THE UNITARY FERMI GAS: TAN RELATIONS

SCALE INVARIANCE AND RF-SPECTROSCOPY

TECHNISCHE UNIVERSITÄT MÜNCHEN

work done together with

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I) The Tan contact thermodynamics $\leftrightarrow$ short-distance physics

entropy as a function of inverse scattering length

$$dS(U,V,N,1/a) = \tfrac{1}{T}\, dU + \tfrac{p}{T}\, dV - \tfrac{\mu}{T}\, dN - \frac{X_{1/a}}{T}\, d\,(1/a)$$

gives $\quad dF(T,V,N,1/a) = -S\, dT - p\, dV + \mu\, dN + X_{1/a}\, d\,(1/a)$

i.e. $X_{1/a}\, d\,(1/a)$ is the work done in an infinitesimal change $d(1/a)$

define a positive, extensive **contact** by $X_{1/a} = -\hbar^2\, C/(4\pi m) < 0$

in a trap $C = \int_{\boldsymbol{R}} \mathcal{C}$ defines an intensive contact density $\mathcal{C}(\boldsymbol{R})$.

$$\frac{\partial F(T)}{\partial(1/a)} = \frac{\partial U(S)}{\partial(1/a)} = -\frac{\hbar^2}{4\pi m} \cdot \int_{\boldsymbol{R}} \mathcal{C}(\boldsymbol{R}) \quad \textbf{Tan adiabatic theorem}$$

$$d\,p(\mu,T,1/a) = n\, d\mu + s\, dT + \tfrac{\hbar^2}{4\pi m}\, \mathcal{C}\, d\,(1/a)$$

Tan relations (Tan '05 Braaten/Platter '08) $\mathcal{C}$ determines

- closed channel fraction $N_b = r^\star \cdot \int_{\boldsymbol{R}} \mathcal{C}(\boldsymbol{R})\,/(4\pi)$ Werner, .. '06

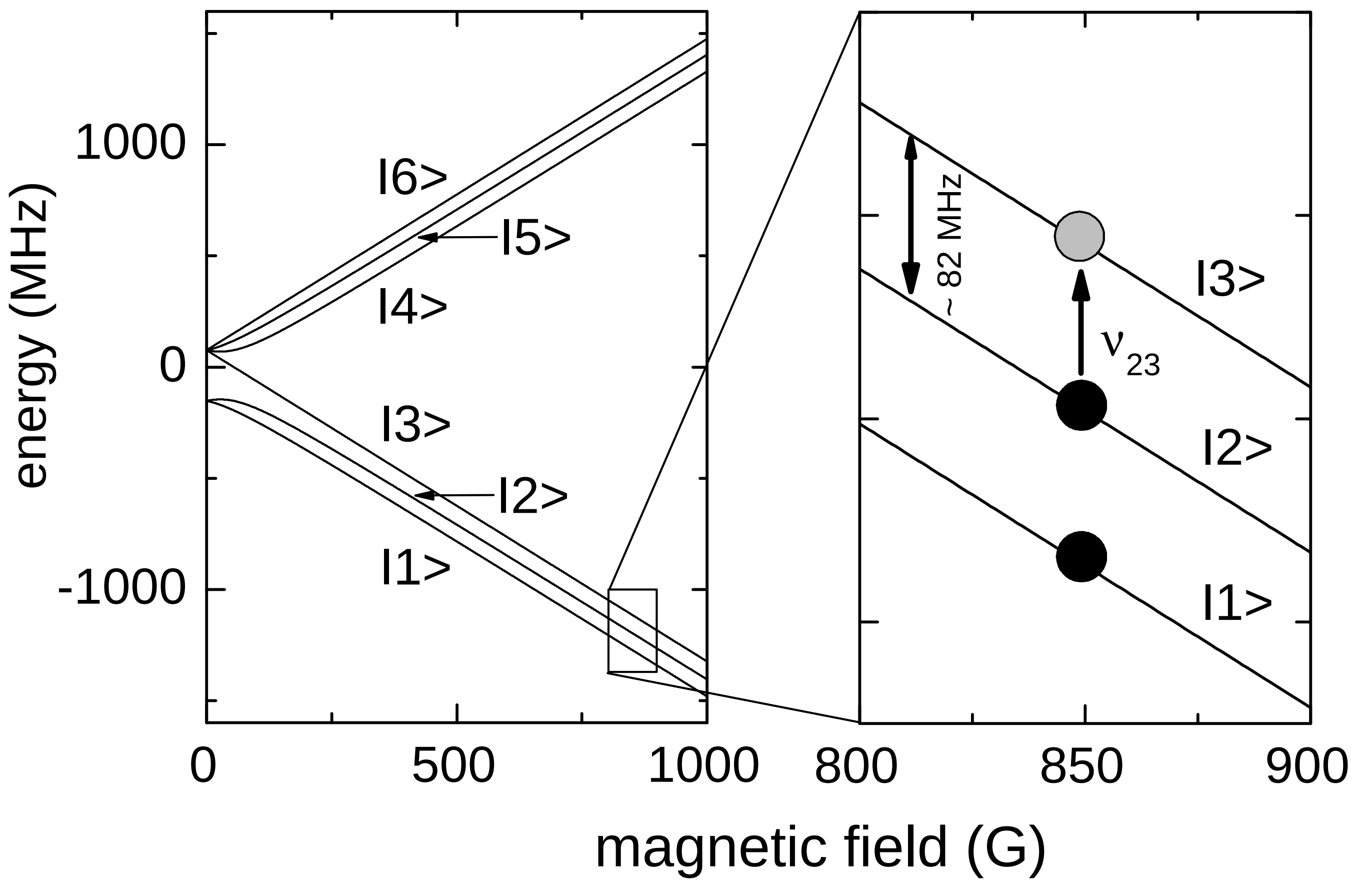
- energy density $\epsilon = \sum_\sigma \int_k \frac{\hbar^2 k^2}{2m}\left[n_\sigma(k) - \frac{\mathcal{C}}{k^4}\right] + \frac{\hbar^2 \mathcal{C}}{4\pi m a}$

