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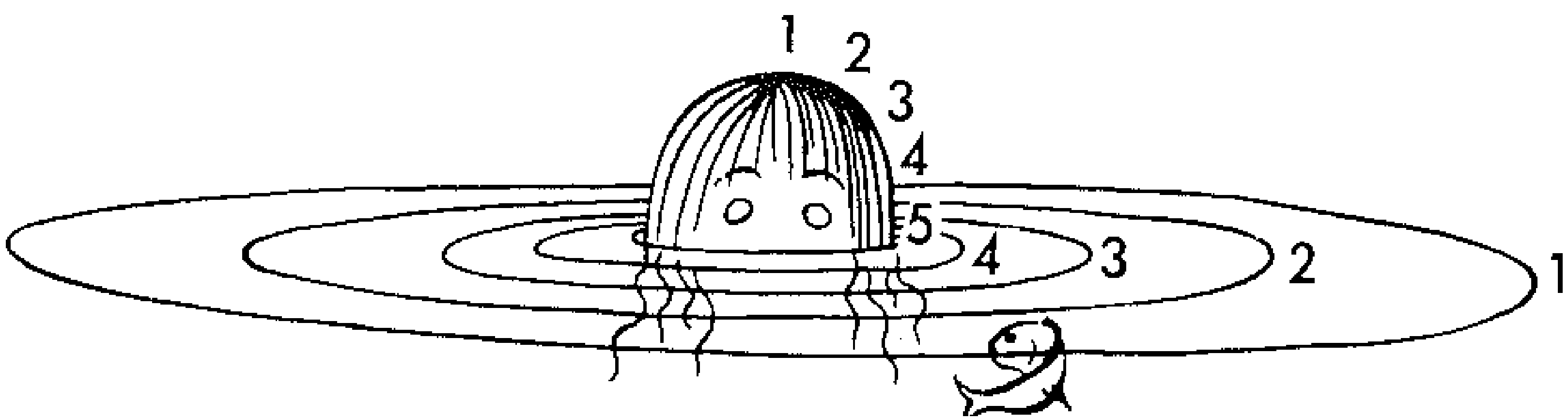
AMERICAN PHYSICAL SOCIETY

A PRIMER

PAGE ONE



SEE, the LADY has FAL-LEN in the WA-TER!

Can she SWIM?

No, but she can FLOAT—Here she comes NOW!

What are those NUM-BERS?

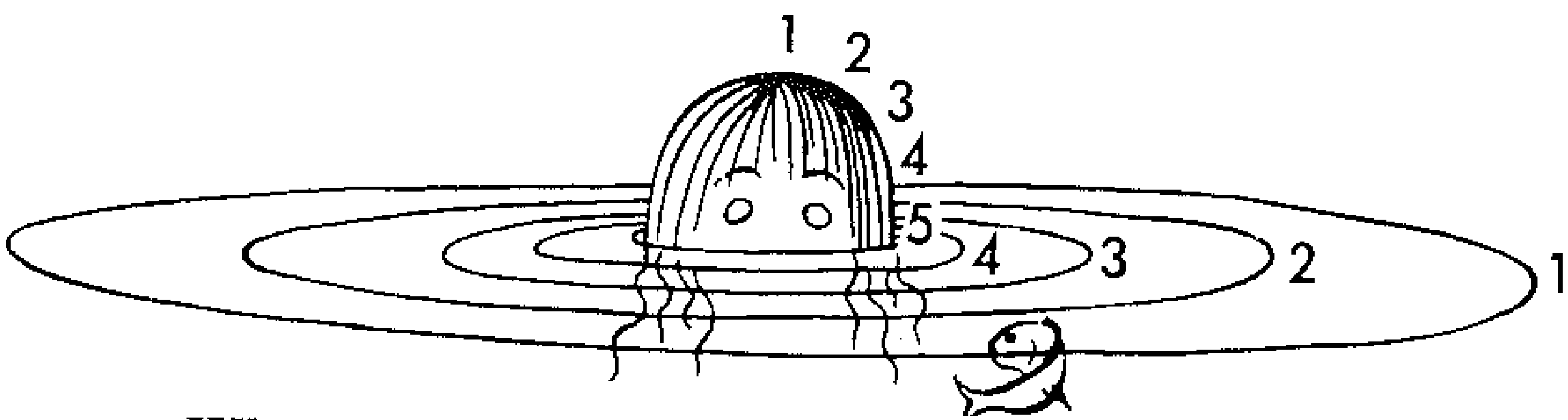
They are the var-ious POINTS on her HEAD—

Why has she got POINTS on her HEAD?

They are IM-AG-IN-ARY: They show which parts of her Head made the Pretty RIP-PLES. See, the RIP-PLES are NUM-BERED.

A PRIMER

PAGE ONE



Why do the Num-bers go BACK-WARD?

Because that is the ORDER they were made in. FIRST the TOP of her HEAD made the OUT-SIDE Rip-ple—That's NUM-BER ONE.

How did it get to be on the OUT-SIDE?

It Started FIRST. Then the circles on her HEAD made the SMAL-LER RIP-PLES, count-ing IN.

I don't see any CIR-CLES on her head.

They are Imag-in-ary, too. As her HEAD comes UP it makes—

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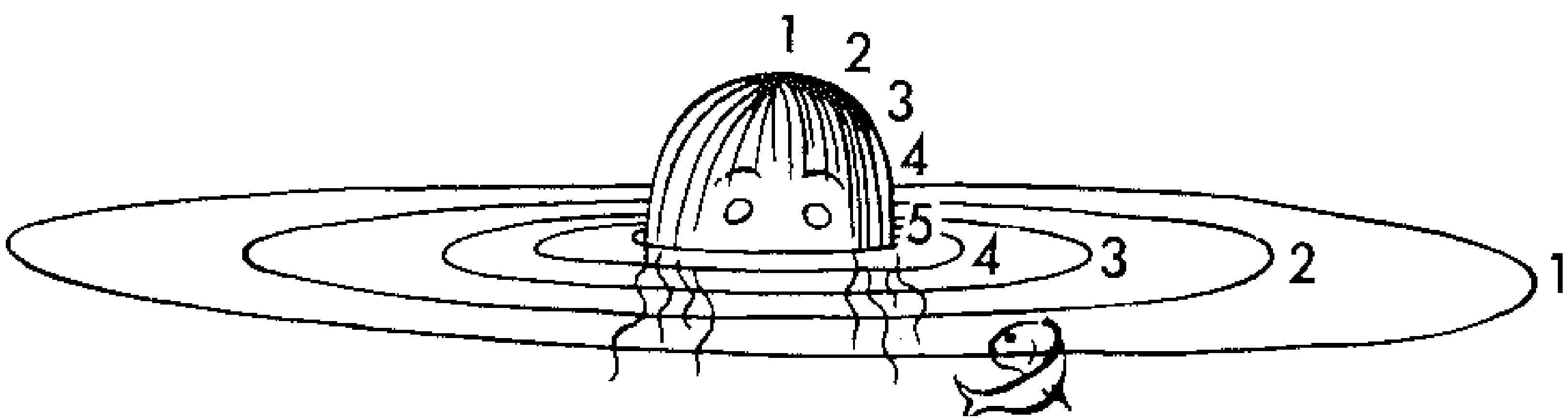
A PRIMER

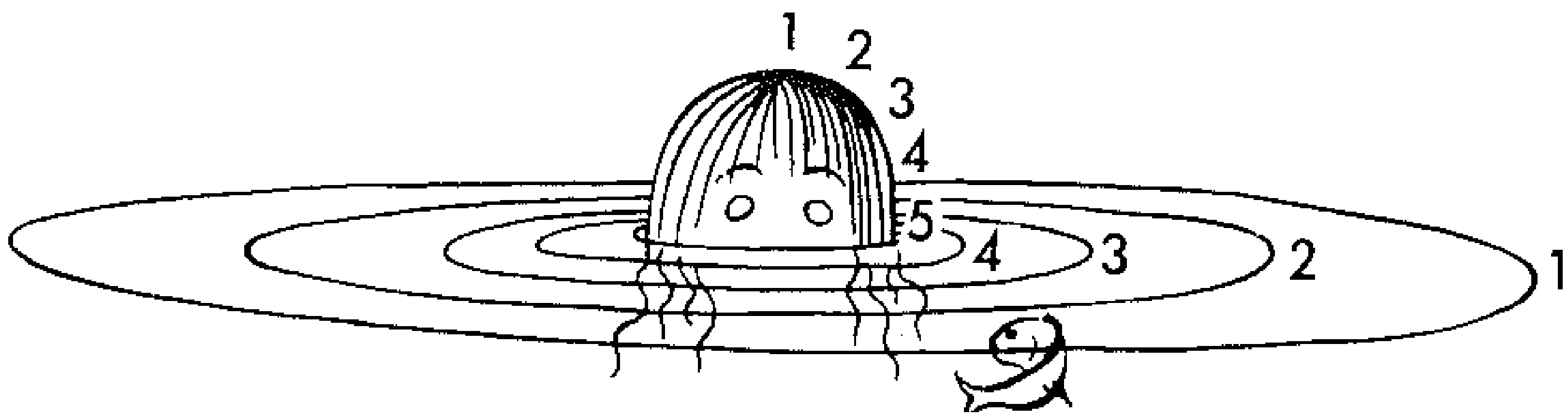
PAGE ONE



What is the FISH doing?

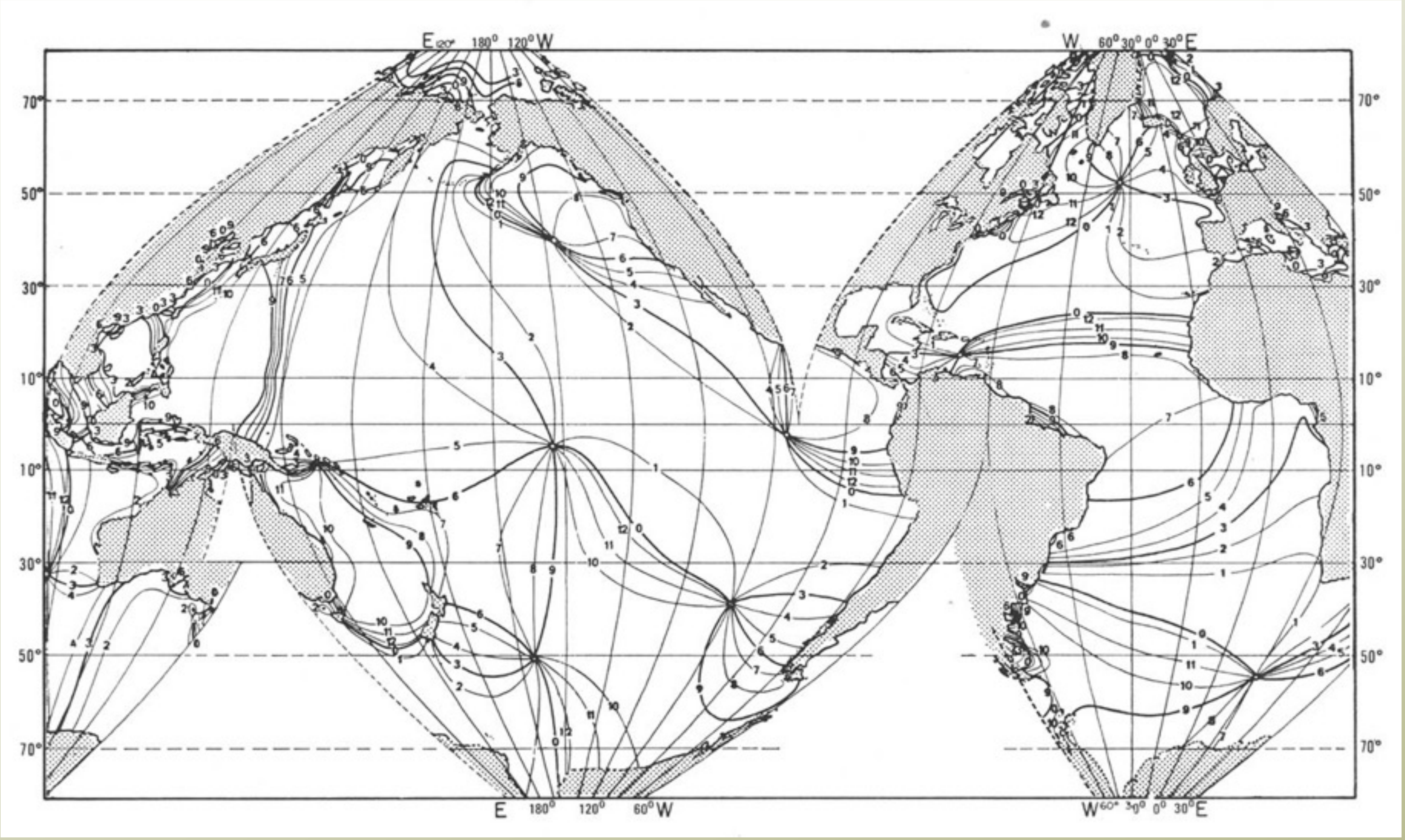
A PRIMER

PAGE ONE



What is the FISH doing?
NOTH-ING.

The Tides



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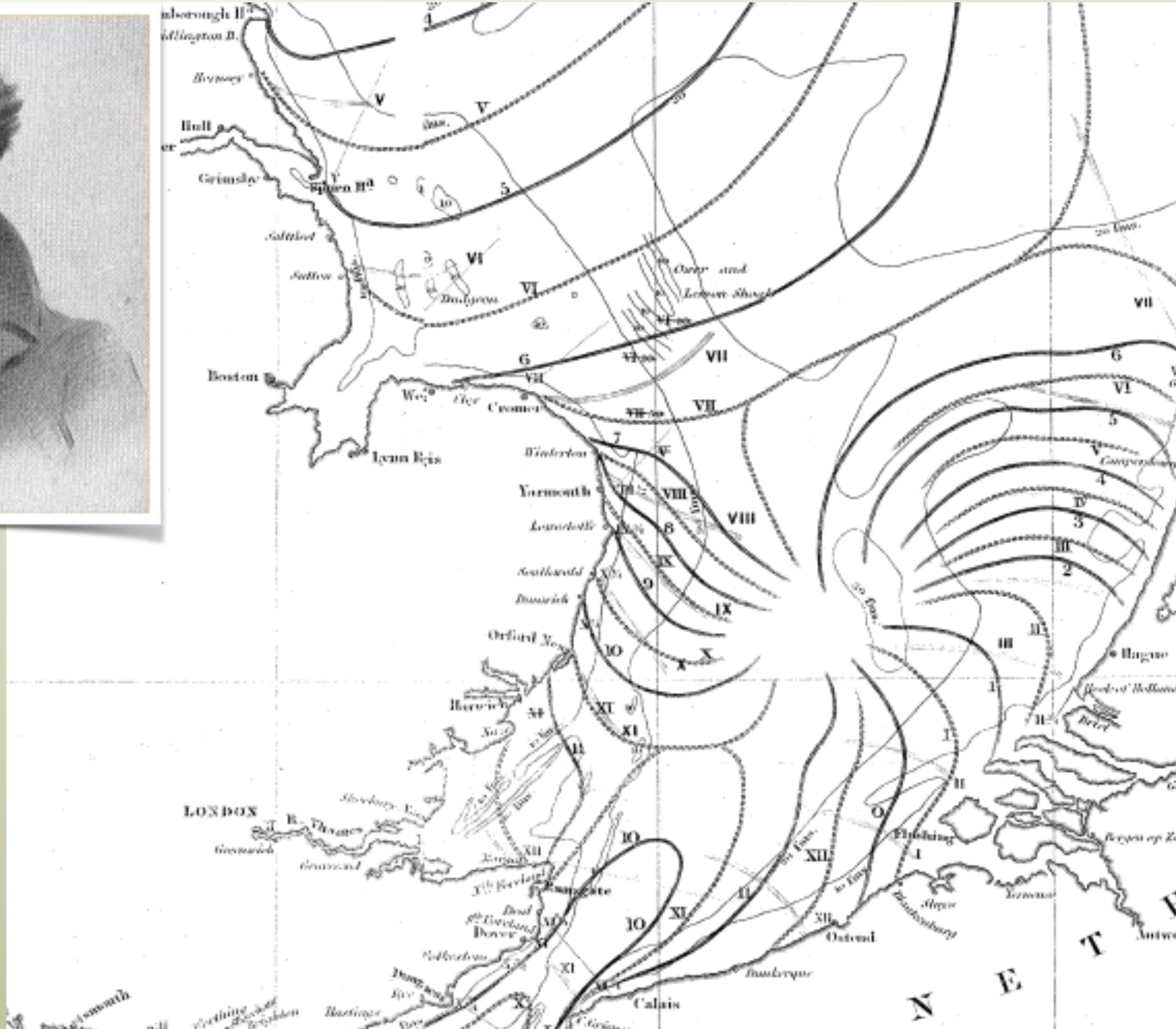
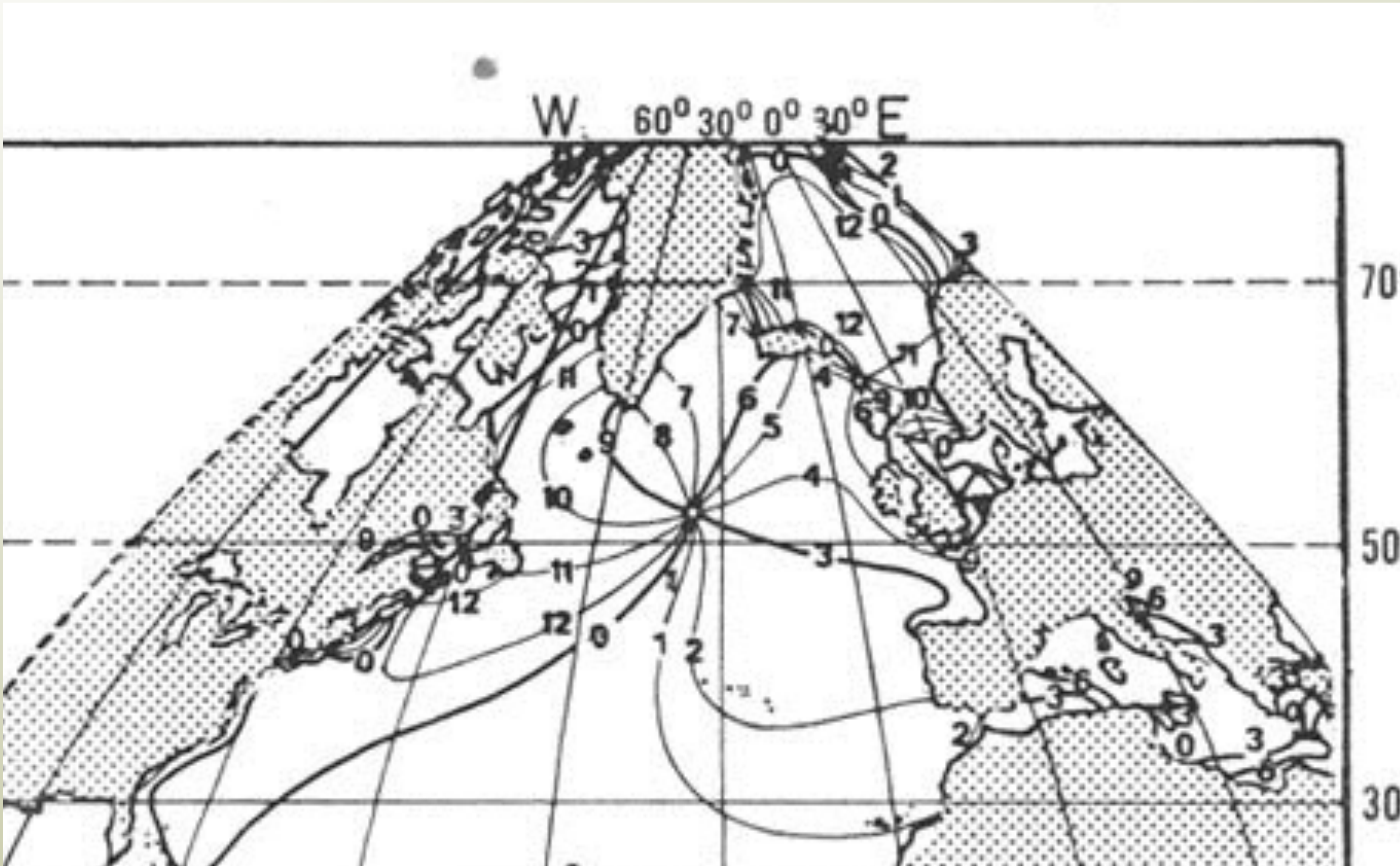
…for it is clear that at a point where all the cotidal lines meet, it is high water equally at all hours, that is, the tide vanishes…



Cotidal Lines



Some History

Violets
Consider how the violets you smell this spring
In your forest-bound garden of rocks
Convey the same surprising scent that Sappho
Smelled some twenty centuries ago.

While empires crumble and epics fade,
The scent of the violet
Drawn from the indifferent dust
Proclaims the same enduring news:
The mute and fragrant gospel of the grass.

Cornel Adam Lengyel

Topology in a Nutshell

Topology in a Nutshell

Topology in a Nutshell

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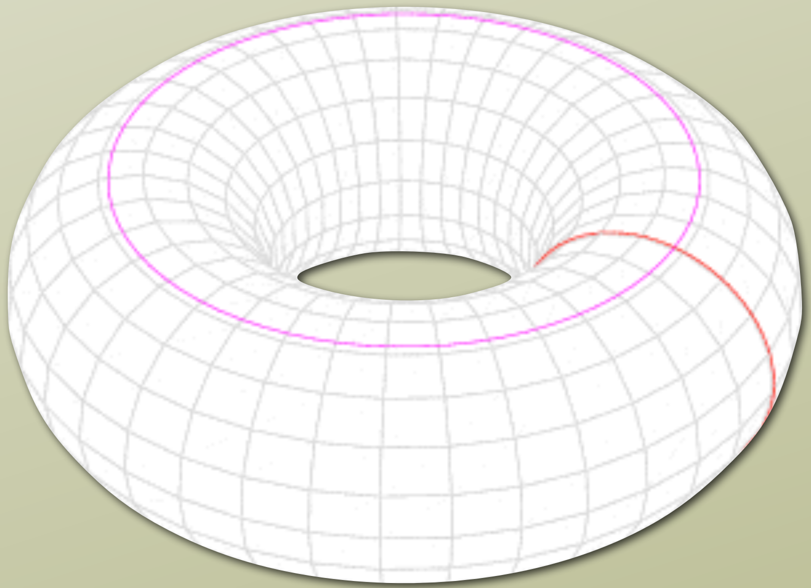
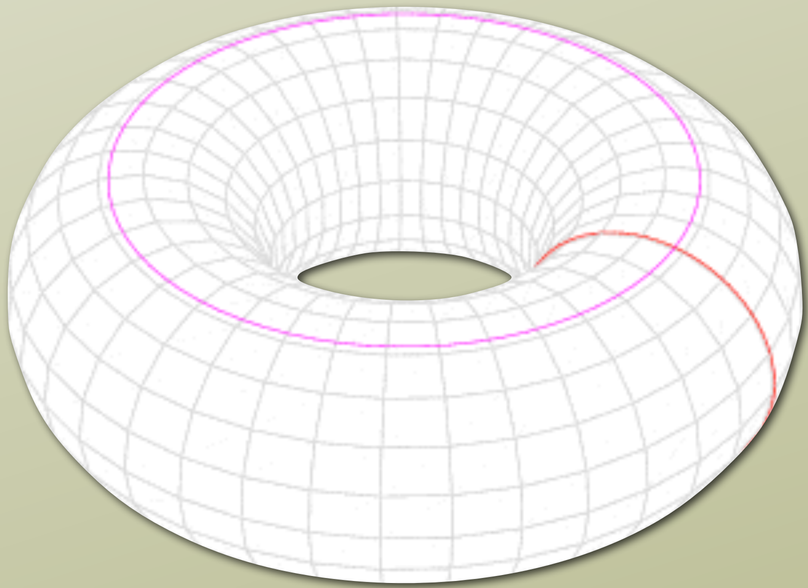
Topology in a Nutshell



Topology Turns Geometry into Counting

Topology in a Nutshell



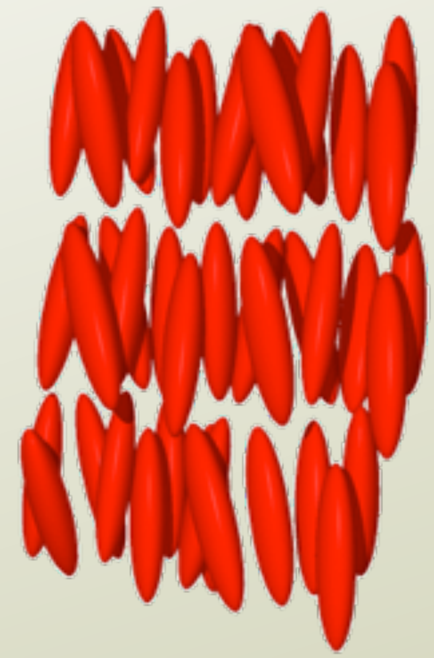
Topology Turns Geometry into Counting

Liquid Crystals

isotropic

nematic

3-D fluids



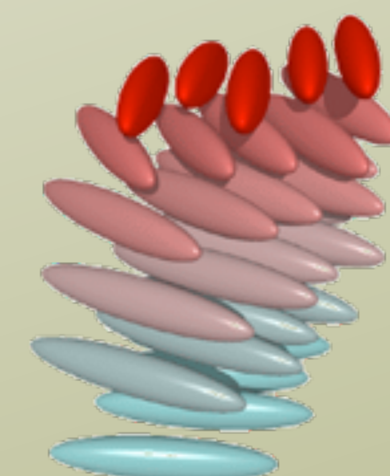
smectic A

smectic C

2-D fluids

Otto Lehman

discotic



cholesteric

Friedrich Reinitzer

http://www2.sfu.ca/chemistry/faculty/Williams/phasetypes.html

Liquid Crystals

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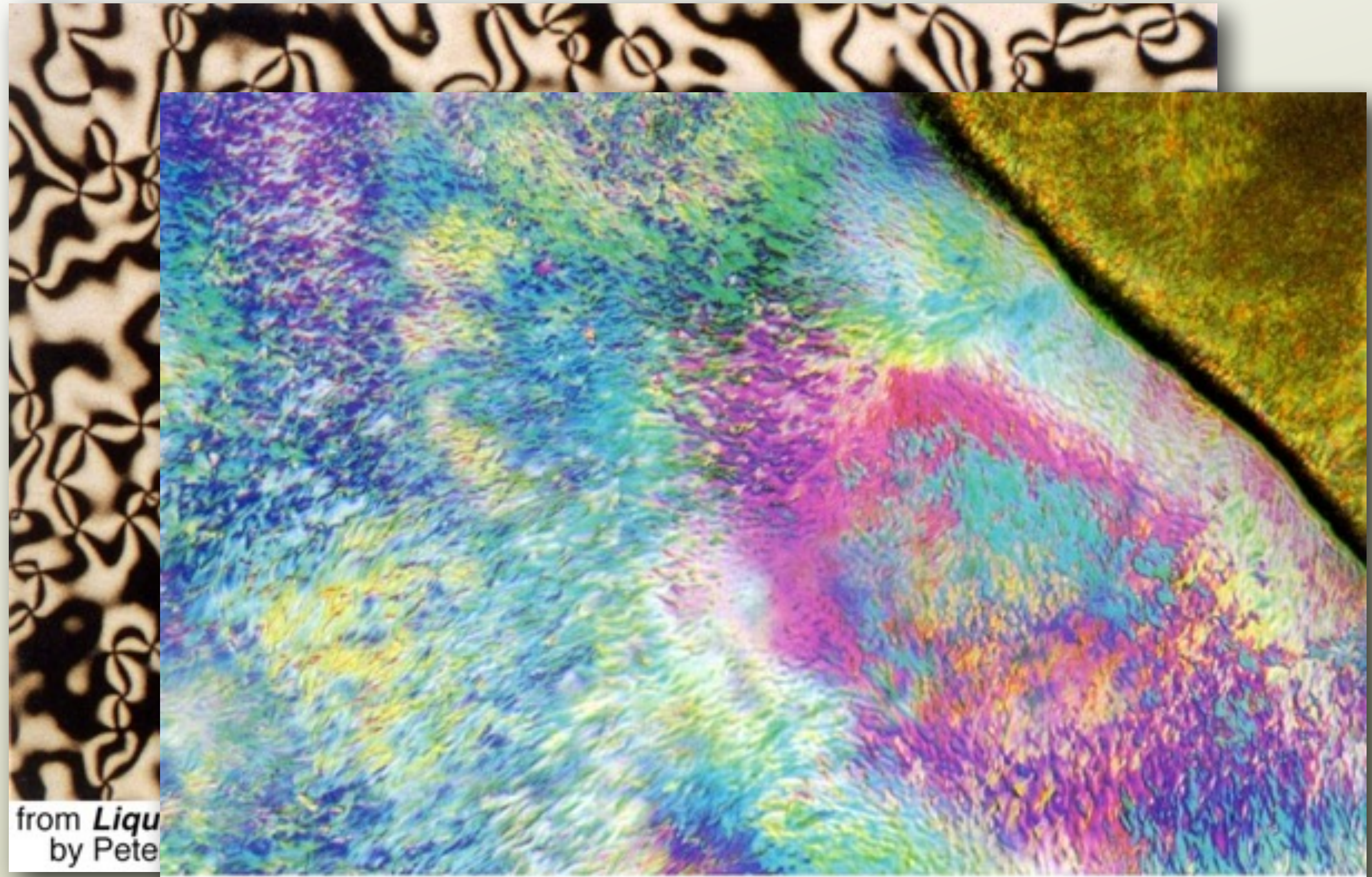
Liquid Crystals

from ***Liquid Crystals : Nature's Delicate Phase of Matter***
by Peter J. Collings, Princeton University Press, 1990

Liquid Crystals



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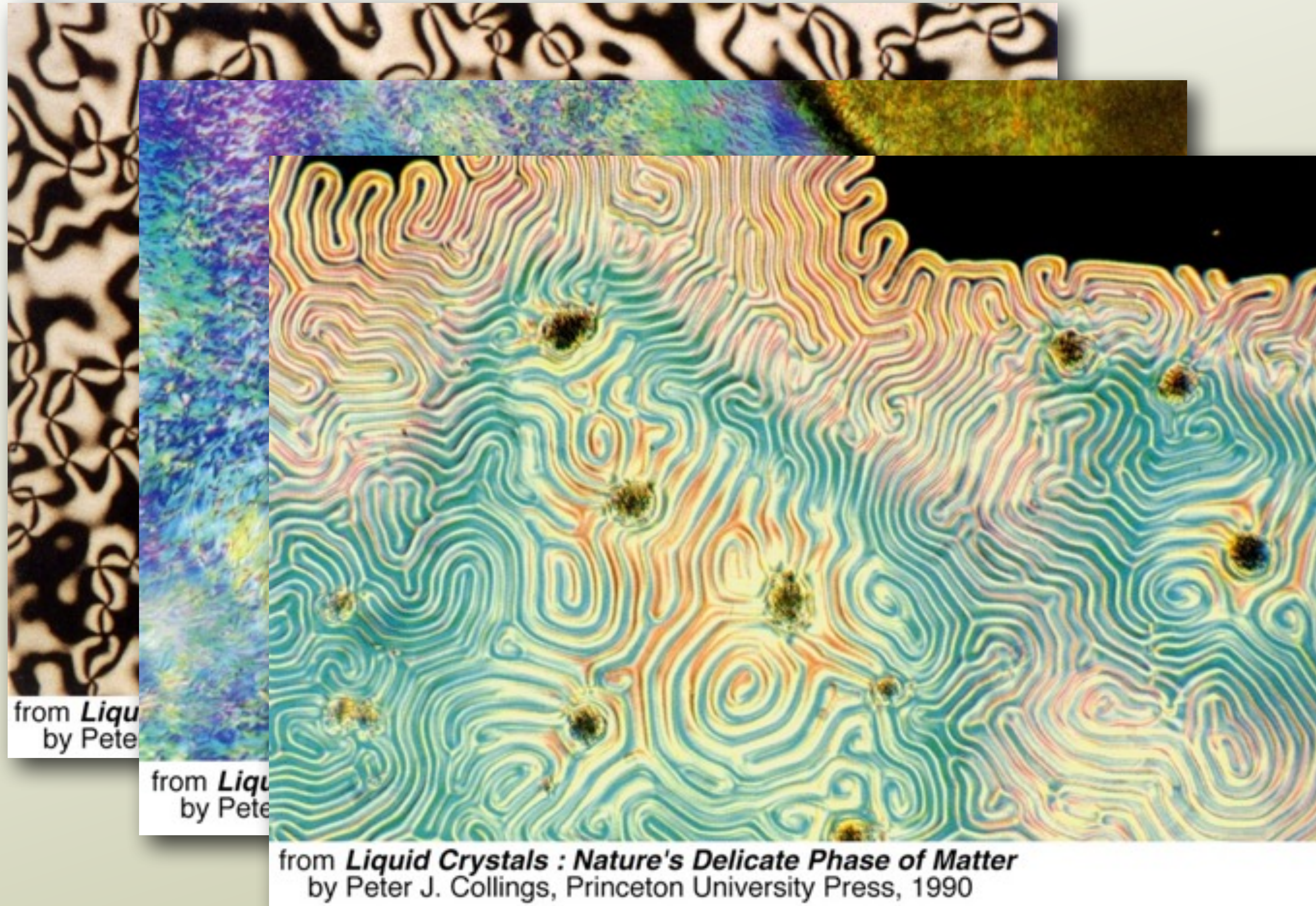

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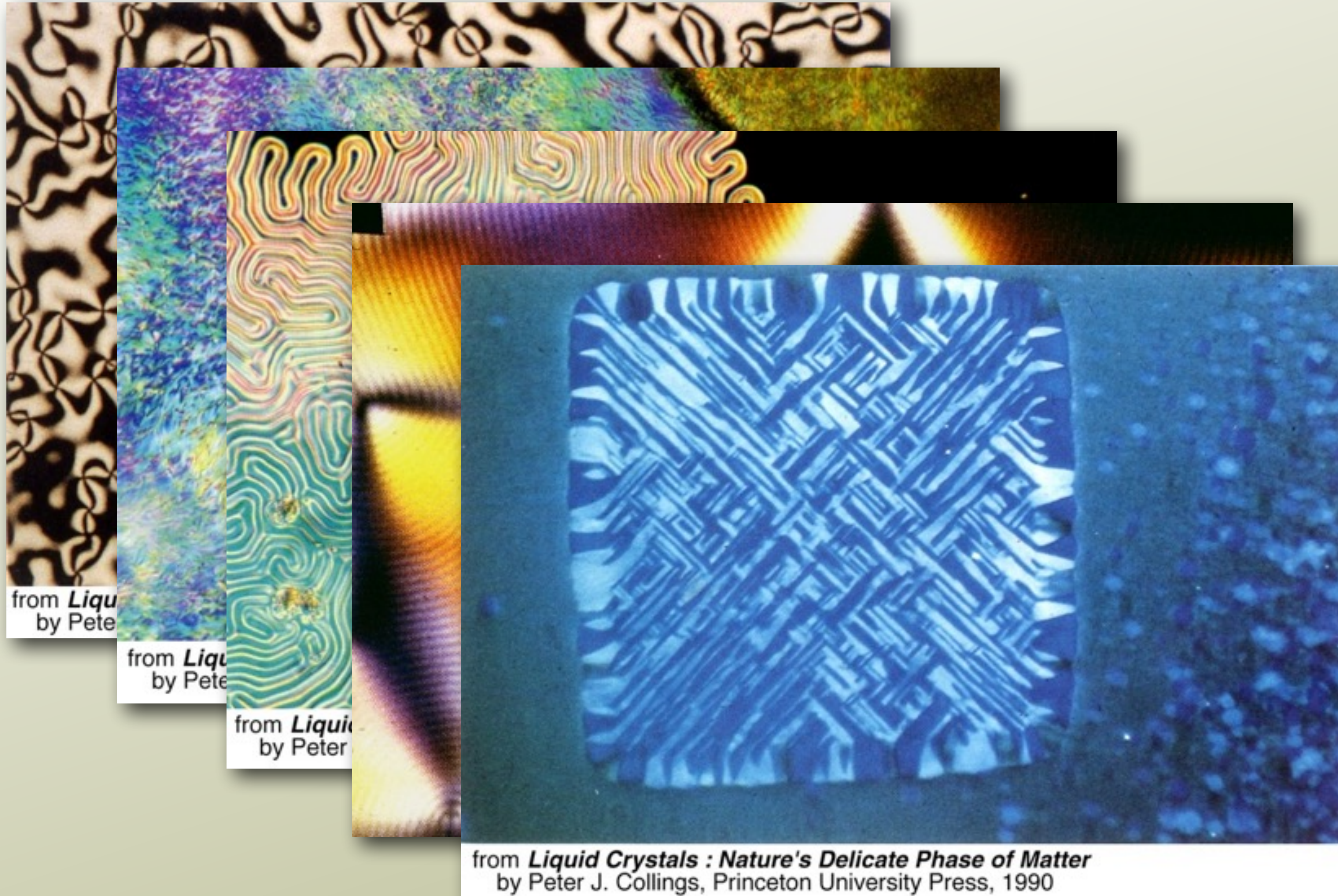
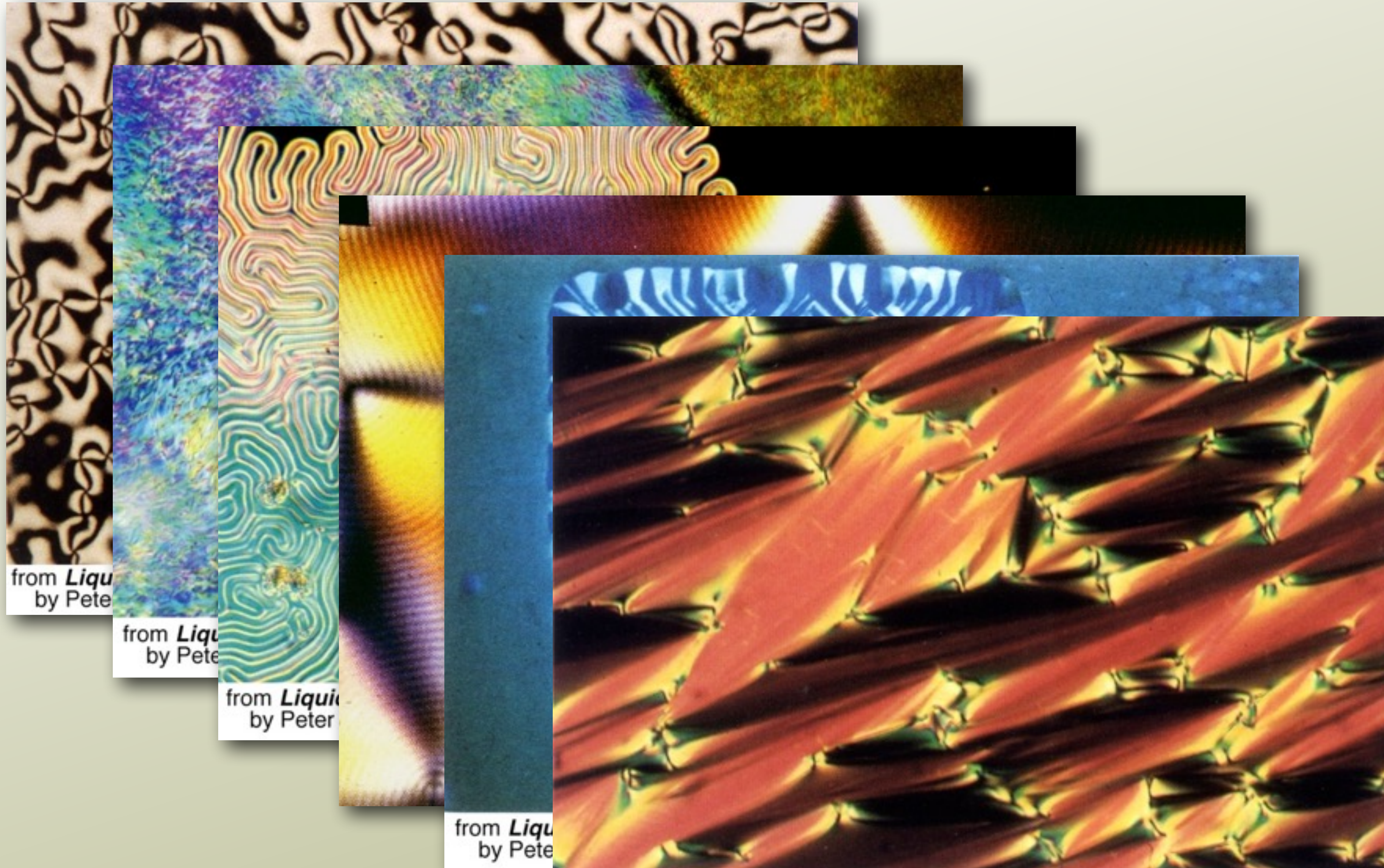


Liquid Crystals



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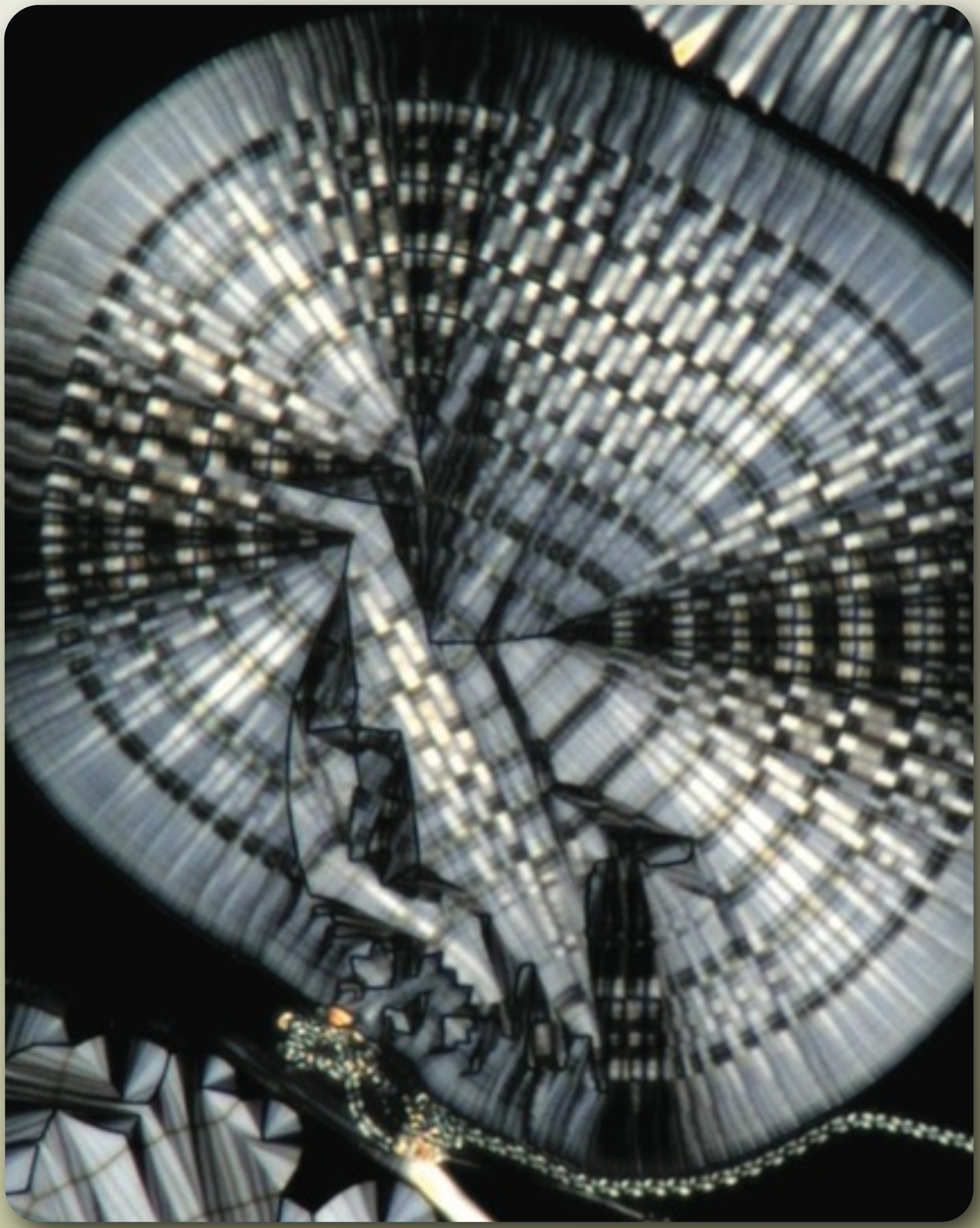
Smectic Liquid Crystals and Lamellae



Photos by Michi Nakata

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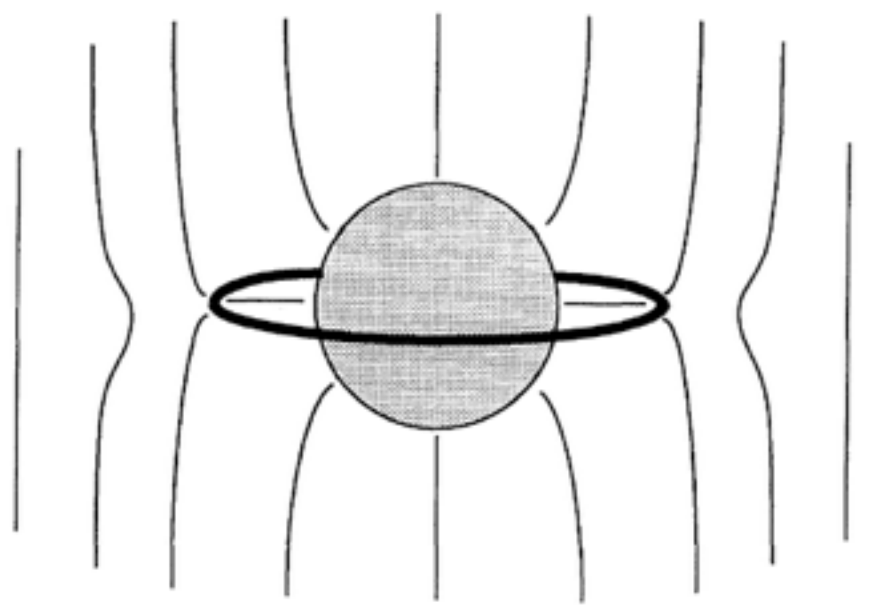
Film by Jean Painleve, Liquid Crystals by Yves Bouligand

Film by Jean Painleve, Liquid Crystals by Yves Bouligand

O Topology

O Topology



Terentjev, PRE **51** (1995) 1330

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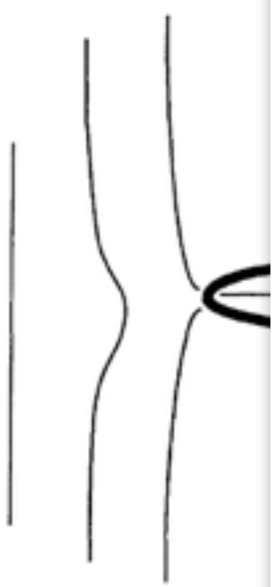

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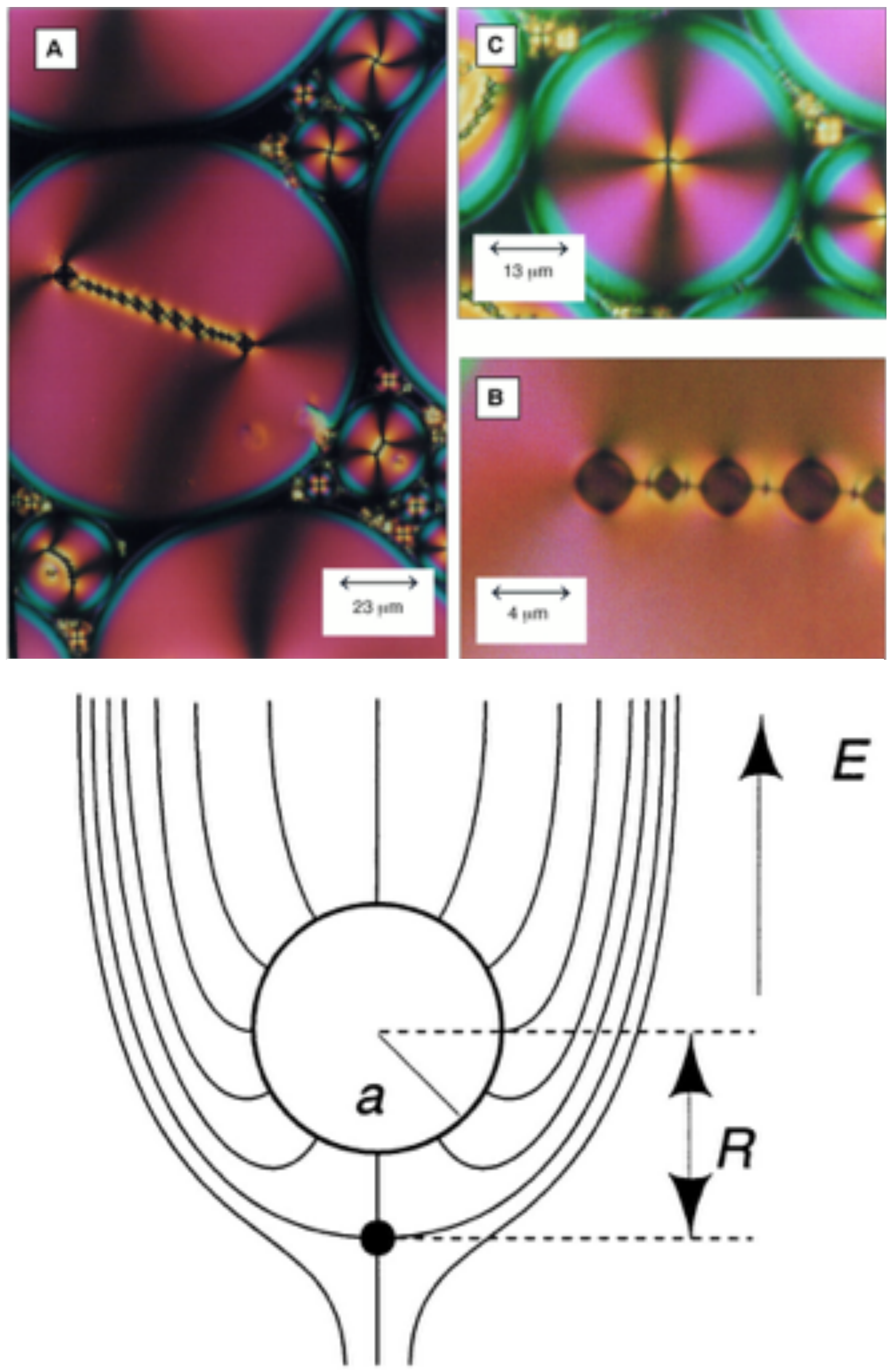

Poulin, *etc*. Science **275** (1997) 1770

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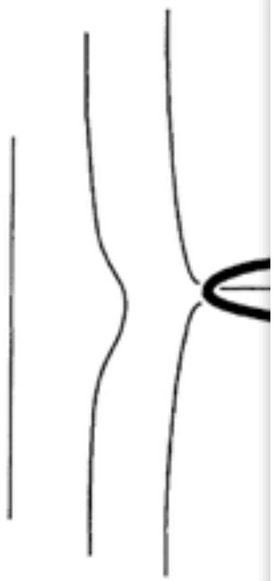

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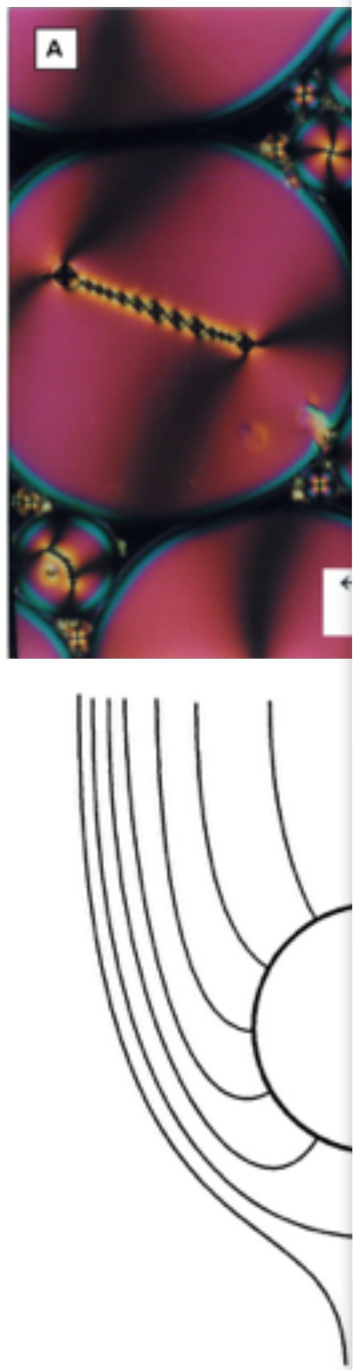

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HIGHLIGHT
Miha Ravnik and Slobodan Žumer
Nematic colloids entangled by topological defects

COMMUNICATION
Andrew J. Parnell *et al.*
Direct visualization of the real time swelling and collapse of a poly(methacrylic acid) brush using atomic force microscopy

1744-683X(2009)5:2;1-K

Ravnik and Zumer, (2009)

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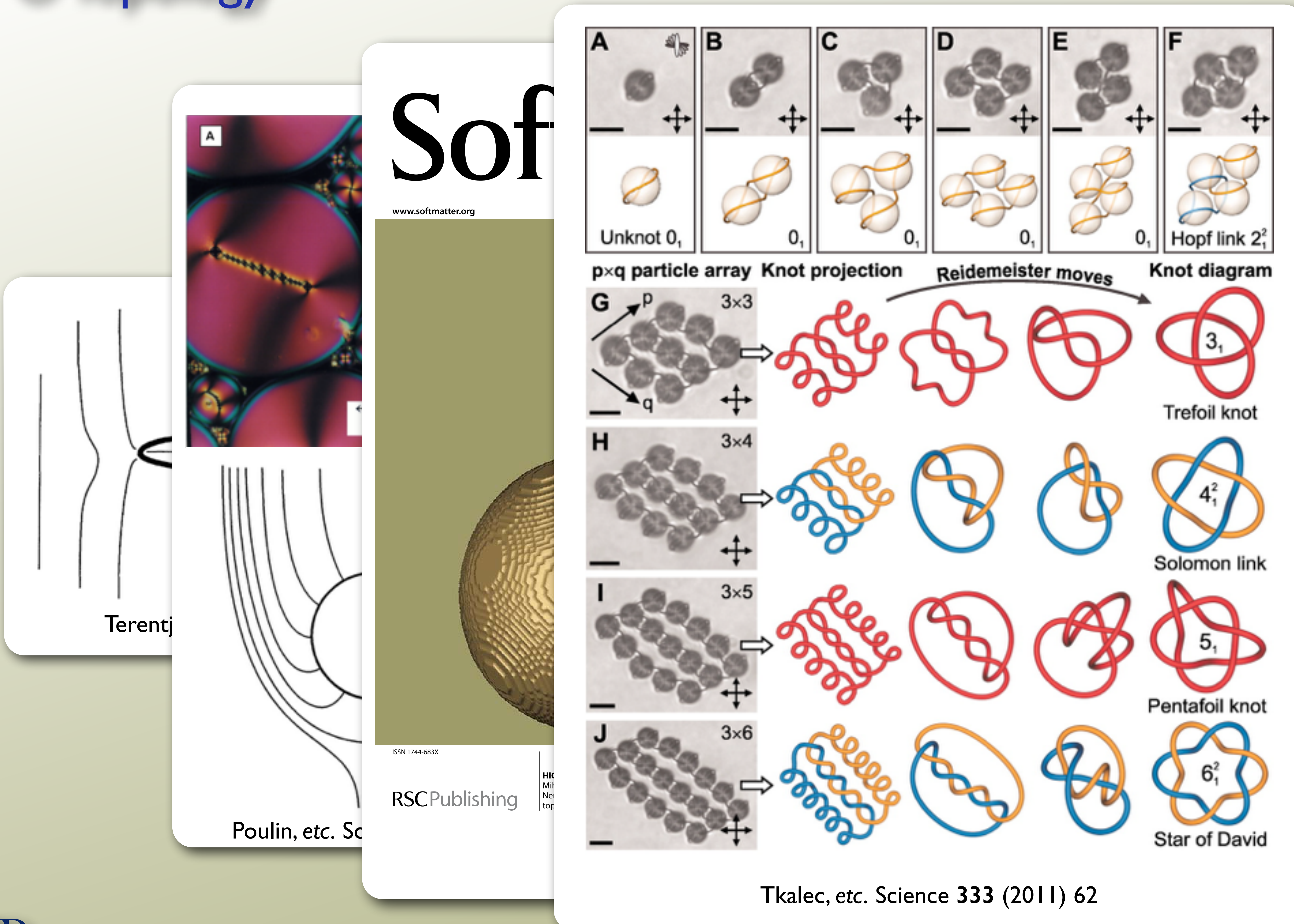

Tkalec, *etc.* Science **333** (2011) 62

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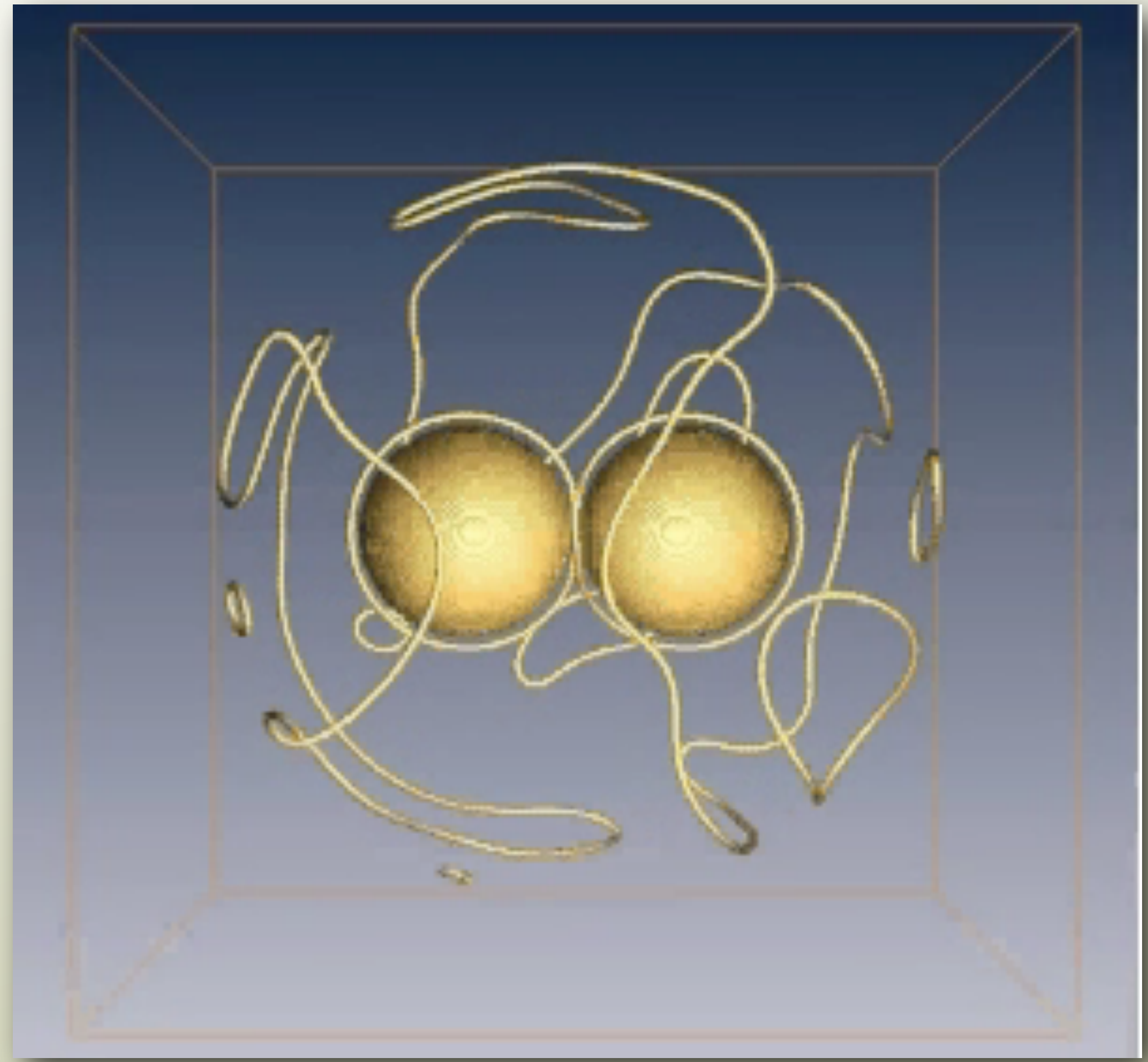

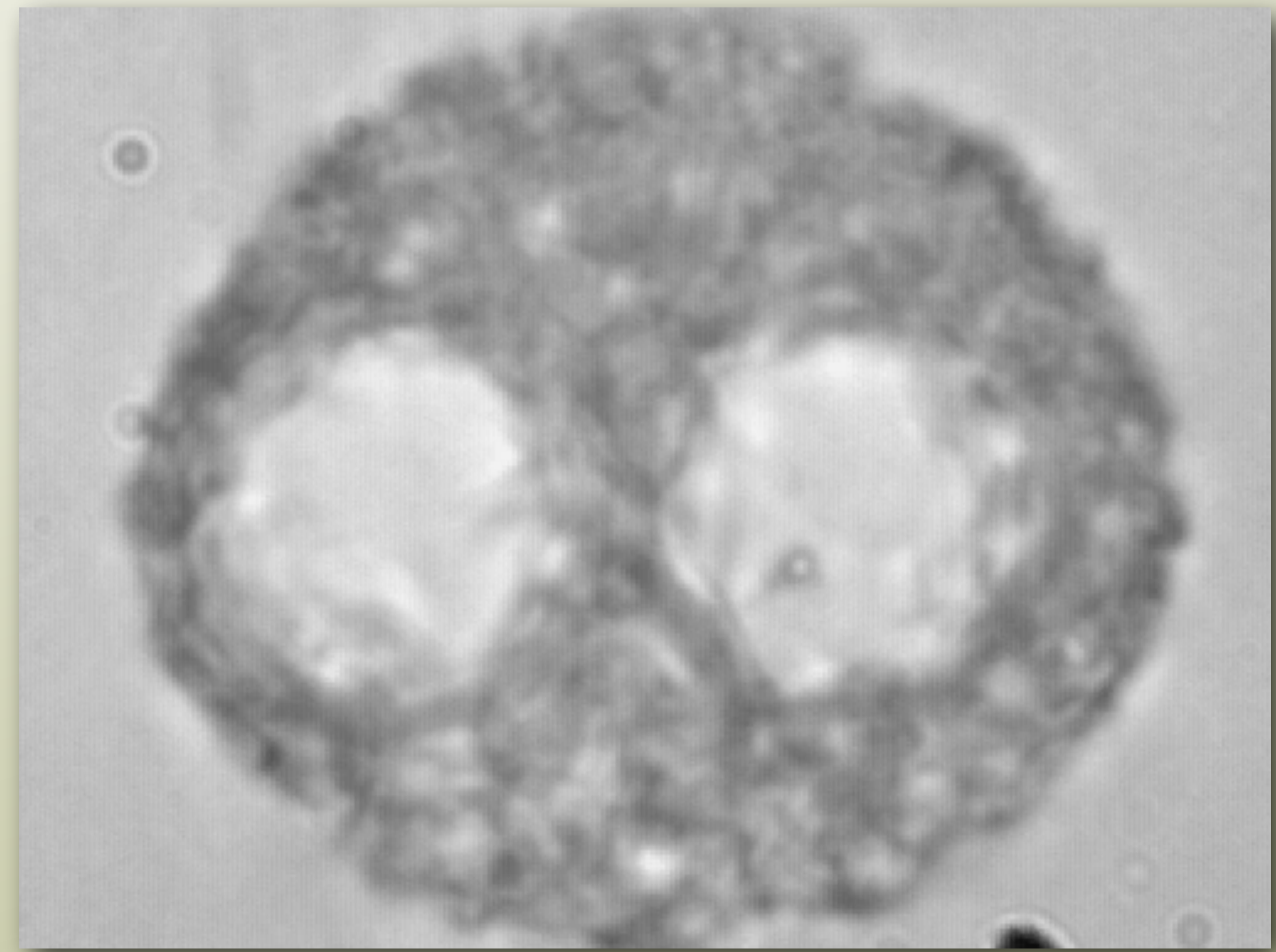

M. Ravnik, M. Škarabot, S. Žumer, U. Tkalec, I. Poberaj, D. Babič, N. Osterman, I. Muševič, *PRL* **99** (2007)

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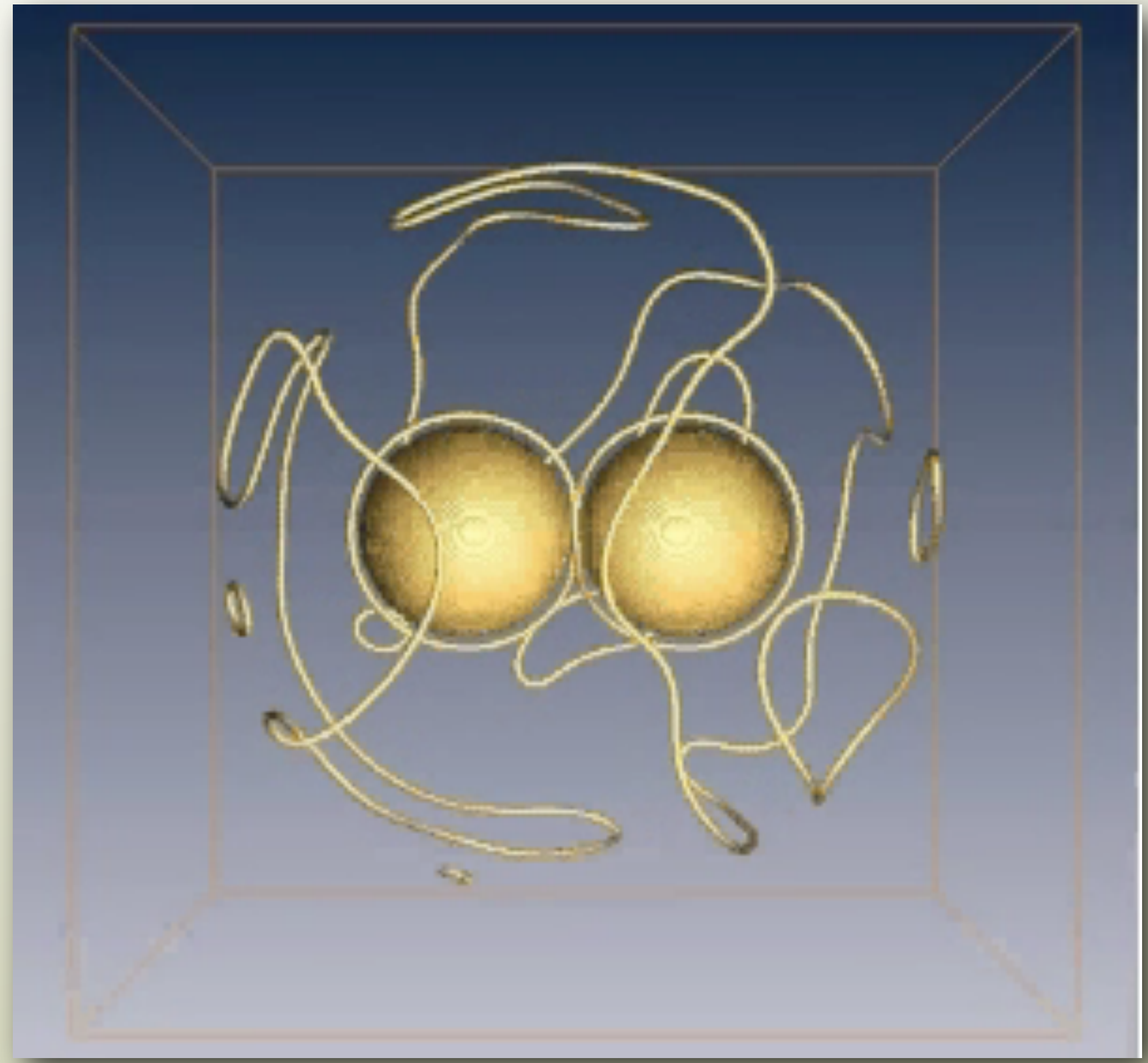

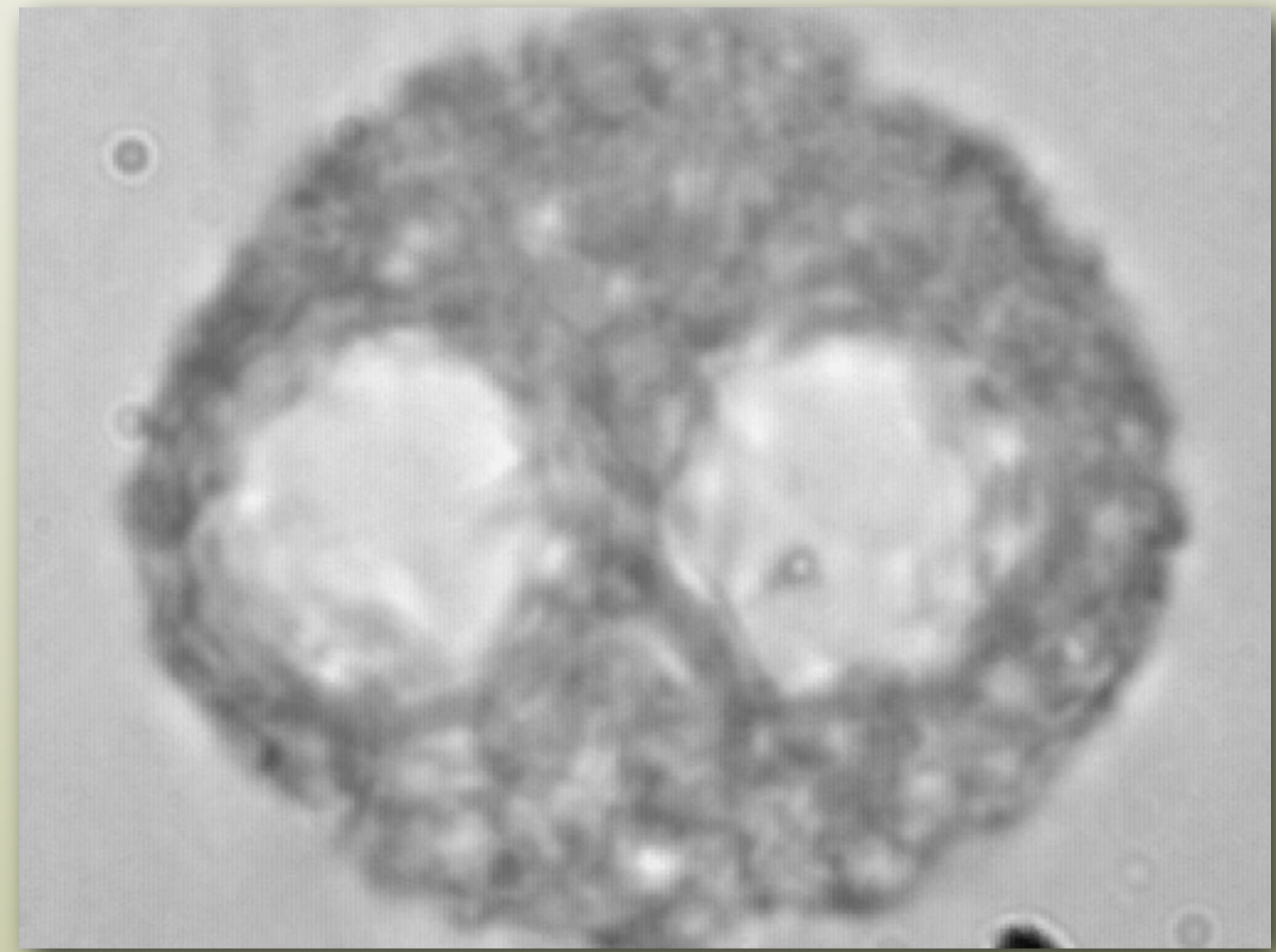

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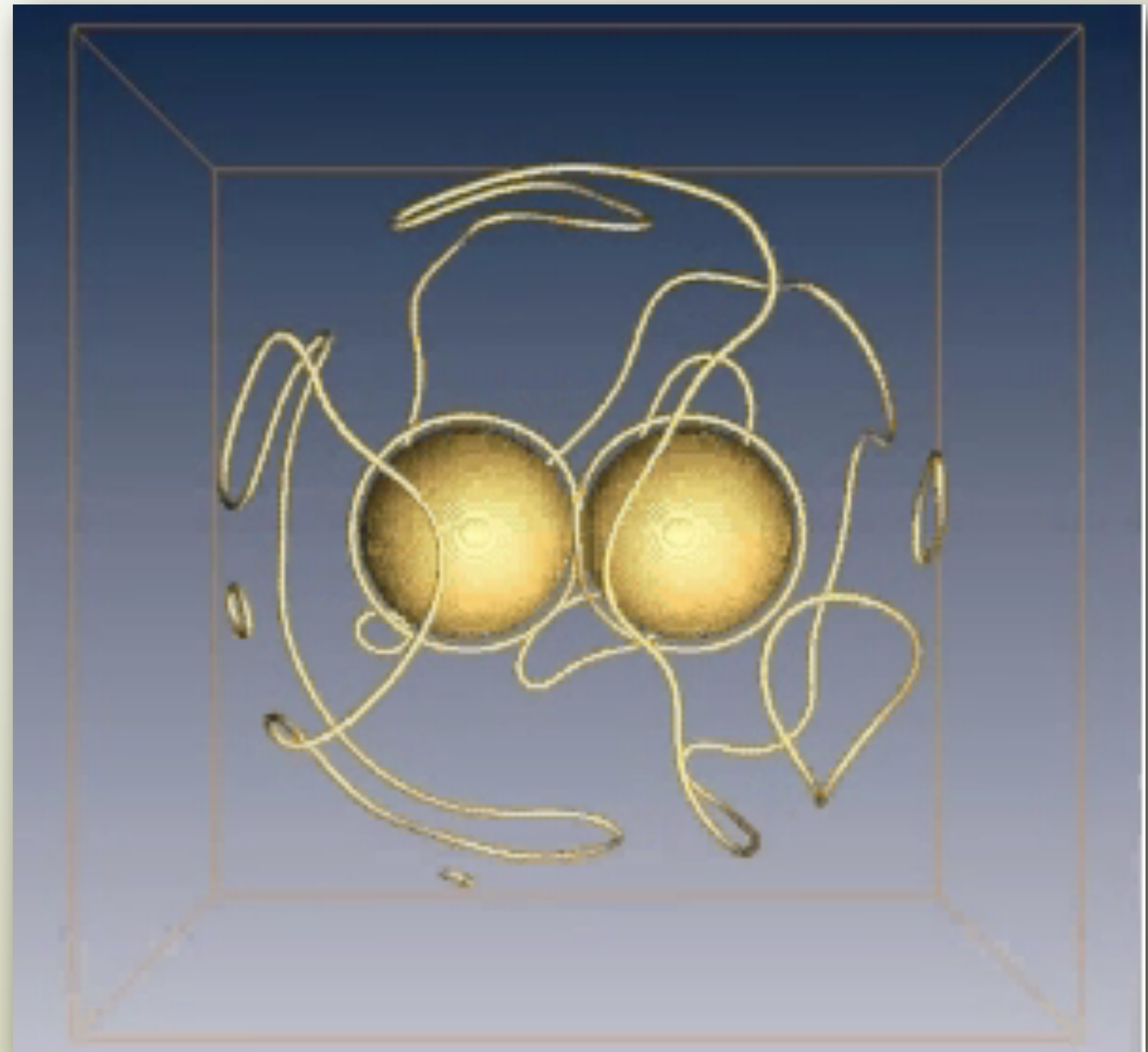

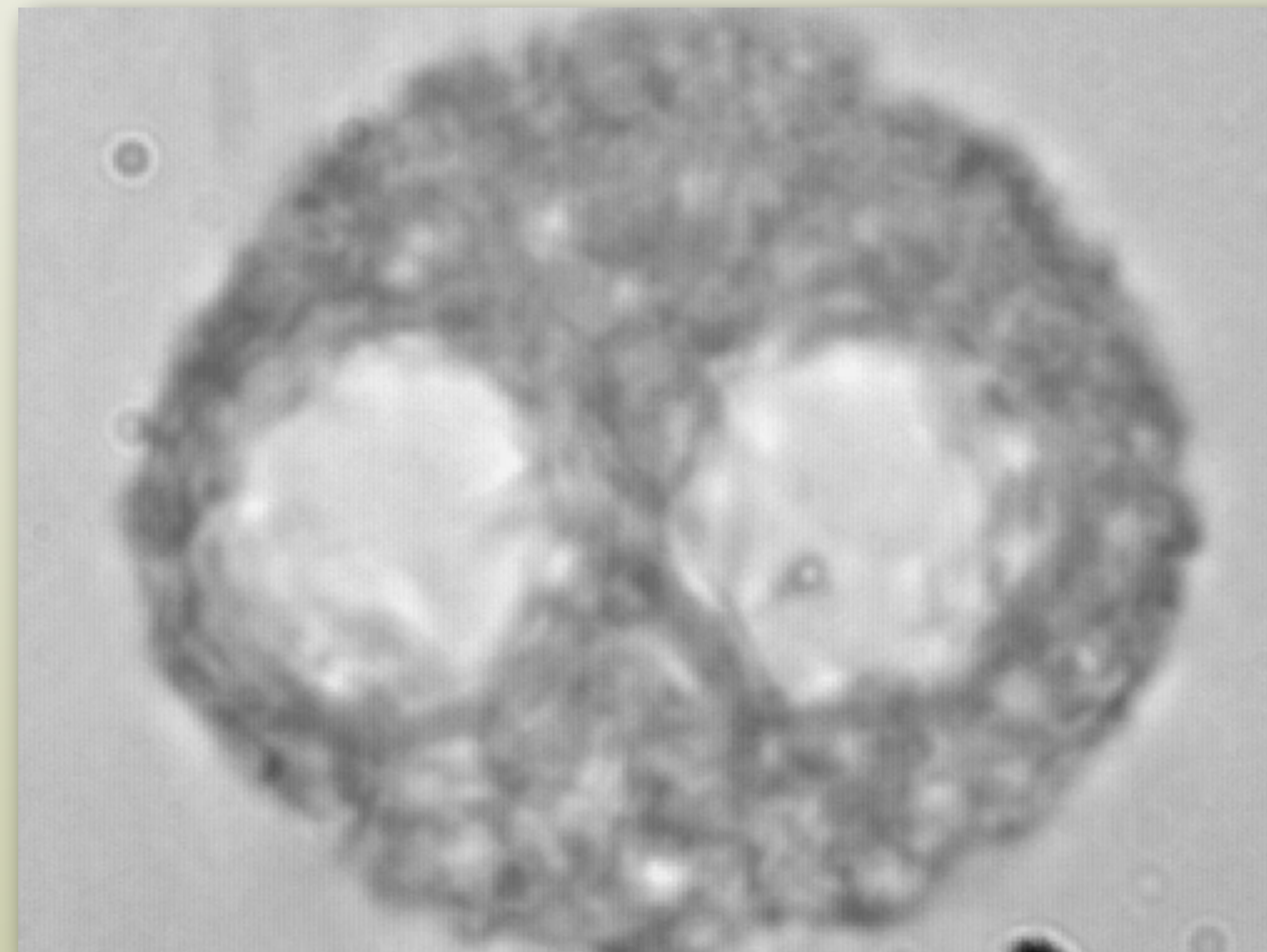

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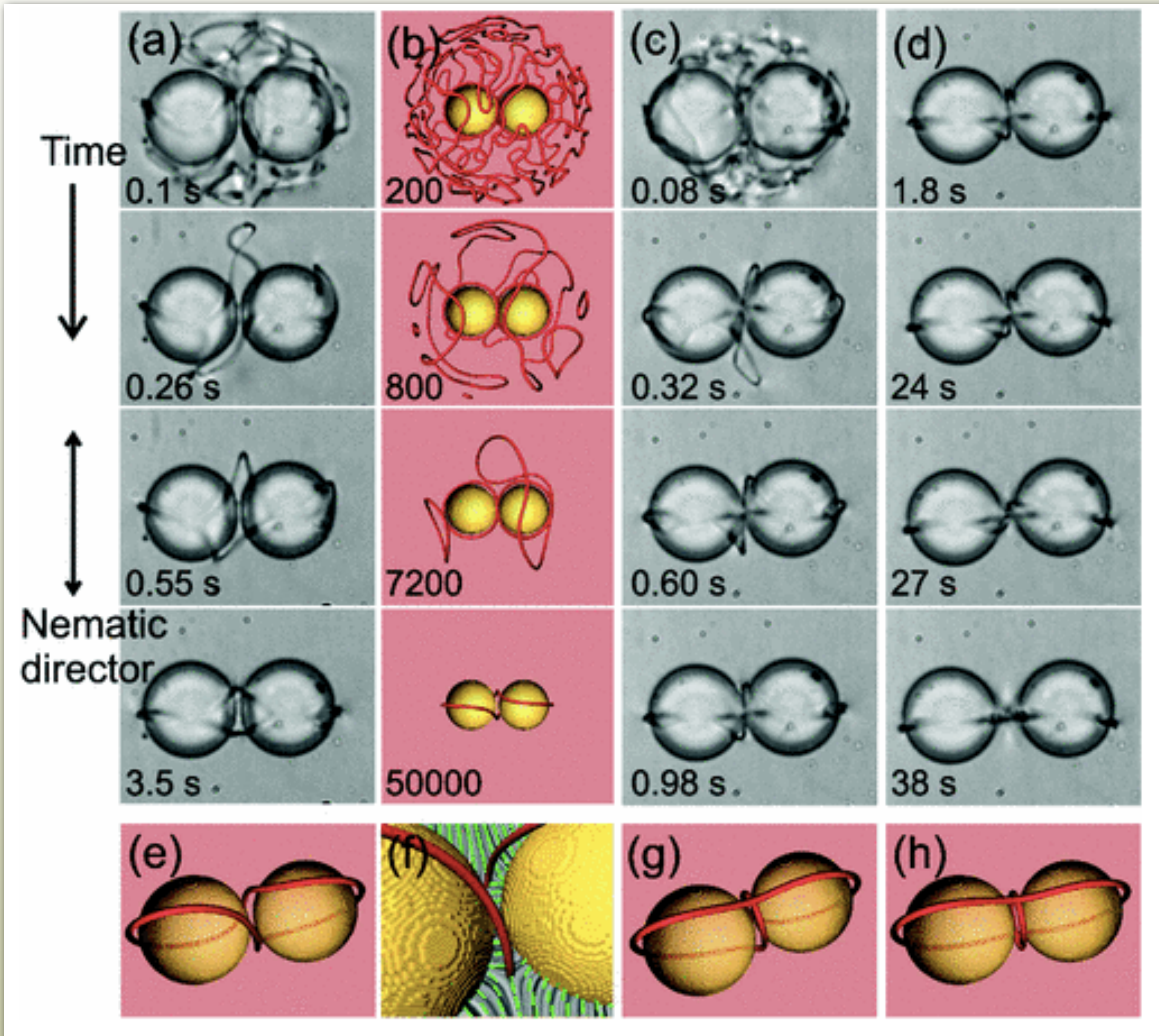

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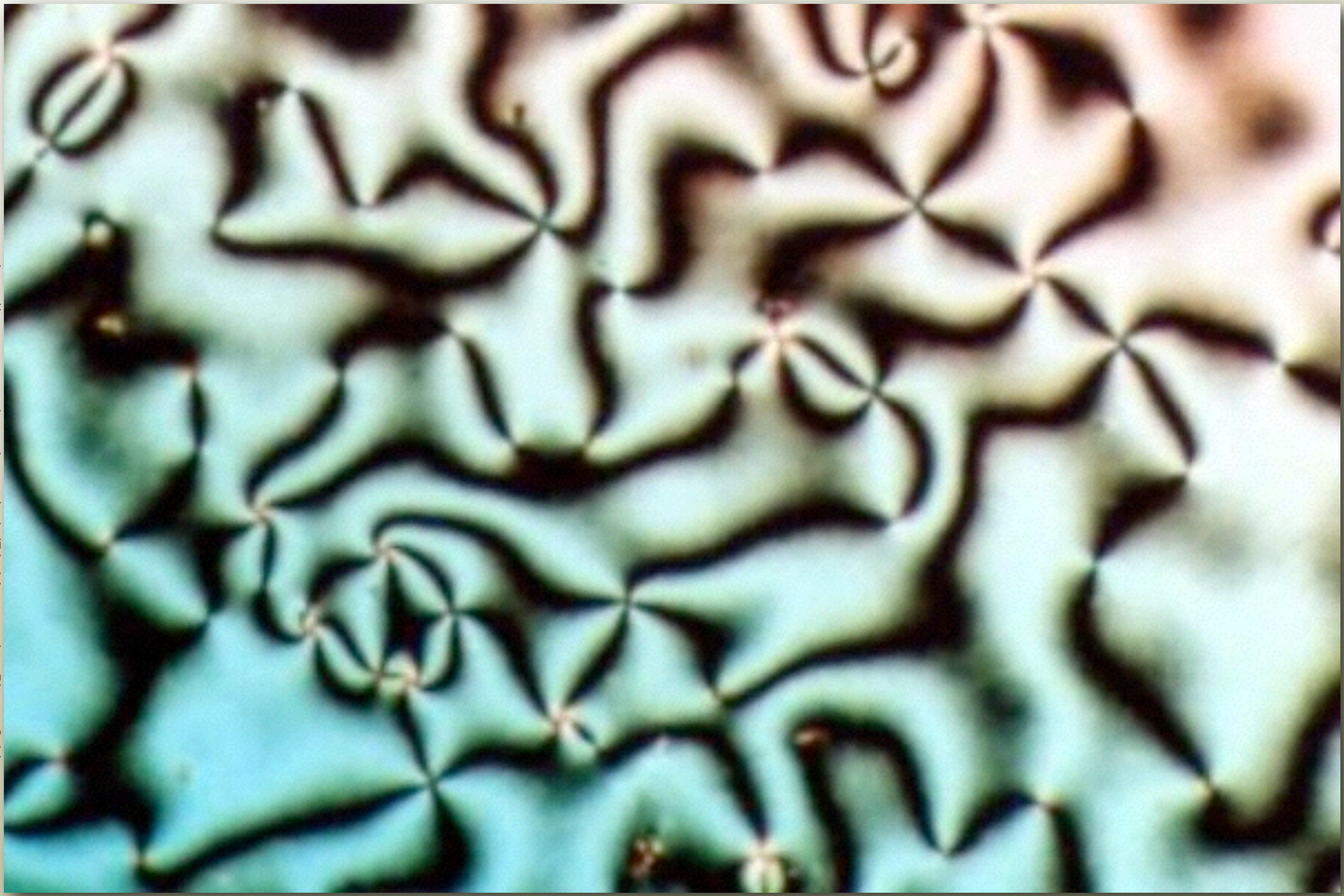
Nematics in Two Dimensions



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Maps from $\mathbb{R}^2\backslash\{0\} \to \mathbb{R}P^1$

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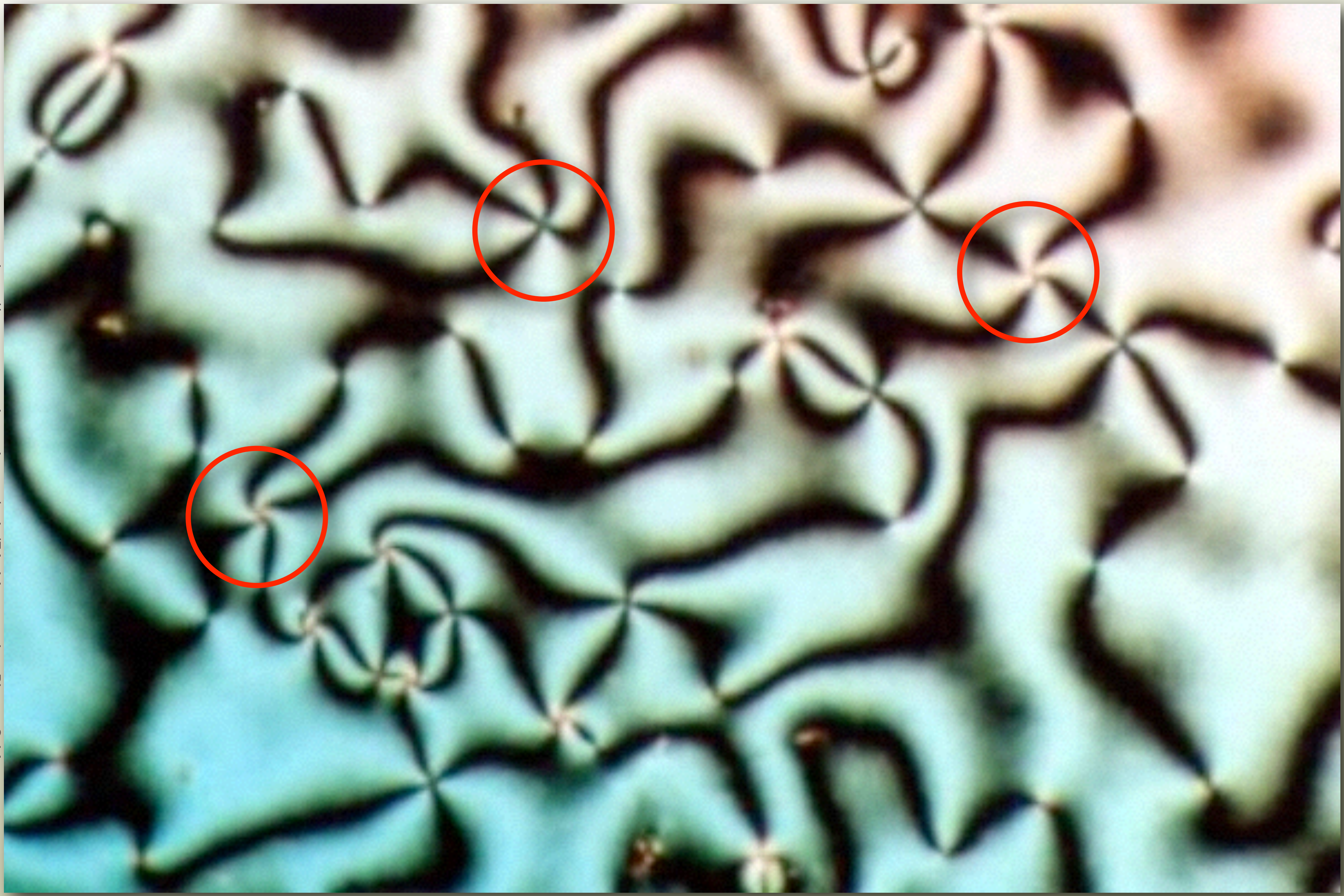
Nematics in Two Dimensions



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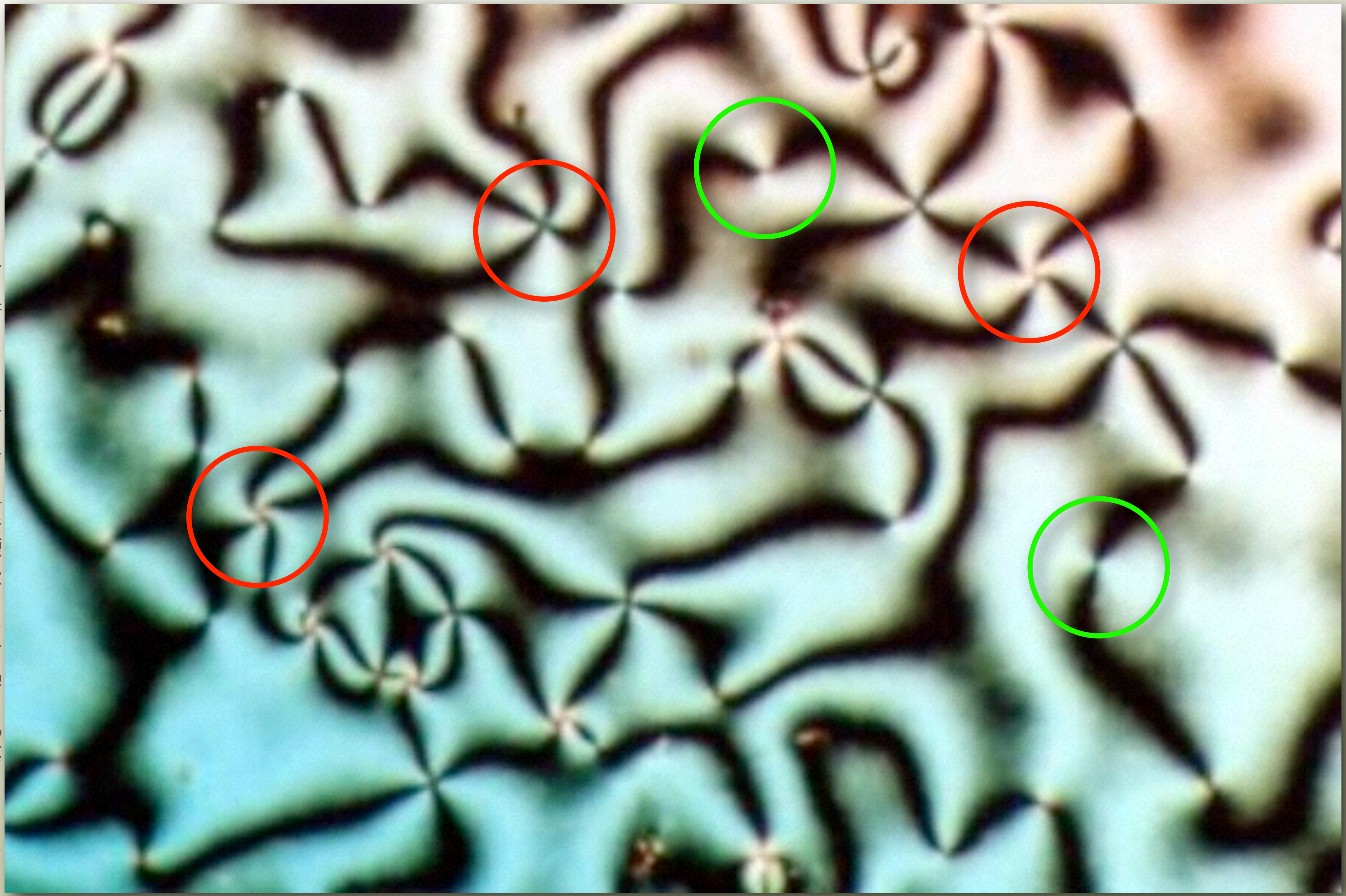

