THE SEARCH FOR A PERFECT FLUID AND TRANSPORT WITHOUT QUASIPARTICLES

work done together with

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Varenna, Lecture 3, July 9, 2014

Scale invariance and non-relativistic CFT's Nishida/Son '07

dilatation operator generates $\boldsymbol{x} \to e^{\lambda}\boldsymbol{x}$ and $t \to e^{2\lambda} t$

$\hat{H}$ is scale-invariant if $\quad i\left[\hat{H}, \hat{D}\right] = 2\,\hat{H} = \frac{d}{dt}\int x_i \hat{\jmath}_i$

momentum balance $\partial_t \hat{\jmath}_i = -\partial_j \hat{\Pi}_{ij} \to$

$\frac{d}{dt}\hat{D} = 2\int \hat{\epsilon} = \int x_i \partial_t \hat{\jmath}_i = -\int x_i\,\partial_j \hat{\Pi}_{ij} = \int \hat{\Pi}_{ii}$

scale invariance implies $\boxed{2\epsilon = \Pi_{ii}}$ trace of stress tensor

conformal transformation $\boldsymbol{x} \to \boldsymbol{x}/(1+\lambda t)$ and $t \to t/(1+\lambda t)$

$\hat{H}$ is conformal-invariant if $i\left[\hat{H}, \hat{C}\right] = \hat{D} = \frac{d}{dt}\int \boldsymbol{x}^2 \hat{n}/2 \to$

breathing mode in a trap at $\boxed{\omega_B = 2\omega_{\text{trap}}}$ Castin/Werner '06

The unitary gas as a quantum critical point   Nikolic/Sachdev '07

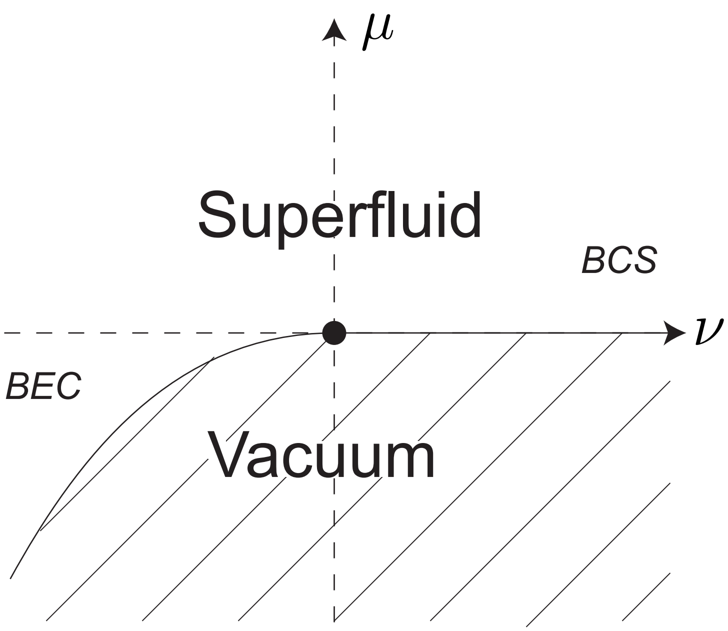
action of free Fermions $S_0 = \int d\tau \int d^d x \left[\psi^\star \partial_\tau \psi + \frac{\hbar^2}{2m} |\nabla \psi|^2\right]$

is invariant under $x \to x e^{-l}, \tau \to \tau e^{-zl}$ and $\psi \to \psi\, e^{dl/2}$ $\to$

dyn. exponent $z = 2$ and $\dim[\psi] = d/2$

chemical potential $\mathcal{L}_\mu = -\mu|\psi|^2$ scales like $\mu \to \mu e^{2l} \to \dim[\mu] = 2$

add zero range interaction $\mathcal{L}_{\text{int}} = u_0\, \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow \to \dim[u_0] = 2 - d$



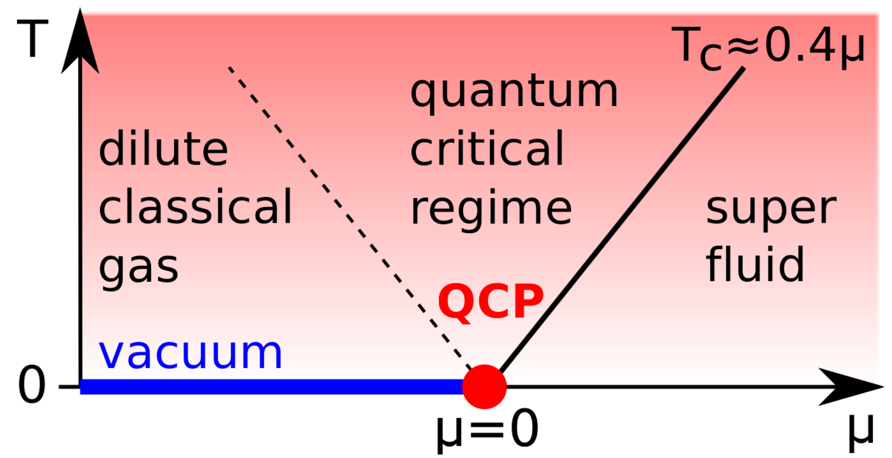
scaling of the dim.less coupling

$u = 2mS_d \Lambda^{d-2} u_0$ under $\Lambda \to \Lambda e^{-l}$

$du/dl = \epsilon u - u^2/2$ with $\epsilon = 2 - d$

unitary Fermions $\rightarrow$ zero density QCP

phase diagram at finite T



no well defined quasiparticles

in the critical regime $|\mu| \ll k_B T$

single length scale λ_T $\rightarrow$ density $n\lambda_T^3 \simeq 3.1$ vanishes $\sim T^{3/2}$

universal amplitude ratio e.g. $s(T) \sim k_B \cdot \lambda_T^{-3}$ and $\eta(T) \sim \hbar \cdot \lambda_T^{-3}$