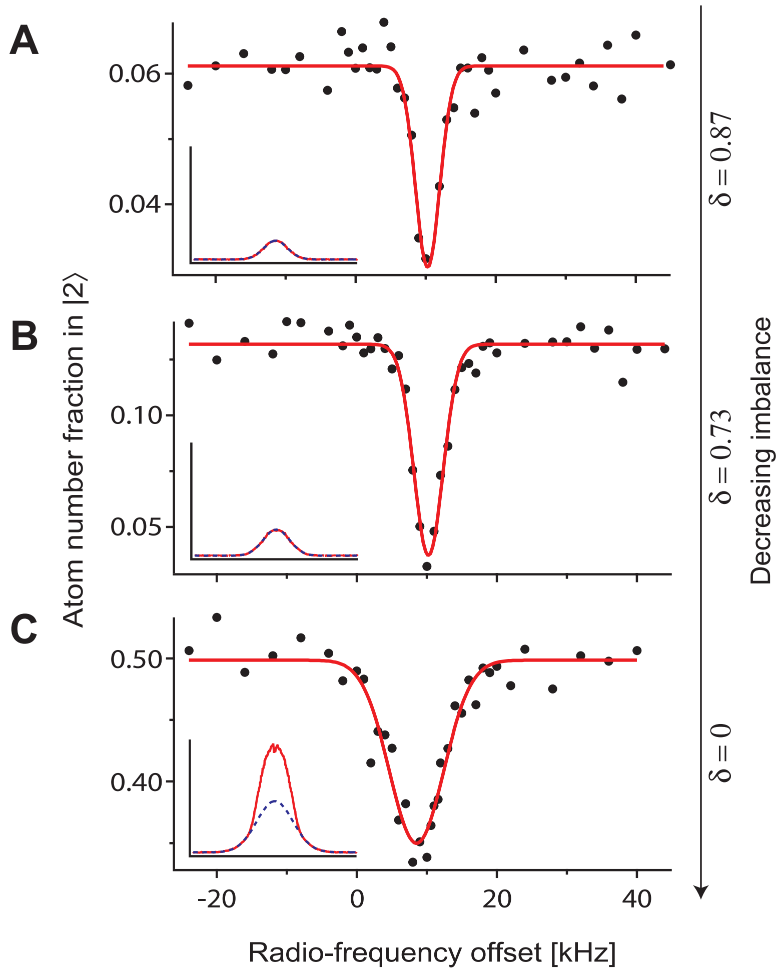
- virial theorem $U = 2\int_{\boldsymbol{R}} V_{\text{ext}}(\boldsymbol{R})\,n(\boldsymbol{R}) - \frac{\hbar^2}{8\pi m a}\int_R \mathcal{C}(\boldsymbol{R})$

- structure factor $S_{\uparrow\downarrow}(q) \to \mathcal{C}/8q$ Hu, Liu, Dr. '08

- **clock-shift and asymptotics of the RF-spectrum**

RF-spectroscopy in Fermi gases Chin et al '04





RF in imbalanced gases

Schunk et al '07

average clock shift

hardly changes from

balanced superfluid to

polarized normal phase

RF-spectrum Punk/Zw. + Baym et al '07

$$I(\omega) \sim \langle \left[\hat{\Psi}_3^\dagger(t) \hat{\Psi}_\downarrow(t), \hat{\Psi}_\downarrow^\dagger(0) \hat{\Psi}_3(0) \right] \rangle_\omega \rightarrow \int_k \mathrm{Im} G_\downarrow^R(\boldsymbol{k}, \xi_k - \omega)$$

clock shift $\hbar\bar{\omega} = \frac{\langle H_{12}' \rangle}{N_2} \left(\frac{\bar{g}_{13}}{\bar{g}_{12}} - 1 \right) = \frac{\hbar^2 \mathcal{C}}{4\pi n_2} \left(\frac{1}{a_{12}} - \frac{1}{a_{13}} \right)$

measures the **contact density** $\mathcal{C} = s \cdot k_{F\uparrow} k_{F\downarrow}^3$

at unitarity $\bar{\omega}(\sigma = 0) = -0.46\, v_F/a_{13}$ $(s \simeq 0.1)$

strongly imbalanced gas $\bar{\omega}(\sigma \lesssim 1) = -0.43\, v_F/a_{13}$

Fermions with zero range interactions

$$\mathcal{H}(\boldsymbol{R}) = \frac{\hbar^2}{2m}\sum_\sigma \nabla\hat{\Psi}^\dagger_\sigma \nabla\hat{\Psi}_\sigma(\boldsymbol{R}) + \bar{g}(\Lambda)\, \hat{\Psi}^\dagger_\uparrow \hat{\Psi}^\dagger_\downarrow \hat{\Psi}_\downarrow \hat{\Psi}_\uparrow(\boldsymbol{R})$$

contact density $\mathcal{C}(\boldsymbol{R}) = \bar{g}^2(\Lambda)\langle \hat{\Psi}^\dagger_\uparrow \hat{\Psi}^\dagger_\downarrow \hat{\Psi}_\downarrow \hat{\Psi}_\uparrow(\boldsymbol{R})\rangle = \bar{g}^2(\Lambda)\,\langle \mathcal{O}_c(\boldsymbol{R})\rangle$

measures probability of Fermions with opposite spin

to be close $\langle \hat{n}_\uparrow\left(\boldsymbol{R}-\frac{\boldsymbol{x}}{2}\right)\hat{n}_\downarrow\left(\boldsymbol{R}+\frac{\boldsymbol{x}}{2}\right)\rangle = \frac{\mathcal{C}(\boldsymbol{R})}{16\pi^2|\boldsymbol{x}|^2} + \dots$

the contact also appears in $G^{(1)}(\boldsymbol{x}\to 0)$ via **OPE** Braaten ... '10

$$\hat{\Psi}^\dagger_\sigma(\boldsymbol{R}+\frac{\boldsymbol{x}}{2})\hat{\Psi}_\sigma(\boldsymbol{R}-\frac{\boldsymbol{x}}{2}) = \hat{n}_\sigma(\boldsymbol{R}) + i\boldsymbol{x}\cdot\hat{\boldsymbol{p}}_\sigma(\boldsymbol{R}) - \frac{|\boldsymbol{x}|}{8\pi} g^2(\Lambda)\, \hat{\Psi}^\dagger_\uparrow \hat{\Psi}^\dagger_\downarrow \hat{\Psi}_\downarrow \hat{\Psi}_\uparrow(\boldsymbol{R})$$

$\rightarrow$ tail in the momentum distribution $n_\sigma(\boldsymbol{k}) \to \mathcal{C}/k^4$