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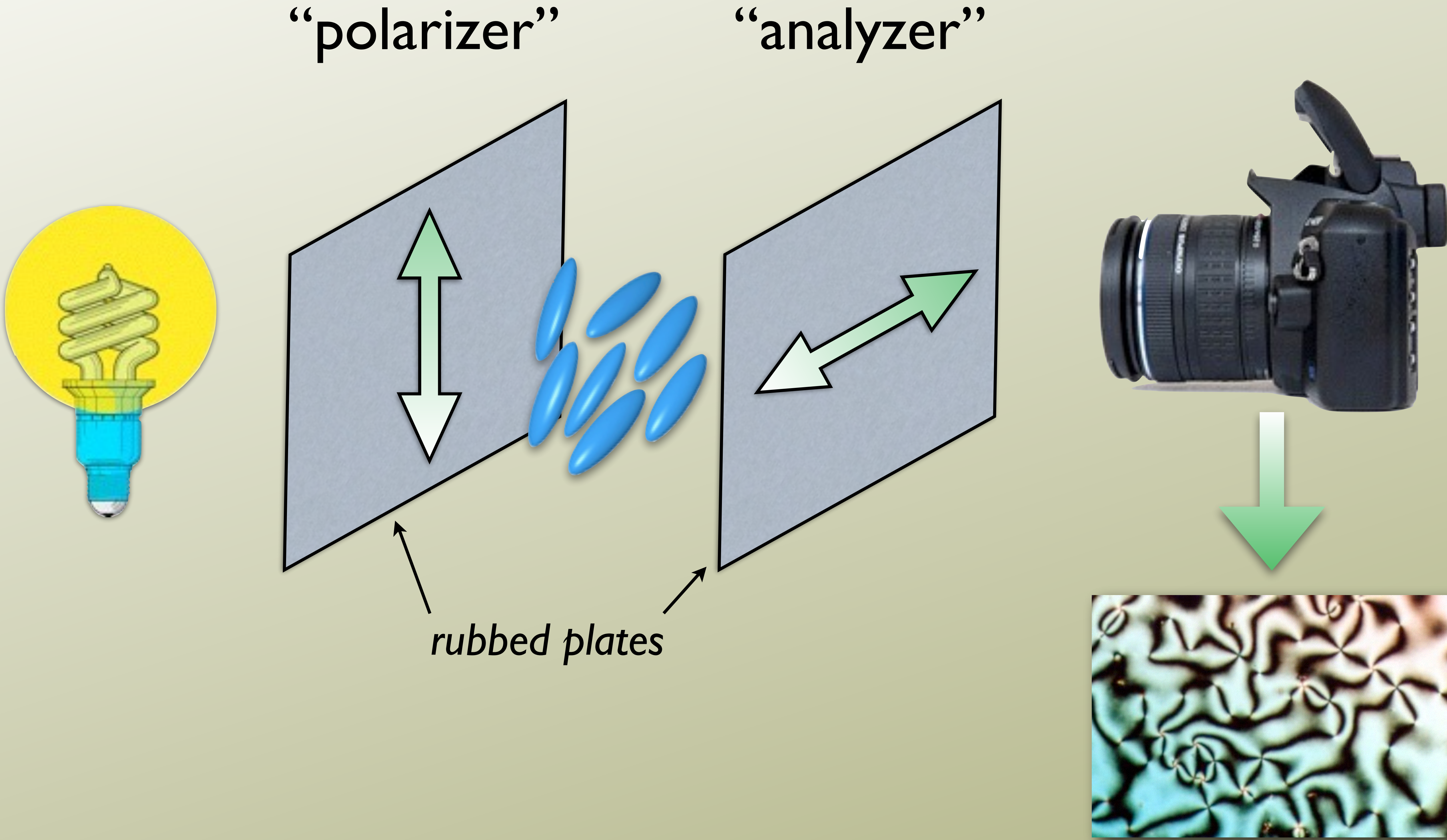
Nematics in Two Dimensions: *What are we seeing?*



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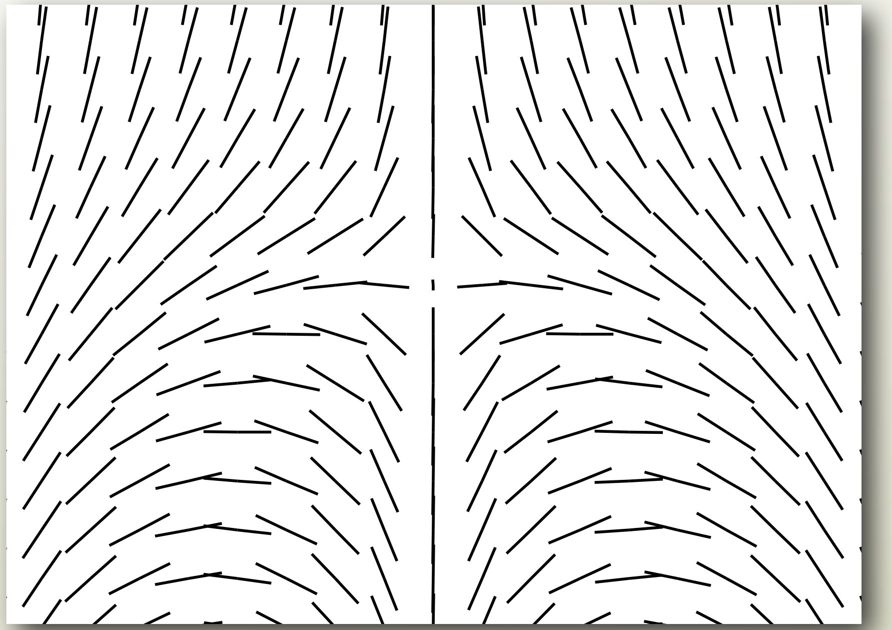

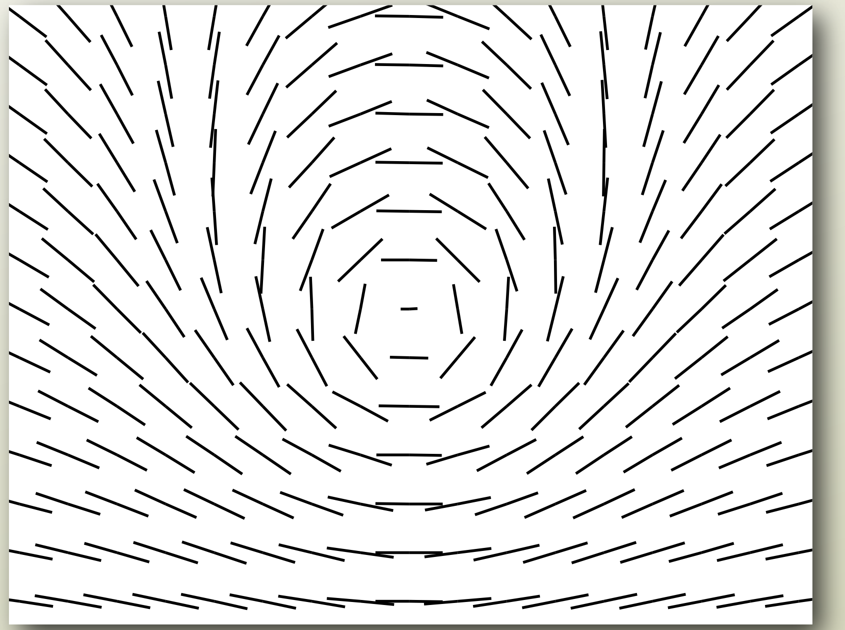

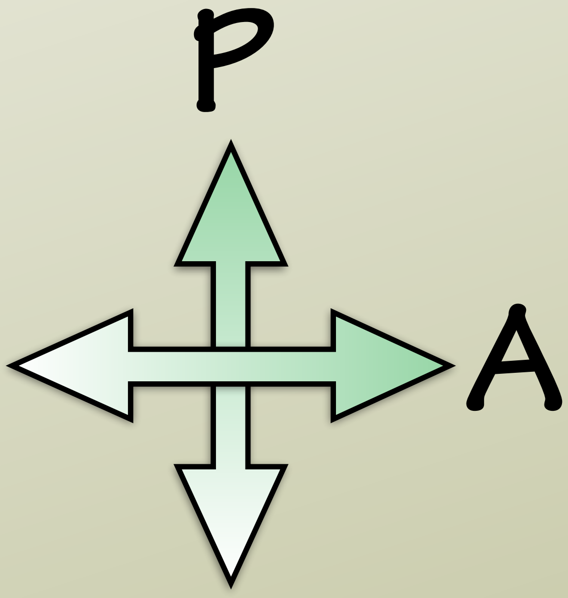

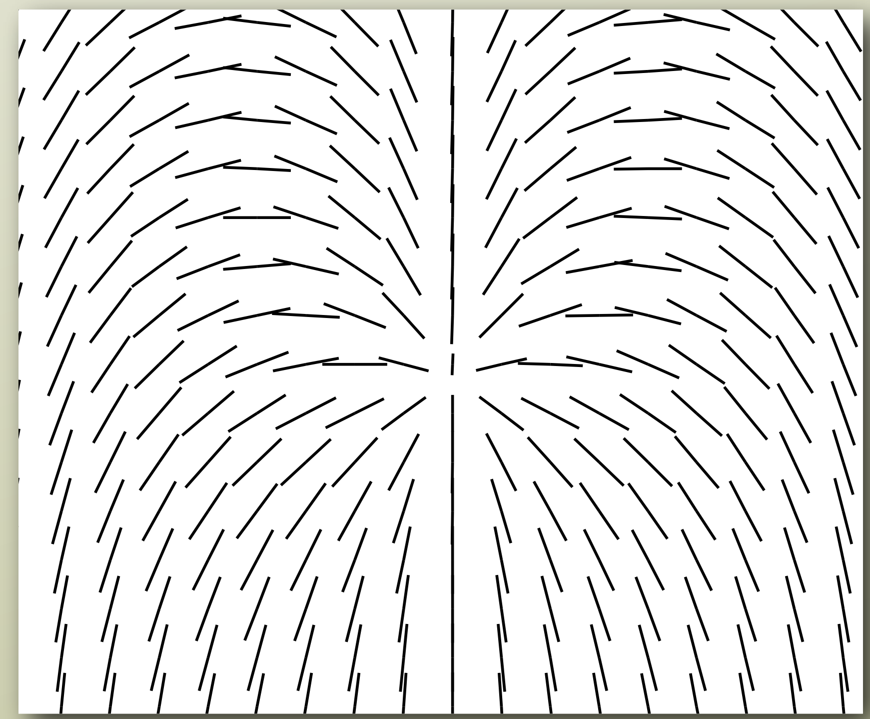

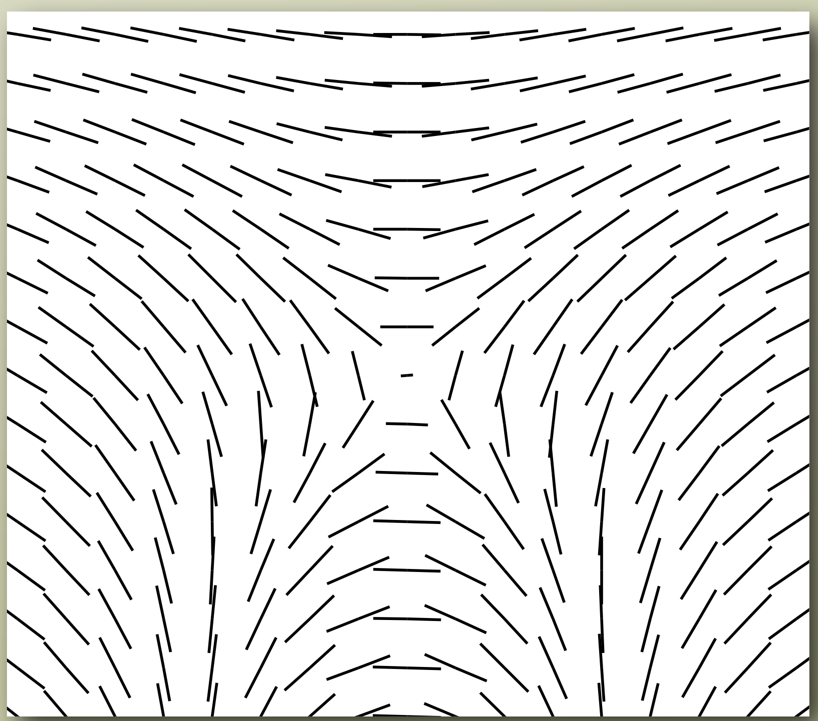

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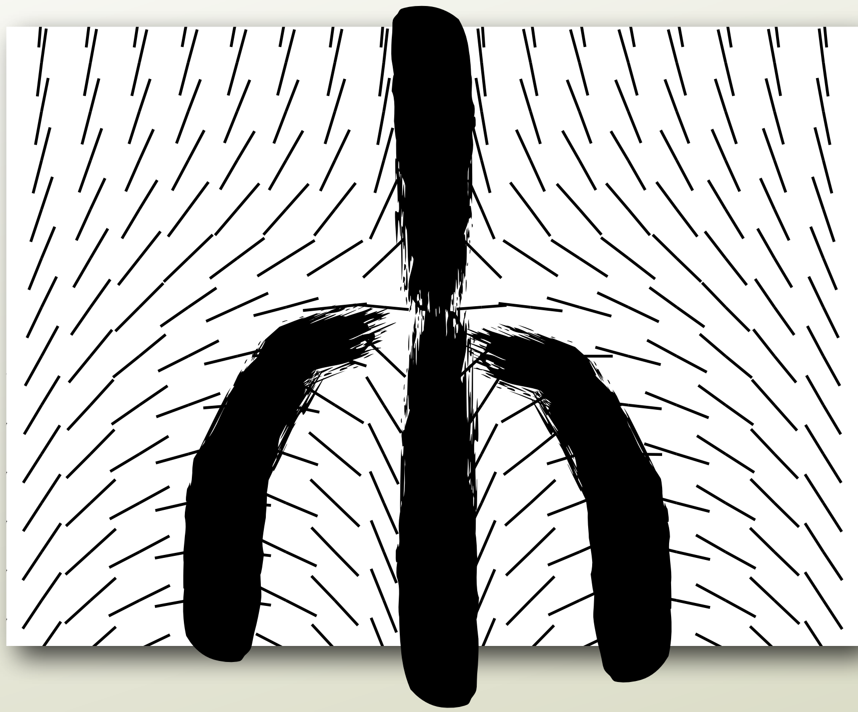
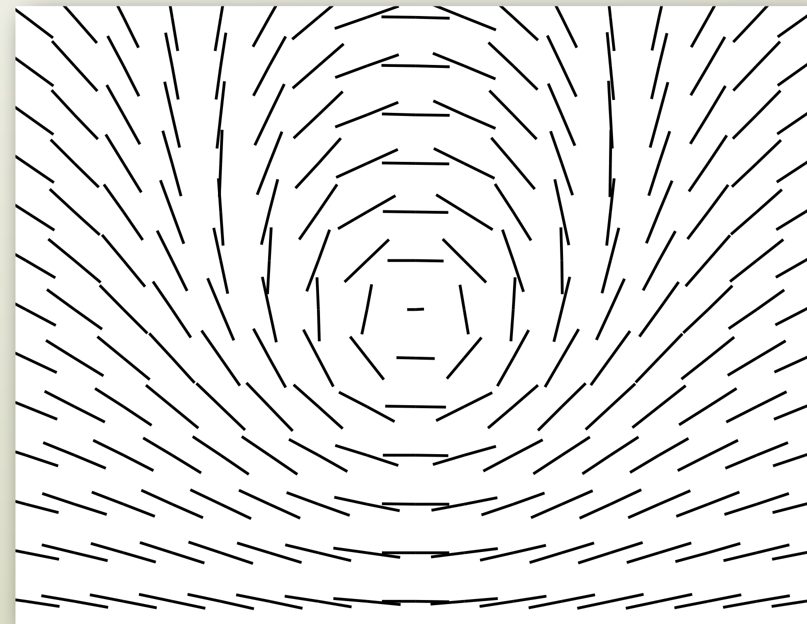
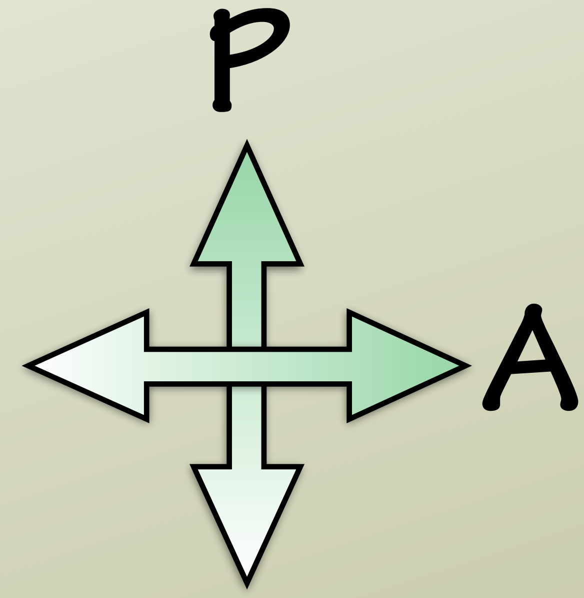
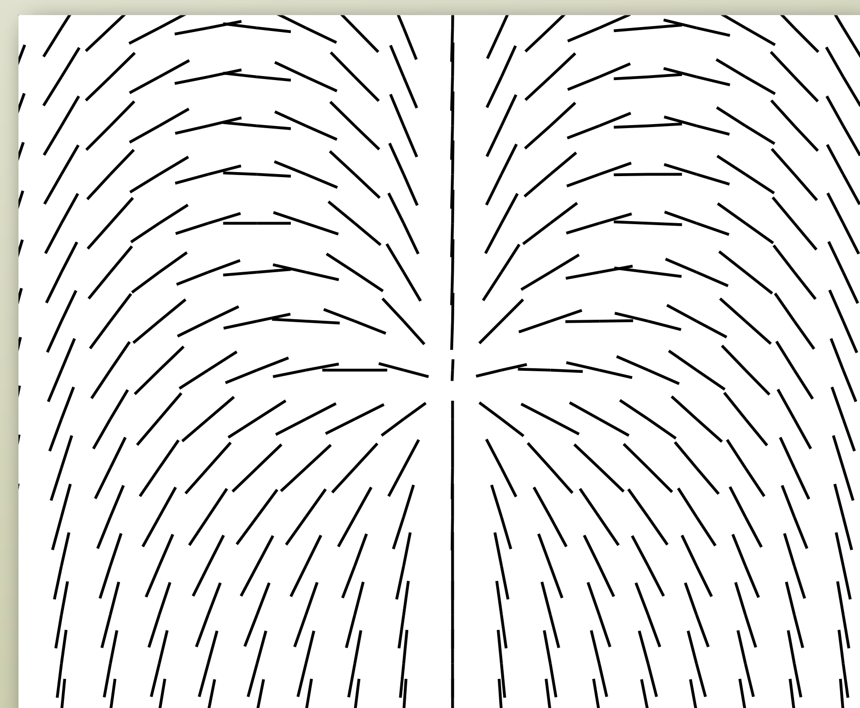
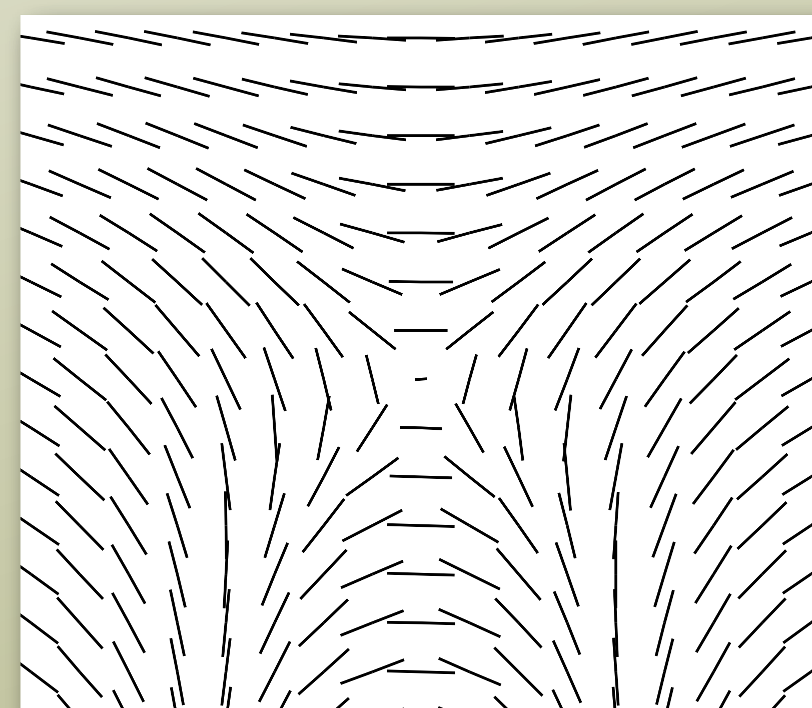




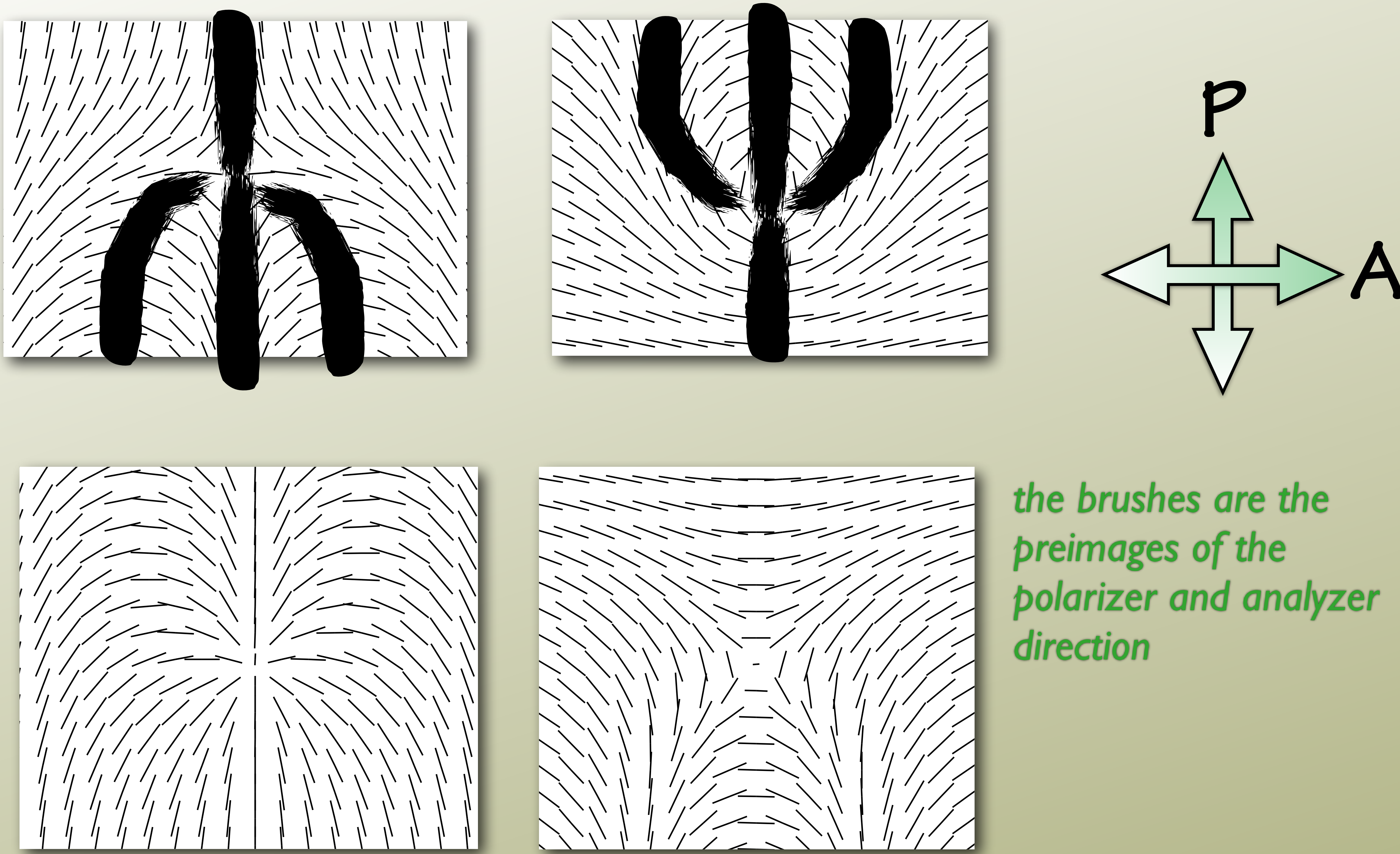






the brushes are the preimages of the polarizer and analyzer direction

Nematics in Two Dimensions: *What are we seeing?*



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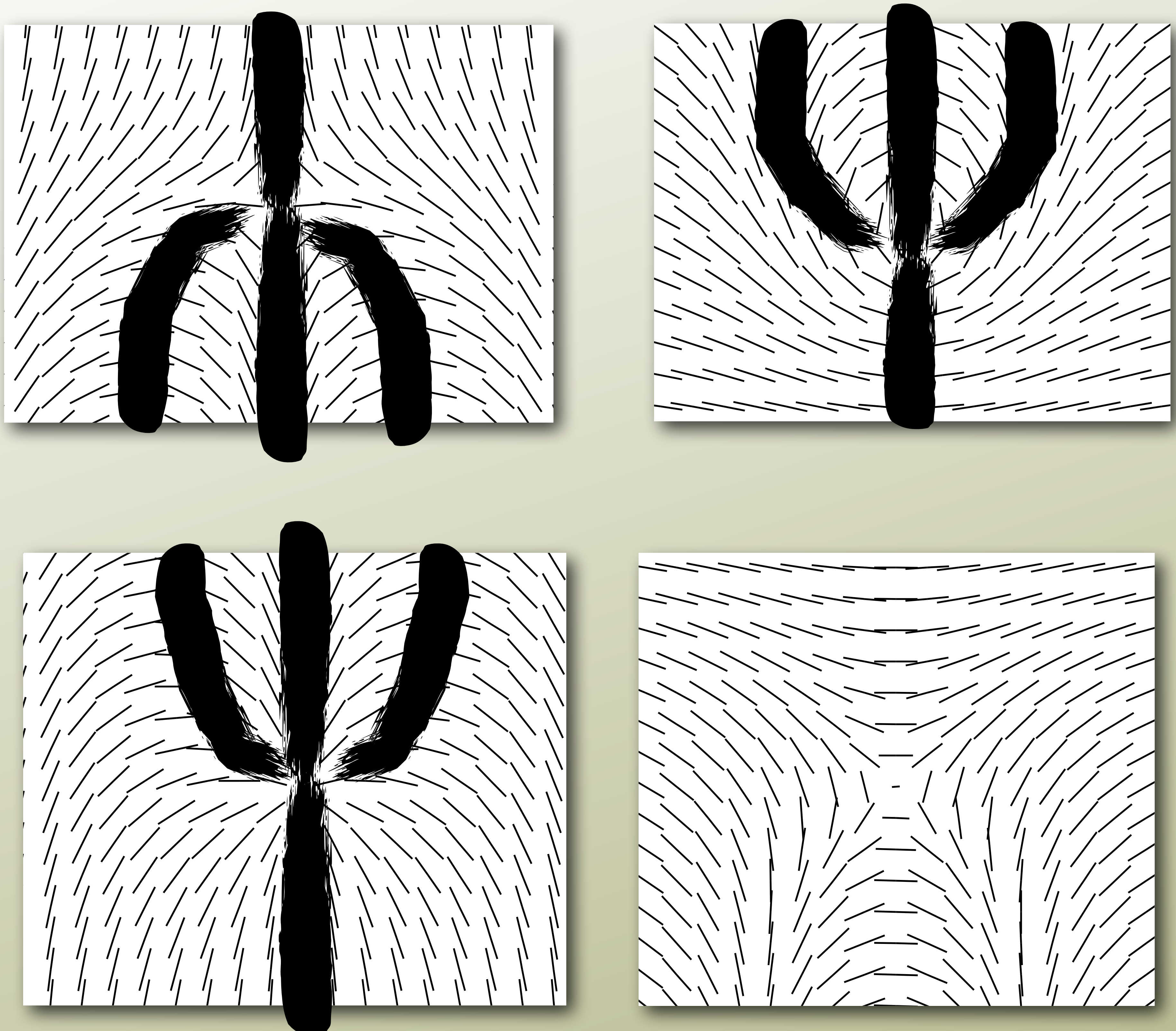

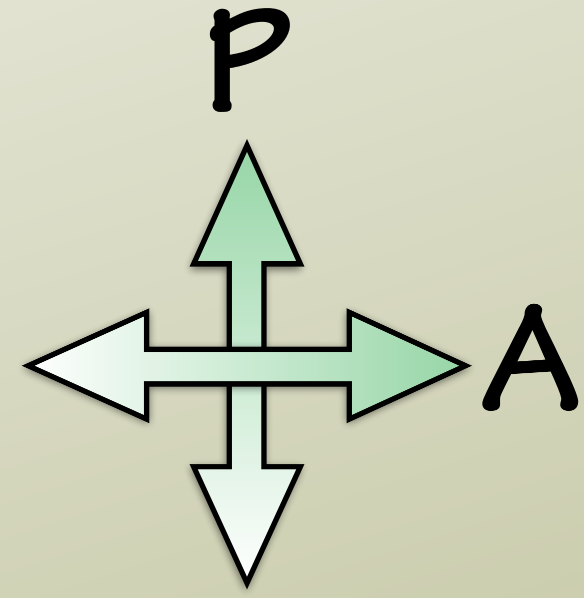

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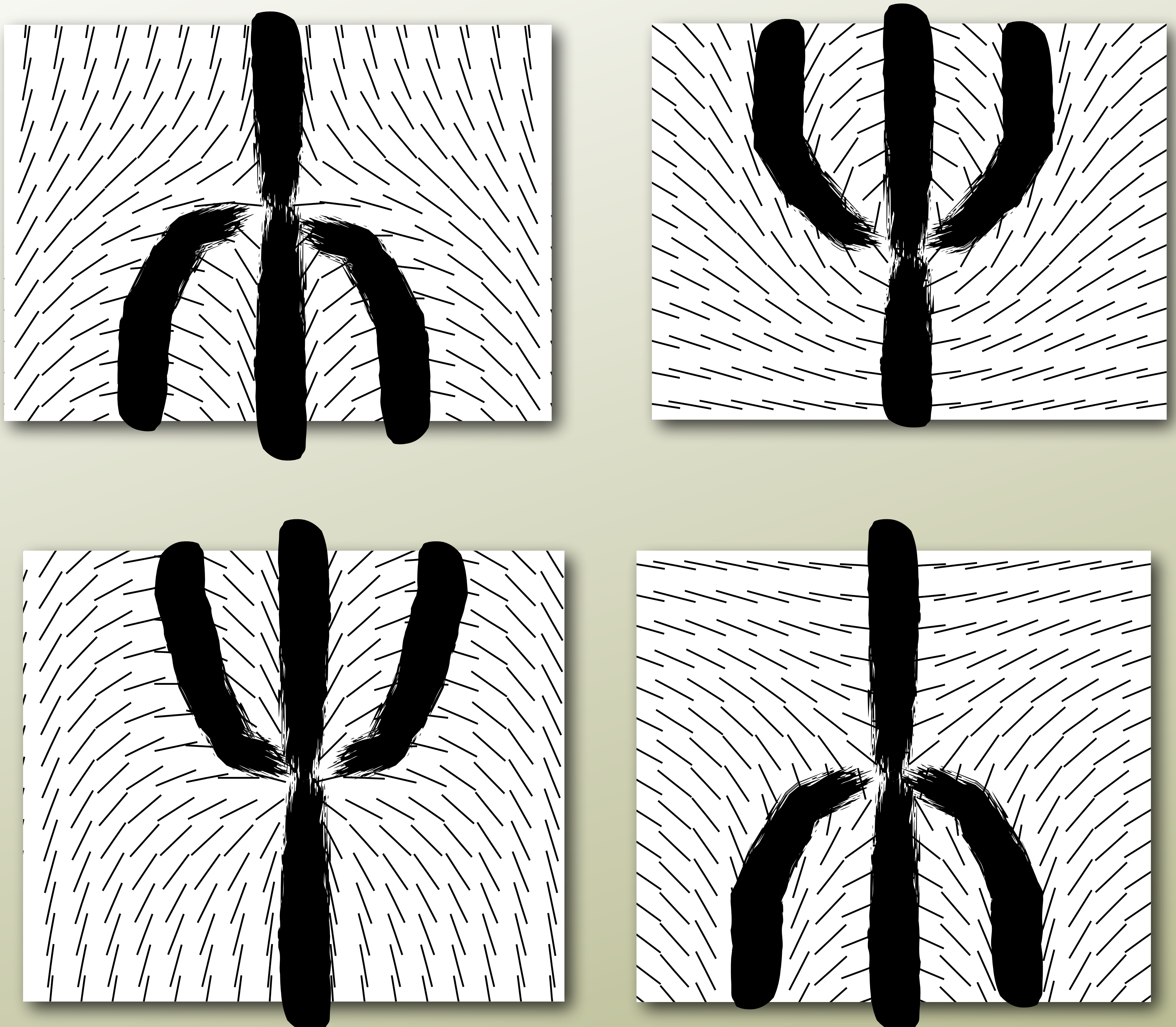

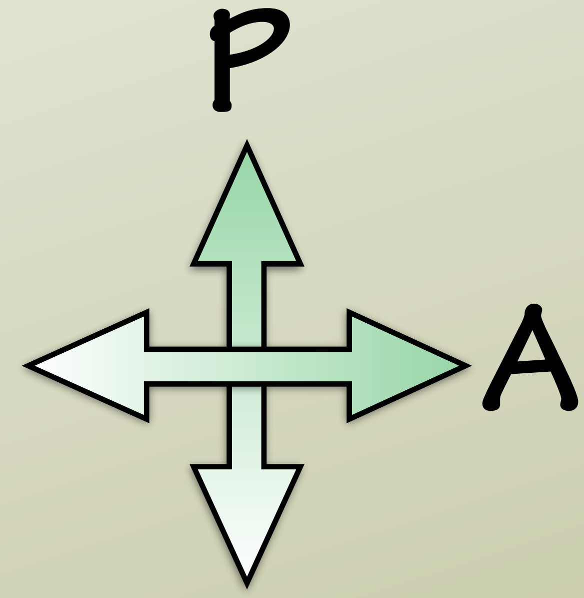

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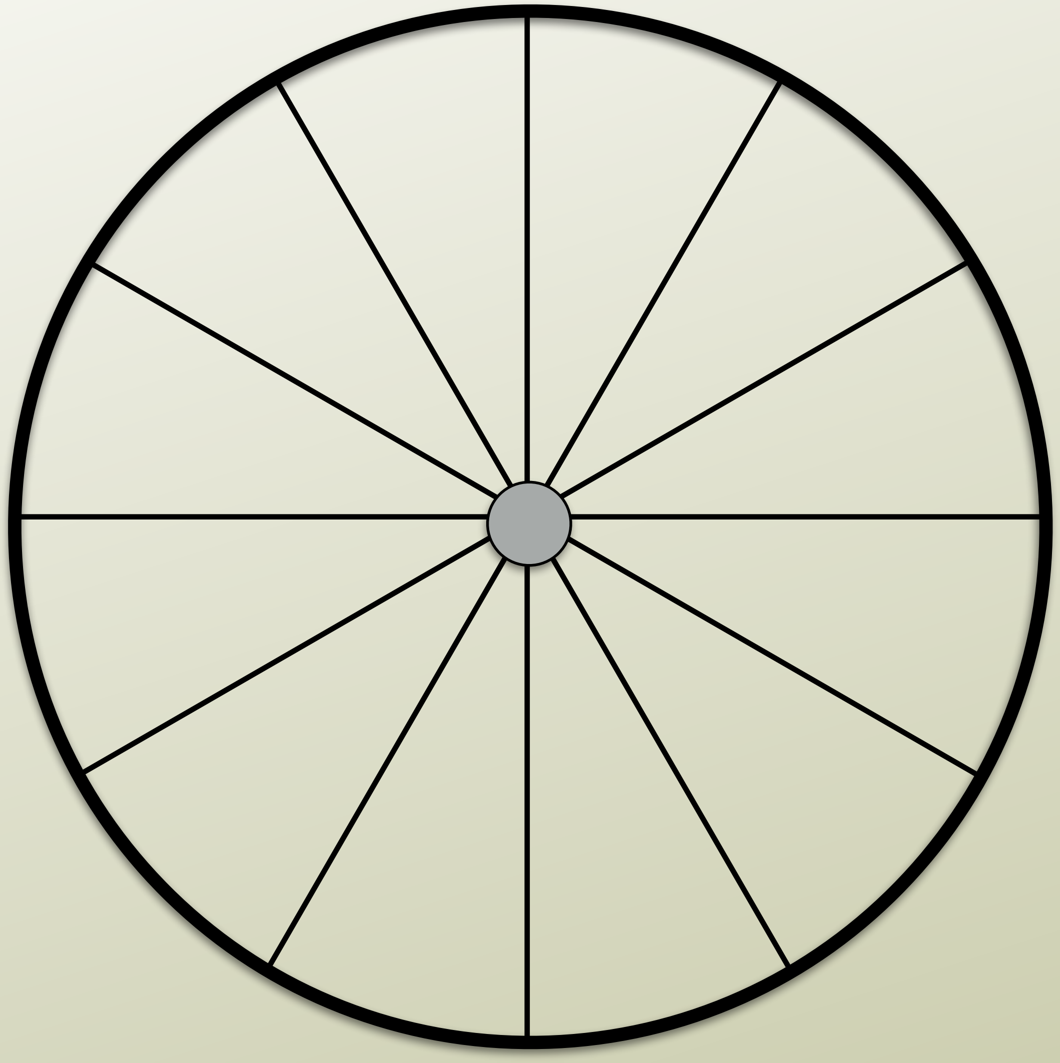
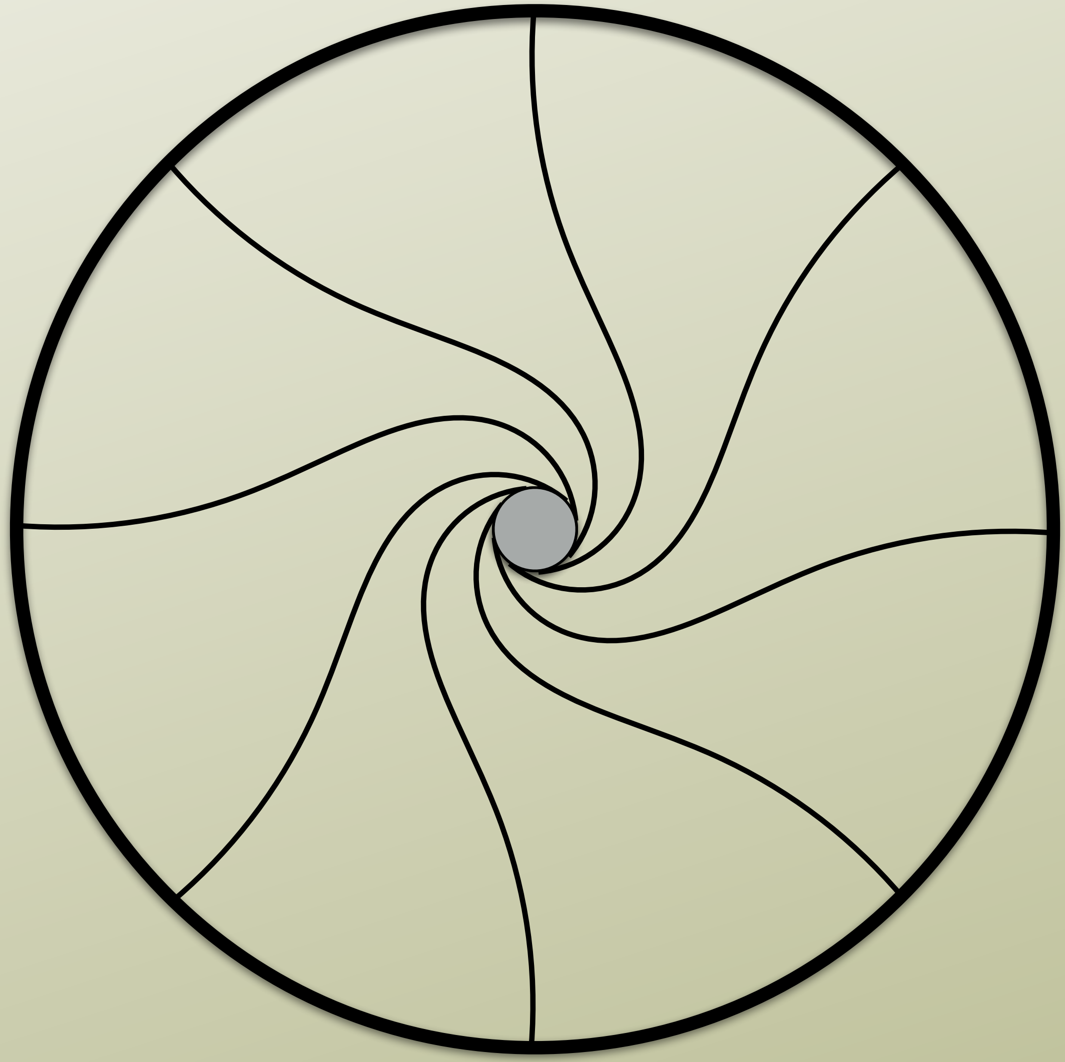
Nematics in Two Dimensions





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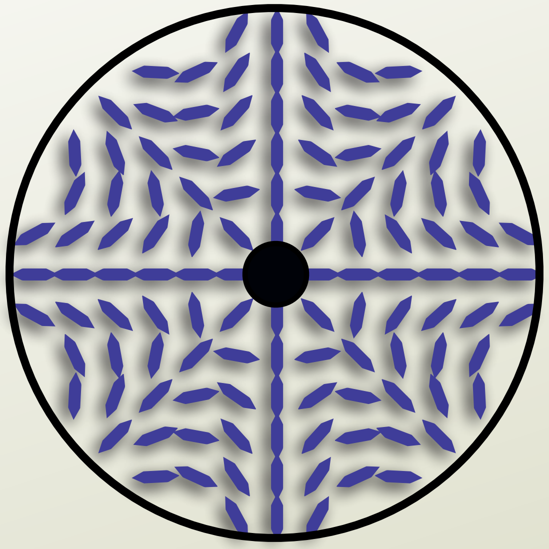
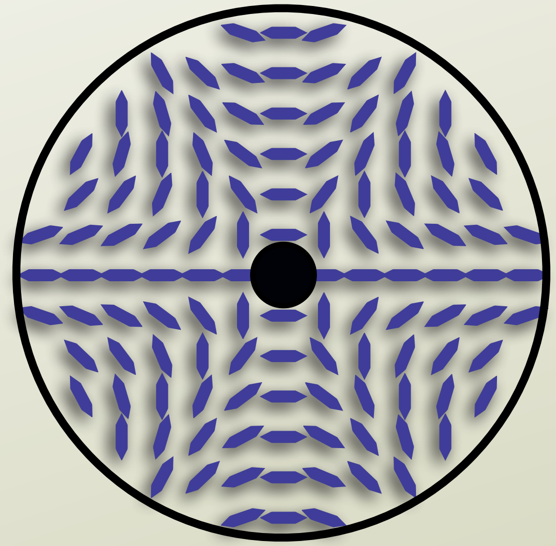
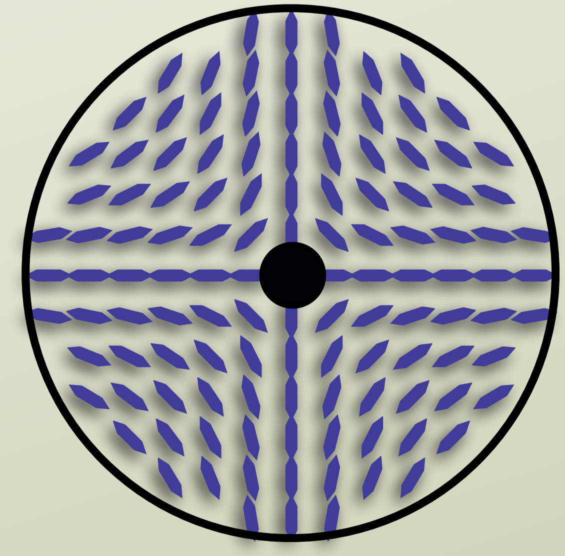
Point Defects in Two Dimensions



$k = -3$



$k = -2$



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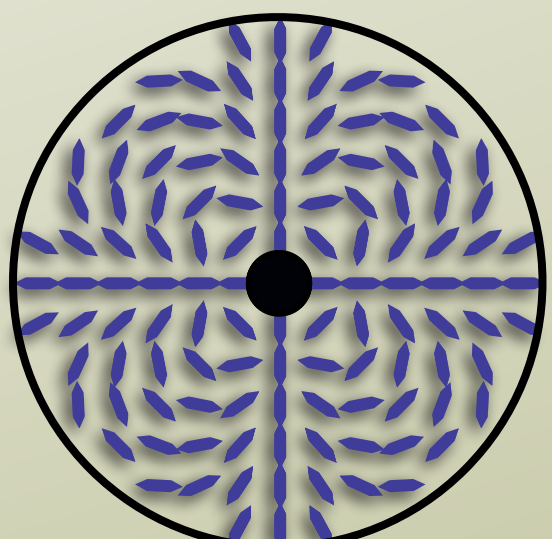
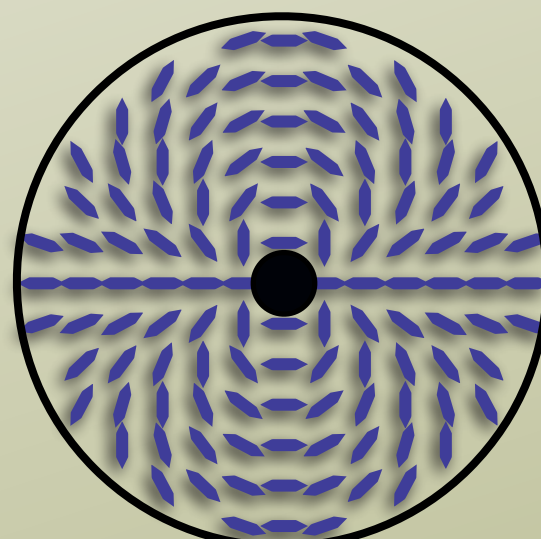
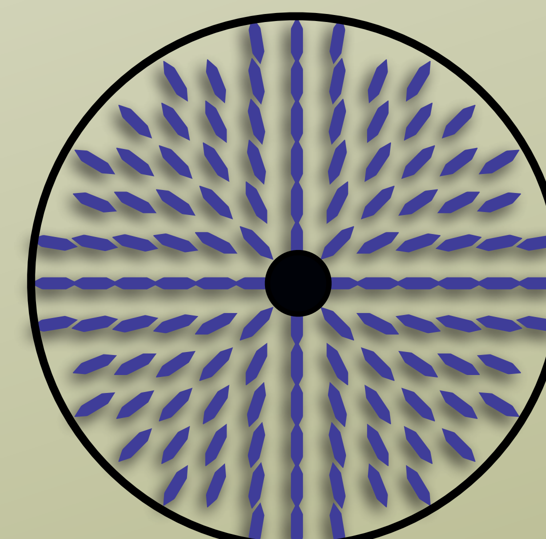


$k = +3$



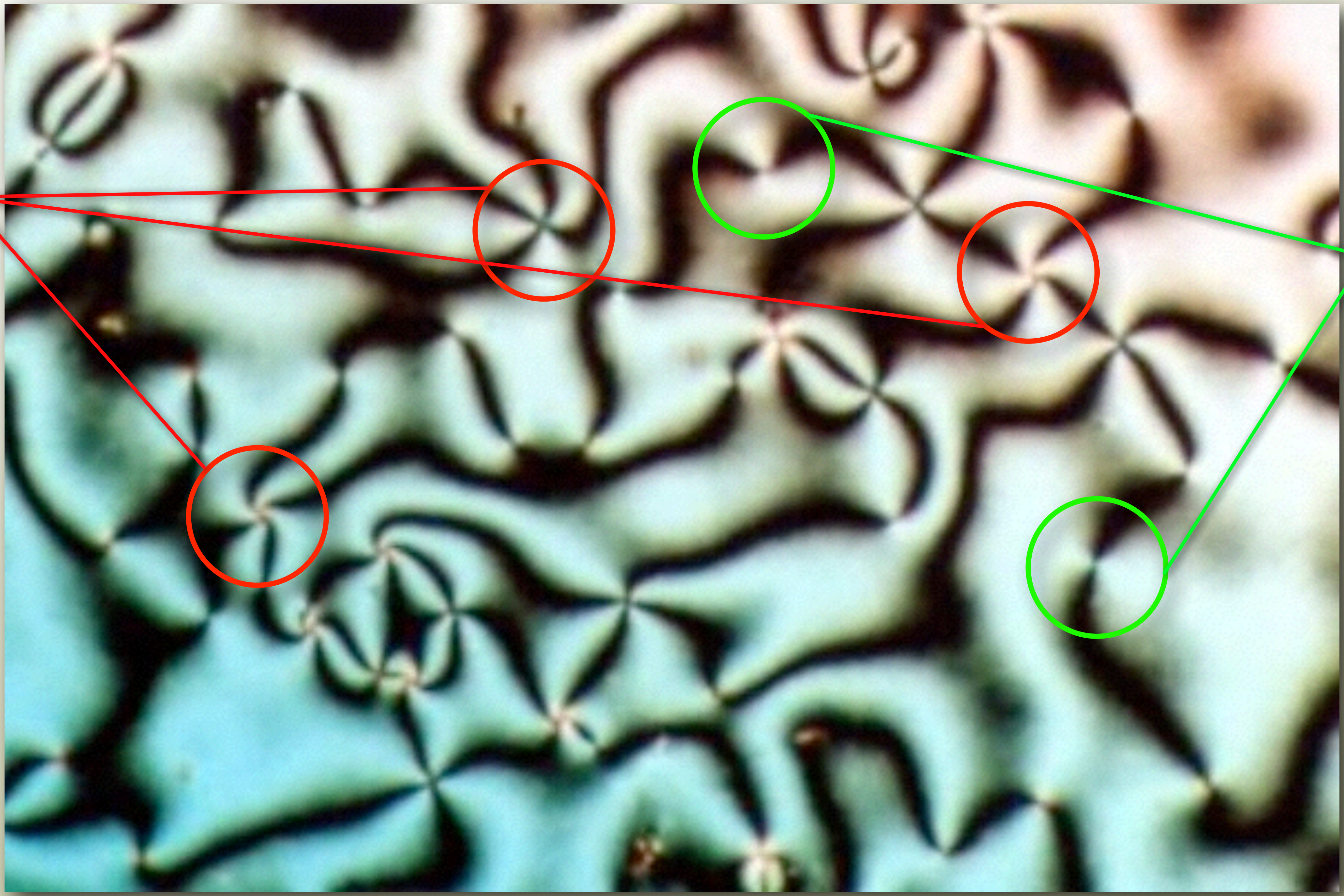
$k = +2$



$k = +1$

$$\mathbf{n} = [\cos(k\phi), \sin(k\phi), 0]$$

Nematics in Two Dimensions



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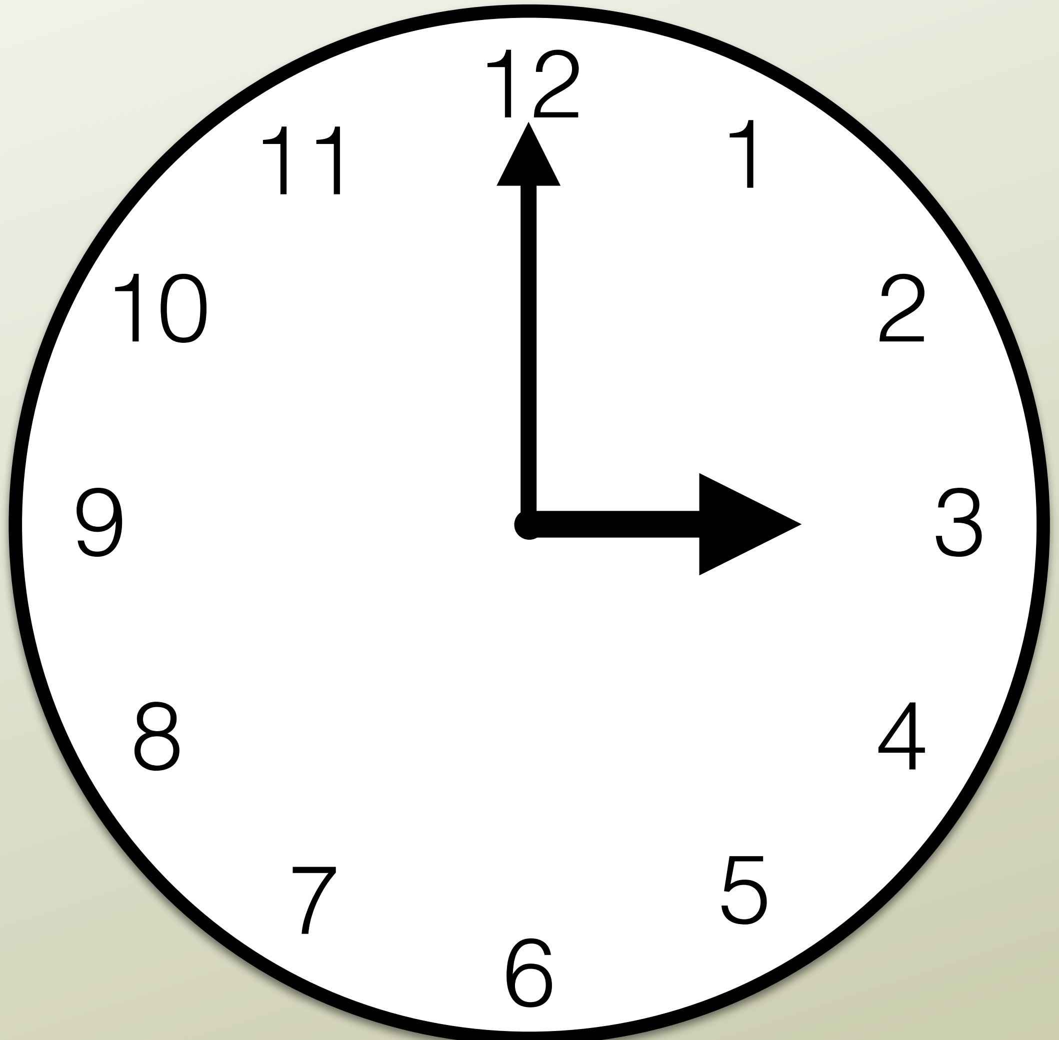
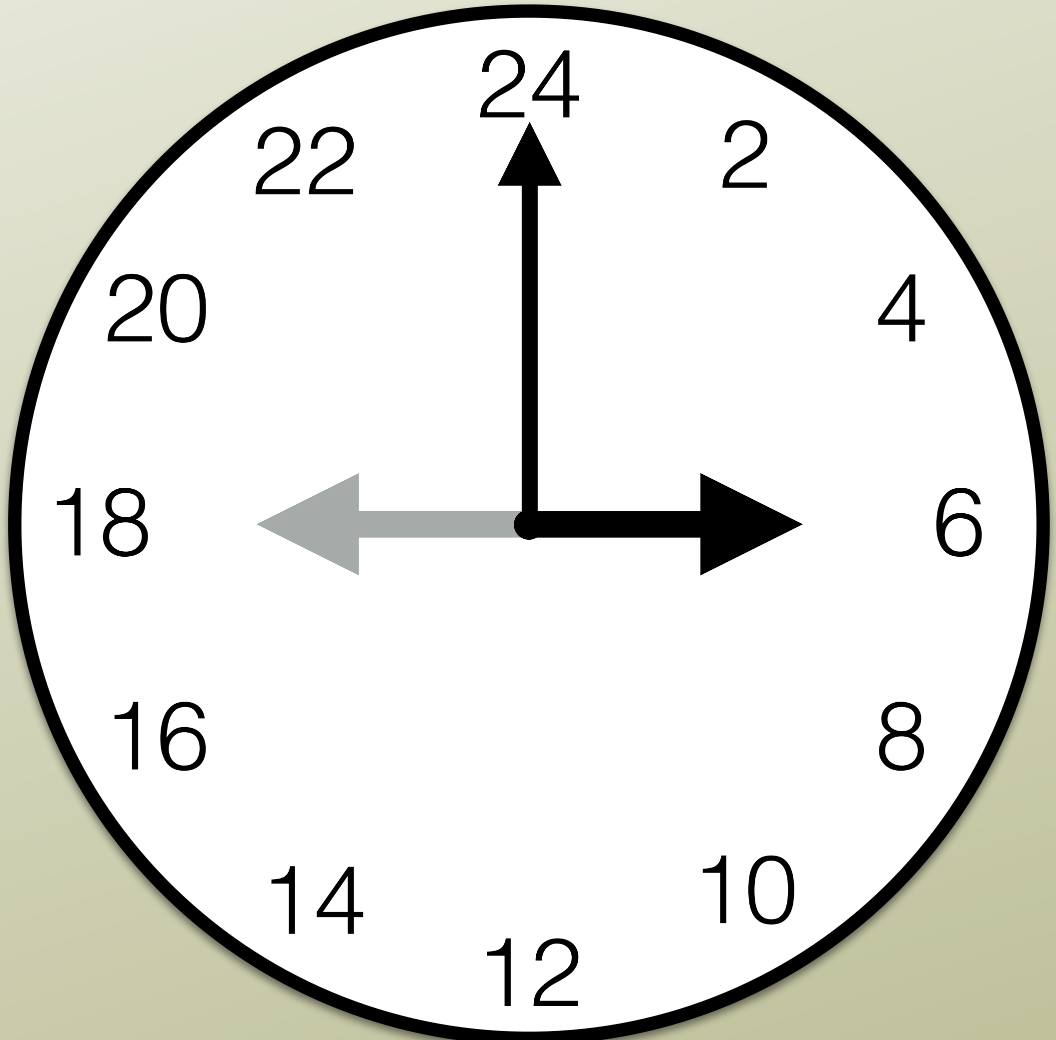
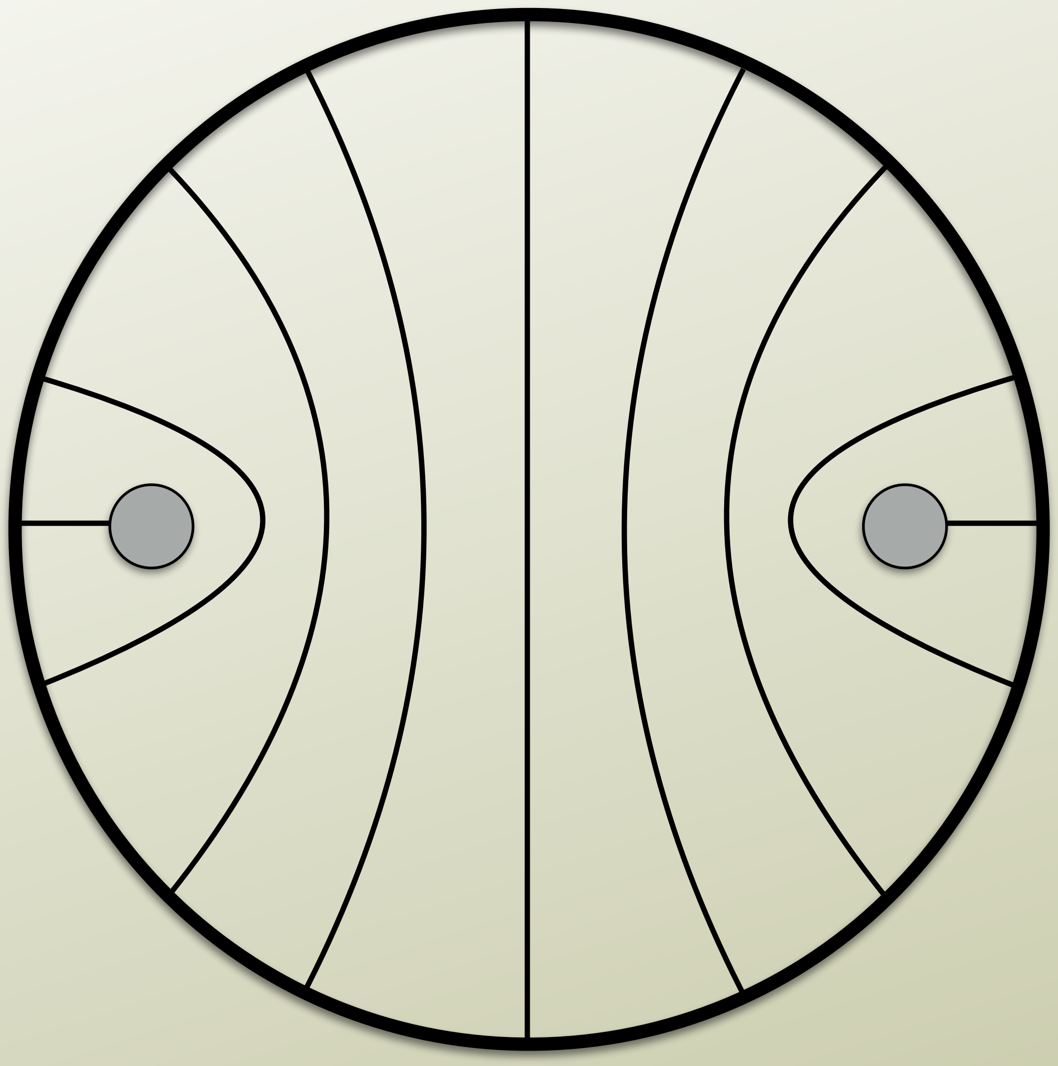
Maps from $\mathbb{R}^2 \backslash \{0\} \to \mathbb{R}P^1$

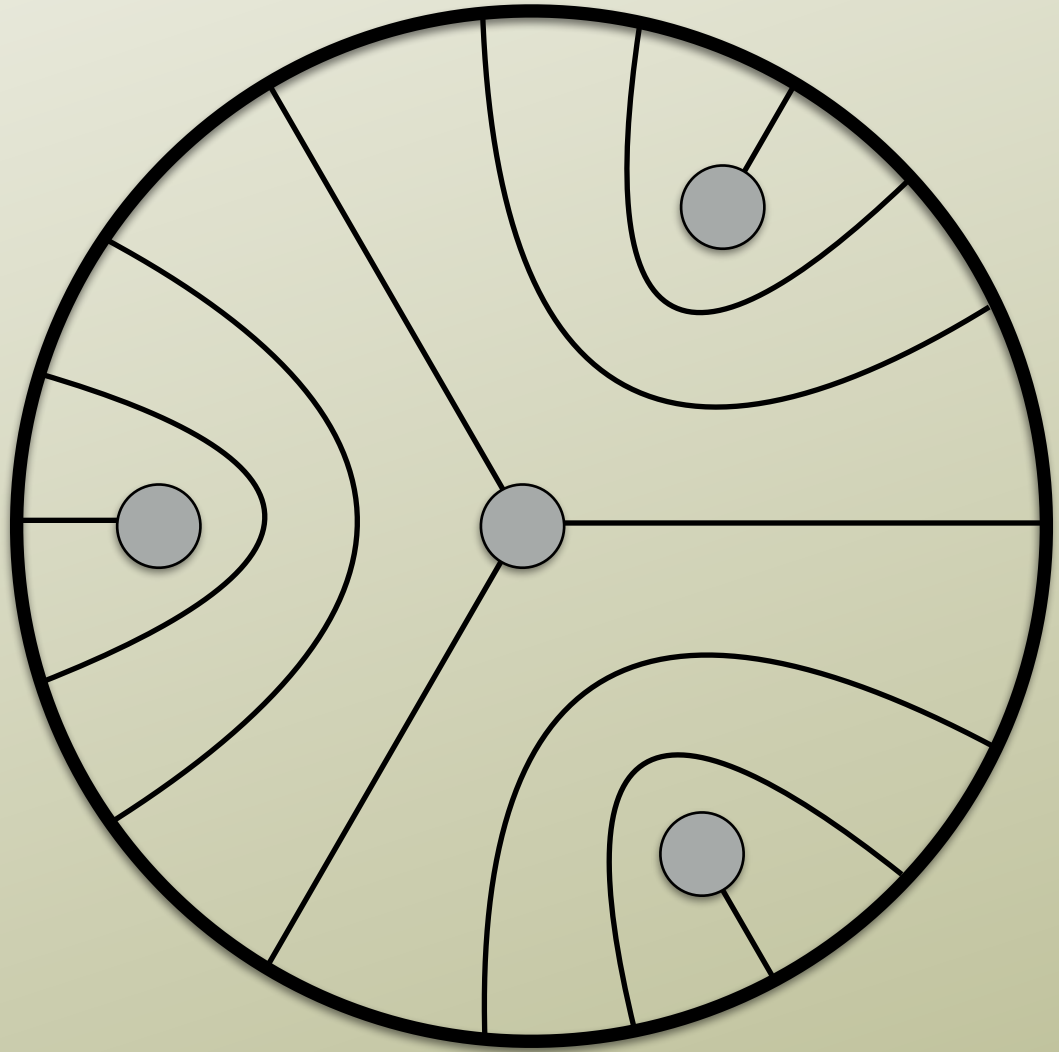
Nematics in Two Dimensions





Nematics in Two Dimensions





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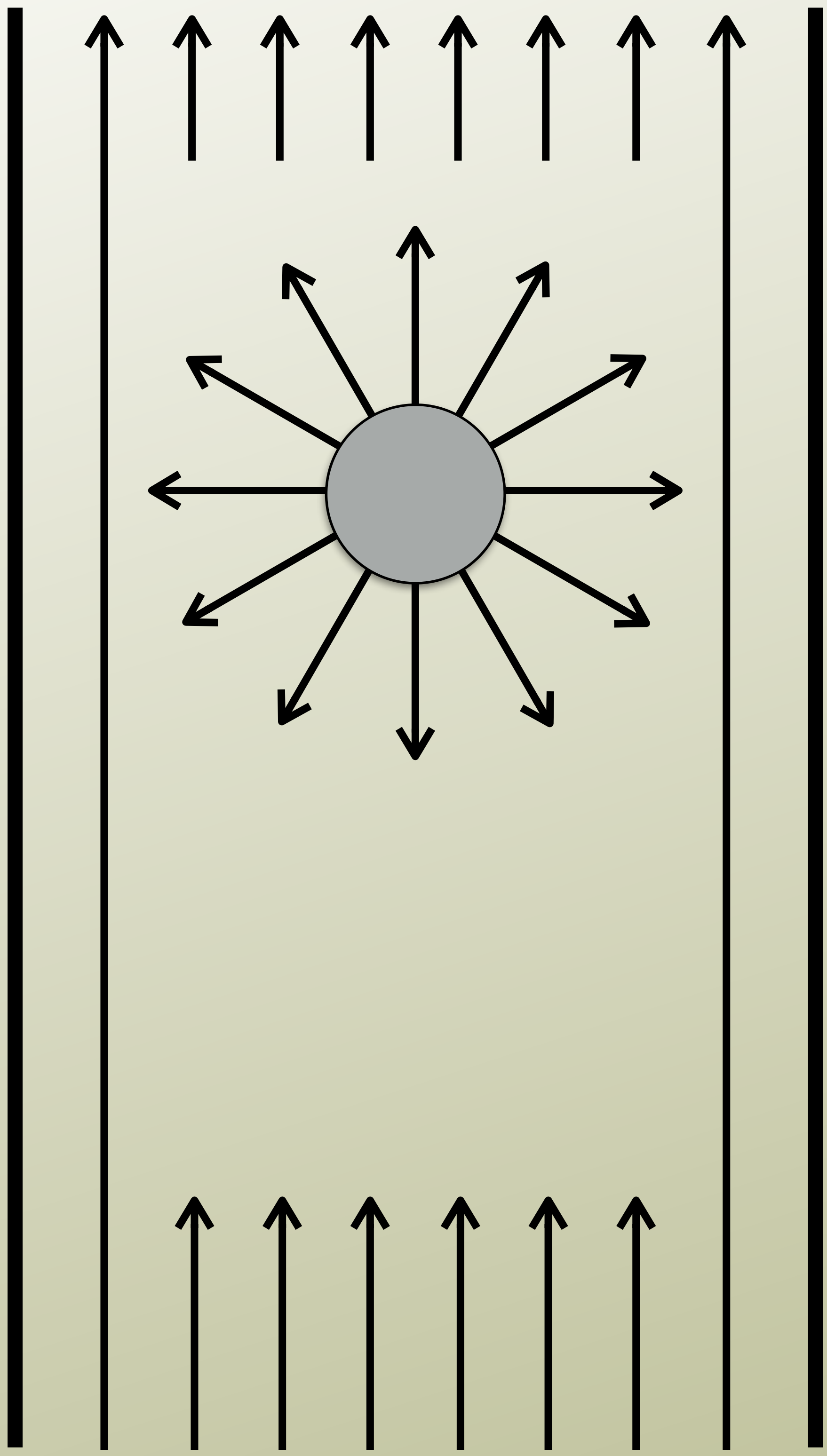
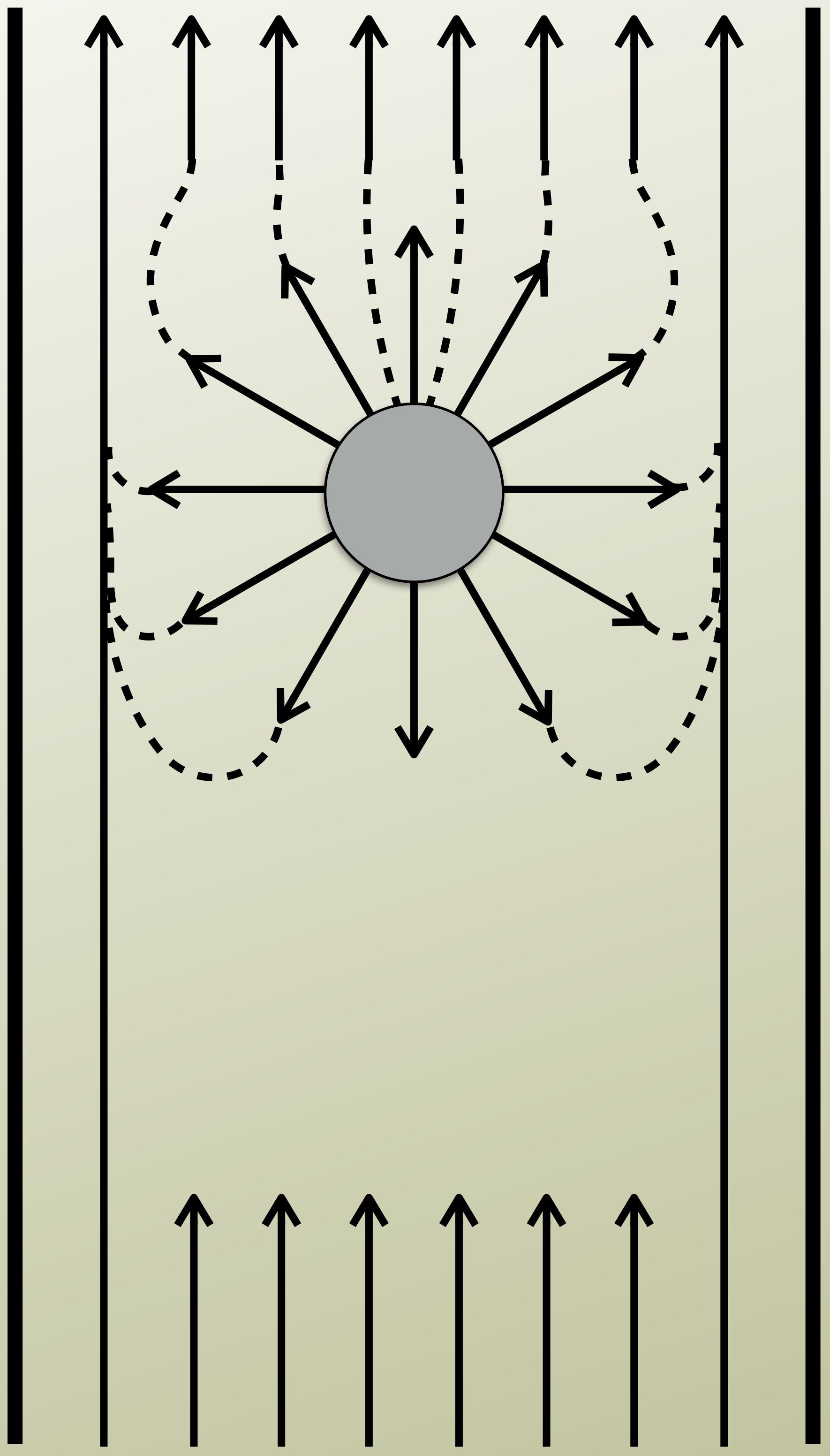
Nematics in Two Dimensions

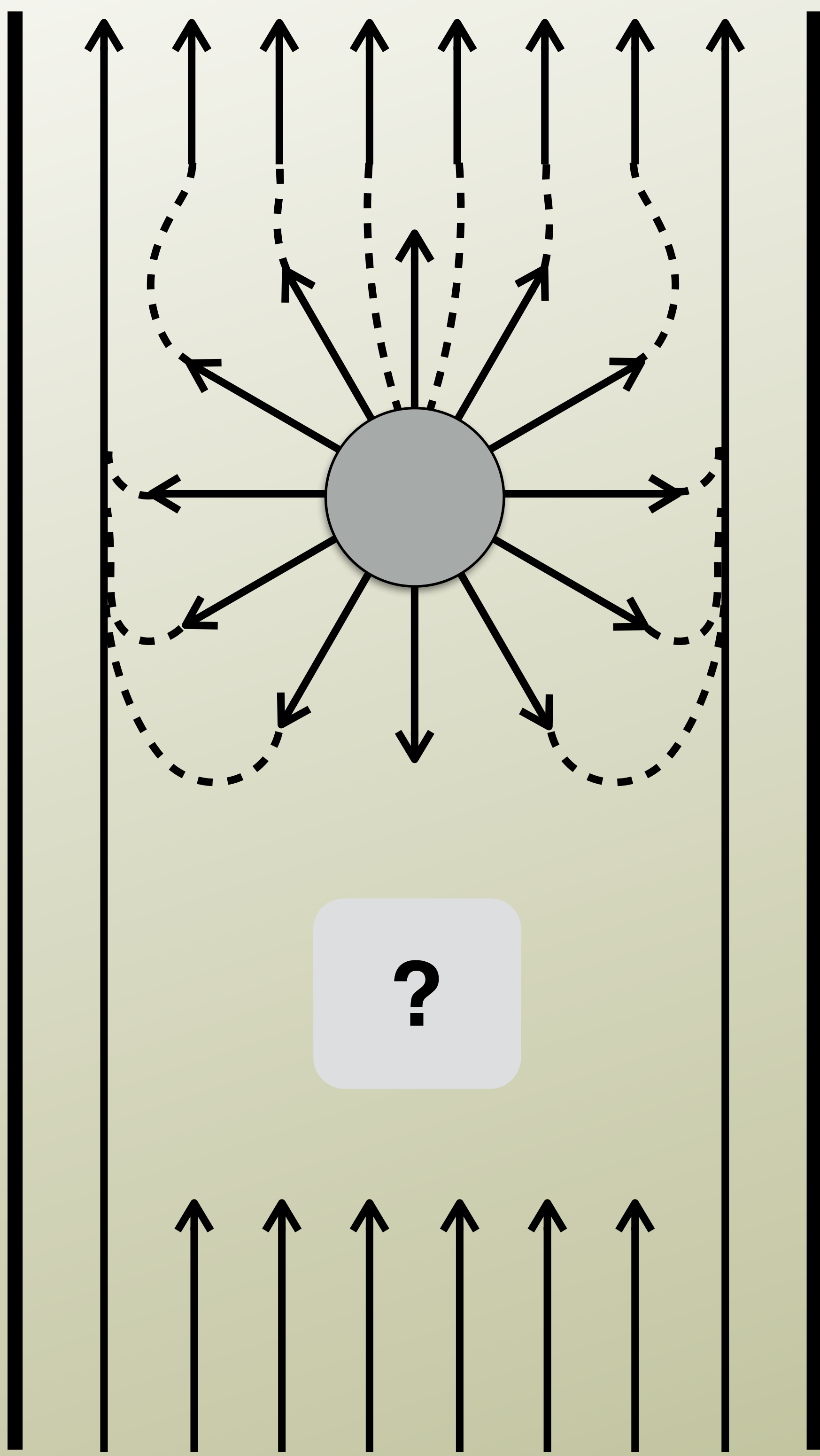


Penn

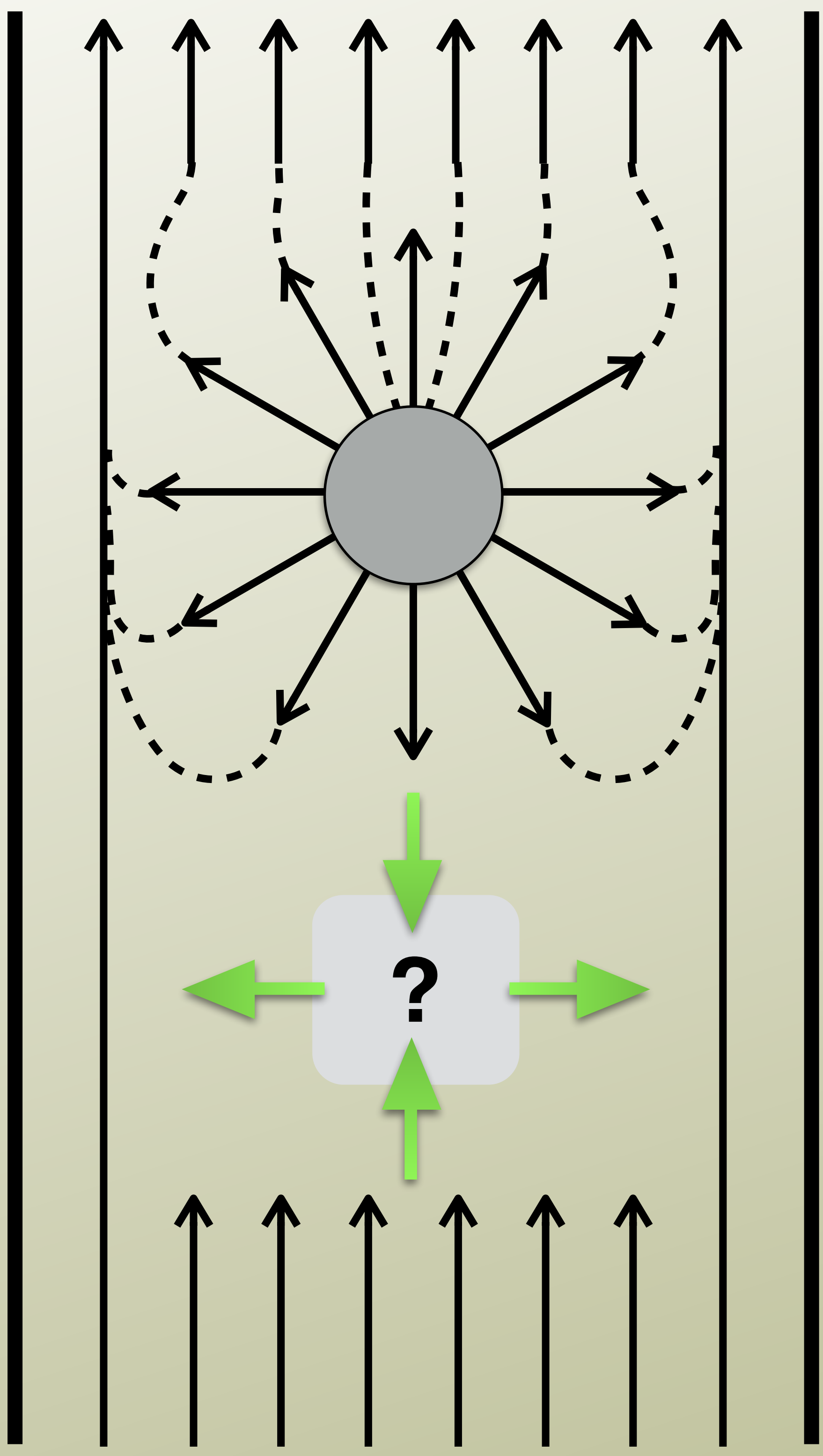
Nematics in Two Dimensions



Nematics in Two Dimensions



Nematics in Two Dimensions



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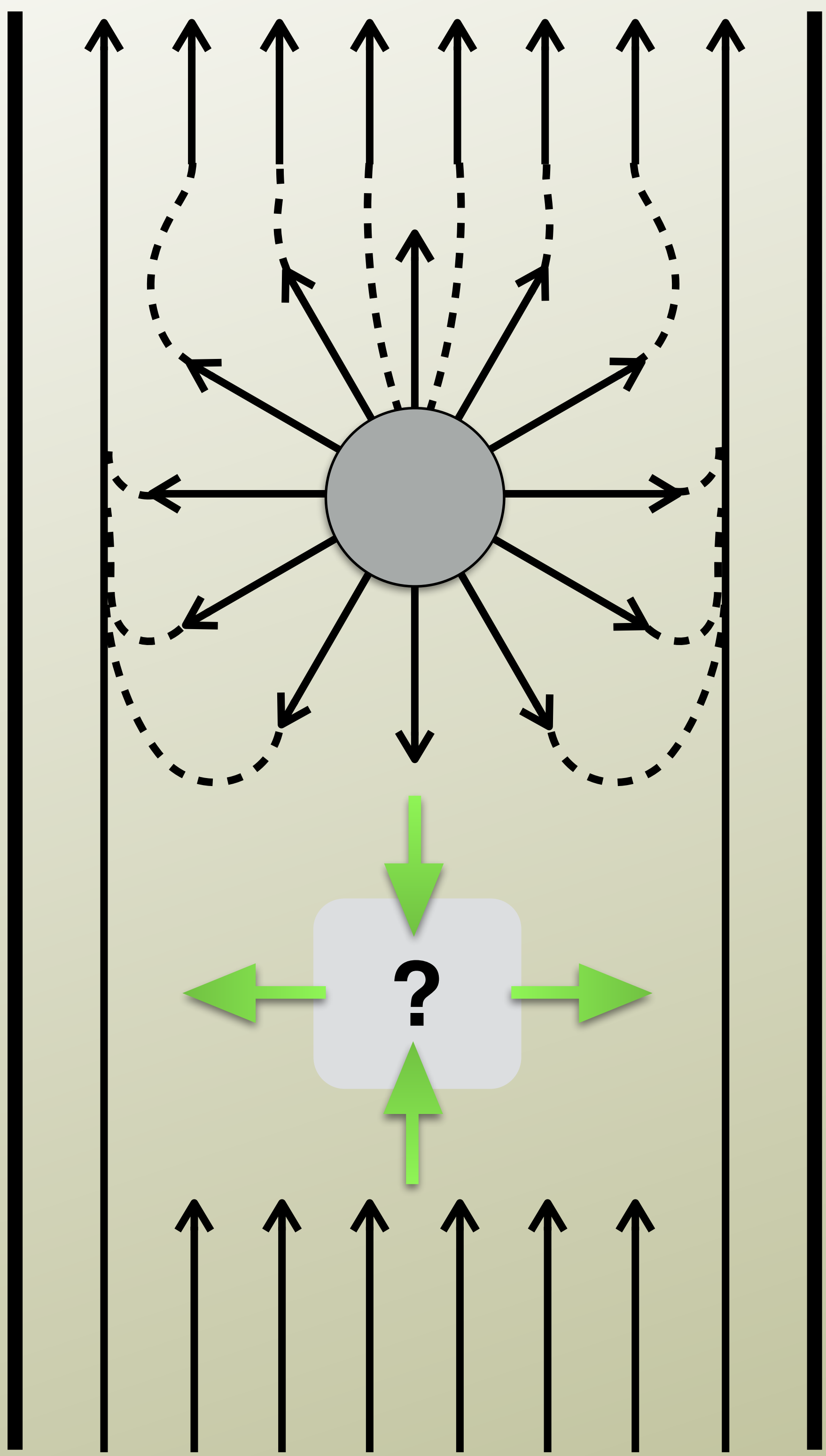
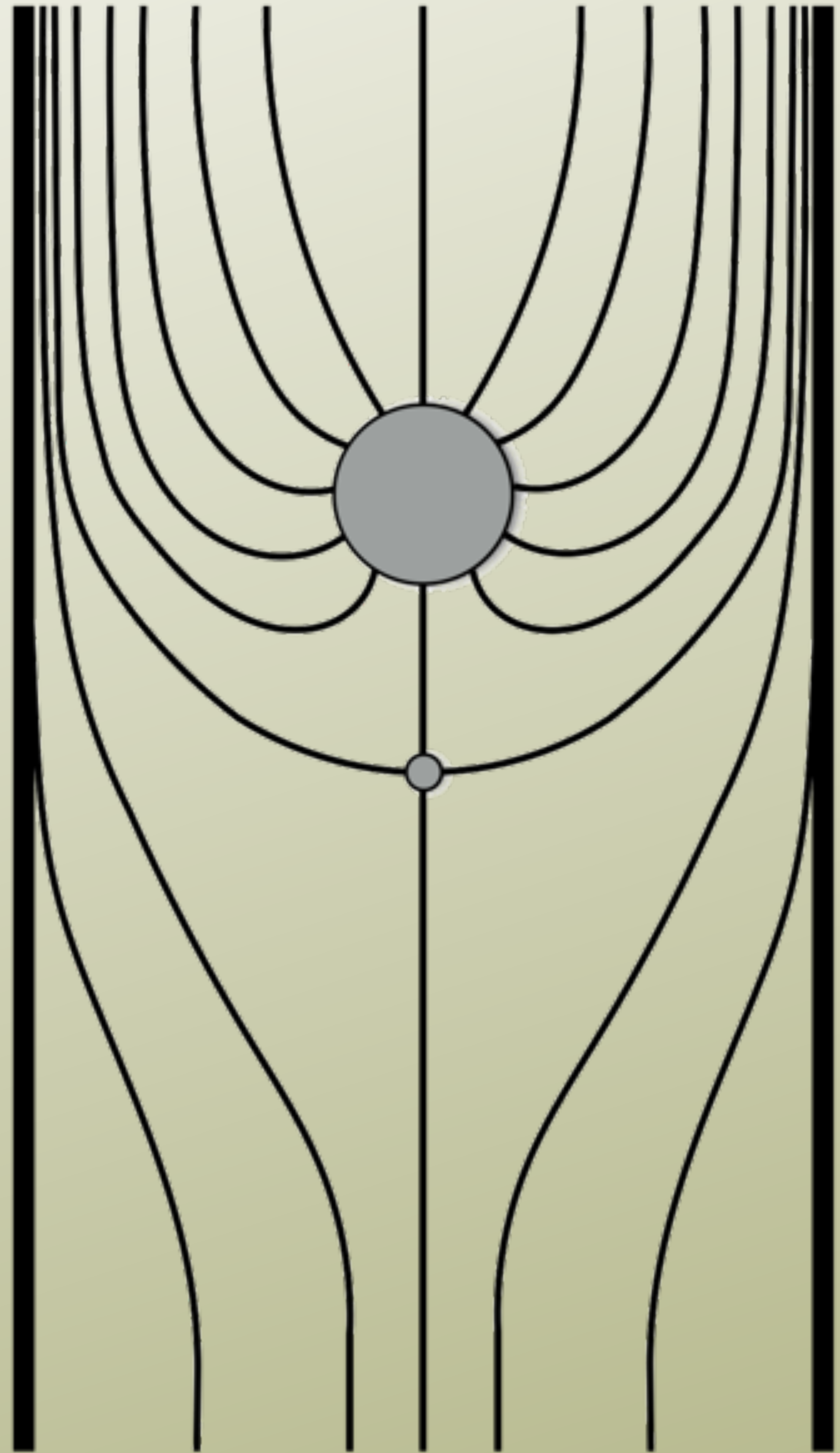
Nematics in Two Dimensions





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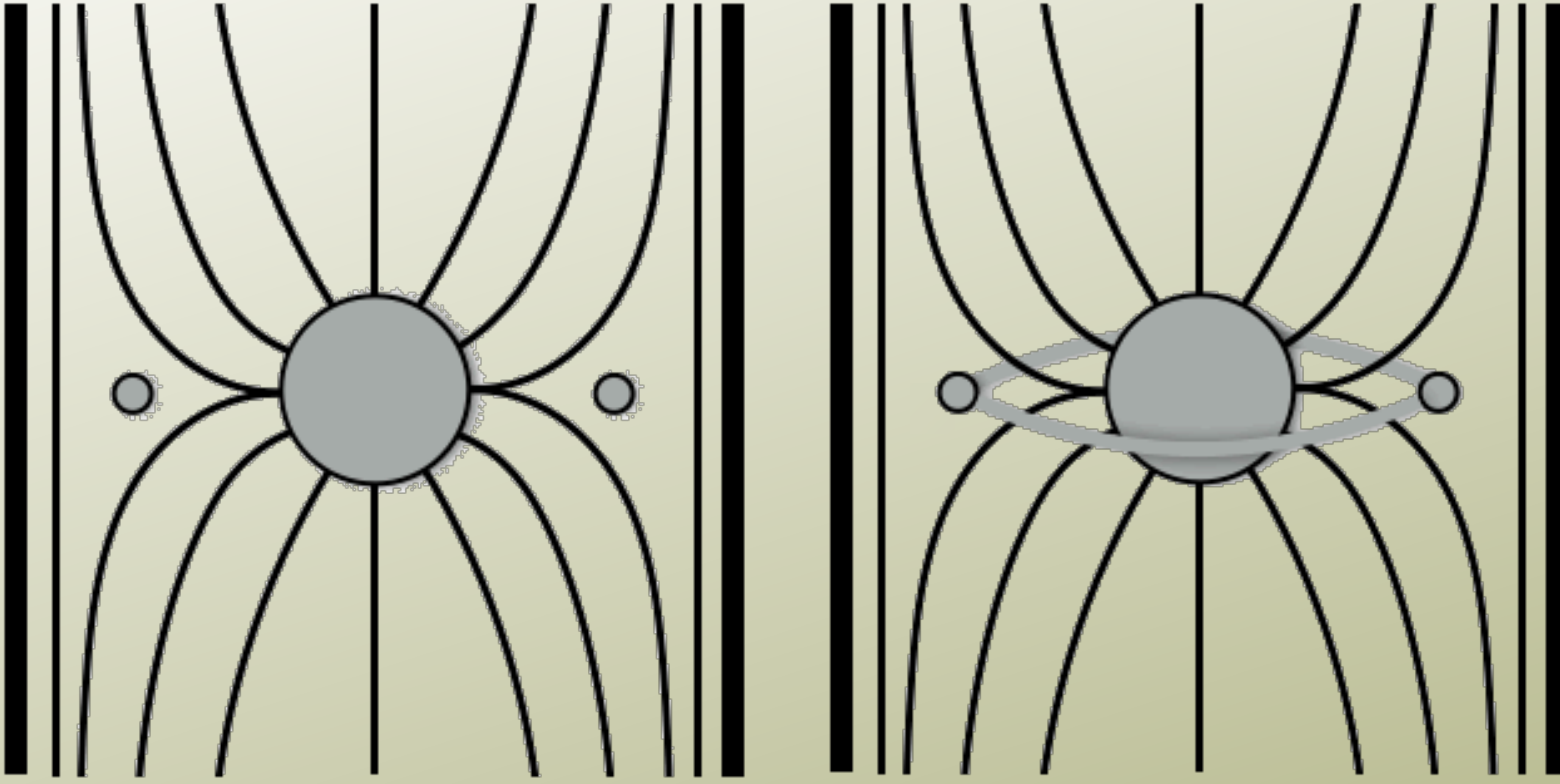
Nematics in Two Dimensions



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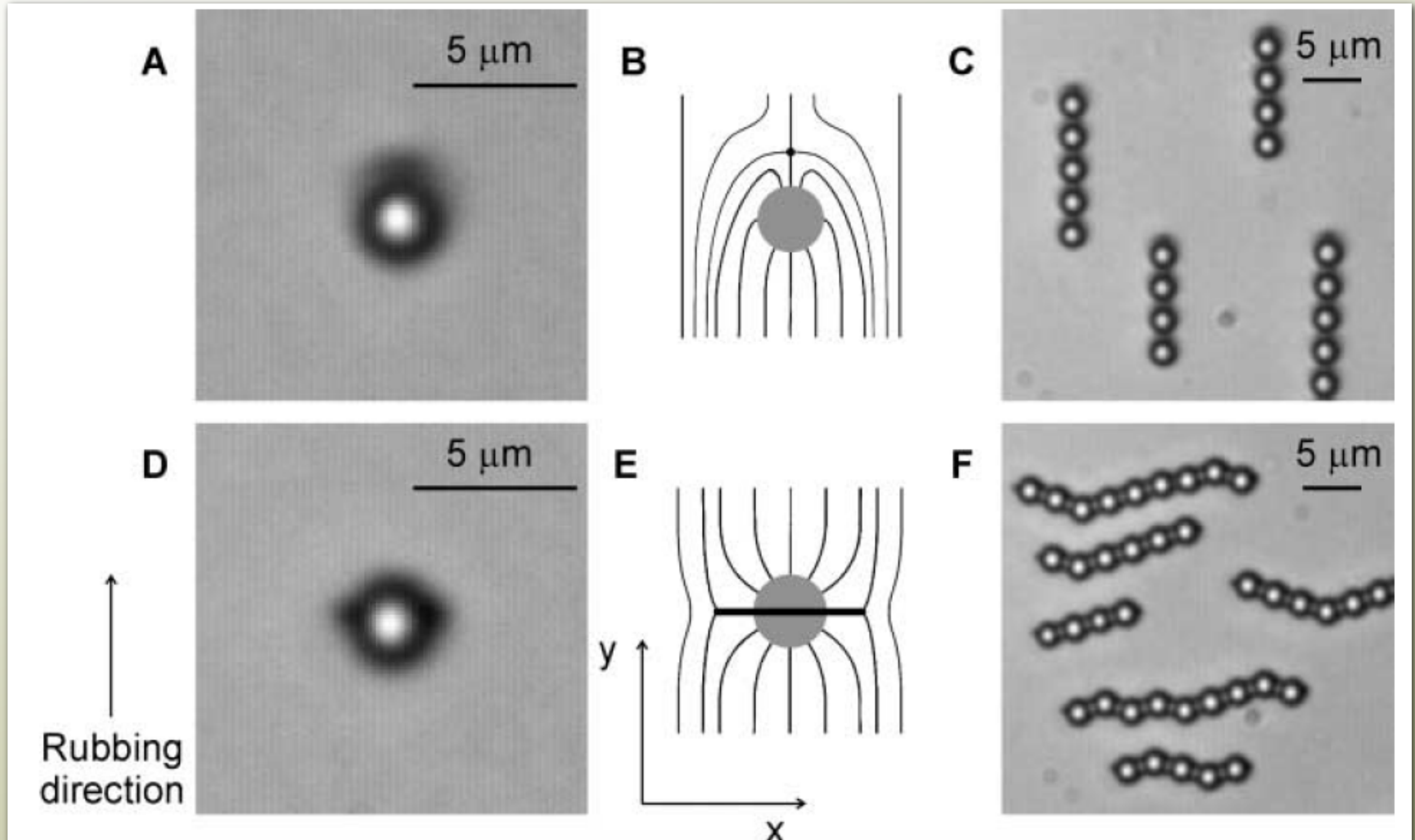
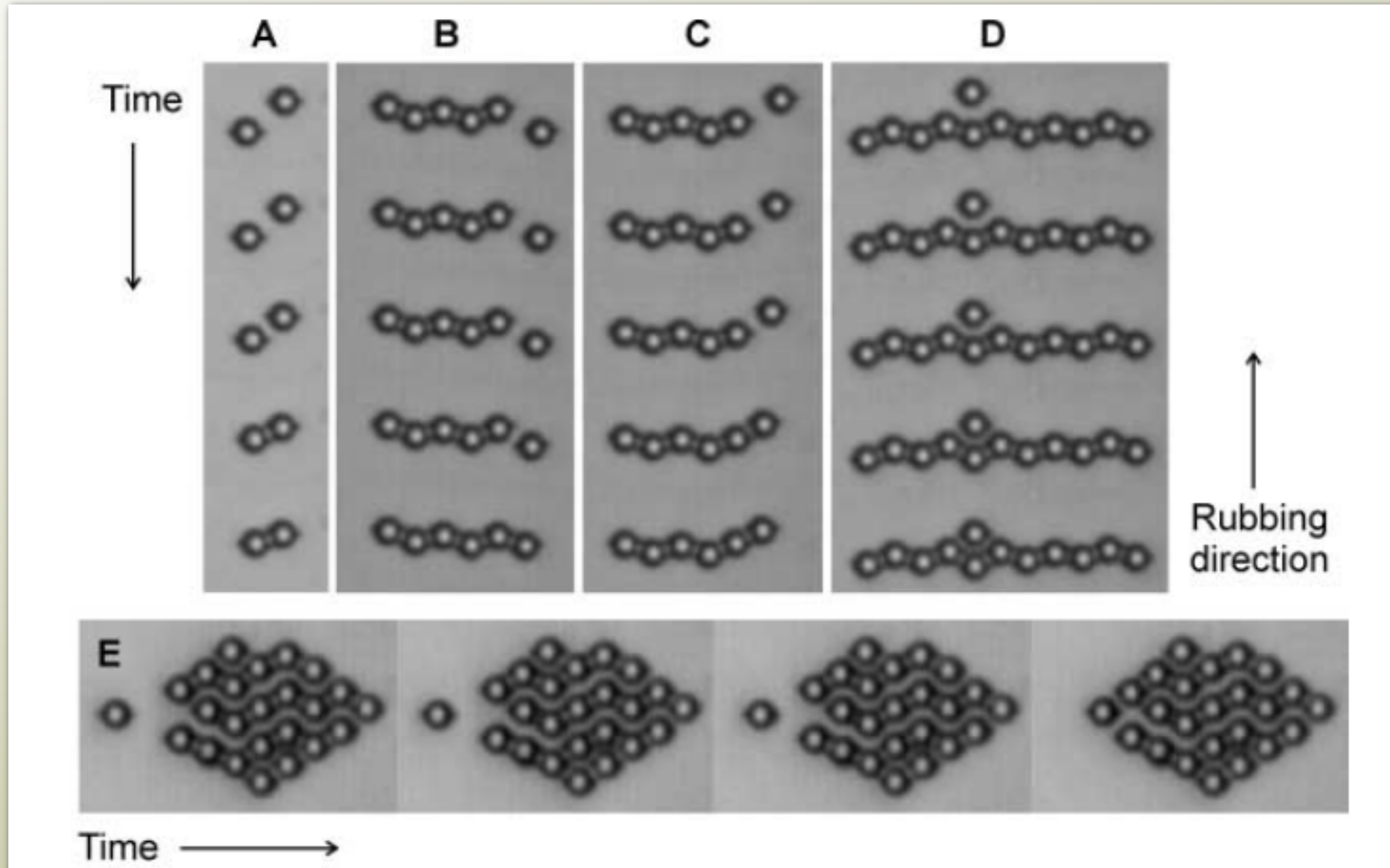
Nematics in Two Dimensions



I. Muševič,[1] M. Skarabot, U. Tkalec, M. Ravnik, S. Žumer, *Science* ***313*** *(2006) 954*.

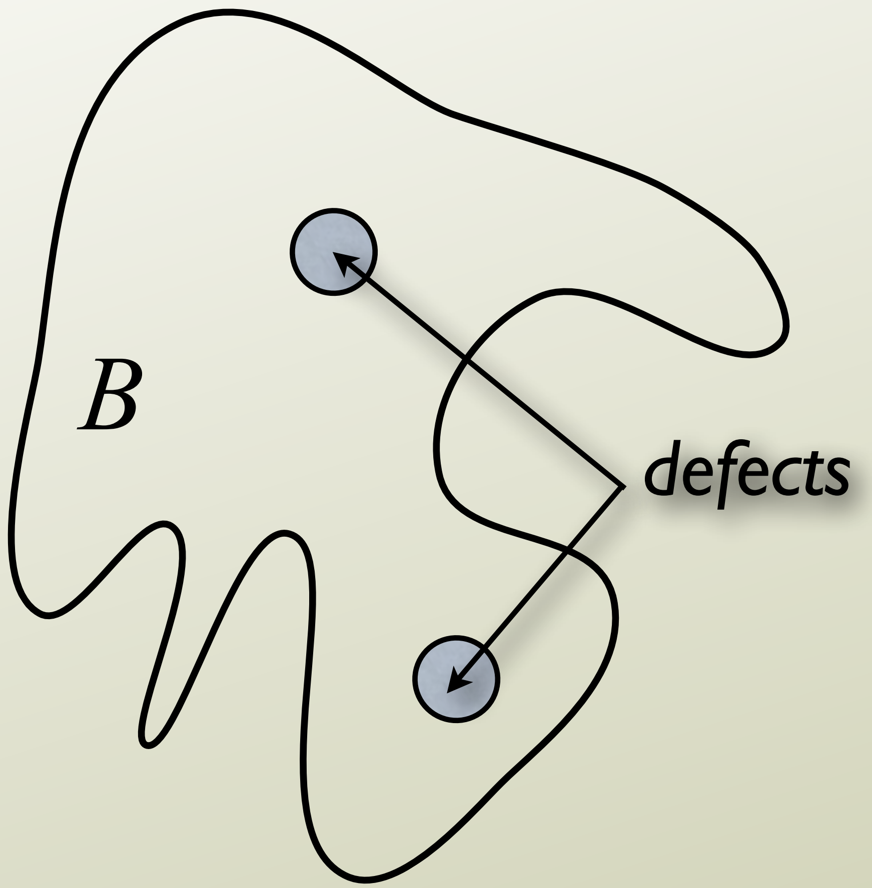
Nematics in Two Dimensions



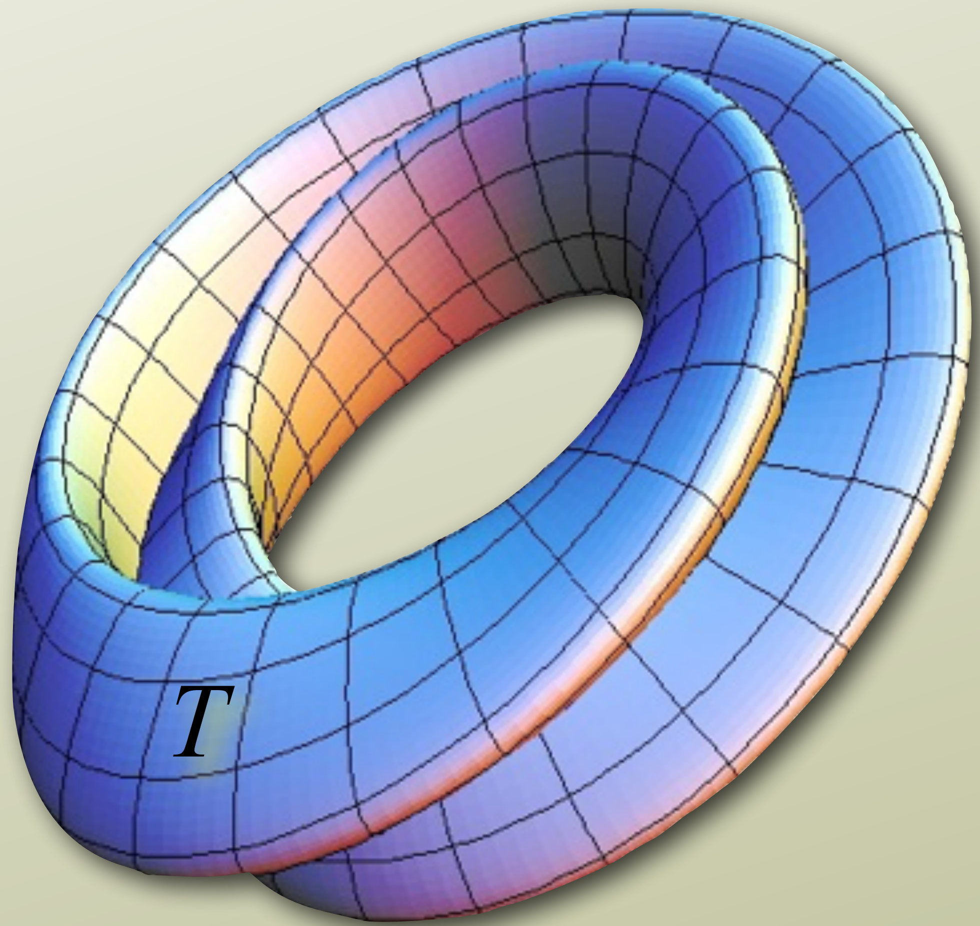
I. Muševič,[1] M. Sǩarabot, U. Tkalec, M. Ravnik, S. Žumer, *Science* ***313*** *(2006) 954*.