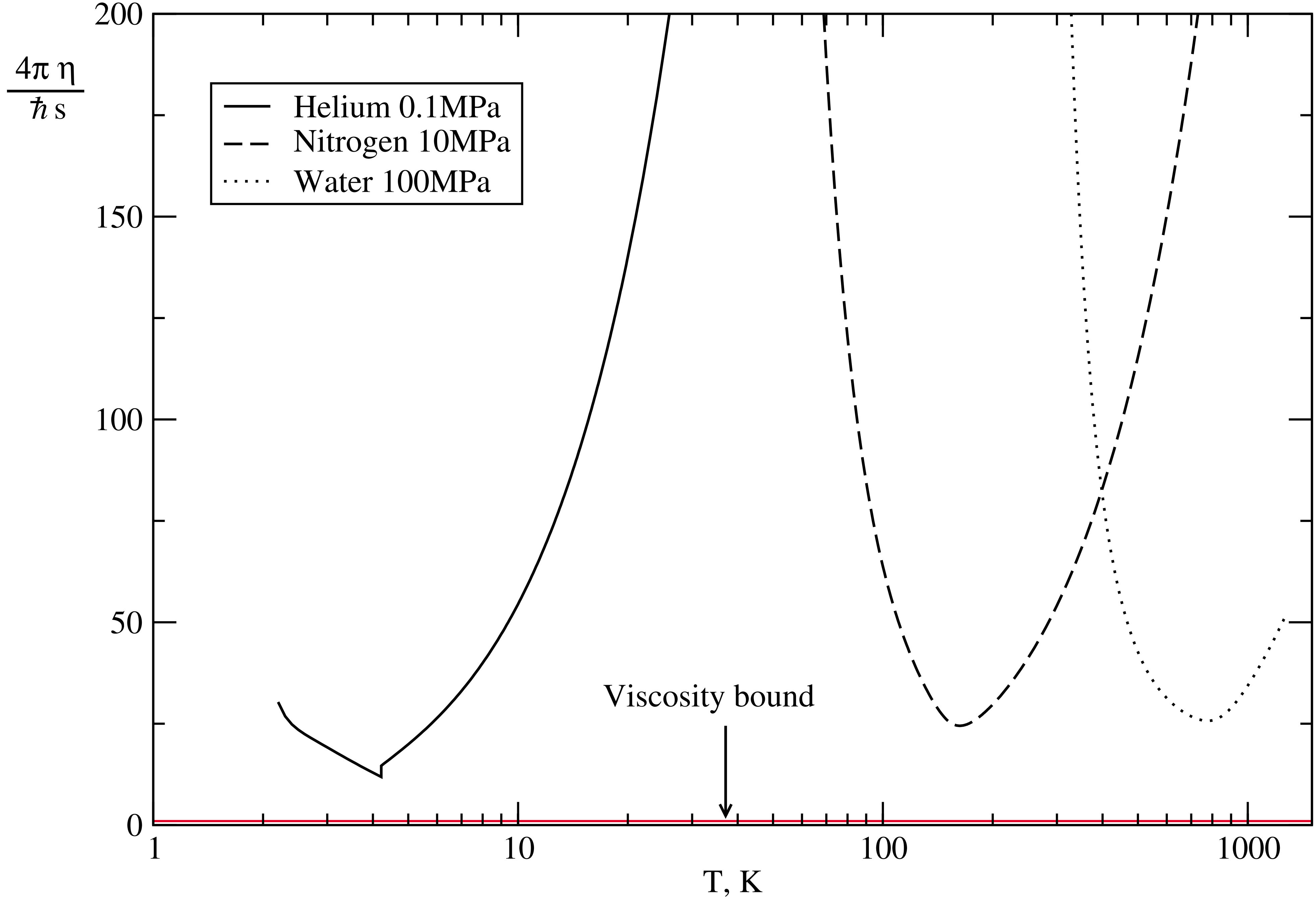
$\rightarrow$ $\eta/s \simeq 0.7\hbar/k_B$ characterizes a **strongly coupled QFT**

dimensional analysis suffices if there is no anomalous dimension !

The unitary gas as a 'perfect fluid'

Definition A fluid is perfect if $\boxed{\frac{\eta}{s} \equiv \frac{\hbar}{4\pi k_B}}$ Kovtun/Son

Starinets '05 (SSYM) **All** known fluids have $\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$!!