II) Fermions at Unitarity or **High -$\mathbf{T}_c$ below** 1 μK

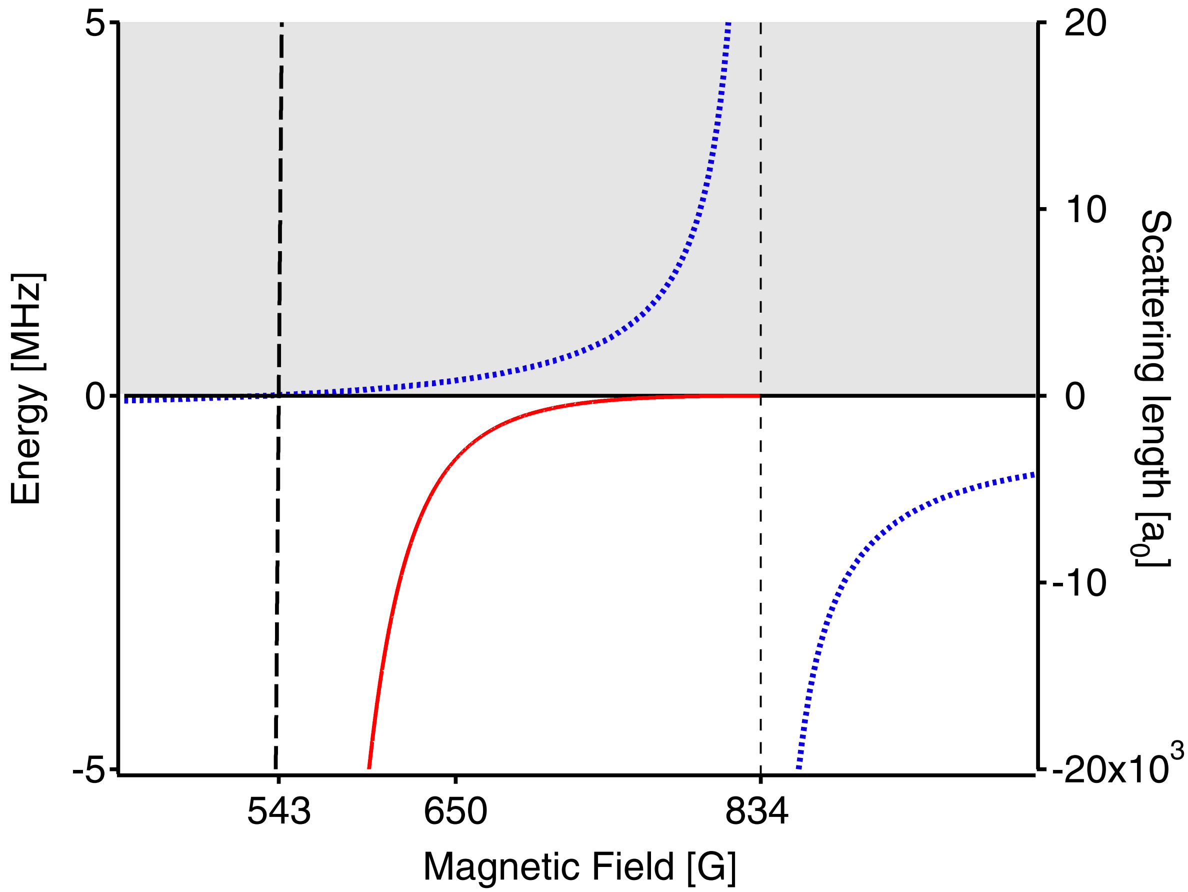
BCS '57 Fermions $\uparrow\downarrow$ with density $n = k_F^3/3\pi^2$ and

attractive two-particle interaction $V_{\uparrow\downarrow}(\boldsymbol{x}) = \bar{g} \cdot \delta(\boldsymbol{x})$

pairs both form and condense at $T_c \sim \exp -\frac{1}{|\bar{g}|N(0)} \ll T_F$

what happens at infinite coupling $g = \infty$?

scattering length in ^{6}Li $\quad gN(0) \to 2k_F a/\pi \to \infty$ at $B_0 = 832.15\,\mathrm{G}$



Fermions remain stable for arbitr. strong zero range attraction

attractive fermions evolve into **repulsive** bosons

$a_{dd} = 0.6\, a > 0$ Petrov/Shlyapnikov/Salomon '03

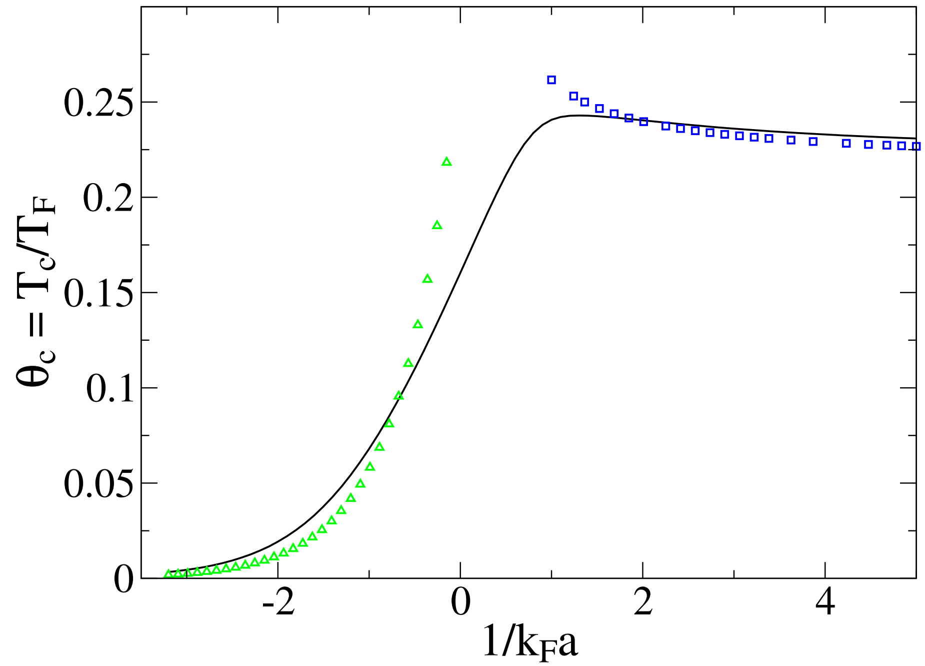
critical temperature

Nozieres/SR '85

Drechsler/Zw. '92

Randeria,.. '93

Haussmann '94



BEC at $T_c = 0.218\, T_F \sim n^{2/3}$ independent of a

Scale Invariance at $a = \infty$ and for $k_F r_e \to 0$ $\quad \boldsymbol{x} \to s\,\boldsymbol{x}$ gives

$\widehat{H} \to \widehat{H}/s^2$ since $\delta_0(k) = \pi/2 - r_e k/2 + \ldots$ becomes indep. of k