Defects and Homotopy: Quick Review

Sample



Ground State Manifold



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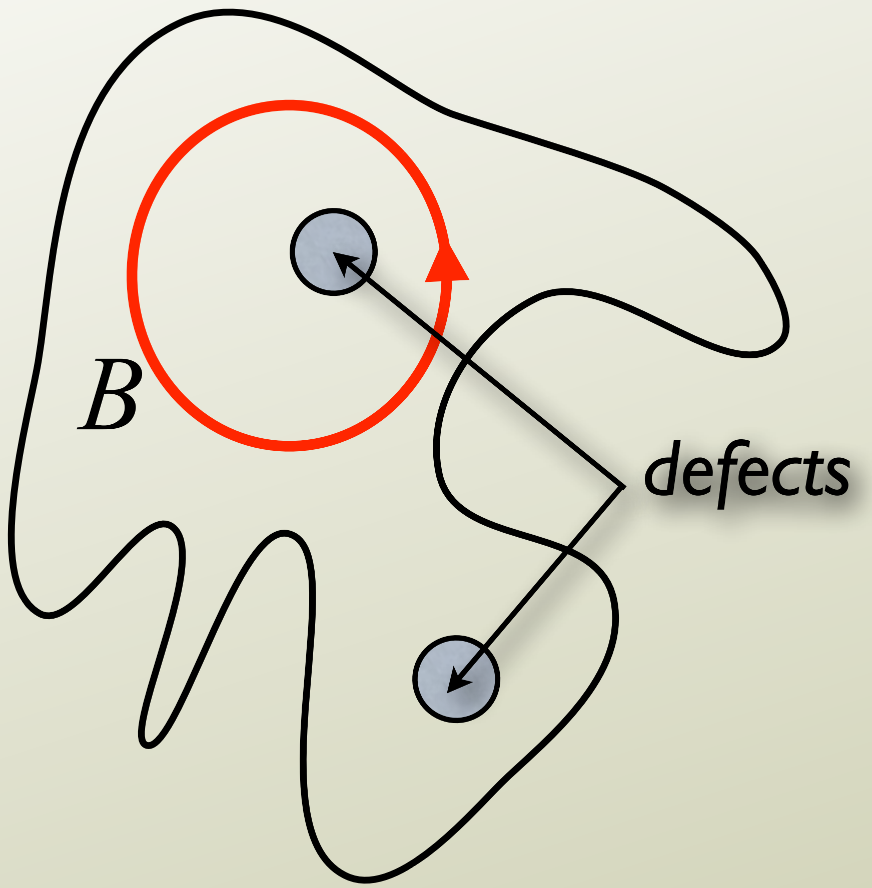
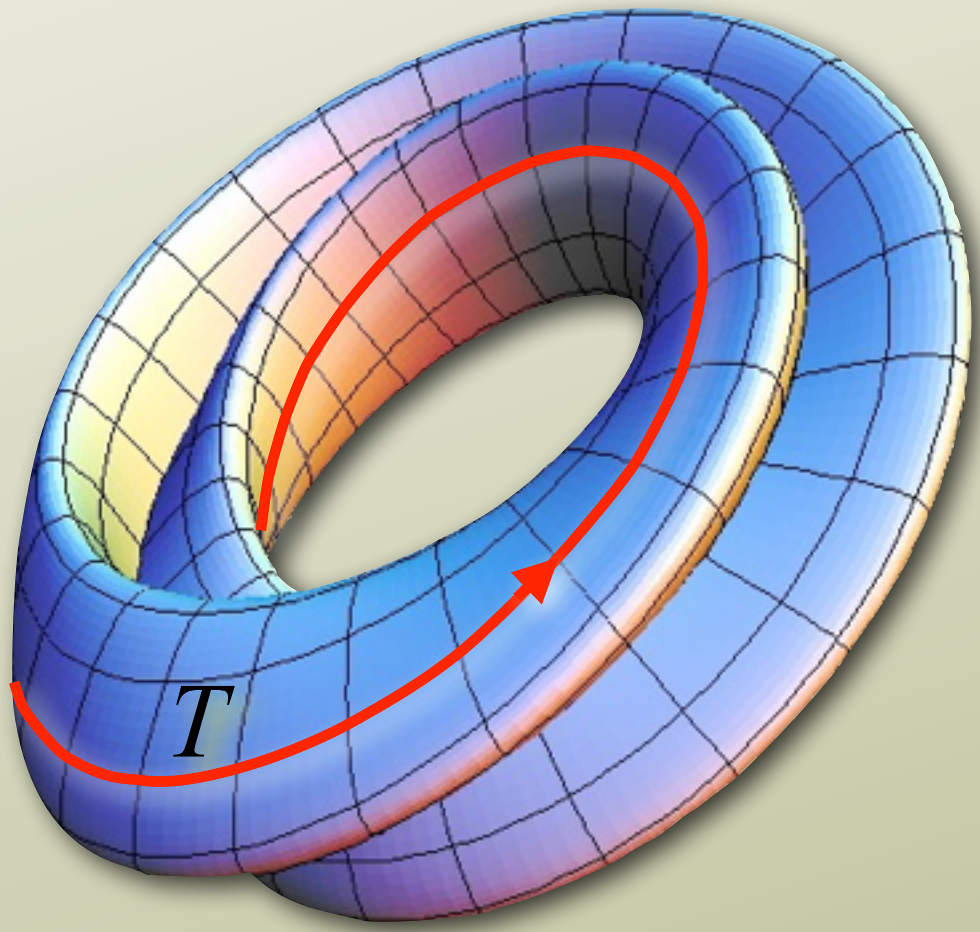
Defects and Homotopy: Quick Review

Sample



Ground State Manifold



Defects and Homotopy: Quick Review

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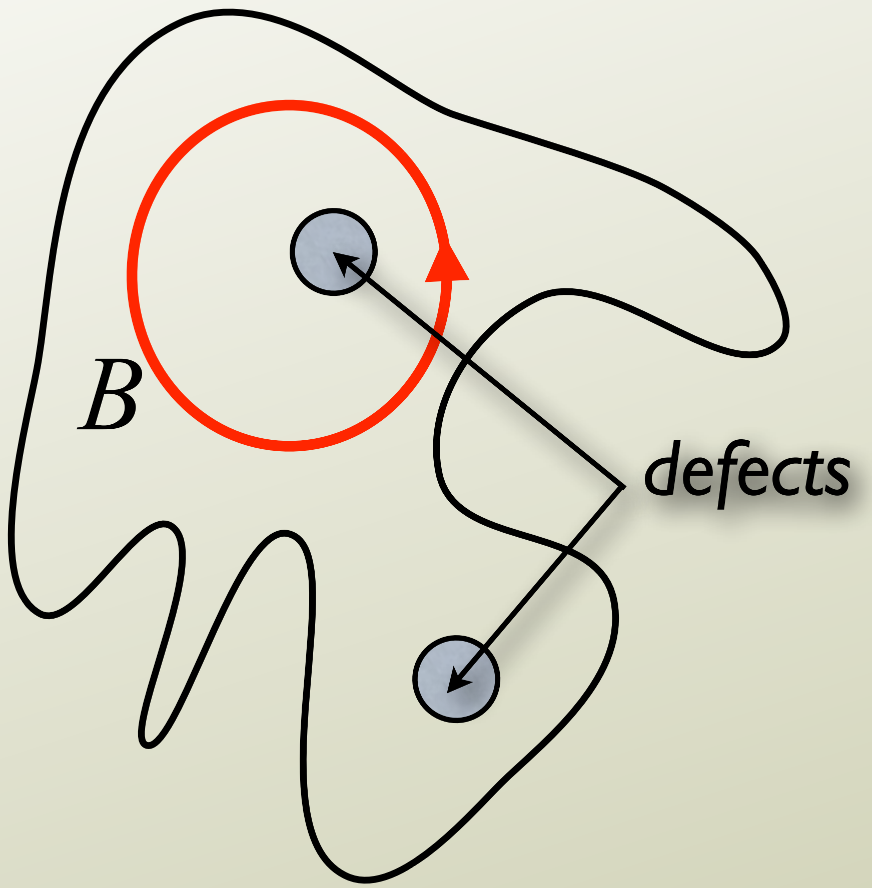
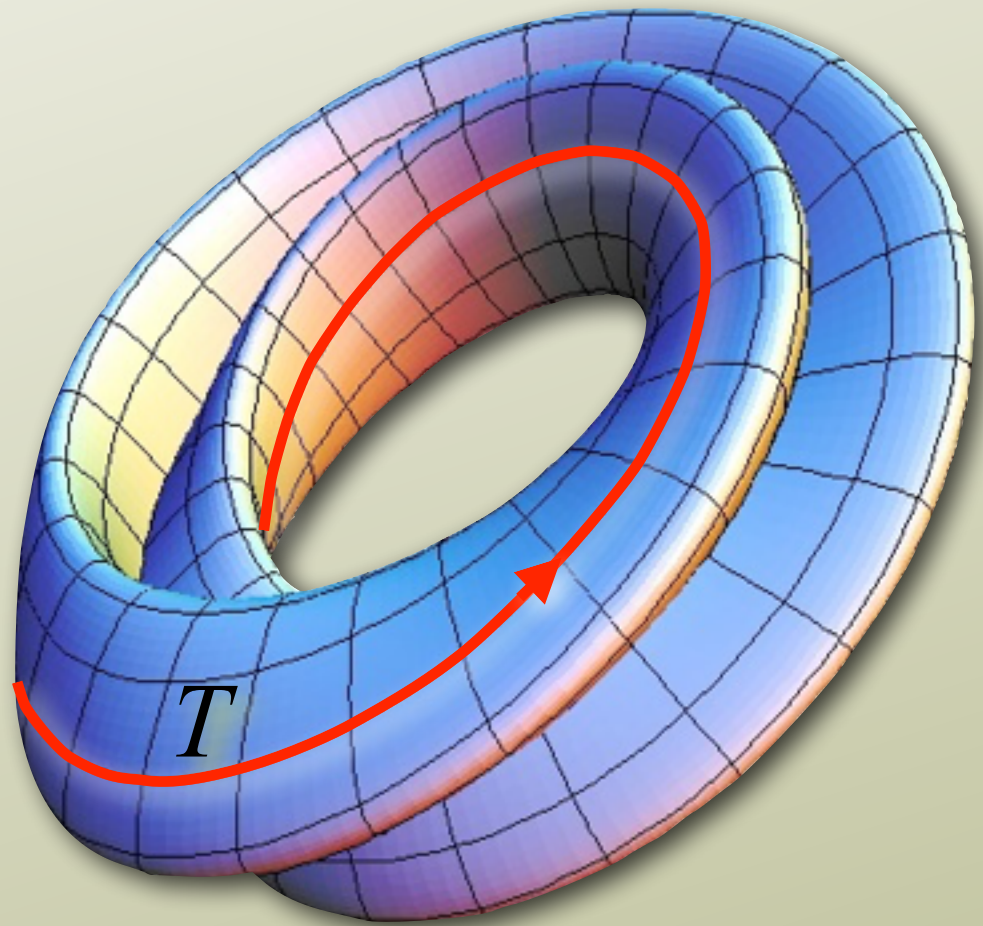


Ground State Manifold



Maps from $\pi_1(B) \to \pi_1(T)$

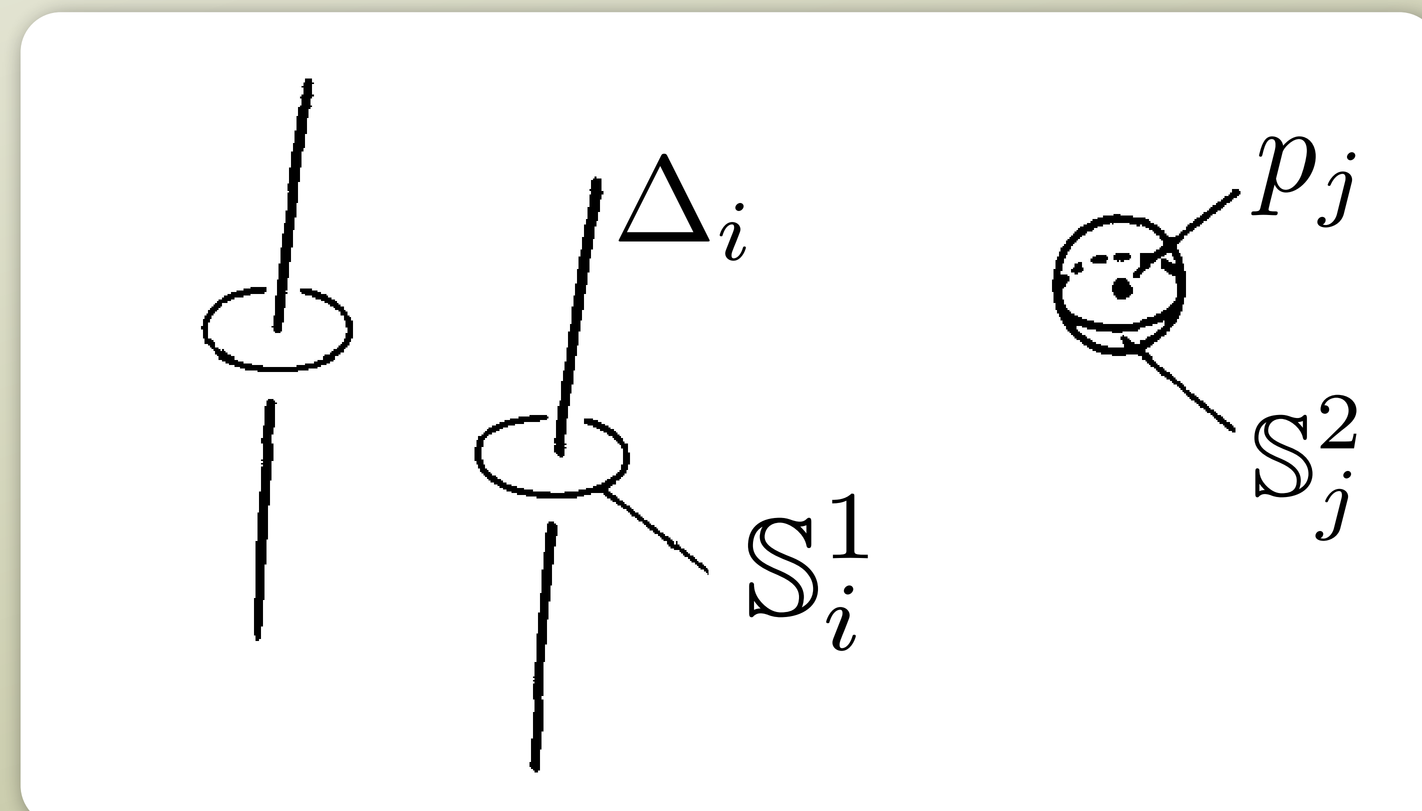
Penn

Seminal Paper on Which My Knowledge is Based

Acta Applicandae Mathematicae **8** (1987), 65–74

Topological Properties of Ordinary Nematics in 3-space

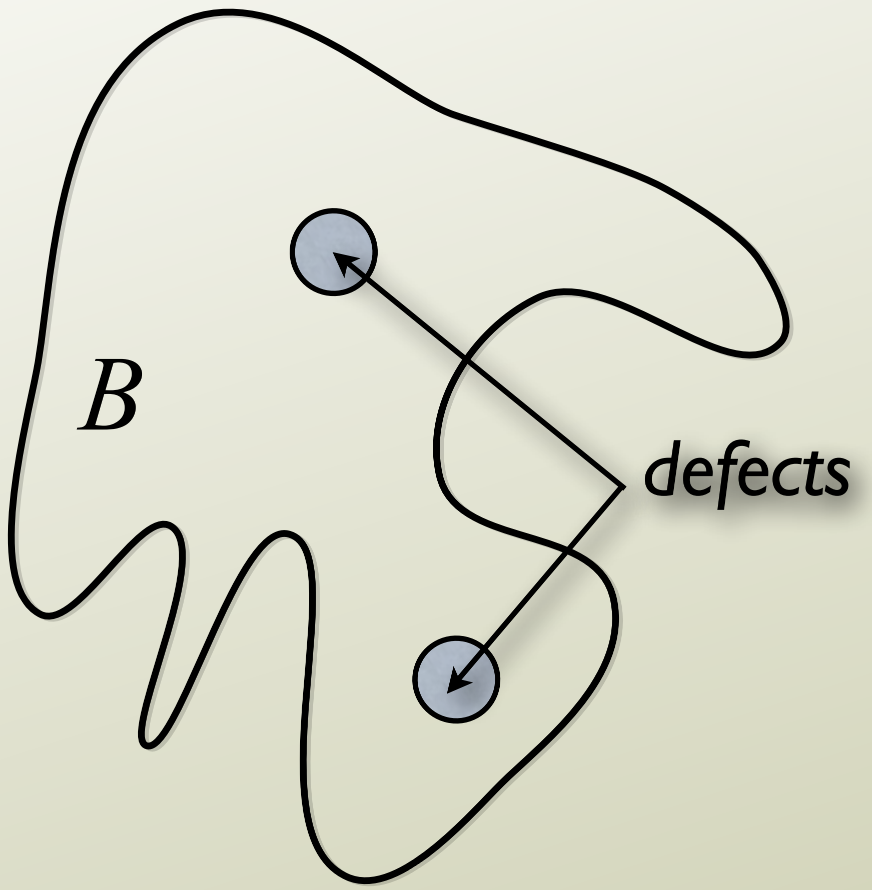
KLAUS JÄNICH
Universität Regensburg, Fakultät für Mathematik, Universitätsstrasse 31, D-8400 Regensburg, West Germany



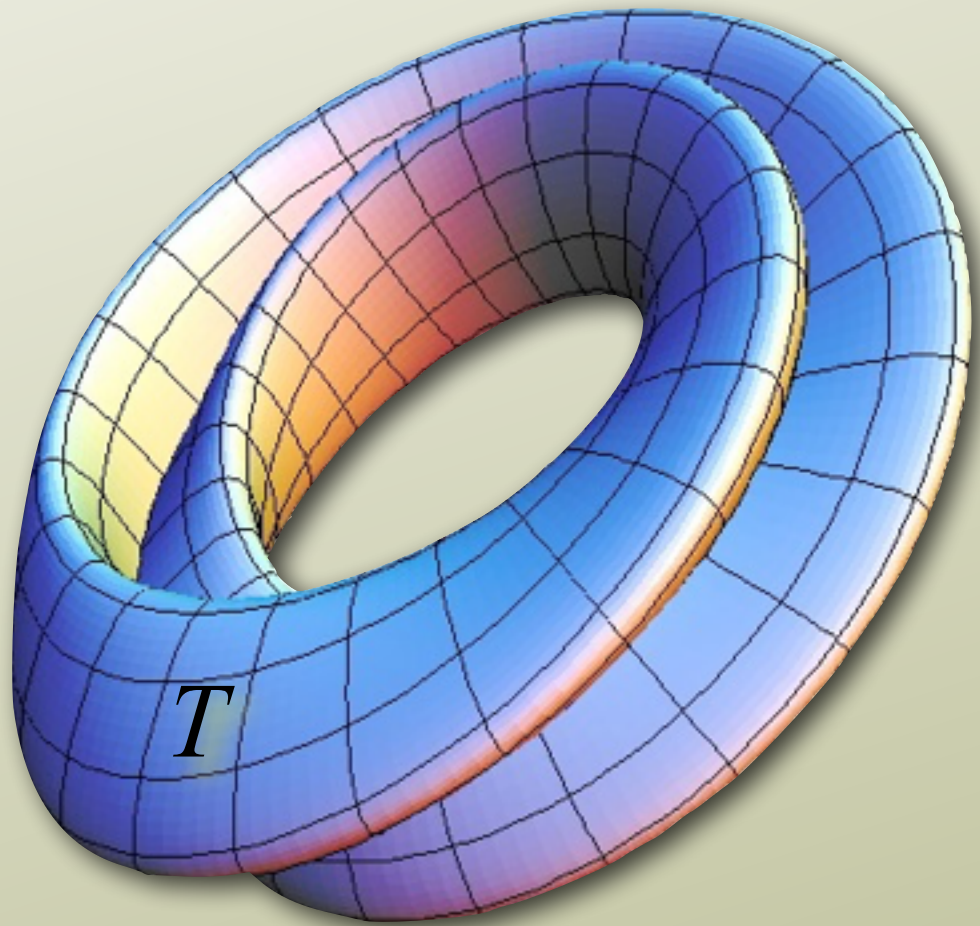
Defect Lines and Defect Points $\Leftrightarrow$ *Different Generalizations*

Second Homotopy Group

Sample



Ground State Manifold



Maps from $\pi_1(B) \to \pi_1(T)$

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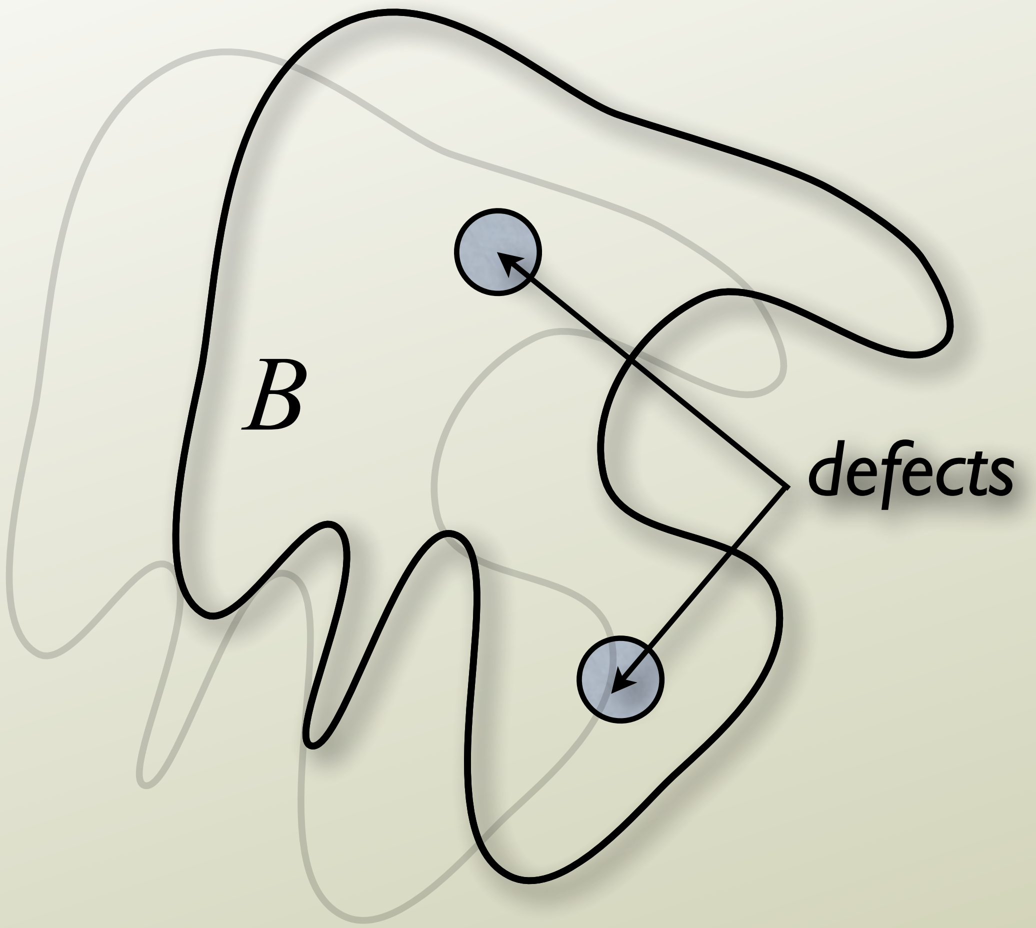
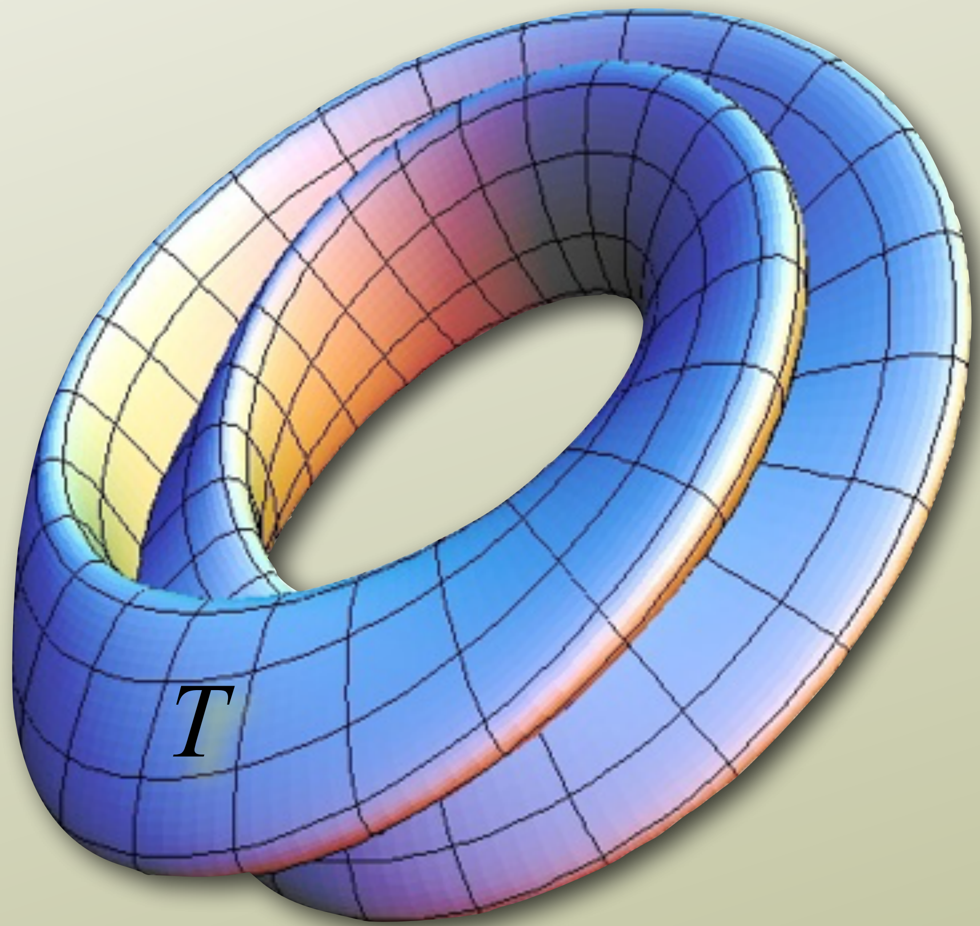
Second Homotopy Group

Sample



Ground State Manifold



Maps from $\pi_1(B) \rightarrow \pi_1(T)$

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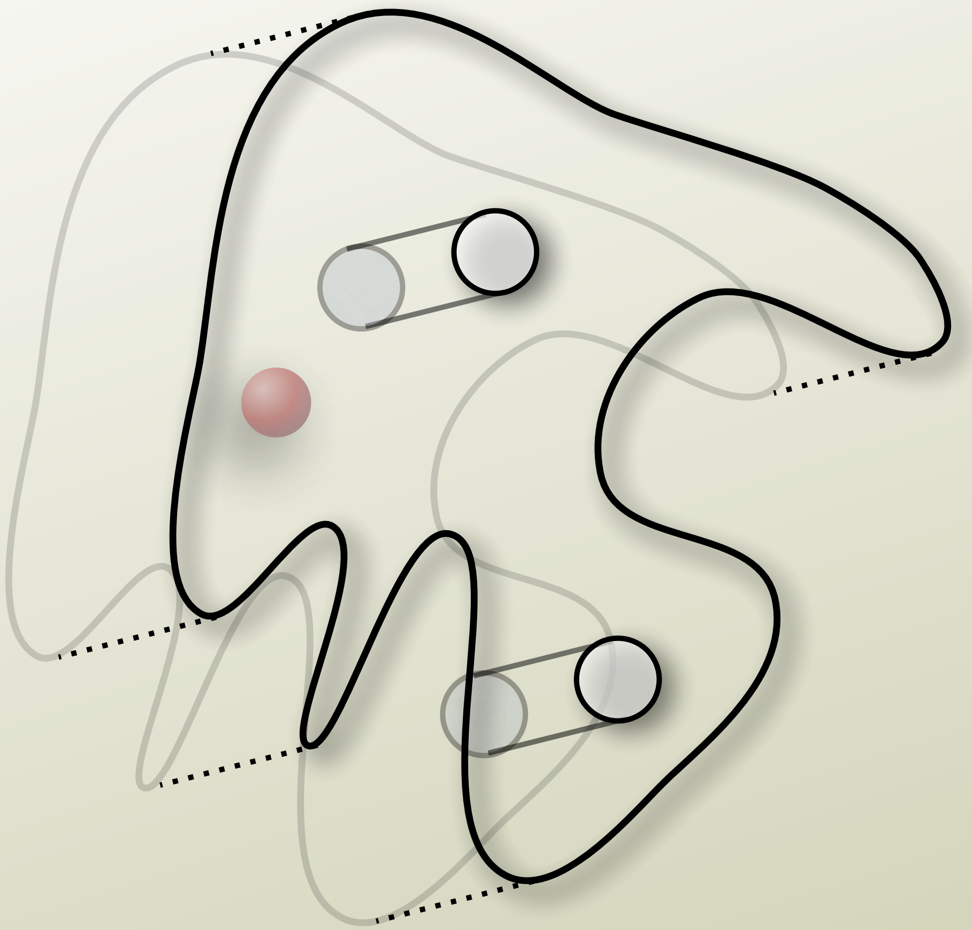
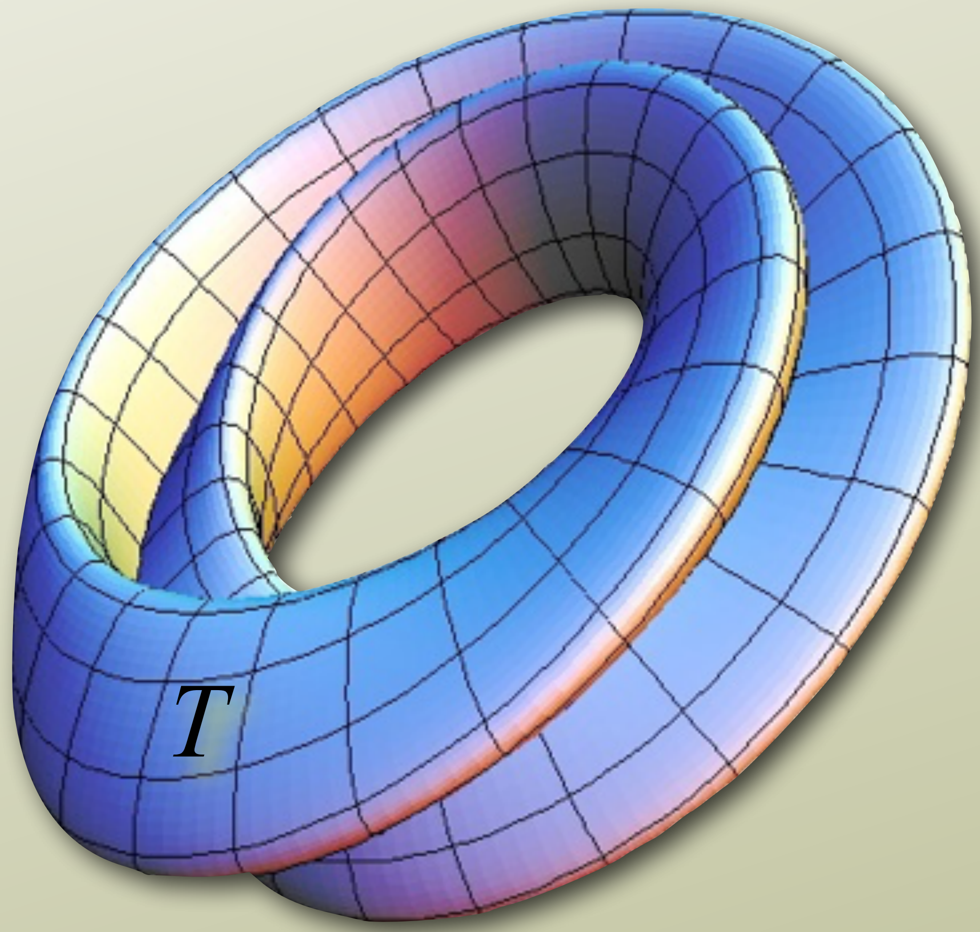
Second Homotopy Group

Sample



Ground State Manifold

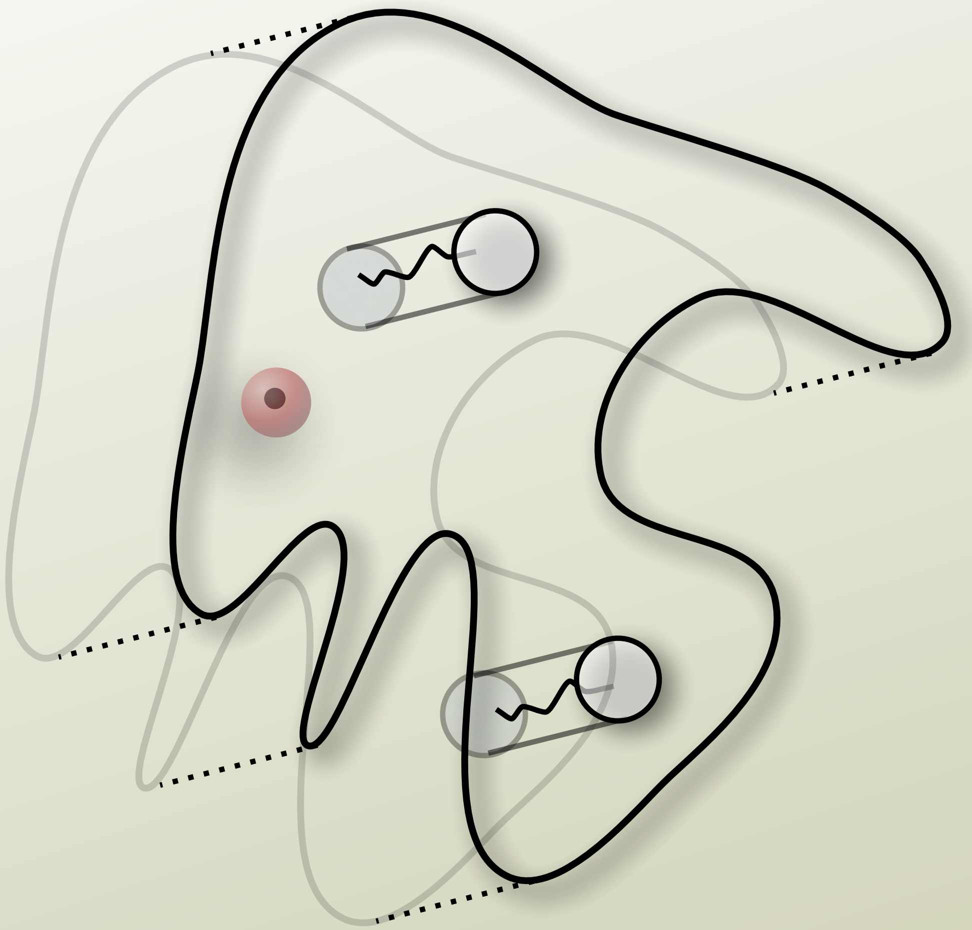


Maps from $\pi_1(B) \rightarrow \pi_1(T)$

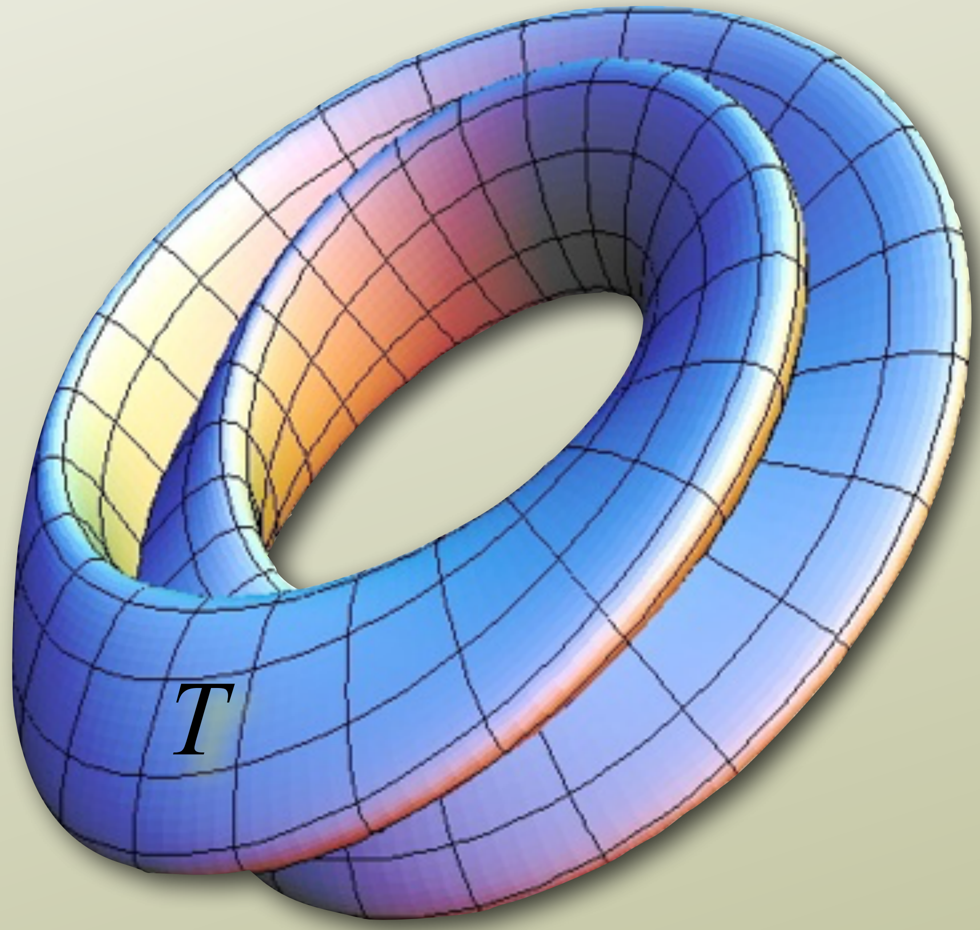
Maps from $\pi_2(B) \rightarrow \pi_2(T)$

Second Homotopy Group

Sample



Ground State Manifold

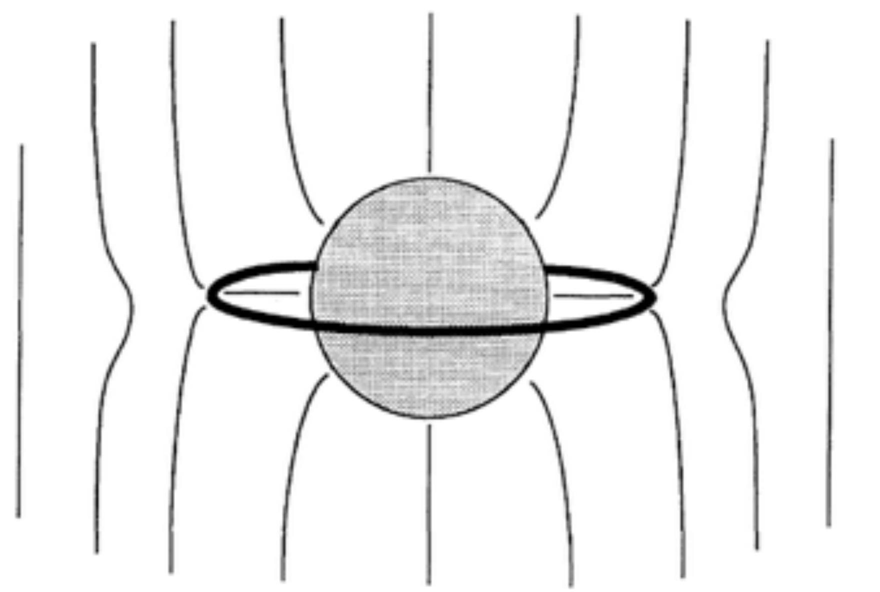


Maps from $\pi_1(B) \longrightarrow \pi_1(T)$

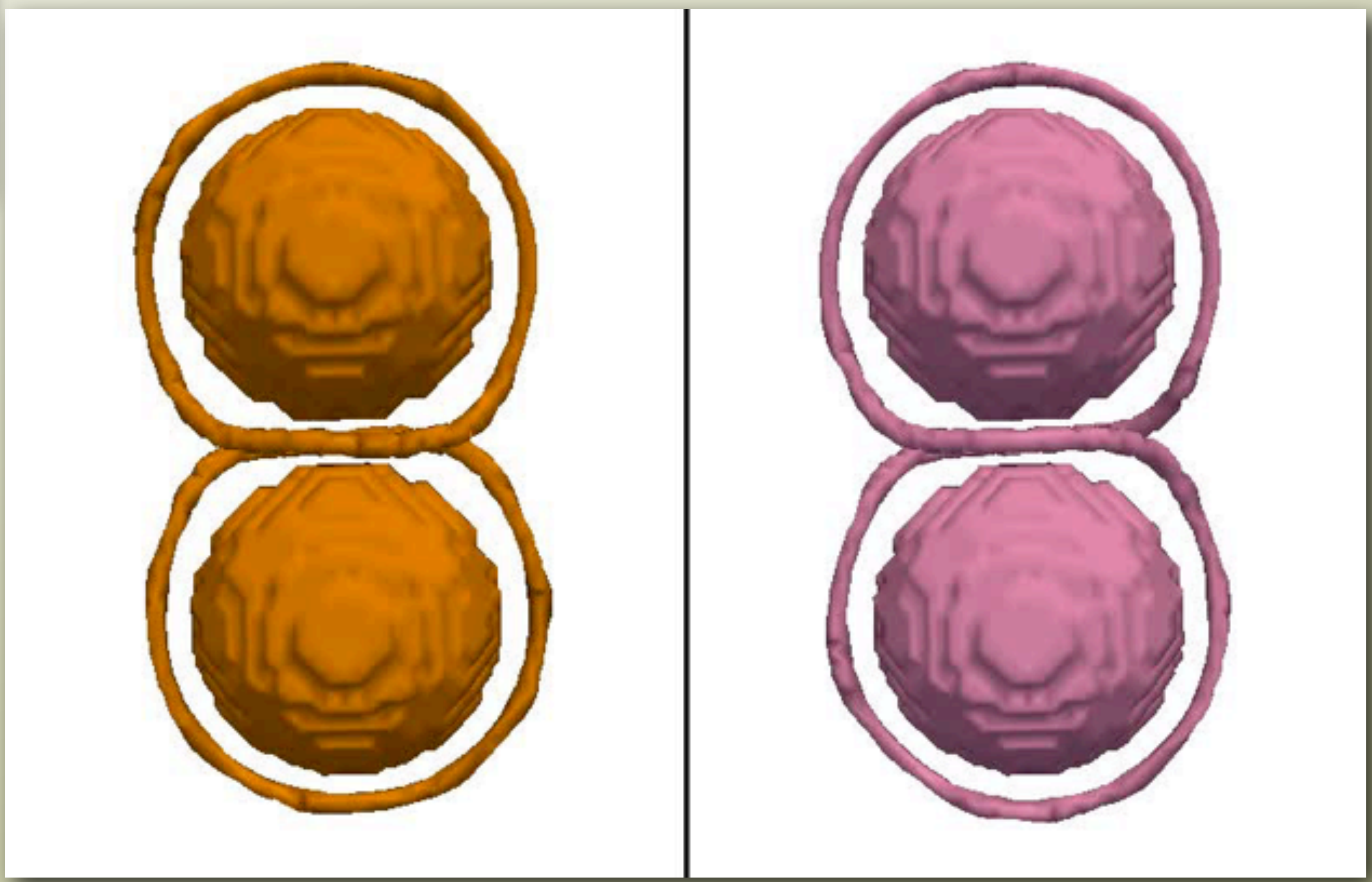
Maps from $\pi_2(B) \longrightarrow \pi_2(T)$

Penn

First Homotopy Group - Line Defects

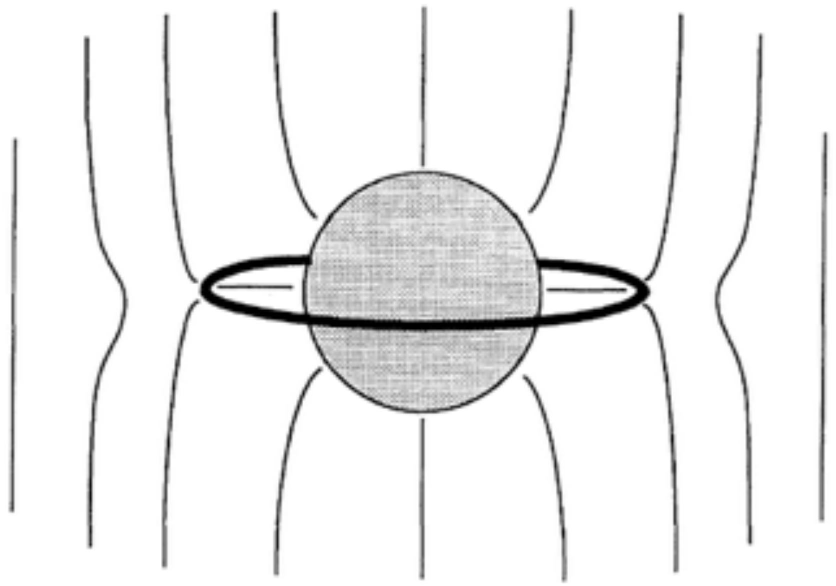


Terentjev, PRE **51** (1995) 1330

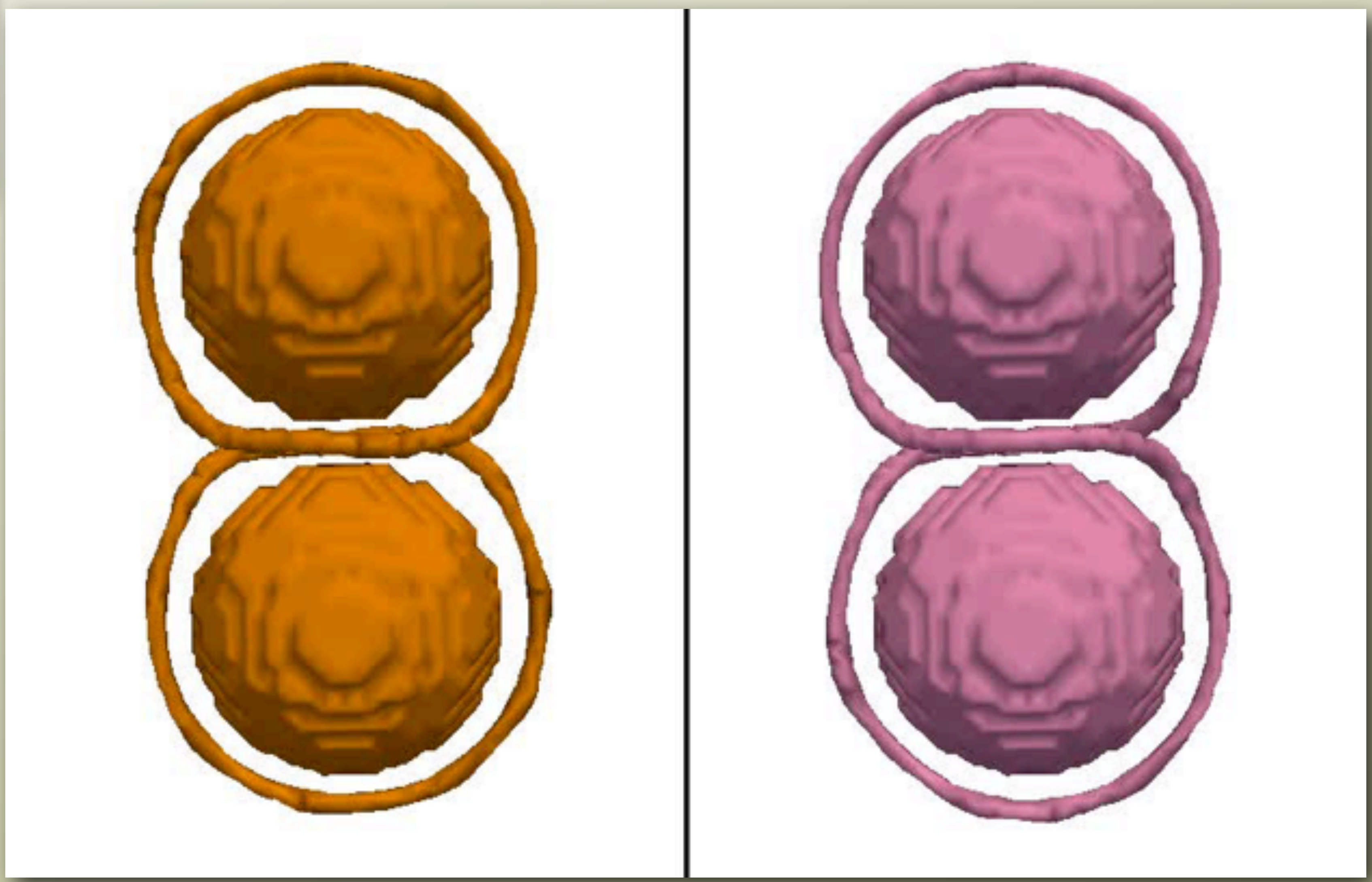


How Do Lines Compensate Points?

First Homotopy Group - Line Defects



Terentjev, PRE **51** (1995) 1330



How Do Lines Compensate Points?

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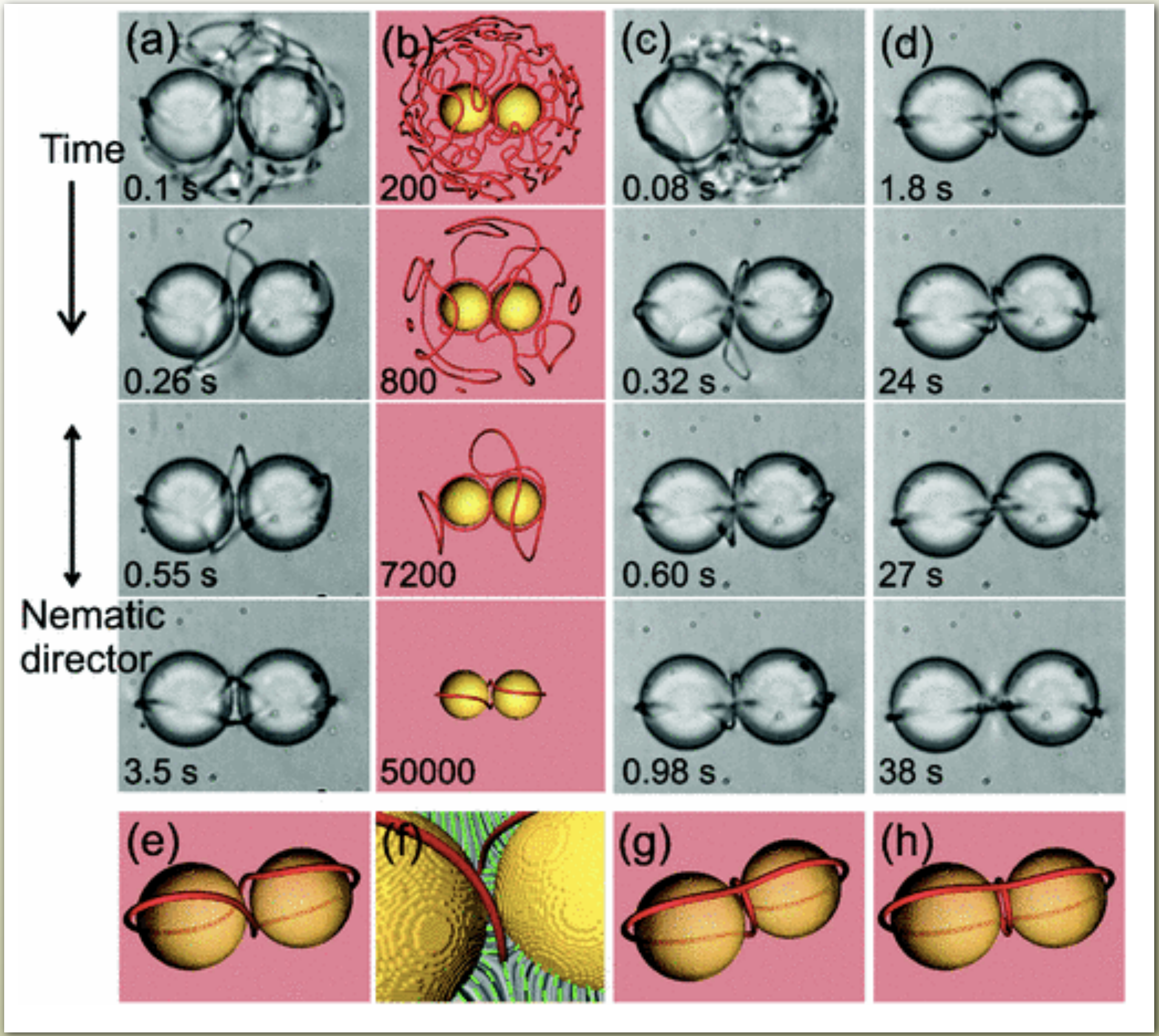
O Topology

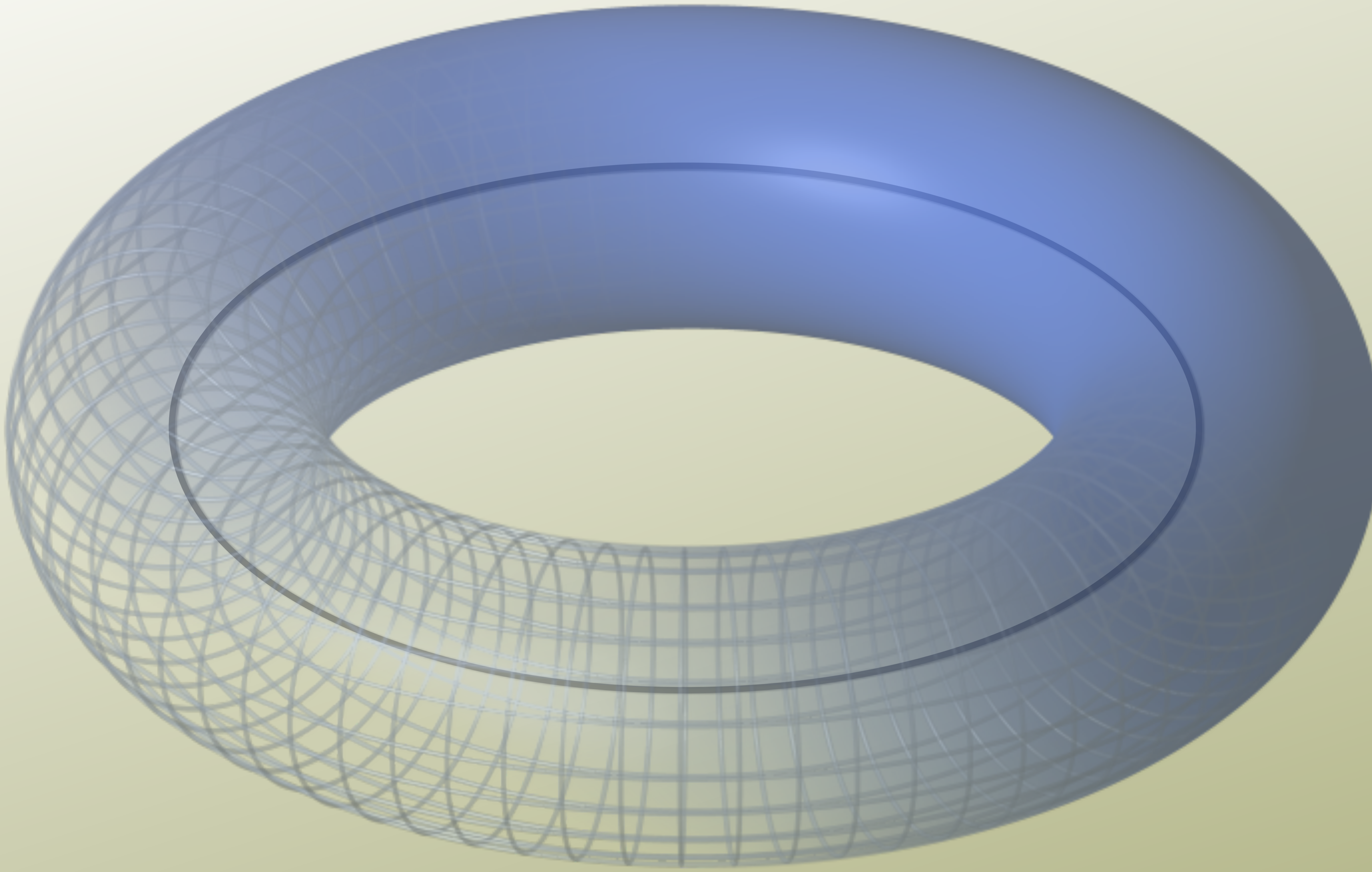


M. Ravnik, M. Škarabot, S. Žumer, U. Tkalec, I. Poberaj, D. Babič, N. Osterman, I. Muševič, *PRL* **99** (2007)

Disclination Loop

http://cis.jhu.edu/education/introPatternTheory/chapters/lie/lie1.html

Disclination Loop



http://cis.jhu.edu/education/introPatternTheory/chapters/lie/lie1.html

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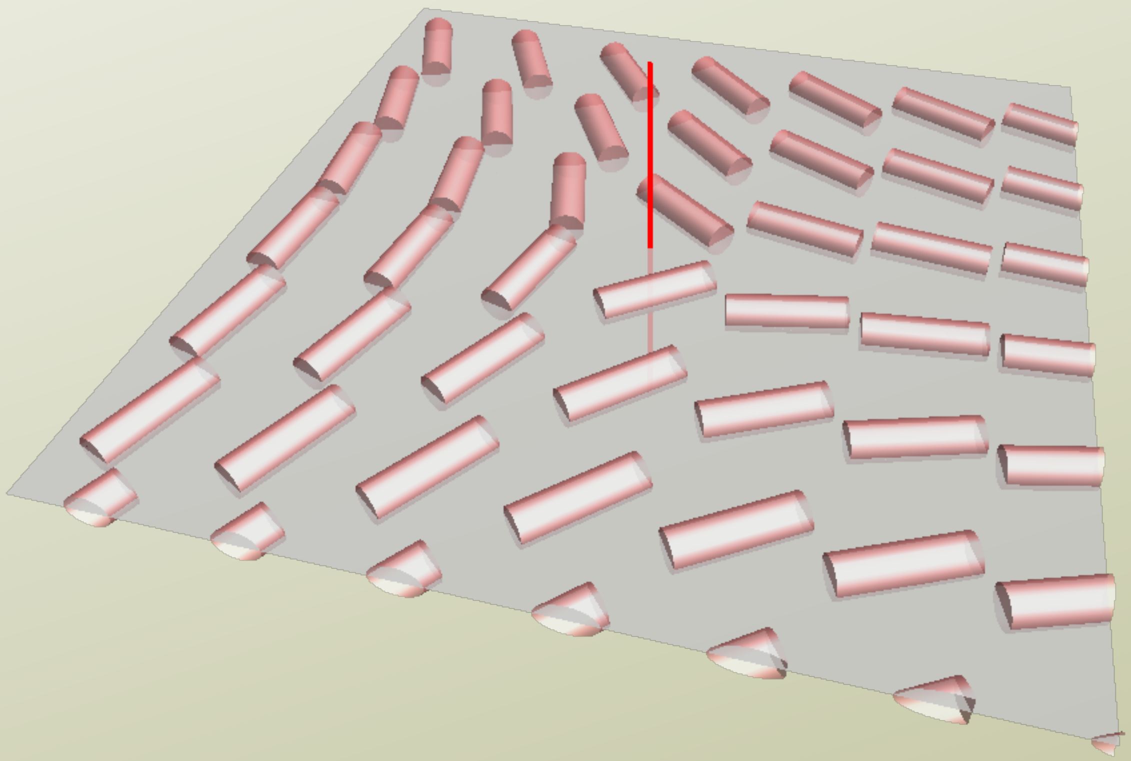
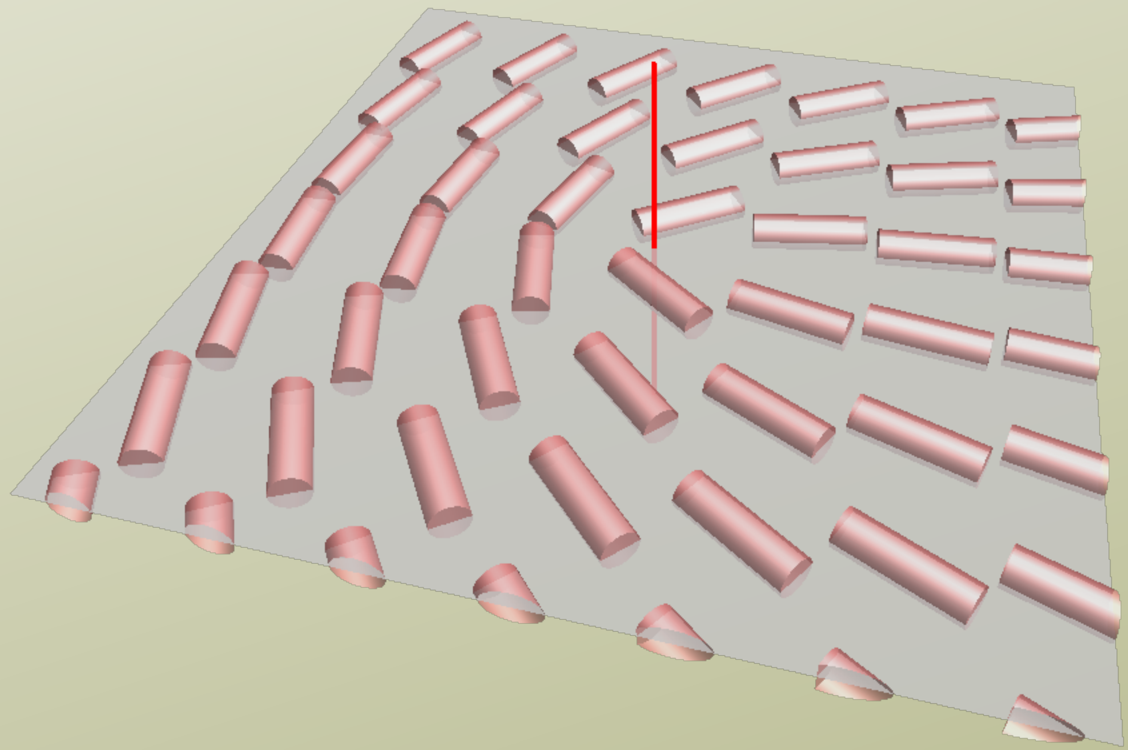
Uniaxial Nematic

- $\pm\frac{1}{2}$ are homotopic in uniaxial nematics

$$\pi_1(\mathbb{RP}^2) = \mathbb{Z}/2\mathbb{Z}$$





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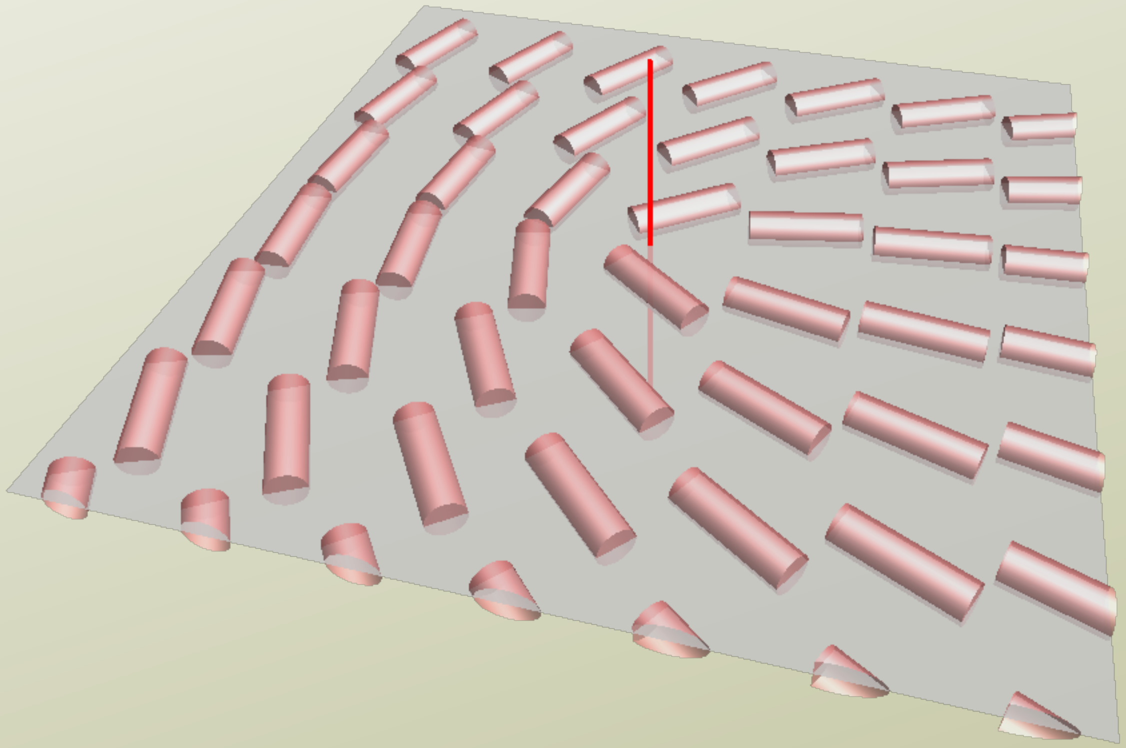

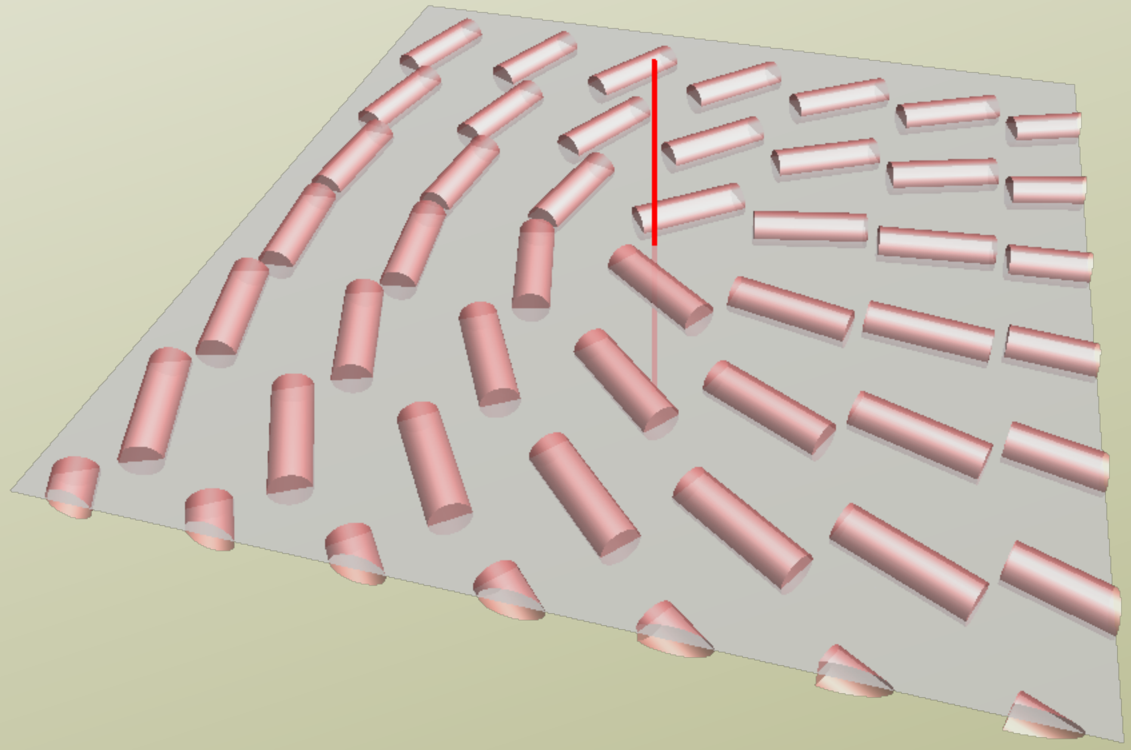

Penn

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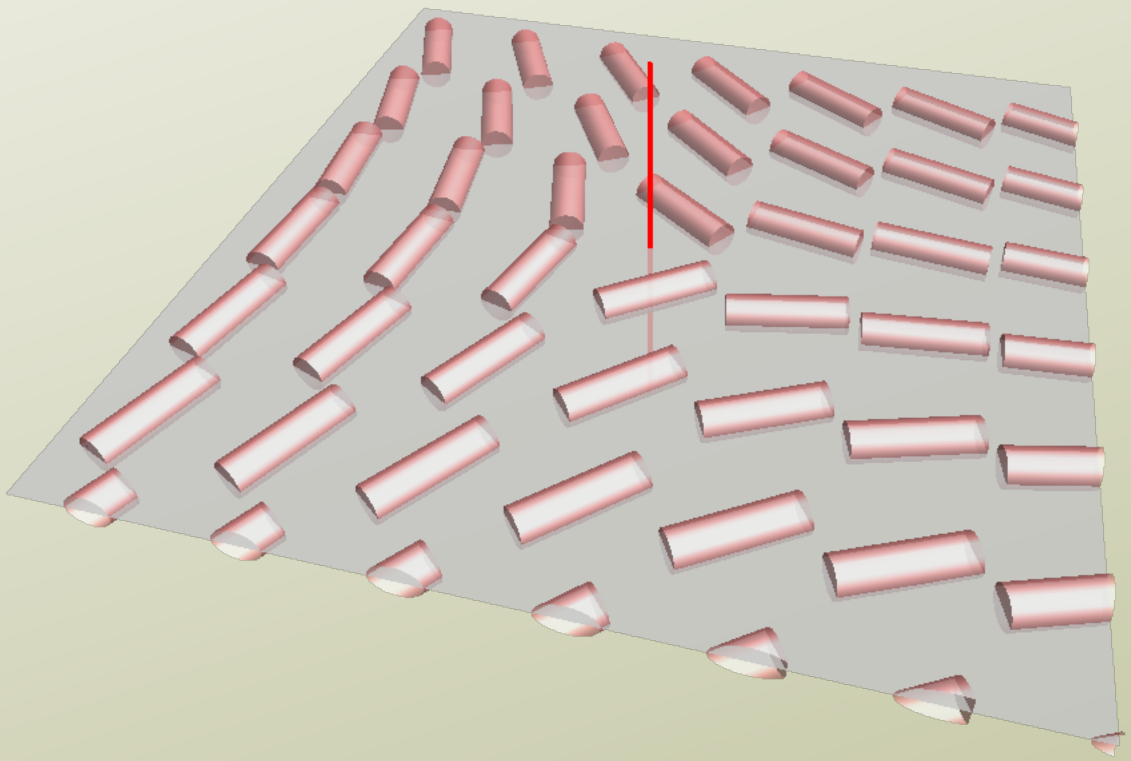

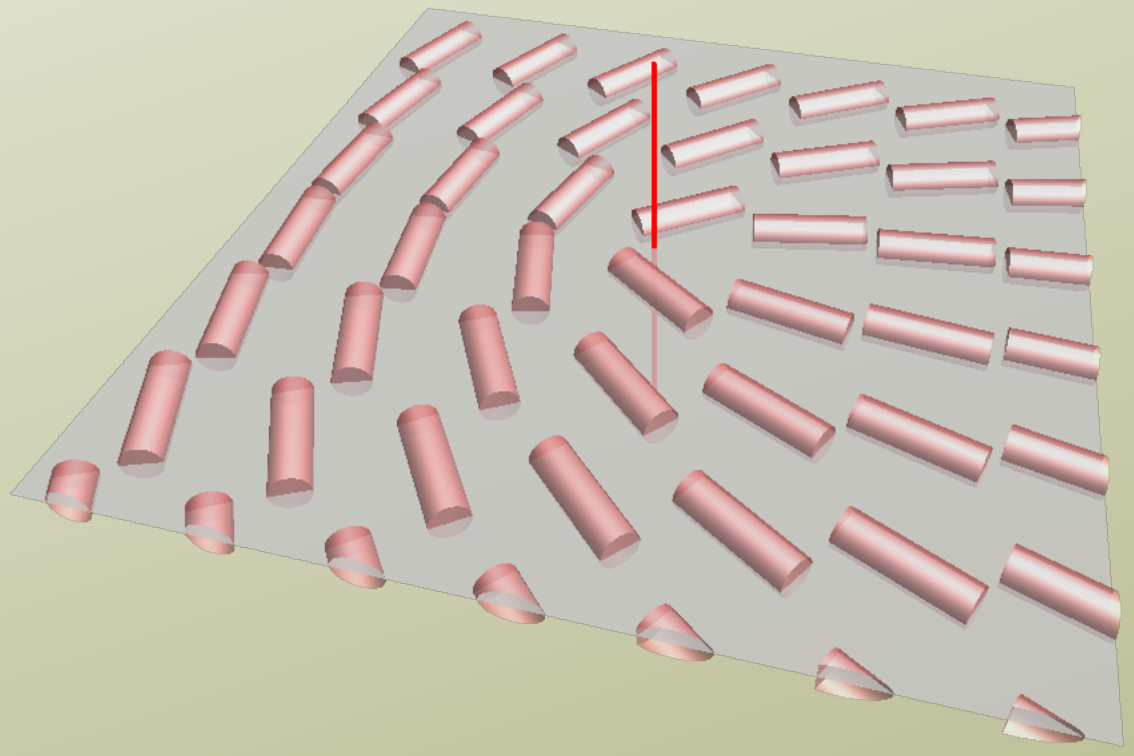

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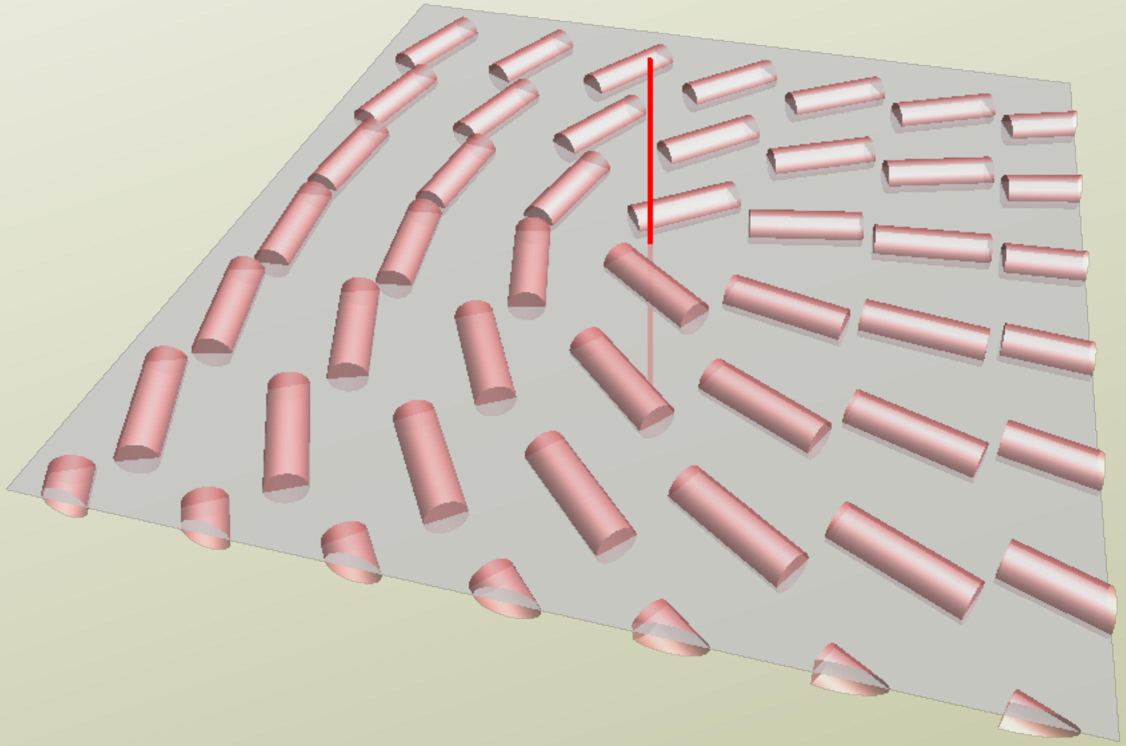

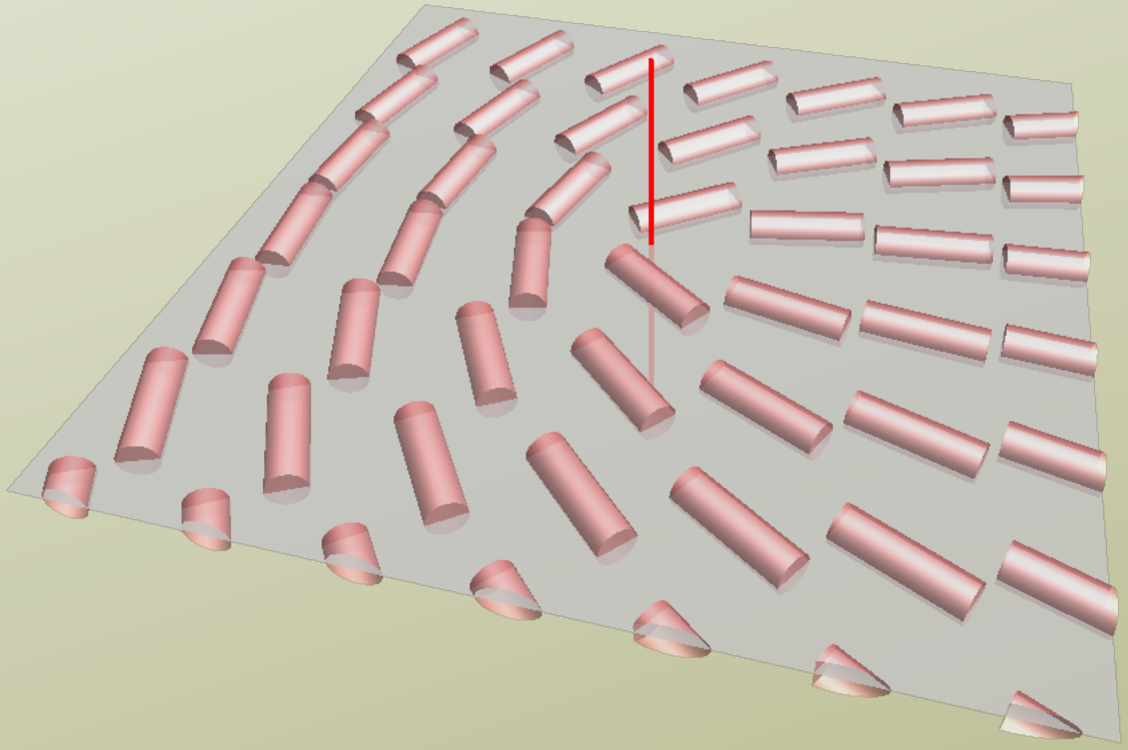

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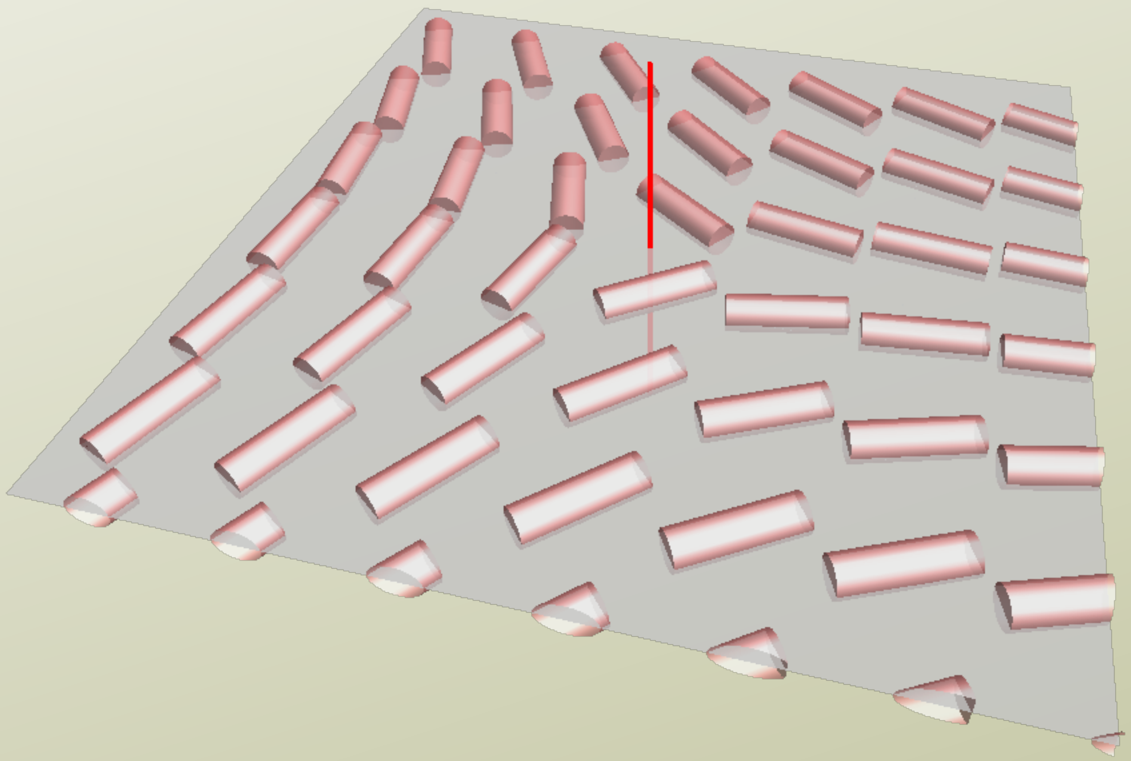

