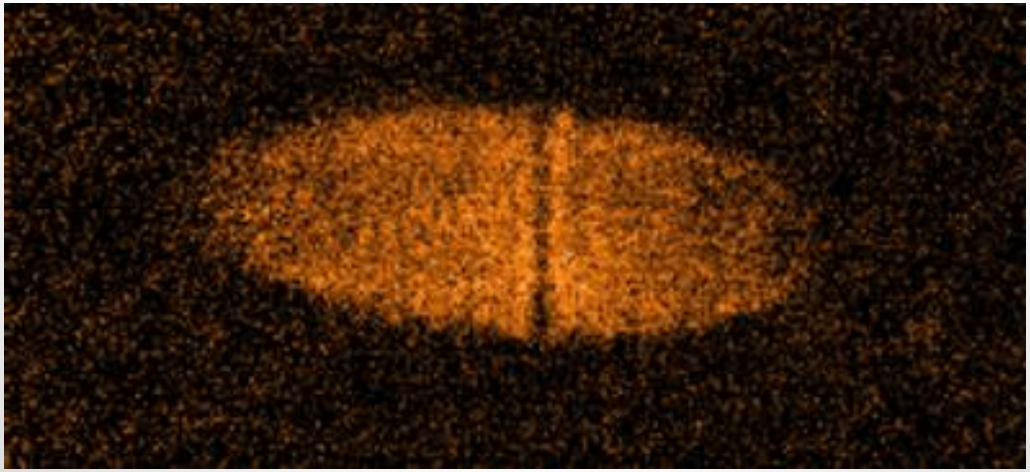
# And now for something completely different

School of Physics E. Fermi – Quantum Matter at Ultralow Temperatures
Varenna, July 9th, 2014

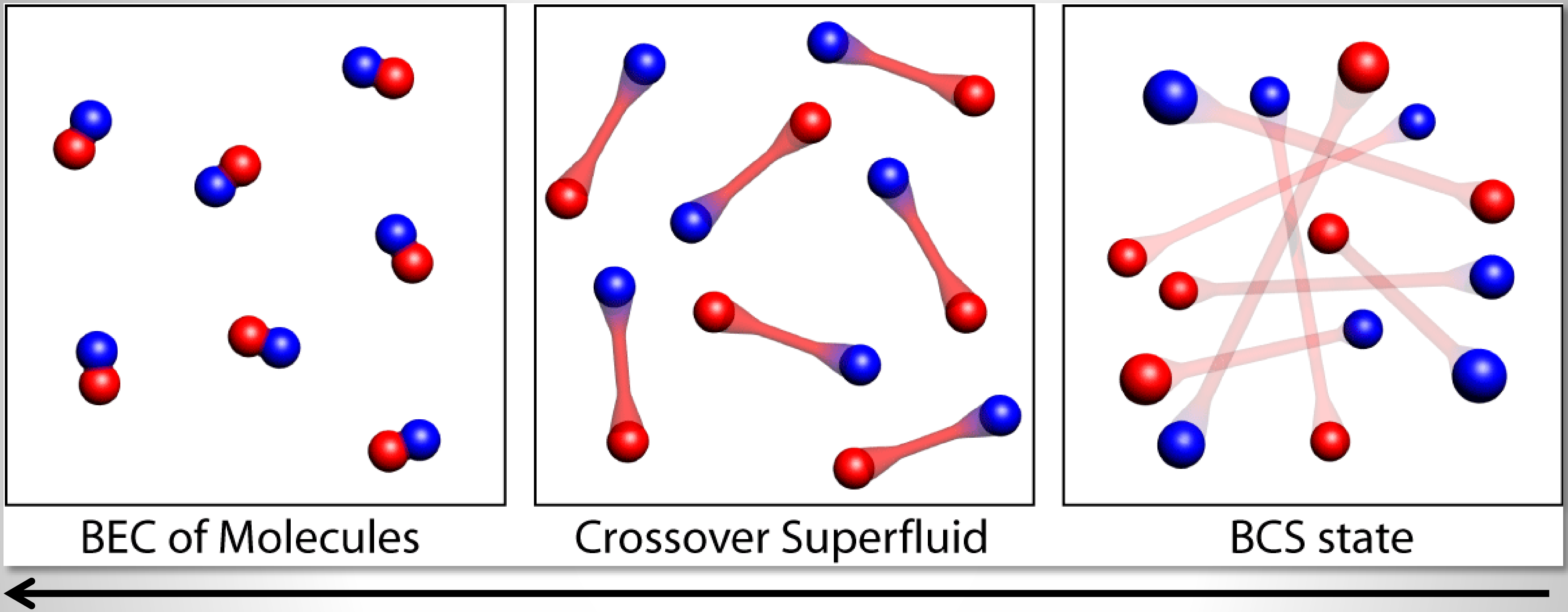
# *Thermodynamics and non-equilibrium dynamics of unitary Fermi gases*

**Martin Zwierlein**

Massachusetts Institute of Technology
Center for Ultracold Atoms



# From BEC to BCS



BEC of Molecules

Crossover Superfluid

BCS state

$\gg 1 \qquad 1 \qquad 0 \qquad -1 \qquad \ll -1$

$$(k_F a)^{-1} = \frac{\text{Interparticle Distance}}{\text{Scattering Length}}$$

Weakly Interacting Bosons

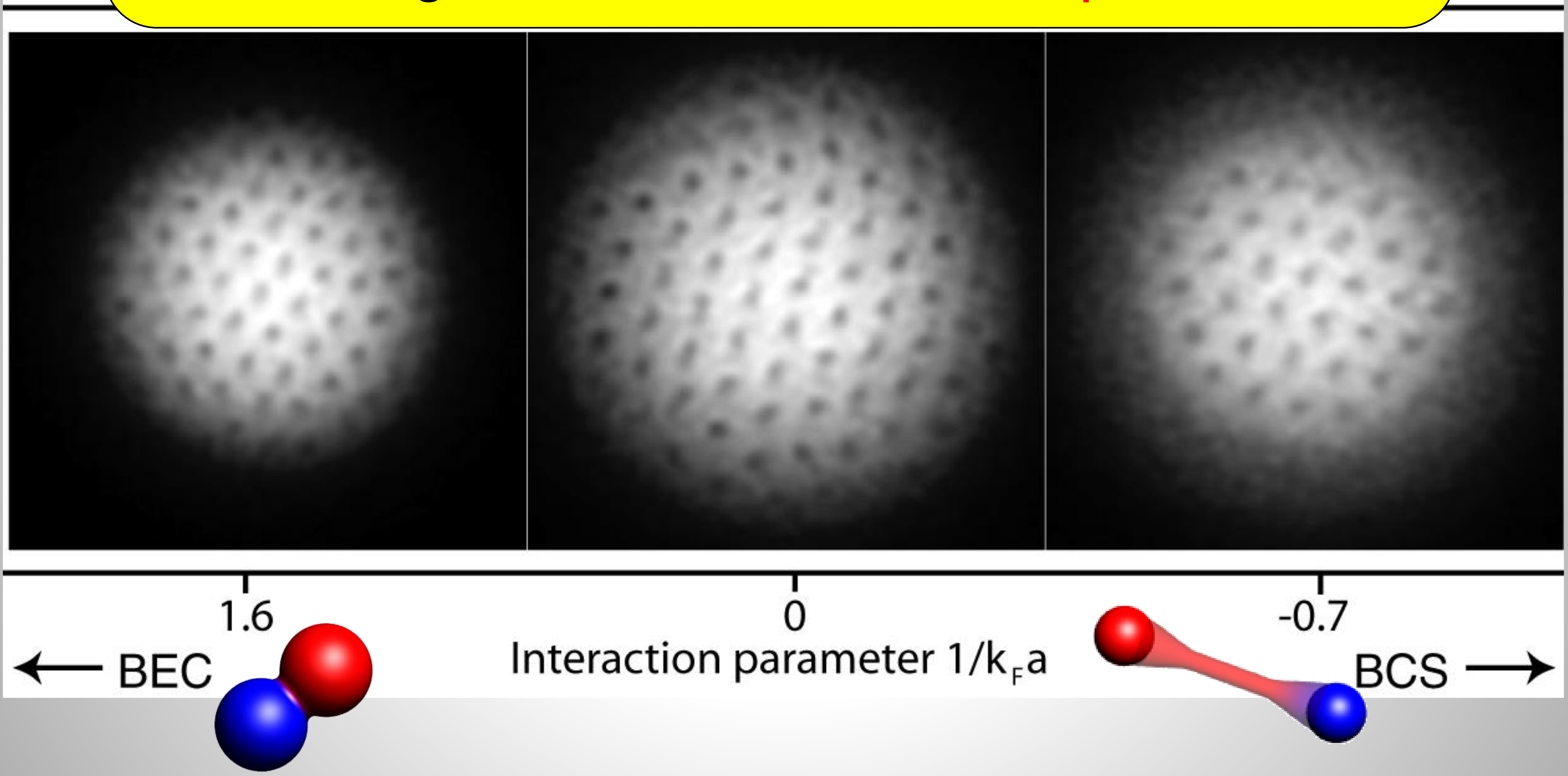
→ Strongly Interacting Bosons

→ Strongly Interacting Fermions

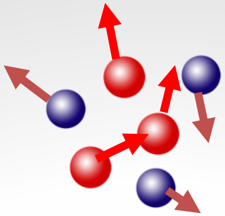
→ Weakly Interacting Fermions

# Vortex lattices in the BEC-BCS crossover

Establishes *superfluidity* and *phase coherence* in gases of fermionic atom pairs

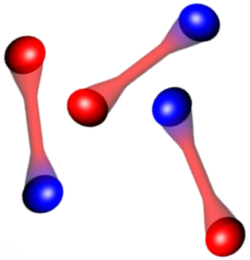


M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle,
Nature 435, 1047-1051 (2005)



Classical gas Equation of State (EoS):

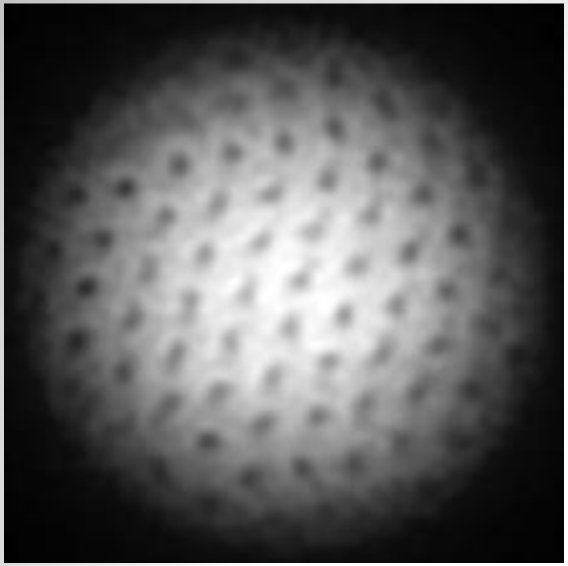
$$P = nk_B T$$

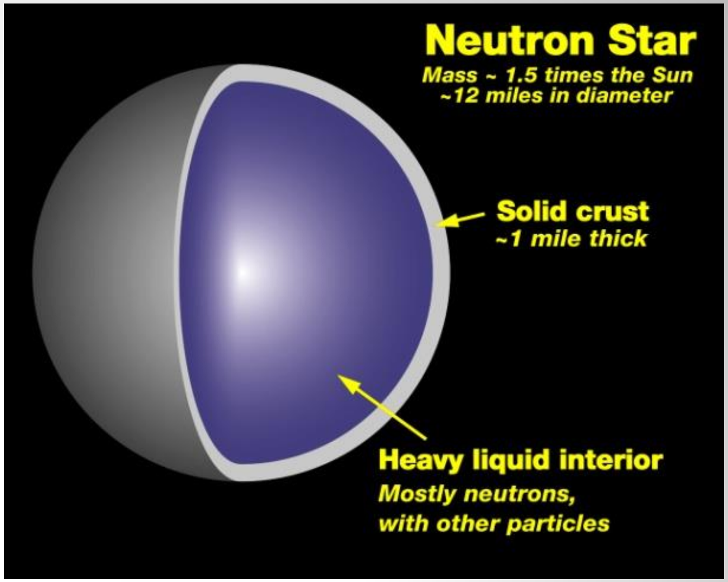


What is the EoS of a strongly interacting Fermi gas?

$$P(n,T)$$

# Relation to equation of state of a neutron star





| Property | Atoms | Neutrons |
|---|---|---|
| Spin | Pseudospin ½ | Spin ½ |
| Interparticle distance $n^{-1/3}$ | 1 μm | 1 fm |
| Density | $10^{13}$ cm$^{-3}$ | $10^{38}$ cm$^{-3}$ |
| Fermi Energy | 1 μK = $10^{-10}$ eV | $10^{12}$ K = 150 MeV |
| Scattering length a | freely tunable | -19 fm |

Both systems lie in universal regime: $a \gg n^{-1/3}$

small print: neglecting effective range

# General Thermodynamic Relations

**Microcanonical Ensemble:** Energy $E(S, N, V, a)$

$$\mathrm{d}E = T\ \mathrm{d}S + \mu\ \mathrm{d}N - P\ \mathrm{d}V$$

$$T = \left.\frac{\partial E}{\partial S}\right|_{N,V,a}$$

Temperature

$$\mu = \left.\frac{\partial E}{\partial N}\right|_{S,V,a}$$

Chemical Potential

$$P = -\left.\frac{\partial E}{\partial V}\right|_{S,N,a}$$

Pressure

$$C = -\left.\frac{\partial E}{\partial a^{-1}}\right|_{S,N,V}$$

Contact

# General Thermodynamic Relations

Grand Canonical Ens.: $\Omega(T,\mu,V,a) \equiv E - TS - \mu N$

$$\boxed{\mathrm{d}\Omega = -S\,\mathrm{d}T - N\,\mathrm{d}\mu - P\,\mathrm{d}V - C\,\mathrm{d}a^{-1}}$$

$\Omega$ is extensive $\quad \Omega(T,\mu,\alpha V,a) = \alpha\,\Omega(T,\mu,V,a)$

$\Rightarrow$ Pressure $P$ is independent of volume

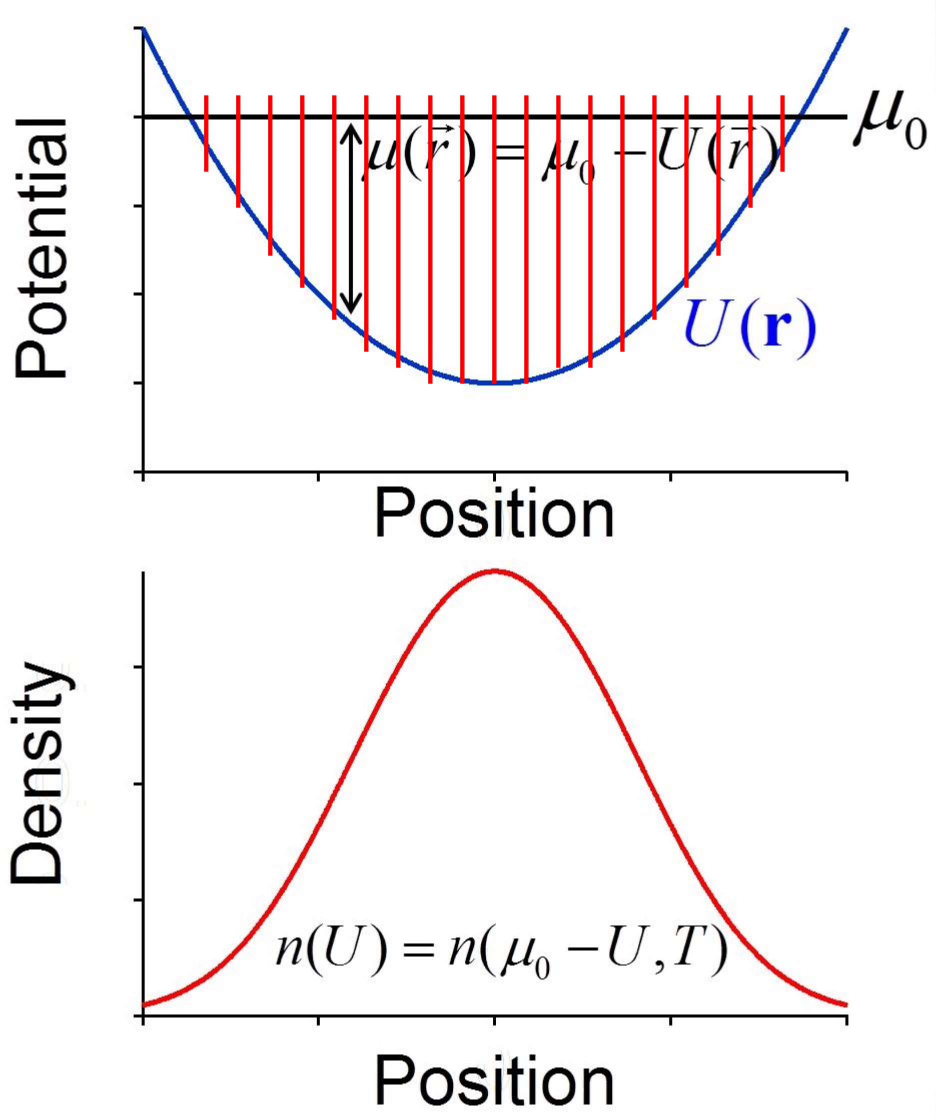
$$P(T,\mu,\alpha V,a) = -\left.\frac{\partial\Omega(T,\mu,\alpha V,a)}{\partial \alpha V}\right|_{T,\mu,a} = -\left.\frac{\partial\Omega(T,\mu,V,a)}{\partial V}\right|_{T,\mu,a} = P(T,\mu,V,a)$$

$$\boxed{P = P(T,\mu,a)} \qquad \boxed{\Omega = -PV}$$

General relation: $\boxed{E = TS + \mu N - PV}$

Gibbs-Duhem: $\boxed{V\mathrm{d}P - S\,\mathrm{d}T - N\,\mathrm{d}\mu - C\,\mathrm{d}a^{-1} = 0}$

# Local Density Approximation



$$P = P(T, \mu, a)$$

$$n = \left.\frac{\partial P}{\partial \mu}\right|_{T,a} = n(T, \mu, a)$$

$$\mu(\vec{r}) = \mu_0 - U(\vec{r})$$

*Local chemical potential*

$$n(\vec{r}) = n(\mu_0 - U(\vec{r}), T, a)$$

*Local density*

Density profile = a scan through the Equation of State

# Measuring the Equation of State

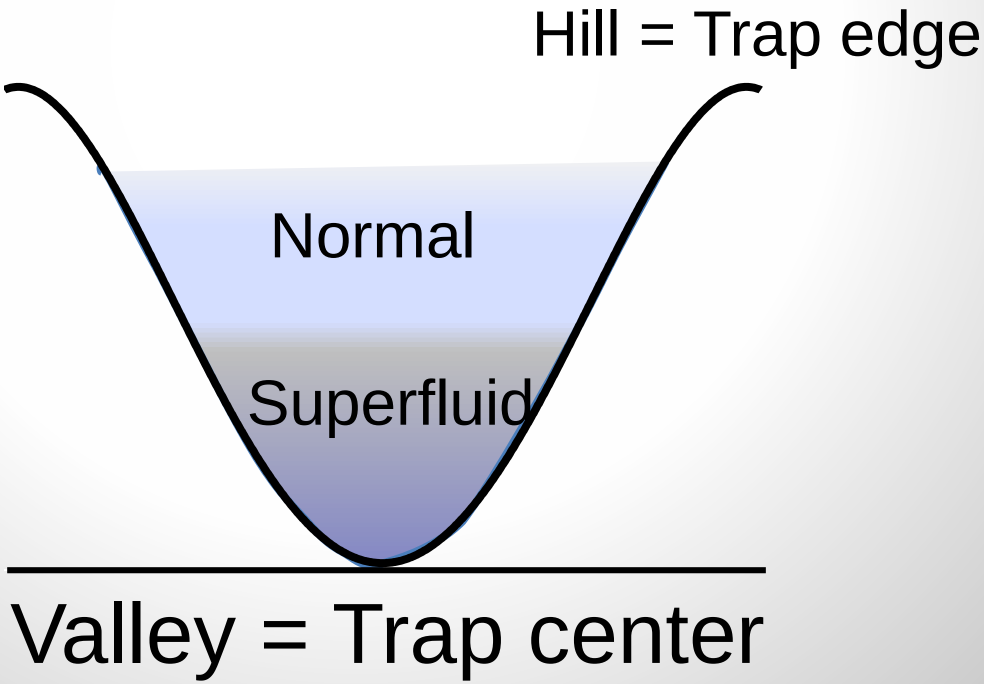
*When climbing a mountain, the air gets thinner...*

Equation of state → density as a function of height

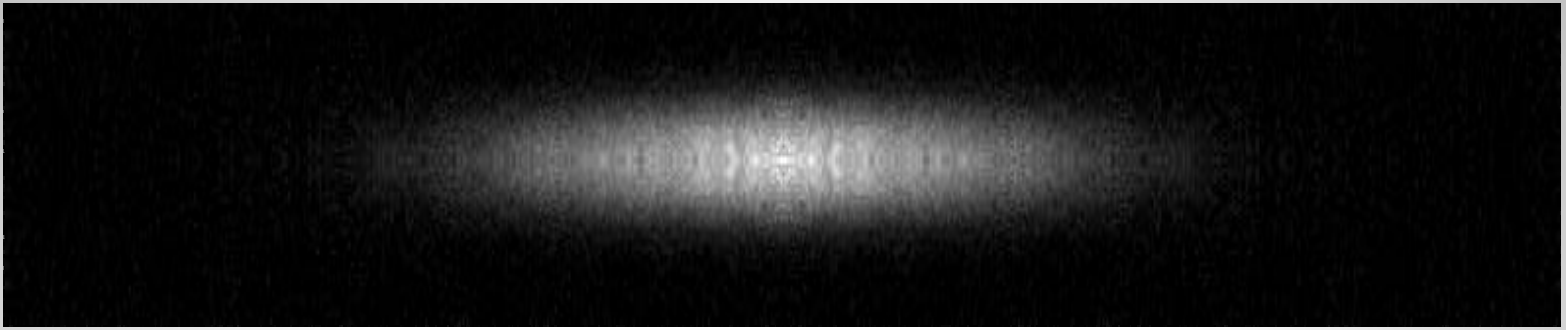
*The inverse works as well!*

Density as a function of height → equation of state

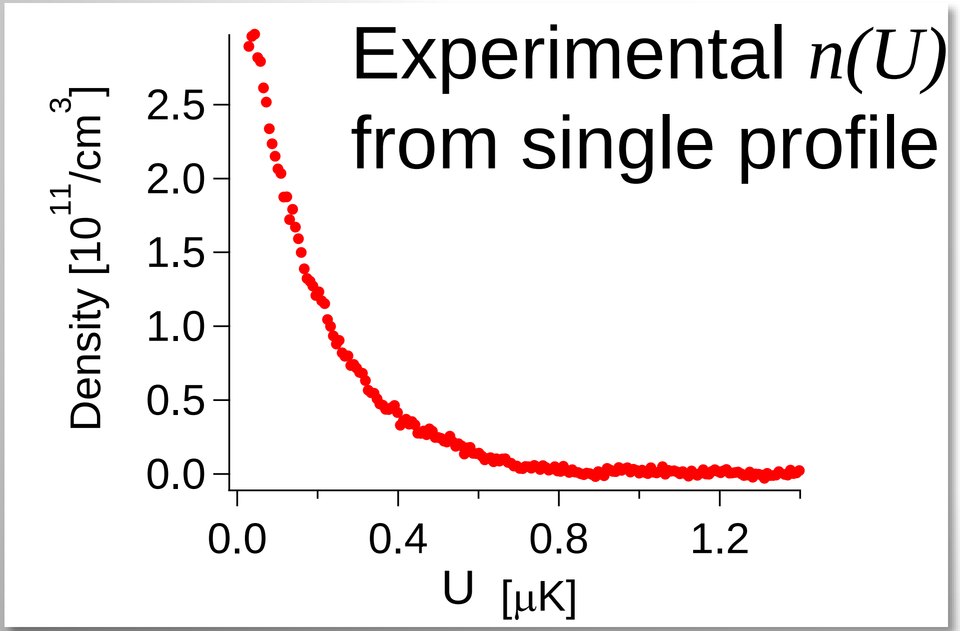
Atoms in our trapping potential $\triangleq$ air in gravitational potential



# Equation of State: Measuring density



Exploiting cylindrical symmetry and equipotential averaging:



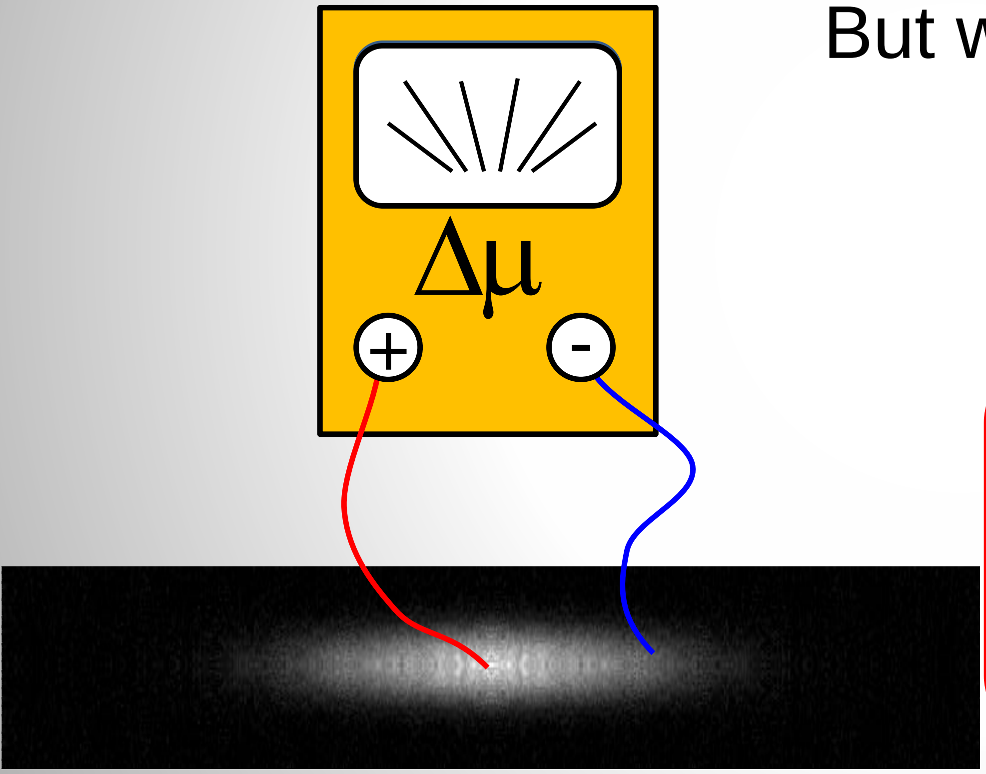
$$n(U) = n(\mu_0 - U, T, a)$$

*Local density*

# How to measure $\mu_0$?

$$n(U) = n(\mu_0 - U, T, a)$$

We cannot locally measure absolute chemical potentials



But we know how μ changes:

$$\mu(\vec{r}) = \mu_0 - U(\vec{r})$$

$$\Rightarrow \; \Delta\mu = -\Delta U$$

$$\mathrm{d}\mu = -\mathrm{d}U$$

Extremely useful differential relation at constant $T$ and $a$

Knowing the function $n(U)$ and $\mathrm{d}\mu = -\mathrm{d}U$ we can generate an “arbitrary” number of new thermodynamic quantities

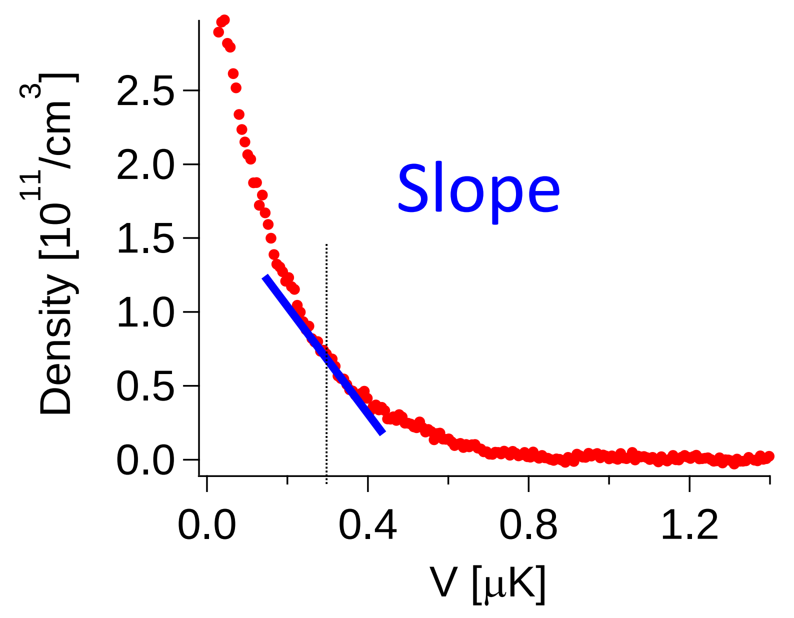
# Compressibility

$$n(U) = n(\mu_0 - U, T, a)$$

$$\boxed{\mathrm{d}\mu = -\mathrm{d}U}$$

Replace μ by the following thermodynamic quantity:

$$\boxed{\left.\frac{\partial n}{\partial \mu}\right|_{T,a} = -\frac{\mathrm{d}n}{\mathrm{dU}}}$$



Gibbs-Duhem

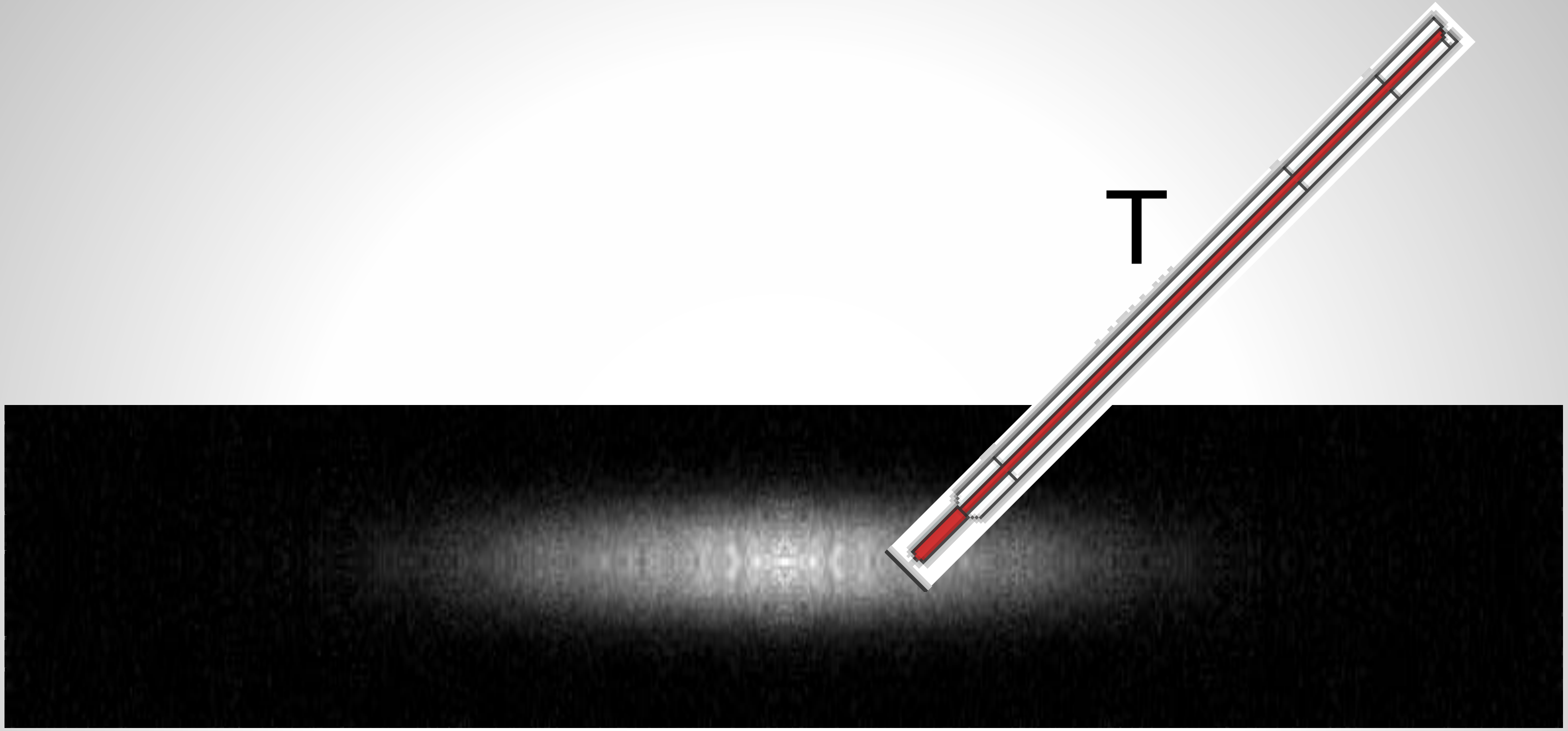
at $T$ = const. and $a$ = const.:

$$\boxed{dP = n\,\mathrm{d}\mu = -n\,\mathrm{d}U}$$

## *Local Compressibility:*

$$\boxed{\kappa = -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_{N,T,a} = \frac{1}{n^2}\left.\frac{\partial n}{\partial \mu}\right|_{T,a} = -\frac{1}{n^2}\frac{\mathrm{d}n}{\mathrm{dU}}}$$

# How to measure T?



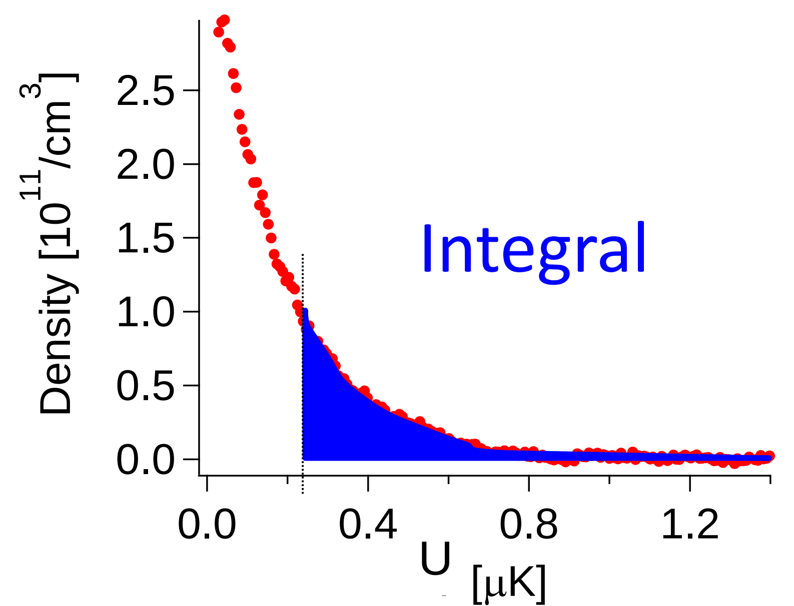
...Not impossible, but difficult, so…

**Don’t!** Use other readily available thermodynamic quantity: Pressure

# Pressure

Replace temperature by following thermodynamic quantity:

$$\boxed{\int_{-\infty}^{\mu_0 - U} n \, \mathrm{d}\mu = \int_U^{\infty} n(U') \, \mathrm{d}U'}$$



Gibbs-Duhem at $T$ = const. and $a$ = const.:

$$\boxed{dP = n \, \mathrm{d}\mu = -n \, \mathrm{d}U}$$

Pressure = weight / unit area of air above you

*For atom trappers: replace m g h → U*

Pressure = integrated density over potential

## *Local pressure*

$$\boxed{P = \int_U^{\infty} \mathrm{d}U' \; n(U', T, a)}$$

# Density, Compressibility, Pressure Equation of State

Instead of

$$\boxed{P(\mu, T, a)}$$

Directly measure

$$\boxed{P(n, \kappa, a)}$$

- General method for sufficiently local interactions
  - works for fermions, bosons, 1D, 2D, 3D, lattice…
- No fit function involved
- No external thermometer
- All other thermodynamic quantities follow

Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

# How to obtain the temperature

Express $P$ using the pressure scale given by density

$$\boxed{P(n,T,a) = P_0\, f\left(\frac{T}{T_F}, \frac{1}{k_F a}\right)}$$

$$x \equiv \frac{T}{T_{\mathrm{F}}}$$

$$y \equiv \frac{1}{k_{\mathrm{F}} a}$$

with $P_0 = \frac{2}{5} n E_{\mathrm{F}}$ $\quad E_{\mathrm{F}} = \mathrm{k_B} T_{\mathrm{F}} = \frac{\hbar^2 k_{\mathrm{F}}^2}{2m}$ $\quad k_{\mathrm{F}} = \left(\frac{6\pi^2}{\#\text{ spin states}} n\right)^{1/3}$

(Note: This is general, i.e. not restricted to fermions, despite the looks. e.g. instead of $T/T_{\mathrm{F}}$, one could equivalently use $n\,\lambda^3$)

I want $$\boxed{\left.\frac{\partial P}{\partial T}\right|_{n,a} = P_0 \frac{\partial f}{\partial x} \left.\frac{\partial (T/T_{\mathrm{F}})}{\partial T}\right|_{n} = \frac{P}{T_{\mathrm{F}}} \frac{\partial \ln f}{\partial x}}$$

I have $$\boxed{\left.\frac{\partial P}{\partial n}\right|_{T,a} \underset{\text{Gibbs-Duhem}}{=} n \left.\frac{\partial \mu}{\partial n}\right|_{T,a} = \frac{1}{n\kappa}}$$

# How to obtain the temperature

I want

$$\left.\frac{\partial P}{\partial T}\right|_{n,a} = \frac{P}{T_{\mathrm{F}}}\boxed{\frac{\partial \ln f}{\partial x}}$$

$$P = P_0\, f\left(\frac{T}{T_F}, \frac{1}{k_F a}\right)$$

$$P_0 = \frac{2}{5} n E_{\mathrm{F}}$$

I have

$$\frac{1}{n\kappa} = \left.\frac{\partial P}{\partial n}\right|_{T,a} = \frac{\partial P_0}{\partial n} f + P_0 \frac{\partial f}{\partial x}\left.\frac{\partial (T/T_{\mathrm{F}})}{\partial n}\right|_{T} + P_0 \frac{\partial f}{\partial y}\left.\frac{\partial (\mathrm{k_F a})^{-1}}{\partial n}\right|_{a}$$

$$= \frac{5}{3}\frac{P}{n} - \frac{2}{3}\frac{P}{n}\frac{T}{T_{\mathrm{F}}}\boxed{\frac{\partial \ln f}{\partial x}}\underbrace{\boxed{-\frac{1}{3}\frac{P}{n}\frac{1}{k_F a}\frac{\partial \ln f}{\partial y}}}_{=0}$$

at $a = \pm\infty$

Or generally if system is scale invariant

# Application to scale invariant gases

$$P(n,T) = P_0\, f\left(\frac{T}{T_F}\right) \Rightarrow \left.\frac{\partial P}{\partial n}\right|_T = \frac{5}{3}\frac{P}{n} - \frac{2}{3}\frac{T}{n}\left.\frac{\partial P}{\partial T}\right|_n$$

In words: For a scale-invariant system, the variation of pressure with temperature is directly related to the variation of pressure with density

- which we can measure

Furthermore, scale invariance implies

$$P(n,\kappa) = P_0\, g\left(\frac{\kappa}{\kappa_0}\right)$$

with $\kappa_0 \equiv \frac{3}{2}\frac{1}{nE_F}$ Compressibility of non-interacting Fermi gas

$$\tilde{\kappa} \equiv \frac{\kappa}{\kappa_0}$$

is a universal function of

$$\tilde{p} \equiv \frac{P}{P_0}$$

# How to obtain the temperature

For scale-invariant system, have shown that

$$\boxed{T\frac{\partial P}{\partial T}\bigg|_{n}=\frac{5}{2}P-\frac{3}{2}\frac{1}{\kappa}}$$

In dim.less form: $\boxed{\frac{T}{T_{\mathrm{F}}}\frac{\mathrm{d}\tilde{p}}{\mathrm{d}(T/T_{\mathrm{F}})}=\frac{5}{2}\left(\tilde{p}-\frac{1}{\tilde{\kappa}}\right)}$

$$\boxed{\frac{T}{T_{\mathrm{F}}}=\left(\frac{T}{T_{\mathrm{F}}}\right)_{i}\exp\left(\frac{2}{5}\int_{\tilde{p}_{i}}^{\tilde{p}}\mathrm{d}\tilde{p}\frac{1}{\tilde{p}-\frac{1}{\tilde{\kappa}}}\right)}$$

Ku, Sommer, Cheuk, Zwierlein Science **335**, 563 (2012)

$\left(\frac{T}{T_{\mathrm{F}}}\right)_{i}$ initial normalized temperature at initial normalized pressure $\tilde{p}_{i}$ that can be chosen to lie in the known Boltzmann or Virial regime

# Application to the Unitary Fermi Gas

$a = \pm\infty$ Unitary Fermi gas is scale invariant

$$\underbrace{P = P_0\, f_P\left(\frac{T}{T_\mathrm{F}}\right), \quad E = E_0\, f_E\left(\frac{T}{T_\mathrm{F}}\right), S = N\mathrm{k_B}\, f_S\left(\frac{T}{T_\mathrm{F}}\right)}$$

S = const. implies T/T$_\mathrm{F}$ = const.