$\to$ pressure $p = 2\epsilon/3$ Ho '04 bulk viscosity $\zeta = 0$ Son '07

ground state $\quad p(\infty) = \xi_s \cdot p_F^{(0)}$ **Bertsch-parameter** ξ_s

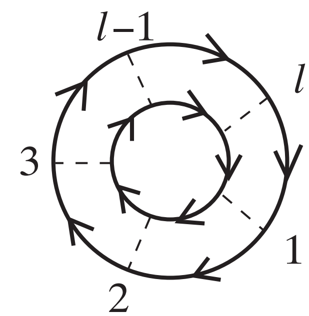
determines cloud size in a trap $\quad R_{TF} = R_{TF}^{(0)} \cdot \xi_s^{1/4}$

universal numbers $\xi_s \simeq 0.36$, $T_c \simeq 0.16\,T_F$, $\Delta_0 \simeq 0.46\,\epsilon_F$

transport: **shear viscosity** $\eta(T_c) \simeq 0.5\,\hbar n$ Shuryak '04

Luttinger-Ward approach two-body int. $V_{\uparrow\downarrow}(\boldsymbol{x}) = \bar{g}(\Lambda)\,\delta(\boldsymbol{x})$

$$\Omega[\widehat{G}] = -k_B T \ln Z = \beta^{-1}\Big(-\tfrac{1}{2}\mathrm{Tr}\{-\ln \widehat{G} + [\widehat{G}_0^{-1}\widehat{G} - 1]\} - \Phi[\widehat{G}]\Big)$$

Ladder-approximation $\Phi[G] = \sum_{l=1}^{\infty}$ 

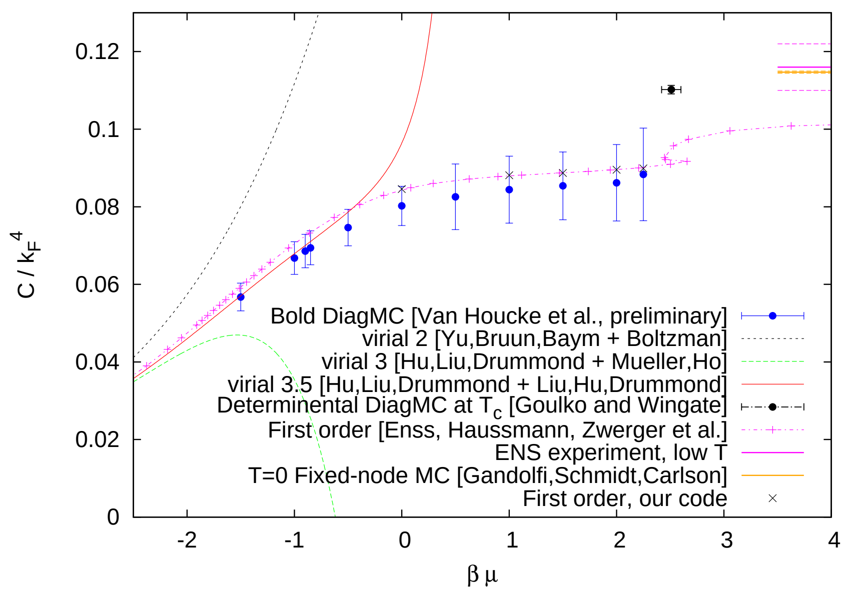
$\delta\Omega[\widehat{G}]/\delta\widehat{G} = 0$ determines both $\mathcal{G}(\boldsymbol{k},\tau)$ and $\mathcal{F}(\boldsymbol{k},\tau)$

Haussmann/Rantner/Cerrito/Zw. '07

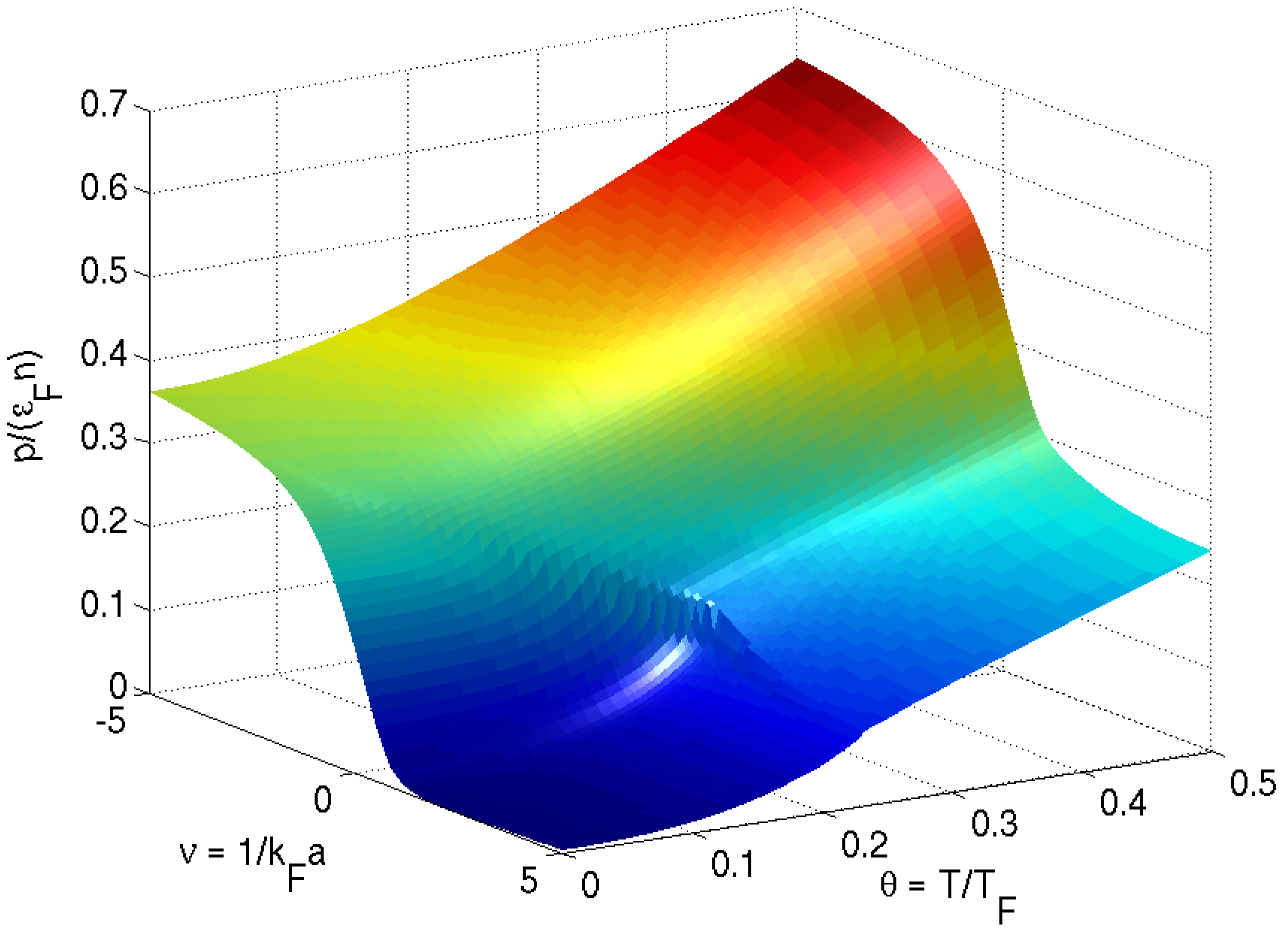
why does Luttinger-Ward work well ?