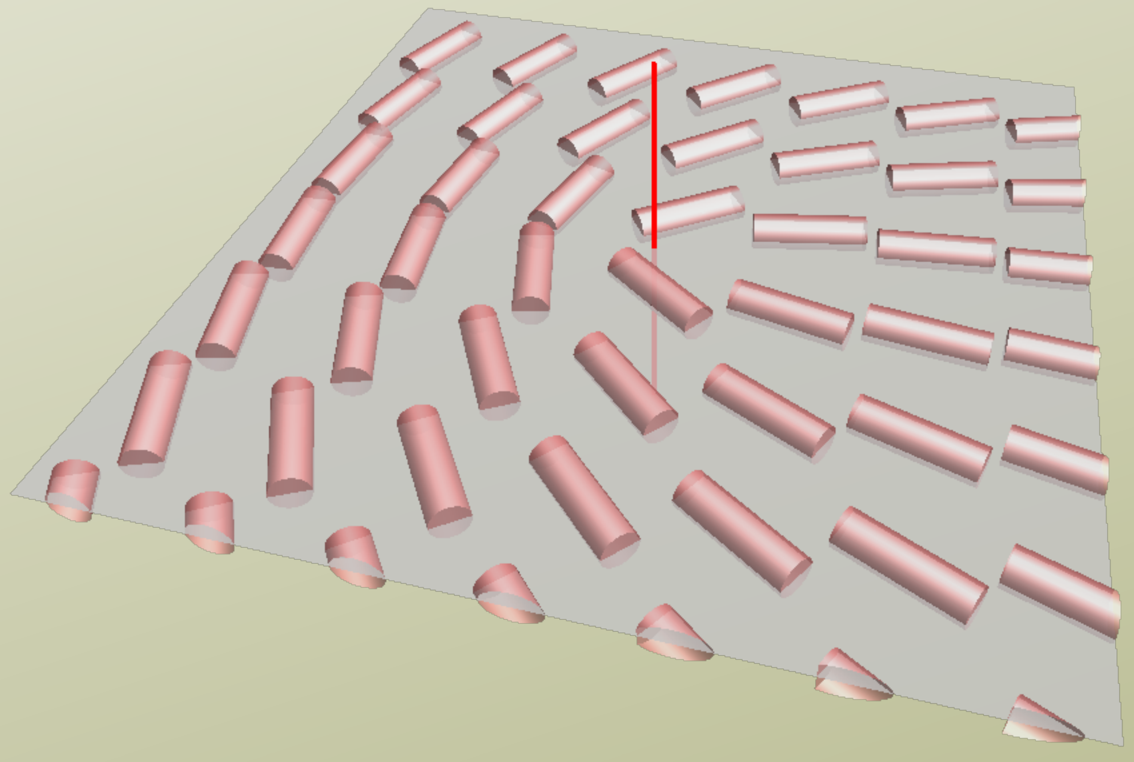

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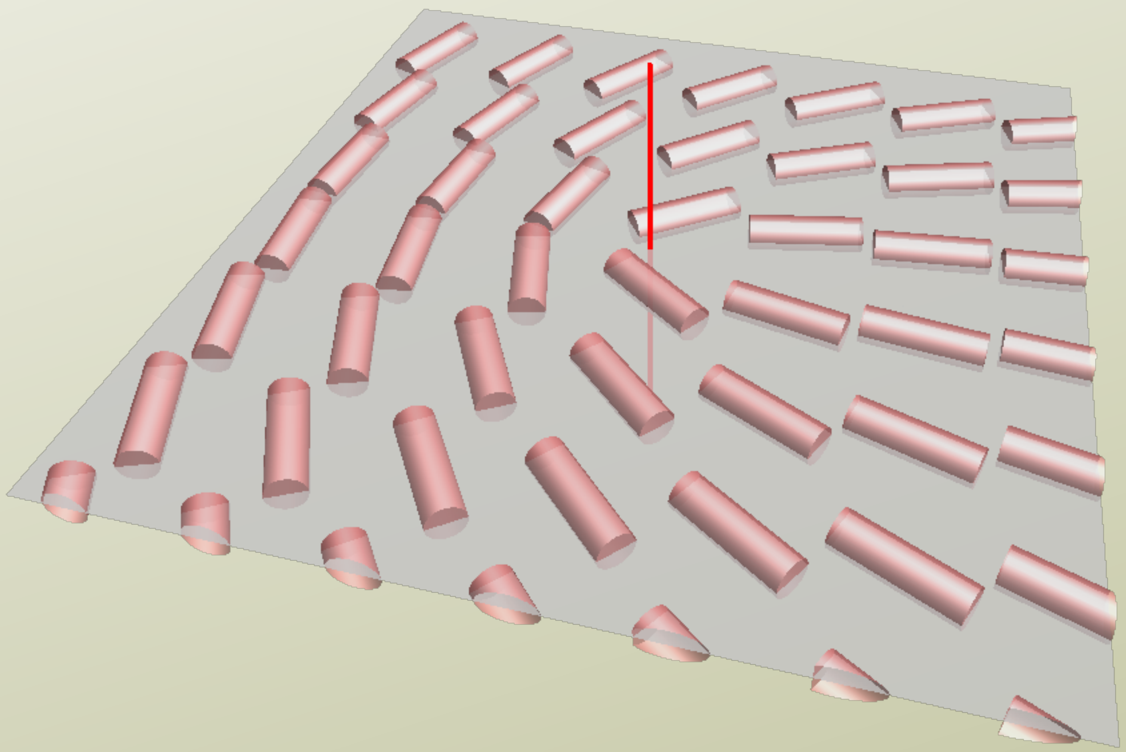
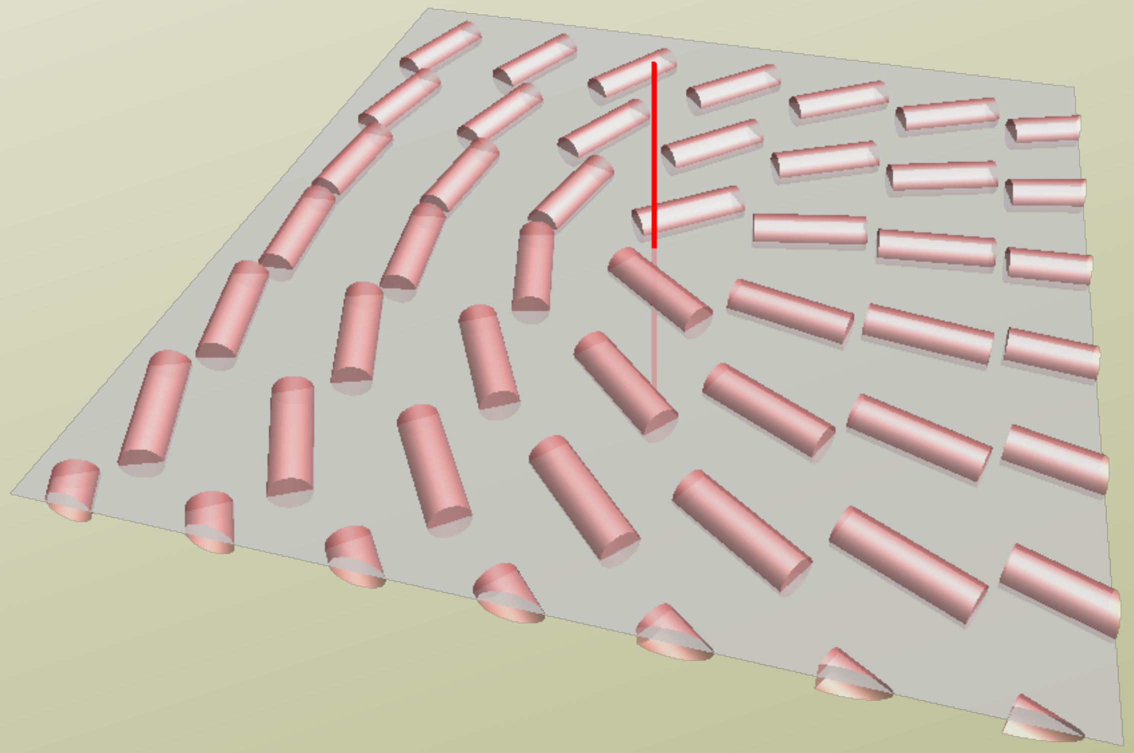
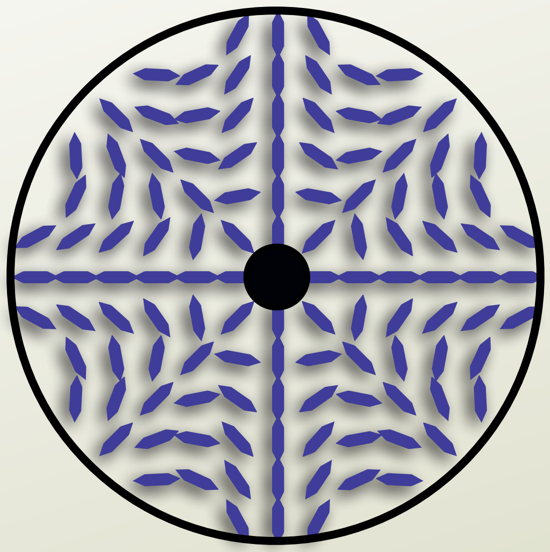


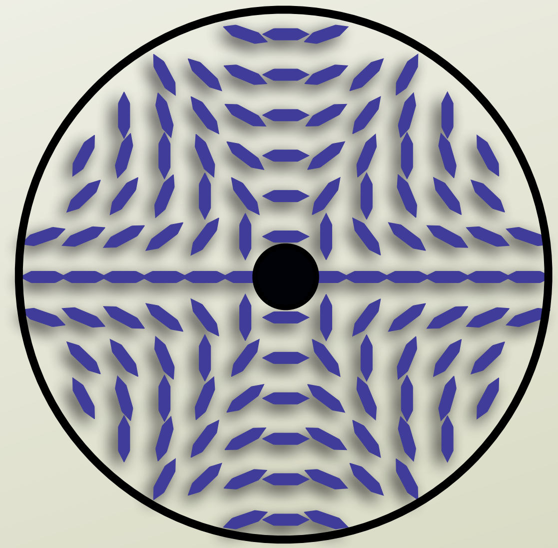


Penn

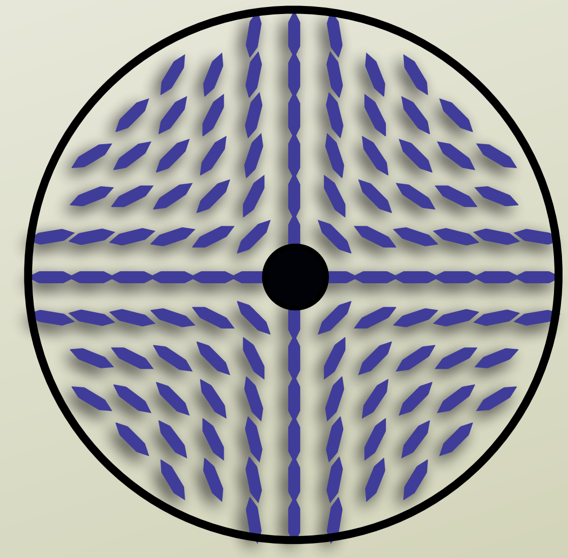
Point Defects in Two Dimensions



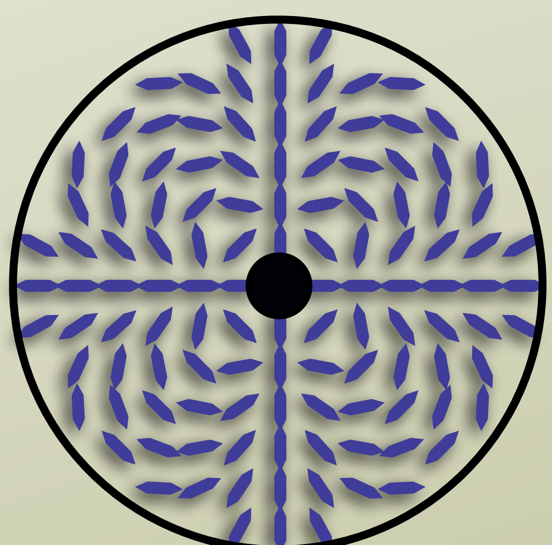
$k = -3$



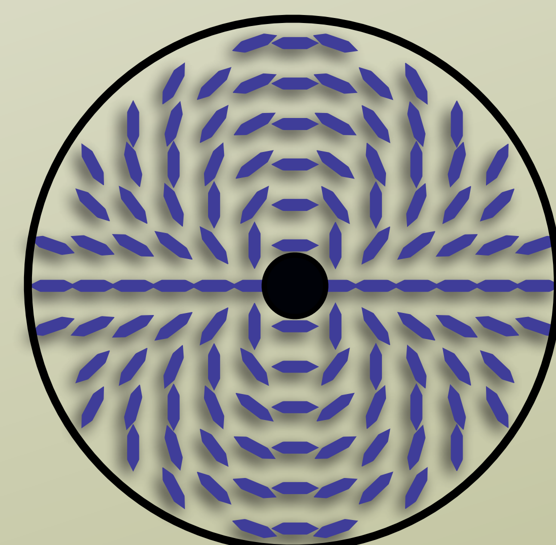
$k = -2$



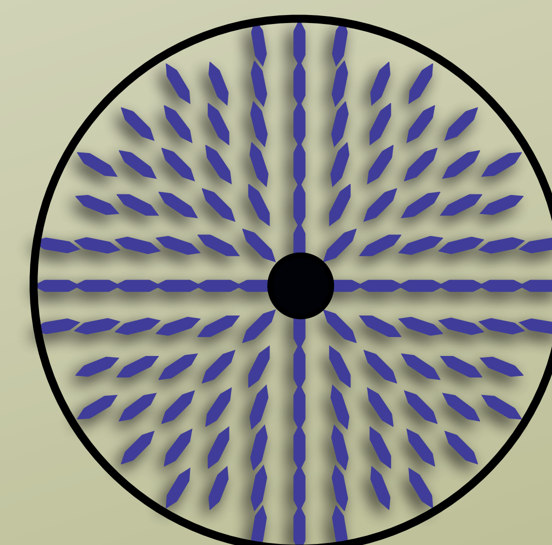
$k = -1$



$k = +3$



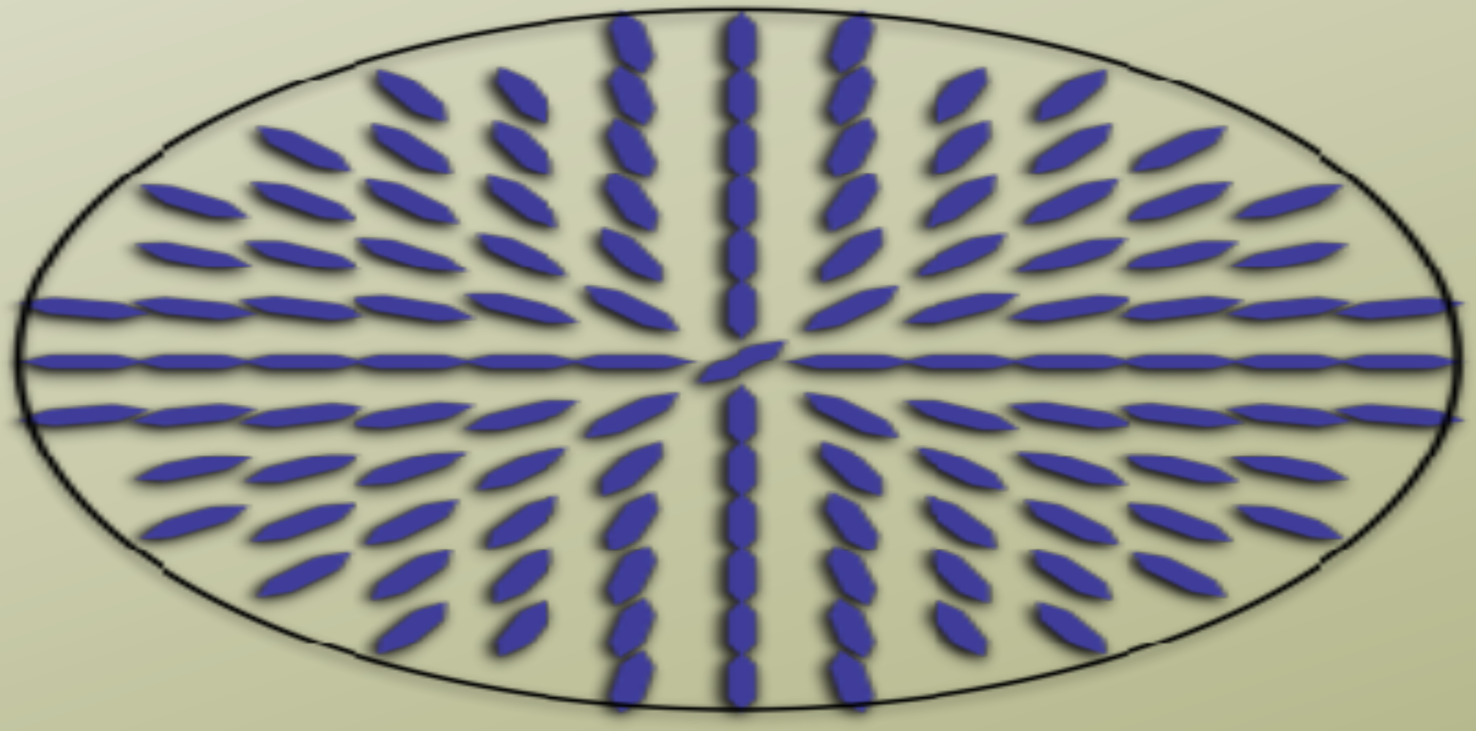
$k = +2$



$k = +1$

$$\mathbf{n} = [\cos(k\phi), \sin(k\phi), 0]$$

ESCAPE!



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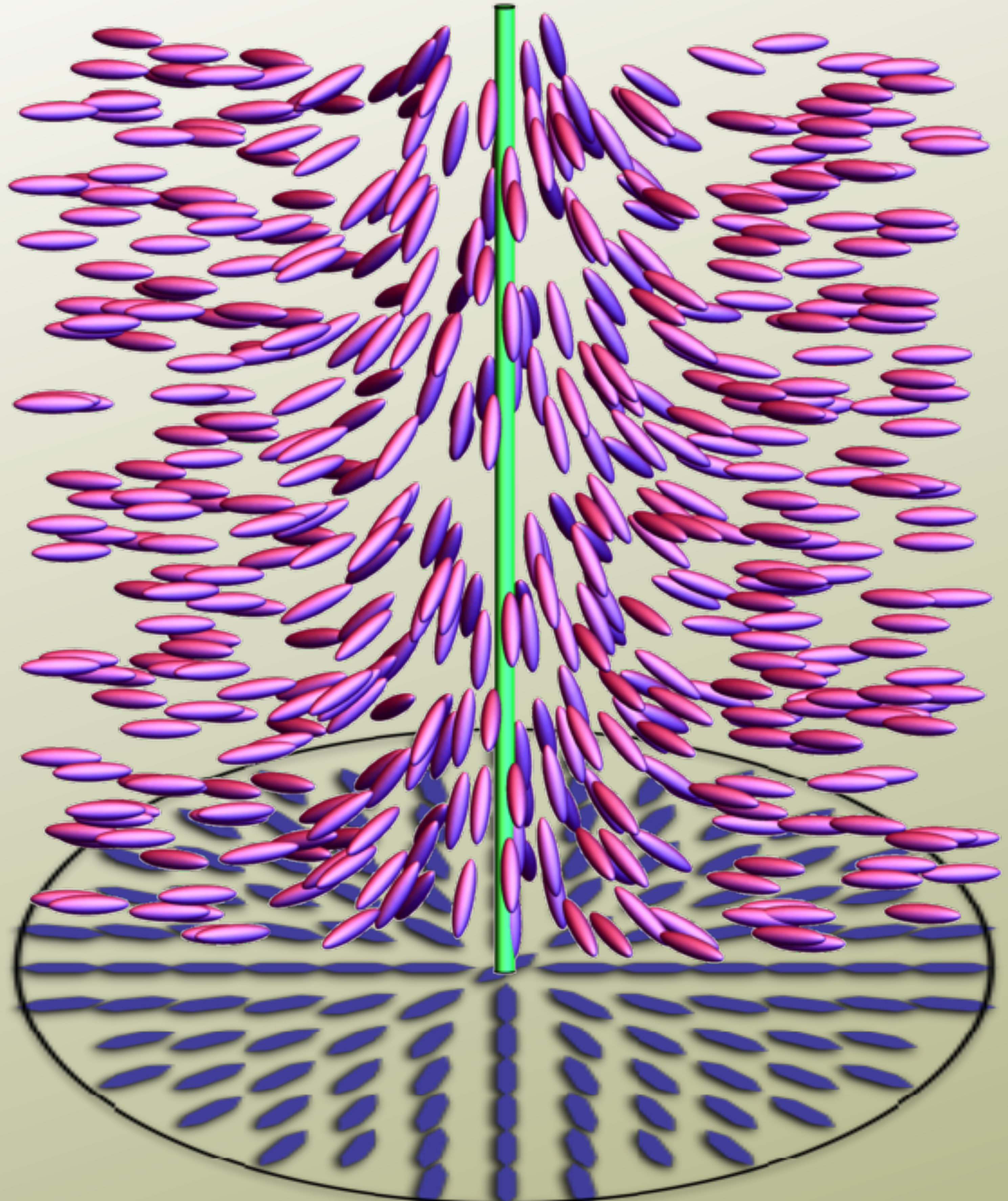

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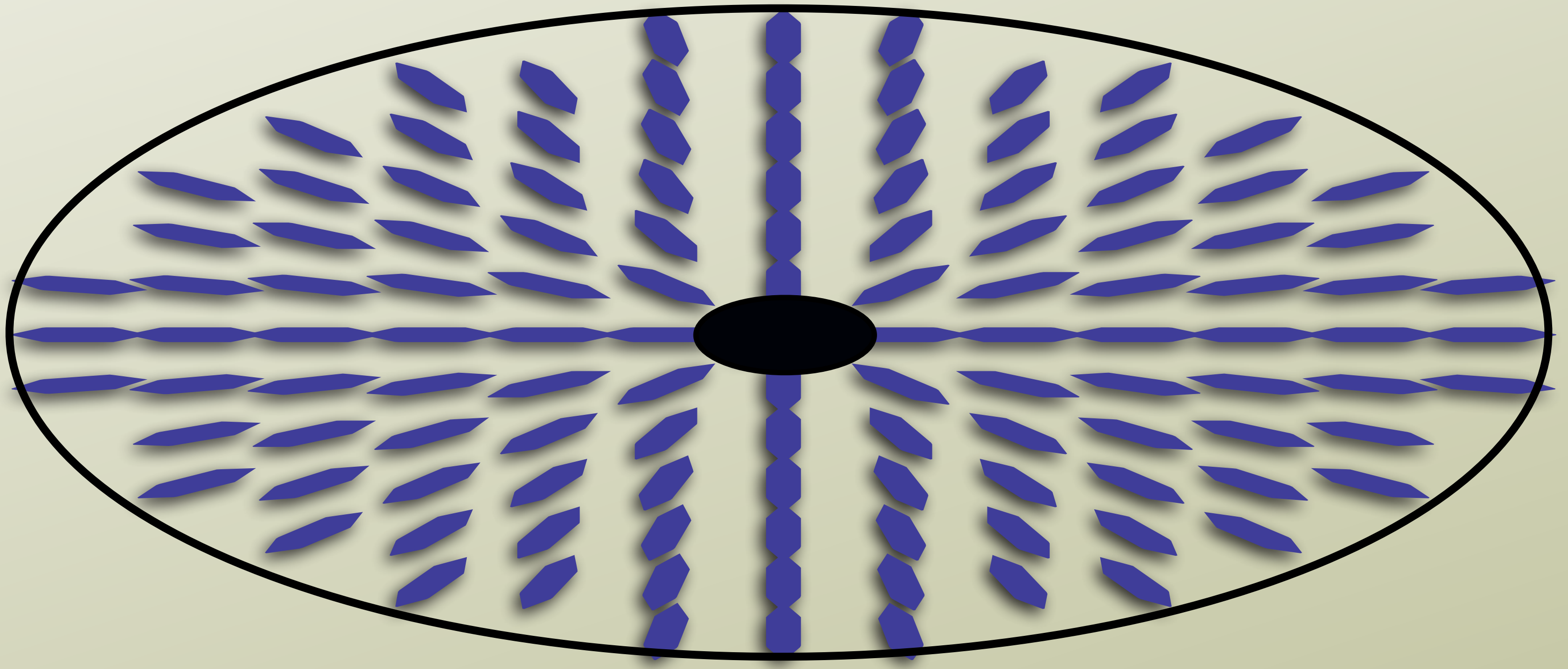
Disclination Loop

- $\pm\frac{1}{2}$ are homotopic in uniaxial nematics
- YES OR NO

Penn

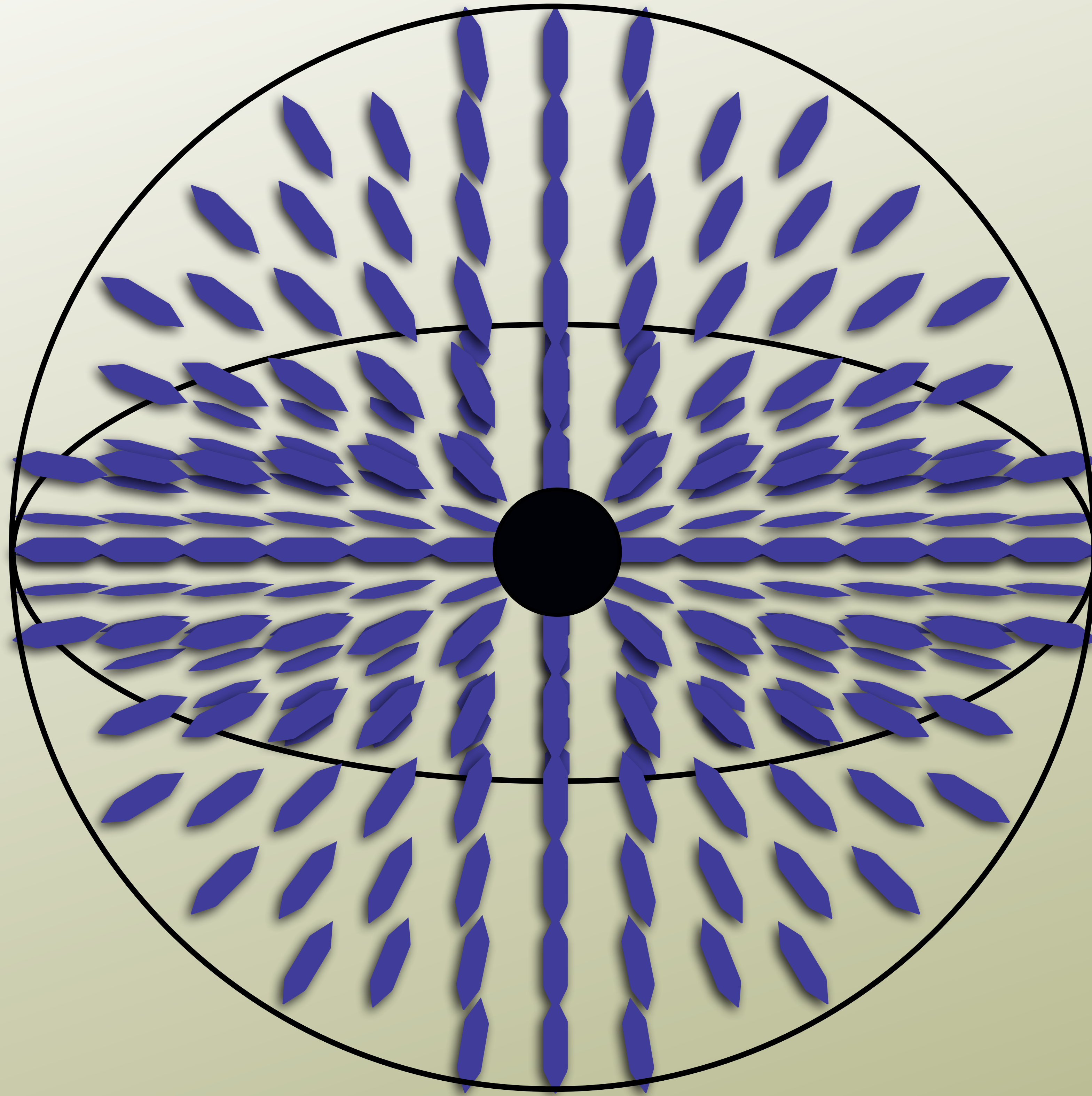
Point Defects in Three Dimensions



hedgehog charge +1

Penn

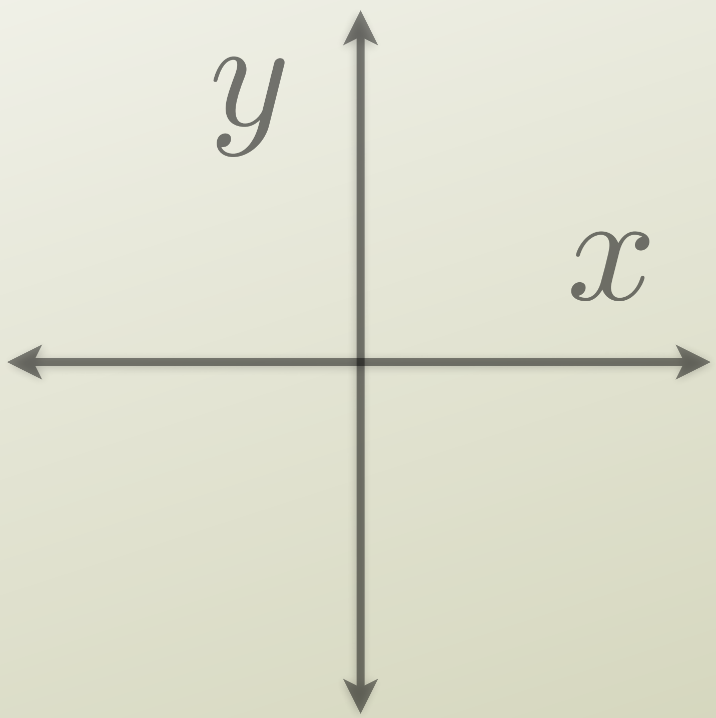
Point Defects in Three Dimensions

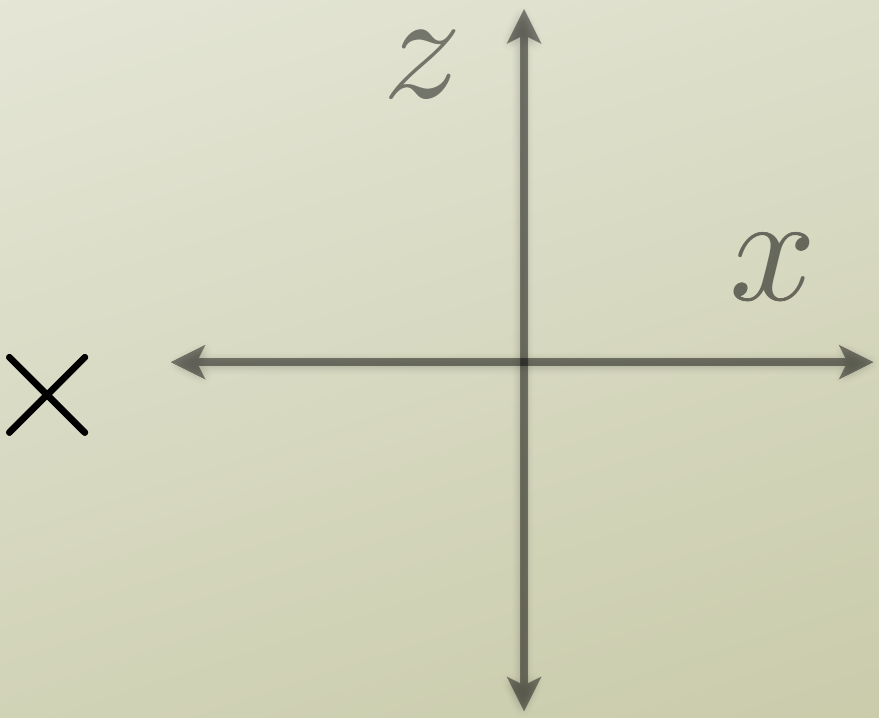


hedgehog charge +1

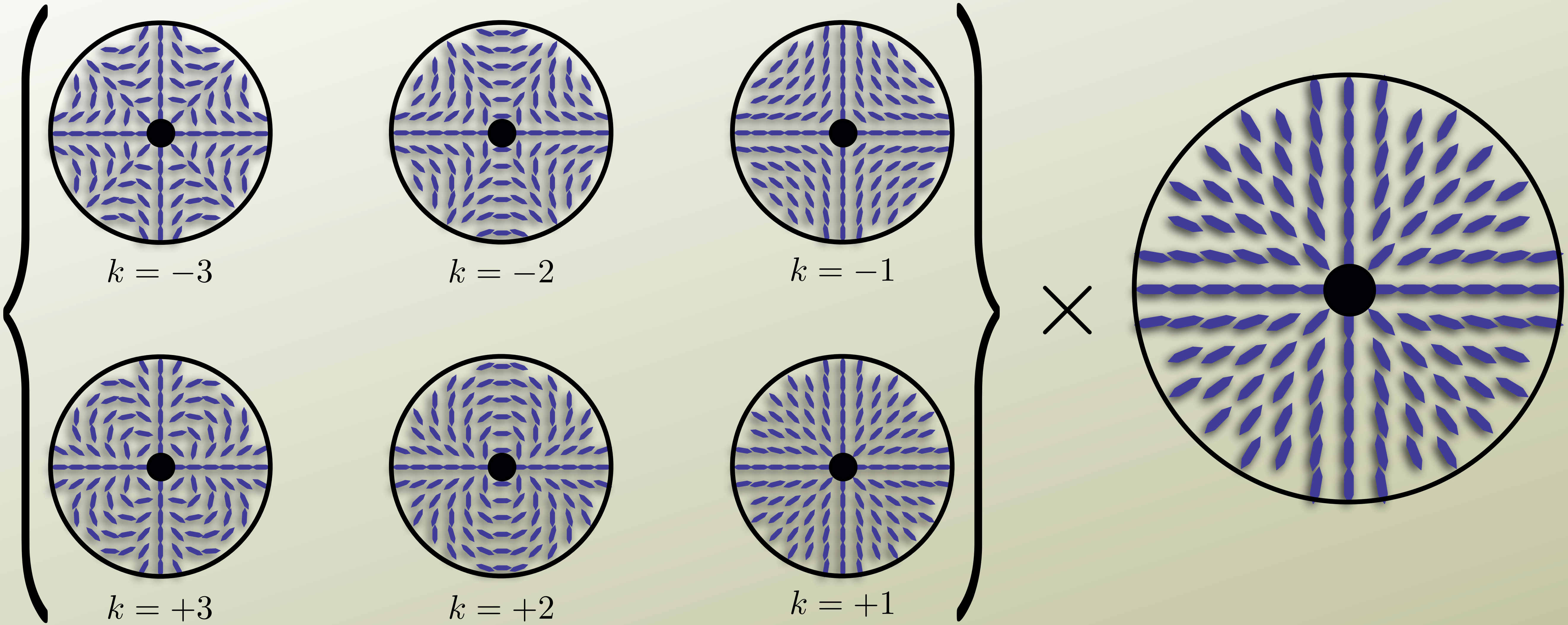
Penn

Point Defects in Three Dimensions





Point Defects in Three Dimensions



$$\mathbf{n} = [\sin\theta\cos(k\phi), \sin\theta\sin(k\phi), \cos\theta]$$ $\Longleftarrow$ charge k

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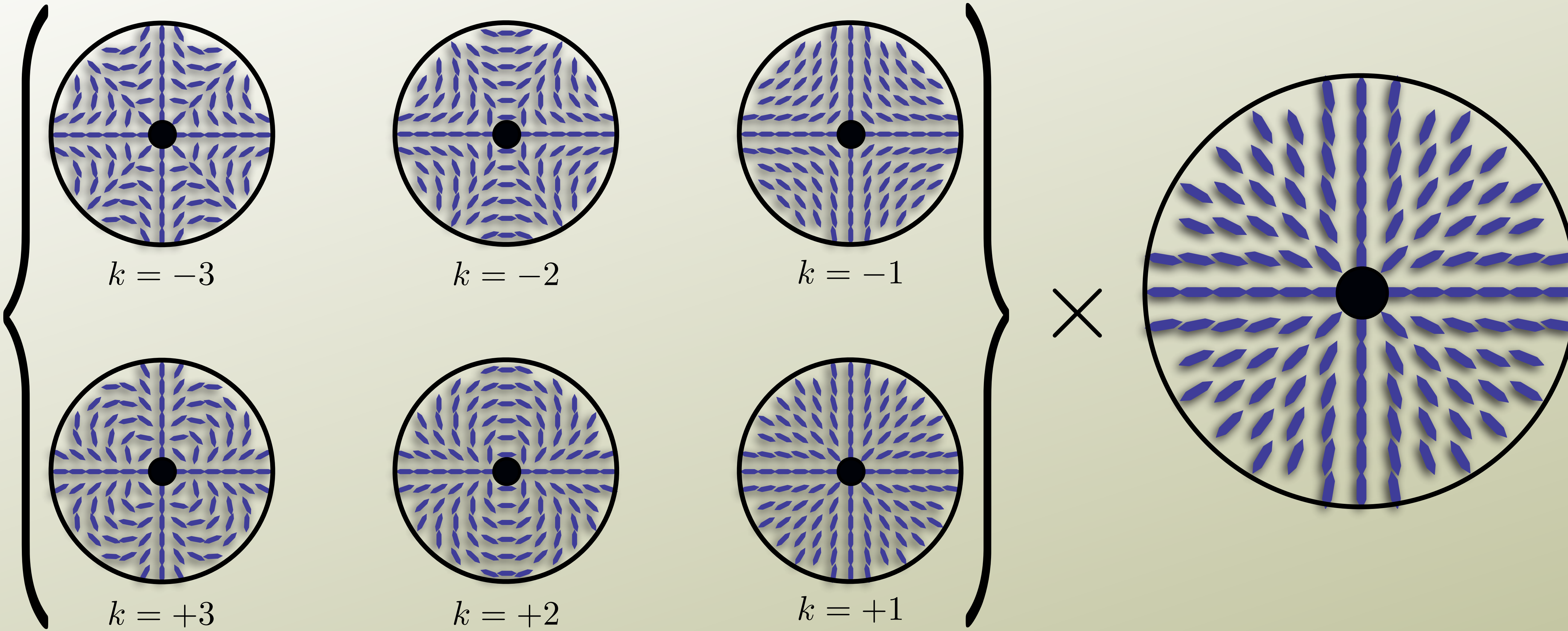
Point Defects in Three Dimensions



$$\mathbf{n} = [\sin\theta\cos(k\phi), \sin\theta\sin(k\phi), \cos\theta] \Leftarrow \text{charge } k$$

180° around $\hat{x}$ $\quad \mathbf{n} = [\sin\theta\cos(k\phi), -\sin\theta\sin(k\phi), -\cos\theta]$

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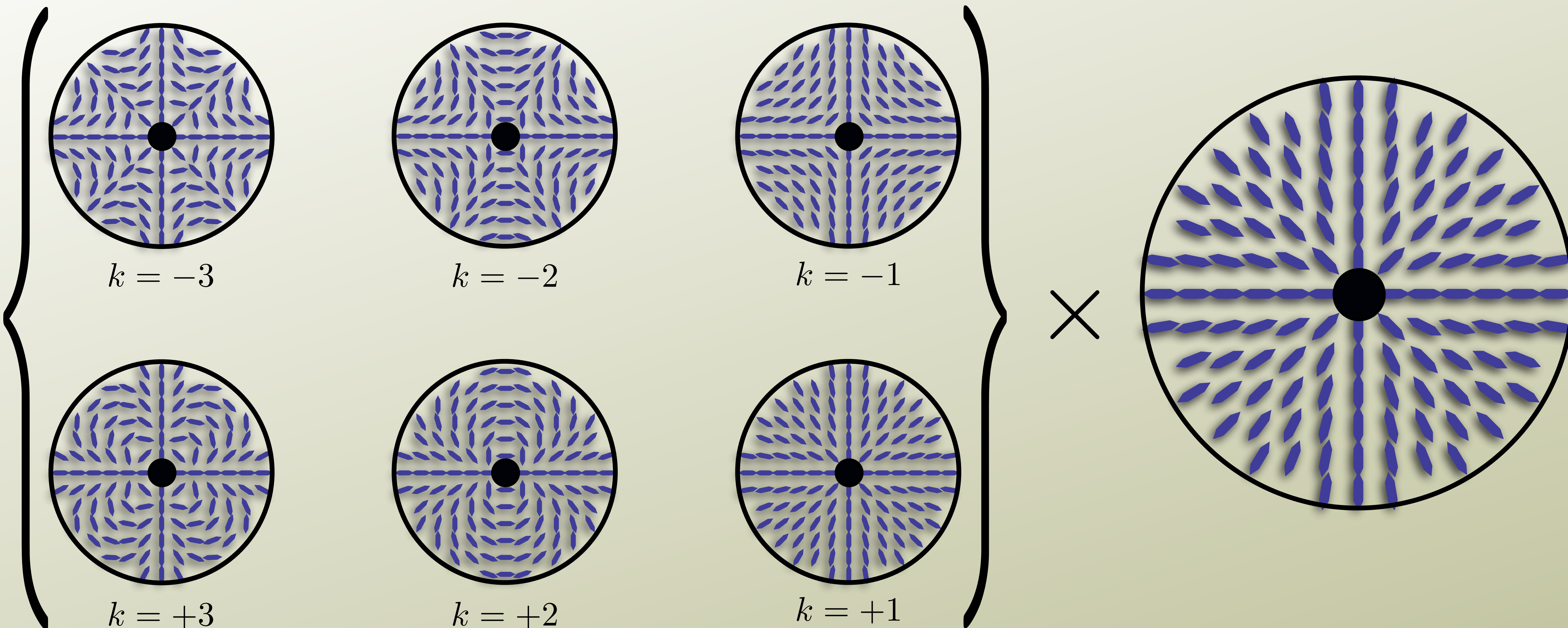
Point Defects in Three Dimensions



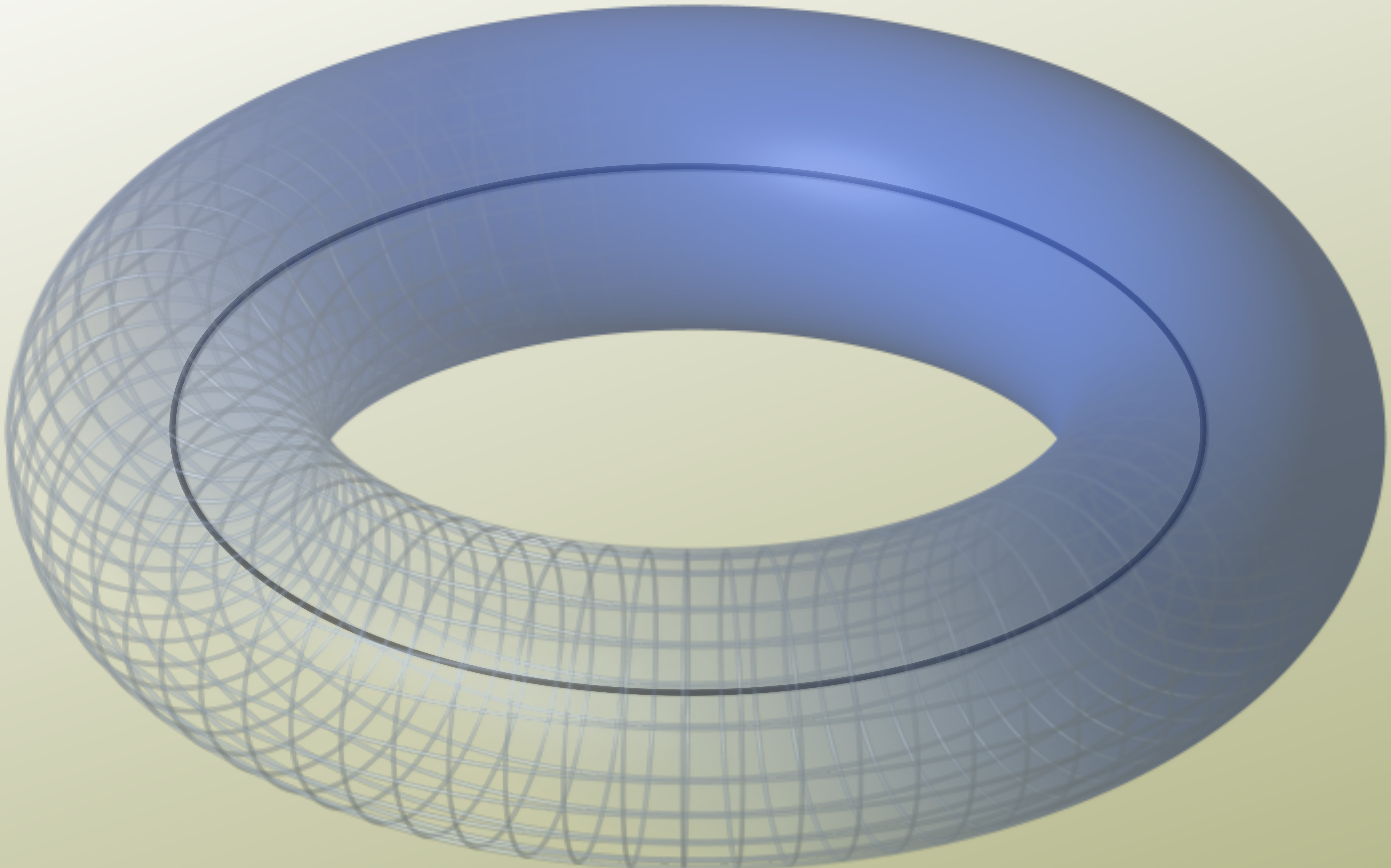
$$\mathbf{n} = [\sin\theta\cos(k\phi), \sin\theta\sin(k\phi), \cos\theta] \Longleftarrow \text{charge } k$$

180° around $\hat{x}$ $\quad \mathbf{n} = [\sin\theta\cos(k\phi), -\sin\theta\sin(k\phi), -\cos\theta]$

$$k \longrightarrow -k \qquad \text{NORTH} \leftrightarrow \text{SOUTH}$$

Based Versus Free Homotopy

Disclination Loop

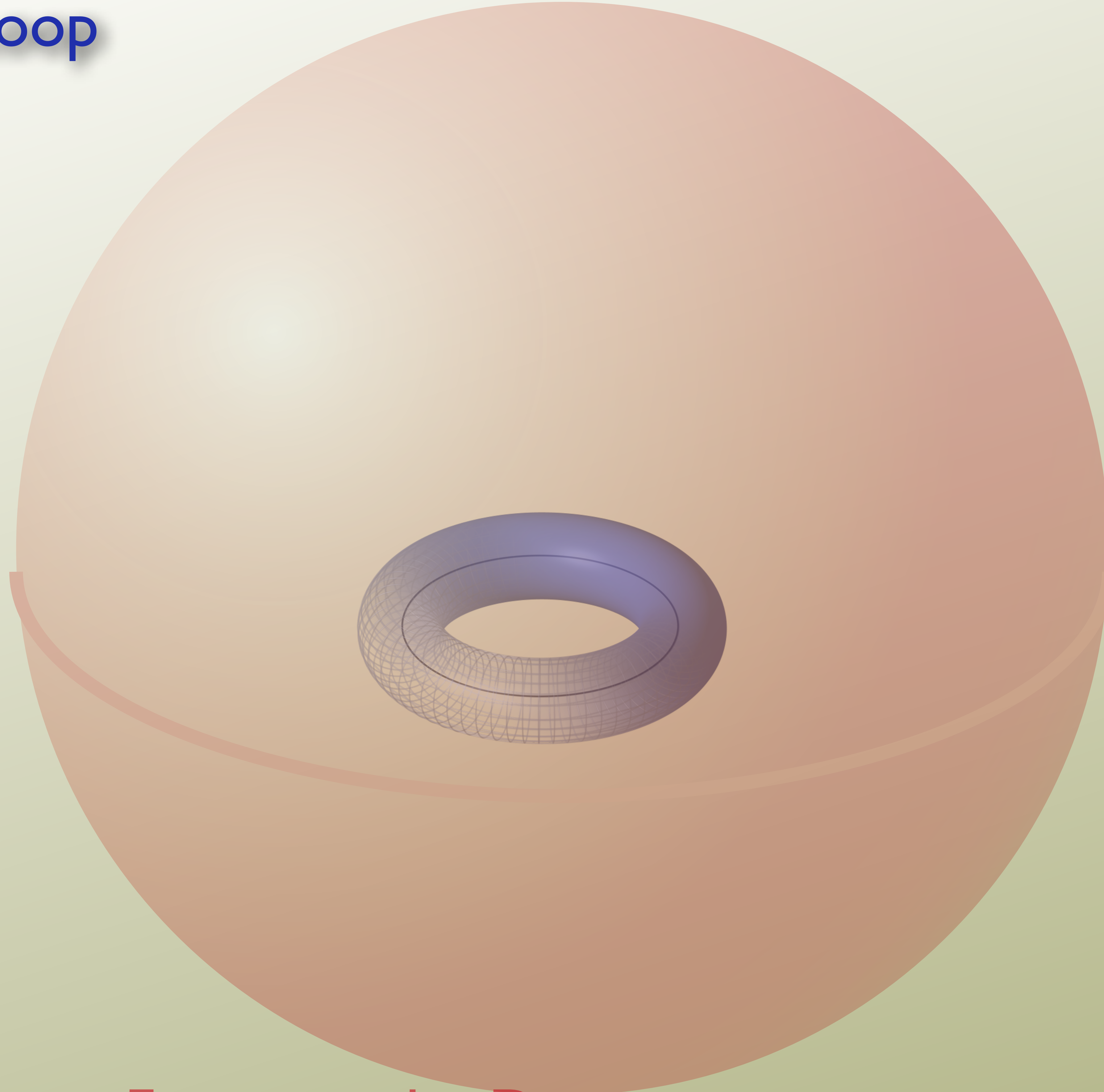


Focus on the Donut

http://cis.jhu.edu/education/introPatternTheory/chapters/lie/lie1.html

Penn

Disclination Loop



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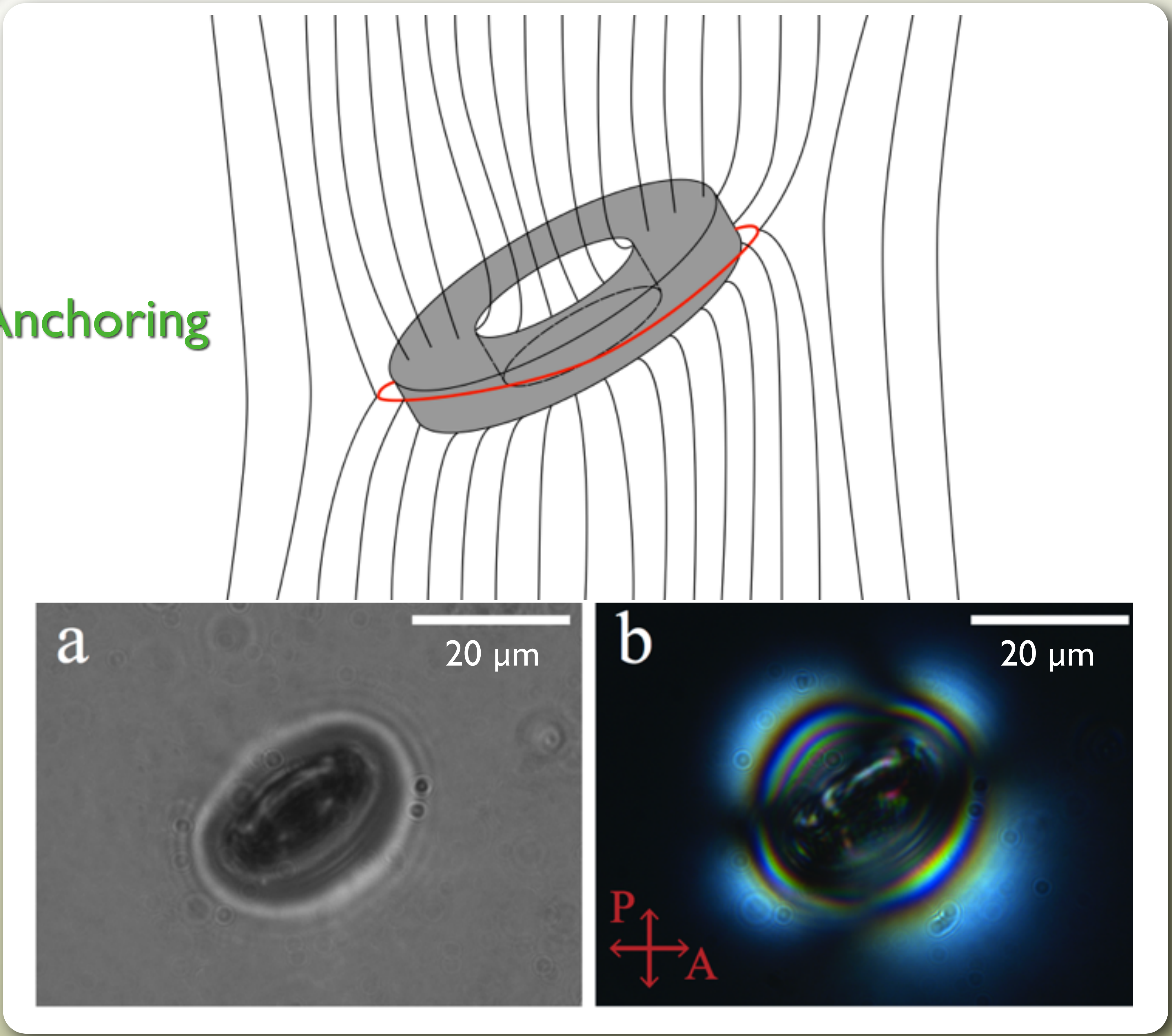
Charged Tori!

Hybrid Anchoring

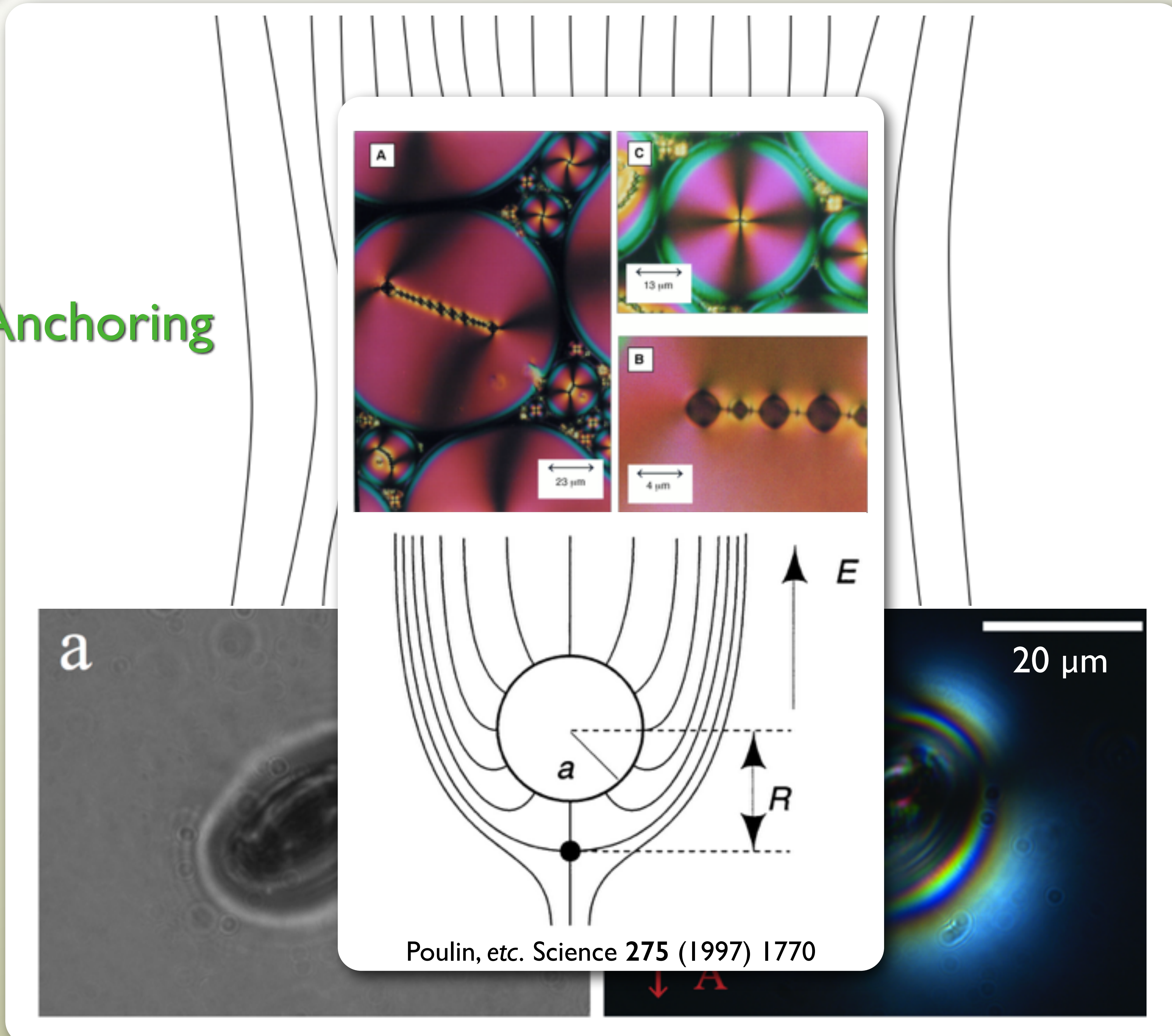


Penn

UPENN MRSEC, Cavallaro *et al. Soft Matter* **9** (2013) 9099.

Charged Tori!

Hybrid Anchoring



Poulin, *etc.* Science **275** (1997) 1770

UPENN MRSEC, Cavallaro *et al. Soft Matter* **9** (2013) 9099.

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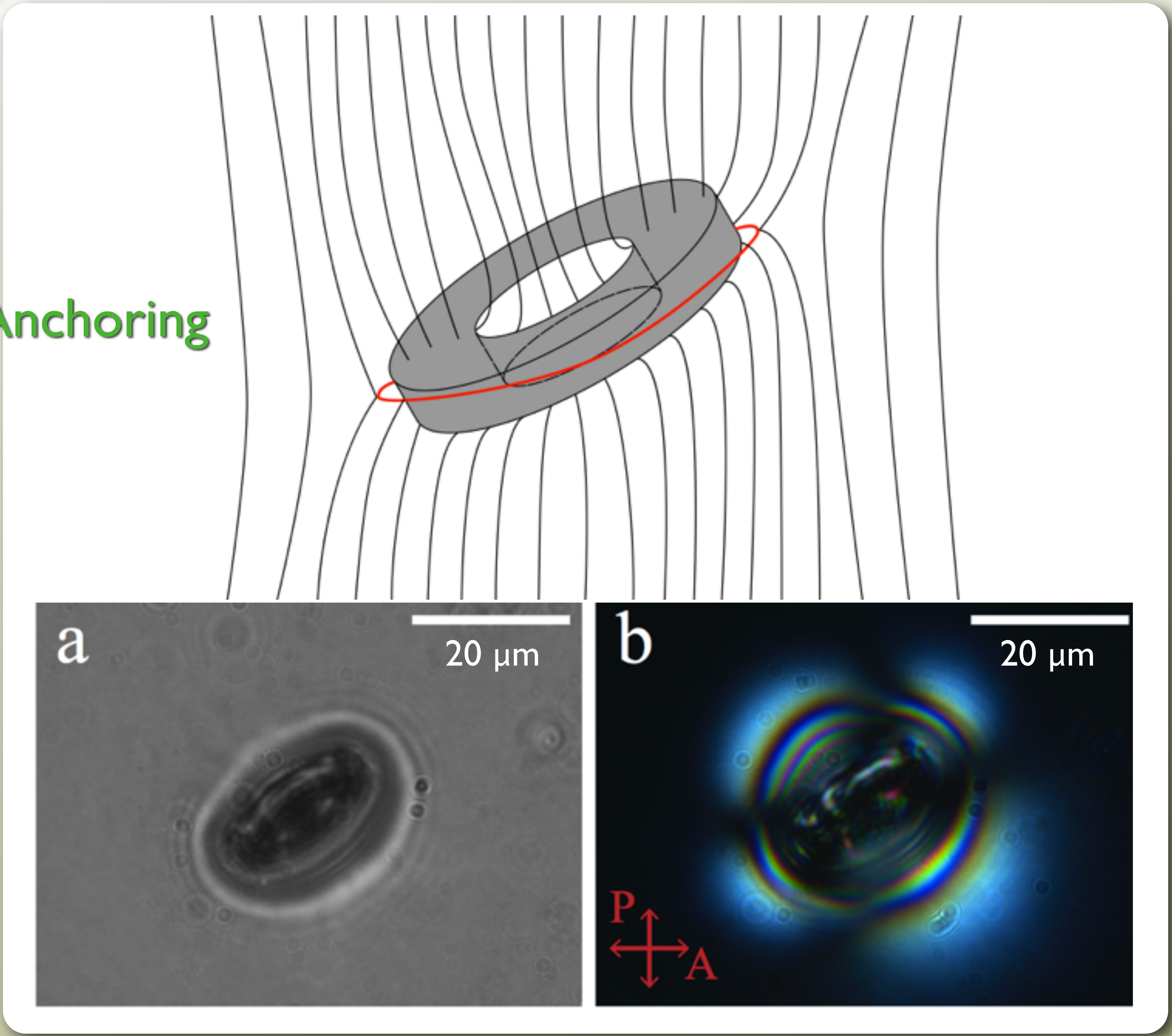

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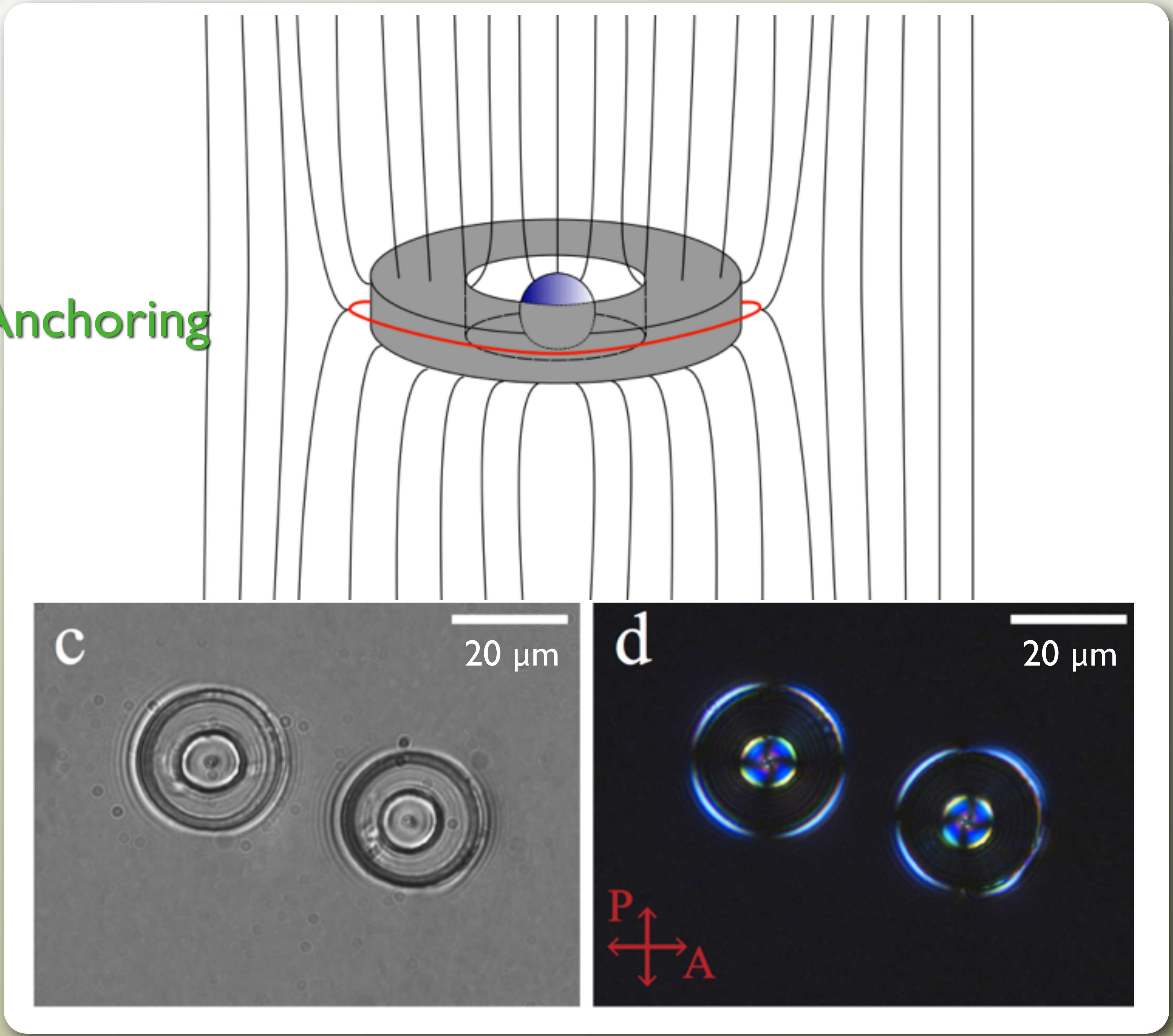

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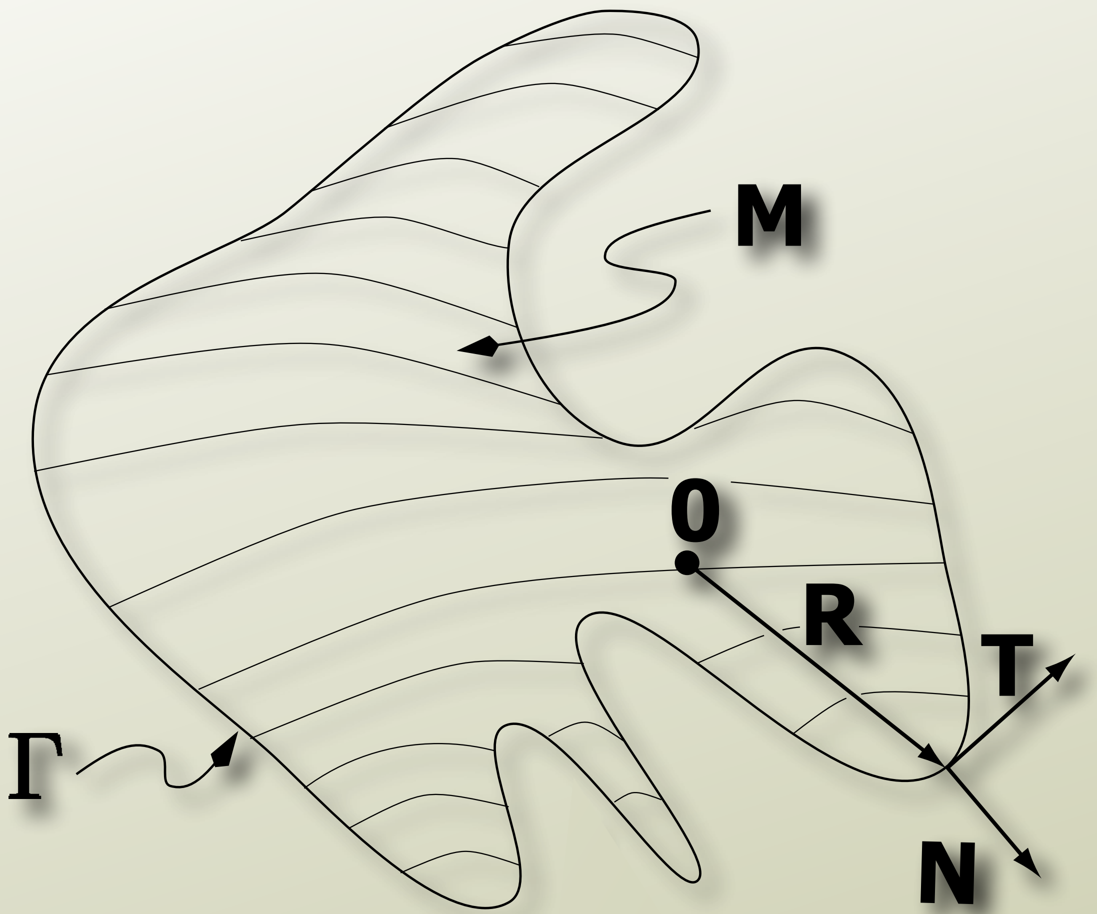
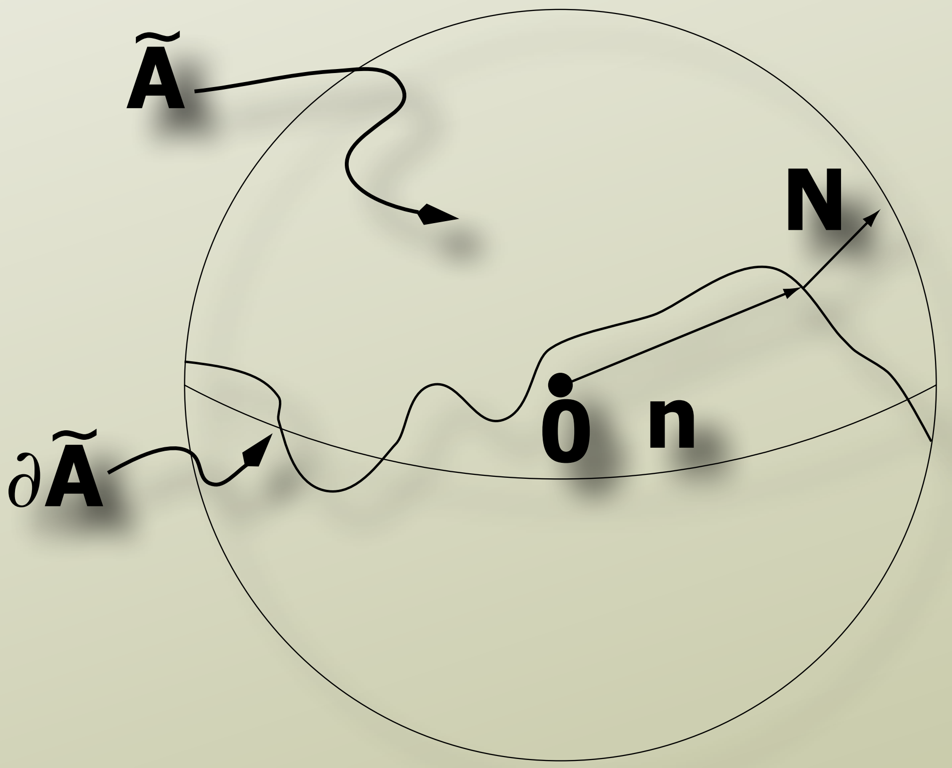
What is the Point Charge?





$$q = \frac{1}{4\pi}\int dx dy\, \mathbf{n}\cdot[\partial_x \mathbf{n}\times\partial_y \mathbf{n}]$$

Penn

B. Senyuk, Q. Liu, S. He, RDK, R.B. Kusner, T.C. Lubensky, and I.I. Smalyukh, *Nature* ***493*** *(2013) 205.*

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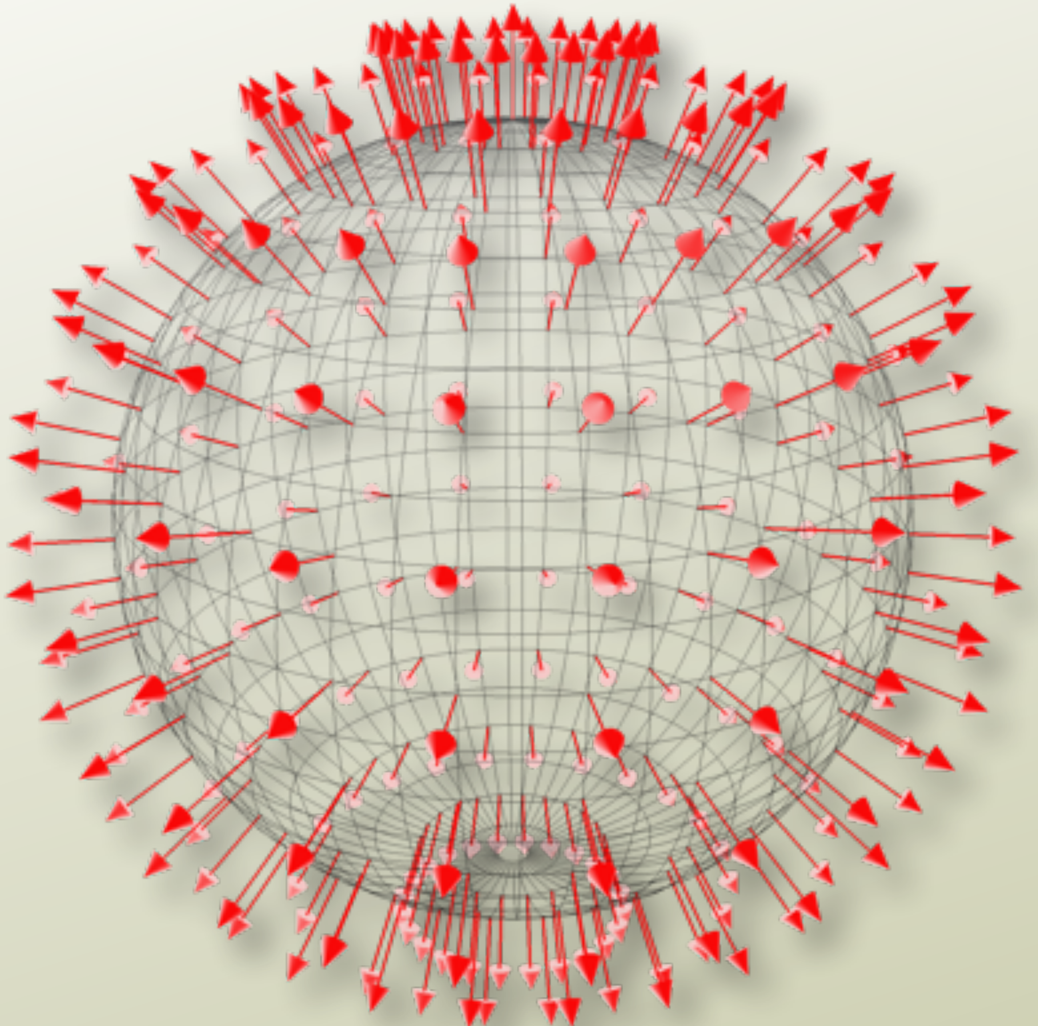

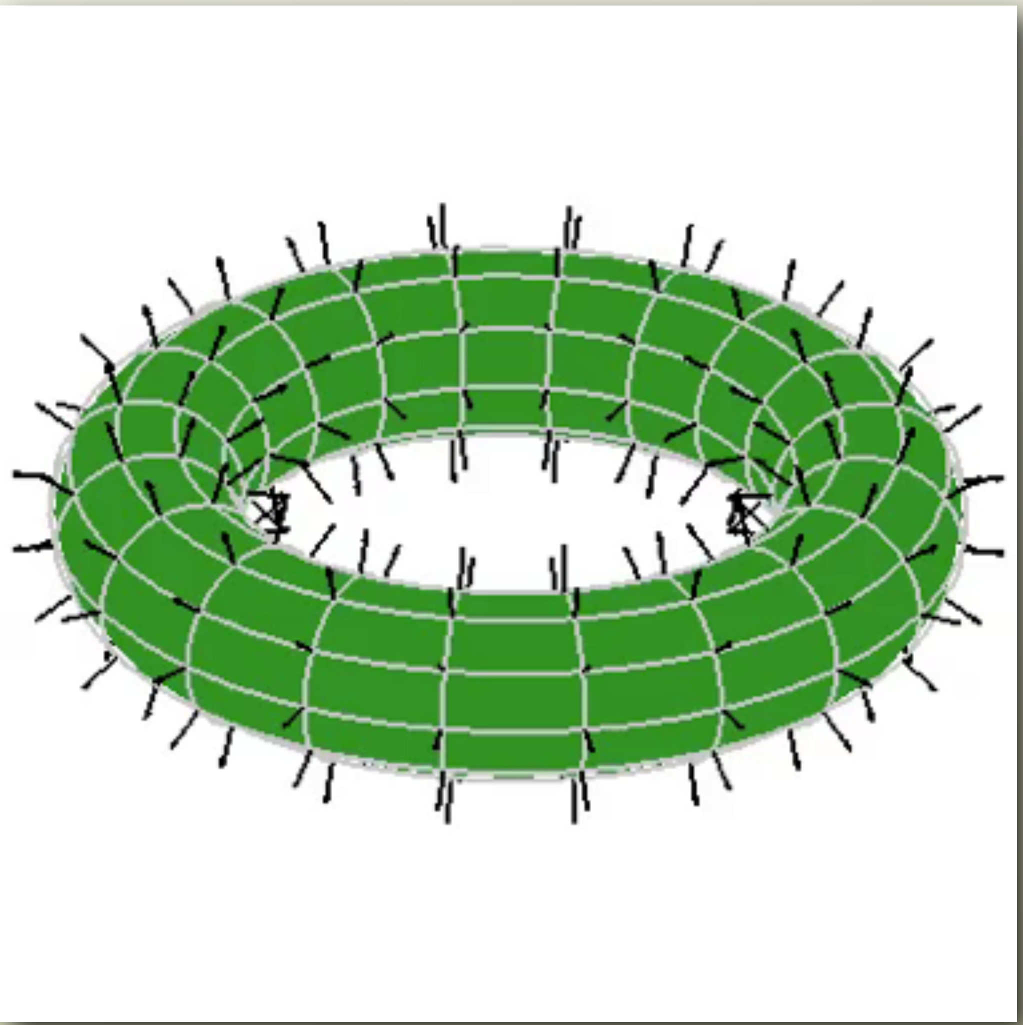

http://www.supermath.info/ZooOfMathematicalCreatures.html

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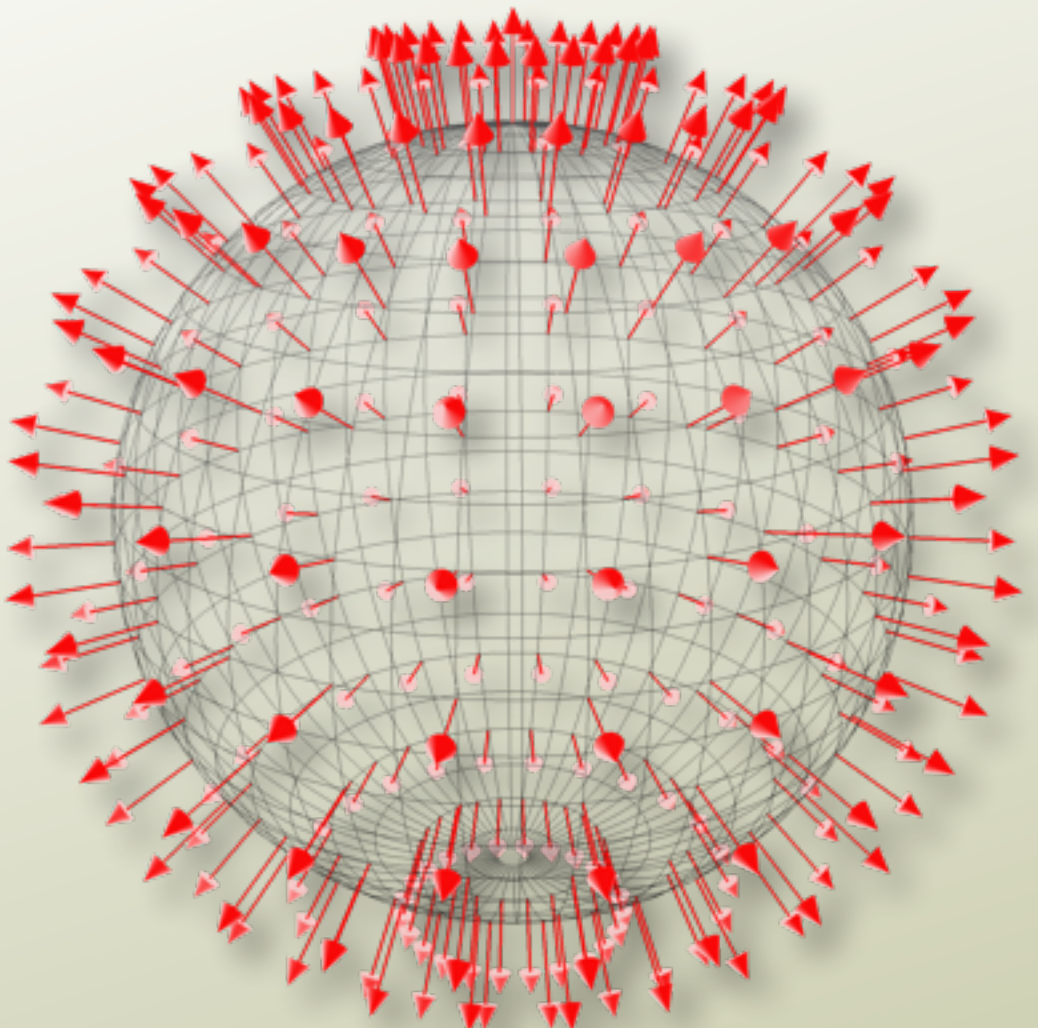

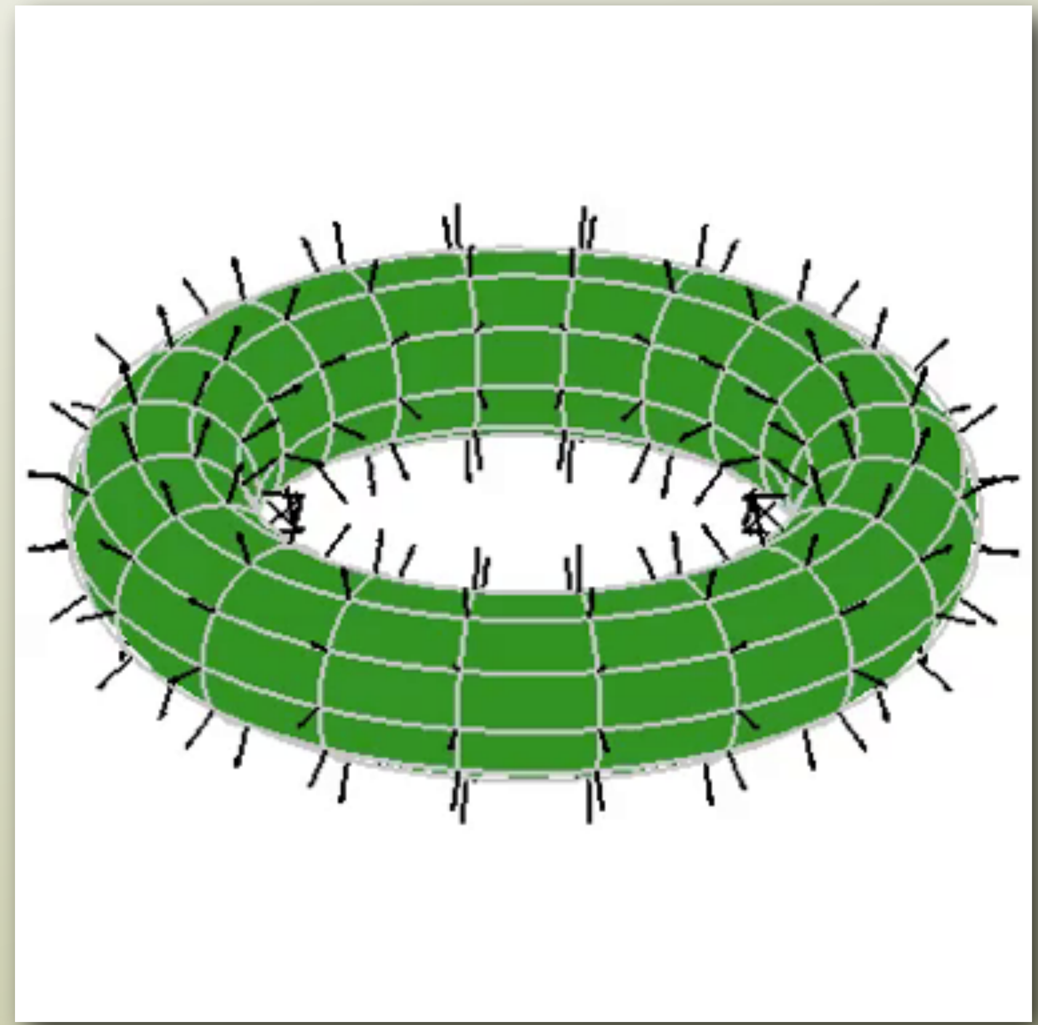

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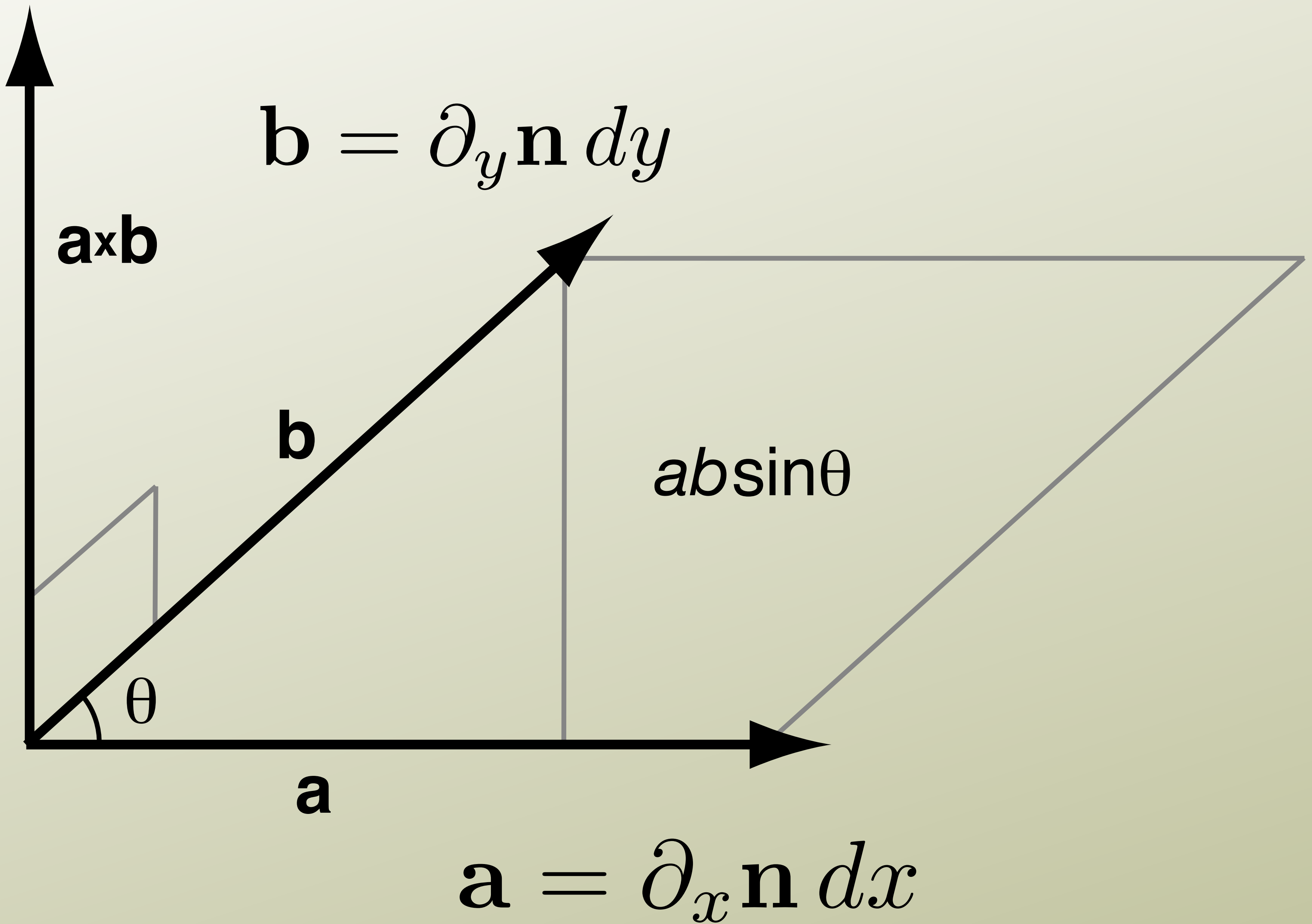
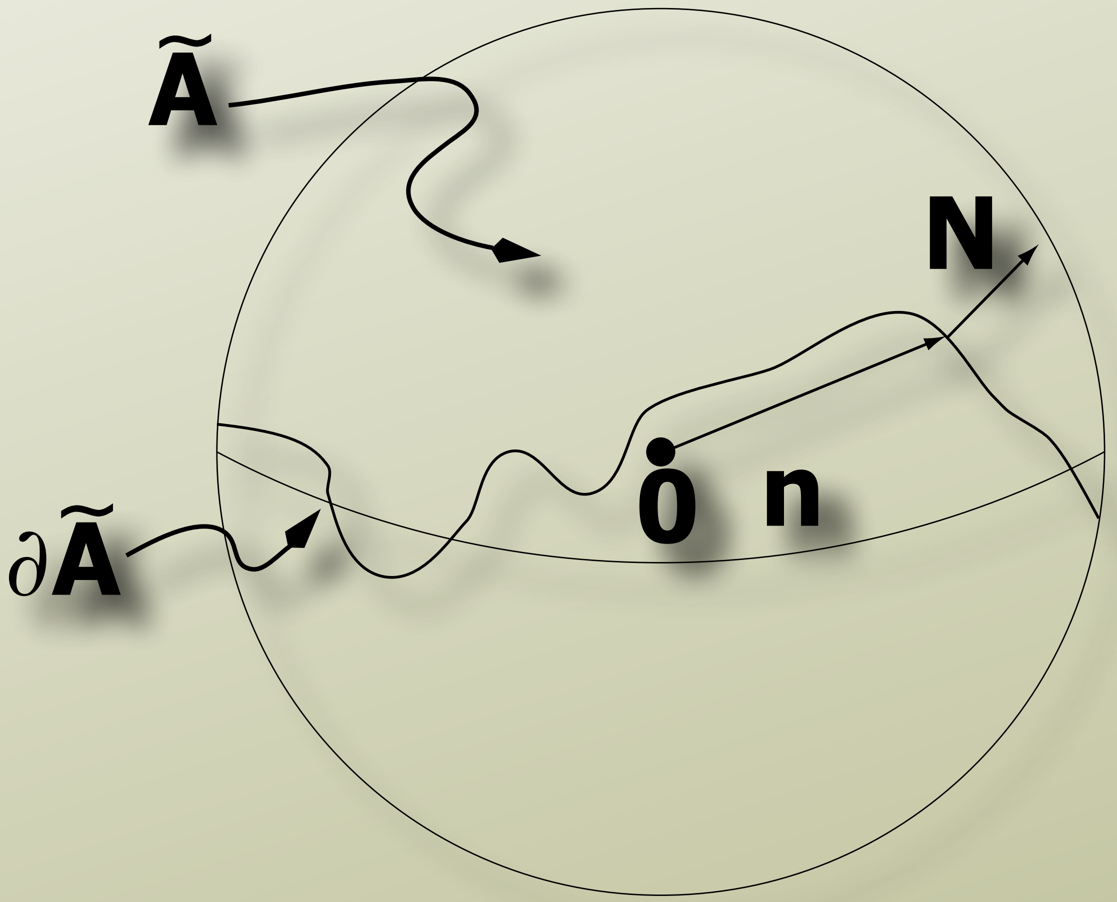


What is the Charge?

$$\mathbf{n} \cdot (\mathbf{a} \times \mathbf{b})$$



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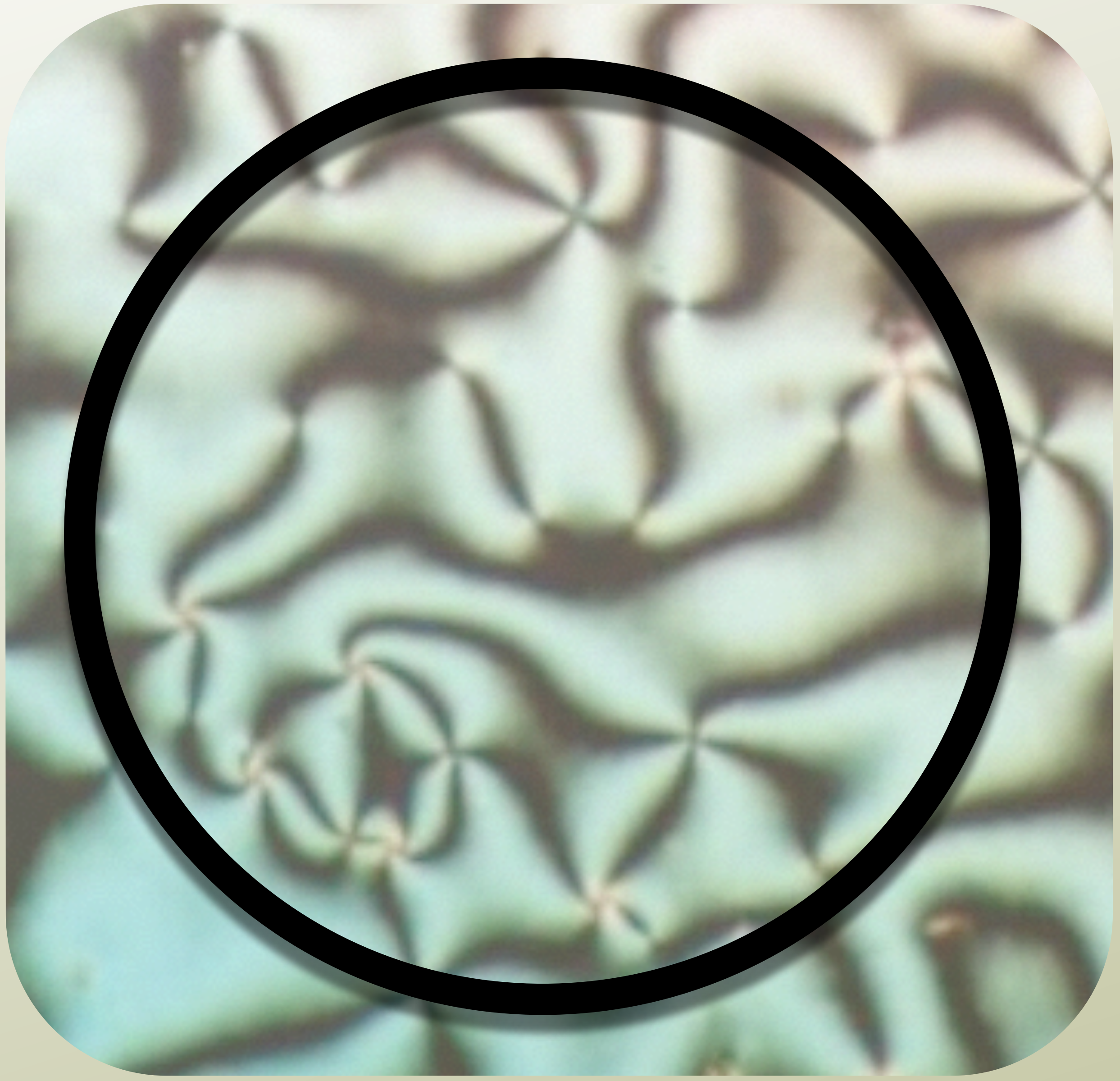
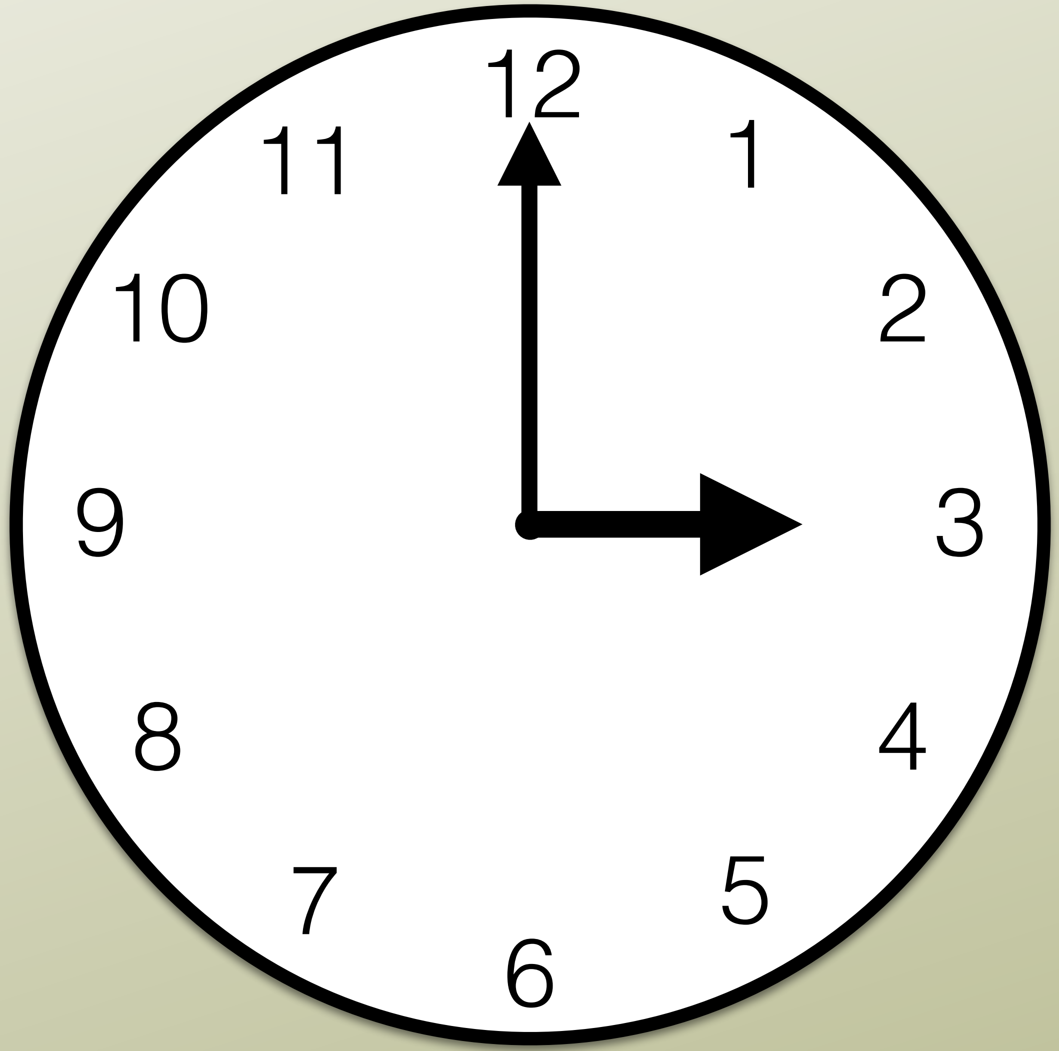
Degree of a Map - Integer