200
150
100
50
0
$\frac{4\pi\,\eta}{\hbar\, s}$
Helium 0.1MPa
Nitrogen 10MPa
Water 100MPa
Viscosity bound
1
10
100
1000
T, K

momentum balance $\partial_t(\rho v_i) + \partial_j \Pi_{ij} = 0$

$$\Pi_{ij} = p\delta_{ij} + \rho v_i v_j - \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij} \cdot \partial_k v_k \right) - \zeta \delta_{ij} \cdot \partial_k v_k$$

positivity $\eta \geq 0$ and $\zeta \geq 0$ due to $dS/dt \geq 0$

liquids thermally activated $\rightarrow \eta(T)$ grows as $T \downarrow$

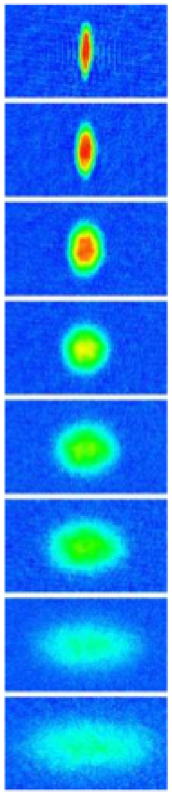
gases $\eta = \frac{1}{3} mn\langle v \rangle \ell \simeq \sqrt{mk_B T}/\sigma(T)$ grows as $T \uparrow$

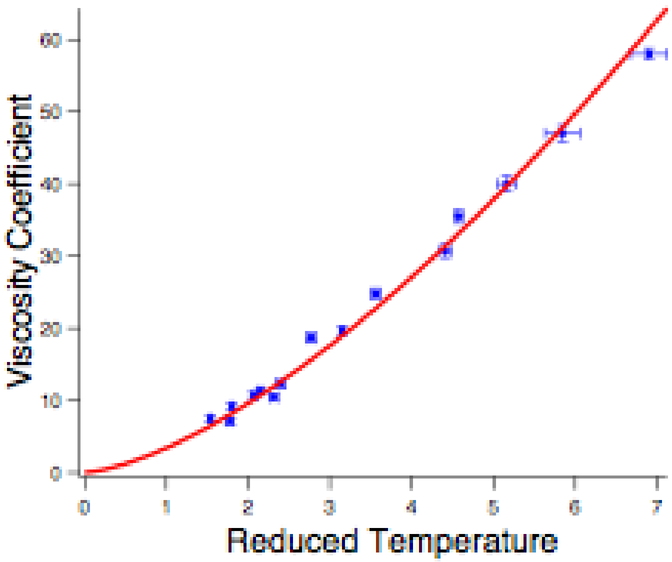
A quantum limit on the viscosity ?

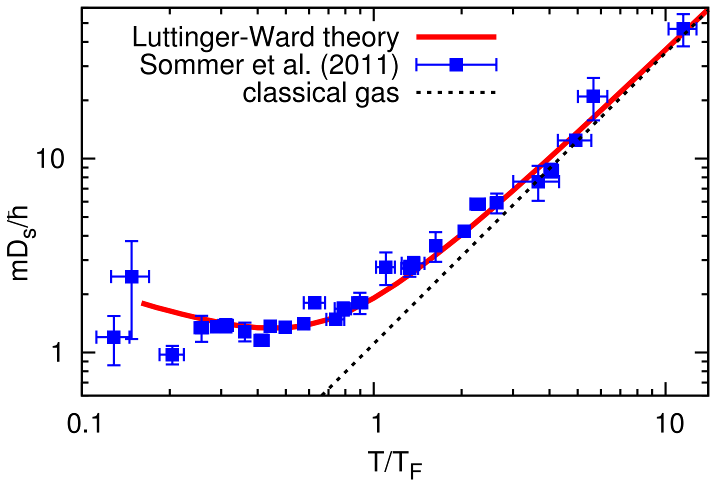
mean free path $\ell \gtrsim n^{-1/3}$ average velocity $\langle v \rangle \gtrsim \frac{\hbar}{m} n^{1/3}$

gives $\eta \geq \alpha_\eta \cdot \hbar n$ e.g. $\alpha_\eta \simeq 0.5$ for ^{4}He at 2K

exp. data on **viscosity** and **spin diffusion**







Cao ... Science **331** (2011) and Sommer ... Nature **472** (2011)

shear viscosity of the unitary gas

Boltzmann-limit $\eta(T \gg T_F) = 2.8\,\hbar n (T/T_F)^{3/2} = 4.2\,\frac{\hbar}{\lambda_T^3}$

(density drops out!), well defined quasipart. $\hbar/\tau_\eta \ll k_B T$

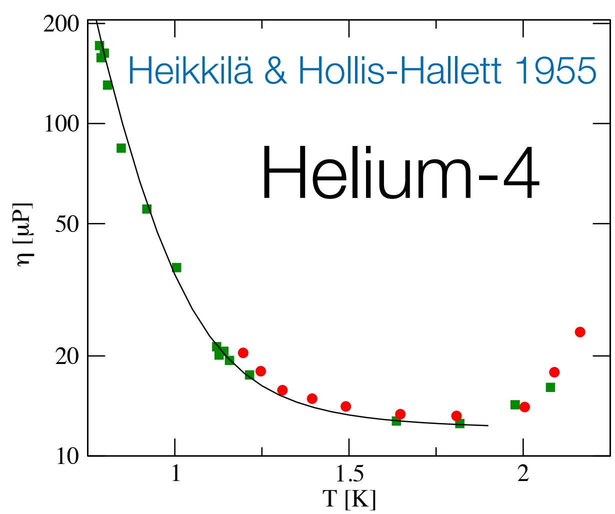
superfluid below $T_c \simeq 0.16\,T_F$ has **finite** viscosity due to