- it is **conserving** $\rightarrow$ all th. dyn. relations are obeyed
- it obeys **scale invariance** at $a = \infty$ $\rightarrow$ $p = 2\epsilon/3$
- it obeys the **Tan relations** e.g. $\dfrac{\partial\, p(\mu, T)}{\partial\,(1/a)} = \dfrac{\hbar^2 \mathcal{C}}{4\pi m}$

contact density $\mathcal{C}$



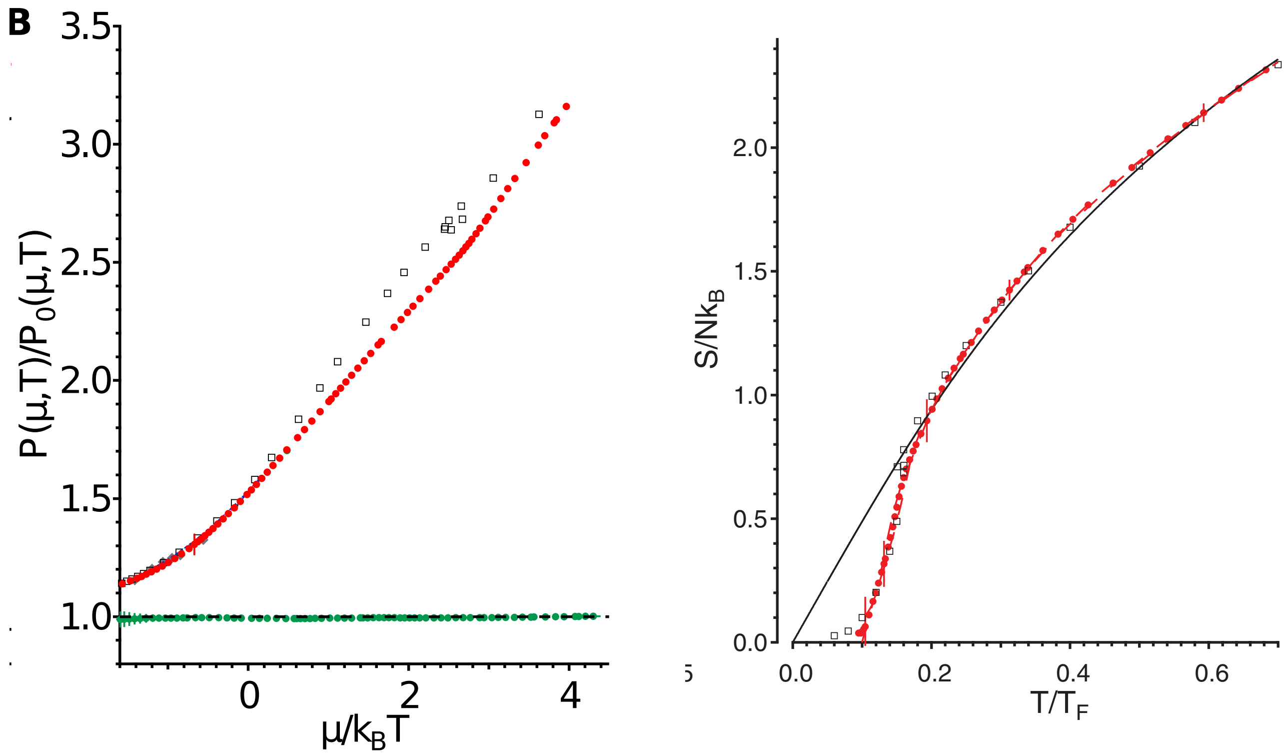
pressure as a function of T/T_F and $1/k_F a$ $(\xi_s = 0.36)$



equation of state from density profiles $n(V_{\text{ext}}) \to n(\mu)$

pressure $P(\mu) = \int^{\mu} n(\mu')$ entropy $S/Nk_B = (P/P_0 - \mu/\epsilon_F)\, T_F/T$

universal scaling function $P(\mu,T)/P_0(\mu,T) = f(\beta\mu) \;\to\; \xi_s^{-3/2} \simeq 4.45$



Ku, .. Zwierlein

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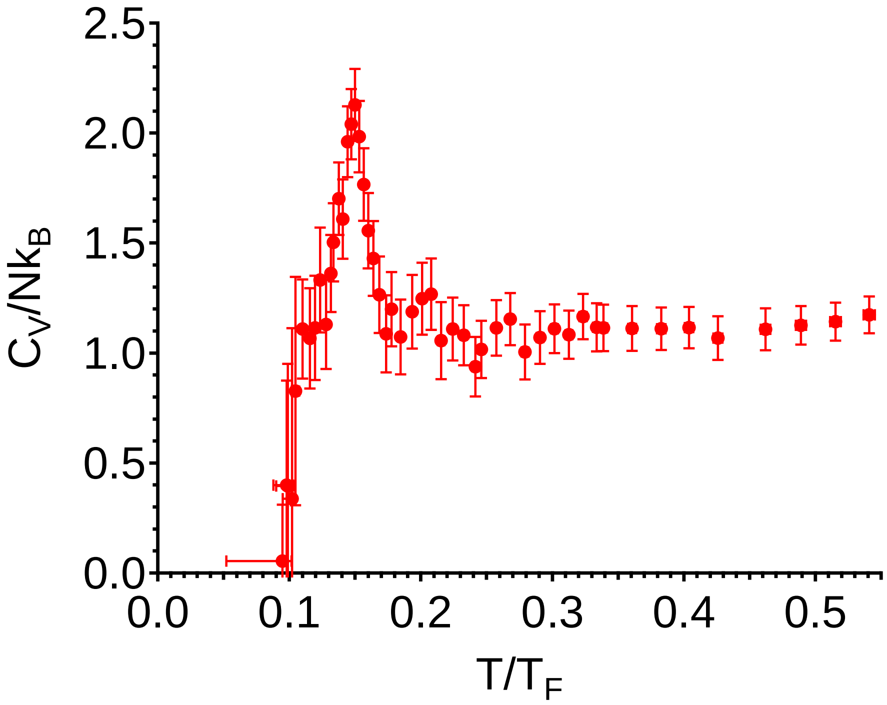
$T_c/T_F \simeq 0.16$ at