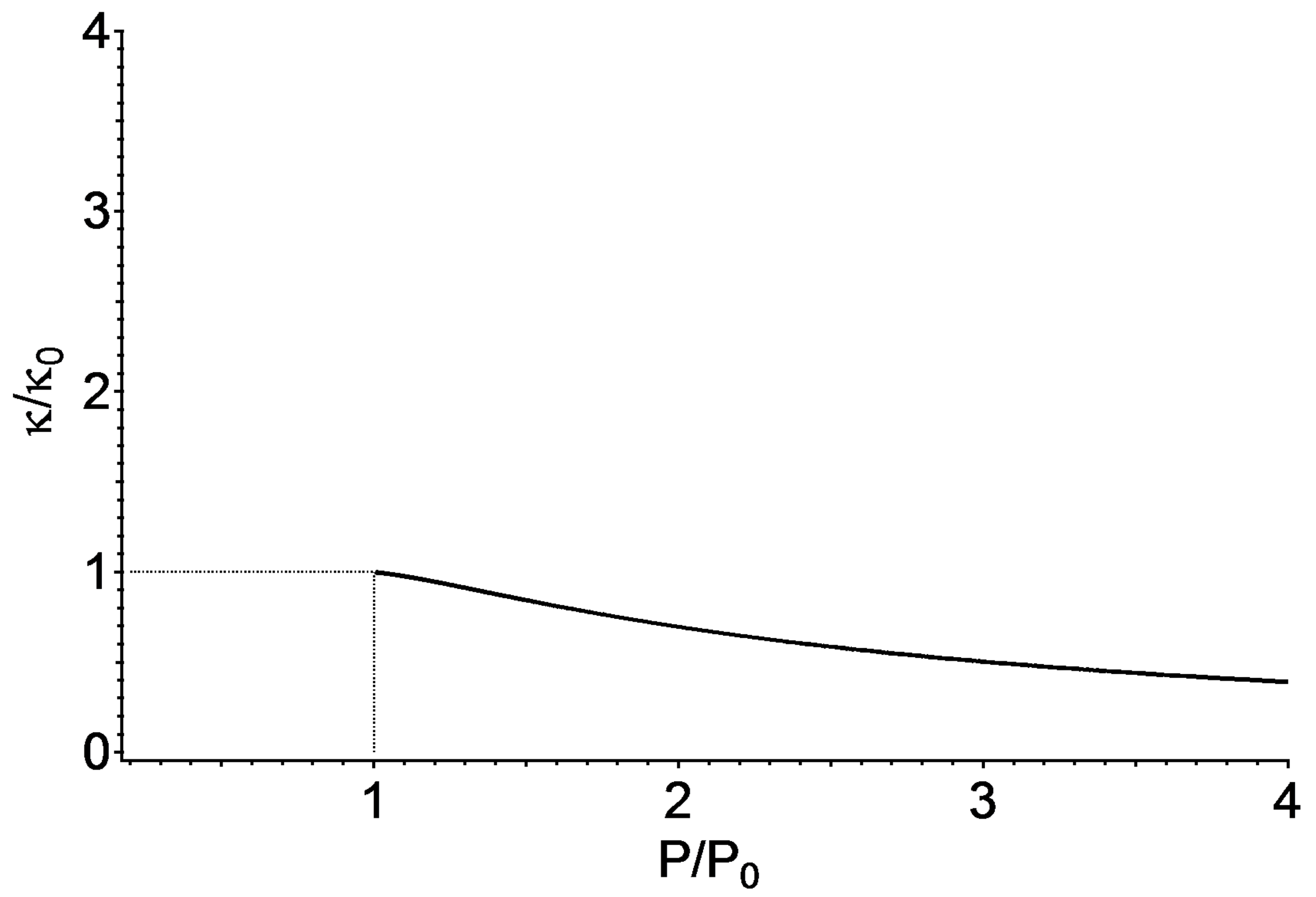
$$\boxed{P = -\frac{\partial E}{\partial V}\bigg|_{S,N} = -f_E \frac{\mathrm{d}E_0}{\mathrm{d}V} = \frac{2}{3}\frac{E}{V}}$$

etc., and generally all dim.less quantities are directly related:

$$\frac{P}{P_0} \quad \frac{E}{E_0} \quad \frac{T}{T_\mathrm{F}} \quad \frac{S}{N\mathrm{k_B}} \quad \frac{\kappa}{\kappa_0} \quad \frac{\mu}{E_F}$$

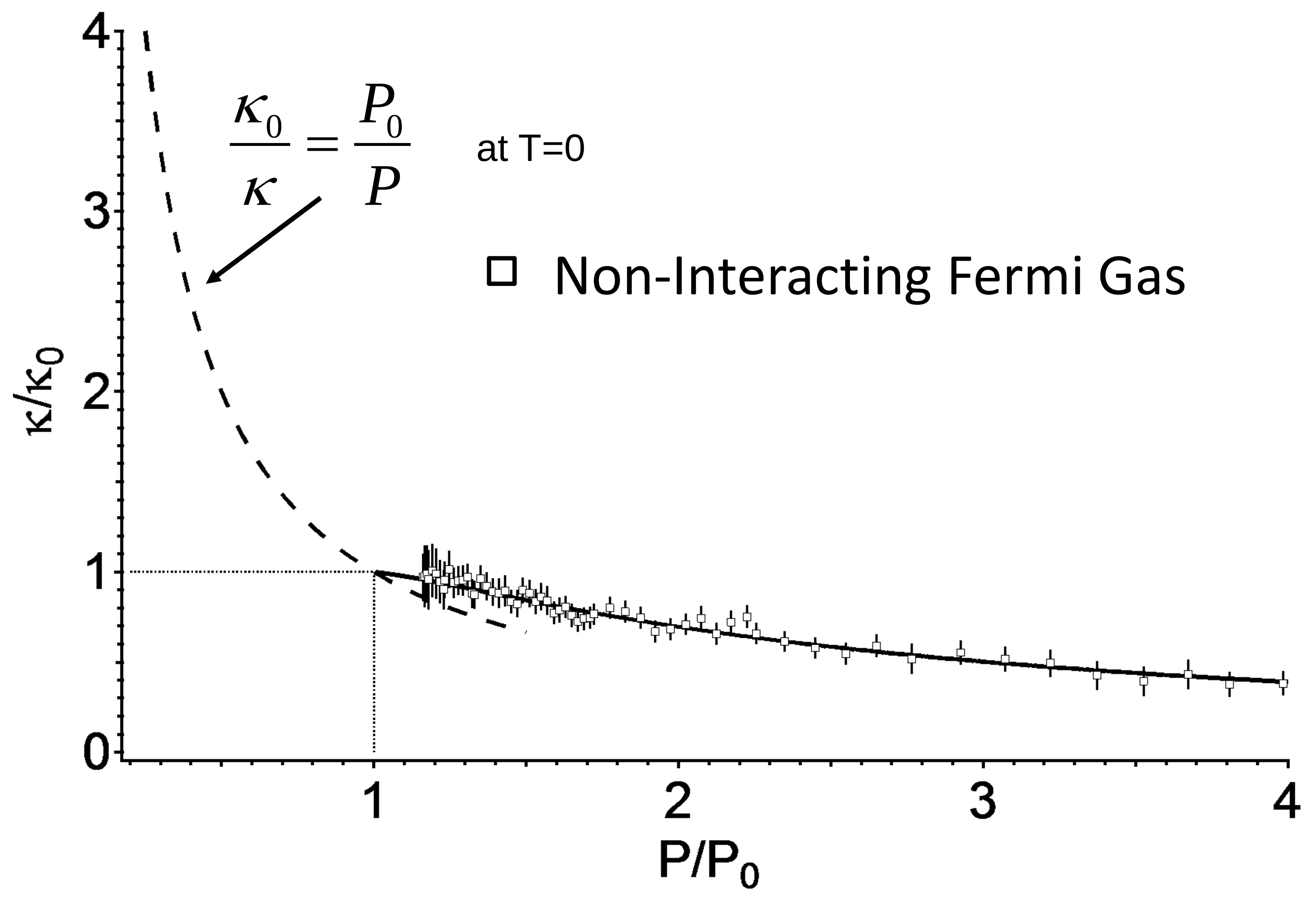
In particular at T = 0: $\boxed{\frac{P}{P_0} = \frac{E}{E_0} = \frac{\mu}{E_F} = \frac{\kappa_0}{\kappa} = \xi}$ Bertsch Parameter

# Compressibility Equation of State



Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

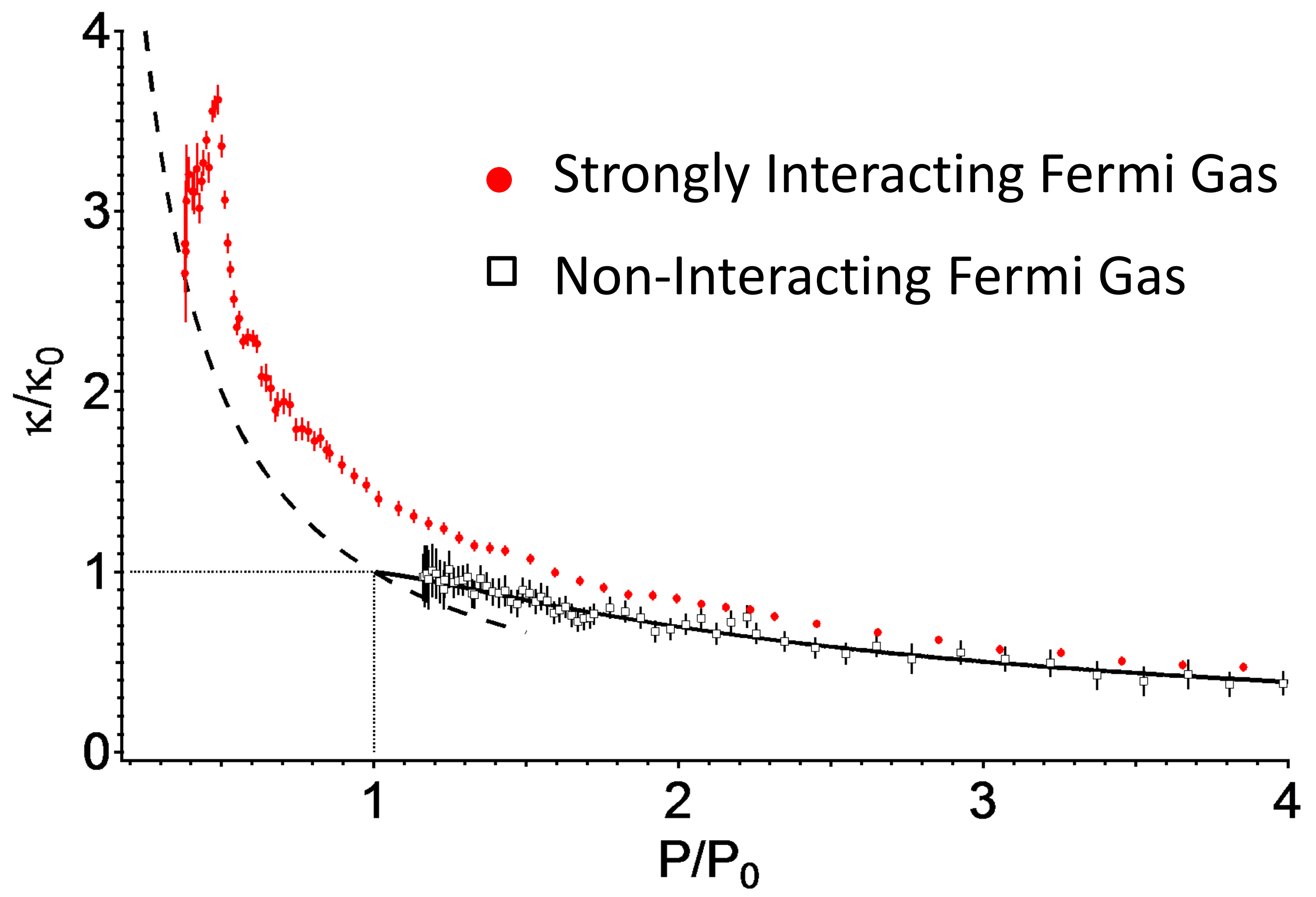
# Compressibility Equation of State



$$\frac{\kappa_0}{\kappa} = \frac{P_0}{P}$$ at T=0

Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

# Compressibility Equation of State



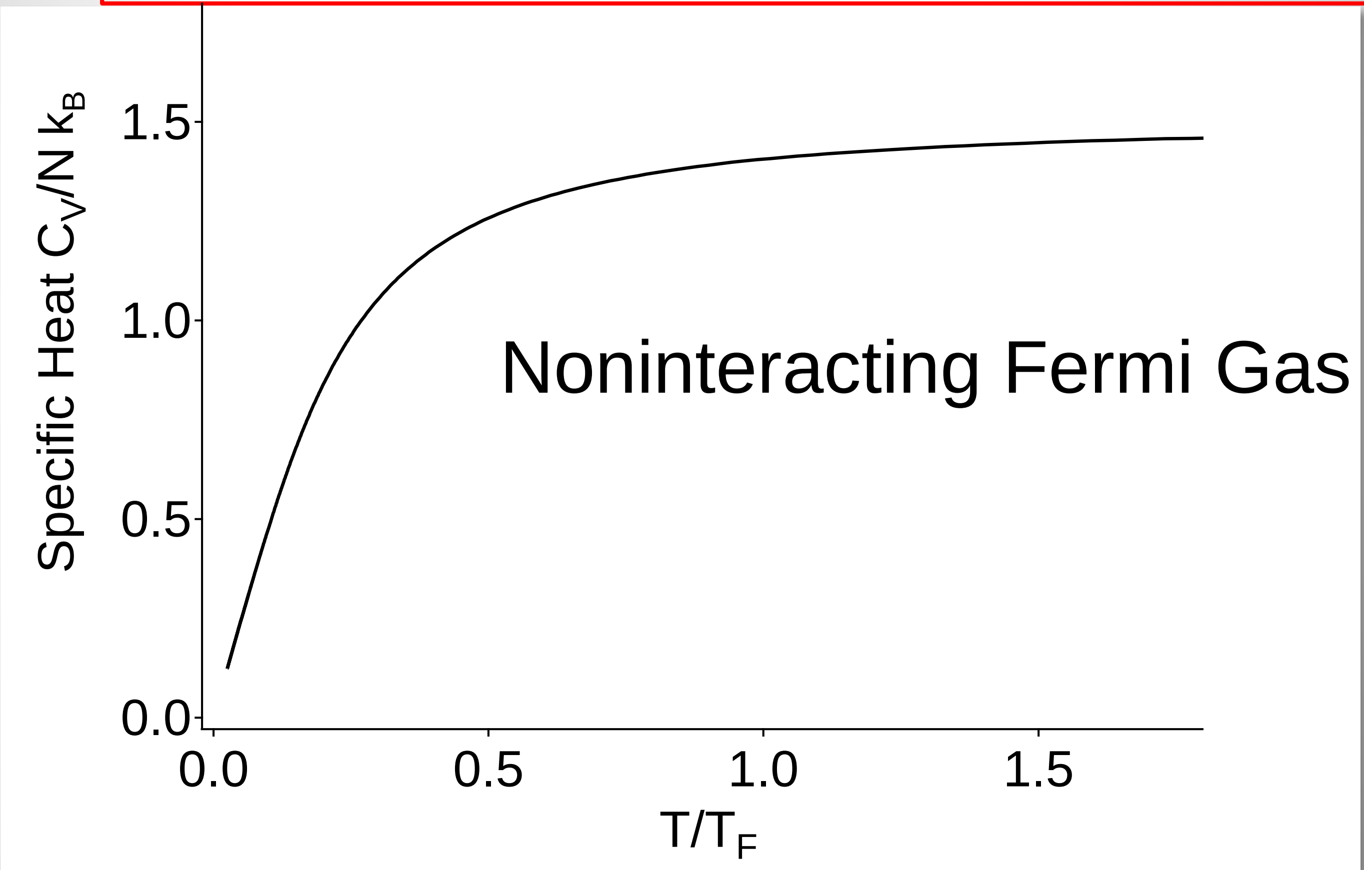
Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

# Heat capacity

For a resonant gas:

$$P = \frac{2}{3}\frac{E}{V}$$

$$\frac{C_V}{Nk_B} = \frac{\mathrm{d}\left(E/Nk_B\right)}{\mathrm{d}T}\Bigg|_{N,V} = \frac{\mathrm{d}\left(P/nE_F\right)}{\mathrm{d}(T/T_F)} = \frac{3}{2}\frac{T_F}{T}\left(\frac{P}{P_0} - \frac{\kappa_0}{\kappa}\right)$$



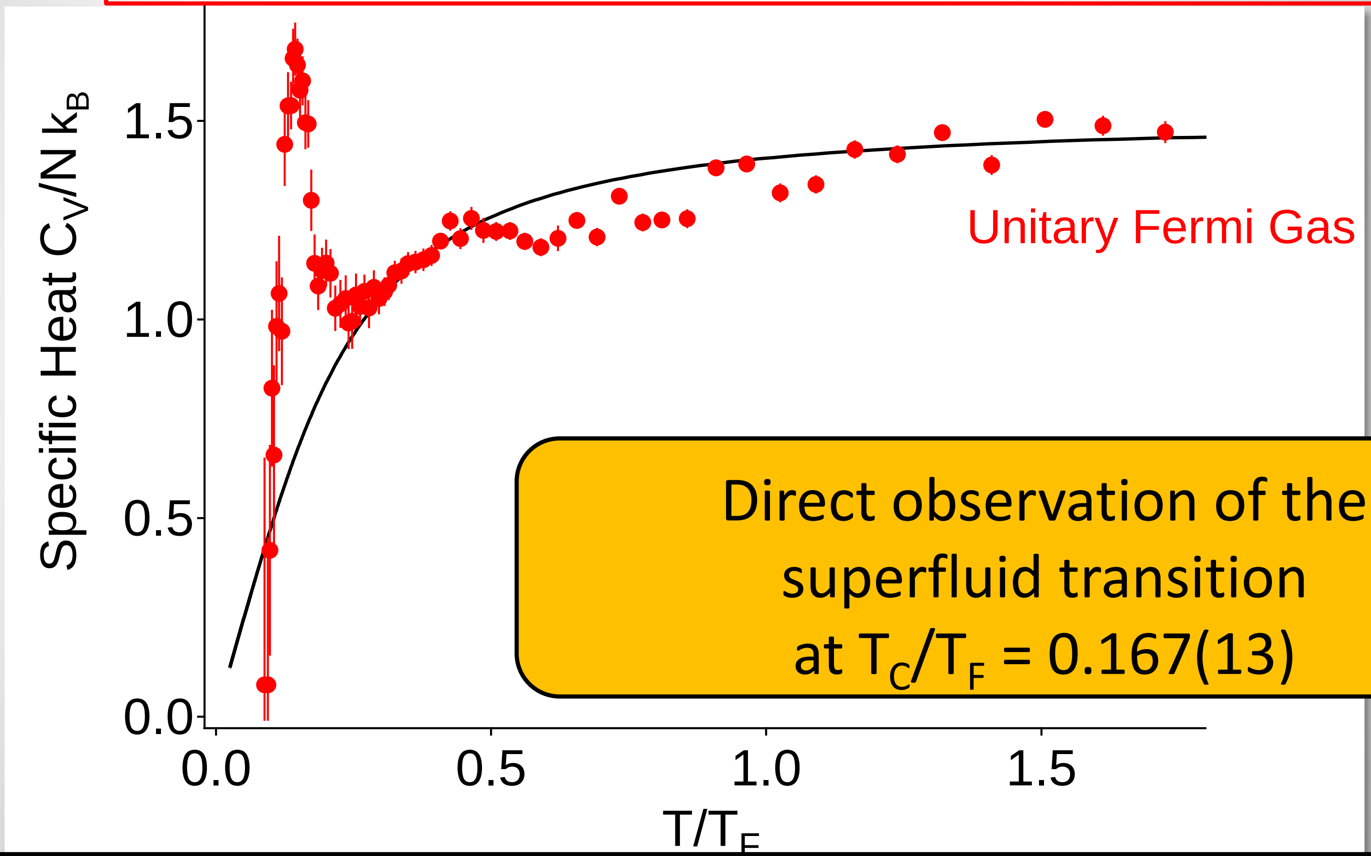
Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

# Heat capacity

For a resonant gas:

$$P = \frac{2}{3}\frac{E}{V}$$

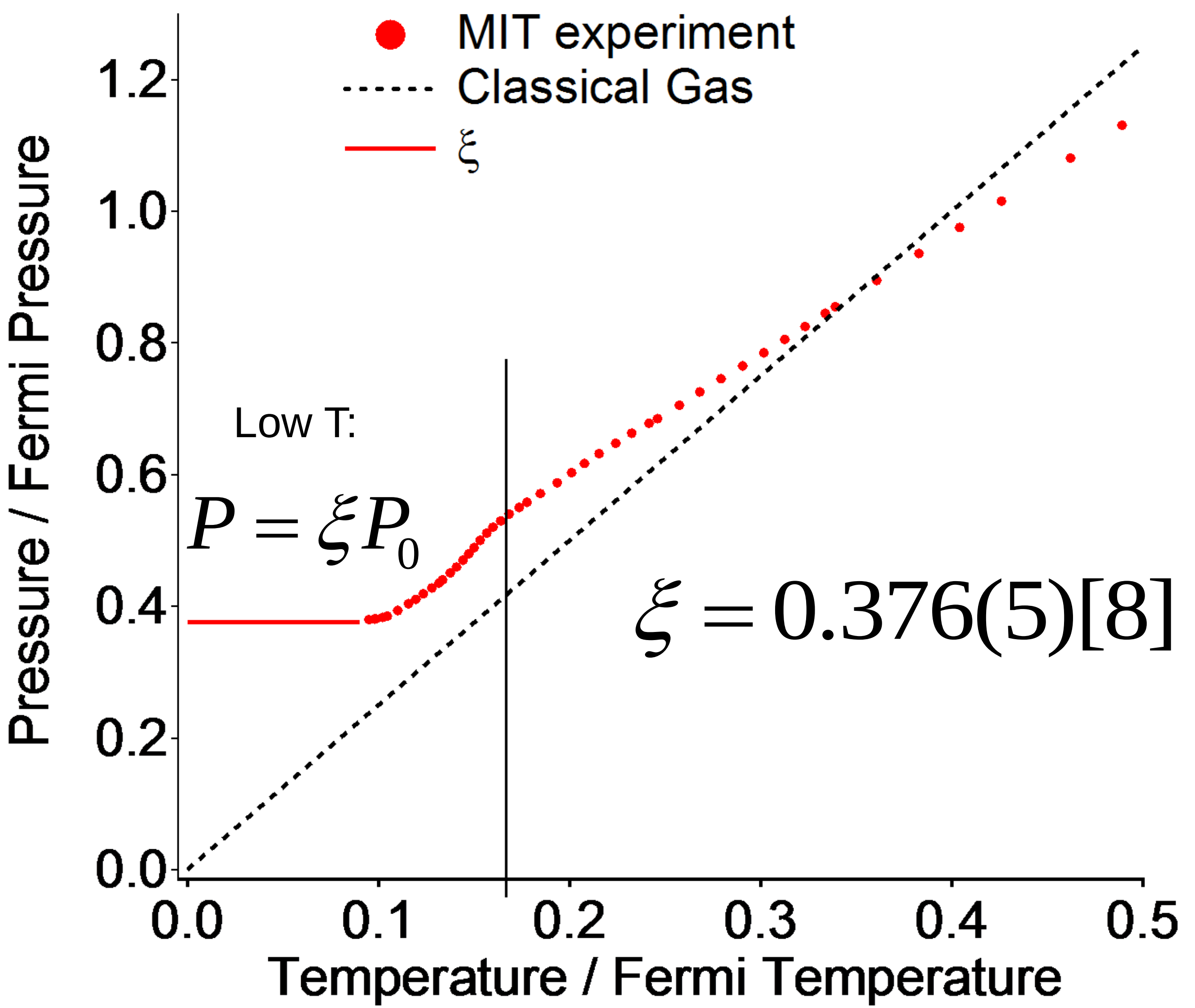
$$\frac{C_V}{Nk_B} = \frac{\mathrm{d}\left(E/Nk_B\right)}{\mathrm{d}T}\Bigg|_{N,V} = \frac{\mathrm{d}\left(P/nE_F\right)}{\mathrm{d}(T/T_F)} = \frac{3}{2}\frac{T_F}{T}\left(\frac{P}{P_0} - \frac{\kappa_0}{\kappa}\right)$$



Direct observation of the superfluid transition at $T_C/T_F = 0.167(13)$

Scaled to the density of electrons in a solid, superfluidity would occur far above room temperature

# Pressure versus Temperature



Low T:

$$P = \xi P_0$$

$$\xi = 0.376(5)[8]$$

Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

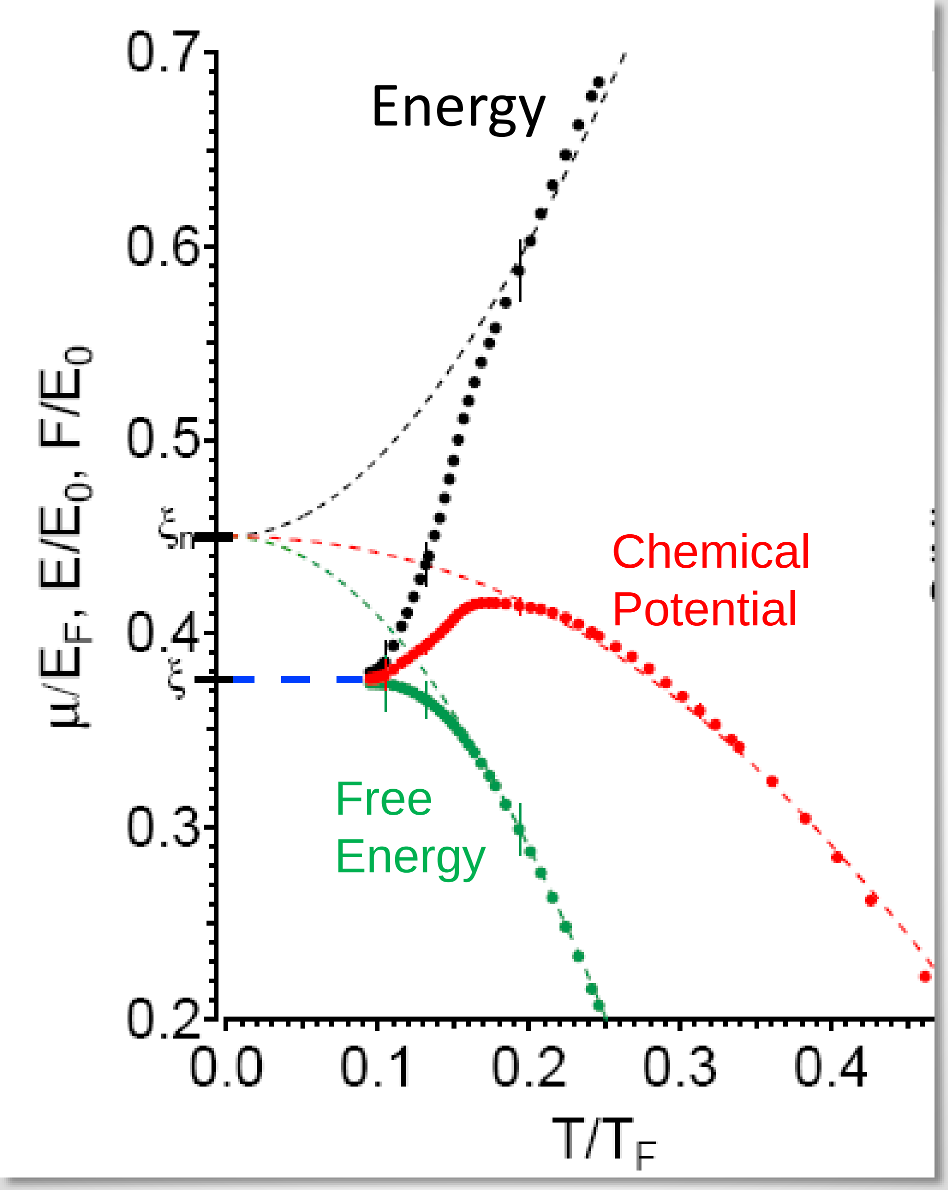
# Obtaining the chemical potential

Invert compressibility relation

$$\kappa = \frac{1}{n^2} \frac{\partial n}{\partial \mu}\bigg|_T = -\kappa_0 \left( \frac{T_F}{T} \right)^2 \frac{d(T/T_F)}{d(\beta\mu)}$$

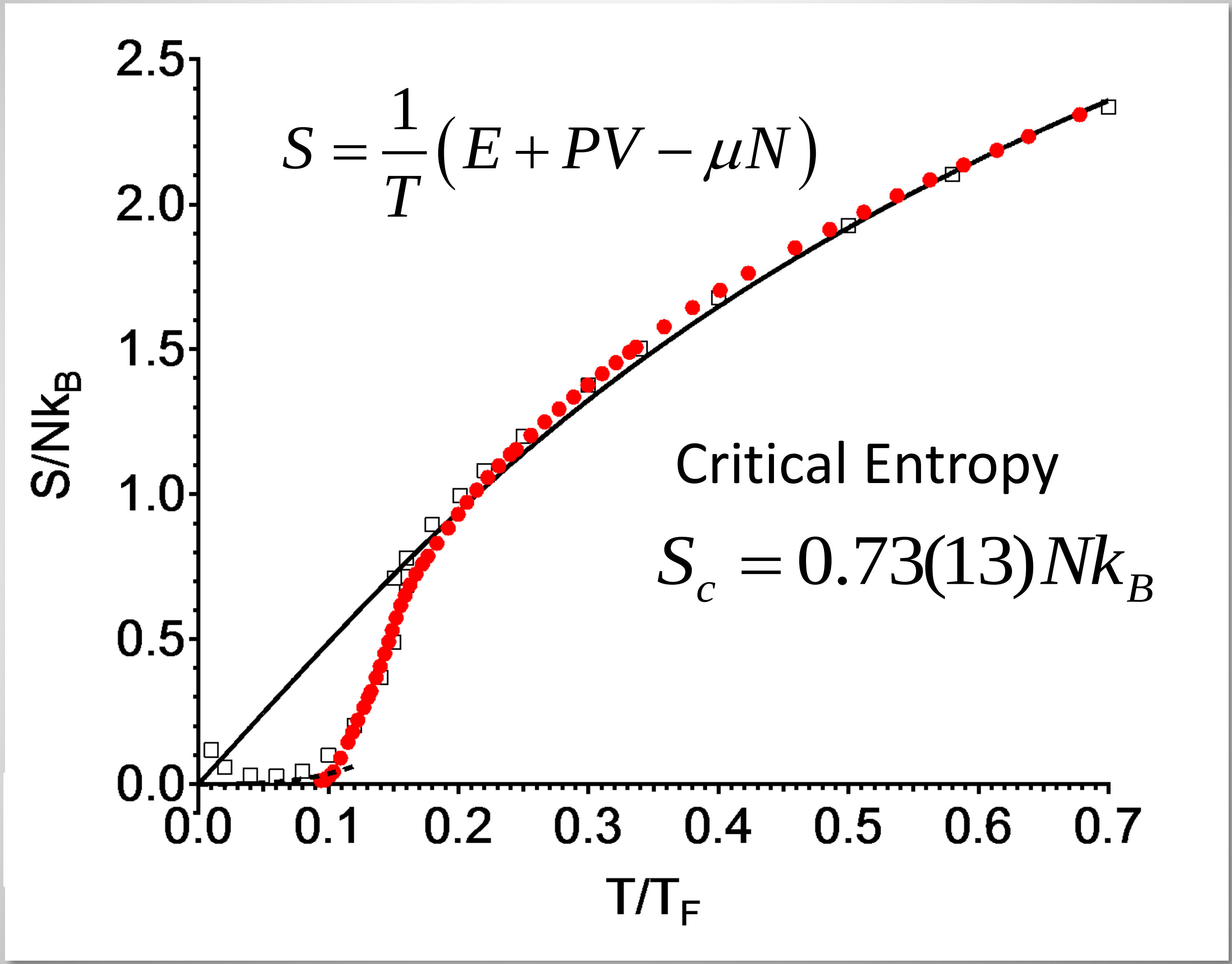
$$\boxed{\beta\mu = (\beta\mu)_i - \int_{T_i/T_\mathrm{F}}^{T/T_\mathrm{F}} \mathrm{d}\left(\frac{T}{T_\mathrm{F}}\right) \frac{\kappa_0}{\kappa} \left(\frac{T_\mathrm{F}}{T}\right)^2}$$

# Energy, Chemical Potential, Free Energy



Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567 (2012)

# Entropy



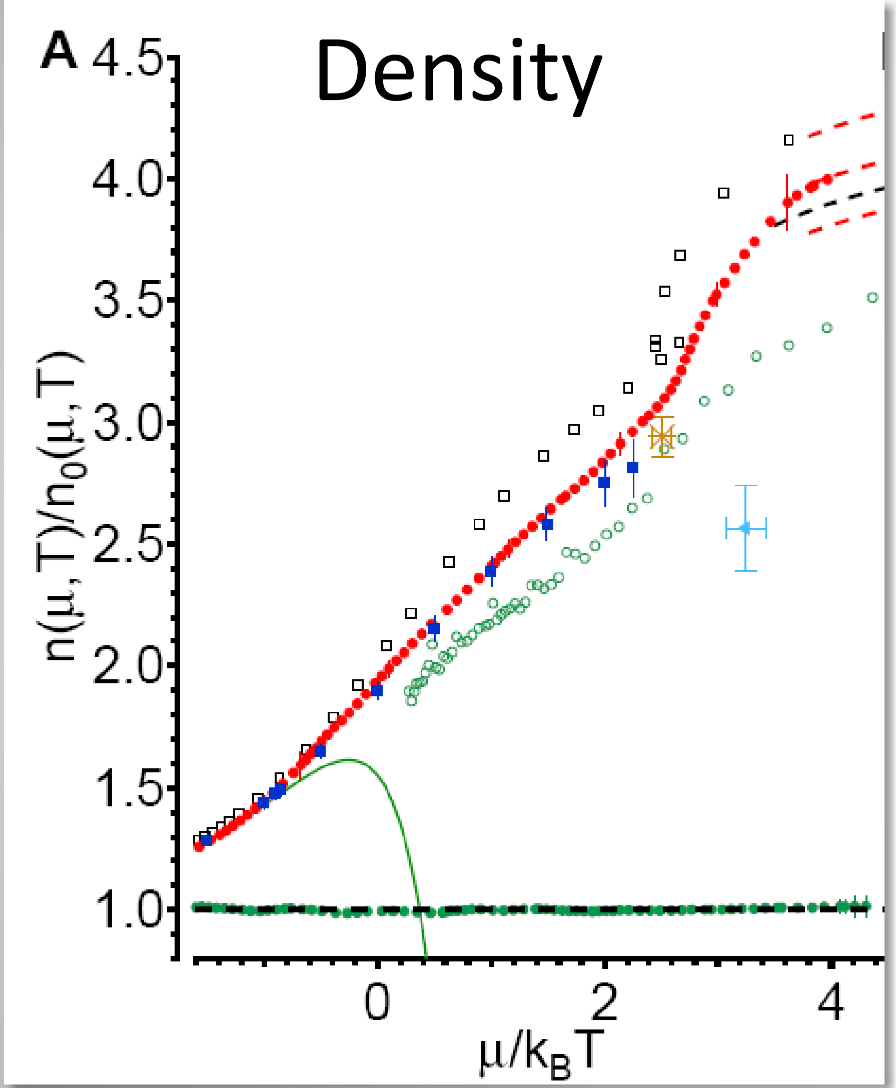
$$S = \frac{1}{T}\left(E + PV - \mu N\right)$$

Critical Entropy

$$S_c = 0.73(13)\,Nk_B$$

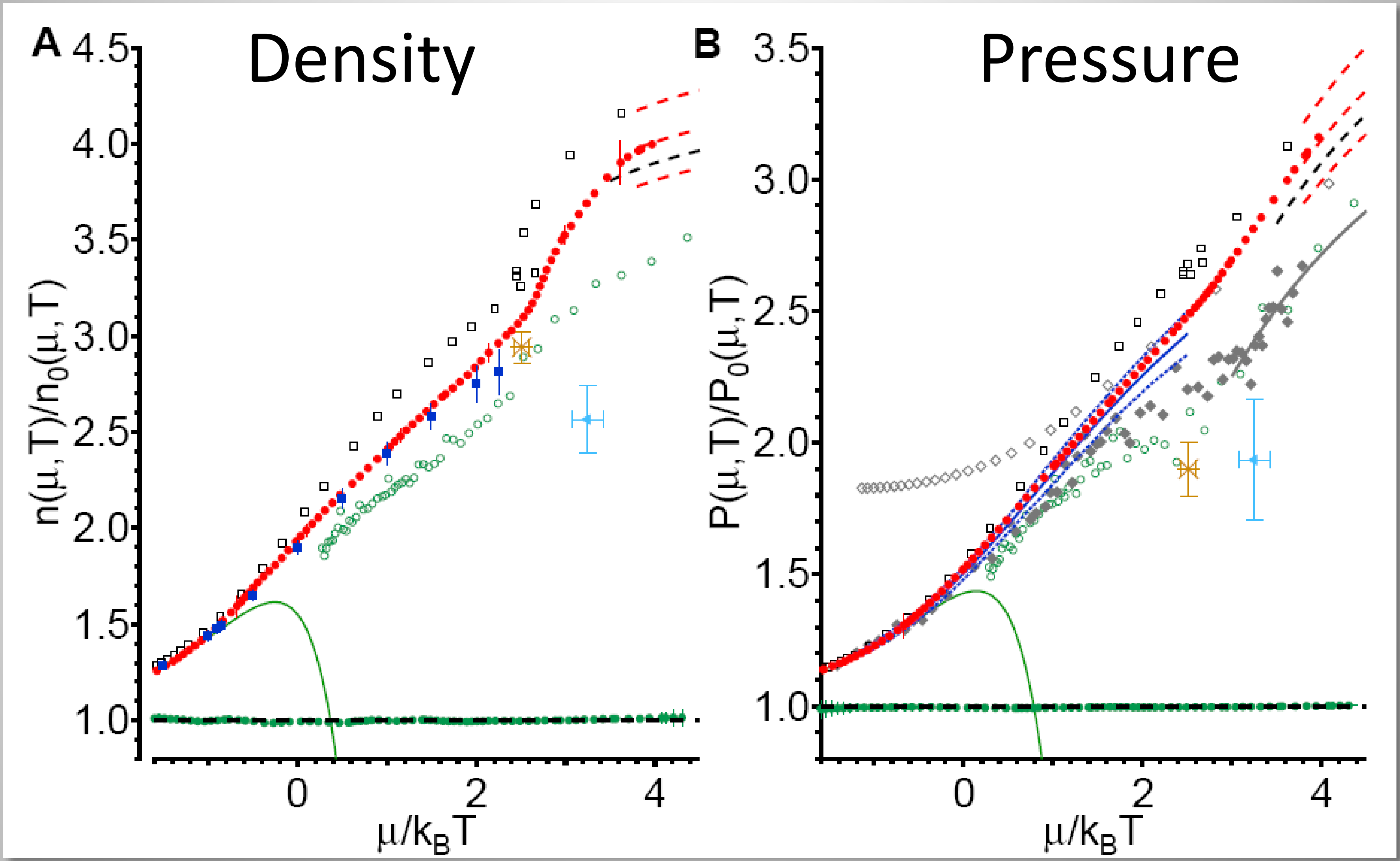
Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein
Science **335**, 563-567  (2012)

# Realization of a Feynman Quantum Simulation



Mark Ku, Ariel Sommer, Lawrence Cheuk, MWZ, Science **335**, 563-567 (2012)
K. Van Houcke, F. Werner, E. Kozik, N. Prokofev, B. Svistunov,
M. Ku, A. Sommer, L. Cheuk, A. Schirotzek, MWZ, Nature Physics **8**, 366 (2012)

# Realization of a Feynman Quantum Simulation

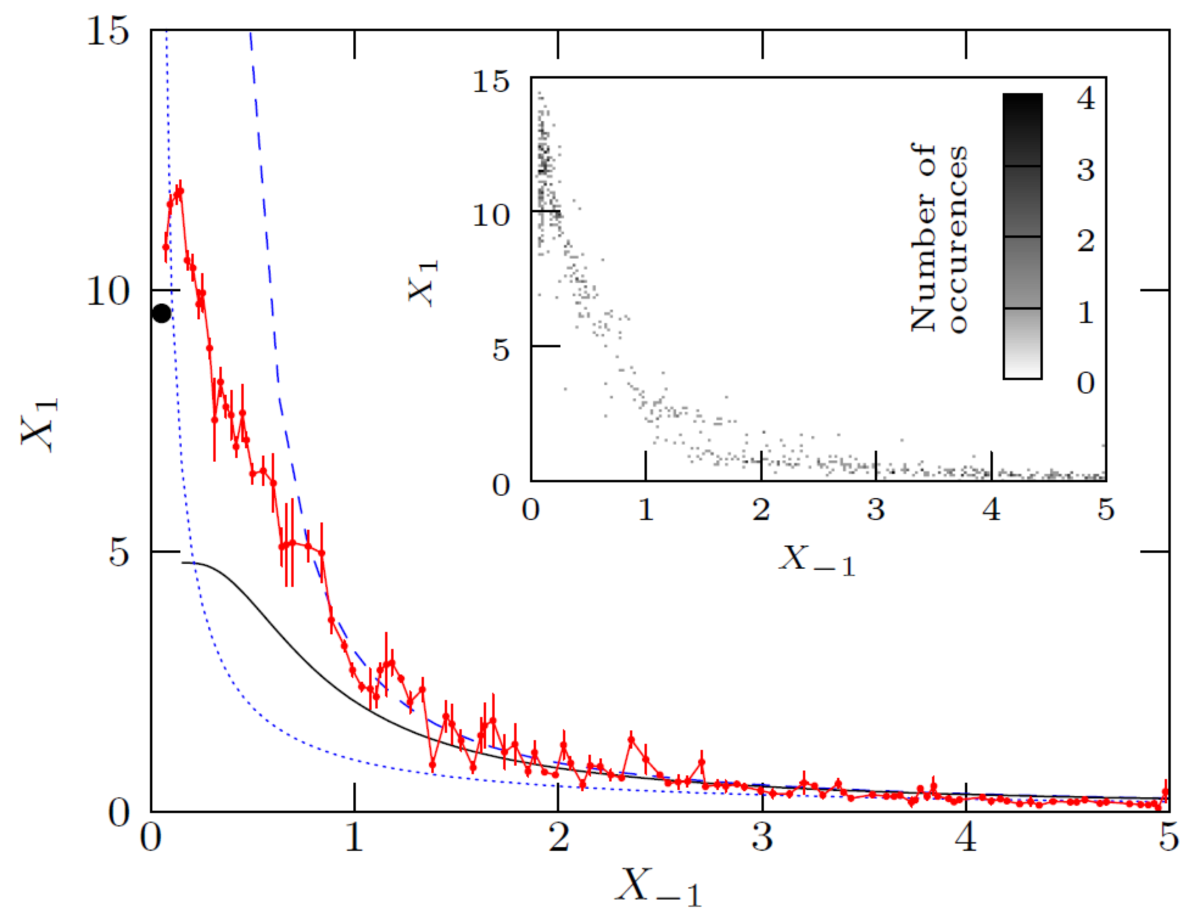


Mark Ku, Ariel Sommer, Lawrence Cheuk, MWZ, Science **335**, 563-567 (2012)
K. Van Houcke, F. Werner, E. Kozik, N. Prokofev, B. Svistunov,
M. Ku, A. Sommer, L. Cheuk, A. Schirotzek, MWZ, Nature Physics **8**, 366 (2012)