a) phonon interactions: $\eta(T) \sim T^{-5}$ as $T \ll T_c$ Rupak/Schäfer '07

b) fermionic qp's: $\eta(T) \to$ const as $T \to 0$ Pethick/Smith '75

minimum is observed in ^{4}He

just below T_λ



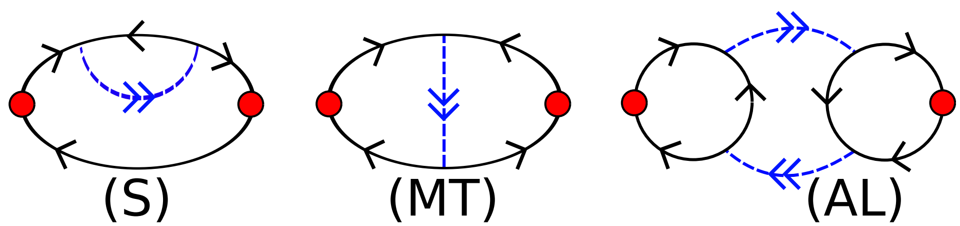
Viscosity in linear response: Kubo formula

- viscosity from stress correlations (cf. hydrodynamics):

$$\eta(\omega) = \frac{1}{\omega}\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t}\int d^3x \left\langle \left[\hat{\Pi}_{xy}(\boldsymbol{x},t), \hat{\Pi}_{xy}(0,0)\right] \right\rangle$$

with stress tensor $\hat{\Pi}_{xy} = \sum_{\mathbf{p},\sigma} \frac{p_x p_y}{m}\, c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma}$ (cf. Newton $\frac{\partial v_x}{\partial y}$)

- correlation function (Kubo formula): Enss, Haussmann & Zwerger Ann. Phys. 2011

η(ω) = 

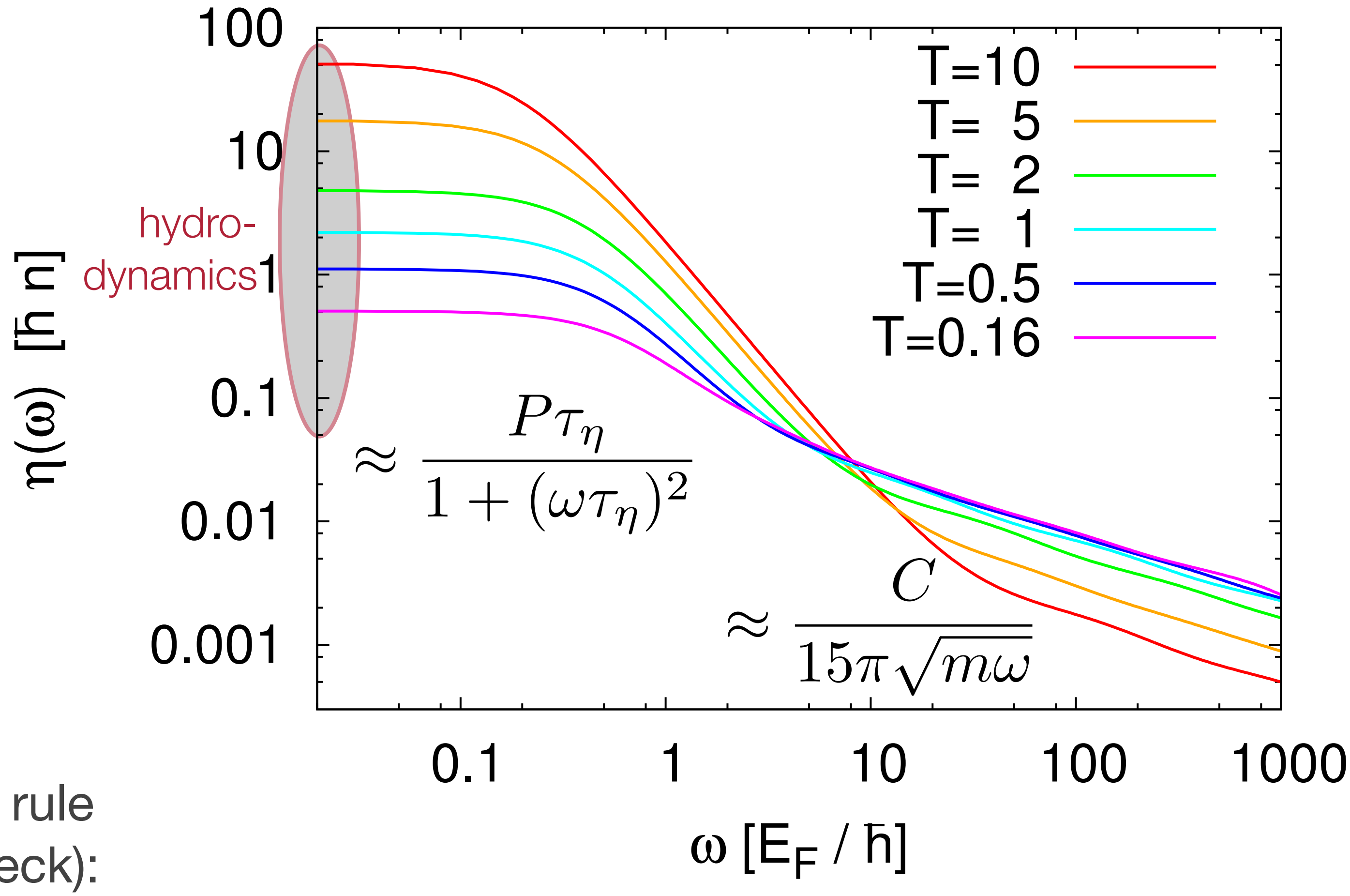
(resummed to infinite order)