$(\mu/k_B T)_c \simeq 2.5$

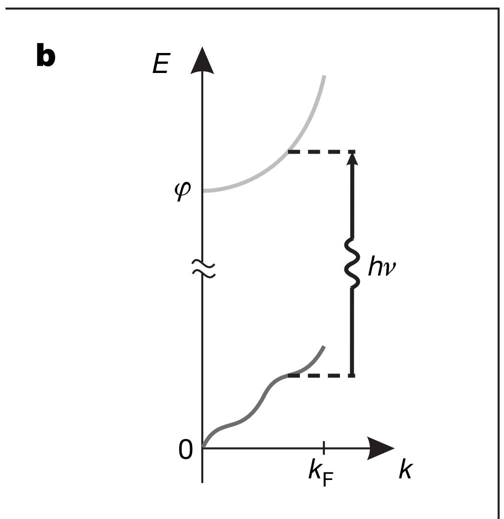
$\xi_s = 0.370(5)$

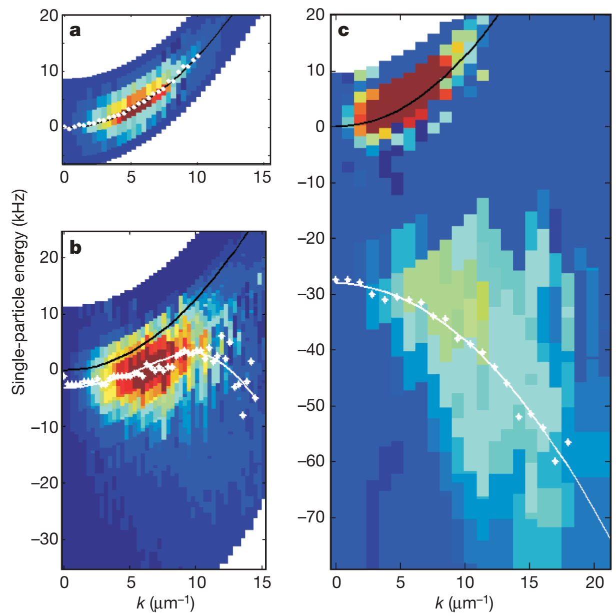
benchmark for **bold diagrammatic MC** van Houcke et al '12

specific heat exhibits a jump $\Delta C/C|_{T_c} \simeq 1.2$ (1.43 in BCS)



II) Momentum resolved RF Stewart,Gaebler,Jin '08





measures hole spectral function $A_{-}(\boldsymbol{k}, \varepsilon_{\boldsymbol{k}} - \hbar\omega)$

$A(\boldsymbol{k},\varepsilon)$ from $\mathcal{G}(\boldsymbol{k},\tau)$ via $\mathcal{G}(\boldsymbol{k},\omega_n) = \int d\varepsilon \, \frac{A(\boldsymbol{k},\varepsilon)}{-i\hbar\omega_n + \varepsilon - \mu}$ (**Maxent**)

momentum integrated rf locally resolved Y. Shin,... MIT '08

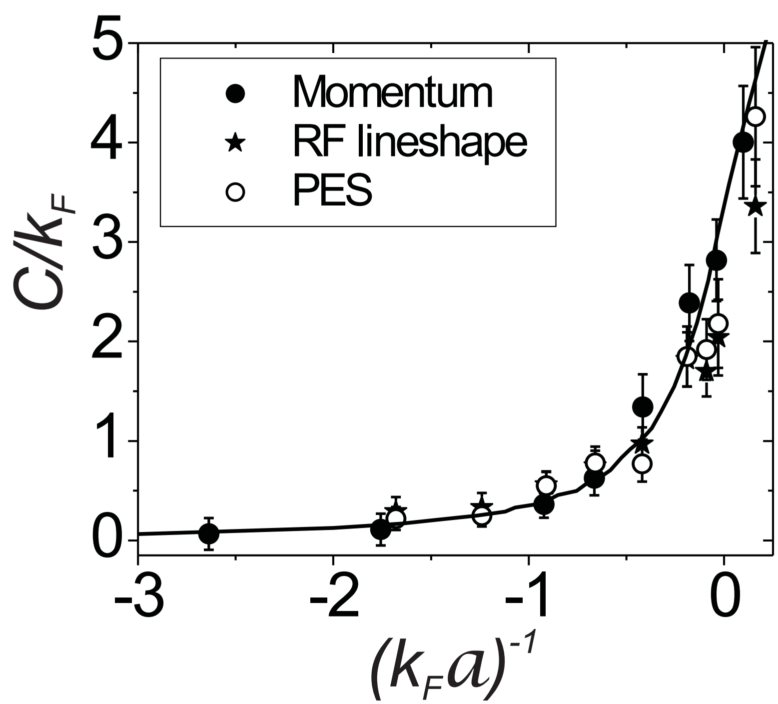
$I(\omega) = \hbar \int_k \; A_-(\boldsymbol{k}, \varepsilon_{\boldsymbol{k}} - \hbar\omega)$ (no final state interactions)

high frequency asymptotics $I(\omega) \to \frac{\mathcal{C}}{4\pi^2} \left(\frac{\hbar}{m}\right)^{1/2} \cdot \omega^{-3/2}$

