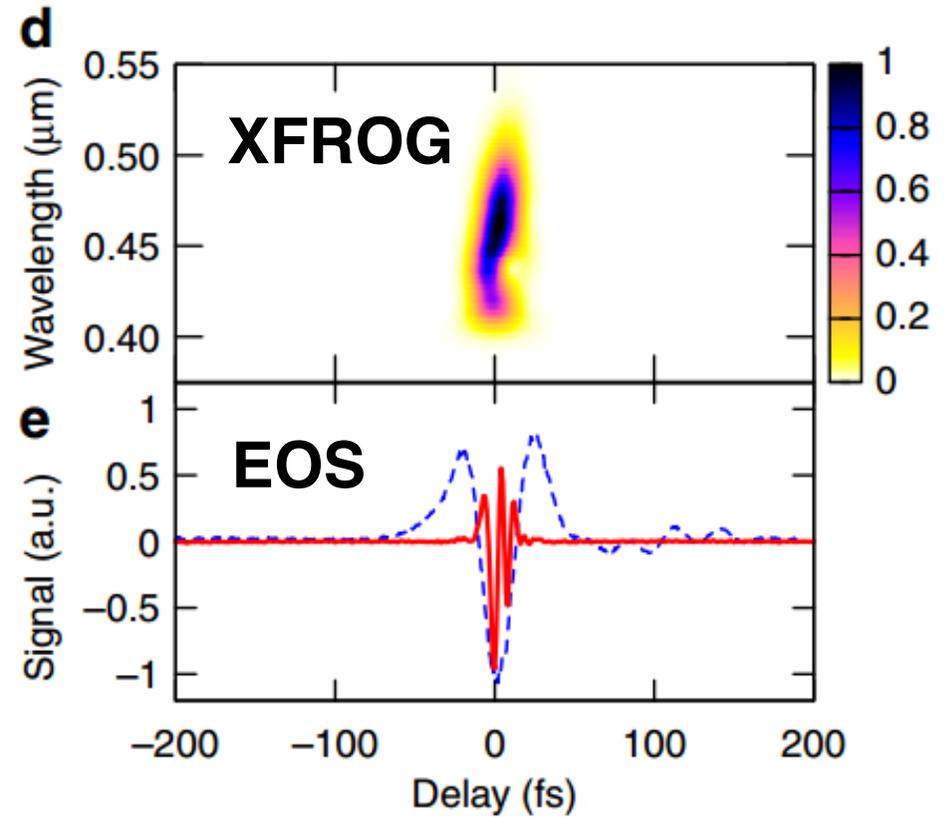
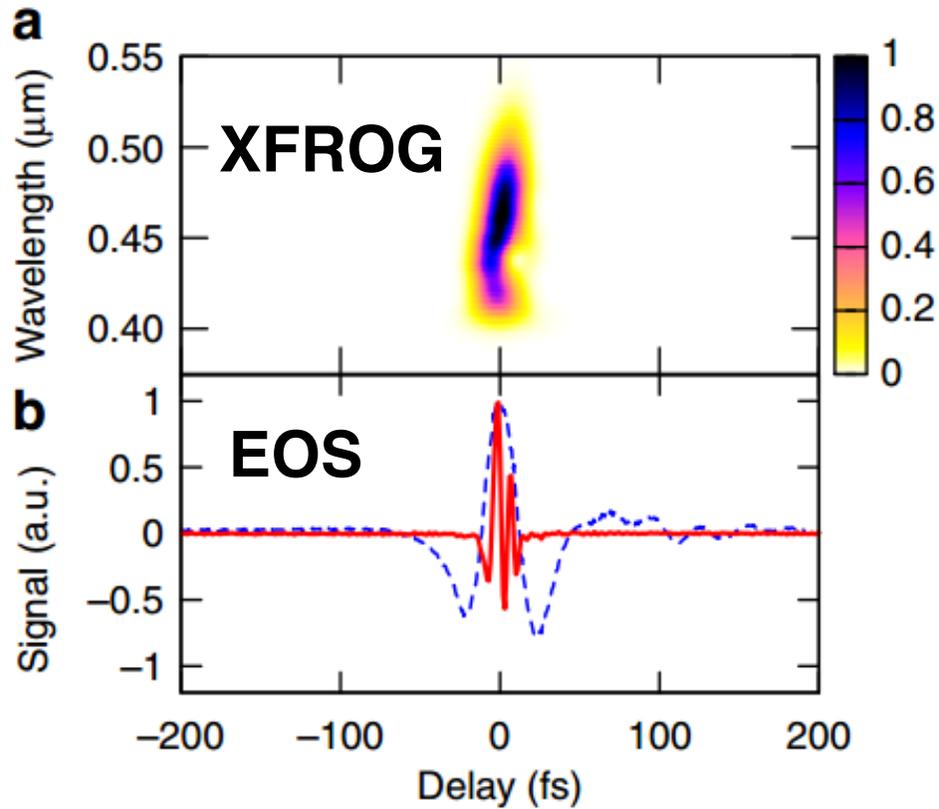
**Figure 1 | Schematic of the system for scanning operation.** BS, 7% beam splitter; HV, high voltage (4 kV); MH, mirror with a hole; OP, off-axis parabolic mirror; P, calcite polariser; PMT, photomultiplier tube.

**Use electro-optic sampling for CEP detection  
Use FROG at the same time**

**FWM and SHG are generated at crossed polarization**



# Combining FROG and CEP measurement



Y. Nomura et al., *Nature Communications* 4, 2820 (2013)