# Application to the 2D Bose Gas

Rémi Desbuquois, Tarik Yefsah, Lauriane Chomaz, Christof Weitenberg, Laura Corman, Sylvain Nascimbène, and Jean Dalibard, arXiv:1403.4030 (2014)

$$\frac{\kappa}{\kappa_0}$$



$$\kappa_0 = \frac{1}{nE_{\text{"F"}}}$$

$$E_{\text{"F"}} = \frac{\hbar^2 n}{m}$$

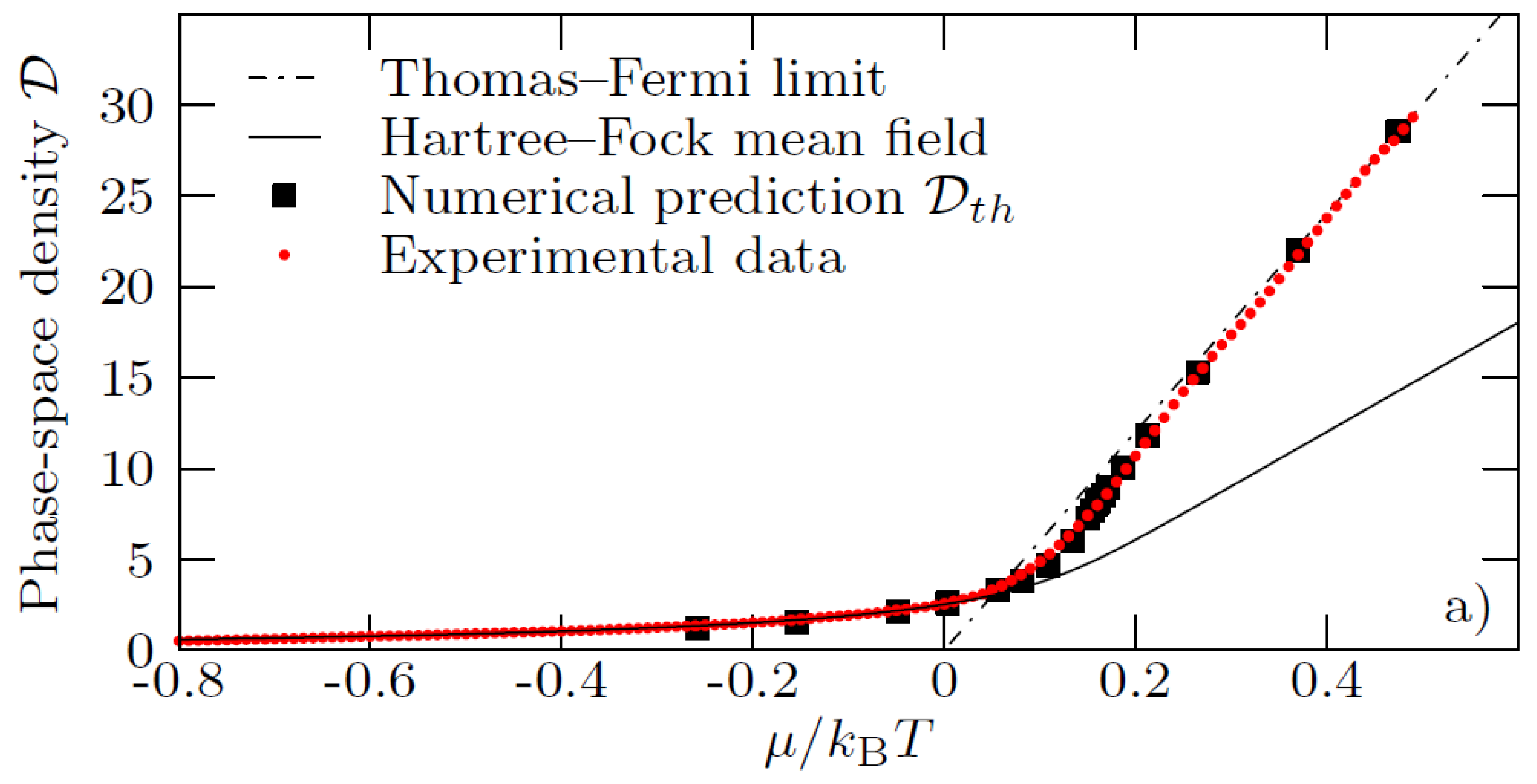
$$P_0 = \frac{1}{2} nE_{\text{"F"}}$$

$$\frac{1}{2}\frac{P}{P_0}$$

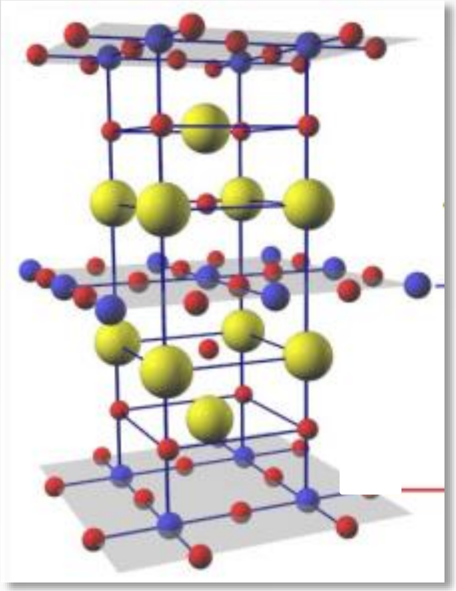
# Application to the 2D Bose Gas

Rémi Desbuquois, Tarik Yefsah, Lauriane Chomaz, Christof Weitenberg, Laura Corman, Sylvain Nascimbène, and Jean Dalibard, arXiv:1403.4030 (2014)

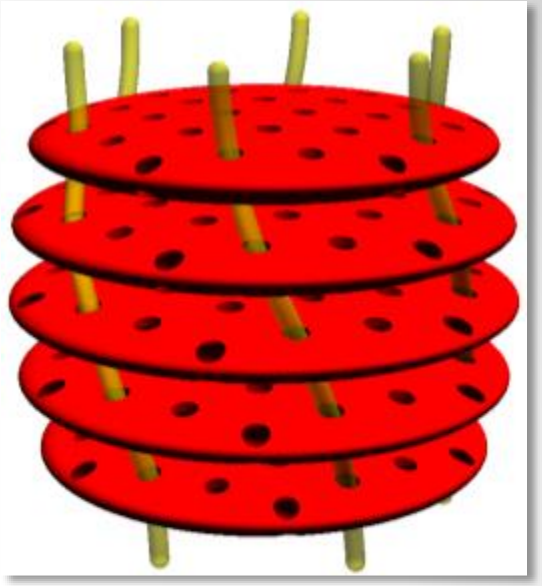
$$D = n\lambda^2 = \left(\frac{T_F}{T}\right)^2$$



# Application to the 2D Fermi Gas



**High-$T_c$ Superconductor with stacks of CuO planes**

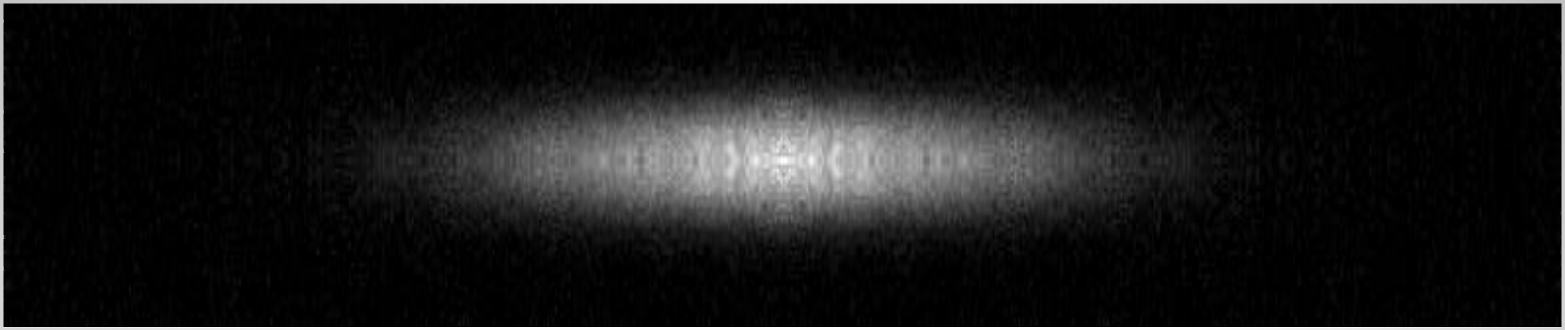


**Stacks of 2D coupled fermionic superfluids**

… to come…

(Preliminary results by Chris Vale, Beijing 2014)

# Fit-free method using the column density



What thermodynamic information does the column density contain?

$$n_{2D} = \int dz\ n(\mu_0 - V(x,y,z),T)$$

Int. by parts:

$$= -\int dz\ z\frac{\partial n}{\partial z} \quad = -\int dz\ z\ \frac{\partial n}{\partial \mu}\frac{\partial \mu}{\partial z}$$

Harmonic trap:

$$V(z) = \frac{1}{2}m\omega^2 z^2$$

$$= m\omega^2 \int dz\ z^2\ \frac{\partial n}{\partial \mu} \quad = \frac{m\omega^2}{2\pi}\int d^3r\ \frac{\partial n}{\partial \mu}$$

$$\boxed{= \frac{m\omega^2}{2\pi}\ \frac{\partial N}{\partial \mu}}$$

Compressibility of the *trapped* gas

# Equation of State of *Trapped* Gases

Compressibility
$$\frac{\partial N(\mu_0 - V, T)}{\partial \mu} = \frac{2\pi}{m\omega^2} n_{2D}(V)$$

Atom number
$$N(\mu_0 - V, T) = \frac{2\pi}{m\omega^2} \int_V^\infty dV' \, n_{2D}(V')$$

Grand potential
$$\Omega(\mu_0 - V, T) = -\int_{-\infty}^{\mu_0 - V} d\mu \, N(\mu, T)$$

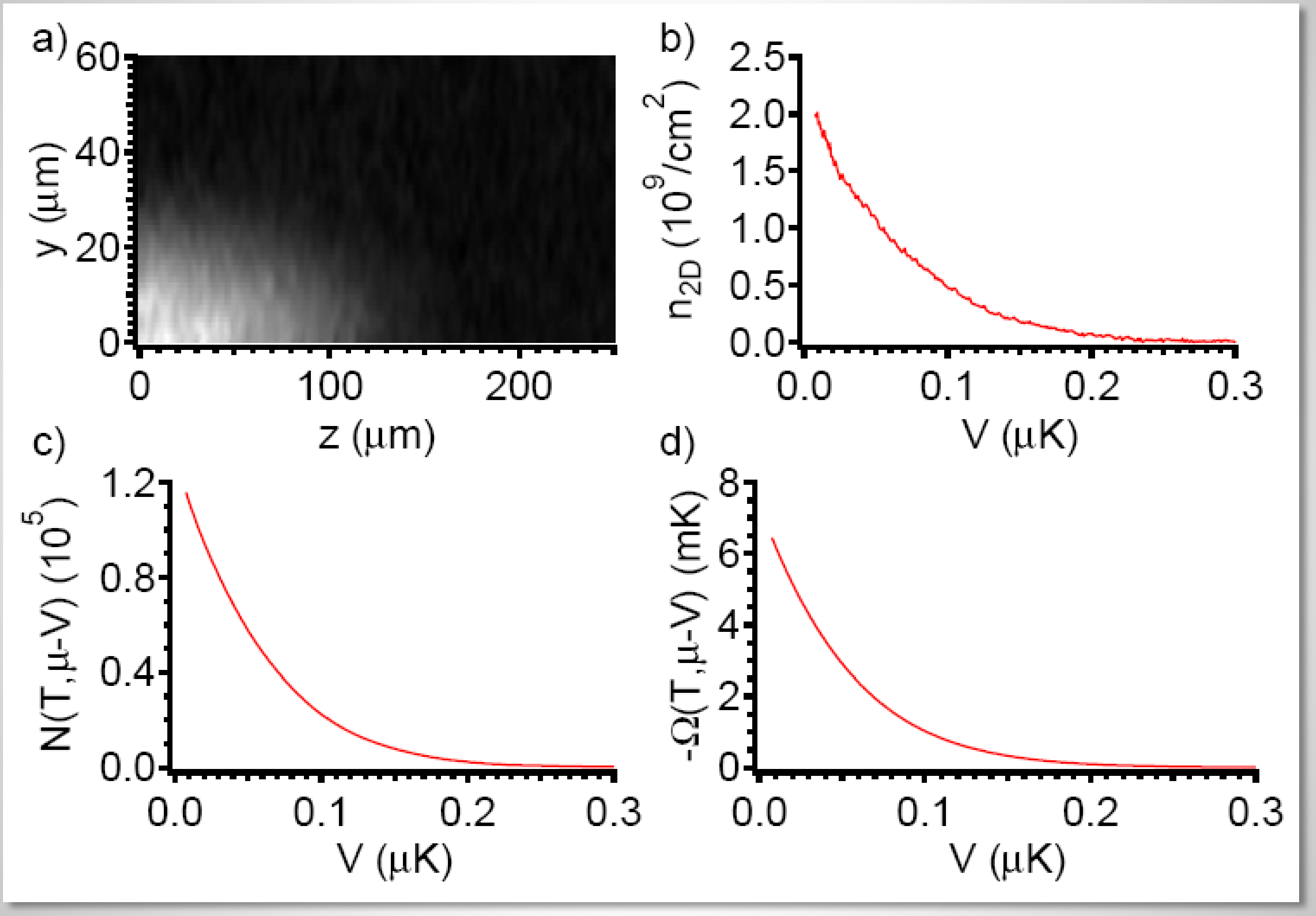
$$= \frac{2\pi}{m\omega^2} \int_V^\infty dV' \int_{V'}^\infty dV'' \, n_{2D}(V'')$$

**Thermometer for harmonically trapped gases (including lattice expts.)**

Mark Ku et al., 2012, unpublished

= Average Potential
$$= \frac{2\pi}{m\omega^2} \int_V^\infty dV'(V' - V) \, n_{2D}(V')$$

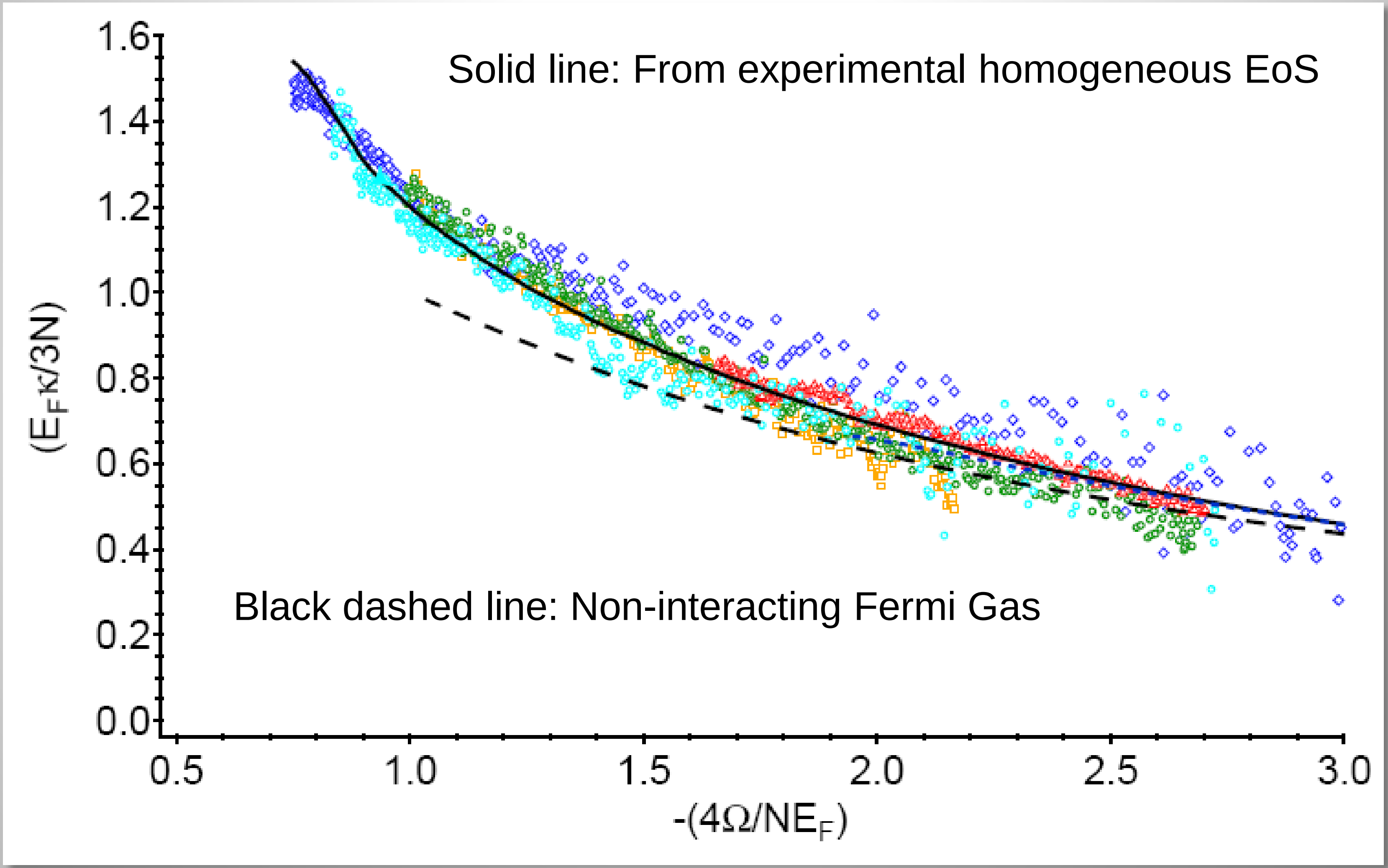
# EoS of trapped, unitary Fermi gas



Mark J. H. Ku, Lawrence W. Cheuk, Martin W. Zwierlein, 2012, unpublished

# EoS of trapped, unitary Fermi gas

Compressibility versus Grand Potential



Solid line: From experimental homogeneous EoS

Black dashed line: Non-interacting Fermi Gas

Mark J. H. Ku, Lawrence W. Cheuk, Martin W. Zwierlein, 2012, unpublished

# Obtaining the temperature

Change in Grand potential with $T/T_F$ involves the *measured* compressibility. Can integrate:

$$\left(\frac{T}{T_F}\right) = \left(\frac{T}{T_F}\right)_i \exp\left(\frac{1}{4}\int_{\tilde{\Omega}_i}^{\tilde{\Omega}} \mathrm{d}\tilde{\Omega}\, \frac{1}{\tilde{\Omega} - \frac{1}{\tilde{\kappa}}}\right)$$

Chemical Potential:

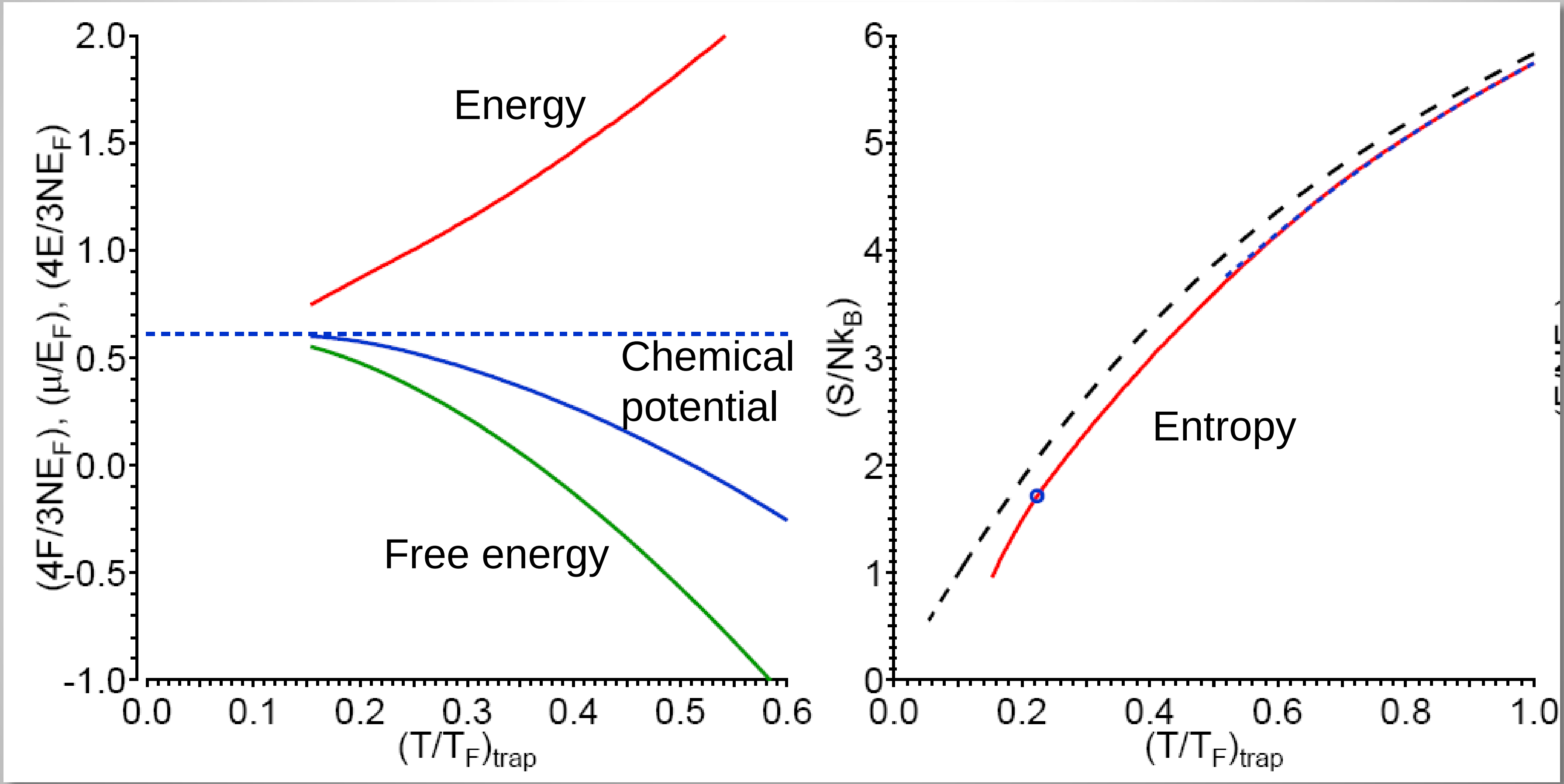
$$\frac{\mu}{k_B T} = \left(\frac{\mu}{k_B T}\right)_i - \int_{\left(\frac{T}{T_F}\right)_i}^{\left(\frac{T}{T_F}\right)} \mathrm{d}\left(\frac{T}{T_F}\right) \frac{1}{\tilde{\kappa}} \left(\frac{T_F}{T}\right)^2$$

Entropy:

$$\frac{S}{k_B N} = \tilde{\Omega}\left(\frac{T_F}{T}\right) - \frac{\mu}{k_B T}$$

# Energy, Chemical Potential, Entropy

Via integration of **experimental** homogeneous equation of state

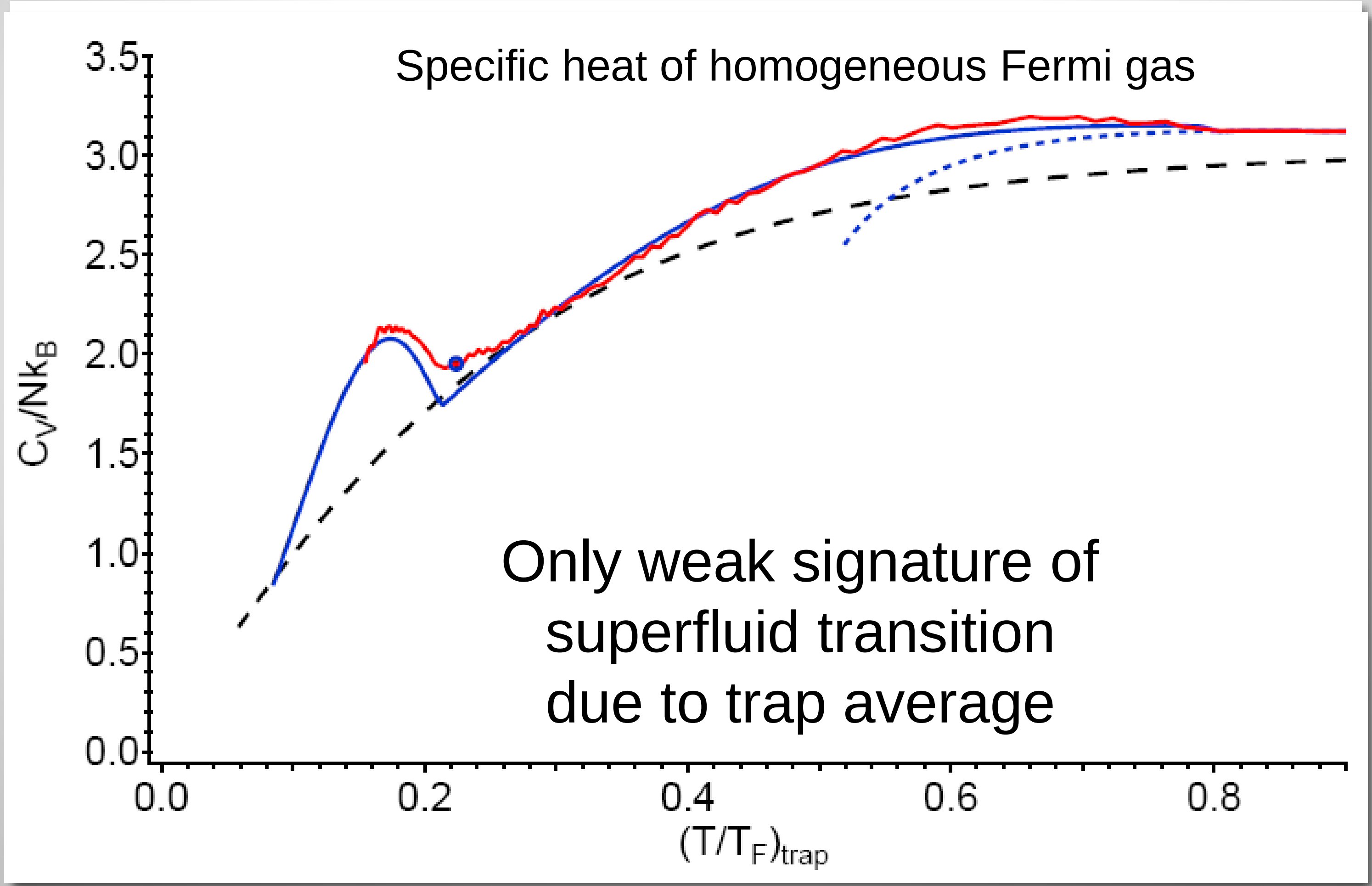


Mark J. H. Ku, Lawrence W. Cheuk, Martin W. Zwierlein, 2012, unpublished

# Specific heat of trapped Fermi gas

Directly follows from the column density and its integrals!

$$\frac{C_V}{Nk_B} = \frac{d(E/N)}{dT} = -3\frac{d(\Omega/NE_F)}{d(T/T_F)} = 4\frac{T_F}{T}\left(\tilde{\Omega} - \frac{1}{\tilde{\kappa}}\right)$$



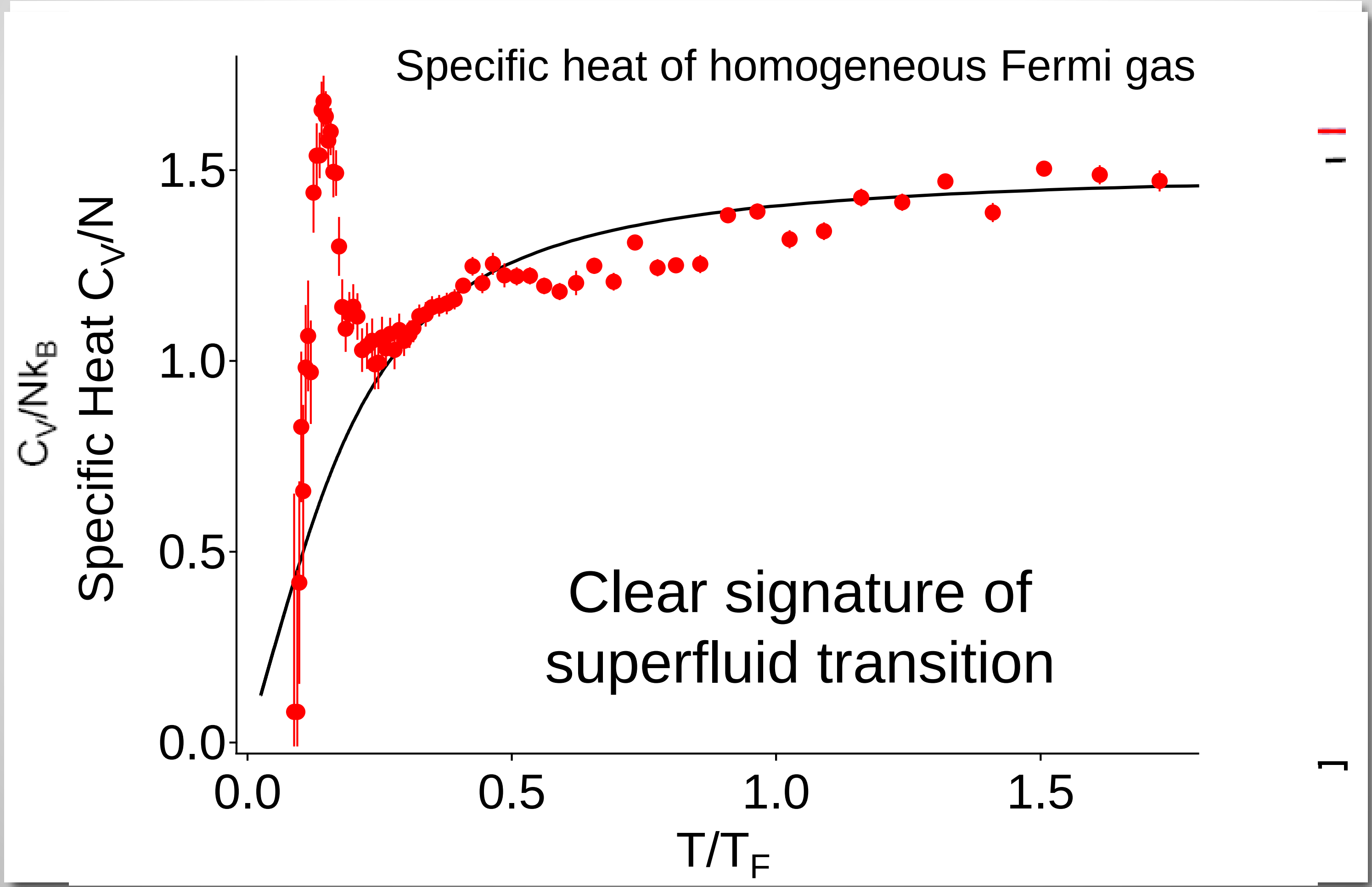
Only weak signature of superfluid transition due to trap average

Mark J. H. Ku, Lawrence W. Cheuk, Martin W. Zwierlein, 2012, unpublished

# Specific heat of trapped Fermi gas

Directly follows from the column density and its integrals!

$$\frac{C_V}{Nk_B} = \frac{d(E/N)}{dT} = -3\frac{d(\Omega/NE_F)}{d(T/T_F)} = 4\frac{T_F}{T}\left(\tilde{\Omega} - \frac{1}{\tilde{\kappa}}\right)$$



Specific heat of homogeneous Fermi gas

Clear signature of superfluid transition

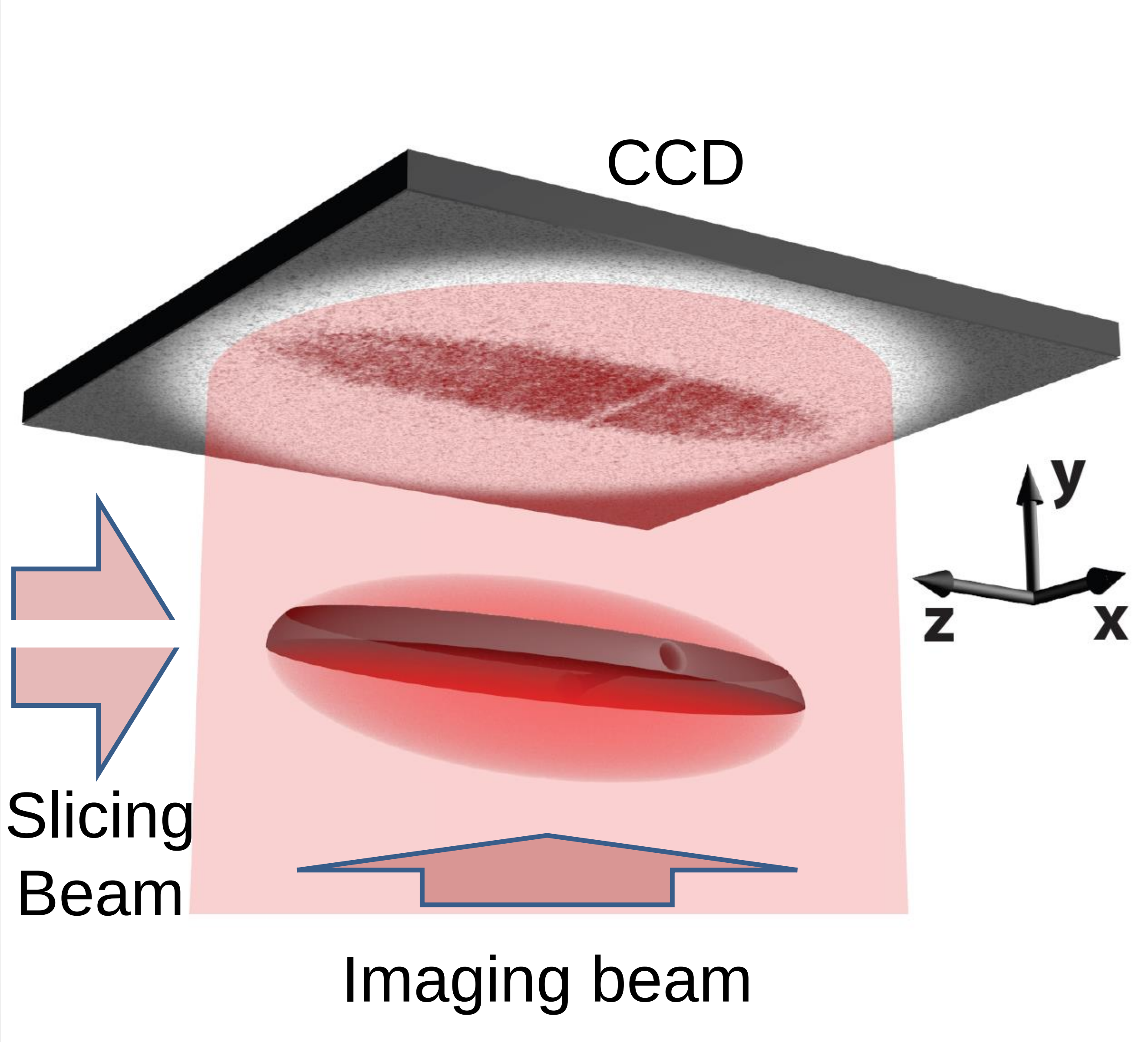
Mark J. H. Ku, Lawrence W. Cheuk, Martin W. Zwierlein, 2012, unpublished

# Application in Superfluid Hydrodynamics

## Motion of a Vortex in a Fermionic Superfluid

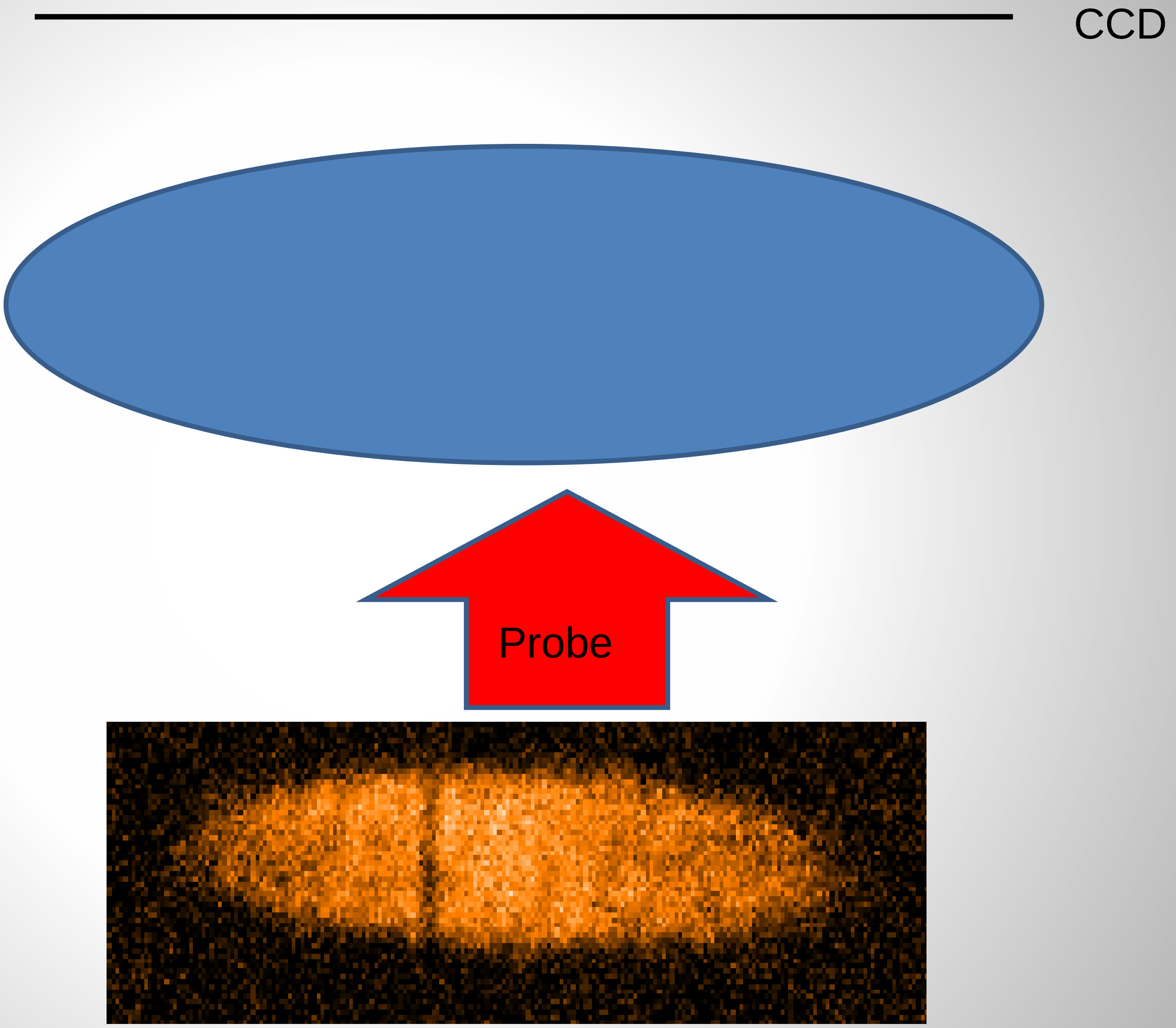
MJHK, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L.W. Cheuk, T. Yefsah,
M. W. Zwierlein, arXiv:1402.7052 (2014), PRL, to appear

# Slicing



M.J.H. Ku, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L.W. Cheuk, MWZ, arXiv:1402.7052 (2014)