consistent with

$n_\sigma(\boldsymbol{k}) \to \mathcal{C}/k^4$

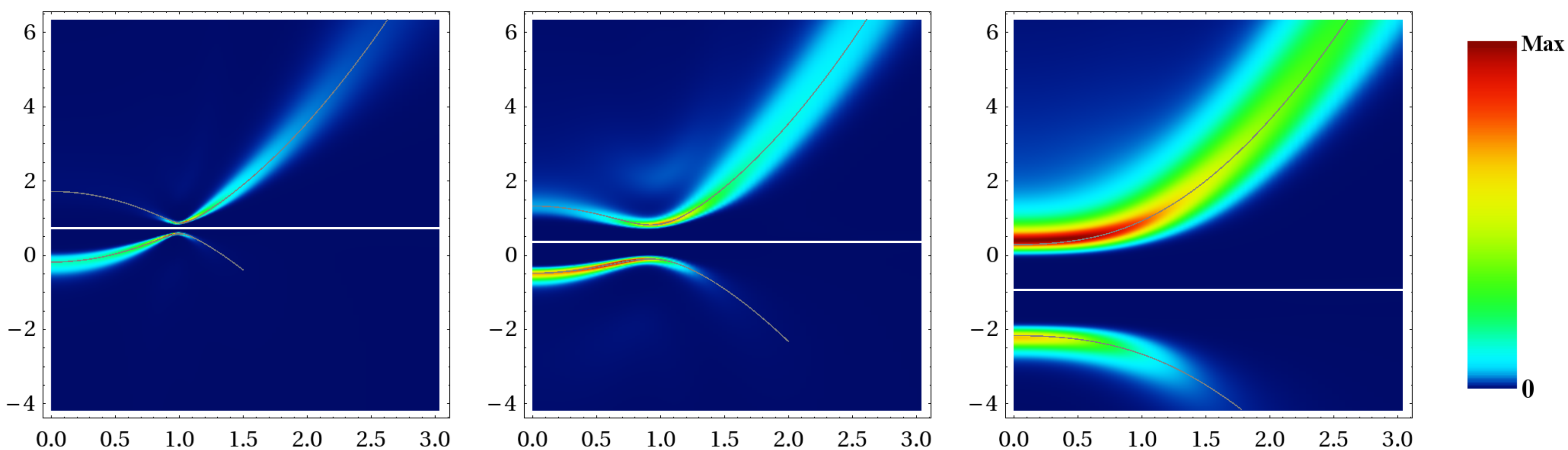
Stewart et al '10



BCS quasiparticles $A_{\mathrm{BCS}}(\boldsymbol{k},\varepsilon)=u_{\boldsymbol{k}}^2\,\delta(\varepsilon-E_{\boldsymbol{k}}^{(+)})+v_{\boldsymbol{k}}^2\,\delta(\varepsilon-E_{\boldsymbol{k}}^{(-)})$

at $E_{\boldsymbol{k}}^{(\pm)}=\mu\pm\sqrt{(\varepsilon_k-\mu)^2+\Delta^2}$ have infinite lifetime

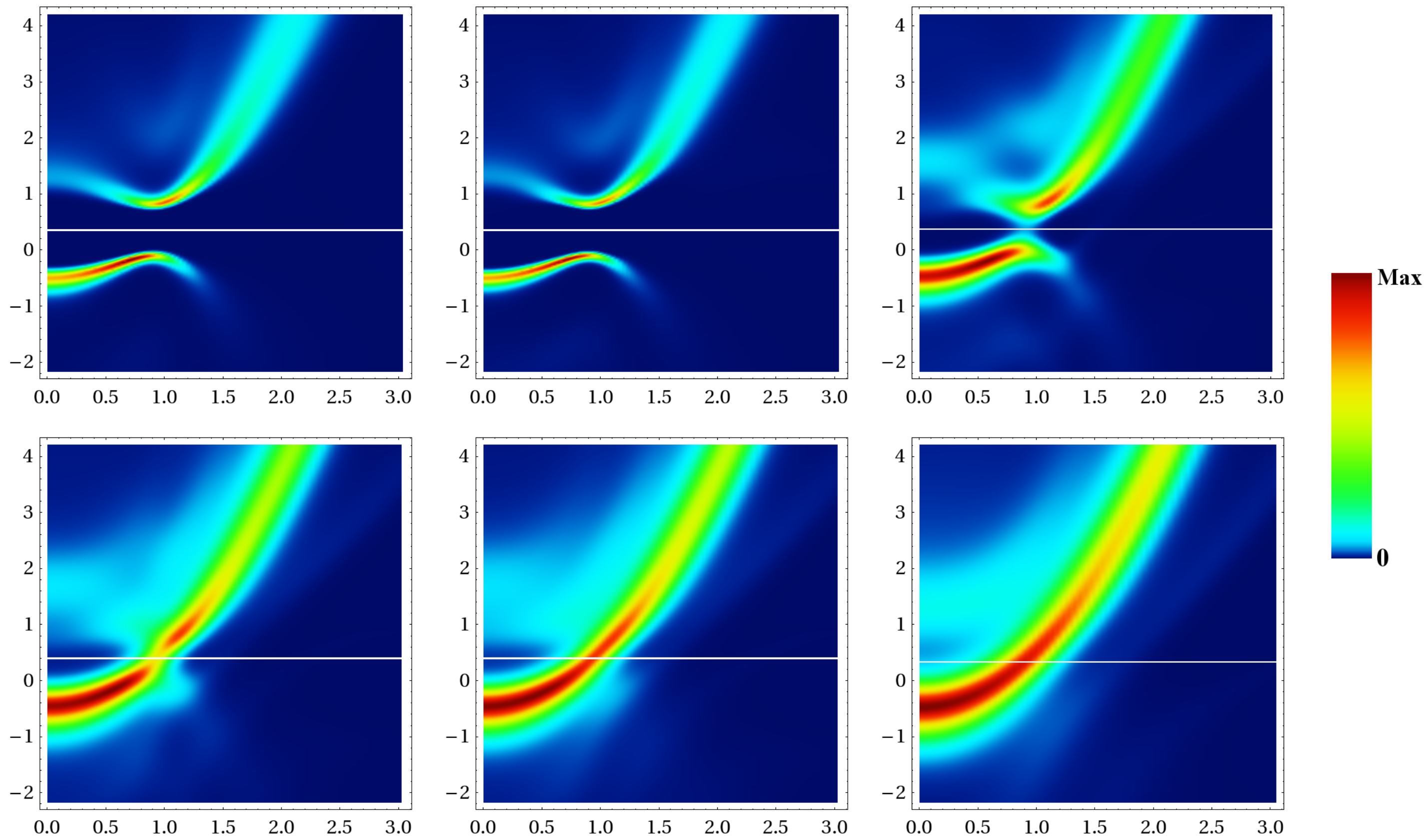
numerical spectral functions $A(\boldsymbol{k},\varepsilon)$ at $T=0$