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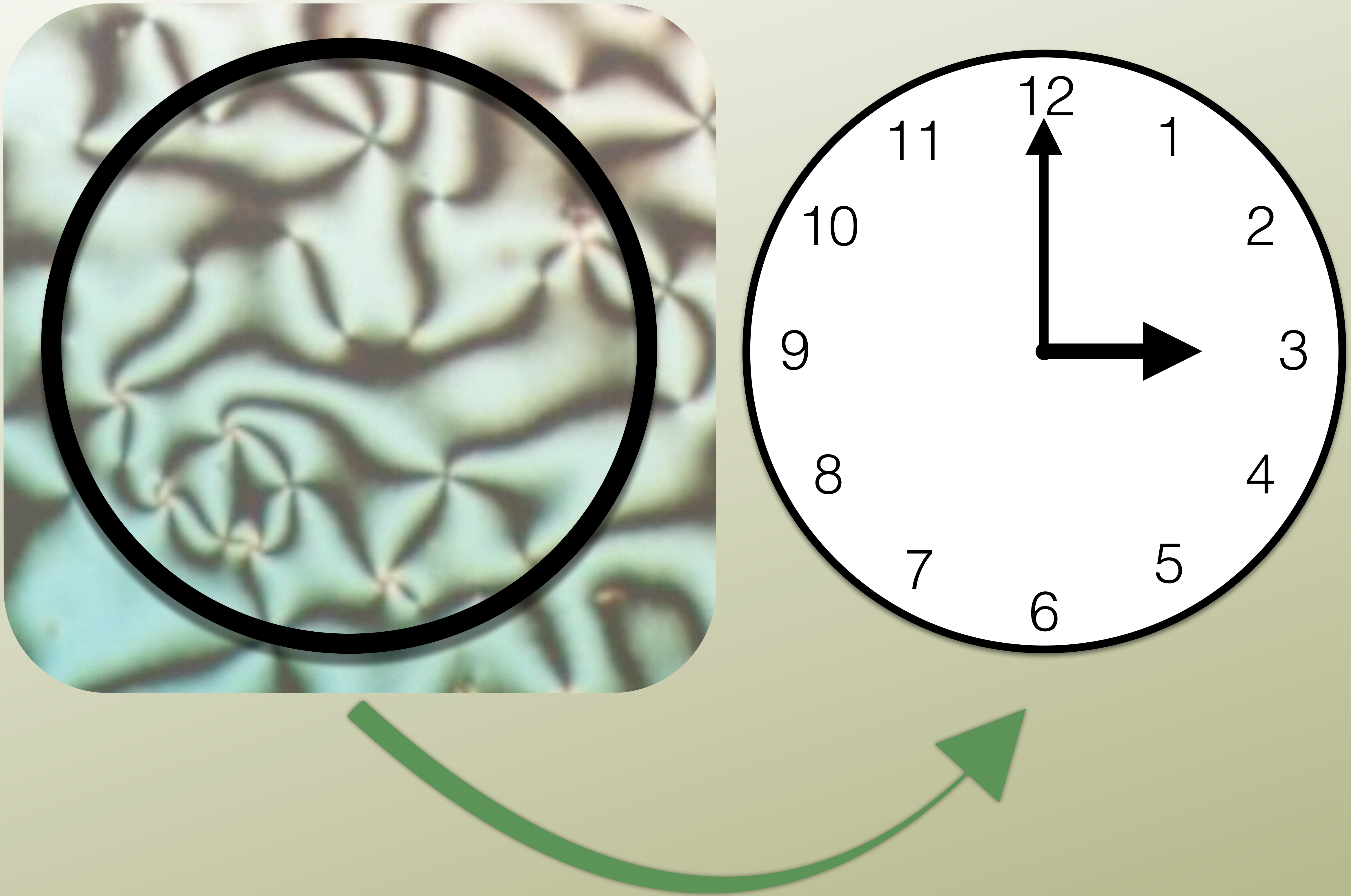

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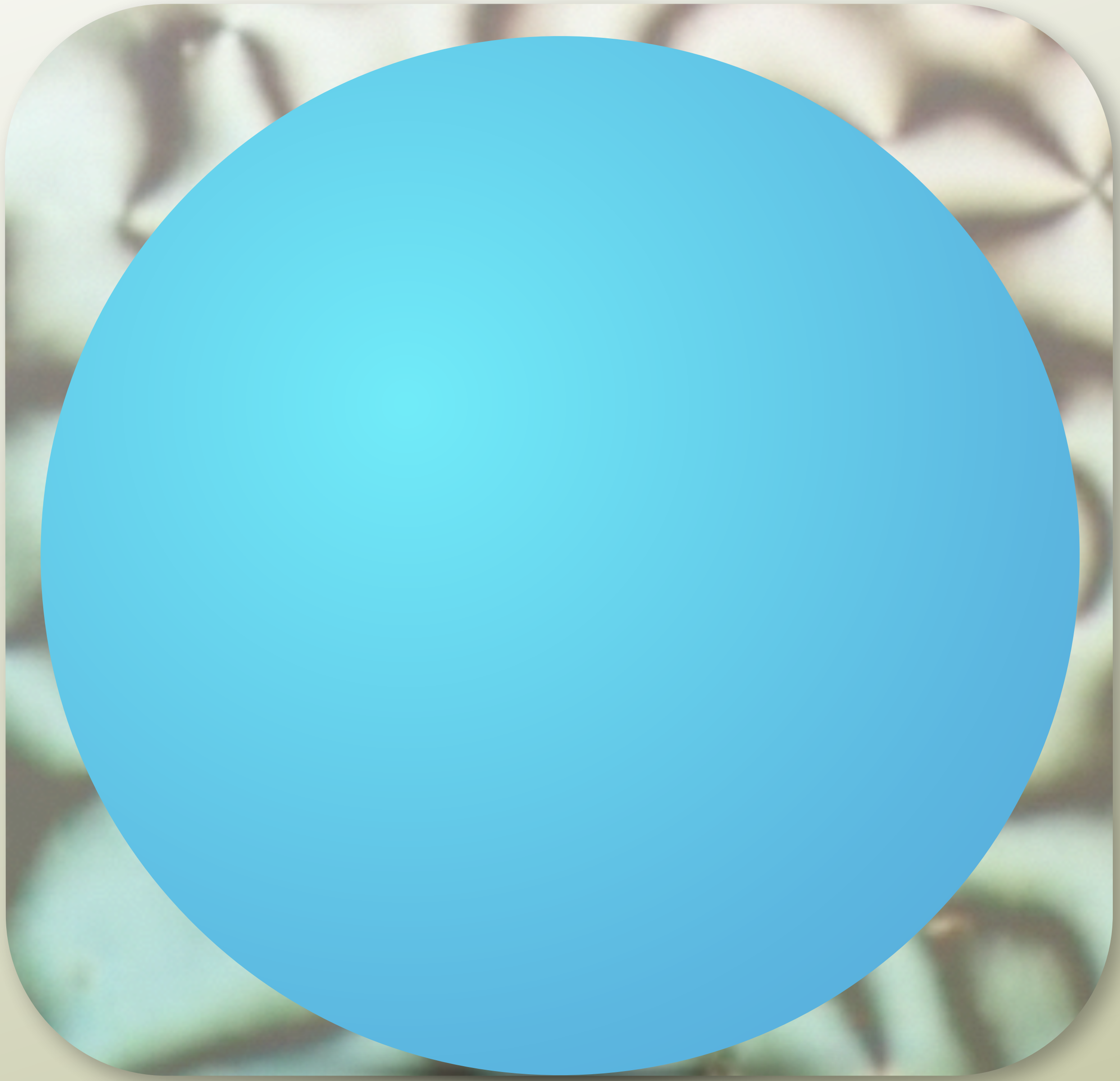

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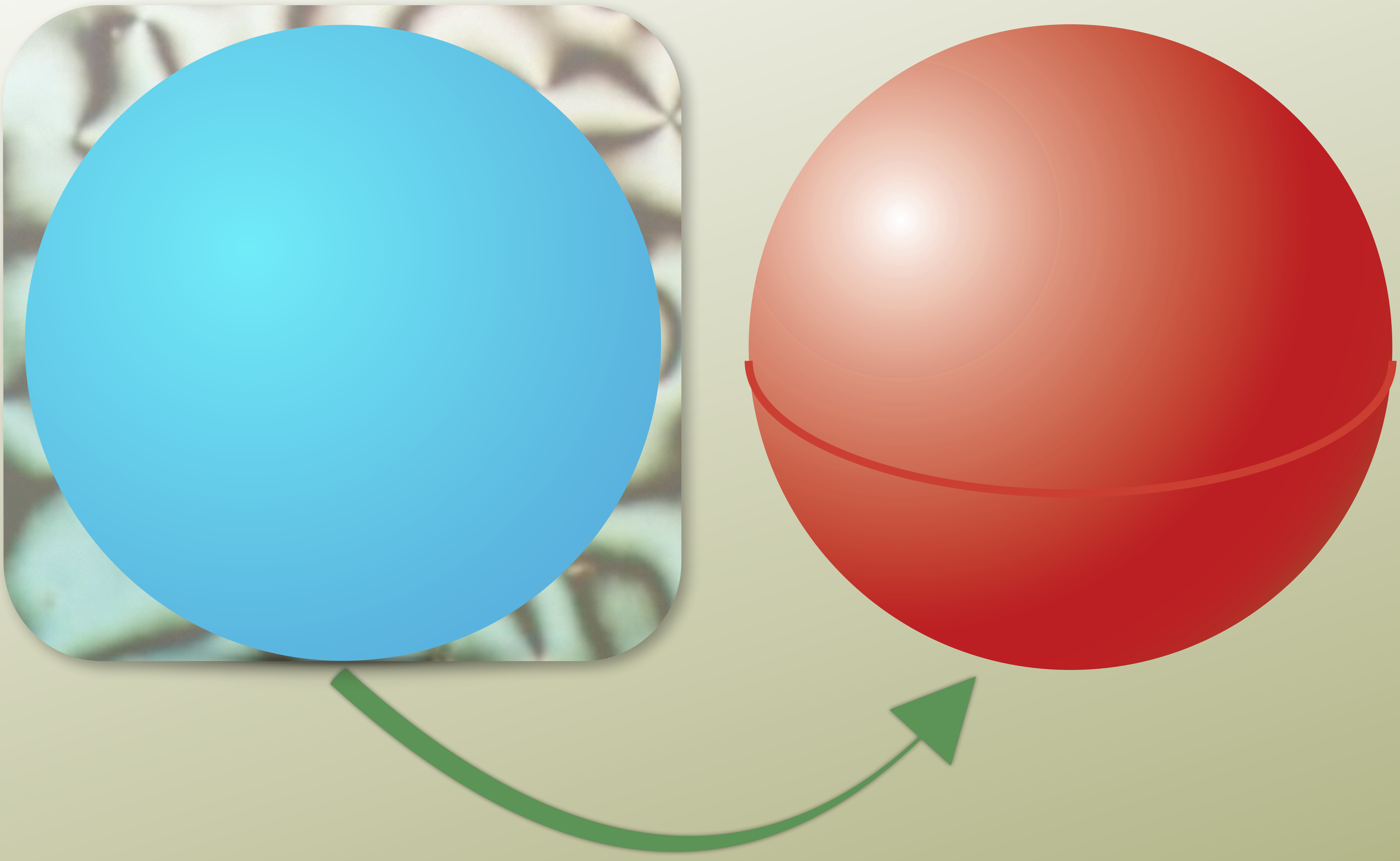

Penn

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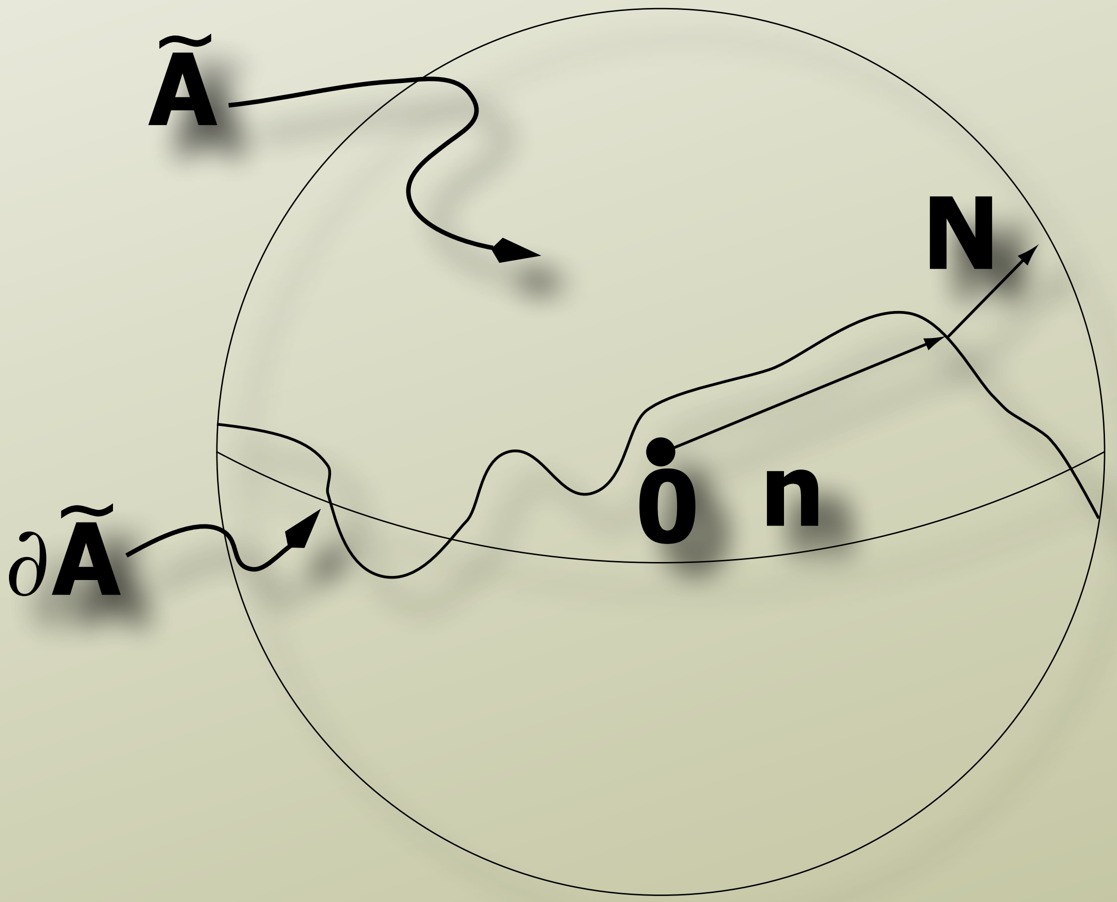

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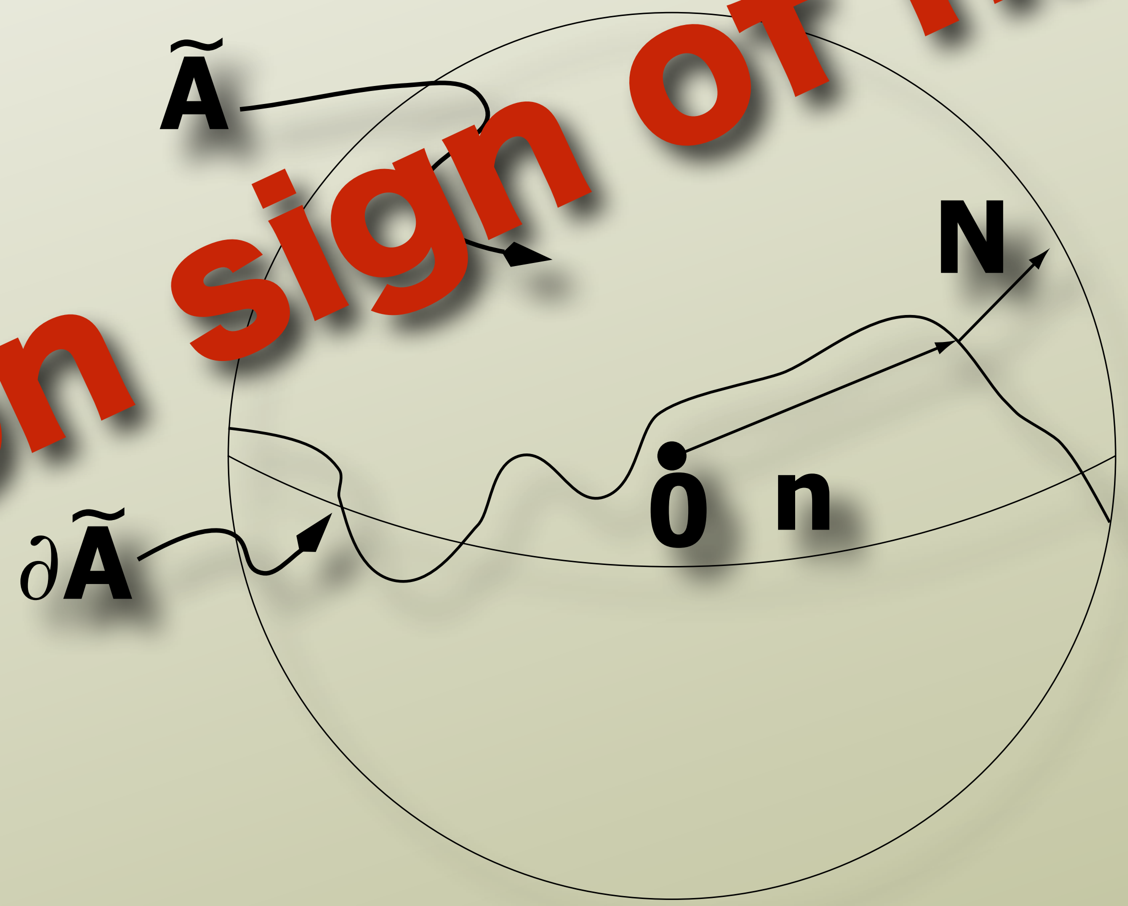
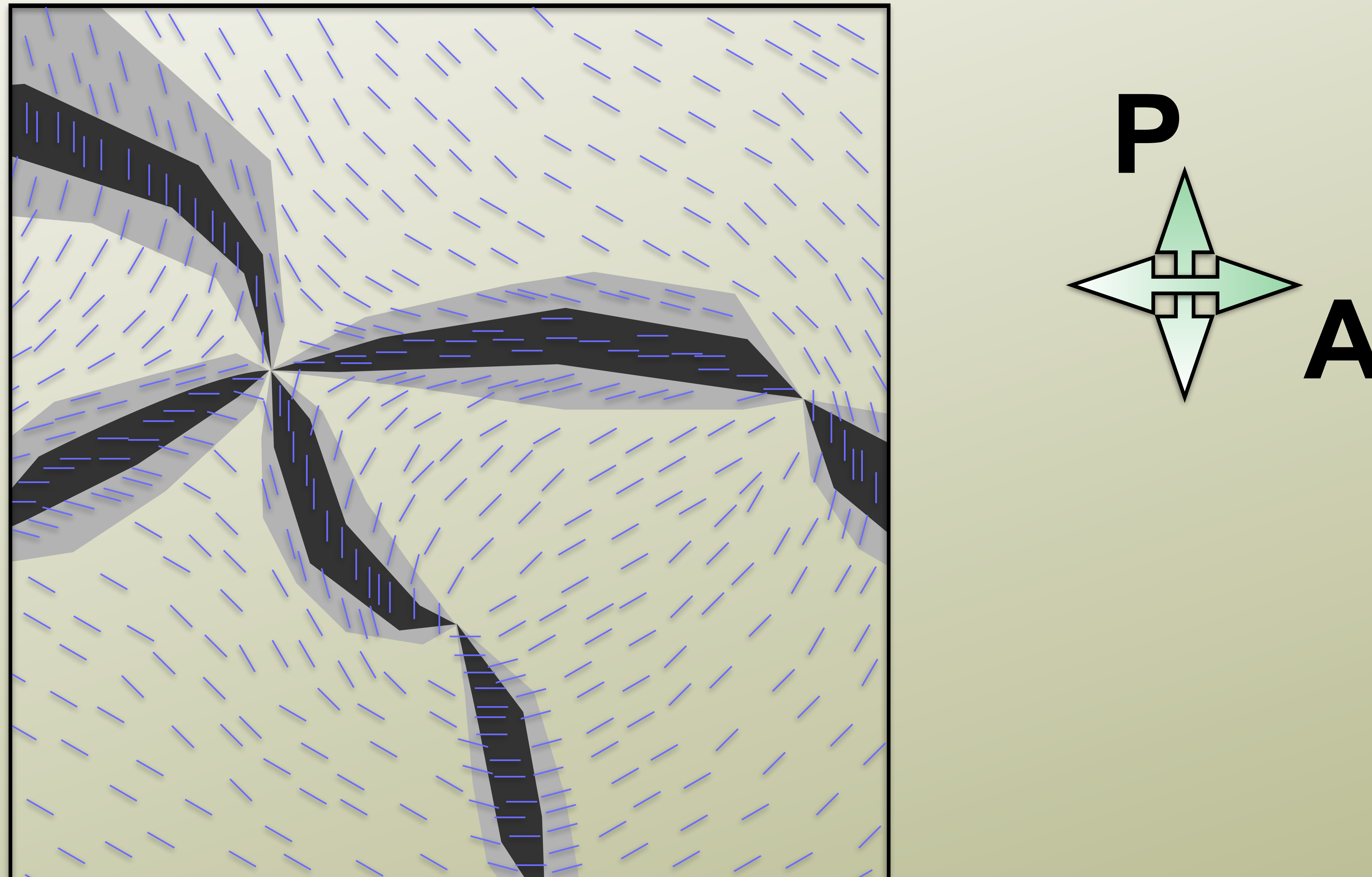


q depends on sign of n!!!

$$q=\frac{1}{4\pi}\int dx\,dy\;\mathbf{n}\cdot[\partial_x\mathbf{n}\times\partial_y\mathbf{n}]$$

B. Senyuk, Q. Liu, S. He, RDK, R.B. Kusner, T.C. Lubensky, and I.I. Smalyukh, *Nature* ***493*** *(2013) 205.*

Nematics in Two Dimensions



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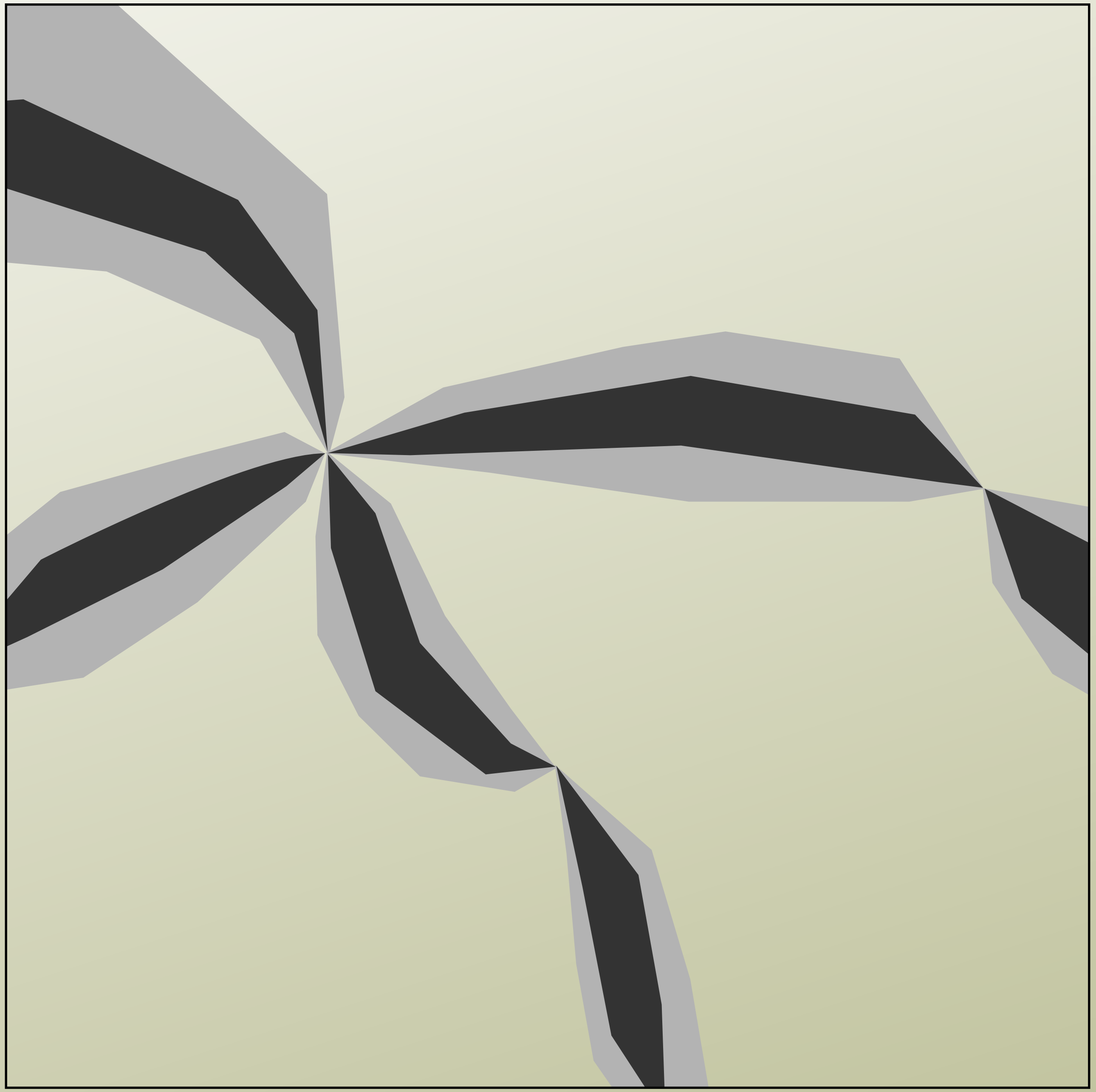
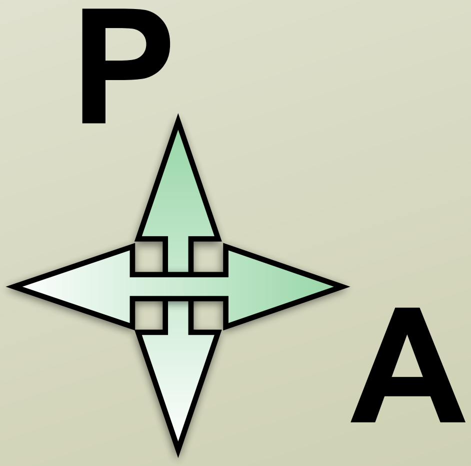


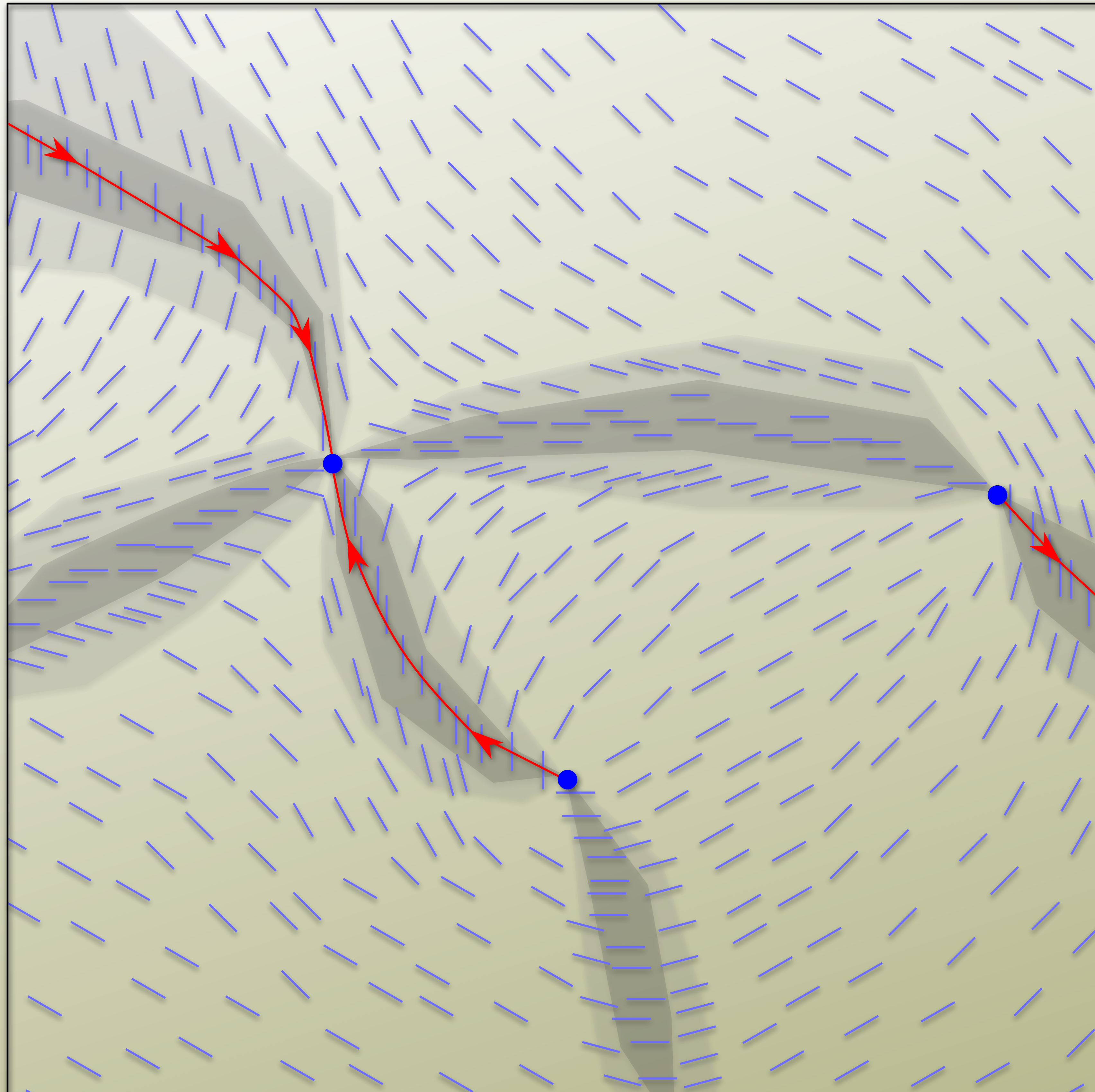


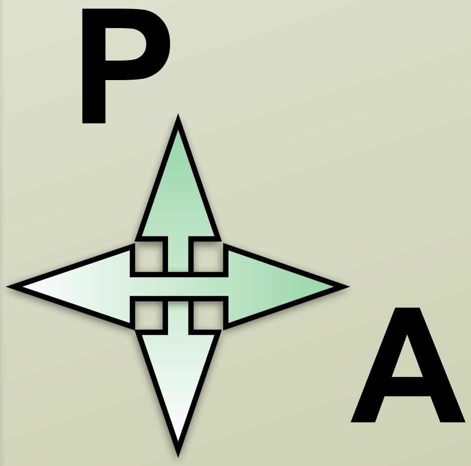
…we just need the black lines

Nematics in Two Dimensions

Arrows point from + to –

encode charges





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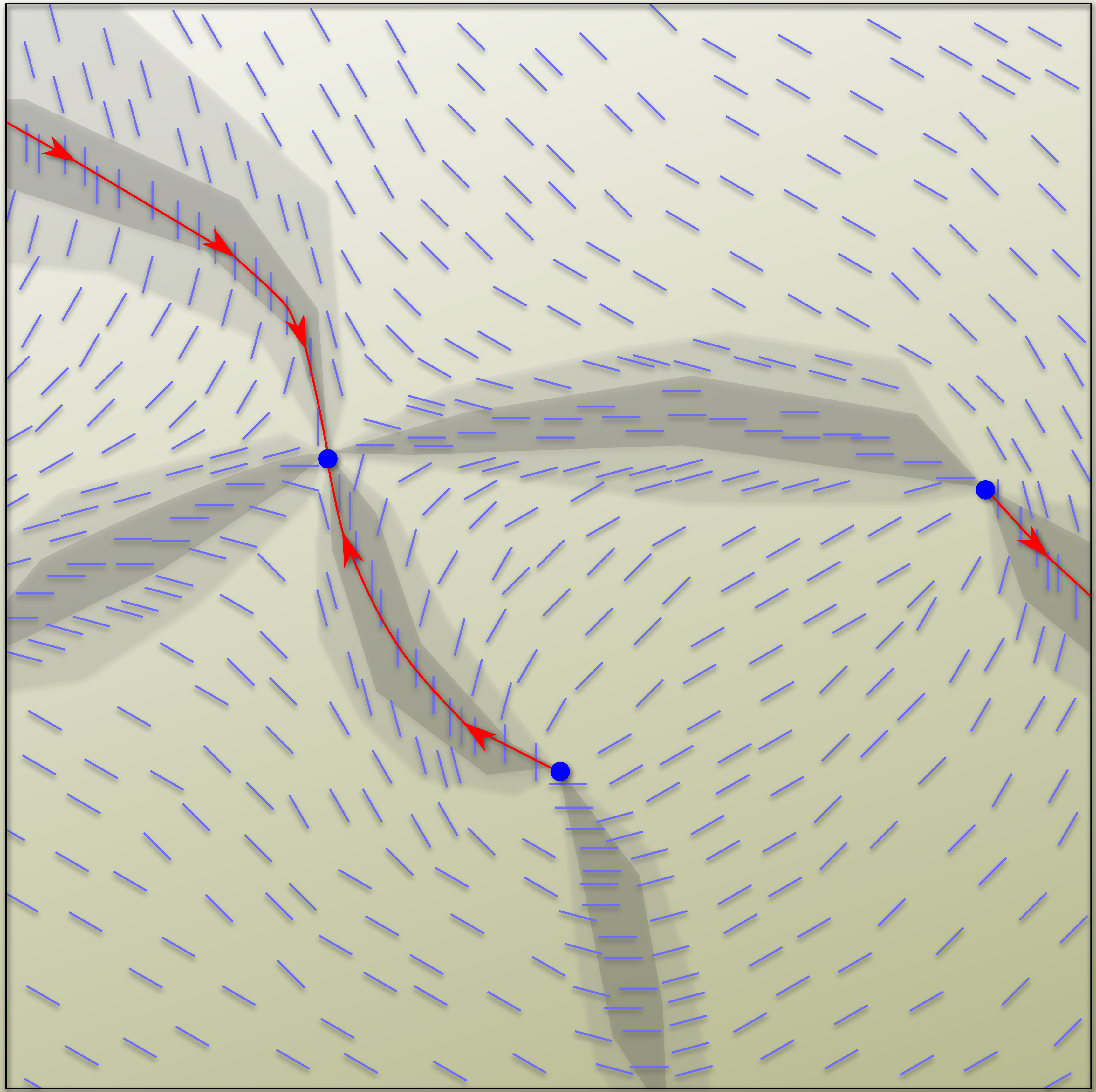

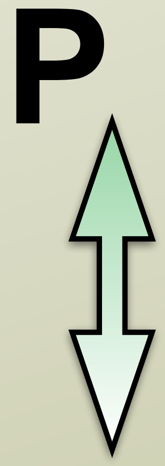

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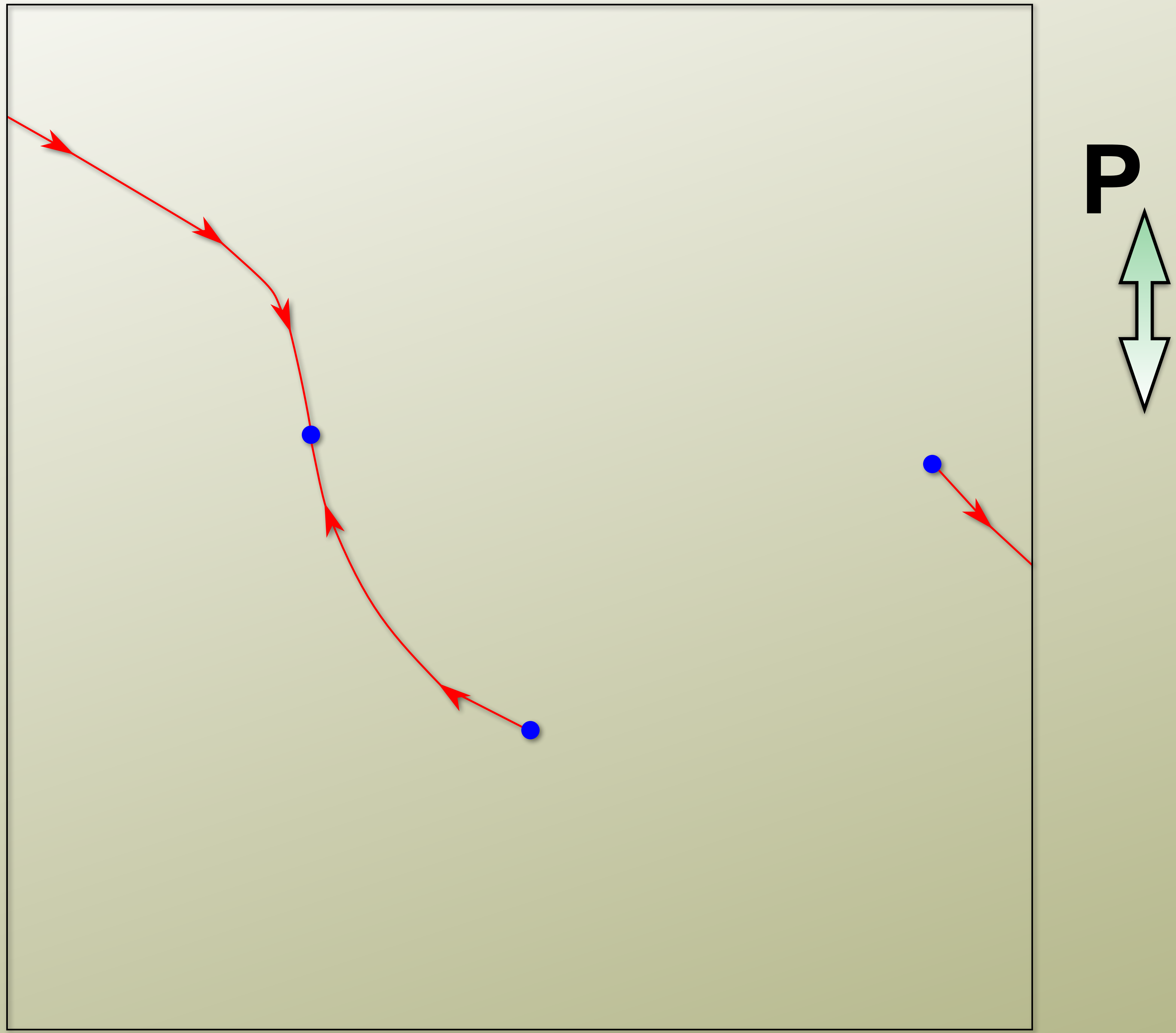

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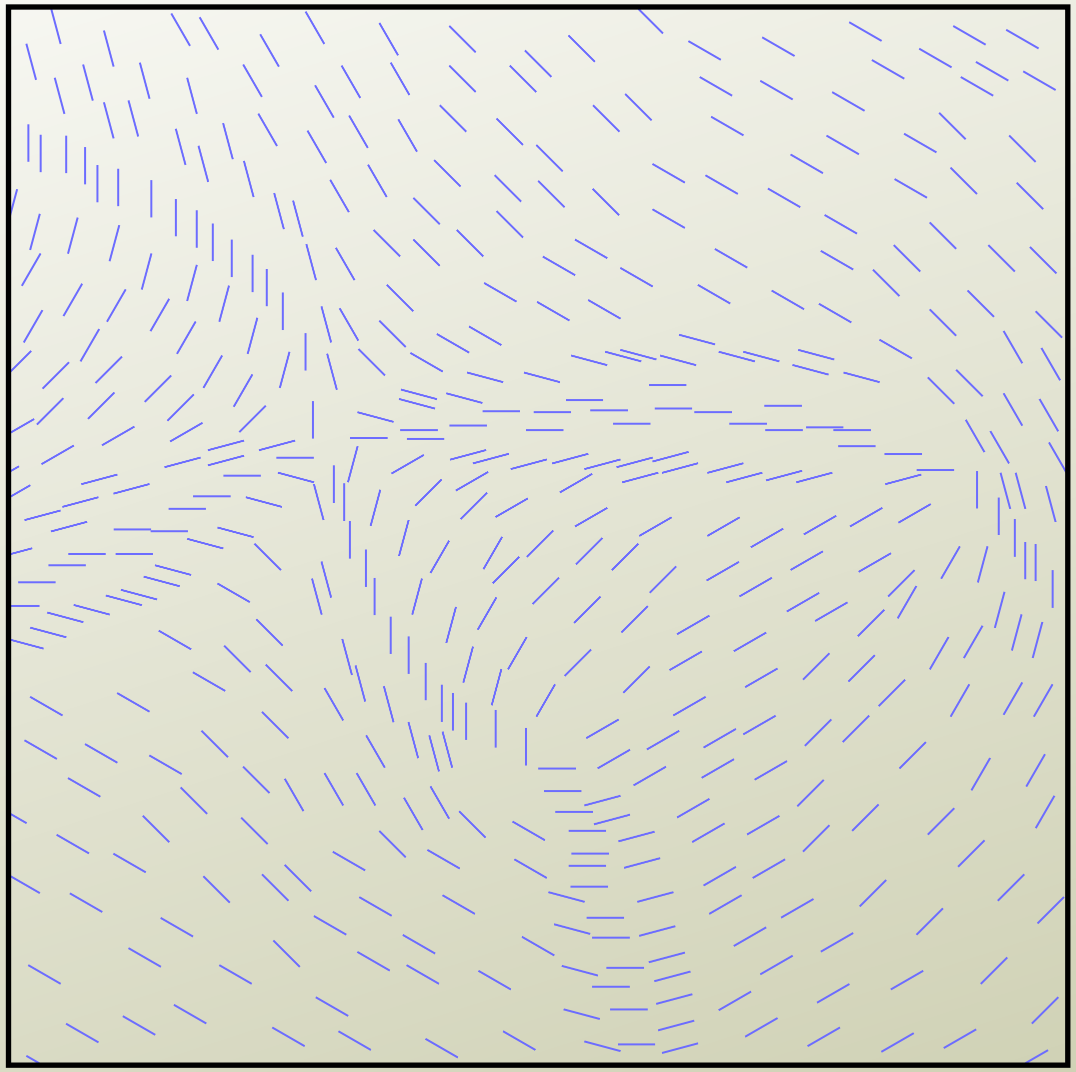
Nematics in Two Dimensions

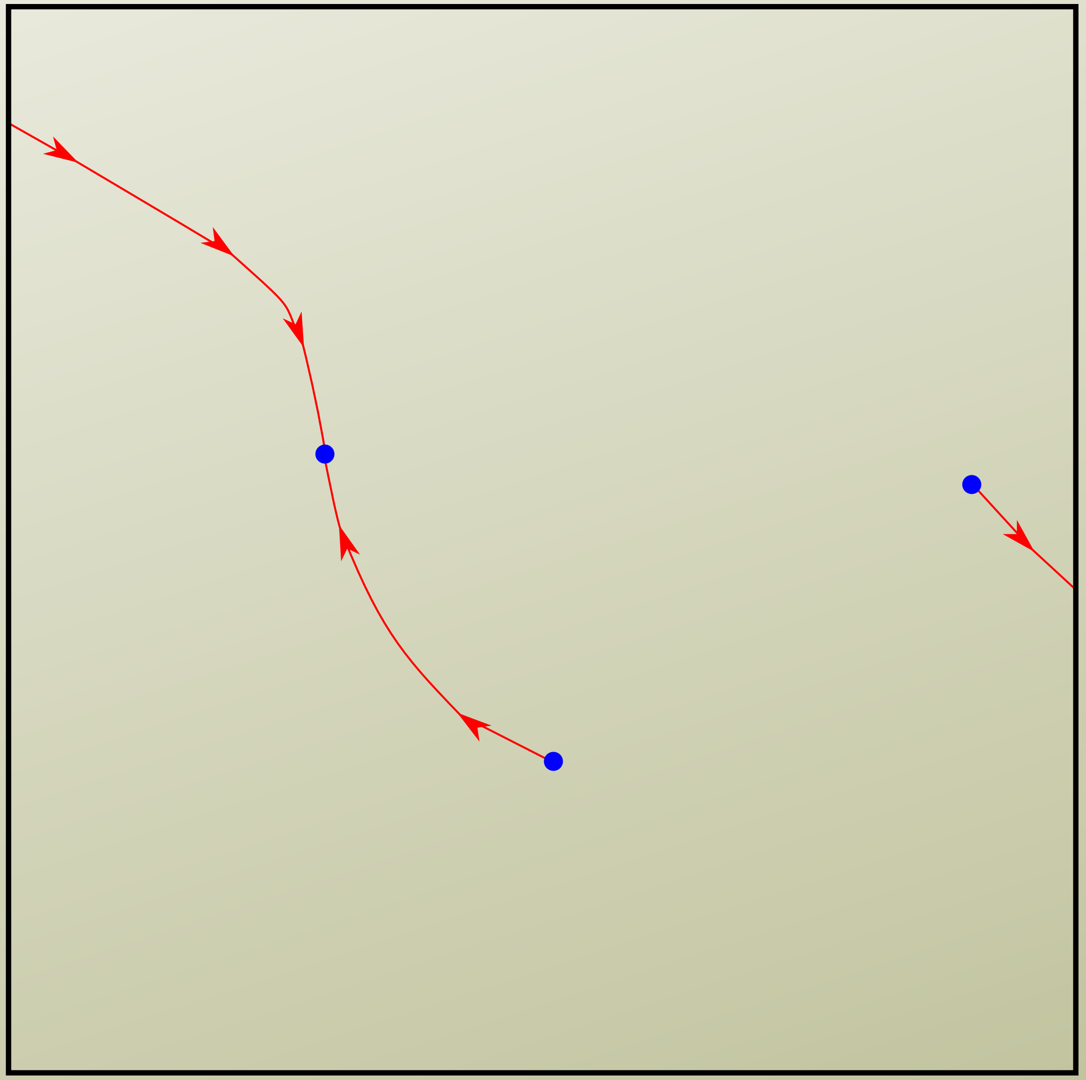
These directed lines carry ***all*** of the topology (Pontryagin-Thom)



choose a special orientation

1 to 1

fill in neighborhoods

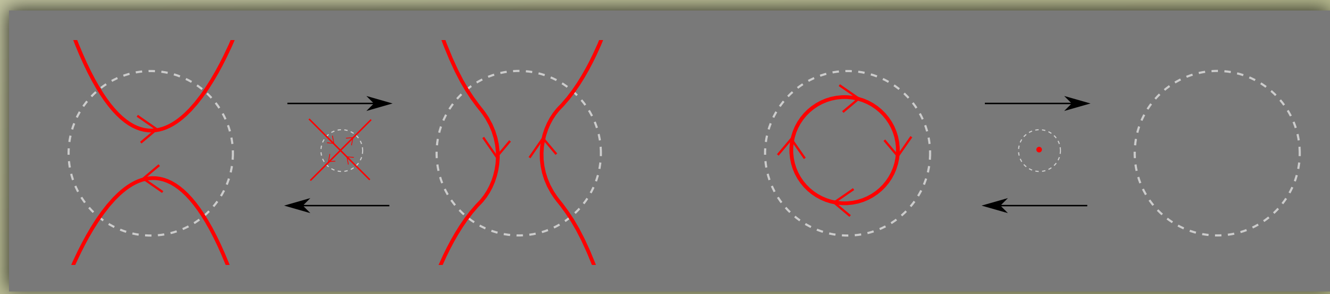


2D nematic director fields

up to smooth deformations

pictures of directed lines

up to switching moves



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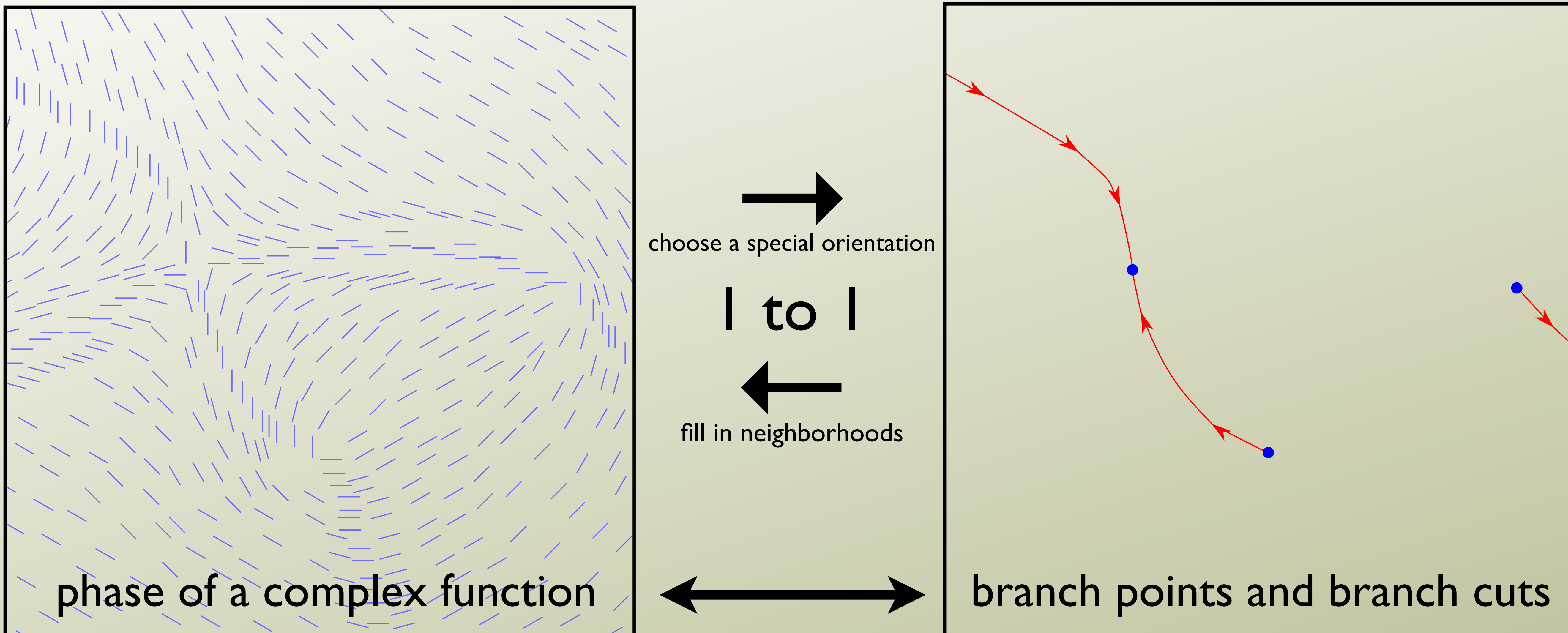

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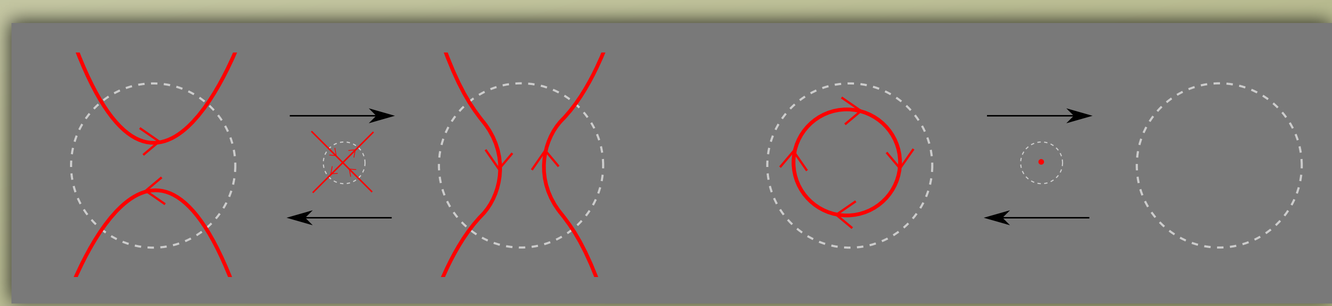
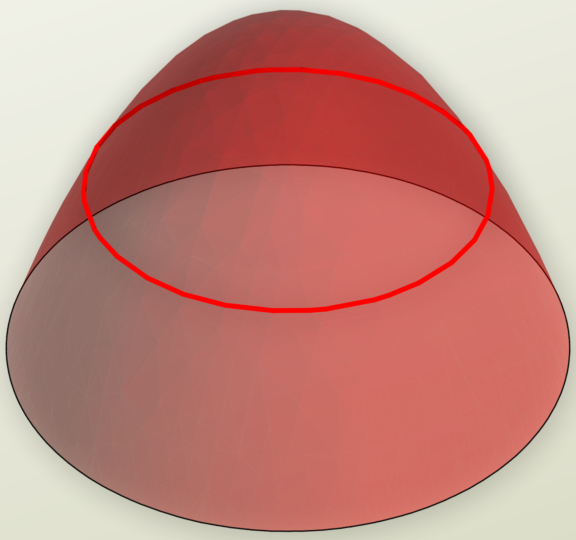
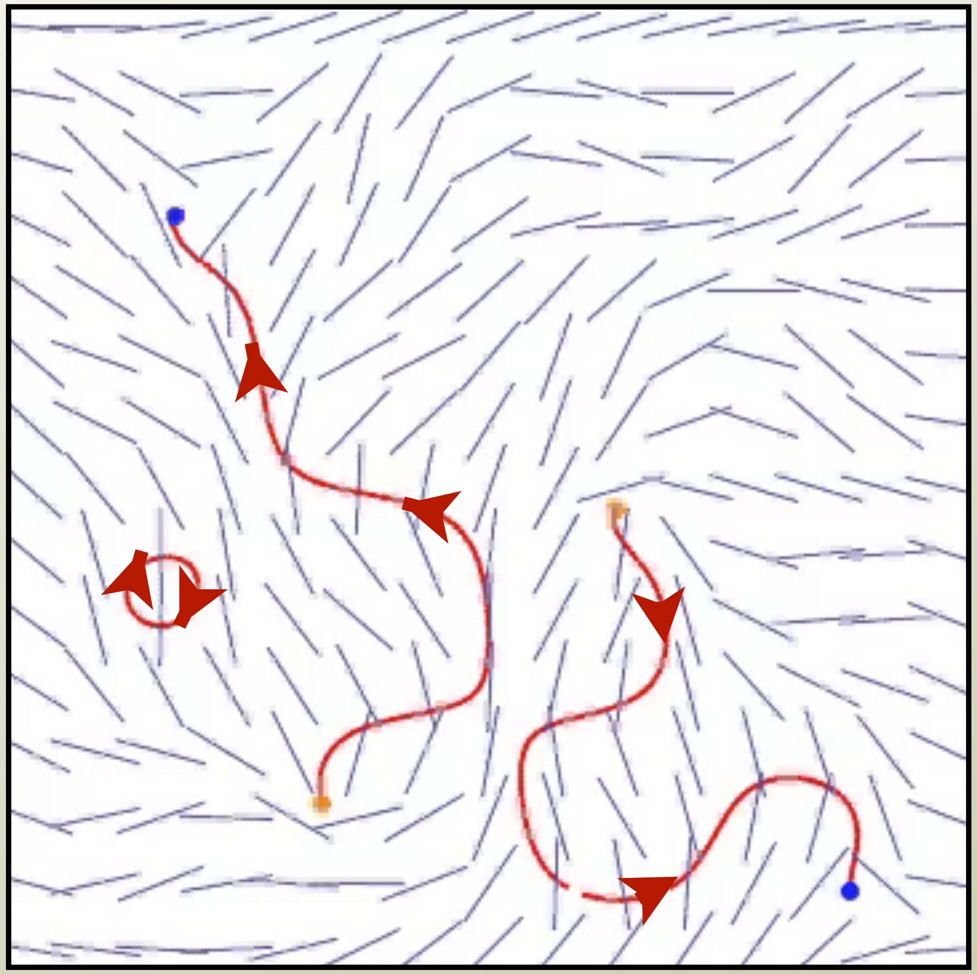
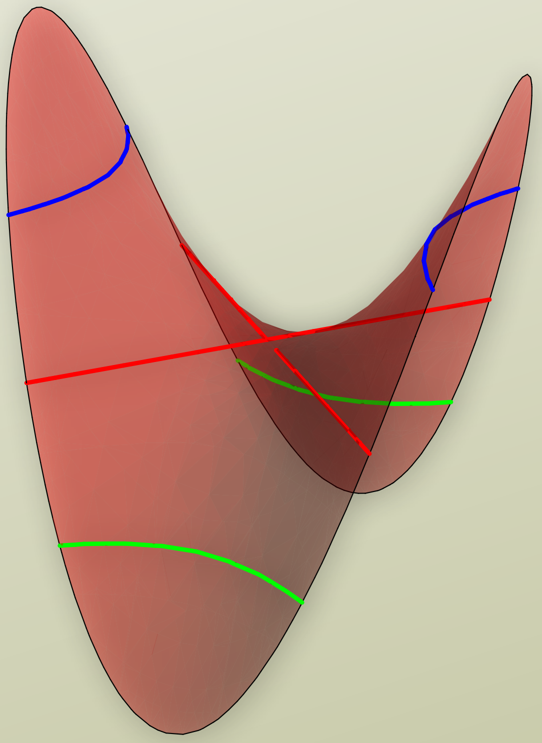
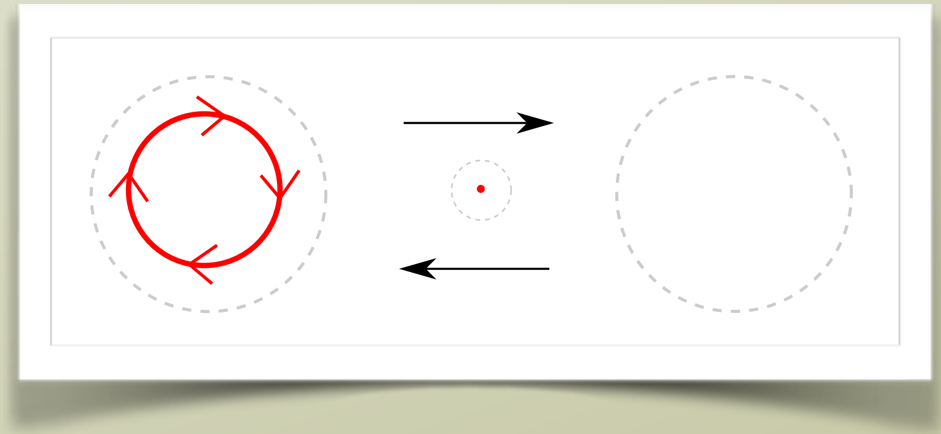


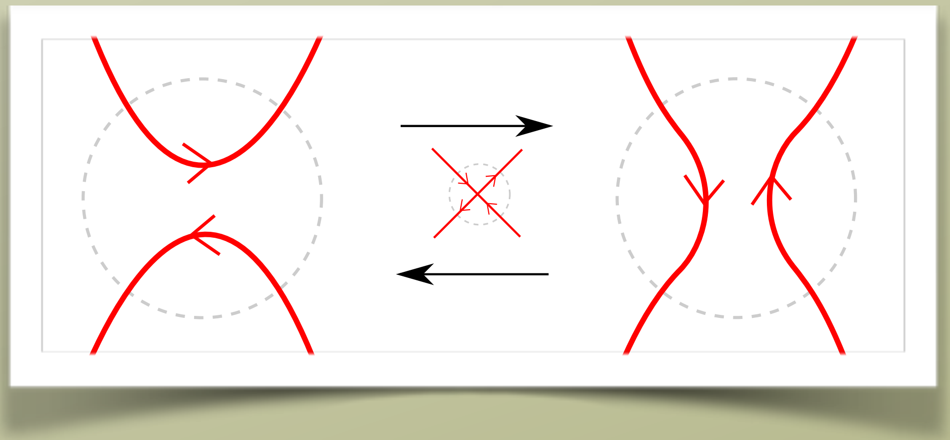
Bordisms











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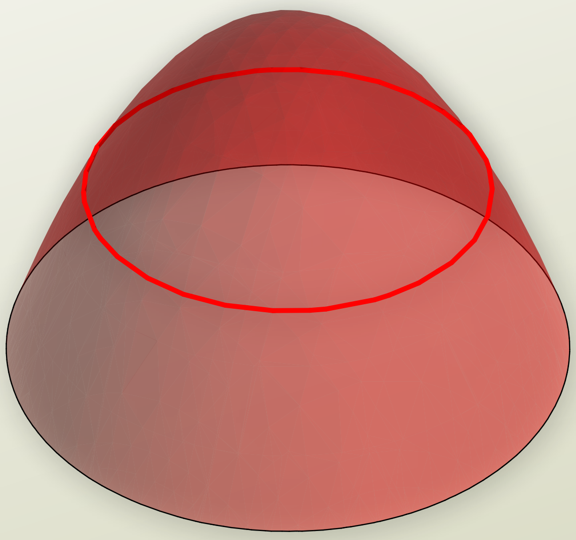

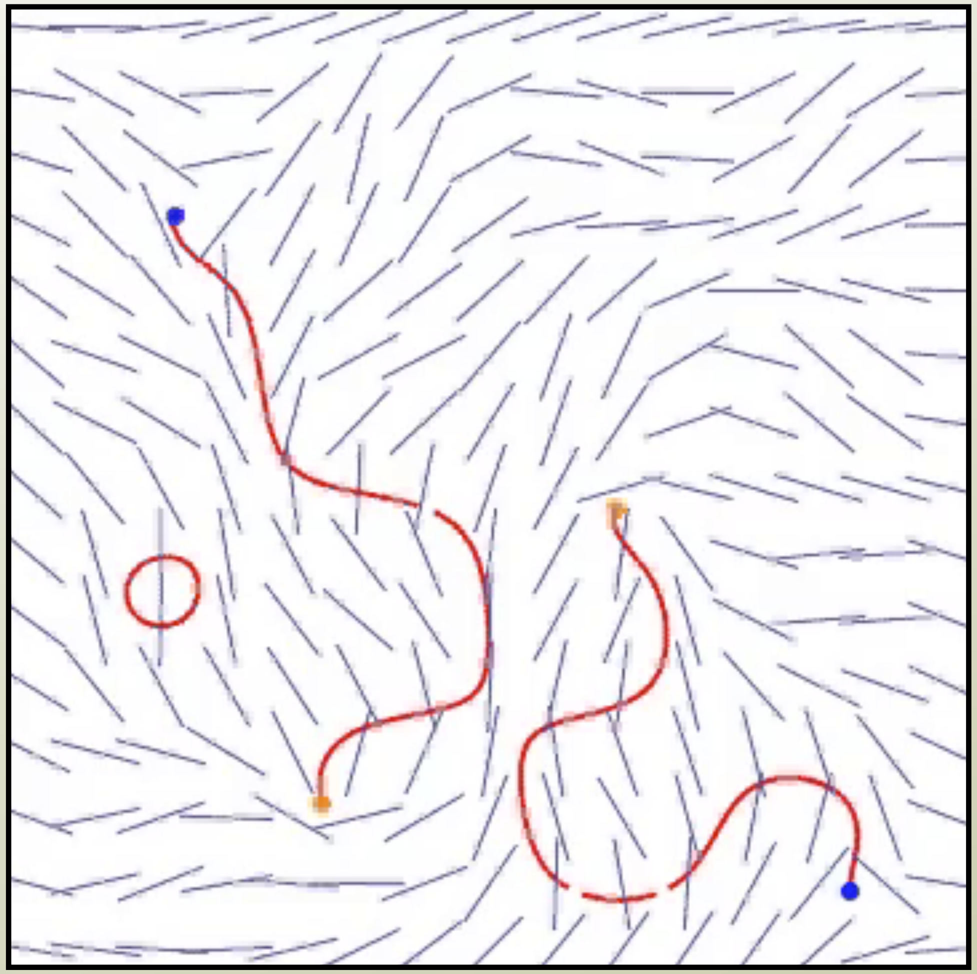

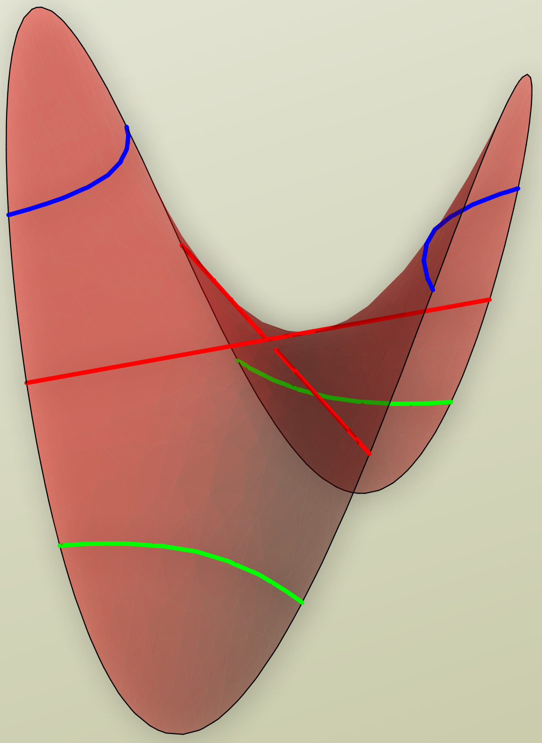

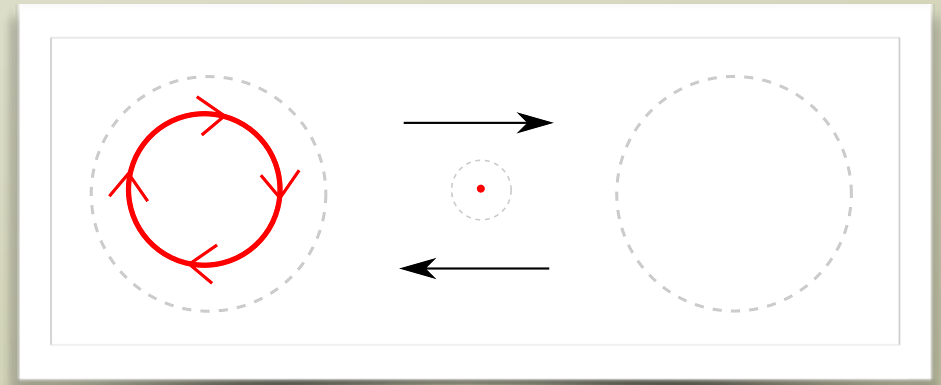

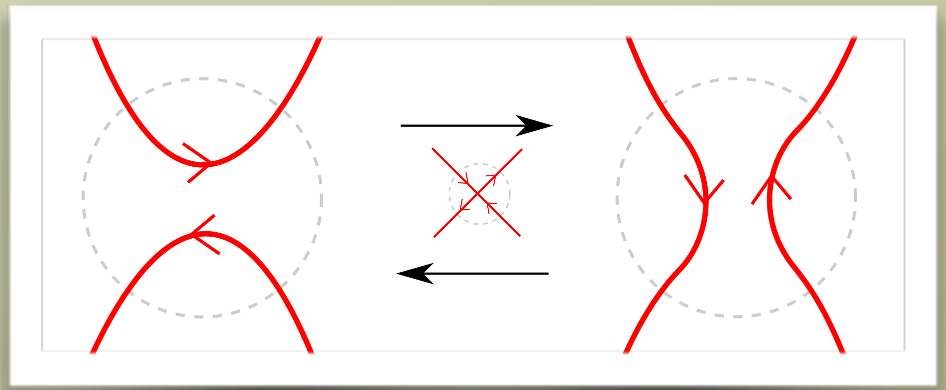

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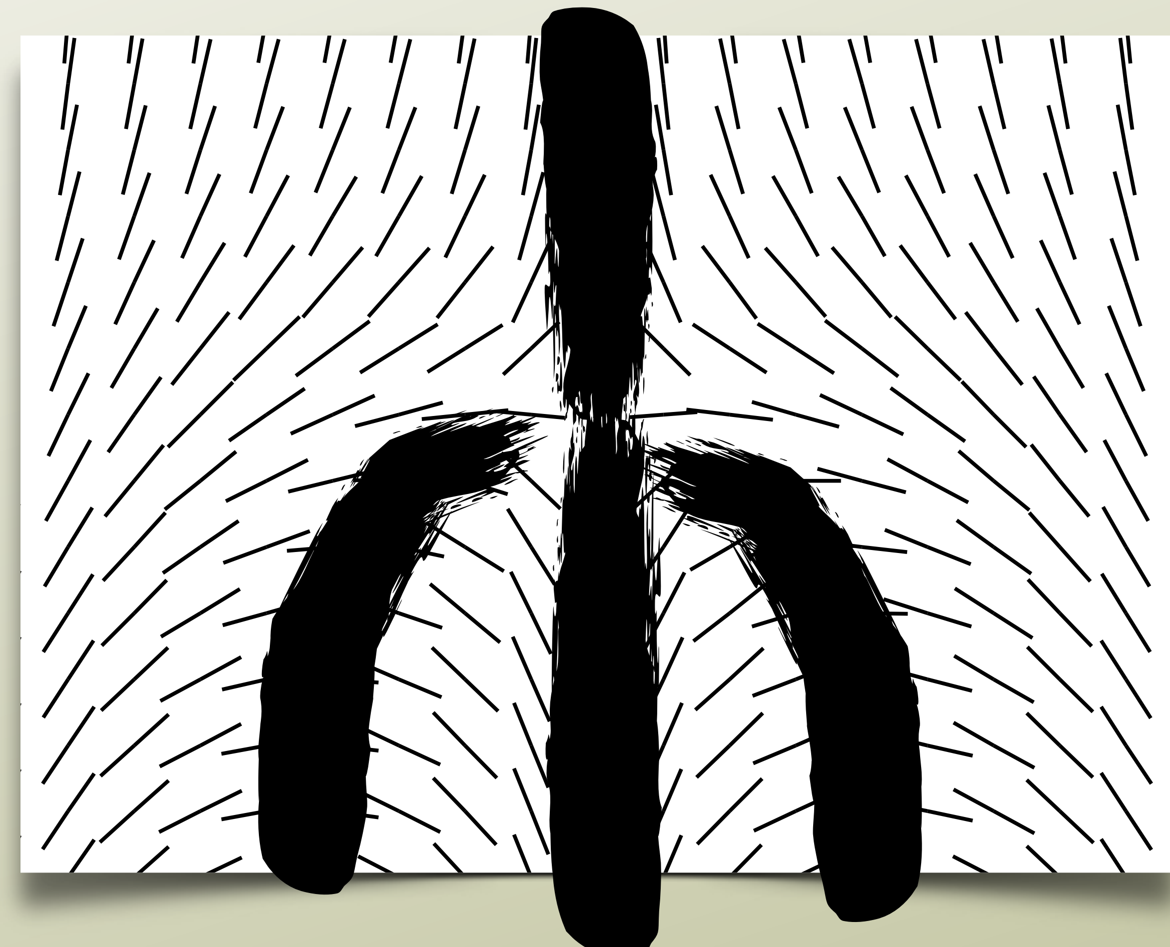
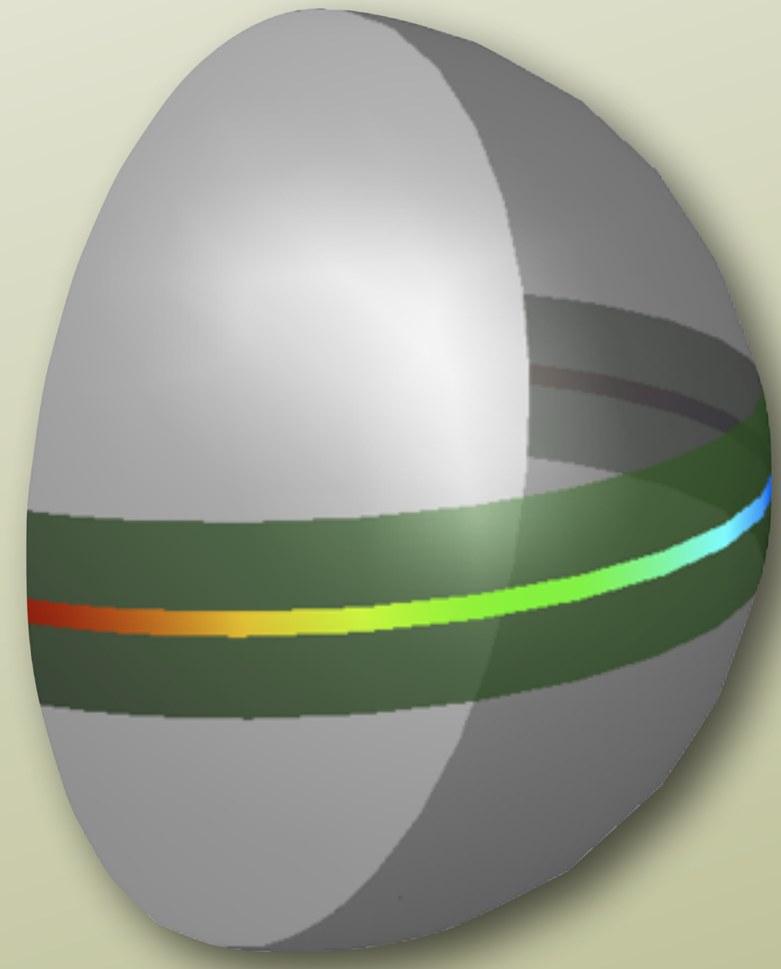
Nematics in Three Dimensions

instead of considering a **single (or two)** orientations, look at a whole curve of orientations:





space of orientations

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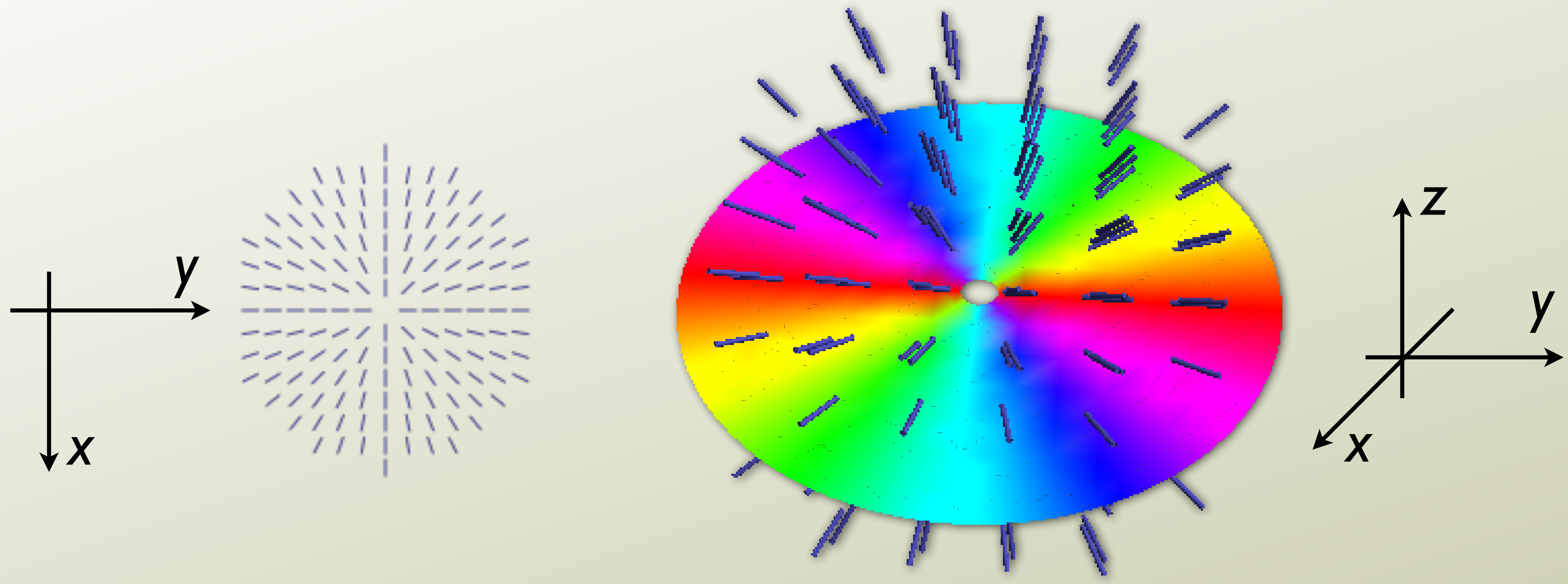
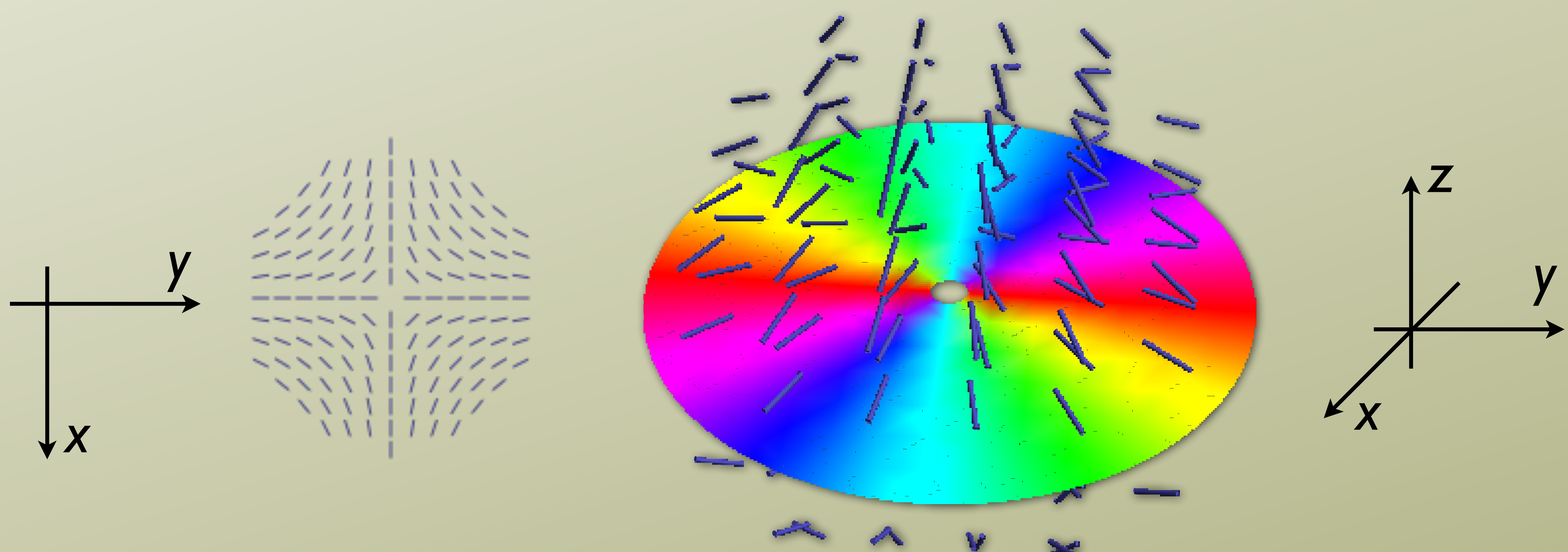
Nematics in Three Dimensions



Point defects in 3D = color phase singularity on a surface



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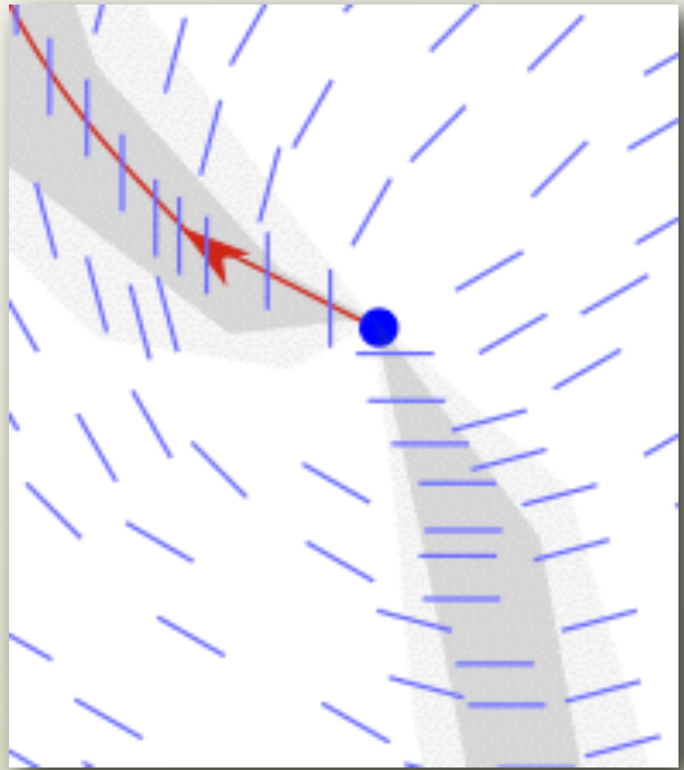
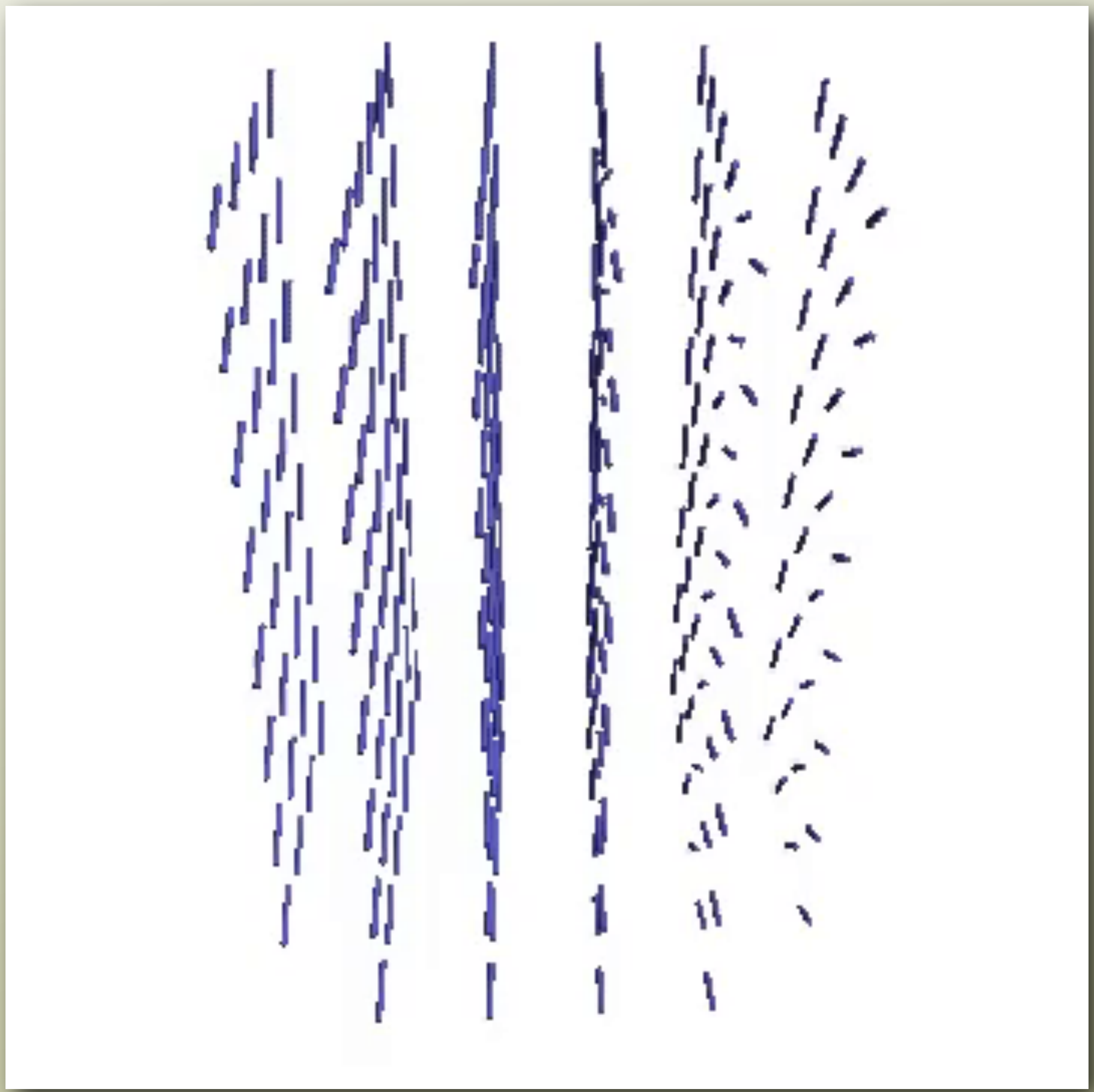
Nematics in Three Dimensions - Bryan Chen's Approach

From 2D to 3D: a visual dictionary of defects



Point defects in 2D = endpoints of lines



Line defects in 3D = boundaries of **surfaces**

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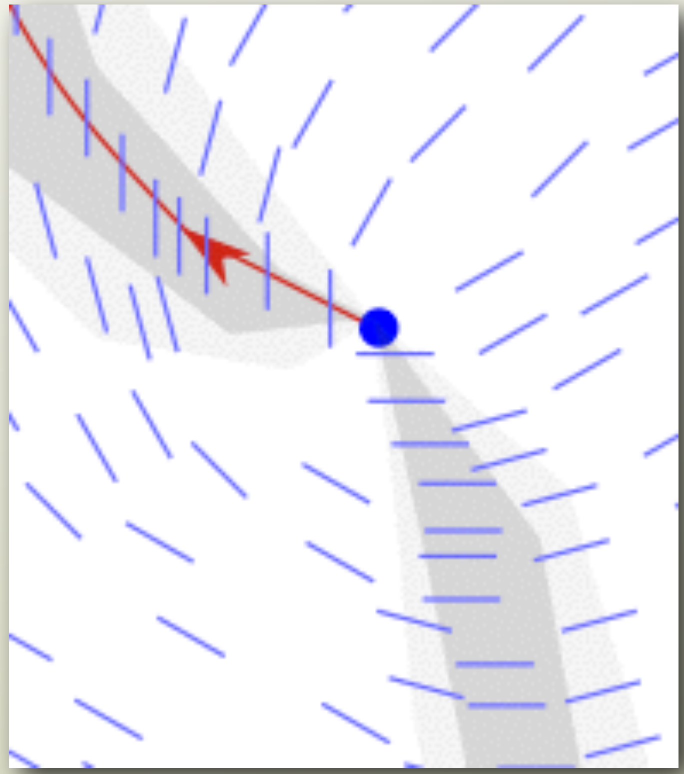

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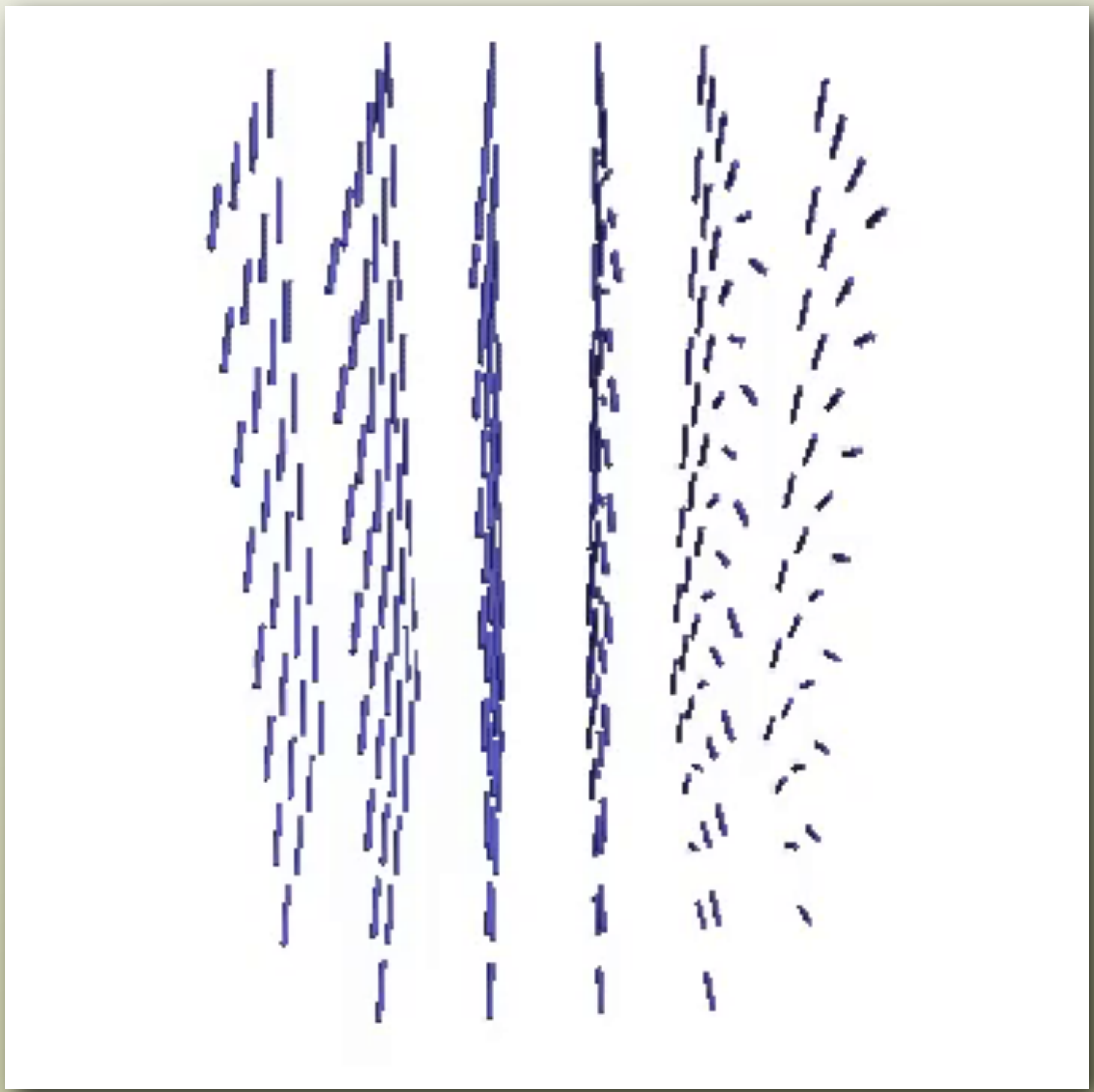

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Nematics in Three Dimensions - The Hedgehog

http://www.buzzfeed.com/babymantis/15-hedgehogs-with-things-that-look-like-hedgehogs-1opu

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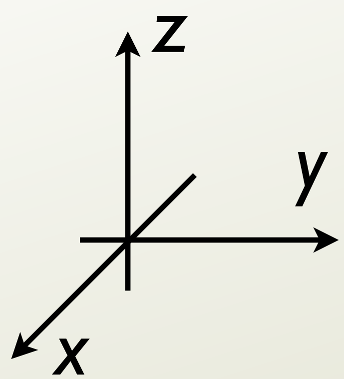