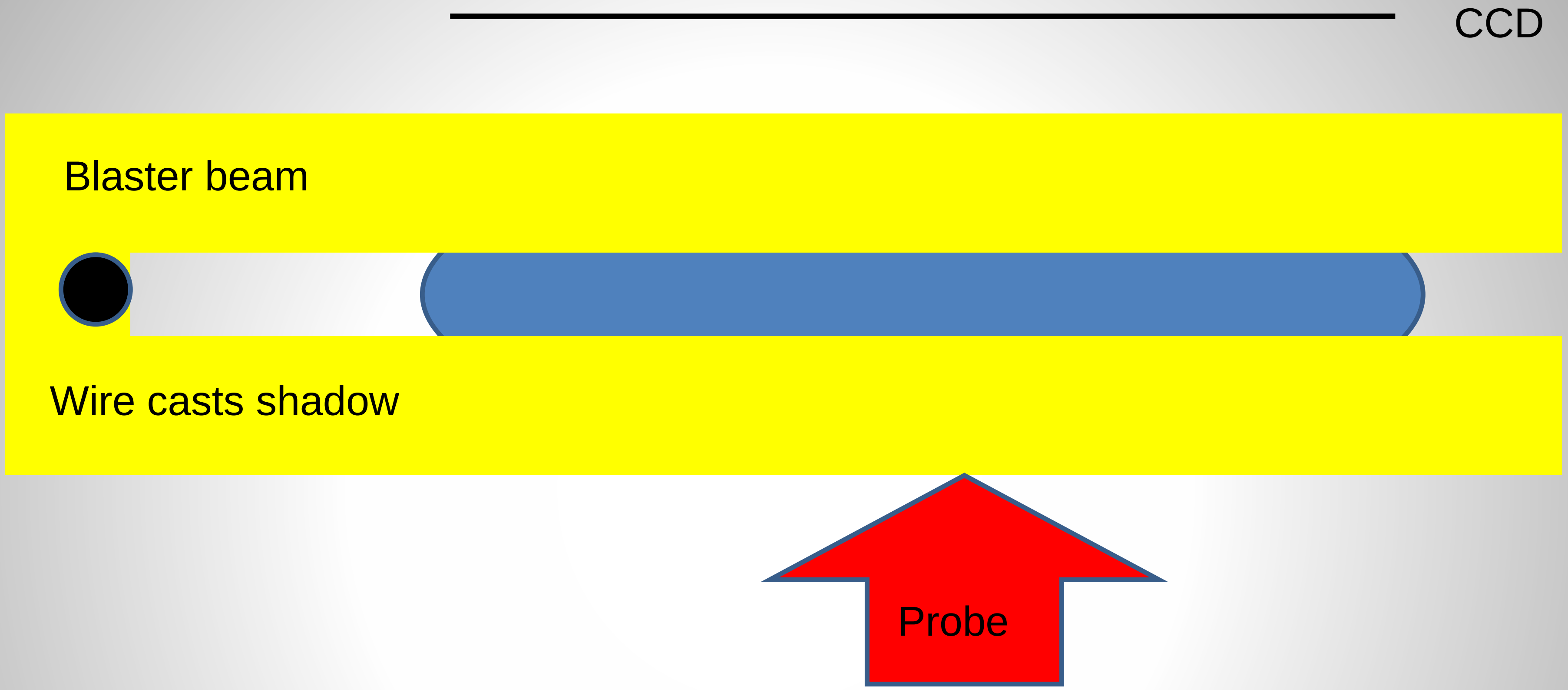
# Tomography: Slicing the Cloud



CCD

Probe

# Tomography: Slicing the Cloud

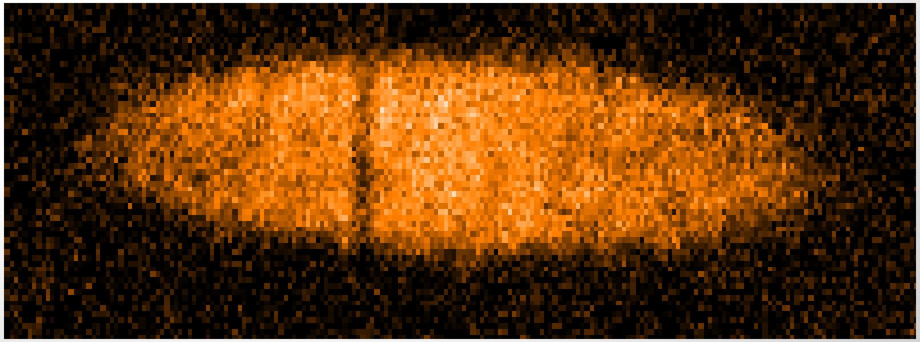


CCD

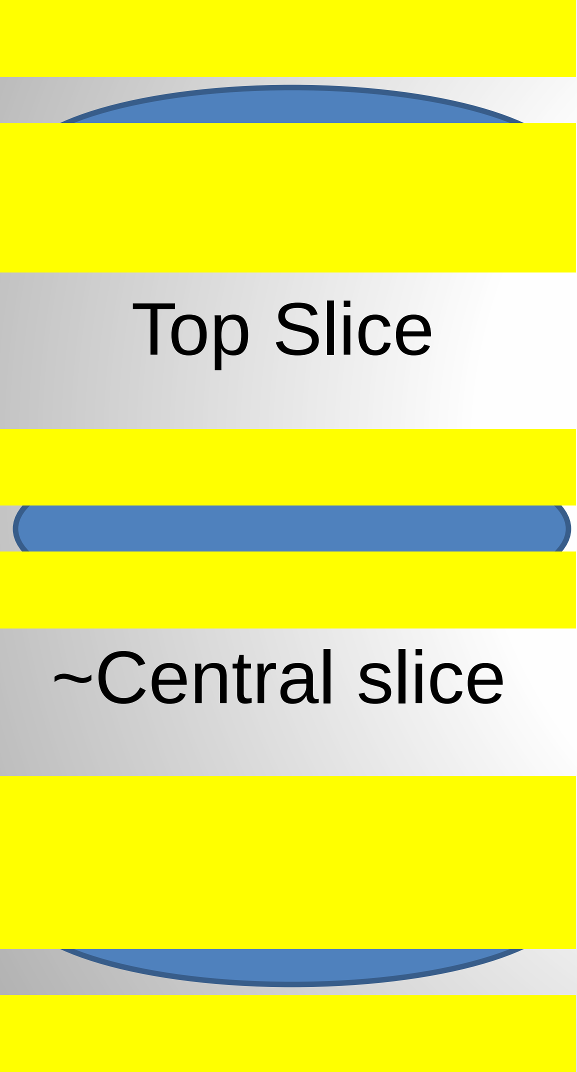
Blaster beam

Wire casts shadow

Probe



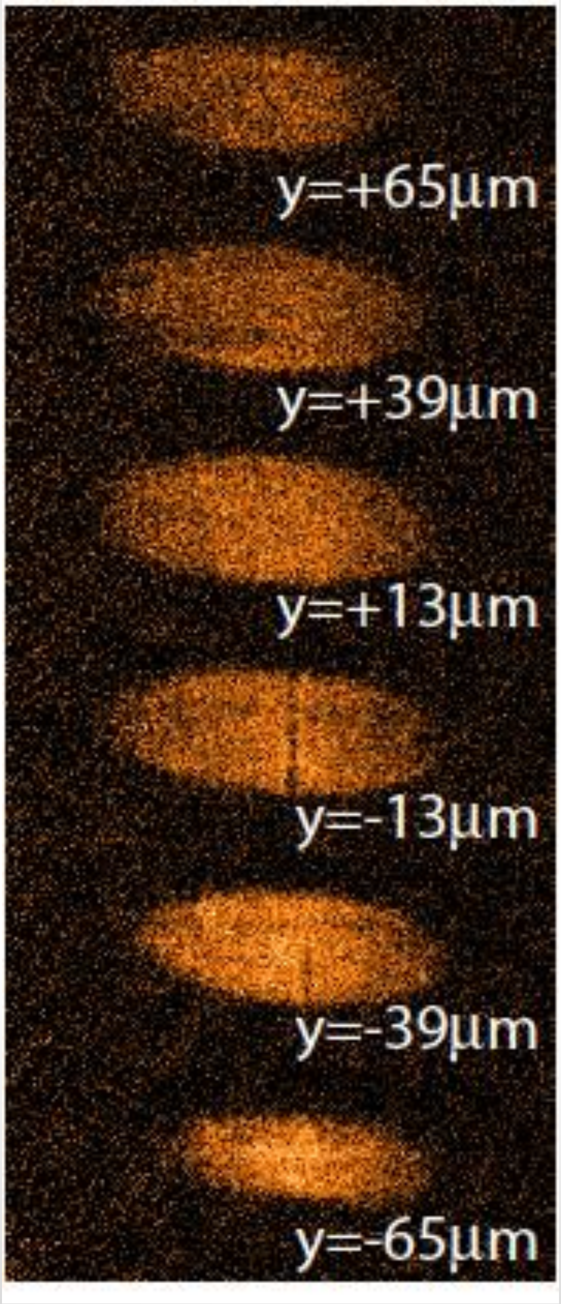
# Tomography: Slicing the Cloud



Top Slice

~Central slice

Bottom Slice

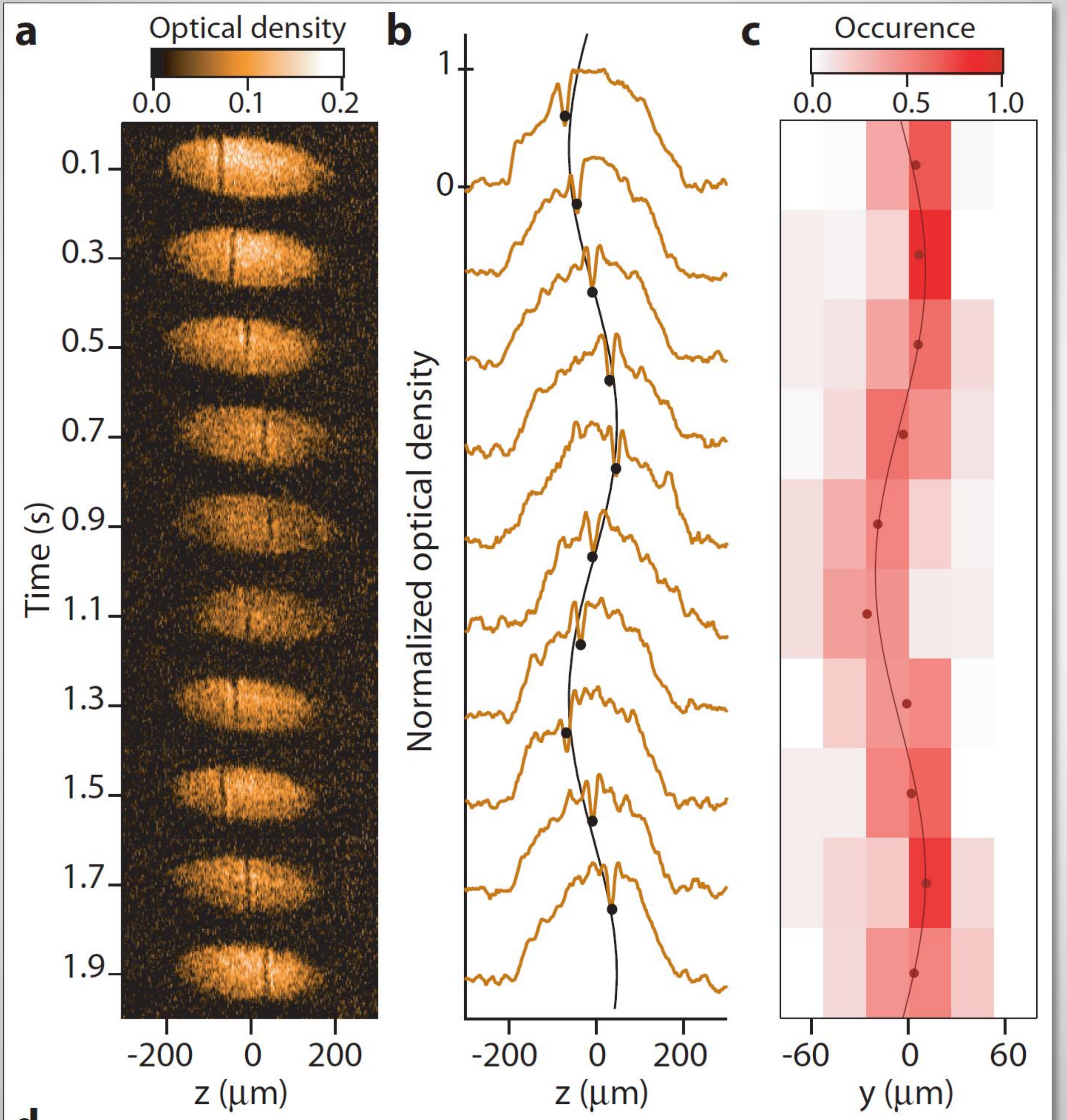


"Solitonic Vortex"
Brand, Reinhardt
PRA 65, 043612 (2002)

Anisotropy
and anharmonicity
due to gravity
→ Vortex aligned in
x-z plane

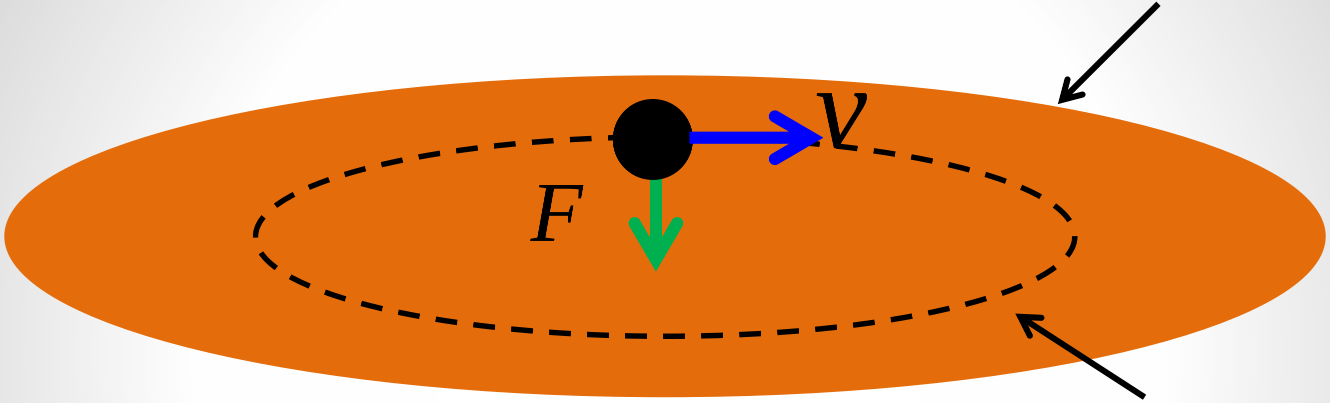
Mark J.-H. Ku, W. Ji,
B. Mukherjee,
E. Guardado-Sanchez,
L. W. Cheuk, T. Yefsah,
MWZ, arXiv:1402.7052
PRL, to appear

# Direct observation of vortex precession



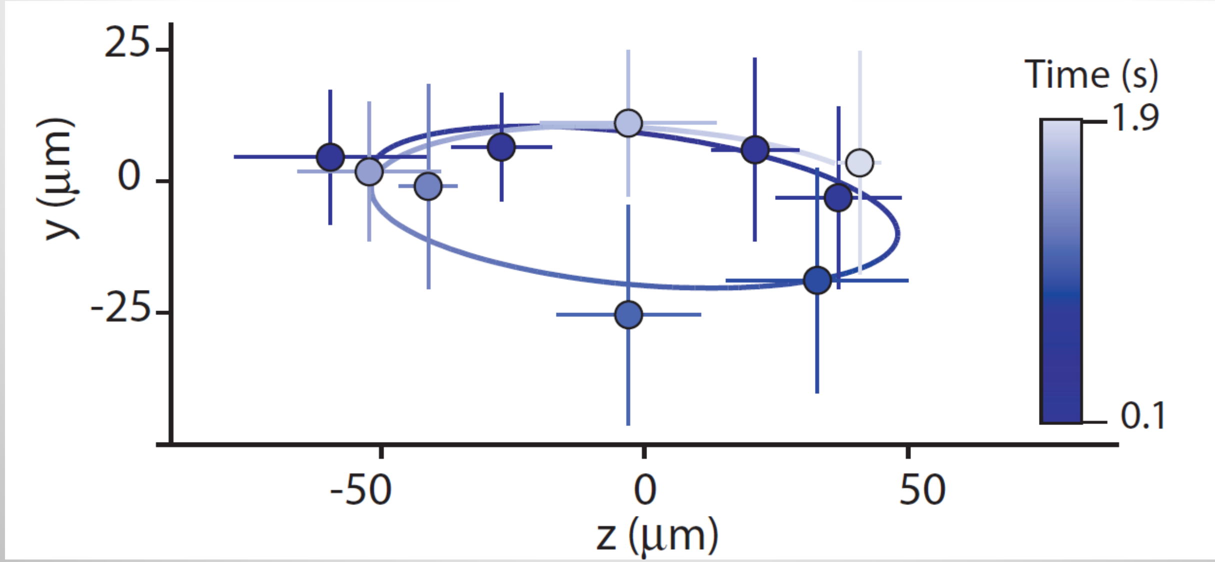
# Direct observation of vortex precession

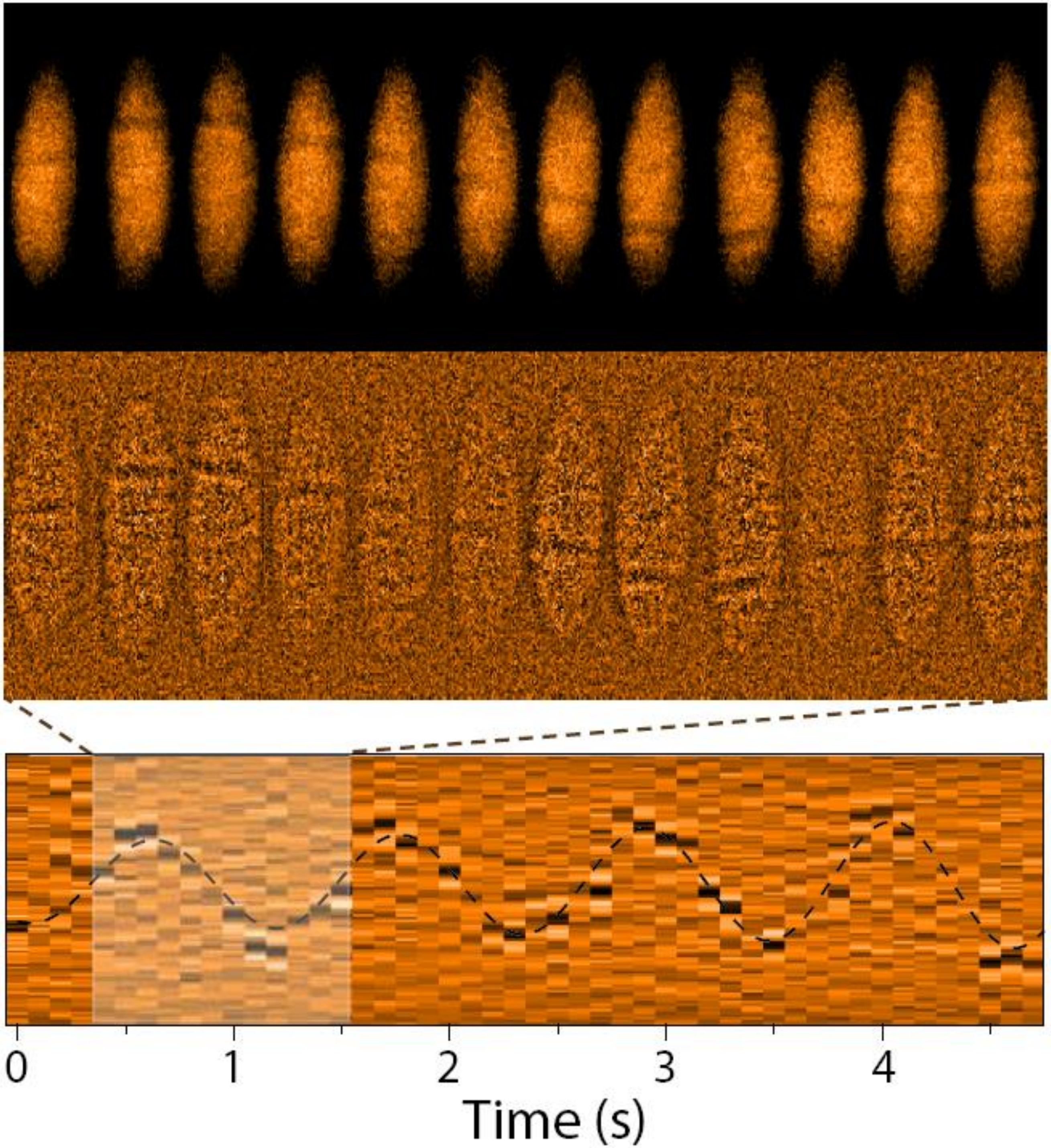
Magnus Force $h\bar{n}_S\vec{v}_V \times \hat{\Omega} = -\nabla E_V$ External Force on vortex



Superfluid

Equipotential contour




0
1
2
3
4
Time (s)

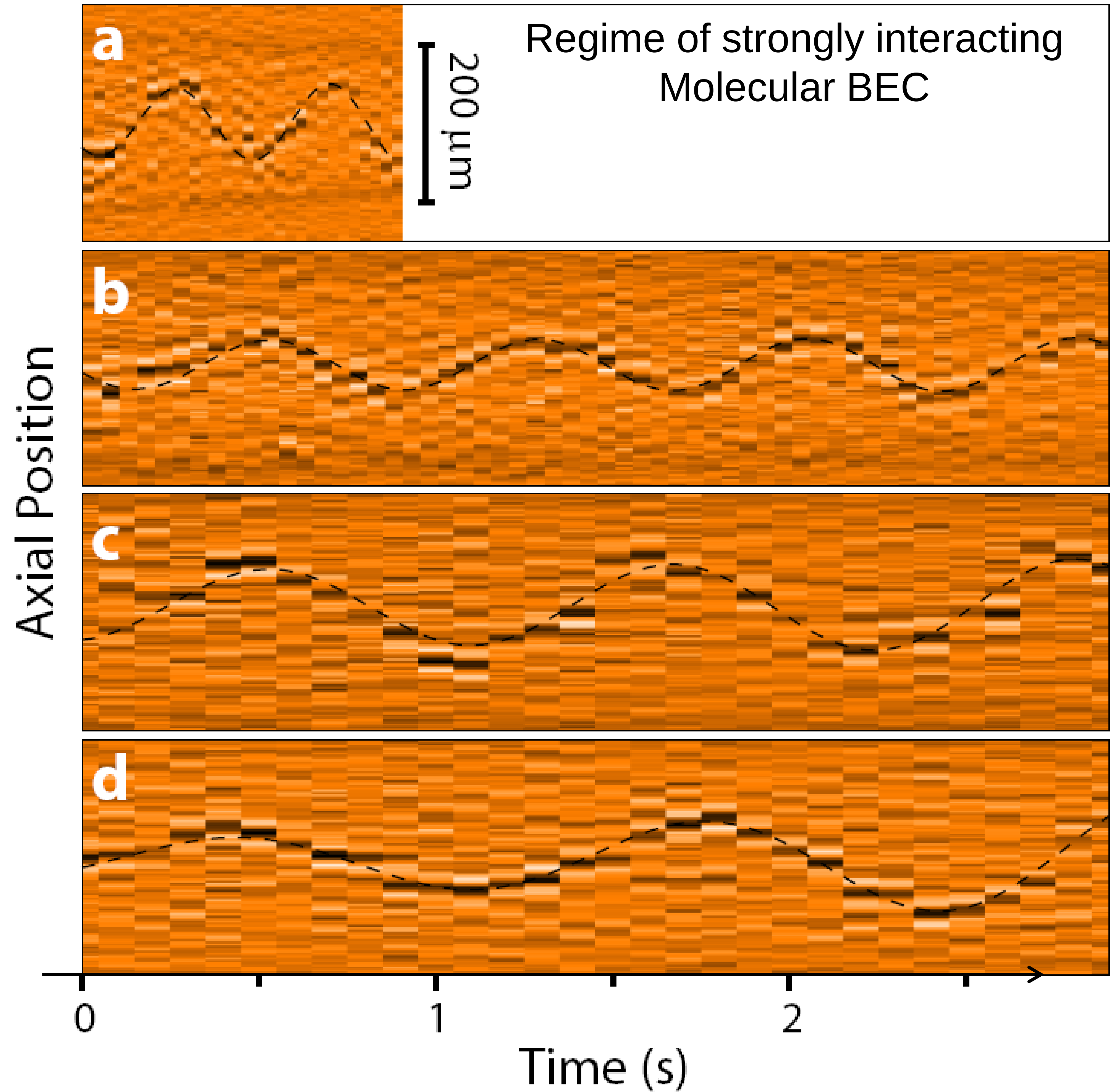
Stronger Interactions

700 G

760 G

815 G

832 G



# Compressibility controls Vortex Period

Magnus Force $h\bar{n}_S\vec{v}_V \times \hat{\Omega} = -\nabla E_V$ External Force on vortex

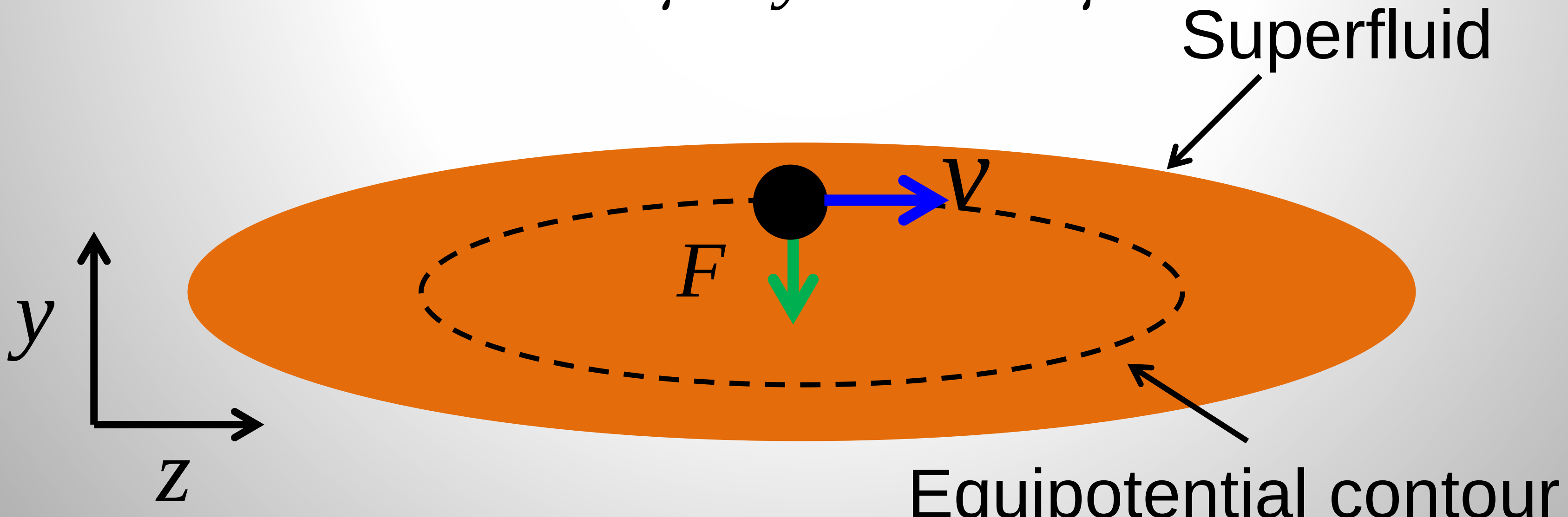
$\hat{\Omega}$ Circulation direction

$$E_V = \frac{\pi\hbar^2 L}{m_B}\bar{\mathrm{n}}(\mathrm{y}_V, \mathrm{z}_V)$$

$$F \sim -\nabla\bar{\mathrm{n}} \sim -\frac{\partial\bar{\mathrm{n}}}{\partial\mu}\nabla V$$

$$v_z \sim -\frac{1}{\bar{\mathrm{n}}}\frac{\partial\bar{\mathrm{n}}}{\partial\mu}\frac{\partial V}{\partial y} \sim -\alpha\frac{\hbar\omega_\perp\omega_z}{\mu}y$$

$$\alpha = \frac{\mu}{\bar{n}}\frac{\partial\bar{n}}{\partial\mu}$$



# Vortex Period in the BEC-BCS Crossover

Magnus Force $h\bar{n}_S \vec{v}_V \times \hat{\Omega} = -\nabla E_V$ External Force on vortex

$$E_V = \frac{\pi\hbar^2 L}{m_B} \bar{\mathrm{n}}(\mathrm{y}_V, \mathrm{z}_V)$$

$$\boxed{\frac{T_V}{T_z} = 4\frac{\mu(\mathrm{r}_V)}{\hbar\omega_\perp}\frac{1}{\alpha L}}$$

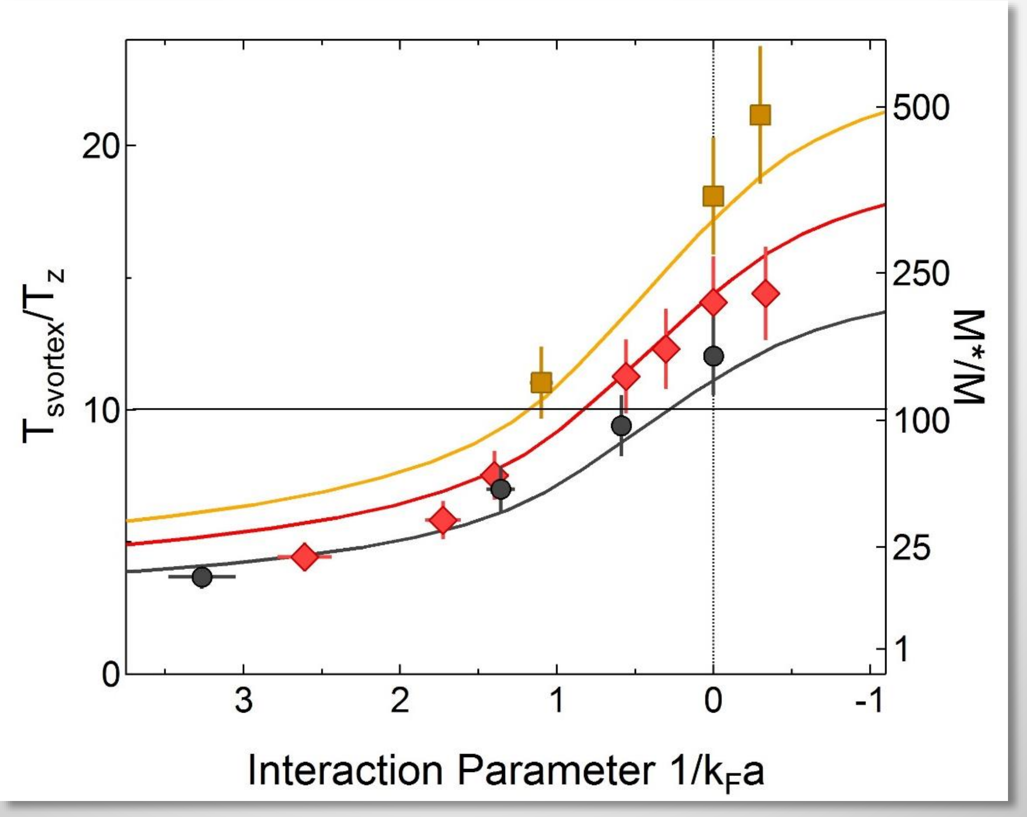
$$\alpha = \frac{\mu}{\bar{n}}\frac{\partial\bar{n}}{\partial\mu}$$

$$= \frac{3}{2}\ \text{(BEC)}$$

$$= 2\ \text{(Unitarity)}$$

Data from:
T. Yefsah, A. Sommer, M. J.-H. Ku, L. Cheuk, W. Ji, W. Bakr, MWZ, Nature **499**, 426–430 (2013)

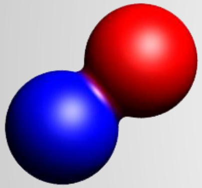
Theory from:
Mark J.-H. Ku, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L. W. Cheuk, T. Yefsah, MWZ, arXiv:1402.7052 PRL, to appear



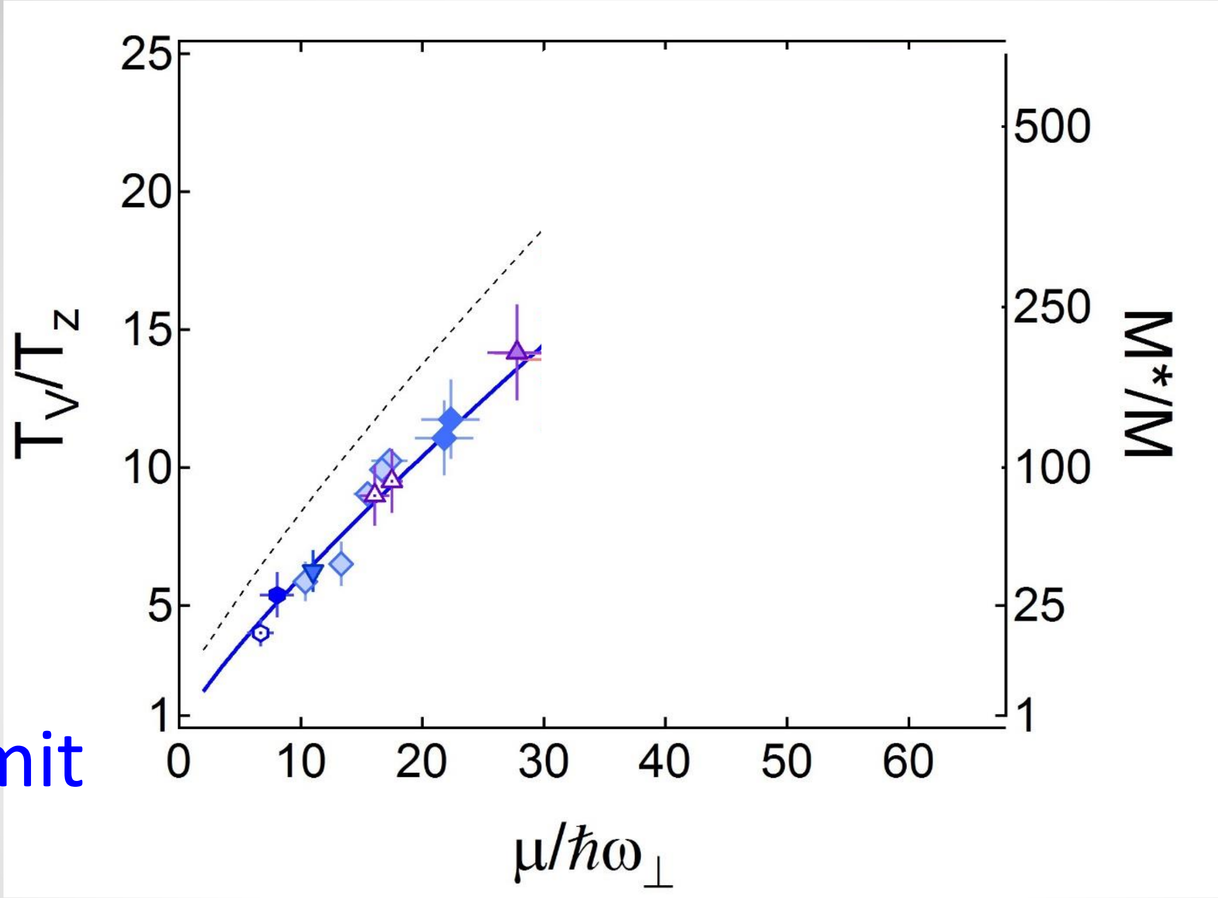
N = 3 x $10^5$

$$L = \log(R/\xi)$$

# Vortex Period in the BEC-BCS Crossover

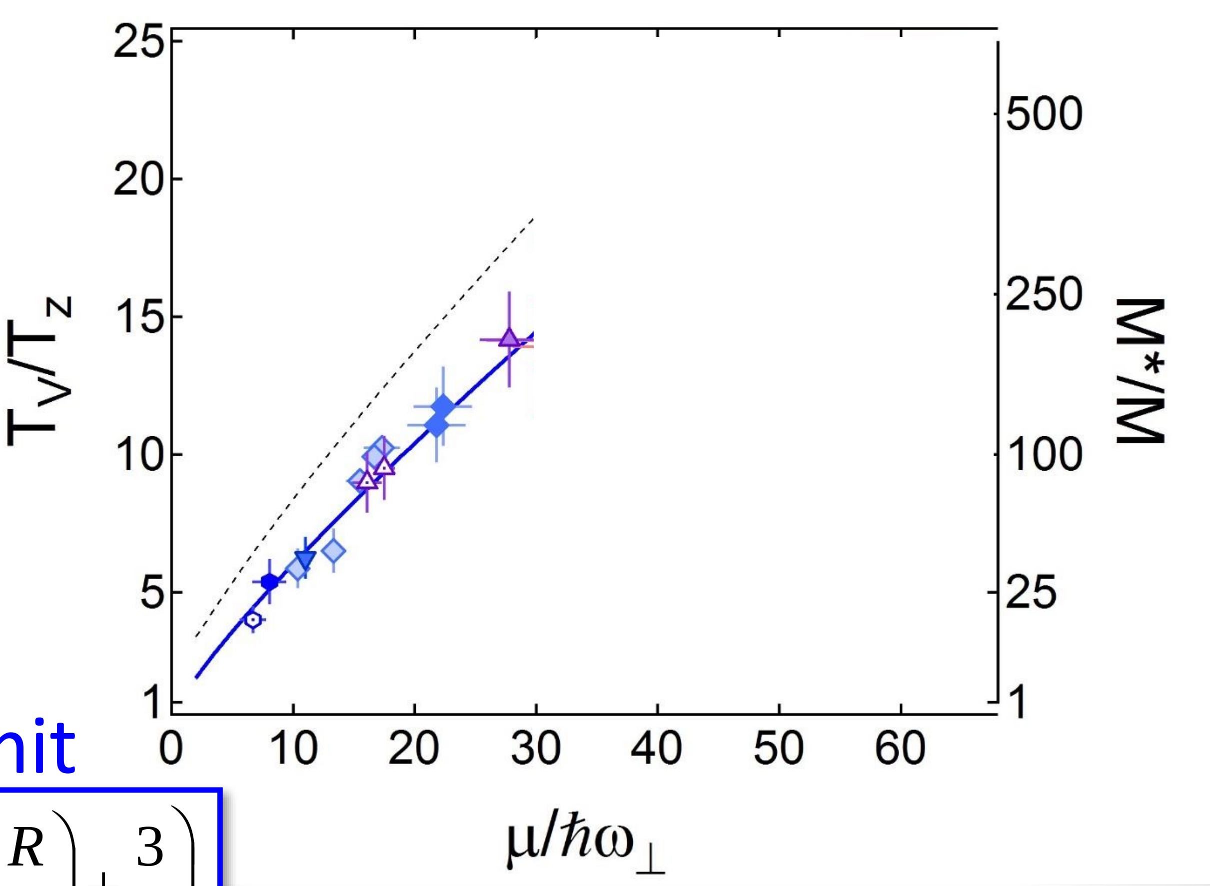


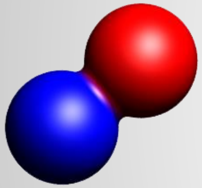
BEC limit



MJHK, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L.W. Cheuk, T. Yefsah, M. W. Zwierlein, arXiv:1402.7052 (2014), PRL, to appear

# Vortex Period in the BEC-BCS Crossover





BEC limit

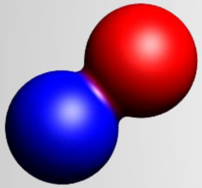
$$\boxed{\frac{8}{3}\frac{\mu}{\hbar\omega_\perp}\left(\ln\left(\frac{R}{\xi}\right)+\frac{3}{4}\right)}$$

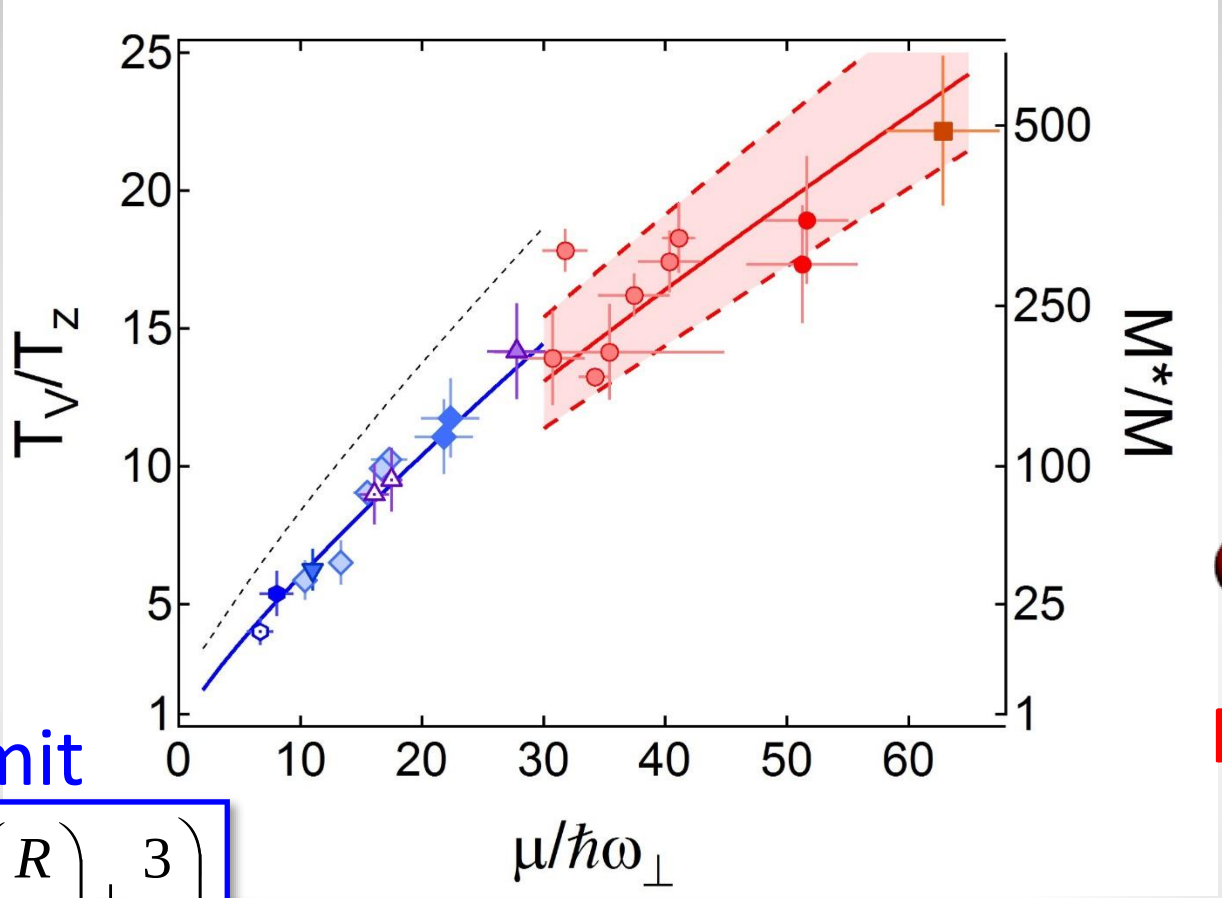
Lundh, Ao, PRA 2000
Fetter, Kim, JLTP 2001

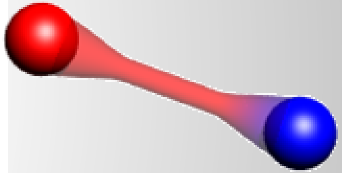
$$\frac{R}{\xi}=\frac{4\mu}{\hbar\omega_\perp}$$

MJHK, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L.W. Cheuk, T. Yefsah, M. W. Zwierlein, arXiv:1402.7052 (2014), PRL, to appear

# Vortex Period in the BEC-BCS Crossover







Resonance

BEC limit

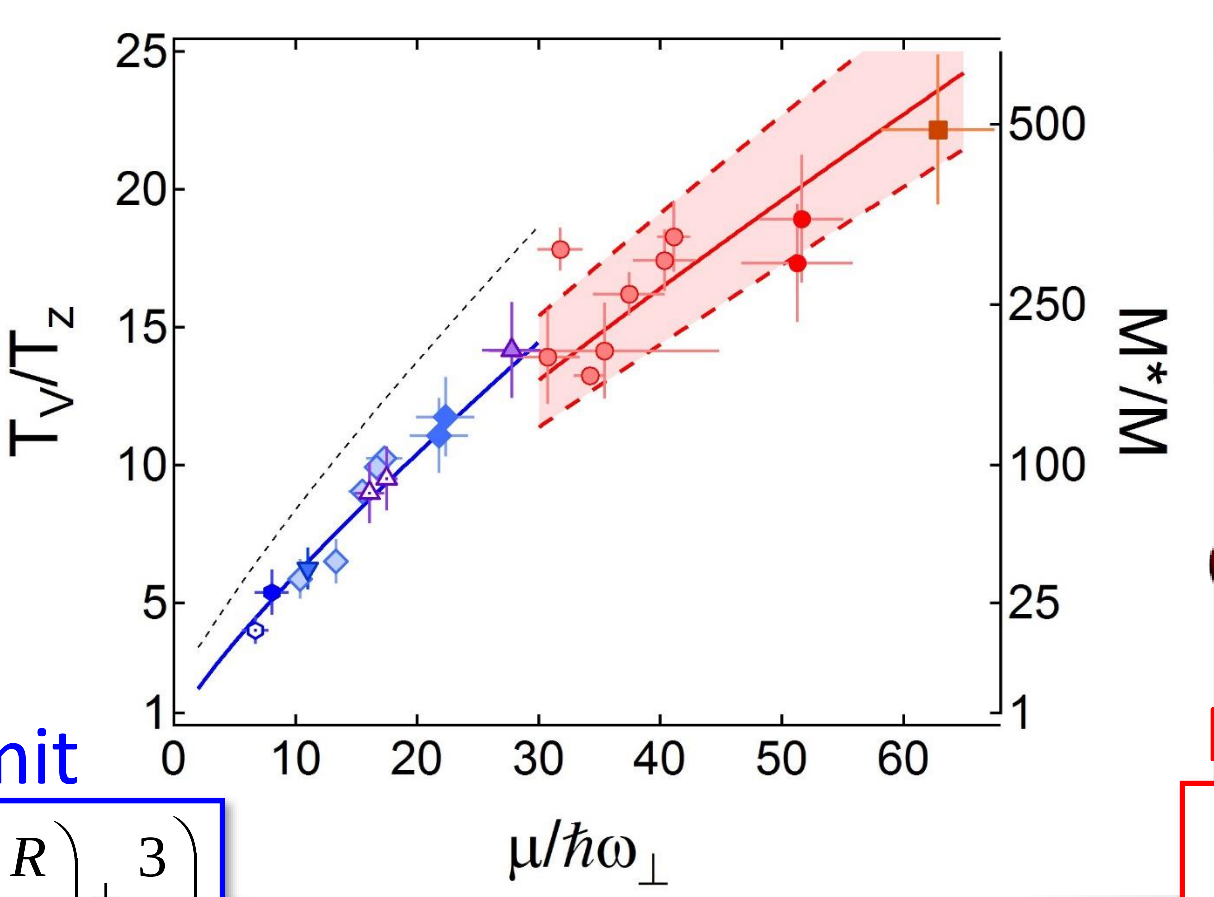
$$\boxed{\frac{8}{3}\frac{\mu}{\hbar\omega_\perp}\left(\ln\left(\frac{R}{\xi}\right)+\frac{3}{4}\right)}$$

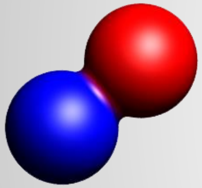
Lundh, Ao, PRA 2000
Fetter, Kim, JLTP 2001

$$\frac{R}{\xi}=\frac{4\mu}{\hbar\omega_\perp}$$

MJHK, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L.W. Cheuk, T. Yefsah, M. W. Zwierlein, arXiv:1402.7052 (2014), PRL, to appear

# Vortex Period in the BEC-BCS Crossover



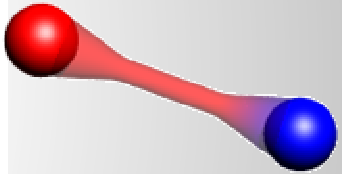


**BEC limit**

$$\boxed{\frac{8}{3}\frac{\mu}{\hbar\omega_\perp}\left(\ln\left(\frac{R}{\xi}\right)+\frac{3}{4}\right)}$$

Lundh, Ao, PRA 2000
Fetter, Kim, JLTP 2001

$$\frac{R}{\xi}=\frac{4\mu}{\hbar\omega_\perp}$$



**Resonance**

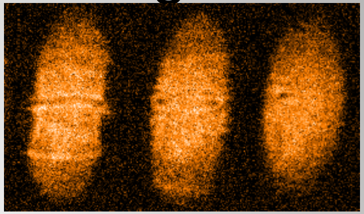
$$\boxed{2\frac{\mu}{\hbar\omega_\perp}\ln\left(\frac{R}{\xi}\right)}$$

$$\frac{R}{\xi}=0.5\ldots2\times\frac{2}{\sqrt{0.37}}\frac{\mu}{\hbar\omega_\perp}$$

MJHK, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L.W. Cheuk, T. Yefsah,
M. W. Zwierlein, arXiv:1402.7052 (2014), PRL, to appear

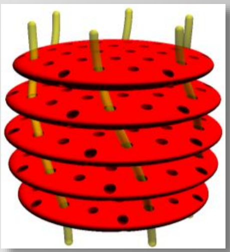
# (Some) Current Topics with Fermi Gases

- Non-equilibrium dynamics of strongly interacting Fermi gases
  Spin and “Charge” (Density) Transport
  Instability Cascade from Solitons to Vortex Rings

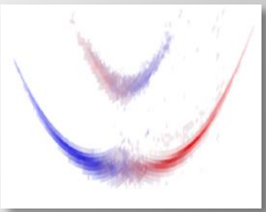


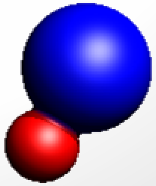
- Fermi gases in lower dimensions
  enhanced role of fluctuations - competing or enhancing superfluidity?