- transport via fermions and bosonic molecules: very efficient description, satisfies conservation laws, scale invariance and Tan relations Enss PRA 2012

- assumes no quasiparticles: beyond Boltzmann kinetic theory, works near Tc

Dynamic shear viscosity

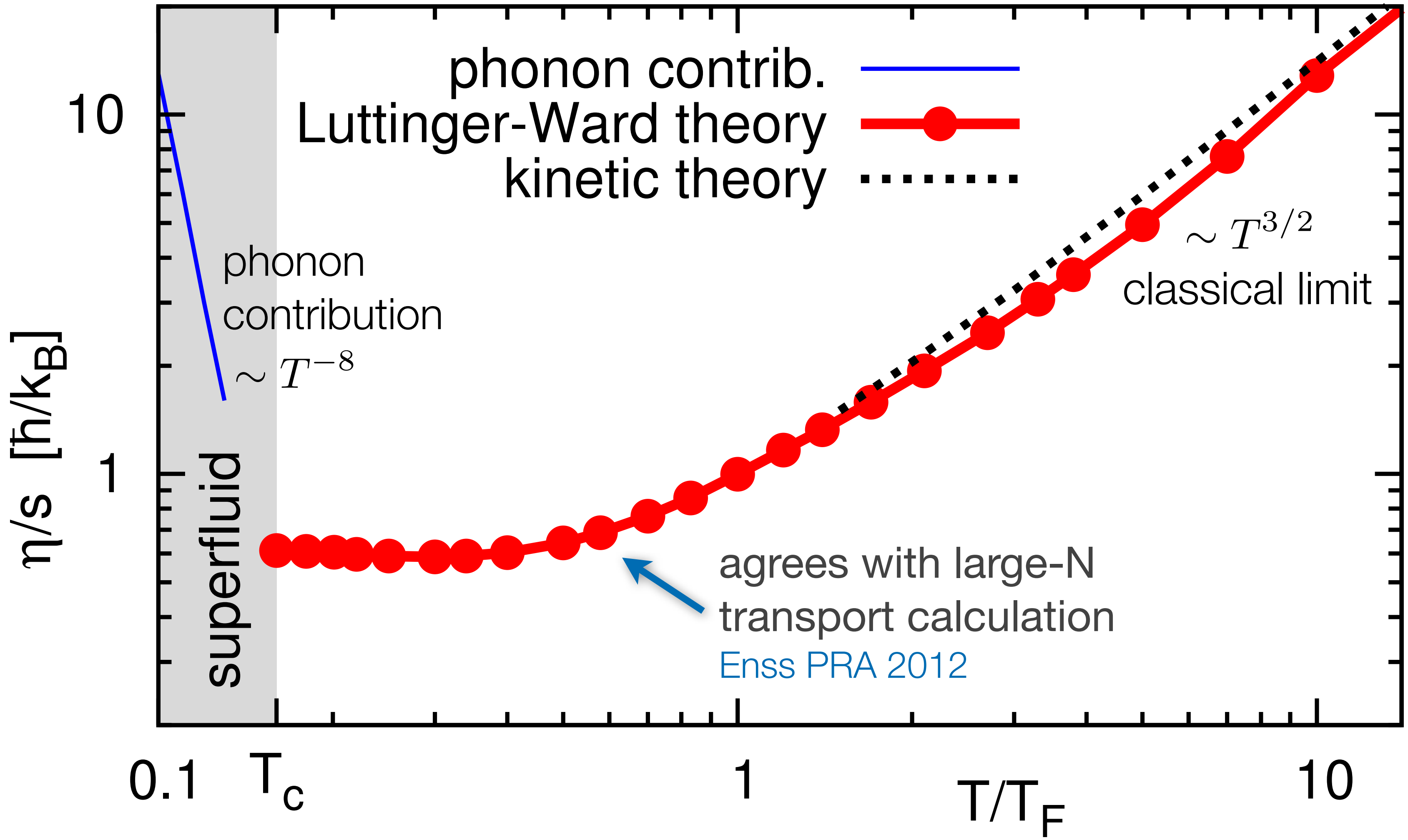
Enss, Haussmann & Zwerger 2011



exact viscosity sum rule
(nonperturbative check):

$$\frac{2}{\pi}\int_0^\infty d\omega\ [\eta(\omega) - \mathrm{tail}] = P - \frac{C}{4\pi ma}$$