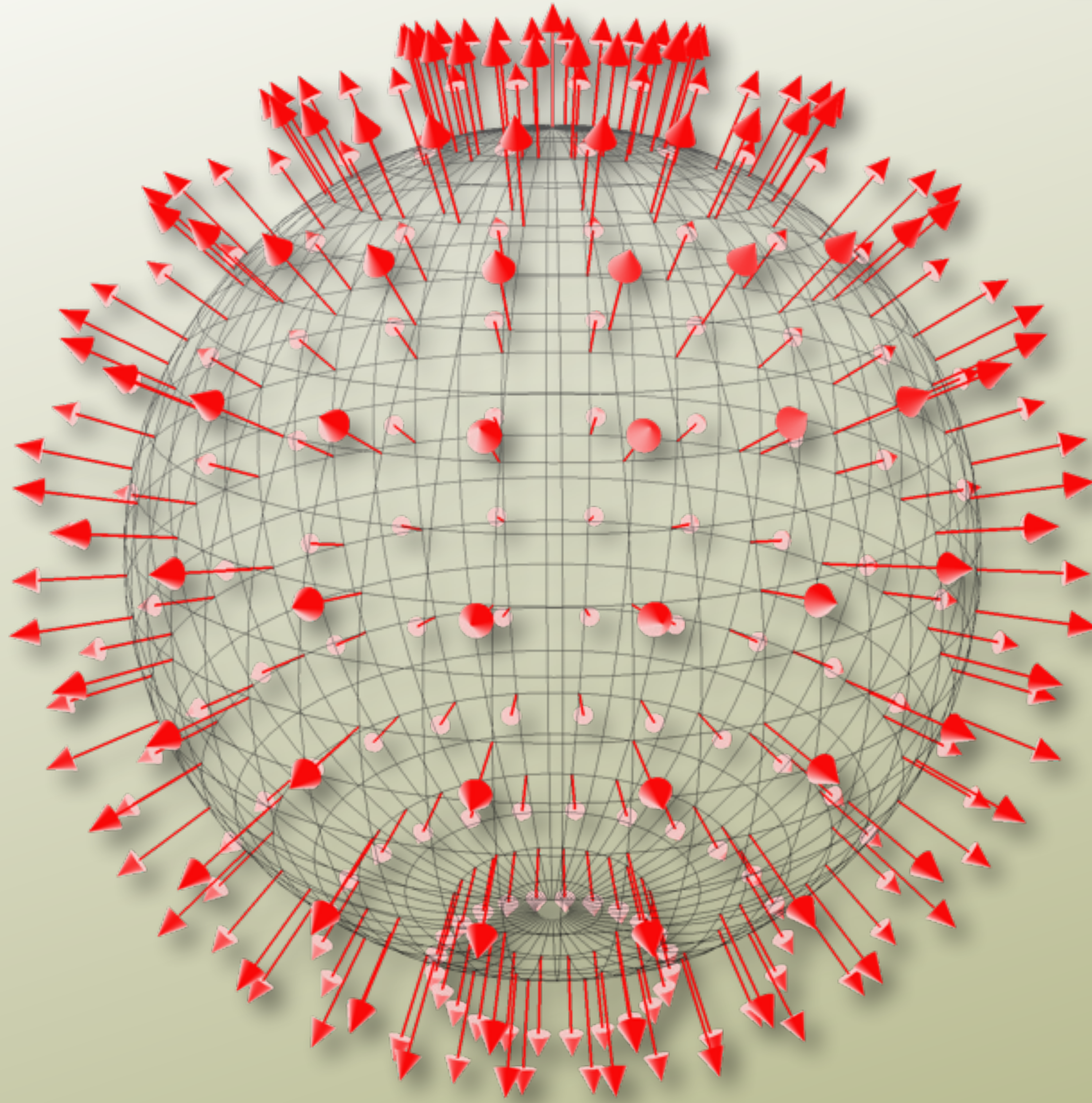

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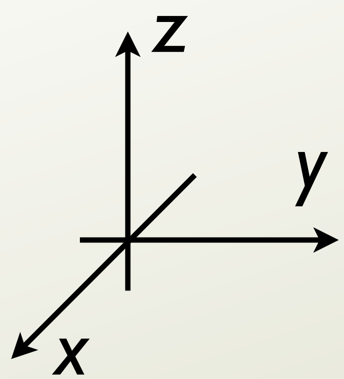

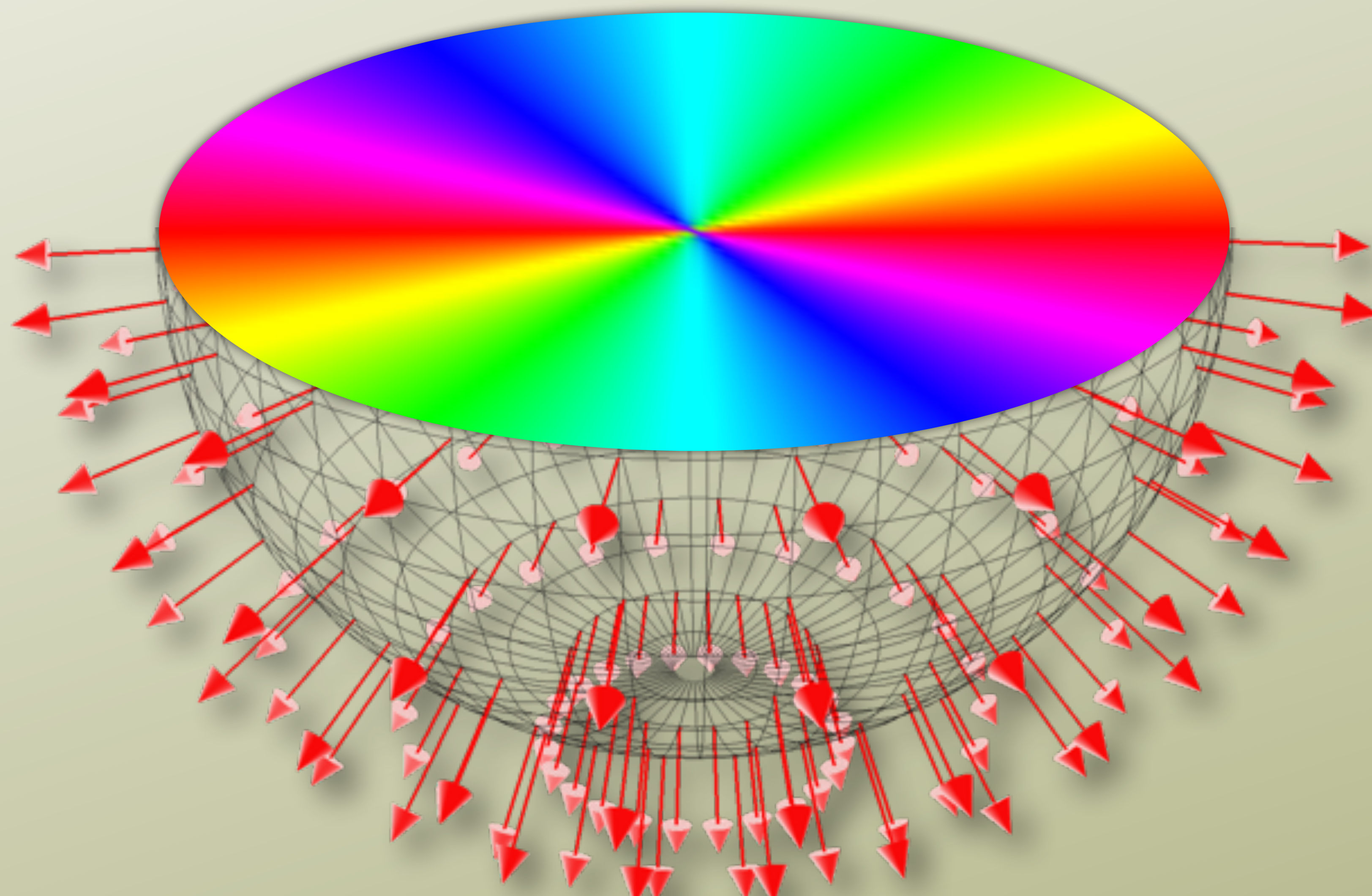

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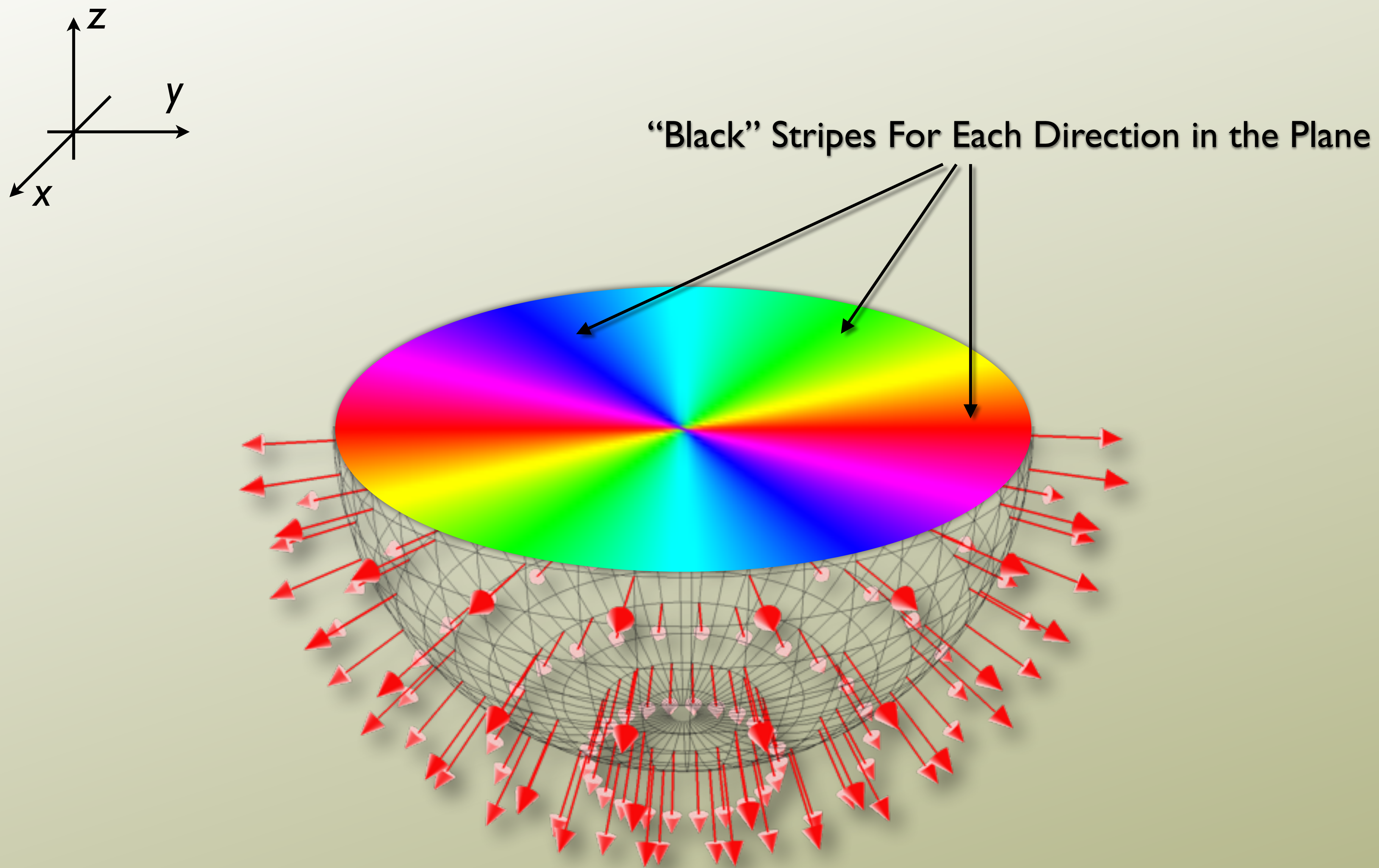

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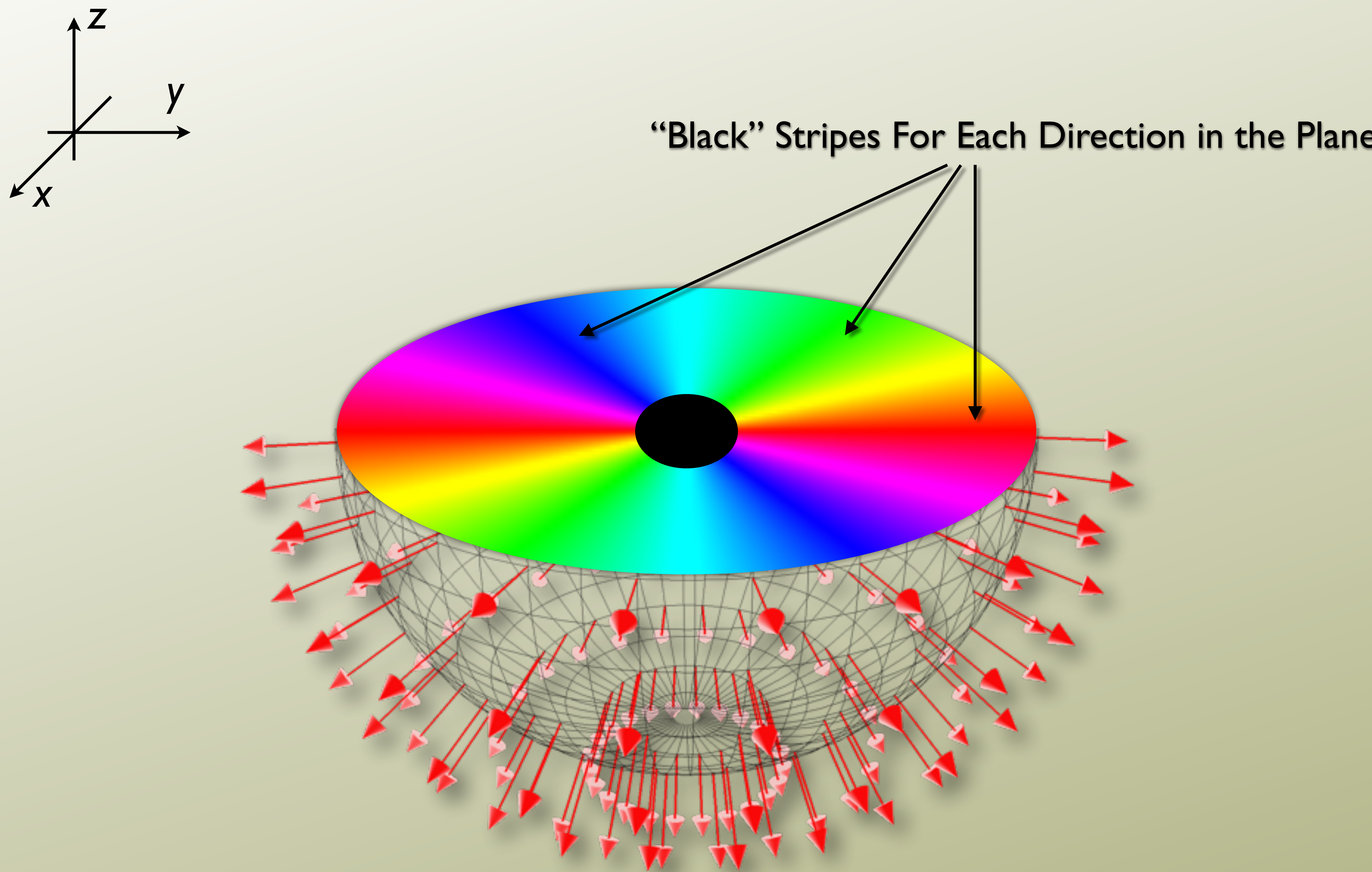

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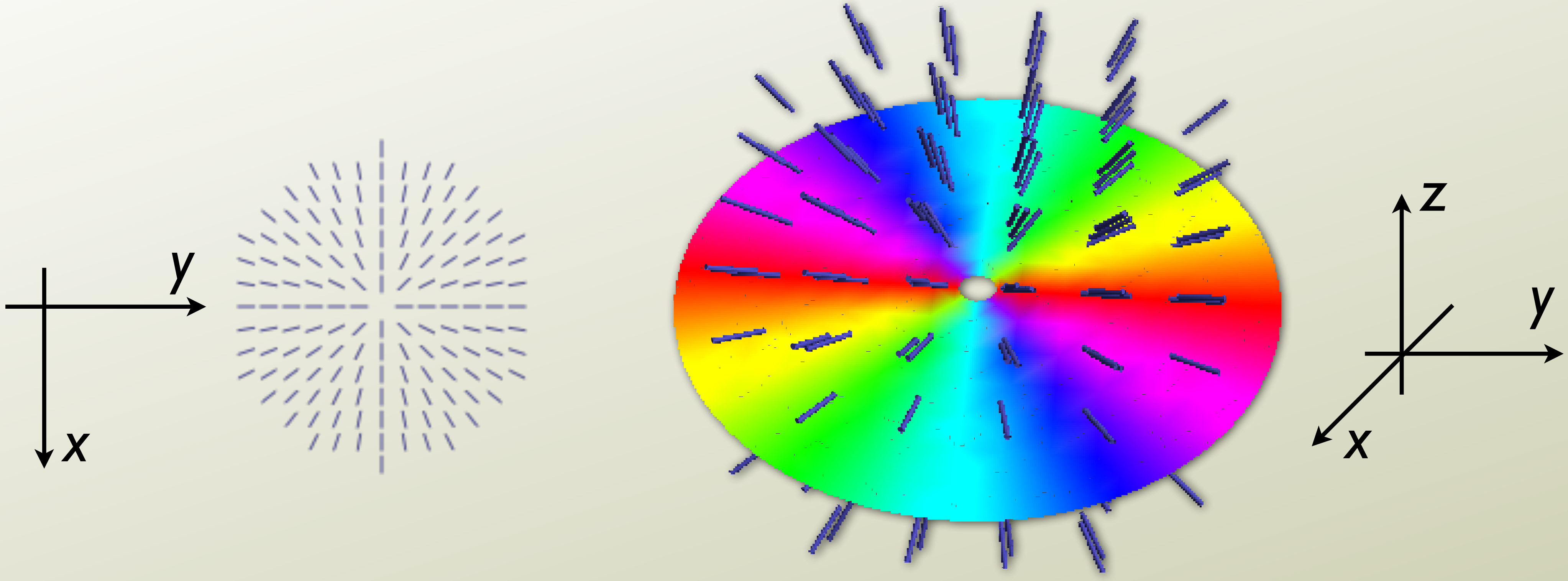

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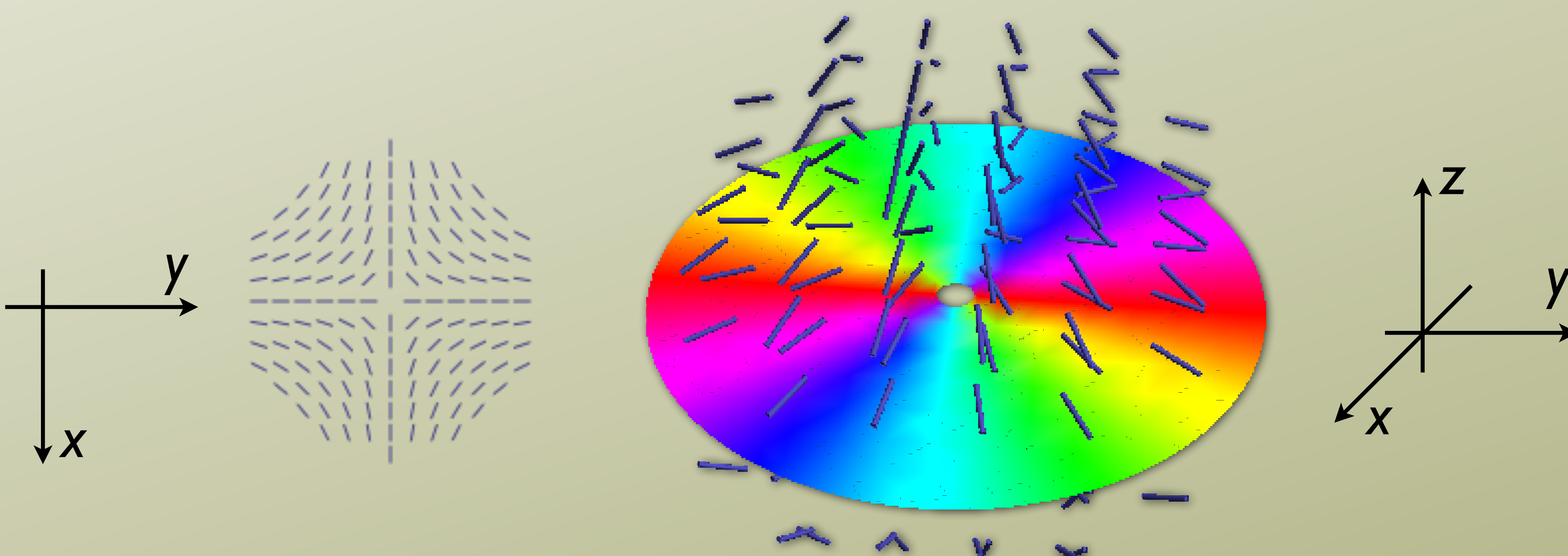

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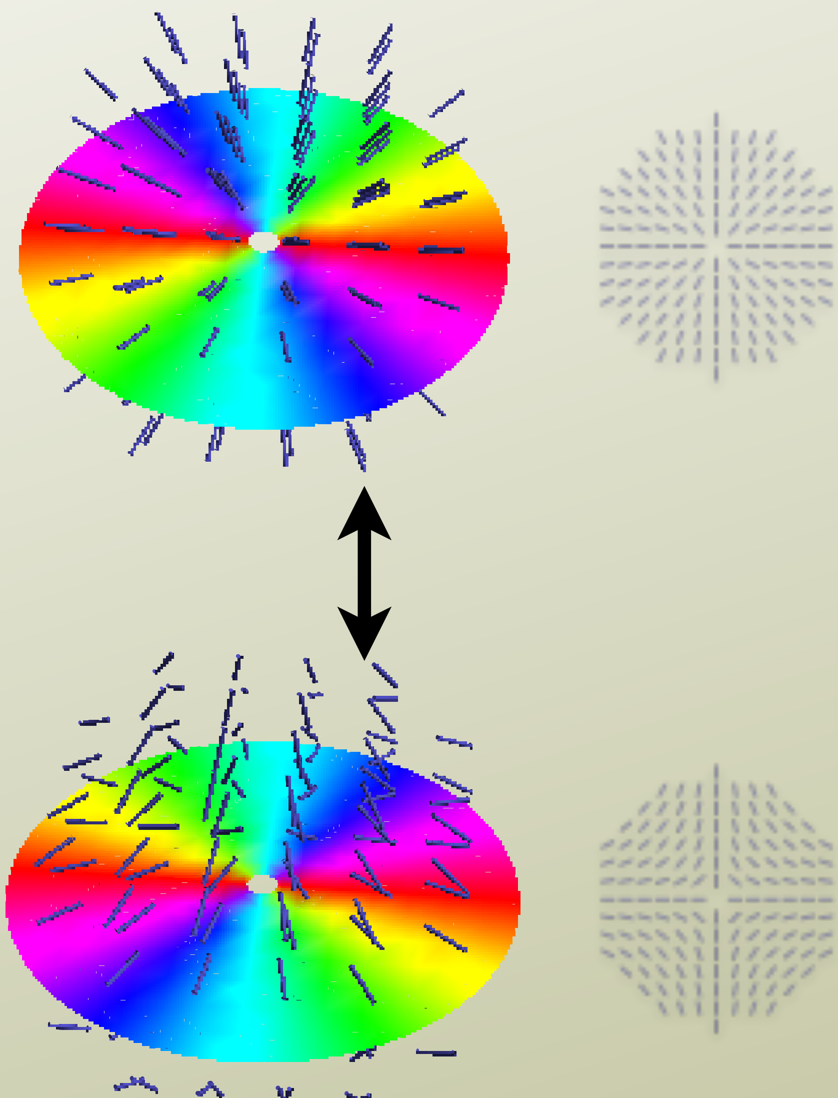
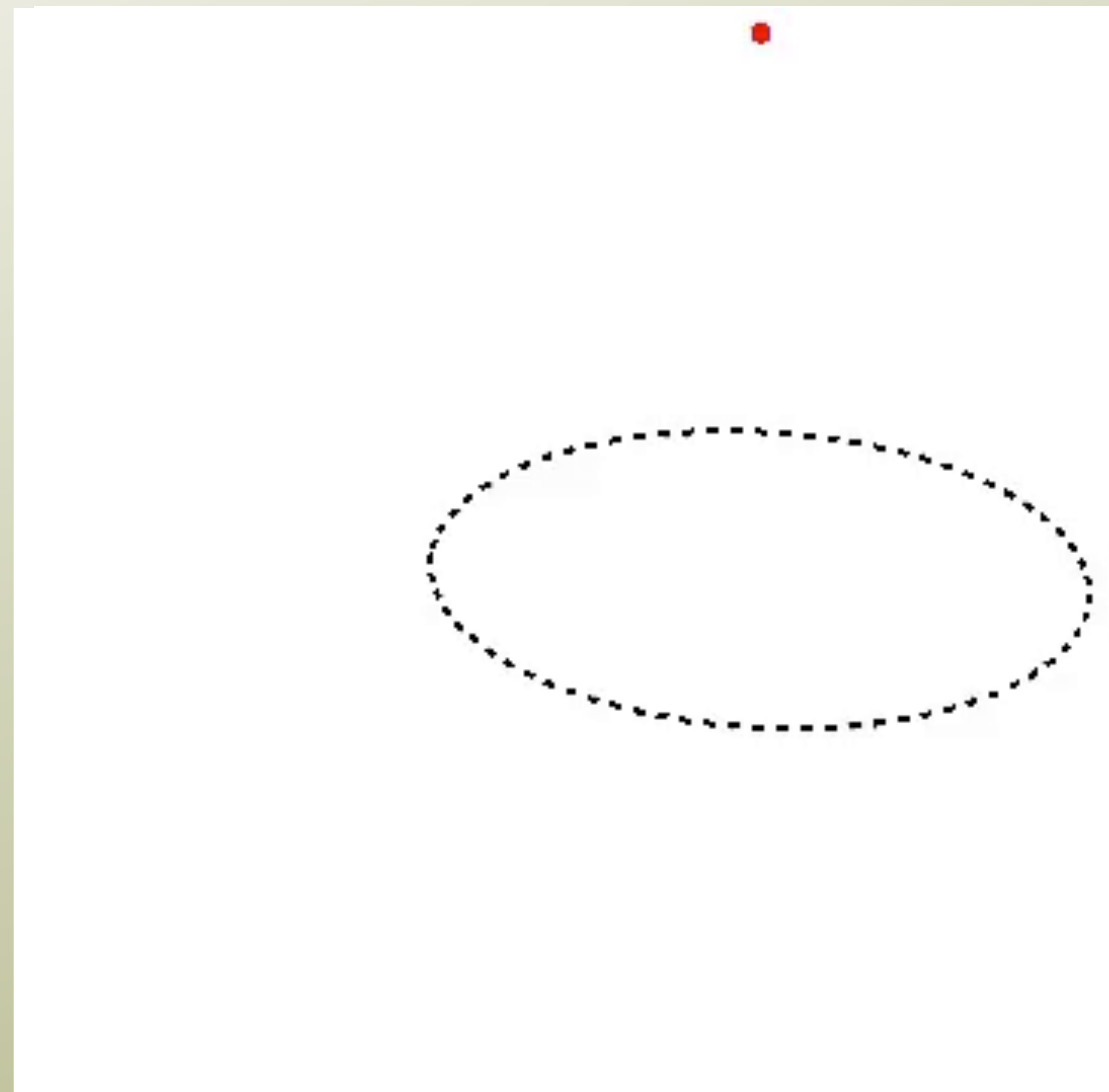
Nematics in Three Dimensions - Bryan Chen's Approach





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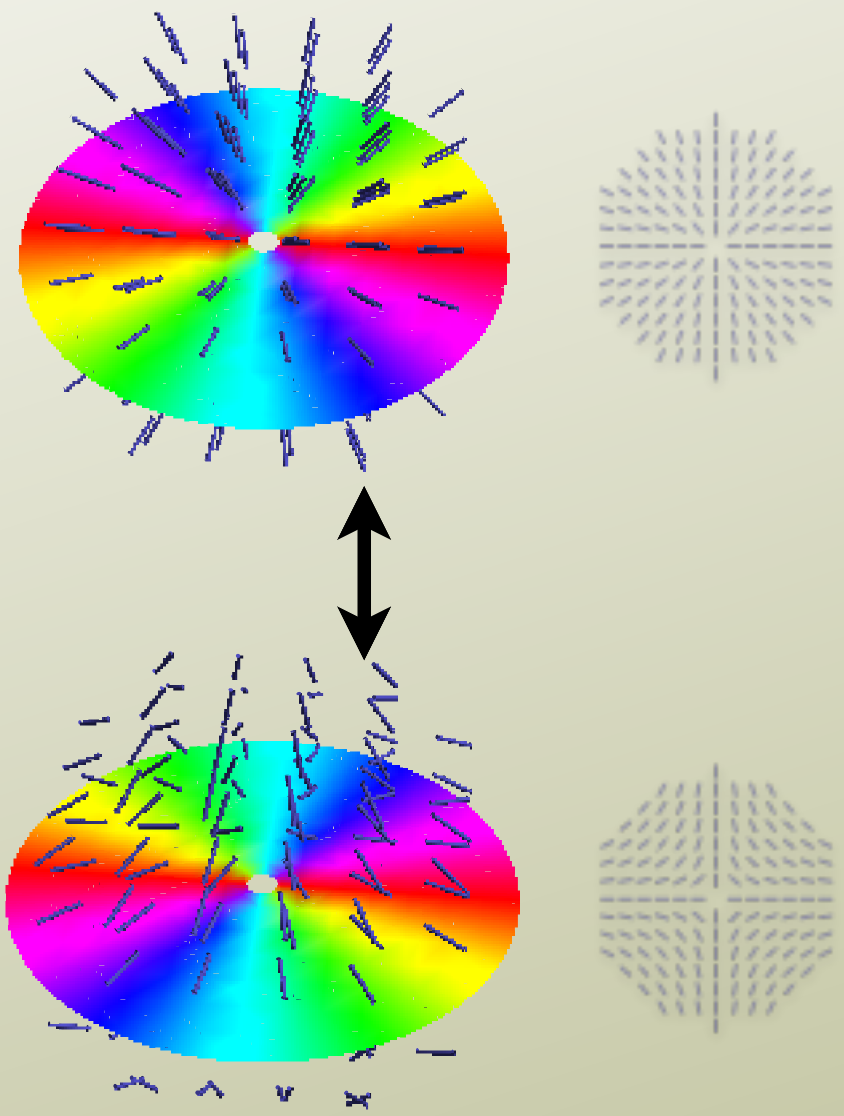

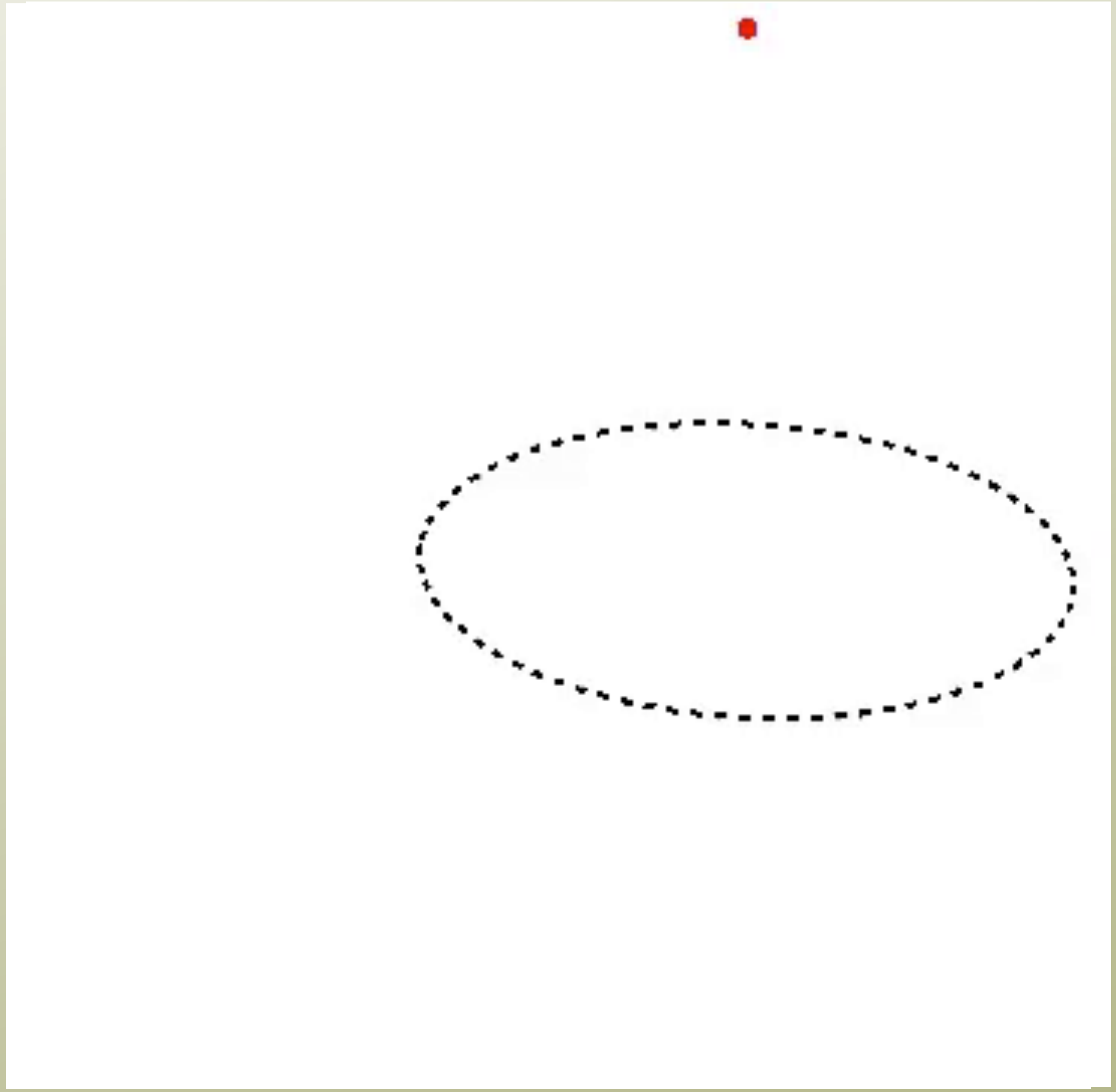

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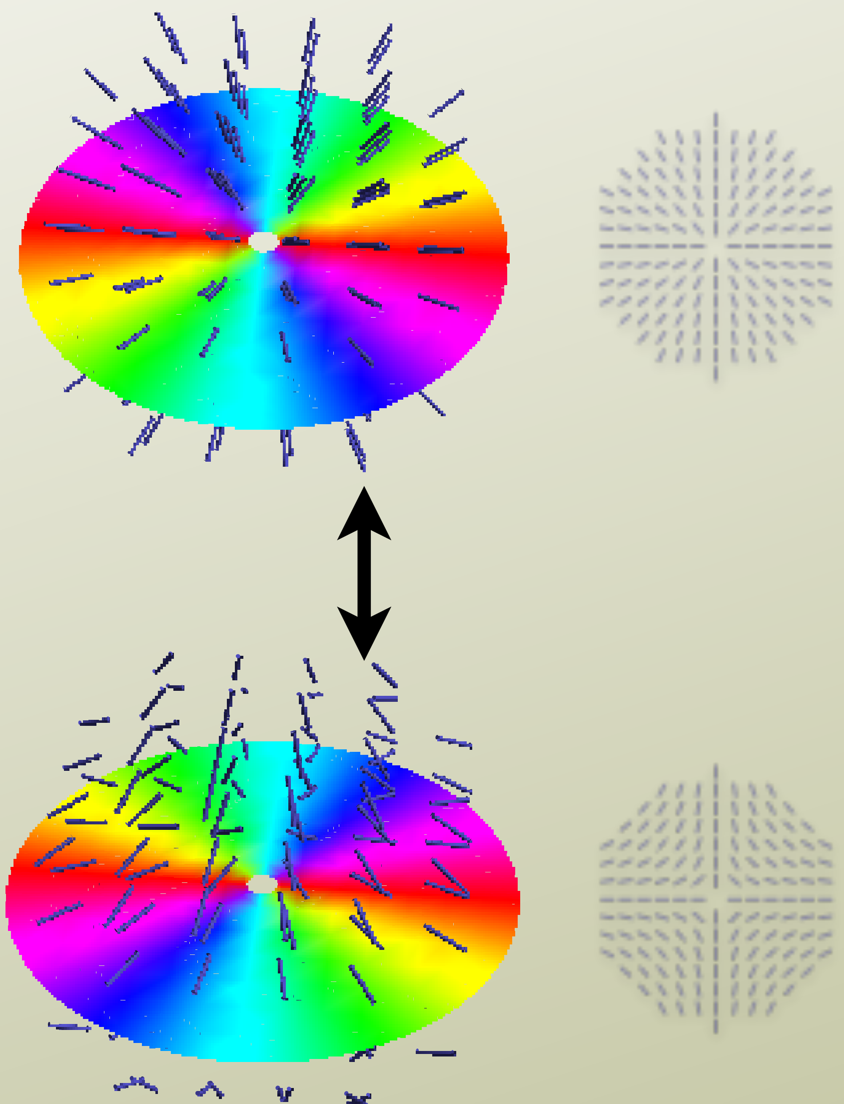
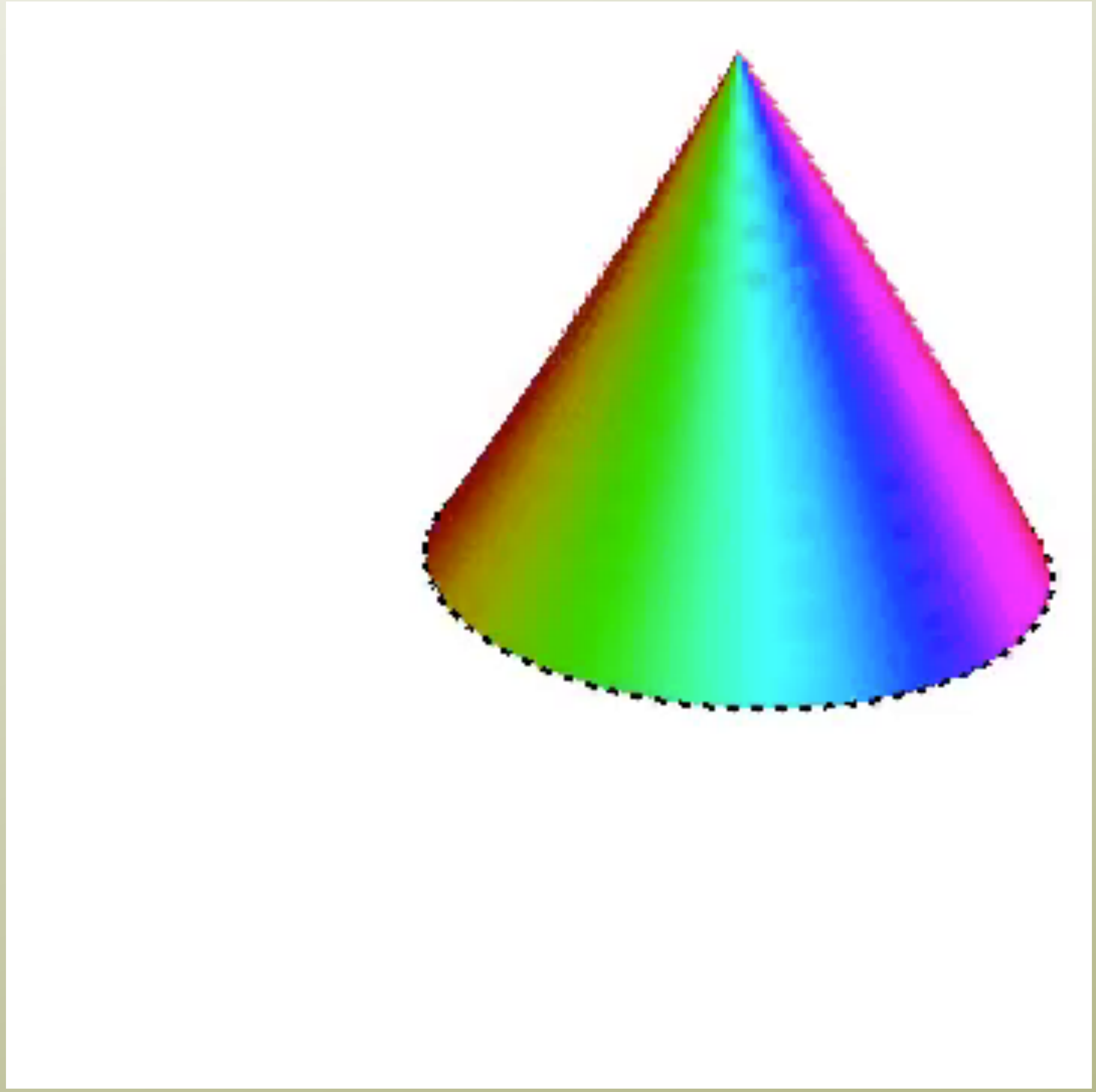
Nematics in Three Dimensions - Bryan Chen's Approach

direction of color winding switches!





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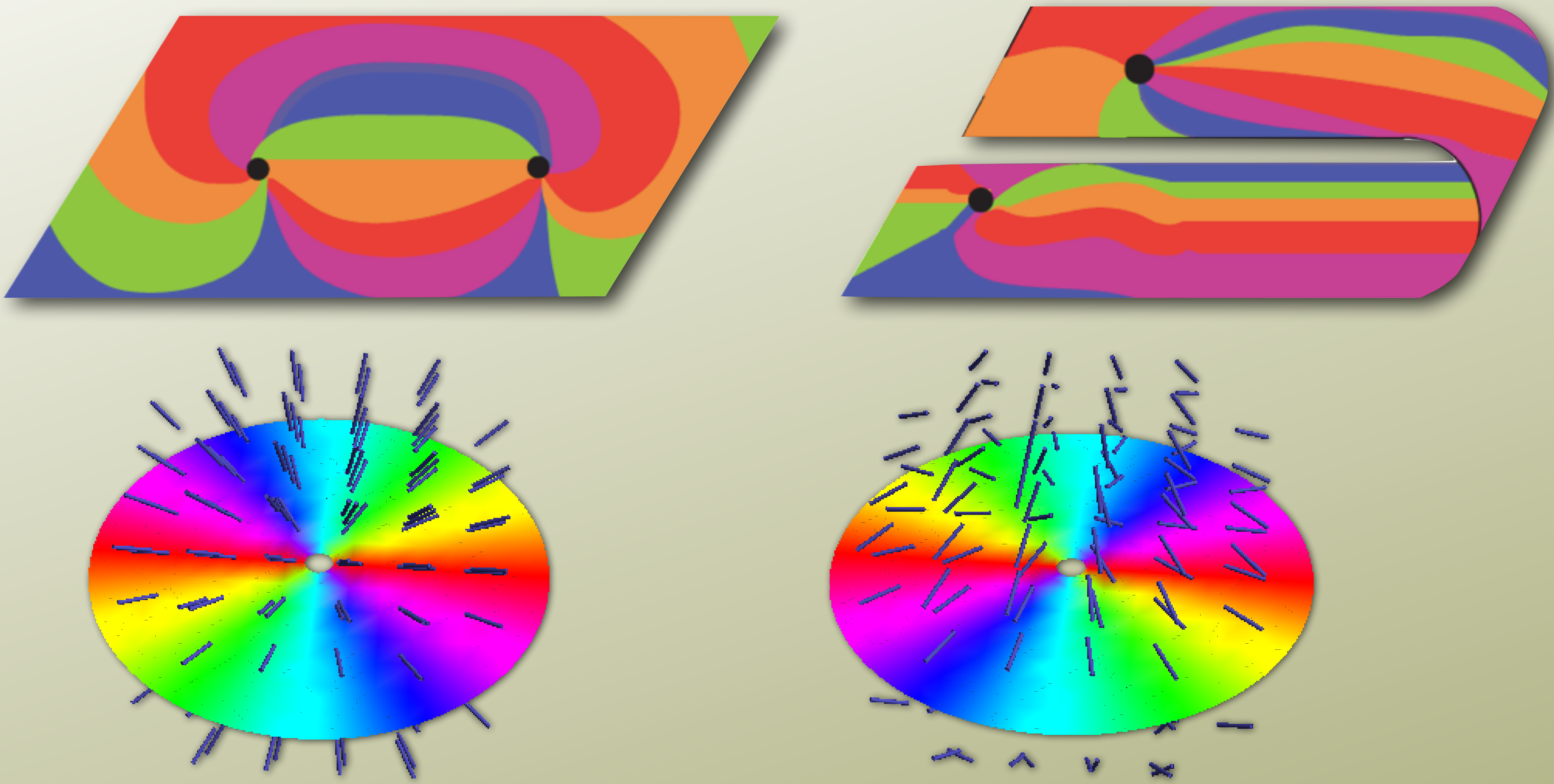

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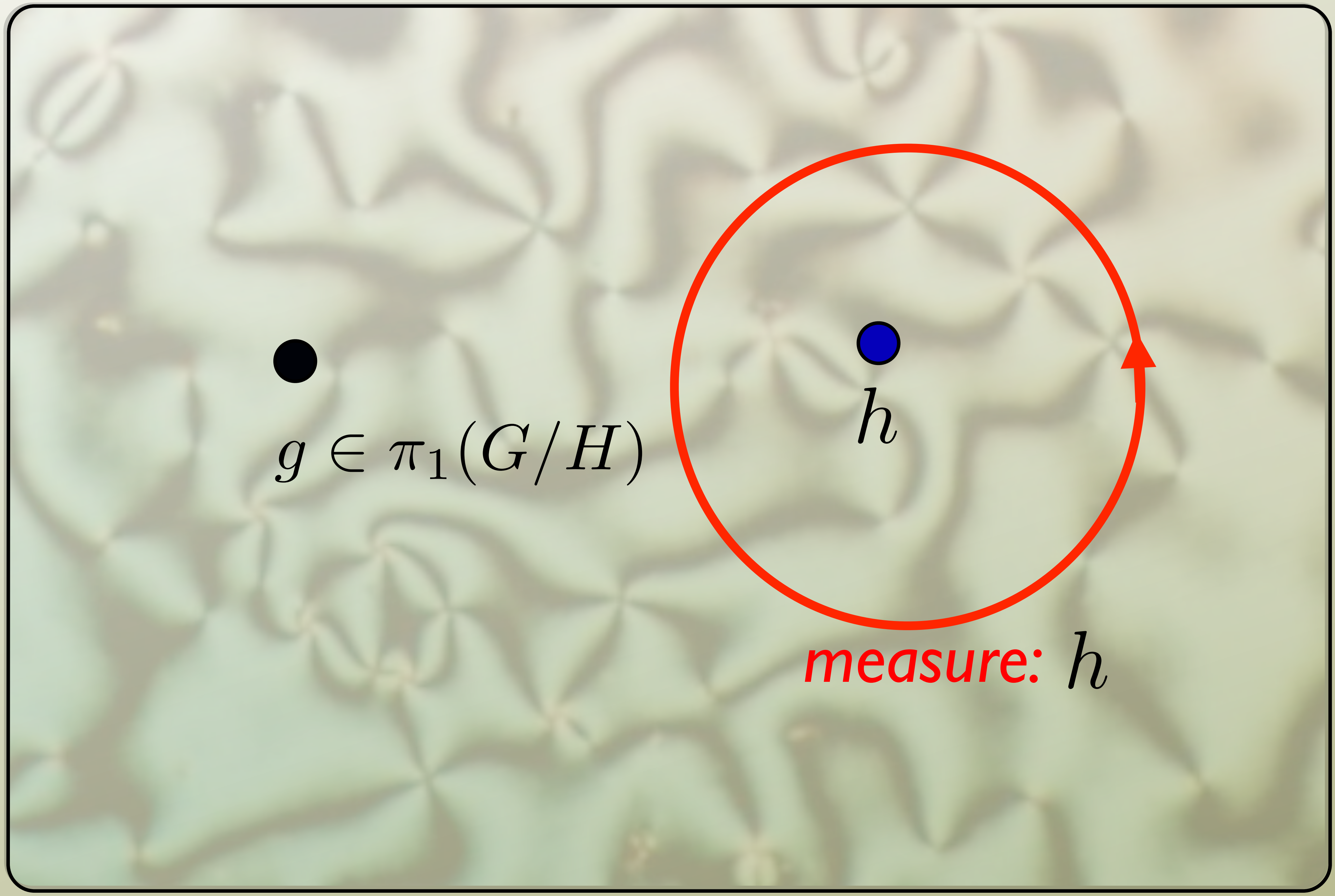
B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

Dynamics Generates Homotopy

Monday



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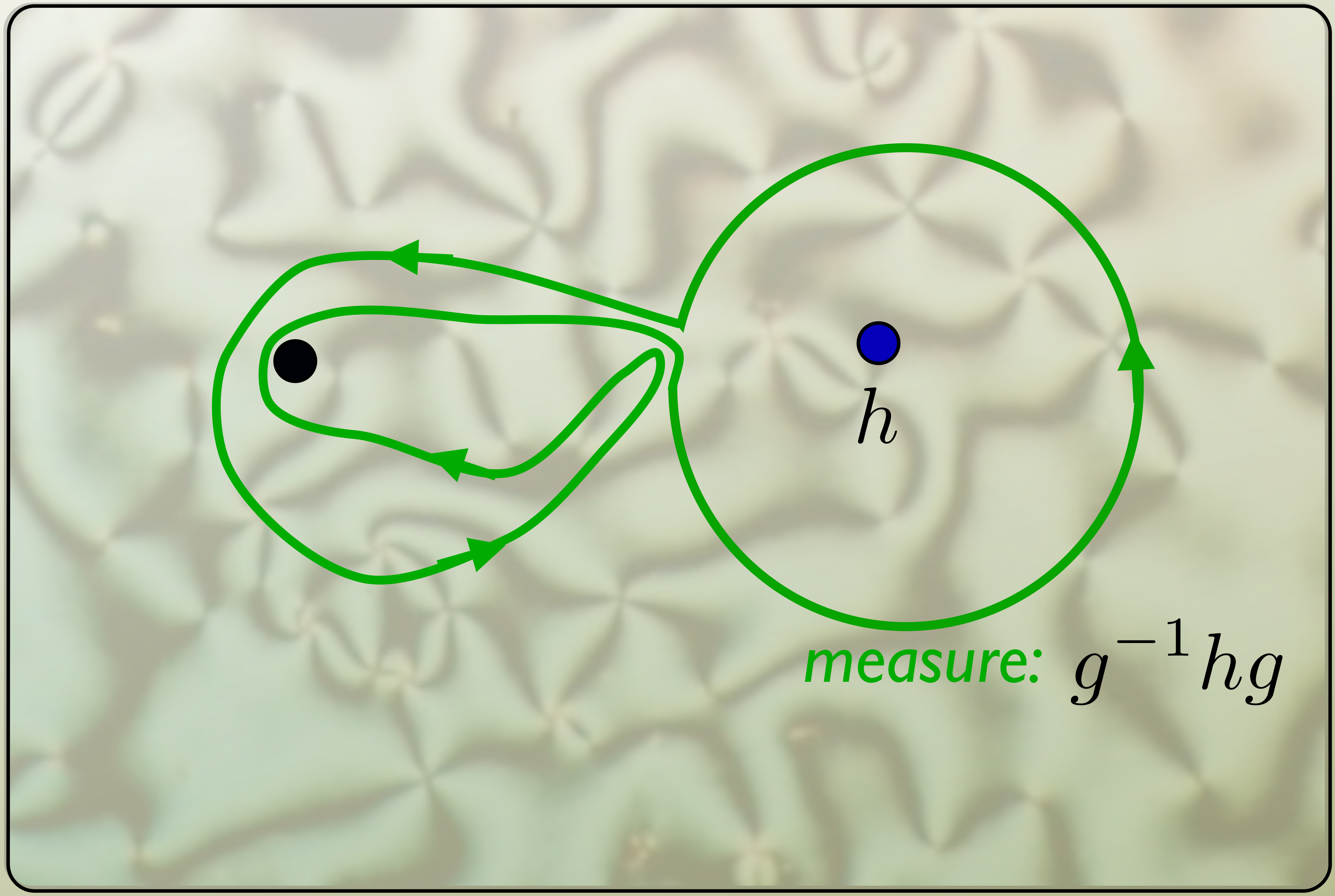
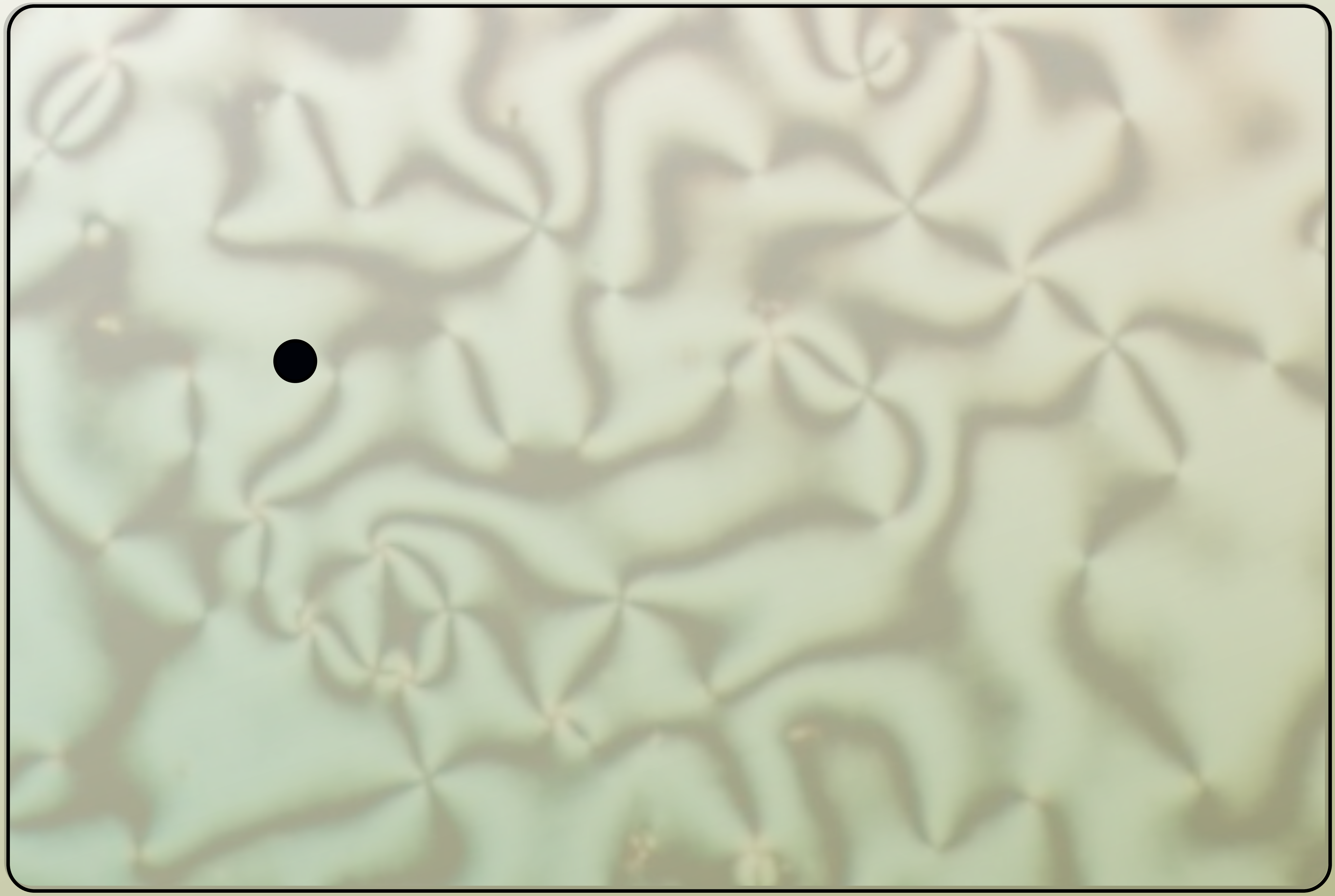
Dynamics Generates Homotopy

Tuesday



Dynamics Generates Homotopy

Wednesday (we are tired of measuring!)



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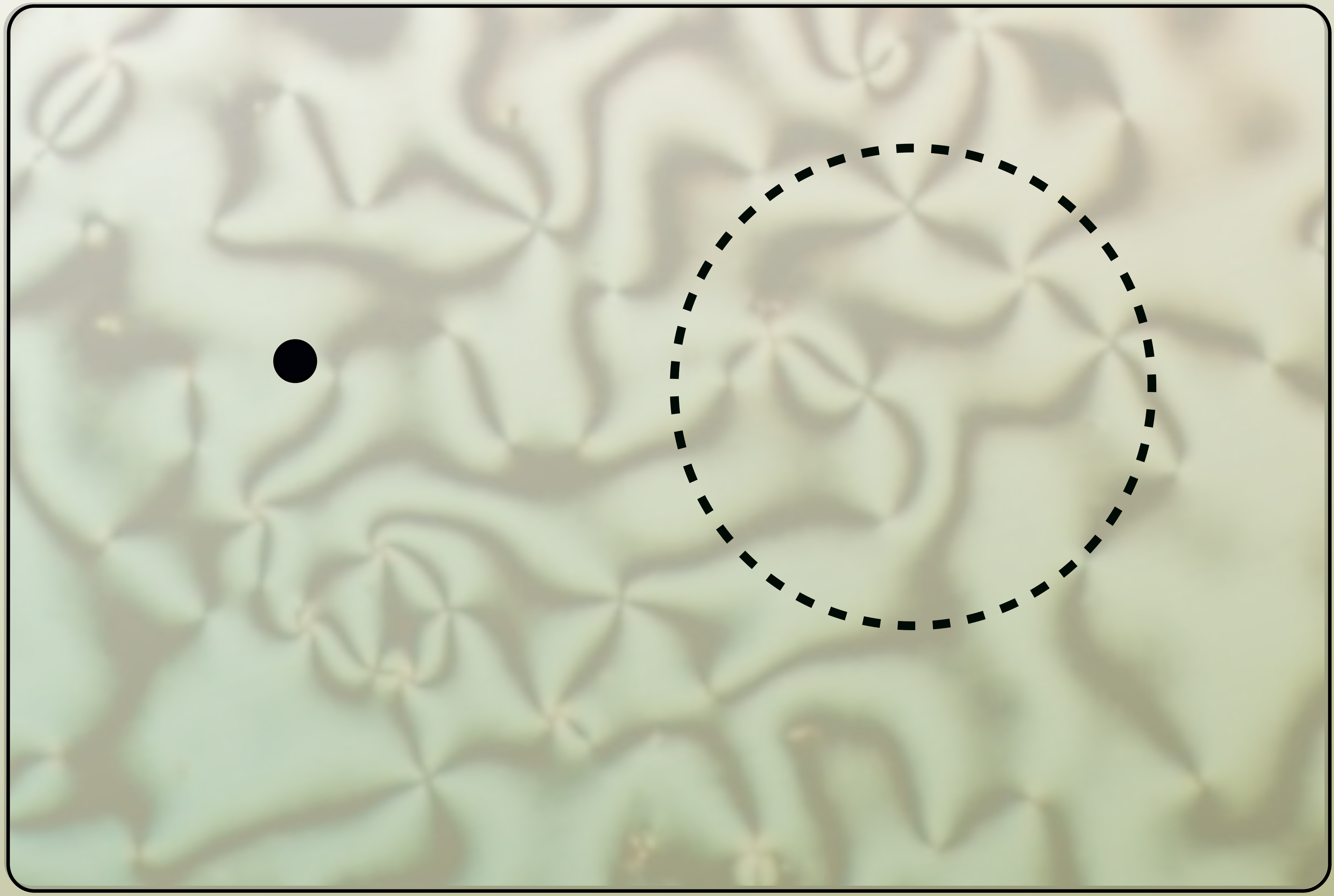

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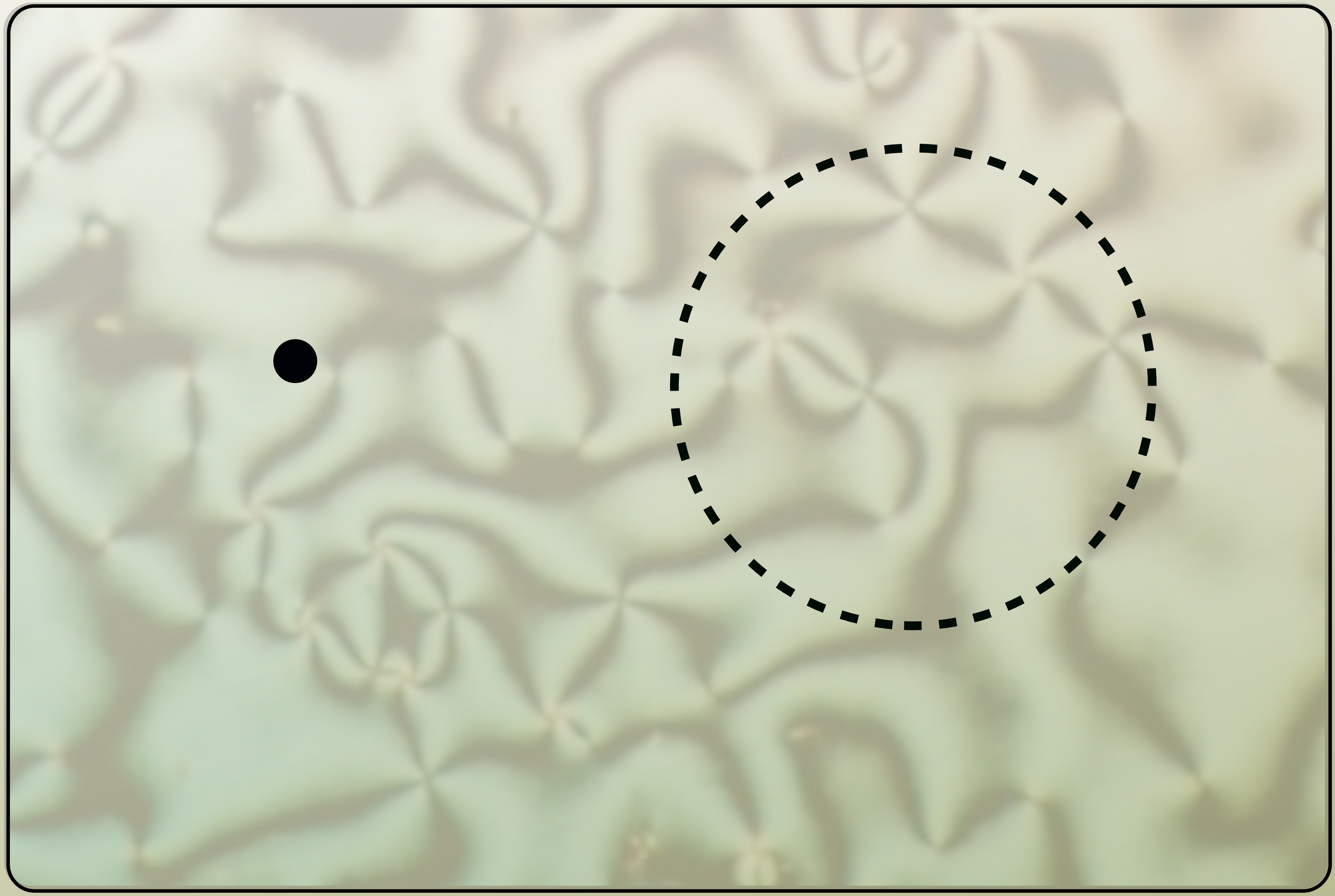

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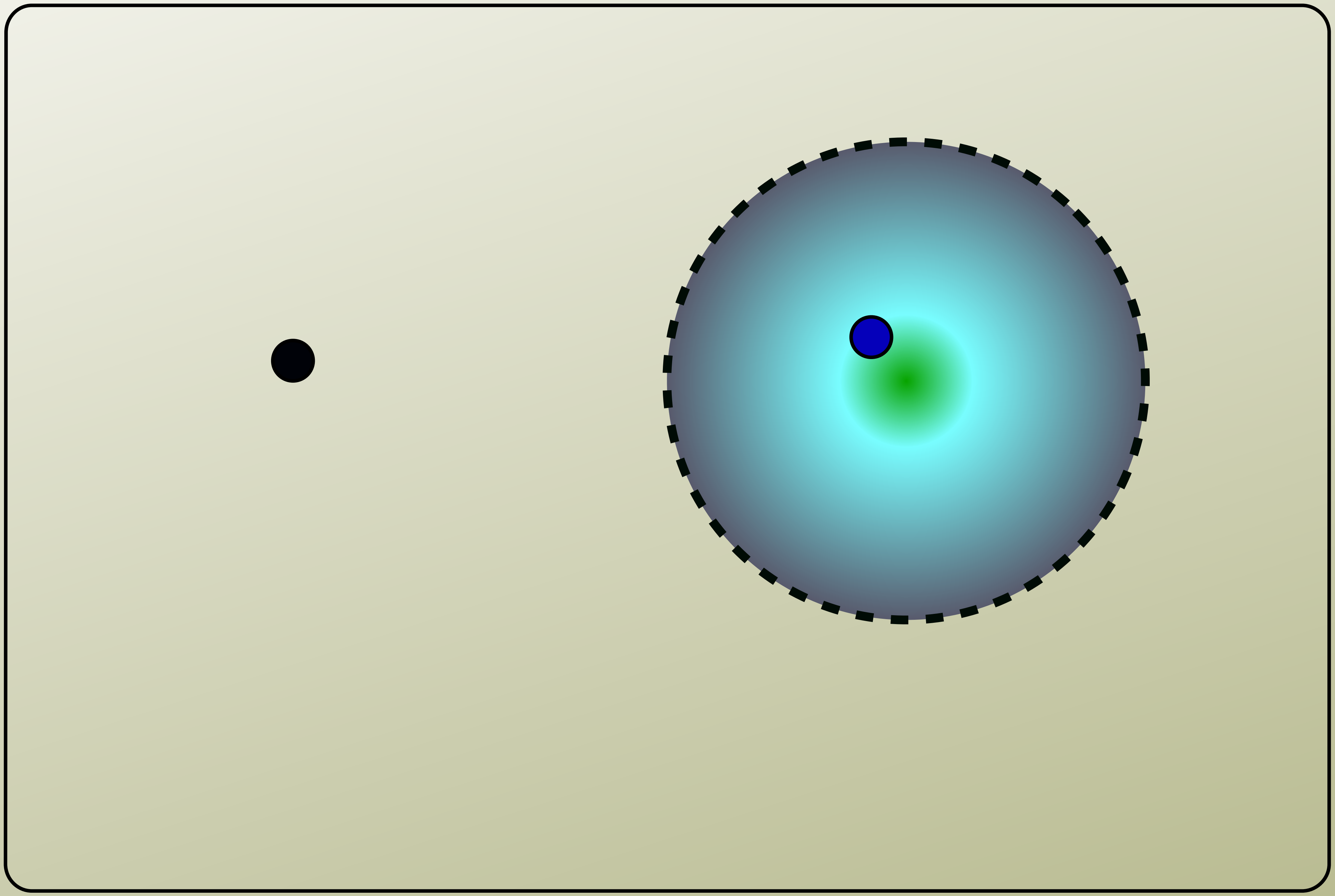

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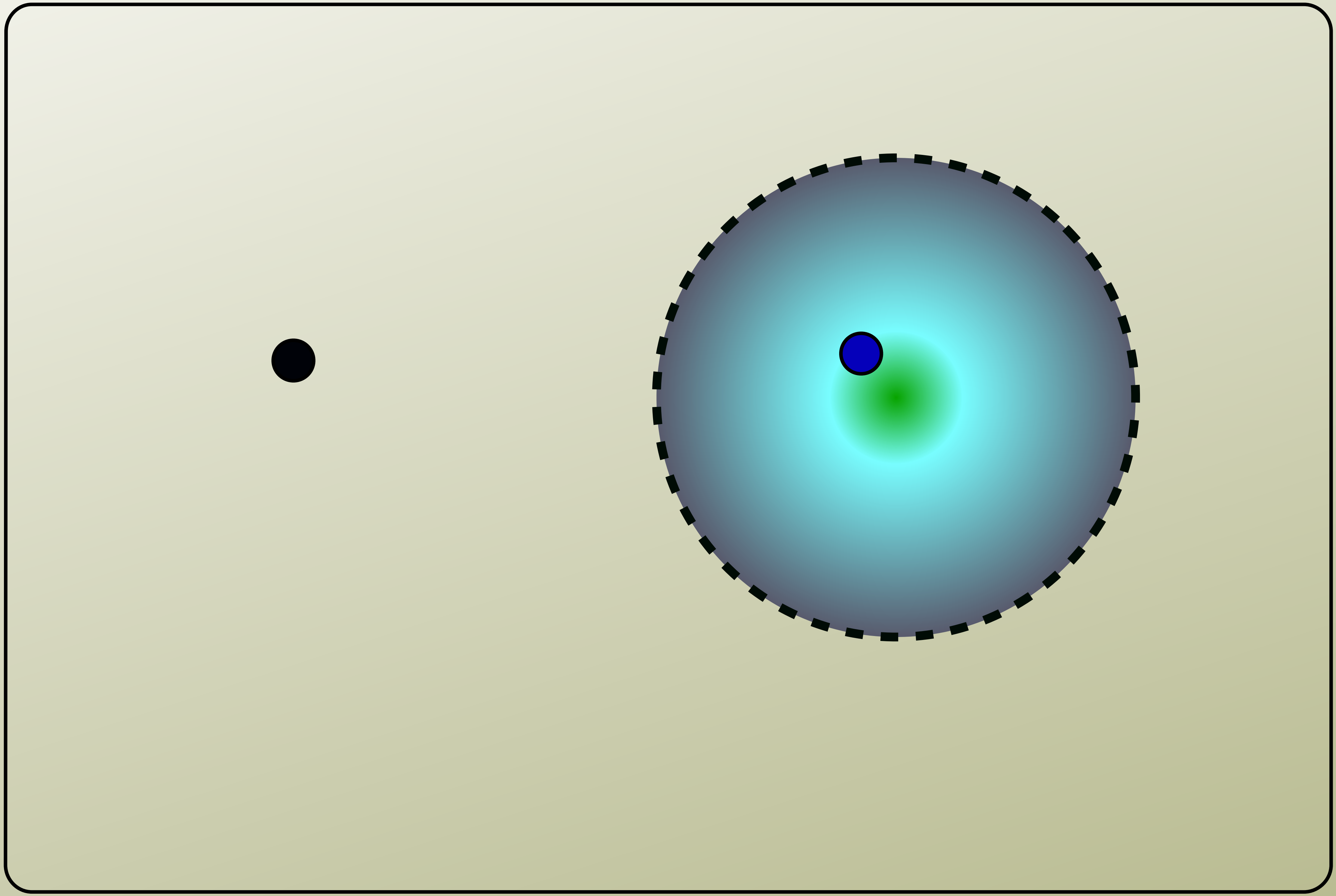
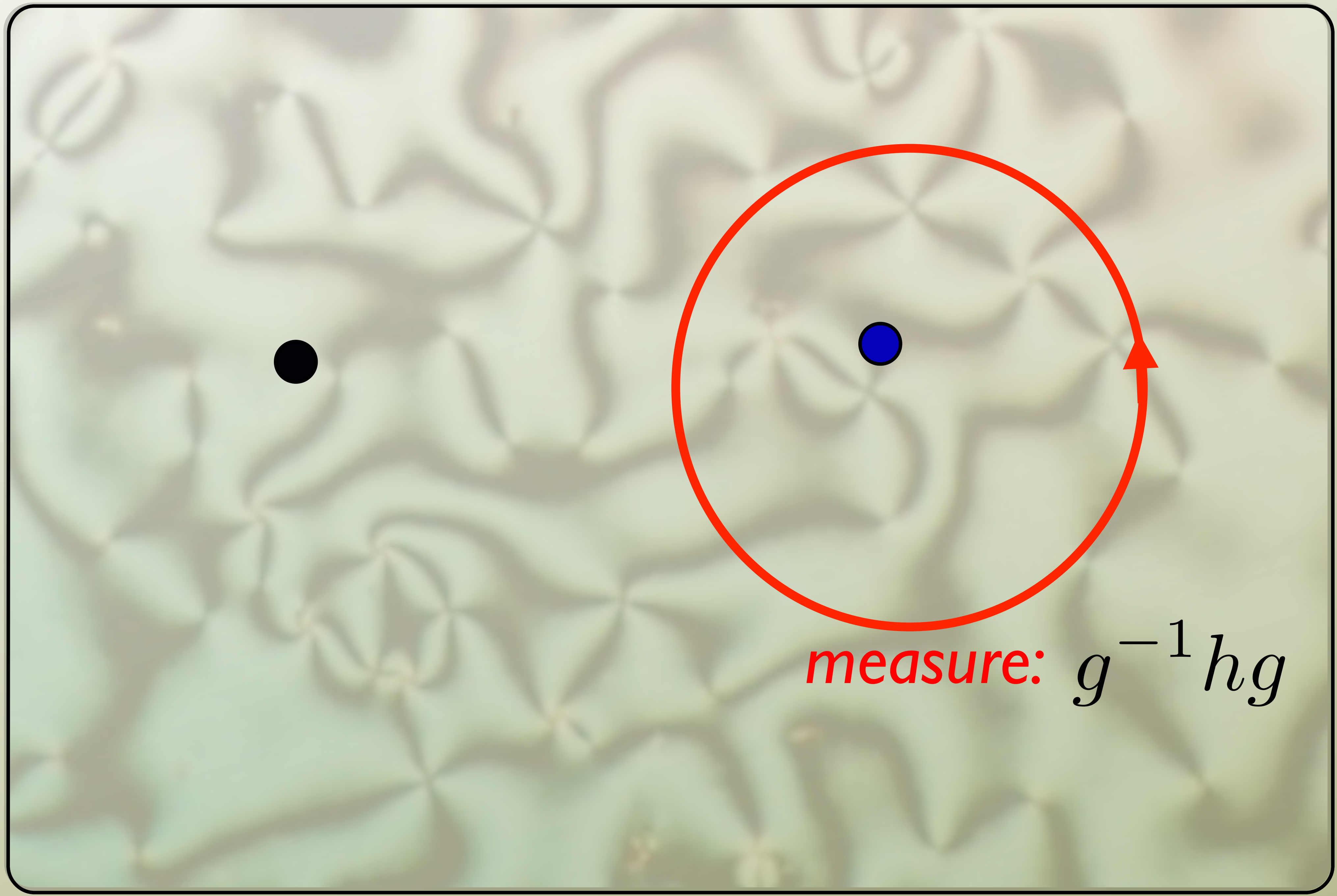


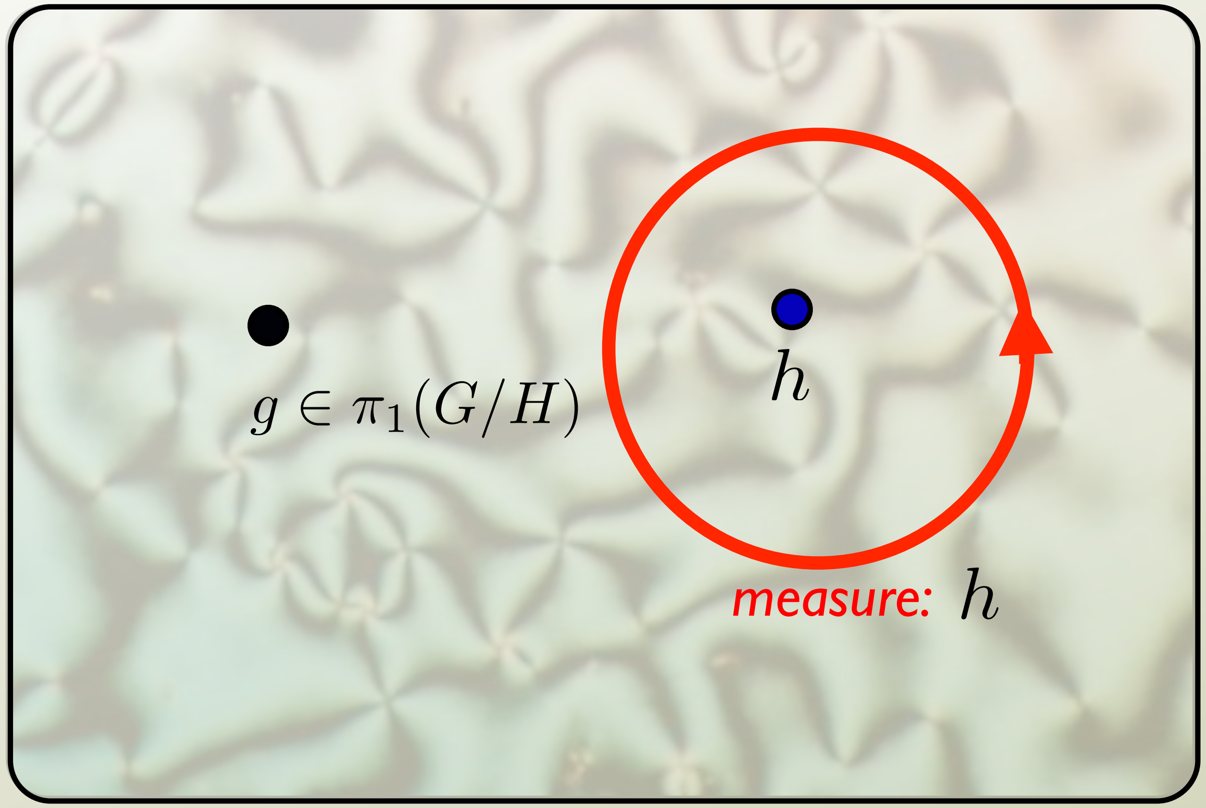
Dynamics Generates Homotopy

Thursday



Change Circles to Spheres!

Monday



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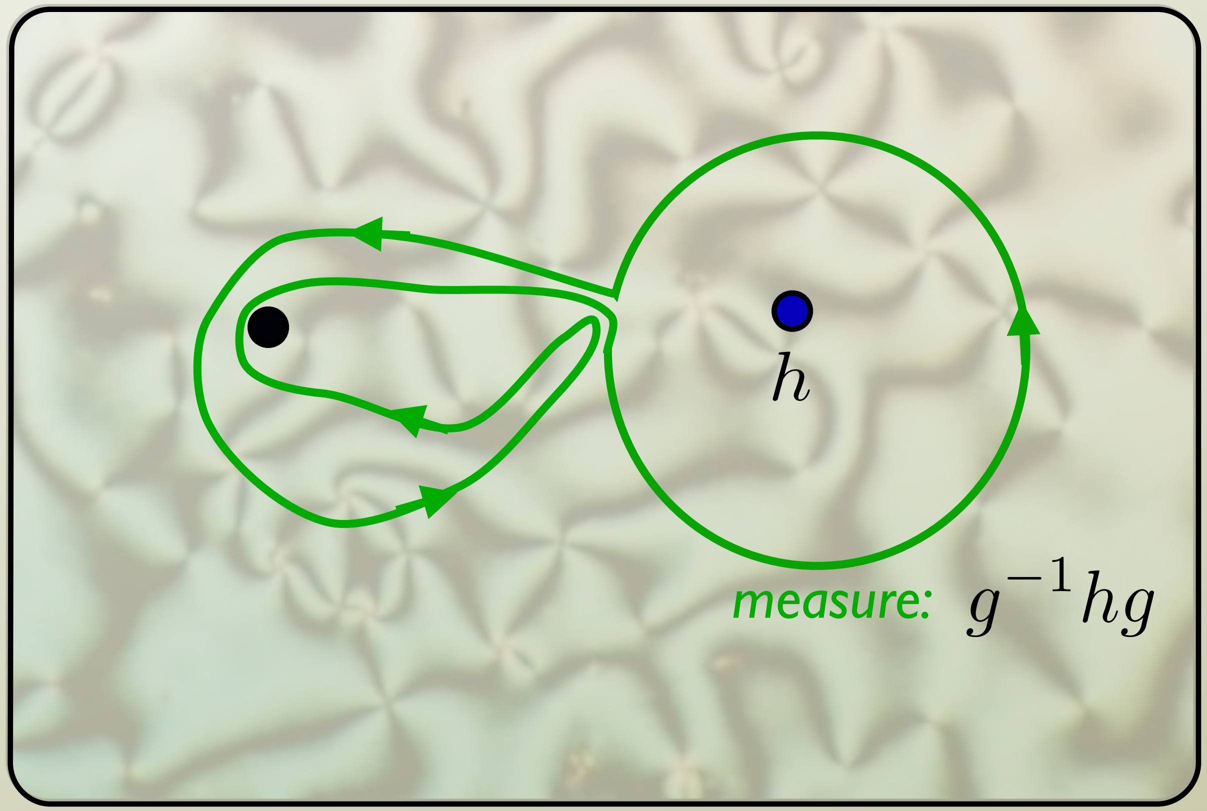

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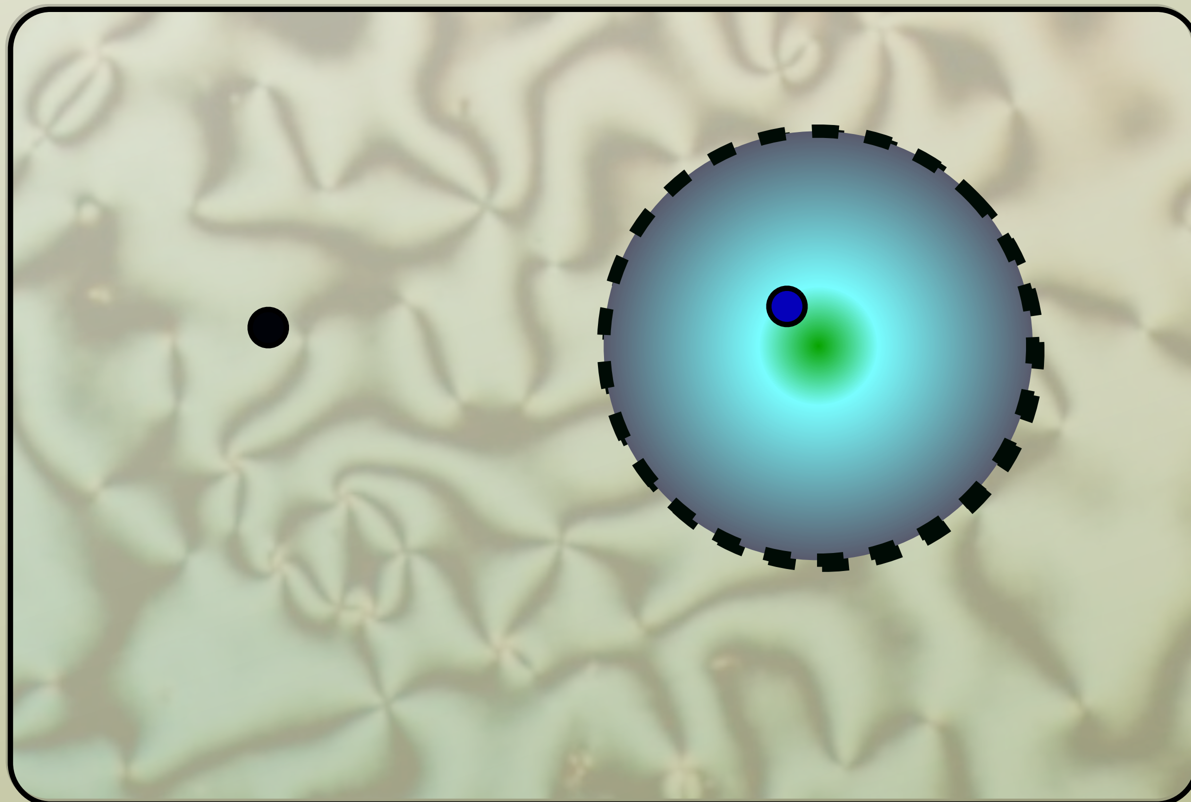

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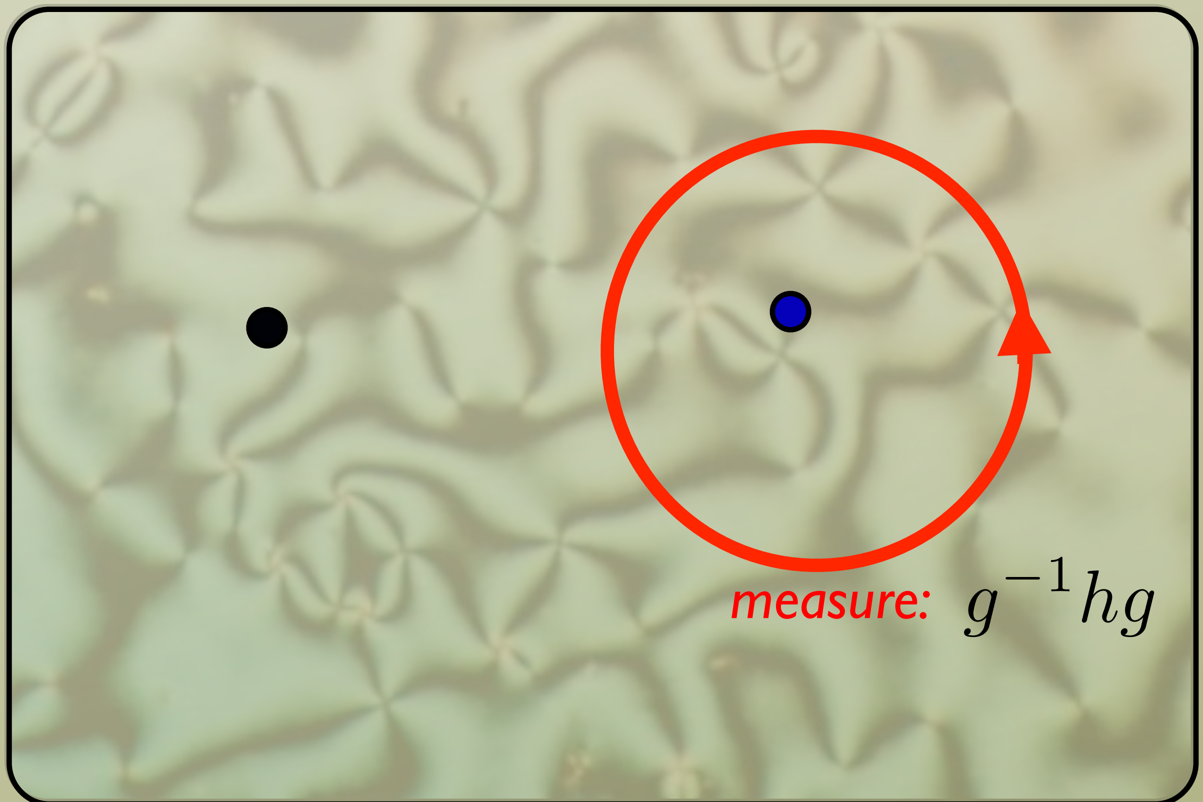

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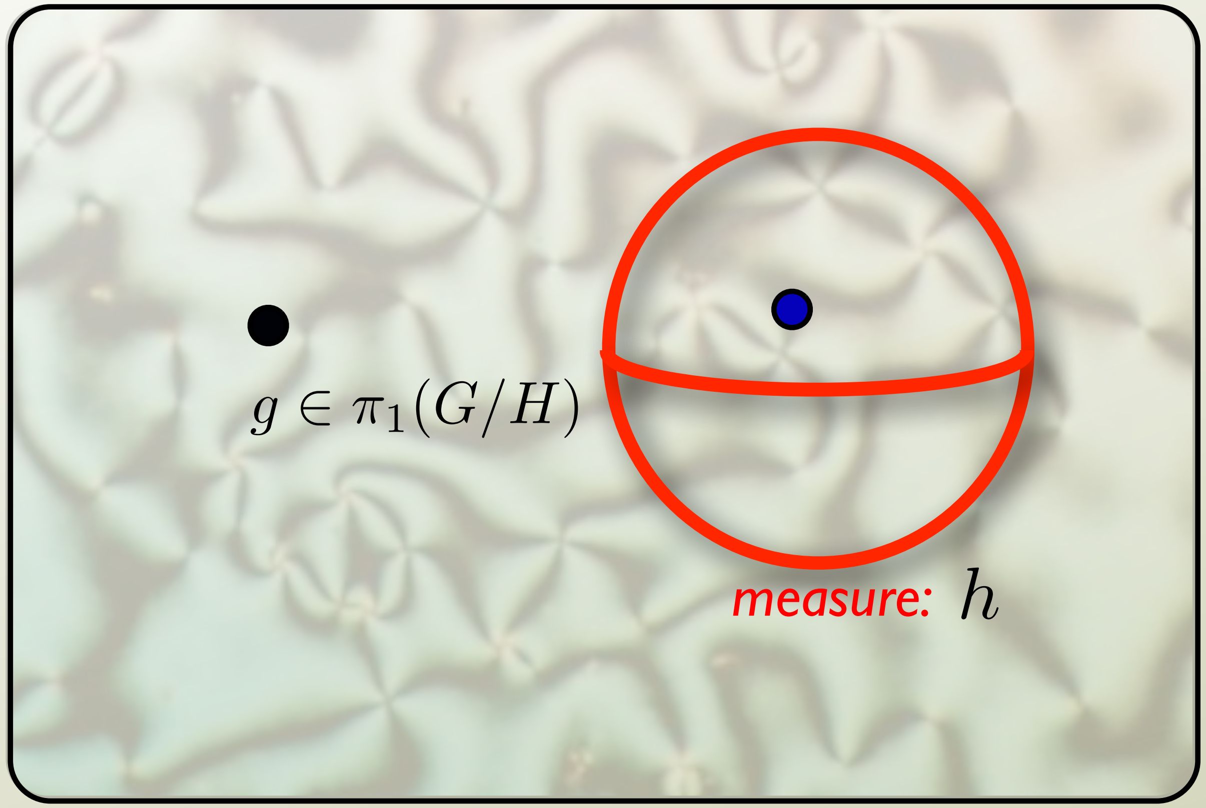

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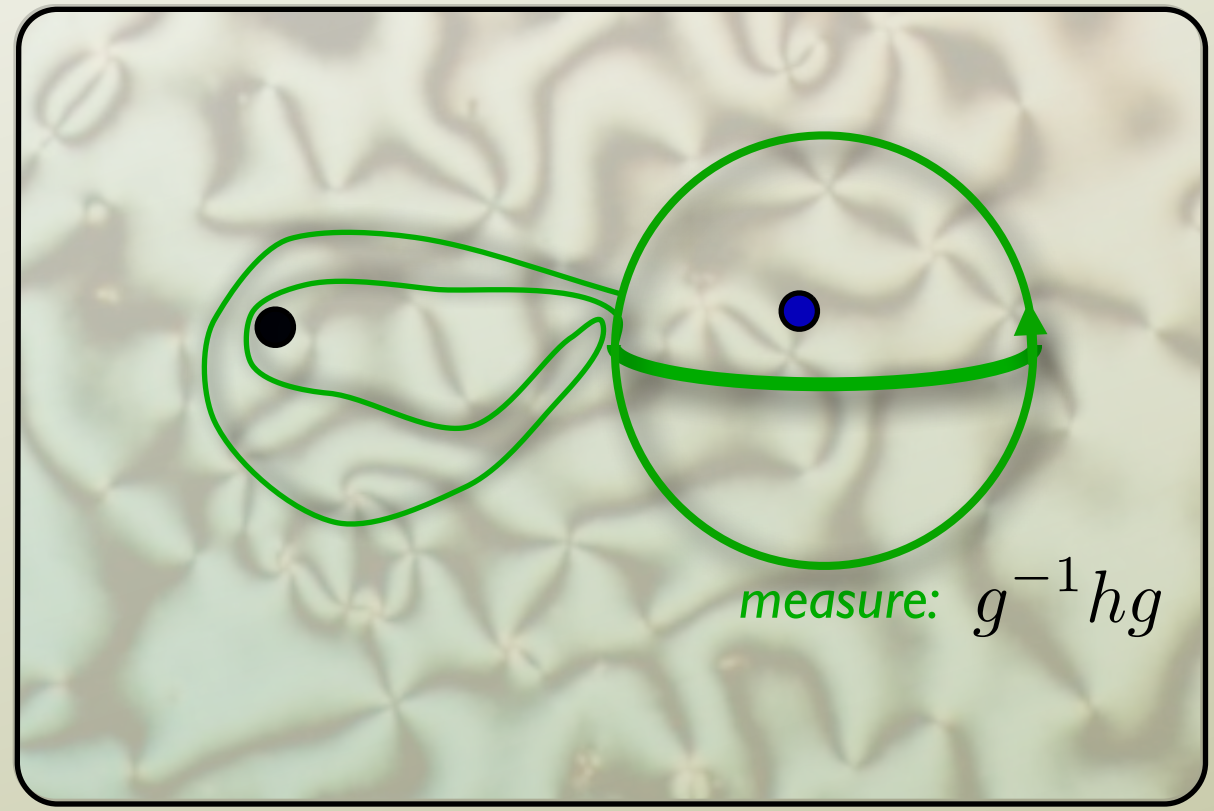

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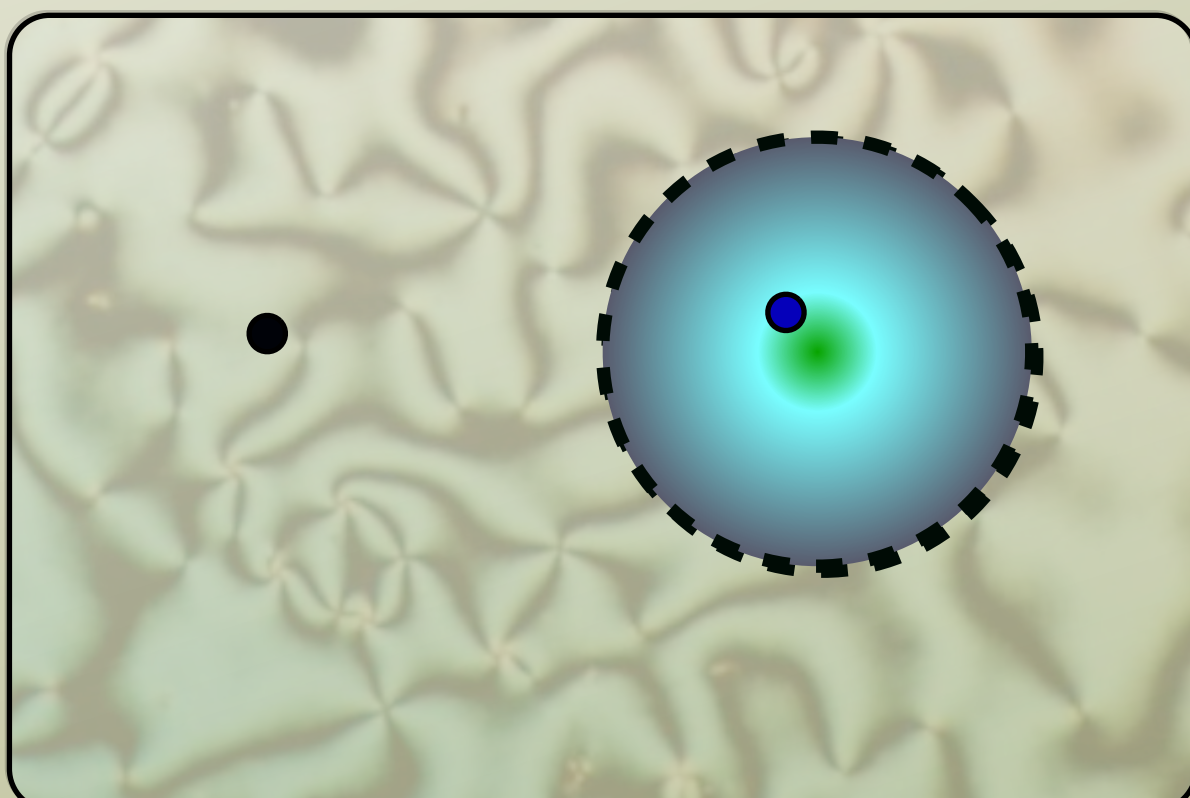

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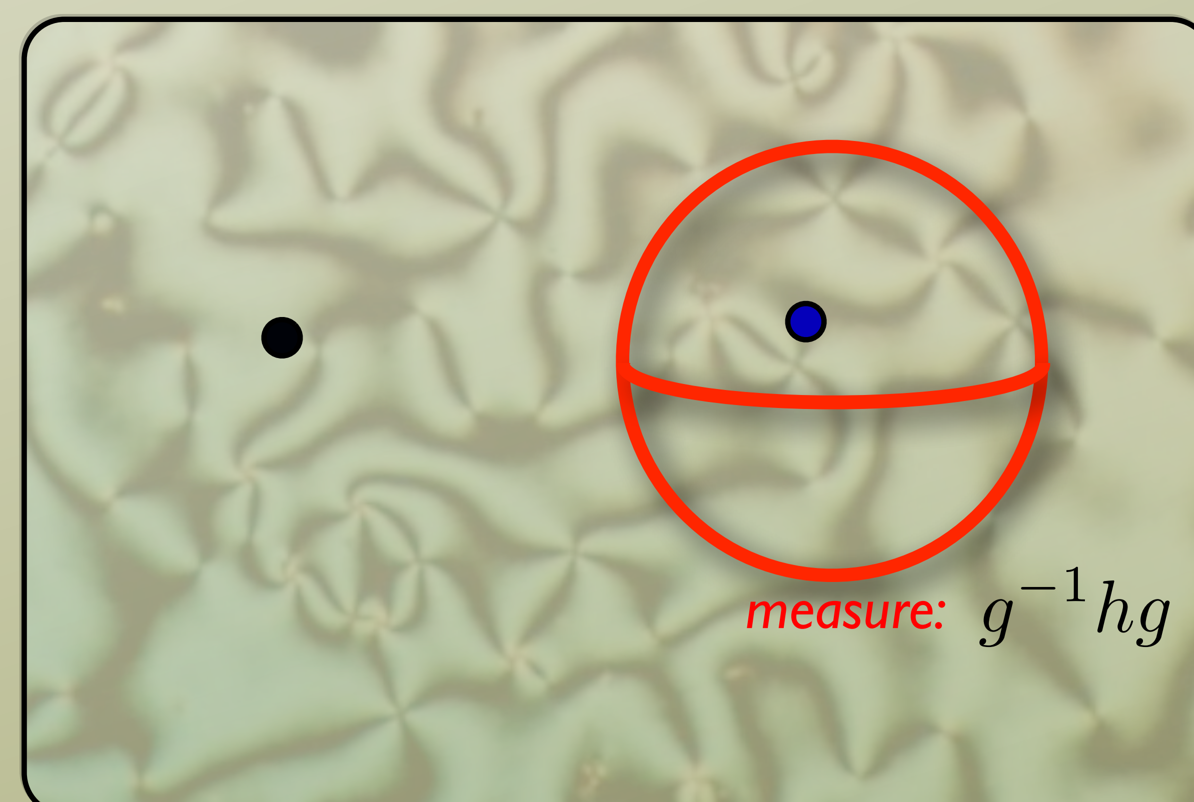
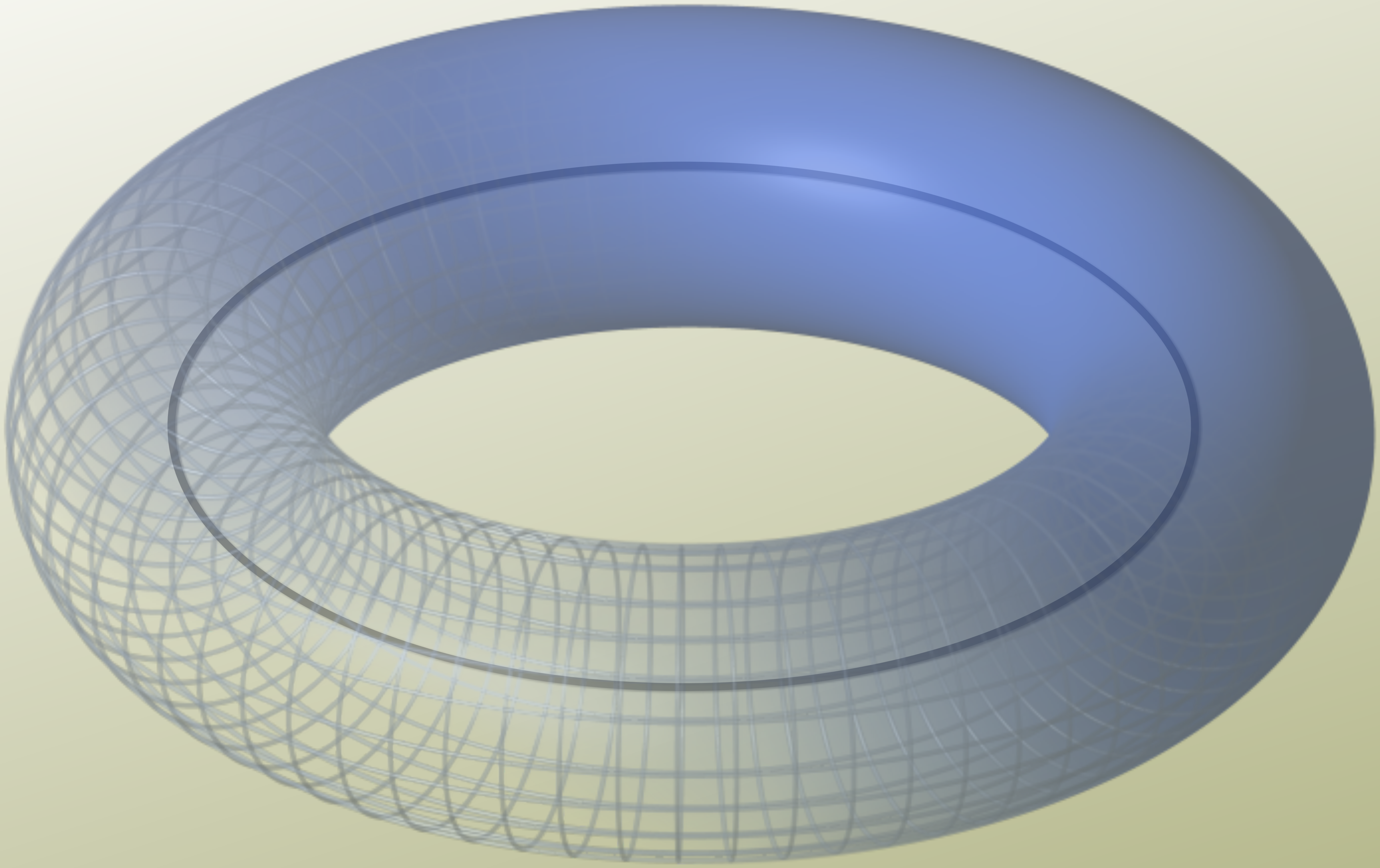


Disclination Loop

http://cis.jhu.edu/education/introPatternTheory/chapters/lie/lie1.html

Disclination Loop



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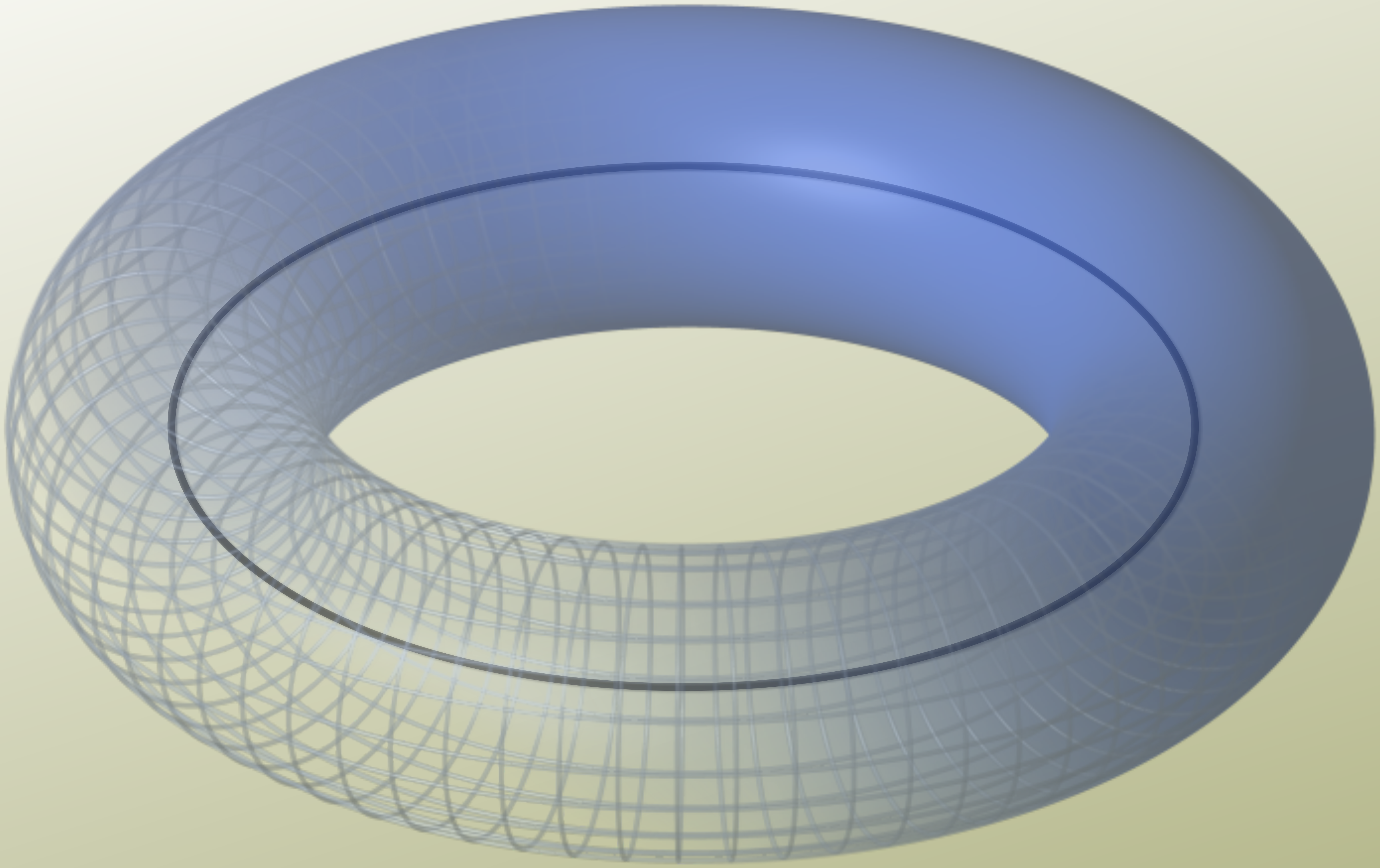
Disclination Loop



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What Information Do We Get From the Texture on The Torus?