$(k_F a)^{-1}=-1$ **unitarity** $(k_F a)^{-1}=+1$

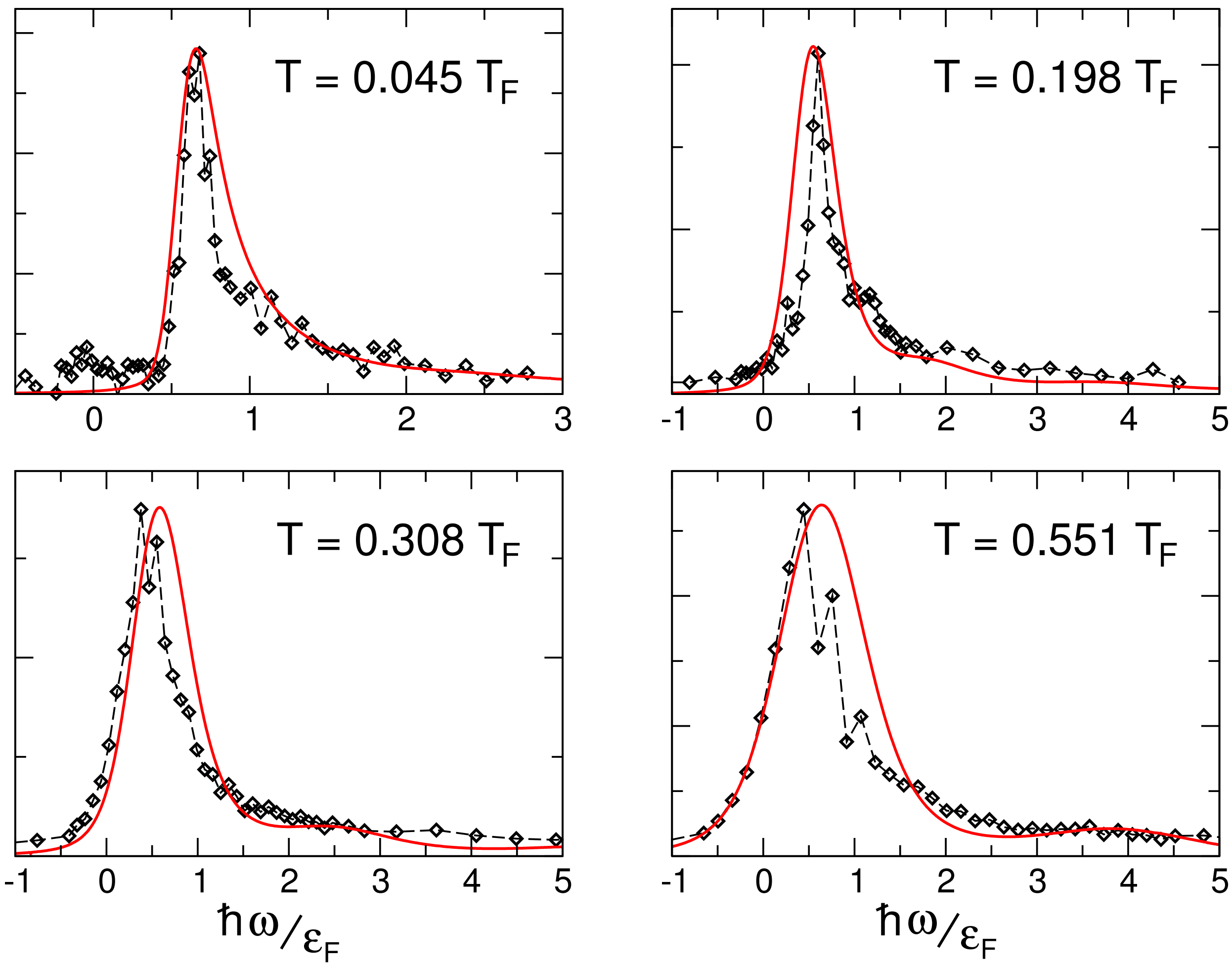
sharp quasiparticles only near excitation minimum $k_0<k_F$

spectral functions at unitarity for $T/T_F = 0.01, ..0.14, 0.16, ..0.3$



no pronounced pseudogap above T_c in spite of backbending

comparison with experiment Haussmann/Punk/Zw. '09



rf-spectra

at unitarity

locally resolved

at trap center

MIT '08

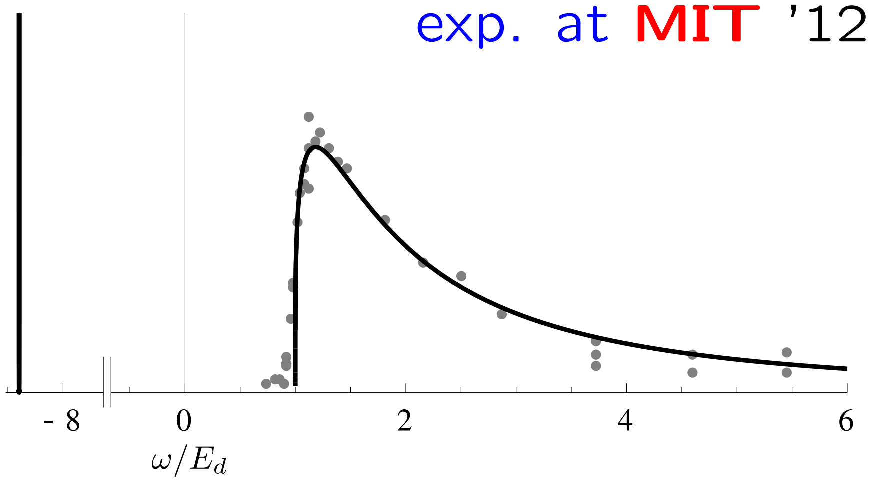
RF-spectroscopy in 2D Langmack/Barth/Braaten/Zw. '12

$V(\boldsymbol{x}) = g_2\, \delta(\boldsymbol{x})$ seems scale invariant Pitaevskii, Rosch '97

scatt. amplitude $f(q) = \dfrac{4\pi}{\ln(1/q^2 a_2^2) + i\pi}$ requires $g_2(\Lambda) = -\dfrac{2\pi}{\ln(\Lambda a_2)}$

broken scale invariance in RF $I(\omega) \to \dfrac{\ln^2(E_d'/E_d)\, \mathcal{C}}{4m\omega^2 \left[\ln^2(\omega/E_d') + \pi^2\right]}$

exp. at **MIT** '12



smooth onset of bound-free

spectrum $\sim \mathcal{C}/\ln^2(\omega - E_d)$

dimer energy at 3D Feshbach-

resonance $E_d = 0.244\, \hbar\omega_z$