- Fermi gases in optical lattices
  Fermi Hubbard model, anti-ferromagnetism, 1D, 2D Fermi gases,
  single-site resolution and addressing

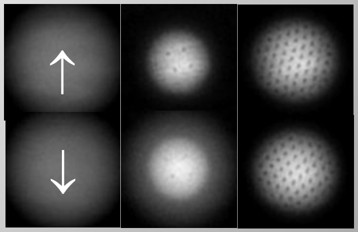


- Fermi gases in non-trivial topology
  Spin-orbit coupled Fermi gases
  topological superfluids - Haldane model…

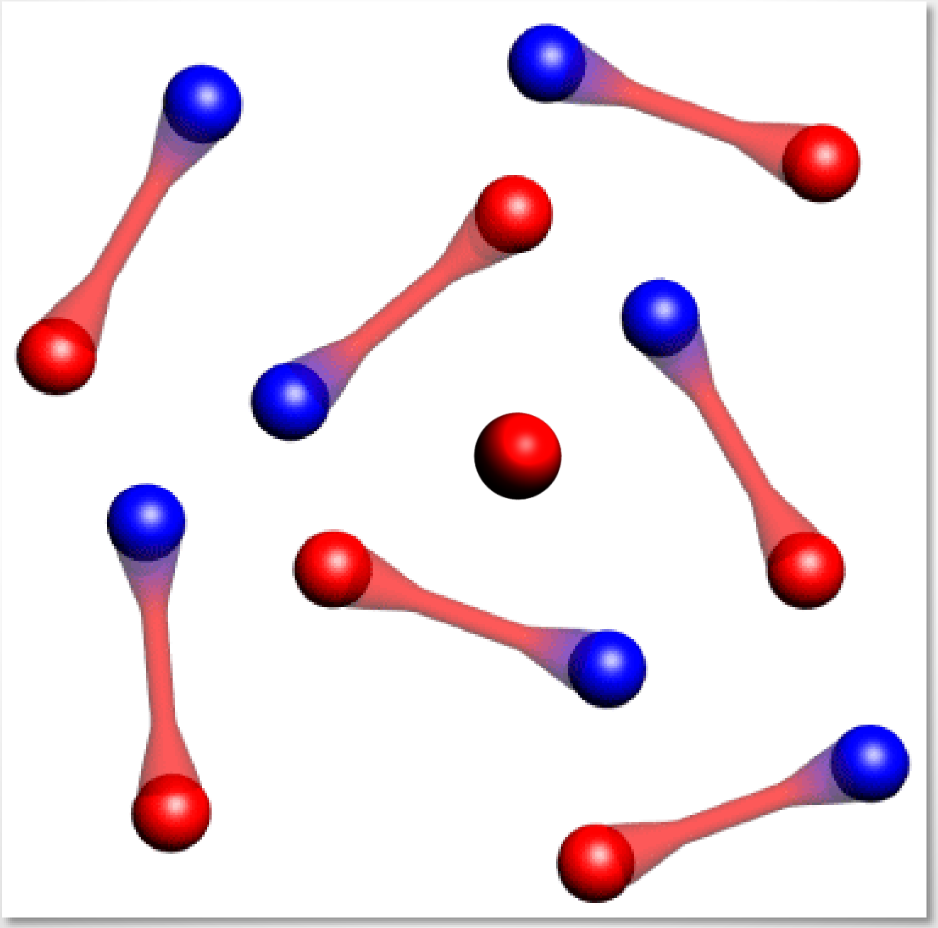




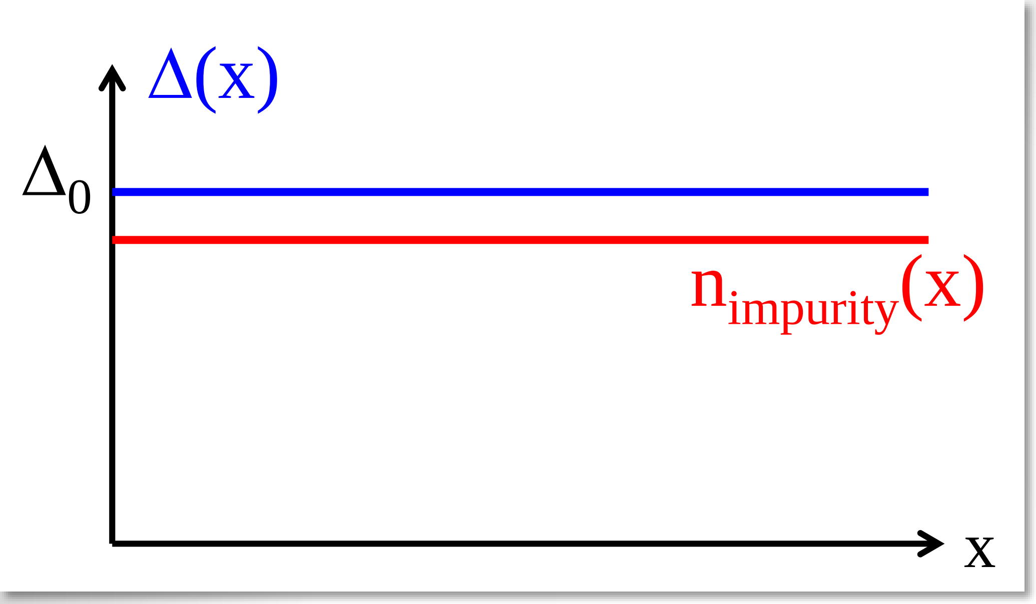
- Dipolar Fermi gases
  Magnetic dipoles (Erbium, Dysprosium)
  Dipolar molecules (KRb, NaK)
- Ground-state of spin-imbalanced Fermi gases



# One excess fermion in the superfluid

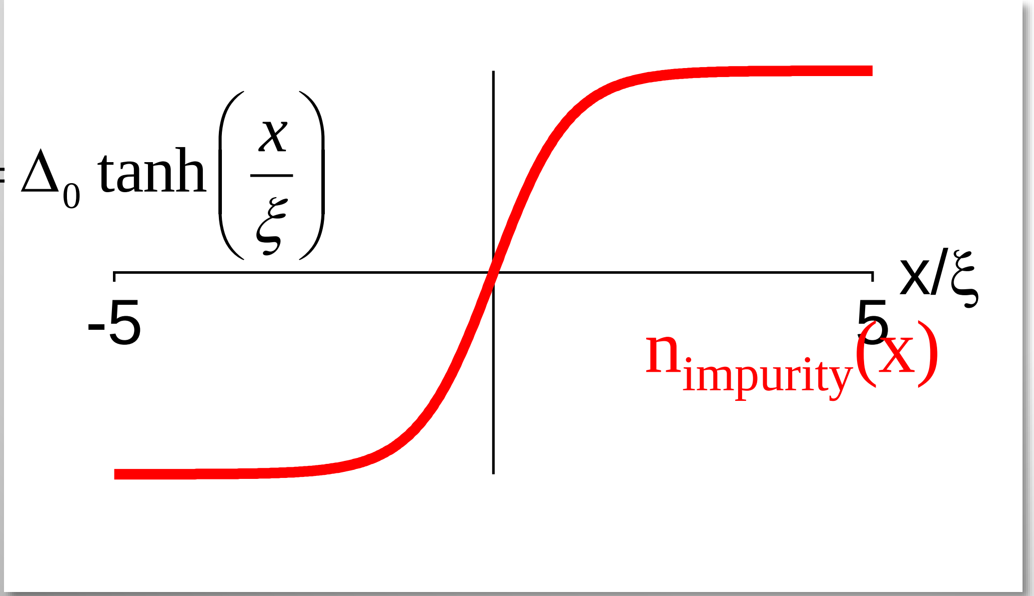


# Answer from Meanfield BCS: Solitons



Energy cost:

$$\Delta_0$$

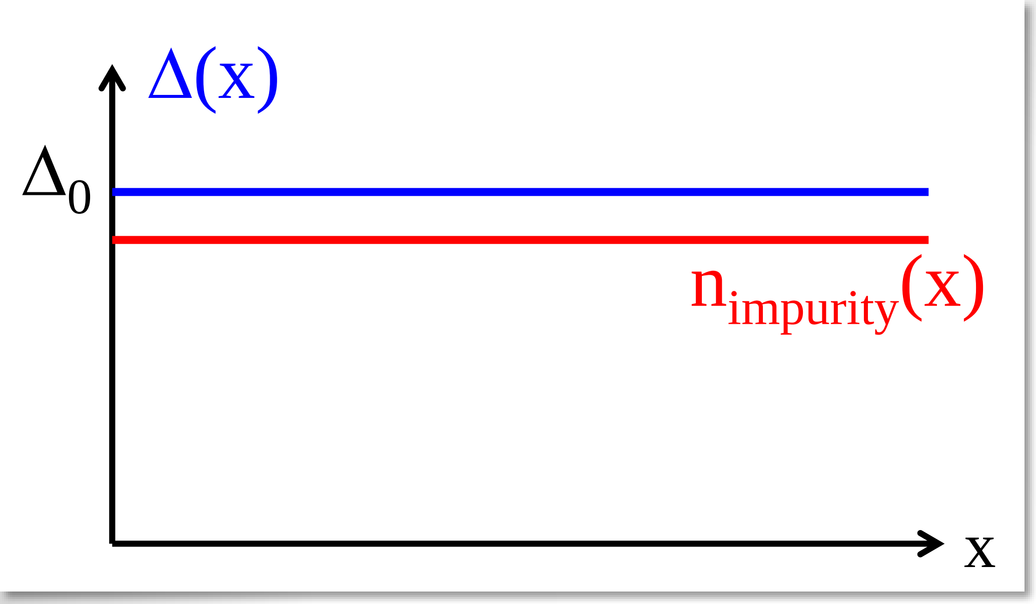


$$\Delta = \Delta_0 \tanh\left(\frac{x}{\xi}\right)$$

Energy cost:

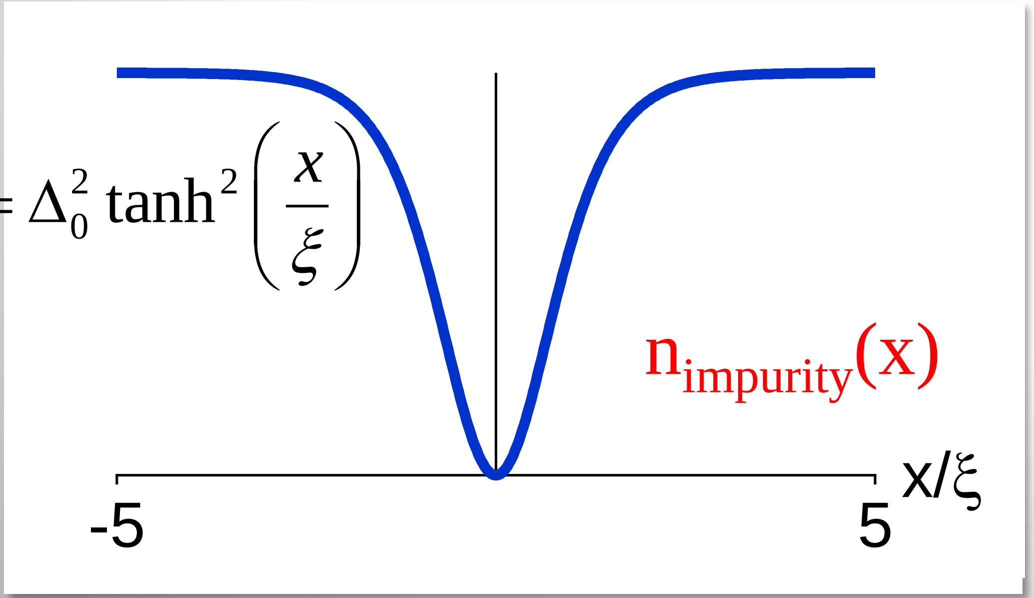
$$\xi \sim \frac{1}{k_F}\frac{E_F}{\Delta_0}$$

# Answer from Meanfield BCS: Solitons



Energy cost:

$$\Delta_0$$

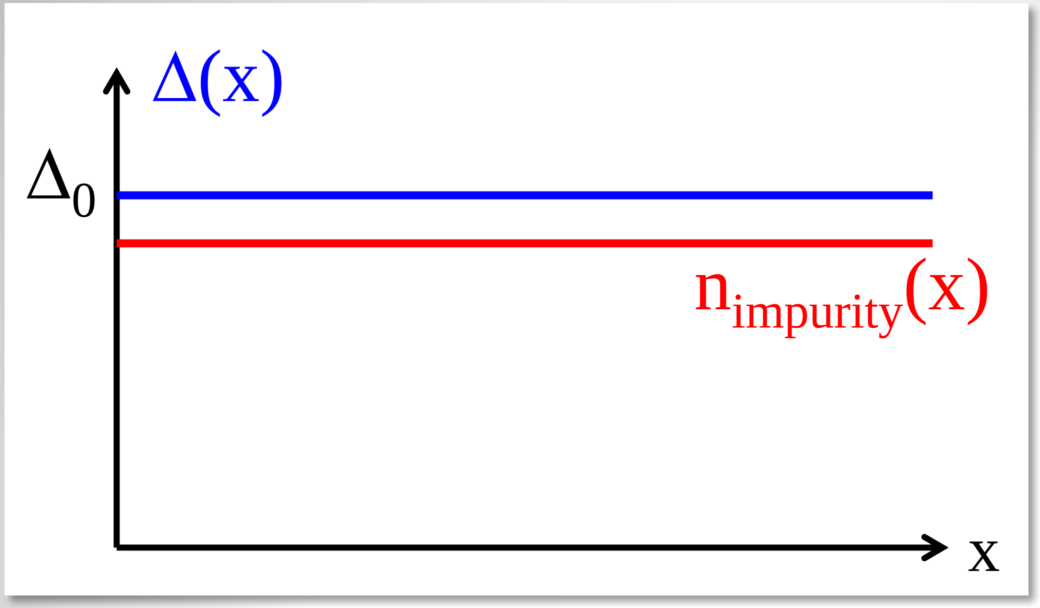


$$|\Delta|^2 = \Delta_0^2 \tanh^2\left(\frac{x}{\xi}\right)$$

Energy cost:

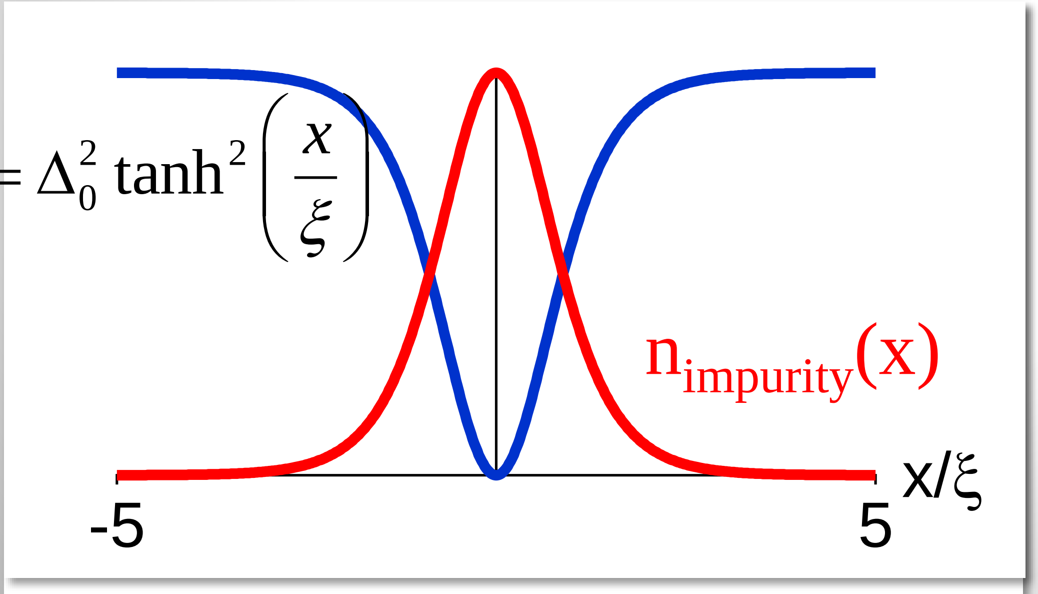
$$\xi \sim \frac{1}{k_F}\frac{E_F}{\Delta_0}$$

# Answer from Meanfield BCS: Solitons



Energy cost:

$$\Delta_0$$



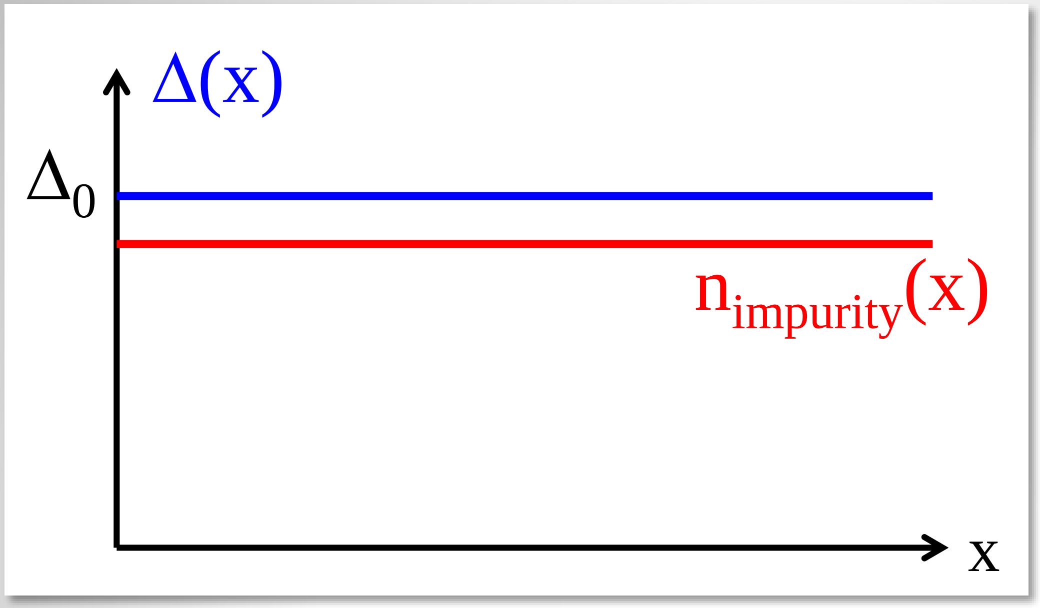
$$|\Delta|^2 = \Delta_0^2 \tanh^2\left(\frac{x}{\xi}\right)$$

Energy cost:

$$\frac{\Delta_0^2}{E_F} n_{1D}\xi \sim \quad \Delta_0$$

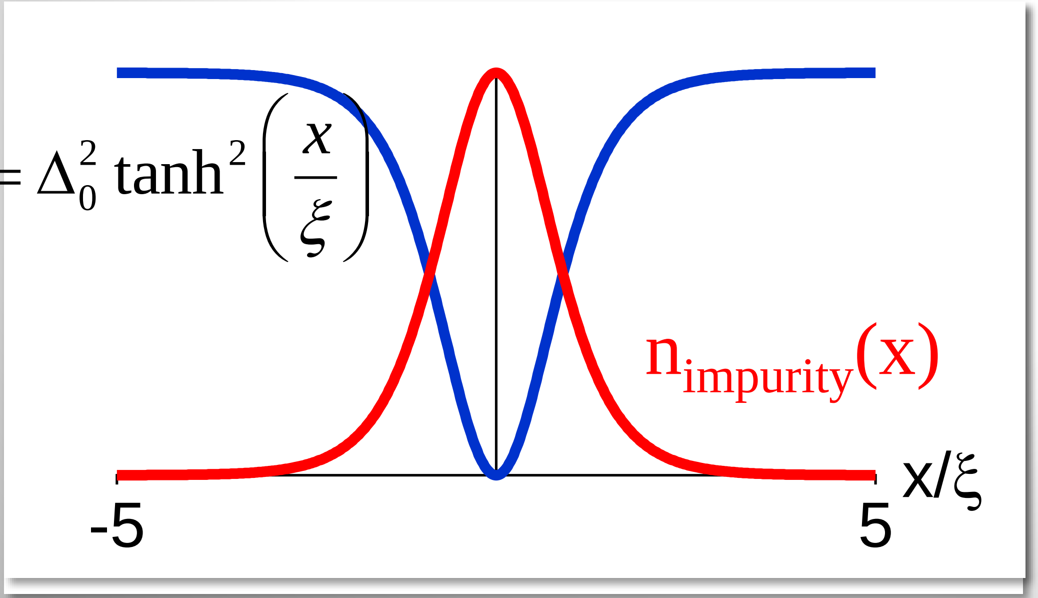
$$\xi \sim \frac{1}{k_F}\frac{E_F}{\Delta_0}$$

# Answer from Meanfield BCS: Solitons



Energy cost:

$$\Delta_0$$



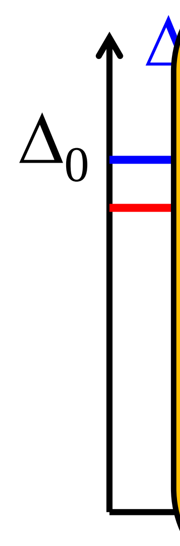
$$\left|\Delta\right|^2 = \Delta_0^2 \tanh^2\left(\frac{x}{\xi}\right)$$

Energy cost:

$$\frac{\Delta_0^2}{E_F} n_{1D}\xi \sim \frac{2}{\pi}\Delta_0$$

$$\xi \sim \frac{1}{k_F}\frac{E_F}{\Delta_0}$$

# Answer from Meanfield BCS: Solitons



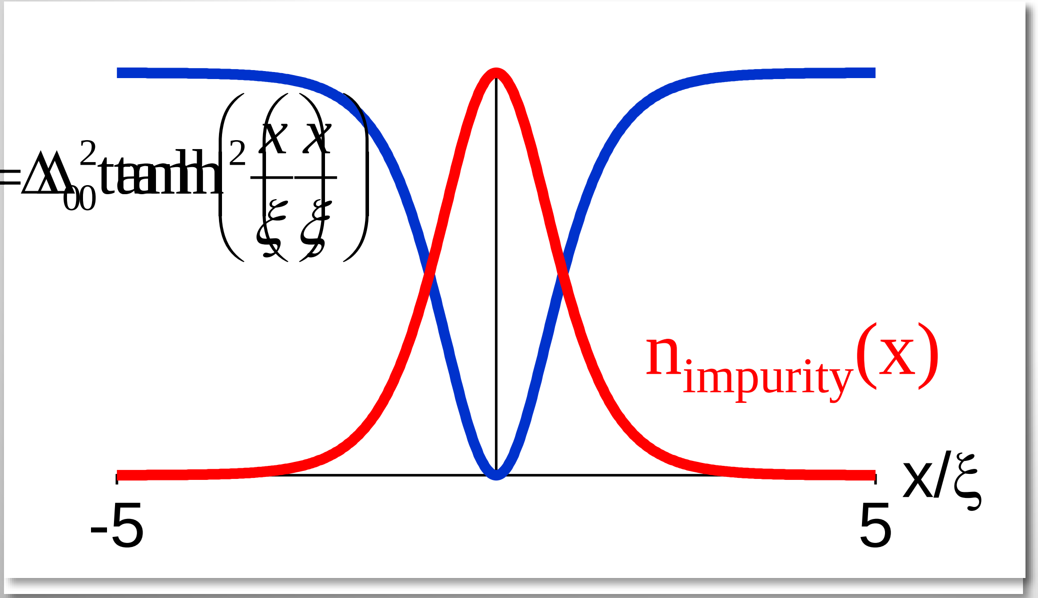
cost:

**Note:**

$$\frac{2}{\pi} = 0.637\ldots < \frac{1}{\sqrt{2}} = 0.707\ldots$$

Energy cost < Clogston limit

Solitonic state is better than phase separated state



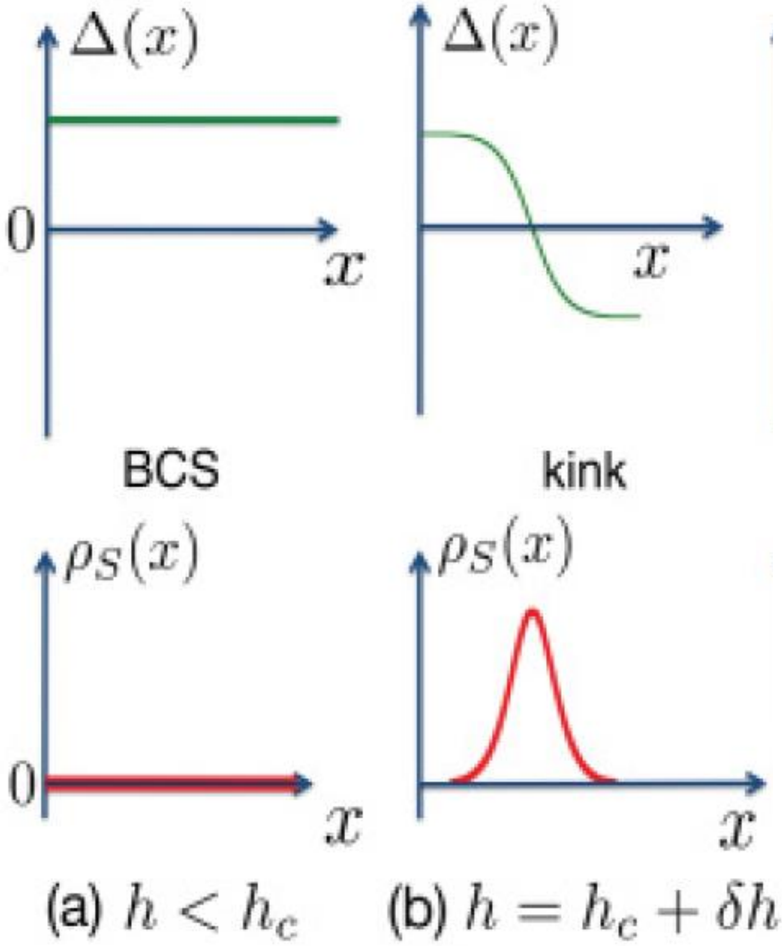
$$|\Delta|^2 = \Delta_0^2 \tanh^2\left(\frac{x}{\xi}\right)$$

Energy cost:

$$\frac{\Delta_0^2}{E_F} n_{1D}\xi \sim \frac{2}{\pi}\Delta_0$$

$$\xi \sim \frac{1}{k_F}\frac{E_F}{\Delta_0}$$

# Solitons as one limit of the FFLO state

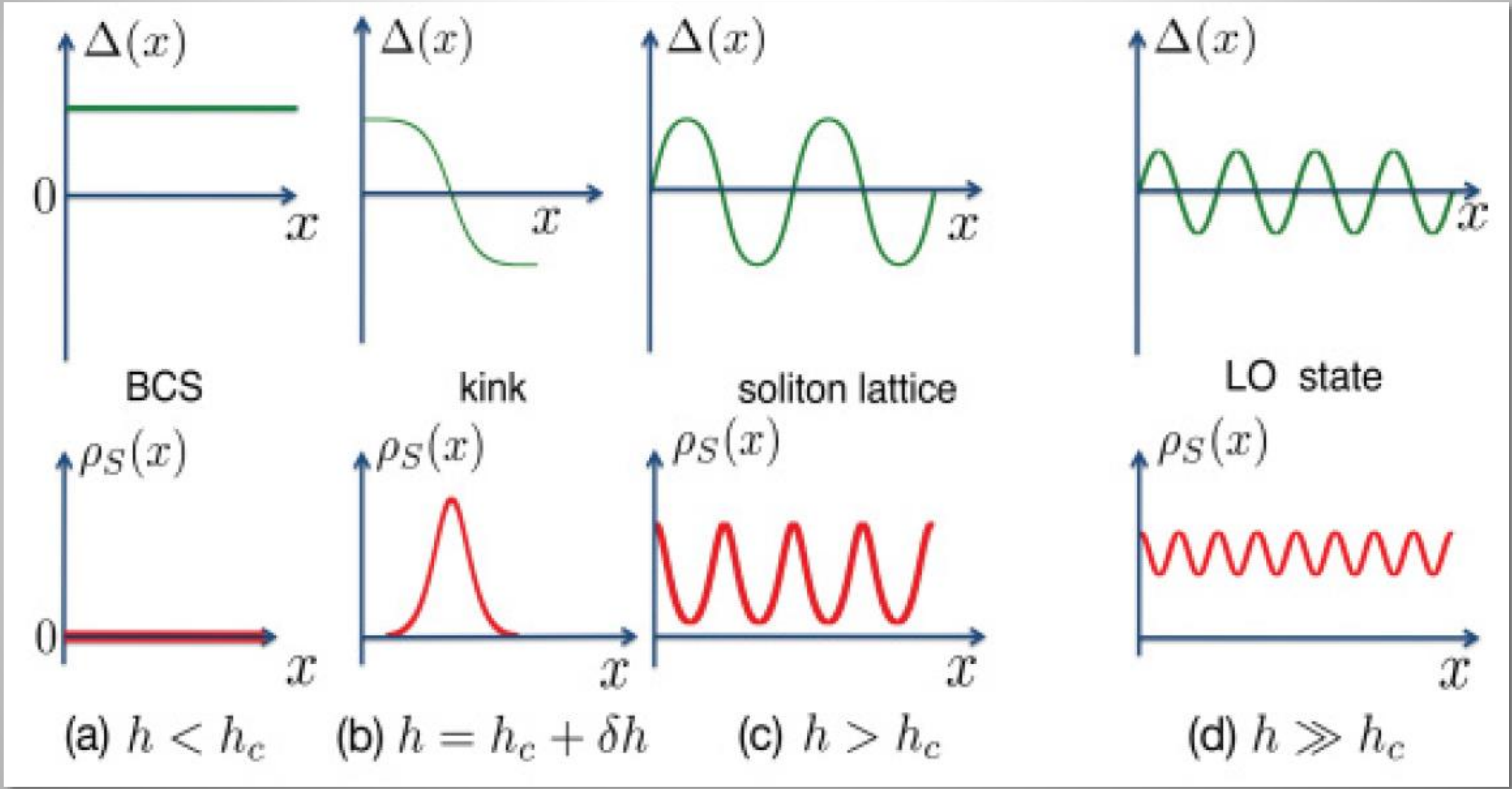


(a) $h < h_c$ (b) $h = h_c + \delta h$

from Lutchyn, Dzero, Yakovenko, PRA 84, 033609 (2011)

See Yoshida, Yip, PRA 75, 063601 (2007), Radzihovsky, PRA 84, 023611 (2011)

# Solitons as one limit of the FFLO state

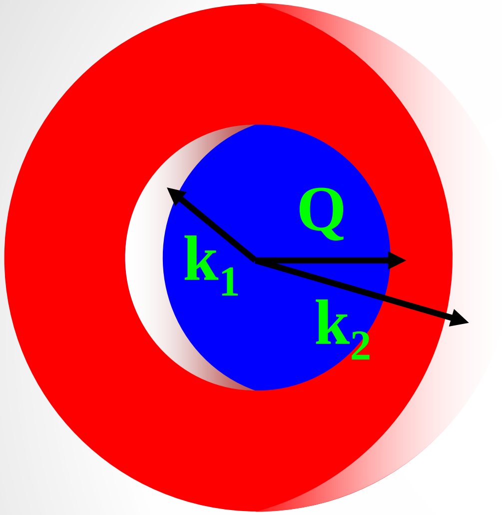


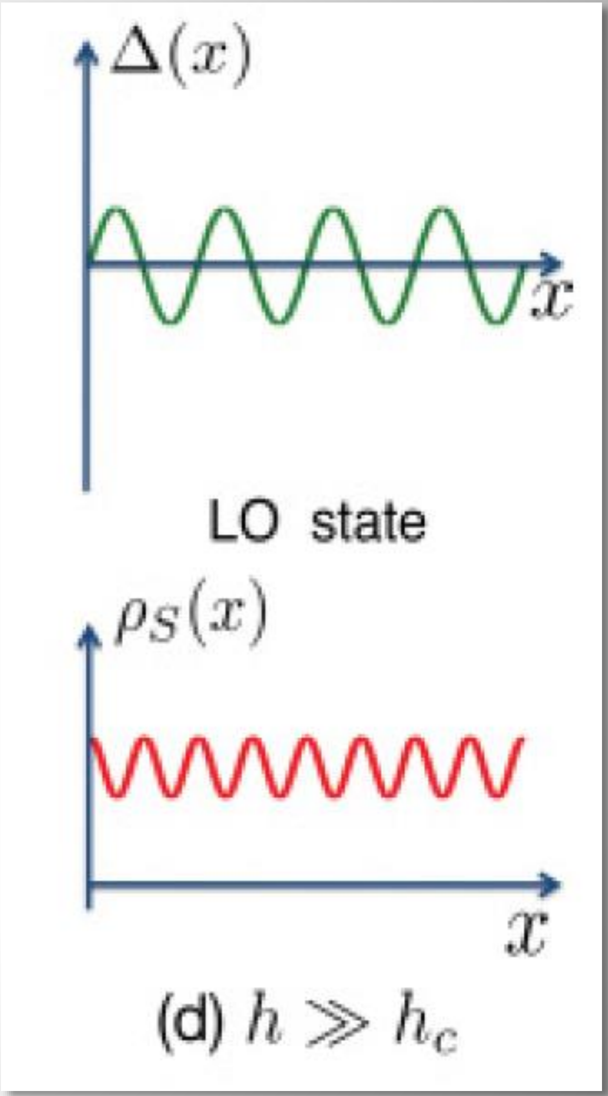
from Lutchyn, Dzero, Yakovenko, PRA 84, 033609 (2011)

See Yoshida, Yip, PRA 75, 063601 (2007), Radzihovsky, PRA 84, 023611 (2011)

# Fulde-Ferrell-Larkin-Ovchinnikov State

*Cooper pairs with non-zero momentum*



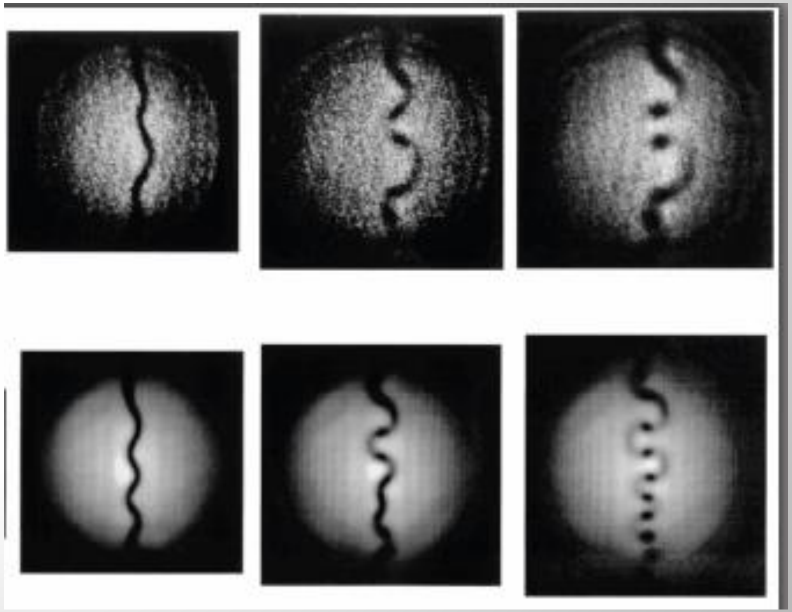


FF $$\Delta(x) = \Delta_0 e^{ikx}$$

LO $$\Delta(x) = \Delta_0 \sin(kx)$$

*A. I. Larkin, Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz.* ***47****, 1136 (1964)*
*P. Fulde, R. A. Ferrell, Phys. Rev. 135, A550 (1964)*

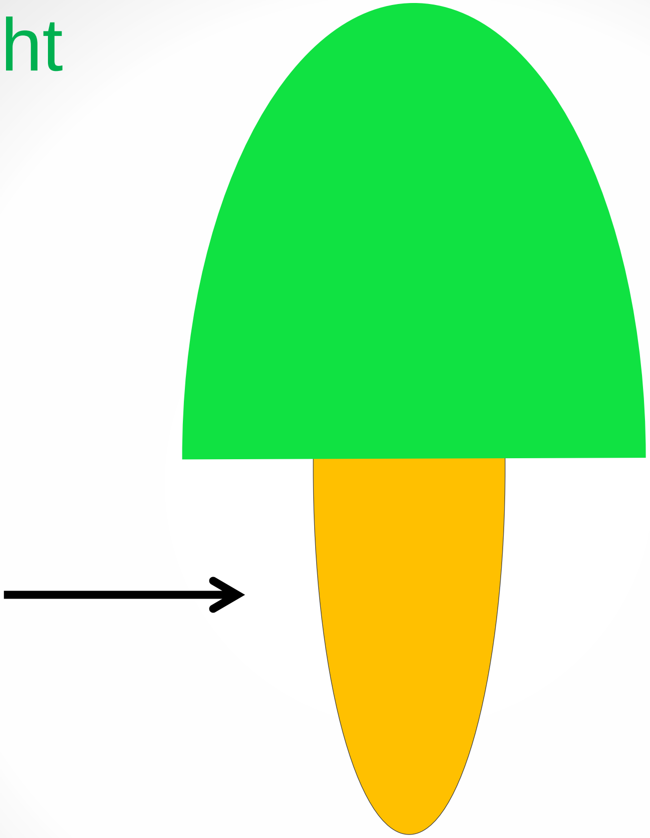
# Solitons



Coupled pendulum, Alex Kasman, Charleston

Optics, Tikhonenko et al., 1996

# Making Solitons by phase imprinting



Pulse
off-resonant light

superfluid

After the pulse:

$$\Delta\varphi = \frac{2Ut}{\hbar}$$

Needs to be fast enough: $t < \frac{\hbar}{\mu} \sim 100\,\mu s$

Solitons in BECs by phase imprinting: Hannover, Hamburg, NIST,...