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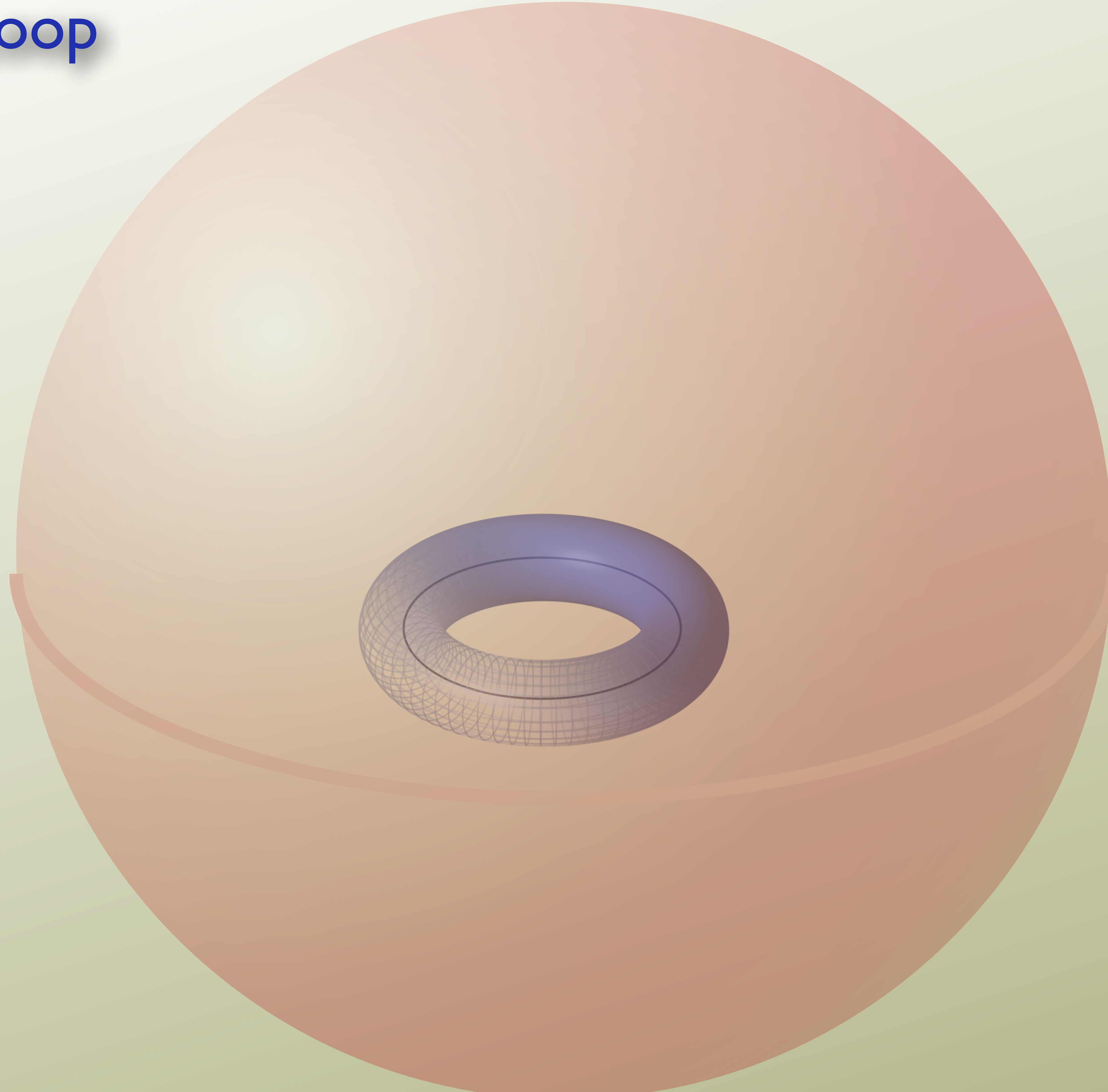
Disclination Loop



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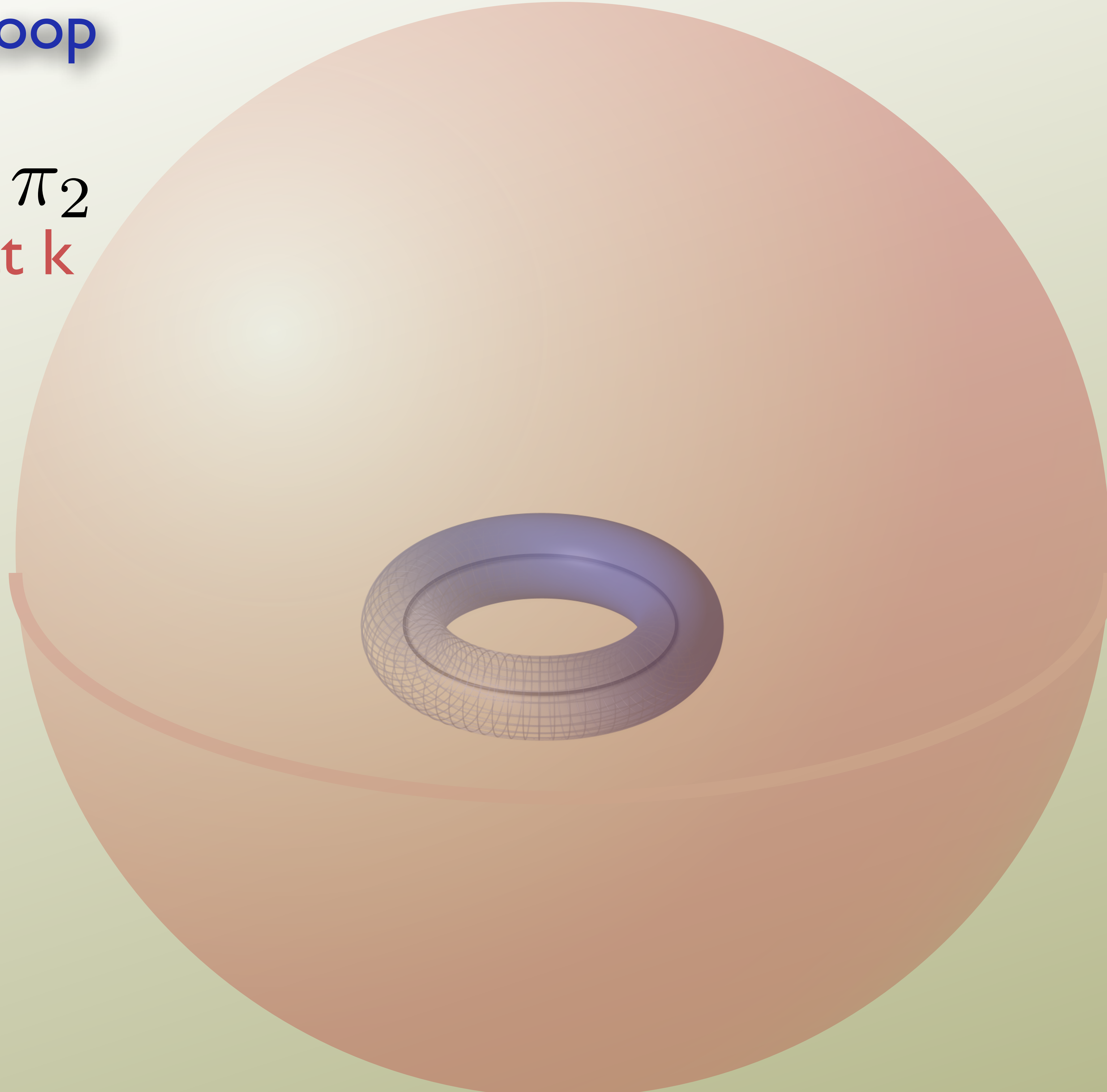
What Information Do We Get From the Texture on The Torus?

Disclination Loop

measure π_2

Charge Fixed at k



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The Torus Gets a Charge!

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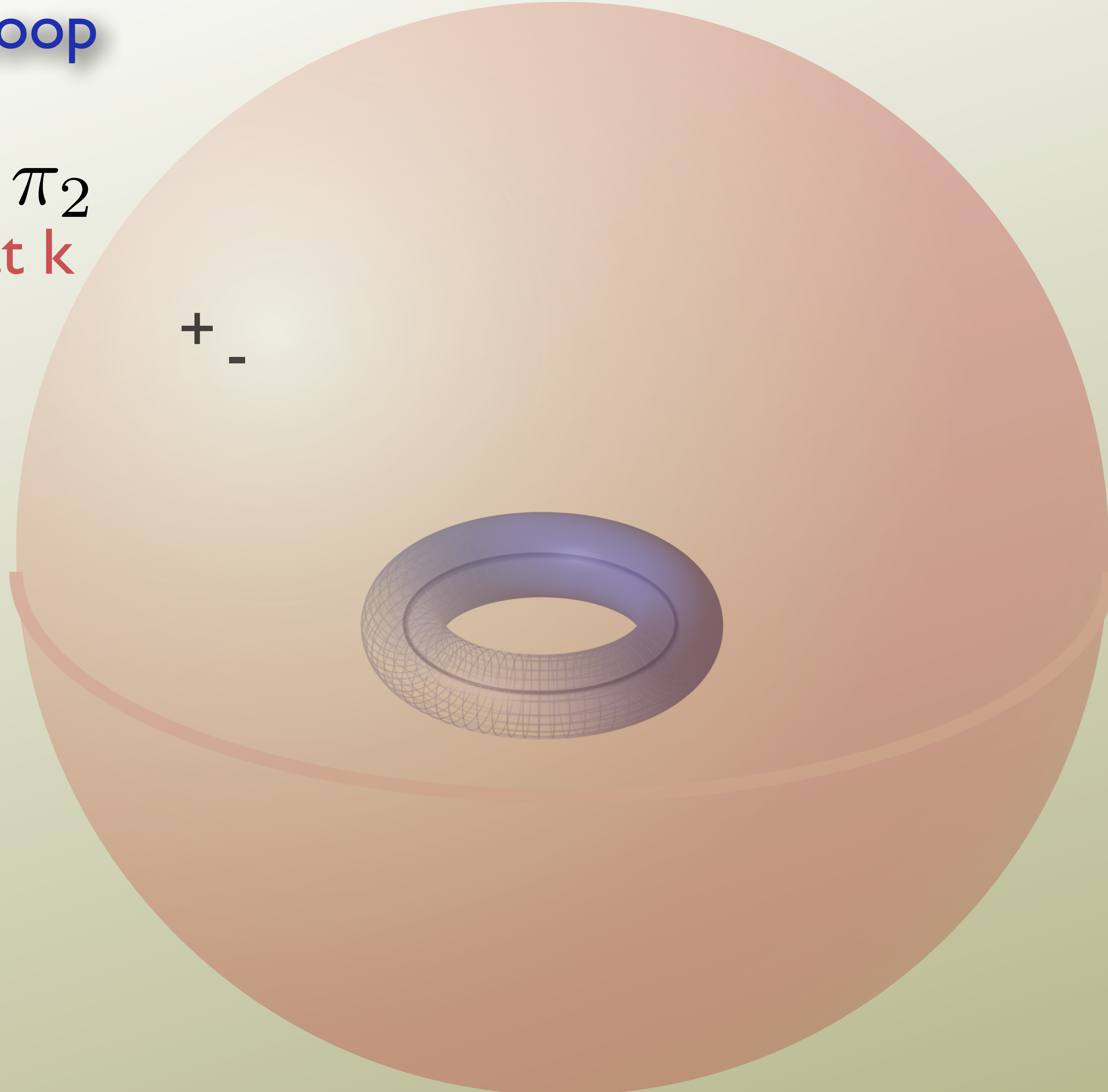
Disclination Loop

measure π_2

Charge Fixed at k



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Disclination Loop

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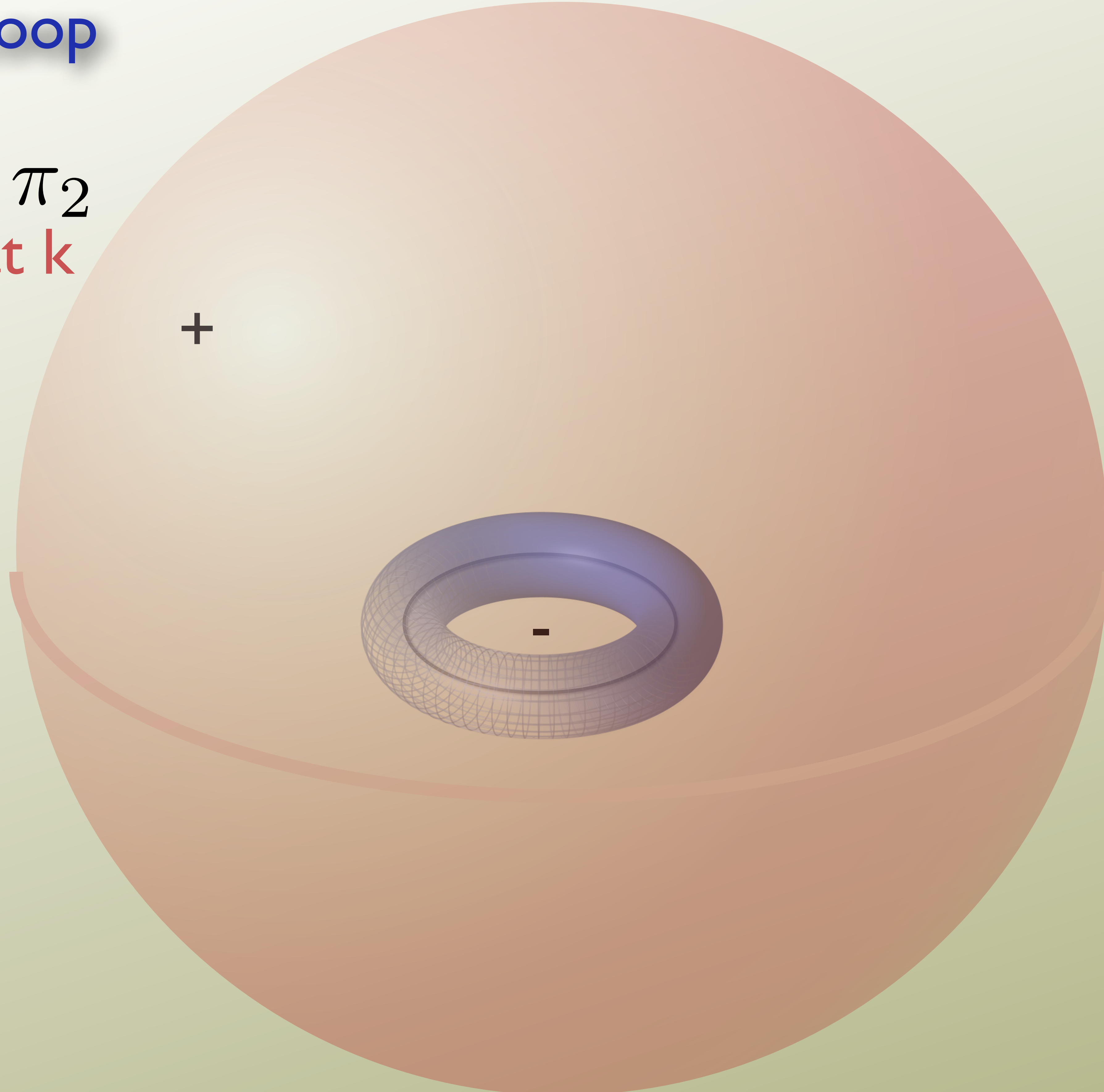
Charge Fixed at k



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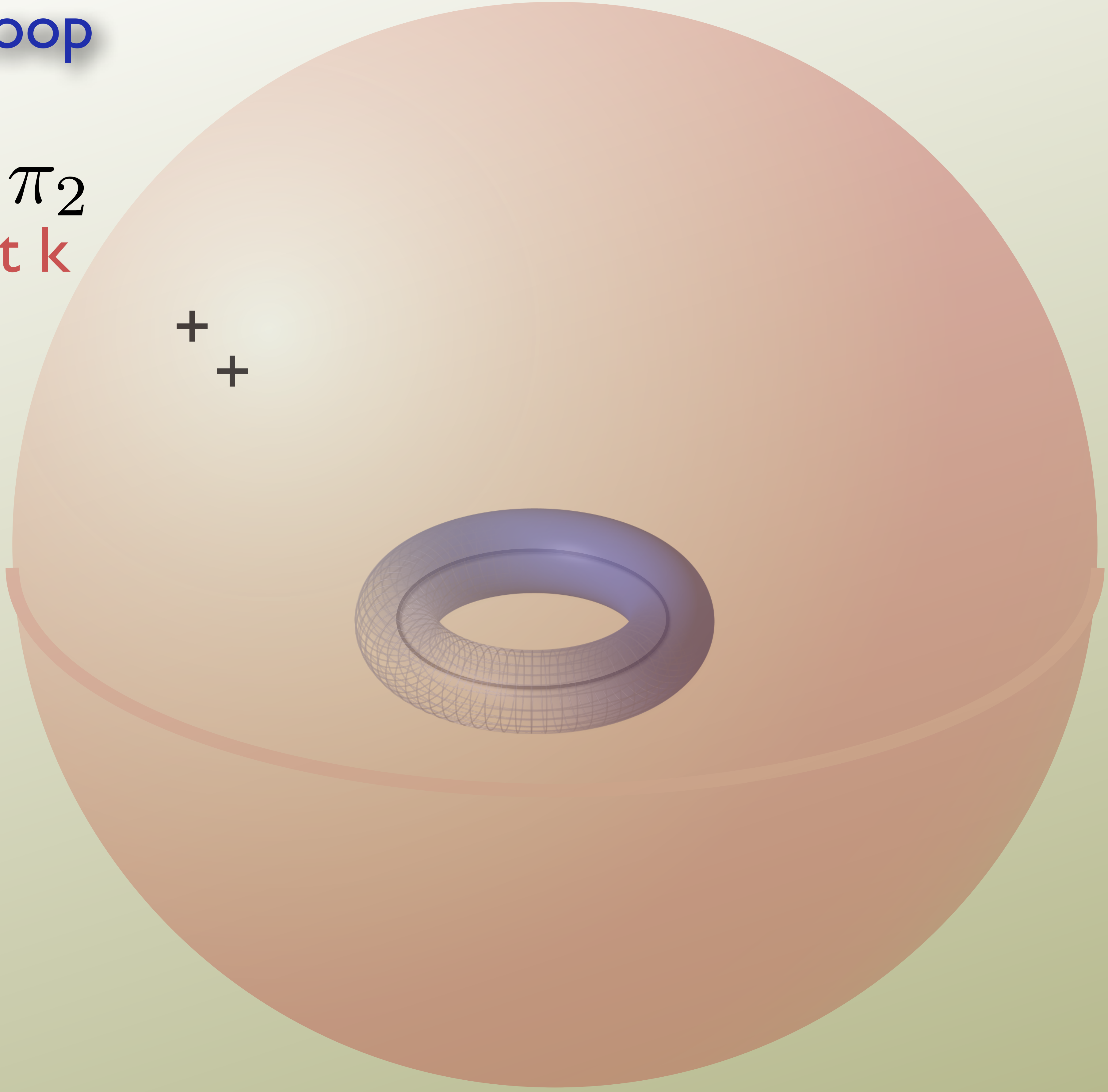

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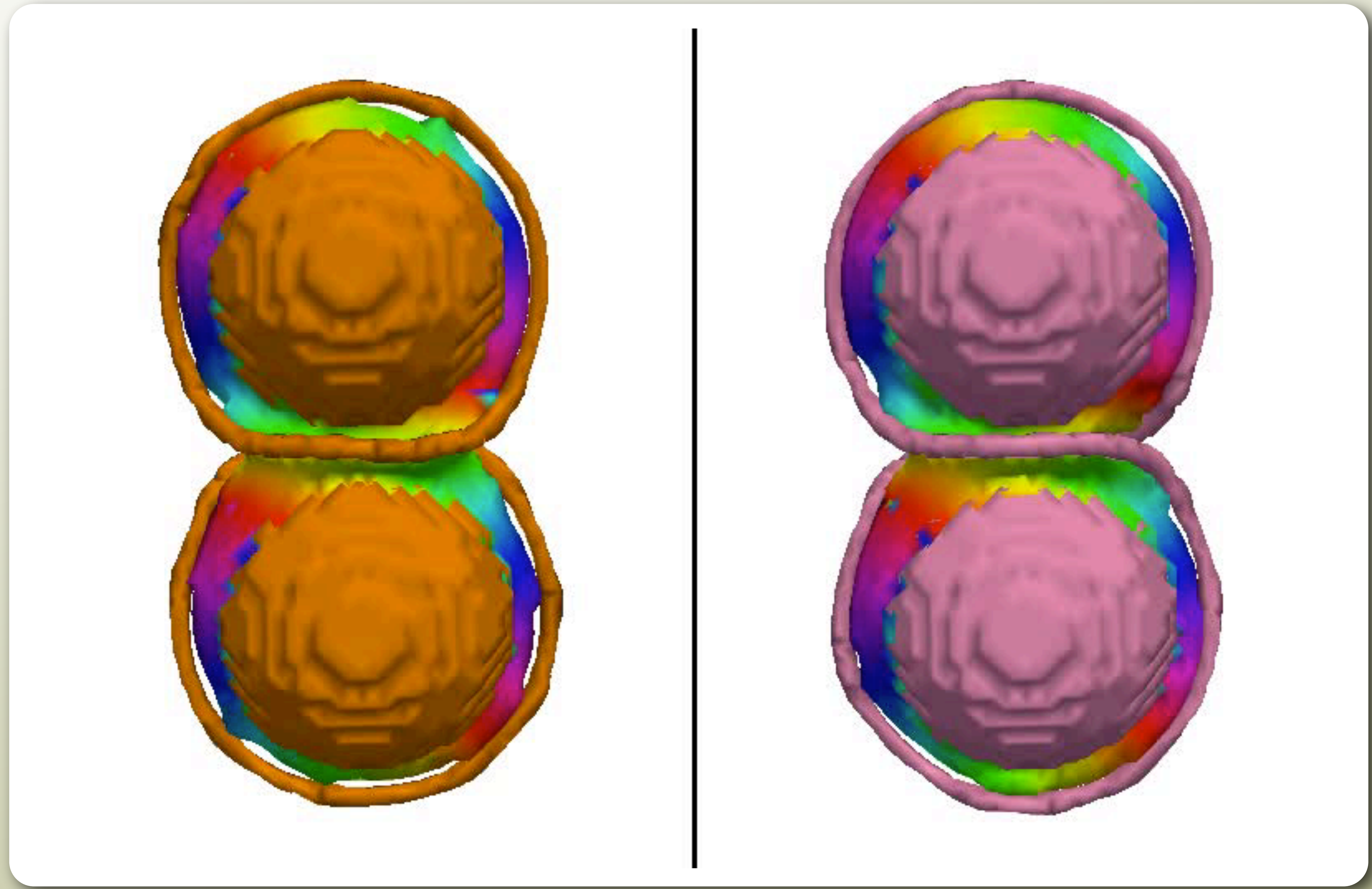
The Torus Gets a Charge!

Charge is Subtle!!



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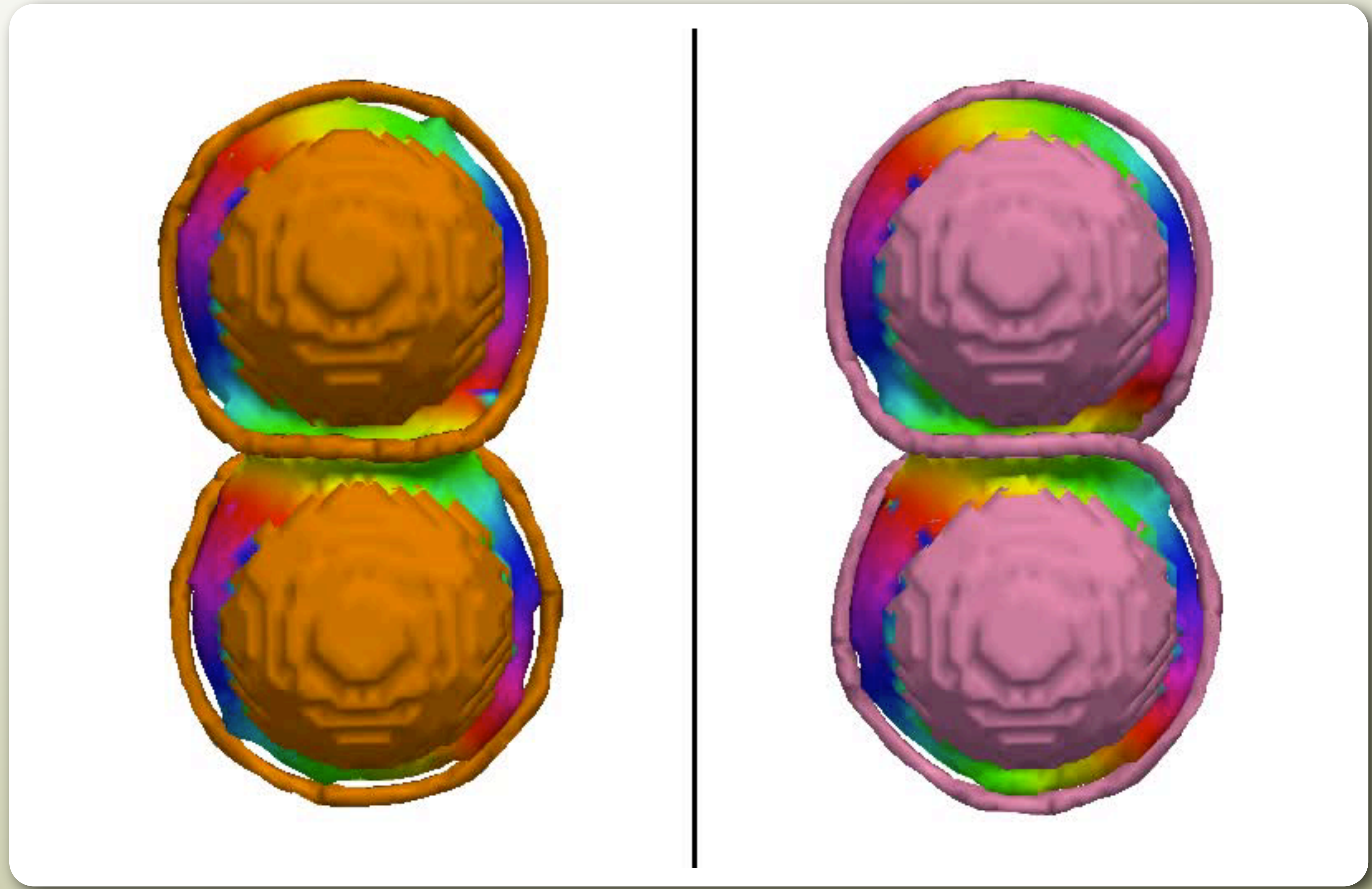

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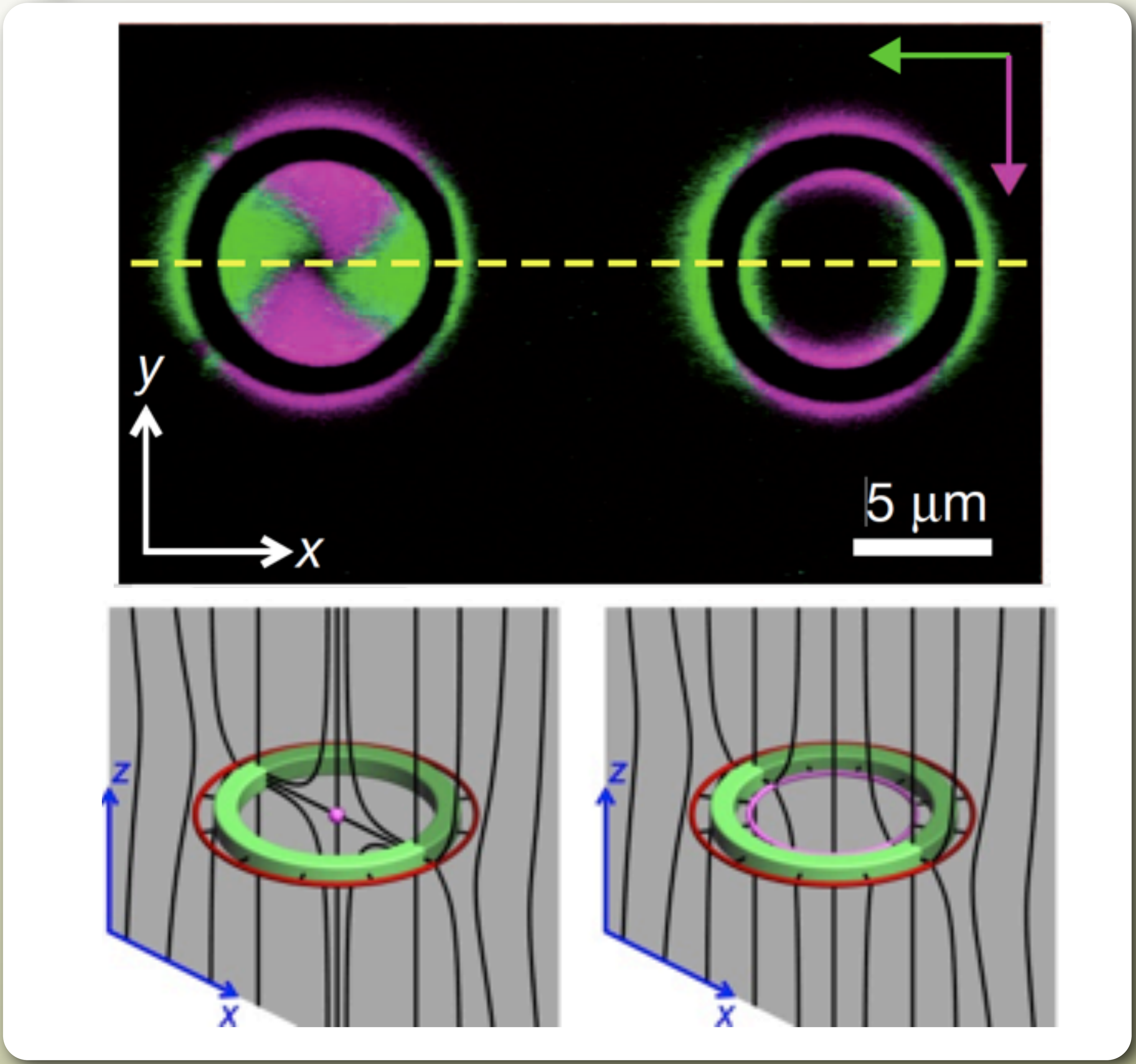
Topological Colloids



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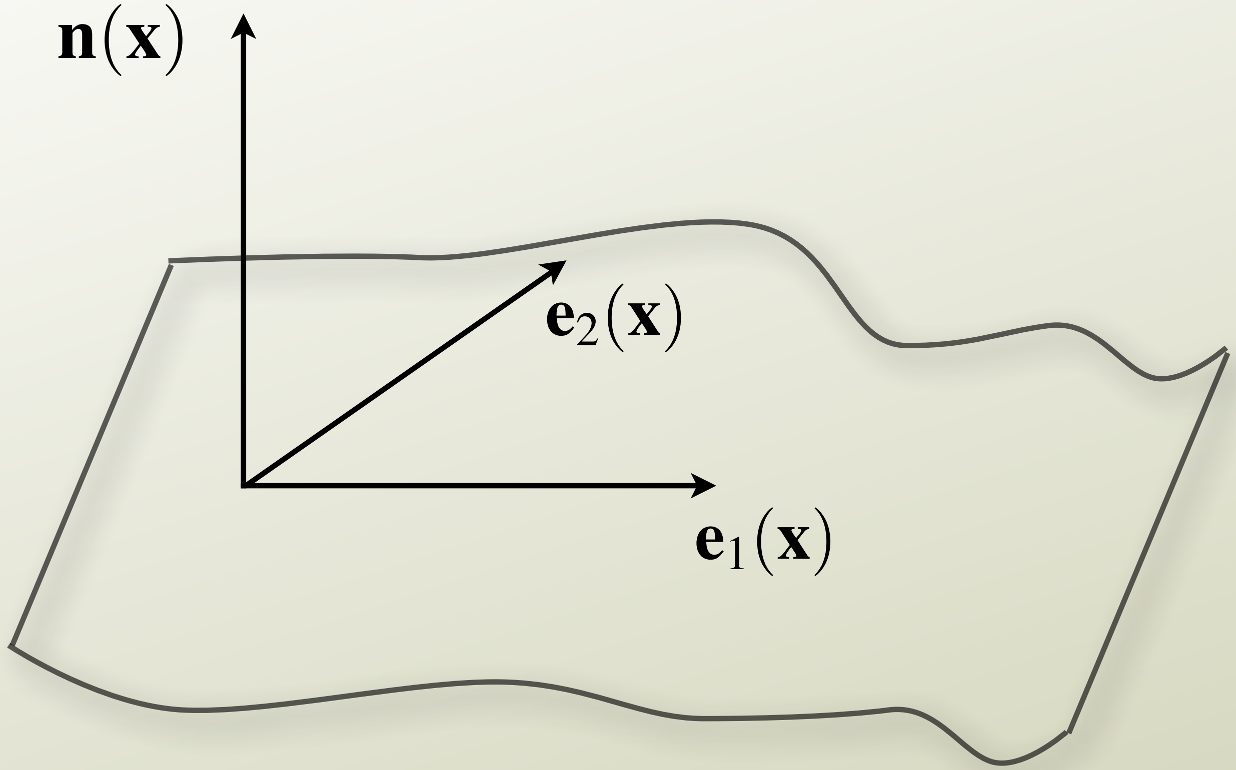
B. Senyuk, Q. Liu, S. He, RDK, R.B. Kusner, T.C. Lubensky, and I.I. Smalyukh, *Nature* ***493*** *(2013) 205.*

The Gauss-Bonnet Theorem



curvature tensor

$$\mathbf{L} = -\begin{bmatrix} \mathbf{e}_1 \cdot [\mathbf{e}_1 \cdot \nabla]\,\mathbf{n} & \mathbf{e}_2 \cdot [\mathbf{e}_1 \cdot \nabla]\,\mathbf{n} \\ \mathbf{e}_1 \cdot [\mathbf{e}_2 \cdot \nabla]\,\mathbf{n} & \mathbf{e}_2 \cdot [\mathbf{e}_2 \cdot \nabla]\,\mathbf{n} \end{bmatrix}$$

$$\mathbf{L} = \mathbf{S}\Lambda\mathbf{S}^T$$

eigenvalues κ_1 *and* κ_2

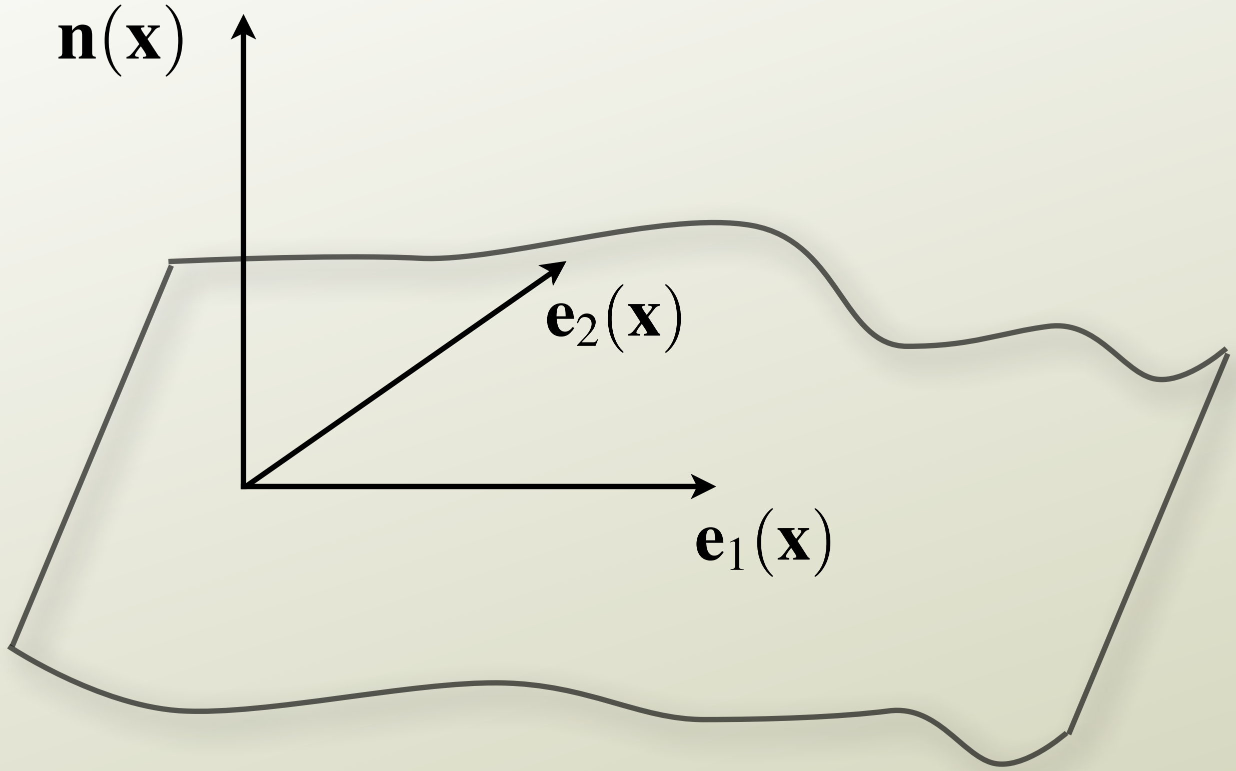
$$\begin{bmatrix} \mathbf{e}_1' \\ \mathbf{e}_2' \end{bmatrix} = \mathbf{S}\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix}$$

principal directions

$$\mathbf{e}_1' \cdot \nabla\mathbf{n} = \kappa_1\mathbf{e}_1'$$

$$\mathbf{e}_2' \cdot \nabla\mathbf{n} = \kappa_2\mathbf{e}_2'$$

The Gauss-Bonnet Theorem



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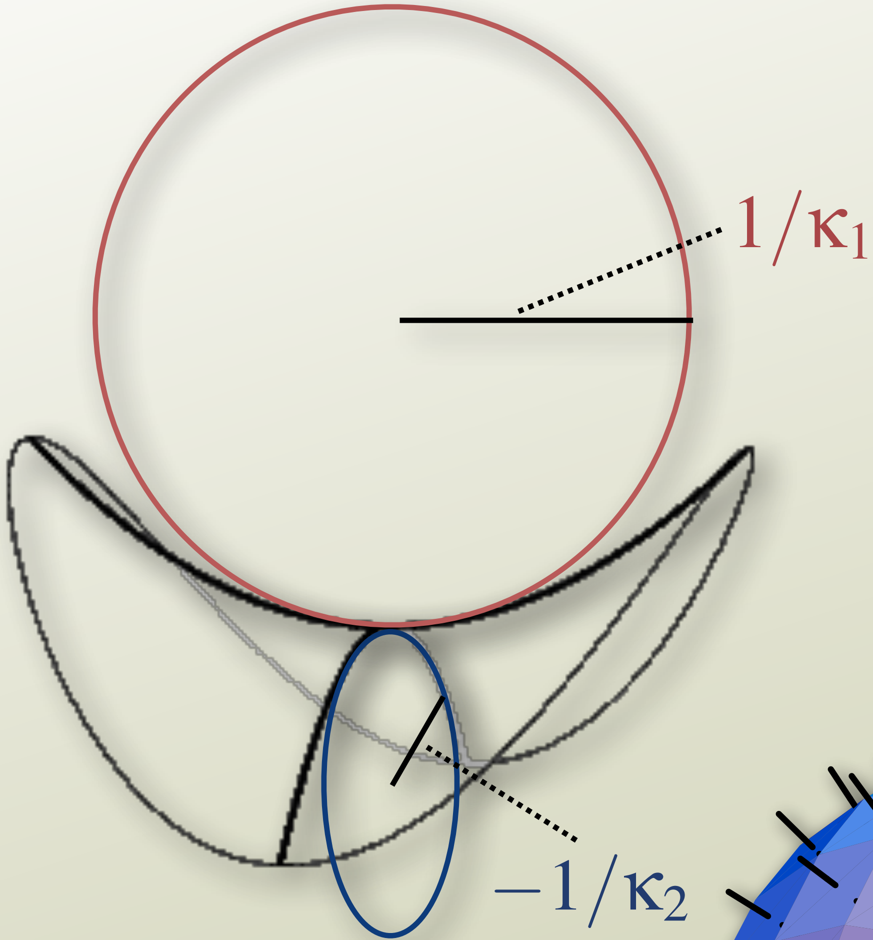
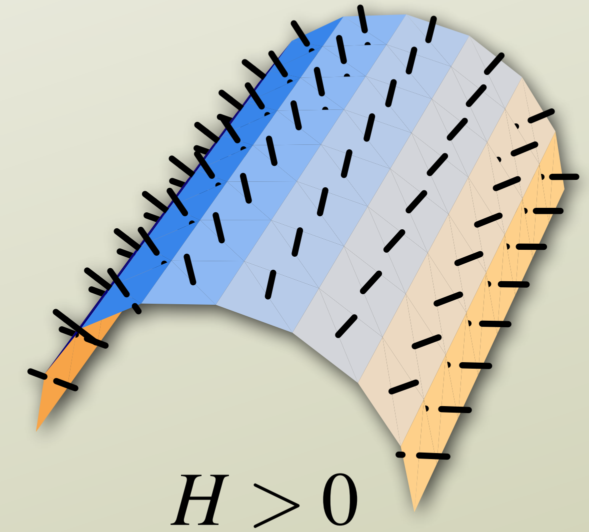
Two Kinds of Curvature



Mean or Extrinsic Curvature

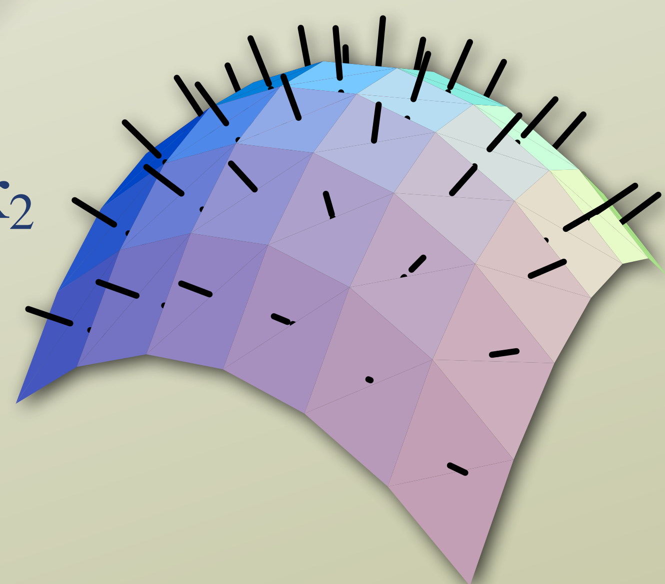
$$H = \frac{1}{2}(\kappa_1 + \kappa_2)$$



$H > 0$
$K = 0$

Gaussian or Intrinsic Curvature

$$K = \kappa_1 \kappa_2$$



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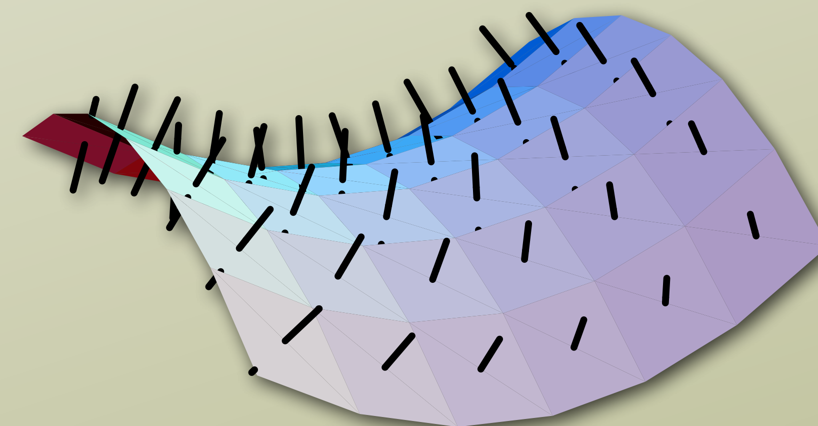
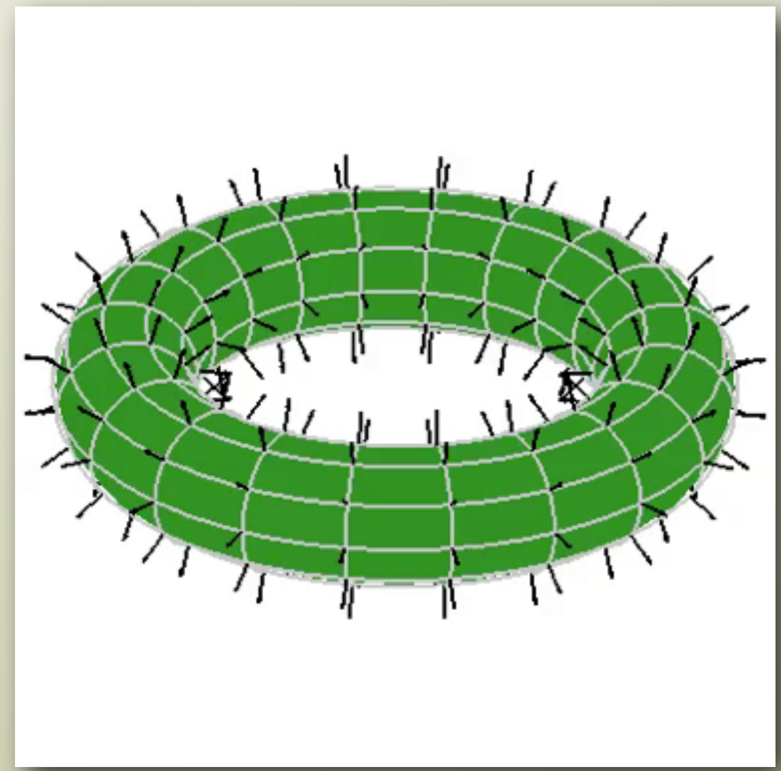


$H = 0$
$K < 0$

Gauss-Bonnet Theorem

$$q = \frac{1}{4\pi} \int dx dy \, \mathbf{n} \cdot [\partial_x \mathbf{n} \times \partial_y \mathbf{n}]$$



http://www.supermath.info/ZooOfMathematicalCreatures.html

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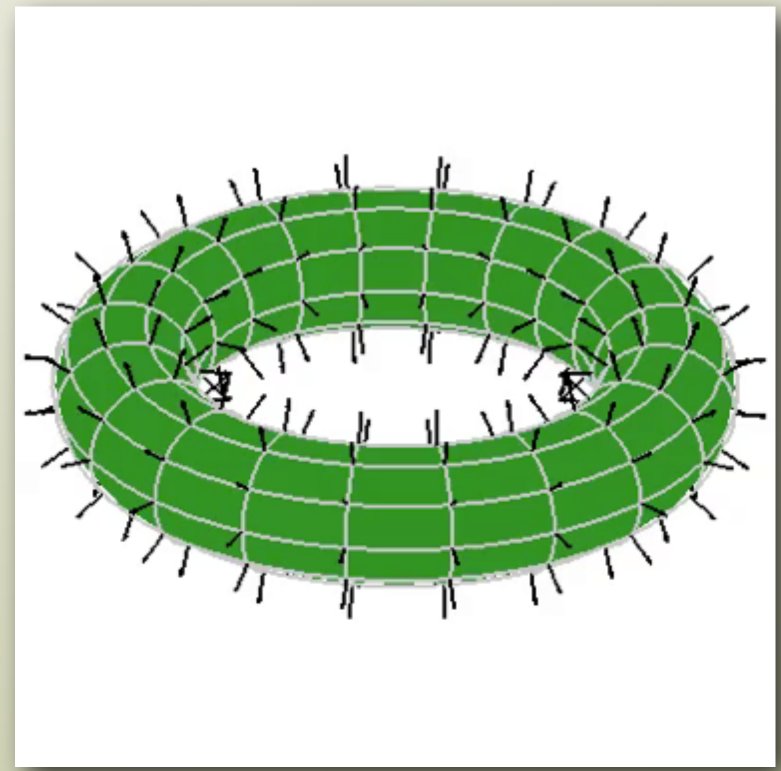

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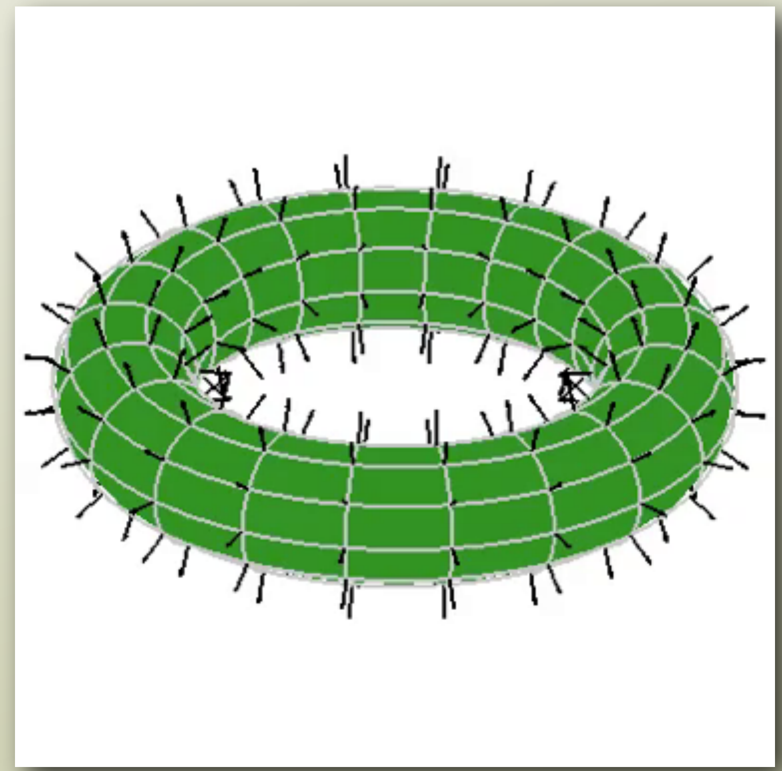

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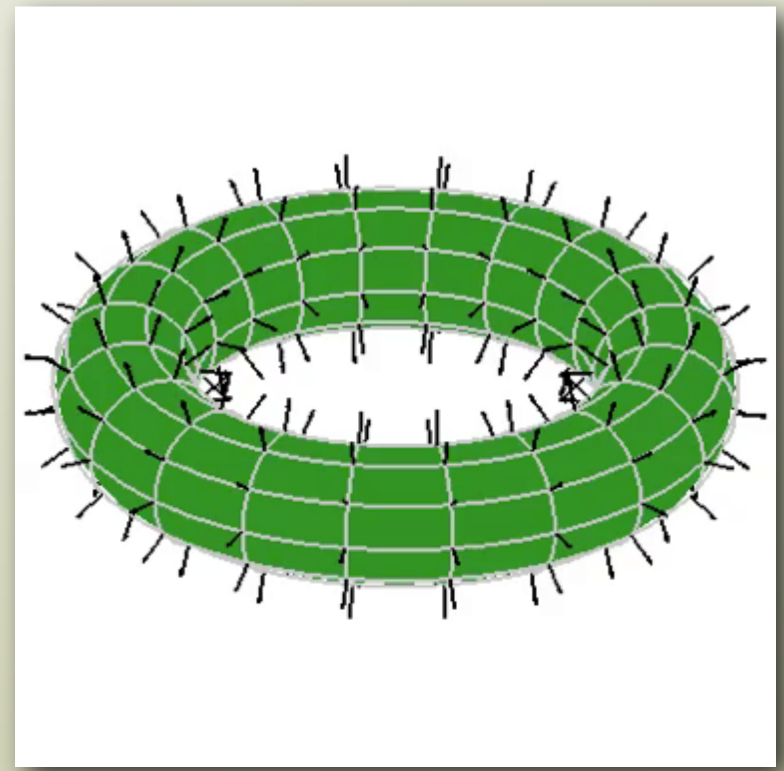
$$q = \frac{1}{4\pi} \int dx_1 dx_2 \, \kappa_1 \kappa_2 = \frac{1}{4\pi} \int dA \, K$$



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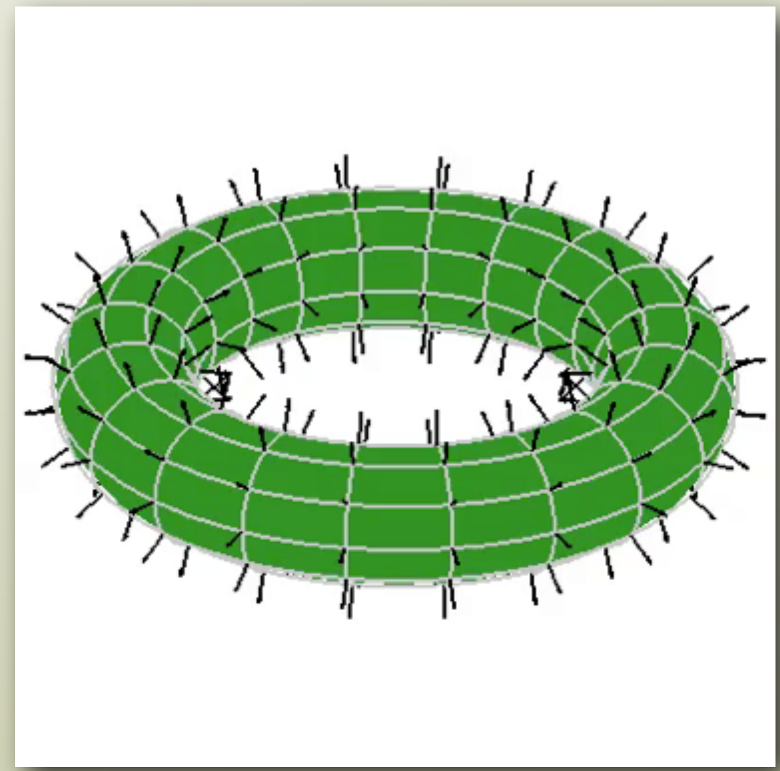

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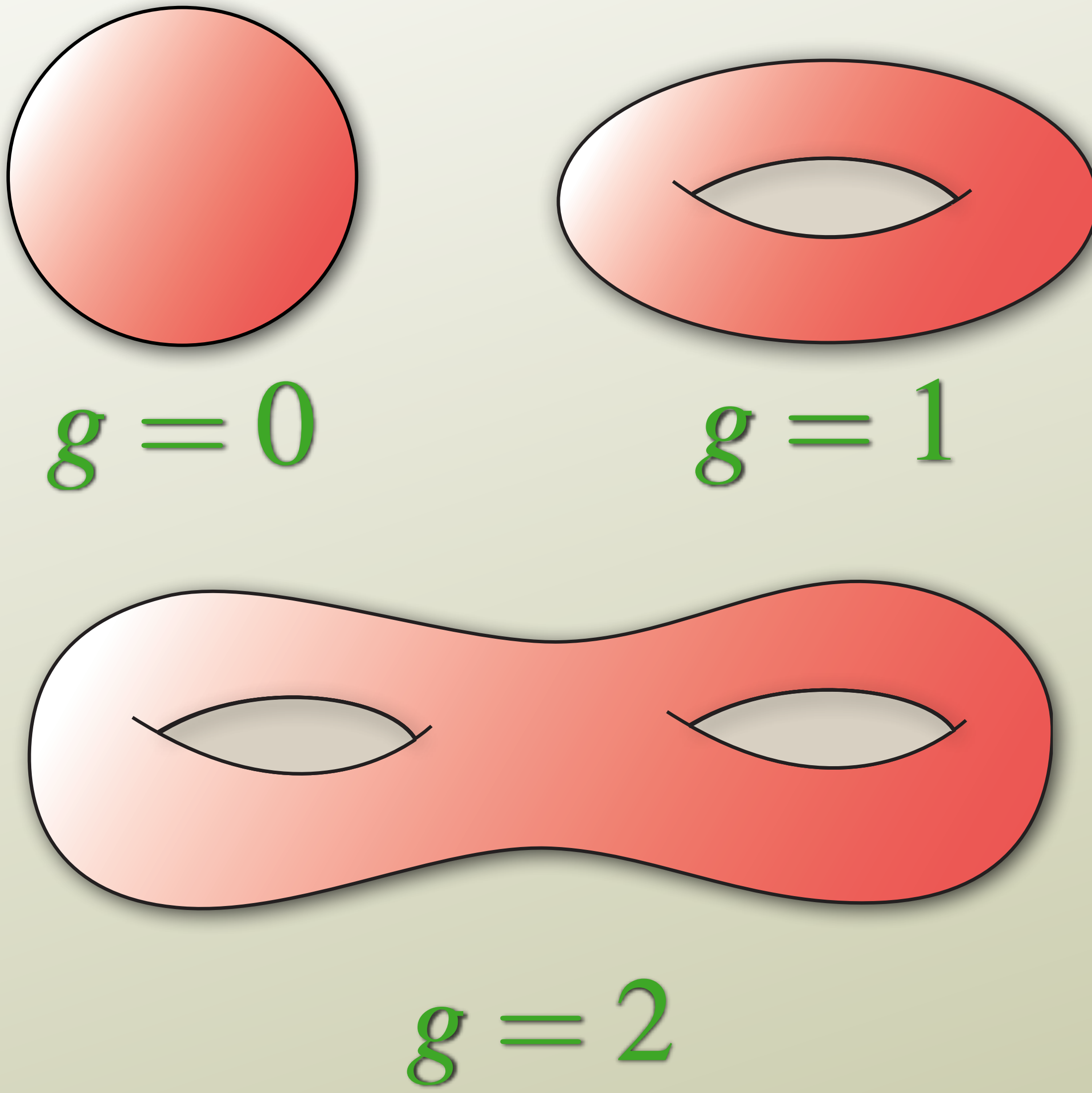
$$\boxed{q = 1 - \text{genus}}$$

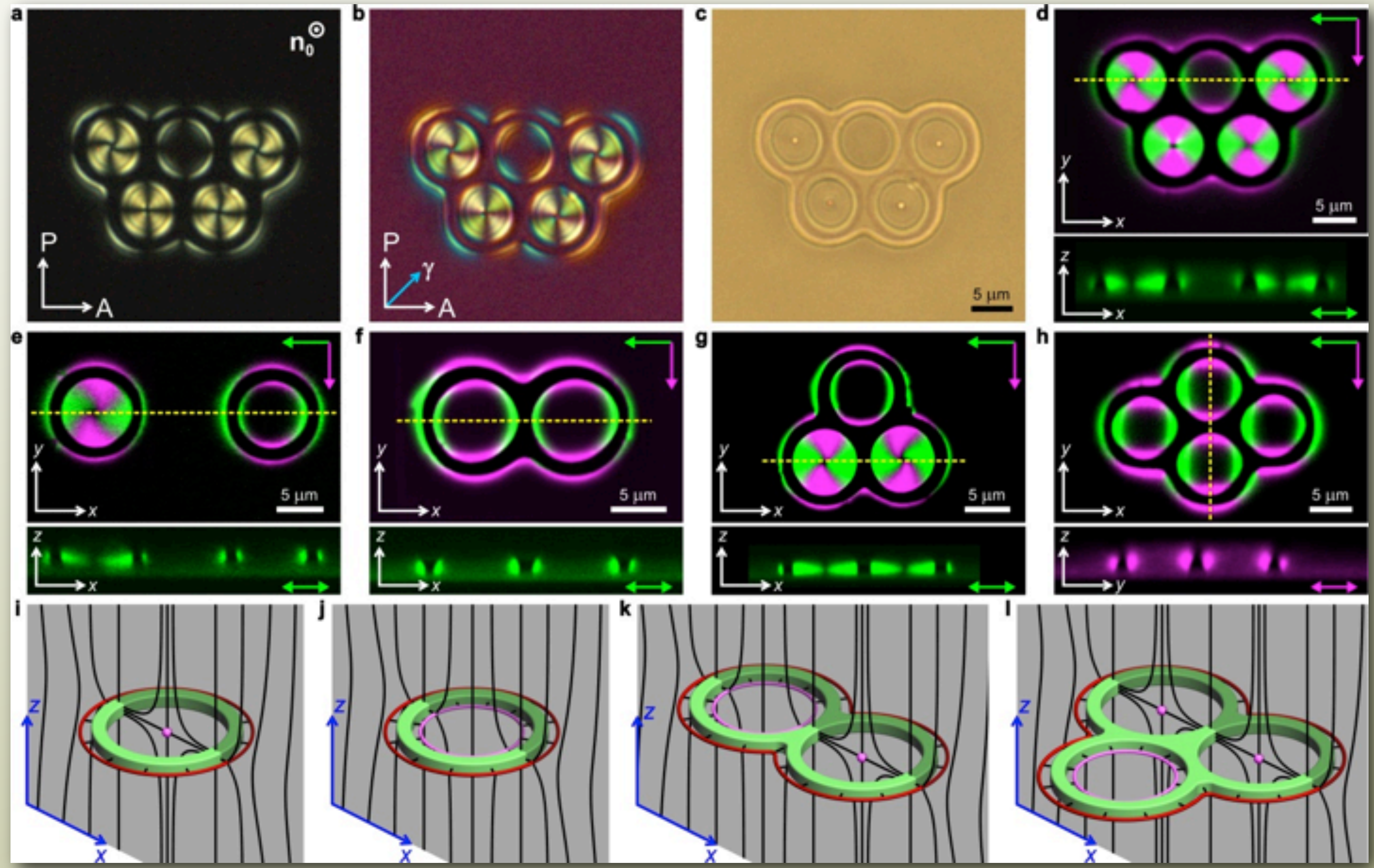
Gauss-Bonnet Theorem



$$\int K dS = 4\pi(1 - g)$$

see, for instance, RDK, Rev. Mod. Phys. 74 (2002) 953

Topological Colloids With Topology!



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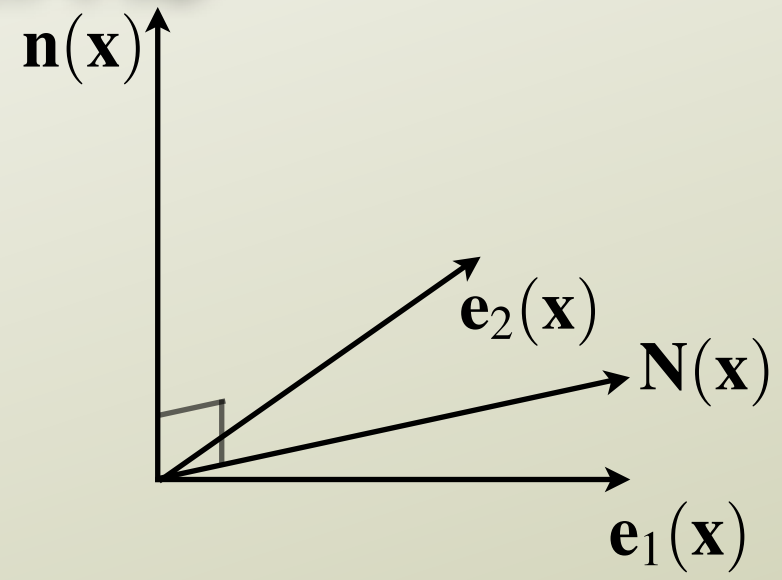
B. Senyuk, Q. Liu, S. He, RDK, R.B. Kusner, T.C. Lubensky, and I.I. Smalyukh, *Nature* ***493*** *(2013) 205.*

Planar Anchoring on a Sphere: Mermin-Ho

$$\mathbf{N}(\mathbf{x}) = \cos[\theta(\mathbf{x})]\mathbf{e}_1(\mathbf{x}) + \sin[\theta(\mathbf{x})]\mathbf{e}_2(\mathbf{x})$$



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N.D. Mermin and T.L. Ho, *Phys. Rev. Lett.* **36** (1976) 594.

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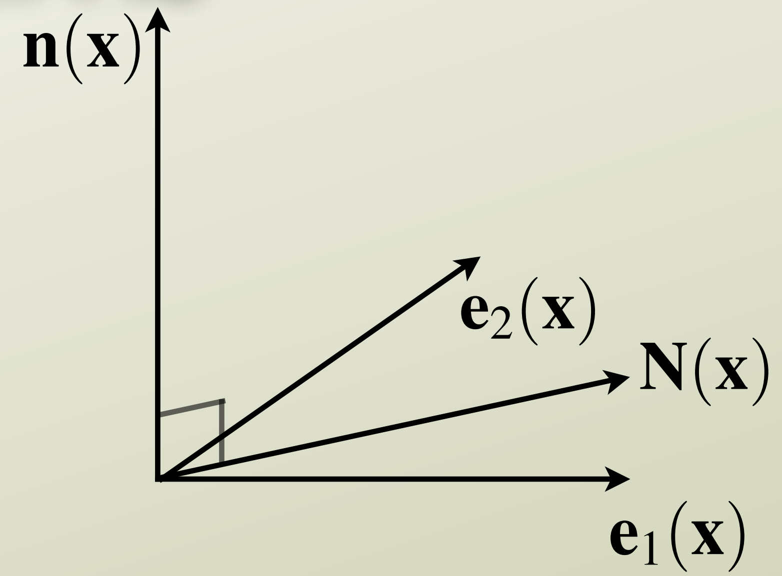

$$\begin{aligned}
\mathbf{e}_1(\mathbf{x}) \cdot \partial_i \mathbf{N}(\mathbf{x}) &= -\sin\theta(\mathbf{x}) \left[\partial_i \theta(\mathbf{x}) - \mathbf{e}_1(\mathbf{x}) \cdot \partial_i \mathbf{e}_2(\mathbf{x})\right] \\
\mathbf{e}_2(\mathbf{x}) \cdot \partial_i \mathbf{N}(\mathbf{x}) &= \cos\theta(\mathbf{x}) \left[\partial_i \theta(\mathbf{x}) + \mathbf{e}_2(\mathbf{x}) \cdot \partial_i \mathbf{e}_1(\mathbf{x})\right] \\
&= \cos\theta(\mathbf{x}) \left[\partial_i \theta(\mathbf{x}) - \mathbf{e}_1(\mathbf{x}) \cdot \partial_i \mathbf{e}_2(\mathbf{x})\right]
\end{aligned}$$

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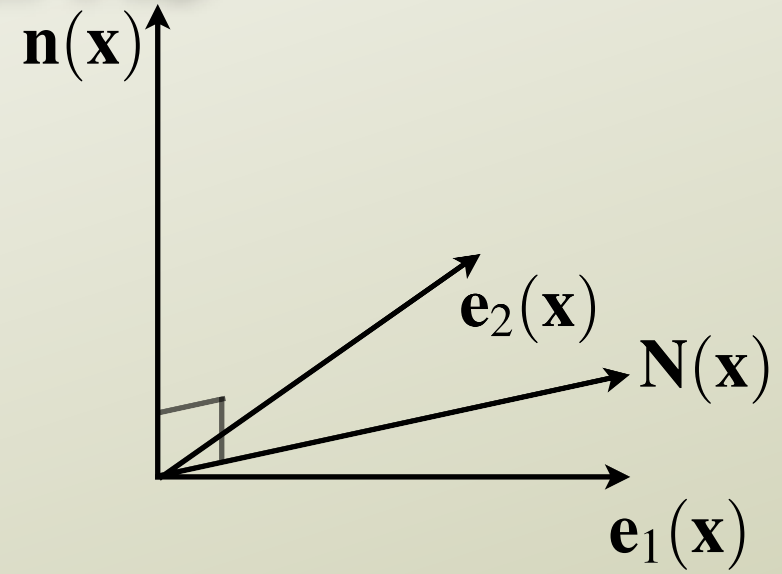
Planar Anchoring on a Sphere: Mermin-Ho



$$\mathbf{N}(\mathbf{x}) = \cos[\theta(\mathbf{x})]\mathbf{e}_1(\mathbf{x}) + \sin[\theta(\mathbf{x})]\mathbf{e}_2(\mathbf{x})$$

$$\begin{aligned}
\mathbf{e}_1(\mathbf{x}) \cdot \partial_i \mathbf{N}(\mathbf{x}) &= -\sin\theta(\mathbf{x})\,[\partial_i\theta(\mathbf{x}) - \mathbf{e}_1(\mathbf{x})\cdot\partial_i\mathbf{e}_2(\mathbf{x})] \\
\mathbf{e}_2(\mathbf{x}) \cdot \partial_i \mathbf{N}(\mathbf{x}) &= \cos\theta(\mathbf{x})\,[\partial_i\theta(\mathbf{x}) + \mathbf{e}_2(\mathbf{x})\cdot\partial_i\mathbf{e}_1(\mathbf{x})] \\
&= \cos\theta(\mathbf{x})\,[\partial_i\theta(\mathbf{x}) - \mathbf{e}_1(\mathbf{x})\cdot\partial_i\mathbf{e}_2(\mathbf{x})]
\end{aligned}$$

$$\nabla\theta_0(\mathbf{x}) = \mathbf{e}_1(\mathbf{x}) \cdot \nabla\mathbf{e}_2(\mathbf{x}) \equiv \Omega(\mathbf{x})$$

$$\begin{aligned}
[\nabla \times \Omega(\mathbf{x})]_i &= \varepsilon_{ijk}\partial_j\,[e_1^\alpha(\mathbf{x})\partial_k e_2^\alpha(\mathbf{x})] \\
&= \varepsilon_{ijk}\,[\partial_j e_1^\alpha(\mathbf{x})]\,[\partial_k e_2^\alpha(\mathbf{x})] + e_1^\alpha(\mathbf{x})\varepsilon_{ijk}\partial_j\partial_k e_2^\alpha(\mathbf{x}) \\
&= \varepsilon_{ijk}\,[\partial_j e_1^\alpha(\mathbf{x})]\,[\partial_k e_2^\alpha(\mathbf{x})] + 0
\end{aligned}$$

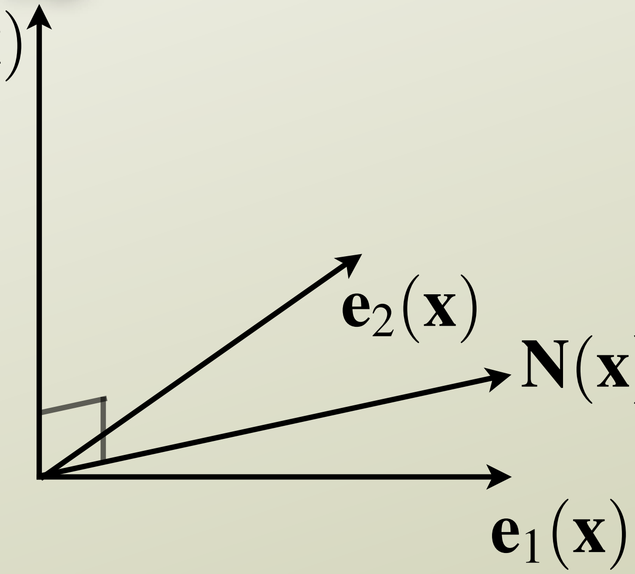
$$\begin{aligned}
\partial_j e_1^\alpha(\mathbf{x}) &= A_j(\mathbf{x}) n^\alpha(\mathbf{x}) + B_j(\mathbf{x}) e_2^\alpha(\mathbf{x}) \qquad & A_j(\mathbf{x}) = -\mathbf{e}_1(\mathbf{x})\cdot\partial_j\mathbf{n}(\mathbf{x}) \\
\partial_k e_2^\alpha(\mathbf{x}) &= C_k(\mathbf{x}) n^\alpha(\mathbf{x}) + D_k(\mathbf{x}) e_1^\alpha(\mathbf{x}) \qquad & C_k(\mathbf{x}) = -\mathbf{e}_2(\mathbf{x})\cdot\partial_j\mathbf{n}(\mathbf{x})
\end{aligned}$$

N.D. Mermin and T.L. Ho, *Phys. Rev. Lett.* **36** (1976) 594.

Planar Anchoring on a Sphere: Mermin-Ho

$$\mathbf{N}(\mathbf{x}) = \cos[\theta(\mathbf{x})]\mathbf{e}_1(\mathbf{x}) + \sin[\theta(\mathbf{x})]\mathbf{e}_2(\mathbf{x})$$

$$\begin{aligned}
\mathbf{e}_1(\mathbf{x})\cdot\partial_i\mathbf{N}(\mathbf{x}) &= -\sin\theta(\mathbf{x})\left[\partial_i\theta(\mathbf{x}) - \mathbf{e}_1(\mathbf{x})\cdot\partial_i\mathbf{e}_2(\mathbf{x})\right]\\
\mathbf{e}_2(\mathbf{x})\cdot\partial_i\mathbf{N}(\mathbf{x}) &= \cos\theta(\mathbf{x})\left[\partial_i\theta(\mathbf{x}) + \mathbf{e}_2(\mathbf{x})\cdot\partial_i\mathbf{e}_1(\mathbf{x})\right]\\
&= \cos\theta(\mathbf{x})\left[\partial_i\theta(\mathbf{x}) - \mathbf{e}_1(\mathbf{x})\cdot\partial_i\mathbf{e}_2(\mathbf{x})\right]
\end{aligned}$$



$$\nabla\theta_0(\mathbf{x}) = \mathbf{e}_1(\mathbf{x})\cdot\nabla\mathbf{e}_2(\mathbf{x}) \equiv \Omega(\mathbf{x})$$

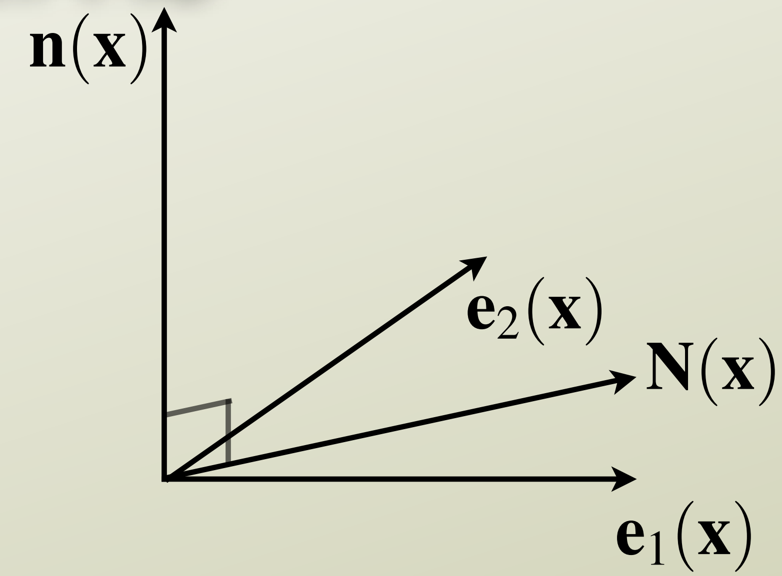
$$\begin{aligned}
[\nabla\times\Omega(\mathbf{x})]_i &= \varepsilon_{ijk}\partial_j\left[e_1^\alpha(\mathbf{x})\partial_k e_2^\alpha(\mathbf{x})\right]\\
&= \varepsilon_{ijk}\left[\partial_j e_1^\alpha(\mathbf{x})\right]\left[\partial_k e_2^\alpha(\mathbf{x})\right] + e_1^\alpha(\mathbf{x})\varepsilon_{ijk}\partial_j\partial_k e_2^\alpha(\mathbf{x})\\
&= \varepsilon_{ijk}\left[\partial_j e_1^\alpha(\mathbf{x})\right]\left[\partial_k e_2^\alpha(\mathbf{x})\right] + 0
\end{aligned}$$

$$\begin{aligned}
\partial_j e_1^\alpha(\mathbf{x}) &= A_j(\mathbf{x})n^\alpha(\mathbf{x}) + B_j(\mathbf{x})e_2^\alpha(\mathbf{x}) \qquad & A_j(\mathbf{x}) = -\mathbf{e}_1(\mathbf{x})\cdot\partial_j\mathbf{n}(\mathbf{x})\\
\partial_k e_2^\alpha(\mathbf{x}) &= C_k(\mathbf{x})n^\alpha(\mathbf{x}) + D_k(\mathbf{x})e_1^\alpha(\mathbf{x}) \qquad & C_k(\mathbf{x}) = -\mathbf{e}_2(\mathbf{x})\cdot\partial_j\mathbf{n}(\mathbf{x})
\end{aligned}$$

$$\begin{aligned}
[\nabla\times\Omega(\mathbf{x})]_i &= \frac{1}{2}\left[e_1^\beta(\mathbf{x})e_2^\gamma(\mathbf{x}) - e_1^\gamma(\mathbf{x})e_2^\beta(\mathbf{x})\right]\varepsilon_{ijk}\left[\partial_j n^\beta(\mathbf{x})\right]\left[\partial_k n^\gamma(\mathbf{x})\right]\\
&= \frac{1}{2}\varepsilon_{\alpha\beta\gamma}n^\alpha(\mathbf{x})\varepsilon_{ijk}\partial_j n^\beta(\mathbf{x})\partial_k n^\gamma(\mathbf{x})
\end{aligned}$$

N.D. Mermin and T.L. Ho, *Phys. Rev. Lett.* **36** (1976) 594.

Planar Anchoring on a Sphere: Mermin-Ho



$$\#\ \text{of Defects} = \frac{1}{2\pi}\int dA\, \mathbf{n}\cdot(\nabla\times\mathbf{\Omega})$$

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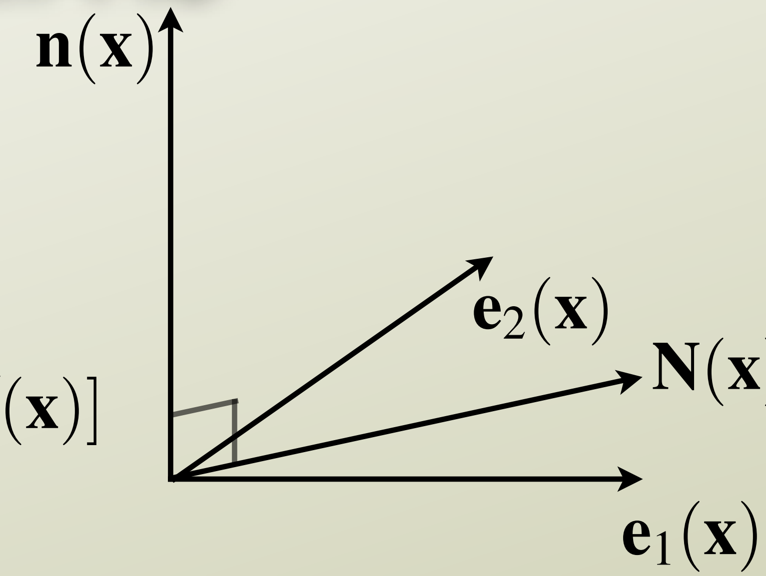
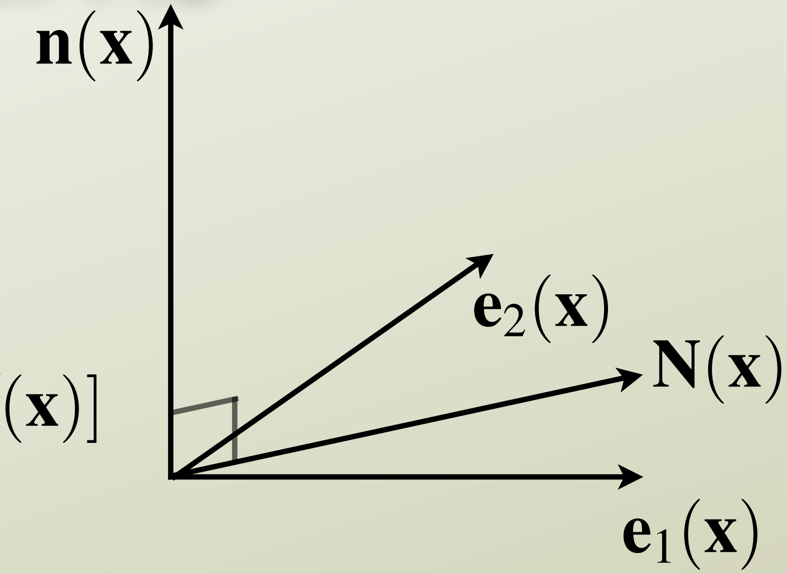
Planar Anchoring on a Sphere: Mermin-Ho



$$
\begin{aligned}
[\nabla\times\Omega(\mathbf{x})]_i &= \frac{1}{2}\left[e_1^{\beta}(\mathbf{x})e_2^{\gamma}(\mathbf{x}) - e_1^{\gamma}(\mathbf{x})e_2^{\beta}(\mathbf{x})\right]\varepsilon_{ijk}\left[\partial_j n^{\beta}(\mathbf{x})\right]\left[\partial_k n^{\gamma}(\mathbf{x})\right] \\
&= \frac{1}{2}\varepsilon_{\alpha\beta\gamma} n^{\alpha}(\mathbf{x})\varepsilon_{ijk}\partial_j n^{\beta}(\mathbf{x})\partial_k n^{\gamma}(\mathbf{x})
\end{aligned}
$$

$$
\#\ \text{of Defects} = \frac{1}{2\pi}\int dA\,\mathbf{n}\cdot(\nabla\times\boldsymbol{\Omega})
$$

Planar Anchoring on a Sphere: Mermin-Ho



$$
\begin{aligned}
[\nabla\times\Omega(\mathbf{x})]_i &= \frac{1}{2}\left[e_1^\beta(\mathbf{x})e_2^\gamma(\mathbf{x}) - e_1^\gamma(\mathbf{x})e_2^\beta(\mathbf{x})\right]\varepsilon_{ijk}\left[\partial_j n^\beta(\mathbf{x})\right][\partial_k n^\gamma(\mathbf{x})] \\
&= \frac{1}{2}\varepsilon_{\alpha\beta\gamma} n^\alpha(\mathbf{x})\varepsilon_{ijk}\partial_j n^\beta(\mathbf{x})\partial_k n^\gamma(\mathbf{x})
\end{aligned}
$$

$$\#\ \text{of Defects} = \frac{1}{2\pi}\int dA\, \mathbf{n}\cdot(\nabla\times\boldsymbol{\Omega})$$

$$\#\ \text{of Defects} = \frac{1}{2\pi}\int dA\, K = 2(1-g)$$

No Colloid

$$F = \frac{1}{2}\int d^3x K_{ijkm}\partial_i n_j \partial_k n_m$$

Derrick's Theorem

J. Math. Phys. **5**, 1252 (1964)

No Colloid

$$F = \frac{1}{2}\int d^3x K_{ijkm}\partial_i n_j \partial_k n_m$$

$$\tilde{x} = \lambda x$$
$$n(x) = \bar{n}(\tilde{x})$$

Derrick's Theorem

J. Math. Phys. **5**, 1252 (1964)

No Colloid

$$F = \frac{1}{2}\int d^3x K_{ijkm}\partial_i n_j \partial_k n_m$$

Derrick's Theorem

J. Math. Phys. **5**, 1252 (1964)

$$\boxed{\begin{aligned}\tilde{x} &= \lambda x \\ n(x) &= \bar{n}(\tilde{x})\end{aligned}}$$

$$\begin{aligned} F &= \frac{\lambda^2}{2\lambda^3}\int d^3\tilde{x}\, K_{ijkm}\tilde{\partial}_i n_j(\tilde{x}/\lambda)\tilde{\partial}_k n_m(\tilde{x}/\lambda) \\ &= \frac{1}{2\lambda}\int d^3\tilde{x}\, K_{ijkm}\tilde{\partial}_i \bar{n}_j(\tilde{x})\tilde{\partial}_k \bar{n}_m(\tilde{x}) \end{aligned}$$

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No Colloid

$$F = \frac{1}{2}\int d^3x K_{ijkm}\partial_i n_j \partial_k n_m$$

Derrick's Theorem

J. Math. Phys. **5**, 1252 (1964)

$$\boxed{\begin{aligned}\tilde{x} &= \lambda x \\ n(x) &= \bar{n}(\tilde{x})\end{aligned}}$$

$$\begin{aligned} F &= \frac{\lambda^2}{2\lambda^3}\int d^3\tilde{x}\, K_{ijkm}\tilde{\partial}_i n_j(\tilde{x}/\lambda)\tilde{\partial}_k n_m(\tilde{x}/\lambda) \\ &= \frac{1}{2\lambda}\int d^3\tilde{x}\, K_{ijkm}\tilde{\partial}_i \bar{n}_j(\tilde{x})\tilde{\partial}_k \bar{n}_m(\tilde{x}) \end{aligned}$$

As $\lambda \to \infty$, $F \to 0$ and the texture shrinks to a point

Topology and Liquid Crystals?