Enss, Haussmann & Zwerger 2011; Enss 2013; cf. Taylor & Randeria 2010

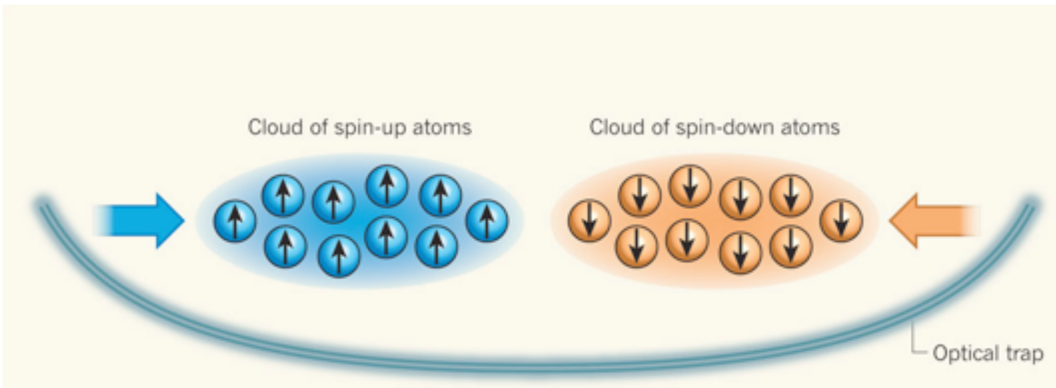


Shear viscosity/entropy of the unitary Fermi gas

Enss, Haussmann & Zwerger 2011

Spin transport with ultracold gases

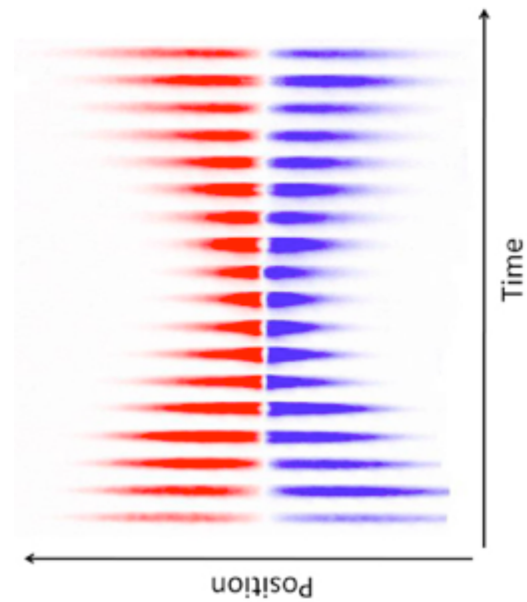
- experiment: spin-polarized clouds in harmonic trap



picture: J. Thomas 2011

- strongly interacting gas [movie courtesy Martin Zwierlein]:

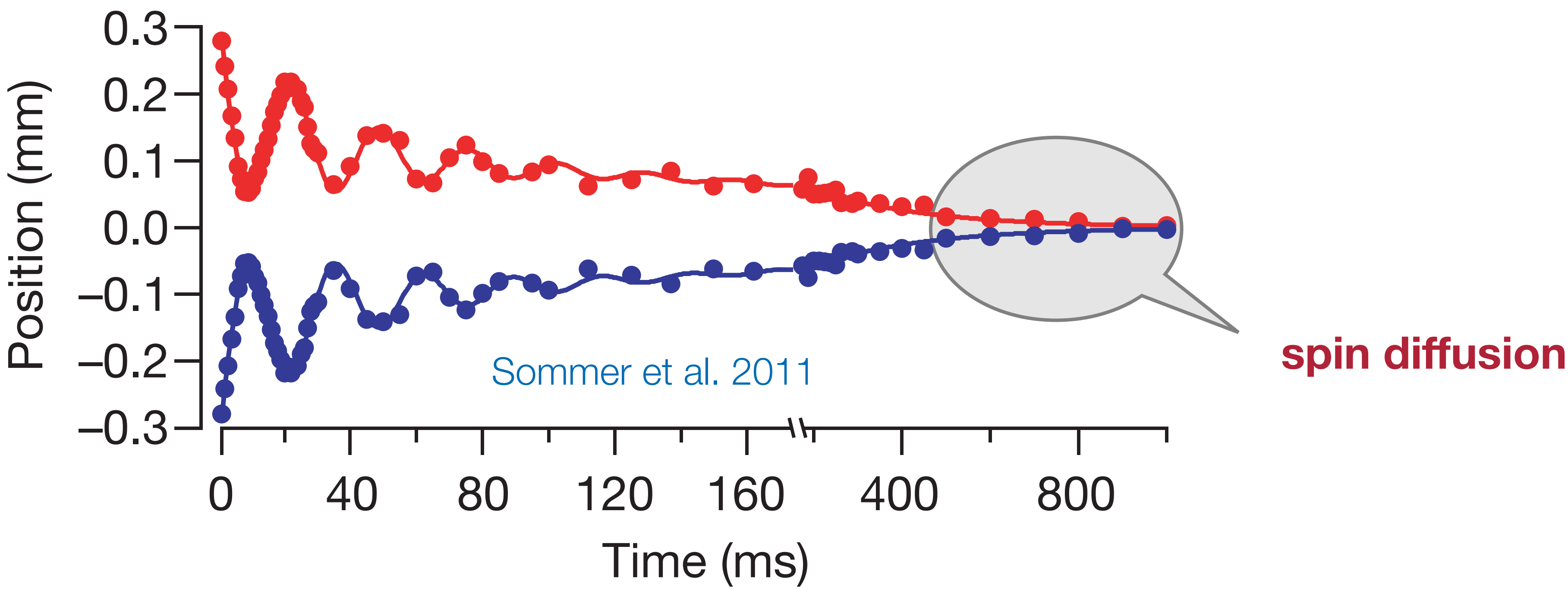
bounce!