# Solitons in 3D are unstable

Komineas, Papanicolaou,
PRA 68, 043617 (2003)

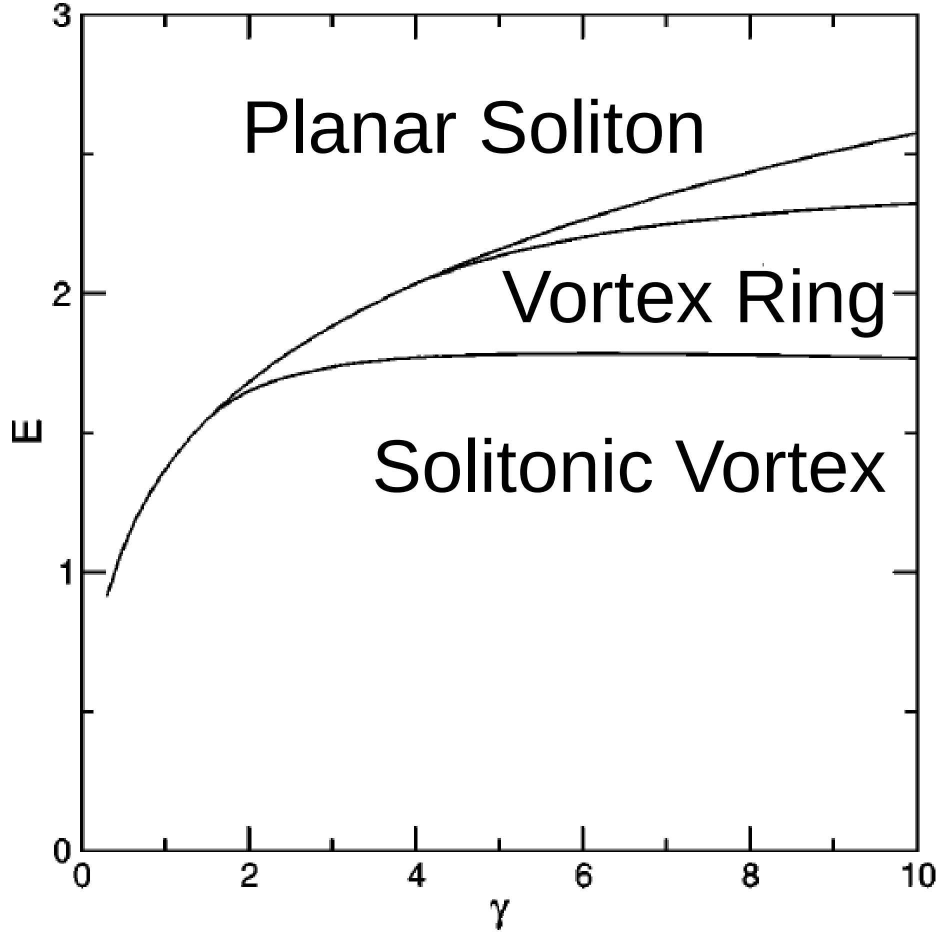
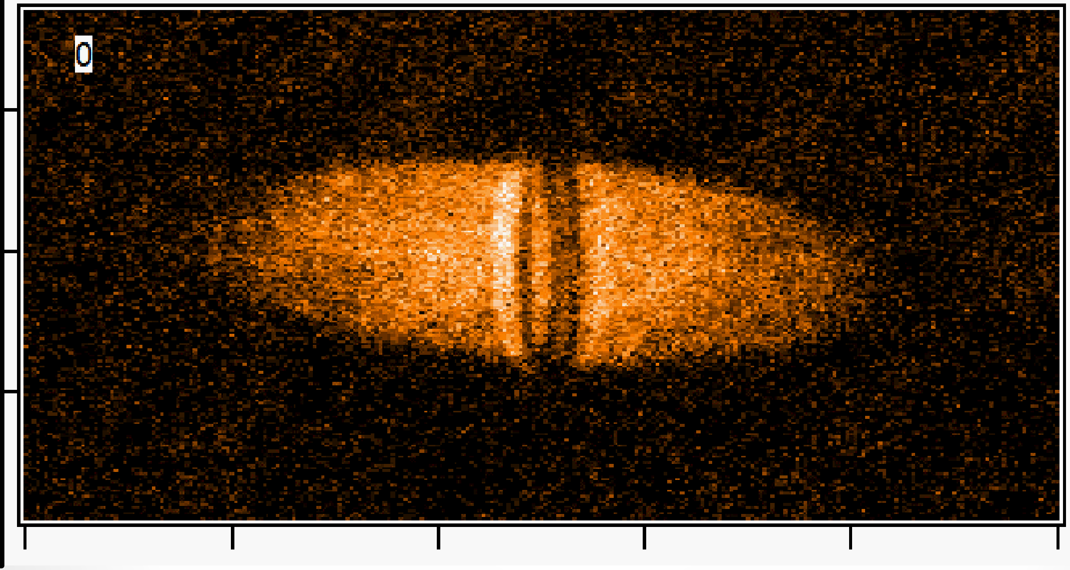


FIG. 1. Excitation energy $E$ in units of $\gamma_\perp(\hbar\omega_\perp)$ as a function of the dimensionless coupling constant $\gamma$, for static solitary waves such as a soliton (S), a solitonic vortex (SV), and a vortex ring (VR). Bifurcations occur at the two critical couplings $\gamma_0 = 1.5$ and $\gamma_1 = 4$.

# Instability Cascade of Solitary Waves in a Unitary Fermi Gas



Planar soliton →

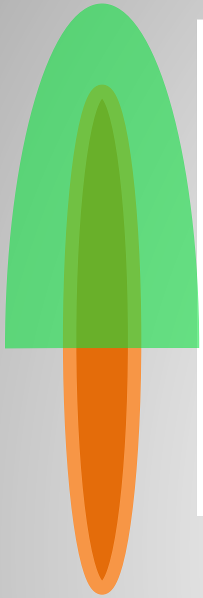
vortex ring →

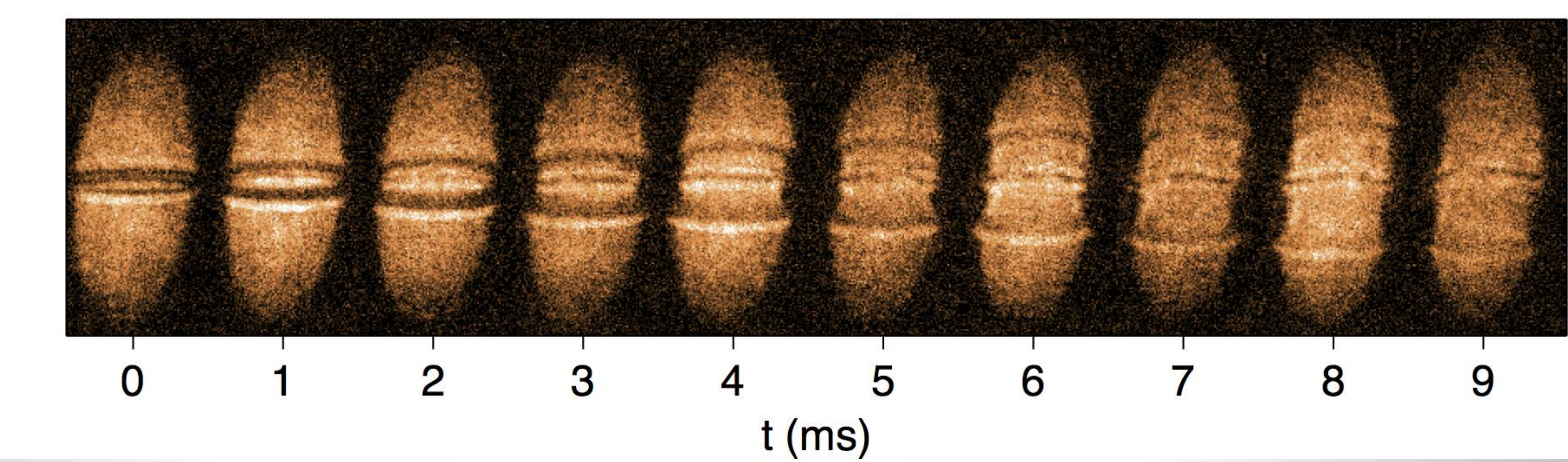
vortex / anti-vortex pair →

solitonic vortex

# Instability Cascade of Solitary Waves in Unitary Fermi Gas

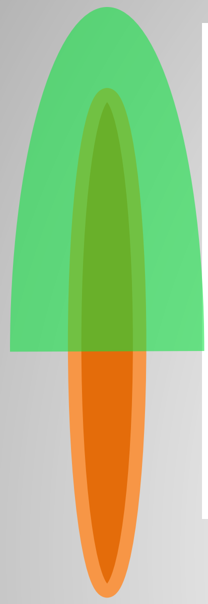
Early time dynamics after imprint (central slice)

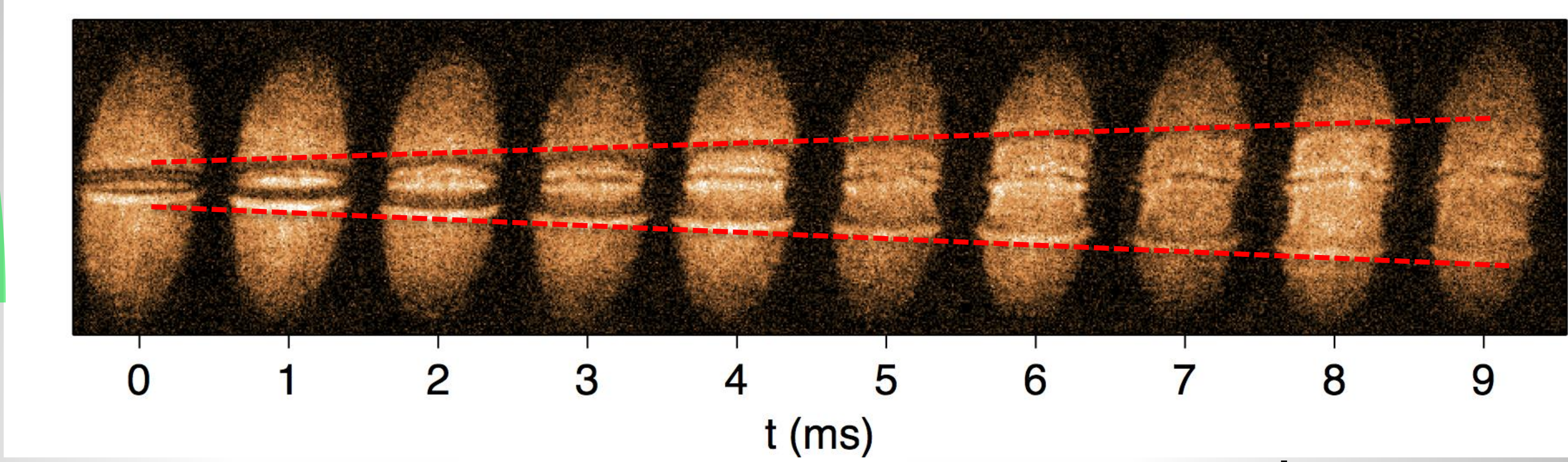




# Instability Cascade of Solitary Waves in Unitary Fermi Gas

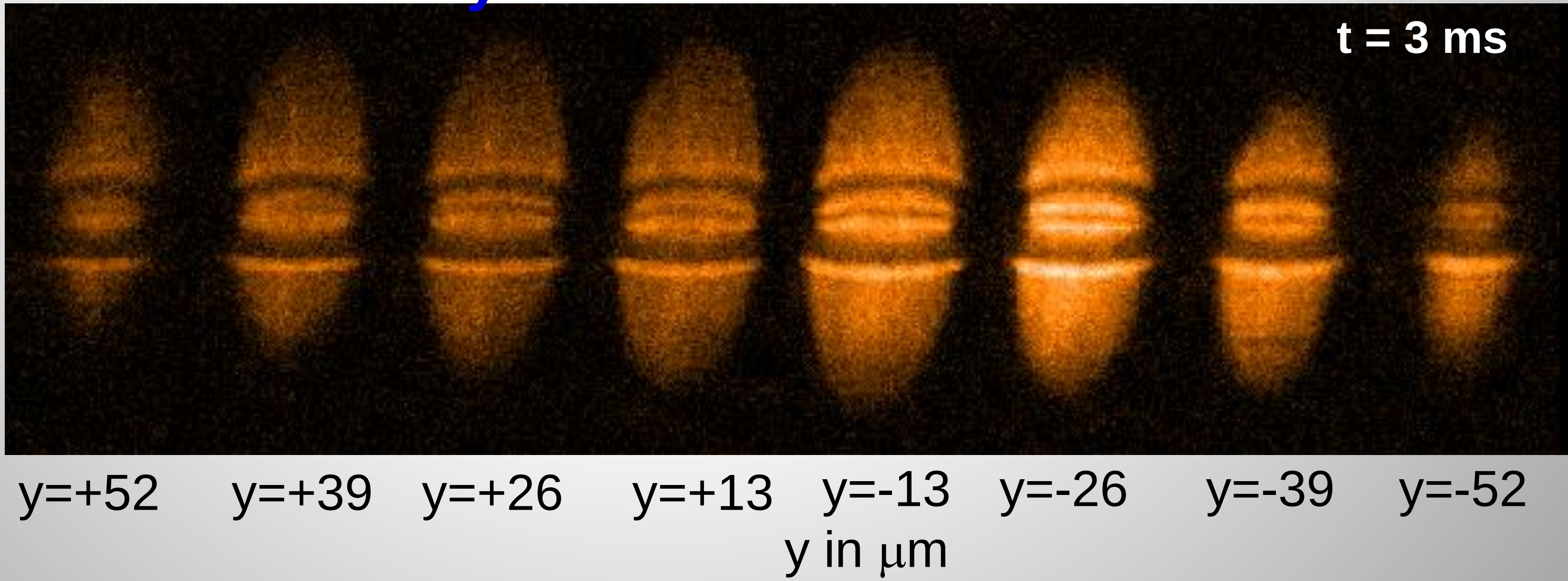
Early time dynamics after imprint (central slice)





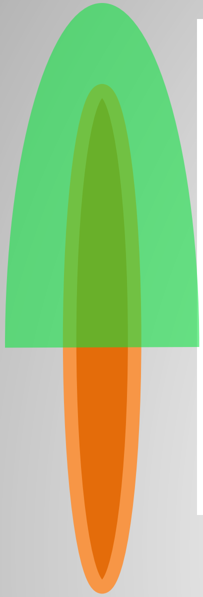
**2 shock wave fronts +**
**1 slow solitary wave**

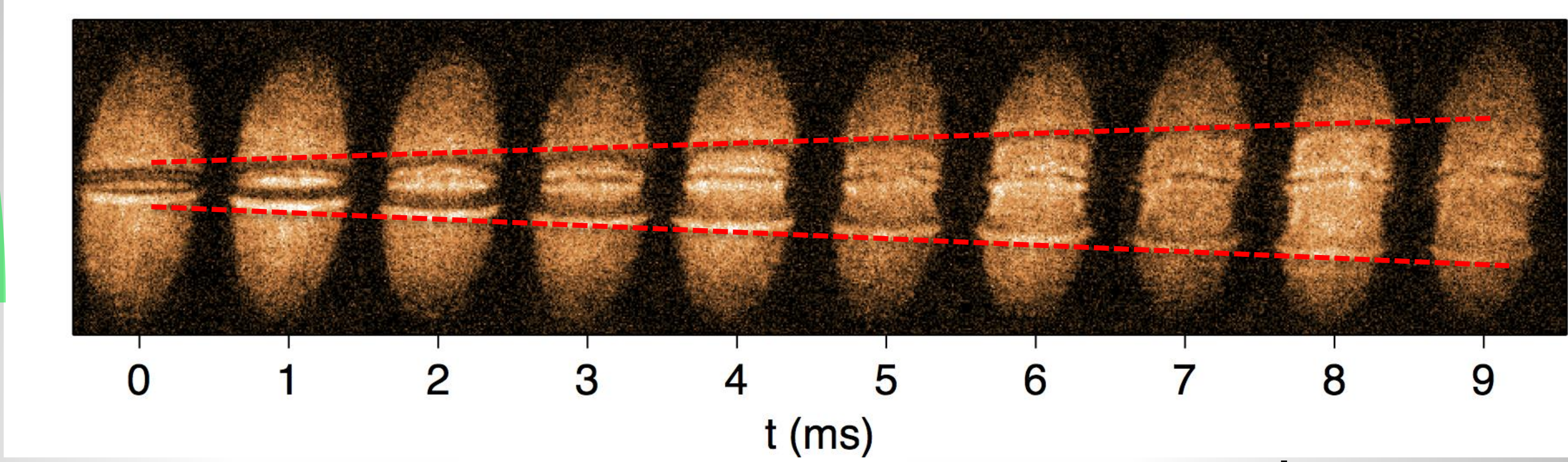
Tomography:
Planar Soliton



# Instability Cascade of Solitary Waves in Unitary Fermi Gas

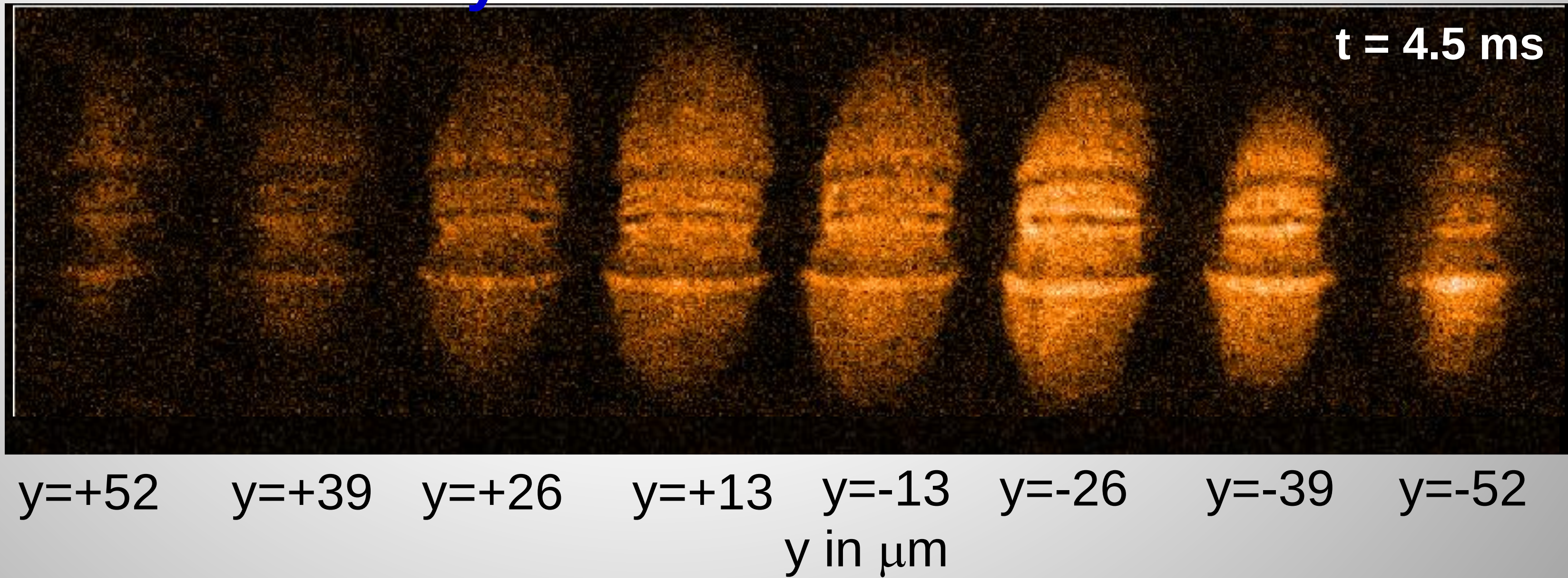
Early time dynamics after imprint (central slice)





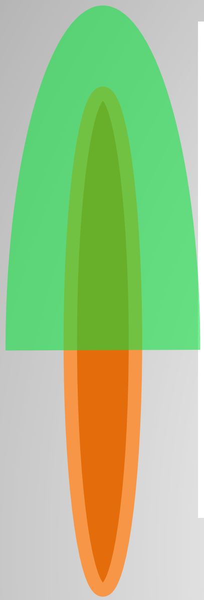
**2 shock wave fronts +**
**1 slow solitary wave**

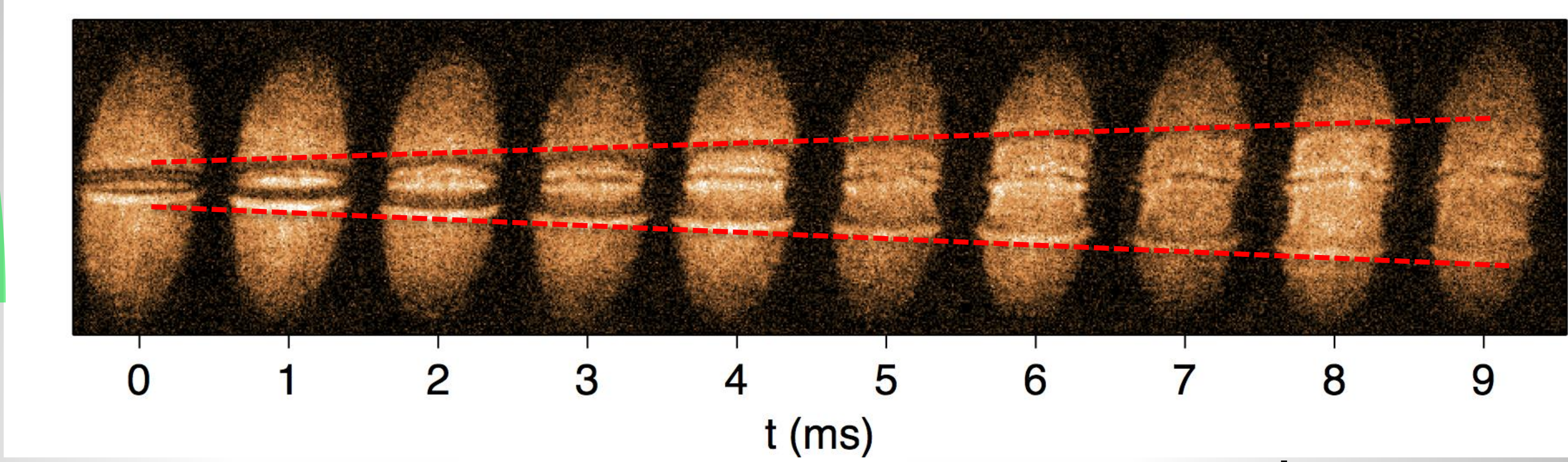
Tomography:
Planar Soliton



# Instability Cascade of Solitary Waves in Unitary Fermi Gas

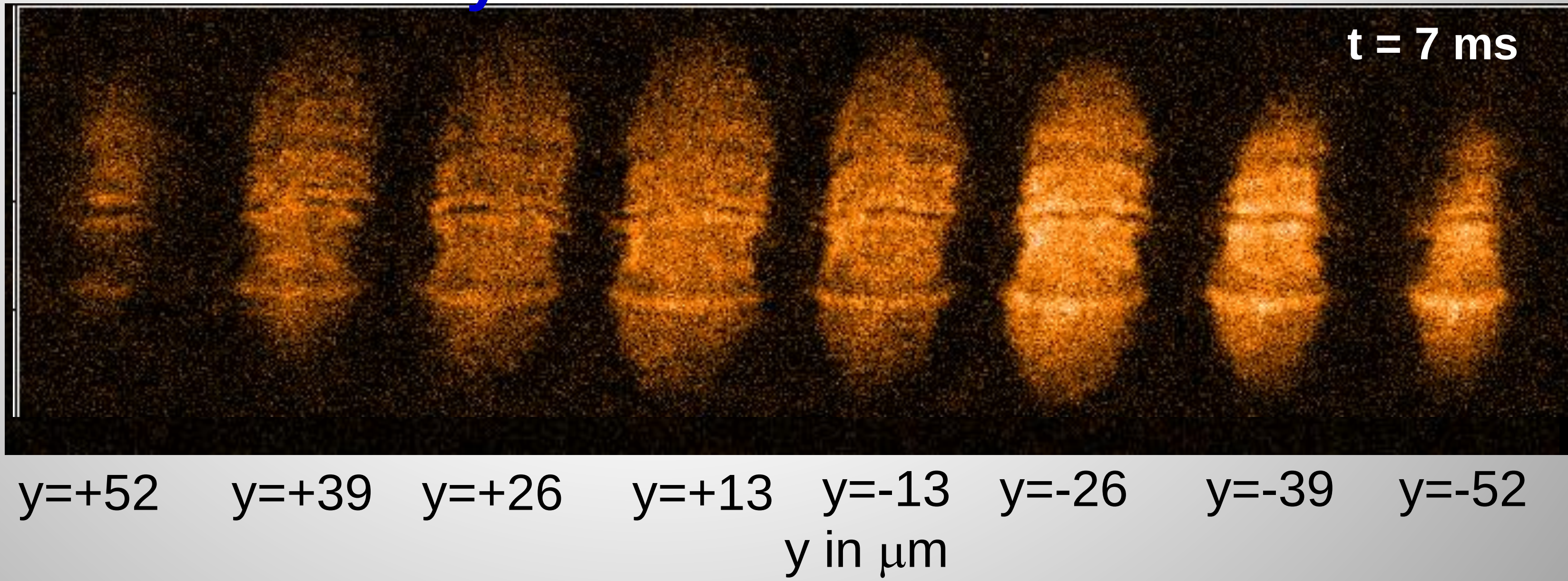
Early time dynamics after imprint (central slice)





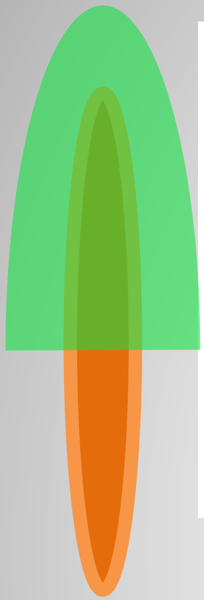
**2 shock wave fronts +**
**1 slow solitary wave**

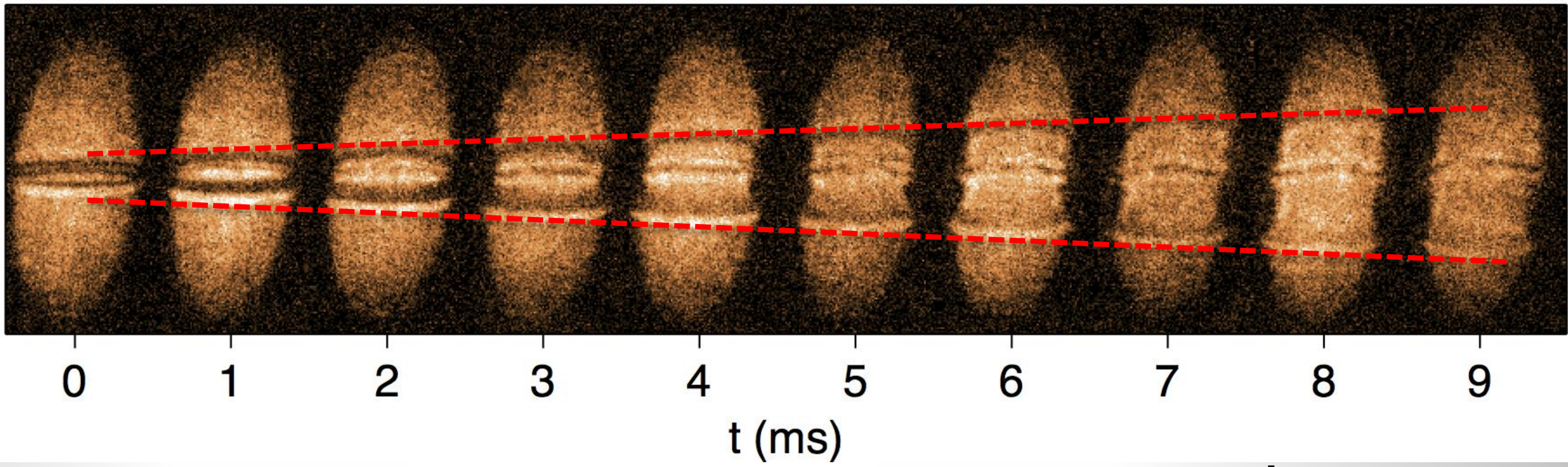
Tomography:
Planar Soliton



# Instability Cascade of Solitary Waves in Unitary Fermi Gas

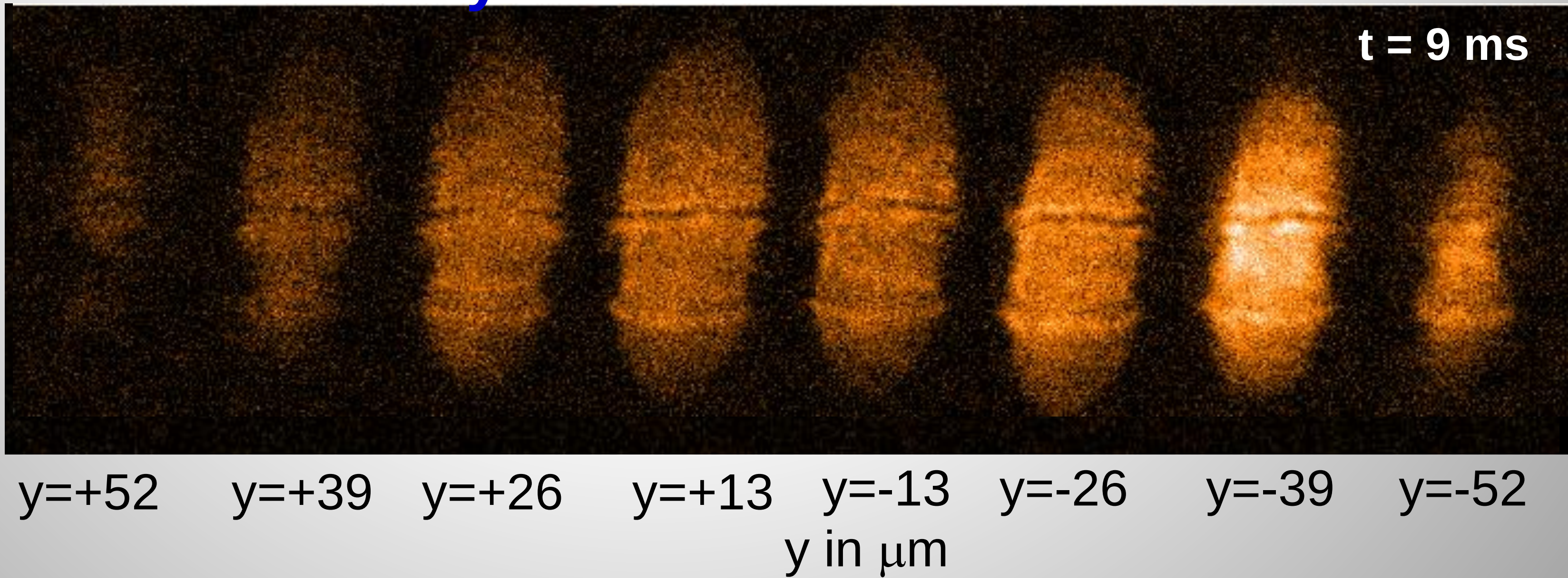
Early time dynamics after imprint (central slice)



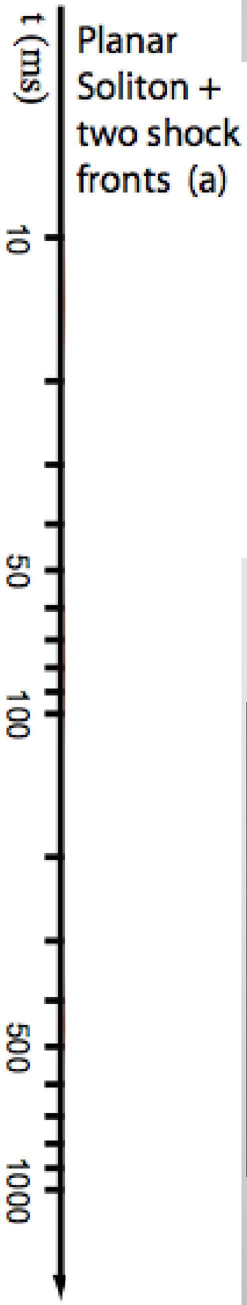


**2 shock wave fronts +**
**1 slow solitary wave**

Tomography:
Planar Soliton

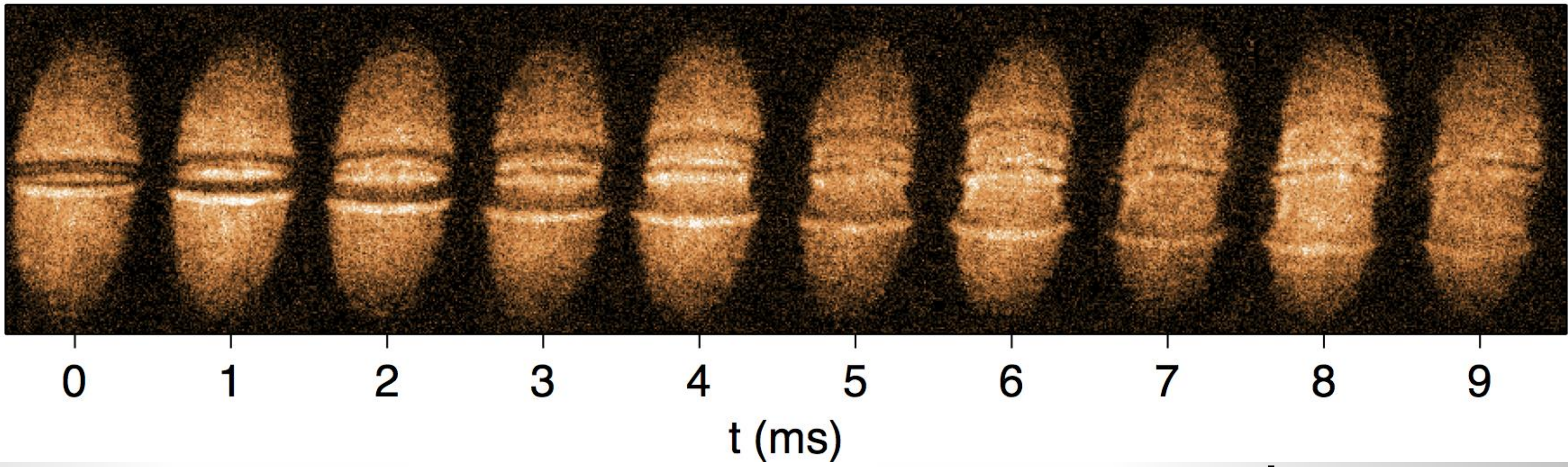


# Instability Cascade of Solitary Waves in Unitary Fermi Gas



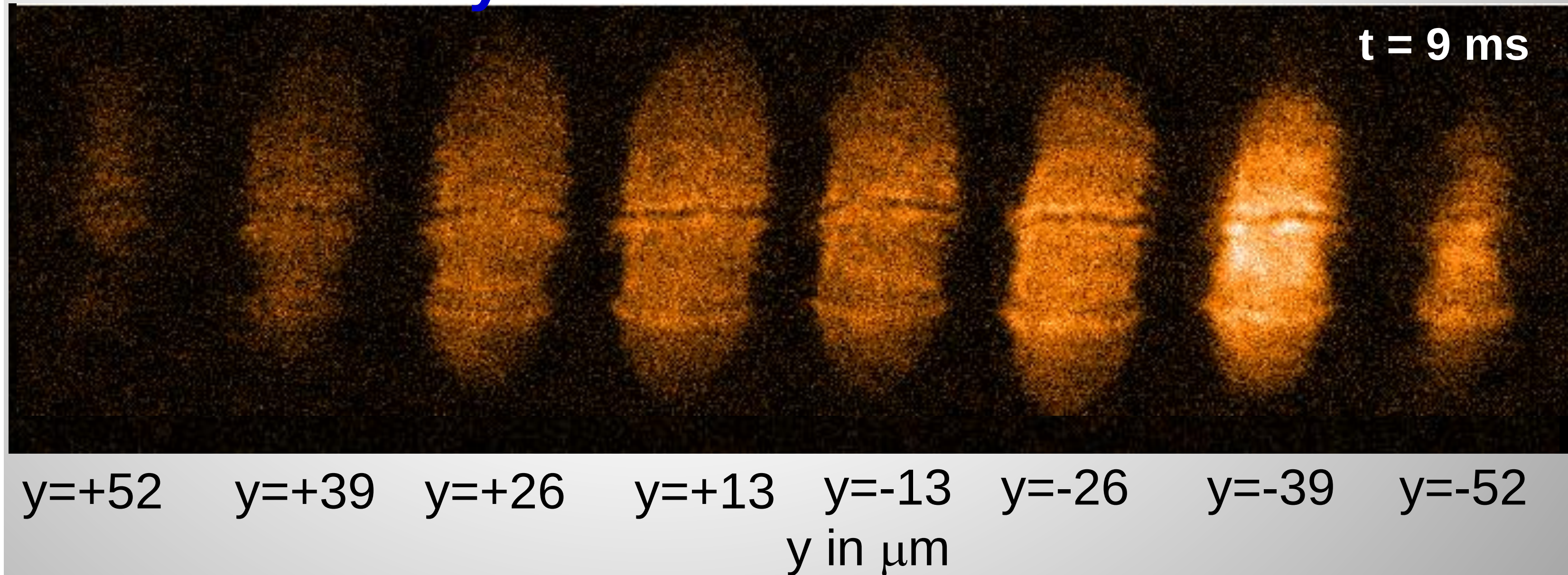
Planar Soliton + two shock fronts (a)

## Early time dynamics after imprint (central slice)

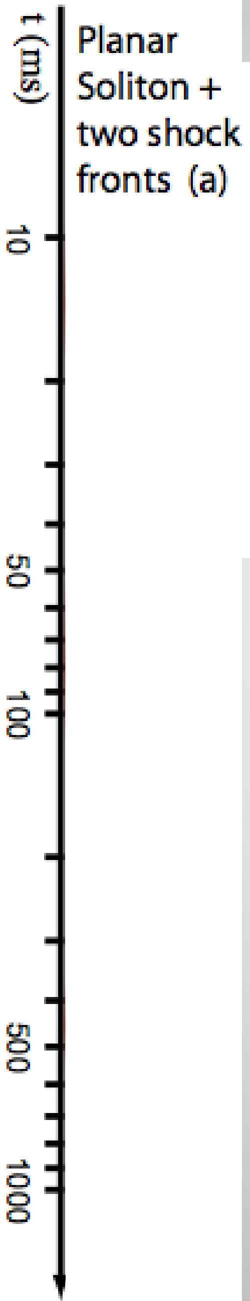


**2 shock wave fronts + 1 slow solitary wave**

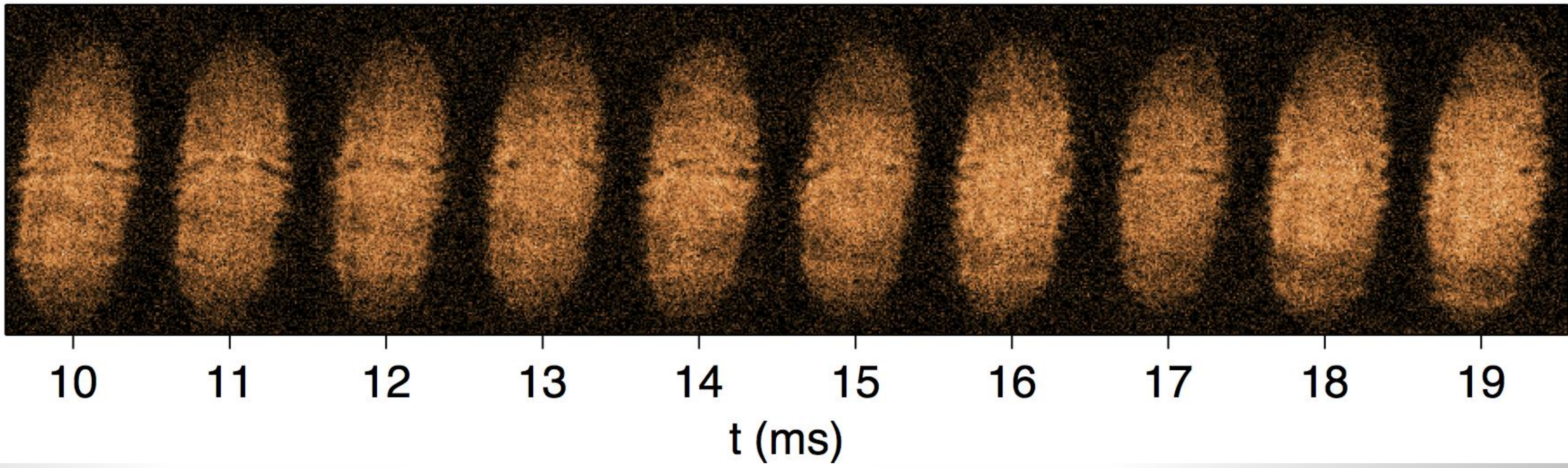
Tomography:
Planar Soliton



# Instability Cascade of Solitary Waves in Unitary Fermi Gas

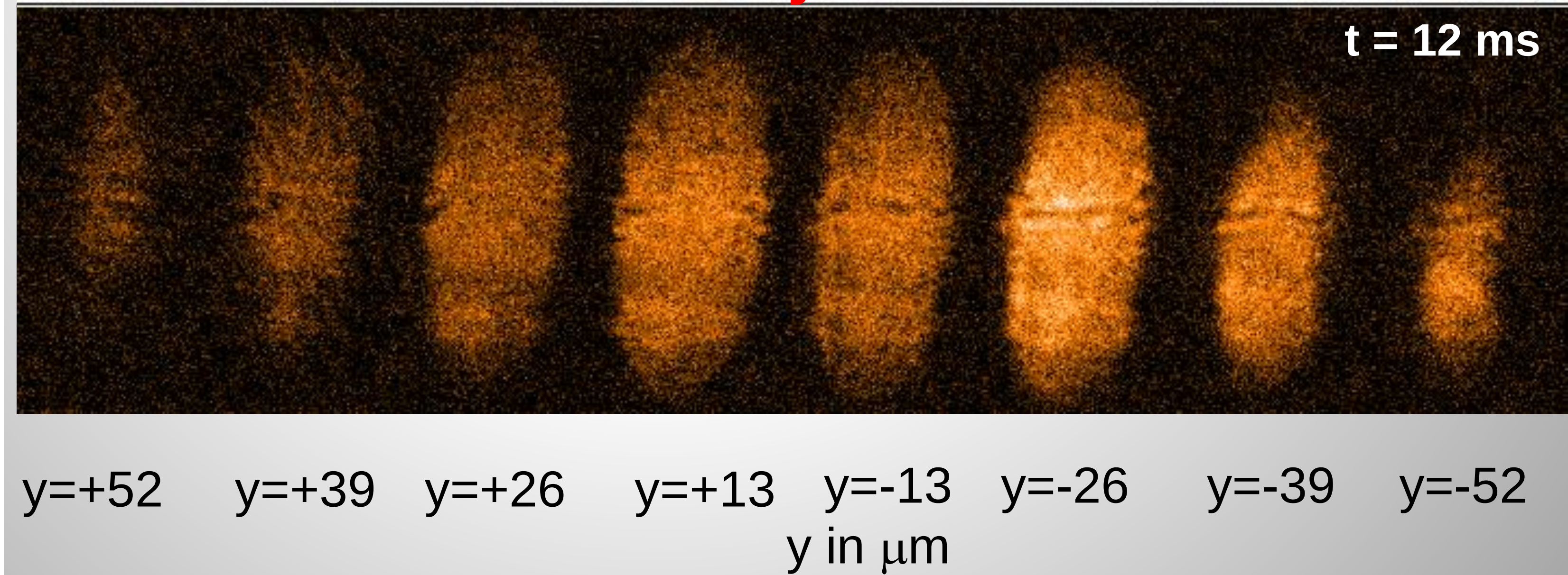


Early time dynamics after imprint (central slice)