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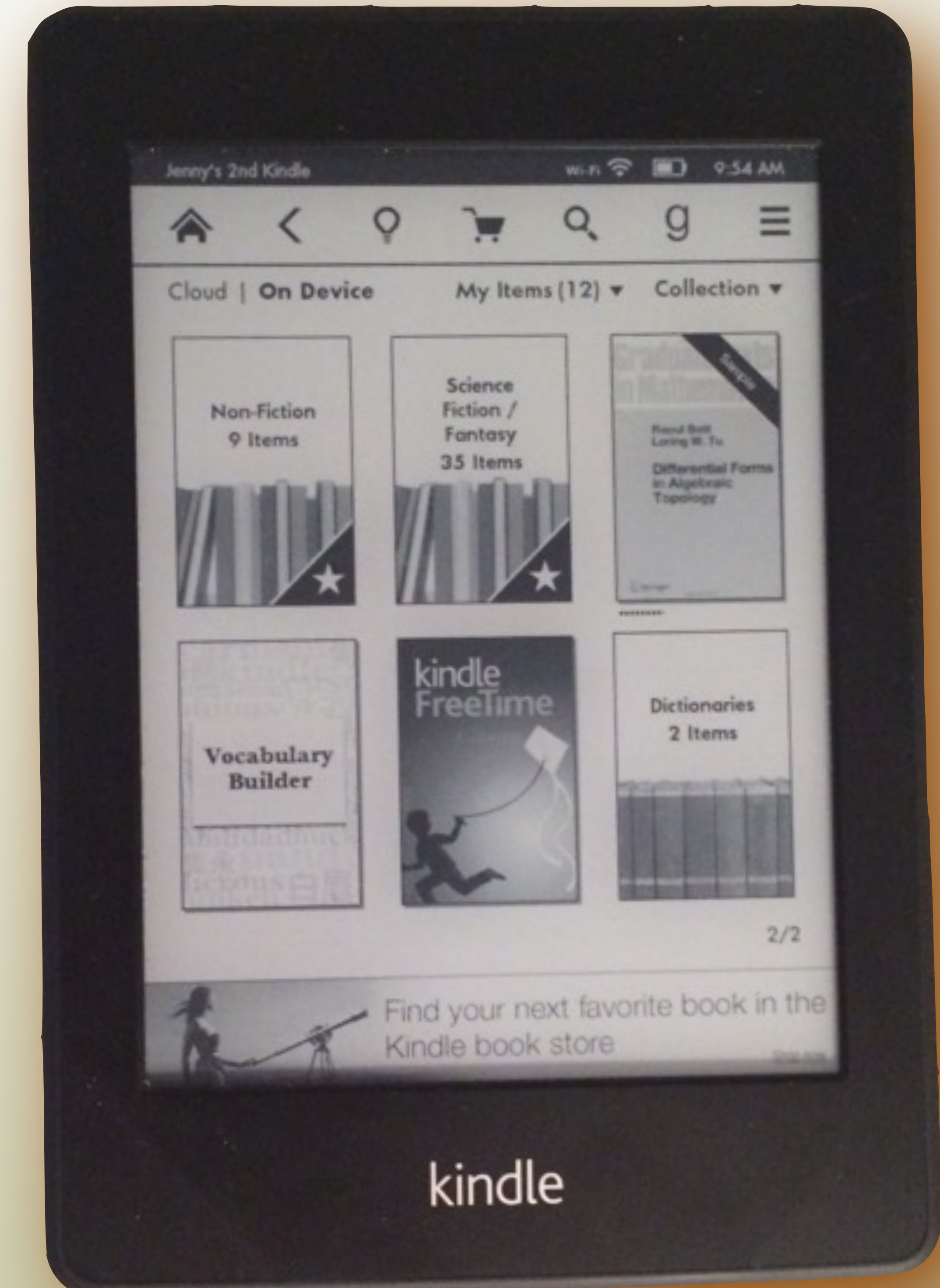
Topology and Liquid Crystals?



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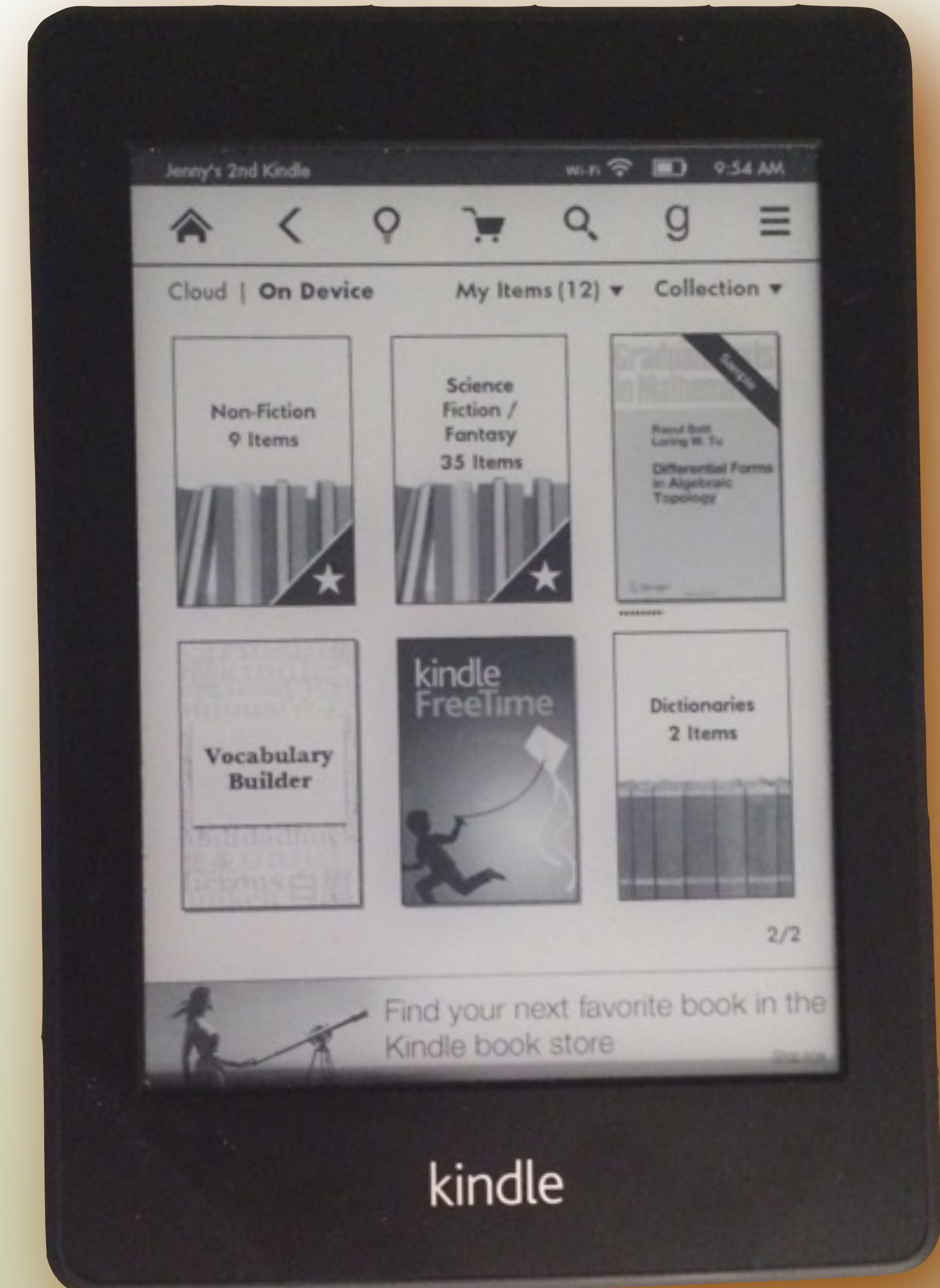
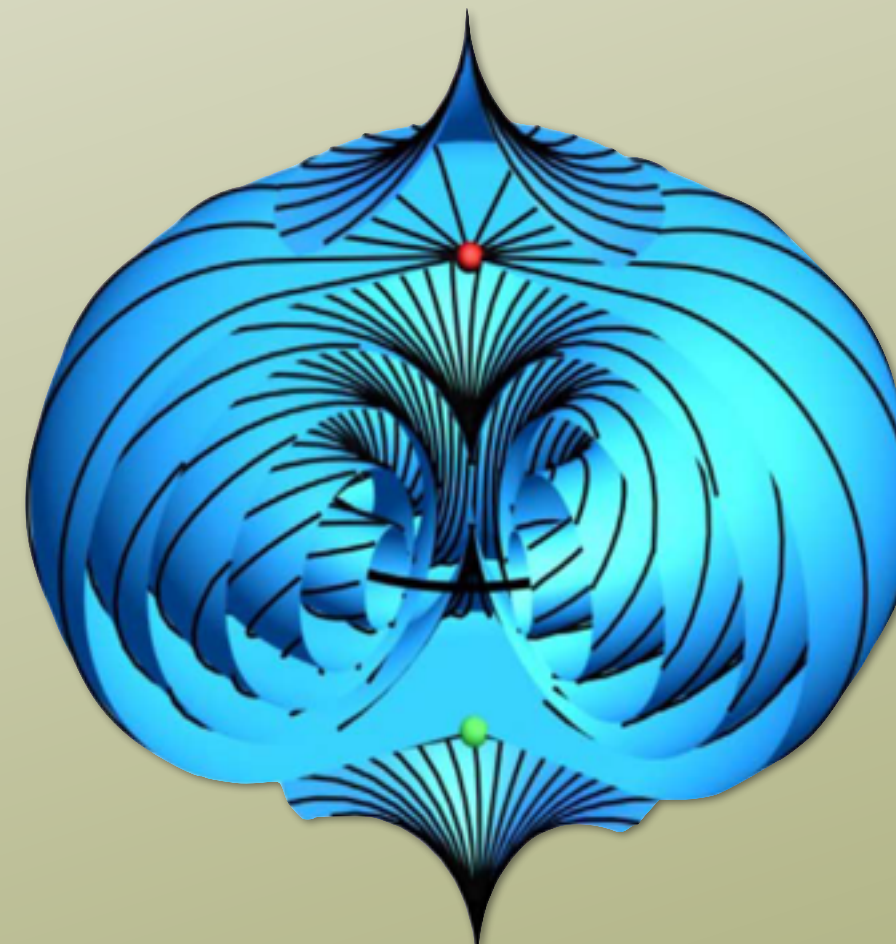
Topology and Liquid Crystals?





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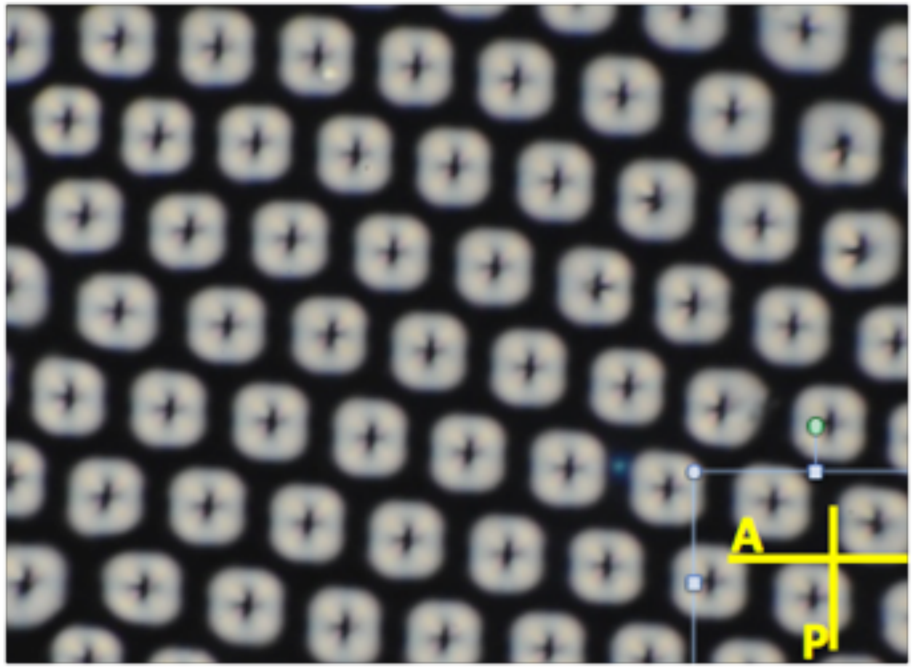
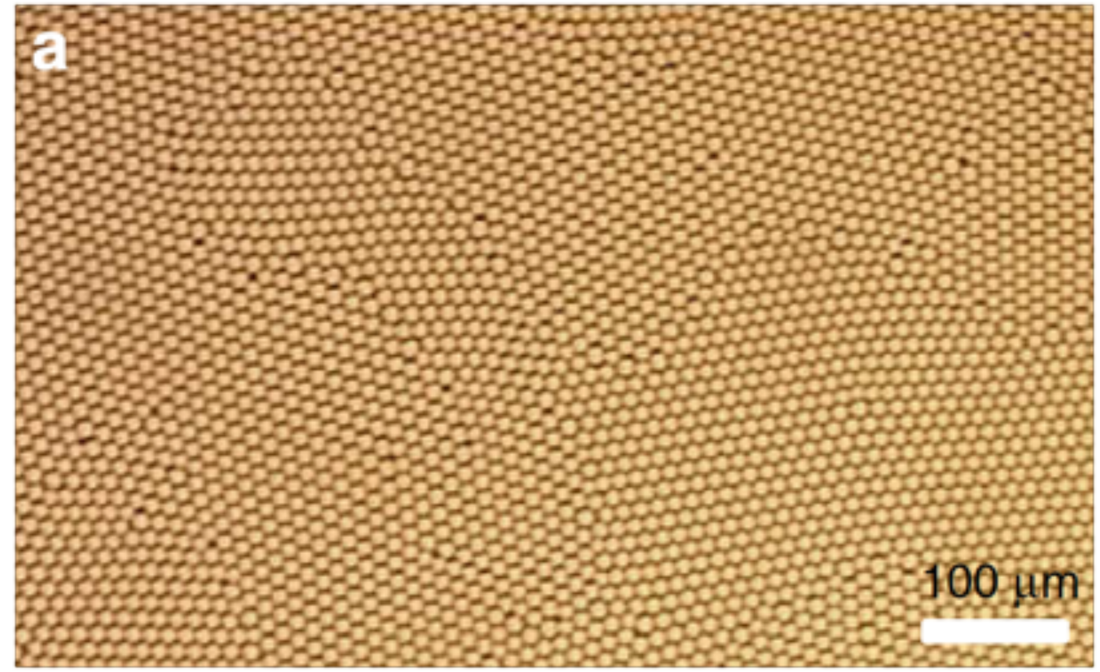
The Toron Display — Passive!





P. J. Ackerman, J. van de Lagemaat, I. I. Smalyukh. *NATURE COMM* 6, 6012 (2015)

I. Smalyukh *et al. Nature Materials* **9**, 139-145 (2010).

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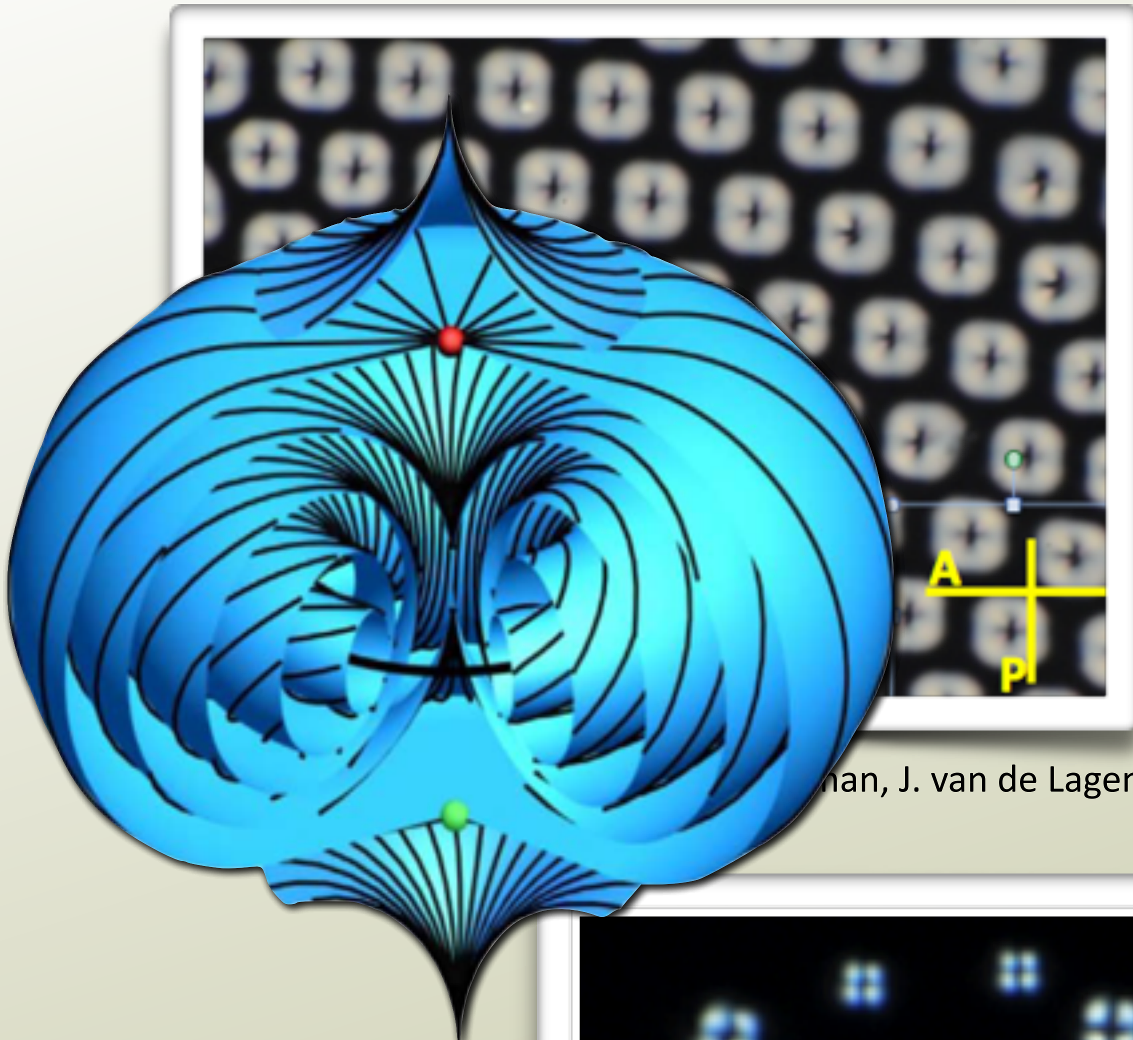

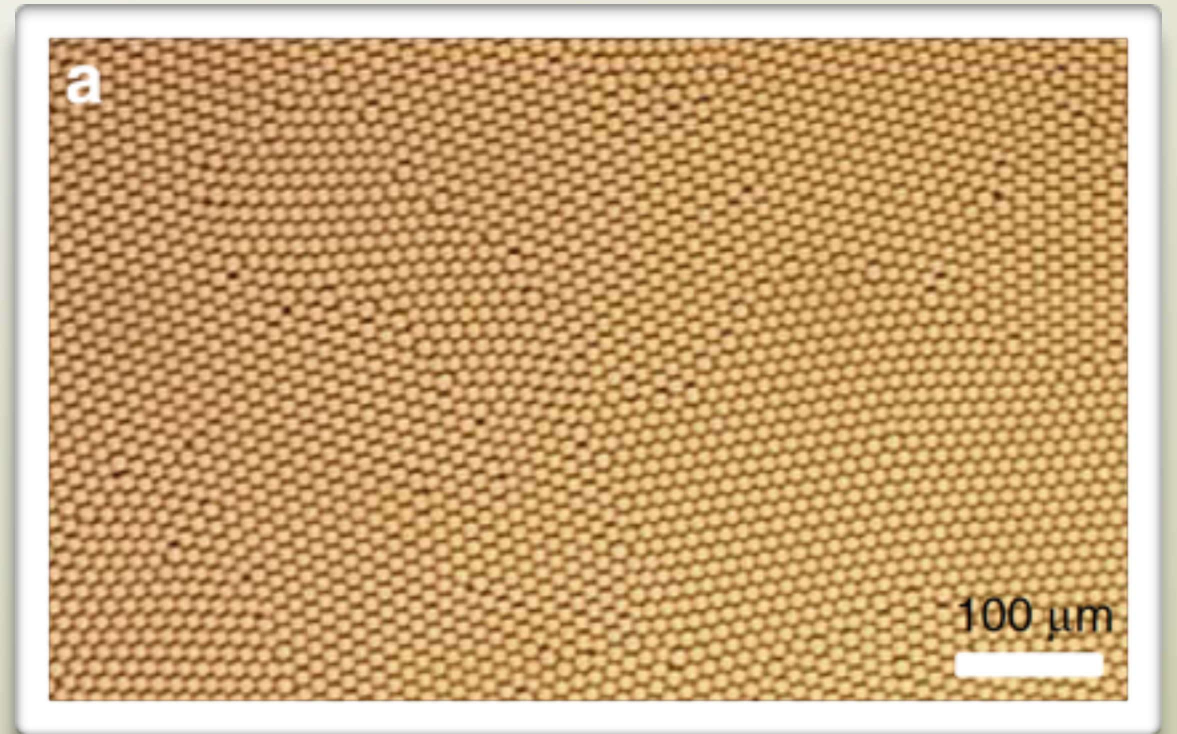

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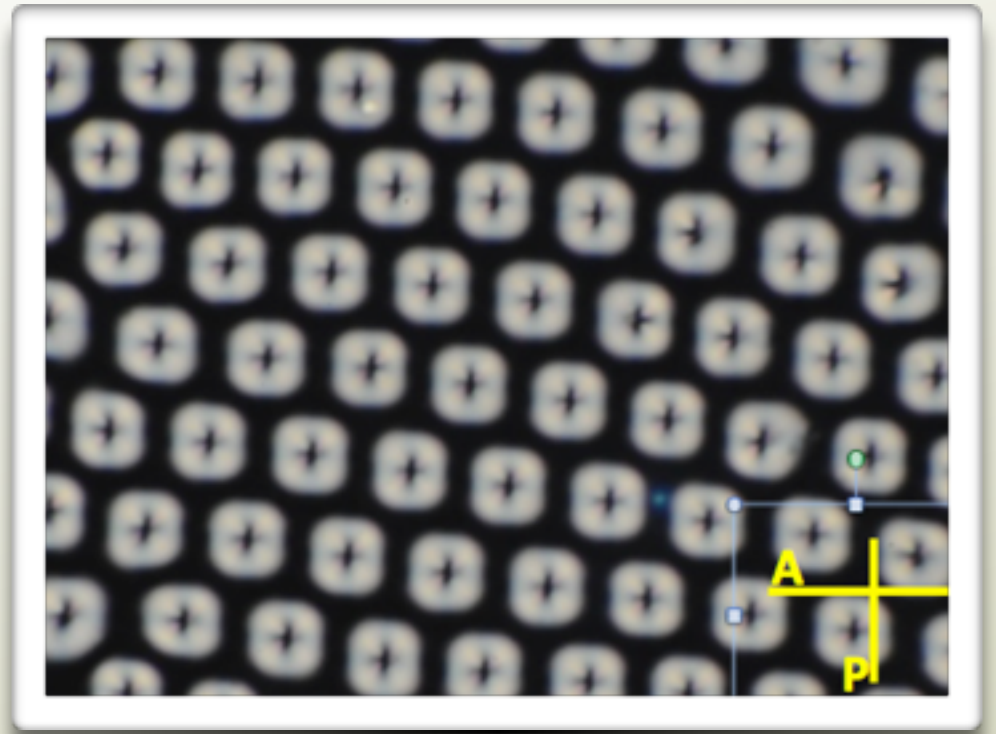

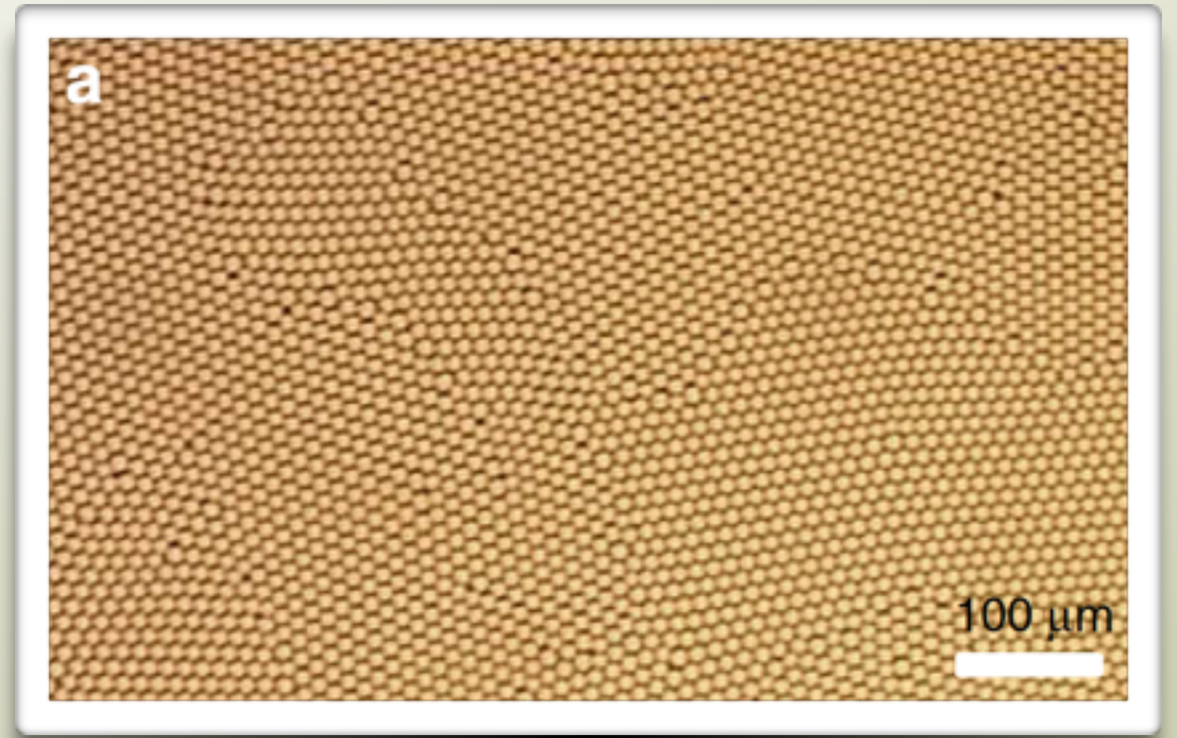

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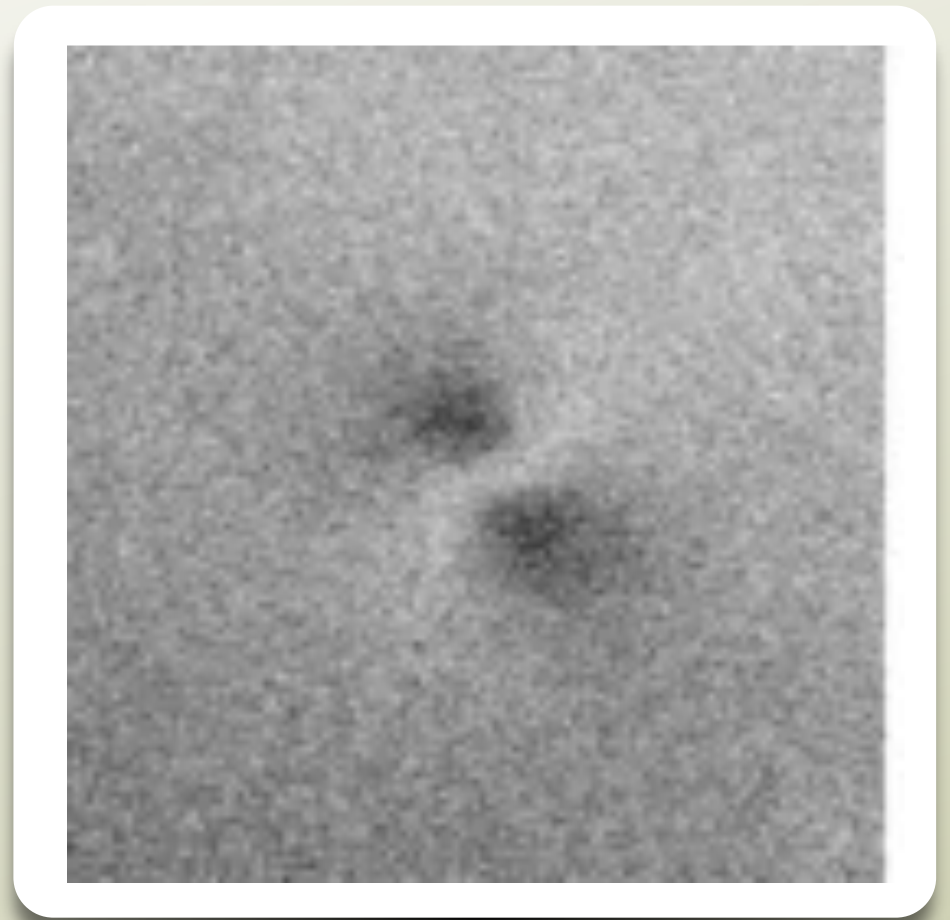
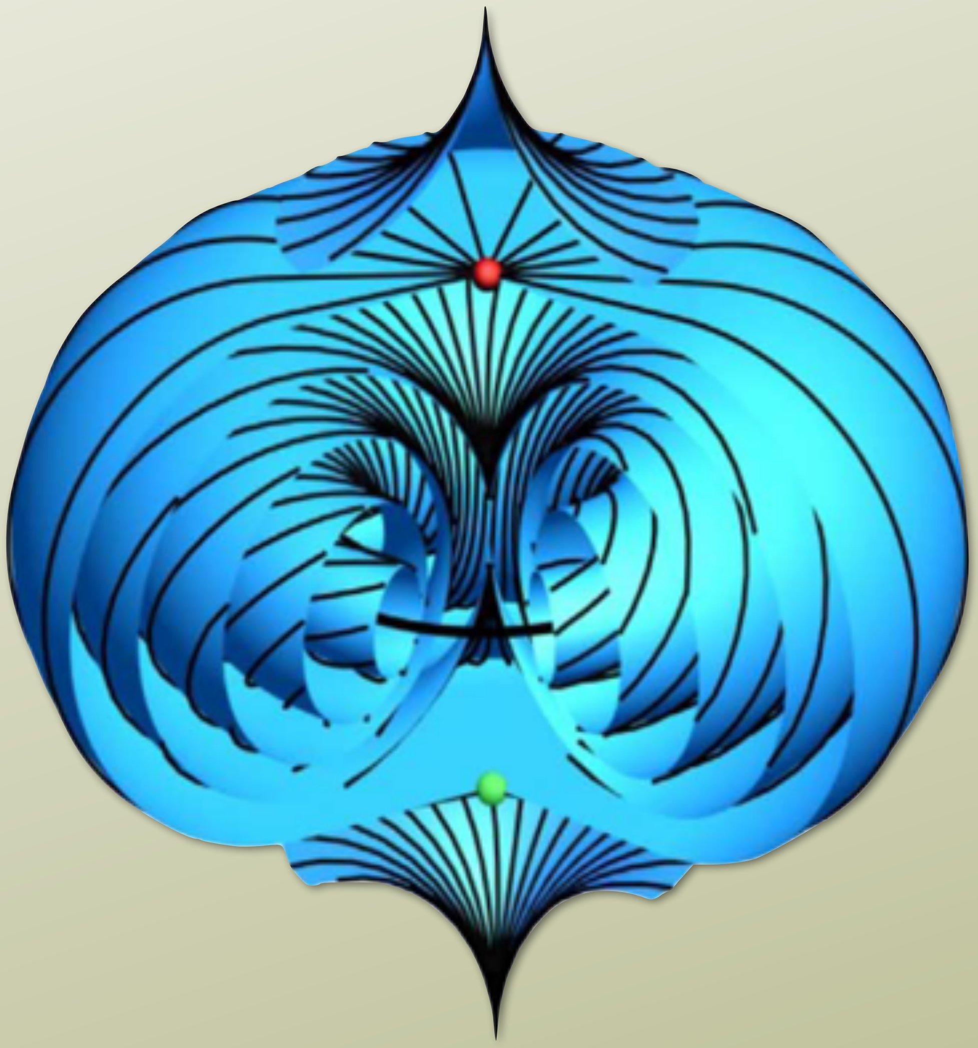
The Toron

Taming the toron: From experimental data...



Imaging technique: Trivedi et al, Optics Express 2010



B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

The Toron

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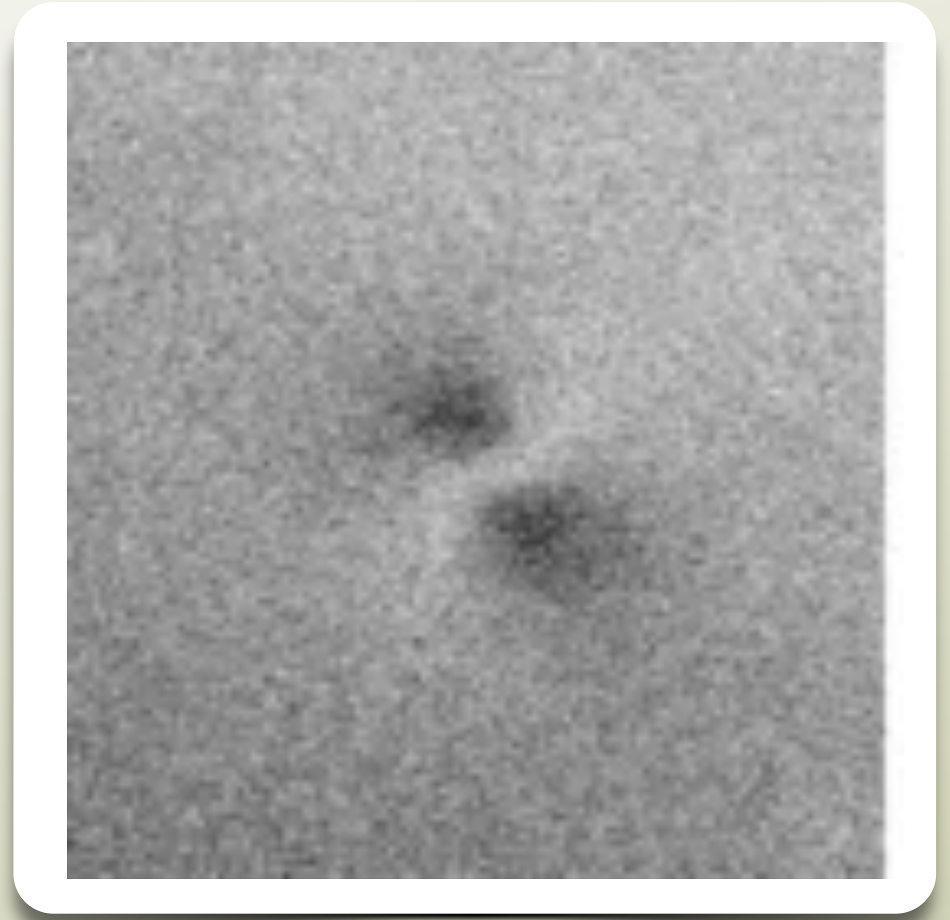

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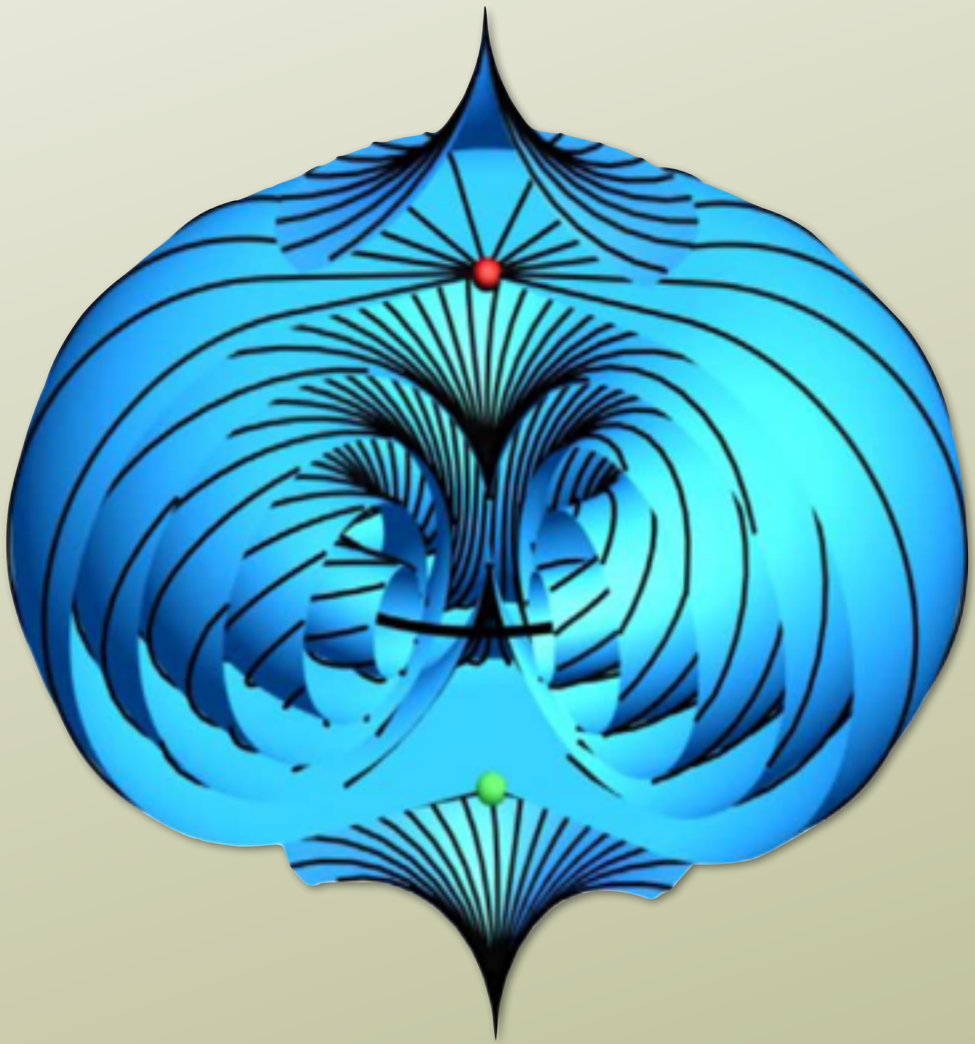
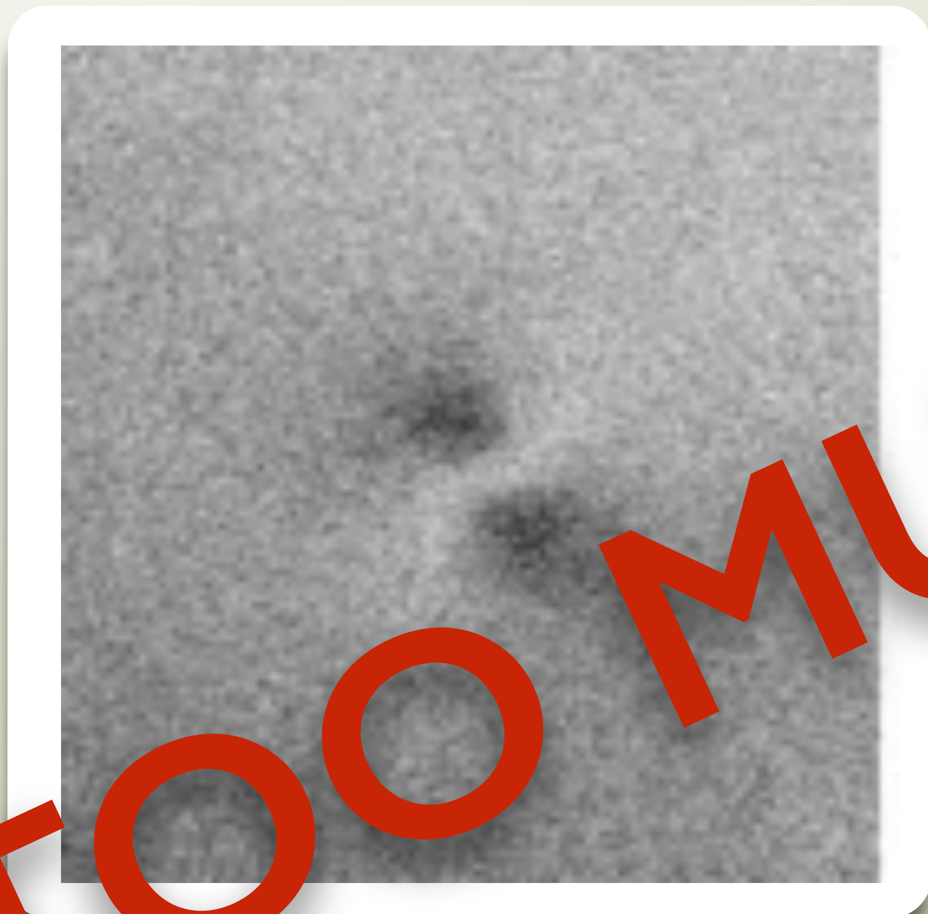


B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

The Toron

Taming the toron: From experimental data...

TOO MUCH DATA



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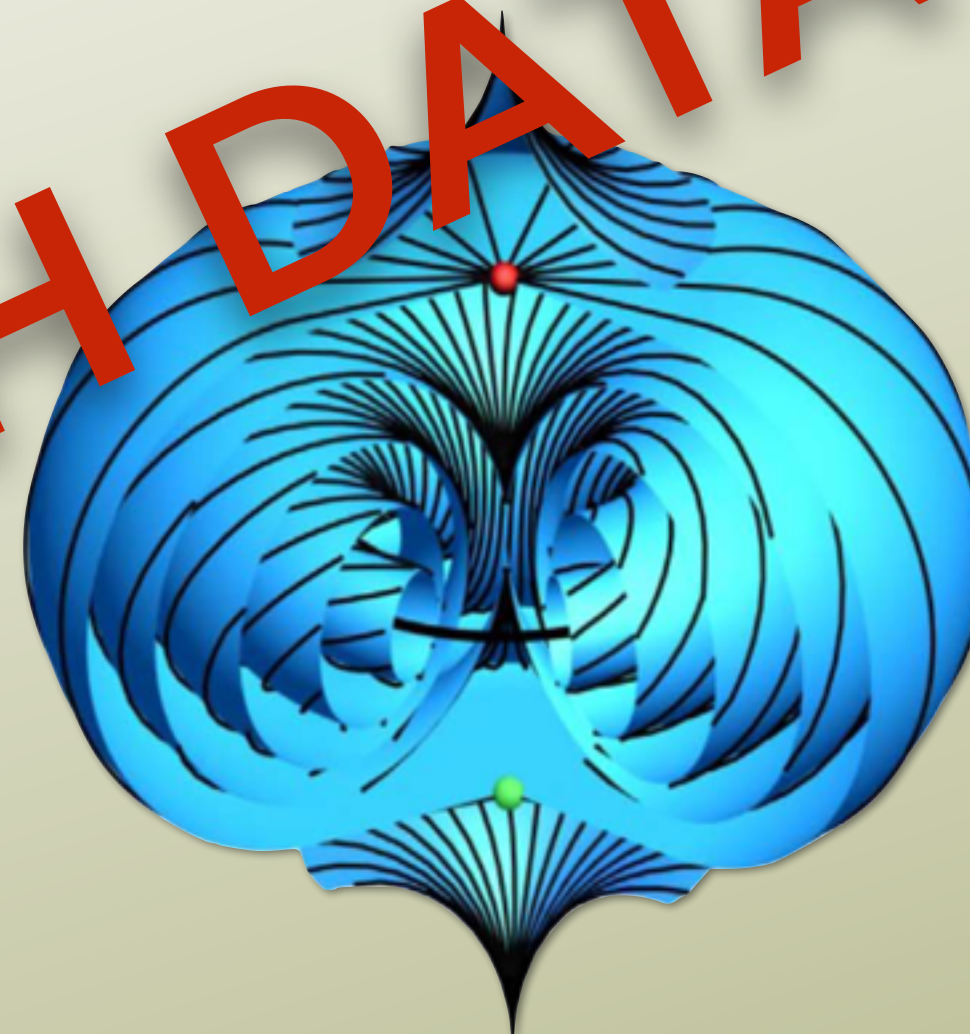

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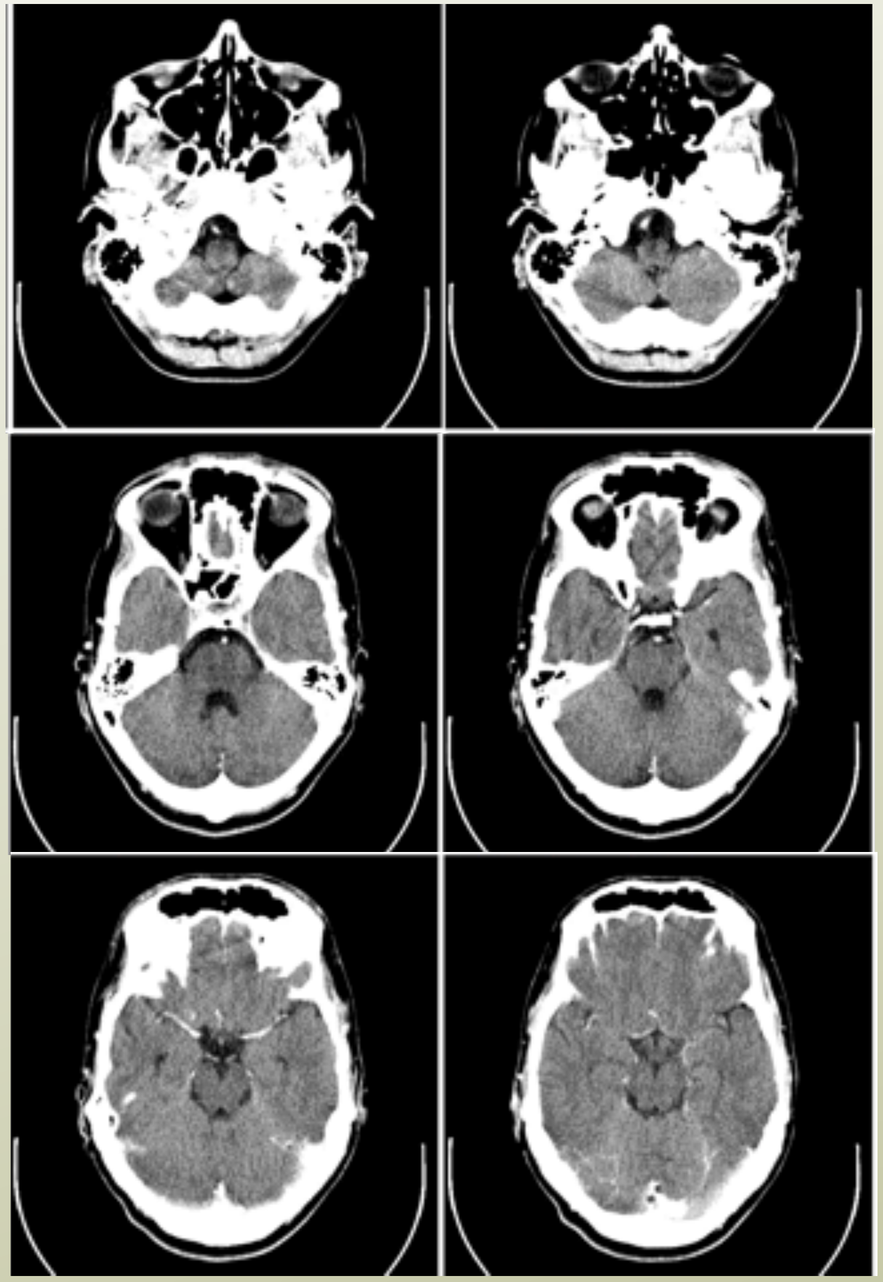

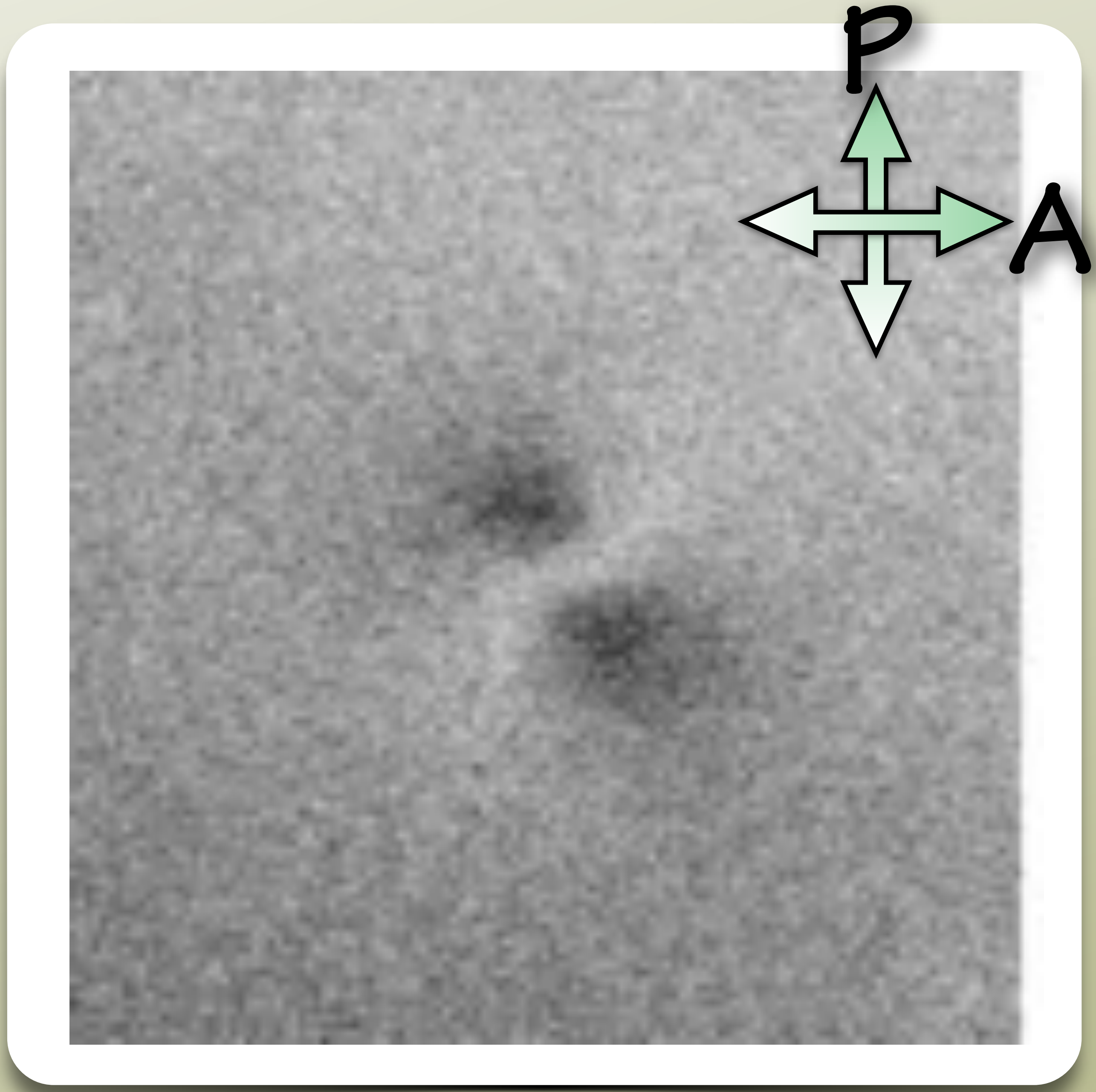

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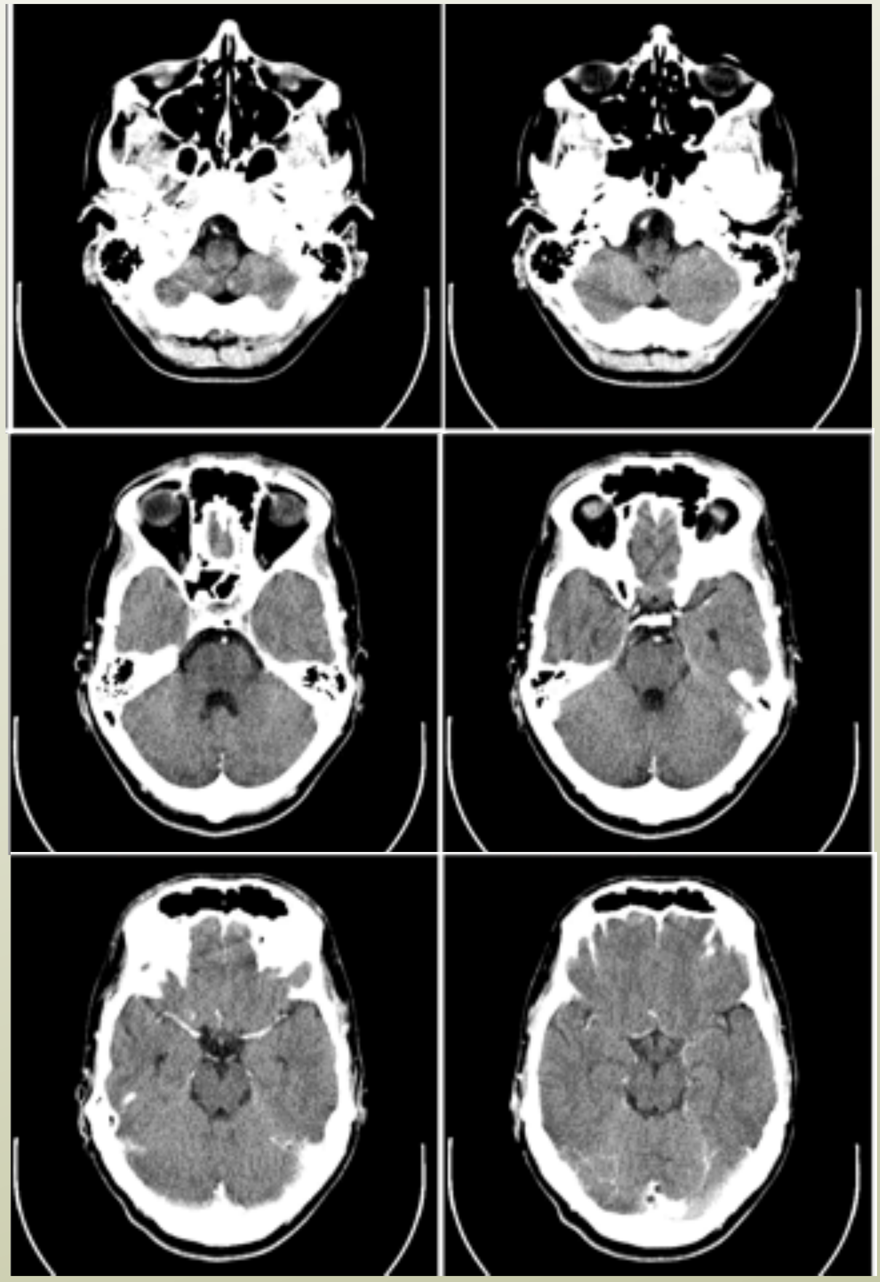

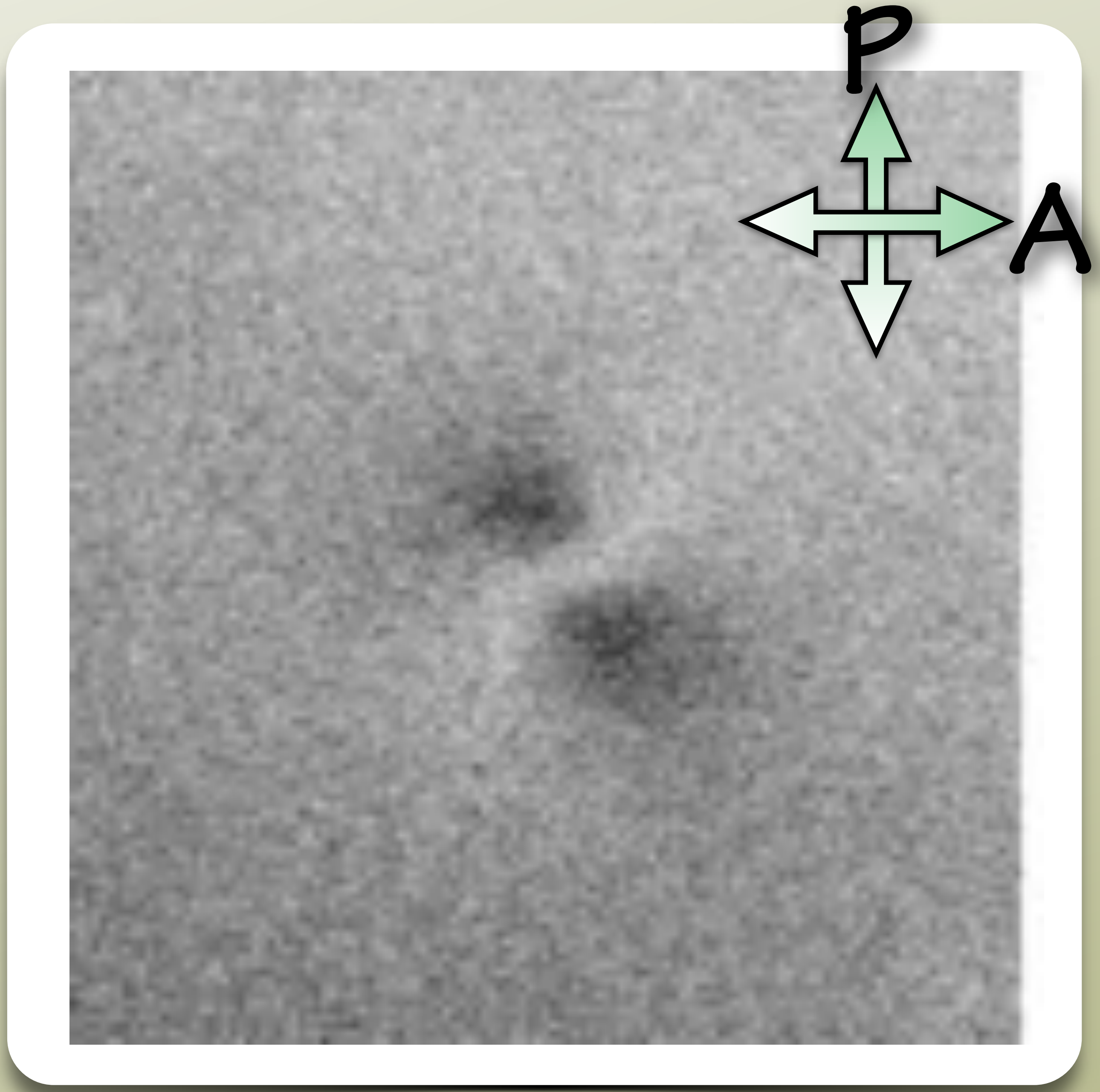

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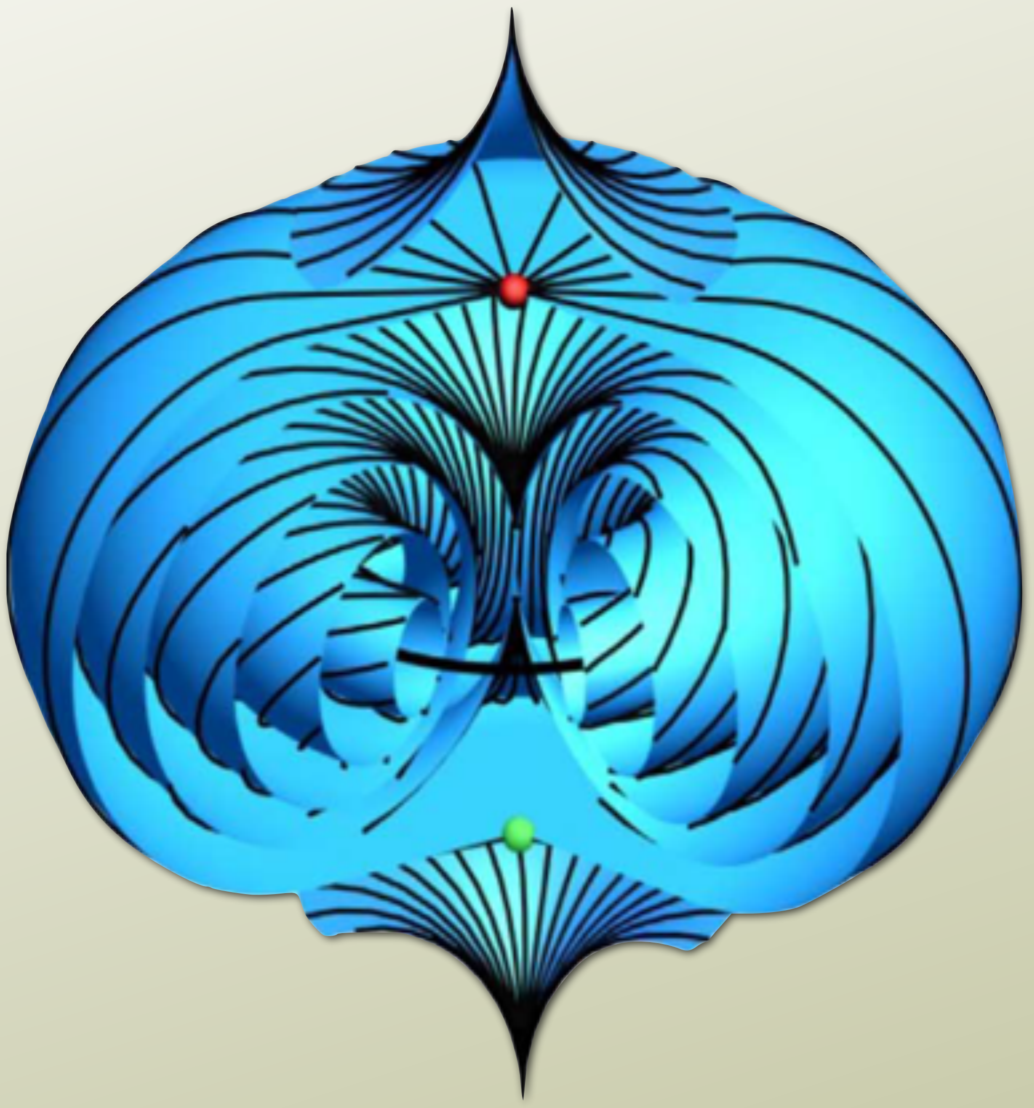
B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

Nematics in Three Dimensions

(Thom construction)



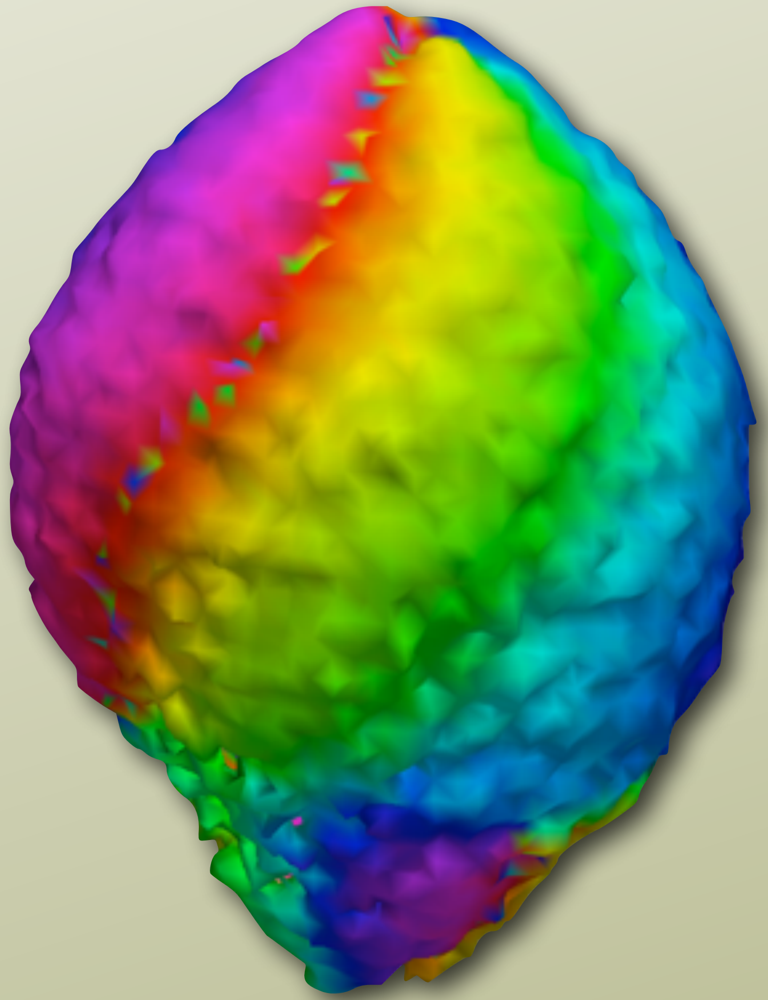
→

take the inverse image

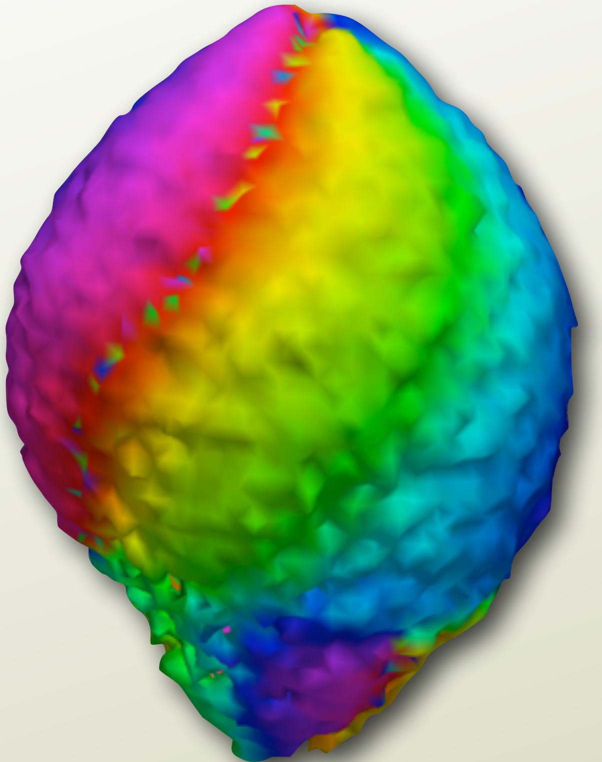
1 to 1

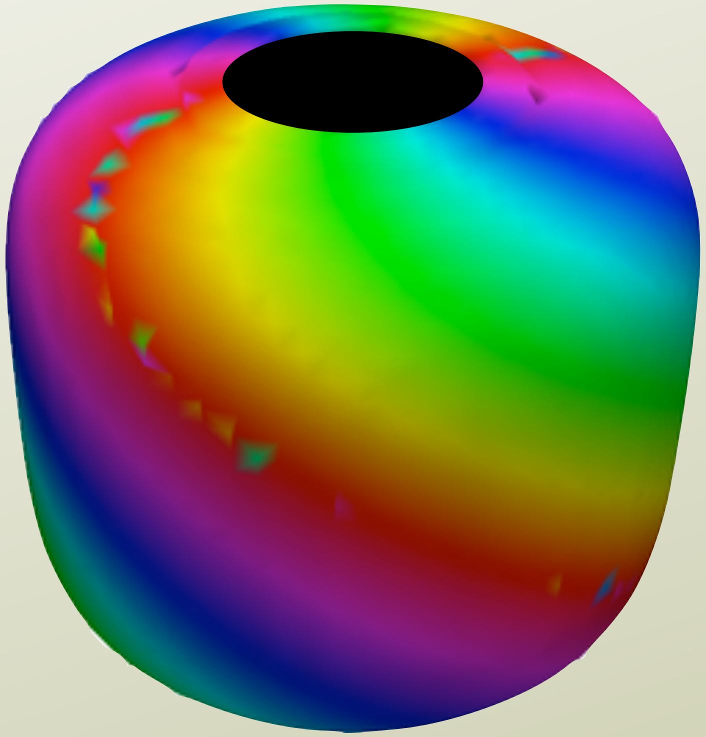
←

fill in neighborhoods



Threading the Needle





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B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

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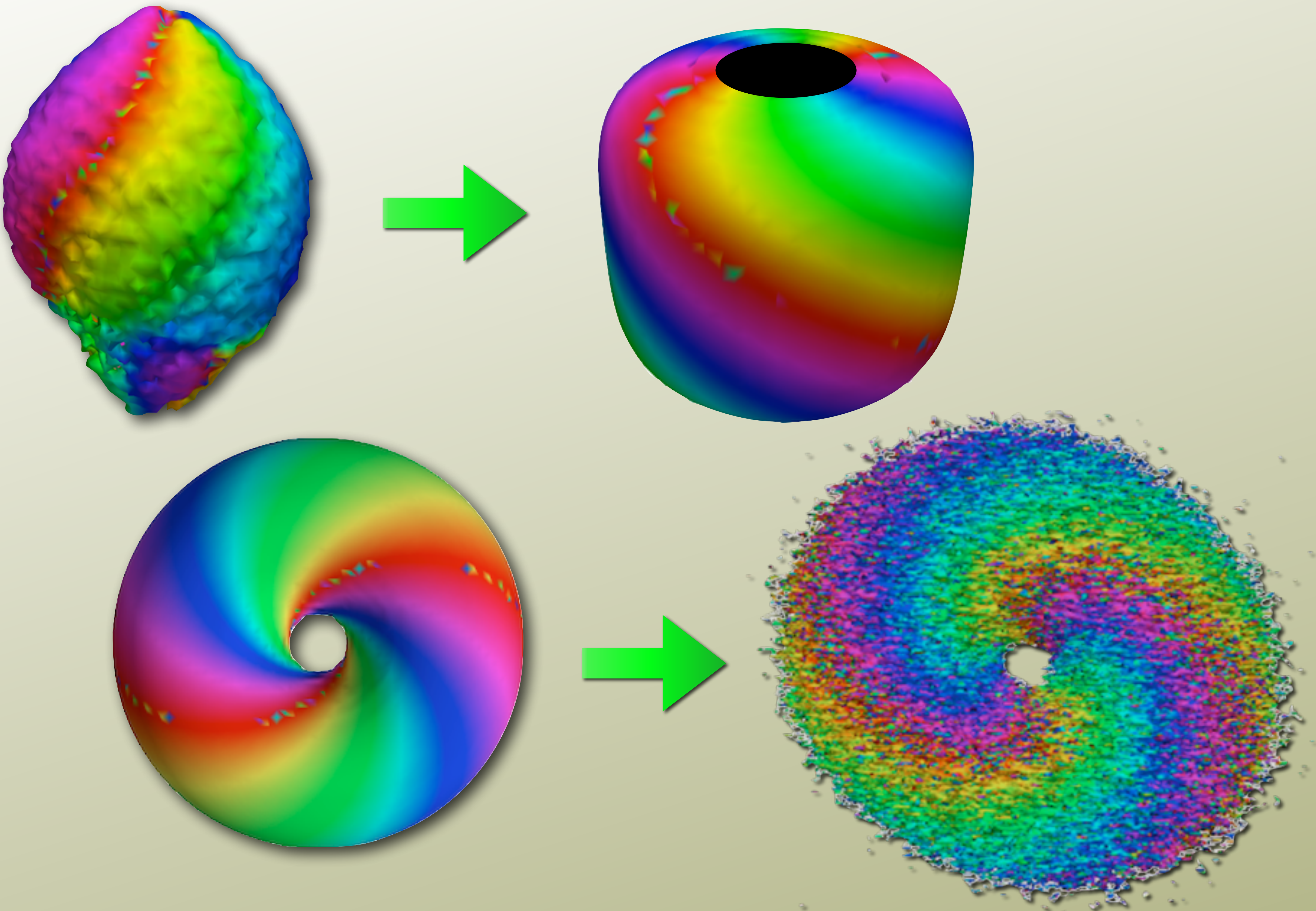

B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

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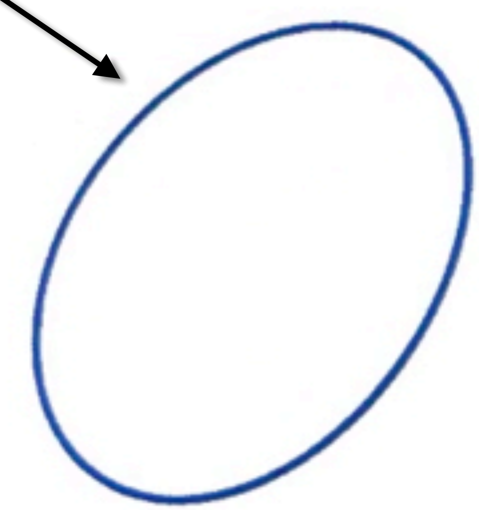
The Hopf Fibration

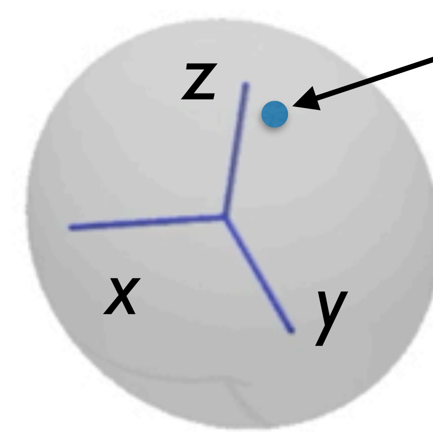
$$S^3 \longrightarrow S^2$$

“Black” Stripe For Each Direction on the Sphere



$\mathbb{R}^3$

Direction of Molecule



$\mathbb{R}P^2$

Courtesy of
Niles Johnson

http://nilesjohnson.net/hopf.html

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The Hopf Fibration

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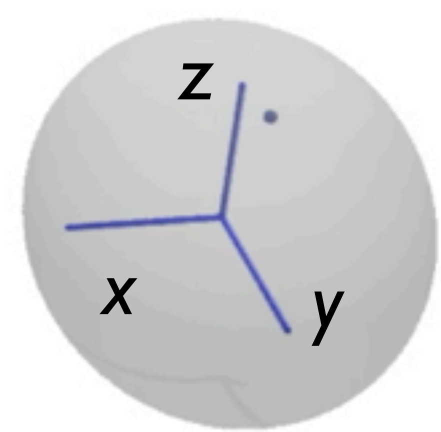
$\mathbb{R}^3$

Direction of Molecule



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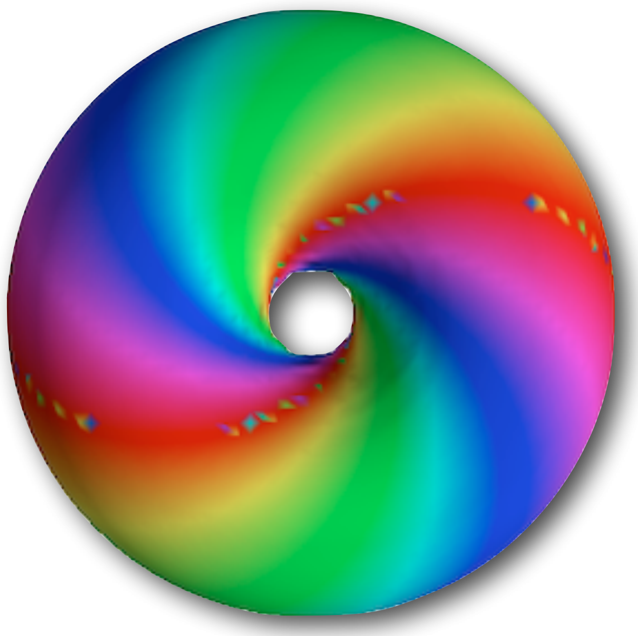
The Hopf Fibration!



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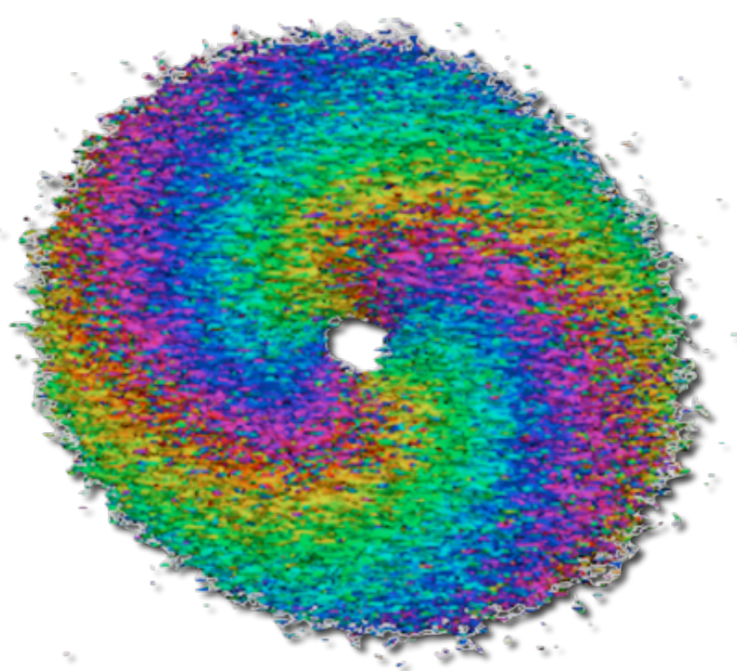
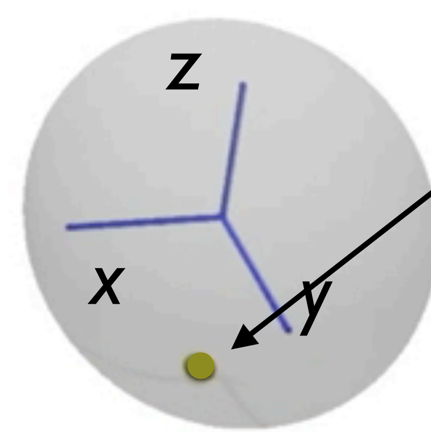
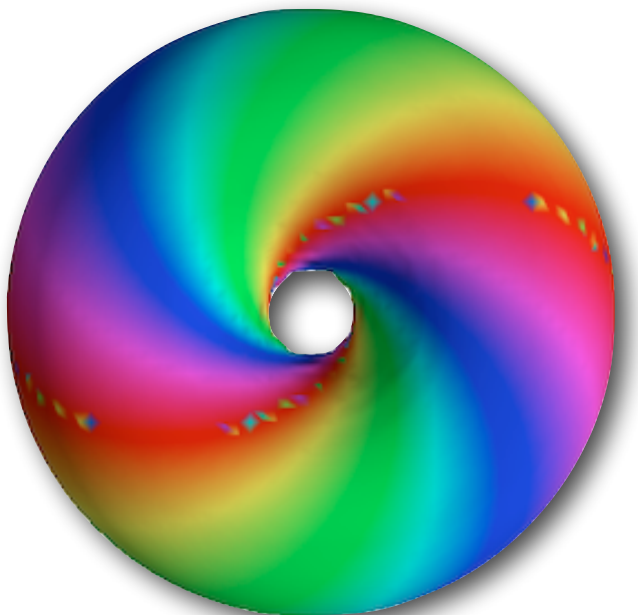


Directions of Molecules



$$\mathbb{R}P^2$$

The Hopf Fibration!



"Black" Stripe For Each Direction on the Sphere

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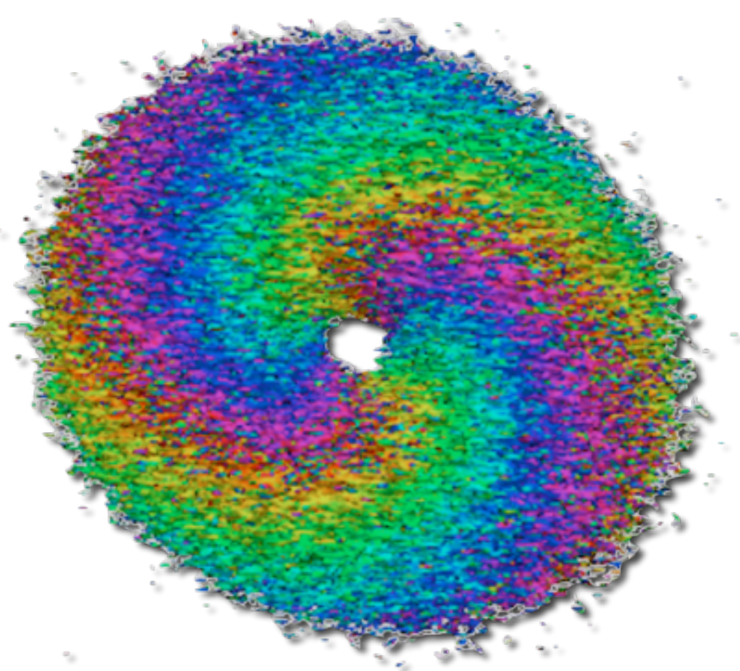
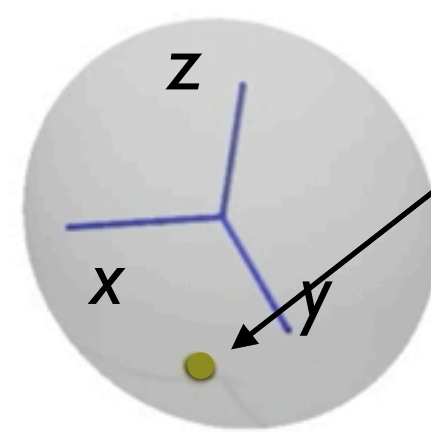
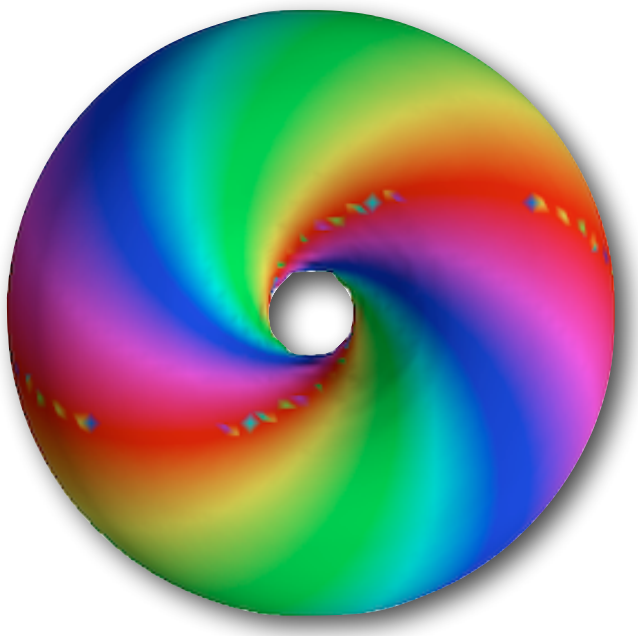


Directions of Molecules

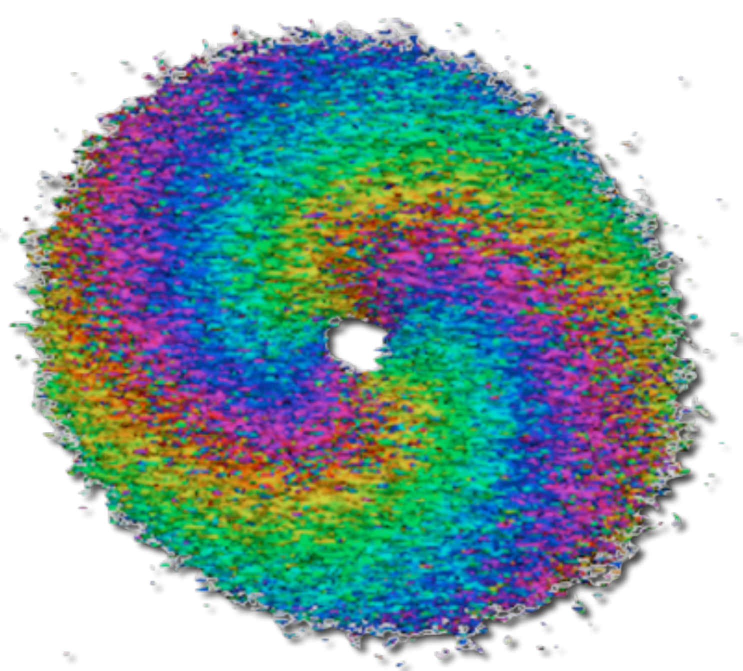


$$\mathbb{R}P^2$$

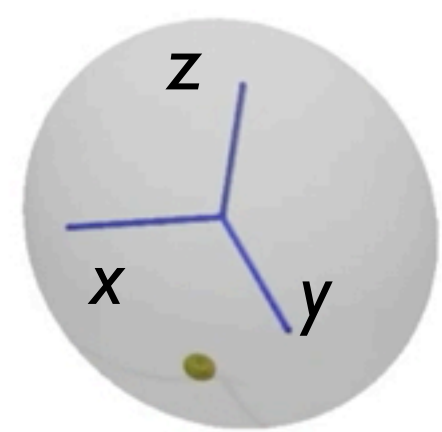
The Hopf Fibration!



$$\mathbb{R}^3$$

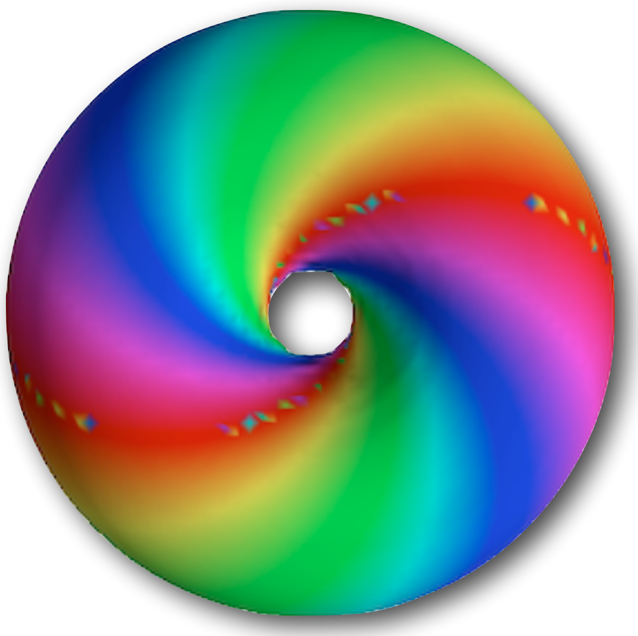


Directions of Molecules



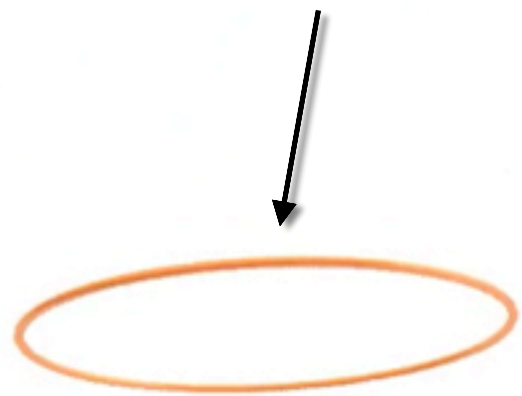
$$\mathbb{R}P^2$$

The Hopf Fibration!

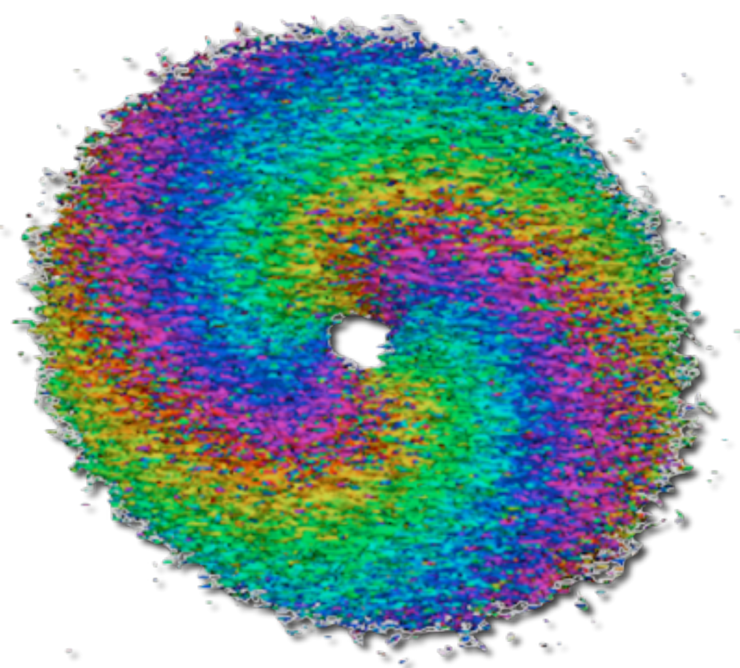


$$S^3 \longrightarrow S^2$$

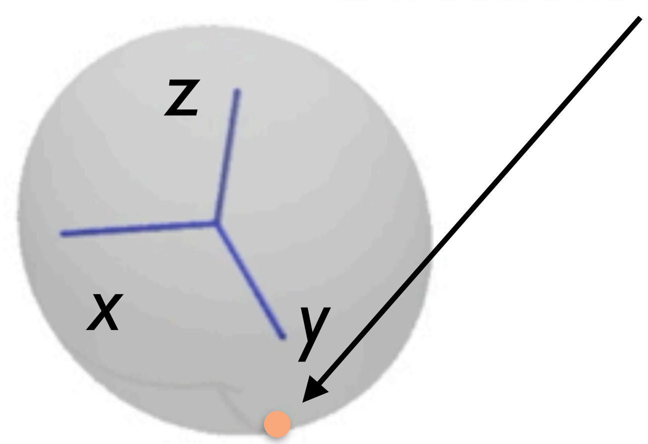
"Black" Stripe For Each Direction on the Sphere



$$\mathbb{R}^3$$

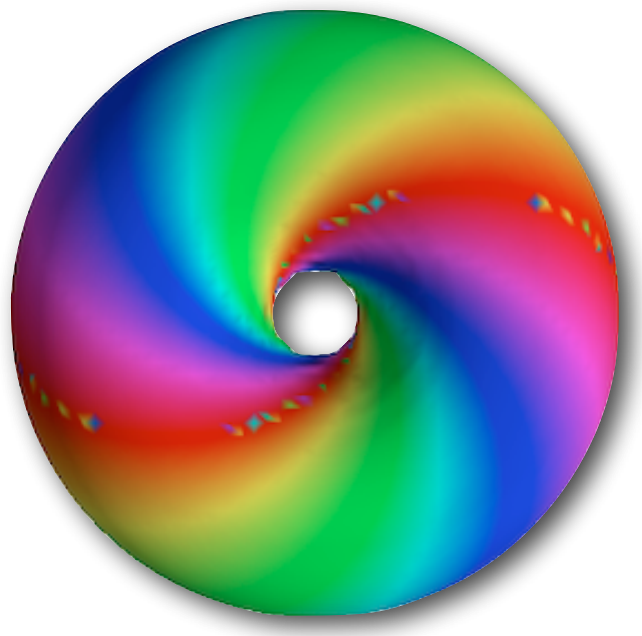


Directions of Molecules

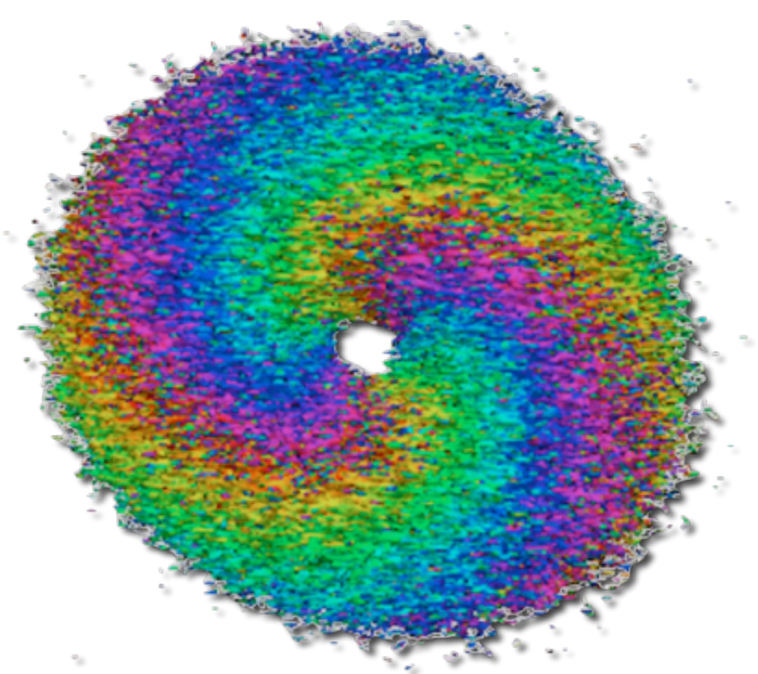


$$\mathbb{R}P^2$$

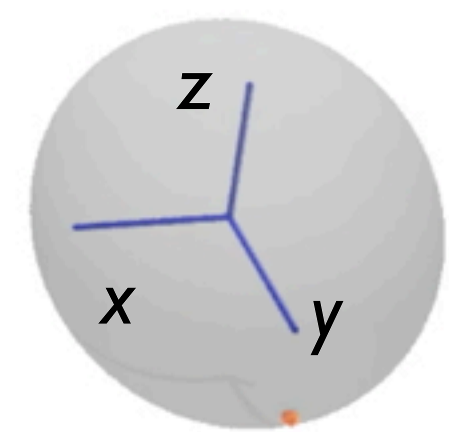
The Hopf Fibration!



$\mathbb{R}^3$



Directions of Molecules



$\mathbb{R}P^2$