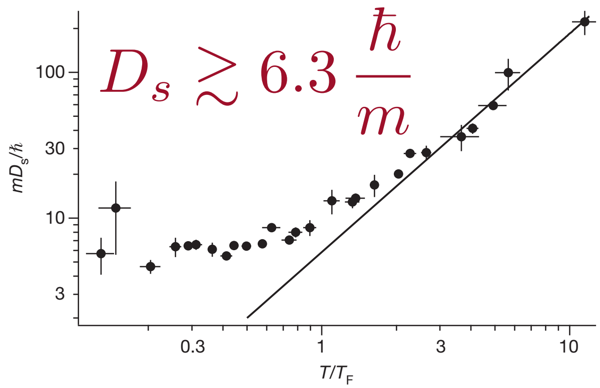
A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Spin diffusion

- scattering conserves total ⬆+⬇ momentum: mass current preserved
  but changes relative ⬆-⬇ momentum: spin current decays



Is there a quantum limit for diffusion?



Sommer et al. 2011

cf. spin Coulomb drag in GaAs quantum wells:

$$D_s \simeq 500\,\hbar/m$$

Weber et al. 2005

- **kinetic theory:** diffusion coefficient $D_s \approx v\ell_{\rm mfp},\ v \simeq \hbar k_F/m,\ \ell_{\rm mfp} \gtrsim 1/k_F$

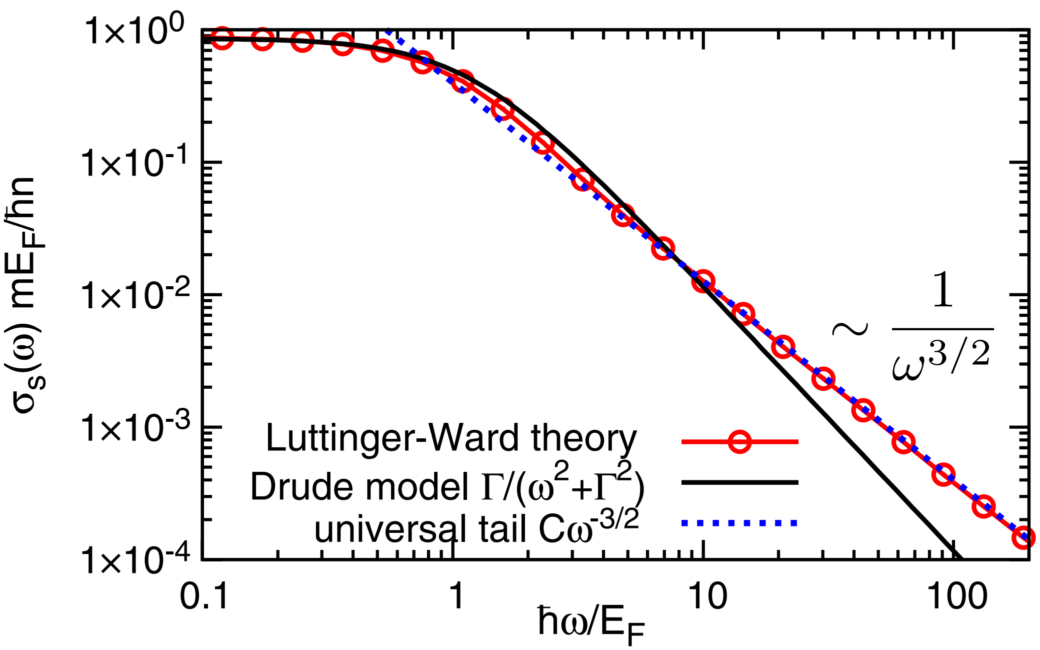
$$\boldsymbol{D_s \simeq \frac{\hbar}{m}}$$ **quantum limit of diffusion**

- spin conductivity from current correlations:

$$\sigma_s(\omega) = \frac{1}{\omega}\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t}\int d^3x\,\big\langle\big[j_s^z(\boldsymbol{x},t), j_s^z(0,0)\big]\big\rangle$$

with spin current operator $j_s(\boldsymbol{x},t) = j_\uparrow(\boldsymbol{x},t) - j_\downarrow(\boldsymbol{x},t)$