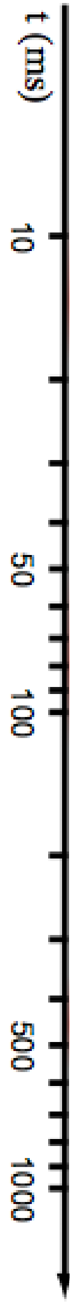


**Bending & breaking of planar soliton – snake instability**

Tomography:

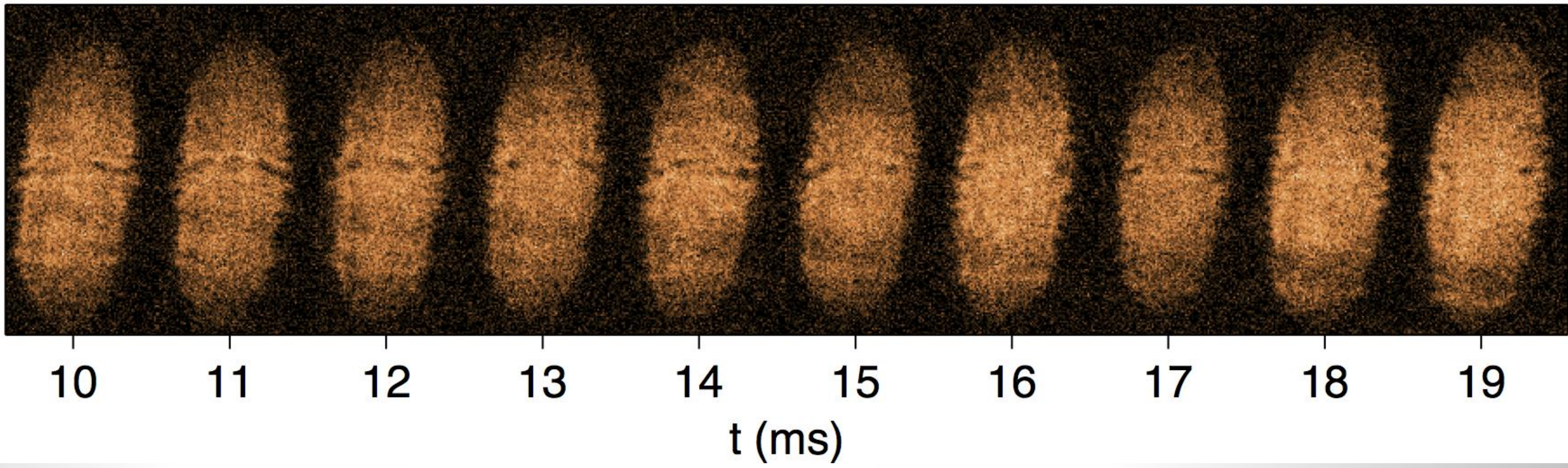


# Instability Cascade of Solitary Waves in Unitary Fermi Gas



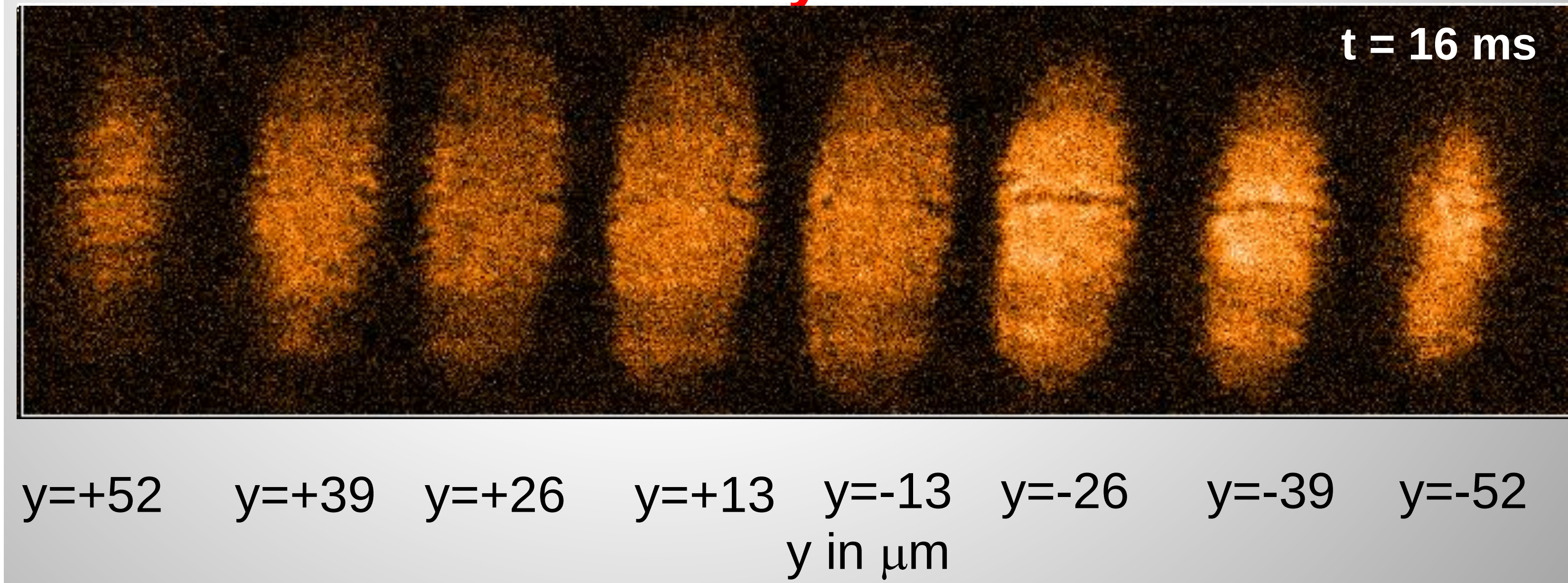
Planar Soliton + two shock fronts (a)

Early time dynamics after imprint (central slice)

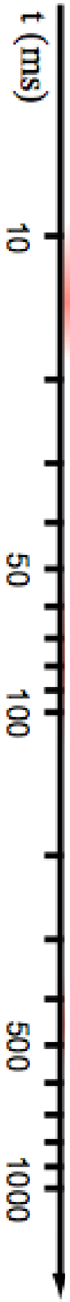


**Bending & breaking of planar soliton – snake instability**

Tomography:



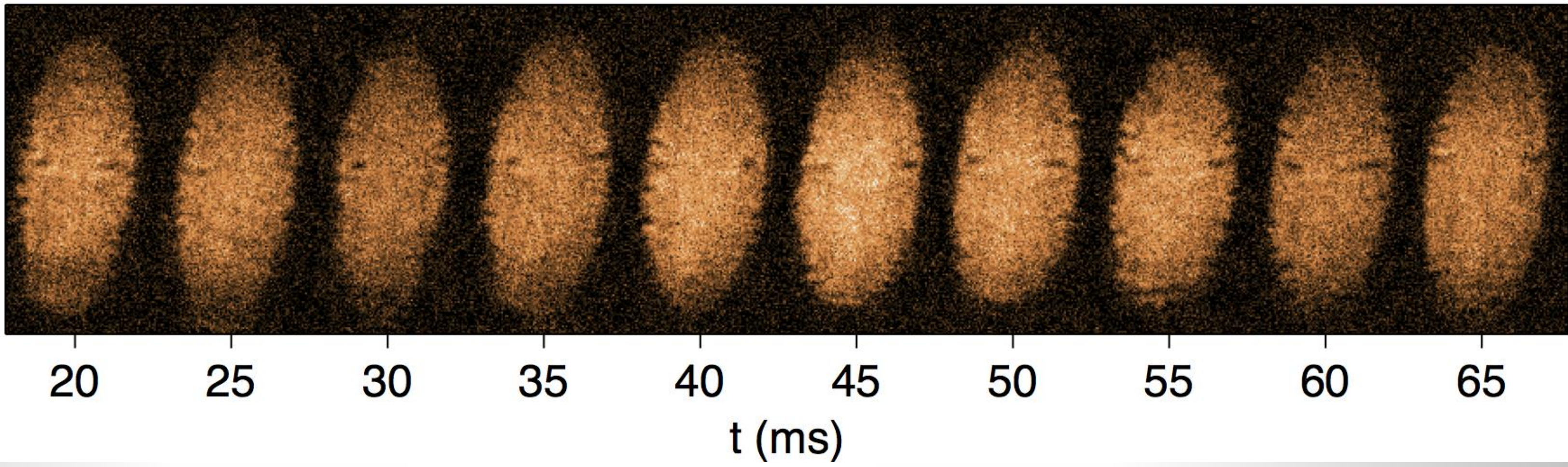
# Instability Cascade of Solitary Waves in Unitary Fermi Gas



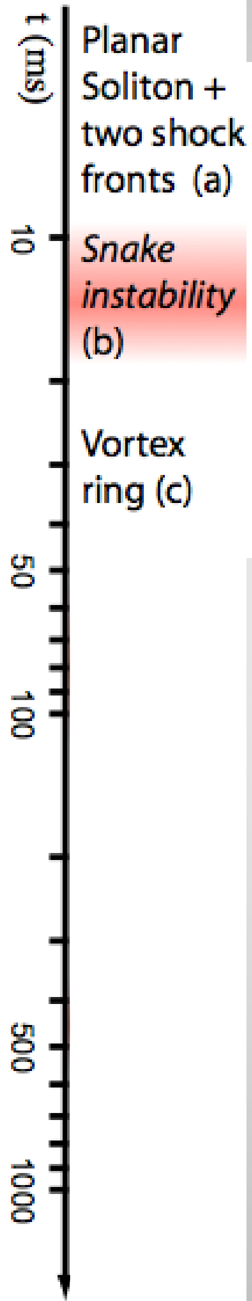
Planar Soliton + two shock fronts (a)

*Snake instability* (b)

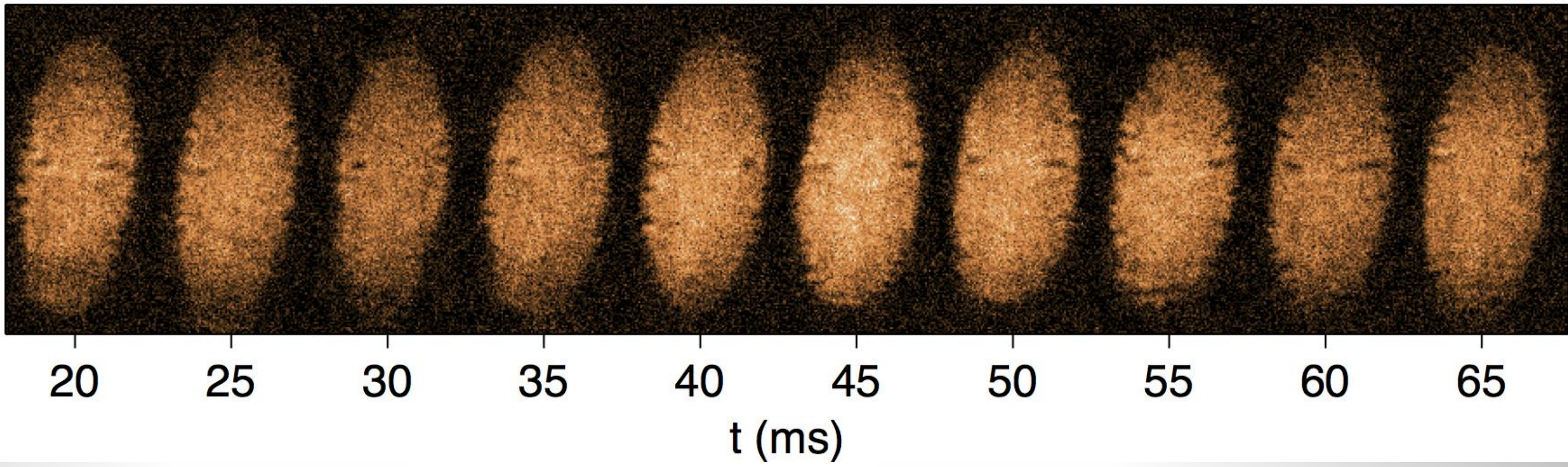
Early time dynamics after imprint (central slice)



# Instability Cascade of Solitary Waves in Unitary Fermi Gas

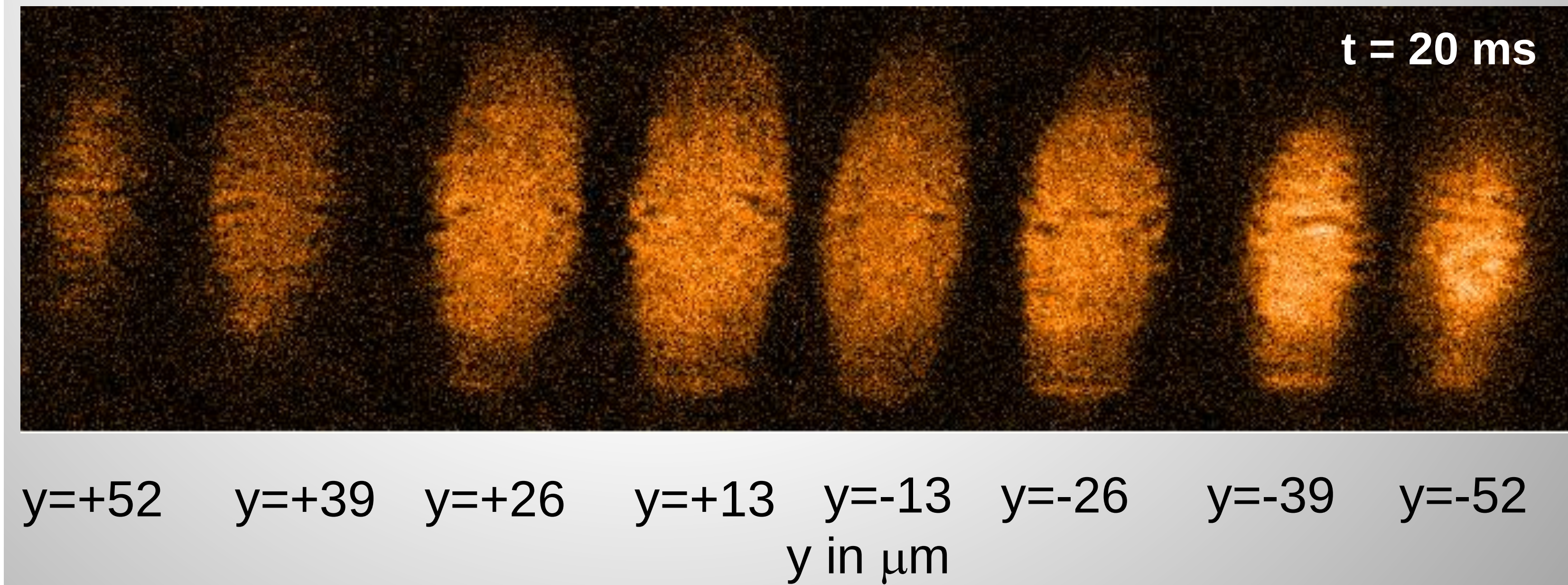


Early time dynamics after imprint (central slice)

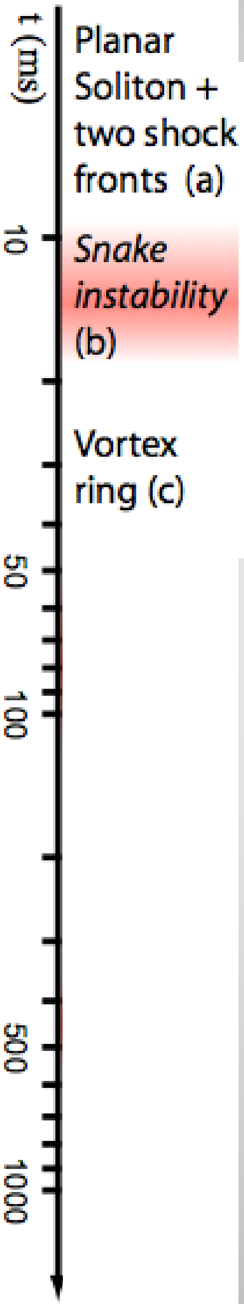


**Vortex ring**

Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas

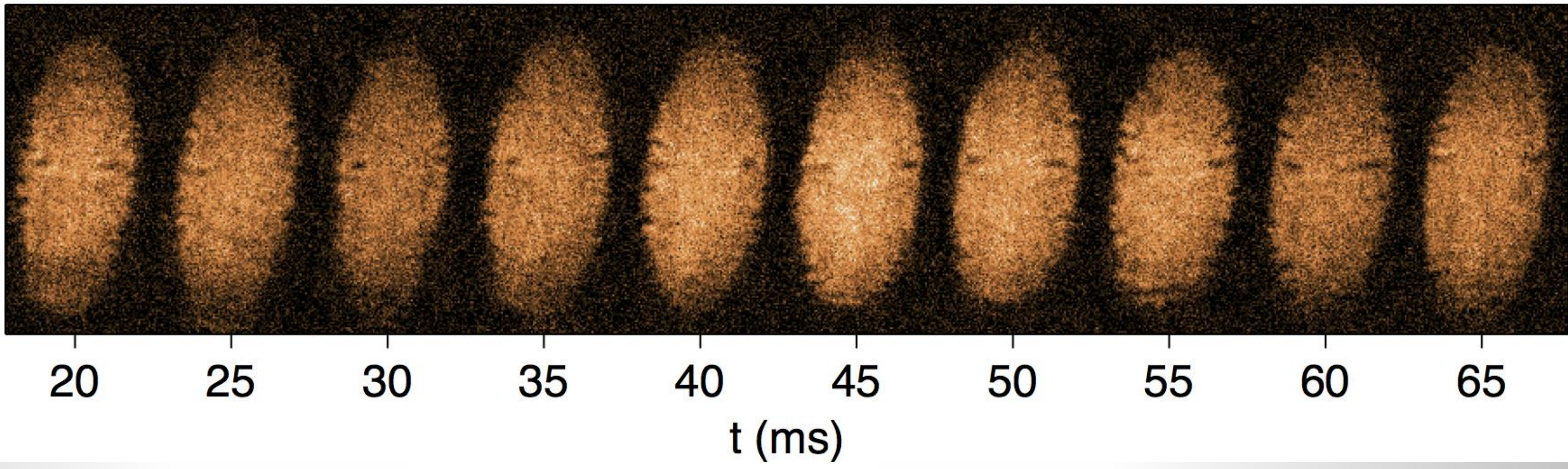


Planar Soliton + two shock fronts (a)

*Snake instability* (b)

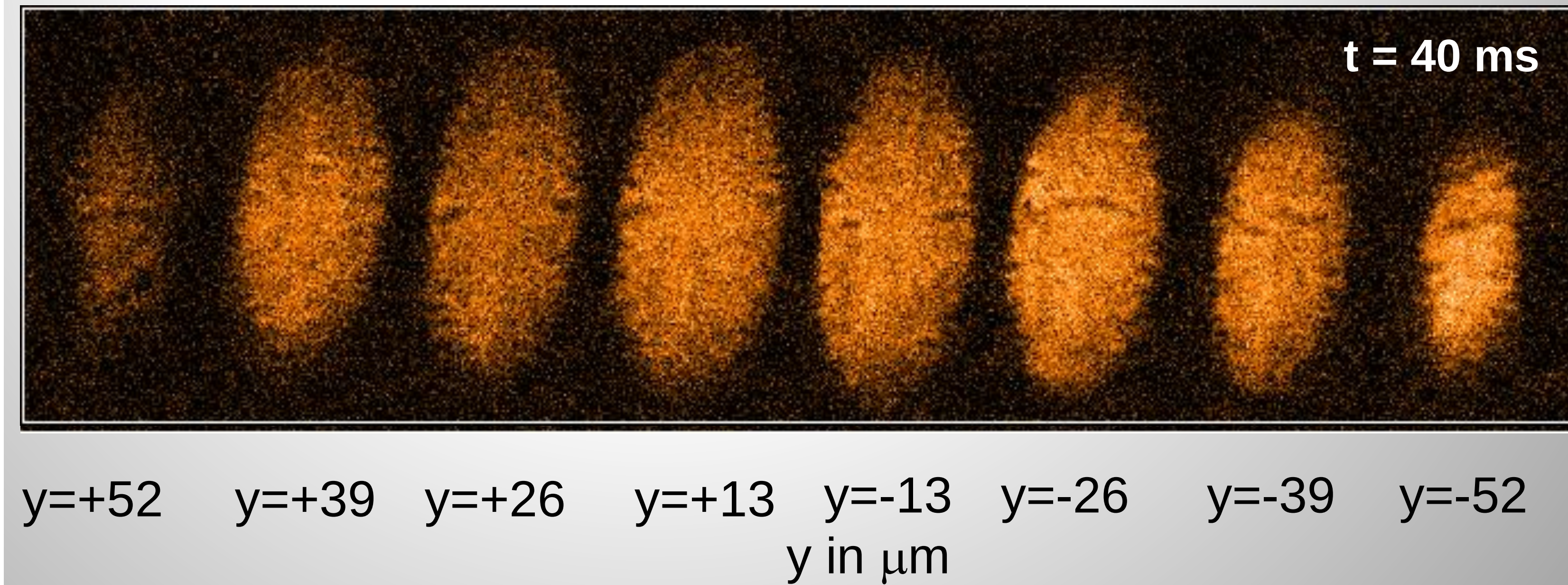
Vortex ring (c)

Early time dynamics after imprint (central slice)

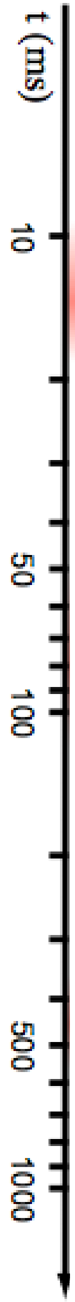


**Vortex ring**

Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas

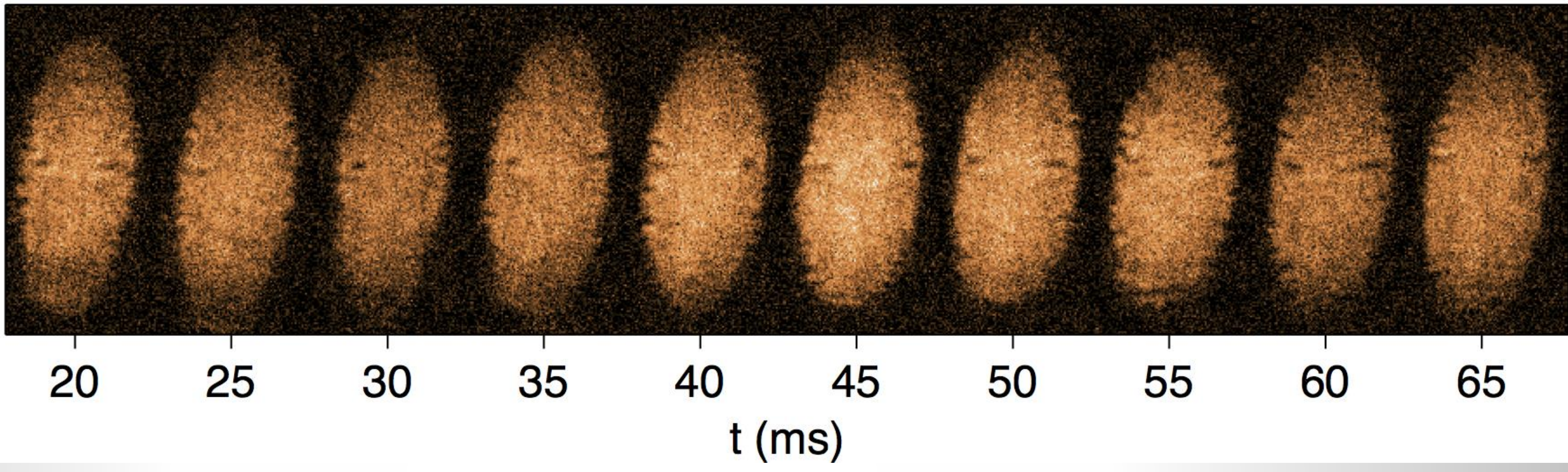


Planar Soliton + two shock fronts (a)

*Snake instability* (b)

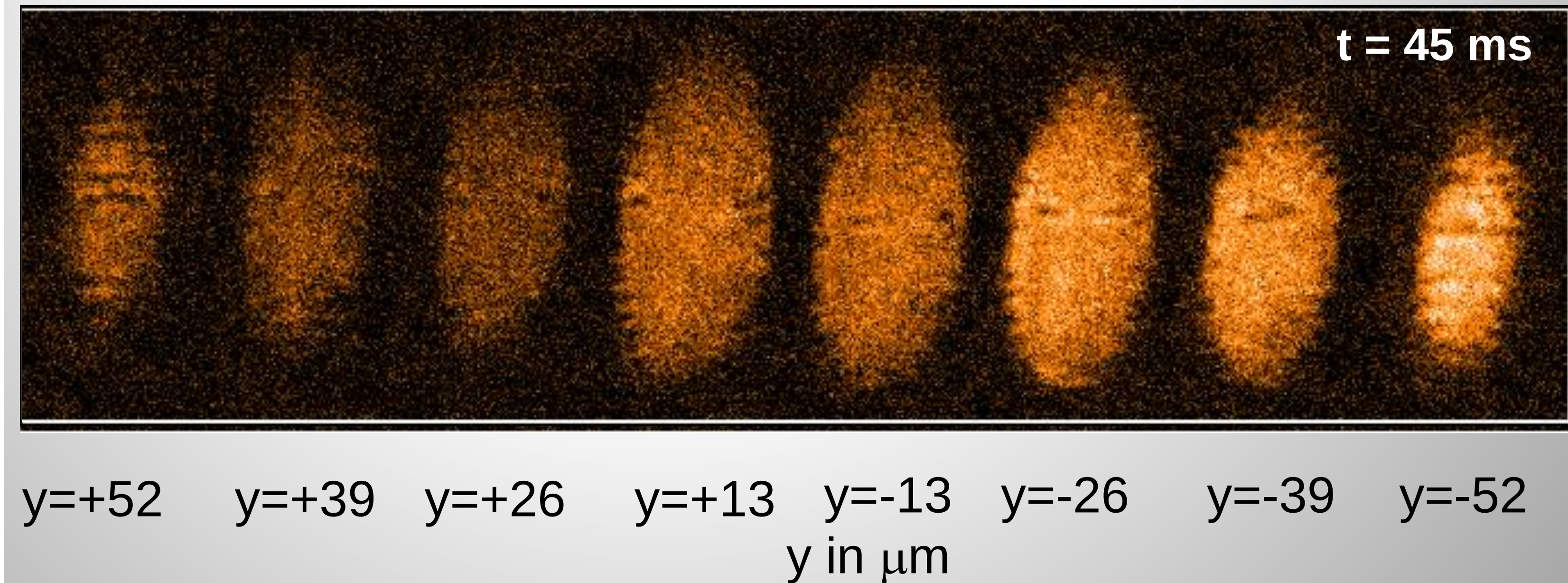
Vortex ring (c)

Early time dynamics after imprint (central slice)

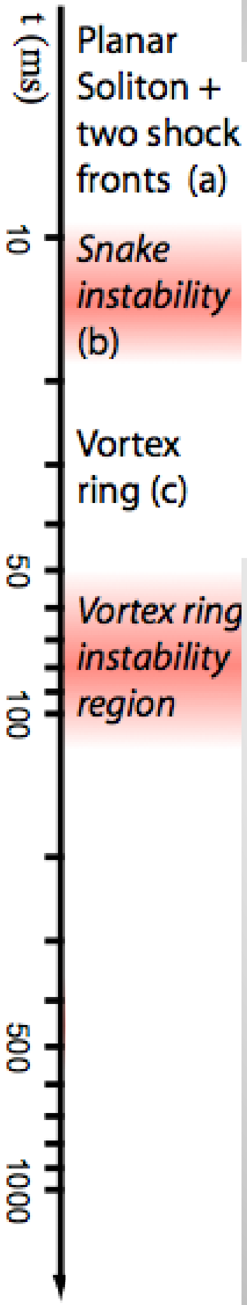


**Vortex ring**

Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas



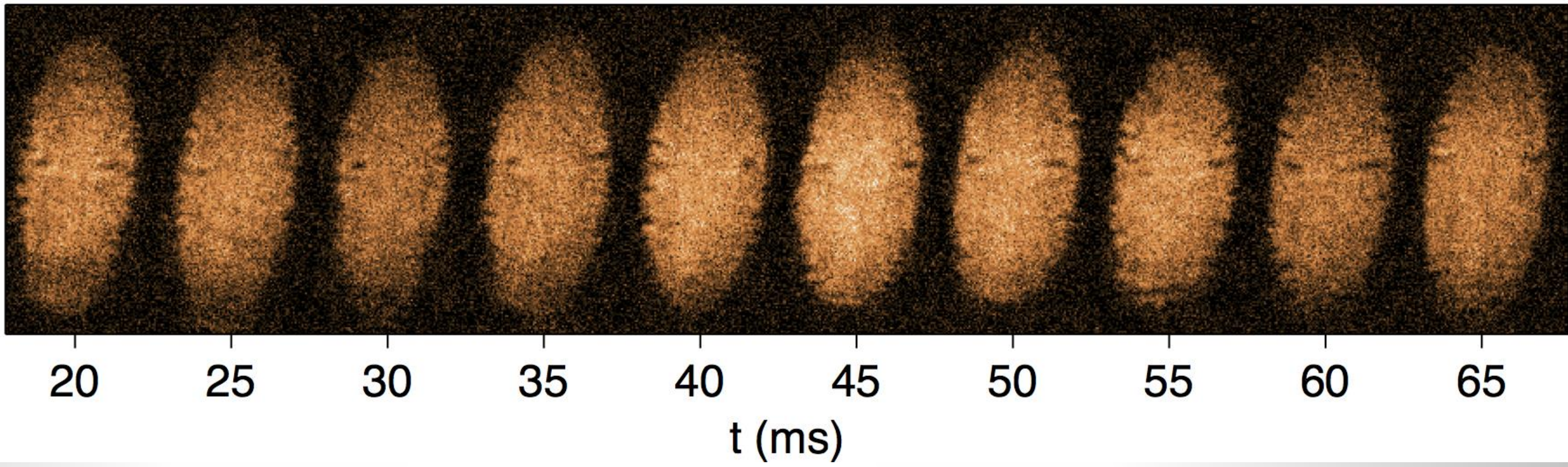
Planar Soliton + two shock fronts (a)

*Snake instability* (b)

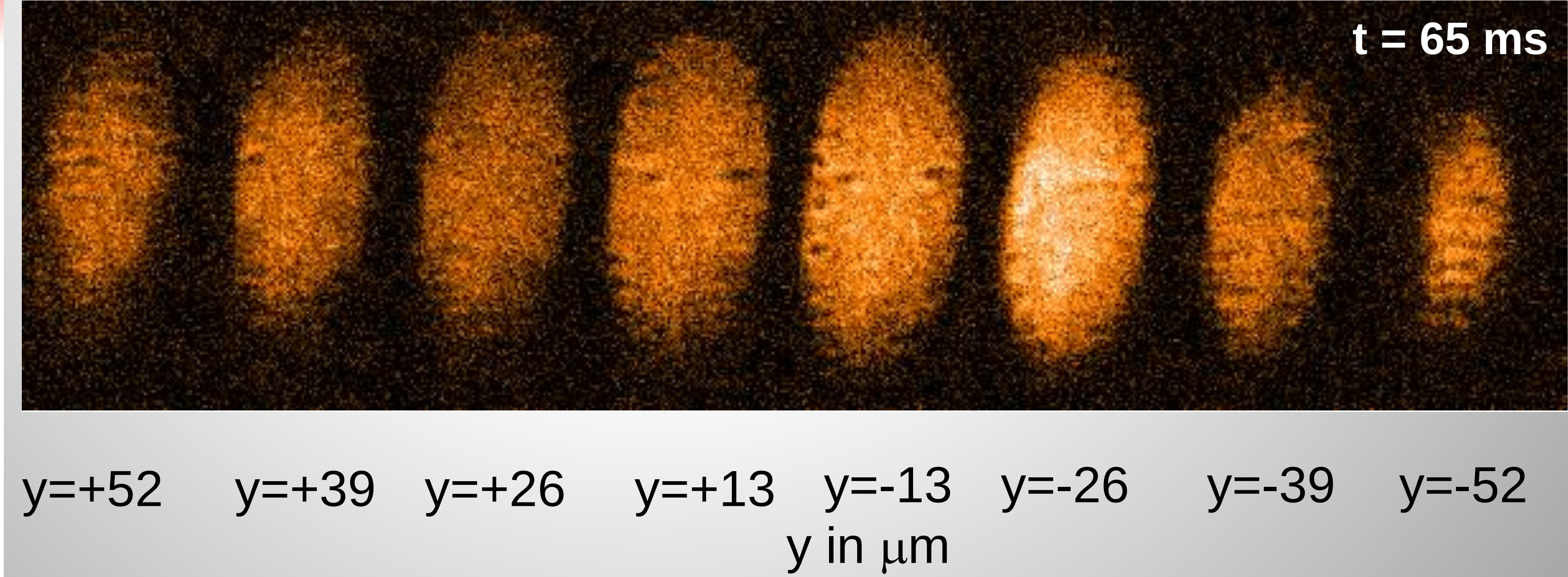
Vortex ring (c)

*Vortex ring instability region*

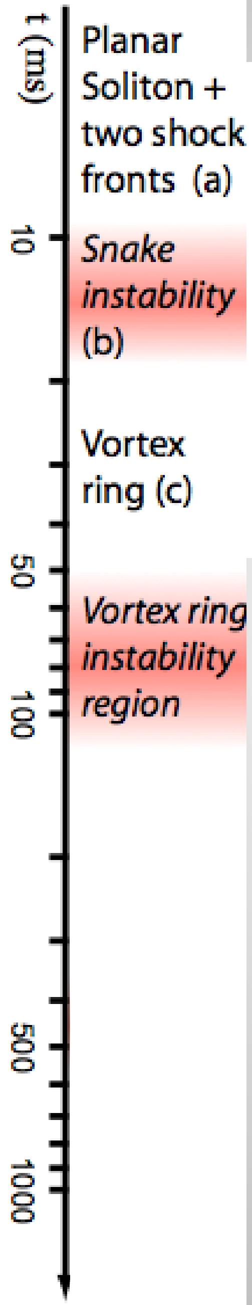
Early time dynamics after imprint (central slice)



Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas



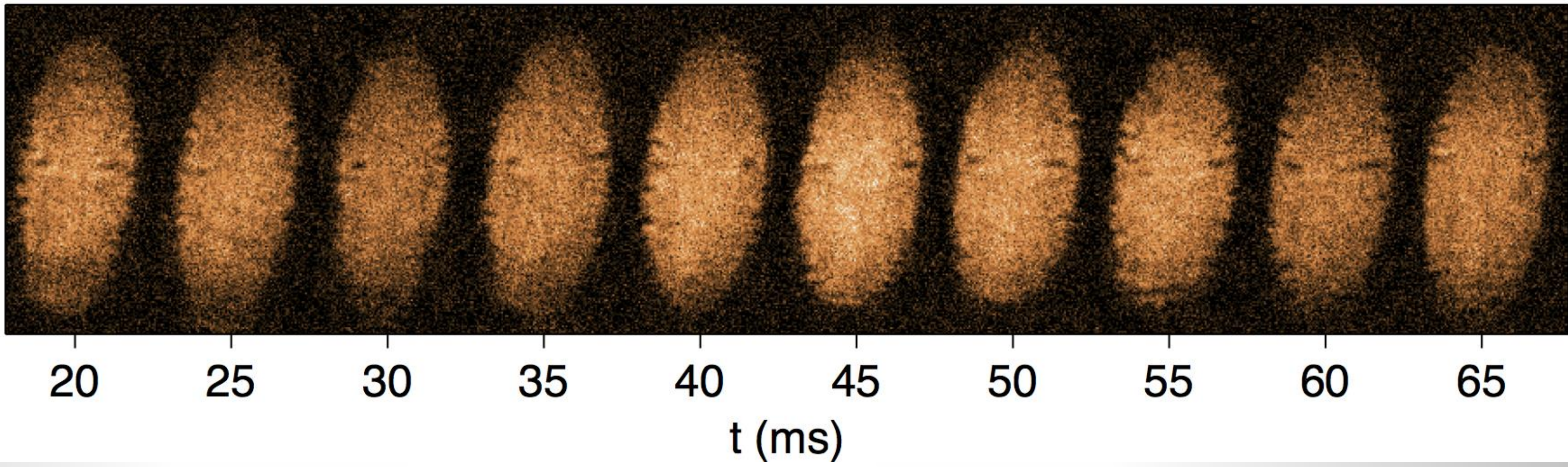
Planar Soliton + two shock fronts (a)

*Snake instability* (b)

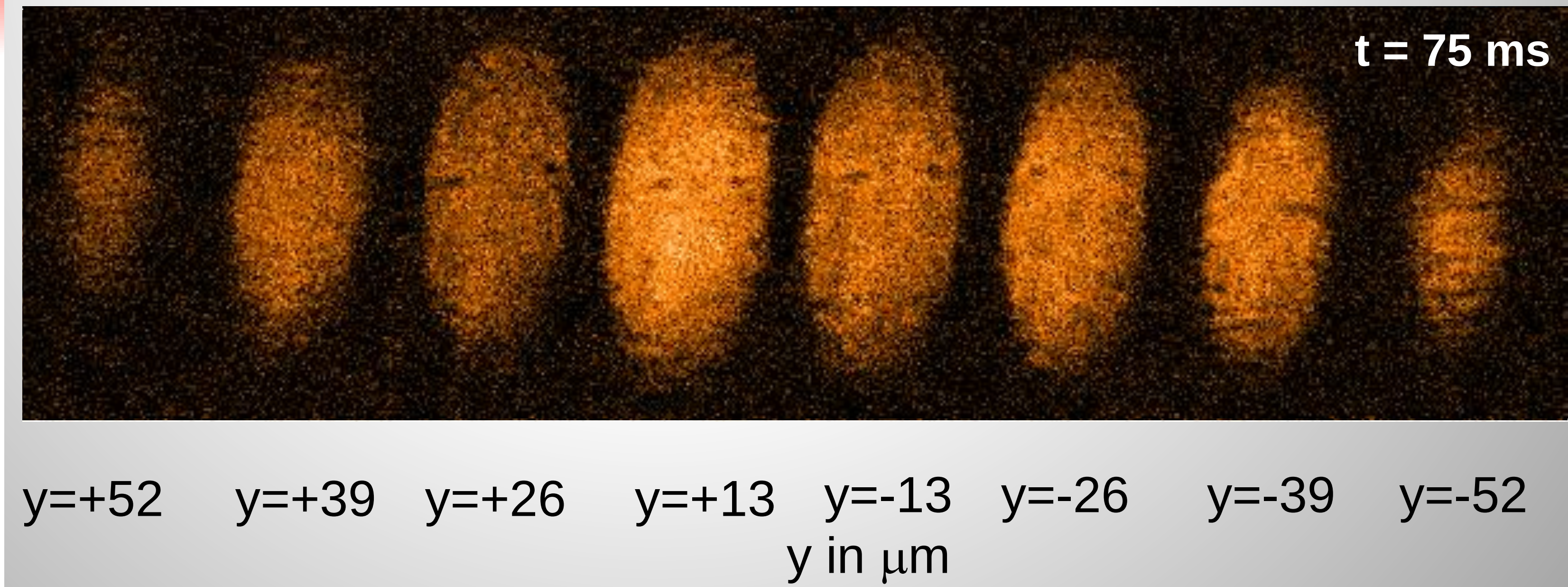
Vortex ring (c)

*Vortex ring instability region*

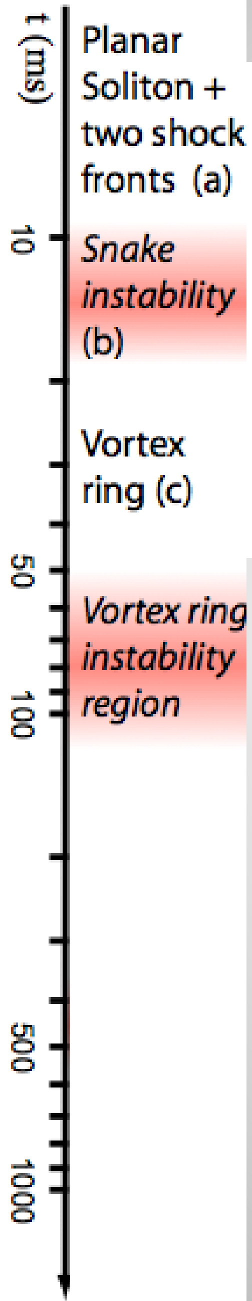
Early time dynamics after imprint (central slice)



Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas



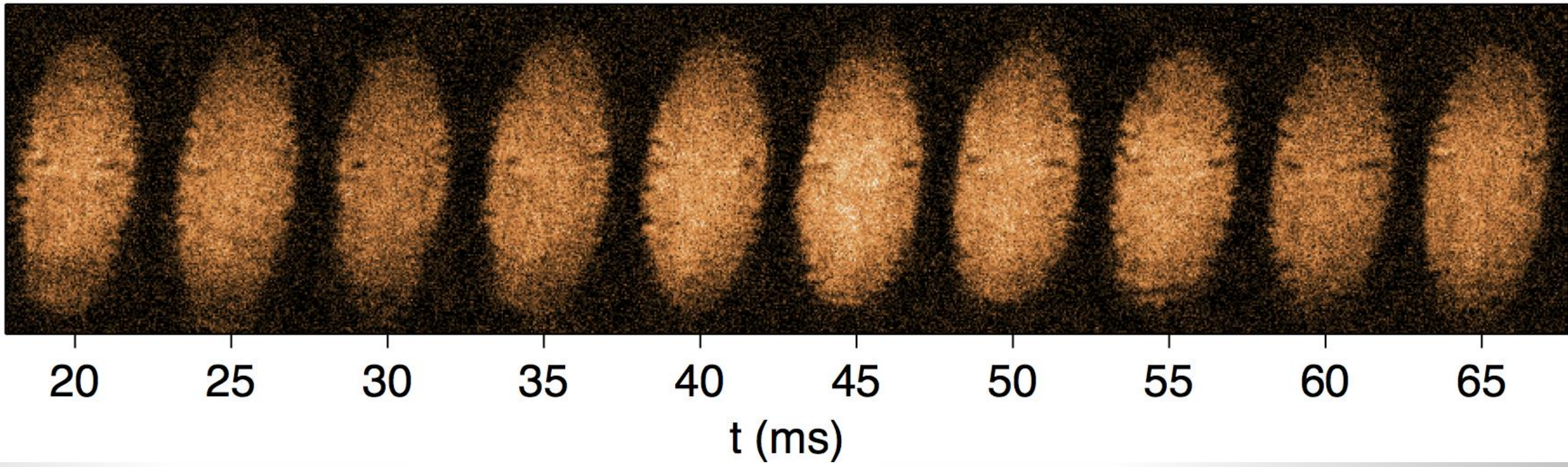
Planar Soliton + two shock fronts (a)

*Snake instability* (b)

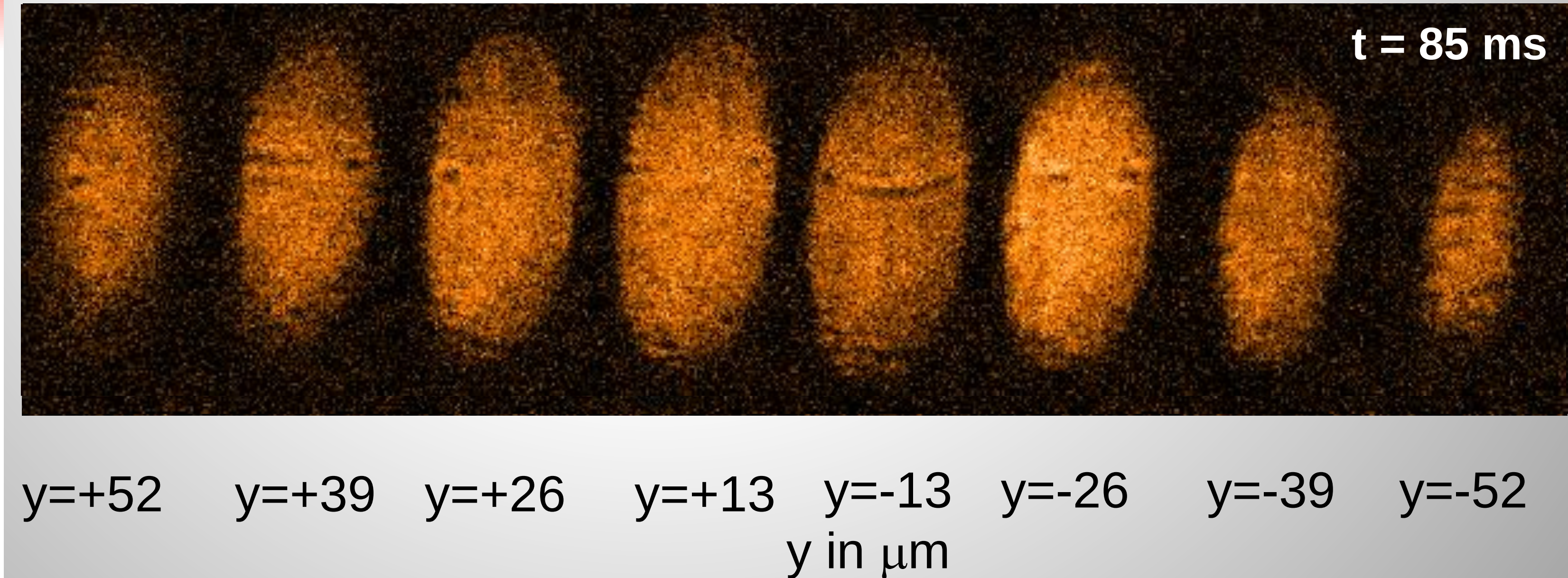
Vortex ring (c)

*Vortex ring instability region*

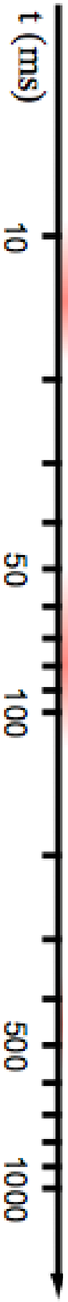
Early time dynamics after imprint (central slice)



Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas



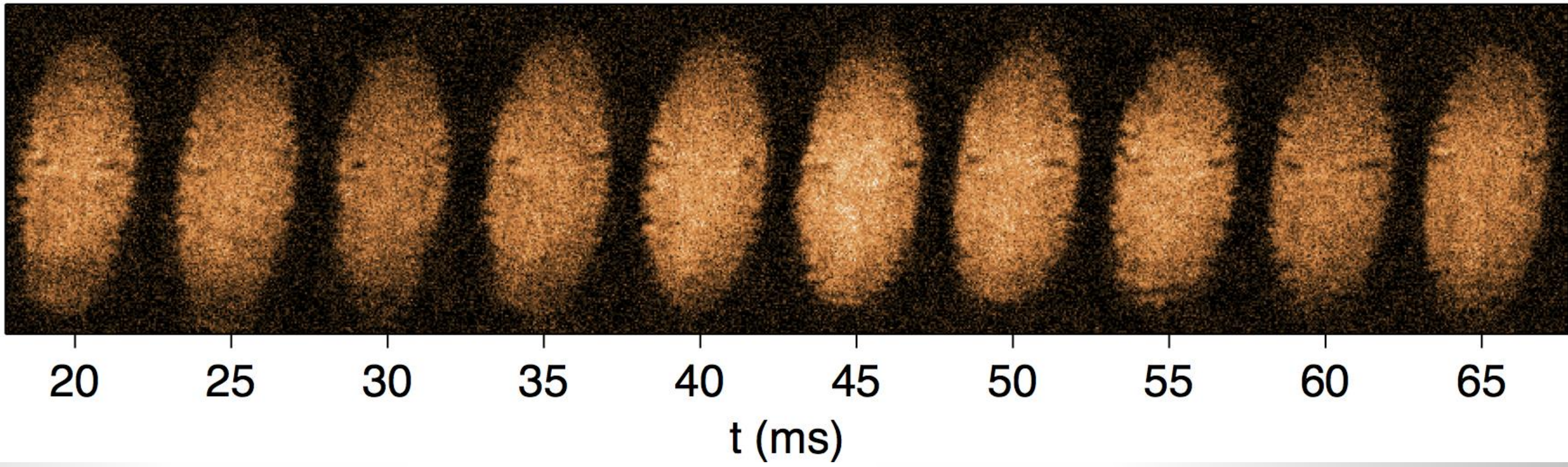
Planar Soliton + two shock fronts (a)

*Snake instability (b)*

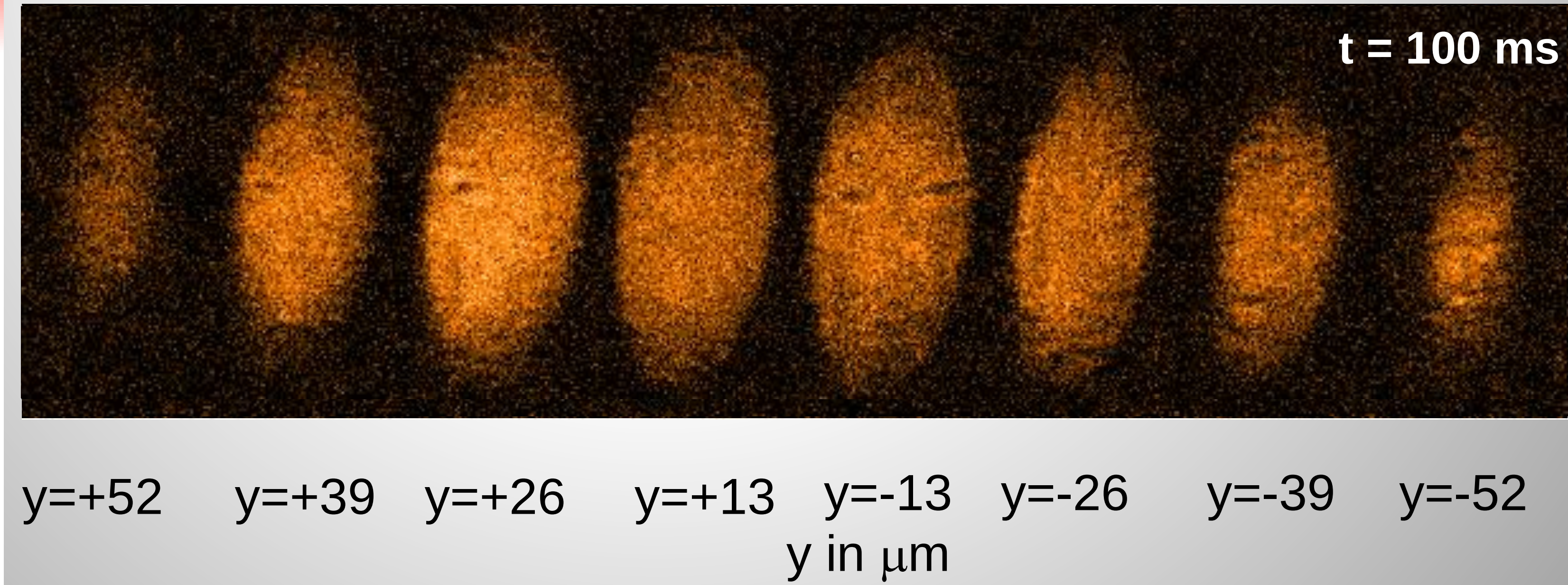
Vortex ring (c)

*Vortex ring instability region*

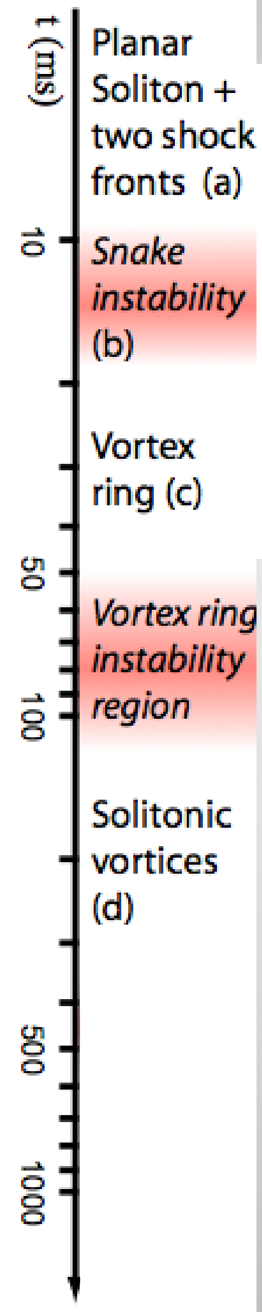
Early time dynamics after imprint (central slice)



Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas



Planar Soliton + two shock fronts (a)

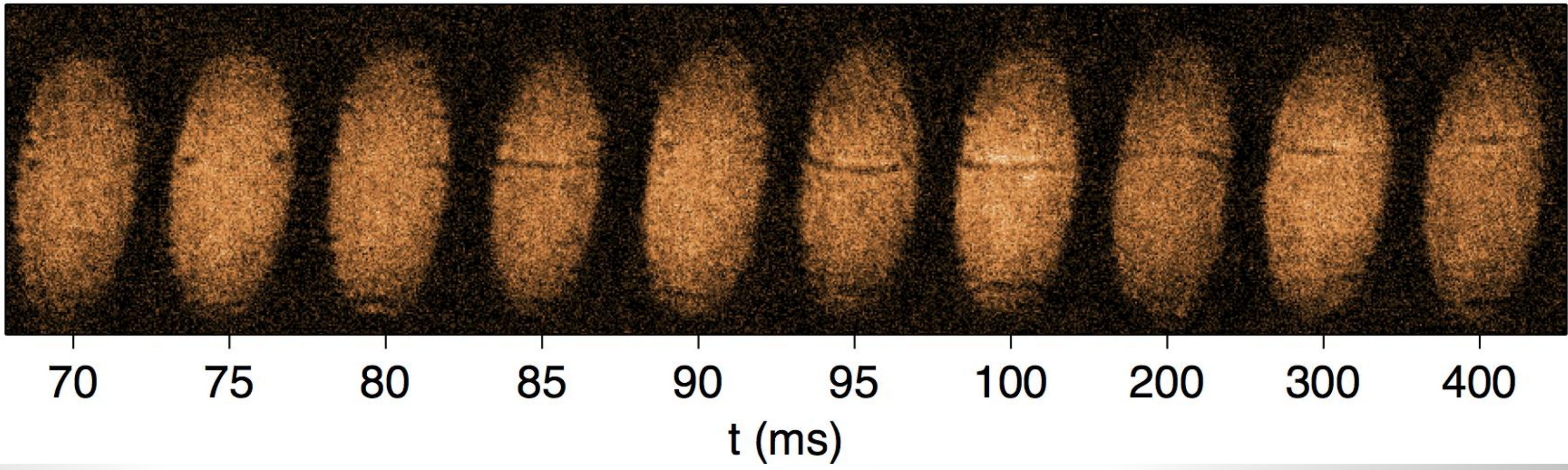
*Snake instability* (b)

Vortex ring (c)

*Vortex ring instability region*

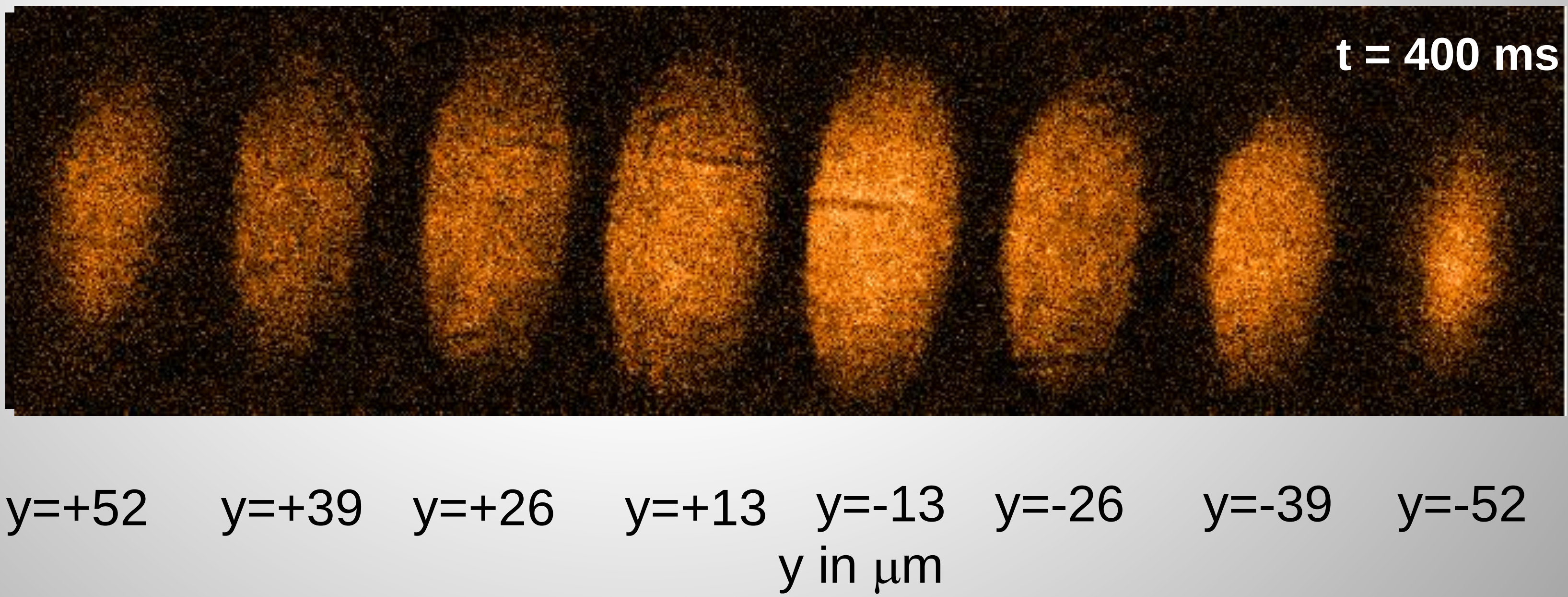
Solitonic vortices (d)

Deterministic generation of solitonic vortex (central slice)

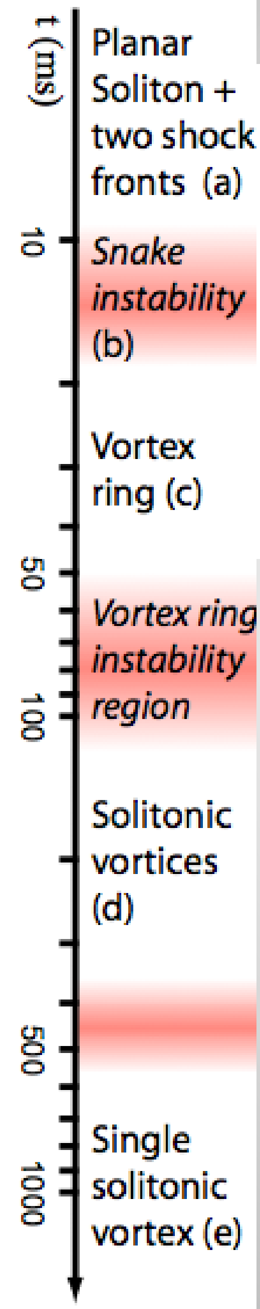


**Solitonic vortices**

Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas



Planar Soliton + two shock fronts (a)

*Snake instability* (b)

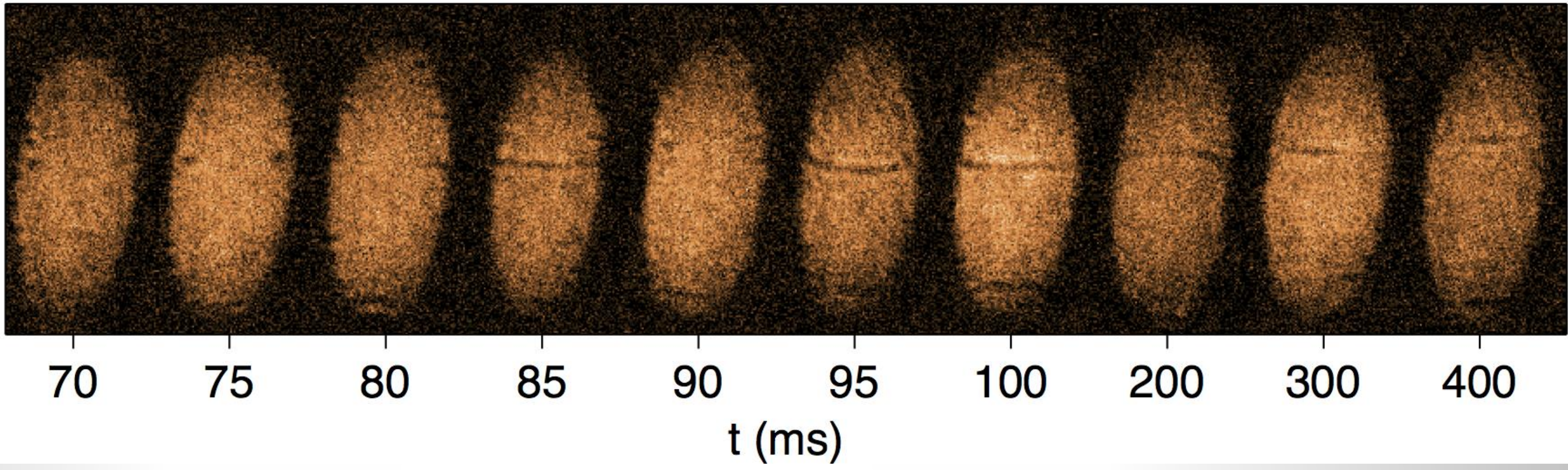
Vortex ring (c)

*Vortex ring instability region*

Solitonic vortices (d)

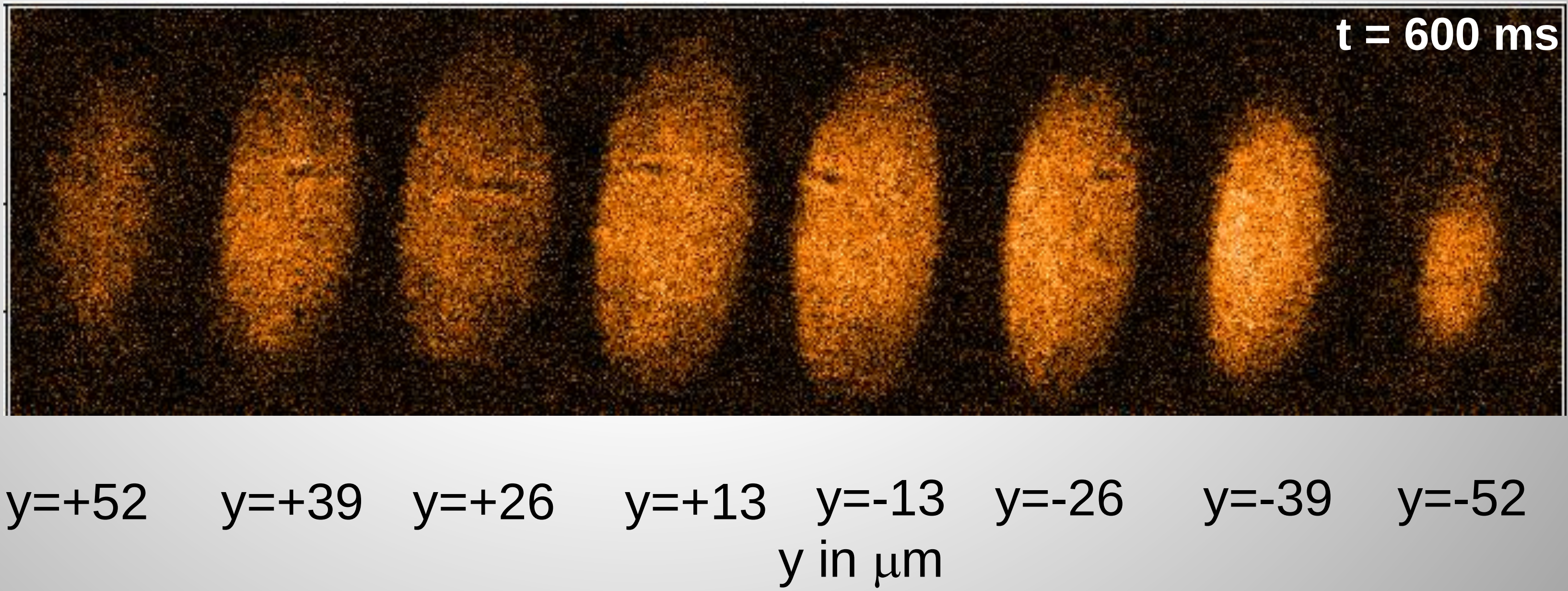
Single solitonic vortex (e)

Deterministic generation of solitonic vortex (central slice)

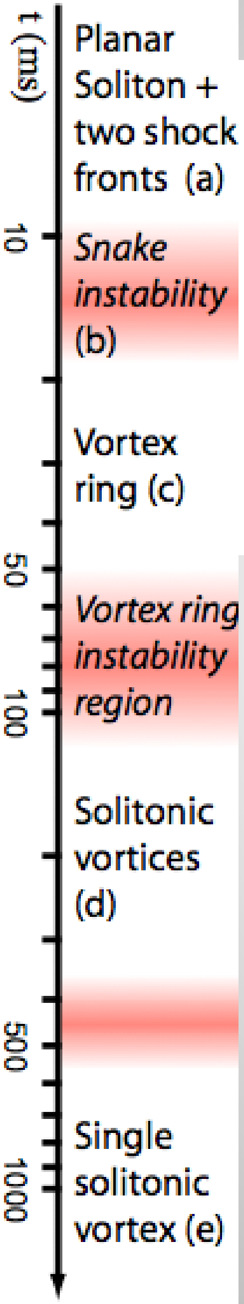


**Single solitonic vortex**

Tomography:



# Instability Cascade of Solitary Waves in Unitary Fermi Gas



Planar Soliton + two shock fronts (a)

*Snake instability* (b)

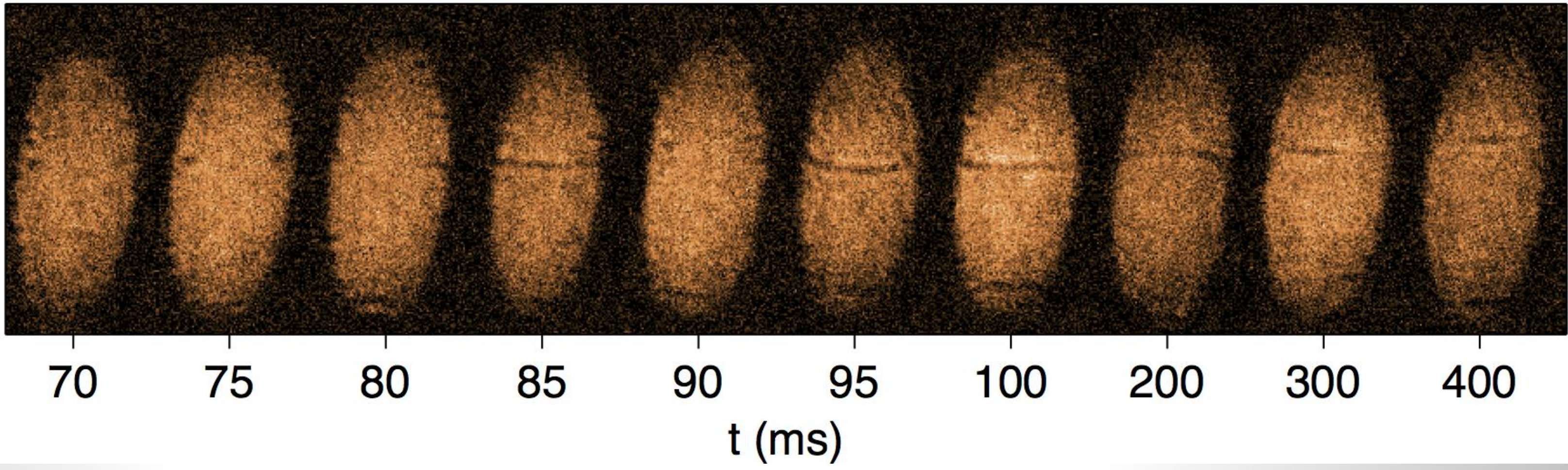
Vortex ring (c)

*Vortex ring instability region*

Solitonic vortices (d)

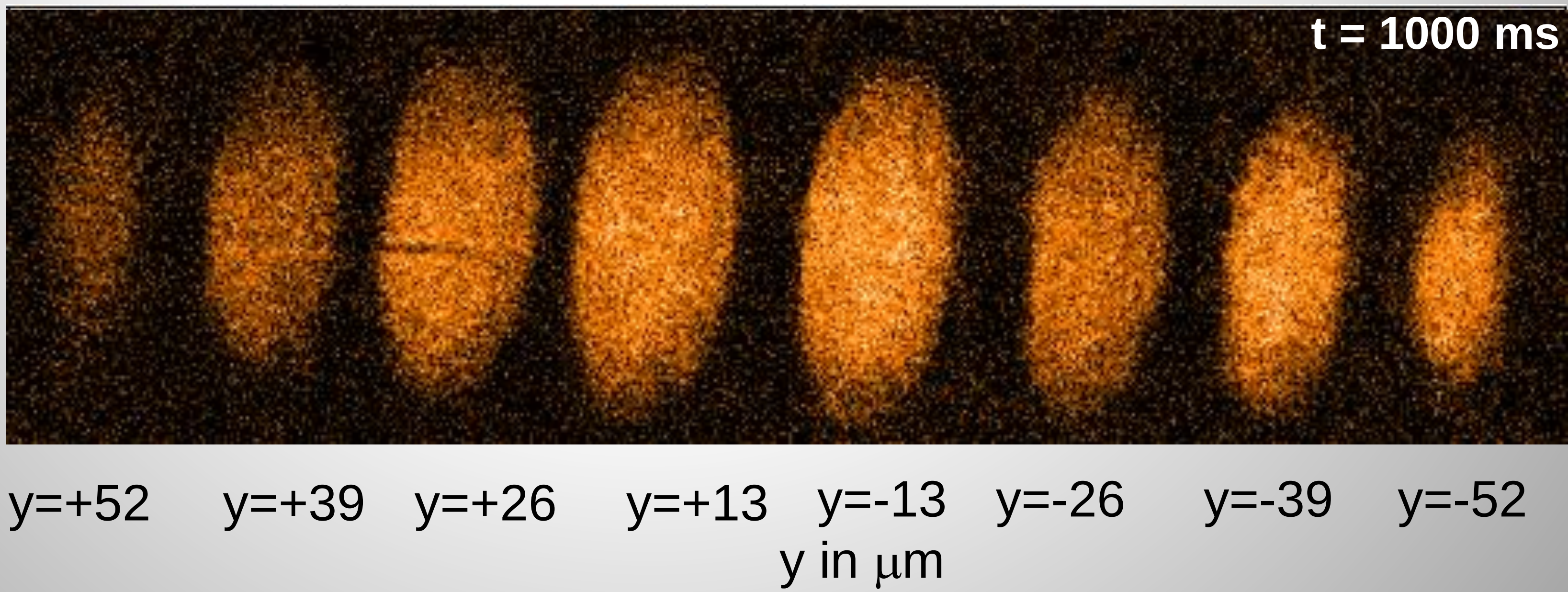
Single solitonic vortex (e)

Deterministic generation of solitonic vortex (central slice)



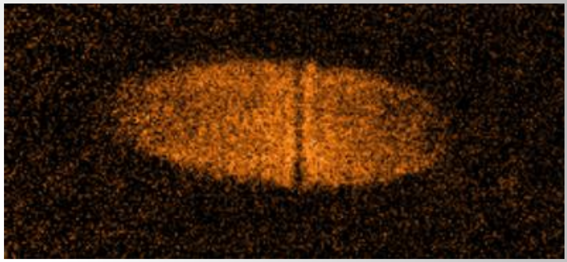
**Single solitonic vortex**

Tomography:

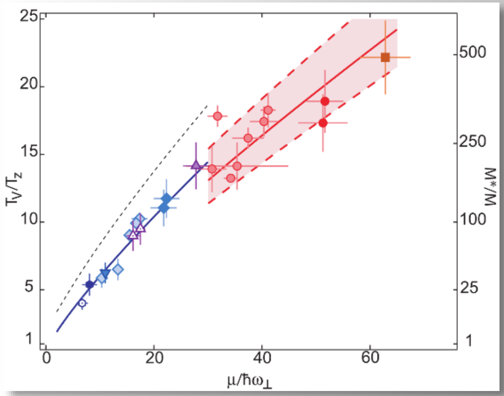


# Summary 2nd part

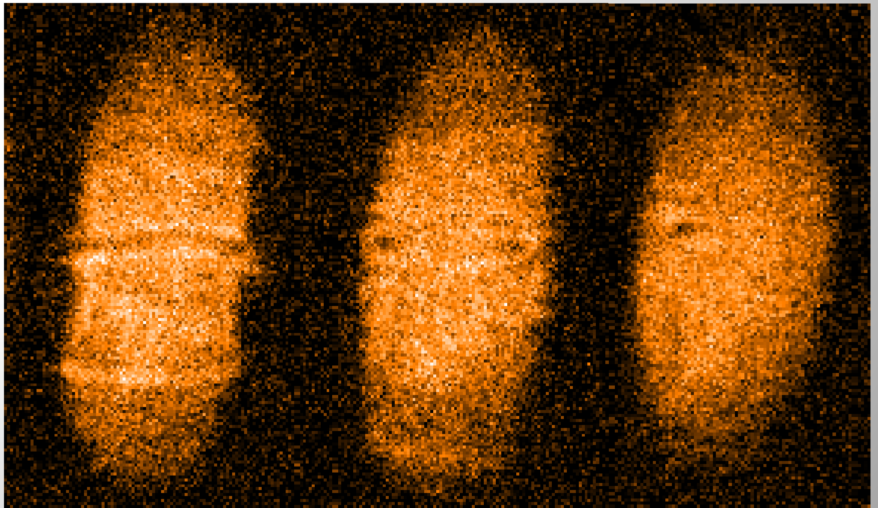
- "On demand" creation of solitary waves in a Strongly Interacting Fermi Gas



- Measurement of the Vortex Period in the BEC-BCS Crossover



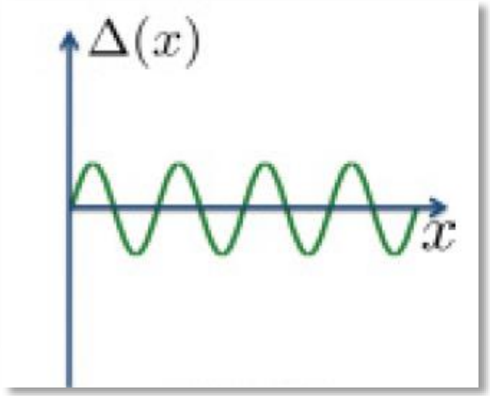
- Instability Cascade: From planar solitons to vortex rings to solitonic vortices



# Outlook

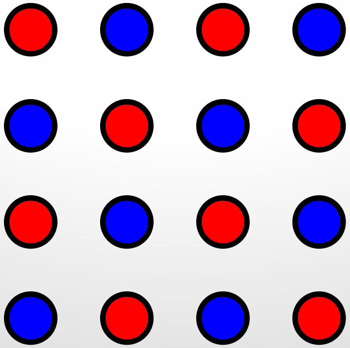
Solitons in the presence of imbalance:

Do excess fermions stabilize the soliton? → LO phase!



If they still decay into vortices:

Ground state of imbalanced Fermi gases might rather be vortex-anti-vortex lattice



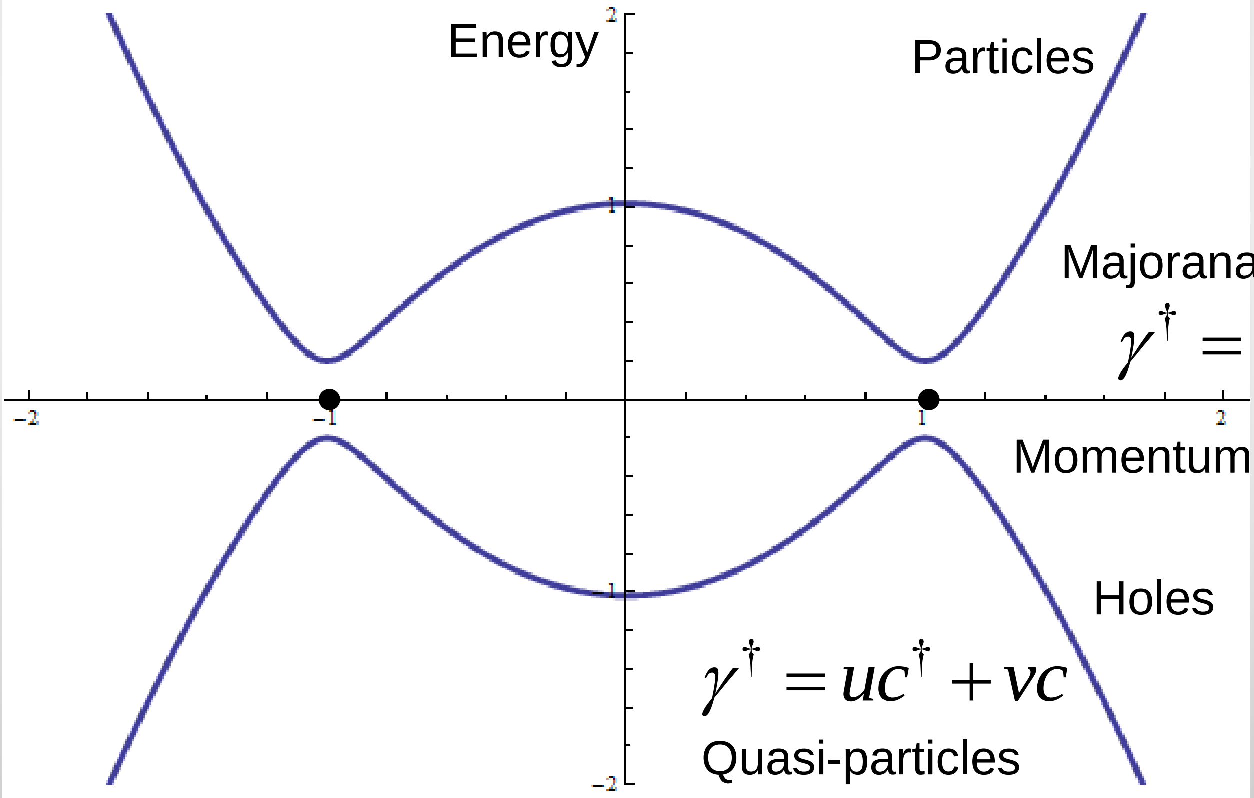
Excess fermions trapped in Andreev states inside vortex cores

# Majorana Fermions in Superfluids

Particle is its own anti-particle

$$\gamma^\dagger = \gamma$$

One place to look for this: Superconductors / Superfluids



Majorana bound states

$$\gamma^\dagger = c^\dagger + c$$

$$\gamma^\dagger = uc^\dagger + vc$$

Quasi-particles

No spin involved → Pairing of fermions of same spin → Need p-wave pairing

# Many-Body Physics

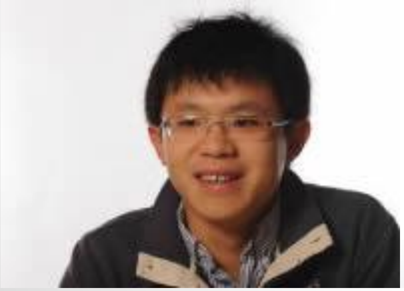
## The (Bright!) Solitons:

Dr. Tarik Yefsah

Ariel Sommer
(PhD 2013)

Mark Ku

Dr. Waseem Bakr
→ Princeton U.



Lawrence Cheuk

Wenjie Ji

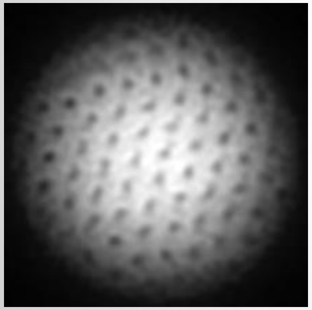
Biswaroop Mukherjee

# Bosons and Fermions

## BEC 1

Fermionic Superfluids

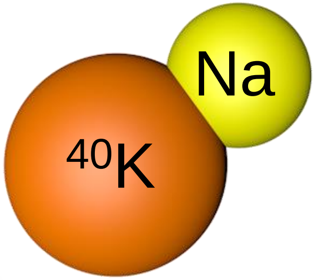
Mark Ku
Wenjie Ji
Biswaroop Mukherjee
Dr. Julian Struck
Dr. Tarik Yefsah

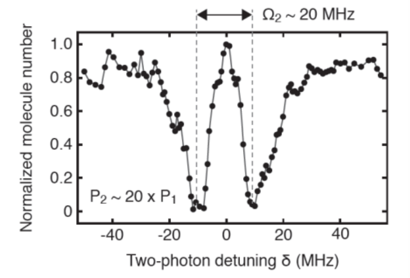


## Fermi 1

NaK Dipolar Molecules

Jeewoo Peter Park
Jennifer Schloss
Qingyang Wang
Dr. Sebastian Will

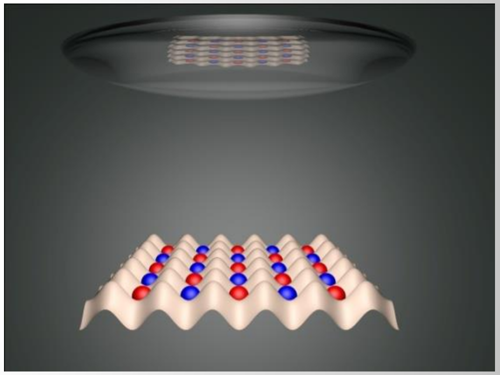


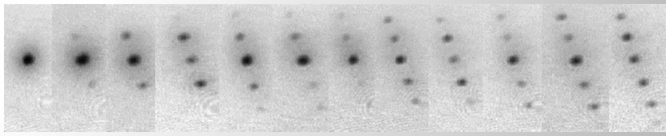


## Fermi 2

Fermi Gas Microscope

Lawrence Cheuk
Melih Okan
Matthew Nichols
Dr. Thomas Lompe





the David & Lucile Packard FOUNDATION
NSF
ONR
OLE OPTICAL LATTICE EMULATOR