Dynamical spin conductivity

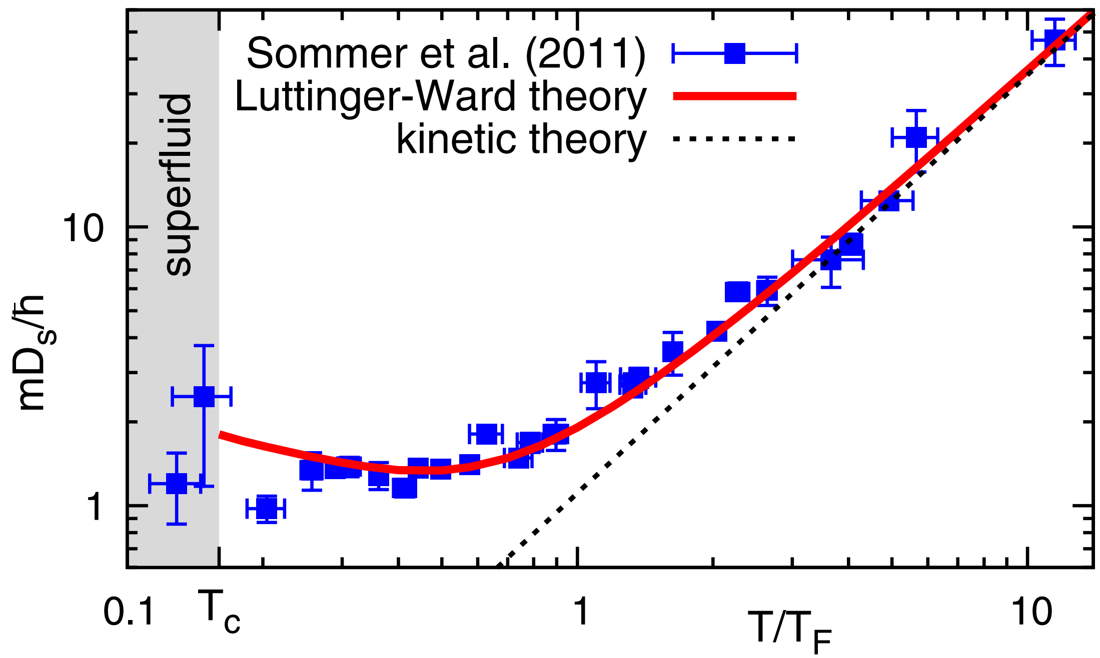


- exact high-frequency tail
  Hofmann PRA 2011;
  Enss & Haussmann PRL 2012

$$\sigma_s(\omega \to \infty) = \frac{C}{3\pi(m\omega)^{3/2}}$$

Spin diffusivity

- obtain diffusivity from Einstein relation, $D_s = \dfrac{\sigma_s}{\chi_s}$



(experiment rescaled from trap to infinite homogeneous box)

minimum $D_s \simeq 1.3\dfrac{\hbar}{m}$

Enss & Haussmann PRL 2012

- Quantum Monte Carlo simulation for finite lattice: $D_s \gtrsim 0.8\dfrac{\hbar}{m}$
  Wlazlowski et al. PRL 2013

Thermal expansion and transport Frank/Zw. '14

expansion $\alpha_p = \kappa_T \left(\frac{\partial p}{\partial T}\right)_V \to \gamma\, \kappa_T c_v$ Grüneisen parameter γ

scale invariance implies $p = 2\epsilon/3 \;\to\; \gamma(T) = 2/3$ is universal

therm. conductivity $\boldsymbol{j}_Q = -\kappa \nabla_{\boldsymbol{r}} T$ at $\boldsymbol{j}_p \equiv 0$

Boltzmann equ. in a $1/N$-expansion $\kappa = 1.89\ldots N^2 T/\lambda_T \sim T^{3/2}$

Prandtl-number $\mathrm{Pr} = \eta\, c_P/\kappa \; = 1$ if a gravity dual exists (Son 08')

leading order in $1/N$ gives $\mathrm{Pr} = 0.630136\ldots + \mathcal{O}(1/N)$

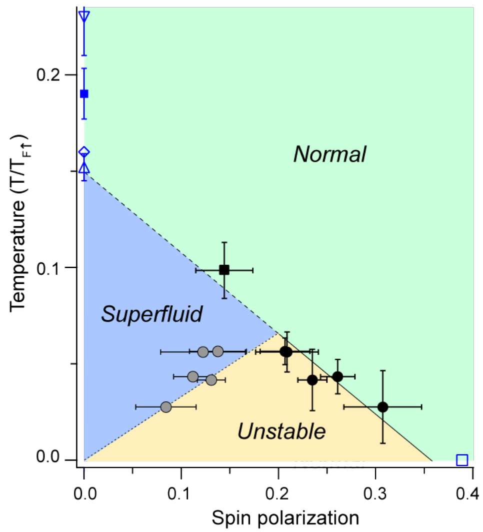
is there a gravity dual of the unitary Fermi gas ?

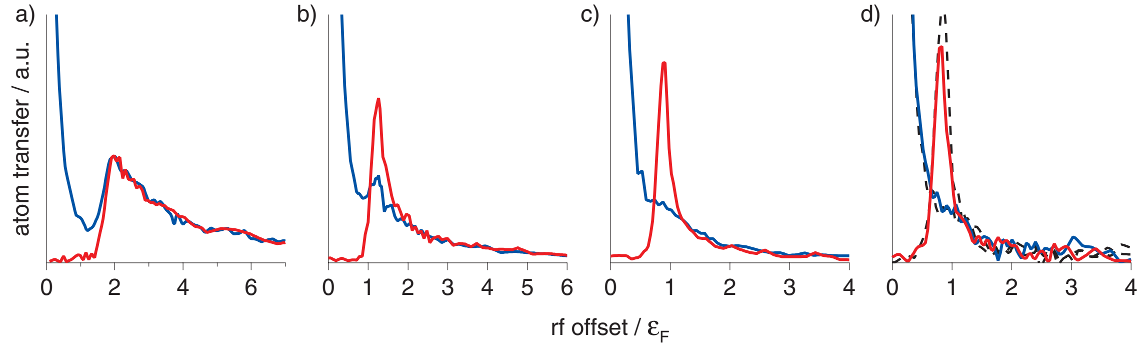
The unitary gas is a benchmark for many-body physics. It

- realizes a high-temperature fermionic superfluid below $T_c/T_F \simeq 0.16$ and a **scale-invariant many-body problem** with universal ratios $p/p_F = \xi_s \simeq 0.37$ or $S/N|_c \simeq 0.7\, k_B$
- exhibits **universal fermionic spectral functions** $A(k/k_F, \varepsilon/\varepsilon_F)$ in k-resolved RF and no pronounced pseudogap above T_c
- is the **most perfect non-relativistic fluid** with η/s close to the KSS bound and quantum-limited spin-diffusion $D_s \simeq 1.3\, \hbar/m$

open problems

- unconventional pairing in imbalanced gases, FFLO, ...
- Fermi and Bose polarons, quantum impurity problems





- transport in the quantum critical regime, solitons, ...